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Dissertation

Situational Analysis of Students' Imagination and Creativity through Mathematical Discourse at Babur School in Dire Dawa

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Situational Analysis of Students' Imagination and Creativity through Mathematical Discourse at Babur School in Dire Dawa

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This is to certify that the thesis, prepared by Yenealem Ayalew Degu entitled: *Situational Analysis of Students' Imagination and Creativity through Mathematical Discourse at Babur School in Dire Dawa* and submitted in fulfillment of the requirements for the degree of Doctor of Philosophy in Mathematics Education, complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

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Yenealem Ayalew Degu, Doctor of Philosophy in Mathematics Education

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Abstract

Mathematics education is a multi-disciplinary field of study which treats a wide range of diverse but interrelated areas. The Philosophy of Mathematics Education as a subfield covers: the status of Mathematics Education as knowledge field; the understanding of, and the meaning we attribute to, mathematics and its nature; purposes and meanings of teaching and learning mathematics; and the relationship between mathematics and society. I recognized that learning mathematics is about discourse, imagination and creativity. The objective of this study was exploring the status of imagination and creativity in a classroom mathematical discourse. Situational Analysis was employed as the method of inquiry to understand the dense complexities of the situation. Data were collected from Dire Dawa city, Eastern Ethiopia using audio-video recordings, field notes, focus group discussions, and documents. Comprehensive data were collected from a grade 12 mathematics classroom at Babur school in the city. The broadly conceived situation and Focused Groups of students within the classroom served as units of analyses. I followed the emic approach to study the socio-cultural and thus meaning makings were supplemented by applying the Ethiopian Orthodox Church interpretive methods. This study has shown that remembering, intuition, ambiguity, visualization, illustration, argumentation, persistence, usefulness, understanding, flexibility, evaluation, fluency, insightful, and originality are common indicators for imagination and creativity. On the other hand, dwellers' "simple" and "harmony" life styles were mathematical elements; they have impacted on students' fluency and persistence. The socio-cultural mathematics in Dire Dawa is described by mathematizing Christianity and making multiple meanings of one plus one. These were either missed opportunities for or impacting factors in engaging students in imagination and developing their creativity. Furthermore, the study revealed that some problems and activities in the text book that could have been opportunities for developing pattern, prediction and generalization were not delivered in the classroom. This study showed the bi-implication of creativity and imagination with grounded theoring. The research would contribute for popularizing Mathematics Education as a discipline in Ethiopia.

Key Terms:- *discourse, imagination, creativity, philosophizing, theorizing, situation, map*

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My supervisor, Dr. Solomon Areaya, had been supporting me throughout my research work. At first, he let me exploit my interest. I had high ambitions and lots of claims with minimal justifications. His questions: “So what?” “What do you like to tell readers?” “What is the take-away from your description?” and “Is it all about [this issue]?” have helped me to think better and write more thoughtfully.

Solomon Areaya, what a mentor?!
He did inspire me and I did aspire.
He let me think of a concern;
As far as up to the sky horizon;
If not, just to touch the top of a tree;
Just agreed; questioned my bravery.
As I thought of in and out the box;
Acquainted with the abstract terms:
Creativity, Imagination and *Discourse*
Any time I had been writing about
He was there and guided me a lot.

I am also indebted to different stakeholders for their contributions. I had special attachment with Emeritus Professor Peter Sullivan of Monash University, Australia. He gave me some thoughts on the preliminary phases of the research proposal. Besides, Dr. Kassa Michael, Dr. Mulugeta Atnafu, and Dr. Solomon Belay of Addis Ababa University (AAU) reviewed the research progresses for endorsement. Thus, I am grateful for their thoughtful questions & suggestions. On the other hand, Dr. Girma Moges of Dire Dawa University (DDU) sketched the map of research site. I have a great respect for him.

I am indebted to individuals and organizations that gave me necessary budget and resource materials used for the research work. The budget required to undertake the field work was covered by AAU and DDU. On behalf of the Department of Science and Mathematics Education at AAU, Dr. Mekbib Alemu provided me audio-video recording materials. I was lucky to have office shared with Challa Regassa; I couldn’t forget the care he had been offering to me. My wife, Dr. Meseret Dessalegne, imported two vital books from abroad: "Imagination and Creativity, 2010" and "Situational Analysis: Grounded Theory after the Interpretive Turn, 2018". Besides, Paul Ernest (Emeritus Professor at Exeter University, UK) sent me one of his books “The Philosophy of Mathematics Education Today, 2018”.

I have special acknowledgment for teachers in Dire Dawa who helped me gather data. To mention some: Girma, Diriba, Yenus, Daniel, Tewodros, Halefom and Yesuf. Moreover, I am very grateful for grade 12A students with whom I conducted the study; their hospitality was amazing.

Let my praise be conveyed to the Almighty God who paves my ways!

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Abbreviations

CI: - creative imagination

CIA: - creative imagery ability

CT: - convergent thinking

DDU: - Dire Dawa University

DIC: - discourse, imagination and creativity

DT: - divergent thinking

$f_a(K, I, E)$: - Noller's formula of creativity as a function of knowledge, imagination & evaluation

FGD: - Focus Group Discussion

Forum: - Mathematics Forum which has been practiced in Dire Dawa

GT: - grounded theory

HIQ: - Hunger Imagination Questionnaire

I & C: - imagination and creativity

RI: - reproductive imagination

S_{12}: - a student who participated in the first FGD and the second member within the group

SA: - Situational Analysis

TCI: - Test of Creative Imagination

TCIA: - Test of creative imagery ability

T.G.o:- Transitional Government of [Ethiopia]

TTCT: - Torrance Tests of Creative Thinking

Chapter 1: Introduction

There were many events and instances that geared me to think of and propose this research. The insights are summarized under and presented in the following subsections.

1.1 Background of the Study

My academic life commenced by pursuing a Bachelor of Education degree in the field of Pedagogical Sciences [PdSc] and Mathematics [Maths] composite [majors] from the Faculty of Education at Bahir Dar University in 2004. The PdSc courses and other supportive courses were common for all of us. On the other hand, we had been joining groups majoring Mathematics or Geography or English or Amharic. Thus, unlike its naming “composite”, Pedagogy and Mathematics were not interwoven each other. For instance, our instructor for the course *Subject Methodology* (PdSc 451) had delivered contents from the Secondary School Mathematics; he left us with an assignment of applying the Pedagogy in Mathematics. That was possible after graduation.

It was after three years of teaching secondary school mathematics that I got acquainted to *Mathematics Education* as a proper field of study. I was among the frontiers in attending postgraduate program in the field at Addis Ababa University. Yet, following my graduation, I could not get job with my field of specialization. In September 2009, I was employed as a Mathematician and had been teaching Mathematics courses at Dire Dawa University. While I was teaching the mathematics courses there, I used to think of students’ prior knowledge. In addition, there were opportunities and exposures that kept me concerned with pre-university mathematics education. To mention one, the *Mathematics Forum* of Dire Dawa and the surrounding whereby I, together with other colleagues, had been organizing Olympiads, trainings and experience sharing events over 40 schools. Through time, I developed research interests in students' aspirations in regard to higher education and their transition from school to university.

The goal of mathematics education at preparatory secondary (grades 11 & 12) level is to enable students acquire the necessary mathematical knowledge and develop skills and competencies needed in their further studies, working life, hobbies, and all-round personal development (Ayalew, 2018; Federal Democratic Republic of Ethiopia, 2010c, 2010d). The goal calls for enabling students to visualize their future. Hence, it is crucial to create appropriate context and resource for effective teaching and learning of mathematics in general and the concept of calculus in particular at preparatory secondary schools in Dire Dawa city (Areaya & Sidelil, 2012). The authors founded that:

> Significant numbers of students in secondary schools of Dire Dawa city are observed to have difficulties and formed misconceptions on basic concepts of calculus. Good numbers of students possess only a limited concept image of . . . limit of function. [Many] students lack the necessary knowledge and skills of representing function using different methods. There are also students with restricted mental image of functions to the extent that they lack knowledge of algebraic manipulation (Page 32).

Here, conceptual understanding, concept image, mental image, representing, using different methods, manipulation are key terms that could instigate to research on.

Since October, 2014, I had been attending at the doctoral program in Science and Mathematics Education of Addis Ababa University. At the beginning of my study, I had two [among others] resourceful instructors: Solomon Areaya and Sileshi Yitbarek. Solomon had been expecting us to work out up to "the sky as the limit; if it could not be touched, there would be a probability of reaching at the top of a tree". He was also saying: “a skill of philosophizing is demanded to earn Doctor of Philosophy degree”. Together, Sileshi and Solomon asked us to write down our own educational philosophies. By then, I mentioned pragmatism and social re-constructivism thoughts as my worldviews[1]. Being a junior mathematics educator, I like to discuss these issues in a way applicable to the field.

Pragmatism is the name of a philosophical position that embeds rational discourse in life and conduct (Rowland, 2000). It allows a researcher study *context*, *actions*, *situations* and *consequences* rather than antecedent conditions (Creswell, 2014). For instance, the overall social context of the mathematics classroom is a form of life including: persons, relationships and roles, material resources, the discourse of school mathematics, including both its content and its mode of communication (Ernest, 1994). Actions and consequences are held within the context. However, situation is more inclusive of the others issues (context, actions and consequences). It has cultural and social dimensions; it is inclusive of social environment (Clarke, Friese, & Washburn, 2018).

[1] **My Personal Philosophy of Education** (December, 2014). By: Yenealem Ayalew, submitted in partial fulfillment of the course Fundamentals of Curriculum Theory and Development in Science and Mathematics Education, Addis Ababa University

For me, Mr. Zumra of the *Awramba* community, in southern Gondar is a ‘sociologist’; the late Premier of Ethiopia, His Excellency Meles Zenawi, was an ‘Economist’, ‘Leader’, ‘Political scientist’, ‘Analysit’, and ‘Author’. Mr. Haile G/Sellassie is an ‘Economist’, ‘Manager’ and ‘Entrepreneur’. These honorable individuals contributed to the people of Ethiopia. Their outstanding performance is due to accumulated experiences via *informal education*. Let me add one more: “Green’s Theorem is named after the self taught English scientist George Green (1793–1841). He worked full-time in his father’s bakery from the age of nine and taught himself mathematics from library books”. The development of science and technology has depended chiefly upon the genius of certain gifted individuals and their experience has owed less to formal education than might be supposed and more to experience of life as a whole. I know many people blaming their twenty years or more formal education; it does not enable them being famous, or influential or even to win their economical demands. We had learnt to fulfill parents’ desire. My philosophy is based up on pragmatism and social re-constructivism thoughts.

By *social re-constructivism*, I refer to Lev Semenovich Vygotsky's social constructivism. Vygotsky attempted to explain human thought in new ways and contributed to the psychological thought (Vygotsky, 1978) by emphasizing on socially meaningful activity as an important influence on human consciousness. My educational practice[2] has been in line with this perspective. It goes to the development of *subjective meanings* of *experiences* by looking for the *complexity of views* than labeling meanings into few categories or ideas.

The two world views have many features in common; their underlying ideas are central to the philosophical foundations of the interpretivist research paradigm. At the end of the course, I published my first ever article on Mathematics Education as a Discipline (Degu, 2015). The implication was stated as: theorizing and philosophizing could be opportunities for mathematics education researchers; it is a good opportunity for frontiers to play lots of roles. Solomon has been my supervisor since my Master's program. I was planning to extend my former Thesis on mathematics curriculum using content analysis. But, he advised me to "think out of the box" and appointed me for a discussion.

I had been thinking about the phrase . . . *Thinking out of the box* is commonly suggested by many individuals as against a thinking bounded in. He demanded me to change the thinking I had. So, a different idea was expected. That is to get rid of rigid ways of

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[2] I remember I had enjoyed organizing social events by the time I attended my undergraduate degree program at Bahir Dar University during 2000-2004. After I graduated there in 2004, I served as one organizing committee of social events while I was teaching secondary school mathematics. Being an enthusiastic young teacher, among other things, I used to advocate *collaborative learning*. The school compound had lots of trees; some which form straight lines where I had facilitated students' *group works*. I had been delivering out-of-class assignments to my students there, by availing myself in person for supervision and guidance. However, this point of view may not be typically reflected in school mathematics (Zawojewski, Lesh, & English, 2003). During 2009-2014, I had been delivering *students' group work* as one strategy particularly for the course Transformation Geometry.

thinking and acting; deviating from the intended lesson to exploit an unexpected situation (Foster, 2015). This is depicted in the figure (a) below. But, how would the idea be unlike the overlooked thought? It makes sense to reconsider the opposite, *thinking inside or on the box*. Figure (b) portrays abstract level of seeing at *other* points or directions. Imagining the point in the cube (box) is disclosing the invisible to visible.

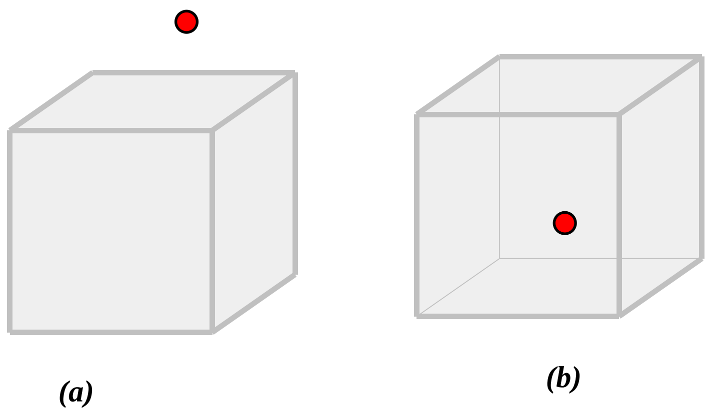

Figure 1.0.1: Thinking out of the Box versus Thinking in/on the Box

The term *thinking outside of the box* is related to imagination (Samli, 2011) or creativity (Glaˇveanu, 2014). Creativity is also related to *thinking inside the box* (P. Stokes, 2014). The later terms are of paramount importance (Whitcombe, 1988) in mathematics and *thinking* is a common thread to both. It is also noted in preparatory secondary (grades 11 & 12) mathematics teacher's guide that the field of mathematics education would help students develop creative and imaginative minds (Federal Democratic Republic of Ethiopia, 2010d). Mathematics is about multiple realities, complexities, and ambiguities (Saiber & Turner, 2009). In essence, doing mathematics is an exercise in imagination. Its essence is creativity (Mann, 2006). In turn, creative activities provide personal, social, and educational benefit (Mitchell, Inouye, & Blumenthal, 2003).

At the beginning of Ethiopian's millennium (2007/8), the Ministry of Education declared a 70:30 intake ratio for Higher Institutions' admission. In a sense that, 70% of the admission should be reserved for training in Science oriented fields and 30% students would join the Social Science & Humanities. In the years 2011, 2012 & 2013, I was permitted to extend my doctoral study; but, the opportunities were limited to Mathematics areas.

Sileshi shared us a comment forwarded to [Ethiopian] science educators by a colleague from Social Science camp. It was stated as: "we are all teaching history". What does it mean and imply? Here is the story in short. The Ministry of education had begun admitting students in higher institutions by applying a 70: 30 ratio of science and technology to social science fields of studies. Many professionals from humanities and social science area were disappointed by the regulation and/or practice. Yet, the aforementioned colleague argued that Ethiopian science teachers are teaching history [of science] too. In a sense, a science or mathematics teacher transfers knowledge built up by former scholars. What value could an educator or teacher adds to the system? Sileshi extended the idea further and claimed that: *having a perspective and engaged in an imagination is needed*. In this millennium, there have been no undergraduate but on/off postgraduate programs in science and mathematics education. It was only Addis Ababa University who stood up persistently, indeed with the effort of few scholars to sustain the field.

Once again, Solomon and Sileshi gave us an assignment on the relation between curriculum and instruction. My response has been updated and reported here. One of the most compelling needs in mathematics education is ensuring that all students have access to high quality, engaging mathematics (Lamon, 2003). Concerns for engaging students; the process of mathematical thinking and reasoning; and importance of productive mathematical conversations might be insufficient (Herbel-Eisenmann, Choppin, Wagner, & Pimm, 2012). So, various ways for teachers to encourage students talk in classrooms (Morgan, 1998) need to be thought. This calls for teacher's role in students' active engagement in the instructional process. In the classroom context, there could be discussions between teacher and students and among students themselves.

1.2 Statement of the Problem

The Government of Ethiopia is promoting team work in many of its sectors. In the schools, the [mathematics] teacher is expected to create favorable conditions for students to come together in groups and discuss on ideas (Federal Democratic Republic of Ethiopia, 2010d). So, it is about a strategy in the teaching and learning mathematics. The "ideas" to be discussed are expected to be mathematics related issues or contents. Yet, the statement brings about two basic questions to be posed at school practices. How a teacher facilitates group learning? What do students gain after discussing on ideas?

Formal education is mainly concerned with students' activities in the classroom. Since they are members of the larger society or community, mathematics learning is expected to be closely linked with daily life (Federal Democratic Republic of Ethiopia, 2010c) by paying attention to the practical application of mathematical concepts, theorems, methods and procedures. In line with this, there were questions and recommendations already pinpointed by scholars in the field.

In 2012, three questions (Zhou & Luo, 2012) were forwarded to go through: (1) What factors influence individual creative contributions at the group level in a specific learning environment? (2) What environmental factors influence group-level creative synergy? (3) From a knowledge-creation view of group creativity, what elements are necessary for building a learning environment conducive to creativity?

A year later, it was reported that only small number of empirical studies on creativity associated with mathematics have been carried out (Leikin, 2013). Then, researchers were invited to examine instructional support and classroom practices that afford the development of mathematical creativity (Aizikovitsh-Udi, 2014). Besides, context provide opportunity or challenge for the students to exhibit and develop their abilities or can enjoy mathematics by observing and feeling its essence and beauty (Kenderov et al., 2009). Yet, context refers to surrounds something, but is not part of it (Clarke et al., 2018). Thus, the phenomena under investigation should be broadly conceived.

As it would be discussed in the coming chapter, the research gap shows that there is a need to re-conceptualize creative imagination (CI); see if the in- and out-side classroom environments are factors for fostering creativity; and apply qualitative view of constructs for imagination and creativity (I & C). Basically, different positions on the relationship between I & C are taken in the literature:

(i) I & C are the same
(ii) Creativity is the application of imagination;
(iii) Creativity occurs without imagination taking place;

Hence, it will be important to study the complex interaction between I & C (Jung, Flores, & Hunter, 2016). A deep conflict between different views about matters which arise in human experience is a philosophical problem (Palma, 1991). Thinking about mathematics inspires metaphysical, epistemological and semiotic questions (Saiber & Turner, 2009). Accordingly, the following concerns need to be addressed in elucidating the relationship between mathematical imagination and creativity (I & C). Do I & C exist in mathematics classroom? How do we know about them to be true? What mathematical language represents I & C? Thus, the complex interaction between I & C demands philosophical clarification supported with empirical evidences.

1.3 Objectives of the Research

The objective of this research was to explore the status of imagination and creativity (I & C) in a classroom mathematical discourse at Babur School in Dire Dawa city. The specific objectives were:

- To explore the environmental factors influencing group-level synergy?
- To investigate the existence of I & C in grade 12 students' small-group interactions and discussions;
- To examine relationship between I & C in grade 12 students' small-group interactions and discussions.

The word "status" could mean condition, position, standing; it also refers to situation of the state of affairs. The classroom practice is in the school context; in turn, it is positioned in the city context.

1.4 Guiding Research Questions

The following were guiding research questions as per the objective of the study.

- What environmental factors influence group-level synergy?
    - How do the socio-cultural aspects in the research milieu impact on the classroom mathematics?
    - What instances of instructions elicited students' mathematical discourses?

- How do individual students contribute to small-group interactions and discussions?

- How do I & C happen in grade 12 students' small-group interactions and discussions?
- How do I & C relate to one another?
  - Which components of imagination fit in to creativity?
  - What aspects of thinking belong to CI?

A "status" study would uncover considering the wider situation although the focus is on the classroom level. So, the external and internal environmental factors would be incidental to the study.

1.5 Significance of the Study

Mathematics education as a discipline is situated at the confluence of an array of diffuse, seemingly incommensurable, and radically divergent discourses. Research claims that have grown out of mathematics education are wide-ranging and antagonistic rather than circumscribed by hidebound disciplinary frames. There has never been a unified, totalizing discipline of knowledge labeled 'mathematics education research', and while it has always been a contested terrain (Valero & Zevenbergen, 2004). I looked into the various dimensions of mathematics education: essence of mathematics, social constructivism, situated learning, sociocultural theory, ethnomathematics, indigenous knowledge, mathematicology, and philosophy of mathematics education.

This research would add values and knowledge to the international Mathematics Education audience. Mathematics Education is at infant stage, especially in Ethiopian context, the findings of this research will be useful in improving the awareness of different stakeholders towards the discipline. So, this research would have contribution for the promotion of the discipline. The methodological approach of the study will provide scope for new issues to emerge in mathematics education and, as a consequence, instigate further studies. The research outcome will hopefully show why and how I & C are needed to be developed in Ethiopia.

1.6 Delimitation of the Study

This research falls under the philosophy of mathematics education. It was conducted in Dire Dawa city at one private school. Although the instructional process of Grade 12 student's mathematics was given due focus, the wider context of the city is also studied. In other words, both in and out of classroom contexts were regarded. The first three chapters of grade 12 mathematics education were covered. The chapters consist of but not limited to the basics concepts of Sequence, Series, Limit, Continuity and Derivative. Hence, the method of inquiry was Clarke's Situational Analysis (SA) which lies in the family of grounded theory (GT).

1.7 Limitations of the Study

This study was conducted at one school; it would have been great if more schools were studied too. Thus, the conclusion of this study would be restricted to the school itself. As the method of inquiry was SA, it brought challenges. The relational and positional maps were geometrical; but, the locations of constructs in maps were not placed with exact measurements. The maps were produced solely by the analyst; it would have subjectivity limitations. In turn, it was an opportunity to seek a perspective towards subjective truth in mathematics education.

Chapter 2: Review of Related Literature

By the preceding chapter, the following issues were introduced. They were: - philosophizing, theorizing, conceptualizing mathematics education, subjective meaning making, context and situation, social constructivism, thinking, imagination and creativity. This section is a brief overview of conceptual orientations of these key elements.

2.1 Mathematics Education: its Essence and Status as a Discipline

The essence of Mathematics Education is concerned with the ontological, epistemological and values of the subject. Most of the researchers and educators try to swim in some parts of mathematics education (Eisenberg, 2014), for instance:

- ✓ *Dispelling math-phobia;*
- ✓ *How to give students non-verbal reinforcement;*
- ✓ *Verbal and social interactions in the classroom;*
- ✓ *Teaching techniques in inner-cities, and still others who worry about teaching techniques in rural areas via long-distance education;*
- ✓ *Trying to understand how the brain works with respect to the sequencing and processing of information;*
- ✓ *Understanding if there are gender differences in learning styles; and*
- ✓ *The role of symbols in mathematics.*

Thus, I am intended to review the definitions of “Mathematics Education” as a full-fledged discipline. Firstly, I looked for its scope in journals directly dedicated to serve as a platform for the subject. Then, later in this subsection, I give the views of notable scholars in the field.

The *Journal for Research in Mathematics Education (JRME)* is devoted to a disciplined inquiry into the teaching and learning of mathematics[3]. In this case, *Mathematics Education* is assumed as teaching and learning of Mathematics. Similarly, the *Mathematics Education Research Journal (MERJ)* provides a forum for publication of research in the teaching and learning of mathematics. As such it promotes a forum for the publication of high quality empirical research and theoretical/philosophical papers that contribute to the knowledge of mathematics education[4].

On the other hand, the journal *For the Learning of Mathematics (FLM)* aims to stimulate reflection on mathematics education at all levels, and promote study of its practices and its theories. It is intended for the mathematics educator who is aware that the learning and teaching of mathematics are complex enterprises about which much remains to be revealed and understood[5]. FLM is more inclusive than what its name entails. Besides, the *International Electronic Journal of Mathematics Education (IEJME)* aims to stimulate discussions at all levels and aspects of mathematics education including empirical, theoretical, methodological, and philosophical works that have a perspective wider than local or national interest[6].

Another journal *Research in Mathematics Education* is publishing original refereed articles on all aspects of mathematics education[7]. Then, what do "all levels" and "all aspects" mean? Again, *REDIMAT - Journal of Research in Mathematics Education* **-** is

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[3] https://www.nctm.org/Publications/journal-for-research-in-mathematics-education/About-the-Journal-for-Research-in-Mathematics-Education/
[4] https://www.springer.com/education+%26+language/mathematics+education/journal/13394
[5] https://flm-journal.org/index.php?do=board_aims&lang=en
[6] https://www.iejme.com/home/aims-and-scope
[7] https://www.tandfonline.com/action/journalInformation?show=aimsScope&journalCode=rrme20

an international space to stimulate scientific discussion on mathematics education drawing on scientific evidence. It publishes original empirical and theoretical work, focusing on scientific research from a diversity of theoretical and methodological approaches[8]. An immediate question could be: if research in Mathematics Education is expected rely on to empirical and scientific approach, then to what extent theory and methods would vary? In other words, does it entertain post-modern and interpretive paradigms? In this regard, *ZDM - Mathematics Education* – surveys, discusses, and builds upon current research and theoretical-based perspectives in mathematics education[9]. In parallel, the monograph series - *Advances in Mathematics Education* - is producing themed issues with lead scholars in the field. It aims to integrate, synthesize and extend papers by orientating issues of relevance towards the future state of the art. Indeed, there are more efforts in defining the discipline.

2.1.1 Interdisciplinary Mathematics Education: A Concern of Philosophy

Since mathematics education is rooted from different disciplines, adoption and adaptation of theories from respective fields would bring plenty of approaches. The sources of theoretical diversity (Bikner-Ahsbahs & Prediger, 2014) can be summarized as follows. They could evolve independently in different cultural circumstances, including traditions of typical classroom cultures, values, but also varying institutional settings (Sriraman & English, 2010). Accordingly, various ways of knowing in the field of mathematics education produce different theoretical views (Degu, 2015). Thus, the diversity is an indicator for the dynamic character of the field and an outcome of the dynamic of theories.

[8] http://hipatiapress.com/hpjournals/index.php/redimat
[9] https://www.springer.com/education+%26+language/mathematics+education/journal/11858

The expansion of theories in use within the mathematics education research community can be categorized as (Sriraman & English, 2010): *Cultural psychology*, including work based on Vygotsky, activity theory, situated cognition, communities of practice, social interactions, semiotic mediation; *Ethnomathematics*; *Sociology*, sociology of education, post structuralism, hermeneutics, critical theory; and *Discourse*, to include psychoanalytic perspectives, social linguistics, semiotics.

The large diversity already starts with the heterogeneity of what is called a *theoretical approach* or a *theory* by various researchers and different scholarly traditions (Bikner-Ahsbahs & Prediger, 2014). That demands to seek a *mathematics education* where the socio-cultural aspect is relevant, the classroom activity is important, and the daily mathematical practices have role. The teaching and learning of mathematics inculcates the key symbols and subject specific languages, and then the meaning making aspects. If we choose the broader view of mathematics education as a discipline, it would lead us to think of all the above categories at a time. A broader view of mathematics education is to consider its roots: discourse, language, sociology, psychology, anthropology, etc. Besides, mathematics and education are indispensable elements of mathematics education.

A study group of interdisciplinary mathematics education (Williams et al., 2016) posed the need for conceptual clarity, and situated the different conceptions of interdisciplinarity in a social, cultural, and historical account of how disciplines in general have arisen. Different theories can be made to network with each other and in particular inform researchers interested in analyzing their data from *multiple perspectives* (Sierpinska, 2016). Yet, addressing interdisciplinary issues lies in the philosophy of mathematics education (Ernest et al., 2016).

2.1.2 Mathematics versus Mathematics Education

A mathematician could be a capability of working with certain key mathematical ideas and their applications. Then, who would be a mathematics educator? In other words, what the interests, foundations, and goals of mathematics education as a field should be? Is it possible to imagine the alignment between mathematics education and mathematics as academic disciplines? The next diagram illustrates the concern better.

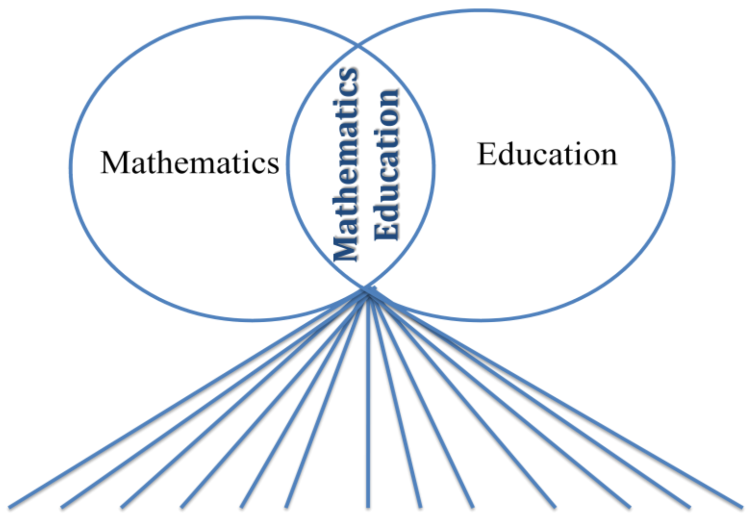

Figure 2.0.1: Foundations of Mathematics Education

What can be inferred is that Mathematics and education are indispensable elements of mathematics education. In turn, both mathematics and education have diversified roots. Thus, mathematics education stems from disciplines as diverse as mathematics, psychology, philosophy, logic, sociology, anthropology, history, women's studies, cognitive science, semiotics, hermeneutics, post-structuralism and post-modernism (Hoyles, Morgan, & Woodhouse, 1999). In order to address issues of mathematics education, we need to consider the above theoretical domains. Thus, Mathematics Education is a multi-disciplinary field of study which treats a wide range of diverse but interrelated areas.

It would be vivid to see that mathematics education is *in* Mathematics; it is also found *in* Education. Mathematics is a living and changing discipline; it is a powerful language for sharing and systematizing knowledge (Hoyles et al., 1999). Yet, how these could be conceived or practiced if the classroom activity is delimited to the implementation of designed textbooks? Therefore, learning mathematics should entertain acquiring unexpected learning too. Equally, we do need to know what the word “education” means and what counts as being educated (Barrow & Woods, 2006). If we are going to further think about it, we must have a conception of mathematics education.

2.1.3 Where is Mathematics in Mathematics Education?

I attended my Master’s degree at the then Department of Mathematics Education of AAU. I was hoping Gebre (name changed), one of my classmates, would score “Excellent” in his final Thesis work. At the Defense, an assessor asked him “where is the mathematics in your research?” The concern has appeared in literature too. For instance, Heid’s (2010) critics “where is the Math?” at the *Journal for Research in Mathematics Education* was a good foreword. The question was repeated as: Where Is the “Math” in “Mathematics Education” These Days (Eisenberg, 2014)?

As depicted in the above Figure, some portion of mathematics and education are *in* mathematics education. Thus, a research in mathematics education would uncover mathematical issues (Heid, 2010). In a sense, mathematics should be an integral part of mathematics education research (Bikner-Ahsbahs, Knipping, & Presmeg, 2015; Heid, 2010). That is, learning mathematics could enable to be aware of the mathematical way of thinking at the world.

The concerns about our understanding of, and the meaning we attribute to, mathematics and its nature are issues of philosophy of mathematics education (Radford, 2018). Then, *what mathematics educational thought constitutes* would be the crucial question. The aforementioned domains provide lenses through which researchers can view the theory and practice of mathematics teaching and learning (Hoyles et al., 1999). Thus, framing mathematics education from broader perspective is essential.

Research in Mathematics education is as a special type of discourse and consistent collection of stories coming from it is known as a theory (Sfard, 2018). In other words, theory plays a powerful role structuring what research is able to say (deFreitas & Walshaw, 2016). Any attempt to explore the role of theory needs to focus on what it means for particular theories to be applied.

The idea of [mathematics] education as an academic discipline in its own right and with its own forms of theory and theorizing is a way in which the academic study of the field has developed in other settings (Biesta, Allan, & Edwards, 2013). Theorizing needs to be encouraged in researchers at all levels (Bryant, 2017). In line with this, I once engaged in theorizing and outlined ten common research areas in Mathematics education (Degu, 2015). Those were:- Philosophy of Mathematics Education, Ethnomathematics, Psychology of Mathematics Education, Mathematics Curriculum, Mathematicology, Pedagogy of Mathematics, Technology in Mathematics Education, Mathematical discourse and communication, Equity in Mathematics Education, and Research Methods in Mathematics Education. It is possible to criticize that the research areas are not exhaustively classified. For instance, to which sub domain does *mathematics teacher education* belong to?

The above report is minor evidence entailing that I was interested in theorizing. Yet, theorizing demands persistence, practice, and self - critical insight to move to higher and ever more effective levels of competence and proficiency (Bryant, 2017). The following sections address some more discussions on theory and theorizing in the academic field of study.

2.2 Philosophy of Mathematics Education

The philosophy of mathematics education is rooted from the nature of mathematics itself (Bendegem & François, 2007; Ernest, 1991) and education (Ernest, 2016). It tackles questions about (1) the purposes and meanings of teaching and learning mathematics; (2) our understanding of, and the meaning we attribute to, mathematics and its nature; and (3) the relationship between mathematics and society (Radford, 2018). Research and theories in mathematics education are analyzed according to the branches of philosophy they draw upon, including metaphysics and ontology, epistemology, social and political philosophy, ethics, methodology, and aesthetics (Ernest, 2018). So, it is beyond studying about the purposes of teaching and learning mathematics. It addresses the following concerns: Mathematics, learning, pedagogy and applying philosophy into Mathematics Education. Since philosophy is concerned with making explicit the nature and significance of concepts, the philosophy of Mathematics Educations also the status of Mathematics Education as a field of knowledge. Basically, its position is to analyze, question, challenge, and critique the claims of mathematics education practice, policy and research (Ernest, 2018). Hence, it is vital to be clear what it means.

2.2.1 Understanding of Mathematics and its Nature: the Notion of Limit in Focus

The field Mathematics education is rooted from and connected to Mathematics and Education. In other words, it is included in the disciplines and at the same time encompasses them. Thus, a practice and research in mathematics education will uncover the understanding, and the meaning making we attribute to, mathematics and its nature. For instance, an instruction or research on the notion of limit will be concerned with definitions, symbols, practical examples, conjectures, and proofs in relation to the concept. In many mathematics curricula, the notion of limit is introduced three times: the limit of a sequence, the limit of a function at a point and the limit of a function at infinity (Fern´andez-Plaza & Simpson, 2016). On the other hand, the concept of derivative is introduced as the limit of slopes of secants of smaller widths. Likewise, Riemann integral is the limit of Riemann sums of finer meshes.

Thus, a student cannot understand the concept of derivatives before understanding the concept of limits (Areaya & Sidelil, 2012). There are crucial terminologies such as: *convergence, divergence, rate of change, greatest lower bound, least upper bound, tends to, approaches but never touch, infinity,* and so on. It can be said that Limit is fundamental to the standard formal foundations of many aspects of Calculus. Besides, students' understanding of limit concerns the use of dynamic imagery (Fern´andez-Plaza & Simpson, 2016) by considering the pattern of numbers or functional values.

There are at least two reasons that necessitate the understanding and meaning making of mathematics. First, Mathematics is characterized by a combination of natural language, symbols (e.g. $\sum$, $\rightarrow$, ∞, $\int$, etc) formulae, models (e.g. Cartesian plane, 3-dimensional space, Poincare model, etc), and visual displays for expressing ideas or concepts.

Consequently, students must learn the ways of thinking and communicating them. Second, though there are arguments, mathematics is a human activity, a social phenomenon, part of human culture, and intelligible in a social context (Ernest, 1991). Nowadays, large parts of the philosophy of mathematics are with descriptions and analyses of mathematical practices (Degu, 2015), the relationship between mathematics and human beings, and socio-philosophical reflections on the role of mathematics for society (Prediger, 2007). It may imply to generate new interdisciplinary approaches, instruments, and models including new knowledge, transcending scientific boundaries to adopt a more holistic approach.

Generally, there are five characteristics of mathematics (Linnebo, 2017): abstraction, idealization, computation, extrapolation and infinity, and proof. Mathematics is concerned with abstract features of actual or possible objects or systems of objects. On the other hand, idealization is common in geometry. For instance, in a circle, each point is *exactly* the same distance from its center. Another characteristic of mathematics is computation. It goes to the algorithmic operations on syntactic signs or other systems of representations. The signs on which we compute can be used to represent objects and their properties. So, computation plays an essential role in connection with term formalism, Hilbert's finitism, and intuitionism. For every numeral, we extrapolate and start to reason about the entire sequence of numerals. Proof is an instrument in mathematical investigations, not the object of these investigations.

An intuitive idea should be expressed in a format which is sufficiently precise to enable deduction, calculation and proof (Sierpinska & Kilpatrick, 1998); that is one important practice in mathematization. The intuitionists' view of mathematical reality leads them to interpret mathematical language regardless of the laws of logic. Intuitionists believe that mathematical objects are our own creation (Bostock, 2009); their approach is important for introducing an idea more informally. For instance, the following definition is taken from grade 12 student's text book (Federal Democratic Republic of Ethiopia, 2010b), page 62.

> Let $y = f(x)$ be a function defined on an interval surrounding $x_0 \in \mathbb{R}$ (but f need not be defined at $x = x_0$). If $f(x)$ gets closer and closer to a single real number L as x gets closer and closer to (but not equal to) x_0, then we say the limit of $f(x)$ as x approaches to x_0 is L. Symbolically, this is written as: $\lim_{x \to x_0} f(x) = L$.

This intuitive definition of the limit of a function at a point contains: language, symbols and visual imagery. The key term is "limit of". What do we mean by "an interval surrounding x_0"? What does "f need not be defined at $x = x_0$" imply? There are more phrases that need to be understood such as: "gets closer and closer to", "approaches to", "a single real number", The symbol x_0 refers to the point at which the corresponding functional values of various real numbers are compared. Here, the numbers are represented by x, variable, and the functional values are denoted by $f(x)$. The comparison of $f(x)$ is assumed to be equal to another fixed number, say L. The idealizations of "as x approaches to x_0" will form a mental (visual) imagery in our mind. Likewise, "$f(x)$ gets closer and closer to . . . L" is abstract by itself.

It is becoming increasingly clear that mathematical thinking is essential to understanding the world around us (Krantz, 2018). While it would be unreasonable to impose a causal requirement on the knowledge of mathematical truths, we still want an informative answer to the question (Linnebo, 2017). Certainly, there is room for intuition in mathematics, and even room for guessing (Krantz, 2018). But, a mathematical situation and/or solve a problem by being very logical; logic makes the process dependable and reproducible. It shows that what we are producing is a verifiable truth. So, thinking and reasoning are realized through semiotic activity where by language, symbolism, and visual imagery function together in mathematical discourse (Bjuland, Cestari, & Borgersen, 2008).

2.2.2 Purposes of and Meanings in the Learning-Teaching of Mathematics

In the classroom, the basic event is instructional process thereby curriculum, teacher and students are key players for learning to happen. The implementation of a curriculum would come up based on teacher's perception and how the student is experiencing it. Then, the organization of tasks, mathematical objects, actions, teacher's and students' activities determine the overall outcome of the instructional process.

Successful instruction results from broader view of a system for dealing with people, process and programs (Ornstein & Hunkins, 2018). In other words, the intended, perceived, implemented and practice curriculum may not be the same. Thus, I believe that a curriculum should be planned with some degree of flexibility to include teacher and learners' experiences as well as emerging learning opportunities. That means, outside of the classroom activity should be planned too. After all, a curriculum should be minimum input for effective instruction; and the flexibility could be exercised by way of implementing the curriculum.

The purposes of and meaning attributed in the teaching and learning of mathematics is another concern in the philosophy of mathematics education. If philosophical inquiry is applied to classroom mathematics, it may aid in the opening of a "wider horizon of interpretations" that includes a potential expansion of students' mathematical experience, and promises to provide bridges for establishing richer, critical, and more meaningful connections and interactions between students' personal experience and the broader culture (Kennedy, 2018). An approach to understanding learning from a sociocultural and interactional perspective implies a focus on development and appropriation of knowledge (Langman & Hansen-Thomas, 2017). Thus, in the teaching and learning of mathematics, there would be a demand for activities that involve explanations and justifications of arguments. These, in turn, require sophisticated use of language linked to mathematics-specific content area learning.

The interaction of interpersonal, cultural–historical, and individual factors are key to human development. The way that learners interact with their worlds transforms their thinking. The meanings of concepts change as they are linked with the world. He considered that social environment is essential for learning. The view links together the contexts of schooling and research mathematics is mathematics centered (Ernest, 1998).

The work of Vygotsky has provided many researchers with theoretical tools for interpreting the social origins of thinking and learning (deFreitas & Walshaw, 2016). Therefore, learning can be viewed as a process by which individuals construct meaning through socially and culturally situated activity (Cobb, Yackel, & McClain, 2000) under the particular character of classroom communication (Steinbring, 2005). Students learn through sharing their ideas, listening to and critiquing the ideas of others, and by having others critique their approaches in solving problems (Smith & Stein, 2011; Sullivan, 2011). It is through joint engagement that ideas are argued over, contested, borrowed, and shared as our understanding is advanced.

Currently, it is being promoted in the Ethiopian educational practice that it relies on *constructivism* ideology. The assumptions are to involve students actively in their learning and to provide experiences that challenge their thinking and guide them to rearrange their beliefs. Advocators of Social constructivism consider that social group learning and peer collaboration are useful. As students model for and observe each other, they not only teach skills but also experience higher learning (Lesh & Doerr, 2003).

The learning and understanding of mathematical concepts by students can be enhanced by the positive interaction between the in- and out-of-school mathematical activities (Tegegne, 2015). Tegegne argued that concepts presented in the textbooks should be related to the students' real world in terms of the tasks and activities, the examples, problems and their solutions, ideas carried, and so on. The daily practices of a classroom provided students with knowledge which would be evolved as they took part in the socially developed and patterned ways (deFreitas & Walshaw, 2016). These are recommendations to closely link mathematics learning with daily life.

2.2.3 The Mathematics We Live By: Mathematics and Society

Much of the mathematics we are using today was developed as a result of modelling real world situations (Federal Democratic Republic of Ethiopia, 2010b). Mathematics shapes the world in which we live; the world in turn shapes the discipline of mathematics (Greenwald & Thomley, 2012). As the societal problems evolve, new mathematical solutions would be created.

A situational perspective focuses on idealization of some aspects of the world; Mathematics is part of the physical world (Poythress, 2015). Thus, a mathematical knowledge is socially and culturally situated (Clarke, Friese, & Washburn, 2015; Clarke et al., 2018; Razfar, 2012). It is a cultural phenomenon; a set of ideas, connections, and relationships that we can use to make sense of the world (Boaler, 2016). Since mathematical practices vary across different communities (Moschkovich, 2007), Mathematics education research would be a local practice in socially and culturally bounded places and communities (Sekiguchi, 1998). In this regard, socio-cultural and situational perspectives are relevant to study the "mathematics" being practiced at a certain milieu.

2.3 Discourse Studies and Implications for Research in Mathematics Education

Mathematical meaning is produced in discourse and we are always trapped between producing or reproducing meaning (Brown, 2001). The term discourse refers to written or spoken communication or debate; a formal discussion of a topic in speech or writing; a text or conversation[10]. It is also considered as a serious and lengthy speech or piece of writing about a topic[11]. Equivalently, it is about a series of discussions on something between people or groups. The concept is common in linguistics and mostly entails as a unit of text used for the analysis of linguistic phenomena that range over more than one sentence. On the other hand, discourse corresponds to the ability to reason or the reasoning process[12].

Since the 1960s, a new field of research has emerged around the concept of *discourse*, known as Discourse Analysis (Angermuller, Maingueneau, & Wodak, 2014). Yet, the

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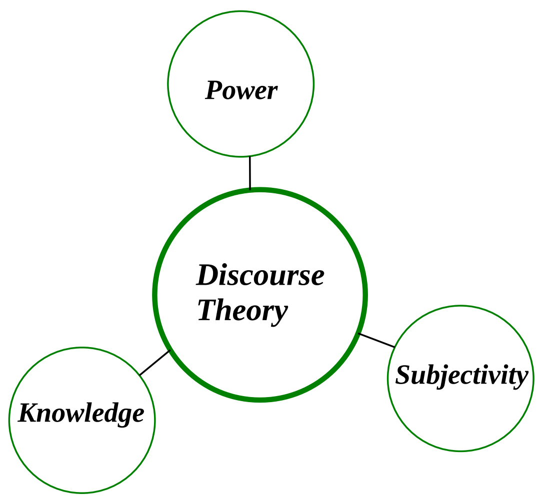
[10] Concise Oxford English Dictionary © Oxford University Press
[11] Encarta Dictionaries 2009 © 1993-2008 Microsoft Corporation
[12] Collins English Dictionary and Thesaurus

concept has become vague, either meaning almost nothing, or being used with more precise, but rather different, meanings in different contexts (Jørgensen & Phillips, 2002). On the other hand, it refers to the language structured according to and used in when people take part in different domains of social life.

Then, if discourse is about a written or spoken communication or conversation, then who is going to be the doer of it? Doesn't he/she determine the extent to which discourse is acceptable or not? And will discourse be the same if it emanates from different sources or people? The coming figure articulates the concern and portrays that discourse is a function of power, knowledge and subjectivity.



Source: The Discourse Studies Reader (Angermuller et al., 2014)

Figure 2.0.2: The Triangle of Discourse Theory: Power, Knowledge, and Subjectivity

The written or spoken communication is highly determined by knowledge. The more knowledgeable is going to be accepted. Most of the time books and reputable journals have been trusted as sources of information. There is power implicitly given to books, a professional or an authority; thus, the discourse they exhibit might be taken for granted.

These discussions imply that discourse is arbitrary and a subjective truth is expected of it. In other words, discourse can considered as the postmodern way of looking at reality.

2.3.1 Discourse in Mathematics Education

The descriptions and analyses of mathematical practices, the relationship between mathematics and human beings, and socio-philosophical reflections on the role of mathematics for society are relevant. Research on local mathematical practices and the implementation of these practices in the formal mathematics curriculum relies on the practical turn within the philosophy of sciences and the social turn in learning theory (François, Mafra, Fantinato, & Vandendriessche, 2018). In turn, they share the combination of scientific practices and learning processes in a given sociocultural environment. In this regard, *knowing* would be located in relations among practitioners, their practice, the artifacts of that practice, and the communities of practice (Lave & Wenger, 1991).

On the other hand, problem-solving process, in many of the most important 21st century contexts, involves teamwork (Zawojewski et al., 2003). Students would learn better together by interaction and sharing of ideas (Sullivan, 2011). Such an attempt of equating learning as emergent and social lies in *situated learning theory* (Lave & Wenger, 1991). Learning and teaching mathematics is a complex process, requiring both students and teachers to know and use a variety of types of knowledge, including knowledge of the language/communication challenges inherent to mathematics learning (Wilkinson, 2015). Recent research suggests that learning can best be understood through analyses of sociocultural context (Langman & Hansen-Thomas, 2017). The issue under consideration is related to “discourse” which is common in, but not limited to, linguistics. Discourse refers to written or spoken communication or debate; a formal discussion of a topic in speech or writing; a text or conversation[13].

Thus, understanding is a dialogical phenomenon, and its achievement a fundamentally social and collaborative process (Zhou & Luo, 2012). By introducing important mathematical ideas and ways of thinking, we could analyze the event in terms of students' distinctive conceptual constructions. This perspective assumes that participants bring multiple perspectives to a situation, that representations and utterances have multiple meanings for participants, and that these multiple meanings are negotiated through interaction (Moschkovich, 2007). Therefore, curriculum should be planned with some degree of flexibility to include outside of the school activity (Ornstein & Hunkins, 2018). After all, the curriculum should be the minimum input for effective instruction; and the flexibility could be exercised by way of implementing the curriculum.

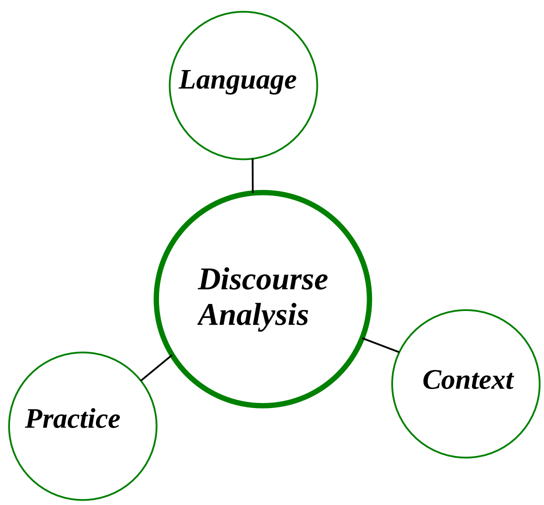
Mathematics is one subject in the school curriculum. The learning and understanding of mathematical concepts by students can be enhanced by the positive interaction between the in- and out-of-school mathematical activities (Tegegne, 2015). As to Tegegne, concepts presented in the textbooks should be related to the students' real world in terms of the tasks and activities, the examples, problems and their solutions, ideas carried, etc. Successful learning takes place when learners solve real life problems through active engagement in activities and extensive communication and collaboration. In this regard, out-of-school practices can be of interest in the formal mathematics curriculum and for the formal school system (François et al., 2018). Then, the pedagogical approaches shall be in accordance with the local knowledge they are imbedded in. It implies to the demand

[13] Concise Oxford Dictionary

for interdisciplinary activities and the integration of mathematical knowledge in a lively and meaningful context. Accordingly, a situated learning process considers the informal learning of out-of-school transmission of local practices.

2.3.2 Classroom Situation or Situated Classroom

In studying the dynamics of a classroom, it is necessary to understand the environment which circumscribes it. Since discourse is about the language use when people take part in different domains of social life (Jørgensen & Phillips, 2002), discourse study would be the study of language in use. It is the study of the meanings we give language and the actions we carry out when we use language in specific contexts (Gee & Handford, 2012). So, language, context and practice are the underlying variables in studying a discourse.



Source: The Discourse Studies Reader (Angermuller et al., 2014)

Figure 2.0.3: The Triangle of Discourse Analysis: Language, Practice, and Context

Mathematics has its own language; the context would mean the in and out of classroom environment. In other words, the context is the situation whereby the classroom discourse is being orchestrated. The third dimension, practice, is concerned with engagement and participation. In this regard, the *Building Thinking Classroom* (Liljedahl, 2016) framework would permeate mathematics classroom practice. Liljedahl forwarded the following remarks on the matter.

1) *The type of tasks used and when and how they are used;*
2) *The way in which tasks are given to students;*
3) *How groups are formed, both in general and when students work on tasks;*
4) *Student workspace while they work on tasks;*
5) *Room organization, both in general and when students work on tasks;*
6) *How questions are answered when students are working on tasks;*
7) *The ways in which hints and extensions are used, while students work on tasks;*
8) *When and how a teacher levels to their classroom during or after;*
9) *Assessment, both in general and when students work on tasks*

Students' questions might lead to explanations and justifications that may be challenged and subsequently defended (Corazza & Agnoli, 2016). Practice can extend to everything that may take place between the participants in interaction; context can be thought of the setting, situation or knowledge available. Such knowledge can be situation-dependent or situation-transcendent, individual or shared by large collectives (Angermuller et al., 2014). But, what does situation mean? Situation is a set of circumstances in which one finds oneself; the location and surroundings of a place[14]. It also refers to the conditions that characterize somebody's life or events in a particular place, country, or society[15]. Besides, it would mean a complex or critical state of affairs in a novel, play, etc[16]. When a classroom situation is a unit of analysis, it would have impact on further development of research in mathematics education (Bussi, 2005). Thus, studying classroom mathematical discourse is a compulsory issue of researching in the field.

[14] Concise Oxford English Dictionary © Oxford University Press
[15] Encarta Dictionaries ® 2009
[16] Collins English Dictionary and Thesaurus

2.3.3 Mathematical Discourse in a Situated Classroom

From social constructivism perspective, there is emphasis for "learning-as-participation" than "learning-as-acquisition". Then, mathematics would be discourse developed by groups of people working together. Accordingly, students must be encouraged and supported directly in their efforts to learn to speak, read, and write mathematics. Yet, the linguistic, symbolic, visual forms of representation of mathematical knowledge are complex but interwoven (Wilkinson, 2015). Thus, discourses replace the metaphor of *learning-as-acquisition* with the metaphor of *learning-as-participation* (Sfard, 2008). Then, we could conceive of mathematics as a discourse developed by groups of people working together (Appelbaum, 1995).

In order to study discourses, we have to research the language and student's actions, interactions, values, beliefs, and uses of objects, tools, and environments within classroom (Gee, 2014). The scenario describes mathematical discourse in situation and social context (Razfar, 2012). So, what does it mean to learn in such a context? And what type of knowledge is expected of students? It would be possible to conceive knowledge as more an issue of the social, historical and political conditions under which writing, speaking and thinking come to count as true or false (deFreitas & Walshaw, 2016). In this sense, learning is a process of participation in communities of practice; it increases gradually in engagement and complexity (Lave & Wenger, 1991). It takes as its focus on the relationship between learning and the social situations in which it occurs.

Student engagement in mathematical discourse begins with the decisions when teachers plan classroom instruction (Kersaint, 2015). There are reasons for drawing students' attention to discourse (Herbel-Eisenmann et al., 2012) which denotes any specific instance of communicating (Kieran, Forman, & Sfard, 2003). Effective mathematical discourse is an iterative process by which students engage in a variety of types of discourse at different cognitive levels. Among the assumed features of a discourse rich classroom is *thinking*. It has a high likelihood of improving the students' understanding of important mathematics (Zoest et al., 2017). Thus, the generation of new ideas that have value is regarded as creative thinking.

2.4 Imagination in the Philosophies of the Mind and Mathematics

We are living in the early years of the twenty first century. These days, it is witnessed that philosophical interest, in imagination, extends broadly from ethics to epistemology, from science to mathematics (Kind, 2016). Imagination is associated with many issues: mental imagery, perception, memory, empathy, and dreaming (Beaney, 2010). It is commonly practiced in aesthetics while there are engagement with music, art, and fiction. Imagination in epistemology and philosophy of science, including learning, thought experiments, scientific modeling, and mathematics (Kind, 2016). That might be the reason for many authors (Carson, 2010; Valett, 1983) to quote Albert Einstein's word as "imagination is more important than knowledge".

So, what does it mean by imagination? It is about forming ideas or mental images[17]. It is the ability to deal resourcefully with unexpected or unusual problems[18]. It is also the creative act of perception that joins passive and active elements in thinking and imposes unity on the poetic material[19]. That leads us to the conception of imagination as it feeds

[17] Concise Oxford English Dictionary
[18] Encarta Dictionaries 2009
[19] Collins English Dictionary and Thesaurus

our ability to ask the big questions, to think large and deep (Gallas, 2003). Such an ability of asking big questions can be identified in a number of different ways.

2.4.1 Definitions and Conceptions of Imagination

The concept *imagination* is an abstract and philosophical. Beneay (2010) synthesized different authors' conceptions of imagination in to seven faculties.

Table 2.0.1: Seven Conceptions of Imagination

No.	Aspect of Imagination	Definition	Illustration
1	Corporal imagination	*Having or processing images in the brain*	Imagine about a vertex of a cube which is not directly observable.
2	Empirical imagination	*the empirical association and reproduction of ideas*	Limit of a function at a point can be computed by considering sample numbers from the neighborhood of the point.
3	Intellectual imagination	*supposition, such as forming hypothesis, conceiving of and engaging in thought experiments, and raising special doubts*	Learning Mathematics gives a power of thinking and reasoning, inductive, analysis, synthesis, originality, generalization, discovery, etc.
4	Productive imagination	*Actively synthesizing our intuitions and for freely producing new ideas or representations.*	A linear mapping f from set A to set B has two criterions: $\mathrm{f(x+y)=f(x)+f(y)}$ and $f(ax)=af(x)$. Then, a student might construct an equation $f(ax+by)=af(x)+bf(y)$ as short cut method.
5	Reproductive imagination (RI)	*reproducing ideas or representations*	Standards such as representation, communication, reasoning and proof, and connection
6	Sensory imagination	*Perceiving images*	If we model $3-space$ using corners of a classroom as axis lines, then our conception is determined by our point of view.
7	Creative Imagination (CI)	*recombining experiences in the creation of new images directed at a specific goal or aiding in the solution of problems*	We usually write "Want To Show", or its abbreviation WTS, while we proof a theorem. Then, we need to be creative in expressing or relating the variables.

Although there are such typologies of imagination, some aspects could be related to each other. For instance, sensory imagination depends on corporal imagination. The ability to perceive space, to imagine objects in different positions; and to characterize shapes, their properties and relationships among their elements is spatial imagination (Vallo, Rumanova, & Duris, 2015). If I position myself at different locations (say at *A* and *B* in the figure below) in a Room, then my perception for the intersection of Walls of the Room would be different.

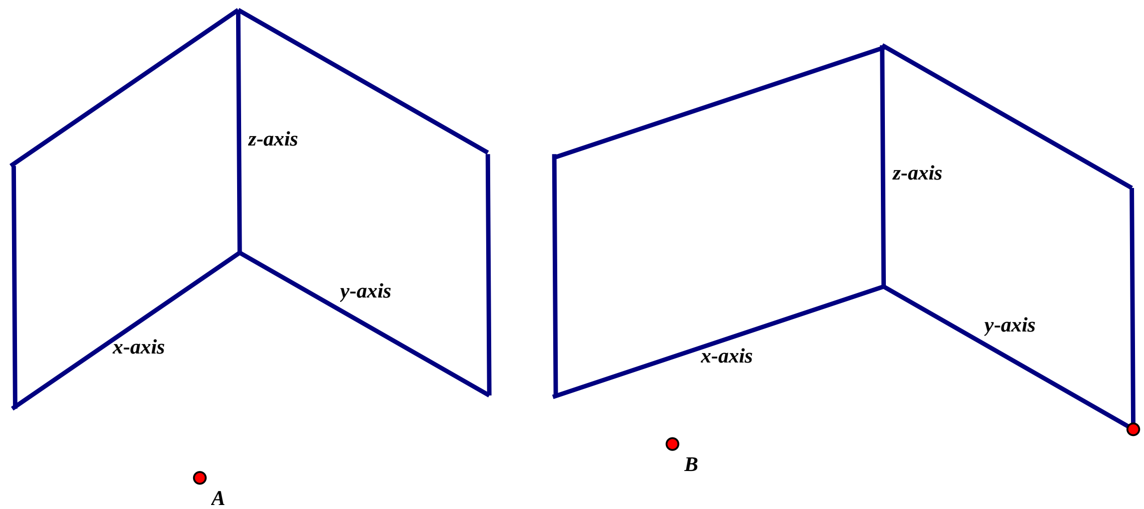


Figure 2.0.4: Perceiving xyz-Space from Different Locations

On the other hand, aspects of imagination can be grouped in to *creative* and *reproductive imaginations* (Liang, Chang, Chang, & Lin, 2012). That means, the two components are regarded inclusively with other aspects of imagination. If the first terminology is more inclusive, it demands some illustration.

The distinguishing attribute of the human animal is its unique ability to imagine and manipulate symbolic thoughts and ideas (Valett, 1983). In the beginning of 20th century, Théodule A. Ribot and Lev S. Vygotsky concurrently introduced the concept of CI (Karwowski, Jankowska, & Szwajkowski, 2017). According to Vygotsky, the development of imagination originates in the child's social dialogue with adults (Smolucha & Smolucha, 1986). In other words, the Vygotskian perspective regards imagination as it is learned through social interactions.

As research and theories developed, attention was drawn to the complexity of CI (Smolucha & Smolucha, 1986). For instance, Karwowski et al (2017) noted the concept refers to the ability of creating and transforming mental representations based on the material of past observations, but significantly transcending them. A brain activity includes re-experiencing of images. Thus, according to this view, the dimensions of CI are: vividness, originality, and transformative ability. When CI is actualized into dynamic form within the human mind, the person begins a transformational process to self-realization (Valett, 1983). In this way, the powerful idealized image becomes real in life.

Considering the pivotal role that creative ideas play in human societies, and creativity's contribution to multiple aspects of human life, understanding the cognitive components underlying creativity has become increasingly fundamental (Palmiero et al., 2016). Thus, teachers must value the development of CI and thinking by providing their students with open, flexible, opportunities for creative development (Valett, 1983). More broadly, imagination involves episodic memory retrieval, visualization, mental simulation, spatial navigation, and future thinking, making it a complex cognitive construct (Jung et al., 2016).

2.4.2 Imagination: Is Mathematics Invented or Discovered?

Mathematics is about multiple realities, relative truths, complexities, and ambiguities. In essence, doing pure mathematics (not merely doing computations) is an exercise in imagination—and imagination, an exercise in abstraction (Saiber & Turner, 2009). For instance, the irrational number π with approximation of $\frac{22}{7}$ is actually a different number $3.1428571428571\cdots$ where the decimal goes on forever without repeating.

Besides, the introduction of $\sqrt{-1}$ or i brought the set of complex numbers. So, a complex number is written in the form of $a + bi$ where $i = \sqrt{-1}$; i is imaginary number. Indeed, there are so many speculations in mathematics: set theory, the discovery of non-Euclidean geometry, topological space, etc.

The question "is mathematics invented or discovered?" has occupied the interest of philosophers and mathematicians for centuries. This will be addressed if we understood the nature and role of imagination in mathematics. There are twofold usages to *RI* and *CI* (Liang et al., 2012) reported earlier. The first refers to reproducing of ideas (Beaney, 2010) or representation of the visual. It considers imagination as the ability to retain a simple impression of objects; and imagination is treated as faculty of representation. It implies the discovery aspect in mathematics, demanding consideration of a rather different set of issues. The later is assumed as the ability to think of novel solutions to problems – in other words, ingenuity (Arana, 2016). It belongs to the ability to arrange objects, and combine them in a thousand ways. It is CI and thus the faculty of invention, and is linked with genius, in particular in mathematics.

2.4.3 Imagination and Learning Mathematics

As discussed above, learning Mathematics is the development of a particular type of imagination (Nemirovsky & Ferrara, 2009). Thus, taking imagination is a critical part of the educational process (Gallas, 2003). The process starts with curriculum planning and development. An Imaginative Curriculum would then include the following features for the students (Jackson, Oliver, Shaw, & Wisdom, 2006).

Being imaginative:- to think in ways that move us beyond the obvious, the known into the unknown, that see the world in different ways or from different perspectives, that take us outside the boxes we normally inhabit and lead to the generation of new ideas and novel interpretations.

Being original: - make a contribution by way of inventing, adapting and transferring on what already exists

Exploring for the purpose of discovery: - experimenting and taking risks, openness to new ideas and experiences typically linked to problem working

Using and combining thinking skills: - critical thinking to aid evaluation, synthesis and intuition to interpret and gain new insights and understandings.

Communication: – this is integral to the creative process (for example, storytelling as a means of communicating meaning within the discipline).

The first feature of an imaginative [reproductive and creative] curriculum is inclusive of others. The second feature which talks about "being original" is in line with the creative aspect of imagination and thus would enable students invent a 'Mathematics". The last three would go to the reproductive version of imagination. Hence, they would contribute for certain mathematical discoveries.

The preparatory secondary school Mathematics Teachers' guides (Federal Democratic Republic of Ethiopia, 2010c, 2010d) sought for stimulating student's inquiry. The role of the teacher is expected to be guiding students construct their own knowledge and skills; discover concepts by themselves; and develop personal qualities that will help them in real life. As the students develop personal confidence and feel comfortable on the subject, they would be motivated to address their material to groups and to express themselves and their ideas with strong conviction. We can extract some features that qualify the curriculum as imaginative. For instance, if students are guided in such a manner, they will be equipped with communication skill and could explore for the purpose of discovery.

2.4.4 Indicators of Imagination

If imagination is beyond knowledge, is it really possible to measure it? I have accessed three validated instruments that serve for measuring imagination.

Hunter's Imagination Questionnaire (HIQ): - it served for assessing imagination over an extended period of time (Jung et al., 2016) via an online portal. The instrument HIQ was designed to: (1) capture aspects of memory retrieval, visualization, simulation, spatial navigation, and episodic future thinking, (2) capture imagination activities over an extended period of time, and (3) ask participants to envision future goals and achievements. Finally, participants were instructed to review all of their ideas and notes from the last four sessions and to consider which appealed to them the most, which ideas they will implement, and which they were most likely to forget or not implement. They were instructed to select three of their best ideas. This was followed by a set of questions ranked on a scale. The questions are presented below.

1. *How passionate or engaged are you with the ideas you generated?*
2. *Have you taken steps to implement any of your ideas?*
3. *How likely are you to implement or continue implementing your ideas in the days and weeks to come?*
4. *How difficult was the first session?*
5. *Did the process become easier or more difficult as you repeated the assessment?*
6. *Please estimate how much time you devoted to thinking about your ideas between sessions.*
7. *Are you satisfied with the number of ideas you generated?*
8. *Did the assessment process help you learn about your own thinking?*
9. *How would you rate your experience of the assessment?*
10. *Please provide an overall assessment of your ideas on a scale of 1–10*

However, the instrument sounds for neurological investigations. How could we employ the questionnaire in a classroom setting?

Liang's *Indicators of Imagination* is a more inclusive method of assessing imagination. There are four indicators of RI: transformation, crystallization, and effectiveness. There also five indicators of creative imagination: elaboration, exploration, intuition, novelty, productivity, and sensibility as indicators of assessing human imagination (Liang et al., 2012). The account of each indicator is presented as follows.

Transformation*: - represents the ability to perform tasks by transforming knowledge across multiple fields of study.*

Crystallization*: - represents the ability of individuals to express abstract ideas by using concrete examples.*

Effectiveness*: - represents the ability of individuals to generate effective ideas to a desired goal.*

Elaboration*: - is the ability of individuals to seek improvement by formalizing ideas.*

Exploration*: - represents the ability of individuals to explore the unknown.*

Intuition*: - represents the ability of individuals to generate immediate associations to a target.*

Novelty*: - represents the ability of individuals to create uncommon ideas.*

Productivity*: - represents the ability of individuals to generate numerous ideas.*

Sensibility*: - represents the ability for individuals to arouse feeling during the creating process.*

Yet, the proposed indicators could not be enough to reach at a wise judgment about demonstrations on imagination.

Test of Creative Imagination (TCI) was developed by Janusz Kujawski in mid 1990s. This paper-pen test, TCI, is comprised of: fluency, originality, elaboration and transformativeness (Gundogan, Ari, & Gonen, 2013). Again, these components are qualitative constructs; and thus, the paper-pen evaluation would be too subjective. The authors (Liang et al., 2012) invited all of the readers to test and validate the indicators proposed in their study, by using the following items.

- *I often come up with new ideas through my intuition*
- *I often help myself imagine through personal emotions*
- *I constantly have ideas toward my designs*
- *I like to explore the unknown world*
- *I often have uncommon ideas compared to others*
- *I improve my thoughts by focusing on formalizing ideas*
- *I often complete my tasks by focusing on effective ideas*
- *I think flexibly and can transfer ideas to multiple fields of tasks*
- *I am good at expressing abstract ideas by using concrete examples*
- *I am good at seeking improvement by logically analyzing ideas*

The authors' thought is that imagination is needed to construct activities, build system, and anticipate conversations and actions that will bring learners' inquiry to fulfillment, enabling growth toward desirable skills and understandings. Yet, since imagination is abstract and a philosophical concern, such test of imagination would be rather considered as just indicators. So, additional evidences would be required to conclude about a learner's imagination.

2.5 Creativity

Mathematics education is a boundary crossing discipline; for instance, the if essence of Mathematics is creativity (Mann, 2006), then creativity would have implication for Mathematics education. Hence, it has to be functional at school and community wide.

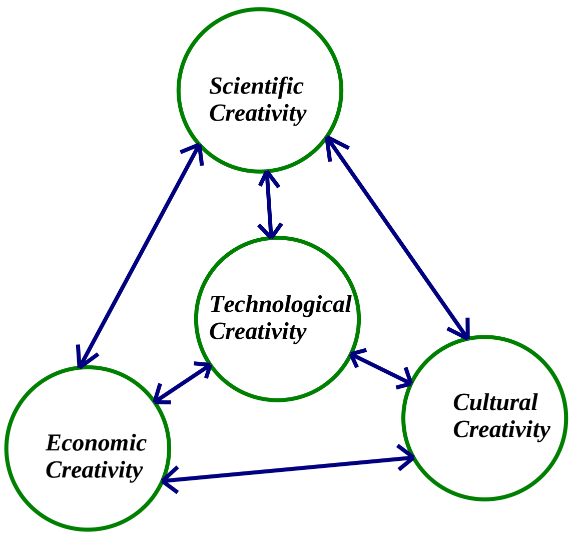
The term creativity is commonly spoken by peoples of different age. The dictionary meaning equates it as activity or engagement[20]; it is the quality of bringing something into existence or resulting in something or making something to happen[21]. It is also

[20] Concise Oxford English Dictionary © Oxford University Press
[21] Microsoft Encarta Dictionaries 2009

related to cleverness, fertility, imagination, imaginativeness, ingenuity, inspiration, inventiveness, originality, productivity, and talent[22].

2.5.1 Conceptions of Creativity

In a vastly changing world, creativity is needed both for adapting to changes in social networks and individuals' lives and for continuing technological and scientific advancements (Leikin, 2013). Personality and motivational variables as well as the socio-cultural environment are sources of creativity (Sriraman, 2004). Creative activities provide personal, social, and educational benefit; and creative inventions which are in turn key drivers of economic development (Mitchell et al., 2003).



Source: Mitchell *et al*, (2003, P.22)

Figure 2.0.5: Domains of Creativity

It is a boundary-crossing discipline that is essential to learning and teaching, social-economic dialogues, academic discourses and cultural practices, as well as technological and digital communications (Corazza & Agnoli, 2016). It is multifaceted (Leikin, 2013; Runco, 2014) and one cornerstone, among others, for innovation and economic progress

[22] Collins English Dictionary and Thesaurus

of nations across the globe (Ayele, 2016; Florida, Mellander, & King, 2015). In explaining the interactions among the domains of creativity, Mitchell *et al* (2003) noted that scientific discovery sometimes drives technological invention; and the pursuit of technological innovation suggests scientific questions and ideas. They also indicated that an entrepreneurial energy may motivate engineering and product innovations, but such innovations may also demand creative strategies for successfully bringing inventions to the market. The above elements focus Scientific/Mathematical, technological, economic, and cultural creativity could result in new knowledge, technological/product innovation, Entrepreneurship and cultural improvements respectively. We have seen that different domains of creativity could lead to variety of innovations. Innovation is the result of using creative acts to produce something unique.

2.5.2 Mathematical Creativity

There is an increase of interest recently in the classroom creativity of mathematics students (Czarnocha, Baker, & Dias, 2018). Yet, research on creativity has been on the fringes of psychology, educational psychology, and mathematics education (Liljedahl & Sriraman, 2006). This would imply that there is no agreed up on and complete theory of creativity (Leikin, 2013; Zhou & Luo, 2012). The nature of mathematics could entail what type of creativity would be expected of in this field. Thus, there is no single, authoritative perspective, or definition of creativity in the field of mathematics education.

Mathematical creativity can be considered as an ability to analyze a given problem from different perspectives, see patterns, differences and similarities, and generate multiple ideas (Uwaezuoke & Charles-Organ, 2016). Again, it is fine to pose: “could one model adequately capture the creativity of children as well as the creativity of adults, both novices and experts in their fields" (Kaufman & Beghetto, 2009)? A creativity exhibited by gifted individuals is extraordinary (Kaufman & Beghetto, 2009). But, the ordinary or everyday creativity is more relevant in a regular school setting (Sriraman, Haavold, & Lee, 2013). An orientation or disposition toward mathematical creativity may be fostered among students of different mathematical abilities (Levenson, 2015).

2.5.3 Developing Student's Creativity

Teaching for creativity relates to the objective of identifying young people's creative abilities, encouraging and providing opportunities for the development of those capacities (Lin, 2011). Specific conditions are needed if creativity is to boom in student's learning. In this regard, the Bloom's Taxonomy of Educational Objectives served as a format for classifying educational goals, objectives, and, most recently, competencies. In this perspective, the revised taxonomy of cognitive process (Krathwohl, 2002) puts creativity at the top remembering, understanding, applying, analyzing, and evaluation. Creativity is assumed as higher order thinking.

The ability to proceed creatively is associated with a number of personality traits: flexibility, problem sensitivity, abundant production of ideas, ability to restructure thoughts, and ability to elaborate on skeleton ideas (Brater, Buchele, Reuter-Herzer, & Selka, 1989). The question then becomes: how can we foster mathematical creativity in the classroom? This is related to process; and, how could we check the increment in the development of creativity? For that matter, there is lack of a consistent definition of creativity. Besides, it can be deduced that many researchers seem to explain the idea of creativeness and few of them tried to demonstrate the value of imagination in the nurturing of creativity. But, how we would know it is not succinctly explained. Besides, creativity is used to be perceived from individual student's growth and development. Anyways, the pedagogical view of creativity is illustrated with different showcases as follows.

Table 2.0.2: Summary of Pedagogical Techniques for the Development of Creativity

Views	Domain	Method	Conclusion
Creativity is connected with problem solving or problem posing (Aizikovitsh-Udi, 2014)	gifted students attending a special program	Tasks with no explicit instruction and no time limit	Three types of reasoning: analytical reasoning, practical reasoning, and creative reasoning;
mathematical creativity is the process for novel (insightful) solutions and new questions (possibilities) (El-Sahili, Al-Sharif, & Khanafer, 2015)	unexpected links in mathematics classrooms	Design problems	"The Unexpected Links" is a component of mathematical creativity; it can deliver to school students.
Teachers help students to contribute towards creating a classroom culture (Foster, 2015)	Exploiting unexpected situations	students working on a new mathematical task	It can lead students into intense and enjoyable mathematical engagement
Creativity has possibilities for the generation of new ideas, a subjective inner life symbolized through imagination, and creative social action (Griffiths, 2014)	Implications for Teacher Education	use of combination of unfamiliar ideas	Encouraging I & C in experienced teachers is a powerful way to make space for creative students, and, possibly, encourage them.
Creativity as a mental ability emerging from a social context. It is about divergent/ imaginative thinking (Hadzigeorgiou, Fokialis, & Kabouropoulou, 2012)	creativity in science and science education	Conceptualizing prior ideas on the matter;	Suggesting activities or strategies; yet, good science education cannot help but foster students' imaginative skills and creativity.
creativity can be nurtured and encouraged in the school environment (Hirsh, 2010)	arts integrated into their mathematics classroom at elementary school	Interview of students	Students access to content, multiple perspectives on a topic, and invites them to think, apply, understand, create, and participate in their learning
creativity benefits from, and enhances, learning (James, 2015)	Managing the classroom for creativity	providing students with freedom, positive challenge, encouragement, work group supports, resources	classroom environments are created that enable students to thrive academically and creatively

Table 2.0.2: Pedagogical Techniques for the Development of Creativity (Continued)

Views	Domain	Method	Conclusion
a reasoning sequence is created or recreated sufficiently fluent and flexible enough to avoid restraining fixations (Jonsson, Norqvist, Liljekvist, & Lithner, 2014)	facilitate students' own construction of solutions	Allowing mathematical struggle in didactical situations (no teacher support) with tasks	Effective in terms of memory retrieval and construction of knowledge; it is more beneficial for students cognitively less proficient.
Creativity as DT (Levenson, 2015)	Teacher's changing perspectives	Tasks with potential different ways of doing	creativity may be promoted among students learning in mixed-ability classes
Everyone has potential to be creative in terms of everyday problem-solving (Lin, 2011)	Creativity and pedagogical practices	examining theoretical assumptions	creative teaching, teaching for creativity, and creative learning are interrelated
Creativity is the end result of the manner in which human cognition works (Mehta, Mishra, & Henriksen, 2016)	Common Manners of Creative mathematics teachers	Looking into the profiles	Creative mindsets persist in their lives and everyday work. It is where the individual, the domain, and the field (their peers) operate in harmony
Imagination is the ability to visualize ideas; Creativity is everything that did not exist before but can be realized (Perrone, 2014)	Design	Interventions	gives students great satisfaction
creativity as both a social and a personal construct (Sriraman et al., 2013)	Giftedness & Everyday creativity	Review of literature	Mathematical creativity is linked to and influenced by ability, beliefs, intelligence, cognitive style, and the classroom environment
Mathematical creativity is an ability to analyze a given problem from different perspectives, see patterns, differences and similarities, and generate multiple ideas (Uwaezuoke & Charles-Organ, 2016)	teaching mathematics with creativity	experimenting, experiencing, hands-on activities and collaboration	When the teacher and students' creative participation is guaranteed, sustaining students' interest and improving their performance in the subject is possible.

The in- and out-side classroom environment is key factor to provide the proper nourishment for the development of creativity. It demands instructional support and classroom practices that afford the development of mathematical creativity. The development of creativity in collaborative work and classroom discourse is not properly considered. If ordinary creativity is more relevant in a regular school setting (Sriraman et al., 2013), then everyday activities need to be promoted.

2.5.4 Qualitative Constructs of Creativity

There have been attempts in quantifying creativity. For instance, *Ruth Noller's creativity formula*[23] as $C = f_a(K, I, E)$ – the interaction between knowledge, imagination, and evaluation generates Creativity. But, *Torrance Tests of Creative Thinking* (TTCT) frequently appears in the literature. The test includes eight activities: ask-and-guess, product improvement, unusual uses, unusual questions, just suppose, picture construction, picture completion, and repeated figures of lines or circles (Kim, 2006). More broadly, the *global creativity index* in terms of talent, technology, and tolerance (Florida et al., 2015) serves as a broad-based measure for advanced economic growth and sustainable prosperity of economic development in rating nations worldwide. Yet, the multidimensional creativity concept demands assessments of products, processes and personal factors (Cropley, 2000). Thus, no one creativity test is enough to characterize the concept.

I started this study just considering *thinking out of the box* and *thinking inside the box*. The first could be compared with divergent or associative thinking where as the later concept might be equipped with convergent or deliberate thinking (Carson, 2010). Carson introduced a model of seven brain activation patterns that would affect an ability to think creatively. They are: openness and cognitive flexibility, mental imagery, divergent or

[23] http://www.creativeeducationfoundation.org/research/noller/

associative thinking, convergent or deliberate thinking, judgmentalism, self-expression, and improvisation or flow.

If we stick on the brain activation patterns, we may lost the social constitutes of I & C. The above seven indicators could not sufficiently describe creativity. The two components DT and mental imagery belong to imagination (Samli, 2011). But, Carson labeled them under creative thinking due to the fact that DT and CT are seemingly opposing yet interrelated processes (Wang & Hou, 2018). So, the creative process encompasses: CT, DT, and insight.

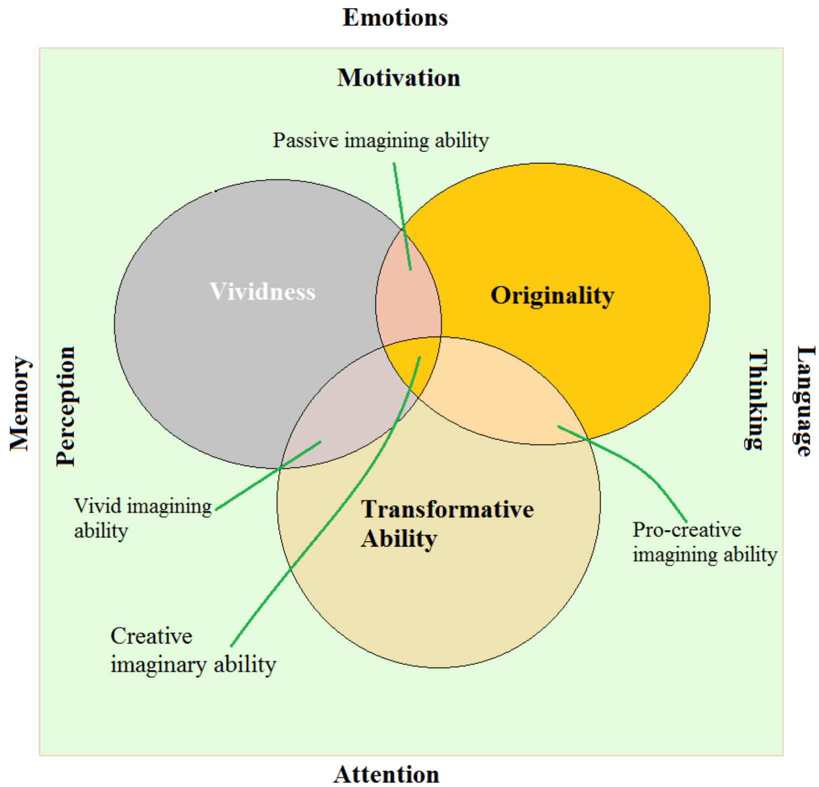
2.6 Interplay of Discourse, Imagination and Creativity (DIC)

Among the characteristics of mathematical creativity are: social interaction, imagery, heuristic, intuition and proof (Sriraman, 2004). Mathematics is considered as a multi-semiotic construction; that is, discourses formed through choices from the functional sign systems of language, mathematical symbolism and visual display (O'Halloran, 2005). In short, Mathematical discourse involves language, symbols and visual images (Markee, 2015). On the other hand, there is a range of reasons for drawing students' attention to discourse (Herbel-Eisenmann et al., 2012). Thus, learning mathematics is an initiation to mathematical discourse. Again, taking part in discourse demands a vivid imagination (Clark & Wege, 2001). In turn, imagination is related to free-thinking and knowledge creation are linked to imagination (Samli, 2011).

Over the decades, creativity and imagination research developed in parallel, but they surprisingly rarely intersected (Jankowska & Karwowski, 2015). The variety of approaches and methods to measure creativity and its components makes difficult to draw clear conclusion about this topic (Palmiero et al., 2016). Since creativity requires the collaboration of imagination and thinking in concepts (Smolucha & Smolucha, 1986), exploring the relationships among creativity and other cognitive domains, such as mental imagery, attention, and problem solving would be of valuable contribution to the existing literature.

Mental imagery fell into disrepute in the mid-twentieth century in both philosophy and psychology. Recently, however, scientists have found ways of studying them, and integrating their study with that of vision (Proudfoot & Lacey, 2010), and following this their study has become respectable again in philosophy too. Interest has centered mainly on visual images, and two main theories are current about what these are, pictorialism and descriptionalism. Pictorialists argue first that images do indeed exist and are used in solving various imaginative problems, and they do so by representing the relevant material in a spatial manner. Pictorialists use experimental data, such as that when subjects are asked to say whether two diagrams are congruent, where this could be found by rotating one of them, the time needed to answer is proportional to the size of the angular rotation required; this, pictorialists claim, suggests that subjects do indeed mentally rotate one of them.

Thus, CI can be assumed as the ability to create and transform representations (Jankowska & Karwowski, 2015). The following figure is impressive and would enable to think I & C simultaneously.



Source: Children's CIA (Dziedziewicz & Karwowski, 2015)

Figure 2.0.6: The Conjunctional Model of Creative Imaging Ability

However, there are cases in which creativity occurs without imagination taking place (Beaney, 2010). More specifically, creative efforts may be independent of images and imagery (Runco, 2014). A balancing argument is presented as follows. I already reported that CI is an aspect of imagination. According to Wang & Hou (2018), creativity is characterized by individual reserve, thinking process, motivation, product and evaluation. The individual reserve includes personality and intelligence. The motivation comes from the surrounding environment, the educational or social requirements and individual need. The evaluation includes the community evaluation or self-evaluation on individual's creative products. Thus, "creative imagining" belongs to creativity. In short, creative imagination belongs to imagination and creativity.

2.7 Critique of the Existing Literature

The essence and characteristics of mathematics include: abstraction, idealization, computation, extrapolation, and application of the concept infinity. In short, Mathematics and its learning are dependent on creativity, imagination and discourse skills. Yet, creativity is sometimes used as a synonym for imagination (Gundogan et al., 2013; Kaufman, 2016) and other times assumed to occur without imagination taking place (Beaney, 2010; Runco, 2014). In response to the later argument, the basic questions to be forwarded could be: aren't we generating new mental images throughout creative work and, conversely, aren't the products of creativity a springboard for imagination itself (Glăveanu, Karwowski, Jankowska, & Saint-Laurent, 2017)? Another argument is whether I & C could be assessed quantitatively or qualitatively. The measuring scales such as $f_a(K, I, E)$, *GCI*, *HIQ*, *TCI* and *TCIA* and *TTCI* entail that the first approach is dominating. Quantifying the notion of creativity is the psychometric approach to studying creativity (Sriraman, 2004). Yet, mathematical I & C could be measured in terms of the fluency, flexibility, and originality of ideas produced (Leikin, 2013; Levenson, 2015) which are in turn qualitative constructs. Again, such terms are relative by themselves. These could also be characterized in qualitative approach. So, approaches that focus on the social and continuous aspects of a creativity will find key roles played by particular interactions and preexisting ideas. It is time for setting out to construct new, more situational way of promoting creativity of students (Corazza & Agnoli, 2016). In conclusion, there is a need to see the in- and out-side classroom environments and apply qualitative view of constructs for imagination and creativity.

Chapter 3: Research Methods and Procedures

I have been seeking a *mathematics education* where the socio-cultural aspect is relevant, the classroom activity is important, and the daily mathematical practices have role. The presentation and analyses of data were framed from the wider context to the school level academic life to classroom discourse.

3.1 Conceptual Framework of the Study

In this study, Vygotsky's social constructivism was favoured as an epistemology and learning theory. The concept Epistemology belongs to philosophy. As it noted in the glossary section of this paper, philosophy is concerned with knowledge, reality, existence, values, reason, mind, language and many more issues. On the other hand, learning Mathematics is assumed as discourse developed by students and the development of imagination. Again, discourse is learning by participation. According to Vygotsky, imagination is learned through social interaction.

Based on the leading questions of this study and the critical review of related literature, a conceptual framework is given below. The framework is guided by the principles of philosophy of mathematics education which regards mathematics, its relation with the society, its teaching and learning issues. That is why the classroom discourse is conceived broadly to include the out-of-classroom contexts.

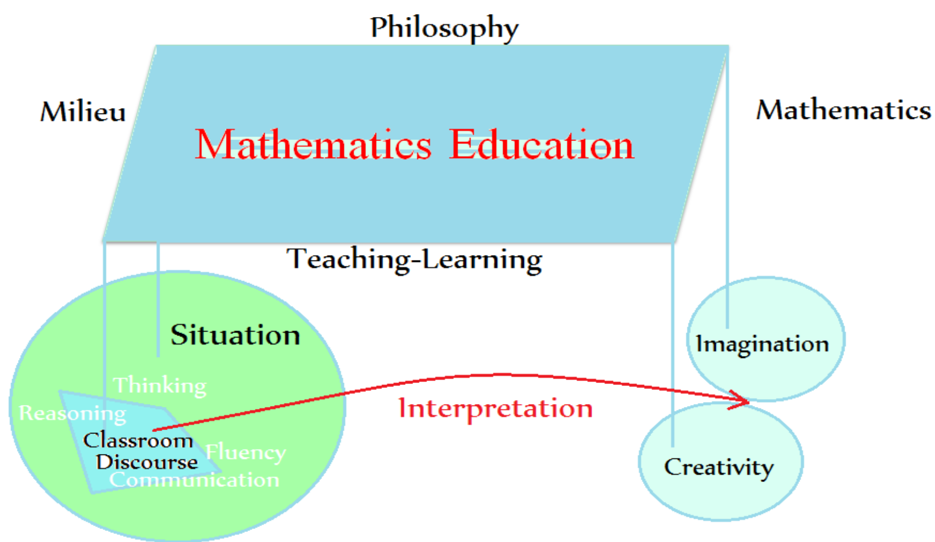


Figure 3.0.1: Conceptual Framework of the Study

In this regard, communication and reasoning are components of discourse. The lengthy speech or writing about a topic is related to fluency. Then, it would be indispensible to consider the forming of and using thoughts as far as discourse is concerned. Thus, the classroom discourse is represented by thinking, reasoning, communication and fluency would be mentioned and discussed.

The directed line segment connects the quadrilateral region with imagination and creativity. Yet, the relationship between imagination and creativity is controversial for the time being and would be explored in this research.

Creativity was operationally defined as the process that students do activities in new way particular to a situation with utility and practical worth. So, the immediate question will be: given a broadly conceived situation, what can students do peculiar to it? Thus, it follows that the scrutiny is situational analysis.

3.2 The Interpretive Research Paradigm

Mathematics is about explorations, conjectures, and interpretations (Boaler, 2016); hence *meaning making* (Prediger, 2007) has great potential in bringing a perspective. For instance, different conceptions of the Euclid's fifth postulate has created disagreement among geometers and, as a result, different forms of geometry[24] had emerged (Degu, 2015). I consider that *meanings are derived from social interaction and modified through interpretation* (Corbin & Strauss, 2008). Accordingly, multiple meanings could be created for the same object. Particularly, mathematical *meaning* is produced in discourse (Brown, 2001). Thinking mathematically, which is an aspect of mathematics education research, is about *interpreting situations* mathematically (Lesh & Doerr, 2003). That demands a mathematics education researcher to advocate the interpretive paradigm which views knowledge production socially and culturally.

Yet, there are various perspectives within the interpretive paradigm (Clarke et al., 2018; Howe, 2002): postmodernism, post-structuralism, and transformationism. On the other hand, it is recommended to actively include various approaches and understand people in their own terms, in their own environments (Howe, 2002).

[24] Euclid's fifth postulate: "that if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than two right angles" has created disagreement among geometers. As a result, different forms of geometry had emerged: Euclidean, Absolute, Elliptical and Hyperbolic geometries. Based on such disparities, for instance, the sum of the interior angles of a triangle is either $180°$, less than $180°$ or greater than $180°$. Absolute geometry takes no stand on such disagreements.

3.2.1 Grounded Theory as a Family of Methods

In 1967, Barney G. Glaser and Anselm L. Strauss published their seminal work *The Discovery of Grounded Theory* [GT]*: Strategies for Qualitative Research* to systematically produce a set of procedures for the generation of theory from qualitative data. It happened during the time that grand theory and the verification of such theories[25] were being practiced. Glaser and Strauss suggested that too much emphasis has been put on verification of the existing theories. Glaser and Strauss's work was intended to underscore the basic sociological activity that only sociologists can do: generating sociological theory.

However, GT has become as one method in qualitative research and applicable in many disciplines. In the one hand, Glaser and Strauss urged sociologists to discover theory systematically from data. In response to the call, many researchers have taken up that challenge as it will be discussed later in this section. On the other hand, the seminal work has been subjected to critique and criticism (Charmaz, 2006; Clarke et al., 2018). Accordingly, extensions of the method had begun emerging. But, the distinction between the Glaserian and Straussian[26] versions frequently reported in the literature.

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[25] They explained that the book's function is to provide a defense against doctrinaire approaches to verification. Attempts to close the gap between theory and research have concentrated principally on the improvement of methods for testing theory, and sociologists, as well as other social and behavioral scientists, have been quite successful in that endeavor. Attempts to close the gap from the "theory side" have not been nearly so successful. In fact, "grand theory" is still so influential and prevalent that for many researchers it is synonymous with "theory"- and so they think of "theory" as having little relevance to their research.

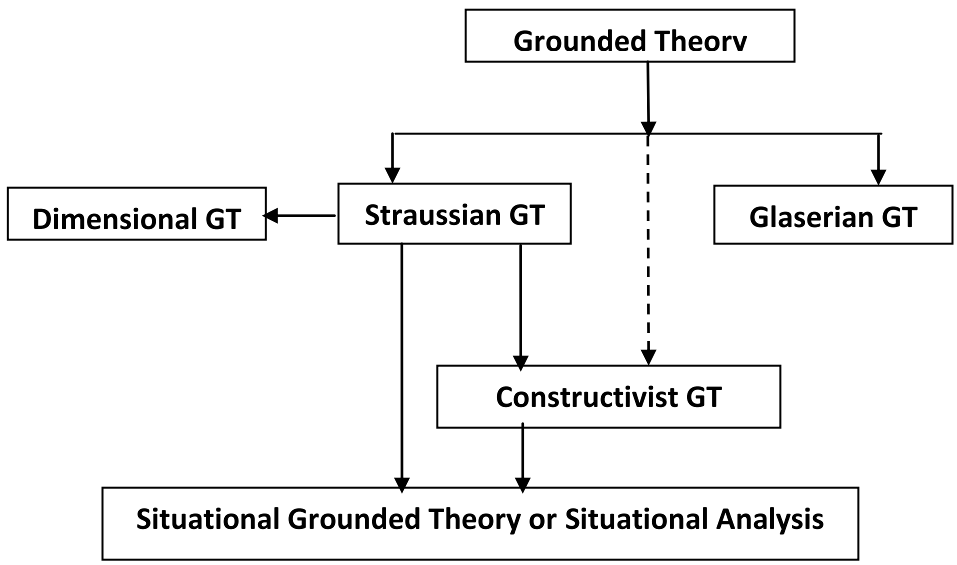
[26] The departure might have resulted in as Glaser and Strauss were already trained differently. Those sociologists were trained, separately, at Columbia University (about middle-range theory and quantitative methodology) and the "Chicago tradition" (down-to-earth qualitative research, a less than rigorous methodology, and an un-integrated presentation of theory). They noted that neither of these traditions nor any other in postwar sociology—has been successful at closing the gap between theory and empirical research.

Table 3.0.1: Comparison between Glaserian and Straussian Grounded Theories

Glaserian GT	Straussian GT
Beginning with an empty mind	Having a general idea of where to begin
Emerging theory, with neutral questions	Forcing the theory, with structured questions
Development of a conceptual theory	Conceptual description (description of situations)
Theoretical sensitivity comes from immersion in the data	Theoretical sensitivity comes from methods and Tools
The theory is grounded in the data	The theory is interpreted by an observer
The credibility of the theory, or verification, is derived from its grounding in the data	The credibility of the theory comes from the rigour of the method
A basic social process should be identified	Basic social processes need not be identified
The researcher is passive	The researcher is active
Data reveals the theory	Data is structured to reveal the theory
Coding is less rigorous, a constant comparison of incident to incident, with neutral questions and categories and properties evolving.	Coding is more rigorous and defined by technique. The nature of making comparisons varies with the coding technique. Codes are derived from 'micro-analysis which consists of analysis data word-by-word'
Two coding phases or types, simple and substantive	Three types of coding: open, axial and selective
Regarded by some as the only 'true' GTM	Regarded by some as a form of qualitative data analysis

Source: Comparisons of the two schools of GT (Jones & Alony, 2011)

The differences can have an important impact in the direction and execution of the primary research. The schools are further extended (Morse, 2009); Dimensional, Constructivist, and Situational grounded theories are the most recognized ones. The Dimensional Analysis is based on the assumption that research findings tell a story and that researchers need a perspective to select items from the data for the story, to create their relative salience, and to sequence them (Corbin & Strauss, 2008). As depicted below, the Dimensional Analysis extends from Straussian camp. Corbin & Strauss (2008) reported that the founder Schatzman worked closely with Strauss, and thus his emphasis fits very nicely with own approach to analysis. In *Constructing GT,* researcher Kathy Charmaz assumed that researchers are not separate from their theories but construct them through their interactions with people, places, and research perspectives (Charmaz, 2006). According to her, neither data nor theories are discovered, but are constructed by the researcher and research participant.



Source: Adapted (Clarke et al., 2018; Morse, 2009)

Figure 3.0.2: Genealogy of Grounded Theory and Situational Analysis

On the other hand, SA was sought to overcome the limitations of classical GT: oversimplifications, lack of reflexivity and its positivist revolution (Clarke, 2005; Clarke et al., 2015; Clarke et al., 2018). The intentions in this method are: to push GT more fully around the interpretive turn, capture the complexities and multiplicities of social life and produce thick analyses; and take into account human, nonhuman and discursive elements.

It is basically based on the principle that *situations* become the fundamental units of analysis. That means, the *situation of inquiry* is to be empirically constructed through the making of the maps and following through with analytical work and memos of various kinds[27]. Yet, the basic question that should be forwarded is "does SA lie in the family of GT?" In other words, does it really extend from Straussian, Glaserian and Constructing ground theories? The concern is addressed in the following section.

3.2.2 Situational Grounded Theory or Situational Analysis

In mathematics, "or" serves as disjunction. So, it is enough to get either *Situational GT* right or *Situational Analysis* true. In this sense, the later is not necessarily equivalent to the former one. According to the latest publication of Clarke et al (2018), SA can be used on its own right in studies centered on analyzing and interpreting situations (P. xxvi) and can use SA without using GT (P.361). Indeed, their analytic foci for GT and SA are different. GT generally focuses on analyzing action and specifying social processes; SA generally focuses on understanding relationalities among heterogeneous elements, social worlds, and debates in situations and their ecologies. It offers a set of analytic tools for researching the messiness and complexity of mathematics classroom (Bikner-Ahsbahs et al., 2015).

[27] The theoretical grounds of Situational Analysis as a theory/method package (Clarke et al., 2018) are: pragmatism; the return to the social in social theory; the early Chicago School conceptions of social ecologies foundational to interactionism; framings of the concept of the situation; Chicago social ecologies as they became contemporary social worlds/arenas frameworks, outgrowing their early geographic constraints to address more widely distributed organizational and institutional phenomena; conceptual toolbox of social worlds/arenas theory; the work of Michel Foucault, the gaze, discourse, formations, disciplining, fields of practice, conditions of possibility and the dispositive; salient nonhuman elements (nonhuman actors, actants or elements pervade social life) in situations of interest explicitly in to account; and the work of Deleuze and Guattari on rhizomes and assemblages. The co-constitution and relationality of rhizomes and assemblages make it vividly clear that neither humans nor nonhumans individually or collectively 'end at our skins'. Therefore, situational analysis can be used as a method assemblage and a topographical analysis that uses bits and pieces of theory to listen to and analyze complexities.

On the other hand, “Situational Analysis” is commonly practiced in business firms, industry, company, and projects by means of SWOT (strength, weakness, opportunity, & threat) analyses. Strength and weakness are related to internal factors where as opportunities and threats are associated with external factors. It includes the following elements (Annan, 2005): an analysis of the state and condition of people and ecosystem (including identification of trends and pressures); identification of major issues related to people and ecosystems that require attention; and analysis of key stakeholders – groups of people and institutions with a right, mandate and/or interest in resources and their management in the geographic area of the potential project.

In a sense, it would be participatory, outward looking, learning from others, using the framework of people and ecosystems, careful research, selecting indicators to describe the state and condition; identifying trends and pressures and responses.

Once again, Clarke and her co-authors frequently refer to SA and rarely use “situational grounded Theory”. But, if SA is really belonging to the GT, coding might be sought. Yes, analysis involves what is commonly termed as coding, taking raw data and raising it to a conceptual level (Corbin & Strauss, 2008). On the other hand, extracting elements from data for the purpose of mapping could be a form of descriptive or coding in the situation of inquiry. Besides, new and innovative approaches continue to grow in qualitative research. Thus, SA is an inclusive of methodological variations in the interpretive research paradigm.

3.2.3 The School of Books

I have been envisioning for having a skill of writing books. So far, I authored two books[28]; and I have more plans. It was just to develop the skill of such writing that I have been immersing in qualitative research paradigm. Through time, I learnt that new and innovative approaches continued to grow in qualitative research (Freeman, 2017). Then, as an advocator of a pragmatic world view, I urged to consider emerging methods. Data gathered in the forms of interviews, observation, audiovisual recordings, field notes, and the like need to be transcribed in to texts or put in images. Then, a document analysis follows.

Document Analysis has been highest level of learning in Ethiopian Orthodox Church education with the name *metsehaf bet* which is equivalent to “The school of books” or “School of Commentaries” (Tikuye, 2014; Tsegaye, 2011). The Church scholars have three different *interpretation methods*:

1) *yandemta tirguame* which means interpretation by alternatives;
2) *net'ela tirguame* which means literal meaning, gives direct meaning; and
3) *ye'mistir tirguame* which means interpretation does not take into consideration the structure of the sentence, but the meaning it conveys (Jemberre, 2012).

To sum up, when each sentence or phrase of a text is interpreted, depending on the content, theological, moral and historical questions are raised, discussed, and developed.

[28] Transformation Geometry: Introduction to Theory and Applications, 2015
Preparatory mathematics for Career Aspirations, 2018

3.3 Research Site

This research was conducted in Dire Dawa city at one Preparatory Secondary School on Grade 12 Natural Science students. I had reasons while I set the research site. I had been serving Dire Dawa University (DDU), a public University in Ethiopia, since September, 2009. In line with duties and responsibilities vested upon higher education institutions, I had been engaged in teaching, research and community services. On the other hand, I have been attending the doctoral program for which this study is a partial fulfillment under the sponsorship of DDU. Thus, I have commitment to continue my services in the area. As a base line, there was a prior study conducted in preparatory secondary schools of Dire Dawa city (Areaya & Sidelil, 2012); the study identified difficulties, challenges, and misconceptions in learning concepts of calculus (limit, continuity, and derivative).

3.3.1 Dire Dawa, Eastern Ethiopia

Dire Dawa is one of two chartered cities in Ethiopia (the other being the capital, Addis Ababa). It includes the city proper and the non-urban areas. The city is 515 kilometers away from the capital city Addis Ababa in east direction. The city can be reached via road, railway and air transportations.

It is the hometown for diverse people from different ethnic and cultural backgrounds. The main ethnic groups are, but not limited to, Oromo, Somali, Amhara, Gurage, Tigrie, Hareri and Hadiya. The widely worshiped religious are Ethiopian Orthodox Christianity, Islam (Muslim), and Protestant.

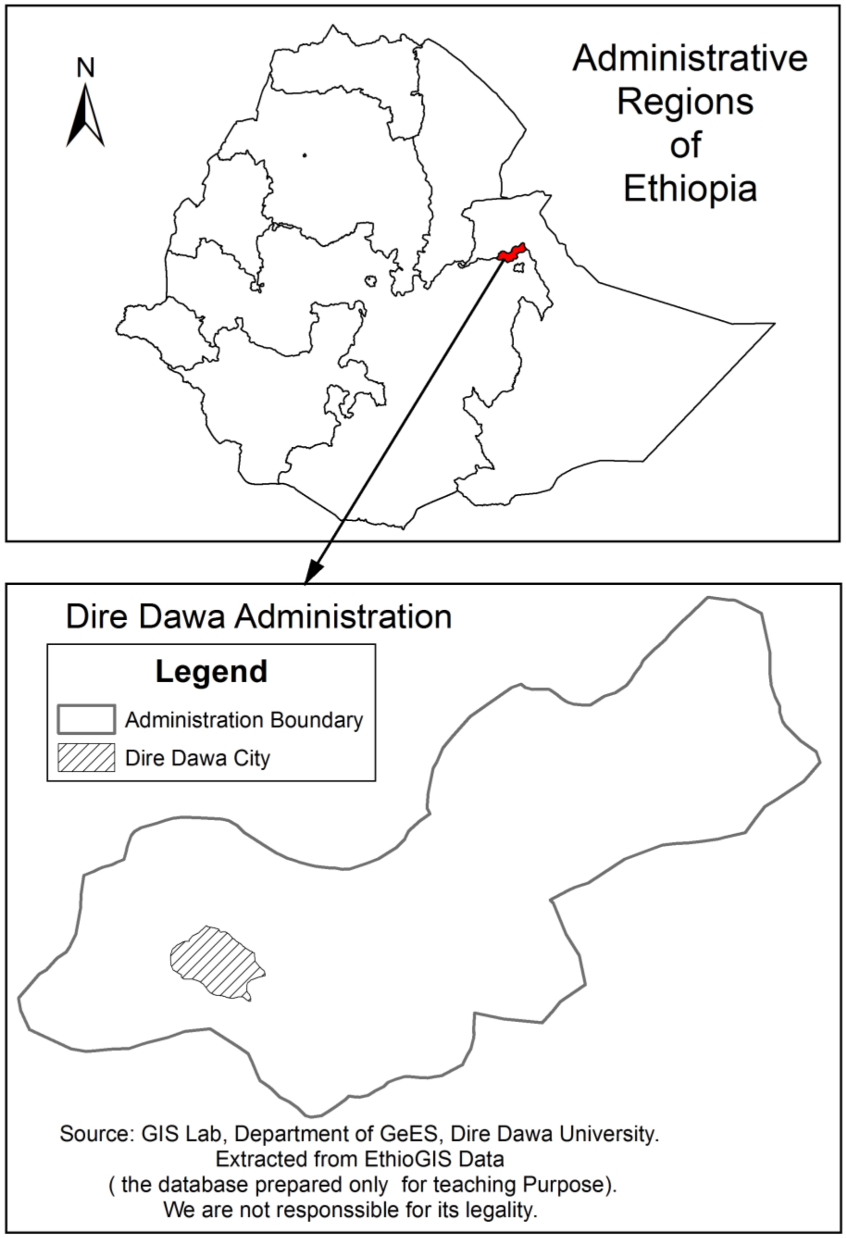


Figure 3.0.3: Study Area

Among the famous villages of Dire Dawa city: Feres-Megala, Connel, Laga-Harrie, Ganda-Korrie, Kezira, Number-One, Greek Camp, Sabiyan, *Sebategna* [Seventh], Ashewa, *Amistegna* [Fifth], and Afate'essa can be mentioned. Since the city had been a destination for national and international community, their legacies are evidenced in some villages. For instance, French people had been living in Kezira, English people were living in Number-one, Greeks in Greek Camp and Arabians were residing in Connel and. There are many hotels in Dire Dawa; some of which are: *Triangle*, Care-*square* and *Circle* (renamed as MM). The milieu is characterized by diversified socio-background.

3.3.2 Shifting from a Government School to a Private School

In the urban of Dire Dawa, there are 3 Government and 6 private/missionary schools which run preparatory secondary (grades 11 and 12) education. From my experience as a coordinator of Mathematics Forum in the area, those schools had been cooperative for collateral works. Nevertheless, as to my information, government schools were comparatively more open to host research and community service activities.

In March 2017, the College of Natural and Computational Science, Dire Dawa University, gave me a formal letter of cooperation written for three schools. I went to a Government school and submitted the letter to the then principal. He appreciated the research agenda and directed the letter to Educational Programs and Organizational vice Principal of the school. I requested her to suggest two or three Grade 11 mathematics teachers who were promoting cooperative learning. A mathematics teacher whom I had known was mentioned; I communicated him.

Since the research targets on students' learning, I had to get students' permission for classroom audio-video recordings. However, I had not yet found the recording materials and the school would be closed for about a month as Grades 10 and 12 students were expected to take model and national examinations. Besides, an extended period of time was demanded to generate rich data for theorizing. So, I decided to postpone the time and informed the teacher the revised plan. However, he told me that he had already selected and informed a class of grade 11 students that “a researcher would come to visit” them. In turn, they were ready to be observed while attending their mathematics classes. I was in dilemma. I needed to consider his attempts and keep his promises. I immediately borrowed an audio recording material from my younger brother. I also bought a cell phone that I thought would enable me capture videos and pictures during my classroom observation and field note. Unfortunately, my wife was critically sick and there was no one to take care of her. Thus, I sustained in my previous decision and postponed the schedule.

By end of September 2017, I went to an already communicated school and contacted the principals and mathematics teachers. As a matter of practice, students were promoted to the next grade level keeping their section. For instance, students promoted from Grade 11”D” were all assigned in 12”D”. But, a different teacher was assigned to deliver mathematics at the targeted class. I found the teaching learning process already started; but, there were a maximum of 10 students who had been attending in each grade 12 classes. This least attendance was attributed to two factors: (1) contemporary state of affairs (political instability) in Eastern Ethiopia; and (2) the school critically failed in promoting students at national examinations, by the previous academic year. So, it seemed that there was high drop out. The principal and Educational programs and Organizational vice principal of the school recommended waiting for some more time.

After two more weeks, the maximum number of students in a grade 12 section was 26. The number is fine to study students’ discourse in depth. Yet, I was not confident enough in generating rigorous data. I heard that the number of grade 12 students per section was expected to be 40. On the other hand, I understood that students didn't complete or cover Grade 11 mathematics contents. This could have impact in their grade 12 mathematics education.

Based on my prior contemplation, I started my data collection at one of the government schools in September, 2017. The school principal and two vice principals were substituted by others. I introduced myself, and submitted a new letter of cooperation to the main principal. The teaching learning process was already started; but, as different teachers told me, maximum of 10 students were attending in each grade 12 classes. The principal and vice principal for Educational programs and Organizational advised me to wait for more time. They forwarded the following reasons:

1) All students should attend class regularly;
2) Students would be given orientation and awareness on contemporary issues of the country;
3) [Since I am conducting on collaborative learning] the students would be organized in to "five in one" grouping; and
4) [I think] the environment was not stabilized.

After two more weeks, the maximum number of students in a section was 26. The number is fine to study students' discourse in depth. Yet, I was not confident to start gathering data as more students were expected to join. In qualitative research, a dynamic decision-making process in-the-midst of the particularities of a data set, situation, aims and desires, rather than a predetermined procedure to follow (Freeman, 2017).

I sought another school in the nearby area as a backup.

I heard from different individuals that the managing director of Babur school encourage students to help each other; this has meaning of cooperative learning. I went to. Then, I continued collecting data concurrently from two classes from both schools. Since the research basic issues, imagination and creativity, could be labeled in to higher order thinking, my preliminary analysis showed that the two schools were incomparable. Besides, the class at government school was 20 periods lagging from the class at private school. Comparatively, there was smooth class environment in the private school. Hence, I sustained collecting data from the private school named as Babur school (pseudo name).

Thus, I began considering Babur (pseudo name) private school as a backup. However, the schools are incomparable in many aspects. It was from the second school that I continued collecting data. That is why I shifted from the government to Babur school[29].

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[29]

The Government School	Babur, the private School
26 students in grade & section 12A	60 students in grade & section 12A
Grade 12A students didn't complete or cover Grade 11 mathematics contents.	Grade 12A students had completed learning Grade 11 mathematics textbook.
Mathematics class is scheduled at 4 periods per week; there are two shifts of class which vary over weeks.	Mathematics class is scheduled at 5 periods per week; there are whole day classes.
There is an assigned vice principal for students "grouping"	There is group leadership for over all activities; no one is assigned for facilitating collaborative learning.
Class became regular very late; the mathematics lesson is 20 periods lag from Babur's schedule.	Class began early; the mathematics lesson is 20 periods ahead of DD's schedule.
Staffs seem not engaged in with motivation for two major reasons. (1) Most teachers hold Master's degree; and their hope to promoted to a lecturer post was denied by Ministry of Education; (2) they seem they are concerned with the socio economic and political conditions of the country. For the last one week, I had been recording only at Babur School due to the fact that the Mathematics teacher from the other school was sent for a meeting out of Dire Dawa.	Teachers are busy engaged in staffroom checking students' exercise book; getting prepared for the next class and so on; talking to students or parents.
The school principals spend much time in their office.	The school principals spend much time in the school compound

3.3.3 Babur School and the Mathematics Classroom Considered

Babur school is a private share company located in Dire Dawa. It has been contributing service in the education sector. It runs pre-primary, primary and secondary education. The school compound is a little bit confined. Yet, it contains two libraries, a science laboratory, IT (information technology) room, basket ball and hand ball grounds, toilet rooms, janitors' office, etc. There are two staffrooms in the compound, one for primary school teachers and the other for secondary level teachers. Actually, the school treats the primary education and upper level education distinctly. There are two buildings dedicated for the two programs. Secondary level classes are found on a Ground plus three $(G + 3)$ building where as the primary students use an octagonal shape $G + 2$ building together with some more blocks on the ground. Besides, the school is constructing a new big building. I had the chance to visit it; the plan is fascinating. I saw the design of some offices, a library, conference room, and kids' corner. I think its finishing time may take at least a year.

The organizational structure of the school is partially explained as follows. The school is led by a managing director, a principal, two vice principals, an administrative assistance, unit leaders, program coordinators, home room teachers and student representatives.

The academic year is divided in to four quarters. In 2017/18, Babur School had two sections of grade 12 students; under natural and social streams. Since the theme of this research is on exploring imagination and creativity from the classroom discourse situation, based on the recommendations of mathematics teachers and indeed from my own experience, I selected the natural science stream students. There were 60 students (30 males and 30 females) in the classroom.

3.4 Data Collection, Management and Analysis

I already considered Adele Clarke and her colleagues' assumption: the researcher needs to know the data (Clarke et al., 2018). They regarded researcher's engagement could be in collecting and spending time with the data and Memoing. I had devoted myself in video recording, attending all the classroom events, listening and transcribing the video records, identifying extant discourses, making observational and/or field notes, planning and leading focus group discussions.

In qualitative research paradigm, the process of analysis had to allow for the unexpected and the possible change of direction in the project itself (Bryant, 2017). Such flexibility was reflected in the course of this research.

3.4.1 Instruments for Data Collection

The basic question is what counts as data (Clarke et al., 2018)? The research was sought to be conducted at a natural setting. Thus, I tried to keep the actual practice of the Mathematics teacher of selected class. Then, to what extent my engagement would be permitted? The following instruments were employed for data collection and their descriptions are briefly given as below.

Audio and Video Recordings: - I did use two video cameras for the action of recording. In order to control the influence the cameras might create, I introduced the recording in to classes in advance. The wide angle camera usually placed at a back corner to capture every action in the class. Another camera served to focus on groups of students with a table mic plugged in. This camera would capture the overall phenomenon of group discussions.

Figure 3.0.4: Setting Up & Collecting Back Video Cameras

The mathematics teacher of the class was a homeroom teacher (leader) too. That means, one of his roles was to take and record the attendance of each student. For that matter, the mathematics lessons were scheduled at every first period of each day except for Thursday that happens in the second period. So, the actions of fixing Cameras & Table mic or collecting them back did not distort the class times.

Focus Group Discussion (FGD):- Focus groups served as important tools for studies in which we begin with a research question and use groups as primary data to (Lune & Berg, 2017). I had been posing questions for the selected group members on how and why did they think, communicate and reason out way they did it.

The table given in Appendix 1 demonstrates the number of participants of the FGDs; each participant has a two digits code placed in subscripts. The first subscript indicates group number; and the second subscript denotes individual student code within a group. For instance, the student S_{12} involved in the first focus group discussion and the second student within the group; she participated four times on the 31^{st} October, 3^{rd} November, 28th November and 29th December. Totally, 10 FGDs were successfully planned and conducted by forwarding open-ended questions to members of each selected group in discussion. Generation of data had passed the following key phases; but, in non-linear approach. These were: video recording, transcribing (noting instances of discourses from videos), planning and conducting FGDs. Besides, student's notes and textbook were referred. Yet, it was not easy to get free time for conducting FGD; thus, the events happened mostly after 4:20pm.

Field Notes:- A qualitative research requires the investigator to spend much time in the field of data collection. Besides, the *analysis and interpretation of the data* would demand deeper understanding of the situation under consideration. Hence, the field note is used as supplementary reference.

Documents:- At the end of audio-video recorded sessions, students' works had been collected and documented for analysis. Besides, I referred the textbook and teacher's guide (Federal Democratic Republic of Ethiopia, 2010b, 2010c) whenever there was a need to compare the plans with the actual practice.

3.4.2 Presentation of Data and the Mapping Process

When we use FGDs, the group itself is the unit of analysis, and the data from one group yields measures about that group as a unit (Lune & Berg, 2017). It is me that decided how data are to be coded and creatively determine the meanings that emerge from data to codes and categories. This is supported in literature (Bikner-Ahsbahs et al., 2015). The main features of the overall process were: data collection, reading through all the data, coding the data, mapping the situation with themes and interpreting the meaning of themes. Mainly, I employed narration and tick description. The figure below portrays the procedure.

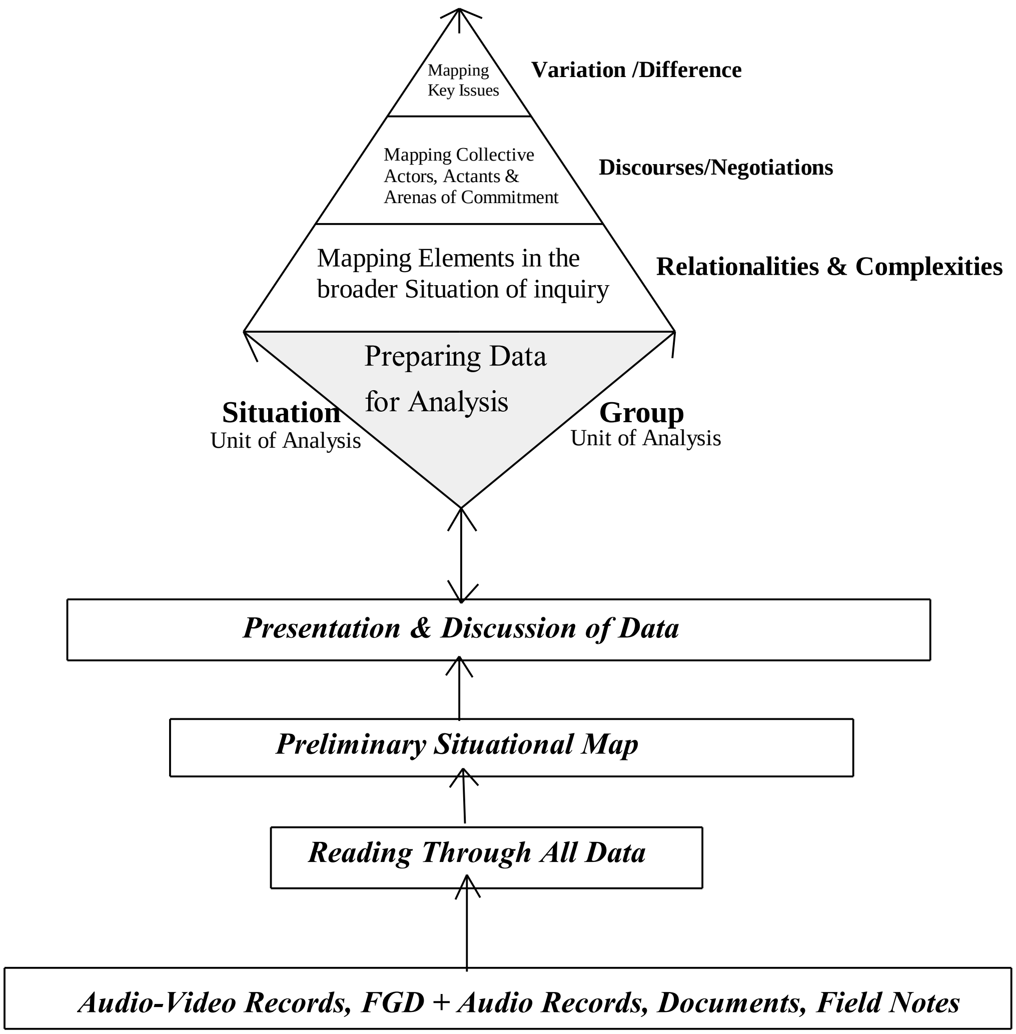

Figure 3.0.5: The Process of Data Organization, Coding, Mapping and Interpretations

Each video and audio recording from the recorded classes were transcribed and filed in chronological order. Day to day during field work, I had been writing the focus group discussions and field notes. Then, I read all the data obtained via videos, FGDs, documents and field notes. I found word file documents suitable to use different text highlight colors which in turn enabled me to identify key themes or repeated concepts.

Once, I select a key word or phrase, it was easy to search or find it from a word document. The transcripts and field notes were analyzed for interaction patterns by considering some points given in a sample analysis. The interpretation of the core themes is made manually. Tick descriptions from different sources of data are made.

In order to establish that findings authentic and credible, a rigorous approach to the research coupled with triangulation and openness at all stages of the process (Bartlett & Burton, 2007) is maintained. The issue of validity for analyses of situated meaning involves considering the fact that any aspect of context can affect the meaning of an (oral or written) utterance (Gee & Handford, 2012). I was always thinking if the research data would enable me to reach at a new insight. The following questions were important in that regard. Will the analysis provide a new conceptual rendering of the data? What is the social and theoretical significance of this work? How will the theory challenge, extend, or refine current ideas, concepts, and practices?

3.5 Ethical Considerations

Access to data and attachment to participants started on an official supporting letter that was written from the College of Natural and Computational Sciences, Dire Dawa University. The managing director of Babur school considered my request very positively. He told me: "because you are professional, you might give us a feedback at the end of your fieldwork". By the following day, I entered to class and introduced myself [in Amharic] and my research concerns (translated) as follows:

> Good morning! My name is Yenealem; I am a lecturer of mathematics at Dire Dawa University and currently attending my postgraduate study at Addis Ababa University. As a partial fulfillment of my graduation, I am expected to carry out a research project. So, I come here to conduct a research; specifically, to video record Mathematics class periods. I already submitted official letter of cooperation to your school; the school and

> the mathematics teacher allowed me to do so. But, I will do it only if I get your permission. I will use the data only for my research purpose; I shall keep documents secured. Do you allow me to video record and study on it?

However, no student spoke any word; nor expressed something. I continued talking.

> . . . In our country, we have many police persons and guards. But, I heard that developed countries like USA, perform most activities are under camera sights. Nothing is secret. In our country, if you go to a Bank, Supermarket, Airport or star level hotels, there are hidden cameras. The high way from Addis Ababa to Adama has lots of cameras. So, we are heading towards that. The education system is guided by strategies and methods which are results of research. If you allow me to conduct my research, directly, you will experience working under camera. Indirectly, you will contribute in bringing a way to improve mathematics education. Do you have any concern?

A student raised his hand asked me: “are you going to study mathematics class or other subjects too?” I responded: “I shall consider only the mathematics periods.” I continued: “I am going to fix the cameras at the corner and I will take a seat so that you will not be disturbed. I shall never waste your single minute; you will continue the way you have been learning and doing.” Finally, the students permitted me to have trial out recording by the following day.

On the next day, students showed me welcoming faces. I took out my instruments and begun to fix them. I requested a student if he is voluntary if I put the *table mic*; he permitted me to do so. Then, I fixed the *neck mic* at the teacher; students were laughing. The trial out went well. Then, I departed from the class by acknowledging the whole and wishing them a bright day.

I met the one who did let me go to the classroom by yesterday. We talked about the successful video recording session. Next, he accompanied me to staff room; there were around six or seven teachers. I said “good morning” loudly; and everybody gave me attention. I introduced my name; I informed them as I begun research in one mathematics classroom. I politely requested them: “may I sometimes share your room?” The accompanying person moves up-down his head seemingly to support my inquiry. They expressed their agreement by saying “no problem”.

The data collection methods involved me to have personal talks and observations of participants for three months at the site. There were times I acted as an assistant teacher and a contributor for the school; but I had no intended intervention. To obtain unbiased data, I tried to form friendship with participants and develop trust with each other. In reporting data, I used pseudo names and code numbers to keep track of what information came from whom without revealing identities.

Throughout the study, I simultaneously read through my source at hand and sought supplementary data. Most of the data, particularly FGDs, were founded in local (Amharic) language. But, the reporting is done in English language. I myself translate the row data. Then, I had consulting a teacher of English language to cross check the translation. He had been sharing me relevant inputs. All information provided by and collected from the participants was used solely for this research.

By 4th January, 2018, I used the mathematics class for a *thank you program* for the students and the teacher. I recalled the time we had together was great. On the following day, students were celebrating “Christmas” which would happen after two days. I was already invited to attend. I availed myself; my informants, those who had been under study, acknowledged my efforts. It was a welfare event for me.

At the end of the field work, many staff members of Babur School told me that I had been sociable with everybody. Yes, I had been always available at school compound everyday; and I perceived my approach was “living in” with students, teachers, principals, school guards, and others. I met all target students in the classroom, some of them in the public library, and few of them on the city road. I spent some time with majority of secondary school teachers in the staffroom, and some of them at a nearby cafeteria.

Chapter 4: Presentation of Data and Discussions

The purpose of this research was exploring the status of and relationship between imagination and creativity in a situated classroom mathematical discourse.

4.1 Reading through All Data

In this section, data collected are organized and discussed under subtopics. The basic issues that immediately captured as I read through all data are depicted as follows.

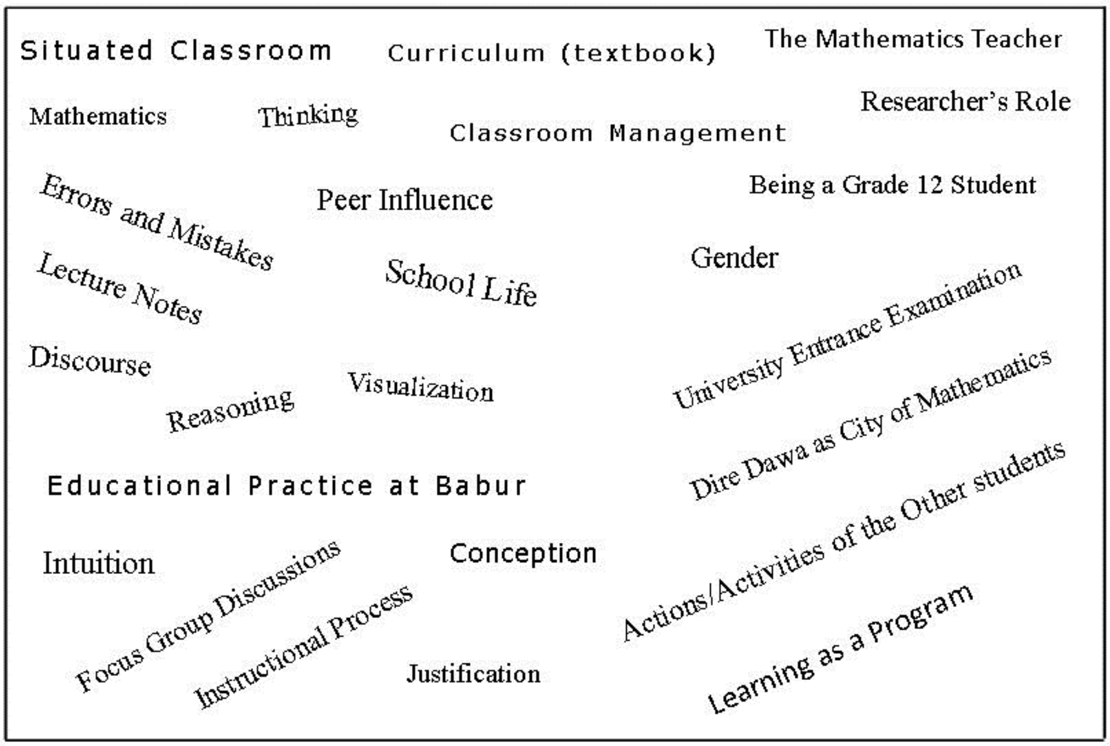

Figure 4.0.1: Preliminary Situational Map

The purpose of doing such a map was to grasp the breadth and complexity of the situation of inquiry. This is provisional and will be developed further after the presentation and discussion of data gathered from the research site. Since I employed SA, the presentation of themes is from broader perspective (macro level) to narrower (micro level) contexts. So, it served as every day reference for deciding how data are to be coded and creatively determine the meanings that emerge from data to codes and categories (Bikner-Ahsbahs et al., 2015). The process and analytic note making throughout the data collection (Clarke et al., 2018) were accomplished by *coding*, *category formation*, and *theme extraction* using techniques such as *pattern matching*. The next action was performed by relational analysis by drawing lines between each of the elements, and to describe the significance of that line. The conclusion is grounded with the dominant discourses, theoretical analysis of the data and the coding. Whenever I had doubt about the coding, the issue was brought to my attention and either the codes or the code definitions were modified to resolve the issue.

4.2 Socio-Cultural Mathematics in Dire Dawa

Among the famous villages in Dire Dawa city include: Number-One, *Sebategna* [Seventh], and *Amistegna* [Fifth]. There are many hotels in Dire Dawa; some of which are: *Triangle*, Care-*square* and *Circle* (renamed as MM).

Mathematically, a triangle is a three sided figure with three edges and three vertices (Lial, Brown, Steffensen, & Johnson, 2004). A square is a four sided figure with all sides equal and interior angles each measures 90°. Circle is the set of all points in a plane that are located at a fixed distance from a fixed point (Ayalew, 2015). In the figure (c) below, X and Y are arbitrary points on the circle whose center is O. Then, the lengths XO and YO are equal.

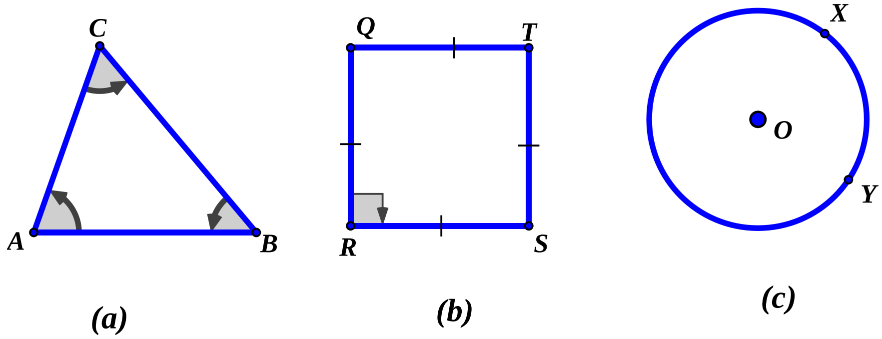

Figure 4.0.2: Triangle, Square & Circle

So, do “number one”, “triangle-hotel”, “care-square”, and “circle hotel” really imply mathematical concepts? What are the underlying mathematical conceptions, if any?

4.2.1 *Number One*, an Urban Village in Dire Dawa

In front: Number one village; to the right is the grave of former Soldiers for Britain

We count natural numbers as $1, 2, 3, \cdots$. The existence of *Amistegna* (5th) and *Sebategna* (7th) depends on *1 (one)*. In mathematics, a sequence is viewed as a set of numbers one comes after another in a given rule**.** Studying about number sequences is helpful to make predictions in the patterns of natural events (Federal Democratic Republic of Ethiopia, 2010b). In the above list of numbers, it seems that: $2 = 1 + 1; 3 = 2 + 1; 4 = 3 + 1;\ \cdots$

We can say that $5 = 1 + 1 + 1 + 1 + 1$; $7 = 1 + 1 + 1 + 1 + 1 + 1 + 1$.

Then, the general formula is: $a_n = a_{n-1} + 1; n \geq 2$. It is a recursion formula. A sequence that is specified by giving the first few terms together with a recursion formula is said to be defined recursively.

But, is 2 necessarily equals to one more added to one?

In other words, how much is $1 + 1$? The next example elaborates the idea.

Example:- How many are one female and one male married to each other?

Net'ela tirguame (literal meaning):- one female plus one male equals to two individuals. For instance, if the husband and wife travel in a Taxi, they pay for two persons. This could be true for Mathematicians use words as words, and they don't want to use the words with their meaning (Kossak & Ording, 2017).

Yandemta tirguame (interpretation by alternatives):- in Gestalt's psychology, the "whole is greater than the sum of the parts" imply that one plus one is greater than two. It is possible to support this argument with different examples. For instance, a work done by the male and female together and independently is not the same.

Let M refers to the male; F corresponds to the female. If M alone complete a task in m days and F alone took f days to complete the same task, then how many days will they take to complete if they work together?

M's one day work is $\frac{1}{m}$ and F's one day work is $\frac{1}{f}$.

Together, they will complete the same task in x days.

Then, they perform $\frac{1}{x}$ or $\frac{1}{m}+\frac{1}{f}$ in each day.

So, we have: $\frac{1}{m}+\frac{1}{f}=\frac{1}{x}$. It follows that $x=\frac{m.f}{m+f}$.

$x=\frac{m.f}{m+f}<\frac{m.f}{f}=m$; and $x=\frac{m.f}{m+f}<\frac{m.f}{m}=f$.

$2x<m+f$; hence, $x<\frac{m+f}{2}$ which is below the average.

In a sense, *doing the same task together is time saving*.

Ye'mistir tirguame (meaning it conveys):- From Genesis (2:25), the husband and wife share many things and are thought united spiritually. "Therefore shall a man leave his father and his mother, and shall cleave unto his wife: and they shall be one flesh" (T. B. S. o. Ethiopia, 2007; *The King James Version of the Holy Bible (*2004). They are assumed to be one.

Therefore, $1+1$ could be less than two, equals to two or greater than two.

$$1+1=2$$
$$1+1>2$$
$$1+1<2$$

Equation 4.1: How much is One plus One?

From other point of view, *number one* could be used to refer best quality or a thing pioneer in some way. Yes, in Dire Dawa city, the area called *Number One* had been given preference by many dwellers for residence.

4.2.2 The Naming of *Triangle* Hotel: Counting, Limit or a Geometrical Concept?

Around Triangle Hotel, Dire Dawa

Grade 12 student's mathematics textbook begins with an opening problem with instruction of imagining student about trees planted on a land of *isosceles triangle* region (page 2) and a class activity based on Pascal's triangle (page 3) demonstrated below.

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
⋰ & & 5 & & 10 & & 10 & & 5 & & \ddots
\end{array}
$$

The Pascal's triangle has $1's$ all the way down each of the two sloping sides. Besides, each number in the middle is got by adding the two numbers directly above it. The pattern is created by taking $1 + 1 = 2$ for granted. However, the class activity is supposed to be facilitated in a way that students discuss with partners and /or teacher (Federal Democratic Republic of Ethiopia, 2010b). But, the limitation is the result of the discussion itself which could be modeled as $1 + 1 > 2$ is not regarded.

In both cases, a triangle is assumed to hold the shape it characterizes.

Triangle hotel is located just around the juncture of the ways to: Dire Dawa International Airport, Ashewa, and Sabiyan. Hence, the *net'ela tirguame* (literal meaning) for the hotel naming could be attributed to this juncture. The assumption could be adapted from the fact that *Arat Killo*, *Amist Killo* and *Sedist Killo* are among the famous areas in Addis Ababa city. They are juncture points for correspondingly 4, 5, and 6 roads.

We can think of *yandemta tirguame* (interpretation by alternatives) too. In June, 2010, Dire Dawa University organized a dinner program for over 300 staffs at Triangle hotel; and I was one organizing committee member. We had two types of menu: with and without meat ingredients. I remember a Muslim colleague expected us brought a food from a Muslim restaurant. His perception was the hotel is a "Christian house". Being coordinators of the event, we forwarded the question to the General Manager of the hotel. The Manager told us the hotel serves all regardless of religion. He mentioned that the owner of Sheraton Addis hotel is Mohammed Hussein Al-Mudin; but, the hotel serves the international community. The two individuals have two perspectives. So, it needs to understand their point of view and see if there is mathematical notion or not.

From the manager's side, I think "star" level hotels are not interested to a particular group of individual. I had known people who assumed the hotel as a "three star" hotel. Then, the "three stars" would mean *counting* an aspect of mathematics.

Let us assume the Muslim customer was right; the hotel is Christian-oriented house. On the other hand, Christianity is the belief in Jesus Christ. For instance, the Ethiopian Orthodox Church sanctions the "Holy Trinity" – the Father, the Son (Jesus Christ), and the Holy Spirit. But, the audience (reader) might be hazy about the Trinity: three in one and one in three. So, how is it possible to relate the naming “Triangle” with “Holy Trinity” provided that all Christians do not have the same belief?

Let me recall what happened to me in beginning of November 2017. I met three Jehovah’s witnesses on my way to field work. I spent some time with them. They believed that the praise and worship being given to the Son is not enough. On the other hand, I was already convinced that the Son is the “Savior of the World”. They claimed and argued that equating the Son and the Father could not be appropriate. Here are the computing truths: the Son is the “Savior of the World” and hence equals to his Father; and the praise and worship being given to the Son should be amplified.

My intention is not addressing such a difficult question, but rather reflecting on what we can discern about the mathematics of Christianity. The point is: what if the praise and worship for the Son is indefinitely amplified? Such a philosophical attempt to define Christ [to be God] can be put as:

$$\lim_{\boldsymbol{p}\to\infty} \boldsymbol{S}(\boldsymbol{p}) = \boldsymbol{F}(\boldsymbol{p})$$

Equation 4.2: Mathematizing Christianity

Where S refers to Jesus Christ (the Son); p is the praise given to S; ∞ is infinity; F represents to God (the Father). *Infinity* is a kind of limit or extrapolation from our mind that we never literally attain it (Poythress, 2015). Yet, a *function* is defined as a relation that uniquely associates members of one set with members of another set, how could S be such a function of p? How do we form the sets of p and $S(p)$? How do we testify the extent of p increasing indefinitely? If so, how do we systematically prove that $S(p)$ tends to $F(p)$?

Nevertheless, we can work with the idea. Instead of granting 'objectivity' by practice of 'assigning numbers', it may be useful to look into what contemporary mathematics can offer to model feelings, social representations, decision making under uncertainty, etc (Rudolph, 2013). My argument was rhetorical; it is a philosophical reasoning by which an agreed up on consensus could be reached.

The other way to get what mathematical concept the naming of "Triangle hotel" conveys is to search for its *ye'mistir tirguame*. Seeking first hand information, I contacted the owner and general manager of Triangle Hotel by 21st November 2017.

The short conversation is presented below.

Yenealem:- . . . would please tell me the reason behind naming "Triangle hotel"?

Mr. Birhane:- the ancient Israel people used to hold money in a paper which is triangle shaped and open from one side. [He took out a paper and demonstrated the paper folding.]

Yenealem:- Aha. . .

Mr. Birhane:- . . . hotel is a business firm. 'Triangle hotel' is brand name in many countries; you may search and check in Google.

Yenealem:- may I see the Logo . . .

Mr. Birhane:- let me show you . . . [called the office assistant and ordered to bring the stump. [Then, he sealed it on a piece of paper; it is depicted in Figure below.]

Figure 4.0.3*:* Logo of Triangle hotel

If we connect the three angles, we get the usual triangle. Since *tri* is a numerical prefix meaning three, the hotel's logo better defines *triangle* directly as *three angles* than the description of "three sided figure".

4.2.3 *Square* as an Exponent and a Four Sided Figure

A Round About and Care-Square Hotel

The Care-square Hotel is located in the western area of Dire Dawa city. It has been recognized by the surrounding community for its Boucher service. I thought the service "care for customers" as a core value in service sector. I personally witnessed that the hotel has good care and services. So, people might consider the *high care* offered there.

So, as its *net'ela tirguame* (literal meaning), "care-square" would mean *care* to the *power* of two. That is, c^2 for c referring to care. Since *care* is relative and varying according to the perception of customers, the satisfaction or service may be put in *function* form.

Let c refers to $care$. Then,

$$f(c) = c^2$$

Equation 4.3: Care-Square

But, this is not a mathematical *proposition* or *statement*. By definition, statement (proposition) is an assertive or declarative sentence which is either true or false but not both (Ayalew, 2018). A *squared care* service could not be judged as “true” or “false” by all customers. We have to get rid of granting to ‘objectivity’ by practice of ‘assigning numbers’ (Rudolph, 2013). So, a different assumption might be forwarded too.

Since the Boucher service at the hotel is closed in Christian fasting days, the naming might be associated with a concept in the religion. For instance, the square could mean the four Gospels: Mathew, Mark, Luke and John. Yet, such assumptions could be right or not; the actual truth would be obtained from the concerning body. The owner of the hotel told me the following:

> In front of (nearby area), a square (round about) was already built by Care Organization. Then, people dwelling in the milieu gave the name *ye Kér adebabay* (Care’s square) to the area. So, we used it as our trade name.
> Mr. Mengiste Alemu, interviewed by 21st November 2017

So, the *ye'mistir tirguame* (meaning it conveys) for “Care square” has nothing with the geometrical concept *square*.

4.2.4 Spiritual & Cultural Values of *Circle* in Architectural Designs

Left to Right: Circle Hotel, African Village hotel, Ethiopian Airlines Ticket Office, Millennium Park, Delt Hospital

Grade 12 student's mathematics textbook included an idea of a circle (page 42) with no center located. It is ok as far as the formal definition of a circle "the set of points which are equidistant from a fixed point" is concerned. Here, the "what" of a circle is emphasized; but, the learning of "how" to make it happen could be more helpful.

The well-known architectural design in Ethiopian Orthodox churches is cylindrical wall and the roof is cone-shaped. Such house buildings are also common in rural areas of the country. However, the trend is not totally attributed to religious backgrounds. For instance, the *Afar* and *Somali* people built such cultural houses.

In Dire Dawa city, African Village hotel has a similar design. However, Circle hotel, Delt hospital and Ethiopian Airlines ticket office have cylindrical walls. The renaming of Circle hotel as MM hotel is due to its expansion and change of owners. Currently, it is has got more buildings with different architectural designs. Therefore, the *net'ela tirguame* (literal meaning) for "circle" could refer to spiritual and cultural values attached to the construction of Ethiopian houses.

On the other hand, the owner of the former "circle hotel" was known for his worship in Ethiopian Orthodox Church. In a circle, the beginning and the end are the same; hence, difficult to distinguish. It is noted in the Revelation (2:8) of the Holy Bible that:

> "I am Alpha and Omega, the beginning and the ending, saith the Lord, which is, and which was, and which is to come, the Almighty" (T. B. S. o. Ethiopia, 2007; *The King James Version of the Holy Bible (*2004).

One may seek to know *ye'mistir tirguame* (what meaning "circle" conveys) from this perspective. I also asked the manager of Circle (renamed as MM) hotel; and he told me that the hotel was founded in 1990 and has been transferred in to four different owners in different times. Its trade name has been changing accordingly. Recently, I saw the name "MM hotel" posted. Yet, residents of Dire Dawa continue calling the location as "circle" due to the house design.

The preparatory mathematics education considered a circle (Federal Democratic Republic of Ethiopia, 2010a) as: "the locus of a point that moves in a plane with fixed distance from point. The fixed distance is called the radius of the circle and the fixed point is called the radius of the circle (P. 78). The concept is expressed in terms of 'point' which is undefined term. Again, it is similar to earlier works of mathematicians (Loney, 1895).

> "Suppose to be a given point in the plane of the paper and that a point F is to move on the paper so that its distance from shall be constant and equal to a. It is clear that all the positions of the moving point must lie on the circumference of a circle whose centre is and whose radius is a" (P.24).

The process of *making a circle* could be understood by considering the common practice of building a cultural house in Ethiopia. When dwellers in rural areas of the country build a traditional house, they decide the *center* and "area" of the house. By "area", they mean *radius* in the curriculum. The procedure is: they first decide a fixed *location* where a pillar will be erected and a rope of some length (fixed *distance*). Then, from the fixed location (center), say O, they stretch a rope of length r. Fixing O, they rotate the other end of the rope and thereby mark all the points. At those points, they build the wall of the house. The locations where the wall is erected can be represented by a circle.

Putting all such points give a continuous curve forming the circle. The definition of the circle could have been stated after students had discovered its essence from their every day experience. Indeed, it is noted there that students can do different activities to conceptualize the term and reach at conclusion.

> The students can also use a piece of string to sketch loci of circles. Tie one end of the string to a nail inserted at the centre of the circle. Attach the other end of the string to a pencil, and, holding the string tight, draw the circle. The radius will be the length of the string (Federal Democratic Republic of Ethiopia, 2010d).

I have presented the two plausible reasons for the naming of Circle hotel: architectural and spiritual. The first option gives more sense. In any ways, the process of making the geometrical object *circle* is referred for better learning.

Table 4.0.1: Applications of Ethiopian Orthodox Church Interpretive Methods

Issues	***Net'ela tirguame***	***Yandemta tirguame***	***Ye'mistir tirguame***
Number one, Amistegna, & Sebategna (#1, 5th and 7th) Is 7th = 5th + 2? *That's a recursive formula* But, what is 1 + 1?	$1 + 1 = 2$ *Yet, it contradicts with the fact that Mathematics is a meaning making.*	❑ Synergy ❑ Gestalt Psychology ❑ Group Work done 1 + 1 > 2. *It entails the value of a Group Learning.*	1 + 1 = 1 < 2
Number One	It is a Mathematical object. *The definition varies within the philosophical dimensions of mathematics*	❑ Best village? Then, $f(1) > f(2)$. *It is ranking.* ❑ First place to be established?	Legacy that promotes Greatness of Britain
Triangle	Juncture point of three roads? *If so, it implies for a spatial Thinking.*	❑ Three Stars? *That's Counting.* ❑ Christianity? *Mathematizing Christianity lies in Sacred Mathematics.*	Trade Mark *Re-conceptualizing Triangle as 3-angles.*
Square	$f(c) = c^2$ *It is qualitative Mathematics*	❑ 4-Sided Figure ? ❑ 4 Gospels?	Kare's Round About
Circle	Architectural Design *It would imply for Process Learning.*	❑ Spiritual Value Beginning = End ❑ Cultural Value?	- - -

4.2.5 What else? *Simplification* and *Harmony* in Dire Dawa City

By 30th December, 2017, I delivered a two hours training on “Pedagogy of Mathematics” at a different (distinct) secondary schools of Dire Dawa. The explicit purpose of the training was to brainstorm mathematics teachers with desirable mathematical actions for students (Sullivan, 2011) and principles of engaging students in learning mathematics. Besides, the implicit purpose of conducting the training was to explore the pedagogical practices in Dire Dawa. There were eight attendants from three schools. During presentation, I have raised the question “Do you know more mathematical concepts tie with the cultural or social contexts of Dire Dawa?” Mulatu, one of the attendants, responded:

> *People of Dire Dawa used to* ***ease*** *and* ***simplify*** *things; they chose simple life. At the same time they solve problems. So, they don’t like and allow situations be complicated. The people create and maintain harmony with conditions and other people.*

Just to restate Mulatu’s idea: people in Dire Dawa are known for enjoying a simple and harmonized life regardless of their origins, language, and religion.

We tend to think of simplicity and complexity as polar and exclusive opposites. Pure Mathematics is characterized by lots of axioms, theorems, and proofs. The real goal of mathematics is to unravel the reasons behind them which in turn perceived as one of simplification (Kossak & Ording, 2017). Although problem solving is related to creativity, I observed many students choosing short cut methods in computing mathematical exercises. An informant, Temesgen, claimed “the generation prefers short cut methods”. Indeed, thinking of a short cut method would have elements of and could be labeled in to imagination or creativity. Once, the mathematics teacher said: “we are entering to a derivative system with short cut method that most of you favour of . . . [but,] it is not enough to just have rote memory of the formula . . . it came out of the limit approach [of derivative].” Let me extend the idea to and supplement it with a focus group discussion recorded and transcribed by 15th December, 2017.

Yenealem:- by yesterday's class, the student S_{14} called a classmate and asked: "which method would be used for the derivative of $f(x) = \frac{1}{\sqrt{x}}$ at 1?" What does it mean?

[They were expected to apply limit approach for the derivative of f at a point $\lim_{x \to 1} \frac{f(x)-f(1)}{x-1}$]

S_{14}:- I was seeking a short cut method of solving the question. . .

Yenealem: Aha! Did you have different methods of doing the question in your mind?

S_{14}: I thought $\frac{1}{\sqrt{x}}$ as $x^{-\frac{1}{2}}$ and brought down $-\frac{1}{2}$ as coefficient of x; but, was unable to know the new exponent of x. We had learnt [differentiating] *using integer powers at grade 11 physics education and $-\frac{1}{2}$ is not integer. That is why I asked him if there was alternative method. He advised me to just use the other one.*

Yenealem: what is "the other one"?

S_{14}: Since we had learnt the limit approach, he thought it . . . I started applying it; but, it was confusing. I asked him again. He considered my concern and agreed.

In the above note, concepts like, thinking, another way, asking, suggesting and agreeing are mentioned. Such words are indicators of discourse.

On the other hand, the word "harmony" has the following meanings: *connection, consent, combination, and order* (Stakhov, 2009). During my classroom observation, I saw the mathematics teacher reshuffling students' sitting arrangement using lottery method. It was my witness that almost all students were sociable; they easily interact each other and with me, the researcher. Towards, the end of my fieldwork, the whole class invited me to attend their Christmas festival held in the classroom. I wondered Muslim classmates were also celebrating the event. We can ask a question: what branch of mathematics studies these concepts? It is Combinatory.

4.3 Expecting High: Is Education really Exhaustive at Babur School?

Babur school was opened before 20 years to serve the needs of the community in Dire Dawa. As to my information, the school has earned praise for its continuous improvement in student test scores and consistently high levels of parent satisfaction over the years.

I saw many quotes posted at different corners in the school compound. Two of them are: "Discipline is Wisdom and Vice Versa" and "He who lives without Discipline dies without honor." In these senses, a discipline refers to a certain profession. The term may imply good behavior too. These could serve as guiding principles of engaging students in their academic life.

I already reported, earlier in the methodology section, that the pace of grade 12 Mathematics instruction is according to the syllabi. Besides, there is team leadership at Babur school. Generally, the school's daily class was scheduled from 7:45am till 11:40am in the morning and from 3:00pm till 4:40pm in the afternoon. Besides, tutorial classes for selected subjects are scheduled after class time.

The following note is presented to demonstrate partially how the school functions.

> *I went to school early in the morning 25 minutes in advance to the formal class time. I had time to stare at school atmosphere. The school compounded seems crowded as it encompasses primary and secondary education levels. There is a great building under construction; hopefully the problem might be solved next year onwards. I saw the school managers coordinate each other; the vice principal, guards, the unit leaders and assigned teachers facilitate and organize students of grade 1 to 12. I had completed setting up of video camera and saved time to watch National Flag ceremony. At* $7{:}45am$,

students were lined up at the ground where the Flag ceremony is being celebrated. First, a student presented a poem from mini-media. Next, children sang "Thank you my Lord" lyrics and the Ethiopian national flag song respectively. They were singing emotionally; that is impressing! (Source: field note taken on 24th October, 2017).

Once during my field work, the managing director and a program coordinator entered to the class I was observing. I already set up the recording cameras. As they entered the class, I left the room assuming that they would talk to students about some issue. I had been waiting for them on the corridor. I was hearing only the voice of the teacher. I did not know what to do. I just spent the period outside the room. Lately, I noticed that the two authorities came to supervise.

I was worried the supervision could bring change in classroom atmosphere. When the time was up, I requested one of them if they were performing it regularly; he told me in short that they had such a habit. Again, I asked the classroom teacher, if the practice had being exercised last year. His response was to the opposite; the supervision had begun two days ago. I started my research 10 days ago; and the school commenced this year's activity a month ago. That means, the supervision is introduced after I started my research in the school. So, my presence as guest and as a researcher might have motivated them to do so.

At the end of the first quarter, parents were invited to come to school in order to watch students' report card. I realized that there was a general discussion with parents about how the school is functioning and how students are progressing. By beginning of second quarter, I was observing a number of parents in the staff room each working day. Most of the time, the vice principal for secondary education deals with them; it seems that that teachers are complaining about some misbehaving students. I already noticed that students and teachers became exhausted to some extent. Therefore, I think, the complaint and the frequent call for parents might be due to such cumbersome feeling of teachers.

Some days later, the vice principal, for the secondary education, was talking with a teacher about parents' meeting. I grasped that parents complained the large class size. (I noted in the previous chapter that I had been observing 60 students per class.) On the other hand, the school wanted parents to follow up their children.

In the staff room, there were many teachers. Three home room teachers had been preparing students' report cards to be delivered in front of parents. I saw a teacher writing scores below 60% using red pen and marks above and including 60% using black pen. I asked him "why…" He told me that the school considers a score below 60% as a failure. I know that the passing mark for a student at a subject is 50% (T. G. o. Ethiopia, 1994). So, I understand that the school expects students to score better. Then, I left the school to a nearby tea room. I spent there for some time; about six teachers came in and went out. There were discussions. I understood that students are getting exhausted of series of classes and continuous tests.

Education is a collaborative task; and parents are stakeholders. Involving stakeholders is a very good practice. By 25th November, 2017, I delivered a training delivered to more than 30 teachers of the school. As I arrived 30 minutes earlier, I had the chance to talk to one of the school guards. We began talking . . . I realized that he read many books. He told me that Israel people used to plan for a century; if they pass away their children would sustain the projects. Being surprised, I thought that the school founders have a 100 years plan. They are building extra rooms even after 20 years of establishment of the school. Contrary to this, I saw some staff members of Dire Dawa public library spending the whole day by talking. They could have been role models for teenagers and the wider community who visit the library. There are alternative resources in the library such as: newsletter, supplementary reference books, life books, history books, and etc. So, they could have been engaged in reading newspapers or periodicals while they supervise the activities of customers.

The other things that have been touching my feeling were the morning *flag ceremony programs*. At the location where all students celebrated the national flag ceremony, I was surprised by students' emotions in singing the songs. Indeed, crowd of primary (grades 1 to 8) students' voice was more audible than their seniors. I repeatedly saw a unit leader on the corridors of the building. He had been bringing plenty number of students down to the ground and insisted them attended the *programs*. That means, few secondary school students who came earlier chose spending at their rooms.

One day, in beginning of December, 2017, the school principal's elder sister passed away. On my way to the mourning house, I met the vice principal for secondary education. We talked about personal life and professional related issues. A point of discussion that could serve as input for this study was teachers' motivation in their routine works. He shared his reflection on the matter that teachers started the academic year with enthusiasm and currently seen exhausted. This could have impact on the motivation and engagement of students on their learning; teaching and learning processes are dynamic.

The following report is another piece of evidence for the educational practice in the school. One day, as I completed conducting a focus group discussion, we began talking other issues. I realized that one of the interviewees, S_{52}, is transferred from a public school.

Yenealem:- . . . what was your reason to transfer from there [public school]?

S_{52}*:- I realized that there is thorough education in private schools and I chose Babur school*

Yenealem:- Aha . . . In Dire Dawa, there are more private [private, missionary and community] than public schools. What do you think is the reason?

S_{52}*:- there is intensive education in private schools. Besides, teachers are more competent here. In addition, the students are intelligent. I thought that could have a positive influence over my courage. That is why I came here.*

Yenealem:- Ok, have you ever heard about a student from a private school and transferred to public school?

S_{52}*:- May be… only if the student is obliged to leave the school for good [based on discipline cases] ...*

Yenealem:- Thank you very much. Have a nice evening!

I remember, by 4th December 2017, few students were on the corridor; a teacher commanded them to enter in to their class room. I realized that students are expected to spend their class time in their seats. In case, a teacher is absent; they need to read or do some academic activities.

On the other day, I heard the vice principal blaming of the librarian of the school. I remember that the conversation happened with available staff members. I just noted the ideas and actions as follows. The vice principal visited the library; he observed some students were inside and noticed that it was class time. So, he commanded the librarian not to permit students enter in Library during the class time. Students are formally allowed to visit the library when there is assignment given demanding reference books or when a teacher could not come for some time.

I knew students spend 7:40-11:40 am and 3:00-4:20 pm; mostly there could be after class tutorials. So, I doubt they spend at and study in the library for extended time. However, I used to see many students in the public library after class till 7:00 pm. I believe that the “whole day” class is helpful to engage students in instructional activities. Yes, the formal curriculum is covered and achievement in tests and examinations. Yet, the academic activities should not be limited to classroom events. Extracurricular activities and library services could support them develop their learning. So, it is difficult to say the educational practice in Babur school is intensive.

4.4 The Instructional Process in a Situated Classroom at Babur School

When I came back to Dire Dawa after three years, by the end of September, 2017, I found the number of private/missionary schools was twofold of the number of Government schools. I spend for about 3 months at Babur (private) Primary and Secondary School collecting data. Each day, I had been meeting some teachers in the staff room even for a while. Occasionally, I go to a nearby “tea room” where few teachers eat breakfast or drink coffee. In one way or another, I had known many teachers of the school; I feel I was not a guest. There was also a public library where most students spend their extra time reading. I had also been available there. These were great opportunities enabling me familiar the school environment.

In the following sections, I first describe the data used in the study and the ways these data were analyzed. I then provide illustrative Classroom discourses from the data set and use them to elucidate the Maps scheme I developed to analyze the data.

4.4.1 The Mathematics Teacher and His Classroom Management

As I met, the mathematics teacher with whom I conducted the research, Demissu (name changed) for the first time in the beginning of October 2017, I sensed that he was friendly to approach. During my classroom observations, I got him easily communicating with his students. Being the homeroom teacher of the class, he has control over students. I saw his students respect him, not only for his knowledge of mathematics and confidence on teaching, but for his overall attitude and actions. In the mean time, he told me that he has been teaching mathematics for more than 10 years. He laid out reasonable expectations for students to follow. I also saw that when students misbehave, his treatment is consistent and fair. Demissu is good at 'teaching'; knows humors; and works exhaustively.

I witnessed that there were occasions that could insist him to facilitate group learning. I was expecting thorough classroom discussion; and, discourse is beyond that. When such a situation arises in the classroom, it can be challenging for the teacher to improvise and craft it into an opportunity for learning mathematics (Foster, 2015). Though the teacher gave chance to many individual students, he preferred lecture method and dominated the activities. Those were opportunities that could engage students in discourse activities. However, he did not favour for informal discussions and collective talks. On Tuesday 5th December 2017, as the teacher entered to class, he gave the duster to a student to clean up the blackboard. The student reacted very politely. What a great discipline! I think that is the fruit of hardworking teachers and well functioning schools.

There could exist inequitable patterns of students' participation in mathematics classroom discourse (Herbel-Eisenmann et al., 2012). I was not happy with the ways he had been structuring interactions to position every student as a contributor to the collective development of mathematical ideas. By 18th December, 2017, I was conducting a focus group discussion. In the mean time, I shared them a doubt to the members.

> *Today's lesson was limited to the derivative of power functions. I am afraid the favored for my research by allotting more than enough time for this section; and hence discussions were facilitated. He could have passed to simple trigonometric functions. . .*

A student S_{14} replied to the contrary:

> . . . *No he was right. Though we are familiar with the formula* [for derivative of simple power functions], *he brought various examples. So, it was good.*

I considered the opinion as feedback in that the students are learning as usual.

4.4.2 Was My Role Affecting or Facilitating?

I used to set up and recollect the instruments earlier or after the mathematics class. So, class time was not affected. Yet, that could not be enough to claim the teaching learning process held at natural environment. It is impossible for the researcher to enter the field with an empty mind (Bikner-Ahsbahs et al., 2015). I had involved in some activities either when the teacher was absent, delivering training up on request, during focus group discussions, or invigilating a test. The good thing is that I had been waiting for extended period of time and that enabled me investigate the real situations. For instance, each day, I met some teachers in the staff room even for a while. Occasionally, I went to a nearby "tea room" where few teachers were eating breakfast or drinking coffee. In one way or another, I had known many teachers of the school; I felt I was not a guest. There is also a public library where most students spend their extra time reading. I was also available there. These all were great opportunities for me to keep things as usual.

Since I employed a qualitative research approach, the investigation has to be held under "real" situations. So, no intervention is required. Yet, I was challenged two times in this regard. By 6th November, 2017, about 25% of the students were absent by the morning session. I did not know what happened in afternoon session. In following day, class started without delay. I saw some seats empty. After some minutes elapsed, the vice principal for secondary education came in and commanded students, who were absent by yesterday, to leave the class. The number of remaining students was almost half. He was upset by their action; requested my opinion on such a behaiviour.

I reacted nothing. The teacher carried on his lesson with staying students. In the mean time, students got puzzled on the concept of "infinite limit".

A student forwarded a question; the doubt was shared by many pupils; and the teacher was answering and explaining over and again. The main argument was: *if* $lim_{x \to a} f(x) = \infty$*, can we say that the limit exists or doesn't it exist?* The concepts *limit at infinity* and *infinite limit* are not clearly distinguished. I found it a meaningful discourse happened that day. The teacher invited me to add or say something; I appreciated the participation of all and proved that a true learning was happening. I picked up a chalk and went to the blackboard. I demonstrated that "spoon feeding" and "confusion" are opposite extremes of learning strategies. I added that "∞ is not a number; and thus, $f(x)$ is not approaching to a number". I reminded them to let me be reserved from such involvement.

By 3rd November, 2017, the classroom teacher did not come to school; his telephone was not functioning. And I already set up the cameras; I waited for him for about ten minutes. Students requested me to cover today's teaching task; they seem eager to be taught by a guest teacher. I was in dilemma; the investigation has to be held under "real" situations and no intervention is expected. If I could not say "ok" as my research is not experimental. Besides, I had no organized thought to be delivered to them. On the other hand, if I said "no", I couldn't imagine what feeling students would develop about me. If they found me a better teacher, they might develop a different perception about their formal teacher which in turn would impact the research. If they found my pedagogy worse, they might demonstrate an uneven behavior. I knew I had to be in conformity with them as I am researching on them.

I decided to talk for some minutes about relevant issues in mathematics learning but not a continuation of the topics which are under way. I lectured about my MATHEMATICS principles (Ayalew, 2018). As they are going to join a university, they could be learning any of the following clusters of fields: Medicine and Health Science; Agriculture and Veterinary Sciences; Technology; Humanities; Enterprise; Maritime; Arts; Teaching/Training; Informatics; Construction; and Science. The first letters give MATHEMATICS. The class was normal. On the other day, the teacher came in to class late. I already set up the cameras; so, I give the following calculation task for the students. They were doing it mentally and on hands on activities.

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{2}}}}$$

The problem is found in earlier grades and could not be directly linked to the then topics under instruction. As soon as the teacher entered class; he directly engaged them on the day's lesson about *continuity of a function at an interval*.

Another challenge happened by 10th November, 2017 when I came to the school just to refer students' home works. All the exercise books were collected and brought to the staff room. I purposely selected and looked in five of them. (I had a list of five students appointed for a focus group discussion.) It could have been great input for the research if it was a collaborative work.

On the other hand, the mathematics teacher was busy with office issues. He requested me to assist him in checking the exercise books. I refused taking the responsibility and promised to involve in matters out of research.

I wanted keeping my role as facilitating the smooth process of the research. For instance, I wrote the following:

> *I had some informal talk with the* [mathematics] *teacher. He teaches at grade 10, 11 and 12 levels. I understood that he has been writing detailed notes on the blackboard at the class I am recording. I thought he does it in order to complement shortage of textbooks. I asked him "don't students have textbook?" He replied: "the* [Dire Dawa administration] *education bureau sells books for the school and the school, in turn, passes it to students* [who demand and afford paying]. *As I am a homeroom teacher, there are lots of books in different subjects beneath my table." I remember I saw the collection there someday. That means, he is practicing some unrealistic behaviors in this targeted class. I asked him why; and his response was short ". . . for your research". I immediately told him not deviate from the usual way of practice as the research should be conducted at the very natural setting. I also added that he is influencing the research the other way by saying: "During the first 15 out of 40 minutes of every class, you had been spending writing note on the blackboard and students copy it to exercise books; and that would limit the classroom discussions. He agreed to maintain the classroom environment as usual*. (Field note: 22nd November, 2017)

Another occasion happened in the same month. One day, I met the managing director of the school on my way to home; he was going out too. He requested me for some opinions about the school environment. However, I was not sure how much I should reflect on the issue. If my opinion could serve as feedback, the managing bodies might intervene on the real "situation" of the school. After a while, I acknowledged the school for allowing me to conduct the research. I told him that I was thinking to share my over all comments at the end of my field work. Yet, he insisted me to disclose my observations.

I thought I should be in harmony with all school community. So, I forwarded my opinion summarized as follows.

> *I found the school well functioning. I came back to Dire Dawa to discover students' group works. Though the school has no such formal programs, I rather keep on observing on the real practice and hope will reach at whatever result it would render.*

I immediately divert the focus of discussion in to other topics.

After a while, he took me back onto the track. He explained on behalf of the school that he requested me to deliver training to its science and mathematics teachers. This was also a difficult question for me at that time. I expressed that I had interest to share my experience and deliver training; and it is my career. (I wished this could happen at the end.) He tried to convince me that the school would pay money for my effort. Then, we agreed to conduct the training with no payment.

I thought different topics that might not influence the research. I listed down topics which up to my potential and experience: teacher's professional identity; engaging in action research; teachers as reflective practitioners; multiple intelligence; teacher as a lifelong learner, etc. Finally, since my research is on classroom setting, I decided to deliver training on and begun being prepared for extra curricula activities.

The training happened Saturday, 25th November, 2017. There were over 30 attending teachers and directors of the school; I was happy of getting teachers of almost all subjects at different grade levels at that event. My training was prepared in local language Amharic[30]. It had a positive impact on the research process; I was able to collect data very easily. It should also be thought on the contrary. For instance, one day, I was in the staff room; I met few teachers there. The vice principal for secondary education sought my opinion on the status of students' interaction; I replied "very good!" Then, he told me that he already informed the mathematics teacher to engage students in discussions. I had decided to reflect. "Yes, there is improvement; if my research had been led by questions like: 'is there a classroom discussion', then, such interventions would have negatively affected the findings of the research.

[30] I started the training by recalling the seven standards for Ethiopian school teachers: know students and how they learn; know the content and how to teach it; plan for and implement effective teaching and learning; create and maintain supportive and safe learning environments; assess, provide feedback and report on student learning; engage in professional learning; and engage professionally with colleagues, parents/care givers and the community. I emphasized on the second competency, school teachers are expected to know their students and how they learn (Ministry of Education, 2012) and curriculum documents such as text books and teacher's guides are the minimum requirements. To know students and give them opportunity to exhibit their talent, the class activity may not be sufficient. I considered that education is a collaborative work where the stake holders are students themselves, teachers, parents and the school principals. Engaging all these partners could not be realized in a classroom context. Thus, out of classroom activities are also relevant. The following questions were forwarded in order to trigger trainees engaged in the training. (1) . . . from the 1960s and current generations, which one is massively brave? (2) . . . from a class of students, how many of them are "intelligent"? (3) Are student's study hours limited in school time and particularly in classroom context? These points served as a springboard for introducing multiple intelligences. So, extra curricula activities are crucial to promote students' over all development. At the end, the attendants acknowledged that the training would "add values" to their practice.

[Basically, it is after the training that the [mathematics] teacher begun involving as many students as possible within a period. My methodology during the training was too participatory; and he might be influenced by my style. I was interested in digging out 'imagination and creativity' in the discussion. So, it was ok.]

He continued: you are doing the research work thoroughly.

I responded: Emm. I should not miss an event to give better explanation about the study.

The following event is also reported here just to give evidence of my endeavors in collecting intensive data. It was the time ahead of the actual class sessions; students gathered at the ground for the national flag ceremony. The mathematics class to be observed was scheduled at the first period. So, I had to set up the cameras before students entered to class. That was just to save their time and minimize intervention . . . I had always been punctual. But, that day, I was a little bit late. So, I was going to the classroom in hurry carrying the instruments needed.

The room was at the second floor of the bigger building . . . I begun moving up the stages of the building just looking down the ground. Unfortunately, a wood crossing the entrance kicked of my forefront. My eyeglass went away. . . I collected the frame and broken lenses. For over 10 years, I had sight problem. On the other hand, I was determined to gather data continuously. Thus, I hardly had the video recording. Up on the completion of the lesson, I took a Taxi and begun looking for an optics center.

4.4.3 School Life, Peer Influence and Being a Grade 12 Student

In Babur school, males and females students learn or play together. The exception I saw was that female used to climb up on the left side stages of the building and males were obliged to take the right side stages to get in to their class. What if males and females step up together in the same side of the building? The fact that females wear skirts would probably initiate a boy to watch upward from behind or a lower stage. This implies that pone way to safeguard students' academic endeavor is overcoming non-academic factors.

This study considers classroom discourse; and the classroom s conceived broadly. In other words, it is assumed that there could be internal or external factors facilitating or hindering the actual discourse. Thus, it is indispensable to report about student's activities and conducts with respect to gender and age, or from other angles. The school administration and classroom management could be such perspectives that could direct the practice of students' discourse.

Three times during my field work, the teacher rearranged the setting places of students by lottery method. Some special cases (e.g. sight problems) were considered. He (and students) told me that the arrangement was according to the school's administrative policy and would serve only for the next two weeks. Let me narrate what happened one day. The teacher ordered students to write down their roll numbers and names on pieces of papers. I saw they know what would happen next, i.e. reshuffling their sitting arrangements. He called up a student to assist him. The teacher and the student facilitated the reordering of students on a randomly basis. Like the preceding similar reshuffling, it takes them almost all the minutes of the period to complete the process. The main challenge was male students were not speedy to leave their former seats and take over the new ones. In addition, the teacher treated students with personal problems. The action of rearranging took much time and there were no actual classes.

It was a great opportunity to see that students were friendly each other. It seems that the majority of students might have spent many years in the same class or at least at the same school. Supporting evidence for the above claim could be the following: there were 60 students in the class; each day, the attendance sheet is given to a student to tick the available and absent students; he just looked at class and easily mark the attendance.

Though the school manager is intended to have students helping each other and being promoted together, there are no formal groups for such collaborative learning. So an important question would be: how informal groups are formed, both in general and when students work on tasks?

I have seen classroom discourses were limited to individual student's reflection and to some extent collective participation of students. Sometimes, the whole students lost engagement on doing the questions written on the blackboard; mathematics lessons turned into talking personal matters sessions. The teacher had been repeatedly regulating the shouting class to keep quiet. He also used to encourage them got engaged in the formal learning.

Most of the time, I had been putting the *table mic* at one armed chair. Based on my personal observation and after I looked at the videos recorded, I invited participants for further reflections. The following picture demonstrates how informal group discussion had been formed.

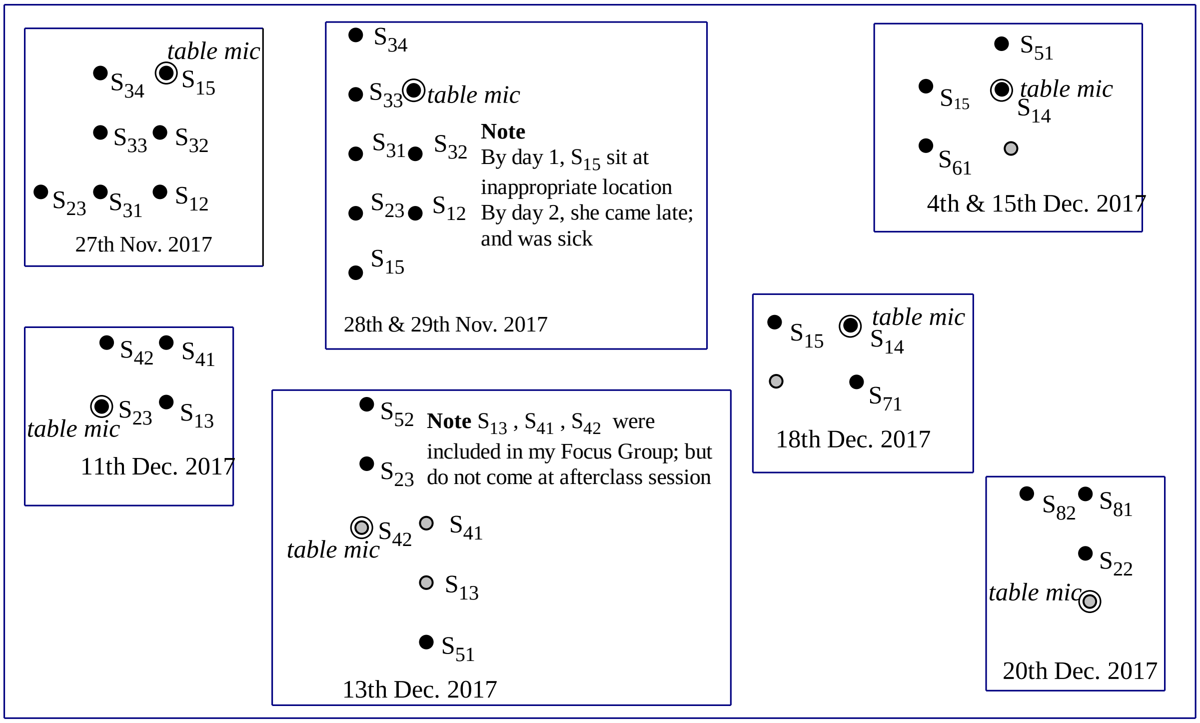

Figure 4.0.4: Sample Groups of Students Considered

Although students sat in matrix (parallel) sitting arrangements within the classroom, group discussions happened irregularly. I used to put the table mic just at an armed chair so that it would gather the nearby sounds from all directions. Yet, unexpected group learnings were observed. For instance, students who sat behind the sit with table mic were observed communicating one another. On the other hand, I had been inviting observed group participants for further communication during focus group discussions. Yet, some were not attending the events. For instance, students S_{42}, S_{41} and S_{13} did not come during after class focus group discussion sessions. I wondered how students S_{23} and S_{51} communicated each other without interrupting the lesson held by 13th December, 2017. Thus, I extended the question as presented under.

Yenealem:- [The whole class was engaged in finding the limit of x minus three divided by the square root of x squared minus six x plus nine as x approaches to three.] . . . S_{51}

and S_{23} sat far apart each other; but they were sharing information. I would like to know if it was not a private talk.

S_{23}*:- I did not have idea on how to divide the polynomials. At that time, the teacher forwarded the question to the whole students;* S_{51} *reported "just dividing . . ." Thus, I asked her the how of it. I had no idea . . . I just divided each term of the numerator by* $x-\frac{1}{3}$ *and could not go on. Then, I asked her.*

Yenealem:- . . . I was surprised; you have communicated each other with an instance time. And your voice was not that much audible . . .

S_{23}*: I have problem in Mathematics. I do the major calculations; but, I miss the silly computations. Thus, she was correcting the errors. We had mutual benefits.*

Although the two students sat far apart each other and there was a buzz sound in the class, they could be able to share ideas and understand accordingly.

The following three stories are show cases for students' life at Babur school.

By the 6th December 2017, the vice principal for secondary education and I had some time; we were drinking coffee together. In the mean time, the vice called a man who was there too. They affectionately greeted each other. They began talking about a student. I lately understood that the student is son of the man and belong to the class I was researching on. I heard the parent acknowledging the vice for the favor he made to the student. I did not understand what the favour was about. The vice principal conveyed the school's message to him. He said: "parents don't understand that students have two kinds of behaviors. Students might be honest at home; but, the school investigated that they have a different school conduct." I thought the school conduct could be attributed to: peer influence, teachers' impact, and school regulations.

The man left us and very soon, we also left to the school. I was curious to know what the favour was about. I raised the case . . . He narrated the story and I reported it as follows.

By the end of last academic year, the student failed to promote to grade 12. He applied to transfer to another school. One day, his father came to school to facilitate the process. On the following day, his mother came and solicited the school to take some reaction which was actually out of the school's policy. However, two reasons over dominate the policy. On the one hand, the student wanted to leave the school because he was going to miss his twelve years classmates. [He had the chance to continue his education with his juniors.] On the other hand, the parents were divorced and which might be source of student's academic failure. Then, the school offered extra tutorials and make up examinations; and many students were benefited. I considered that was a great consideration.

Such students tick friendship might have come from strong social relationships in the school. For instance, on Friday, 15^{th} December, 2017 and the time is 20 minutes past 4 pm in the afternoon. I went to the classroom to conduct a focus group discussion with targeted students. The rest classmates could not leave the room though the bell rang; and at the corridor, they continued talking soundlessly. One of my key informants, S_{15}, asked me permission and spent some time on the corridor. After she came back, I intentionally commented: "it is unusual to see students delay of leaving the room". Then, she replied the following.

S_{15}*:-* "*We are organizing a carnival to be held one day in next week.*"

Yenealem:- what is that? And what are you going to perform . . .?

S_{15}*:- it is a fund rising program . . . the graduation committee is organizing many events.*

Yenealem:- wow! What are you going to do for the graduation day?

S_{15}*:- I would be acting for Modelling.*

S_{61}:- *we are contributing money for T-Shirts and bulletins. The designs are already selected. So far, we have celebrated Flag-day, culture-day, black and white-day . . . all such stories would be included in the bulletin*.

Some days, later, by 20th December, 2017, the classroom observation and focus group discussion held in the first two periods. Unfortunately, the participants were all males. So, just as a warm up activity, I began the session by asking them the following questions. *What does it feel being a grade 12 student? When your grade level was at elementary, what had you been thinking for grade 12 students? In this school, females achieve better than males do. What are the reasons behind?* A student responded: *we are senior students. It is time to get prepared for another higher education. Thus, it is special*. Another student added: *I think females are studying 24 hours; most of the time, males engage in 'school life' like loving or having a girl friend and the like . . . but, females are serious at their education*. According to these informants, females spend much time for academic concerns than male students do.

By 1st January, 2018, I was expecting to conduct another classroom observation which would have been held during the first period. As usual, I had fixed the video cameras and began waiting for the students. As the flag ceremony was over, students of different grade levels were getting into their classrooms. But, the students I am researching on were staying on the ground. I lately realized that they were penalized to spend outside the classroom. I asked a teacher why they were ordered to do so. He told me that they had a "carnival" by Saturday. Yet, they left to home without re-ordering back the chairs they used. After a while, they were allowed to come in. As they took their sits, their homeroom and mathematics teacher yelled at them. He reacted:

I am very ashamed on you; the school had trust and honor on you . . . I remember, by the preceding event, we kept the school properties ordered. What happened . . .?

I have forwarded the following remarks too.

. . . I know the school has confidence and pride on you. Besides, your juniors, grade 11 students, grade 9 students, and lower grade level students would take you as their models. So, you have a responsibility. Don't you think…?

Some of them shake their heads; and I considered it as regret.

At the beginning of this section, I noted that the classroom under investigation was conceived broadly. This study revealed that students' class activities are influenced by friendship, the status of being promoted to youth, gender disparity.

4.4.4 Male versus Female Student's Engagement

In the preceding section, it was reported that 12th grade female students were more engaged than males in their academic life. One testimonial is that 7 out top ten achievers of the class were females by the end of the first quarter of the academic year. Besides, I observed females were more interactive than males in class activities. This was witnessed by the number of participants involved in focus group discussions.

It is my observation that females help each other and males do prefer individual reflection; seek the teacher to check their individual attempts; or otherwise keep reserved. I already quoted opinion of a male student: "I think females are studying 24 hours; most of the time, males concern for 'school life'; but, females are serious at their education". I had a similar discussion with a teacher in the staff room. I remember he said:

. . . In this school, females are more active participants than males. Since grade 12 males are at 'fire stage', they may not be kin to listen to and positively react with teacher's instructions.

This truth is also verified by females themselves during a day I had only three female participants for a focus group discussion. As the session was over, I forwarded two questions to them. *What is special in being a grade 12 student? I highly appreciated most of the best achieving students are females; is it the same in lower grade levels or not?*

Hawa (name changed) replied: "by last year, we were highly exhausted. For instance, Mathematics was wide. So, for this year we are relatively relaxed." The other girl continued: "in this school, females are known for holding ranks."

Yet, from the above statements, one could pose two more questions. If grade 12 male students were at "fire stage", would not be the case for females of the same grade level? Does it mean an affirmative action is needed for males than girls? In fact, these are beyond the concerns of this research. But, the facts entail who and why engage in classroom discourses.

4.4.5 Learning as a Program: Expectations and Examinations

This research is conducted exclusively on grade 12 students. The SA of the classroom discourse is the study of the phenomenon in learning mathematics. So, what does the concept learning mean? I remember a special classroom instance that happened 18 years ago while I was a grade 12 student. Our English language teacher forwarded a question to one of our classmates, Mekonnen (name changed): "write a sentence that begins with learning." I did not know why the teacher asked him such a "sentence construction" at that stage. We had been learning English [as a second] language. Mekonnen replied: "Learning is burning." The teacher was upset. But, Mekonnen could have been exhausted of learning for at least the last 12 years in class.

By the mid of the first quarter, the teacher gave orientation about the coming mid-semester examination. It would worth 65%; the item types multiple choices and already set from units 1 and 2 of grade 11 and 12. [The second quarter mid examination was also objective type.] Then, he wrote down list of questions that resemble model test.

Students reacted: four units [coverage] are too much.

He replied by saying: “what’s new? The national [entrance] examination would be set out of 20 units”. Then, three questions came in to my mind.

- Why did he interrupt the regular class for preparing students for the coming mid-examination?
- Why only “multiple choice” questions?
- Why grade 11 contents are included in grade 12 examination?

I looked in to grade 11 and 12 text books. Indeed, understanding “limits of functions” (Federal Democratic Republic of Ethiopia, 2010b) require knowledge of *relations* and *functions* and *rational functions* and *their graphs* (Federal Democratic Republic of Ethiopia, 2010a). The other way of addressing prior knowledge could have been delivering diagnostics test before *limit* and *continuity* begun. Finally, the listed out questions left to students as home work tasks; he told them to submit by the next day.

Most of the time, students spend thousands of hours in classrooms learning sets of procedures and rules that they will never use, in their lives or in their work (Boaler, 2016). The collective nature of mathematical learning is as it emerges and unfolds moment by moment in the classroom (Martin, Towers, & Pirie, 2006).

> I left the school and went to a nearby tea room. I spent there for some time; about six teachers came and went. There were discussions. I understood that students are getting exhausted. I also realized that the top score in mathematics at the section I am observing is 100%. However, 5 marks were added as bonus for the incorrect

> questions included in the final examination. So, bonus mark is a means of motivating students learn better. (Field Note 7th December, 2017).

By 6th November, 2017, about one fourth of the students were absent for possibly two reasons. By next week, there would be mid-semester examination in all subjects; so, students might be absent in order to get prepared for the examination. It was the 27th day of the second month in Ethiopian calendar. For orthodox Christians, it was time for the *Annual Celebration of the Savior of the World*; and, there existed a Church in the nearby area. The home room teacher shared the second assumption; but, I gave more weight for the first. I have also another justification.

By the 6th Dec. 2017, after demonstrating examples and revised the basic *limit formulae*, the teacher gave the students two class works. Immediately, being the homeroom teacher of the class, he distributed the "report cards". Students get loose of doing the class activity for extended minutes. I personally started looking at some cards. I was surprised by a student who scored 100% in one subject; I highly appreciated her. But, a student sitting next to her disregarded the achievement; he said: "there is a student who scored 120 out of 100". What did it imply?! I remember the teacher twice propagated that the class works had marks [subjected to checking for bonus marks]. I saw students participated and reacted better. I thought teachers might have given bonus marks for extra questions too in the final examination.

Of course, there were times that the mathematics teacher did present his lessons at conceptual level. For instance, he tried to link "understanding the rate of change" and "slope of a function at a point" with grade 11 physics education. To check the extent, I contacted the concerned physics teacher; and he told me that he taught "rate of change".

Yenealem:- Is there a derivative concept or formula in grade 11 physics . . .?

He replied:- it [the physics education] does not use differentiation formula; but, the entrance examination mostly demands derivative formula.

Yenealem:- It does not matter since the entrance examination [including physics] is administered after students learn 'derivative' at grade 12 mathematics . . . students have been recalling their prior knowledge of derivative. Did you teach them . . .?

The physics teacher:- I only showed them the short formula $n \cdot x^{n-1}$ *for differentiating* x^n.

Yenealem:- thank you . . . do you have the text book please?

He moved his head. I borrowed the textbook (Federal Democratic Republic of Ethiopia, 2010e); the important idea is noted as follows.

> Average acceleration is the change in velocity, divided by the total time taken for the change in velocity. Expressed mathematically, average acceleration is: $a_{av} = \frac{v_2 - v_1}{t_2 - t_1}$. As the difference in velocity decreases, the two points get much closer and the average accelerations tends towards the instantaneous acceleration, which is the acceleration at a point. Expressed mathematically; this is: $a_{ins} = \frac{\Delta v}{\Delta t}$ as $\Delta t \to 0$.

I have also consulted Grade 12 Mathematics textbook. All have interrelated concept. Yet, the dominating argument is that learning at private school is towards national examination. That is due to the fact that a private school is both a service institute and business firm. In Dire Dawa, there are more private schools; hence, there would be strong competent on the market. One means of promoting a school could be its efficiency on regional and national examinations.

I remember the second mid quarter mathematics test was administered in delay. The teacher informed me that students insisted him to overdue the schedule. He said: "they would like to score better marks; so, they demanded more time for reading and preparation."

One day in that week, I met a teacher around Shell, a village in the city, whom I knew since the last 7 years while he was teaching mathematics (part-time) at private and (full-time) government schools. We talked about different issues. In the mean time, I instigated him to compare students in private and government schools. He commented: "It is hard to mange private school students; they disturb while you teach. The only mechanism of keeping students on track is by conferring them with marks. Besides, teachers gain incentives for additional tutorials." I understood that marking is a great factor for smooth functioning of instructions. I also thought that parents could have shared their interest for lifelong learning to the school.

4.4.6 Failure to Go beyond the Minimum Learning Competencies

The preceding sub-section was devoted for reporting expectation and examination trends on grade 12 students' learning. Students at this stage are expected develop competencies needed in their further studies, working life, hobbies, and all-round personalities (Federal Democratic Republic of Ethiopia, 2010c, 2010d).

By the third of November, 2017, as I completed class observation, I begun collecting and packing my instruments. A student, whose name was Ferhan (pseudo name), asked me if there was anything he could help me; I accepted his kind support. In the mean time, I just forwarded a question to him: "which field you aspire to study at a university level?" His response was immediate: "Medicine!" I continued asking how he got interest in the field. My assumptions were: he could have heard others promoting the field; or he had relatives already in the area and influenced him to think about *Medicine*. His response was rather amazing. "It is my interest; and I want to work on cancer cases; radiotherapy and chemotherapy are being used. I want treat cancer in a different way." I continued inquiring: “Do you know someone suffered from Cancer? And how did you know about the treatment mechanisms?” After he listened to me, he told me that he knew a relative victim of cancer; then he had the opportunity to ask a doctor on the matter. His information source includes reading from the internet. That’s intrinsic motivation. What a wonderful and ambitious boy he is?!

When I returned back to staff office, I shared Ferhan’s astonishing experience to teachers who availed themselves there. One teacher was wondering; and another person was condemning Ferhan’s activities. I was eager in listening to their arguments. The later teacher continued to explain:

> “Indeed, Ferhan has great potential; he reads more books; yet he is not scoring better. There are many students who score much more than him. I remember a similar student… he was a ‘researcher’; he had been spending his time investigating on things in his surroundings. Finally, he could pass the national examination . . . he was on the way get mental disorder. With the psychological treatment and financial support of his family, he began working a vocational work. Yes! He is not doing right; high school is not the appropriate stage to [be engaged in] research. He needs to pass the exam in order to join Medicine field… he needs to be competent enough to be admitted.”

It is difficult to criticize the teacher’s argument. But, it reminds me to think of many concerns: priority for examination, absence of extracurricular activities, and inclusive education. The ability to imagine oneself carrying out activities in the future is an important aspect of both creative cognition and creative achievement (Jung et al., 2016). This is the most interesting scenario for “imagination and creativity” researcher. I felt that students, like Ferhan, have to be guided properly.

The preparatory secondary (grades 11 and 12) school mathematics education considers that students learn best by trying to make sense of something on their own with the teacher as a guide to help them along the way (Federal Democratic Republic of Ethiopia, 2010c). So, it is appropriate to understand learners within their *world* (social world, mathematical world, classroom context, etc). Students have to be acquainted with forms of cooperative work between peer groups (Federal Democratic Republic of Ethiopia, 2010a). Though there are plenty of group works in included grade 11 mathematics textbook, only few are suggested in 12th grade mathematics education. Still, classroom discourse is included in the preparatory mathematics curriculum. Group discussion is suggested as one instrument for continuous assessment. It is left for the teacher to do so.

The preparatory (second cycle of) secondary education is assumed to enable students choose subjects or areas of training which will prepare them adequately for higher education and for the world of work (T. G. o. Ethiopia, 1994). Grade 12 students' score at national University Entrance Examination (UEE) has been a determining criterion in order to be admitted to a university and field of study you are interested in (Ayalew, 2018). Currently, Addis Ababa Science and Technology University, Adama Science and Technology University and St. Paul's Hospital Millennium Medical College and particular Departments (e.g. Architecture) are demanding students to pass additional Entrance Examinations. That would influence students and the school itself to be examination oriented.

I believe that the current program at any educational level has to be linked with prior experiences and future aspirations. A failure to achieve secondary school education, for instance, may be attributed to admission of incompetent students from a lower grade level. On the other hand, Addis Ababa Science and Technology, Aksum, Bahir Dar, and Dire Dawa universities are offering Mathematics summer outreach services for top achieving secondary students. Another endeavor is also being made by the Ministry of

Science and Technology; in collaboration with the Ministry of Education, it awards best achieving/creative school student and teachers in Science and Mathematics area. In line with this gap, additional effort should also be made to well prepare preparatory school students for further studies (Walelign, 2014). Towards this end, educators need to develop the sorts of learning dispositions needed for learners and their work futures. Yet, that demands students to be encouraged to collaborate in investigations that *go beyond the standard curriculum* and creatively use the ingredients of the particular context.

4.4.7 Elements of Imagination and Creativity Missed in the Classroom

This research focused on the instructional processes whereby the first three units of grade 12 mathematics education are covered.

The first Unit is entitled as “Sequence and Series” is designed to enable students: revise the notions of sets and functions; grasp the concept of sequence and series; compute any terms of sequences from given rule; find out possible rules (formulas) from given terms; identify the types of sequences and series; compute partial and infinite sums of sequences; and apply the knowledge of sequence and series to solve practical and real life problems. The second Unit “Introduction to Limit and Continuity” assumes students would: understand the concept of "limit" intuitively; find out limits of sequences of numbers; determine the limit of a given function; determine continuity of a function over a given interval; apply the concept of limits to solve real life mathematical problems; and develop a suitable ground for dealing with differential and integral calculus. After the completing the third Unit “Introduction to Differential Calculus”, students are expected to: describe the geometrical; determine the differentiability of a function at a point; find the derivatives of some selected functions; apply the sum, difference, product and quotient rule of differentiation of functions; and find the derivatives of power, polynomial, trigonometric, exponential and logarithmic functions.

The sequence of numbers forms a pattern and expecting the following number equips us with prediction skill. If we add the terms of an infinite sequence, we get series of partial sums which form another sequence. The limits of sequences, limits of functions at a point and limits of functions at infinity are related concepts. Grade 12 students are introduced to techniques of derivatives using limit notions which are grounded on examples and informal descriptions (Federal Democratic Republic of Ethiopia, 2010b). Derivative is the limit of slopes of secants of smaller widths (Fern´andez-Plaza & Simpson, 2016).

I am not going to provide a complete evaluation of the syllabus nor do report how far is its implementation in the classroom. Rather, I tried to show the instructional practices in favour of discourse, imagination and creativity. I consider a curriculum is a minimum framework for a professional teacher to exercise his/her roles. I have discussed that the classroom teacher advocates formalism. However, the key learning experiences start up (opening problems), group work and activities were not practiced in the actual classroom. I hereby presented them in order to show how much these could have been opportunities for meaningful discourse, imagination and creativity to happen.

Historical Notes in the Textbook

In grade 12 student’s mathematics textbook, brief historical notes about great mathematicians Leonardo Fibonacci, Carl Friedrich Gauss, Leonhard Euler, Sir Isaac Newton and Gottfried Wilhelm Leibniz are presented. Fibonacci produced numbers that have many interesting properties such as the birth rates of rabbits and the spiral growth of leaves on some trees. He is known by Fibonacci series where a number is equal to the sum of two preceding numbers (Page 8). Particularly, the patter created by the leaves of a tree is an aspect of imagination. Gauss's teacher, at his elementary school, asked him to add all the integers from 1 to 100 (Page 21). He introduced new way of doing it:

$$\begin{matrix} 1+ & 2+ & 3+ & \dots + & 100 \\ 100+ & 99+ & 98+ & \cdots + & 1 \end{matrix} = 101+101+\cdots+101=\frac{100\times101}{2}$$

The impression here is that a student is also a source of knowledge. So, the teacher could have mentioned the story to encourage students think differently. Another scholar, Euler computed irrational number e using the sequence $\left(1+\frac{1}{k}\right)^k$ for $k>1$ (Page 96). On the other, the 17th–century thinkers Newton and Leibniz simultaneously invented the branch of mathematics called calculus (Page 111). The lesson that could be drawn is that whenever two or more students bring new ideas concurrently, they would be acknowledged equally. Yet, none of the notes were narrated in the classroom. However, it is possible to argue that the notes could be left as reading assignments for students.

Opening Problems Not Delivered in the Classroom

The main presentation of the "Sequence and Series" unit begins *opening problem* (Federal Democratic Republic of Ethiopia, 2010b).

> *A farmer has planted certain trees on a piece of land. The land is in the form of an isosceles triangular region with base 100 m and height 50 m. The trees are grown up in different rows as shown in Figure below. In each row, the distance between any two adjacent trees is 5 m. The distance between any two consecutive rows is 5 m, too.*

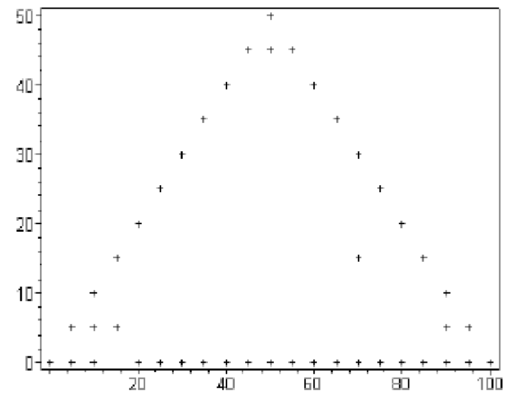

Figure 4.0.5: An Opening Problem for Pattern Making & Imagination

> ***a.*** *How many rows of trees are there on the piece of land?*
> ***b.*** *How long is each row?*
> ***c.*** *How many trees are there in each row?*
> ***d.*** *What is the total number of trees on the piece of land?*
>
> *To solve problems like this and many others, a detailed study of sequences and their sums (called series) is required.*

Students could form mental images of visual representations of patterns of the trees. This aspect belongs to imagination. The problem demands recalling the given information and interacting with the figure above now and again. That is discourse and of course thinking.

Again, the following problem is taken from mathematics text book for grade 12 students. It seems a population or demography question. However, the emphasis is not the list of parents and ancestors; the *total number* is requested.

> *As we know, each of us has parents, grandparents, great grandparents, great – great grandparents and so on. What is the total number of such relatives you have from your parents to your tenth grandparents?*

A student could create a mental picture of his relatives up to the tenth grandfather and mother.

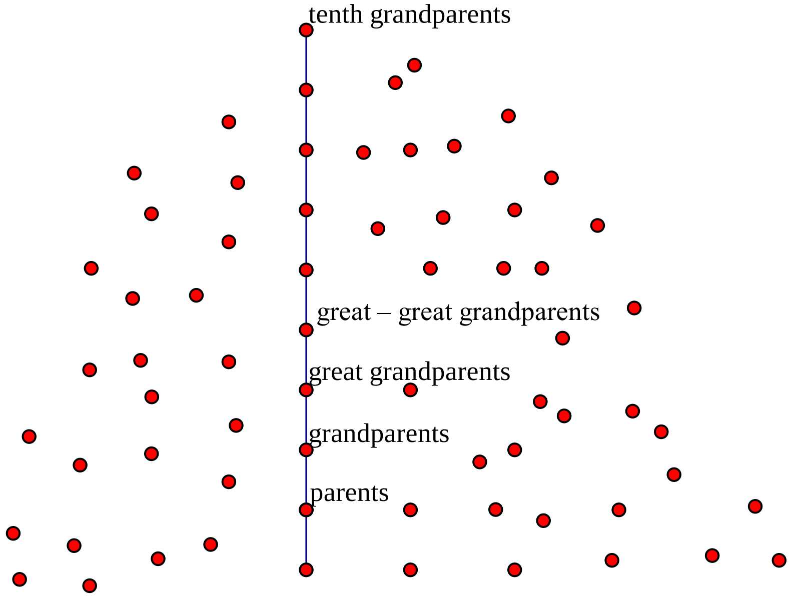

Figure 4.0.6: A Missed Opportunity: Mental Images of Relatives from the Textbook

Of course, difficult to keep all memorized. He or she might immediately plot it in a piece of paper. In any ways, the image formed is geometrical. The ultimate goal is to count all.

Another start up is found on page 9 of the textbook; presented as follows.

100 students registered to take an exam were given cards with numbers ranging from 1to 100. There were four exam rooms: R_1, R_2, R_3 and R_4. Students with card numbers

a. $1, 4, 7, 10, \ldots$ *must be in* R_1*;* ***b.*** $2, 5, 8, 11, \ldots$ *in* R_2*;*

c. $3, 9, 15, \ldots$ *in* R_3*;* ***d.*** $6, 12, 18, 24, \ldots$ *in* R_4.

The numbers in each room continue with a constant difference.

1. *Find the total number of students in each room.*
2. *Are there any students assigned to different rooms simultaneously? If so, which card numbers?*
3. *Are there students who are not assigned? If so, which card numbers?*

Students are expected sit for national examination; so, they could give more sense for the above problem. Such exercises would enable students learn how to make predictions, decisions and generalizations from given patterns. Likewise, the second unit of the textbook (Federal Democratic Republic of Ethiopia, 2010b) begin with the following problem (P. 42).

Imagine that a regular polygon with $n-sides$ is inscribed in a circle.

1. *As $\boldsymbol{n}$ gets large, what happens to the length of each side of the polygon?*
2. *What will be the limiting shape of the polygon as n goes to infinity?*
3. *Will the polygon ever get to the circle?*

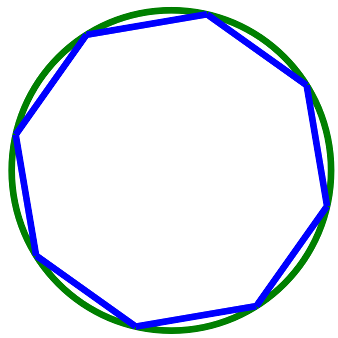

Figure 4.0.7: An Opening Problem from the Textbook

The above problem constitutes the concept "imagine". This would have been a good opportunity to think of inscribed regular triangle, square, pentagon, hexagon, 20-gon, 100-gon, and so on. The students could revise the upper bound and be introduced with the limits of sequences of numbers. The third question might have enabled them to compare the [perimeter of] an inscribed regular polygon and the [circumference of] circle. That would in turn result in gaining a different perspective about the number π.

Usually, π is considered as the ratio of the circumference of the circle with its diameter. But, comparing the perimeters of inscribed polygons with the circumference of the circle is another means of approximating π. Thus, a new learning as *process* [approximating π] than as a product [π as a constant ratio] would be entertained.

(The second Opening Problem in Unit 2) Limits of Sequences (page, 51)

Consider the terms of the sequence $\left\{\frac{1}{n}\right\}$.

1. *List the terms of* $\left\{\frac{1}{n}\right\}$ *that satisfy the condition* $0 < \frac{1}{n} < 10^{-2}$.
2. *Find the smallest natural number N such that* $0 < \frac{1}{n} < 10^{-5}$ *for all* $n \geq N$.

This problem could be opportunity for students engaged in imagination. Writing all the terms in $0 < \frac{1}{n} < 10^{-5}$ might be tiresome. Just to mention some: $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \cdots, \frac{1}{10000}, \cdots, \frac{1}{\mathbf{N}}$. But, students might look in to the patterns and predict **N**.

Group Works Suggested in the Textbook but Not Practiced in the Classroom

The only group work is located at pages 91 – 92 (Federal Democratic Republic of Ethiopia, 2010b). For this task, the teacher is advised to select students at random to sketch a graph that satisfy the given condition and ask the rest of the students whether or not the given condition is satisfied (Federal Democratic Republic of Ethiopia, 2010c).

1. *Discuss the following points by drawing graphs and producing examples.*
   Are there maximum and minimum values if
   i. *the function on [a, b] is not continuous?*
   ii. *the function is continuous on (a, b)?*
   iii. *the function is not continuous but defined on an open interval?*
2. *Let f be continuous on [a, b]. Answer the following points in terms of f (a) and f (b). Use graphs to illustrate your answers.*
   i. *Find the minimum and the maximum values of f (x) when f is an increasing function.*
   ii. *Find the minimum and maximum values of f (x) when f is decreasing.*
3. *Discuss the following statements using the intermediate value theorem.*
   i. *Among all squares whose sides do not exceed 10 cm, is there a square whose area is* $11\sqrt{7}cm^2$ *,* $11\sqrt{17}\ cm^2$ *?*
   ii. *Among all circles whose radii are between 10cm and 20 cm, is there a circle whose area is* $628cm^2$*?*
   iii. *There was a year when you were half as tall as you are on today.*

The students are asked to draw graphs which would have contributed for the development of mathematical thinking and reasoning. On the other hand, producing examples demands the student forward alternative ideas. When students assume the function as "continuous on $[a, b]$", they would be visualizing the smooth curve in their mind. The comma in "$11\sqrt{7}cm^2$,$11\sqrt{17}\ cm^2$" serves as logical conjunction. That means, the question is double and students might understand and solve the problem in case by case approach.

Relevant Activities Not Played in the Classroom

If ordinary creativity is more relevant in a regular school setting (Sriraman et al., 2013), then everyday activities need to be promoted. There are totally 35 activities suggested in the textbook; and none of them were presented in the class. The activities are believed to *open class room discussion* and introduce unfamiliar terms and create an opportunity for students to explore relevant rules and methods.

In conclusion, though the use of challenging, unfamiliar and open-ended problems were proposed in the mathematics textbook, there is little evidence observed about their implementation in the classrooms.

4.5 Sample Conversations from the Focus Group Discussions

Under this section, sample conversations from each of the ten focus group discussions (FGDs) are presented and discussed. The FGDs were conducted by looking the video records, planning for further insights and arranging sessions.

The First Focus Group Discussion

The first focus group discussion was conducted by 31st October 2017 based on two lessons observed. This was my first time formal focus group discussion (FGD) held in the afternoon. I had appointed five students whom I saw discussing ideas during the two periods. Yet, it was impossible to consider them as one group. By previous day, four of them were involved in discourse in pair or with one another. By the following, all of the five participated in some way.

Planning the first FGD:- the instances of discourses during the lessons are reported as follows. By the first day, students didn't respond to teacher's question about the domain of a function. There was a "buzz sound. Then, the teacher plotted corresponding curves for $x < 2$ and $x \geq 2$. The ends of the two curves had a hole and a point. Discussions among students commenced; yet, students around the *table mic* were seemingly confused in recognizing $f(2)$. After a while, the teacher briefed students about "undefined" and "does not exist" concepts. Some students were observed quiet. Besides, the time given for students to do the activity was less than a minute. By the following day, no student raised a question or even a reacted for teacher's note and very brief explanation on "let $y = f(x)$ be a function defined on interval surrounding $x_0 \in \mathbb{R}$ (but f need not be defined at $x = x_0$)". One of the examples was $\lim_{x \to 0} \frac{|x|}{x}$. The teacher used the graph of $|x|$ to support the above operation. Students were collectively talking while the teacher was working out the limit. The class activities were not completed within the time bound.

In the same day, five students with respective codes: S_{11}, S_{12}, S_{13}, S_{14}, S_{15} were involved in the focus group discussion and part of the conversation is presented below. At the end some insights are extracted.

Conducting the first FGD:- I came earlier in the afternoon to lead the session. I met the home room and mathematics teacher in staff room. Grade 12A students were exercising some physical fitness activities. I had been waiting for five of them for FGD. Yet, by assuming that they are "free", the mathematics teacher wanted to call for them. I resisted as I saw them enjoying their engagement. The bell rung; and students went fast to their regular classroom; the teacher and I accompanied them. They closed the door until they took off "sport" clothes; and we waited outside the room. As they finished wearing their uniforms, they allowed us to enter.

I began the discussion by giving orientation summarized as follows.

> "As I told the whole class students, the purpose of having this kind of discussion is to supplement the video records. There were two cameras set at the back corners; I could not catch quiet sounds that a student talked to a nearby classmate. As long as you cooperate for my research, I want you enable me access full data. This focus group discussion happens at your extra time which you might have needed it to study in library or support your family. So, I pay each of you 37.5 birr per hour."

I gave them an agreement in two copies; they signed and took one copy. I continued:

> "…we shall me twice a week until the seating arrangement of students is reshuffled by the homeroom teacher. When that happens, I shall continue with other students."

Finally, I invited them to raise any concern; one student asked me whether the videos would be secured or not. I confirmed that I should obey to research ethics which totally prohibits exposing their personal data to others. Besides, I told the classroom events are normal and of academic activities.

Then, I showed them the video recorded and the conversation commenced.

Yenealem:- (I paused the playing video) *some of you were having conversations each other and your teacher. Whatever your discussion was about, it would be great input for the research. Would you tell me, please?*

$\boldsymbol{S_{13}}$**:-** [she took out a pen and paper and wrote: $\lim_{x\to -2}\frac{x^2+x-2}{x+2}$. All of us observed it.] *I substituted* -2 *in the equation and it was undefined. Then, I asked* $\boldsymbol{S_{11}}$ [pointing her figure to $\mathbf{S_{11}}$] *why it happened? And she told me that we need to factorize and simplify. Finally, I got the idea clear.*

$\boldsymbol{S_{11}}$**:-** *my case was different. I just factorized the numerator, cancelled* x *by* x *and the result was not the same as Helen (name changed).*

Yenealem:- which question you are telling us about? Who is Helen?

$\boldsymbol{S_{14}}$**:-** *it's me teacher [raised her hand]. She was doing at* $lim_{x\to 1}\frac{x}{x^2-1}$.

Yenealem:- so, what happened?

$\boldsymbol{S_{15}}$*:- I think she factorized* x^2-1 *as* "$x.x-1$". *And the cancellation could have been possible if the factorization was like* $x(x-1)$.

Yenealem:- what you did then [looking at $\mathbf{S_{11}}$]*?*

$\boldsymbol{S_{11}}$*:- I feel happy.*

Yenealem:- I would also like to know if $\boldsymbol{S_{12}}$ *could tell us her emotions.*

$\boldsymbol{S_{12}}$**:-** *the question* [to evaluate $lim_{x\to 1}\frac{x}{x^2-1}$] *was unlike other examples. I asked the teacher to help me; he was busy by going to many students. He ordered me to draw the graph of the function [*$f(x)=\frac{x}{x^2-1}$*]. I feel up set.*

Yenealem:- why?

S_{12}:- *he would have assisted if I were Helen (**S_{14}**) and I am not tough student. [Helen is best performing student.]*

As we can understand from the above data, following teacher's presentation, students were reacting with side talks or collective reflection in buzz sound. There was some confusion on visual representation of a discontinuous function. At first, I got a manipulative work from a student S_{13}. She reported that she found an answer that she felt wrong. She sought a justification from S_{11} and finally took correction. Similarly in another computation, S_{11}'s result was different as compared to S_{14}'s attempt. The third student S_{15} had thought how S_{14} was performing on the task; she recognized the mistake of factorizing $x^2 - 1$ as $x.x - 1$. Such collaborative learning enabled S_{11} felt happy. That is emotion.

Therefore, there were important aspects of thinking, communication and reasoning obtained from the first focus group discussion. To mention some: confusion, visual representation, manipulative work, identifying mistake, feeling of the wrong, seeking justification and taking corrections.

The Second Focus Group Discussion

The planning of the second FGD was due to some incidents which are reported as follows. The teacher wrote and explained a definition and a theorem about *limits of polynomial functions*; but, he listed down *limits of rational functions*. That means, his oral and written explanations were different. Indeed, before he started demonstrating any of the examples [about limits of rational functions], he recalled *sequence limit properties* and justified how these could be implemented. I heard a "buzz" sound while the teacher was working out $\lim_{x \to -3} \frac{x^3+3x^2-x-3}{4x^3+12x^2-x-3}$. Then, the teacher invited a student S_{21} to demonstrate the techniques of solving $\lim_{x\to 1} \frac{x-1}{\sqrt{x}-1}$ on the blackboard. The "class" gave attention to the elaboration and involved in answering queries. Towards the end of the session, Marta was invited by the teacher to work out one problem; she begun demonstrating $\lim_{x\to 0} \frac{\sqrt{5-x}-\sqrt{5}}{x}$. She was baffled; the "class" was seemingly unhappy about the technical errors she was committing; two of the participants told her about something.

So, the intention of having a second FGD was to investigate the implicit discourses orchestrated in the instruction process. During the FGD, five students S_{11}, S_{12}, S_{13}, S_{14}, S_{15} and I sat in circular forms. Video recorded on 1st Nov. 2017 is displayed for some time so that participants of FGD memorize the event; besides, they all took out their (exercise books) notes. Part of the conversation is presented below.

Yenealem:- let us start our discussion. . . how did you use the definition and theorem given to "evaluate" questions under "examples"? Did they help you? Or, did not look in to them while you were solving questions?

Participants: [quiet, no response]

Yenealem:- ok, once again. . . the teacher wrote and explained the definition and the theorem about polynomial functions. Before he started working out any of the examples, he recalled the sequence limit properties and justified how these could be applied for computing limits of functions. Right?!

Participants: collectively say "Yes!"

Yenealem:- . . . based on your [collective] request, the teacher manipulated the limit of x cubed plus three x squared minus x minus three the whole divided by four x cubed plus twelve x squared minus x minus three [as x approaches to]. So, which explanation helped you to cope up with the solution of the question?

S_{14}*: we have referred to the sequence properties. . .*

Yenealem:- good, keep on telling me . . . [I invited all of them to talk]

S_{12}*: since the teacher informed that f of -3 is zero, we noticed that x plus 3 is a factor of "four x cubed plus twelve x squared minus x minus three". Then, we divide it [four x cubed plus twelve x squared minus x minus three] by $x+3$ [to get the other factor].*

S_{12}:- I had another method in mind and asked my friend [pointing to S_{14}] why he [classroom teacher] did use only one factor. She told me that $(x+3)$ is used to get the other factor.

S_{14}: I give her another example x^2+x-2, we find two numbers whose sum is 1 [coefficient of x] and whose product is -2 [products of 1, the coefficient of x^2, and the constant term, 2]. Finally, we [both] reached at $(x+2)(x-1)$.

Yenealem:- would you [the student S_{12}] *please share us what was your thought?*

S_{12}:- I just group the denominator in to two. [I showed her a confused face.]

S_{14}: [involved and wrote down $(4x^3+12x^2)+(-x-3)$ on a piece of paper].

S_{12}:- yes, I factorized four x square from the first [two terms] and minus 1 from the second [$-x-3$].

Yenealem:- did you get a different answer?

S_{14}:- it was the same.

S_{12}:- [after a while] *yes! I related this [her grouping of terms technique] method with Gashe's* [classroom teacher] *approach.*

The following words or phrases could be taken as key elements of thinking, communication and reasoning. I recognized there was a collective request; the student S_{14} reported "we have referred to. . ." S_{12} was saying "… we noticed… we divided…" Having a different method in mind, S_{12} asked S_{14} to explain "why?"

In return, S_{14} shared an example with S_{12} and involved her in finding *two numbers whose sum is 1 and whose product is* -2. Finally, the two participants reached at a common result. Then, S_{14} was engaged in hands on activity; she employed an alternative method and reached at the same result. This was approved by S_{12}.

Yenealem:- so far so good! That's what I need; we are having a nice interview. Let us continue. .. [Pointing to] *Desta* (name changed, the student S_{21}) *was demonstrating* $lim_{x\to 1}\frac{x-1}{\sqrt{x}-1}$ *on board. And you had, simultaneously, conversations each other. Could you tell me what your discussion was all about?*

***S_{13}**:- . . . first, he multiplied it by the square root of x plus 1*

[Desta wrote: $\lim_{x\to 1}\frac{x-1}{\sqrt{x}-1}\times\frac{\sqrt{x}+1}{\sqrt{x}+1}$]

***S_{15}**:- I was not clear why he* [the student S_{21}] *did that; I was asking "why?" Fortunately, Miki* (pseudo name) *has heard my voice and told me that we cannot substitute* 1 *unless the denominator is rationalized*

[Miki was not among the participants of the focus group discussion]

$\boldsymbol{S_{11}}$*:- at that time . . . I multiplied* $x-1$ *by* $\sqrt{x}+1$ *as* $x\sqrt{x}-1$*; but, Desta did differently as* $x(\sqrt{x}+1)-1(\sqrt{x}+1)$.

Yenealem:- do you think the multiplication was needed.

$\boldsymbol{S_{12}}$*:- no! he simplified . . . but returned it back to the original step. He was multiplying the factors in the numerator.*

Yenealem:- yes, I remember; just tell us how it happened.

S_{12}*:- he rationalized the denominator but make it* [the expression $\frac{x-1}{\sqrt{x}-1}$] *complex. The good thing was that the teacher advised him not to do so. Then, limit of square root of* x *plus on was left.*

Yenealem:- how was the overall process; I mean how did Desta manipulate the task?

Participants:- it was fine! He is intelligent! He did it well!

From the above conversation, I noticed that different students were engaged in co-action independently while Desta was demonstrating the computation on the board. For instance, Miki reacted to a student's doubt or question by saying "why?" and another student, S_{11}, reported she had a different thinking at a step. On the other hand, S_{12} did evaluate Desta's rationalization as complex.

The Third Focus Group Discussion

For the third focus group discussion session, four students were involved. First, I gave them orientation on the need to have a FGD. Videos recorded on 6th and 7th of Nov. 2017 were displayed for some time so that participants of FGD memorize the event; besides, they all took out their (exercise books) Notes. By the first lesson, it was after 23 minutes elapsed that a student coded as S_{24} shared an opinion. The teacher explained; but, she continued arguing. He invited the whole class to justify her query. Mainly, "undefined" was heard from mass as an answer. At the 28th minute, a student coded as S_{23} was talking to S_{24}. And a student in the nearby seat whom I coded S_{22} was staring at their discussion. Another student sitting far from S_{22}, S_{23} and S_{24} was actively involved on that particular discussion. I gave him code is S_{21}. The teacher once again requested if the question was clear t the whole class. The second day lesson was not that much interactive. In the mean time, the teacher wrote and explained the definition limit at infinity; but, much time was devoted for and lots of ideas were forwarded to the conceptualization of $\lim_{x\to\infty} f(x) = \pm\infty$. A student whose code is S_{21} was challenging the teacher and the whole class.

Yenealem:- let us start our discussion. . . by yesterday, S_{24} asked a question and that brought some dialogue. By calling the name of S_{24}, would you please remind us what your question was about?

S_{24}:- the question was to find the limit of x minus three divided the square root of x squared minus six x plus nine as x approaches to three.

Yenealem:- Eh?

S_{24}: Since . . . when x is placed in the denominator, it brings a zero value. And it cannot be zero. . . that has to be changed in other form. The teacher simplified the expression and got x minus three over absolute value of x minus three. Now to know the limit as x approaches to three, necessarily, 3 should have substituted . . . the teacher used two forms: when x is greater than 3 and when x is less than 3. For x is greater than 3, it becomes 1; and for x is less than 3, it becomes negative 1. That means, he [the teacher] cancelled $x-3$ by $x-3$; he didn't insert 3 in the equation. So, the effect of 3 is not considered. If we are not using 3, we are referring to x approaches to infinity.

Yenealem:- Have you got S_{24}'s idea?

S_{22}: . . . she [S_{24}] has said 'we have to rationalize the denominator'. The teacher once told us that 'if the denominator becomes zero and the whole term is undefined; we need to rationalize and simplify…' But, she . . . if we don't use 3, why do we write 3 there.

Yenealem:- let me ask S_{24} once again. Before you raised a question and before the teacher replied to your concern, you had been trying by yourself. I saw your writing $\frac{x-3 \quad -3x-3}{\sqrt{(x-3).(x-3)}}$ on your exercise book. What was your thought?

S_{24}: I used $-3x-3$ in order to factorize x^2-6x+9. I was finding two number whose sum is

-6 and whose product is 9. I found -3 and -3.

Participants: [keep quiet for a while.]

Yenealem:- let me take you back to your teacher's question; after the teacher and S_{24} had some conversation. Then, the teacher requested the class: "Is there anyone who did understand S_{24}'s idea?" What did you think by that time? For instance, S_{24} and S_{24} were discussing. Or, what did you do?

S_{23}: She [S_{24}] suggested: "if the denominator gets undefined, why don't we leave it?" I was telling her Gashe [the teacher] once advised us to try further in such cases.

S_{22}: [he was attentively following S_{23}'s report.]

Yenealem:- so what did you [both] conclude?

S_{23}: the time was very short; and couldn't complete our discussion. We immediately shifted to teacher's lecture.

S_{21}: [He might be rethinking the question, the answer or the whole conversation.]

Yenealem:- [By pointing my hand to S_{21}] *do you have anything to share?*

S_{21}:- I had been rethinking the mistakes I committed earlier; so, I didn't heard any of the discussions.

Yenealem:- I remember you worked out one exercise on the blackboard. [After the lesson was started, sooner the $\lim_{x \to 3} \frac{x-3}{\sqrt{x^2-6x+9}}$ was posed, the teacher recalled an exercise left unsolved by the other day.]

S_{21}:- yes! I was asking myself my over all procedures for that question.

S_{22}: by the time she [pointing towards S_{24}], raised her question, I was just waiting for teacher's explanation.

Two occasions ignited interactive classroom events. The student S_{24} had misconception on the difference between limit of a function at 3 and the value of the function at 3. S_{24} did not tell us why she considered inserting 3 in the equation; but S_{23} justified a reasonable assumption. Their teacher taught them 'if the denominator becomes zero and the whole term is undefined; we need to rationalize and simplify…' This is *synthesis imagination*.

On the other hand, the case where *limit does not exist* and *infinite limit* brought a hot discussion. The student S_{21} thought *infinite limit* as the limit exists. He was comparing:

$lim_{x\to+\infty} f(x) = \infty$; $lim_{x\to-\infty} f(x) = \infty$ with that of $lim_{x\to a^+} f(x) = L$; $lim_{x\to a^-} f(x) = L$.

The story is given below.

Yenealem:- so far so good. Let continue reflecting on today's class. The topic was "limit at infinity". How did you get the topic itself? How did you understand the teacher's explanation about it?

Participants: [keep quiet for a while.]

Yenealem:- [Emm…] *or, the teacher asked you all: "when do we say 'limit does not exist'?" and some students from the back bench replied to the question. Did they meet your expectation?*

S_{24}*: he [the teacher] had already written that if the limit of* a_n *as* x *approaches to infinity is* k*, then it exists. So, it was easy to understand the idea [concept].*

Yenealem:- how did you [all] get the expression: $lim_{x\to\infty} f(x) = \pm\infty$*?*
I think S_{21} *could tell us something. . . [He had been in dialogue with the teacher.]*

S_{21}*:- my question was: "if both limits were positive [infinity], can't we say the limit exist?" if we have positive infinity and positive infinity answers . . . that means we get the same value. So, we should say "the limit exists". There was a similar idea… it will contradict with that concept. It came in to my mind that if the limit [as* x *approaches to, say* a*] from the right and left sides are the same, we said the limit exists.*

Yenealem:- Good! What did you [all] think while S_{21} *was asking the question?*

S_{22}*:- I had the same idea. I thought if the limit values are* $+\infty$ *and* $+\infty$ *[as* x *closes to* $+\infty$ *or* $-\infty$*], then, the limit exists.*

S_{24}*:- we can talk about the limit. . . but, it does not mean that the limit exist.* f *of* x *continues.*
[Remark: if $lim_{x\to\infty} f(x) = \pm\infty$ *was expressed differently, like as* $x \to \infty$*, then* $f(x) \to \infty$ *or* $f(x) \to -\infty$*, we could have seen other conceptions too.]*

Yenealem:- Ok! We are completing our discussion. Let you talk any time you want to share ideas. Don't wait for my inquiry; feel free. I hope . . . by next time . . . we would have better conversation.

S_{23}*:- his [*S_{22}*] question became my question too.*

Yenealem:- That's is enough for today. See you another time.

The misconception exhibited was treating ∞ as a number. Participants had the same assumption. Indeed, the limit values of the function were same $+\infty$ and $+\infty$ as x closes to $+\infty$ or $-\infty$. Yet, it does not show the existence of two sided limit.

The Fourth Focus Group Discussion

I had no time to get well prepared for running this event; it happened earlier than the time I scheduled. Yet, I had attended the two lessons in person; so, I remembered the whole process and managed facilitating the group interview. As usual, I did put the *table mic* at a desk before students came in to class. A student S_{31} sits on her [armed] chair which unfortunately carries the *mic*. There are four students involved in group discussions; for the sake of confidentiality, I give them codes: S_{31}, S_{32}, S_{33}, S_{34}. Their sitting position was around the center of the class. I saw three more students were participating in the discussion. Since these students were once FGD members, I keep their previous code: S_{12}, S_{15}, S_{23}. Most of the time, the student, S_{33} was dominating the discussion. A day before, I informed my third time focus group of students that we would have interview. I was thinking to base my interview on today's class too. Yet, their sitting arrangement was changed and little discussion happened. I also see other students sitting in a different place. The home room teacher claimed that he would reshuffle the sitting arrangement every two weeks. But, during the last two weeks, there were first quarter final examination and then feedback activities.

Yenealem:- We will be talking about yesterday and today's lessons. By yesterday, following the example demonstrated by the teacher, there was good discussion. What was your point of discussion?

$\boldsymbol{S_{12}}$:- *on the first example given, f of x absolute of x over x, he demonstrated it at x equals to -3; he [the teacher] said continuity exist at -3. She [S_{33}] explained at x [equals to] one; she said it [continuity] does not exist. Then, . . .*

$\boldsymbol{S_{34}}$:- *. . . then, we were arguing on that. I mean, we asked . . .*

$\boldsymbol{S_{12}}$ *and* S_{34}:- *we asked her [S_{33}] about it, and she explained for us.*

Yenealem:- what was your question?

$\boldsymbol{S_{34}}$:- *our question was: 'at x is equal to 1, does continuity exist or not [for $f(x) = \frac{|x|}{x}$]?'*
She showed us that continuity does not exist. Then, we did it by ourselves and found the same answer. However, the teacher explained that the function can be continuous at $x = 1$.

Then, we asked her for the second time. Then, we do the given [the rest] class work.

Yenealem:- let $\boldsymbol{S_{33}}$ *talks about it.*

$\boldsymbol{S_{33}}$*:- the teacher gave us at* x *equals* 1. *But, I checked it [the continuity] at* $x = -1$. *They [the group members] tried on* $x = 1$. *That means, it was incorrect . . . the agreement . . . it was after the teacher informed that the given point is* $x = 1$, *the discussion got finished.*

Yenealem:- so, did they give you correction or forward a question?

$\boldsymbol{S_{33}}$*:- the teacher ordered us to check [continuity] at* $x = 1$*; and they did it at* x *equals to* 1. *But, I solved the example at* $x = -1$.

$\boldsymbol{S_{12}}$*:- . . . she did it at* $x = -1$ *and explained for us that way. After the teacher informed us* [the class] *that the function is continuous at* $x = 1$, *she explained the continuity at* $x = 1$.

There were attempts in recalling of information, bringing different ideas together, asking, answering, extending another student's thought, argument, exercising based on demonstration, agreement, and evaluation.

The Fifth Focus Group Discussion

The teacher recalled "slope of straight line"; he also mentioned grade 9 physics education. He introduced new concepts in advance: slope of curve, local maximum and local minimum, absolute maximum and absolute minimum, and Integral Calculus. The teacher forwarded another question: "how do we measure a volume of an irregular cylinder?"A student suggested inserting the cylinder in a container holding water and seeing the difference in volume; and different students continue discussing. The teacher had recommended the issue will be handled in unit 5 after *Riemann sum*, *definite integral* and *bounded interval* are introduced. Students were asked to draw a partition-time graph. My focus group members were S_{13}, S_{41}, S_{42}, and S_{23}; the first two males and the other two females. Since S_{41} and S_{42} came to the interview session for the first time, orientation was given.

Yenealem:- the teacher forwarded: "how do we measure a volume of an irregular cylinder?" Many students responded differently to this question. What was (were) your idea?

S_{13}:- *we were partitioning the cylinder [which was not drawn on the blackboard] in parts . . . then is possible to apply formula for the parts.*

S_{23}:- *just to insert cylinder in water . . . then the water rise up or float down. It is possible to know the volume [of the cylinder] by considering the volume of risen up water*

S_{41}:- *it is applicable for volume . . . but, if it were area, we divided it [the shape or polygon] into squares.*

Yenealem:- . . . S_{13}*, how did you get into partitioning . . .*

S_{13}:- [smiling] *I was just confused. I applied the 'square' idea to compute volume. . .*

Yenealem:- but, to some extent, you were right. . .

S_{13}:- *the teacher later told us to partition* [the cylinder] . . .

S_{41}:- *no, I agree with the idea of "inserting the cinder in water"; it was best idea.*

[They continue arguing around the bush; S_{23} joined their discussion.]

S_{23}:- *may be . . . we may learn this in* [the future] *calculus.*

[So far, and during the classroom time, S_{42} kept silent; but, she was attending all.]

Yenealem:- good . . . S_{42}*, how did you get all those discussions?*

S_{42}:- *I had the same idea with them.*

Yenealem:- but, S_{13} *and* S_{23} *forwarded very different opinions . . .*

S_{42}:- *when we measure the volume* [of the cylinder], *we add it water . . .*

Yenealem:- ok, we are having a nice time. . . The teacher mentioned other new topics . . . like Riemann sum, definite integral . . . he was saying "we will learn these issues in unit 5". So, how did you get the introduction?

S_{41}:- *I think, we could use them to find the volume of an irregular shape.*

S_{23}:- *he was just giving as brief. . .*

Yenealem:- so, you hoped you would learn them ahead of time . . . and you did not bother . . [They have a facial expression entailing agreement.]

From the above conversation, we can see that the following concepts were included. At first, different ways of measuring volume of an irregular cylinder; S_{13} got confused; getting in track of discussion; argument continued; taking side on argument; and facial expressions.

Yenealem:- next, he let you draw the graph of a partition-time. What did you do?

S_{13}:- [she take out her exercise book] *I drawn it like this . . .*

S_{23}:- *she [S_{13}] asked me to check her work . . .*

Yenealem:- Hadn't you sketched . . .?

S_{23}:- *she already finished . . . and I just looked at her work.*

S_{13}:- *[she plotted the $x-$ and $y-$ axises at the back page of her exercise book]. I assume t as $x-axis$ and v as $y-axis$.*

S_{41}:- *teacher [me, the interviewer], it was simple. When the table is given, it is easy to draw the points on the graph . . .*

Yenealem:- but, I remember the table was given later . . .

S_{41}:- *aha . . .*

[S_{23} and S_{42} were engaged in side talk; I waited for a while and asked them to share their discussion.]

S_{23}:- *I told her to correct the graph she sketched on her exercise book.*

Yenealem:- what happened to the graph?

S_{23}:- *when she connected the points, she had drawn a curve which should have been a straight line. I just demonstrated her . . .*

S_{41}:- *she had a straight line before Gashe [the mathematics teacher] showed us the graph. Then she made it curve.*

Yenealem:- so, did she change her sketch of "straight line" into "curve" accordingly?

S_{41} *and* S_{42}:- *yes! [It seems that the teacher was imperfectly sketched the graph.]*

The participants were involved in: looking at notes, screening notes, side talk, correcting a partner's work, and accepting suggestions.

The Sixth Focus Group Discussion

This session was arranged following the lesson on *differential of a function at a point*.

I observed, during the lesson that when 18 minutes elapsed, the teacher demonstrated how $\lim_{h\to 0}\frac{f(x+h)-f(h)}{h}$ and $\lim_{x_0\to 0}\frac{f(x)-f(x_0)}{x-x_0}$ would be equivalent. From the very beginning, $\lim_{h\to 0}\frac{f(x+h)-f(h)}{h}$ should have been written as $\lim_{h\to 0}\frac{f(x_0+h)-f(h)}{h}$. Next, he wrote $\lim_{h\to 0}\frac{f(x+x-x_0)-f(x_0)}{h}$; in turn individual student's reflection was created. Many students were engaged in computing problems; some write on their hands; some others write on the back pages of their exercise book and then copy it to the main writing section. After the 30th minute of the period, students were solving an example individually. The teacher continued giving hints. At the 33th minute, there was also a collective response "$80/27$"at the whole class level.

I had six participants S_{51}, S_{13}, S_{41}, S_{42}, S_{23}, and S_{52} involved in the focus group discussion. I already told them to bring their exercise books; now they take out their Notes.

Yenealem:- the morning class was very good; specially at the beginning, different ideas had been raised. I would be happy if two more students were here with us to share their thoughts. Anyway… the teacher wrote limit of f of x minus f at x note [the whole] divided by x minus x note . . . and limit of f of x plus h minus f at h . . .

S_{51}*:- . . . f of $x+h$ minus f of x_0 over x minus x_0 . . . the limit as x approaches to x_0.*

Yenealem:- ok, . . . then, in addition to these expressions, he [the teacher] recalled the slope formula. He was also demonstrating the equivalence of the first two formulae. Then, different ideas were forwarded. It was S_{51} that expressed her thought. What was that?

S_{51}*:- he [the teacher] already told us that as $x\to x_0$, then h equals 0. That means, as $h\to 0$, x is equal to x_0. Since x and x_0 are equal, we can interchange them.*

S_{52}: in order to cancel . . .

Yenealem:- ok, eh. . .

S_{51}: . . .we can substitute h by $x - x_0$.

Then [she write on the paper I gave her], $lim_{h\to 0}\frac{f(x+x-x_0)-f(x_0)}{x-x_0}$.

We get . . . $lim_{h\to 0}\frac{f(2x-x_0)-f(x_0)}{x-x_0}$.

Since $x = x_0$, we have $lim_{h\to 0}\frac{f(2x-x)-f(x_0)}{x-x_0} = lim_{h\to 0}\frac{f(x)-f(x_0)}{x-x_0}$.

Yenealem:- . . . then, you were referring back at your notes. What were you finding?

S_{51}: firstly, he [the teacher] did not tell us that they [x and x_0] are equal. I wrote f of $2x$ minus f of x note . . . and I was confused. That is why I was looking at my exercise book in search of any relevant formula.

The participants were engaged in mental activities. The student S_{51} elaborated teacher's statement and continued reasoning out the idea. Then, S_{52} began to extend the computation; but, S_{51} took over the opportunity to speak. She drafted her ideas on the paper and shared it with us. She tried to be logical in her argument.

Let me add more conversations.

Yenealem:- Mahlet (name changed; a classmate) was speaking that x is greater than x_0 . . .

S_{51}: . . . because in $f(x) - f(x_0)$, $f(x)$ is written at the left. Hence, $f(x)$ is greater than $f(x_0)$.

Yenealem: so, did you [all] share Mahlet's idea?

S_{51} and S_{52}: yes we did.

Yenealem: who did suggest multiplying by negative . . .

S_{51}: it was me . . . since $h = x - x_0$, I said that we can substitute h by $x_0 - x$ and then multiply $f(x) - f(x_0)$ by negative . . . it yields the same result.

Yenealem: . . . let S_{52} say something. . .

S_{52}: before the teacher told us and I was sure about the equality of x and x_0, I had the same understanding with S_{51}.

S_{51}: it works too for multiplying by negative . . .

Yenealem: . . . didn't you have a different way? Did you accept that $x = x_0$?

S_{23}: yes . . .

S_{51}: Since he [the teacher] said . . . $h = x - x_0$, when h approaches to 0, [we can consider] x_0 is the same as x and x equals to x_0.

Yenealem: so far so good . . . and. . . S_{23} and S_{62} were discussing . . . who started it?

[Long pause]

S_{52}: it was me. We were assisting each other about solving the long division . . .

Yenealem: which division?

S_{52}: [after he referred at her exercise book] . . . to differentiate x cubed minus $9x$ at x [equals to] $\frac{1}{3}$ . . . and asked Tigi for more hint.

Yenealem:- aha . . . so, what did Tigi tell you?

S_{52}:- we independently tried up to some steps. Then, she showed us how to divide x cubed minus $9x$ minus f of one over three by $x - \frac{1}{3}$.

S_{23}: I worked out half way . . . but, there is no factor to be cancelled. Then, we did it together.

Yenealem:- . . . the teacher invited the class [all students] to report at each step while he was solving the division on the blackboard . . . many students were collectively saying "negative eighty over twenty seven". Did you both discovered $-\frac{80}{27}$?

S_{23}: $-\frac{80}{27}$ is the value at the numerator which is found by inserting $\frac{1}{3}$ in [place of] x. Yes, we reached at that step. Our problem was the long division.

I just recalled Mahlet's thought; and S_{51} completed it. Besides, she added the reason behind; the justification is not presented on behalf of Mahlet. They agreed with Mahlet's argument and continued to support it with good reason.

The Seventh Focus Group Discussion

Four students S_{51}, S_{14}, S_{15} and S_{61} were selected for the 7^{th} round focus group discussion as they were observed interacting in the actual lessons. The events are described and reported in the following two paragraphs.

At the beginning of the first lesson, the group members were talking personal issues; but, S_{15} asked S_{14}: "have you worked out [question number] 'D'?" Yet, they continued talking the personal concerns. As a revision of the previous lesson, the teacher requested the class to compare "slope of tangent line" and "Slope of secant line". He also explained the equation of a straight line using slope point form. At the 15th minute, the teacher wrote till the bottom of the blackboard; and S_{15} complained "teacher, it's not observable. . ." He didn't hear her voice; and, many students shared complaining. Then, he had re-written it. I think the students were lagging behind the lecture or are visual learners. After demonstrating examples, the teacher let the class to do the remaining examples. Very soon, S_{15} spoke: "Gashe [the teacher], let you do [question number 2] 'B'!"

In turn, he replied: "what happened? What are you going to do then?" Then S_{14} and S_{15} were talking; S_{61} was attending their conversations.

He recognized that $(-2,7)$ was incorrectly given; he excused the class for the technical errors. S_{14} called another student; and spoke "which method of derivative we should apply for question '2C'?".

At the beginning of the second lesson, the teacher introduced the various notations for derivatives. $f'(x) = \frac{dy}{dx}; f'(x_0) = \frac{dy}{dx}\Big|_{x=x_0}$.

After 6 minutes of the lesson, S_{51} was sharing ideas with the one who sat next to her; S_{14} had asked another student: "do we write . . .?" At the same time, S_{15} was looking at her friends' activities. Around the 10th minute of the lesson, the teacher wrote [on the blackboard] an example for "differentiability [of a function] on an interval". Part of his text was: dif$\underline{^{le}}$. The students were confused and asking each other about this representation.

At the mid of the period, the teacher wrote $\left.\frac{dy}{dx}\right|_{x=2}$; and forwarded the question "what does it mean?" to the class. No one replied fast. Around the 25th minute, the teacher observed S_{14}'s work; they were talking each other. At the same time, S_{61} was looking at the conversations. After the teacher left S_{14}, the student S_{15} asked S_{14} to share the discussion made.

Around the 30th minute of the lesson, the teacher wrote: $\lim_{x \to x_0} \frac{f(x)-f(x_0)}{x-x_0} = f'(x_0)$. Then, he asked the class: "when do both are equal?" A student Kebe (name changed) responded: "when the limit exists!" But, the whole class discussion continued for a while. The teacher took more 7 minutes after the bell rang.

Yenealem:- *by yesterday, you didn't immediately immerse in to the teaching learning process. There were some personal talks in your group. In the mean time* S_{15} *requested* S_{14} *the question: "have you done 'D'?" But, you didn't wait for her answer. Do you remember . . .?*

S_{15}*: . . . before I asked her, I already asked the teacher to do [question] '2B' . . .*

Yenealem:- *no, that dialogue came later. . . if you forget that. . . [she felt guilty] . . . never mind . Let jump to the next issues . . . the teacher raised: "slope of tangent line" and "slope of secant line". He forwarded questions like: "do you remember . . . do you forget?"*

[They kept quiet]

Yenealem:- *Had you recognized their difference . . . their slopes . . . why they were mentioned together . . . you have learnt these things by Monday or Tuesday?*

What was your conception by yesterday? Did you remember them . . . or did you find them as new ideas? I appreciate if you speak loudly.

[S_{15} and S_{14} were talking each other; and I waited until they completed their discussion.]

Yenealem:- ok, let you share what you have been discussing right now . . .

[S_{15} laughing; S_{51} smiling]

S_{14}*:- . . . about tangent line . . .*

Yenealem:- I like if S_{15} *talks first as she commenced the discussion . . . What did you ask her?*

S_{15}*:- I advised her: "speak out what you understood".*

S_{14}*:- it is easy to calculate the slope of secant line; but, it is tangent line . . . I was confused and didn't know what to do.*

Yenealem:- didn't you understand "slope of a tangent line" by Monday . . .

S_{14}*:- Had we learnt that?*

From the above conversation, some elements of classroom discourse include asking, reflecting, reactions with laughing or smiling, and answering. The student S_{14} sought to know what to apply for calculating the slope of secant line; but, had no clear idea. The focus group discussion continued; more conversations are presented below.

S_{15}*:- he [the teacher] taught about derivative.*

Yenealem:- when the teacher introduced the concept of derivative . . .

[S_{15} and S_{14} were talking each other; S_{61} looking at their interactions; and I waited until they completed their discussion.]

S_{14}*:- we have learnt a tangent line with respect to a circle by last year. But, I was not clear by Monday.*

Yenealem:- ok. . . the teacher continue to recall equation of a straight line using slope point formula. Was that clear for you all?

[They confirmed with their facial expression.]

Yenealem:- . . . and when the teacher talked off point of tangency, S_{15} *was writing on her hand. What . . .?*

S_{15}*:- I was thinking back; I was doing question number 1 . . .* f *of* x *equals to* x *squared at* 1.

Yenealem:- so, did you plot the graph of the tangent line or . . .

S_{15}*:- I just computed the question.*

Yenealem:- you mean calculating the derivative at the given point . . .?

S_{15}*:- yes . . . then, he* [the teacher] *sketched the slope. I mean the graph. Then, I asked question "B".*

As it was reported earlier, there were new discussions happened during the focus group discussion sessions. For instance, while the 7th FGD session S_{15} and S_{14} had side talks and S_{61} was looking at their interactions. Since my intention was to investigate what and how individuals communicate within a group, I waited until they completed their discussion and requested them to share with the rest participants. The very crucial aspects of this research were disclosed in that event. The student S_{15} was writing on her hand; S_{14} associated learning a tangent line with previous lesson on circle; they attended teacher's illustration and then, S_{15} asked a question.

A discussion was also initiated by an individual's interaction with lecture note; it brought a concept that worth much. The following conversation is a show case.

Yenealem:- [pointing to S_{15}*] I think, you did not see what the teacher wrote at bottom of the blackboard. Therefore, you asked him. . . Then, many students repeated your question; wasn't enough to hear his voice?*

S_{15}*:- [laughing] I heard; but, we follow his speech and write simultaneously. We have to see what he has demonstrated.*

S_{14}:- it is mathematics; so, we have to observe . . .

Yenealem:- S_{51}, was that your question too?

S_{51}:- no, even I was criticizing them [group members] that sound could be enough.

Yenealem: what was your reaction, S_{61}?

S_{61}: I could not see too.

Yenealem:- [do] you prefer to look . . . not sufficient hearing . .   .?!

S_{61}: yes! [we all smiled]

The above data entail the type of learning styles practiced by many mathematics students. Two students S_{15} and S_{61} were visual learners where as S_{51} was an auditory learner. However, S_{51} expected her partners to adapt for and being able to master hearing.

Below are some more conversations; strategies such as short cut method and alternative methods are presented there.

Yenealem:- so far so good; we are conducting a fruitful interview. Ok . . . next, the teacher wrote this notation [I showed them $\frac{dy}{dx}\Big|_{x=2}$]. Do you remember it? How did he read it?

S_{14}: dy over dx . . .

S_{15}: . . . [evaluated] at x equals to 2.

Yenealem: Sure! And he asked "what does it mean?" Do you remember it?

S_{51}: after we solve, we substitute 2.

S_{15}: we evaluate at 2.

Yenealem: . . . then, the teacher observed while S_{14} was working out . . . what feedback did he give?

S_{14}: I was solving the question in example 1. I had doubt on my solution. And I asked him "it will be four cases"; he told me that it would be two cases and informed me I forgot

substituting the given number in place of x.

Yenealem: am not clear. Would you tell me again?

S_{14}*: absolute value of* $t-x$ *is positive* $t-x$ *or negative of* $t-x$*. . .*

S_{15}*: [writing on a piece of paper]* t *minus* 3 *in absolute again minus absolute value of* x *minus* 3 *the whole over* t *minus* 3 *as* x *approaches to* x.

$$\lim_{t \to x} \frac{|t-3|-|x-3|}{t-x}$$

S_{14}*: . . . then, he completed it on the blackboard.*

Yenealem: while you were working on the question, S_{15} *requested you to explain.*

S_{15}*: yes, I did not understand it; how could we simplify the absolute value. . . and she explained it to me. Then, I understand it.*

Yenealem: eh, you returned back soon. How did you get her explanation in that instance?

S_{15}*: I looked at her worked out solution . . .*

S_{14}*: she had already gone the basic steps. . .*

Yenealem: [turning my face to wards S_{51}*] how did you get the question?*

S_{51}*: I was solving it. He [the teacher] already told us the technique . . . in finding the limit of an absolute value [function], we need to check from the right and left [side limits].*

Yenealem: [looking to wards S_{61}*] which lesson was good for you? Yesterday's or today's . . .?*

S_{61}*: I like more today's class.*

Yenealem: thank you very much! Have a nice evening!

From the above conversations, we can see that the teacher guided the student S_{14} to the right track. She attempted a question; but, had doubt on her solution. While S_{14} and I talk to each other, the participant S_{15} was manipulating on a piece of paper. Sooner, S_{15}

shared her works with us; but, S_{14} interrupted her. Among the participants of the day, S_{15} and S_{14} had better dialogues; this was also observed in the actual classroom session.

The Eighth Focus Group Discussion

In the class, around the beginning of lesson, the group members could not totally engaged early in the actual process; S_{71} was copyingS_{14}'s note; and S_{14} was taking notes of other subject. Later around the 16th minute, S_{71} was taking note from the blackboard while the teacher was explaining the contents. Just at 23rd minute, S_{14} looked at and commented on S_{15}'s note. She said: "make it derivative". When 27 minutes elapsed, S_{15} asked S_{14} how had she computed [question number] "i". Then, S_{71} joined them; she was observing. But, after 3 minutes and after Mahlet (name changed; classmate) demonstrated the derivative of $f(x) = x^{e-3}$, she requested S_{15} to rework "i" once again. During the last 2 minutes, many students were circulating their exercise books.

Yenealem:- when as I saw the video, S_{15} was asking S_{14} about differentiating x^5 . . .

S_{15}*:- first I asked her how to differentiate x^5; she informed me to bring down 5 [as a coefficient] and subtract 1 from 5 [the difference will be exponent of x]. Yet, I doubt differentiaing x^{99}; thus, I asked her once again.*

Yenealem:- [looking towards S_{15}] S_{14} was looking at and commenting on your note. I heard her saying: "make it derivative". . .

S_{15}*:- I was just simply computing derivative values . . . she observed me . . . and advised me to put this [pointing or showing by her finger] symbol . . .*

Yenealem:- prime?

S_{15}*:- yes.*

Yenealem:- [looking towards S_{71}] if you remember, S_{15} asked S_{14} how had she computed [question number] "i". Then, you joined them and had been observing. But, after 3 minutes and after Mahlet (name changed; classmate) demonstrated *the derivative of $f(x) = x^{e-3}$, you requested S_{15} to rework "i" once again. Did not you understand the first time explanation of the same problem?*

S_{15}*:- when S_{15} asked S_{14} . . . I tried to follow up but could not get their idea. That is why I asked the same question later . . .*

Yenealem:- ok, I got the point. . . during the last minutes, many of you were circulating their exercise books . . .

S_{14}*:- the teacher was checking out class works; we gave the exercises books for Feven (name changed; classmate) to pass them to the teacher.*

It was evidenced that students were engaged in group learning by ways of commenting, asking, reworking an exercise, using fingers to show a symbol and follow a dialogue. The practice of *rework* has to do with reproduction imagination.

The Ninth Focus Group Discussion

This session was organized within an hour from the actual lesson. First, let me brief the instances of discourses observed during the instruction. The teacher wrote down basic formulae for derivates of a^x; e^x; $\ln x$; and $\log_a x$. Then, he listed down lots of functions to be computed based on derivative formulae. After a while, he demonstrated the derivative of $f(x) = \sqrt{5^x}$ as $f'(x) = \left(5^{\frac{1}{2}}\right)^x \ln 5^{\frac{1}{2}} = \frac{\left(5^{\frac{1}{2}}\right)^x \ln 5}{2}$. But, many students were not clear on how to consider 2 as denominator. Very soon, some students asked him work out the derivative of $f(x) = e^{x+3}$; he recalled and apllied the formula $y - f(a) = f'(a)(x - a)$. In the mean time, the teacher brought a question from last year Entrance Examination. Many students forwarded a wrong answer. At this time, S_{22} was moving his fingers; and he seemingly having mental calculation. Towards the end of the lesson, S_{81} and S_{82} were discussing . . . The participants were coded as S_{22}, $S_{81}$and S_{82}; all of them were males. Part of the discussion is given below.

Yenealem:- ... some students asked the teacher to work out [the derivative of $f(x) = e^{x+3}$ under question number] 1(d). Did any of you have the same question?

S_{22}*:- I was confused on the first example. In* $ln\, 5^{\frac{1}{2}}$*, I did not recognize that* $\frac{1}{2}$ *will brought in to the coefficient of* $ln\, 5$*. Then, I understood I should have applied the common logarithm concept . . .*

Yenealem:- What about [question number 1] "d"? Students raised the question before they tried it. Was it difficult . . .?

S_{81}*:- the given formula was for* e^x*. I think they assumed* e^{x+3} *as totally different.*

S_{82}*:- I already tried the problem . . . at first, I had no question; but, when he demonstrated the solution, I realized I had computed a different thing.*

Yenealem:- what did you do?

S_{82}*:- I just differentiated the exponents . . .*

Yenealem:- please show us what you committed in this paper.

S_{82}:- [He writes on paper.] *I considered* e^{x+3} *as* $e^x \cdot e^3$ *and I derivated* $x + 3$.

When the student was unable extend the derivative formula applied to e^x to a similar expression e^{x+3}, there is lack of adaptive reasoning. Students were engaged in mental calculation; that is great. But, many of them had perceptions about differentiating e^{x+3} as a "difficult" exercise to do. That is why they waited for the teacher to solve it.

The next conversations contain students' misconceptions, agreements and disagreements.

Yenealem:- . . . the teacher began working out question number 2; no student asked him to do so. He used the formula for tangent line: y *minus* f *of* a *is equal to* f *derivative at* a *in to* x *minus* a*. Next, he solved another question; what was . . . the question . . .?*

S_{81}*:- . . . find the equation of the line tangent to the graph of* $y = e^x\ at\ (1, e)$

Yenealem:- thank you! Finally, he reached at $y = ex$*. How did you [all] get this question . . . the techniques used . . .?*

S_{81}*:- It was simple; we had a similar experience.*

We knew the formula $y - f(a) = f^{'}(a)(x - a)$ of the tangent line.
The teacher wanted to show the derivative of $f(x)$.

Yenealem:- let S_{22} tell us something in this regard. . .

S_{22}:- I did not give it attention. I was thinking about the logarithm of x to the base $\frac{1}{x}$.
And, he told reminded me the formulae we had written.
Then after . . . I could not catch the essence of the question. I lately understand it by asking him [pointing to S_{81}].

Yenealem:- what was your thought? Did you find equation of the tangent line? Or, you just differentiated the function . . .

S_{22}:- I was confused . . . I just asked him and he showed me the way. That was all.

The participant S_{22} was genuine to report that his attention was diverted. Yet, he was engaged in learning process; he was "thinking out of the box" but not in sense explained earlier. Finally, he lost the question under discussion. Thus, he asked for and gained from S_{81} some explanation.

Yenealem:- ok . . . next, the teacher brought . . . $f(x) = \pi^2$. You were moving your fingers on the air. What did you do?

S_{22}:- I did not assume π as a constant. I computed $f^{'}(x)$ as 2π.

[the other two participants had the same misconception].

Yenealem:- towards the end of the lesson, S_{81} and S_{82} were discussing . . .

S_{81}:- we were working out question number 3.

$$\left[\begin{array}{c}\text{Find the equation of the tangent line to the graph of} \\ \mathrm{f(x)} = \ln \mathrm{x} \ \text{at} \ \left(\frac{1}{\mathrm{e}^3}, -3\right)\end{array}\right]$$

Yenealem:- what did you do? Who was writing . . .?

S_{81}:- we had some disagreement . . . when S_{82} evaluated the derivative of f at e, he obtained a different answer.

Yenealem:- ok, let S_{82} tell us . . .

S_{82}:- I calculated the derivative of $\frac{1}{e^3}$ *. . . then, he told me to find the derivative of* $\ln x$ *and evaluate at* $\frac{1}{e^3}$ *. . .*

Yenealem:- aha! What happened next?

S_{81}:- he forgotten the idea of $f(a)$ *. . . and it was necessary.*

Yenealem:- so, you had two different ideas. What was your conclusion then?

S_{81}:- he considered my explanation; and, we agreed on my solution.

Yenealem:- ok, that was great. And I have completed my questions. Thank you.

The interesting aspect of the discourse narrated above is that there was disagreement on ideas that would take to solution. S_{82} and S_{81} had different answers for the same problem; they shared ideas and finally S_{82} agreed with S_{81}.

The Tenth Focus Group Discussion

There were four participants S_{91}, S_{12}, S_{92} and S_{93} involved in the 10th round focus group discussion. This session was held at the "department" office where trainings are facilitated. The office is too narrow. I and two students (S_{92} and S_{93}) held chairs; and the rest two S_{91}, S_{12} sat on the table available. Out of the four participants, S_{91} is male.

As usual, I began the event by informing the participants that I would not be concerned about their private talks. I promised to keep data safe and advised them to feel free.

Yenealem:- did you easily understand Example #1?

$$\left[Let\ h(x) = sin(3x+1)\,;\ evaluate\ h'\left(\frac{\pi-2}{6}\right).\right]$$

S_{93}:- . . . I did not have any idea of letting $f(x)$ *and* $g(x)$*; but, I could have solve it by myself in other times. He [the teacher] considered* $f(x) = \sin x$ *and* $g(x) = 3x + 1$*. It was after that I catch the way to go forward.*

Yenealem:- The teacher was saying "don't be doubtful" when he calculated $h'\left(\frac{\pi-2}{6}\right)$.

Did you really hesitate to take his answers?

S_{12}:- *we understand what the teacher explains . . . the formula* $h'(x) = 3\,cos(3x+1)$. *However, I was somehow confused why he substituted* $\frac{\pi-2}{6}$ *in place of* x.

S_{93}:- *. . . because we were asked to evaluate* h *derivative at* $\frac{\pi-2}{6}$ *. . . that's why . . .*

S_{92}:- $\frac{\pi-2}{6}$ *was given. My problem was . . . I could not work out before he [the teacher] demonstrated* $h(x) = f\big(g(x)\big)$.

S_{91}:- *I did not understand the question until he gave us a hint.*

S_{93}:- *at the beginning . . . the formula did not seem applicable for the example. I was expecting* $g(x)$ *and* $f(x)$ *to be directly given. But,* $sin(3x+1)$ *was given. . .*

The above conversations are exclusively directed to understanding of composite function. The students had learnt the derivative of such a function using chain rule formula (Federal Democratic Republic of Ethiopia, 2010b) stated below.

> Let g be differentiable at x_0 and f be differentiable at $g\,(x_0)$. Then, $f \circ g$ is differentiable at x_0 and $(f \circ g)'(x_0) = f\,'(g\,(x_0)).\,g\,'(x_0)$.

The student S_{93} had *analysis* problem; she did not interpret the given function in terms of $g(x)$ and $f(x)$. The teacher encouraged them to try $h'\left(\frac{\pi-2}{6}\right)$. Yet, the participant S_{12} did not notice that $\frac{\pi-2}{6}$ substituted x_0. It was after teacher's illustration about concept that S_{12} and S_{92} had clear conception. This could be additional evidence to prove that students wait for their teacher. It might not be their fault; his teaching style could be decisive. The next conversation is a good testimony.

Yenealem:- the teacher let you work out set of questions. He immediately began explaining. Was there any student who demanded him to solve . . .?

S_{93}:- *no one, he did it by his initiative.*

S_{12}:- *the teacher already gave us a hint . . . he told us there are three functions existing in* *the composition . . . sin* x*;* $\sqrt{x}$*; and* $x^2 + 4x + 1$.

S_{92}:- *using the formula . . .*

If the teacher gives class work or exercise to students but does not allocate sufficient time, the tasks would be no more opportunities for students to construct and co-construct knowledge. Scaffolding has to have some limit. After all, the students are being youth; so, independent but collaborative works should be appreciated.

4.6 Classroom Mathematical Discourse

There is a range of reasons for drawing students' attention to discourse (Herbel-Eisenmann et al., 2012) which denotes any specific instance of communicating, whether diachronic or synchronic, whether with others or with oneself, whether predominantly verbal or with the help of any other symbolic system (Kieran et al., 2003). Learning mathematics is defined as an initiation to mathematical discourse.

4.6.1 Waiting for the Mathematics Teacher's Initiation

The underlying perspective in preparatory secondary mathematics education (Federal Democratic Republic of Ethiopia, 2010c, 2010d) is to approach it for stimulating student's inquiry. The role of the teacher is expected to be guiding students construct their own knowledge and skills; discover concepts by themselves; and develop personal qualities that will help them in real life. As the students develop personal confidence and feel comfortable on the subject, they would be motivated to address their material to groups and to express themselves and their ideas with strong conviction. We can extract some features that qualify the curriculum as imaginative. For instance, if students are guided in such a manner, they will be equipped with communication skill and could explore for the purpose of discovery.

If most students are asked what they think their role is in mathematics classrooms, they will tell it is to get questions right (Boaler, 2016). In that sense, Mathematical discourse in the classroom is conceptualized as initiation–response–evaluation (Drageset, 2015). It seems a general pattern and the teacher has an important role in collective mathematical learning (Levenson, 2011). Suggestions have been provided of ways for teachers to encourage students talk in classrooms (Morgan, 1998). For instance, concerns for engaging students; the process of mathematical thinking and reasoning; and importance of productive mathematical conversations might be insufficient when we account of teacher-directed discourse (Herbel-Eisenmann et al., 2012). This will remind us to seek for a students' active participation in the process. In Ethiopian context, the teacher is advised to encourage students to apply high-level reasoning, and values to their daily life and to their understanding of the social, economic, and cultural realities of the surrounding context. This in turn helps the students to actively and effectively participate in the wider scope of the development activities of their nation (Federal Democratic Republic of Ethiopia, 2010c).

Let me support the assumption with data gathered on the 7th of November 2017.

Yenealem:- Have you got S_{24}'s idea?

S_{22}*:- . . . she [S_{24}] said 'we have to rationalize the denominator'. The teacher once told us that 'if the denominator becomes zero and the whole term is undefined; we need to rationalize and simplify…' But, she . . . if we don't use* 3, *why do we write* 3 *there.*

Let me restate S_{22}*'s* idea: we are not guided by $x \to 3$. It seems that S_{22} reported his prior thought as S_{24} reacted to the question forwarded.

Yenealem:- let me ask S_{24} once again. Before you raised a question and before the *teacher replied to your concern, you had been trying by yourself. I saw your writing* $\frac{x-3 \quad -3x-3}{\sqrt{(x-3).(x-3)}}$ *on your exercise book. What was your thought?*

S_{24}*:- I used* $-3x-3$ *in order to factorize* x^2-6x+9.
I was finding two number whose sum is -6 *and whose product is* 9.
I found -3 *and* -3.

Participants: [keep quiet for a while.]

Yenealem:- let me take you back to your teacher's question; after the teacher and S_{24} *had some conversation. Then, the teacher requested the class: "Is there anyone who understood* S_{24}*'s idea?" What did you think by that time? For instance,* S_{23} *and* S_{24} *were discussing. Or, what did you do?*

S_{23}*: She [*S_{24}*] suggested: "if the denominator gets undefined, why don't we leave it?" I was telling her Gashe* [the teacher] *once advised us to try further in such cases.*

S_{22}*: [he was attentively following* S_{23}*'s report.]*

Yenealem:- so what did you [both] *conclude?*

S_{23}*: the time was very short; and couldn't complete our discussion. We immediately shifted to teacher's lecture.*

If fluency is flow of ideas, S_{24} was fluent but not perfect in thoughts. This could be an evidence to disprove the claim "she questioned the question". Or, it shows how much she was thinking before she reflected it to the class. After the teacher and S_{24} had some conversation, the teacher requested the class. . . He was playing the role of initiation because no one was involved in the discussion except the teacher and the girl. The teacher explained; but, it was she that continued arguing. The whole class heard saying "undefined" as an answer. That was too brief reply. The teacher did not give students enough time to reflect; and they tended to listen to his lecture.

4.6.2 In Response to an Individual Student's Class Participation

By communicating with each other, students can develop a cascade of inscriptions which emerges through successive iterations of discussion and refinement (Cobb et al., 2000).

One day, a student Desta (name changed) was solving a class work on the blackboard. He followed a participatory method; the "class" gave him attention and involved in answering queries. On the same day, Marta (name changed) was invited to the stage to work out a problem; but, was reluctant to solve the question further. She was baffled; the "class" was seemingly unhappy about technical errors; two of the participants told her something.

Yenealem:- how did you get Marta's presentation? Two of you told her something. Do you remember it?

S_{11}*:- the teacher should have not invited her to solve the problem. I wanted him to do it. It was a difficult question. At least, he is a teacher; he's better than us.*

S_{12}*:- she was confused and committed mistakes in factorizing the terms.*

Yenealem:- so, what did you tell her.

S_{12}*:- we told her to assume* $\sqrt{5-x}-\sqrt{5}$ *as* $a-b$ *of the difference of squares* a^2-b^2.

Yenealem:- Did she applied it?

S_{12}*:- she was running out of time.*

Yenealem:- by the way, how did you manage the time given to evaluate a question?

S_{11}*:- Oh, the teacher did not give us enough time to solve a problem.*

It can be inferred that students demanded the teacher to solve "difficult" questions. Yet, they had their own important mathematical idea.

4.6.3 Interaction with Lecture Notes

By 7th November, 2017, as the class was over, I took out recording instruments to the corridor and began packing. A teacher was going to the next room; he stopped his walk for a while and stared at my activity. Then, he greeted me and joked at the "movie" [video] being recorded. In turn, I reacted: "which subject he was delivering?" He showed me a physics textbook; he continued: "we are ordered to bring textbooks to classrooms so that students who are deprived of it would know it". But, the home room teacher denied the scarcity of textbooks.

> "The [Dire Dawa administration] education bureau sells books for the school and the school, in turn, passes it to students demanding it. Any student can buy textbooks from the school. As I am a homeroom teacher, there are lots of books in different subjects beneath my table."
>
> Ato Demissu, Mathematics teacher whom I had been working with

I remember I saw collection of different books beneath his table. Yet, I had never seen students holding their mathematics textbook. He used to stick on the textbook during most of the lessons. Much time had been devoted to copy notes on the blackboard and paste on exercise books. Sometimes, students might not be aware of the text itself. The following conversation testifies the fact.

Yenealem:- thank you for coming for this FGD session. We shall talk on today's [mathematics] *class. The topic of the lesson was "Derivatives of Power, Simple Trigonometric, Exponential and Logarithmic Functions." And we learnt about power functions; right?*

[The participants confirmed by saying "yes" collectively.]

First, the teacher wrote definition of derivative of power function.

There was a notation $\boldsymbol{w\ r\ t}$ *included in the definition . . .*

[**w r t** corresponds to "with respect to"]

S_{15}*:- there was also* x*; there is* $w\ r\ t$ *. . .*

Yenealem:- Have you recognized the meaning [it conveys]? What does it mean?

[Short pause]

S_{14}*:- he* [the teacher] *wrote it on the other day too. Yet, I have never questioned what it means.*

S_{71}*:- I don't know.*

S_{15}:- *I liked to ask . . . but, I let it. . .*

Yenealem:- I raised just know how much you sense to the silly things. Anyway, it means "with respect to". [They shake their heads; it seems they say "Aha!"]

However, there were many occasions that students questioned confusing phrases or ideas noted on the board. On another occasion, students were questioning such an abbreviation.

Yenealem:- *you had question about the notation* "$\text{diff}^{\underline{le}}$" . . .

S_{15}:- *he* [the teacher] *didn't hear me . . .*

Yenealem:- yes, and many students were repeating your question. I think you are familiar with some short forms of words; like $def^{\underline{n}}$ *which denotes "definition". Couldn't you extend such abbreviations . . .?*

S_{15}:- *it was new for me and I could not read it.*

Yenealem:- [joking . . .] *did it seem Arabic writing?* [They smiled.]

The teacher had double roles: writing notes on the board and explaining the contents. Similarly, students did write and listen to lectures. I had been collecting their exercise books for analyzing the contents. I copied more than 12 days notes of assumed participants of FGDs. I have observed comparatively the same notes taken by a group of students considered for a FGD. Indeed, there were disparities on reports class works which actually reflect individual attempts. Some of them completed class works while others did little. That means, students depend on teacher's formally delivered notes; they did not included ideas forwarded during explanation sessions or side talks. Yet, since I was attending the mathematics lessons, I saw many students writing at the back pages of their exercise books when they were requested to do class works. Just to show what, copies of four students' one day notes are attached at the end of this thesis. I felt that I should have copied samples of all notes in the exercise books in order to have better perceptions of their reaction with the material or lecture notes.

4.6.4 New Interactions during Focus Group Discussions

The next conversation was part of a focus group discussion. The discussion was sought reflections on a previous session.

The teacher demonstrated $lim_{x\to 0} \frac{|x|}{x}$ using the idea $|x| = \begin{cases} x, x > 0 \\ 0, x = 0 \\ -x, x < 0 \end{cases}$.

Yenealem:- Shall we talk about $lim_{x\to 0} \frac{|x|}{x}$ *. . .? The teacher sketched the graph of* $|x|$ *on the blackboard; and he presented* $|x|$ *as a piece wise function. He considered the right side and left side limits. He also told students the limit sequence* $lim_{n\to\infty} k$ *is* k *itself. You were all attending seriously. Is there anyone who would tell us how the limit value gets "does not exist"?*

S_{13}:- *my assumption was different. I took* $|x|$ *as* x *for* x *is greater or equal to* 0 *and* $-x$ *for* x *is less than* 0*. But, he wrote . . . [waving her hands as if she is manipulating the formula]*

S_{14}:- *I just remember the sigma function and the answer was right.*

Yenealem:- what's is that?

S_{14}:- [She begin to draw on her hand; I gave her a paper; her graph is like the one below.]

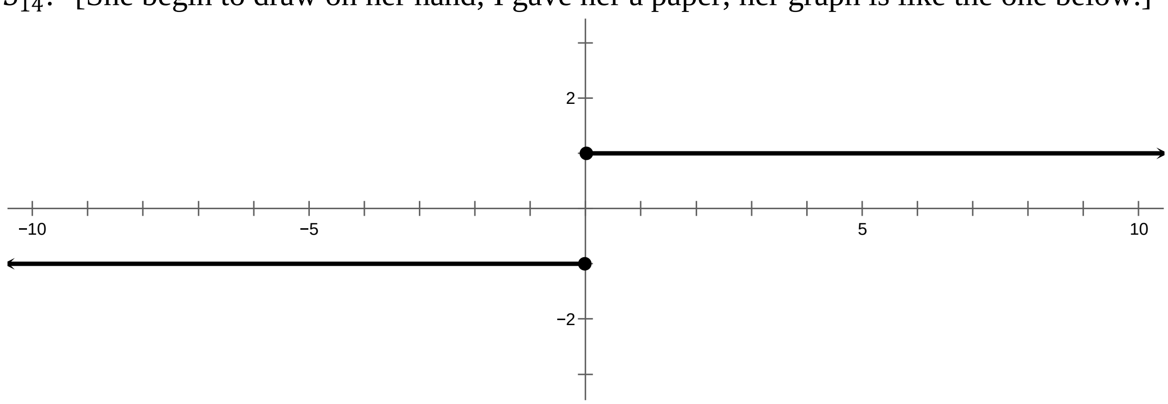

Figure 4.0.8: A Sketch of Sigma Function Drawn During Focus Group Discussion

S_{11}:- *it was clear for teacher.*

She drawn the sketch first on her hand and then on a piece of paper. Other participants of the FGD were attending her drawing. This did not happen during the regular class time. It seems that she had such a thought in her mind during the classroom instruction. She applied the concept of a piece wise function. To evaluate the limit sequence $lim_{n\to\infty} k$ is k it self.

In seeking ways to engage students in learning mathematics, the key decision that the teacher makes is the choice of task (Sullivan, 2011). Sullivan further explained that the most common tasks in textbooks are those that offer students opportunities to practice skills or procedures. From the above event, we can see that the teacher demonstrated the idea that ignited students' thinking. However, the student's thinking and reasoning were reflected later on and shared to others.

4.6.5 Mathematical Errors, Unrelated Justifications and Misconceptions

In September, 2016, I met a colleague at Dire Dawa University's annual meeting and we shared many ideas. He had a critic on primary school environmental science curriculum. His comment is summarized as follows. Students learn that *Dechatu* is a river in Dire Dawa. But Dire Dawa is a desert area where water flows over the river only if there is heavy rain in the nearby or surrounding highlands. Then, his doubt was: the kids may assume a river as a dry gorge which gets wet once or twice a year. Education has to be linked to students' daily lives. Educational practice and research are socially and culturally situated (Sekiguchi, 1998). Actually, the way it will be communicated matters. So, beyond telling "Dechatu is a river", students have to be informed how a river is formed. Then, they would develop a sense of water resources.

Mathematical errors could be opportunity for inquiry based learning or causes of misconception. Students with a growth mindset take on hard work, and they view mistakes as a challenge and motivation to do more (Boaler, 2016). My focus is on the process; hence, I consider some cases observed in mathematics classes elaborated in a focus group discussion. One day, the teacher tried to demonstrate:

$$\lim_{h\to 0}\frac{f(x+h)-f(h)}{h}=\lim_{x_0\to 0}\frac{f(x)-f(x_0)}{x-x_0}$$

From the very beginning, $\lim_{h\to 0}\frac{f(x+h)-f(h)}{h}$ should have been written as $\lim_{h\to 0}\frac{f(x_0+h)-f(h)}{h}$. As he continued wring and discussing $\lim_{h\to 0}\frac{f(x+x-x_0)-f(x_0)}{h}$, students began reflecting on it.

Yenealem:- the morning class was very good; specially at the beginning, different ideas had been raised. I would be happy if two more students were here with us to share their thoughts. Anyway… the teacher wrote limit of f of x minus f at x note [the whole] *divided by x minus x note . . . and limit of f of x plus h minus f at h . . .*

S_{51}*: f of $x+h$ minus f of x_0 over x minus x_0 . . . the limit as x approaches to x_0. . .*

Yenealem:- ok, . . . then, in addition to these expressions, he [the teacher] recalled the slope formula. Then, he was demonstrating the equivalence of the first two formulas. Then, different ideas were forwarded. It is S_{51} that expressed her thought. What was that?

S_{51}*: he* [the teacher] *already told us: as $x\to x_0$, then h equals* 0.
That means, as $h\to 0$, x is equal to x_0. Since x and x_0 are equal, we can interchange them.

S_{52}*: in order to cancel . . .*

Yenealem:- ok, eh. . .

S_{51}*:- . . .we can substitute h by $x-x_0$.*

Then [she was writing on the paper I gave her], $lim_{h\to 0}\frac{f(x+x-x_0)-f(x_0)}{x-x_0}$.

We get . . . $lim_{h\to 0}\frac{f(2x-x_0)-f(x_0)}{x-x_0}$.

Since $x = x_0$, *we have* $lim_{h\to 0}\frac{f(2x-x)-f(x_0)}{x-x_0} = lim_{h\to 0}\frac{f(x)-f(x_0)}{x-x_0}$.

Yenealem:- . . . then, you were referring back at your notes. What were you finding?

S_{51}*:- firstly, he [the teacher] did not tell us that they [*x *and* x_0*] are equal. I wrote* f *of* $2x$ *minus* f *of* x *note . . . and I was confused. That is why I was looking at my exercise book in search of any relevant formula.*

Yenealem: but, Mahlet (name changed; a classmate) was speaking that x *is greater than* x_0 *. . .*

S_{51}*:- yes . . . because in* $f(x) - f(x_0)$, $f(x)$ *is written at the left.*

Hence, $f(x)$ *is greater than* $f(x_0)$.

Yenealem: so, did you [all] *share Mahlet's idea?*

S_{51} *and* S_{51}*: yes we did.*

Yenealem: who did suggest multiplying by negative . . .

S_{51}*:- it was me . . . since* $h = x - x_0$, *I said that we can substitute* h *by* $x_0 - x$ *and then multiply* $f(x) - f(x_0)$ *by negative . . . it yields the same result.* She wrote down the following on a paper I gave her:

$$\lim_{h\to 0}\frac{f(x+h)-f(x_0)}{h} = \lim_{x\to x_0}\frac{f(x+x_0-x)-f(x)}{x_0-x} = \lim_{x\to x_0}\frac{f(x_0)-f(x)}{x_0-x}$$

She multiplied both the numerator and denominator by -1.

Yenealem: . . . let S_{52} *say something. . .*

S_{52}*:- before the teacher told us. . . and . . . I was sure about the equality of* x *and* x_0*, I had the same understanding with* S_{51}.

S_{51}*:- it works too for multiplying by negative . . .*

Yenealem:- . . . didn't you have a different way? Did you accept that $x = x_0$*?*

S_{23}*:- yes . . .*

S_{51}:- Since he [the teacher] said . . . $h = x - x_0$, when h approaches to 0, [we can consider] x_0 is the same as x and x equals to x_0.

Yenealem:- let me check what you have written on your exercise book.
[I could not get a note on this regard.]

S_{23}:- he just tried to proof it; and we did not copy it.

Many good teachers have told students that mistakes are useful and they show that they are learning (Boaler, 2016). Mistakes or unrelated justification in the classroom could be sources of meaningful learning if guided properly. In this regard, I witnessed that the mathematics teacher admitted mistakes and gave corrections up on modification.

4.6.6 Actions/Activities of the Others in a Group

By 30th October 2017, the teacher plotted the corresponding coordinates for a hole and point as portrayed below.

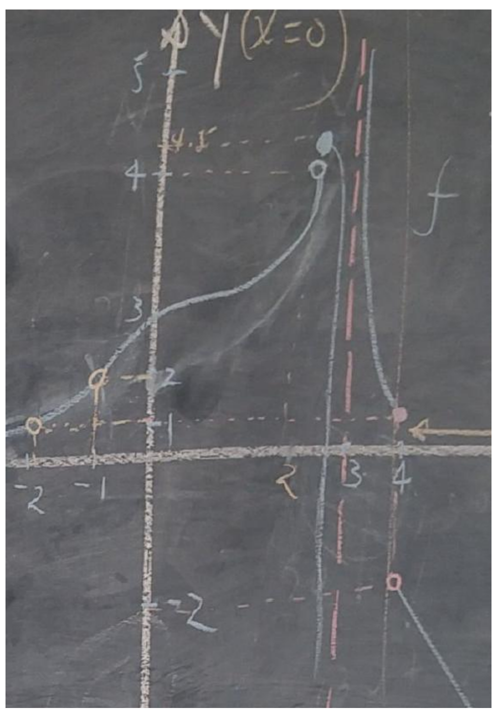

Figure 4.0.9: A Sketch Used for Demonstrating Domain of a Function

Discussions among students commenced; yet, my focused students were seemingly confused in recognizing $f(2)$. "What number does $f(x)$ approach to as x approach: (a) $-\infty$? (b) -2? (c) -1 from the right?" The question is not presented from the simple one. It might create doubt; students were seen waiting their teacher's explanation. In the mean time, the teacher informed students that "undefined" and "does not exist" are not the same. Some students shock their head; shows doubt.

I already defined students' communication as a collectively performed patterned activity in which an individual's action is followed by another individual's re-action (Sfard, 2008). When students express different opinions for or against a given task, mathematical object, concept, etc, there would be reasoning. Then, a mathematical idea would enable to thinking (Zoest et al., 2017). Group Creativity is a function of individual creative behavior contribution, the interaction of the individuals involved, group characteristics, group processes, and characteristics of group task (Zhou & Luo, 2012).

Coaction is a mathematical action that can only be meaningfully interpreted in light of, and with careful reference to, the interdependent actions of the others in the group (Martin et al., 2006). Improvisational performance may be tied into creativity in the classroom in the sense that participants are not following a regulation (Levenson, 2011). An improvisational group, where co-acting is constantly occurring, can be seen as a kind of complex system (Martin et al., 2006). Students rarely think that they are in math classrooms to appreciate the beauty of mathematics, to ask deep questions, to explore the rich set of connections that make up the subject, or even to learn about the applicability of the subject (Boaler, 2016). Co-acting is a process through which mathematical ideas and actions, initially stemming from an individual learner, become taken up, built on, developed, reworked, and elaborated by others, and thus emerge as shared understandings for and across the group, rather than remaining located within any one individual (Martin et al., 2006). Interaction might be carried out with an implication of two or more different sets of ideas being compatible with one another.

4.6.7 Mathematical Intuition

The following conversation is in line with this fact.

Yenealem:- . . . you had an activity based on a graph demonstrated on the black board. I want to focus on portion of the graph around the "hole" and "point". Do you remember?

S_{11} *and* S_{13}*:- shake their heads. I considered it as "yes" answer.*

Yenealem:- stand up and pointed to the blackboard where the graph were plotted.

S_{11}, S_{12}, S_{13}, S_{14} *and* S_{15}*:- [They were looking at blackboard and seemingly visualizie its location.]*

Yenealem:- [I used my figure to demonstrate the idea below and asked the following.] Would you tell me how you have understood "hole" and "point" on the graph?

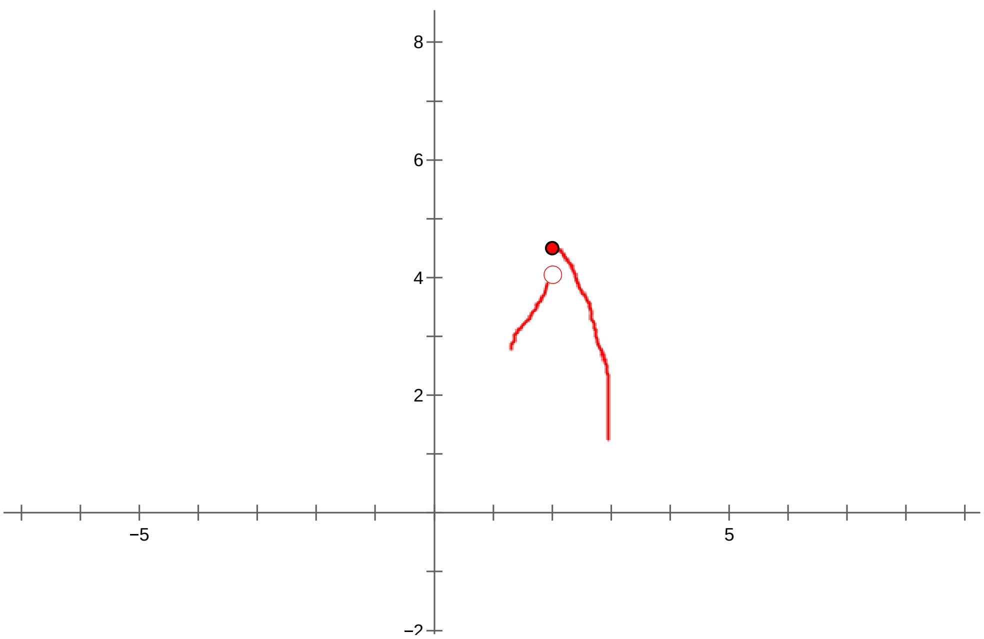

Figure 4.0.10: A Student's Conception of a "Hole" and a "Point" on a Graph

S_{14}*:- I was confused a little bit and thought the teacher would explain it.*

S_{13}:- *teacher* [they call me “Teacher” or “Gashe”], *I was in hurry copying all the note, the graph and examples from the blackboard. So, I had not thought over the issue.*

S_{11}:- *I was assuming that the hole and the point refer to 4 [on the $y-$ axis]..*

4.7 The Situational, Social Worlds/Arenas & Positional Maps

By reading through all the data, a preliminary situational map was developed and portrayed in the previous chapter. The map was a good reference for the presentation and discussion of data in different themes. In turn, more elements came across the discussion of data.

The cartographic mappings were done in different phases. First, I took hold of the broadly conceived situation of inquiry, and created a disorganized (messy) situational map to identify all the analytically pertinent elements within the situation of inquiry. Then, I displayed each element onto an organized situational map by adapting the categories proposed by Clarke (2005) and Clarke *et al* (2018).

4.7.1 Abstract Situational Maps

The analyses of broadly situated classroom and group [of students] units resulted in the inclusion of additional elements to be interpreted. As the name implies, the messy version and abstract level situational map contains many elements of the situation. Some concepts, such as “intuition” and “extending an idea”, are close to the theme of the research while issues such as “university” and “school life” are seemingly beyond the scope of the research. However, studying collective discourse involves the *whole* of the conversation as it evolves while still appreciating the *threads* of contributions that make it up (Armstrong, 2017). Hence, potentially analytically pertinent elements are included.

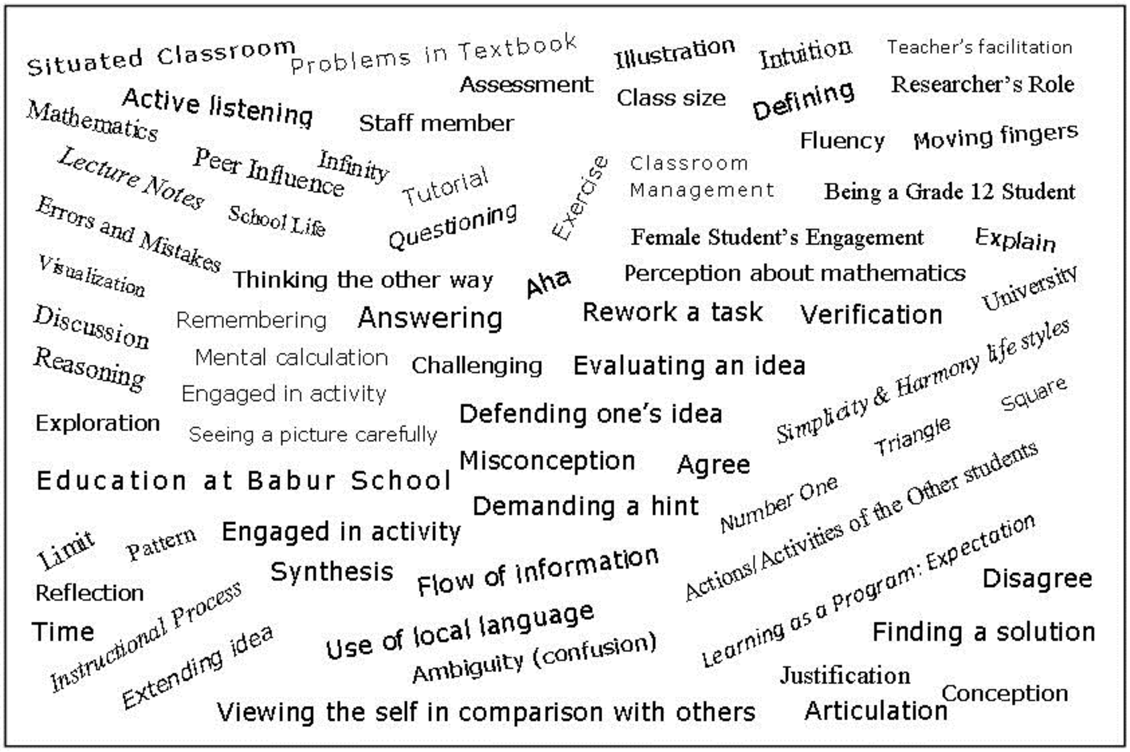

Figure 4.0.11: Abstract Situational Map – Messy Version

This "abstract" map is a synopsis of sample data detailed in sections 4.2 through 4.6. For instance, under the discussion of socio-cultural mathematics in Dire Dawa, the urban village: *Number One*, *Amistegna* (5^{th}) and *Sebategna* (7^{th}) brought a recursion formula which in turn is a pattern. The naming of *Triangle* Hotel could be a learning opportunity for counting, limit at infinity or a geometrical concern. On the other hand, the spiritual & cultural values vested up on *Circle* are excellent examples for bringing a shift in the curriculum and instructional process. That would contribute for promoting the process model of curriculum development. On the other hand, the life styles *Simplification* and *Harmony* are themselves mathematical concepts; and, hence, included in the abstract situational map.

Yet, the ultimate goal of having such an inclusive approach is to disclose the status and relationship of imagination and creativity. So, how does the messy and abstract level map be helpful in making a meaning out of the data? It would serve as spring board to go forward and a reminder to return to them analytically. The next step is organizing the messy version in a way it would give sense. The elements are grouped in to different categories; the illustration is given below.

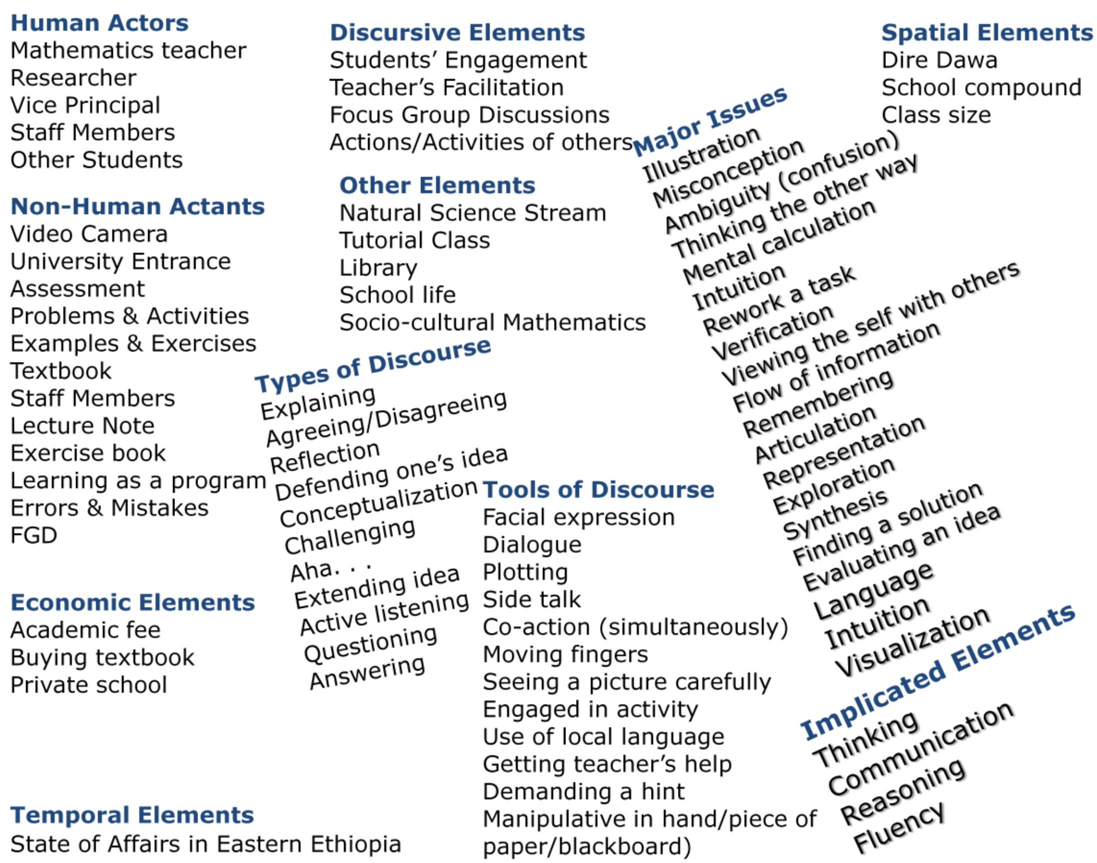

Figure 4.0.12: Abstract Situational Map - Ordered Version

However, the ordered version is not self explanatory; further justification is needed why some elements are included in the map. For instance, what roles did "socio-cultural mathematics" and "state of affairs in Eastern Ethiopia" play for the analysis? The mathematizing Christianity, the spiritual values of a circle and simple and/or harmonized life styles are concerns for socio-cultural mathematics. On the other hand, the school had involved in supervising the research and instructional processes. This could be attributed to the then state of affairs in the area. Yet, the impact what the elements rendered would be scrutinized in the coming sections. The relationship among the main actors/actants in the situation is described below. The representation would be built up in social worlds map and refined in positional map.

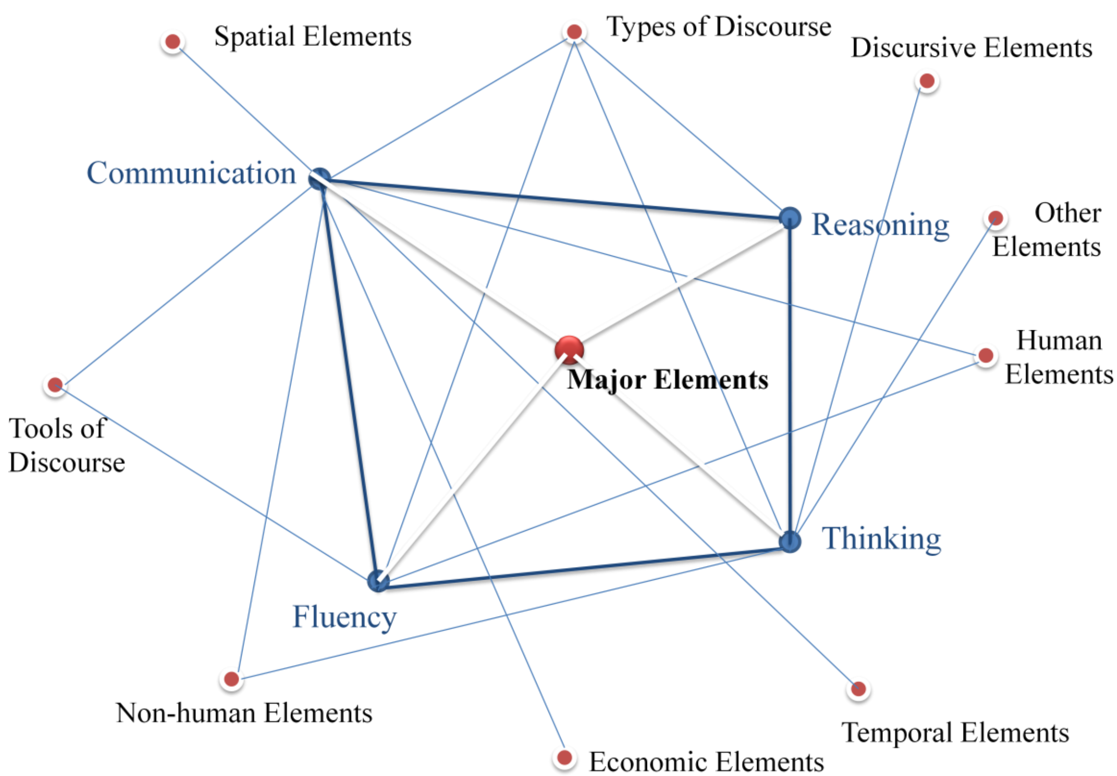

Figure 4.0.13: Abstract Relational Map

The "relational" map is delivered at "abstract" level; the line lines relating two or more constructs are not directional. This is not a matter of cause and effect or factor analysis exploration. The overall process is a way of refining towards the key variables.

The major issues such as illustration, misconception, ambiguity, thinking the other way, mental calculation, rework a task, verification and the like had relationships with each of the implicated elements. The implicated variables include:- thinking, reasoning, intuition, communication, language, fluency, visualization and focus group discussions. The remaining other ordered elements had also influenced the implicated elements, positively or negatively, in one way or another. Such influences were reported particularly in sections “Is Education really Exhaustive at Babur School?” and “The Instructional Process in a Situated Classroom at Babur School”.

Just to add few words on the abstract relational map, the following case is reported. By Wednesday, 6th Dec. 2017, while the vice principal was rounding classes on the corridor, he observed that the [Mathematics] teacher had been inviting students to participate and they in kept quiet. He entered to class and commented their passive presence; and then, the teacher advised them to smile. Then, the classroom condition gets improved.

4.7.2 Mapping the Social Worlds/Arenas

Classroom mathematical discourses were observed during the following instances: when students wait for their teacher‘s initiation, respond to an individual student‘s class participation, and copy Lecture Notes from the blackboard. Besides, more discourses had also emerged as results of some mathematical errors, unrelated justifications given by the teacher or a classmate student. Above all, actions/activities of the others in a group had been triggering additional thinking and communications. Since, the Focus Group Discussions were unstructured new interactions had been orchestrated.

In earlier subsections, I have discussed the situations of Dire Dawa, private school, the goal of preparatory secondary education, the learning scheme, the mathematics as a source of knowledge, peer influences, and other issues relevant to students’ discourse.

The comprehensive view of the aforementioned themes with respect to students classroom discourse is presented in the next diagram.

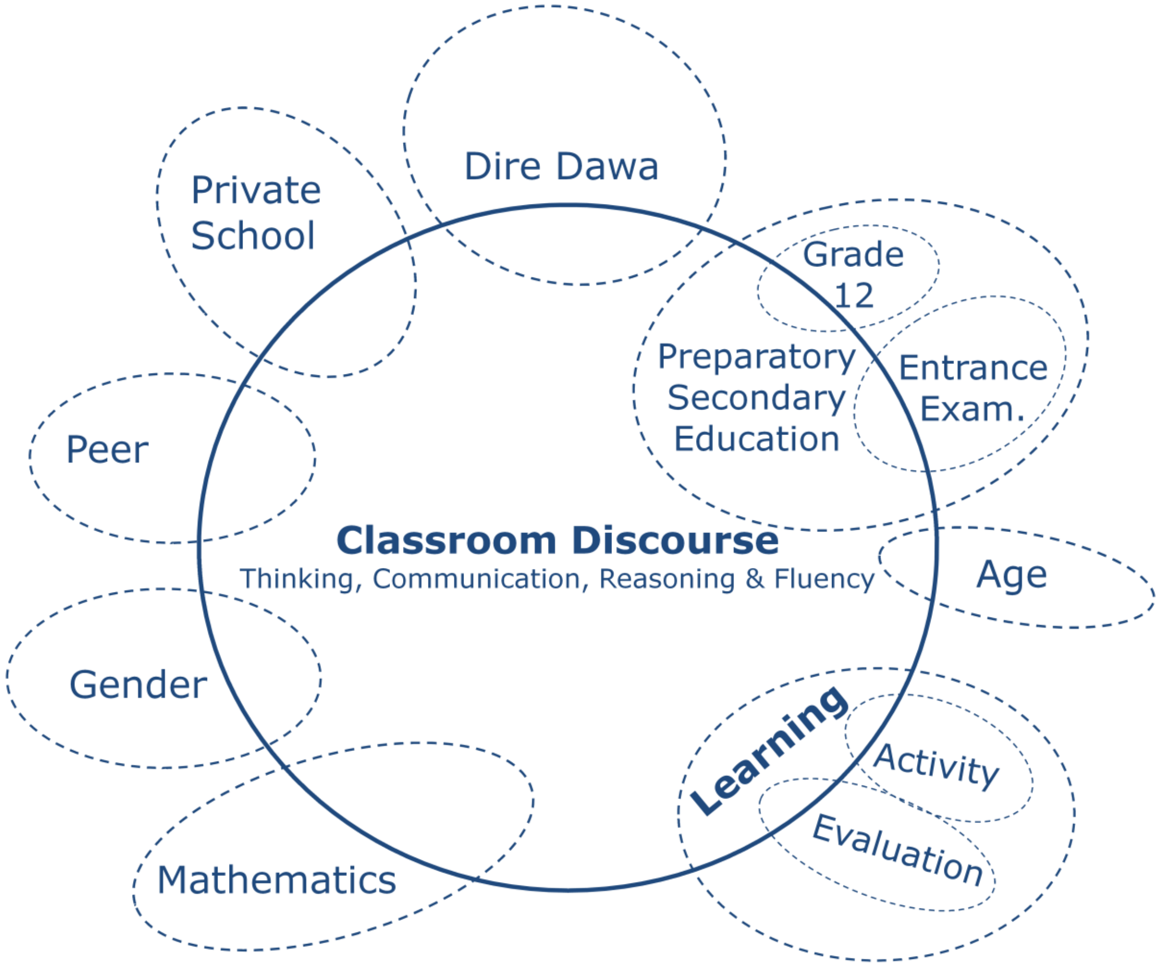

Figure 4.0.14: Social World Maps in Students' Classroom Discourse Arena

The above map was developed from the previous section; the core theme is classroom discourse. But, what makes a mathematics classroom different from any other classroom is the nature of norms (Güven & Dede, 2017). Investigating how mathematics is learned and taught from a sociological perspective demands analyzing the classroom culture. Besides, mathematics itself is a different world. Yet, it is impossible to clearly identify the origin of the resources students use in the classroom for a contribution that sounds mathematical (Moschkovich, 2007). I have shown that there are portions of mathematics missed in the classroom at Babur school. The students' classroom discourse would not be exactly in line with the underlying principles of the preparatory education program.

On the 30th December 2017, I delivered training on "Pedagogy of Mathematics" under *Mathematics Forum* at the school I started collecting data. The explicit purpose of the training was ethical clearance from the school. Besides, the training was organized to explore the pedagogical practices in Dire Dawa and call for Mathematics Forum. There were eight attendants from different schools; and for the purpose of confidentiality, I gave them the following pseudo names: Abiyu Abyot Ahmed, Gemechis, Haftu, Mulatu Temesgen, and Sisay. During presentation, I tried to brainstorm the mathematics teachers with desirable mathematical actions for students (Sullivan, 2011) and principles of engaging students in learning mathematics. In the mean time, I raised some questions seeking their reflections; selected conversations are reported below.

Yenealem:- What challenges of teaching have you faced so far?

Abiyu:- we could not cover all the contents in the textbooks. We want to arrange additional classes in the opposite shift; but, the school has no extra or free rooms.

Temesgen:- I am always lagging behind other [mathematics] *teachers* [teaching at the same grade level]. *I have ever asked them how they accomplished . . . my problem is speed.*

Abyot: mathematics and economics textbooks [for grade 11 students] *are very loaded with lots of contents, examples and exercises . . . it needs more time. . .*

Yenealem:- I have observed either start up activities or opening problems especially in grade 11 and 12 textbooks. How do you handle such tasks?

Gemechis:- yes, we are trained in SMASEE [strengthening mathematics and science education in Ethiopia] *that it* [start up activity] *is active learning. We are oriented not to give definitions to students; but, to assist them reach at such a conclusion.*

Abyot: it takes time. . .

Ahmed:- opening problem directs the future of a unit topic.

Mulatu:- I lately recognized that "word problems" are vital for developing reasoning and logical thinking. Yet, we do not enable our students work out such problems by themselves.

Temesgen:- . . . the main problem is that students' background is not inviting us to do so. Students are not trained that way in lower grade levels. If we try to give them such a task, I don't know . . . the generation prefers "short cut methods". How could we . . .?

Yenealem:- Don't you engage students in doing homework or taking notes from their textbooks? Do students bring textbooks in to class?

Abiyu:- some students bring textbooks. Of course, if we order them to bring, they could . . . we order them [students] *to discuss in groups. However, it would be complicated.*

Temesgen:- I don't give homework. One day, I met one of my students serving at a grocery after class. So, how do I expect her to do home works?

Yenealem:- but, she is not the only student. . . can't you give homework for the rest of students and treat her with alternative ways?

Temesgen:- hmm . . .

Abyot:- the students demand us to do all the tasks.

These could give us only general pictures of the pedagogical practices of mathematics in Dire Dawa. Fragmented descriptions about the scenario were given directly or indirectly in the preceding chapter.

4.8 Thinking, Reasoning, Communication and Fluency

There were short times given for students to reflection and share ideas at group level. Besides, the students were expecting their teacher explain, give hint or demonstrate as many examples as possible. As a result, fragmented ideas had been forwarded during group discussions. The meaning making and conceptual understanding have been observed in individual student level. In the preceding chapter, particurly under section 5.4, I posed the question “how do students engage in mathematical discourse?” The supporting thems were: waiting for the teacher’s initiation; in response to an individual student’s class participation; interaction with lecture notes; and new interactions during focus group discussions. The mathematical errors, unrelated justifications and misconceptions were sources of discoursing. Accordingly, actions or activities of the others in the group brought lots of co-actions. That is why concepts like co-imagination and co-creativity came in the front line of dicussion.

Therefore, it is possible to identify terms that are related to continuity of ideas, meaning making and conceptual understanding, and use of language. The misconception and ambiguity (confusion), use of local language (Amharic), getting teacher’s help or demanding a hint, time management, viewing the self in comparison with others and flow of information could be labeled in such categories.

Thinking is assumed as observable student‘s action that provides evidence to make reasonable mathematical idea. That means, we are concerned with the useful aspect of thinking; we are looking for a mathematical thinking. When a student moves his/her finger, he/she is doing mental calculation. More actions could be remembering, exploration, engaged in intuition, seeing a picture more carefully, engaged in activity, and having a (positive or negative) perception about mathematics, thinking the other way could be evidences for worth valuing thinking. The strategies of active listening, questioning and answering might reflect that a student’s thinking or would belong to communication. Unlike the previous indicators, groups of students were observed exhibiting the later three ones. While one asks a question, he/she was sometimes challenging the teacher or other classmates. In some ways, participants were evaluating a justification or extending an idea. Individuals experience "Aha. . ." which would in turn help them correct their prior understandings.

Students were also seen representing figures on pieces of papers or at their hands. Co-action was the feature of many participants. Individual had been communicating each other with facial expressions or side talks. In any cases, there were short dialogues. The characteristics like defining, explaining, finding a solution, illustrating a concept using figures, arguing (agreeing/disagreeing), reflecting on an idea, articulation/synthesis, defending one's idea, rework an example or exercise, and verification of solutions could be labeled the reasoning.

4.9 Imagination versus Creativity

In this study, different views on relationship imagination and creativity were considered. The two extremes are: imagination provides pathways to creativity (Carson, 2010) and creativity occurs without imagination taking place (Beaney, 2010; Runco, 2014). The role for imagination in creativity is not exclusive to the rich creativity of artists and scientists, but indeed seems to characterize the minimally creative behavior that we all enjoy (D. Stokes, 2016). Imagination and creativity are related and interdependent (Beaney, 2010; Glăveanu et al., 2017; Samli, 2011). Such arguments instigated me to disclose the common features (if any) of the two concepts. The following sections address the issue.

4.9.1 Indicators of Imagination and Co-Imagination

This research took a position of viewing discourse as student's use of language, visual images, meaning making, interaction with material, conversation, participation in an activity, or classroom discussion. Thus, imagination would involve DT, mental picture and spatial representations. Then, who is going to engage students in imagination? And how we know about the engagement? One simple way is when they are interested and involved in considering and thinking different possibilities.

When students ask questions or move their hands, they are manipulating something in their mind. It might be mental calculation or spatial representation. Thinking the other way or extending an idea is like thinking out of the box. Thus, co-action goes to co-imagination. The new concept *co-imagination* would mean having a DT, mental picture or special representation while others are engaged in such activities. Questioning and answering seem turn by turn activities. If thinking and imagination are unplanned instances, and of course processes, then there could exist concurrent imaginations that happen simultaneously. A student might rethink his own or other's idea. That means, when a student imagines the other also imagines.

The following conversations were transcribed from a focus group discussion. The teacher wrote and explained the definition limit at infinity; but, much time was devoted for and lots of ideas forwarded to the conceptualization of $\lim_{x\to\infty} f(x) = \pm\infty$. A student whose code is S_{21} was challenging the teacher and the whole class.

Yenealem:- [By pointing my hand to S_{21}] *do you have anything to share us?*

S_{21}:- *I had been rethinking the mistakes I committed earlier; so, I didn't heard any of the discussions.*

Yenealem:- I remember you worked out one exercise on the blackboard.

S_{21}*:- yes! I was asking myself my over all procedures for that question.*

S_{22}*: by the time she* [pointing towards S_{24}], *raised her question, I was just waiting for teacher's explanation.*

Indeed, while the class discussion continued, the student S_{21} was revising the procedures he followed and rethinking his mistakes committed. He did it as his class mates and the teacher were exchanging different mathematical ideas. The following conversations also convey the same message.

Yenealem:- so far so good. [Emm…] *the teacher asked you all: "when do we say 'limit does not exist'?" and some students from the back bench replied to the question. Did they meet your expectation?*

S_{24}*: he [the teacher] had already written that if the limit of* a_n *as* x *approaches to infinity is* k*, then it exists. So, it was easy to understand the idea [concept].*

Yenealem:- how did you [all] *get the expression:* $lim_{x\to\infty} f(x) = \pm\infty$*?*
I think S_{21} *could tell us something. . .* [He had been in dialogue with the teacher.]

S_{21}*:- my question was: "if both limits were positive* [infinity]*, can't we say the limit exist?" if there are positive infinity and positive infinity answers . . . that means we get the same value. So, we should say "the limit exists". There was a similar idea… it will contradict with that concept. It came in to my mind that if the limit* [as x approaches to, say a] *from the right and left sides are the same, we said the limit exists.*

Yenealem:- Good! What did you [all] think while S_{21} *was asking the question?*

S_{22}*:- I had the same idea. I thought if the limit values are* $+\infty$ *and* $+\infty$ [as x closes to $+\infty$ or $-\infty$]*, then, the limit exists.*

S_{24}*:- we can talk about the limit. . . but, it does not mean that the limit exist.*

While S_{21} was asking the question, S_{22} had the same (the limit exists) and S_{24} took a different perspective (it does not mean that the limit exist).

4.9.2 Characteristics of Creativity and Co-Creativity

Most creativity researchers refer to two key determinants (Kaufman, 2016): creativity must represent something different, new, or innovative; and it should be something unplanned and surprising. Yet, it is not enough to just be different; creativity must also be appropriate to the task at hand. A creative response is useful and relevant; it should also be of high quality. The ability to be creative is associated with flexibility, problem sensitivity, generation of ideas, ability to restructure thoughts, create new perspectives, and take prudent risks (Brater, Buchele, Reuter-Herzer & Selka, 1989; Van Gundy, 2005). Promoting creativity involves the development of characteristics such a self-motivation, confidence, curiosity and flexibility, all of which need to be supported by a flexible learning context (Das, Dewhurst & Gray, 2011).

So, which elements of thinking and reasoning are indicators of creativity? If creativity is unplanned, my informants were seen reacting to mathematical errors and misconceptions. Their intuitions were also discussed; and these could be evidences. But, the actions of others in the group or the class were determining aspects to develop co-actions and hence co-creativity. If creativity represents something new or different, many informants were heard of saying “I had a different solution” or “my case was different”. But, I claimed *creativity* is the process that students do activities in new way particular to a situation with utility and practical worth. Indeed, this operational definition encompasses the two basic determinants discussed above. I viewed the concept as the action in *new way* and *particular to* a situation which could not always be planned. The utility or the practical worth of such an action could be valued in relation to the academic discourse under consideration.

Ok, which elements of thinking and reasoning are indicators of creativity? The kinds of learning and understanding we may see occurring when a group of learners work together on a piece of mathematics (Martin et al., 2006). The growing interest in group creativity reflects an underlying assumption that the exchange of ideas that occurs in a group setting is more likely to result in a wider range of ideas (Mannix, Neale, & Goncalo, 2009). If different ideas emerge from a group, they could be opportunity for each member to develop different perspectives and co-create knowledge.

Ensuring a meaningful learning setting would be evidenced by the characteristics of creativity. The phenomenon of creativity in classroom discourse can be labelled in to different levels. I have presented and described the ten group discussion. There were sessions that I boldly reported that they reflected discourse rich environment. For instance, there was good discussion observed on among focus group number 3. On the other hand, participants of the first focus group discussion did not respond to teacher's question "what's the domain of the function?" except there was a "buzz" sound; that means, whole class talking. No student raised a question or even a reaction for teacher's note and very brief explanation on "let $y = f(x)$ be a function defined on interval surrounding $x_0 \in \mathbb{R}$ (but f need not be defined at $x = x_0$)".

There are different levels suggested to label creativity. For instance, "low", "intermediate" and "high" categories (Aizikovitsh-Udi, 2014). However, it was already recommended in Treffinger et al (2002) that assessment of creativity could be judged as: "not yet evident", "emerging", "expressing" or "excelling". The first level is when the [students'] group work does not reveal characteristics or behaviours of creativity. As the name implies, "emerging" corresponds to limited evidence of creativity. On the other hand, when group work indicates signs of creativity, the report would be “expressing”. The higher level, “excelling” is for data of consistently presence of creativity.

My intention to study groups of students for their common creative attempts in the absence of such an environmental support (Sriraman et al., 2013) was so hard. In this research, groups and group learning have been considered as units of analyses. Yes, the scenarios in each of the nine groups or ten focus group discussions were presented and discussed towards the end of the preceding chapter. Yet, the basic limitations of labelling are attributed to many factors: there was no organized group learning; some hot conversations were created during focus group discussions rather than actual lessons; there were unsystematic flow of ideas in the classroom; participants’ sitting arrangements were unlike.

4.9.3 Components of Imagination and Creativity on Same Board

The *major issues* in the situation of inquiry were identified and depicted in the Abstract Situational Map. They are outlined as: Illustration, misconception, ambiguity (confusion), thinking the other way, mental calculation, intuition, rework a task, verification, viewing the self with others, flow of information, remembering, articulation, representation, exploration, synthesis, finding a solution, and evaluating an idea.

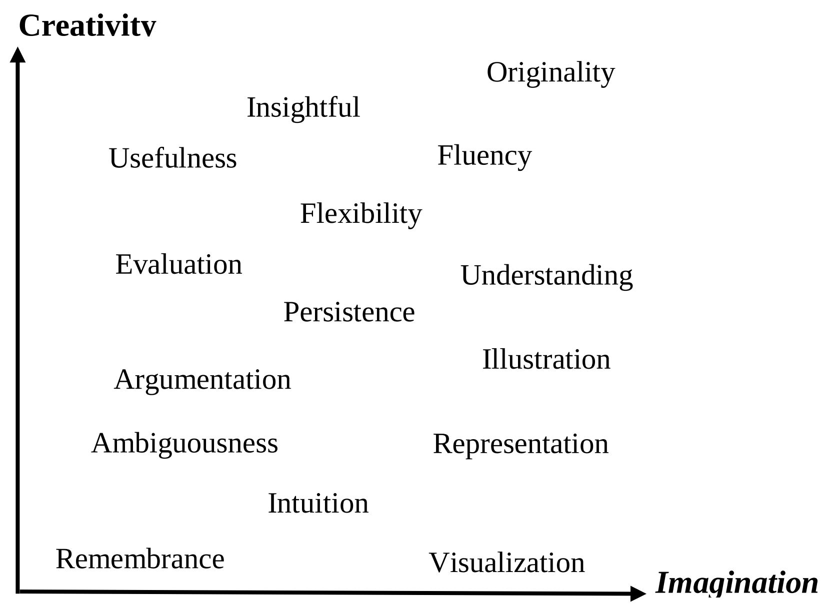

Figure 4.0.15: Positional Map of Imagination and Creativity

Some indicators of imagination and creativity are located at lower levels. The classroom activities were teacher centered phenomenon. Besides, many students were observed waiting for the teacher or other classmates assist them in explaining vague ideas. The vague ideas would be sources of ambiguity. That, in turn, enables students to develop an intuition. Then, it is possible to question, rework a task, and argue for or against an idea. So, *illustration* and *understanding* are higher level constructs. The *visualization* and *representation* aspects of imagination are placed at the bottom right.

The features of creativity such as unique to a situation, utility and practical worth are located to near to the vertical left. On the other hand, arguing, evaluation, insight, and flexibility are located on the left top. These elements have the tendency to fall in to creativity; but with little contribution to imagination. That means: - ideas are generated by making use of imagination and gaining insights.

On the other hand, mathematical creativity is not a comprehensible event which somehow leads to sudden insights. Students were heard saying “Aha…” as they found a hint or an idea input for solving the problem given. And even if creative insights cannot be produced, the ground for creativity can be prepared. A solution based on a concept learned in a different context would be considered original but may not as original as a solution that was unconventional and totally based on insight (Levenson, 2015).

At highest level, seeking methods which support finding or devising new and relevant approaches to solve difficult problems *speculative philosophy* (Paravicini, Schnieder, & Scharlau, 2018). At the beginning of section of this article, imagination was viewed as representation of the visual. But, imagination is also the ability to think of novel solutions (insight) to problems (Arana, 2016). Hence, originality is the most relevant component of imagination and creativity.

4.10 Towards Subjective Truths in Mathematics Education

Social studies develop hermeneutically; that is, its activity is inherently interpretive (Lerman, 2018). For instance, we analyze questionnaires and draw conclusions of what those completing questionnaires are saying; interpretations are embedded in and dependent on language.

The philosophy of mathematics has been part of the philosophical discussion of meaning; and basic steps in the interpretation of meaning have been inspired by a discussion of meaning in mathematics (Kilpatrick, Hoyles, Skovsmose, & Valero, 2005). In the interpretive paradigm, the basic principle is meaning making. Mathematical expressions can be and often are interpreted when applying mathematics in the real world (Baber, 2011). This research was conducted under the interpretive paradigm. Parts of the results were the following three plausible mathematical expressions:

$$\begin{cases} 1+1=2 \\ 1+1>2 \\ 1+1<2 \end{cases}$$

$$f(c)=c^2$$

$$\lim_{p\to\infty} S(p)=F(p)$$

In the conventional mathematics approach, it would be difficult to testify the truth values of such conclusions. I have also mathematized the life styles in Dire Dawa such as *simple life* and *harmony*. Besides, I proposed relational and positional maps; but, I had two doubts.

- Whether I need to use factor analysis or not in determining causal relationships of the concepts?
- Where should I locate the indicators of imagination and characteristics of creativity?

So, is *another mathematics education* possible (Boylan & Coles, 2017)? The main goal of mathematics is not to work with numbers but to reason about objects, properties, values, and so on (Baber, 2011). That means, Mathematics is not just about quantities, numbers, or numerical properties of various objects. It is beyond that! Qualitative mathematics formalizes notions of quantity and relationships at a more abstract level of detail than mathematics as traditionally used in science and engineering (Forbus, 2008). Mathematicians spend very little of their time calculating with numbers; they spend most of their time reasoning about abstract things (Baber, 2011).

When we consider the question of meaning with respect to mathematics education, the issue becomes even more complex (Kilpatrick et al., 2005). On the one hand, we may claim that an activity has meaning as part of the curriculum, while students might feel that the same activity is totally devoid of meaning. On the other hand, discussion of whether meaning has to do with reference or use is conceptual and not easily explained by such means.

So, how could meanings are to be given to the following mathematical conceptions? The concepts are:- one plus one, care squared, limit of praise, simple life, harmony, the geometrical representation of the relationship imagination and creativity. Or, what are the criterions to consider them as mathematical ideas and of qualitative values? Any account of qualitative value must address three issues (Forbus, 2008):

What is the set of values used? Most qualitative value systems identify a finite set of values. In some systems the set of values is described statically, while in others the set of values varies dynamically, providing variable precision. *How can they be reasoned with?* Traditional values can be plugged into equations and used to derive new values. Most qualitative systems support some form of propagation of value information through qualitative relationships, enabling information about one part of a system to be used to infer information about other parts. Some qualitative systems support more equation-like algebraic manipulations. *How can they be generated from other sources of information?* Often scenario descriptions are automatically derived from sensor data or other noisy, limited accuracy numerical information.

The set of values for "praise given to Christ" could be very least, minimum, medium, maximum or extreme. Just as qualitative values can be viewed as levels of abstraction over the underlying realm, the "praise" can be viewed as abstractions over the relationships of traditional mathematics. Thus, $\lim_{p\to\infty} S(p) = F(p)$ can be a qualitative mathematical relationship. The art is in selecting a level of representation that is appropriate for a given task, both in terms of the information available and in terms of the reasoning required (Forbus, 2008). The other value mentioned above was "care'; it may be labeled into "poor", "satisfactory", "good", "very good", "excellent" or "extraordinary".

The previous two mathematical relationships can be reasoned with subjective meanings and arguments. The formulae can be generated while lots of opinions are gathered. Generally, such qualitative values could be inculcated while mathematics education is associated with the daily life students.

Above all, the key questions to be forwarded might be: (1) what relevance does studying such relationships hold? (2) Is it possible to bring such qualitative mathematical relations in to classroom context? If we focus on formal mathematics education only, there would be no need to spend talking about the above relationships. Research on local mathematical practices and the implementation of these practices in the formal mathematics curriculum relies on two theoretical traditions (François et al., 2018). The first of which is the **practical turn** within the philosophy of sciences; the second, the **social turn** in learning theory. Lerman (2018) argued that didactics [the science of learning and teaching mathematics] can only propose value judgments and normative prescriptions that can inform teachers of mathematics on the provision of study processes, and teachers will take up, or not, those judgments and prescriptions and interpret and apply them as they see fit.

I believe that high regard has to be rendered to informal education. School Mathematics learning is supposed to be realized by closely linking it with daily life, relating theory with practice. A situated learning process considers the student as a whole person (François et al., 2018; Lave & Wenger, 1991) as is the case in the out-of-school transmission of the local practices. I already considered learning is a process by which meaning is constructed through socially and culturally situated activity (Cobb et al., 2000). Our work is richer and more complex when recognized as a social science (Lerman, 2018). A reflective teacher would ultimately interprets research findings, government prescriptions or whatever, for the classroom, raising questions and doing research, and that includes teacher research, are essential for our field and for all those engaged in teaching and learning.

Chapter 5: Summary, Conclusion and Implications

Spinning into the social constructivism and pragmatism world views, I found discourse and interpretivism paradigms indispensable to consider. I regarded, in mathematics education, that the socio-cultural aspect is relevant, the classroom activity is important, and the daily mathematical practices have role in the instruction process. This implies that the learning of mathematics is not limited in classroom events.

5.1 Looking Back: Rationales of the Study and Research Methods

I consider mathematics education comprises of "mathematics" and "education"; thus, mathematics is an integral part of mathematics education research. Indeed, it has multi-disciplinary foundations. Such a broader view is a concern of philosophy. Thus, the philosophy of Mathematics Education uncovers the general philosophy, mathematics, milieu, and pedagogy.

Learning school mathematics is supposed to be realized by closely linking mathematics with daily life and relating theory with practice. Mathematics would help students develop creative and imaginative minds; its main essence is creativity. Yet, there has been much less concern to offer clarification of the relationships between imagination and creativity.

The main objective of the research was to ***explore the status of imagination and creativity in a classroom mathematical discourse***. The method of inquiry – Situational Analysis – endorses studying mathematical discourse (Bikner-Ahsbahs et al., 2015) and relationship between controversial variables (Clarke et al., 2018). Everyday mathematics can be investigated from a local perspective. Thus, I employed the Ethiopian Orthodox

Church interpretive methods to support the situational analysis. Since the classroom discourse was conceived broadly, the situation of inquiry has included the social- and cultural- mathematics at Dire Dawa level, the school context and the instructional process held at classroom. Accordingly, Situational, Social worlds/arenas, and positional maps were framed and discussed from broader to narrower points of views.

5.2 Addressing the Guiding Research Questions

The mathematics textbook for grade 12 students addresses issues on discourse, imagination and creativity. Yet, key learning experiences from textbook such as: start up, group work and activities were not implemented in the actual lessons. On the other hand, some mathematical errors noted on the blackboard, unrelated justifications given by the teacher or a student and misconceptions reflected during the class activity enabled students engaged in better mathematical discourse.

5.2.1 What Environmental Factors Influence Group-level Synergy?

I showed up inclination to the collaborative learning and socio-cultural perspectives. A formula $1 + 1 > 2$, which came out of this study, describes a group synergy. Since the study was conducted at a crowded classroom where group discussions emerge randomly and informally, it was difficult to model students' group synergy.

How did Socio-cultural Aspects in the Research milieu Impact on the Classroom Mathematics?

By analyzing the socio-cultural mathematics in Dire Dawa, I brought three mathematical systems: $2 \leq 1 + 1 \leq 2$, $\mathrm{f(c)} = \mathrm{c}^2$ where c refers to care, and $\lim_{\mathrm{p} \to \infty} \mathrm{S(p)} = \mathrm{F(p)}$ where S refers to Christ, p is the praise given to S, ∞ is infinity, and F represents to God. The formulae are missed opportunities for engaging students in imagination and developing their creativity. Besides, it introduces new concepts: the re-conceptualization of triangle. On the other hand, “simplification” and “harmony” life styles have impacts on students’ learning. In a sense, many students were observed choosing “short cut” methods of solving problems which in turn limit their skill of fluency. On the other hand, “harmony” implies to easily agreeing on others’ idea; so, students’ argument could not be strong. In other words, they could not persist in their thoughts.

What instances of instructions elicited students’ mathematical discourses?

The instances of students’ discourse were generated: by the Mathematics Teacher’s initiation; in response to an individual student’s class participation; interaction with teacher’s notes; new interactions during Focus Group Discussions; against the mathematical errors, unrelated justifications and misconceptions; actions/activities of the other students in the group; and students’ intuition.

How do individual students contribute to small-group interactions and discussions?

As a professional, the classroom mathematics teacher was very experienced and capable one. Yet, he didn’t efficiently use occasions that could insist him facilitate group learning. Unlike my philosophical position, he did not favor for informal discussions and collective talks. He had been leading interactions to position every student as a contributor to the collective development of mathematical ideas. That is why most students were waiting for him to kick off interactions. In short, the classroom interactions and discussions could have been richer.

Yet, the discourses observed were characterized by students’ engagement in actively listening, answering and questioning, sharing, explaining, and challenging ideas. The modes of discourses were following a student's class participation; interaction with lecture notes; reaction with teacher and sometimes with themselves. A tool that testified a student's intra-communication (with himself or herself) was moving fingers, gestures or such "Aha!" sounds. These were evidences for mental calculation. The issue of "engaging into imagination" denotes to students' use of language, visual images, meaning making, interaction with material, conversation, participation in an activity, or classroom discussion. Thus, students' *engaging into imagination in mathematical discourse* was assumed for the how of students' engagement in such instances. The question is addressed part by part as follows.

5.2.2 How did Imagination and Creativity Occur in Grade 12 Students' Small-group Interactions and Discussions?

If creativity represents something new or different, many informants were heard of saying "I had a different solution" or "my case was different". But, I claimed *creativity* is the process that students do activities in new way particular to a situation with utility and practical worth. The actions of other students in the group or class were determining aspects to develop co-actions and hence co-creativity. For instance, individual members were seen questioning, answering, explaining, thinking the other way, extending idea, defending one's idea, verification of answers, rework an example, or extending idea. When different ideas emerge from a group, the ideas were opportunities for each member to develop same or different perspectives and co-create knowledge.

The concept *imagination* is a qualitative construct; and so, indirect method of assessment was considered as essential tool. When students asked questions or moved their hands, they were manipulating something in their mind. It was valued as mental calculation or spatial representation. The qualities of sensibility (active listening) and concentration (seeing the picture more attentively), visualization, intuition, finding a solution, exploration, extending an idea were identified. These could be signs for the capability of conceiving, initiating, transforming imagination (Hsu & Peng, 2014). The co-actions exercised referred to co-imagination. The new concept *co-imagination* would mean having a divergent thinking (DT), mental picture or special representation while others are engaged in such activities.

5.2.3 How did Imagination and Creativity relate to one Another?

Initially, this study presupposed "imagination" as *thinking out of the box* and "creativity" was regarded as *thinking in the box*. So, both imagination and creativity are aspects of thinking. The other versions of thinking are: reasoning, communication, fluency and thinking too. All of them were implicated out of the major issues as such intuition, verification, exploration, etc. Then, some indicators of imagination and characteristics of creativity were drawn.

Which Components of Imagination Fit in to Creativity?

The positional map of *imagination* and *creativity* (illustrated in Chapter 4) portrayed that the terms are related to each other and interdependent. Thus, common indicators for imagination and creativity were identified as: remembering, intuition, ambiguity, visualization, illustration, argumentation, persistence, usefulness, understanding, flexibility, evaluation, fluency, insightful, and originality. Concepts related to representation, visualization and illustration best describe imagination than creativity. Still, they contribute to the creative behaviors of students.

What Aspects of Thinking belong to Creative Imagination?

"Recombining experiences…" requires remembering, intuition, and synthesis; "…creation of new…" demands exploration, thinking the other way, finding a solution; and "…at a specific goal…" would imply representation, verification, or usefulness. Thus, these are the aspects of thinking which belong to creative imagination.

5.3 Implications

The curriculum is open for implementing creative pedagogy and using examples from everyday mathematics. Thus, the socio-cultural mathematics in Dire Dawa is good input. The study introduced the re-conceptualization of triangle, co-imagination, and co-creativity. This 'theorizing' research revealed that discourse, imagination and creativity are foundations for mathematics education. In turn, originality is the highest common construct for imagination and creativity. On the other hand, a theory is a creative tool, an inventive approach to make meaning, as well as being an intervention into current cultural practices (deFreitas & Walshaw, 2016). Some more issues of GT like theoretical sensitivity, inductive process or inducing a theory, and abduction require skills of imagination and creativity. Yet, the role of researchers' creativity and imagination in the implementation of GT methods has rarely been emphasized and should be the subject of further reflection (Pozzebon, Petrini, Mello, & Garreau, 2011). The geometrical representation of imagination and creativity has scaling limitation; and thus insisted me to consider subjective truth in mathematics education.

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Appendix: Codes of Individuals involved in FGDs

No.	Event (Session) Code	Sex	Involved in FGDs Session 31-Oct.2017 #1	03-Nov.2017 #2	07-Nov.2017 #3	28-Nov.2017 #4	11-Dec.2017 #5	13-Dec.2017 #6	15-Dec.2017 #7	18-Dec.2017 #8	20-Dec.2017 #9	29-Dec.2017 #10	Frequency
1	S_{11}	F	X	X									2
2	S_{12}	F	X	X		X						X	4
3	S_{13}	F	X	X			X						3
4	S_{14}	F	X	X					X	X			4
5	S_{15}	F	X	X		X			X	X			5
6	S_{21}	M			X								1
7	S_{22}	M			X						X		2
8	S_{23}	F			X	X	X	X					4
9	S_{24}	F			X								1
10	S_{31}	F				X							1
11	S_{32}	M				X							1
12	S_{33}	F				X							1
13	S_{34}	M				X							1
14	S_{41}	M					X						1
15	S_{42}	F					X						1
16	S_{51}	F						X	X				2
17	S_{52}	M						X					1
18	S_{61}	F							X				1
19	S_{71}	F								X			1
20	S_{81}	M									X		1
21	S_{82}	M									X		1
22	S_{91}	M										X	1
23	S_{92}	F										X	1
24	S_{93}	F										X	1
	Total		5	5	4	7	4	3	4	3	3	4	42

Glossary

(Source: Oxford English[31], Microsoft Encarta[32] & Collins[33] Dictionaries)

Ambiguousness: the state of having difficulty to understand or classify because of assuming more than one possible meaning.

Argumentation: the action of reasoning systematically in support of something; the process of debating or discussing something; the process of reasoning methodically

Communication: (the means of) the sharing or exchanging or sending or receiving information; a sense of mutual understanding; the study of speech, gesture, etc.

Creativity: the activity or engagement; the quality of bringing something into existence or resulting in something or make something happen; the ability to use the imagination to develop new and original ideas or things, especially in an artistic context.

Group Creativity:- is a function of individual creative behavior contribution, the interaction of the individuals involved, group characteristics, group processes, and characteristics of group task.

Create: to make or bring something into existence; invest with a title of nobility; complain; produce.

Creative: the use of imagination or original ideas in order to create new ideas or things; making imaginative use of the limited resources available.

Discourse: written or spoken communication or conversation; a serious and lengthy speech or piece of writing about a topic; language, especially the type of language used in a particular context or subject; the ability to reason or the reasoning process

Ethiopian Orthodox Church interpretation method:- is posing, discussing and developing content, theological, moral and historical questions on a sentence or phrase in search of alternatives meanings; direct meaning or meaning it conveys.

Evaluation: forming an idea of the amount or value of; the act of considering or examining something in order to judge its value, quality, importance, extent, or condition

Flexibility: readiness to be adapted in several ways; capability of adapting to new situation or being changed according to circumstances

Fluency: speaking easily or accurately; flow of ideas; the ability to speak or express something effortlessly and correctly

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[31] Concise Oxford English Dictionary **© Oxford University Press**

[32] **Encarta Dictionaries 2009 © 1993-2008 Microsoft Corporation**

[33] Collins English Dictionary and Thesaurus

Illustration: provision of explanatory pictures to accompany a printed, spoken, or text; the state of clarifying or explaining something by giving examples or making comparisons

Imagination: the ability to form ideas, thoughts, and images in the mind, especially of things never seen or experienced directly; the ability to deal resourcefully with unexpected or unusual problems, circumstances.

Imagine: form image (concept) of something in mind; think, believe, guess, assume or suppose to exist.

Imaginative: being good at thinking of new ideas or at visualizing things that have never been seen or experienced directly: relating to the ability to form images and ideas in the mind, or to think of new things.

Insight: the ability to perceive clearly or deeply; a penetrating and often sudden understanding, as of a complex situation or problem

Intuition: the ability to understand something immediately; without the need for conscious reasoning; the state of being aware of or knowing something without having to discover or perceive it, or the ability to do this.

Mathematics: the study of the relationships among number, quantity, shapes and space by using a specialized notation; is concerned with operations and processes involved in the solution of a problem or study of some scientific field.

Mathematize: to regard or treat something in, or reduce it to, mathematical terms.

Originality: the quality of newness that exists in something not done before or not derived from anything else; the ability to think creatively and depart from previous forms

Persistence: the quality of continuing firmly in a course of action despite problems or difficulties

Philosophy: concerned with making explicit the nature and significance of ordinary and scientific beliefs and investigating concepts by means of rational argument of ethics, logic, knowledge and existence.

Philosophize: to consider, explain, or deal with something from a philosophical standpoint

Reason: a cause, explanation, or justification of something; a premise of an argument in support of something; to think, understand, and form judgments logically, clearly and coherently

Remembrance: the action of keep something necessary or advisable in mind

Representation: the action or an instance of signifying or symbolizing something; a visual description of something.

Situation would mean the state of affairs, circumstances or conditions that characterize events in a particular place or society.

Situated Classroom:- is a class where learning is broadly viewed in relation to its surrounding cultural aspects and social environment.

Situational Analysis (SA):- is analyzing and interpreting situations by way of understanding the elements in it, their relations and their ecologies.

Positional Map:- is a lay out the major positions taken, and not taken, in discussions, debates, and extant discourse materials in the situation of inquiry vis-à-vis particular axes of difference, concern, and controversy about important issues.

Situational Map:- It is a lay out of the major human, nonhuman, discursive, affective, geopolitical and other elements in the research situation of inquiry and provoke analysis of relations among them.

Social Worlds/Arenas Map:- is a lay out social worlds, organizations, institutions, etc and the arena(s) of commitment and discourkse with which they are engaged in ongoing negotiations in the situation of inquiry

Theory: the body of facts, rules, ideas, propositions, principles, and techniques related by logical arguments to explain and predict a wide variety of connected phenomena.

Theorize: to speculate or form a theory or theories about something; to conceive of something in a theoretical way

Thinking: forming of and using thoughts or rational judgment; to ponder a matter or problem

Understanding: comprehension; the power of abstract thought; the ability to perceive and explain the meaning or the nature of something

Usefulness: being used for practical purpose or in several ways; doing something that is of some value or benefit; effectiveness.

Visualization: form a mental image of; imagine; the creation of a vivid positive mental picture of something such as a desired outcome to a problem.