Addis Ababa University
Addis Ababa Institute of Technology
School of Electrical and Computer Engineering

Feedback linearization with LQR control approach for quadrotor trajectory tracking

A thesis submitted to School of Graduate Studies, Addis Ababa Institute of Technology, Addis Ababa University in partial fulfillment of the requirement for the Degree of Master of Science in Electrical Engineering (Control Engineering)

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Declaration

I declare that the work entitled "Feedback linearization control approch for quadrotor and trajectory tracking" is my original work and has not been presented for any degree in any university or college and sources of material used for the thesis is well acknowledged.

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Betsega Yosef

Abstract

A quadrotor is a type of helicopter which is lifted and propelled by four rotor.Quadrotor's are used for variety application due to reason of mechanically simple,small size and low inertia.But controlling quadrotor is a challenging task because of non linearity and underactuacted character of the dynamics.
In this thesis work,the dynamic model of the quadrotor is derived,by using Newton-Euler formulation.Then a nonlinear control approach which is a feedback linearization technique is used .
Feedback linearization technique is used to linearize both the translational and rotational dynamics.Here during linearization an auxiliary control inputs arises and those linear control inputs are found through linear control technique called linear quadratic regulator.Constant trajectory tracking and rectangular trajectory tracking is also obtained by controlling the attitude and the position simultaneously.
The quadrotor dynamic model and the controller design is developed using MATLAB simulation and the dynamics model is tested with step input for each control inputs.The effectiveness of the proposed controller is tested with disturbance and without disturbance through MATLAB simulation.It is observed that,on the constant tracking simulation result the step input response have good settling time response.It achieve the desired 100m position on the settling time of x 19.42 sec, y 19.5 sec and z 11.4 sec .

Keywords : Quadrotor,nonlinear control, Feedback Linearization controller(FBL), Auxiliary control,Linear quadratic regulator controller(LQR),MATLAB simulation

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List of Abbreviations

AR	Augmented reality
b	Thrust coefficient
CM	center of mass
COG	center of gravity
d	Drag coefficient
DJI	Da-Jiang Innovations
FBL	Feed back linearizaton
GPS	Global positioning system
ITAE	integral time absolute error
KV	Kilo volt
L	Length of propeller
LQR	linear quadratic regulator
LQT	Linear quadratic tracking
M	mass
MIMO	Multi - input Multi- output
PID	Proportional integral derivative controller
R	Rotation matrix

Chapter 1

Introduction

1.1 Background of Study

The research and development on robotic filed rapidly grow due to need to replace and aid human being with robots. Robotics system represented by complete human like system which is programmed to do a routine task and take limited decisions [1]. The advanced robotic systems design different vehicles with higher capabilities to take hard decisions and to operate in dangerous circumstance [2].

Unmanned Aerial Vehicles (UAV) are considered highly advanced robotics system and used for different applications. Initially, UAVs were introduced during world war I for pure military tasks and since then they have been developing for wide range of applications, like agricultural spraying, surveillance, power line fault detection, traffic monitoring,mapping and town planning, collecting data, monitoring isolated territories and other various activities[3].

UAVs have two main configurations, fixed-wing aircraft and rotary-wing aircraft. The fixed -wing aircrafts have the capability of flying at higher altitude and it travels long range but lack of maneuverability capabilities. On the other hand, rotary-wing aircraft are more flexible,cheap and lighter in weight, capable of hover over targets, taking off and landing in limited space range[4].

This thesis studies a rotary-wing aircraft type quadrotor with nonlinear control approach. A quadrotor is lifted and propelled by four rotors and the motion of the quadrotor controlled by varying the relative speed of each rotor to change the thrust and torque produced by each rotor. Form control point of view feedback linearization plus linear quadratic regulator (LQR) controller is implemented for better system performance. Feedback linearization technique linearizes the non-linear dynamics of the quadrotor and the linearized dynamics of quadrotor is controlled by using LQR controller.Different trajectory paths are generated to test the performance of the controller.

1.2 History of Quadrotor

The gyroplane No.1 built by Louis and Jacques Breguet in 1907, it was the first human carrying helicopter. This helicopter has four long girders made of welded steel tubes and arranged in the form of horizontal cross. Each rotor consisted of fabric covered biplane type blades, four light, giving a total of 32 small lifting surface. The helicopter makes a flight by a man stationed at the extremity of each of the four arms supporting the rotors[5]. It was the first helicopter to make free flight, but it has a lot of drawbacks on it. But the Breguet-Richet aircraft was uncontrollable and also not steerable in a horizontal plane.

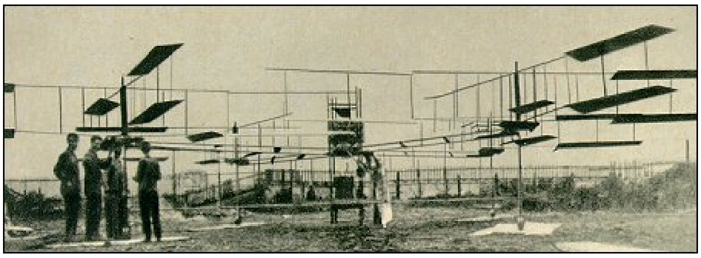


Figure 1.1: The Breguet-Richet No 1-quadrotor

Then the French engineer and helicopter designer Etienne Oehmichen was the first scientist who experiment with rotorcraft designs in 1920[6]. His first successful flight with a helicopter took place on 18 February 1921. Among the six designs he tried, his second multicopper had four rotors and eight propellers. The Oehmichen used a steel-tube frame, with two-bladed rotors at ends of the four arms. The angle of these blades could be varied by warping. Five of the propellers, spinning in the horizontal plane, stabilized the machine laterally. Another propeller was mounted at the nose for steering. The remaining pair of propellers was for forward propulsion. In 1924 his quadcopter makes a distance of 360m (1,181ft) setting a world record. In the same year he flew a 1km (0.62miles) circle in 7m and 40s.

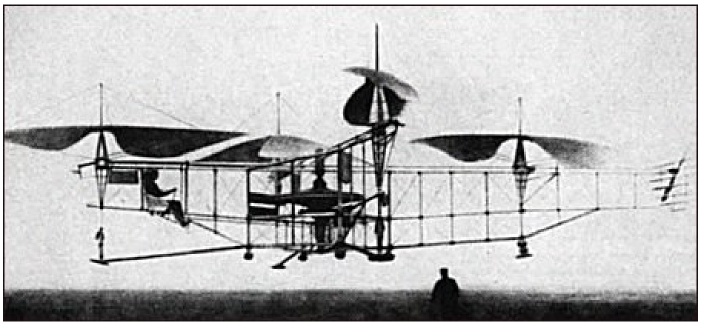


Figure 1.2: Oehmichen No .2 Quadrotor

Around the same time Dr. George de Bothezat built and tested his quadcopter for the US army, completing a number of test flights before the program was scrapped. Dr. George de Bothezat developed the massive six bladed rotors at the end of X-shaped structure. Two small propellers with variable pitch were used for thrust and yaw control. But the design is too much complex, have control difficulties and pilot workload due to such reason the us army reportedly only capable of forward flight in a favorable wind and also Making large rotor blades, 4 or 5 m long or even longer was a huge problem[7].

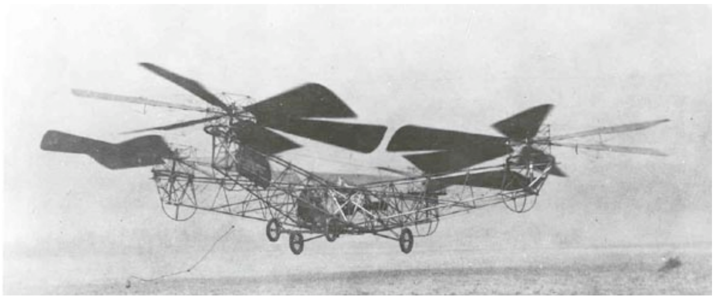


Figure 1.3: de Bothezat Quadcopter

In 1956 convertawings model A quadrotor was designed by Dr. George de Bothezat and Ivan Jerome design featured two engines driving four rotors with wings added for additional lift in forward flight. No tail rotor was needed and control was obtained by varying the thrust between rotors. A quadrotor model was successful demonstrate forward flight. But it was hard for the pilot to fly because of the workload of trying to control the thrust of all propellers at once[8].

Figure 1.4: Model A quadrotor

In 1958 Curtis Wright VZ-7[9] is designed by Curtis wright company. The VZ-was of exceedingly simple design, essentially consisting of a rectangular central airframe to which four vertically-mounted propeller were attached in a square pattern and the control mechanism was also very simple; directional movement was controlled by varying the thrust of each individual propeller, with additional yaw control provided by movable vanes fixed over the engine exhaust. The craft were capable of hovering and forward flight and proved relatively stable and easy to operate. But the design proved consistently incapable of meeting the altitude and speed requirements specified by army.

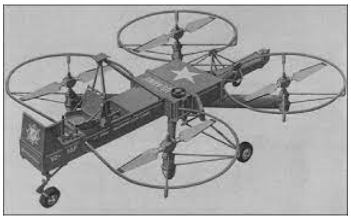


Figure 1.5: Curtis Wright VZ-7 helicopter

After Curtis Wright VZ-7 helicopter the current development of quadrotor become faster. Even the past 15 years many quadrotor companies are built like DJI Phantom and Parrot AR. The new version quadrotors are light in weight, easy controlling mechanism with joystick and use advanced electronics for flight control which we are using nowadays.

1.3 Statement of The Problem

The Quadrotor dynamics is highly nonlinear and underactuated, due to such character it's difficult to control it. As it was mentioned on the literature review section, in the past decade many control techniques are developed and applied to quadrotors. such us backstepping controller, PID controller and LQR controller[10]. They have been proven to stabilize such system about a desired point and track desired trajectories.Compering linear controllers with nonlinear controllers , linear controllers doesn't lead to good performance in terms of disturbance rejection, model uncertainties and internal dynamics. Still researcher exploring new control techniques for better performance in disturbance rejection, stabilization, model uncertainties and reference tracking[11, 12].

In this thesis, nonlinear controlling technique, feedback linearization plus linear controlling technique,linear quadratic regulator controller (LQR) is used for solving serious performance problems that affect the system.

1.4 Objectives of The Study

1.4.1 General Objective

To design feedback linearization technique through input-out feedback linearization technique plus linear quadratic regulator controller in order to get the best performance of quadrotor.

1.4.2 Specific Objectives

The specific objective of this thesis is:

- To formulate the kinematics and dynamics equation of the quadrotor .
- To Derive both rotational and translational motion of dynamics.
- To establish feedback linearization plus LQR controller for quadrotor to track the desired trajectory .
- To demonstrate the performances of the system using MATLAB®/SIMULINK®

1.5 Methodology

The following methodologies have been used to complete this thesis:

- The first stage is literature review which is reading books,articles and forums.
- The second stage is mathematical modeling which includes the formulation of mathematical relations of quadrotor dynamics.specifically,the translational dynamics which gives the determining translational variables values required to achieve the desired positions and rotational dynamics also which gives the determining rotational variables values required to achieve the desired Euler angles.
- The third stage is controller design.A feedback linearization plus linear quadratic regulator (LQR)controller has been designed.
- The fourth stage is simulation.MATLAB 2018b version simulation tools has been utilized for modeled dynamics and controller design.
- The last stage analysis and interpretation the result and followed by recommendation.

1.6 Significance of the Research

One of the major considerations of control system design is achieving the control goal with minimum amount of energy and time. This thesis contributes the application of feedback linearization plus LQR controller for quadrotor to control the system dynamics and the linear controller LQR has robustness against plant uncertainty and disturbance. Another significance of this thesis is the generation of linear model of quadrotor from nonlinear mathematical model of the quadrotor by using feedback linearization control approach.The simplified linear dynamic model done by transforming the original system model into equivalent models of simpler form. This makes the system to easily controlled and to achieve the control goals.

Finally, this research study will be the first to apply feedback linearization plus linear quadratic regulator controller for the application of transmission line inspection.Therefore,it will be a very essential source of knowledge for students and researchers for future studies.

1.7 Thesis Organization

The thesis is organized into five chapters including this introduction. The rest of the thesis is organized as follows.

Chapter 2 presents literature review.

Chapter 3 introduce the mathematical model of quadrotor and derivation of equations of motion is done by using Newton-Euler formulation.

Chapter 4 addresses the design control scheme, feedback linearization controller plus linear quadratic regulator (LQR) controller.

Chapter 5 presents both controller simulation setup, simulation result,application area and discussion

Chapter 6 presents conclusion, recommendation and future work of the thesis .

Chapter 2

Literature review

In this thesis, by R.Boona and J.F.Camino [11]Newton -Euler equations are used for modeling the translational and rotational equations of motion respectively. A feedback linearization is applied to the rotational and translational dynamics. After linearization LQR technique is used to converge the tracking error to zero by controlling the gains that stabilize the dynamics of the tracking error.The trajectory simulation has two parts time -varying reference trajectory and constant reference trajectory.It compares both reference tracking based on magnitude of the control input and the influence of the controller gains on control effort is investigated by using a constant reference trajectory.

HAOQUAN YE [13] have used the Newton -Euler formulation for modeling. Backstepping technique is used to stabilize the quadcopter. The controller scheme is divided into subsystem and Feedback linearization is used for altitude subsystem.The attitude subsystem is obtained by adding an integrator technique to minimize the tracking error and stabilize the system. The simulation demonstrate the S-Function quad model and S-function calculate the real controller and the virtual controller. The controller design makes the system stable and disturbance rejection capability is higher.

In the literature by F. SABATION [14] mathematical model of quadrotor dynamics is derived by using Newton's and Euler laws. Feedback linearization controller is used to linearize the dynamics of the quadrotor. This paper compares the nonlinear control strategies with linear control and it recommend that nonlinear controller strategies are good than linear controller. Based on achieving good performance system and disturbance rejection capability. Future work is enhancing this project by applying nonlinear controller techniques like adaptive feedback linearization, sliding mode control for the real quadrotor to obtain best performance.

Ruth Tesfaye [4] Newton-Euler formulation is used to derive the defining equation of motion of the quadrotor at hovering position. Linear PID controller and LQR controller is used as controller strategies. Four independent PID controller are used to stabilize the craft at hovering position and also it shows the effect LQR on finding the controller gain that minimize the undesired deviation from the desired set point. By comparing the controller LQR have better settling time and have good disturbance rejection ability and stabilize the quadrotor.

EMRE CAN SUICMEZ [15] the mathematical model is done by using Newton - Euler formulation. Backstepping controller method and LQT (Linear quadratic tracking) controller method are used to track desired trajectories accurately. Linear quadratic tracking controller is modeled by using time varying optimal gains as state feedback controller. LQT controller could track relatively complex trajectories more accurately and efficiently compared to LQR and backstepping controller.

Duy Dam [16] the mathematical model is derived by using Newton -Euler equation for both translational and rotational dynamics. Full state feedback controller and PID controller is applied to the quadrotor in order to achieve angle stabilization and height stabilization. By applying full state feedback controller, the desired feedback matrix K is founded and the error also converges to zero with the desired feedback matrix gain. The simulation demonstrates by varying different gain values for both pole placement and PID controller in order to see the effect of each gain to the response of the closed -loop system.

S.Bouabdallah, A.Noth and R.Siegwart[17]the mathematical model of quadrotor dynamics is derived by using Newton's and Euler laws.PID and LQ controllers have been used to control a micro quadrotor.LQ controller was developed without considering the actuators dynamics and also it has a better performance than PID controller.Future work enhance control with position controller and to develop a fully autonomous vehicle.

H Lee and HJ Kim [18]Newton-Euler and Lagrange-Euler equations were used for modeling the translational and rotational equations of motion respectively.It used linear and nonlinear controllers ,and it compares the performance difference between them.From linear controller PID and LQR has been used to control the autonomous quadrotor flight.Also from non linear controller feedback linearization, backstepping and sliding mode controller compered each other and a backstepping controller give us global stability than others.

Chapter 3

Mathematical model of quadrotor

3.1 Overview

A quadrotor is driven with four rotors attached at the ends of arms under a symmetric frame. Quadrotor have two main configuration which are plus (+) and cross(x) configuration. The cross configuration is chosen for this thesis due to reasons,it provides more stability than plus configuration and guarantees no camera arc obstruction for a camera installed on a quadrotor.On the cross structure each propeller is connected to the motor. All the propeller axes of rotation are fixed and parallel.The propeller generates an upward thrust force that lift the quadrotor.Here the difference of their rotating speed it causes unbalanced torque applied to the center of mass and due to the unbalanced torque the attitude also changed. In order to control the attitude and position of quadrotor it should have to assign the rotating speed of all propellers separately and configure the total thrust and torque about their positions on the body frame of the quadrotor.

3.2 Assumptions taken

- The quadrotor is a rigid structure and has a symmetric structure.
- The center of mass of the quadrotor and the origin of the body frame coincides.
- Rotational matrix defined to transform the coordinates from body to earth co-ordinates using Euler angles ϕ- roll angle, θ- pitch angle, ψ -yaw angle.
- Thrust is proportional to the square of rotor speed ($f_i = b\,\omega_i^2$).
- Torque is proportional to the square of rotor speed ($\tau = d\,\omega_i^2$).

3.3 working principle of quadrotor

Quadrotor has two reference frames, the body frame and the inertial frame. The reason why two frames are needed is that the quadrotor has many sensors, like gyroscope and accelerometer, that give readings with respect to body frame. Whereas it has other sensors, like GPS and magnetometer, that give readings with respect to inertial frame.

3.3.1 Inertial frame and body frame

The inertial frame is used to describe the absolute position of the quadrotor on the earth. This frame represented by North-East-Up coordinates system. The origin of the inertial frame is chosen to be a fixed point on the ground surface and Z_E is pointing upward against the gravity, this will consider all the force and the torque that act on the quadrotor.
The body frame is fixed on the quadrotor and its origin is in the center of gravity (COG) of quadrotor. Since the quadrotor has a symmetrical shape with a central core and four identical rotors attached at its arms, the quadrotor COG is located at the core center.
On the other hand,the inertial frame is generally represented by North-East-Up coordinates system. The origin of the inertial frame is chosen to be a fixed point on Earth.

- Body -fixed reference frame(O_B, X_B, Y_B, Z_B)
- Inertial reference frame (O_E, X_E, Y_E, Z_E)

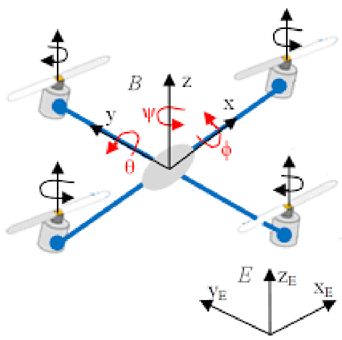


Figure 3.1: Inertial frame to body frame coordinate system ref[19]

Based on the above frame of reference, lets define the position as $[x, y, z]^T$ and the angular position as $[\phi, \theta, \psi]^T$.Here the translational position indicates the location of the quadrotor with respect to the inertial frame and the angular position is defined by Euler angles which is Roll angle (ϕ) refers to the rotating angle of the quadrotor around X_B,pitch angle (θ) refers to the rotating angle of the quadrotor around Y_B and the yaw angle (ψ) refers to the rotating angle of the quadrotor around Z_B .

3.3.2 Basic movements of quadrotor

There are four basic movements which is allow the quadrotor to reach a certain altitude and attitude.

1. Throttle

   Throttle is generated by increasing the speed of all four rotors simultaneously. This action will increase the thrust of the quadrotor without producing any net movement.

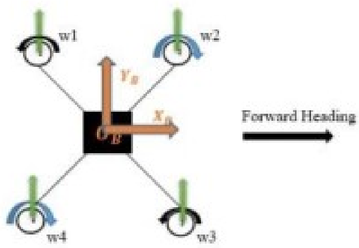


Figure 3.2: Throttle(vertical) movement

2. Roll

   The Roll movement is generated by increasing or decreasing the speed of the rotor on the left and by decreasing or increasing the speed on the right. The torque with respect to the body frame X -axis is generated where the quadrotor turns with a roll angle and the thrust is keep constant.

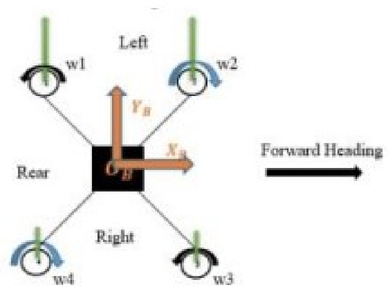


Figure 3.3: Roll movement

3. pitch

   The pitch movement is generated by increasing or decreasing the speed of rotor on the rear and by decreasing or increasing the speed of the rotor on the front. The torque with respect to the body frame Y -axis is generated where the quadrotor turns with a pitch angle and the thrust is keep constant.

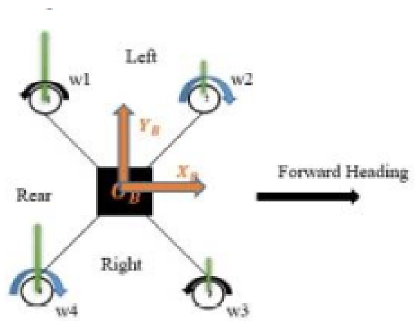


Figure 3.4: pitch movement

4. Yaw

   The yaw movement is generated by increasing or decreasing the speed of the rotor on the rear front and by decreasing or increasing the speed of the rotor on the left right. The torque with respect to body frame Z-axis is generated where the quadrotor turns clockwise (CW) or counter clockwise (CCW) and the total thrust is the same as in hovering.

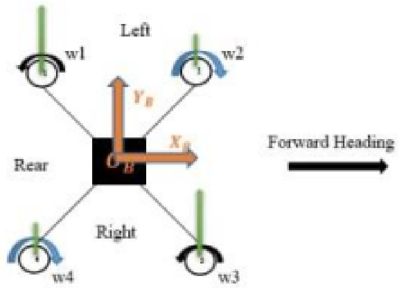


Figure 3.5: Yaw movement

3.3.3 Reference frame alignment

A quadrotor is an underactuated system which is characterized by having more degrees of freedom than actuators. This mean quadrotor has six degree of freedom and four actuators.so in order to express its position and orientation in space the motion of the quadrotor need to be defined with two reference frames.

Body-fixed reference frame (O_B, X_B, Y_B, Z_B)

- O_B coincides with center of quadrotor
- Linear velocity represented with variables($V^B = [u, v, w]^T$)
- Angular velocity represented with variables ($\omega^B = [p, q, r]^T$)
- Force(F^B)
- Torque (τ^B)

Inertial reference frame (O_E, X_E, Y_E, Z_E)

- Linear position ($L^E = [X, Y, Z]^T$)
- Angular position($A^E = [\phi, \theta, \psi]^T$)

From the above representation L^E is determined by the coordinate of vector between O_B and O_E respect to the inertial frame and A^E is orientation of body frame respect to the inertial frame.

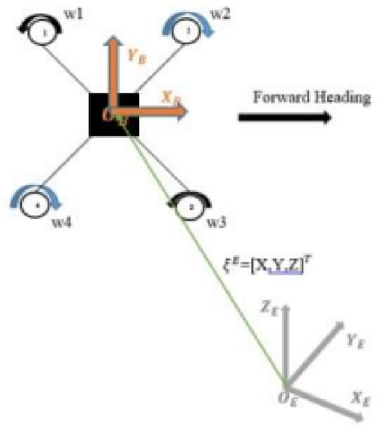


Figure 3.6: orinetation of body frame respect to inertial frame

3.4 Forces acting on quadrotor

3.4.1 Gravitational force

The force of gravity acting on the vehicle can be represented in the body frame using Euler angles F_g^E and F_g^B represents the force of gravity in inertial and body frame of reference respectively. To get force of gravity in body frame of reference the force of gravity in inertial frame should be multiplied with rotational matrix R.

$$F_g^B = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \tag{3.1}$$

$$F_g^B = R * F_g^E = F_g^B = \begin{bmatrix} mg\sin(\theta) \\ -mg\sin(\phi)\cos(\theta) \\ -mg\cos(\phi)\cos(\theta) \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.2}$$

3.4.2 Gyroscopic Force

Gyroscopic effect produced by the propeller rotation in clockwise and counter clockwise direction. Due to this rotation an overall imbalance will happen when the algebraic sum of the rotor speeds is not equal to zero. The gyroscopic effect is proportional to the roll and pitch rates. The following equation defines the overall propellers speed.

$$F^B gro = -\sum_{n=1}^{4} J_T p \left(w^B x \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) (-1)^k \omega_k \tag{3.3}$$

$$F^B gro = J_T p \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ q & -q & q & -q \\ -p & p & -p & p \\ 0 & 0 & 0 & 0 \end{bmatrix} \omega_k = J_T p \begin{bmatrix} 0 \\ 0 \\ 0 \\ -q \\ p \\ 0 \end{bmatrix} \omega_r \tag{3.4}$$

$$\omega_k = [\omega_1 \;\; \omega_2 \;\; \omega_3 \;\; \omega_4]^T, \omega_r = -\omega_1 + \omega_2 - \omega_3 + \omega_4 \tag{3.5}$$

3.4.3 Force generated by the propeller

The propeller shaft of the quadrotor is fixed and due to this the lift direction is constant in body coordinate system and also the total force generated by the propeller system is equal to the thrust u_1. It's force exerted due to the body of the propeller.

$$F^B = \begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix} \tag{3.6}$$

3.5 Equation of motion

Newton -Euler formalism is used to describing the equation of motion of dynamic system of the quadrotor. The dynamics of the quadrotor is represented with inertial reference frames and body-fixed reference frame. Rotational matrix is used to relate the motions expressed in body-fixed reference frame with respect to the Inertial reference frame. The rotational matrix is the composition of Euler angles with respect to the axes of rotation.

The composition of elemental rotation represented with rigid body in 3-dimensional space and expressed in three consecutive elementary rotations about z, y and x axis.

The rotation about z axis (yaw-ψ)

$$R_z(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.7}$$

The rotation about y axis (pitch-θ)

$$R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \tag{3.8}$$

The rotation about x axis (Roll-ϕ)

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \tag{3.9}$$

Finally, the obtained rotational matrix of the body-fixed reference frame relative to inertial reference frame it become as shown below:

$$R_z(\psi)R_y(\theta)R_x(\phi) = \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi c_\theta & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix} \tag{3.10}$$

Where the abbreviation $c_y = \cos(y)$ and $s_y = \sin(y)$
The rotational matrix R is used to transform linear quantities from body to inertial frame. The transformation of linear velocity is defined as shown below.

$$\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix} = R \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{3.11}$$

Another matrix is transfer matrix which is used to relate the body angular rates to inertial rates. That mean it relate the angular velocity $\omega^B = [p, q, r]^T$ to the angular position $A^E = [\phi, \theta, \psi]^T$.

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin(\theta) \\ 0 & \cos(\phi) & \cos(\theta)\sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi)\cos(\theta) \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{3.12}$$

$$T^{-1} = \begin{bmatrix} 1 & 0 & -\sin(\theta) \\ 0 & \cos(\phi) & \cos(\theta)\sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi)\cos(\theta) \end{bmatrix} \tag{3.13}$$

$$T = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)(\theta) \\ 0 & \cos(\phi) & \cos(\phi) \\ 0 & \sin(\phi)\sec(\theta) & \cos(\phi)(\theta) \end{bmatrix} \tag{3.14}$$

3.5.1 Newton-Euler Equation

Newton-Euler method is used to model the flight dynamics of the quadrotor. This method applies newton first law and newton second law of motion and the representation of translational and rotational dynamics depends on the position vector, orientation vector, angular velocity and linear velocity[20].
Where:

- Position vector: $\xi = [X, Y, Z]^T$
- Orientation vector: $\eta = [\phi, \theta, \psi]^T$
- Angular velocity: $\omega = [p, q, r]^T$
- Linear velocity:$[v^B]^T = [u, v, w]^T$ and $\begin{bmatrix} rollrate \\ pitchrate \\ yawrate \end{bmatrix} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$

From newton second law, the forces acting on quad-copter can be expressed as follows:

$$\sum F = ma \tag{3.15}$$

$$F = m\frac{dv}{dt} = m(\frac{dui}{dt} + \frac{dvj}{dt} + \frac{dkw}{dt}) \tag{3.16}$$

Applying the chain rule on the force equation results as shown below:

$$F = m\frac{dv}{dt} = m\left(\frac{du}{dt}i + \frac{di}{dt}u + \frac{dv}{dt}j + \frac{dj}{dt}v + \frac{dw}{dt}k + \frac{dk}{dt}w\right) \tag{3.17}$$

$$m\left(\frac{du}{dt}i + \frac{dv}{dt}j + \frac{dw}{dt}k\right) = m\dot{v} \tag{3.18}$$

$$m\left(\frac{di}{dt}u + \frac{dj}{dt}v + \frac{dk}{dt}w\right) = coriolsforce \tag{3.19}$$

$$F = m\left(\frac{du}{dt}i + \frac{dv}{dt}j + \frac{dw}{dt}k\right) + m\left(\frac{di}{dt}u + \frac{dj}{dt}v + \frac{dk}{dt}w\right) \tag{3.20}$$

$$m\left(\frac{du}{dt}i+\frac{dv}{dt}j+\frac{dw}{dt}k\right)=m\dot{v} \tag{3.21}$$

$$m\left(\frac{di}{dt}u+\frac{dj}{dt}v+\frac{dk}{dt}w\right)=coriols\ \ force \tag{3.22}$$

$$m\dot{v}=(\omega\ \ x\ \ i)u+(\omega\ \ x\ \ j)v+(\omega\ \ x\ \ k)w=w^{B}xv^{B} \tag{3.23}$$

The coriols force is an apparent deflection of the path of the quadrotor that moves with in a rotating coordinate system.That mean the quadrotor does not actually deviate from it's path but it appears to do so because of the motion of the coordinate system.
where :

$$F=m\ddot{\xi}+w^{B}xv^{B} \tag{3.24}$$

$$F=\begin{bmatrix}F_x\\F_Y\\F_z\end{bmatrix}=\begin{bmatrix}m&0&0\\0&m&0\\0&0&m\end{bmatrix}\begin{bmatrix}\dot{u}\\\dot{v}\\\dot{w}\end{bmatrix}+\begin{bmatrix}0&mw&-mv\\-mw&0&mu\\mv&-mu&0\end{bmatrix}\begin{bmatrix}p\\q\\r\end{bmatrix} \tag{3.25}$$

Where the rotational dynamics can be formulated as follows by taking the developed net torque and reaction forces into consideration. Rotating torque on body frame is equal to rate of change of momentum at inertial frame.

$$\frac{dL}{dt}=\tau \tag{3.26}$$

where:

$$L=I*\omega,\qquad I=moment\ \ of\ \ inertia \tag{3.27}$$

$$\tau=L+\omega xL.....Euler\ \ equation \tag{3.28}$$

$$\tau=\frac{d(I_xp\quad I_yq\quad I_zr)}{dt}+[p\quad q\quad r]^{T}x[I_xp\quad I_yq\quad I_zr] \tag{3.29}$$

$$\tau=\begin{bmatrix}\tau_x\\\tau_y\\\tau_z\end{bmatrix}=\begin{bmatrix}I_x\dot{p}\\I_y\dot{q}\\I_z\dot{r}\end{bmatrix}+\begin{bmatrix}p\\q\\r\end{bmatrix}x\begin{bmatrix}I_xp\\I_yq\\I_zr\end{bmatrix} \tag{3.30}$$

$$\begin{bmatrix}\tau_x\\\tau_y\\\tau_z\end{bmatrix}=\begin{bmatrix}I_x&0&0\\0&I_y&0\\0&0&I_z\end{bmatrix}\begin{bmatrix}\dot{p}\\\dot{q}\\\dot{r}\end{bmatrix}+\begin{bmatrix}0&I_zr&-I_yq\\-I_zr&0&I_xp\\I_yp&-I_xp&0\end{bmatrix}\begin{bmatrix}p\\q\\r\end{bmatrix} \tag{3.31}$$

The generalized force vector λ that we agreed for a rigid body subject to external force which is $[F^{B}\ \ \tau^{B}]^{T}$ applied at the center of mass with respect to the body frame.

$$\lambda=[u\quad v\quad w\quad p\quad q\quad r]^{T}=[v^{b}\ \ \omega^{b}]^{T} \tag{3.32}$$

$$\begin{bmatrix} F_B \\ \tau_B \end{bmatrix} = [F_x \ \ F_y \ \ F_z \ \ \tau_x \ \ \tau_y \ \ \tau_z]^T \tag{3.33}$$

$$\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} + \begin{bmatrix} 0 & mw & -mv \\ -mw & 0 & mu \\ mv & -mu & 0 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{3.34}$$

Let matrix

$$M_B = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \quad , C_B(v) = \begin{bmatrix} 0 & mw & -mv \\ -mw & 0 & mu \\ mv & -mu & 0 \end{bmatrix} \tag{3.35}$$

But based on this thesis work the inertial frame is preferable to establish the dynamic model.Then the Newton second law and Euler's equation on the inertial frame for both translational and rotational dynamics become as shown below.

$$\begin{bmatrix} F_E \\ \tau_E \end{bmatrix} = \begin{bmatrix} m_I & 0 \\ 0 & I \end{bmatrix} = \begin{bmatrix} \dot{v}^E \\ \dot{\omega}^E \end{bmatrix} + \begin{bmatrix} 0 \\ \omega^E x I \omega^E \end{bmatrix} \tag{3.36}$$

The total force dynamics for translational part include the gravity and the thrust force. Then the total force dynamic of the translational part can be expressed as shown below:

$$F_e = mI \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = G + RT \tag{3.37}$$

$$m = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = -mg \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi c_\theta & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix} \tag{3.38}$$

Then by dividing both side by mass the translational dynamics will be obtained as shown below :

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + \frac{u_1}{m} \begin{bmatrix} c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi s_\theta c_\phi - c_\psi s_\phi \\ c_\theta c_\phi \end{bmatrix} \tag{3.39}$$

Finally, the system equation for translational dynamics become :

$$\ddot{x} = (\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi) \tag{3.40}$$

$$\ddot{y} = (\sin\psi \sin\theta \cos\phi - \cos\psi \sin\phi) \tag{3.41}$$

$$\ddot{z} = (\cos\theta \cos\phi) \frac{u_1}{m} - g \tag{3.42}$$

Analogous to the developed translational dynamics, the rotational dynamics can be formulated by considering the Euler's equation and the developed net torque and reaction forces into consideration.The external torque equal to the sum of the angular acceleration of inertial $I\dot{\omega}^E$, the centripetal force $\omega^E x I \omega^E$ and gyroscopic torque J_{TP}.

$$\tau = I\dot{\omega}^B + \omega^E x I \dot{\omega}^E + J_{TP} \tag{3.43}$$

$$\begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} + \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} x \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} + J_{TP} \begin{bmatrix} -\dot{\theta} \\ \dot{\phi} \\ 0 \end{bmatrix} \omega_r \tag{3.44}$$

By dividing both side by moment of inertia about the center of mass, then we can obtain the equation for rotational dynamics.

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{1}{I_{xx}} & 0 & 0 \\ 0 & \frac{1}{I_{yy}} & 0 \\ 0 & 0 & \frac{1}{I_{zz}} \end{bmatrix} + \begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} + \begin{bmatrix} \left(\frac{I_{yy} - I_{zz}}{I_{xx}}\right) \dot{\theta}\dot{\psi} \\ \left(\frac{I_{zz} - I_{xx}}{I_{yy}}\right) \dot{\phi}\dot{\psi} \\ \left(\frac{I_{xx} - I_{yy}}{I_{zz}}\right) \dot{\theta}\dot{\phi} \end{bmatrix} + \begin{bmatrix} \left(\frac{J_{TP}}{I_{xx}}\right) \dot{\theta} \\ \left(\frac{J_{TP}}{I_{yy}}\right) \dot{\phi} \\ 0 \end{bmatrix} \omega_r \tag{3.45}$$

Finally, the system equation for rotational dynamics become:

$$\ddot{\phi} = \left(\frac{I_{yy} - I_{zz}}{I_{xx}}\right) \dot{\theta}\dot{\psi} + \frac{\tau_\phi}{I_{xx}} - \left(\frac{J_{TP}}{I_{xx}}\right) \dot{\theta}\omega_r \tag{3.46}$$

$$\ddot{\theta} = \left(\frac{I_{zz} - I_{xx}}{I_{yy}}\right) \dot{\phi}\dot{\psi} + \frac{\tau_\theta}{I_{yy}} - \left(\frac{J_{TP}}{I_{yy}}\right) \dot{\phi}\omega_r \tag{3.47}$$

$$\ddot{\psi} = \left(\frac{I_{xx} - I_{yy}}{I_{zz}}\right) \dot{\phi}\dot{\theta} + \frac{\tau_\psi}{I_{zz}} \tag{3.48}$$

The maneuvers executed by the quadrotor are resultants from the manipulation of the thrust and drag moment created by the four rotors. The thrust provided by the rotor is proportional to the product of the thrust coefficient and the square of the rotor speed and the drag moment also proportional to the product of the drag coefficient and square of the rotor speed. Specifically expressed as:

$$F = b\,\omega_i^2 \tag{3.49}$$

$$\tau = d\,\omega_i^2 \tag{3.50}$$

The control inputs were chosen as to take an action for roll, pitch, yaw movements and it's driven by the rotational torque resulting in changes in attitude. Specifically expressed as:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \omega \end{bmatrix} = \begin{bmatrix} b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \\ bl(\omega_1^2 + \omega_2^2 - \omega_3^2 - \omega_4^2) \\ bl(\omega_1^2 - \omega_2^2 - \omega_3^2 + \omega_4^2) \\ d(-\omega_1^2 + \omega_2^2 - \omega_3^2 + \omega_4^2) \\ \omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2 \end{bmatrix} \tag{3.51}$$

3.6 Definition of state variables

Let's define the 12 state variables which are mapped as following in order to convert the system expression of translational and rotational acceleration into nonlinear state space model.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \\ x_{11} \\ x_{12} \end{bmatrix} = \begin{bmatrix} \phi \\ \dot{\phi} \\ \theta \\ \dot{\theta} \\ \psi \\ \dot{\psi} \\ x \\ \dot{x} \\ y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix} \tag{3.52}$$

3.7 Quadrotor Non-linear state space model

With the above defined state variables and control input the nonlinear state space model of quadrotor is obtained as shown below.The gyroscopic effect which is generated by the propeller is nullified on this thesis work because of two reasons.The first reason is that the total effect is too small and the second reason is to simplify the nonlinear state space model in the following form:

$$\dot{x} = f(x) + g(x)u \tag{3.53}$$

$$y = h(x) \tag{3.54}$$

Where $x = (x_1, x_2....x_{12})^T$is state vector,$u = (u_1, u_2, u_3, u_4)^T$is the input vector,$y = (y_1, y_2.....y_{12})^T$is the output vector of the system and $f(x), g(x)$ and $h(x)$ the nonlinear functions.

Here the nonlinear state space model is extracted in terms of state vector and input vector which can be expressed as follows $\dot{x} = f(x, u)$:

$$\dot{x_1} = x_2 \tag{3.55}$$

$$\dot{x_2} = a_1 x_4 x_6 + b_1 u_2 \tag{3.56}$$

$$\dot{x_3} = x_4 \tag{3.57}$$

$$\dot{x_4} = a_2 x_2 x_6 + b_2 u_3 \tag{3.58}$$

$$\dot{x_5} = x_6 \tag{3.59}$$

$$\dot{x_6} = a_3 x_2 x_4 + b_3 u_4 \tag{3.60}$$

$$\dot{x_7} = x_8 \tag{3.61}$$

$$\dot{x_8} = \frac{u_1}{m}(\cos x_1 \sin x_3 \cos x_5 + \sin x_1 \sin x_5) \tag{3.62}$$

$$\dot{x_9} = x_{10} \tag{3.63}$$

$$\dot{x_{10}} = \frac{u_1}{m}(\cos x_1 \sin x_3 \sin x_5 - \sin x_1 \cos x_5) \tag{3.64}$$

$$\dot{x_{11}} = x_{12} \tag{3.65}$$

$$\dot{x_{12}} = \frac{u_1}{m}(\cos x_1 \cos x_3) - g \tag{3.66}$$

Where the constant a_i and b_i refers to:

$$a_1 = \left(\frac{I_{yy} - I_{zz}}{I_x x}\right) \tag{3.67}$$

$$a_2 = \left(\frac{I_{zz} - I_{xx}}{I_y y}\right) \tag{3.68}$$

$$a_3 = \left(\frac{I_{xx} - I_{yy}}{I_z z}\right) \tag{3.69}$$

$$b_1 = \left(\frac{1}{I_{xx}}\right) \tag{3.70}$$

$$b_2 = \left(\frac{1}{I_{yy}}\right) \tag{3.71}$$

$$b_3 = \left(\frac{1}{I_{zz}}\right) \tag{3.72}$$

Where from the above equation the obtained $f(x)$ and $g(x)$ is expressed as:

$$f(x) = \begin{bmatrix} x_2 \\ a_1 x_4 x_6 \\ x_4 \\ a_2 x_2 x_6 \\ x_6 \\ a_3 x_2 x_4 \\ x_8 \\ 0 \\ x_{10} \\ 0 \\ -g \end{bmatrix} \qquad g(x) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & b_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & b_2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_3 \\ \frac{u_x}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{u_y}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{u_z}{m} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3.73}$$

Where the constant u_i refers to:

$$u_x = (\cos x_1 \sin x_3 \cos x_5 + \sin x_1 \sin x_5) \tag{3.74}$$

$$u_y = (\cos x_1 \sin x_3 \sin x_5 - \sin x_1 \cos x_5) \tag{3.75}$$

$$u_z = (\cos x_1 \cos x_3) \tag{3.76}$$

Chapter 4

Control system design

4.1 Feedback linearization control

In recent decades, most researches are attracted alot by feedback linearization control design approach for solving control system problems. The central idea is to algebraically transform nonlinear systems dynamics into fully or partly linear ones by cancelling the nonlinearities [14]. There are two feedback linearization techniques, those are input-output linearization and input -state linearization techniques. In this thesis input-output linearization techniques are used. Here the main point of using input-output linearization techniques is in order to linearize the map between the transformed input v and the actual output y. Input-output linearization techniques have a repeated differentiation of the output variables until the input terms appear and at the last derivative on the r^{th} term the input will appear. The appeared input variable helps in obtaining a mapping between the transformed input and the output variables [21].
The first step in feedback linearization techniques was input transformation from the nonlinear state space model. which means transforming the input state variable into z coordinate. The input transformation is given by:

$$z_1 = x_{11} = z \tag{4.1}$$

$$z_2 = x_{12} = \dot{z} \tag{4.2}$$

$$z_3 = x_1 = \phi \tag{4.3}$$

$$z_4 = x_2 = \dot{\phi} \tag{4.4}$$

$$z_5 = x_3 = \theta \tag{4.5}$$

$$z_6 = x_4 = \dot{\theta} \tag{4.6}$$

$$z_7 = x_5 = \psi \tag{4.7}$$

$$z_8 = x_6 = \dot{\psi} \tag{4.8}$$

$$z_9 = x_7 = x \tag{4.9}$$

$$z_{10} = x_8 = \dot{x} \tag{4.10}$$

$$z_{11} = x_9 = y \tag{4.11}$$

$$z_{12} = x_{10} = \dot{y} \tag{4.12}$$

The second step is derivate the output (y) through lie derivative until the input terms appears. The control scheme has two control loops, namely the inner loop that control the altitude and attitude of the system and the outer loop that controls the position. Let's first consider the inner loop of the quadrotor system as an output equation and applying the lie derivative.
The output state to be controlled are:

$$y = h(x) = [z, \phi, \theta, \psi]^T \tag{4.13}$$

Steps which is followed that, the derivative of the output y can be expressed in terms of lie derivative and the i^{th} derivative of (y) is expressed in terms lie derivative as shown below:

$$\dot{y} = \frac{\partial h}{\partial x} = [f(x) + g(x)u] = L_f h(x) + L_g h(x)u \tag{4.14}$$

$$y^{(i)} = L_f^{(i)} h(x) + L_g L_f^{(i-1)} h(x)u \tag{4.15}$$

Finally, the above equation can be linearized through dynamic inversion, choosing the control Law as:

$$u = \frac{1}{L_g L_f^{(i-1)} h(x)} [-L_f^{(i)} h(x) + v] \tag{4.16}$$

Then the control law transforms the non-linear system into linear system with $v \in R^n$ an auxiliary control input.

$$y^{(i)} = v \tag{4.17}$$

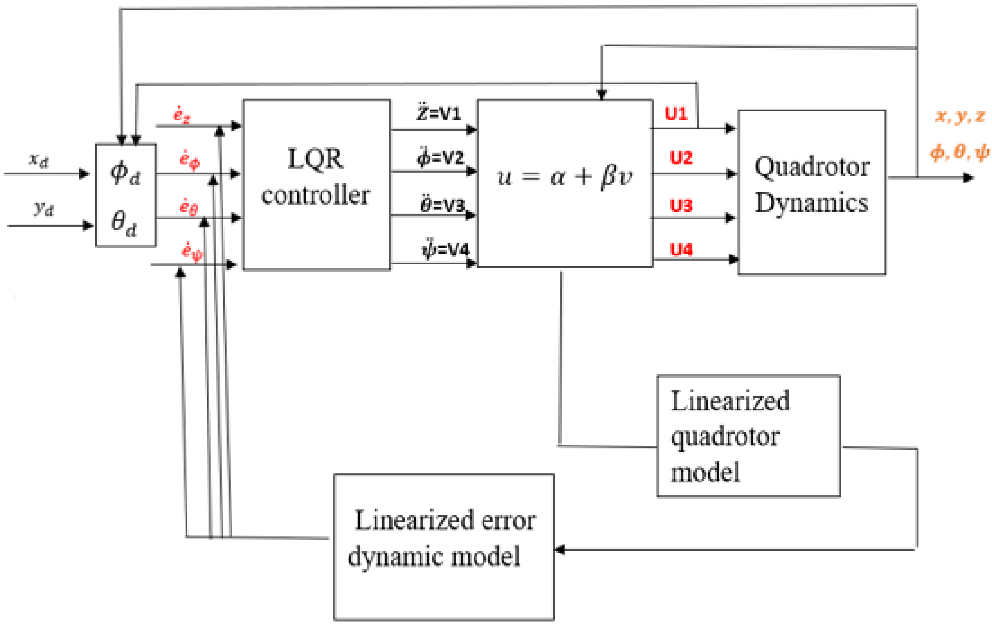


Figure 4.1: control scheme

By following the above steps appling the lie derivative to the output equation and it's expressed as:

For the altitude

$$y = z \tag{4.18}$$

$$\dot{y} = \dot{z} = x_{12} \tag{4.19}$$

$$\ddot{y} = \ddot{z} = \dot{x}_{12} \tag{4.20}$$

$$\ddot{y} = \ddot{z} = \dot{x}_{12} = \frac{u_1}{m}(cosz_3 \;\; cosz_5) - g = v_1 \tag{4.21}$$

The **attitude parts** are as shown below:

For phi angle

$$y = \phi \tag{4.22}$$

$$\dot{y} = \dot{\phi} = x_2 \tag{4.23}$$

$$\ddot{y} = \ddot{\phi} = \dot{x}_2 \tag{4.24}$$

$$\ddot{y} = \ddot{\phi} = \dot{x}_2 = a_1 z_6 z_8 + b_1 u_2 = v_2 \tag{4.25}$$

For pitch angle

$$y = \theta \tag{4.26}$$

$$\dot{y} = \dot{\theta} = x_4 \tag{4.27}$$

$$\ddot{y} = \ddot{\theta} = \dot{x}_4 \tag{4.28}$$

$$\ddot{y} = \ddot{\theta} = \dot{x}_4 = a_2 z_4 z_8 + b_2 u_3 = v_3 \tag{4.29}$$

For yaw angle

$$y = \psi \tag{4.30}$$

$$\dot{y} = \dot{\psi} = x_6 \tag{4.31}$$

$$\ddot{y} = \ddot{\psi} = \dot{x}_6 \tag{4.32}$$

$$\ddot{y} = \ddot{\psi} = \dot{x}_6 = a_3 z_4 z_6 + b_3 u_4 = v_4 \tag{4.33}$$

Where for the inner loop:

$$z = [\dot{z} \;\; \ddot{z} \;\; \dot{\phi} \;\; \ddot{\phi} \;\; \dot{\theta} \;\; \ddot{\theta} \;\; \dot{\psi} \;\; \ddot{\psi}]^T \tag{4.34}$$

$$v = [v_1 \;\; v_2 \;\; v_3 \;\; v_4]^T = [\ddot{z} \;\; \ddot{\phi} \;\; \ddot{\theta} \;\; \ddot{\psi}]^T \tag{4.35}$$

Lie derivative applied again for the outer loop equation and it's expressed as:
Where for the outer loop:

$$z = [\dot{x} \;\; \ddot{x} \;\; \dot{y} \;\; \ddot{y}]^T \tag{4.36}$$

$$v = [v_5 \;\; v_6]^T = [\ddot{x} \;\; \ddot{y}]^T \tag{4.37}$$

For x -position

$$y = x \tag{4.38}$$

$$\dot{y} = \dot{x} = x_8 \tag{4.39}$$

$$\ddot{y} = \ddot{x} = \dot{x}_8 \tag{4.40}$$

$$\ddot{y} = \ddot{x} = \dot{x}_8 = \frac{u_1}{m}(\cos z_3 \;\sin z_5 \cos z_7 + \sin z_3 \sin z_7) = v_5 \tag{4.41}$$

For y -position

$$y = y \tag{4.42}$$

$$\dot{y} = \dot{y} = x_{10} \tag{4.43}$$

$$\ddot{y} = \ddot{y} = \dot{x}_{10} \tag{4.44}$$

$$\ddot{y} = \ddot{y} = \dot{x}_{10} = \frac{u_1}{m}(\cos z_3 \sin z_5 \sin z_7 - \sin z_3 \cos z_5) = v_6 \tag{4.45}$$

The small angle approximation is applied on the dynamics as shown below.Let assume that $\psi = 0$ with $\theta \sim 0$ and $\phi \sim 0$.Which implies :$\sin\theta = \theta, \cos\theta = 1, \sin\phi = \phi$ and $\cos\phi = 1$

$$\ddot{z} = \dot{x}_{12} = \frac{u_1}{m} - g \tag{4.46}$$

$$\ddot{x} = \dot{x}_8 = \frac{u_1}{m}\theta \tag{4.47}$$

$$\ddot{y} = \dot{x}_{10} = \frac{u_1}{m}\phi \tag{4.48}$$

The new linearized coordinates in state space model expressed as:

$$\dot{z} = Az + Bv \tag{4.49}$$

$$y = cz \tag{4.50}$$

Then the linearized state space model for inner loop become:

$$\dot{z_1} = z_2 \tag{4.51}$$

$$\dot{z_2} = v_1 \tag{4.52}$$

$$\dot{z_3} = z_4 \tag{4.53}$$

$$\dot{z_4} = v_2 \tag{4.54}$$

$$\dot{z_5} = z_6 \tag{4.55}$$

$$\dot{z_6} = v_3 \tag{4.56}$$

$$\dot{z_7} = z_8 \tag{4.57}$$

$$\dot{z_8} = v_4 \tag{4.58}$$

The linearized state space model for outer loop become:

$$\dot{z_9} = z_{10} \tag{4.59}$$

$$\dot{z_{10}} = v_5 \tag{4.60}$$

$$\dot{z_{11}} = z_{12} \tag{4.61}$$

$$\dot{z_{12}} = v_6 \tag{4.62}$$

Where the linear auxiliary control input expressed as:

$$v = [v_1 \;\; v_2 \;\; v_3 \;\; v_4]^T \tag{4.63}$$

$$v_1 = \frac{u_1}{m} - g \tag{4.64}$$

$$v_2 = a_1 z_6 z_8 + b_1 u_2 \tag{4.65}$$

$$v_3 = a_2 z_4 z_8 + b_2 u_3 \tag{4.66}$$

$$v_4 = a_3 z_4 z_6 + b_3 u_4 \tag{4.67}$$

$$v_5 = \frac{u_1}{m}\theta \tag{4.68}$$

$$v_6 = -\frac{u_1}{m}\phi \tag{4.69}$$

WhereA, B, C for inner loop refers to:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.70}$$

$$c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{4.71}$$

Where A, B, C for outer loop refers to:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{4.72}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{4.73}$$

From the control law of feedback linearization ,each controllers can obtain though the dynamic inversion and by choosing appropriate linear inputs v with linear quadratic regulator (LQR) controller we can shape the response of the system based on the control law u.

The control law is written based on the general form:

$$u = \beta^{-1}(-\alpha + v) \tag{4.74}$$

Considering the control law, the obtained control inputs are written as follow:

Altitude control

$$u_1 = m(v_1 + g) \tag{4.75}$$

Attitude control

Here on the attitude control roll ,pitch and yaw angles are controlled and also u_2 and u_3 controls the positions of x and y.

$$u_2 = \frac{1}{b_1}(v_2 - a_1 z_6 z_8) \tag{4.76}$$

$$u_3 = \frac{1}{b_2}(v_3 - a_2 z_4 z_8) \tag{4.77}$$

$$u_4 = \frac{1}{b_3}(v_4 - a_3 z_4 z_6) \tag{4.78}$$

position control

For controlling the dynamics x, y position the desired roll angle ϕ_d and desired pitch angle θ_d is calculated as shown below:

$$\phi_d = -\frac{m}{u_1}[\ddot{y}_d + k_{2\phi}(\dot{y} - \dot{y}_d) + k_{1\phi}(y - y_d)] \tag{4.79}$$

$$\theta_d = \frac{m}{u_1}[\ddot{x}_d + k_{2\theta}(\dot{x} - \dot{x}_d) + k_{1\theta}(x - x_d)] \tag{4.80}$$

4.2 Linear Quadratic Regulator

A linear control design techniques is used in order to get appropriate linear auxiliary control input v. LQR is an optimal control technique used to determine the control signal which derives the quadrotor states to the desired value and the objective is to find state feedback law which minimizing the cost function with right choice of weighting matrices Q and R.

Before design the LQR controller the linearized error dynamics should be designed from the linearized dynamics which comes from feedback linearization.

The error dynamics are defined as:

$$e_z = z - z_d \tag{4.81}$$

$$e_\phi = \phi - \phi_d \tag{4.82}$$

$$e_\theta = \theta - \theta_d \tag{4.83}$$

$$e_\psi = \psi - \psi_d \tag{4.84}$$

Where $z_d(t)$,$\phi_d(t)$, $\theta_d(t)$ and $\psi_d(t)$ are the desired twice differentiable trajectories from the coordinates z, ϕ, θ and ψ respectively.
The acceleration of above errors is given by:

$$\ddot{e_z} = q_z - \ddot{z_d} \tag{4.85}$$

$$\ddot{\phi_z} = q_\phi - \ddot{\phi_d} \tag{4.86}$$

$$\ddot{e_\theta} = q_\theta - \ddot{\theta_d} \tag{4.87}$$

$$\ddot{e_\psi} = q_\psi - \ddot{\psi_d} \tag{4.88}$$

Defining new auxiliary inputs $\hat{q}_z = q_z - \ddot{z_d}$, $\hat{q}_\phi = q_\phi - \ddot{\phi_d}$, $\hat{q}_\theta = q_\theta - \ddot{\theta_d}$ and $\hat{q}_\psi = q_\psi - \ddot{\psi_d}$ the dynamics of the errors in matrix form are given by:

$$\dot{e} = Ae + B\hat{q}, \qquad \hat{q} = -ke = v \tag{4.89}$$

where

$$e = \begin{bmatrix} e_z \\ \dot{e_z} \\ e_\phi \\ \dot{e_\phi} \\ e_\theta \\ \dot{e_\theta} \\ e_\psi \\ \dot{e_\psi} \end{bmatrix} \qquad \hat{q} = \begin{bmatrix} \hat{q_z} \\ \hat{q_\phi} \\ \hat{q_\theta} \\ \hat{q_\psi} \end{bmatrix} \tag{4.90}$$

$$k = \begin{bmatrix} k_{1z} & k_{2z} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & k_{1\phi} & k_{2\phi} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{1\theta} & k_{2\theta} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & k_{1\psi} & k_{2\psi} \end{bmatrix} \tag{4.91}$$

Then a static state feedback $q = -Ke$ can be defined, where the matrix K is calculated to minimize the cost function which is:

$$J = \frac{1}{2}\int_0^\infty (e^T Q e + \hat{q}^T R \hat{q})dt, \quad Q \geq \ \ R > 0 \tag{4.92}$$

where

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} \eta_z & 0 & 0 & 0 \\ 0 & \eta_\phi & 0 & 0 \\ 0 & 0 & \eta_\theta & 0 \\ 0 & 0 & 0 & \eta_\psi \end{bmatrix} \tag{4.93}$$

The gain matrix value of K is obtained by solving the Riccati's algebraic equation performed with MATLAB using the LQR function:

$$k = lqr(A, B, Q, R) \tag{4.94}$$

On the weighting matrices R the parameter $\eta_z, \eta_\phi, \eta_\theta, \eta_\psi$ is selected with time convergence of the dynamic errors with minimum cotrol effort.That mean poorly selection of Q and R it may cause inaccurate tracking ,take large control effort,instability. Where, the control input results in:

$$\hat{q_z} = \ddot{z_d} + \hat{q_z} = \ddot{z_d} - k_{2z}\dot{e_z} - k_{1z}e_z \tag{4.95}$$

$$\hat{q_\phi} = \ddot{\phi_d} + \hat{q_\phi} = \ddot{\phi_d} - k_{2\phi}\dot{e_\phi} - k_{1\phi}e_\phi \tag{4.96}$$

$$\hat{q_\theta} = \ddot{\theta_d} + \hat{q_\theta} = \ddot{\theta_d} - k_{2\theta}\dot{e_\theta} - k_{1\theta}e_\theta \tag{4.97}$$

$$\hat{q_\psi} = \ddot{\psi_d} + \hat{q_\psi} = \ddot{\psi_d} - k_{2\psi}\dot{e_\psi} - k_{1\psi}e_\psi \tag{4.98}$$

The parameter $k_{1z}, k_{2z}, k_{1\phi}, k_{2\phi}, k_{1\theta}, k_{2\theta}, k_{1\psi}$ and $k_{2\psi}$ are the gain matrix which is calculated with Riccati's algebraic equation performed with MATLAB using the LQR function.The gain matrix make the dynamic errors to convergences to zero.

4.3 Over all control structure with reference trajectory

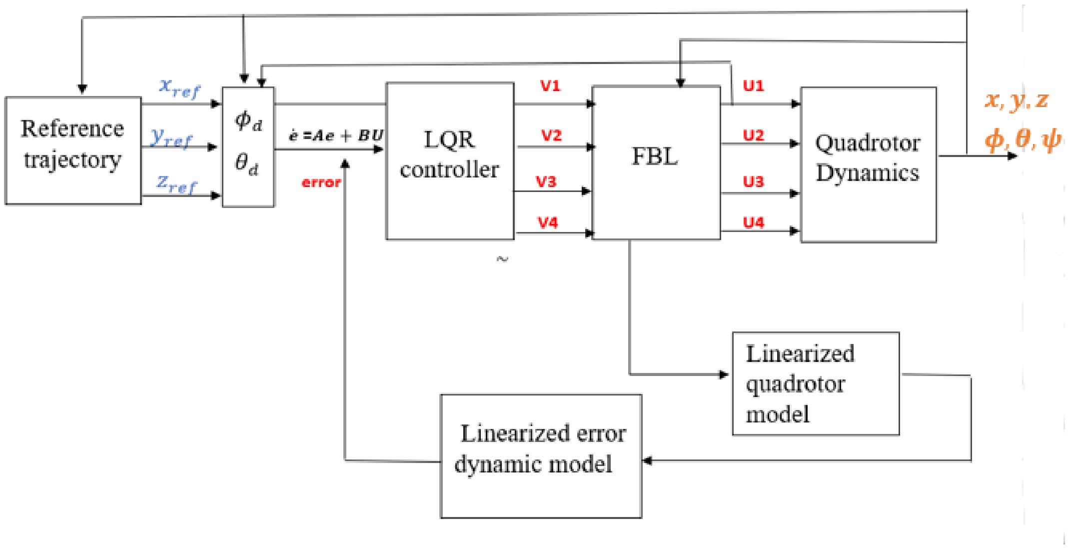


Figure 4.2: Over all control structure with reference trajectory

The over all control structure have two controller loops.The first loop or the inner loop controls the altitude (z) and the attitudes(ϕ, θ, ψ).The outer loop controls the x and y positions by controlling desired roll angle and pitch angle.

Chapter 5

Simulation Results and Discussion

5.1 Quadrotor dynamics simulation model

Here in this section, the quadrotor dynamic model which has been studied in the pervious section is implemented based on the principle of operation and the equations of motion which has been formulated based on Newton-Euler formulation. The dynamics model of the quadrotor has been implemented using MATLAB function block and the output of this block are state derivatives which are shown with a scope for both translational and rotational dynamics, where the integrator is used to output the state vector which is used in the dynamics.

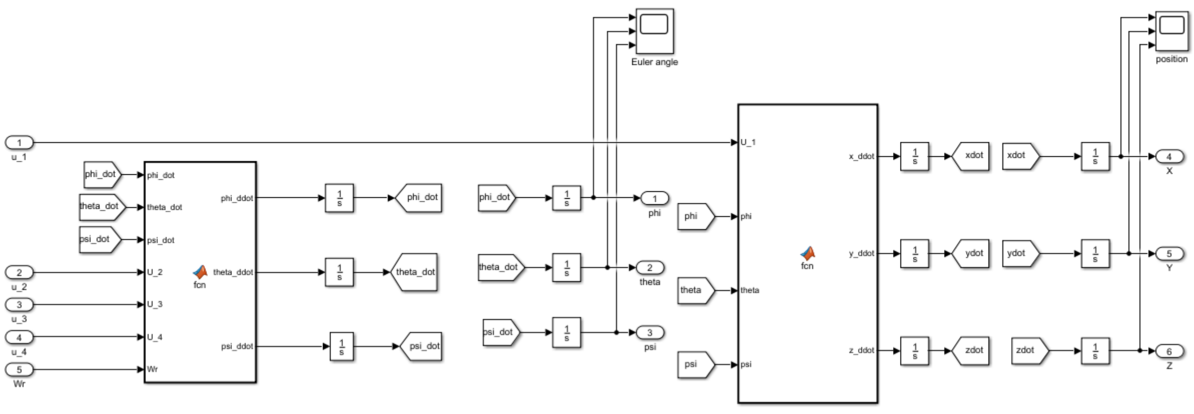


Figure 5.1: Illustrate the quadrotor dynamics with MATLAB/simulink function block

5.2 Simulation parameter specification

Here in this section, the parameter specifications which is used for MATLAB simulation is presented . Table 5.1 presents the parameter specification for quadrotor model simulation.

Parameter	**Values**	**Units**
m	1	Kg
g	9.81	$\frac{m}{s^2}$
L	0.25	m
I_{xx}	0.0081	$Kg.m^2$
I_{yy}	0.0081	$Kg.m^2$
I_{zz}	0.0142	$Kg.m^2$
J_{TP}	$1.04x10^{-4}$	$Kg.m^2$
b	$5.42x10^{-5}$	$N.s^2$
d	$1.1x10^-6$	Nms^2

Table 5.1: parameter specification for quadrotor model ref[22]

5.3 Step response for Quadrotor dynamic model

The step input is used to test the quadrotor dynamic model in order to see the principle of operation of quadrotor and how the dynamics responding faithfully to the step input command. On the dynamical model, the step inputs are given to u_1, u_2, u_3 and u_4 as inputs.Fig 5.2 shows the quadrotor dynamic model with step input in MATLAB function block.

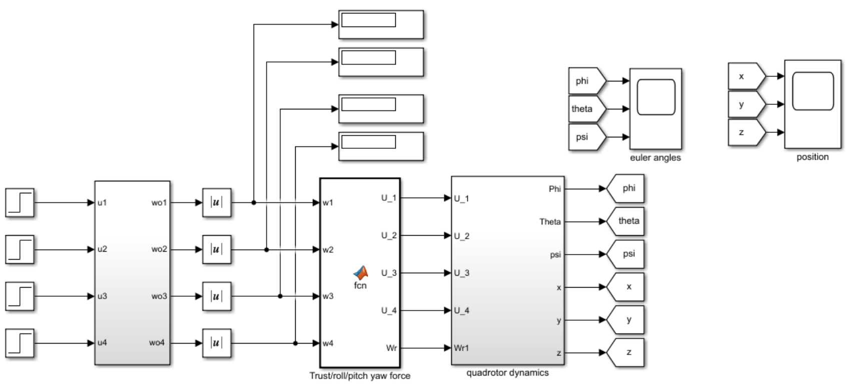
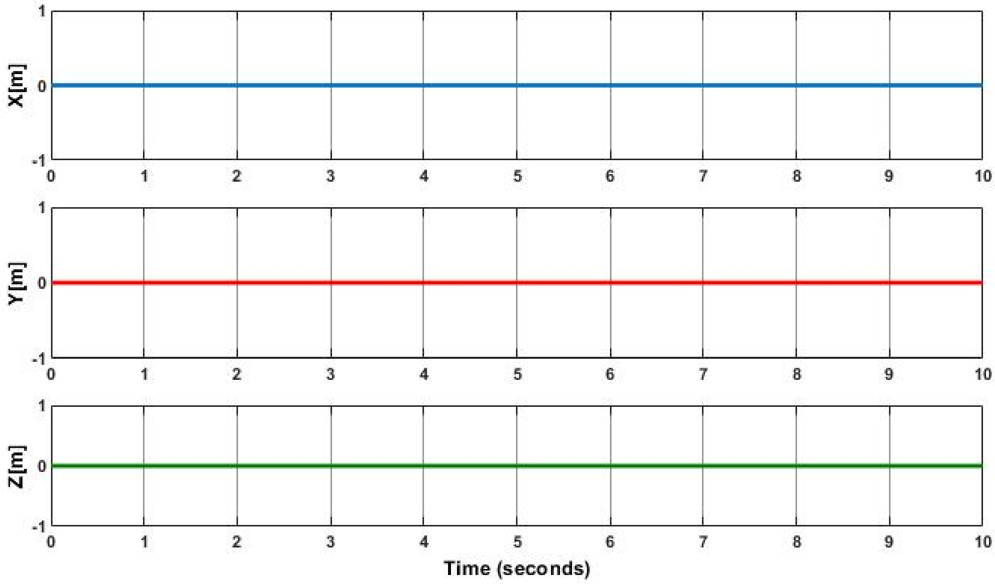
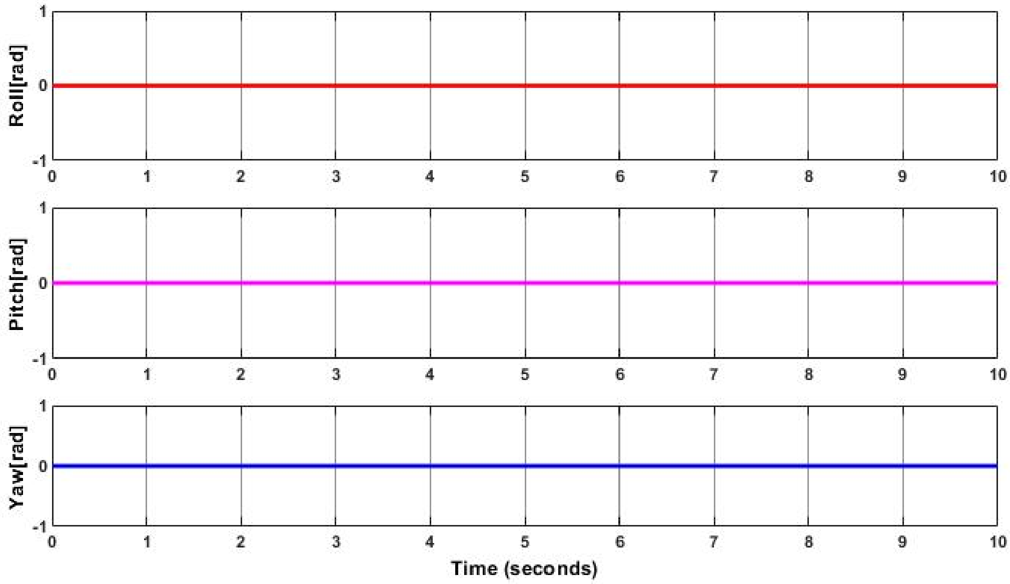


Figure 5.2: quadrotor dynamic model with step input in MATLAB function block

5.3.1 Simulation result for step input

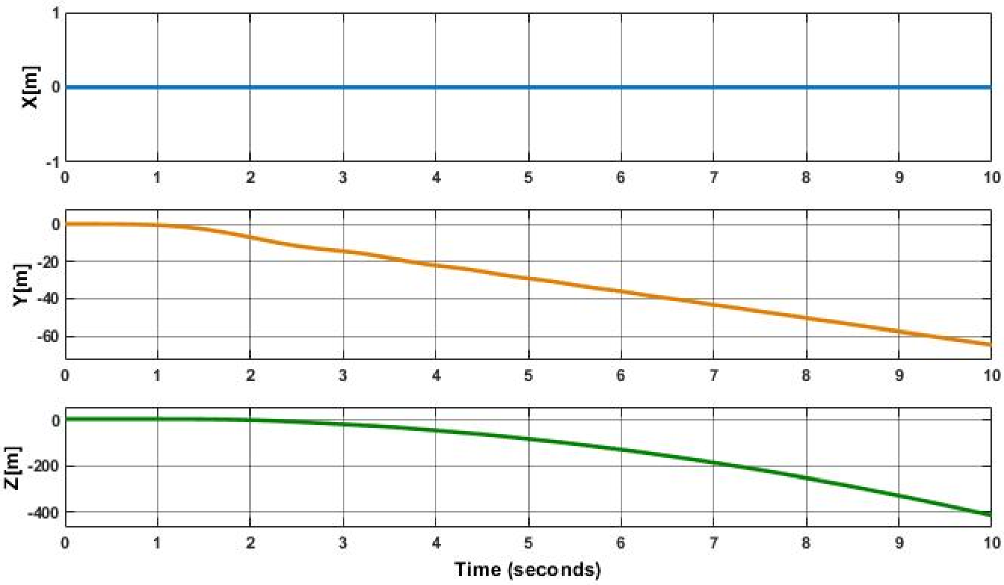


(a) Position

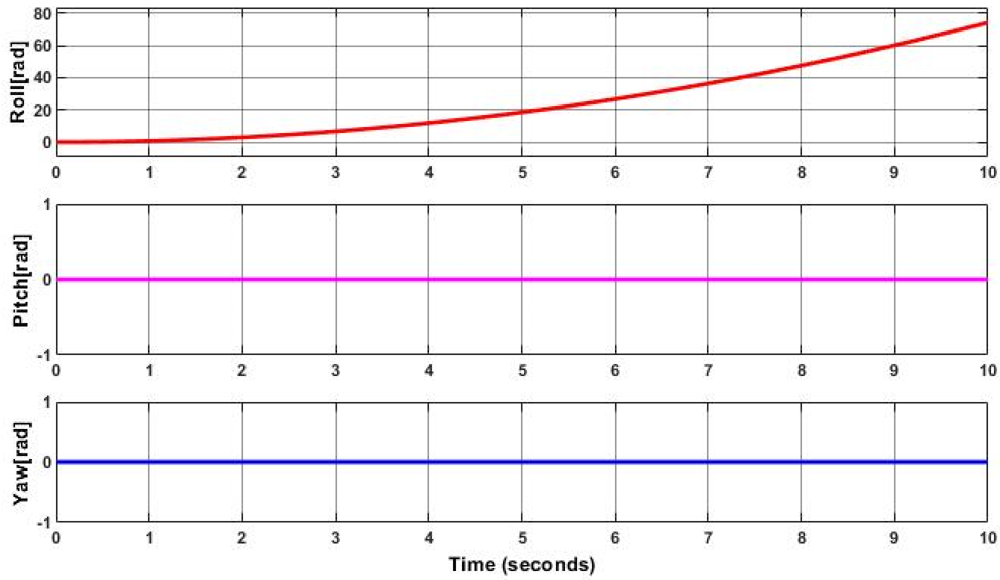


(b) Angle

Figure 5.3: Step response at Z=0m (hovering condition)

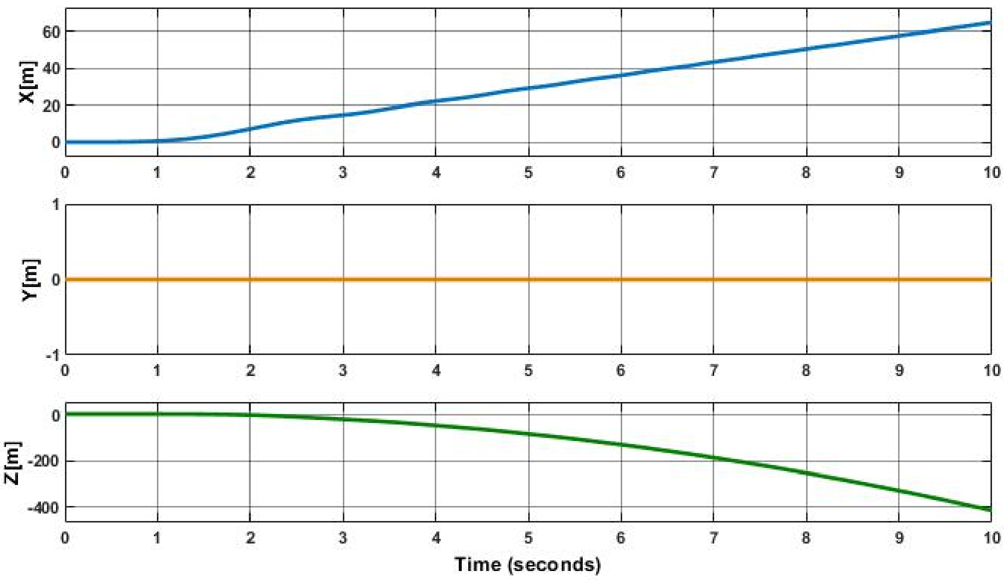


(a) Position

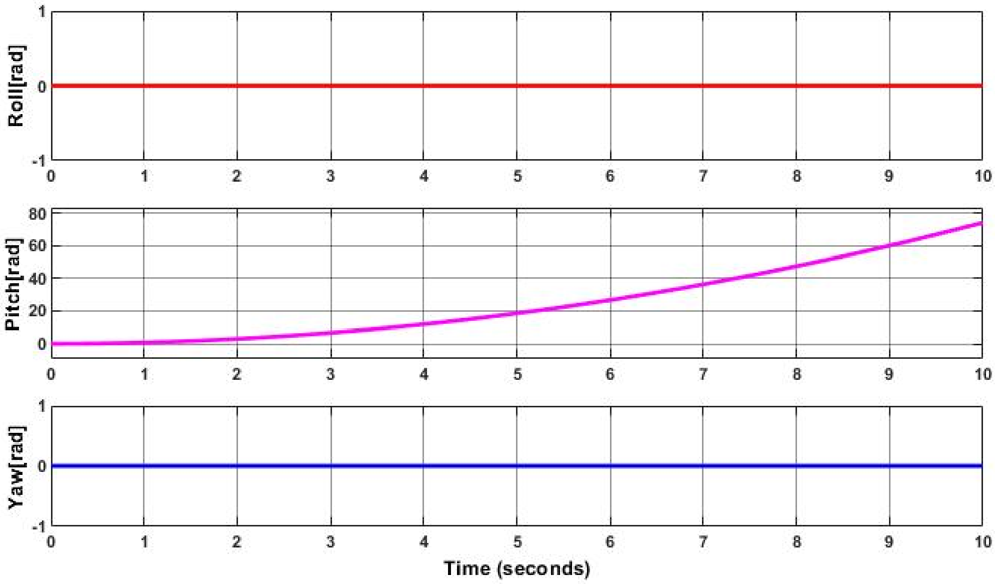


(b) Angle

Figure 5.4: Step response for u_2 (Roll)

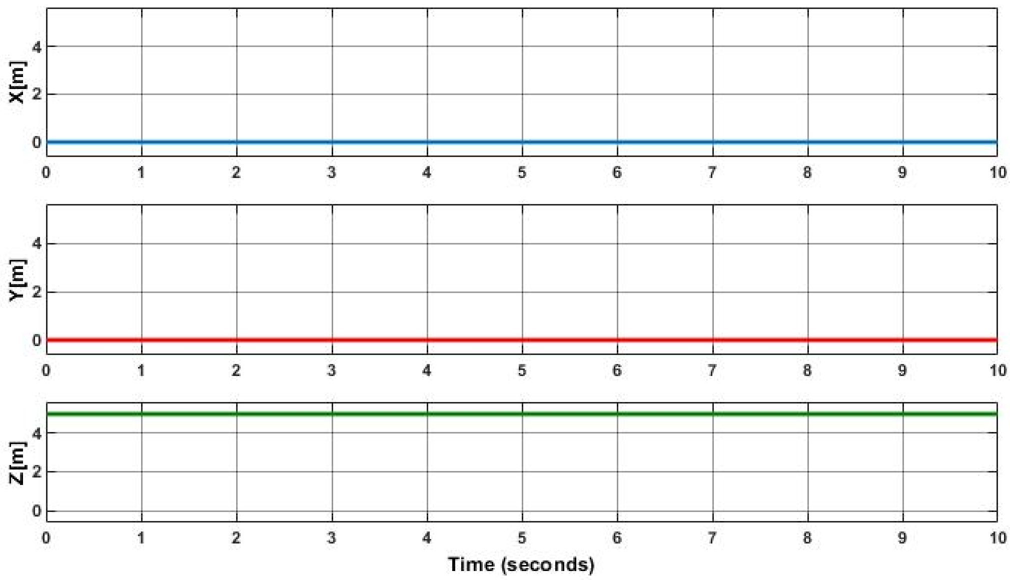


(a) Position

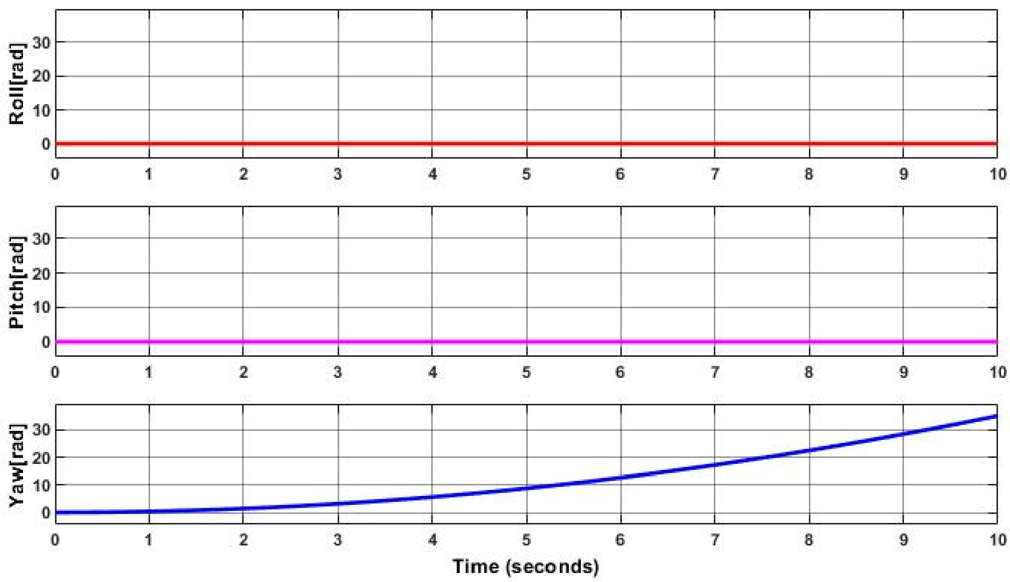


(b) Angle

Figure 5.5: Step response for u_3 (pitch)



(a) position



(b) Angle

Figure 5.6: Step response for u_4 (yaw to counter clockwise and hover at Z=5 m)

5.4 Simulation configuration of controller design with quadrotor dynamics

The controller design section consists the feedback linearization technique and linear quadratic controller(LQR) .The feedback linearization technique canceling the non linearities and convert in to linear dynamics model .On the Simulink block the feedback linearization consists the input transformation,state transformation and the auxiliary inputs. After linearization the linear quadratic controller(LQR) is used to control the linear quadrotor dynamic model,here LQR is used in order to get the appropriate linear control input V (auxiliary inputs). On the Simulink block the LQR controller consists the error dynamics which is feed to the LQR controller and the new auxiliary inputs interms of error dynamics .The gain matrix K calculated with Riccatic equation which is performed with MATLAB code.
The function block in front of the LQR controller is the desired roll angle and desired pitch angle which is used to control the x and y position.

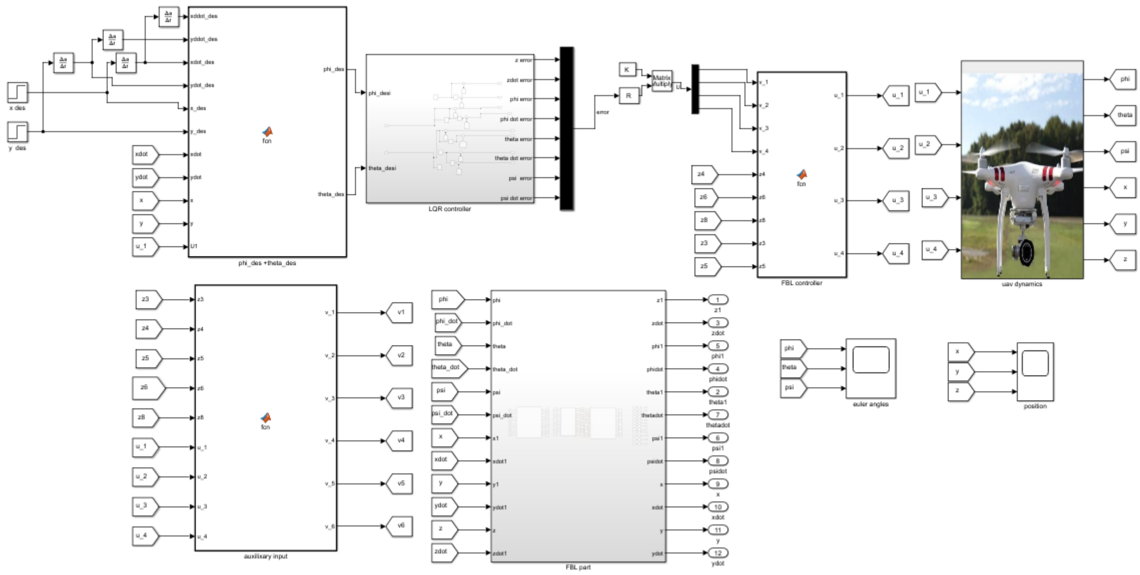


Figure 5.7: Simulation configuration of controller design with quadrotor dynamics

5.5 Over all simulation configuration

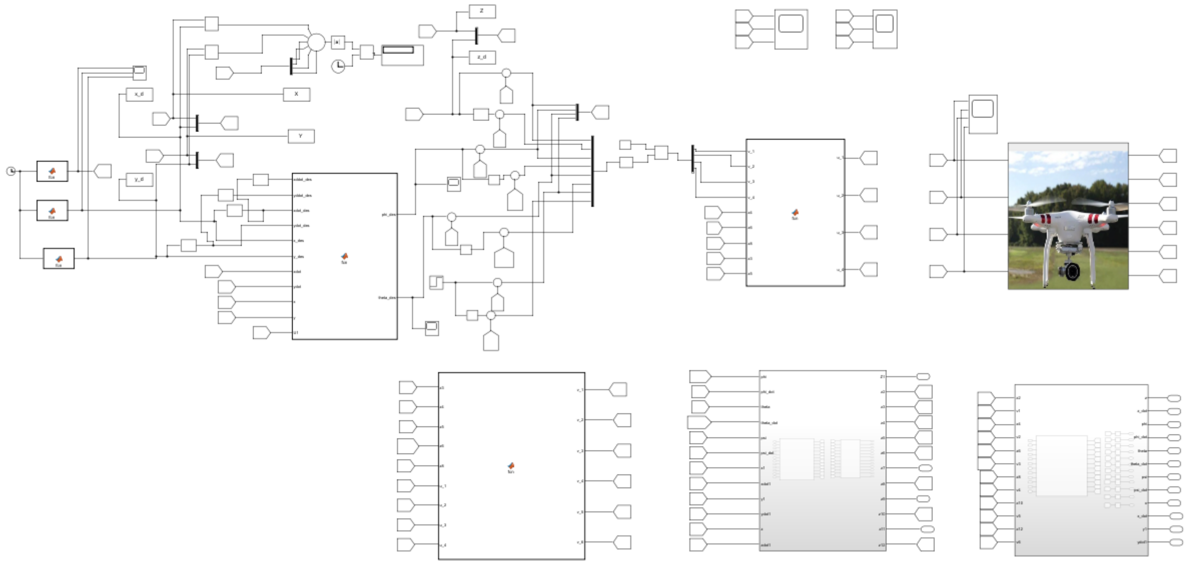


Figure 5.8: Over all simulation configuration

5.6 Constant tracking simulation result

The constant tracking is performed for x,y,z at 100 m as shown below:

$$\begin{bmatrix} x_d \\ y_d \\ z_d \end{bmatrix} = \begin{bmatrix} 100m \\ 100m \\ 100m \end{bmatrix}$$

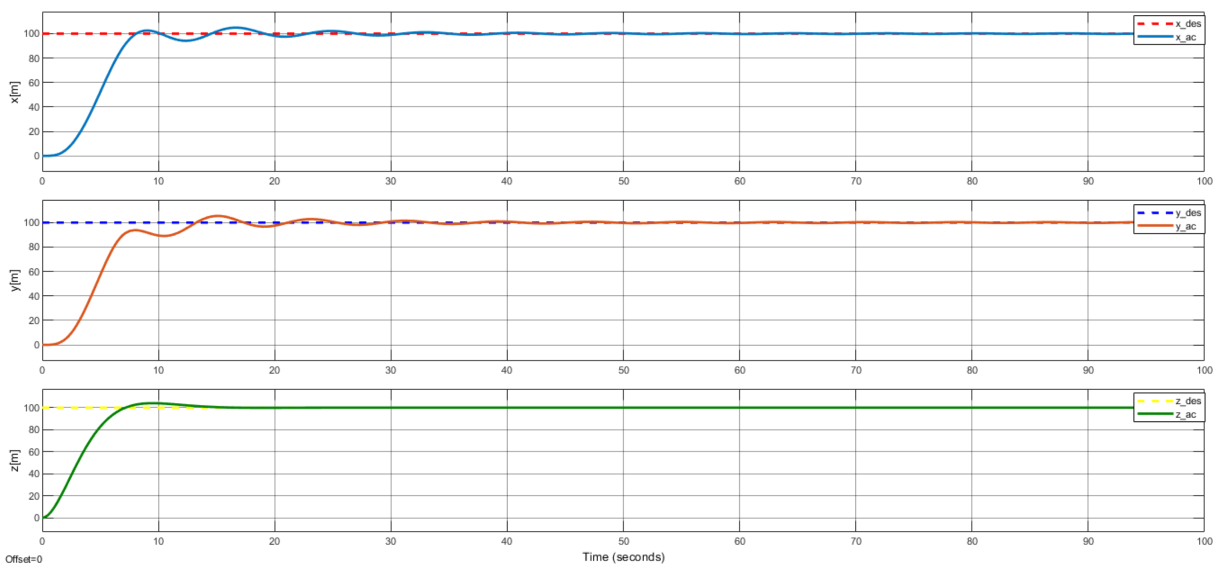


Figure 5.9: constant tracking for position

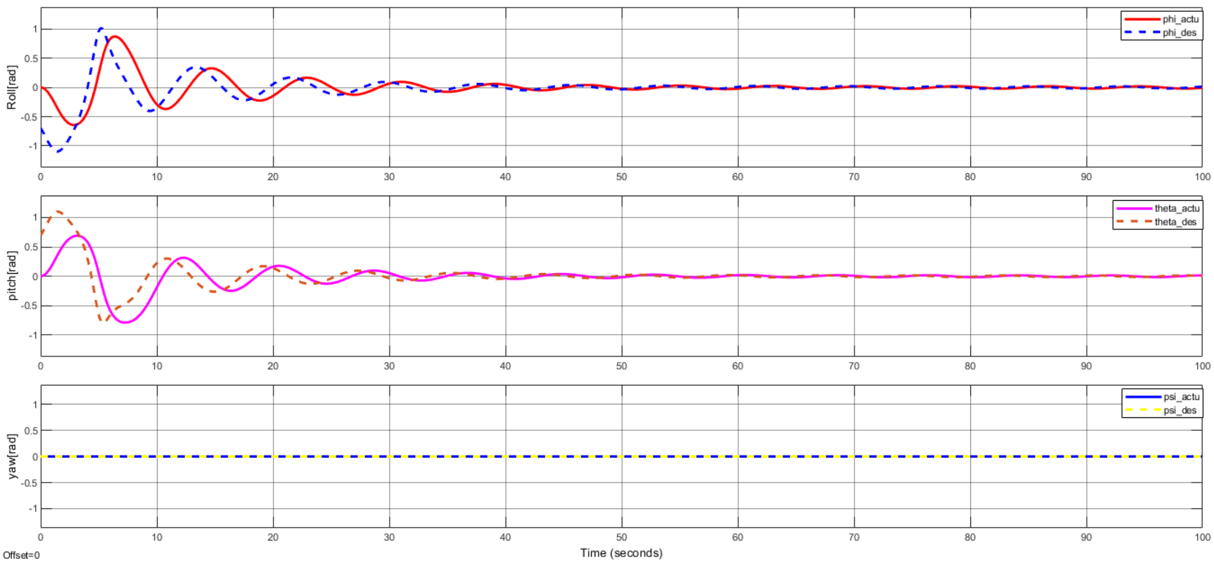


Figure 5.10: constant tracking for angle

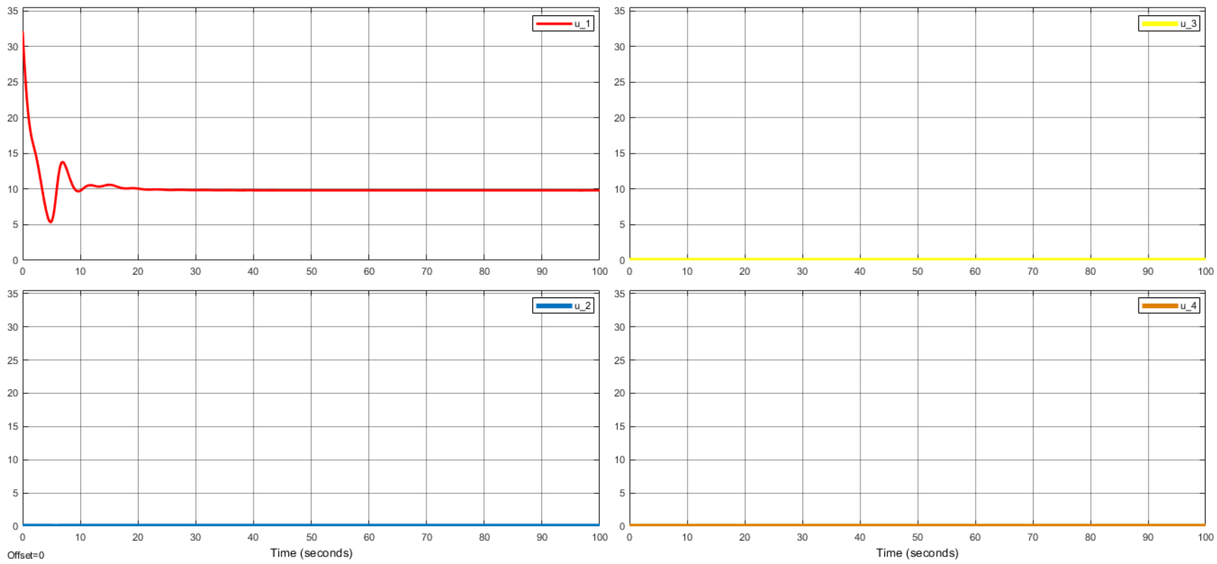


Figure 5.11: Constant tracking control input

The value of Q,R and K matrix for constant tracking are

$$Q = 5 \quad R = 100 \quad K = \begin{bmatrix} 0.2236 & 0.6762 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2236 & 0.6762 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.2236 & 0.6762 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 1.7321 \end{bmatrix} \tag{5.1}$$

5.7 Rectangular path trajectory tracking result

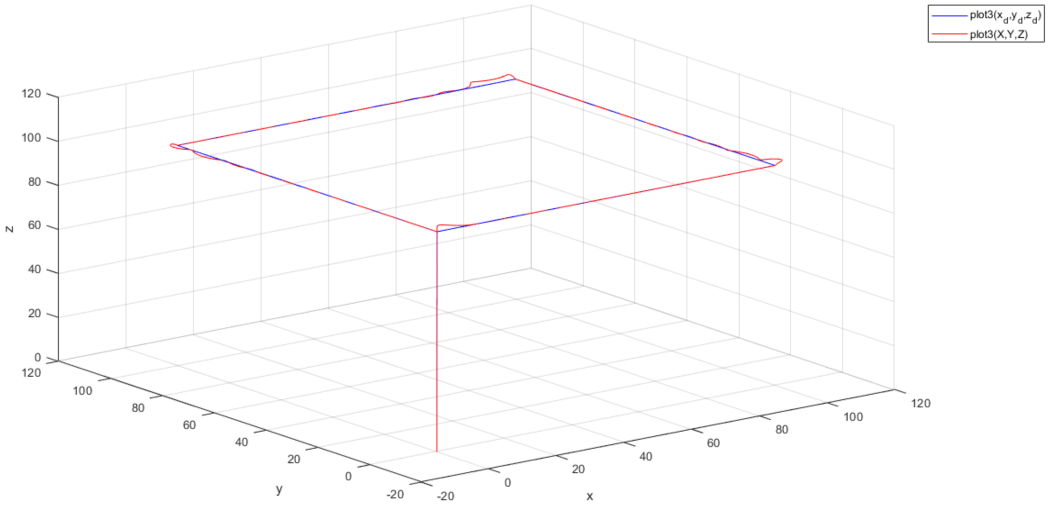


Figure 5.12: Rectangular path trajectory tracking

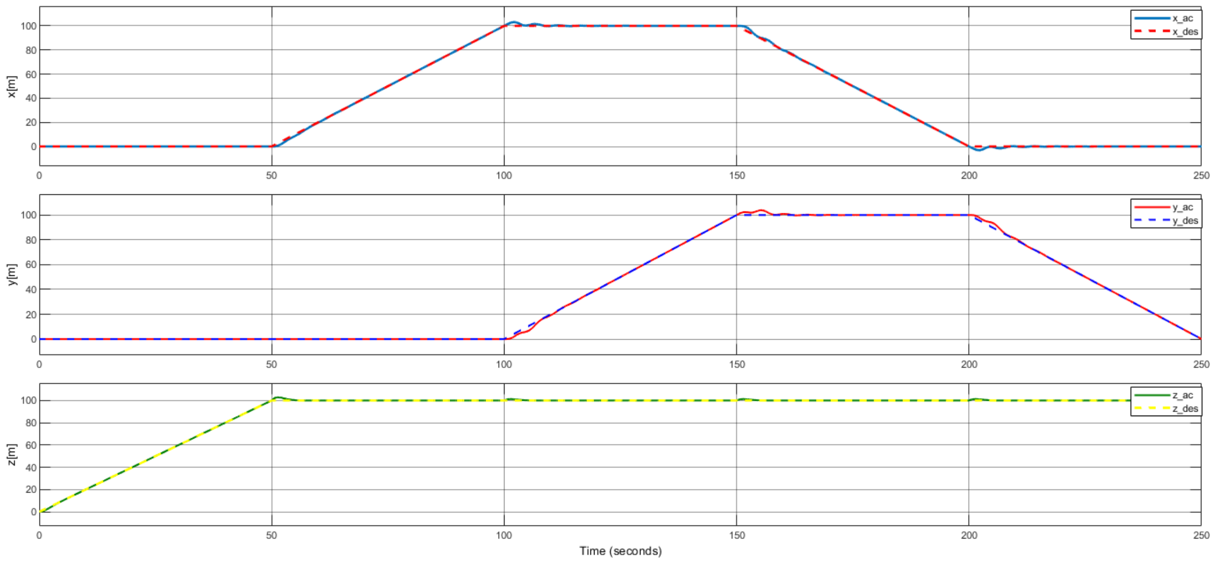


Figure 5.13: Rectangular path trajectory tracking position

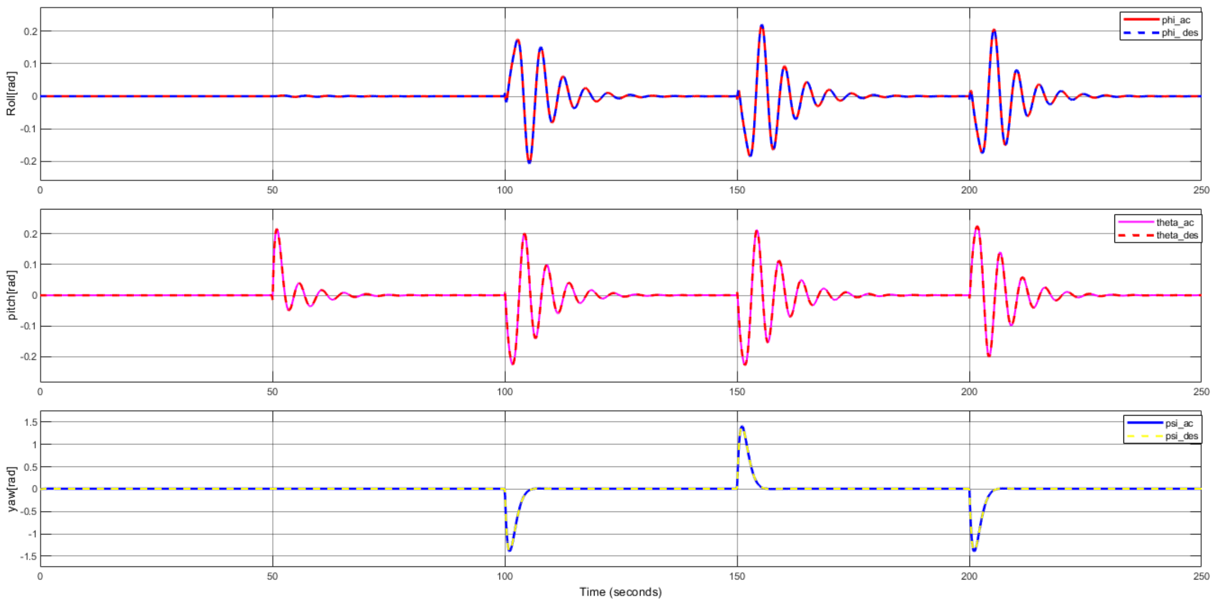


Figure 5.14: Rectangular path trajectory tracking angle

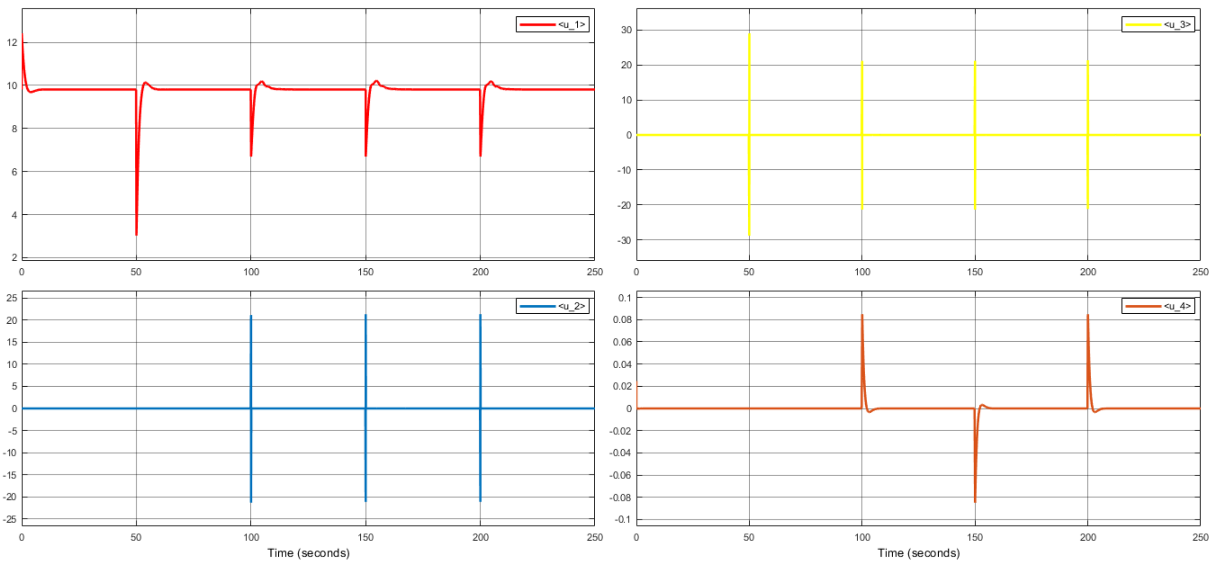


Figure 5.15: Rectangular path trajectory tracking control input

The value of Q,R and K matrix for rectangular path trajectory tracking are

$$Q = 40 \quad R = 100 \quad K = \begin{bmatrix} 0.6325 & 1.2903 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.6325 & 1.2903 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6325 & 1.2903 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 1.7321 \end{bmatrix} \tag{5.2}$$

5.8 Rectangular path trajectory tracking under disturbance

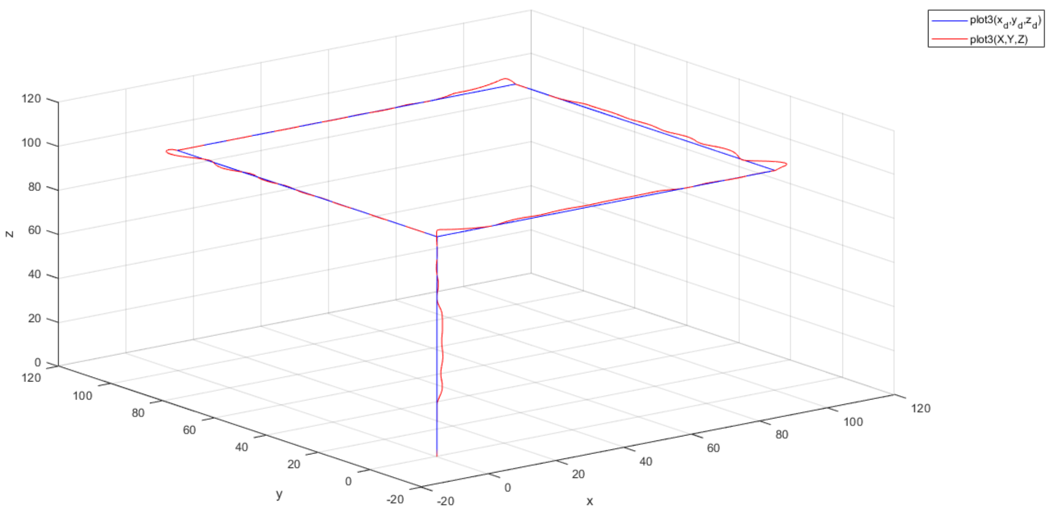


Figure 5.16: Rectangular path trajectory tracking under disturbance

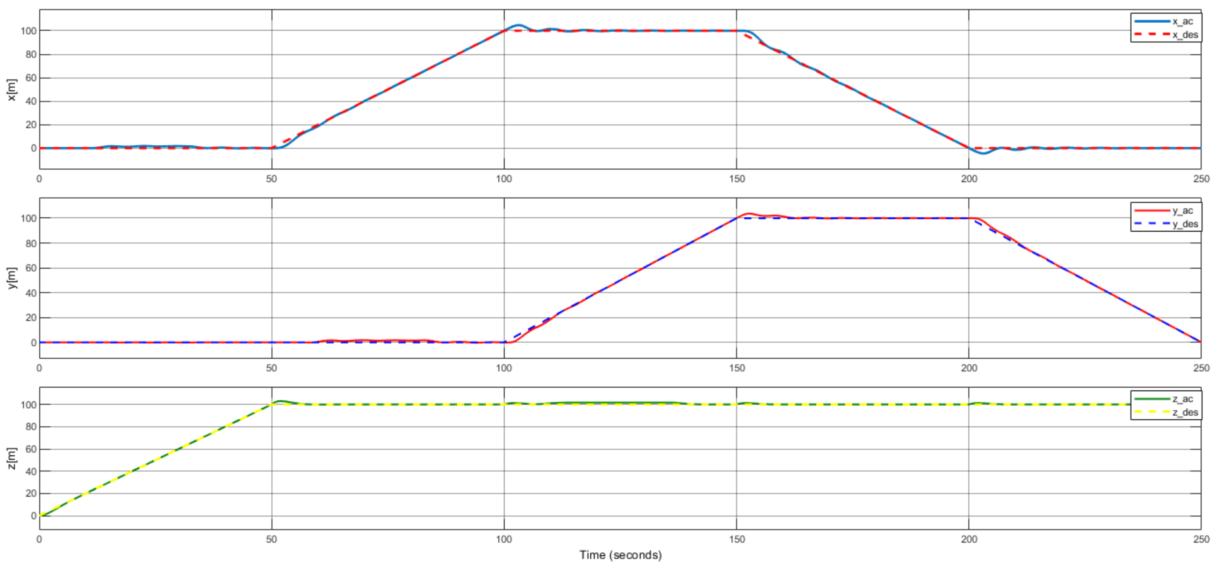


Figure 5.17: Rectangular path trajectory tracking under disturbance position

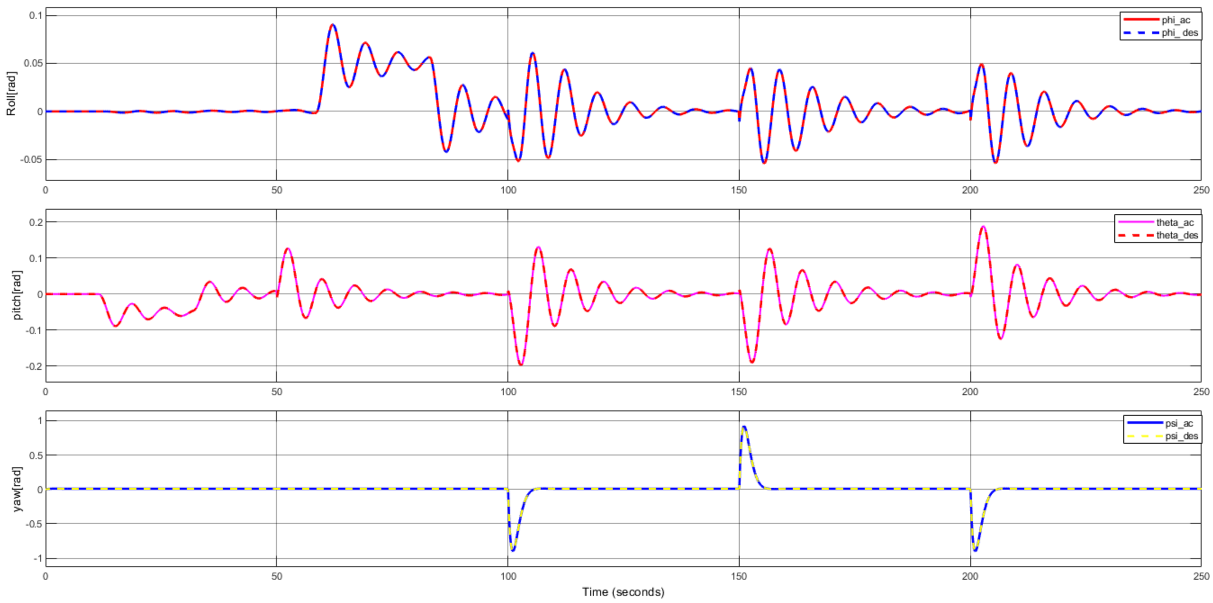


Figure 5.18: Rectangular path trajectory tracking under disturbance angle

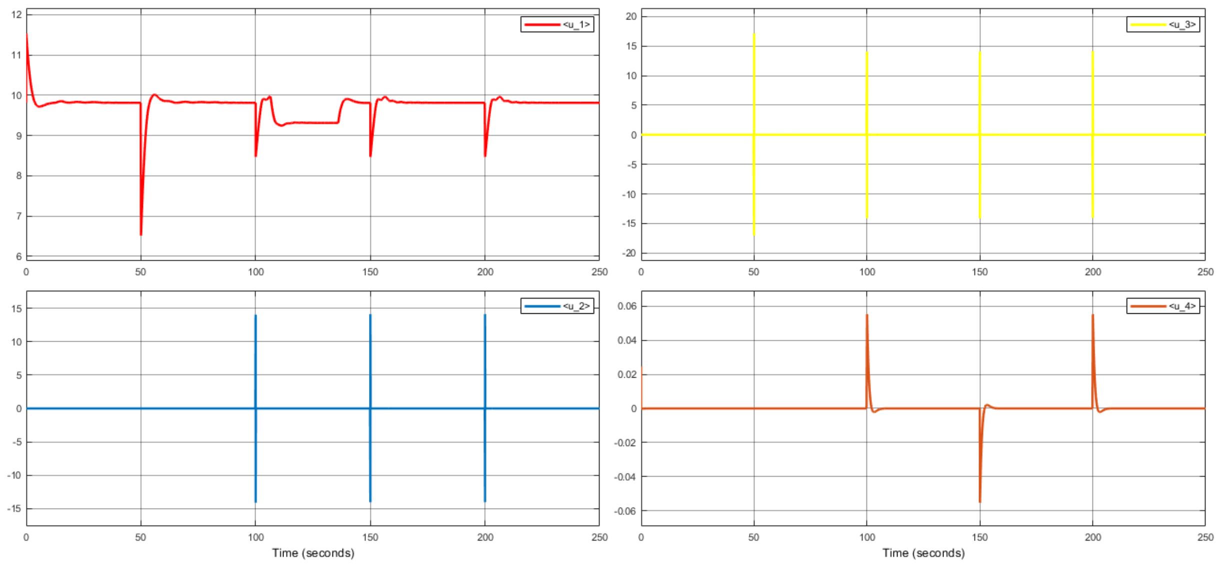
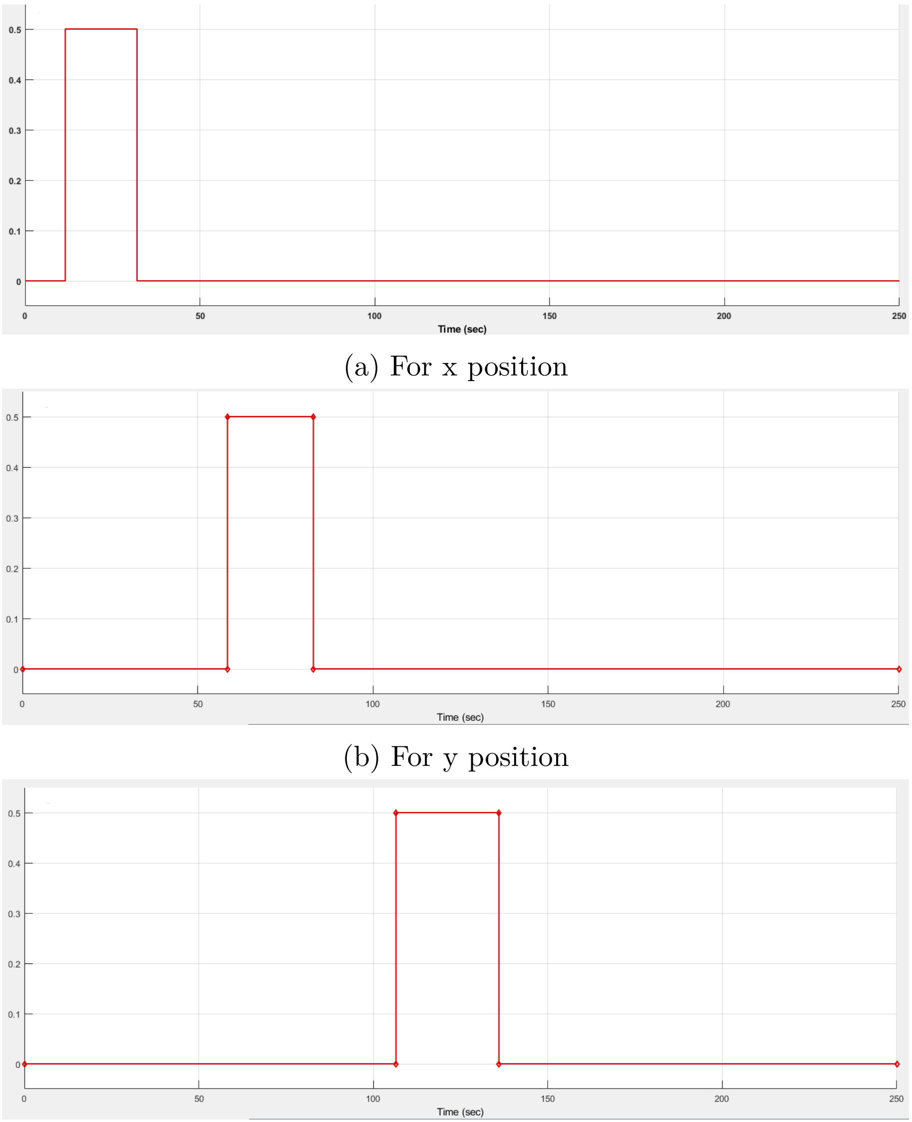


Figure 5.19: Rectangular path trajectory tracking under disturbance control input

The disturbance type which is added to the dynamic model is a wind disturbance.Where on the simulink block the disturbance is added on the dynamic equation of x,y and z function block.The disturbance time sequence plot is expressed as:



(a) For x position

(b) For y position

(c) For z position

Figure 5.20: Wind disturbance signal for x,y,z in different time sequence

- For x the disturbance starts at t=11.5 sec and stops t=32 sec
- For y the disturbance starts at t=58.5 sec and stops t=83 sec
- For z the disturbance starts at t=106.5 sec and stops t=135 sec

5.9 Additional trajectory tracking paths

The additional trajectory's are done in order to check the performance of the controller.For the circular path the desired trajectory for x,y,z are $x = \sin(0.5t)$,$\cos(0.5t)$ and $z = 4m$. Also for the infinity path the desired trajectory for x,y,z are $x = \cos(t)$,$y = \dfrac{\sin(2t)}{2}$ and z=4m.

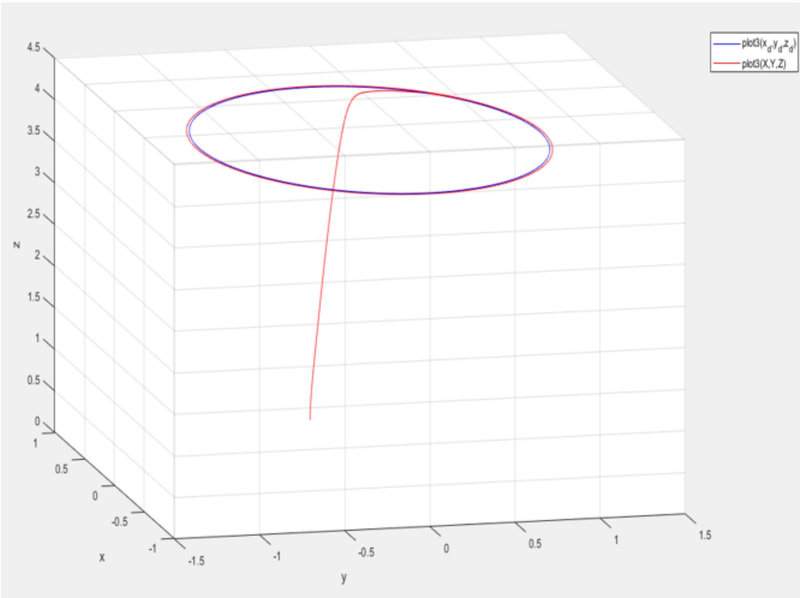


Figure 5.21: circle trajectory tracking

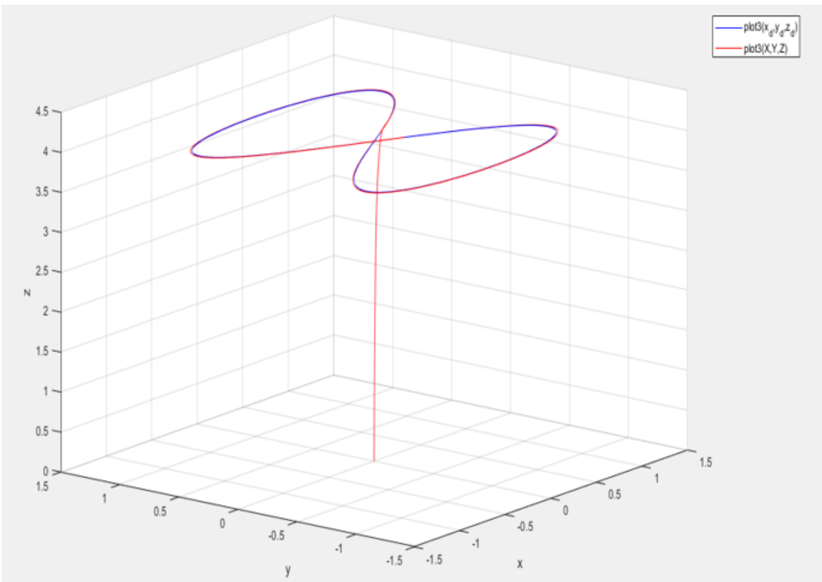


Figure 5.22: infinite trajectory tracking

5.10 Application area

5.10.1 Transmission line inspection

In the past years, transmission lines was inspected manually with human beings.As the technology advances the inspection ways also changed.With modern inspection technology,helicopters equipped with high-resolution cameras or by direct visual examination carried out by highly skilled staff climbing over de-energized power lines. However, the visual inspection is time-expensive ,costly and it risky because of having human being inside the Helicopter.The most common frequently encountered risk is a collision to the transmission lines and towers. The collision causes an accident to the inspector person up to death , also the physical size of the helicopter it also create a pressure to the transmission line because of long propellers and helicopters cannot cover narrow mountainous areas .Due to such problems quadrotor is the best modern technology for inspection of transmission lines.
In this thesis work quadrotor is used for transmission line inspection for the inspection of 132 kv transmission line.

5.11 132 kv transmission line standard parameters

Paramter	**Value**
Tower height, m	19
span length, m	250
Conductor thickness, m	90
Conductor air to ground clearance , m	7
safe flight zone (horizontal), m	2.9
safe flight zone (vertical), m	4.6

Table 5.2: 132 kv transmission line standard parameters

The 132 kv transmission line standard parameters data Source is taken from hawassa electric power corporation .

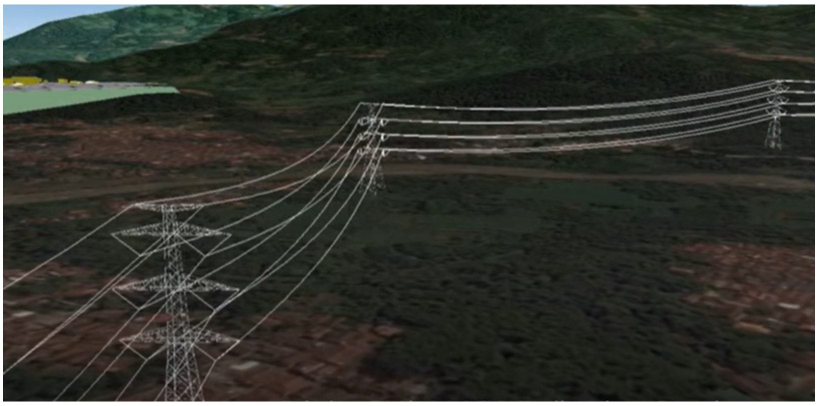
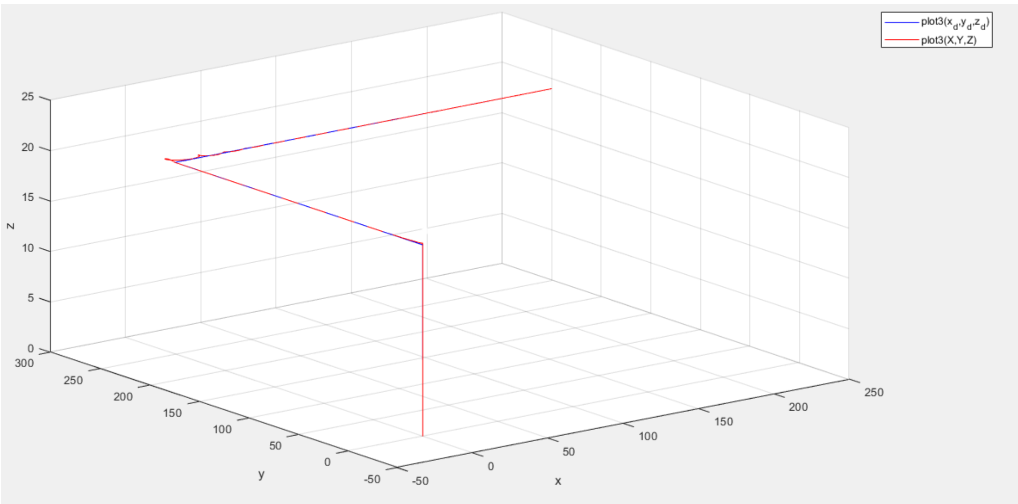
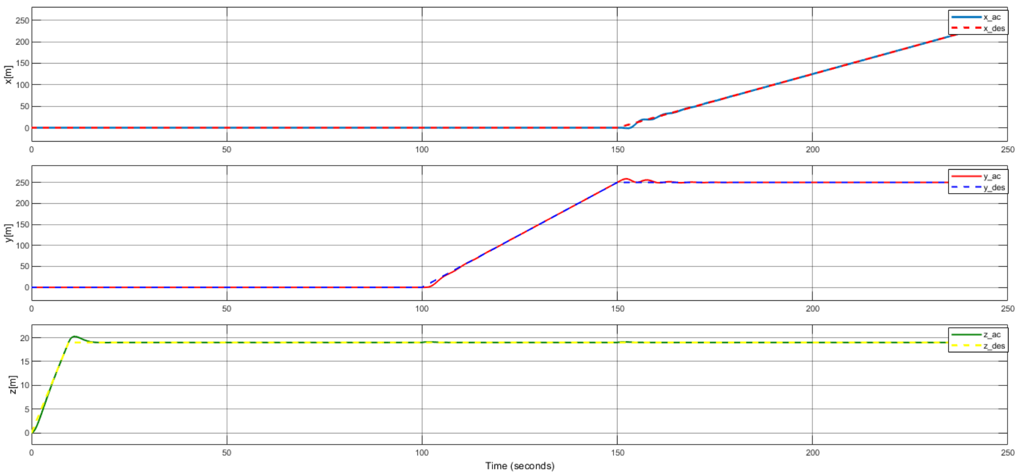


Figure 5.23: Transmission line inspection trajectory tracking line design

On figure 5.23 the trajectory tracking line design is taken straight line with some angle.The MATLAB simulation is implemented based on standard parameters of 132 kv transmission line as shown below:



(a) Transmission line inspection trajectory tracking



(b) Position

Figure 5.24: Transmission line inspection trajectory tracking

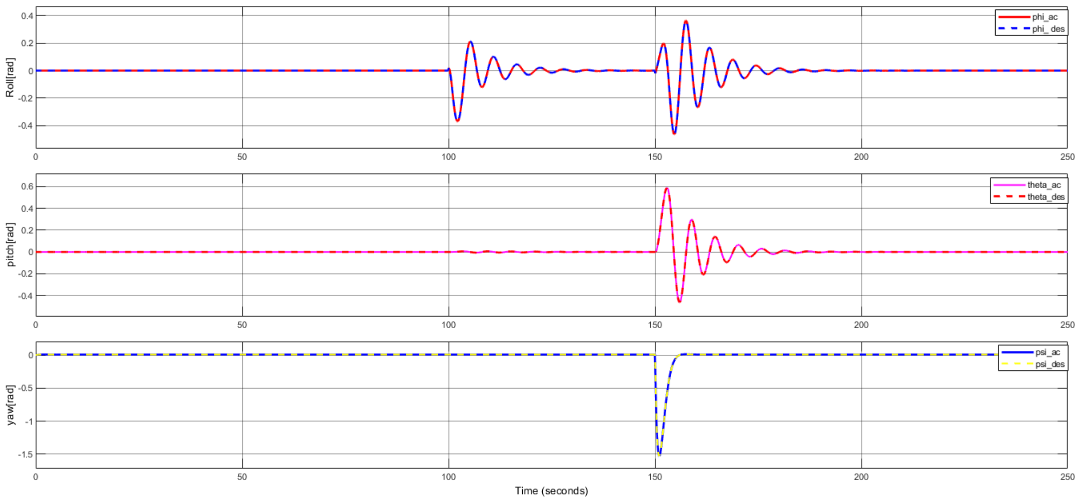


Figure 5.25: Angle

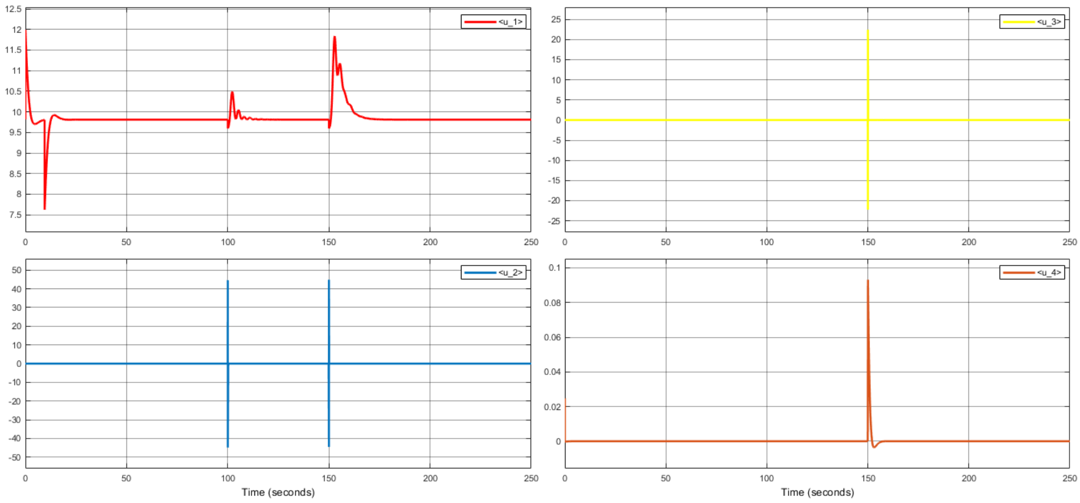


Figure 5.26: Control input

The value of Q,R and K matrix for transmission line inspection trajectory tracking are

$$Q = 25 \quad R = 110 \quad K = \begin{bmatrix} 0.4767 & 1.0866 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4767 & 1.0866 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4767 & 1.0866 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.0000 & 1.7321 \end{bmatrix} \tag{5.3}$$

5.12 Discussion

The simulation result illustrate that feedback linearization plus LQR control scheme is able to control the quadrotor and track the desired rectangular trajectory in a finite time.The feedback linearization technique it cancel the non-linearity of the quadrotor dynamics .Then after linearization a linear control technique which is a linear quadratic regulator is used .The Linearized state space model which is used to the LQR controller the state matrix A ,B is controllable and the A,C matrix is observable. On the simulation result each gain matrix's which is got from LQR controller it have a big roll for the achieved trajectory tracking.

The LQR controller is a robust controller which over came the external wind disturbance with minimum control effort .On the LQR controller the weighting matrix Q and R determines the out put result of the controller.The result of increase Q matrix it makes the response fast and smooth . Also increase the R matrix it results minimum control effort to control the quadrotor.

On the constant tracking simulation result the step input response have good rise time and settling time.It achieve the desired 100m position on the settling time of x 19.42 sec, y 19.5 sec and z 11.4 sec . Also on the application area the trajectory tracking and the constant tracking performance for each positions has less tracking error responses to achieve the desired trajectory which is shown on the simulation result of integral time absolute error.

Chapter 6

Conclusion and Future Works

6.1 Conclusion

In this thesis work, a general over view of quadrotor history has been discussed. The mathematical model of quadrotor dynamics and the differential equations where derived by using Newton-Euler formulation.A feedback linearization technique has been formulated in order to linearize the non linearity characteristics of the quadrotor dynamics or in other word it transform original quadrotor dynamic model into equivalent model of a simpler form.
Once the dynamics has been linearized a linear controller,the LQR has been derived.Before design the LQR controller the linearized error dynamics has been derived from the linearized model.Also the gain matrix value K has been calculated with Riccati's algebraic equation using MATLAB .The gain matrix K it minimize the cost function .

According to the simulation results,the proposed feedback linearization approach produces a good trajectory tracking performance.Finally a wind disturbance where introduced to explore more about the controller performance.
Generally summarizing the work done on this thesis is:

- Formulating the mathematical model of the quadrotor by using Newton-Euler formulation
- Formulating the FBL plus LQR controller
- Trajectory generation
- Regenerate transmission line trajectory path with 132 Kv transmission line standard parameters
- Inclusion of Wind disturbance to the quadrotor dynamics

- Simulate the above formulation using MATLAB and SIMULINK

6.2 Recommendation

The use of quadrotor for transmission line fault detection hampered with number of limitations that can be addressed by more accurate motion control.That mean tracking transmission line is an inherently repetitive task .It needs a precise actions like location of equipment and moving between transmission line needs a flexible motion.Therefore,in order to fix such problem ,it recommended to use iterative learning control(ILC) algorithm should be developed for high-performance trajectory tracking.

6.3 Future work

Actually this thesis has normally accomplished the targeted mission;but not all things have been done.
A transmission line are exposed to thunderstorms, thermal deviations, ice, rain, pollutions like volcanic gases and sour rains.A severe environment that may lead to material fatigue, oxidation and corrosion.To over come such problems the transmission line inspect and maintains of equipment should be periodically.On this thesis work ,as a beginner minimum coverage of transmission line is done.How about long coverage transmission line and how can inspect and maintain them periodically.As a suggestion using GPS technology with image processing algorithms will solves the problems.My future work plan is to optimize more energy or to achieve the desired objective set point of the controller with minimum control effort by using different optimization mechanism and compering them each other and adding GPS technology with image processing algorithms to be more safe and to take intimidate action on the problem.Using other controller to compare their performance and to know which one is the best controller. Finally as motivation all those challenges are not impossible,they just require more hard work.

Appendices

Appendix A

Input-output Feedback linearization

Consider a system with $X \in R^n$,input$u \in R$ and output $y \in R$ where the dynamics are given by

$$\dot{x} = f(x) + g(x)u \tag{A.1}$$

$$y = h(x) \tag{A.2}$$

Where f(x),g(x) and h(x) are non linear function.Our system is Multi-input Multi-output(MIMO) system.So, the Lie derivative of the single output y can be expressed as:

$$\dot{y} = \frac{\partial h}{\partial x} = [f(x) + g(x)u] \tag{A.3}$$

Lie derivative is defined as the derivation of **h** along the trajectory of state **x** and it's expressed as:

$$\dot{y} = \frac{\partial h}{\partial x} = [f(x) + g(x)u] = L_f h(x) + L_g h(x)u \tag{A.4}$$

If the control input don't appear the first derivative $L_g h(x) = 0$,then we have

$$\dot{y} = y^1 = L_f h(x) \tag{A.5}$$

then for the MIMO system higher order derivatives can be expressed as:

$$y^{(2)} = L_f^{(2)} h(x) + L_g L_f h(x)u \tag{A.6}$$

$$y^{(3)} = L_f^{(3)} h(x) + L_g L_f^{(2)} h(x)u \tag{A.7}$$

$$y^{(4)} = L_f^{(4)} h(x) + L_g L_f^{(3)} h(x)u \tag{A.8}$$

$$y^{(i)} = L_f^{(i)}h(x) + L_g L_f^{(i-1)}h(x)u \tag{A.9}$$

then after appearance of the control input at the output equation $L_g L_f^(i-1)h(x)u \neq 0$,then the equation (A.9) can be linearized with dynamic inversion,choosing the law as:

$$u = \frac{1}{L_g L_f^{(i-1)}h(x)}[-L_f^{(i)}h(x) + v] \tag{A.10}$$

On the state region where the inverse $\dfrac{1}{L_g L_f^{(i-1)}h(x)}$ exists.In this Region the Feedback linearized model become:

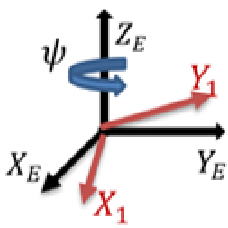
$$y^{(i)} = V \tag{A.11}$$

Appendix B

Rotation matrix

Rotational matrix is the composition of elemental rotations of Euler angles about the axis of a certain coordinate system.

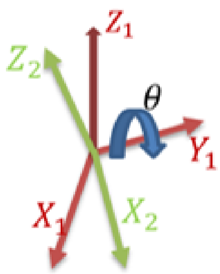
The three basic rotational matrix rotational sequence is expressed as follow i.e first it rotates about the z-axis,then it rotates about y-axis and finally it rotates about x-axis.The three consecutive rotations are shown below based on the chapter two relation ship of inertial frame with body frame.



$$\cos(\psi) = \frac{X_E}{X_1}, \sin(\psi) = \frac{Y_E}{X_1}, \cos(\psi) = \frac{Y_E}{Y_1}, \sin(\psi) = \frac{-X_E}{Y_1} \tag{B.1}$$

$$\begin{bmatrix} X_E \\ Y_E \\ Z_E \end{bmatrix} = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \tag{B.2}$$

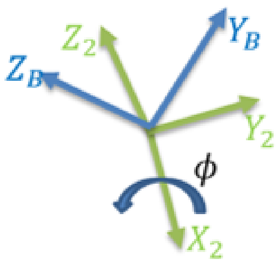
$$R_z(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{B.3}$$



$$\cos(\theta) = \frac{X_1}{X_2}, \sin(\theta) = \frac{-Z_1}{X_2}, \cos(\theta) = \frac{Z_1}{Z_2}, sin(\theta) = \frac{X_1}{Z_2} \tag{B.4}$$

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} \tag{B.5}$$

$$R_y(\theta) = \begin{bmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \tag{B.6}$$



$$\cos(\phi) = \frac{Z_1}{Z_2}, \sin(\phi) = \frac{-Y_1}{Z_2}, \cos(\phi) = \frac{Y_1}{Y_2}, \sin(\phi) = \frac{Z_1}{Y_2} \tag{B.7}$$

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} X_B \\ Y_B \\ Z_B \end{bmatrix} \tag{B.8}$$

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi) & \cos(\phi) \end{bmatrix} \tag{B.9}$$

$$R_z(\psi)R_y(\theta)R_x(\phi) = \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi c_\theta & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix} \tag{B.10}$$

Appendix C

Transfer matrix

The transfer matrix is used to relate the angular rate from the inertial frame to body frame. The proof for the transfer matrix expressed as shown below:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}_I = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + R_{\phi}^{-1} \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + R_{\phi}^{-1} R_{\theta}^{-1} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} = T \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}_B \tag{C.1}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}_I = \begin{bmatrix} \dot{\phi_B} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{\phi} & s_{\phi} \\ 0 & -s_{\phi} & c_{\phi} \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\theta_B} \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{\phi} & s_{\phi} \\ 0 & -s_{\phi} & c_{\phi} \end{bmatrix} \begin{bmatrix} c_{\theta} & 0 & -s_{\theta} \\ 0 & 1 & 0 \\ s_{\theta} & 0 & c_{\phi} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi_B} \end{bmatrix} = T \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}_B \tag{C.2}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}_I = \begin{bmatrix} \dot{\phi_B} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & c_{\phi} & -s_{\phi} \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\theta_B} \\ 0 \end{bmatrix} + \begin{bmatrix} s_{\theta} & s_{\phi} c_{\theta} & c_{\phi} c_{\theta} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi_B} \end{bmatrix} = T \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}_B \tag{C.3}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}_I = \begin{bmatrix} 1 & 0 & -\sin(\theta) \\ 0 & \cos(\theta) & \sin(\phi)\cos(\theta) \\ 0 & -\sin(\theta) & \cos(\phi)\cos(\theta) \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}_B \tag{C.4}$$

$$T = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & \cos(\phi) \\ 0 & \sin(\phi)\sec(\theta) & \cos(\phi)\sec(\theta) \end{bmatrix} \tag{C.5}$$

$$T^{-1} = \begin{bmatrix} 1 & 0 & -\sin(\theta) \\ 0 & \cos(\phi) & \cos(\theta)\sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi)\cos(\theta) \end{bmatrix} \tag{C.6}$$

Appendix D

Program for Linear quadratic requlator

```
clc                    % to clear the command window
clear                  %  to clear the work space

A=[0 1 0 0 0 0 0 0;
   0 0 0 0 0 0 0 0;
   0 0 0 1 0 0 0 0;
   0 0 0 0 0 0 0 0;
   0 0 0 0 0 1 0 0;
   0 0 0 0 0 0 0 0;
   0 0 0 0 0 0 0 1;
   0 0 0 0 0 0 0 0];
 B=[0 0 0 0;
   1 0 0 0;
   0 0 0 0;
   0 1 0 0;
   0 0 0 0;
   0 0 1 0;
   0 0 0 0;
   0 0 0 1];
C=[1 0 0 0 0 0 0 0;
   0 0 1 0 0 0 0 0;
   0 0 0 0 1 0 0 0;
   0 0 0 0 0 0 1 0];
D=0;
sys=ss(A,B ,C ,D);
t=0:0.01:50;
step(sys,t);
Q = zeros(8,8);
Q=C'*C;
Q(1,1)= 1;  Q(3,3) =1;  Q(5,5) =1;
 %z_pos          %phi_pos          %theta_pos

Q(2,2) = 1;    Q(4,4) = 1;  Q(6,6) = 1;
  %zdot_pos     %phidot_pos       %thetadot_pos
Q(7,7) = 1;   Q(8,8) = 1;
  %psi            %psidot
 R=[1 0 0 0;...
```

```
   0 1 0 0;...
   0 0 1 0;...
   0 0 0 1];
co=ctrb(A,B);
rank(co)
ob=obsv(A,C);
rank(ob);
nx=8;
ny=4;
[K, S, E]=lqr(A,B,Q,R);%calculates the optimal gain matrix K  % the state-feedback law
    u = -Kx minimizes the quadratic cost function
Acl=A-B*K;
sysss=ss(Acl,B,C,D);
step(sysss,t);
```

Appendix E

Program for 3D plot

```
clc                     % to clear the command window
clear                   %  to clear the work space
t=0:1:10;
figure
D=plot3(x_d,y_d,z_d,'b');
hold on
A=plot3(X,Y,Z,'R'); grid on ;
xlabel('x')
ylabel('y')
zlabel('z')
legend ([D,A],'plot3(x_d,y_d,z_d)','plot3(X,Y,Z)');
```

Appendix F

Rectangular Path trajectory tracking generation MATLAB code

```
clc                    % to clear the command window
clear                  %  to clear the work space
function z1 = fcn(t)
if  t<=50
    z1=2*t;
elseif (50<=t)&&(t<=100)
      z1=100;
else (100<t) && (t<=150)
      z1=100;

end
end                        %For the alttiude trajectory
function x1 = fcn(t)
if  t<=50
    x1=0;
elseif (50<=t)&&(t<=100)
      x1=2*(t-50);
elseif  (100<=t)&&(t<=150)
       x1=100;
elseif (150<=t)&&(t<=200)
      x1=-2*(t-200);
 else
       x1=0;
end
end                         %For x Trajectory
function y1 = fcn(t)
if  t<=50
    y1=0;
elseif (50<=t)&&(t<=100)
       y1=0;
elseif  (100<=t)&&(t<=150)
      y1= 2*(t-100);
elseif  (150<=t)&&(t<=200)
       y1=100;
```

```
 elseif  (200<=t)&&(t<=250)
      y1=-2*(t-250 );

 else
    y1=0;
end
end                     %For y Trajectory
function z1 = fcn(t)
if  t<=50
    z1=2*t;
elseif (50<=t)&&(t<=100)
      z1=100;
else (100<t) && (t<=150)
      z1=100;

end
end
```

Appendix G

Transmission line inspection trajectory tracking generation MATLAB code

```
function z1 = fcn(t)
if  t<=9.5
    z1=2*t;
elseif (9.5<=t)&&(t<=100)
      z1=19;
else
   (100<=t)&&(t<=250);
      z1=19;

end
end               %For the alttiude(tower height)
function x1 = fcn(t)
if  t<=100
    x1=0;
elseif (100<=t)&&(t<=150)
       x1=0;
 elseif  (150<=t)&&(t<=250)
       x1= 2.5*(t-150);
 else
    x1=0;
end
end              %For x Trajectory
function y1 = fcn(t)
if  t<=100
    y1=0;
elseif (100<=t)&&(t<=150)
      y1=5*(t-100);
elseif  (150<=t)&&(t<=250)
       y1=250;
else
       y1=0;
end      %For y Trajectory
end
```

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