ADDIS ABABA UNIVERSITY-ADDIS ABABA INSTITUTE OF TECHNOLOGY SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

COMPARISON OF EQUIVALENT FRAME ANALYSIS RESULTS WITH FINITE ELEMENT ANALYSIS RESULTS FOR FLAT PLATE SLAB SYSTEM

A Thesis in Structural Engineering

By Lelissie Kudama

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A_c = Area of slab at critical section

E_c = Modulus of elasticity of concrete

EI = Flexural stiffness of compression member

E_{cs} = Modulus of elasticity of slab concrete

E_{cb} = Modulus of elasticity of slab beam

$F_{c'}$ = Compressive strength of concrete

G = Shear modulus of slab concrete

I_e = Effective moment of inertia for computation of deflection

I_g = Moment of inertia of gross concrete section, neglecting reinforcement

I_{cr} = Moment of inertia of cracked section transformed to concrete

K_c = Column stiffness

K_s = Slab stiffness

K_t = Torsional stiffness of the slab edge and spandrel beams, if present

l_n = Length of clear span in direction that moments are being determined

l_1 = Span length in bending direction

l_2 = Span length in transverse direction

M_o = Total static moment

V_g = Total shear at the connection due to gravity load

α = Effective beam width factor

β = Coefficient accounting for cracking

χ = Reduction factor for gravity loads

l_1 = Span length in bending direction

Q_m = the factor for the load distribution

f_b = the factor for the decrease in rotational flexibility

A_m = the factor for the geometries and the boundaries.

α_f = ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels (if any) on each side of the beam

α_{f1} = α_f in direction of l_1

α_{f2} = α_f in direction of l_2

Abstract

Flat slabs can be analyzed by finite element method or equivalent frame method . Computer application of frame analysis method requires that each equivalent frame geometry, material characteristics and member properties be defined effectively. Two simplified methods exist, namely the effective beam width method and the equivalent column method, in which the effect of the slab is accommodated by appropriate modification of the beam width or the column stiffness, respectively. In this study an idealized flat plate slab system under gravity loading only is analyzed using equivalent frame method and finite element analysis method. Effective beam width modeling method is used for equivalent frame analysis method. Comparisons are made for slab moment, column moment and reaction force results with nonlinear finite element analysis results. The results suggest that two dimensional equivalent frame method, that consider nonlinearity effects, is effective in determining slab moments only. Three dimensional equivalent frame method underestimate slab and column moments. Linear finite element analysis method can be used to analyze slab and columns by modifying the stiffness of the slab only. Generally, Consideration of nonlinearity effect is important when analyzing flat plate slab system, both for the slab design to be economical and for the column design to be safe. It is recommended to use load pattern while analyzing columns of flat plate slab system building.

1. Introduction

1.1 General

Cano and Klinger (1988) mention two types of modeling approaches, equivalent frame approach and finite element approach, for analyzing flat slab structures. Equivalent frame approach is based on effective slab width or equivalent column concept. The effective slab-width procedure was originally developed for analyzing two-way slab-column systems for static lateral loads (Pecknold 1975). In this procedure, an effective width factor α_i is obtained such that a slab of width $\alpha_i l_2$ with a uniform rotation at the column support would allow the column located at the center of the panel to rotate the same amount as the original column. The columns are modeled in a conventional manner and the slab is modeled as a beam of effective width $\alpha_i l_2$ with depth equal to the original slab thickness. The effective width factor α_i is calculated based on the equivalent elastic stiffness of the interior slab-column connections [8].

Several researchers have worked on the development of effective slab width models that could be used to define the stiffness of an equivalent beam in a standard frame analysis program for the evaluation of moments and shears in a slab–column frame subjected to combined vertical (Gravity) and lateral loading. Darvall and Allen [7] used finite-element analyses to define an effective slab width. More recent work by Luo and Durrani [8] and [9], Hwang and Moehle [10], and Dovich and Wight [11] based their effective slab width and stiffness models on experimental results for reinforced concrete slabs. Hueste and Wight [12] proposed modifications of these effective slab width and stiffness models for post-tensioned slabs, based on observed damage in a post-tensioned slab–column frame system following a major earthquake. MacGregor recommends simple rules for modeling two dimensional slab-column frames subjected to gravity loading [1]. Hence, the goal of this research is to clarify the effectiveness of selected flat slab analysis methods, especially equivalent frame method (using effective beam width modeling method) as compared to nonlinear finite element analysis method for flat plate slab system subjected to gravity loading.

1.2 Objectives

The general objective of this thesis work is to compare equivalent frame analysis results with finite element analysis results for flat plate slab system, with specific objectives of:-

- Clarifying the importance of considering non-linear effects while analysing flat plate slab system.
- Evaluating the effectiveness of two dimensional and three dimensional equivalent frame methods as compared to nonlinear finite element method.
- Evaluating the effectiveness of linear finite element analysis method as compared to nonlinear finite element analysis method.

1.3 Methodology

To achieve the objectives of this study, literature review is carried out on flat slab modeling and analysis methods. To evaluate the effectiveness of selected analysis methods, comparison is done with non-linear finite element analysis results. The structural analysis is carried out using SOFiSTiK V14 and ETABS V9.5 software.

2. Literature Review

2.1 Review of Current Flat Slab Analysis Methods

2.1.1 Direct design method

Direct design method consists of a set of rules for distributing moments to slab and beam sections to satisfy safety requirements and most serviceability requirements simultaneously [5]. Three fundamental steps are involved as follows:

1. Determination of the total factored static moment:

$$M_O = \frac{q_u l_2 l_n^2}{8} \tag{2.1}$$

Where l_n is length of clear span in direction that moments are being determined.

2. Distribution of the total factored static moment to negative and positive sections:

- In an interior span, total static moment, M_o shall be distributed as follows:
  Negative factored moment 0.65
  Positive factored moment..................................0.35
- In an end span, total factored static moment, Mo, shall be distributed as follows:

Table 2-1 Percentage of end span total factored static moment

	(1)	(2)	(3)	(4)	(5)
			Slab without beams between interior supports		
	Exterior edge unrestrained	Slab with beams between all supports	Without edge beam	With edge beam	Exterior edge fully restrained
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0	0.16	0.26	0.30	0.65

3. Distribution of the negative and positive factored moments to the column and middle strips and to the beams, if any. The rules given for assigning moments to the column strips, beams, and middle strips are based on studies of moments in linearly elastic slabs with different beam stiffness tempered by the moment coefficients that have been used successfully.

3.1 Distribution to Column strip

- Column strips shall be proportioned to resist the following portions in percent of interior negative factored moments:

Table 2-2 Percentage of interior negative factored moments

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

- Column strips shall be proportioned to resist the following portions in percent of exterior negative factored moments:

Table 2-3 Percentage of exterior negative factored moments

ℓ_2/ℓ_1		0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45

Linear interpolations shall be made between values shown, where β_t is calculated in Eq. (2.5) and C is calculated in Eq. (2.6).

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} \tag{2.2}$$

$$C = \sum(1 - 0.63\frac{x}{y})\frac{x^3y}{3} \tag{2.3}$$

Where supports consist of columns or walls extending for a distance equal to or greater than (3/4) l_2 used to compute M_o, negative moments shall be considered to be uniformly distributed across l_2.

- Column strips shall be proportioned to resist the following portions in percent of positive factored moments:

Table 2-4 Percentage of positive factored moments

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

- For slabs with beams between supports, the slab portion of column strips shall be proportioned to resist that portion of column strip moments not resisted by beams [5].

3.2 Distribution to beams

- Beams between supports shall be proportioned to resist 85 percent of column strip moments if $\alpha_{f1}l_2/l_1$ is equal to or greater than 1.0.
- For values of $\alpha_{f1}l_2/l_1$ between 1.0 and zero, proportion of column strip moments resisted by beams shall be obtained by linear interpolation between 85 and zero percent [5].

3.3 Distribution to middle strip

- That portion of negative and positive factored moments not resisted by column strips shall be proportionately assigned to corresponding half middle strips.
- Each middle strip shall be proportioned to resist the sum of the moments assigned to its two half middle strips.
- A middle strip adjacent to and parallel with a wall-supported edge shall be proportioned to resist twice the moment assigned to the half middle strip corresponding to the first row of interior supports [5].

Limitations of Direct Design method

- There shall be a minimum of three continuous spans in each direction.
- Panels shall be rectangular, with a ratio of longer to shorter span center-to-center of supports within a panel not greater than 2.
- Successive span lengths center-to-center of supports in each direction shall not differ by more than one-third the longer span.
- Offset of columns by a maximum of 10 percent of the span (in direction of offset) from either axis between centerlines of successive columns shall be permitted.
- All loads shall be due to gravity only and uniformly distributed over an entire panel. The unfactored live load shall not exceed two times the unfactored dead load.
- For a panel with beams between supports on all sides, Eq. (2.7) shall be satisfied for beams in the two perpendicular directions [5].

$$0.2 \leq \frac{\alpha_f l_2^2}{\alpha_{f2} l_1^2} \leq 5 \qquad (2.4)$$

Where α_{f1} and α_{f2} are calculated in accordance with Eq. (2.8).

$$\alpha_f = \frac{E_{cb} I_b}{E_{cs} I_s} \qquad (2.5)$$

2.1.2 Equivalent frame method

1) Effective beam width procedure

Effective beam width procedure was developed to analyze two-way slab system. It can be used for analysis of flat slab or flat plate structures. When a slab-column joint is subjected to rotation, slab rotation along the transverse direction does not remain constant. Rotation of the slab near the connection is more as compared to the distant portion of the slab (Figure 2.1). The slab is modeled in a simplified manner using slab-beam member having width equal to some fraction (α) of slab panel width (l_2). The value of α will be such that effective beam subjected to uniform rotation will cause the same amount of moment as for the actual slab subjected to varying rotation [2].

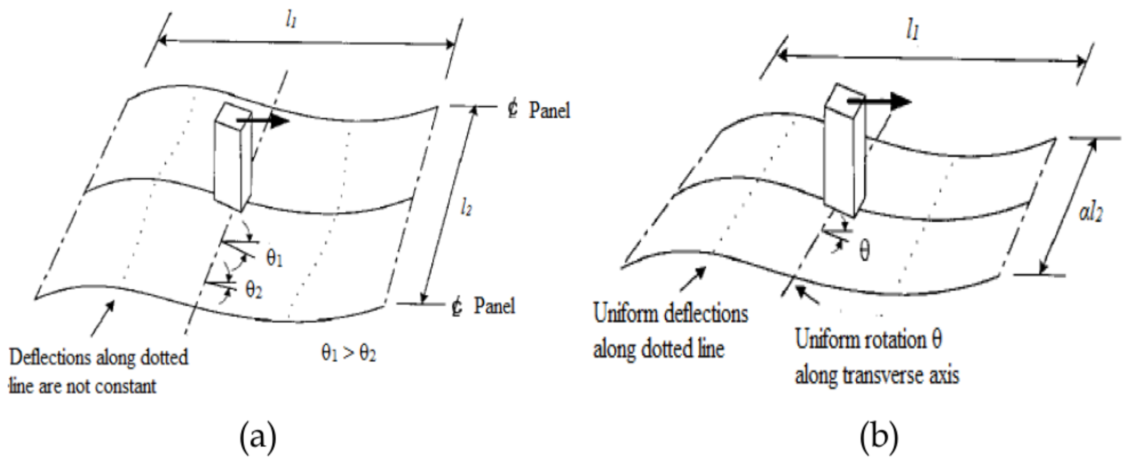


Figure 2-1 Concept of effective beam width model: (a) slab-column element; (b) effective beam-column element

The first step in building a slab–column frame model is to select an effective beam width that is a fraction of the total slab width, l_2 (avg). A wide range of α values have been suggested by various researchers, but MacGregor prefers to simply use α=0.5 for all positive-bending regions and for negative-bending regions at interior supports. For negative-bending regions at exterior supports, the effective slab width depends on the torsional stiffness at the edge of the slab. If no edge beam is present, then an α value of 0.2 is recommended. If an edge beam is present and has a torsional stiffness such that β_t is greater than or equal to 2.5, then the recommended α value is 0.5. If the value of β_t is between 0.0 and 2.5, a linear interpolation can be used to find an α value between 0.2 and 0.5.

For low values of α, the effective slab width should not be taken to be less than the column width, c_2, plus one-half the column total depth, c_1, on each side of the column (Figure 2.2). For slab–column frames along a column line at the edge of a floor plan, the effective slab widths are reduced accordingly. A sample of effective slab-width models for one exterior and one interior column line of the flat-plate floor system is shown in Figure 2.3 [1].

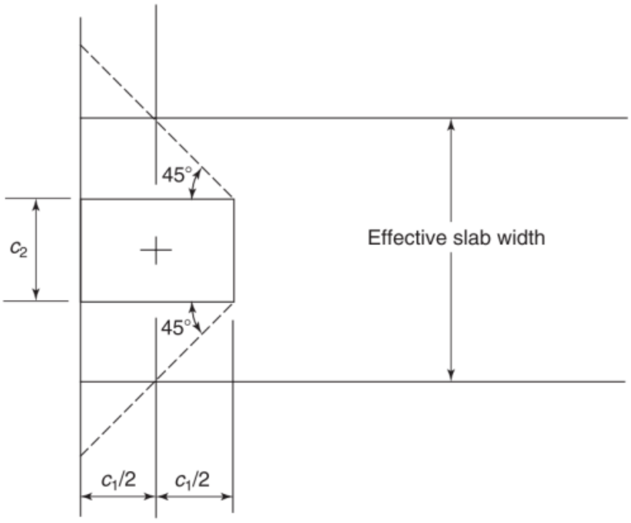


Figure 2-2 Minimum value for effective slab width at exterior slab-to-column connections.

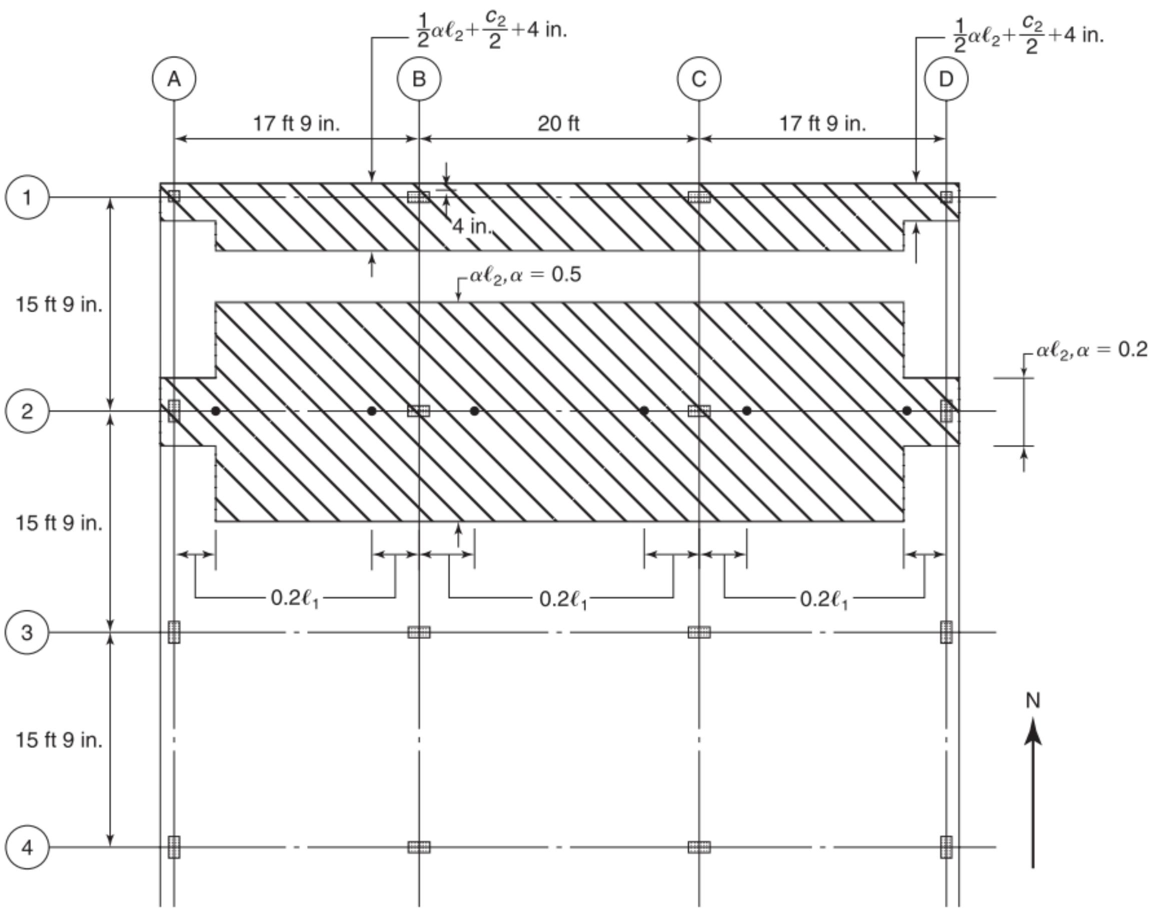


Figure 2-3 Effective slab width, αl_2 and location for intermediate nodes along the span.

As indicated in Figure 2.3, the negative-bending region at the exterior connection is assumed to extend over 20 percent of the span. MacGregor recommend that the same assumption be used for negative-bending regions at all interior and exterior connections. This assumption essentially creates extra node points within the span and becomes important when assigning cracked-stiffness values to the positive-and negative-bending regions. After the effective slab width has been established, the gross moment of inertia for the slab-beam can be calculated [1].

A final modification is to be made to the slab-beam stiffness to account for flexural cracking. In general, the cracked moment of inertia for a slab-beam section, I_{cr}, is some fraction of the gross moment of inertia for that section. Because slabs normally have lower reinforcement ratios than beams, their cracked-section moment of inertia is usually a smaller fraction of the gross moment of inertia than for a typical beam section. However, because large portions along the slab-beam span will remain uncracked and the flexural cracks that do occur usually will not propagate over the entire width of the slab, an effective moment of inertia, I_e, needs to be defined for different portions of the slab-beam span. Commonly, a factor β is used to define the effective moment of inertia as some fraction of the gross moment of inertia ($I_e = \beta I_g$). For all positive-bending regions of the slab, MacGregor recommends to use a β factor equal to 0.5. Because larger moments typically occur near interior connections, and in order to not overestimate the slab-to-edge beam-to-column stiffness at an exterior connection, whether or not an edge beam is present, the author recommends a β factor of 0.33 for all negative bending regions.

A summary of the recommended α and β values to be used to represent the flexural stiffness of slab-beam elements is given in Table 2.5. For analysis of post-tensioned slabs, Hueste and Wight [12] and Kang and Wallace [13] have recommended the use of a βvalue equal to 0.67 because of the reduced flexural cracking expected in a post-tensioned slab [1].

Table 2-5 α and β values for the flexural stiffness of slab-beam elements

Region of the Slab	α-Value (For Effective Width $\alpha\,\ell_2$)	β-Value (For $I_e = \beta\, I_g$)
Positive-bending regions	0.5	0.5
Negative-bending regions (interior columns)	0.5	0.33
Negative-bending regions (exterior columns)	0.2 to 0.5 (function of edge beam stiffness)	0.33

For a gravity load analysis of an equivalent frame, representing a two-way floor system, the slab-beam elements can be assembled with column elements that extend one story above and one story below the floor system , as permitted by ACI Code Section 13.7.2.5 [1].

2) Transverse torsional member procedure

In flat slab-column connections, a fraction of unbalanced moments are transferred through flexure and remaining portion is transferred through torsion. Transferring of moments by means of torsion is modeled using transverse torsional members.

a) ACI equivalent frame method

This method subdivides the three dimensional structures into a series of planer frames, centered on column lines in longitudinal as well as in transverse direction. Load transfer system in this method involves three distinct interconnected elements:

i) Slab-beam member

ii) Column

i) Torsional member

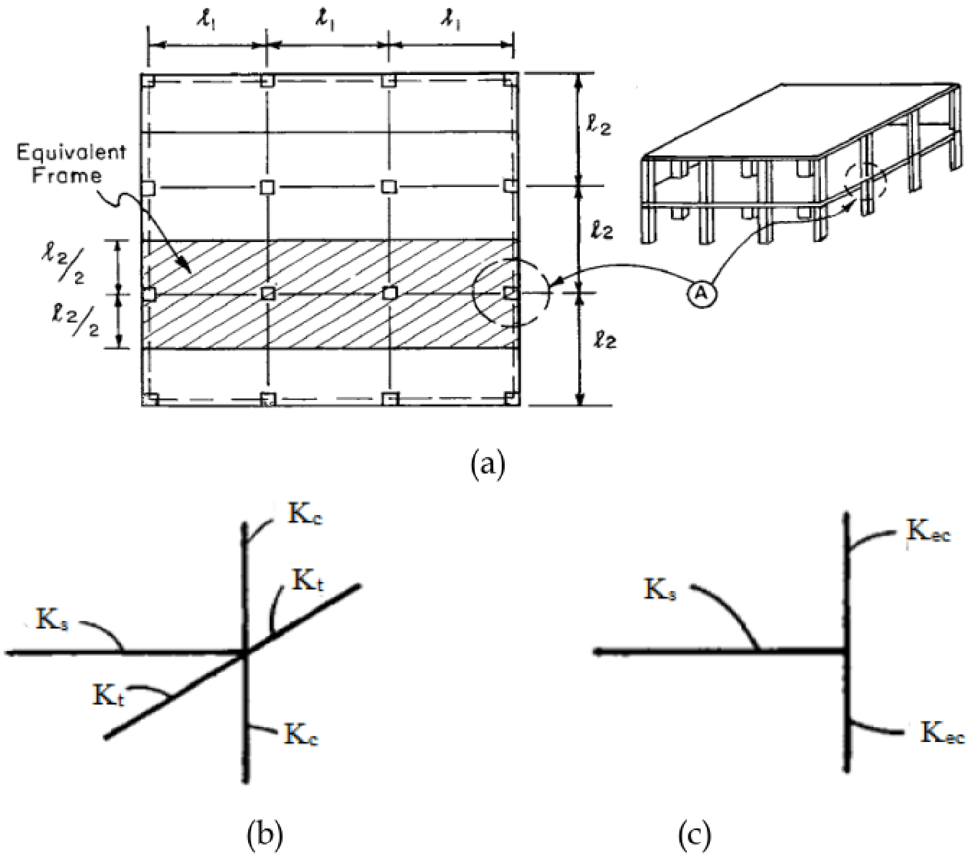


Figure 2-4 Member configuration assumed in ACI: (a) Definition of equivalent frame; (b) Members of 3-D structure, Detail A; (c) Members of ACI EFM, Detail A

Slab beam member is supported on equivalent column (K_{ec}), which represents the flexibility of both column and torsional member. Stiffness of the equivalent column section is calculated as:-

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_t} \tag{2.6}$$

b) Extended equivalent column method

In extended equivalent column method (Figure 2.6), slab is represented by beam element. Arrangement of column elements incorporate the column flexibility and torsional flexibility of attached torsional members [2].

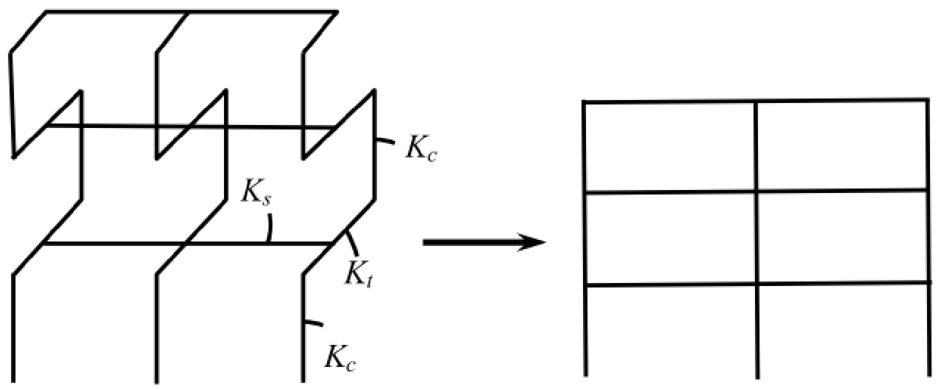


Figure 2-5 Extended Equivalent column method [Cano and Klinger, 1988]

c) Extended equivalent beam method

In extended equivalent beam method [Figure 2.7], column is modeled directly. Slab flexibility and torsional flexibility of attached torsional members are incorporated by arrangement of slab-beam elements. Both extended equivalent methods, developed by Vanderbilt, represent the flat-slab system by planer frames, which can be analyzed conventionally. Both methods include the effect of torsional flexibility under lateral and gravity load [2].

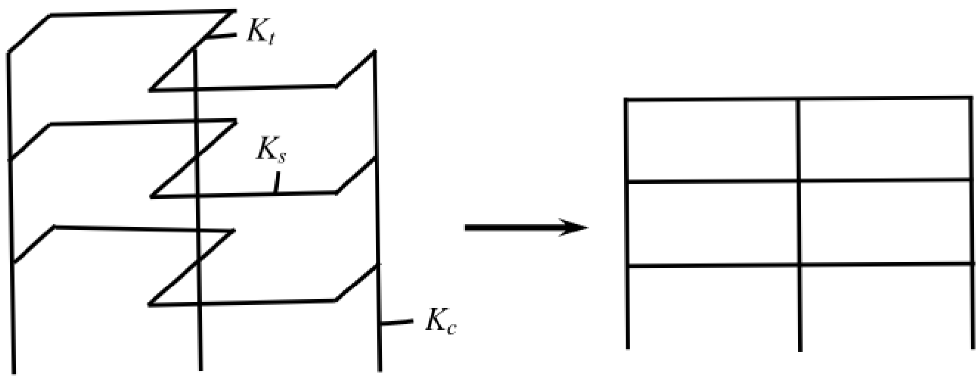


Figure 2-6 Extended Equivalent beam method [Cano and Klinger, 1988]

d) Explicit transverse torsional member method

Cano and Klinger (1988) proposed explicit transverse torsional member method to overcome the drawbacks of other methods. Model of slab-column frame is shown in Figure 2.7. Conventional columns are connected indirectly by two conventional slab-beam elements, each with half the stiffness of the actual slab-beam. The indirect connection, made using explicit transverse torsional members, permits the modeling of moment leakage as well as slab torsional flexibility. While the resulting frame is non-planar, this is not a serious complication. Because the transverse torsional members are presented only for the analytical model, their lengths can be taken arbitrarily, as long as the torsional stiffness is consistent. In explicit transverse torsional member model, gross member properties are used for slab-beams and column. Area, moment of inertia, and shear area are calculated conventionally. For computer input, the torsional stiffness K_t of the transverse torsional members is calculated by:

$$K_t = \sum \frac{9E_{cs}C_t}{l_2\left(1 - \frac{c_2}{l_2}\right)} \tag{2.7}$$

Where E_{cs} is the modulus of elasticity of slab concrete, c_2 is the dimension of the column in the transverse span of the framing direction, l_2 is the transverse span of framing, and C_t is the torsional constant [2].

Using the arbitrary length L for the torsional members, the torsional stiffness J is then calculated by expression:

$$J = K_t L / G \tag{2.8}$$

Where G is the shear modulus of slab concrete.

Column stiffness K_c is independent of K_t and is calculated conventionally, using actual column moment of inertia between the slabs and an infinite moment of inertia within the slabs. Slab stiffness K_s is calculated conventionally with the full transverse span (l_2). ACI 318 [13] also recommended that the effects of column capitals and drop panels can be included in the model by increasing the moment of inertia of that portion between the center of the column and face of column, bracket, or capital by the factor

$1/(1-c_2/l_2)^2$. This increasing is to account for the increased flexure stiffness of the slab-column connection region.

The explicit transverse torsional member model has several advantages. Structural modeling is simple and direct, requiring very few hand computations. Also, computed member actions in the slab-beams and transverse torsional members can be used directly for design of slabs and spandrels, respectively. This model can be developed even for the true three-dimensional analysis of slab system under combined gravity and lateral loads. Two sets of equivalent frames, each running parallel to one of the building's two principal plan orientations, can be combined to form a single three-dimensional model. This single model can be used to calculate actions in all members (slabs, columns, and spandrels) under many combinations of gravity and lateral loads as desired.

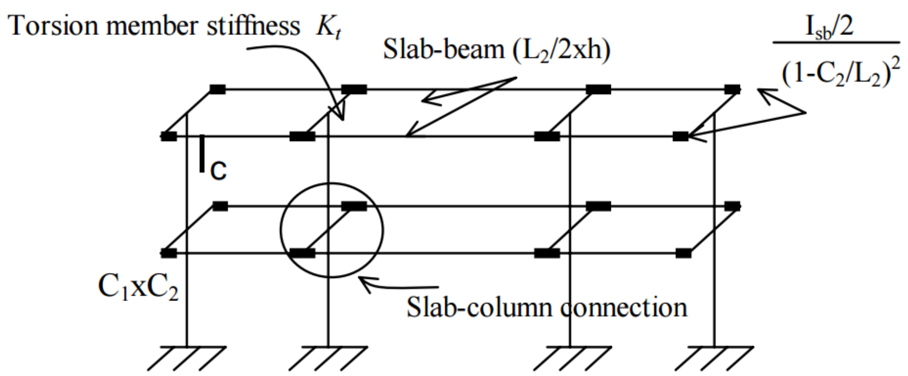


Figure 2-7 Three dimensional equivalent frame model using explicit transverse torsional member method [Cano and Klinger, 1988]

Because this method gives reasonable results, it does not require special computer programs and permits easy consideration of cracking in both slab-beams and transverse torsional members. The explicit transverse torsional member method is proposed as a powerful method for analyzing two-way slab systems under combinations of gravity and lateral loads. Using this method, an entire three-dimensional building can be analyzed at once, under combinations of gravity and lateral loads in different plan directions. If beam end shears for the equivalent frames running in each direction are corrected to avoid doubling the column axial loads, a single computer model, including the effects of cracking, can be used to calculate lateral drifts and all member actions (slabs, walls, and columns). Calculated slab actions can be assigned to column and middle strips, and slab design can easily be completed. No hand calculations are required for load combinations. The explicit transverse torsional member method is believed to present current ACI equivalent frame method concepts in a form that is both powerful and convenient for design purposes [2].

Equivalent frame analysis using computers

The classic equivalent-frame method was derived by assuming that the structural analysis would be carried out by hand using the moment-distribution method. Thus, tables were developed to evaluate fixed-end moments, stiffnesses, and equivalent column stiffnesses for use in such an analysis. If standard frame analysis software based on the stiffness method is to be used, the torsional member (and the resulting equivalent-column stiffness) defined in the classic equivalent-frame method will need to be incorporated into the stiffness of either the slab-beam or column elements. The general research direction has been to modify the stiffness of the slab-beam element by defining an effective slab width to reduce the element stiffness, particularly at connections. The frame analysis results for gravity loading, obtained using the modified slab-beam elements, should be in reasonable agreement with those obtained from the classic equivalent-frame method [1].

2.1.3 Yield line method

The yield line theory is based on the upper bound theorem of the theory of plasticity. This means in principle that a load is found which is high enough to make the slab fail, i.e. the safely in the ultimate limit state is equal to or lower than the intended value. If the theory is correctly applied the difference between the intended and the real safety is negligible, but there exists a great risk that unsuitable solutions may be used, leading to reduced safety factors, particularly in complicated cases like irregular slabs and slabs with free edges. When comparing the strip method and the yield line theory it should be noted that the strip method is a design method, as a moment distribution is determined, which is used for the reinforcement design. The yield line theory is a method for check of strength. When the yield line theory is used for design, assumptions have to be made for the moment distribution, e.g. relations between different moments. In practice the reinforcement is often assumed to be evenly distributed, which as a matter of fact may not be very efficient [4].

The yield line method used to determine the limit state of slab by considering the yield lines that occur in the slab as a collapse mechanism.

2.1.4 Strip method

The strip method is based on the lower bound theorem of the theory of plasticity, which means that it in principle leads to adequate safety at the ultimate limit state, provided that the reinforced concrete slab has a sufficiently plastic behavior. This is the case for ordinary under-reinforced slabs under predominantly static loads. The plastic properties of a slab decrease with increasing reinforcement ratio and to some extent also with increasing depth. The solutions should give adequate safety in most cases, possibly with the exception of slabs of very high strength concrete with high reinforcement ratios. As the theory of plasticity only takes into account the ultimate limit state, supplementary rules have to be given to deal with the properties under service conditions, i.e. deflections and cracks. The lower bound theorem of the theory of plasticity states that if a moment distribution can be found which fulfills the equilibrium equations, and the slab is able to carry these moments, the slab has sufficient safety in the ultimate limit state. In the strip method this theorem has been reformulated in the following way: Find a moment distribution which fulfills the

equilibrium equations. Design the reinforcement for these moments. The moment distribution has only to fulfill the equilibrium equations, but no other conditions, such as the relation between moments and curvatures. This means that many different moment distributions are possible, in principle an infinite number of distributions. Strip method encourages the designer to vary the reinforcement in a logical way, leading to an economical arrangement of steel as well as a safe design.
Torsional moments complicate the design procedure and also often require more reinforcement. Solutions without torsional moments are therefore to be preferred where this is possible. Such solutions correspond to the simple strip method, which is based on the following principle: In the simple strip method the load is assumed to be carried by strips that run in the reinforcement directions. No torsional moments act in these strips. The simple strip method can only be applied where the strips are supported so that they can be treated like beams. This is not generally possible with slabs which are supported by columns, and special solution techniques have been developed for such cases. One such technique is called the advanced strip method. This method is very powerful and simple for many cases encountered in practical design, but as hitherto presented it has had the limitation that it requires certain regularity in slab shape and loading conditions. It has here been extended to more irregular slabs and loading conditions.

An alternative technique of treating slabs with column supports or other concentrated supports is by means of the simple strip method combined with support bands, which act as supports for the strips. This is the most general method which can always be applied and which must be used where the conditions that control the use of other methods are not met. It requires a more time-consuming analysis than the other methods [4].

2.1.5 Finite element analysis method

Finite element analysis is a powerful computer method of analysis that can be used to obtain solutions to a wide range of structural problems involving the use of ordinary or partial differential equations. FE solvers can either use linear or non-linear analysis.

The structures having irregular types of plans with which the equivalent frame method has limitations in analysis can be analyzed without any difficulties by the FEM. FEM is a powerful tool used in the analysis of flat slabs. Most finite element programs are based on elastic moment distribution and material that obey Hooke's Law, but reinforced concrete is an elasto-plastic material and once it cracks its behavior is non- linear. As a consequence the support moments tend to be overestimated and the deflection of the slab is under estimated. Currently, one of the main criticisms of the FEM analysis is its reliance on the elastic solutions that result in high peaked support moments over the column. These support moments are unlikely to be realized under service loads due to cracking and thus the service span moments will be correspondingly increased.

Note that, before any analysis is carried out using computer software, it is always good practice to carry out some simple hand calculations that can be used to verify that the results are reasonable. Some possible checks are:-

1. Calculate $wL^2/8$ for a span and then check that the FE results give the same value between the peak hogging (average) and sagging moments, considering Nichol's principle. A discrepancy of 20% is acceptable; outside of this limit needs further investigation to determine the reasons.

2. Is the span/depth or height to depth ratio in line with standard practice, if not why?

3. Use direct methods from the code and compare, if these vary why?

4. Static equilibrium check: compare total load with total reactions [3].

Linear finite element analysis

This is currently the most widely used method of finite element analysis, which is less sophisticated than non-linear finite element analysis. Reinforced concrete (RC) is treated as an elastic isotropic material, which it evidently is not, and a number of assumptions have to be made to allow this method to be used.

Non-Linear finite element analysis

Many FE packages are capable of carrying out non-linear (iterative) analysis, but this is useful only for reinforced concrete design where it can be used to model the cracked behavior of concrete. Non-linear analysis is used for RC design because as the slab is loaded it will crack and this affects its stiffness. The program first carries out an analysis with uncracked section properties; it can then calculate where the slab has cracked, adjust the material properties and run the analysis again. This process continues until the variation in section properties between runs reaches a predetermined tolerance [3].

Types of non-linearity

1. **Material nonlinearity**

Material nonlinearity is associated with the inelastic behavior of a component or system. Inelastic behavior may be characterized by a force-deformation (F-D) relationship, also known as a backbone curve, which measures strength against translational or rotational deformation. The general F-D relationship shown in Figure 2.10 indicates that once a structure achieves its yield strength, additional loading will cause response to deviate from the initial tangent stiffness (elastic behavior). Nonlinear response may then increase (hardening) to an ultimate point before degrading (softening) to a residual strength value [16].

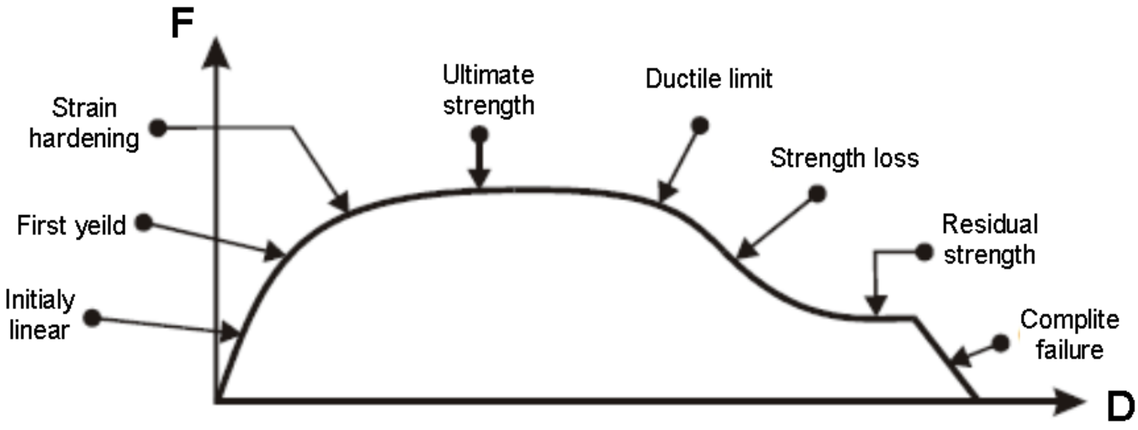


Figure 2-8 Force-Deformation relationship

A variety of F-D relationships may characterize material nonlinearity, including the following:

- Monotonic curve
- Hysteretic cycle
- Interaction surface

Monotonic curve

A monotonic curve is provided when a load pattern is progressively applied to a component or system such that the deformation parameter (independent variable) continuously increases from zero to an ultimate condition. The corresponding force based parameter (dependent variable) is then plotted across this range, indicating the pattern of material nonlinearity.

Some examples of monotonic F-D relationships (and their associated physical mechanism) include **stress-strain (axial)**, moment curvature (flexure) and plastic hinging (rotation).

Serviceability parameters may then be superimposed onto the nonlinear F-D relationship to provide insight into structural performance. Property owners and the general public are very much interested in performance measures which relates to daily use. Therefore it may be useful to introduce such limit states as immediate- occupancy

(IO), life safety (LS) and collapse-prevention (CP) which indicate the correlation between material nonlinearity and deterministic for structural damage sustained [16].

Hysteretic cycle

When the F-D relationship is developed for a component or system subjected to cyclic loading, hysteretic loops are produced. When modeling hysteretic dynamics, the fiber hinge is best applied. Hysteresis is useful for characterizing dynamic response under application of a time-history record.

Depending on structural geometry and materials, a hysteretic cycle may follow one of many different possible patterns. Four possible hysteretic behavior types are illustrated in Figure 2.11 [16]:

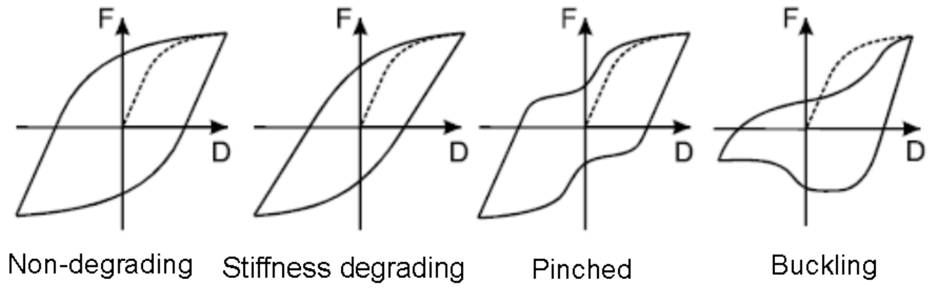


Figure 2-9 Hysteresis loop types

Interaction surface

An interaction surface is developed for a structural element when the combined relationship between various strength parameters is plotted. Von Mises, Mroz, or another such plasticity theory may be used to develop a 2D or 3D surface which represents a performance envelope for a given limit state. Behavior exceeds the limit state when the performance measure is outside the envelope. An example may be a 3D P-M2-M3 interaction surface describing the yielding of a column under combined axial, strong-axis and weak-axis bending. These three performance measures interact in a way which may be plotted to create a 3D ellipse. A response measure outside of the P-M-M envelope would indicate that the column has yielded [16].

2. Geometric Nonlinearity

Also known as P-Delta effect, involves the equilibrium and compatibility relationships of a structural system loaded about its deflected configuration. Of particular concern is the application of gravity load on laterally displaced multi-story building structures. This condition magnifies story drift and certain mechanical behaviors while reducing deformation capacity.

P-Delta effect typically involves large external forces upon relatively small displacements. If deformations become sufficiently large as to break from linear compatibility relationships, then Large-Displacement and Large-Deformation analysis become necessary. The two sources of P-Delta effects are described as follows [16]:

- **P-δ effect,** or P-"small-delta", is associated with local deformation relative to the element chord between end nodes. Typically, P-d only becomes significant at unreasonably large displacement values or in especially slender columns. So long as a structure adheres to the slenderness requirements pertinent to earthquake engineering, it is not advisable to model P-d, since it may significantly increase computational time without providing the benefit of useful information. An easier way to capture this behavior is to subdivide critical elements into multiple segments, transferring behavior into P-∆ effect (Powell 2006) [16].
- **P-∆ effect,** or P-"big-delta", is associated with displacements relative to member ends. Unlike P-d, this type of P-Delta effect is critical to nonlinear modeling and analysis. P-∆ may contribute **to loss** of lateral resistance, ratcheting of residual deformations, and dynamic instability (Deierlein 2010). To consider P-∆ effect directly, gravity load should be present during nonlinear analysis. Application will cause minimal increase to computational time and will remain accurate for drift levels up to 10% (Powell 2006) [16].

Finite element analysis using SOFiSTiK

SOFiSTiK V14 is very versatile and powerful finite element analysis and design software. It has numerous features. It is possible to use SOFiSTiK for building design, bridge design, non-linear analysis (ULS and SLS) and seismic analysis. Other features that witness the power of this software include: robust analysis (first and second order), construction stage manager and flexible definition of materials.

Nonlinear analysis using SOFiSTiK

Non-linear effects can be analyzed only with iterations. This is done in SOFiSTiK with static analysis module, ASE (General Static Analysis of Finite Element Structures), with a modified Newton method, considering constant stiffness matrix. The advantage of the method is that the stiffness matrix does not need to be decomposed more than once and that the system matrix remains always positive definite. The speed of the method is increased through an accelerating algorithm written by Crisfield. In this method new displacements and thus stresses are determined after every iteration step. It can be checked, whether plasticizing, cracks or any other non-linear effects have occurred at any elements. The plasticized elements generate different nodal loads compared to those of the linear analysis. These nodal loads which were generated by the elements are not anymore in equilibrium with the external nodal loads (after the first iteration step). The remaining residual forces are applied as additional loading during the next iteration step. Additional deformations and a new stress state which is general is closer to equilibrium result. The maximum residual force is printed for every iteration.

2.2 Research Review on Effective Slab Width Modeling Method

2.2.1 Introduction

Several researchers have worked on the development of effective slab width models that could be used to define the stiffness of an equivalent beam in a standard frame analysis program for the evaluation of moments and shears in a slab–column frame subjected to combined vertical (Gravity) and lateral loading. Pecknold [6] and Darvall and Allen [7] used classical plate theory and finite-element analyses to define an effective slab width. More recent work by Luo and Durrani [8] and [9], Hwang and Moehle [10], and Dovich and Wight [11] based their effective slab width and stiffness models on experimental results for reinforced concrete slabs. Hueste and Wight [12] proposed modifications of these effective slab width and stiffness models for post-tensioned slabs, based on observed damage in a post-tensioned slab–column frame system following a major earthquake [1].

2.2.2 Effective slab width prediction for flat slabs under gravity loading

In 1975, Packnold studied the effective slab-width of a typical interior panel by using elastic plate theory and a standard Levy type solution. The slab rotation at the center of the original slab was equated to the rotation of a beam subjected to the same moment [15]. The expressions for slab effective width for two types of ideal interfaces between slab and column, rigid column and flexible column, are:

$$\alpha_i = \frac{\dfrac{c_2}{\left(1-v^2\right)l_2}}{f_B + 6\sum_{1}^{\infty}\left(\dfrac{1}{m\pi}\right)^3 Q_m A_m} \tag{2.9}$$

Where: $A_m = \dfrac{a_m \sinh a_m^- - \sinh a_m\left(2\sinh\left(a_m - a_m^-\right) + a_m^- \cosh\left(a_m - a_m^-\right)\right)}{2\left(\sinh a_m\right)^2}$

$a_m = \dfrac{m\pi l_2}{l_1}$ $a_m^- = \dfrac{m\pi c_2}{1}$; v= poisson's Ratio, and

For rigid columns:

$$Q_m = \sin\frac{m\pi c_1}{l_1} + \left(1-\frac{c_1}{l_1}\right)m\pi\cos\left(\frac{m\pi c_1}{l_1}\right) \text{ and } f_B = \left(1-\frac{c_1}{l_1}\right)^3 \text{, and}$$

For flexible columns:

$$Q_m = \frac{3\left(\frac{\sin\left(\left(m\pi c_1\right)\right)/l_1}{\left(m\pi c_1\right)/l_1} - \cos\left(\left(m\pi c_1\right)/l_1\right)\right)}{\left(\left(m\pi c_1\right)/l_1\right)\left(\frac{c_1}{l_1}\right)} \text{ and } f_B = 1-\frac{9c_1}{8l_1}+\frac{3}{10}\left(\frac{c_1}{l_1}\right)^2$$

Where:-

$1/1\text{-}v^2$: the effect of the Poisson's ratio

Q_m: the factor for the load distribution

f_b: the factor for the decrease in rotational flexibility,

A_m: the factor for the geometries and the boundaries.

C_1, C_2, l_1, l_2: the geometries of connections

The effective width factor α_i is calculated based on the equivalent elastic stiffness of the interior slab-column connections. The flexural strength of the slab, which is an important factor in seismic response calculations, is not considered in this approach and the use of the same effective slab width for both interior and exterior connections is not appropriate.

In 1977, Allen and Darvall used a Fourier series technique and published the effective beam width values that are identical with Packnold's results. In a later study (this reference was not available, but was reported by Vanderbilt and Moehle), Allen compared the finite element analysis results with his previous values, and concluded that his previously published values were too high. Then he developed a refined Fourier technique to generate effective beam width values. These revised values are up to 13% smaller than the original ones [19].

In 1987 Banchik employed finite element method to calculate the effective beam width coefficients [19]. The variation of the coefficients for an interior frame and exterior frames is represented as shown below:

$$\alpha = 5.\frac{c_1}{l_1} + \frac{1}{4}.\frac{l_1}{l_2}, \text{ for interior frame} \tag{2.10}$$

$$\alpha = 3.\frac{c_1}{l_1} + \frac{1}{8}.\frac{l_1}{l_2}, \text{ for exterior frame} \tag{2.11}$$

Where, C_1, C_2, l_1, l_2: the geometries of connections.

2.2.3 Effective slab width prediction for flat slabs under gravity and lateral loading

According to (Luo and Durrani 1995, Interior connection) an equivalent beam model for interior connections is proposed based on the test results of 40 interior connections, in which columns are modeled conventionally, and the effective slab width is determined as a function of column and slab aspect ratios and the magnitude of the gravity load as shown below:

$$\alpha_i = \chi \left(\frac{1.02\left[(C_1/l_2)\right]}{0.05+0.002\left[(l_1/l_2)\right]^4-2\left[(C_1/l_1)\right]^3-2.8\left[(C_1/l_1)\right]^2+1.1\left[(C_1/l_1)\right]} \right) \tag{2.12}$$

Where: $\chi = \left[1-0.4\left(V_g / 4A_c\sqrt{f_c'}\right)\right]$, is a reduction factor accounting for the effect of gravity load.

V_g = total shear at the connection due to gravity load

A_c= area of the slab critical section

F_c'= compressive strength of concrete

The proposed approach is verified with selected experimental results and is found to be practical and convenient for analyzing flat-slab buildings subjected to gravity and lateral loading.

According to (Luo and Durrani 1995, Exterior connection), an equivalent beam model for exterior connections is proposed based on test results of 41 exterior connections, the ultimate moment-transfer capacity is found to be a combination of the torsional capacity of the slab edge and flexural capacity of the slab portion framing into the front face of the column. An equivalent beam model is proposed for exterior connections that gives a better prediction of the unbalanced moment at connections and lateral drift of flat-slab buildings. The effective slab width is determined as follows:

$$\alpha_e = \chi \frac{K_t}{K_t + K_s} \tag{2.13}$$

Where: χ , the reduction factor for gravity load can be assumed to be the same for both interior and exterior connections.

K_t= torsional stiffness of the slab edge and spandrel beams, if present

$K_s = \frac{(4E_{cs}I)}{l_1}$ is the flexural stiffness of the slab framing into the exterior connection.

From the test results of Luo and Durrani we can conclude that the effective slab width and stiffness of the exterior connections is significantly different from those of the interior connections. Recognition of this fact is important in accurately predicting the lateral drift and unbalanced moments at connections in flat-slab buildings.

Hwang and Moehle (2000) recommended that the uncracked effective stiffness for a model with rigid joints, for ratios of c_2/c_1 from ½ to 2 and a slab aspect ratio l_2/l_1 greater than 2/3, be determined using an effective beam width represented as

$$b_{\text{int}} = 2C_1 + l_1/3$$

Where b_{int} is the effective width for interior frame connections, C_1 and l_1 are the column dimension and slab span parallel to the direction of load being considered and C_2 and l_2 correspond to orthogonal directions. For exterior connections half the width defined for the interior connection will be used. Effects of cross section changes, such as slab opening, are to be considered. One way to accomplish this is to vary the value of the effective beam along the span [10].

To account for cracking, a stiffness reduction factor β has been proposed by Hwang and Moehle 2000 for non-pre-stressed slabs and is given by;

$\beta = 4c / l > 1/3$

Where c and *l* are the column dimension and slab span parallel to the load direction.

For the determination of the width of the concealed beam, or unbalanced moment transfer width or the slab-beam, there are several different considerations.

2.3 Summary

Mac Gregor recommends a summary of simple modeling rules that could be used to define the stiffness of an equivalent beam in standard frame analysis program. He recommends α (effective beam width factor) and β (coefficient accounting for cracking) values based on the researches on slab-column frame system subjected to gravity loading and combined gravity and lateral loading. For the values of α and β, Refer to Table 2.5. In addition, for lateral stability analysis of frames with slender columns, the effective moment of inertia of the column sections should be taken as 70 percent of the gross moment of inertia [1].

3. Comparison of Equivalent Frame Analysis Results with Finite Element Analysis Results

3.1 Description of the building

3.1.1 General

In this chapter a flat plate slab system is analyzed by equivalent frame method and finite element analysis methods. Comparison of reaction force, slab and column moments is done based on Non-linear analysis results. The analysis is limited to gravity loading, restraining the frame laterally both in x and y direction. For comparison purpose, one interior and exterior two dimensional frame systems are selected from the three dimensional building system.

The idealized flat plate slab system has 0.28m slab thickness with a uniform 2.8m story height. All the columns are 40x40cm square columns. Dead load of 2kN/m^2 and live load of 2kN/m^2 are considered according to Eurocode 2, for residential buildings.

3.1.2 Structural system

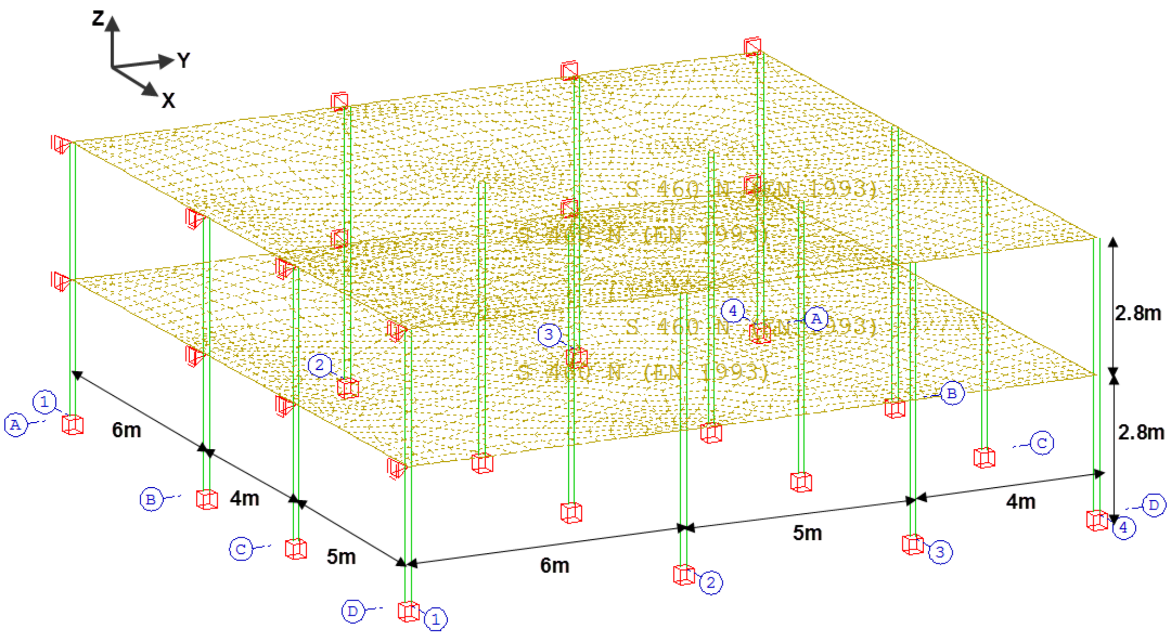


Figure 3-1 Three dimensional model of idealized flat slab

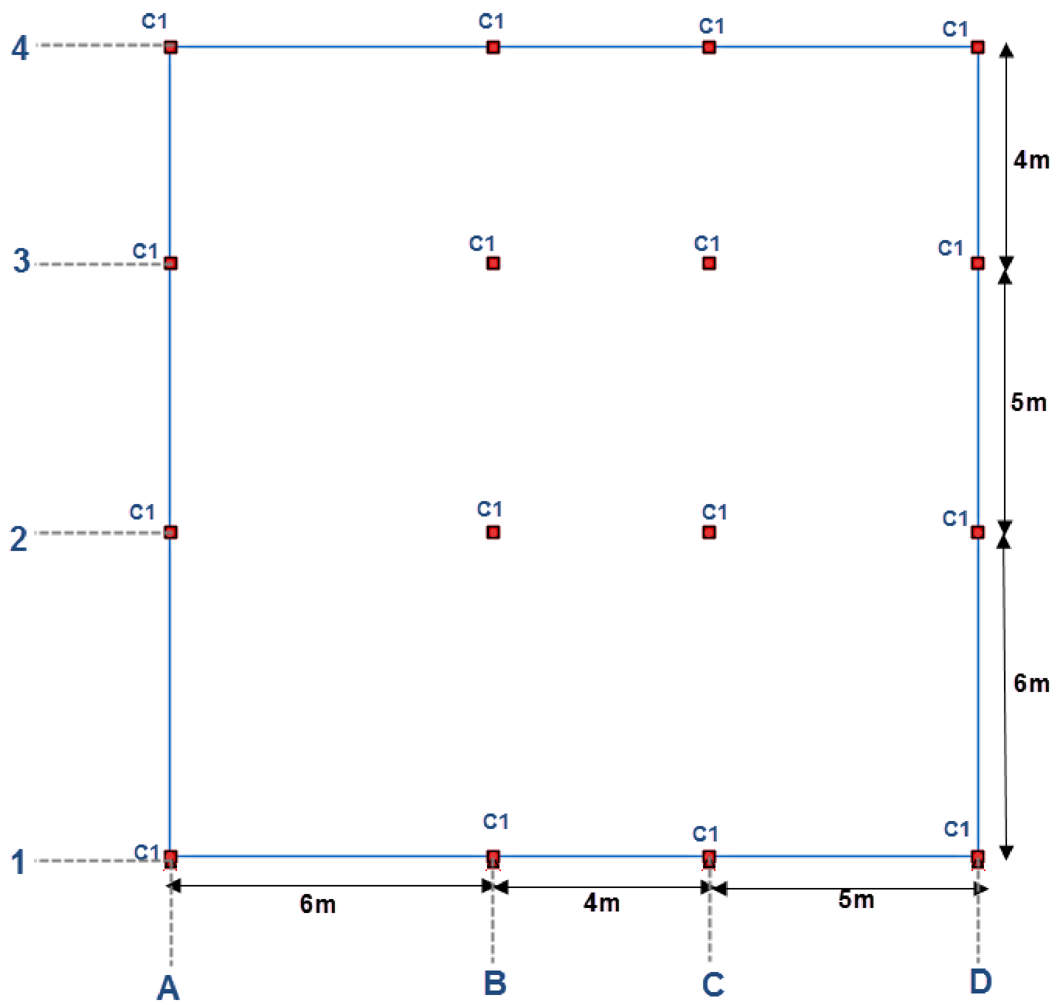


Figure 3-2 Plan view of idealized flat slab

3.1.3 Material

Id	Material	Material class	E [kN/mm^2]	ρ [kg/m3]	α [o/oo]	v
1	Concrete	C25/30	31.5	2350	0.01	0.2
2	Reinforcing Steel	S460	210	7850	0.012	0.3

3.1.4 Loads

- 25kN/m^3 is taken as the unit weight of the columns and slabs.
- A SID (super imposed dead load) of 2kN/m2 and Live load of 2kN/m2 is taken considering the building as a residential building, according to Eurocode 2.

3.1.5 Load combination

For ultimate load combination, 1.35 x (g_k + b_k)+ 1.5(q_k) is considered according to Eurocode 2.

3.2 Nonlinear Finite Element Analysis

- The analysis is done using SOFiSTiK V14 software.
- Nonlinear analysis in ULS is selected in order to consider the crack effect of the slab. Due to the cracked section of the slab we will have reduced flexural slab stiffness.

<u>Way of extracting slab moment analysis results (sample)</u>

Step 1: Bending moment (area element)

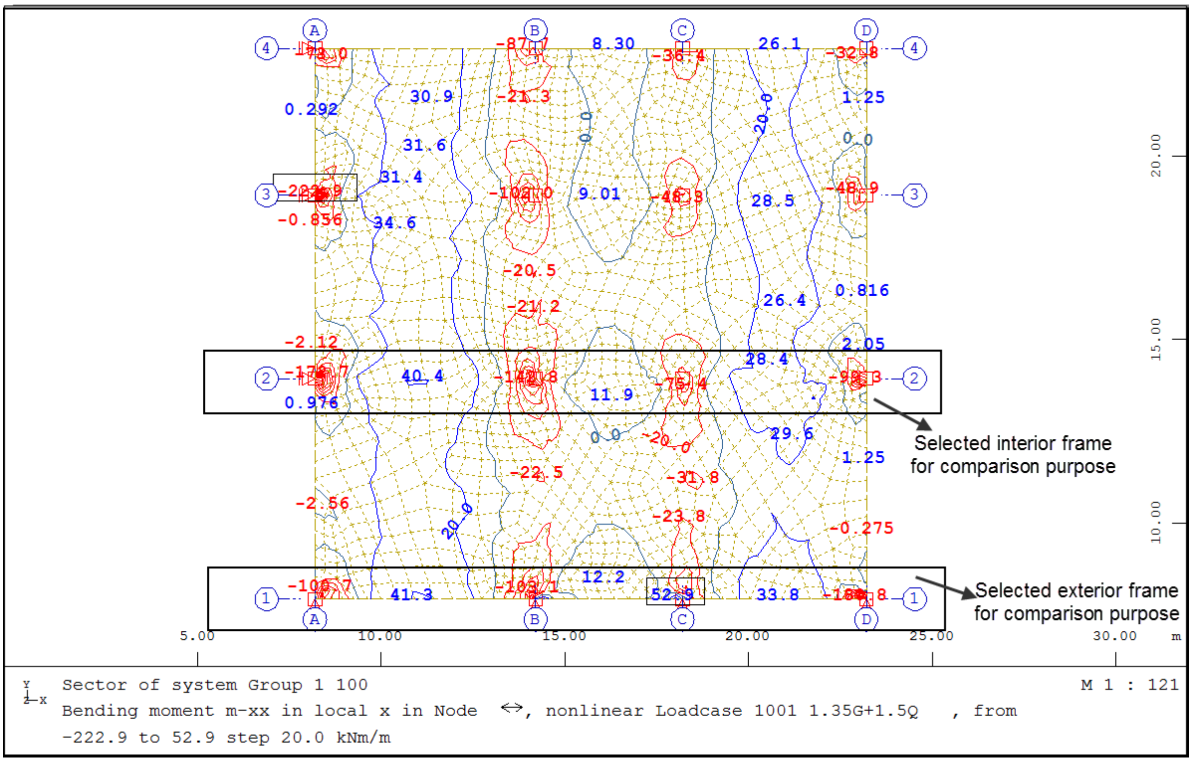


Figure 3-3 Slab bending moment, load case 1001-Mxx in local x direction

- Load case 1001 reperesents nonlinear load case in ULS, with $1.35G_k+1.5Q_k$ load combination.

Step 2: Bending moment per cut, considering interior frame

Since we are going to compare it with equivalent frame analysis results, we have to take moment result per cut. For the interior frame, a cut width of 5.5m is considered, taking half width of the span in both sides (l_2).

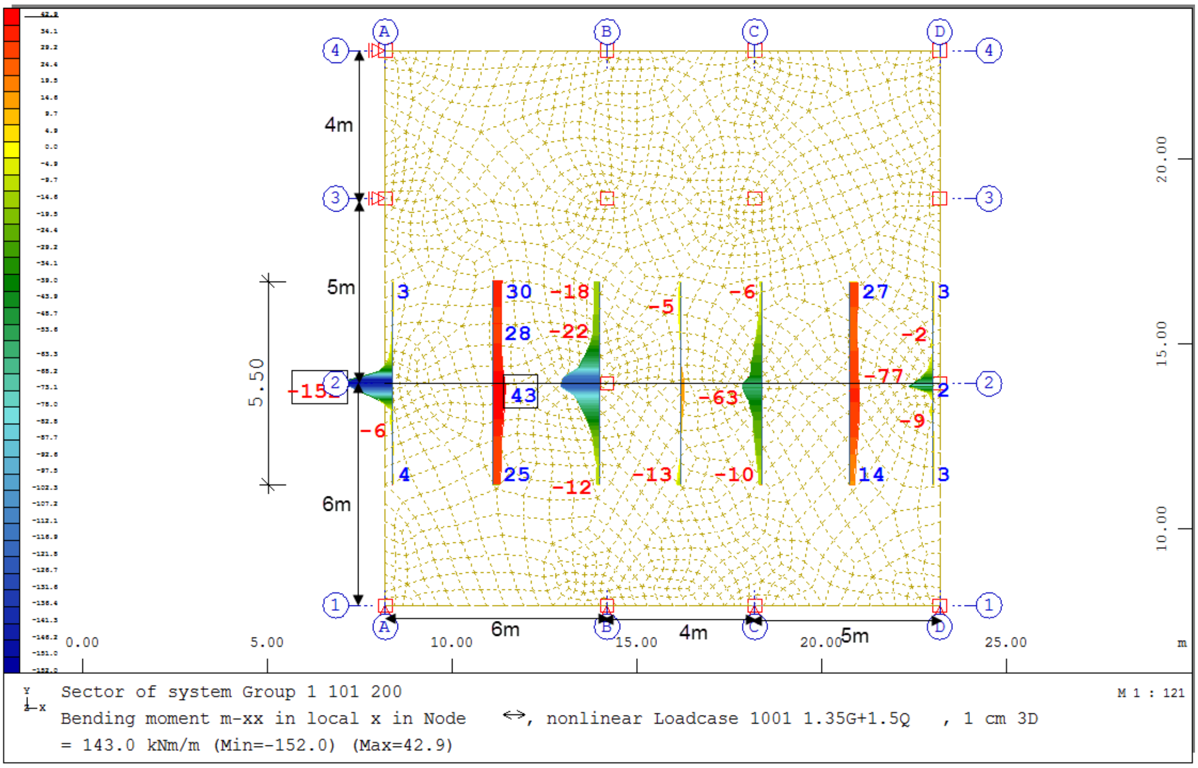


Figure 3-4 Slab bending moment per 5.5m cut length

Step 3: Bending moment per average (constant) cut

For ease of work, bending moment per average (constant) cut is considered. This will help us to calculate the moment per specified width. As it was shown in figure 3-5, for exterior span positive moment on axis 2AB, M=31kNm/m, the corsponding moment per 5.5m width will be M= 31kNm/m x 5.5m = 171kNm.

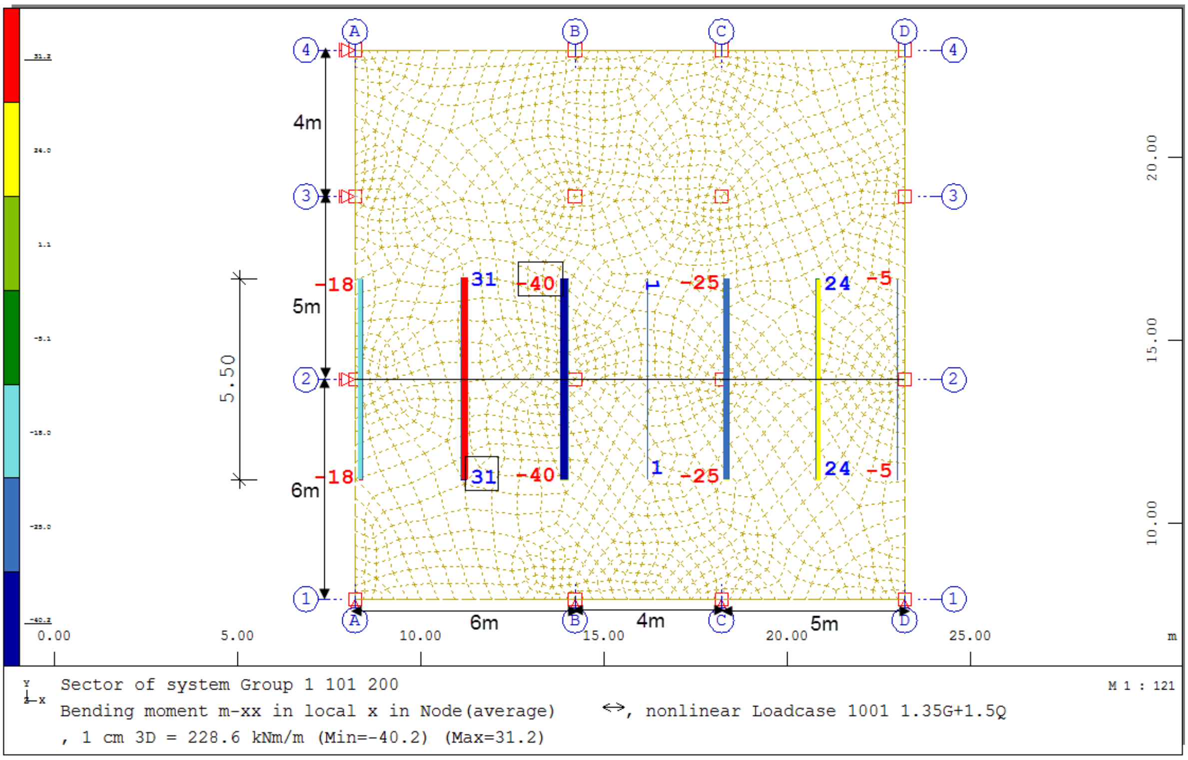


Figure 3-5 Slab bending moment per average (constant) cut

The analysis is done using SOFiSTiK V14 software. Extraction of slab bending moment results is the same as nonlinear finite element analysis method except, we have to consider load case 2101 and 2102 to obtain bending moment MAX-Mxx in local x and bending moment MIN-Mxx in local x respectively.

3.2.1 Linear Finite Element Analysis with Uncracked Section Capacity

In this method reinforced concrete is treated as an elastic isotropic material, considering the stiffness of uncracked slab and column sections.

3.2.2 Linear Finite Element Analysis, Considering 10%EI of the Slab

In this method cracked slab section and uncracked column sections are considered. Reduction of slab stiffness is considered by taking 10% of the elastic modulus of the slab.

3.2.3 Linear Finite Element Analysis, Considering 33%EI of the Slab

In this method cracked slab section and uncracked column sections are considered. Reduction of slab stiffness is considered by taking 33% of the elastic modulus of the slab.

3.2.4 Linear Finite Element Analysis, Considering 50%EI of the Slab

In this method cracked slab section and uncracked column sections are considered. Reduction of slab stiffness is considered by taking 50% of the elastic modulus of the slab.

3.3 Three Dimensional Equivalent Frame Analysis

The analysis is done using Etabs V9.5, Considering Equivalent beam width modeling method. α and β values are taken from Mac Gregor's recommendation, Refer table 2.1.

3.3.1 Structural system

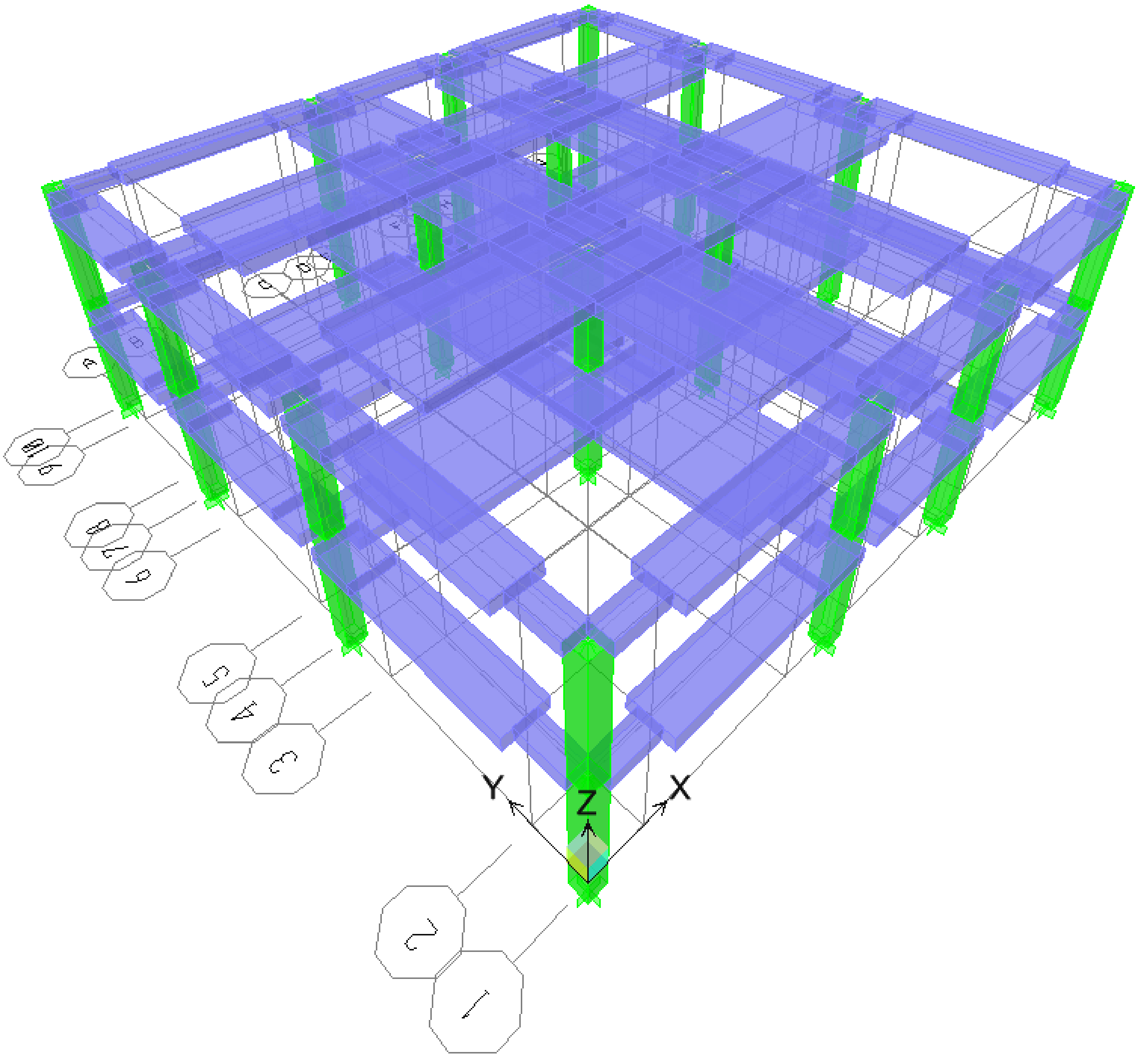


Figure 3-6 Three dimensional equivalent frame system for idealized flat slab system

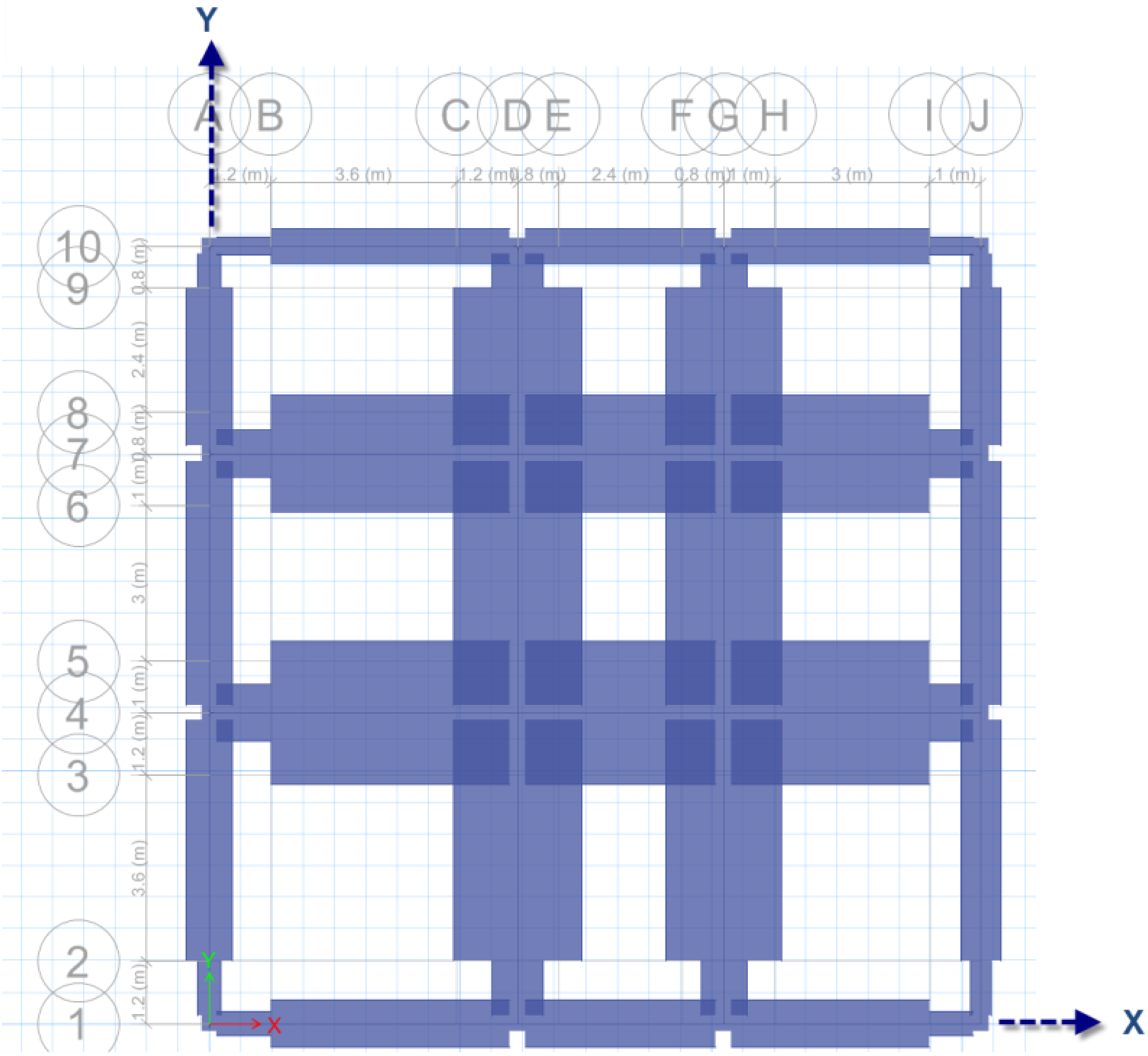
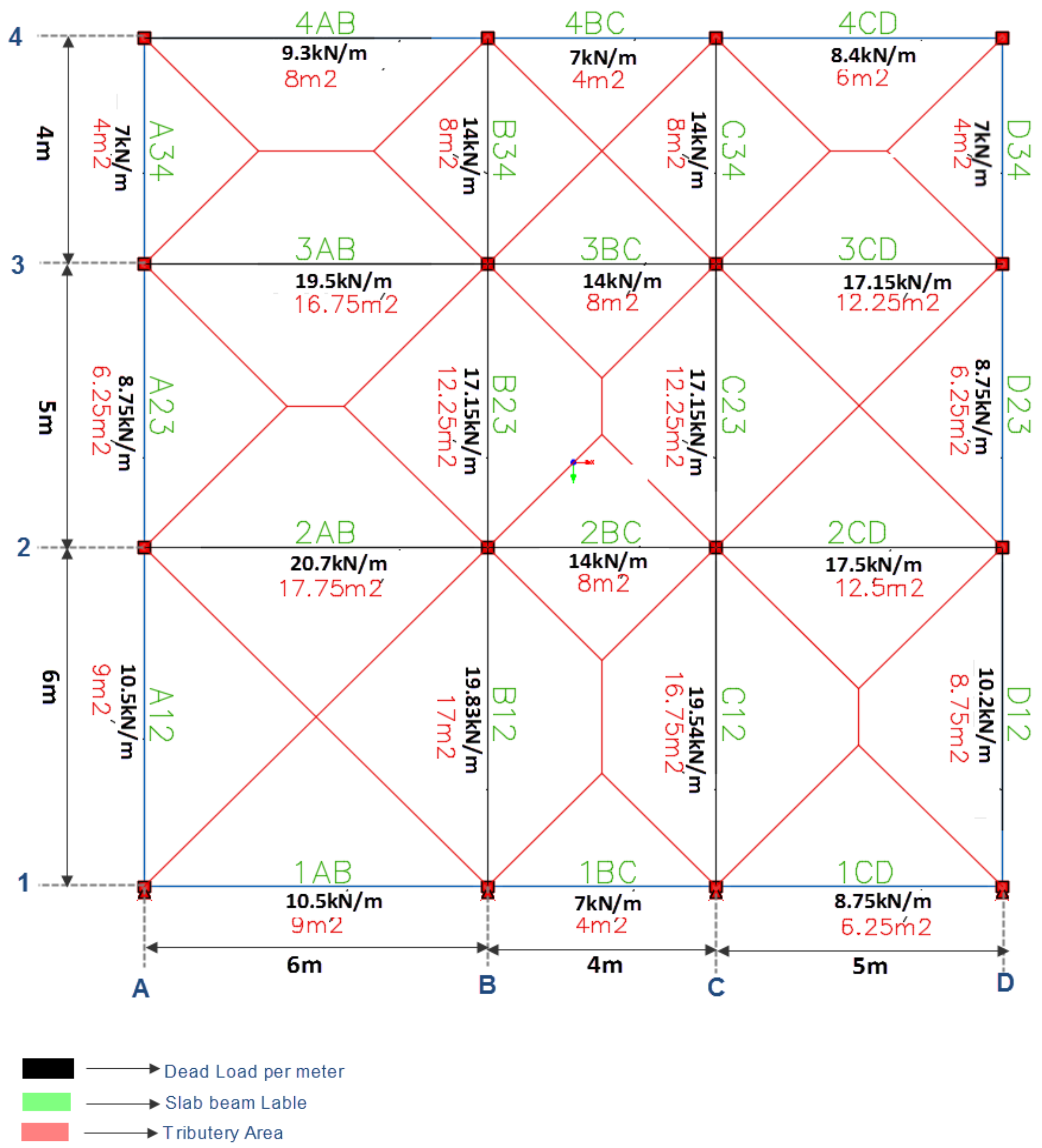


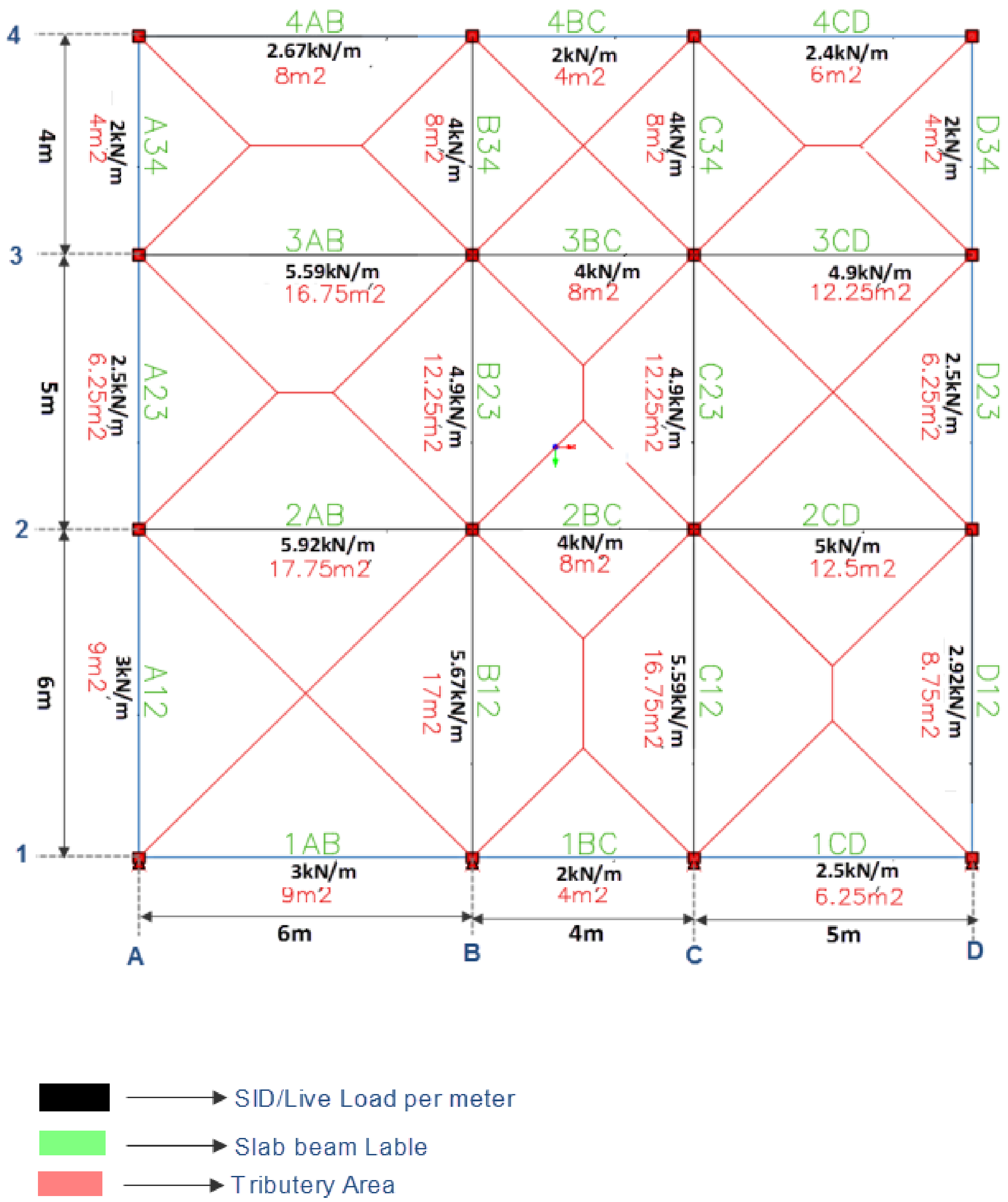
Figure 3-7 Plan view of idealized flat slab for three dimensional equivalent frame analysis

3.3.2 Loading system

Dead Load (Self weight)



SID (supper imposed dead load) and Live Load



3.4 Two Dimensional Equivalent Frame Analysis

The analysis is done using Etabs V9.5 software for selected exterior and interior frame systems, considering Equivalent beam width modeling method. α and β values are taken from MacGregor assumptions for the determination of effective slab width, Refer table 2.5.

3.4.1 Structural system

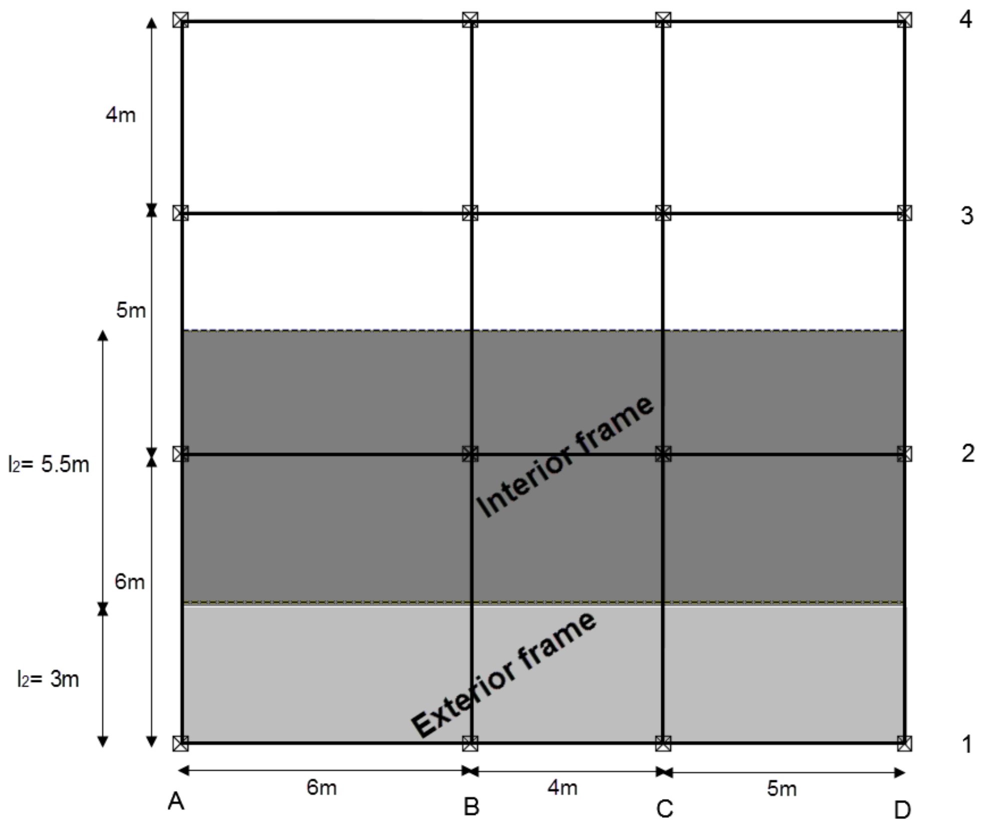
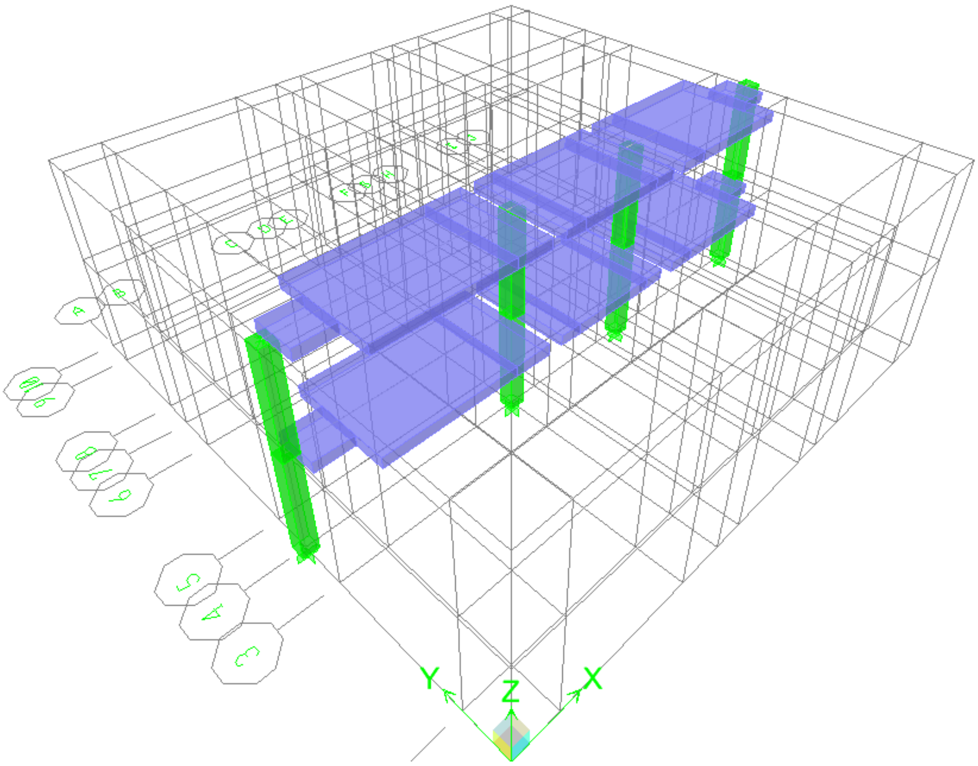
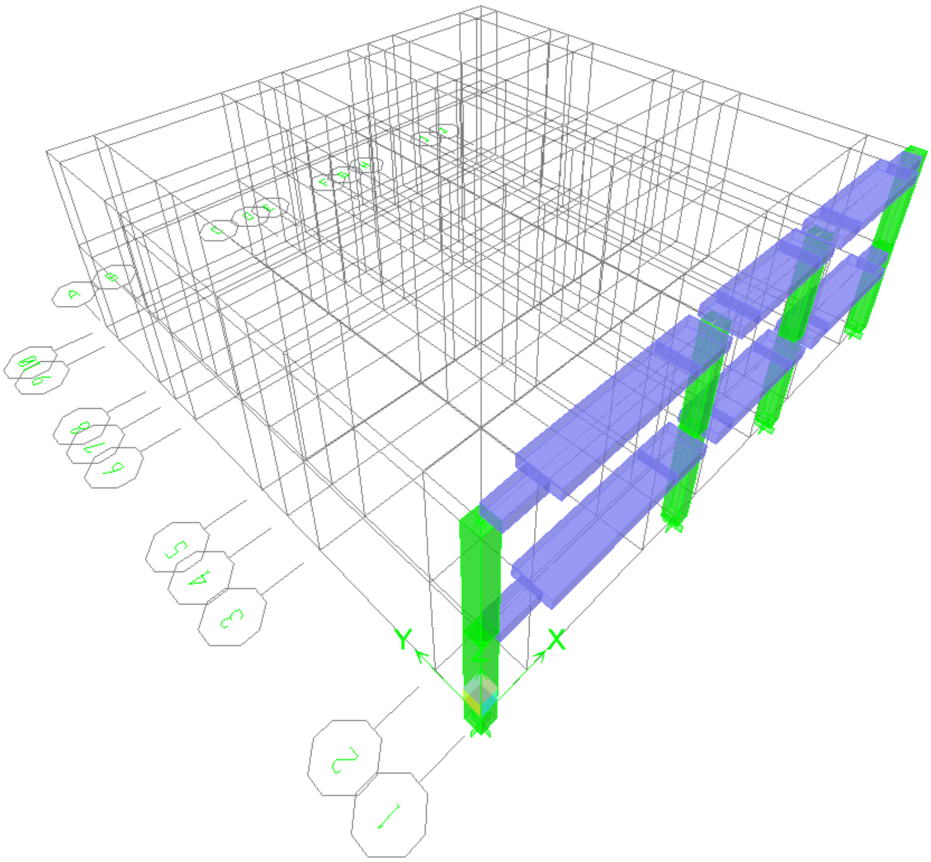


Figure 3-8 Selected interior and exterior frames for analysis



Interior frame model in 3D view



Exterior frame model in 3D view

3.4.2 Loading system

Interior frame:

Dead load= 0.28mx 25kN/m^3x 5.5m =38.5kN/m

SID= 2kN/m^2x 5.5m =11kN/m

Live load= 2kN/m^2x 5.5m =11kN/m

Exterior frame:

Dead load= 0.28mx 25kN/m^3x 3m =21kN/m

SID= 2kN/m^2x 3m =6kN/m

Live load= 2kN/m^2x 3m =6kN/m

3.5 Comparison of Analysis Results

3.5.1 Slab moment

Table 3-1 Slab moment results for exterior frame

Exterior frame										
No.	Floor level	Analysis method		Exterior span 1 (on axis 1AB)			Interior span (on axis 1BC)	Exterior span 2 (on axis 1CD)		
				Support moment (kNm)	Span moment (kNm)	Max.support moment (kNm)	Span moment (kNm)	Max. support moment (kNm)	Span moment (kNm)	Support moment (kNm)
1	First floor ceiling slab	Non-linear finite element analysis		-36	93	-93	12	-75	66	-9
2		3D Equivalent frame analysis		-35	46	-51	9	-28	27	-18
3		2D Equivalent frame analysis		-69	91	-104	30	-70	64	-43
4		Linear finite element analysis	Uncracked x-section	-12	123	-69	9	-45	87	-6
			Considering 10%EI of the slab	-15	117	-75	9	-48	81	-9
			Considering 33%EI of the slab	-15	117	-72	9	-48	84	-9
			Considering 50%EI of the slab	-12	120	-72	9	-45	84	-6
1	Ground floor ceiling slab	Non-linear finite element analysis		-45	93	-96	12	-75	72	-15
2		3D Equivalent frame analysis		-36	45	-53	10	-29	26	-19
3		2D Equivalent frame analysis		-72	89	-106	32	-71	62	-46
4		Linear finite element analysis	Uncracked x-section	-15	117	-78	9	-48	84	-9
			Considering 10%EI of the slab	-15	114	-81	9	-51	81	-9
			Considering 33%EI of the slab	-15	117	-81	9	-51	81	-9
			Considering 50%EI of the slab	-15	117	-81	9	-51	84	-9

Table 3-2 Slab moment results for interior frame

Interior frame										
No.	Floor level	Analysis method		Exterior span 1 (on axis 2AB)			Interior span (on axis 2BC)	Exterior span 2 (on axis 2CD)		
				Support moment (kNm)	Span moment (kNm)	Max.support moment (kNm)	Span moment (kNm)	Max. support moment (kNm)	Span moment (kNm)	Support moment (kNm)
1	First floor ceiling slab	Non-linear finite element analysis		-99	171	-220	6	-138	132	-28
2		3D Equivalent frame analysis		-58	98	-96	11	-53	59	-31
3		2D Equivalent frame analysis		-106	181	-185	42	-123	127	-64
4		Linear finite element analysis	Uncracked x-section	-44	231	-215	-6	-138	160	-22
			Considering 10%EI of the slab	-61	215	-231	-6	-149	154	-33
			Considering 33%EI of the slab	-55	220	-225	-5	-143	154	-28
			Considering 50%EI of the slab	-50	226	-220	-5	-143	154	-28
1	Ground floor ceiling slab	Non-linear finite element analysis		-105	171	-209	11	-132	132	-44
2		3D Equivalent frame analysis		-64	93	-101	17	-56	55	-35
3		2D Equivalent frame analysis		-117	172	192	51	-128	120	-74
4		Linear finite element analysis	Uncracked x-section	-55	215	-215	-6	-143	154	-28
			Considering 10%EI of the slab	-61	209	-226	6	-154	149	-28
			Considering 33%EI of the slab	-61	209	-220	5	-149	154	-28
			Considering 50%EI of the slab	-55	215	-220	5	-149	154	-28

- Two dimensional equivalent frame analysis gives as fair results as compared to non-linear finite element analysis results, refer to table 3-1 and 3-2.
- Three dimensional equivalent frame analysis gives as an underestimated support and span moments, due to the assumption of load distribution mechanism.

3.5.2 Column moment

Table 3-3 Column moment results for exterior frame (top end)

Exterior frame column top end moments (kNm)							
No.	Floor level	Analysis method		Exterior column on axis 1A	Interior column on axis 1B	Interior column on axis 1C	Exterior column on axis 1D
1	First floor ceiling slab	Non-linear finite element analysis		77	-42	33	-29
2		3D Equivalent frame analysis		41	-36	17	-23
3		2D Equivalent frame analysis		78	-55	30	-53
4		Linear finite element analysis	Uncracked x-section	38	-18	12	-25
			Considering 10%EI of the slab	42	-22	15	-29
			Considering 33%EI of the slab	41	-21	14	-28
			Considering 50%EI of the slab	40	-20	14	-27
1	Ground floor ceiling slab	Non-linear finite element analysis		24	-32	-12	-39
2		3D Equivalent frame analysis		13	-14	5	-8
3		2D Equivalent frame analysis		26	-21	10	-19
4		Linear finite element analysis	Uncracked x-section	13	-7	4	-9
			Considering 10%EI of the slab	12	-11	15	-13
			Considering 33%EI of the slab	12	-8	4	-10
			Considering 50%EI of the slab	12	-8	4	-10

Table 3-4 Column moment results for interior frame (top end)

Interior frame column top end moments (kNm)							
No.	Floor level	Analysis method		Exterior column on axis 2A	Interior column on axis 2B	Interior column on axis 2C	Exterior column on axis 2D
1	First floor ceiling slab	Non-linear finite element analysis		161	-74	34	-78
2		3D Equivalent frame analysis		66	-49	25	-39
3		2D Equivalent frame analysis		125	-80	46	-84
4		Linear finite element analysis	Uncracked x-section	75	-33	19	-53
			Considering 10%EI of the slab	91	-52	32	-66
			Considering 33%EI of the slab	86	-45	28	-62
			Considering 50%EI of the slab	83	-42	25	-59
1	Ground floor ceiling slab	Non-linear finite element analysis		48	-49	15	-57
2		3D Equivalent frame analysis		23	-23	10	-15
3		2D Equivalent frame analysis		44	-37	17	-32
4		Linear finite element analysis	Uncracked x-section	28	-16	8	-21
			Considering 10%EI of the slab	27	-21	7	-27
			Considering 33%EI of the slab	28	-18	8	-28
			Considering 50%EI of the slab	28	-18	9	-22

- Columns take more moment in case of nonlinear finite element analysis method and two dimensional equivalent frame analysis methods as compared to linear finite element analysis method and three dimensional equivalent frame analysis methods.
- When we compare linear and nonlinear finite element analysis results, columns attract more moment in case of nonlinear finite element analysis method as

compared to linear finite element analysis method. The reason is, when we consider a cracked slab, the stiffness of the slab will be reduced and more moment will be transferred to the columns.

3.5.3 Reaction forces

Table 3-5 Reaction force results for interior frame

<table>
<tr><th colspan="7">Exterior frame, Reaction force (kN)</th></tr>
<tr><th rowspan="2">No.</th><th rowspan="2" colspan="2">Analysis method</th><th>Exterior column</th><th>Interior column</th><th>Interior column</th><th>Exterior column</th></tr>
<tr><th>on axis 1A</th><th>on axis 1B</th><th>on axis 1C</th><th>on axis 1D</th></tr>
<tr><td>1</td><td colspan="2">Non-linear finite element analysis</td><td>-268</td><td>-458</td><td>-447</td><td>-208</td></tr>
<tr><td>2</td><td colspan="2">3D equivalent frame analysis</td><td>-261</td><td>-448</td><td>-396</td><td>-217</td></tr>
<tr><td>3</td><td colspan="2">2D equivalent frame analysis</td><td>-261</td><td>-470</td><td>-416</td><td>-216</td></tr>
<tr><td rowspan="5">4</td><td rowspan="5">Linear finite element analysis</td><td>Uncracked x-section</td><td>-253</td><td>-462</td><td>-391</td><td>-216</td></tr>
<tr><td>Modified based on MacGregor's recommendation</td><td>-306</td><td>-598</td><td>-486</td><td>-271</td></tr>
<tr><td>Considering 10%EI of the slab</td><td>-253</td><td>-462</td><td>-392</td><td>-216</td></tr>
<tr><td>Considering 33%EI of the slab</td><td>-254</td><td>-462</td><td>-391</td><td>-216</td></tr>
<tr><td>Considering 50%EI of the slab</td><td>-254</td><td>-462</td><td>-391</td><td>-216</td></tr>
</table>

Table 3-6 Reaction force results for Interior frame

Interior frame, Reaction force (kN)						
No.	Analysis method		Exterior column on axis 2A	Interior column on axis 2B	Interior column on axis 2C	Exterior column on axis 2D
1	Non-linear finite element analysis		-523	-978	-834	-435
2	3D equivalent frame analysis		-495	-877	-774	-415
3	2D equivalent frame analysis		-473	-872	-763	-392
4	Linear finite element analysis	Uncracked x-section	-518	-1047	-881	-440
		Modified based on MacGregor's recommendation	-660	-1379	-1189	-549
		Considering 10%EI of the slab	-516	-1048	-877	-439
		Considering 33%EI of the slab	-516	-1048	-878	-439
		Considering 50%EI of the slab	-517	-1048	-879	-439

- When we compare area element analysis results (nonlinear and linear finite element analysis method) with line element analysis results (2D and 3D equivalent frame method), we can observe that almost 60% of the load is attracted to interior frames and 40% to exterior frames in case of area element analysis, whereas equal load distribution is there in case of line element analysis methods.

4. Conclusion

Slab moment results

For flat plate slab systems subjected to gravity loading only, two dimensional equivalent frame analysis gives as fair result as compared to nonlinear finite element analysis results. Three dimensional equivalent frame analysis underestimates slab moments as a result of triangular load distribution mechanism.

In linear finite element method, due to the consideration of uncracked section, we will get overestimated slab moment results and underestimated column moment results as compared to nonlinear finite element analysis results. An approximate analysis results can be obtained by reducing the modulus of elasticity of the slab by 10-50% to account crack effects.

Generally, we can conclude that consideration of nonlinear effect is crucial.

Column moment results

Two dimensional equivalent frame method involves representation of three dimensional slab system by a series of two dimensional frames that are then analyzed for loads acting in the plane of the frames. In this case, we may get different moment results in X and Y direction for a specific column. A column may be exterior column in X direction and interior column in Y direction for planer frame analysis. As a result, one cannot rely on the results of two dimensional equivalent frame analysis results for the determination of column moments. To solve this, it is advisable to modify three dimensional equivalent frame analysis method by modifying effective beam width factor (α) and coefficients accounting for cracking (β).

In addition, it is advisable to consider load pattern for the determination of column moments.

Reaction force results

Reaction force results of finite element analysis method, for interior and exterior frame are higher and lower than equivalent frame analysis method respectively. Almost 60% of gravity load is carried by interior columns and the rest 40% is carried by exterior columns, in finite element analysis method. Whereas, the columns share gravity load equally in case of equivalent frame analysis methods.

5. Recommendation

Further studies can be carried on three dimensional equivalent frame analysis method for flat slab analysis by modifying the effective beam width factor.

References

1. James G. MacGregor and James K. Wight, 2005, "Reinforced Concrete Mechanics and Design", prentice hall, Singapore.
2. Professor Yogendra Singh, "MODELLING AND ANALYSIS OF MULTISTORY BUILDINGS," Department of Earthquake Engineering, IIT Roorkee
3. O Brooker, "How to design reinforced concrete flat slabs using Finite Element Analysis"
4. Proffessor A. Hillerborg, "Strip Method Design Handbook."
5. Building Code Requirements for Structural Concrete (ACI 318-08) and Commentary.
6. Pecknold, D., "Slab Effective Width for Equivalent Frame Analysis," ACI Journal, Proceedings, Vol. 72, No. 4, April 1975, pp. 135–137.
7. Darvall, P. and Allen, F., "Lateral Load Equivalent Frame," ACI Journal, Proceedings, Vol. 74, No. 7, July 1977, pp. 294–299.
8. Luo, Y. H. and Durrani, A. J., "Equivalent Beam Model for Flat-Slab Buildings-Part I: Interior Connections," ACI Structural Journal, Vol. 92, No. 1, January–February 1995, pp. 115–124.
9. Luo, Y. H. and Durrani, A. J., ""Equivalent Beam Model for Flat-Slab Buildings-Part II: Exterior Connections," ACI Structural Journal, Vol. 92, No. 2, March–April 1995, pp. 250–257.
10. Hwang, S. J. and Moehle, J. P., "An Experimental Study of Flat-Plate Structures under Vertical and Lateral Loads," Report No. UCB/SEMM-90/11, Department of Civil Engineering, University of California at Berkeley, January 1996.
11. Dovich, L. and Wight, J. K., "Effective Width Model for Seismic Analysis of Flat Slab Systems," ACI Structural Journal, Vol. 102, No. 6, November–December 2005, pp. 868–875.
12. Hueste, M. B. and Wight, J. K., "Evaluation of a Four-Story Reinforced Concrete Building Damaged During the Northridge Earthquake," Earthquake Spectra, EERI, Vol. 13, No. 3, pp. 387–414.

13. Kang, T. H. K. and Wallace, J. W., "Dynamic Responses of Flat Plate Systems with Shear Reinforcement," ACI Structural Journal, Vol. 102, No. 5, September–October. 2005, pp. 763–773.
14. Yuanhui Luo, "Evaluation, Modeling, and Retrofit of Flat-Slab Buildings subjected to Seismic Loading," A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy.
15. R.S More, "Analysis of flat slab," V.S Sawant, International Journal of Science and Research (IJSR).
16. CSI KNOWLEDGE BASE.
17. Powell, G. H. (2010). Modeling for Structural Analysis: Behavior and Basics (1st ed.).
18. Deierlein, G. G., Reinhorn, A. M., and Willford, M. R. (2010). Nonlinear Structural Analysis For Seismic Design, NEHRP Seismic Design Technical Brief No. 4., NIST GCR 10-917-5. Gaithersburg, MD: National Institute of Standards and Technology.
19. Banchik, C. A., "Effective Beam Width Coefficients for Equivalent Frame Analysis of Flat-Plate Structures," ME thesis, University of California at Berkeley, Calif., May 1987, 56 pp.

DECLARATION

I hereby declare that the work presented in this thesis is my original work and has not been presented for a degree in any other University and that all sources of material used for the thesis have been duly acknowledged.

________________________ ________________________

Lelissie Kudama Date

(Candidate)

This is to certify that the above declaration made by the candidate is correct to the best of my knowledge.

________________________ ________________________

Dr. Esayas G/Youhannes Date

(Thesis Advisor)