OPTICAL MEASUREMENT OF MICROSCOPIC TORQUE BY USING *HE-NE* LASER

By

Tamirat Abebe Desta

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The undersigned hereby certify that they have read and recommend to the School of Graduate Studies for acceptance a thesis entitled "**Optical Measurement Of Microscopic Torque by Using *He-Ne* laser**" by **Tamirat Abebe Desta** in partial fulfillment of the requirements for the degree of **Master of Science in Physics**.

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Supervisor: ______________________
Dr. Araya Asfaw

Examiners: ______________________
Dr. Mulugeta Bekele

____
Prof. A. V. Gholap

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Author: **Tamirat Abebe Desta**

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To my father Abebe Desta

and my mother Tsehay Tessema

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Abstract

In recent years there has been an explosive development of interest in the measurement of forces at the microscopic level, such as within living cells [1,2,3], as well as the properties of fluids and suspensions on this scale, using optically trapped particles as probes. The next step would be to measure torques and associated rotational motion[4]. We demonstrate an optical system that can apply and accurately measure the torque exerted by the trapping beam on a rotating birefringent particle (calcite). Here we used a He-Ne laser beam of wave length of $633nm$ and power of $35mW$. By taking this laser beam we get an optical torque of $9.983pN.\mu m$ and the frequency of the rotating particle is $112.16Hz$ and $136.94Hz$ for RCP and LCP beam. The laser-induced torque acting on an optically trapped microscopic birefringent particle can be used for these measurements. Here we present a method for simple, robust, accurate, simultaneous measurement of the rotation speed of a laser trapped birefringent particle, and the optical torque acting on it, by measuring the change in angular momentum of the light passing through the particle. This method does not depend on the size or shape of the particle or the laser beam geometry, nor does it depend on the properties of the surrounding medium. This could allow accurate measurement of viscosity on a microscopic scale.

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Introduction

Optical torques have been measured previously; two methods have been used. Firstly, if a particle with known birefringent properties has a simple and accurately known size and shape, ideally a flat disc, the torque can be calculated from the beam power [5]. However, particles of more complex shapes will often be used in experiments or encountered in samples; for example, spherical particles are ideal for making measurements of viscosity. Secondly, torques have been determined by measuring the rotation speed in a medium of known viscosity [6,7]. This method cannot be used if the aim is to measure an unknown viscosity. This method would also fail if there were other torques acting on the particle, or if the viscous drag is affected by nearby walls or other particles, or if the particle is not rotating. Previous methods for measuring rotation speeds, based on the periodic variation of backscattered light [8], can also have problems. Very regular or rotationally symmetric particles provide insufficient variation or variation at an increased frequency. Consideration of the basic physical processes giving rise to the torque gives a new method for measuring the torque and rotation speed that overcomes all of these problems. Since optical torques and forces are very small, microscopic particles are ideal for the observation and application of optical torques and rotation. Such microscopic particles will typically be confined within a laser trap. Strongly focussed laser light incident on a transparent particle, usually in a liquid medium, will produce a gradient force acting on the particle towards the region of highest irradiance. If this gradient force near the focus is stronger than scattering and absorption forces, the particle will be trapped at the beam focus, where the irradiance is highest. This technique of three-dimensional confinement and manipulation is called laser micro-manipulation, trapping, or optical tweezers.

Optical tweezers have made critical contributions to the creation of a vibrant field of biophysical research single molecule manipulation of nucleic acids and protein complexes [1,2]. It would be equally useful to have a handle that could be used to rotate

biological structures and to precisely measure the associated torque. Here we demonstrate angular trapping and torque detection using nominally spherical but anisotropic quartz particles. The torque acting on the particle and its deviation from the trap direction are determined by direct measurement of the change in angular momentum of the transmitted beam. The ability to measure instantaneous torque is of great importance, since it will facilitate precise measurement of the torque generated by particles as they rotate. The wide bandwidth and accuracy of our detection scheme allow us to measure Brownian rotational motion of the trapped particle and to use feedback to control the applied torque or particle angle.
An optical trap is based on the attraction of small particles is regions of high intensity in a tightly focused laser beam. Optical torque will result if there is a transfer of angular momentum from the beam to the trapping particle. Since the optical torques result from the change in angular momentum of the beam, it is in principle possible to measure the trapped torque by measuring the momentum of the scattered light[21].
Several other techniques have been demonstrated for rotating microscopic particles. These include use of circularly polarized beams or combinations of beams to rotate spherical particles [6-8], use of linearly or circularly polarized light to orient or apply torque to birefringent calcite particles [11]. The technique we demonstrate here is similar to that employed by Friese et al., but with the important advantage that angular trapping is combined with a detector allowing instantaneous measurement of the torque acting on the particle. Here our set up was built by using simple materials not standardized materials, because of this the alignment takes long time. The resolutions of torque measurement and angular confinement are limited by rotational Brownian motion of the particle.

This thesis organized in five chapters, the first chapter is the introduction part which includes the historical development of light and the nature of light. Also it includes the derivation of electromagnetic wave equation from the four Maxwell equations. And the second chapter describes the polarization of light, different types of polarized light and its propagation. When light propagates through birefringence crystal especially calcite, it transfers angular momentum to the particle. In the third chapter consists: angular momentum of the beam is derived, birefringence crystal, and optical torques are discussed. On fourth chapter, optical trapping of a birefringent crystals, the measurement of optical torques and the experimental set up. The final chapter consists a brief summery and conclusion based on the experimental results.

Chapter 1

Light and Its Property

1.1 Historical Development of Light

The prevailing view of the nature of light has changed several times in the past three centuries. Each time the answer to the question "what is light?" has assumed more fundamental importance in the physicists picture of the universe.

The Iraqi-born scientist Al Hazem, circa 900 AD, wrote that when a ray of light passed from air into glass, part of it reflected and the part that was transmitted bent sharply as it entered the glass. You can see the same phenomenon today by observing the image of a pencil partially immersed in a glass of water. Refraction (literally bending) describes this change in direction of a wave as it changes speed when crossing from one medium into another. Unlike reflection, which can occur for both waves and particles, refraction is most conveniently explained by the wave nature of light. For example, when a light wave crosses an optical interface into a denser medium at an angle, (i) the wave fronts slow down, so (ii) the wave crests bunch together, and therefore (iii) the direction of propagation of the wave must change. Note that the time-dependent part of the wave (i.e. its frequency) must be continuous across the interface, so does not change. Our ears and eyes are frequency detectors, which is why sounds and colors of objects do not change when you are swimming under water. While changes in wave speed and wavelength require sensitive equipment to detect, the bending of a light beam is easy to observe and measure.

The role of vision in our daily lives is so strong we almost think that seeing something is synonymous with understanding it. Our brains are indeed built with a highly developed neural system for processing the sensation of light. Much of our technology for acquiring information, from books to computer monitors and nowadays even cameras built into cell phones caters to our visual sensory system. Thus light and the physical effects that modify how light propagates are topics of enormous technological importance. Interestingly, much of the practical applications of light can be achieved by first adopting the "wrong" model for light's true nature. The everyday experience that light travels in straight lines, reinforced by the phrase "out of sight, out of mind", suggests that light travels like bullets that need an open path to reach their destination. This so-called corpuscular view of light was held by Isaac Newton, whose convictions carried such weight that more than a hundred years passed before the view was successfully challenged both experimentally and theoretically. The corpuscular view can be related to the idea of light traveling along rays, and simple rules for describing how rays must be altered at the interfaces between different materials gives rise to the practical science of geometric optics. From the design of optical instruments to the rendering of images in computer games, ray tracing provides a powerful tool for developing technologies that use light.

In the seventeenth century Kepler had discovered total internal reflection and arrived at the small angle approximation to the law refraction in which case the incident and transmission angles are proportional. The law of refraction also empirically discovered in this century. This was one of the great moments in optics.

The phenomenon of diffraction, that is, the deviation from rectilinear propagation that occurs when light advances beyond an obstruction was first noted by Professor Francisco Maria Grimaldi (1618-1663) at the Jesuit college in Bologna.

The first study the colored interference patterns generated by thin films was discovered by Robert Hook (1635-1703). He proposed the idea that light was a rapid vibratory motion of medium propagating at a very great speed.

Isaac Newton described light as a stream of particles, partly, because it "travels in a straight line ". He observed in a triangular glass prism and conclude that white light was composed of a mixture of a whole range of independent colors. He maintained that the corpuscles of light associated with the various colors excited the aether to characteristic vibrations.

At about the same time that Newton was emphasizing the emission theory in England, Christian Huygens (1629-1695), on the continent was greatly extending the wave theory. Unlike Hook and Newton, Huygens correctly concluded that light effectively slowed down on entering more dense media. He was able to derive the laws of reflection and refraction and even explained the double refraction of calcite, using his wave theory. And it was while working with calcite that he discovered the phenomenon of polarization.

The wave theory of light was reborn at the hands of Dr.Thomas Young (1773-1829), one of the truly great minds of the century. He was able to explain the colored fringes of thin films and determined wave lengths of various colors using Newton's data.

Augustine Jean Fresnel (1788-1827), synthesized the concepts of Huygens's wave description and the interference principle. Fresnel was able to calculate the diffraction patterns arising from various obstacles and apertures and satisfactorily accounted for rectilinear propagation in homogeneous Isotropic media, thus dispelling Newton's main objection to the undulatory theory. He also showed that the light transmitted by Huygens blocks of calcite crystal is polarized and that light waves therefore can not be longitudinal compression waves as Huygens had thought but must be transverse waves oscillating at right angles to their direction of propagation.

Early in the nineteenth century the notion that light consists of waves, a view already expressed by Christian Huygens in the seventeenth century came in to ascendance. A decisive experiment performed in 1803 by Thomas young, a London physicist, demonstrated that a monochromatic beam of light passed through two pin holes would set up an interference pattern.

In the nineteenth century, a powerful new understanding of the nature of light emerged. Careful experiments and new analytical methods in theoretical physics showed that light, when interacting with matter at small enough scales, showed clear evidence of wavelike properties. Within a few decades, the great synthesis of the ideas of electricity and magnetism by Maxwell showed what the waves actually were: oscillating and traveling electric and magnetic fields. Thus the field of physical optics was created, and the consequences of this level of understanding have been extremely important to understanding the limitations and optimizing the performance of optical imaging systems (cameras, microscopes, telescopes, etc.). This is often referred to as the "diffraction theory of imaging." The field also gave rise to a new class of instruments whose function depended wholly on exploiting the wavelike properties of light. The most important class of such instruments is the interferometer. Another important outcome of this understanding of light as waves is the unique method of imaging called holography. Ironically, the twentieth century brought the theory of light "full circle" when it became evident that the energy transmitted by light and other forms electromagnetic radiation was, in fact, quantized into packets called "photons". A successful atomic-level description of the interaction between light and matter requires this quantum view of the nature of light. The elegant field theories of Maxwell needed to be reconciled to the new experimental evidence of the quantum nature of light, and this resulted in the theory called quantum electrodynamics. Meanwhile, the combined quantum view of atoms and light yielded an enormously important new type of physical device: the laser.

This elucidation of the wave nature of light fit nicely in to the electromagnetic theory of light propounded later in the century by James clerk Maxwell. In Maxwell equations light is described as a rapid variation in the electromagnetic field surrounding a charged particle, the variations in the field being generated by the oscillation of the particle. As such varying field, light takes its place beside a number of other forms of radiant energy that were discovered in the nineteenth century. The different kinds of electromagnetic radiation- radio waves on one side of the spectrum of visible light and x-ray on the other correspond to different rates of variation of the field. Thus in Maxwell's theory light appears not as an independent element in nature but rather as an aspect of the fundamental phenomenon of propagation.

The momentous developments in physics in the last century have re-opened and then resolved the old-wave particle controversy. Where as the association of light with electromagnetism remains valid, the interpretation of this connection has changed. It has been shown that such wave properties as interference and polarization, so well demonstrated by light, are also exhibited under suitable circumstances by the sub atomic constituents of matter, such as electrons. Conversely, it has been shown that light, in its interaction with matter, behaves as though it is composed of many individual bodies called photons, which carry such particle-like properties as energy and momentum.

As a result of these developments most physicists today would answer the question "what is light?" as Newton would have "light is a particular kind of matter". The difference between light and bulk matters are now thought to flow from relatively in essential differences between their constituent particles. Particle of both kinds-of all kinds exhibit wave properties.

1.2 The Nature and property of Light

1.2.1 The nature of light

The perception of light is the principal means by which we know the world. From the single celled creatures we see under a microscope, to the most distant stars seen in a telescope, it is light that informs us. The incredible variety of forms and the structure of our universe are revealed by light. Yet the nature of light eluded scientists and natural philosophers through most of recorded history. Only since the mid-nineteenth century have physicists begun to understand this most fundamental natural phenomenon. The understanding physicists have today required the development of electromagnetic theory, relativity, and quantum mechanics. Even with the full array of techniques of modern physics there remains unsolved problems having to do with the interaction of light with matter.

Electromagnetic radiation travels through space as electric and magnetic energy. At times the energy acts like a wave and at other times it acts like a particle, called a photon. As a wave, we can describe the energy by its wavelength, which is the distance from the crest of one wave to the crest of the next wave. The wavelength of electromagnetic radiation can range from miles (radio waves) to inches (microwaves in a microwave oven) to millionths of an inch (the light we see) to billionths of an inch (x-rays). The wavelength of light is more commonly stated in nanometers (nm).

When electrons in a molecule or a gas are excited, they rise to a higher energy level within that atom or molecule. After a period of time, the electrons return to their normal energy level and emit the difference in energy as electromagnetic radiation. The energy emitted is frequently in the visible spectrum. When light strikes an object, the light can be transmitted, absorbed, or reflected. In many cases, all three occur. Transmission, absorption, or reflection can be determined by the wavelength of the light. For example, a piece of clear glass will transmit all the wavelengths of light striking the surface of the glass. If the glass is colored, some wavelengths are absorbed and some wavelengths are transmitted. If there are small particles in the glass, some of the wavelengths may be absorbed, some transmitted, and all reflected. In this case we would describe the glass as both colored and opaque. A piece of colored paper reflects some wavelengths, absorbs some wavelengths, and transmits no light. If light strikes the surface of a transmitting object at an angle other than straight on, the light will be bent as it enters and exits the object. This property of light allows a lens to focus the light rays on a surface, such as the surface of the film used to photograph an object. Additionally, the short wavelengths are bent more than the long wavelengths. It is this property of light that produces a rainbow. As the light enters a water droplet, the light is bent. The light then reflects on the back of the water droplet. Then, as the light exits from the water droplet, the light rays bend again. Because the short wavelengths are bent more than the long wavelengths, the wavelengths of light are spread across the sky and we see the rainbow.

1.2.2 Properties of Light

What is light? This question has been debated for many centuries. The sun radiates light, electric lights brighten our darkness, and many other uses of light impact our lives daily. The answer, in short, is light is a special kind of electromagnetic energy. The speed of light, although quite fast, is not infinite. The speed of light in a vacuum is expressed as $c = 2.99 \times 10^8 m/s$. Light travels in a vacuum at a constant speed, and this speed is considered a universal constant. It is important to note that speed changes for light traveling through non-vacuum media.

For most purposes, we may represent light in terms of its magnitude and direction. In a vacuum, light will travel in a straight line at fixed speed, carrying energy from one place to another. Two key properties of light interacting with a medium are:

- It can be deflected upon passing from one medium to another (refraction).
- It can be bounced off a surface (reflection).

Electromagnetic waves share six properties with all forms of wave motion: Polarization, Superposition, Reflection, Refraction, Diffraction, Interference.

1.3 Maxwell's equations

The propagation of light through a medium can best be understood in terms of electromagnetic waves in the behavior of the electric field $\vec{E}$ and magnetic fields $\vec{H}$, which are independent of one an other can be described by a total of four equations.

When an electric or magnetic field is changing with time a field of the other kind is induces in adjacent regions of space. We are led (as Maxwell was) to consider the possibility of an electromagnetic disturbance, consisting of time-varying electric and magnetic fields that can separate from their sources that is from charges and currents, and can propagate through space even when no matter is present in the region. So this continuous inter-conversation of the field preserves them and an electromagnetic perturbation propagates in space.

The existence of electromagnetic waves had been predicted by Maxwell as a result of a careful analysis of the basic equations of electromagnetic field. Maxwell proved in 1865 that the electromagnetic waves propagate in free space with the speed of light so the light waves are very likely to have electromagnetic nature.

In 1887 Hertz actually produced electromagnetic waves, and verified Maxwell's theory [9]. The development of our knowledge of electromagnetic waves is a beautiful example of the close relationship between theory and experiment in the evolution of physical ideas.

Consider now vacuum or neutral(not charged $\rho = 0$) insulator, that is $\vec{J} = 0$ Suppose that the medium is homogeneous and isotropic and not ferromagnetic. The permittivity ϵ and the permeability μ_0 is constant.

The first equation is derived from the Amperes law, it denotes that the electrical current (both the conductive current J and the displacement current $\frac{\partial \vec{D}}{\partial t}$) induces the magnetic field. The second equation is derived from the Faraday's law; it denotes that the variation of the magnetic field induces the electrical field. The third and the fourth equations are derived from the Gauss theorem, one for the magnetic field, and the other for the electrical field.

The Maxwells equations (Governing the electromagnetic fields) for the case of free space can be expressed in the following form:

$$\vec{\nabla} \cdot \vec{D} = \rho_f \tag{1.3.1}$$

$$\vec{\nabla} \cdot \vec{H} = 0 \tag{1.3.2}$$

$$\vec{\nabla} \times \vec{D} = -\frac{\partial \vec{H}}{\partial t} \tag{1.3.3}$$

$$\vec{\nabla} \times \vec{H} = \vec{J_f} \tag{1.3.4}$$

These four equations are sufficient to determine the electromagnetic field completely if the free charge density ρ_f and the free current density $\vec{J_f}$ are given and the electric properties of the medium are known. The synthesis of electromagnetic phenomena represented in these four relatively simple equations remains one of the greatest achievements of physics. They are the starting point from which all electromagnetic effects can be explained including the nature of light.

For isotropic linear medium, $\vec{B} = \mu\vec{H}$ and $\vec{D} = \epsilon\vec{E}$, the Maxwell's equations become

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho_f}{\epsilon_0} \tag{1.3.5}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \tag{1.3.6}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\mu}{\epsilon}\frac{\partial \vec{B}}{\partial t} \tag{1.3.7}$$

$$\vec{\nabla} \times \vec{B} = \mu\vec{J}_f + \mu\epsilon\frac{\partial \vec{E}}{\partial t} \tag{1.3.8}$$

If there is no external charge densities and currents, then $\vec{J}_f = \sigma_C\vec{E}$

where $-\ \sigma_C$ is the conductivity of the material. Then equation(1.2.8) becomes

$$\vec{\nabla} \times \vec{B} = \mu\sigma_C\vec{E} + \mu\epsilon\frac{\partial \vec{E}}{\partial t} \tag{1.3.9}$$

where$-\ \mu$ is permeability of the material and ϵ is permittivity of the material

1.4 Electromagnetic wave equations

Here we wish to demonstrate, following Maxwell, that light behaves like an electromagnetic wave,that is,a propagating disturbance involving time and space variations of coupled electric and magnetic fields.

To derive the electromagnetic wave equation in its most general form, we must again consider the presence of some medium. Since the field with in the material has been altered, we can define a new field quantity, the displacement $\vec{D}$:

$$\vec{D} = \epsilon_0\vec{E} + \vec{P} \tag{1.4.1}$$

From this,

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} - \frac{\vec{P}}{\epsilon_0} \tag{1.4.2}$$

Where $\vec{P}$ is the polarization vector, which is a measure of the over all behavior of the medium, or it is the resultant electric dipole moment per unit volume.

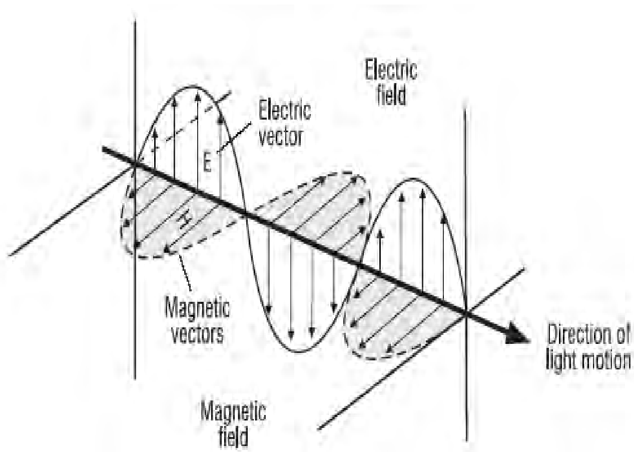

Figure 1.1: Propagation of electromagnetic wave

For a homogeneous,linear and isotropic medium $\vec{P}$ and $\vec{E}$ are in the same direction and are mutually proportional. It follows that $\vec{D}$ is also proportional to $\vec{E}$, i.e. $\vec{D} = \epsilon\vec{E}$ We can also define a magnetic polarization or magnetization vector $\vec{M}$ as the magnetic dipole moment per unit volume. In order to deal with time influence of the magnetically polarized medium, we introduce an auxiliary vector $\vec{H}$, traditionally known as the magnetic field intensity.

$$\vec{H} = \frac{\vec{B}}{\mu_o} - \vec{M} \tag{1.4.3}$$

For a homogeneous, linear(non-ferromagnetic) and isotropic medium,$\vec{B}$ and $\vec{H}$ are parallel and proportional

$$\vec{H} = \frac{\vec{B}}{\mu} \tag{1.4.4}$$

Consider the rather general environment of a linear (non-ferroelectric and non- ferromagnetic), homogeneous and isotropic medium, which is physically at rest. By making use of the constructive relations, we can write Maxwell's equations as shown above. If these equations are some how to yield a wave equation, we had a best form some second derivatives with respect to the space variables. By taking the curl of equation (1.2.7) and (1.2.9), we can obtain

$$\begin{aligned} \vec{\nabla} \times \vec{\nabla} \times \vec{E} &= -\frac{\partial}{\partial t} \vec{\nabla} \times \vec{B} \\ \vec{\nabla} \times \vec{\nabla} \times \vec{B} &= \mu\sigma(\vec{\nabla} \times \vec{E}) + \mu\epsilon \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E}) \end{aligned} \tag{1.4.5}$$

By substituting equation (1.2.8) and (1.2.7) in to the above two equations and by applying the vector product identity , that is

$$\begin{aligned} \vec{\nabla} \times \vec{\nabla} \times \vec{E} &= \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} \\ \vec{\nabla} \times \vec{\nabla} \times \vec{B} &= \vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{B} \end{aligned} \tag{1.4.6}$$

Since from Maxwell's equation we have $\vec{\nabla} \cdot \vec{B} = 0$ and for free charge density $\rho_f = 0$, then the Maxwell's equation $\vec{\nabla} \cdot \vec{E} = 0$. Then finally the simplified form of the above equation will be

$$\begin{aligned} \nabla^2 \vec{E} &= \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B}) \\ \nabla^2 \vec{B} &= -(\mu\sigma\vec{\nabla} \times \vec{E}) + \mu\epsilon \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E}) \end{aligned} \tag{1.4.7}$$

And for non conducting media $\sigma = 0$ and for non-free current density $\vec{J}_f = 0$, then the above two equations become

$$\begin{aligned} \nabla^2 \bar{B} - \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2} &= 0 \\ \nabla^2 \vec{E} - \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} &= 0 \end{aligned} \tag{1.4.8}$$

These two equations are the electromagnetic wave equations. The solution of the above equations have septile complex form

$$\begin{aligned} \vec{E} &= E_0 \exp[i\vec{k} \cdot \vec{r} - i\omega t] \\ \vec{B} &= B_0 \exp[i\vec{k} \cdot \vec{r} - i\omega t] \end{aligned} \tag{1.4.9}$$

Where E_0 and B_0 are complex vector amplitudes independent of $\vec{r}$ and t. When light passes from one medium to other medium the electromagnetic field must satisfy boundary condition.

Chapter 2

Polarized Beam

2.1 Polarization

When an elementary light wave interacts with matter, its electric field causes electrons within the substance to vibrate at the wave's frequency. These vibrating electrons then re-radiate the absorbed energy as new electromagnetic waves in all directions. Although the scattered light has the same frequency as the incident wave its polarization depends on the new direction of propagation. In general, therefore, when light interacts with matter its polarization may be changed. The main mechanisms by which this happens are :

- by passing through dichroic, birefringent and optically active materials;
- by scattering and by reflection;

Since light is a transverse electromagnetic wave thus far we have considered only cases where the electric field vector resided in a fixed plane. This plane is referred to as the plane of vibration and the light is said to be plane polarized. Since the field vectors are perpendicular to the propagation direction, there is a degree of freedom involving their orientation.

An oscillating vector field must instantaneously have a unique direction. For the plane electromagnetic wave in an isotropic medium this direction must be perpendicular to the wave normal; if it remains constantly in the same direction, the wave is said to be plane polarized. On the other hand, if the direction changes randomly, the wave is said to be unpolarized or randomly polarized.

Depending on the phase difference between two oscillating electric field vectors, we can have a linear circular and elliptical polarization. Two plane polarized waves can combine to produce an other wave which is also plane polarized.

We have three kinds of light, that is, unpolarized, partially polarized and completely polarized light.

2.2 Unpolarized Light

One fundamental characteristic of electromagnetic radiation remains to be discussed, is polarization, and it is rooted in the transverse nature of electromagnetic waves. To see different state of polarization, the nature of unpolarized light is defined initially.

Ordinary light is not polarized. Looking along a ray of light, the electric vectors make all angles with the vertical. Light that is plane polarized in the vertical plane has only vertical electric vectors. The plane of polarization is the plane that includes both the vibration direction and the ray path. Light may be polarized by crystals, by polarizing filters, and by reflection. Reflected light is partially polarized, favoring the vibration direction perpendicular to the plane of the ray path (including both the incident and reflected rays).

Strictly monochromatic light is fully polarized, though, it may range any where from linear through elliptical to circular polarization. Natural light on the other hand, is generally found to be unpolarized, for the phase relation between x and y components, is un correlated and phase difference varies rapidly. Because of this the mean value of phase difference become zero.

If their angle of polarization:

$$\alpha = \tan^{-1}[\frac{\vec{E}_y}{\vec{E}_x}] \tag{2.2.1}$$

2.3 Polarized Light

An important property of optical waves is their polarization state. A vertically polarized wave is one for which the electric field lies only along the z-axis if the wave propagates along the y-axis (Figure 2.1).

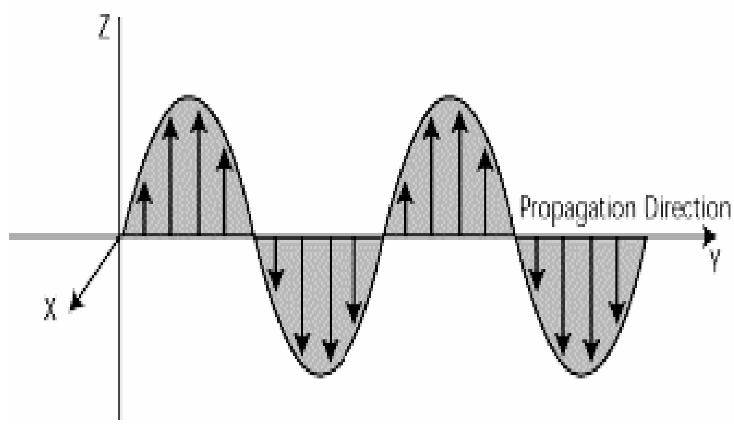

Figure 2.1: Linearly polarized light in the vertical direction

Similarly, a horizontally polarized wave is one in which the electric field lies only along the x-axis. Any polarization state propagating along the y-axis can be superposed into vertically and horizontally polarized waves with a specific relative phase. The amplitude of the two components is determined by projections of the polarization direction along the vertical or horizontal axes. For instance, light polarized at 45^0 to the x-z plane is equal in amplitude and phase for both vertically and horizontally polarized light.

2.3.1 The general Equation of Ellipse

Here, a cartesian system of coordinates with the x-axis in the direction of propagation is considered. The y-axis parallel to the optical vector of one wave, and the z-axis parallel to the optical vector of the other wave. The optical vectors of the two waves are represented by expression of the type

$$\begin{aligned} \vec{E}_y &= A_y \cos(\omega(t - \frac{x}{v}) + \psi_1) \\ \vec{E}_z &= A_z \cos(\omega(t - \frac{x}{v}) + \psi_2) \end{aligned} \tag{2.3.1}$$

The functions $\vec{E}_y$ and $\vec{E}_z$ also represent the components of the resultant optical vector $\vec{E}$ along the y and z axes. At a given point in space, this vector varies with time in both length and direction. Its tip describes a curve, of where Eq.(2.3.1) is the parametric equation. To determine the shape of this curve, we need only t between the two equations. For this purpose we denote $\psi = \psi_2 - \psi_1$, the phase difference between the two oscillations can be define as the origin of time by putting $\omega t' = \omega(t - \frac{x}{v}) + \psi_1$. So Eq.(2.3.1) become

$$\begin{aligned} \vec{E}_y &= A_y \cos(\omega t') \\ \vec{E}_z &= A_z \cos(\omega t' + \psi) \end{aligned} \tag{2.3.2}$$

from this we can get

$$\vec{E}_z = A_z \cos(\omega t') \cos\psi - A_z \sin(\omega t') \sin\psi \tag{2.3.3}$$

From this equation, one can obtain

$$\frac{\vec{E}_z}{A_z} - \frac{\vec{E}_y}{A_y} \cos\psi = -\sin(\omega t') \sin\psi \tag{2.3.4}$$

Squaring both sides of Eq.(2.3.4) then the following result will be found

$$[\frac{\vec{E}_z}{A_z} - \frac{\vec{E}_y}{A_y} \cos\psi]^2 = \sin^2(\omega t') \sin^2\psi = [1 - [\frac{\vec{E}_y}{A_y}]^2] \sin^2\psi \tag{2.3.5}$$

and, after few reductions it is possible to get the coming expression

$$\frac{E_y^2}{A_y^2} + \frac{E_z^2}{A_z^2} - 2\frac{E_y E_z}{A_y A_z} \cos\psi = \sin^2\psi \tag{2.3.6}$$

This is the general equation of ellipse. Depending on the values of ψ, the equation can represent line, circle or ellipse. The sign of ψ determines the sense of rotation. The tilting angle ψ is defined as

$$\tan(2\psi) = \frac{2ab\cos(\phi)}{a^2 - b^2} \tag{2.3.7}$$

Ellipticity e is the ratio of the length of the semi-minor axis of the ellipse b to the length of the semi-major axis a. The right handed and left-handed circularly polarized states correspond to $e = 1$ and $e = -1$ respectively. Linearly polarized light has ellipticity $e = 0$ magnitude.

2.3.2 Plane Polarized Light

In light and all other kinds of electromagnetic waves, the oscillating electric and magnetic fields are always directed at right angles to each other and to the direction of propagation of the wave. In other words the fields are transverse, and light is described as a transverse wave. Since both the directions and the magnitudes of the electric and magnetic fields in a light wave are related in a fixed manner, it is sufficient to talk about only one of them, the usual choice being the electric field. Now although the electric field at any point in space must be perpendicular to the wave velocity, it can still have many different directions; it can point in any direction in the plane perpendicular to the wave's direction of travel.

Any beam of light can be thought of as a huge collection of elementary waves with a range of different frequencies. Each elementary wave has its own unique orientation of its electric field; it is polarized. If the polarizations of all the elementary waves in a complex beam can be made to have the same orientation all the time then the light beam is also said to be polarized. Since there is then a unique plane containing all the electric field directions as well as the direction of the light ray, this kind of polarization is also called plane polarization. It is also known as linear polarization. However, the usual situation is that the directions of the electric fields of the component wavelets are randomly distributed; in that case the resultant wave is said to be randomly polarized or unpolarized.

It is quite common to find partially polarized light which is a mixture of unpolarise (completely random polarizations) and plane polarized waves, in which a significant fraction of the elementary waves have their electric fields oriented the same way.

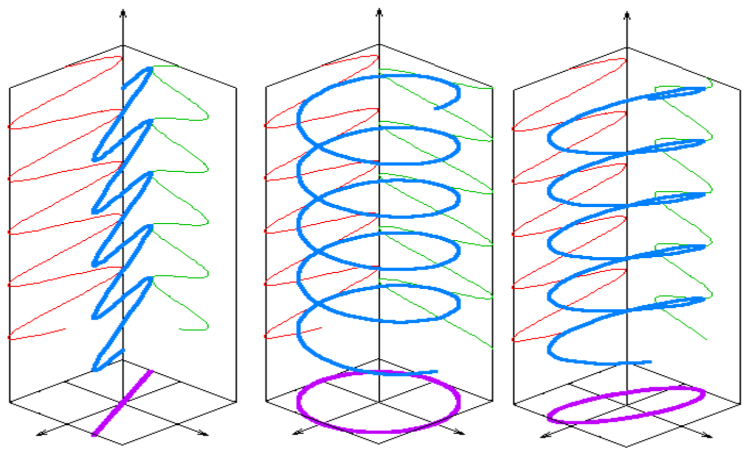

Figure 2.2: Linearly, Elliptically and Circularly polarized light

Since Eq.(2.3.6) is the equation of an ellipse, and one can conclude that the tip of the vector representing the resultant optical distance at a given point in space describes an ellipse in the perpendicular to the direction of propagation. Therefore, this facts expressed by saying that, the wave is elliptically polarized.

Note that at any instant of time, the magnitude $\vec{E}$ of the resultant optical disturbance is giving by the equation:

$$\vec{E}^2 = \vec{E}_y^2 + \vec{E}_z^2 \tag{2.3.8}$$

It follows that the intensity (I) of the elliptically polarized wave equals the sum of the intensities (I_y) and (I_z) of the two linearly polarized vibrating in the perpendicular planes xy and xz:

$$I = I_y + I_z \tag{2.3.9}$$

Thus Eq.(2.3.1) and Eq.(2.3.2) shows that $\vec{E_y}$ varies from $+A_y$ to $-A_y$, and $\vec{E_z}$ varies from $+A_z$ to $-A_z$. Hence, the ellipse represented by these equations, or by Eq.(2.3.7) is inscribed in a rectangle with sides of length $2A_y$ and $2A_z$ see fig.2.1.

If the two oscillations are in phase,i.e., if ψ equals zero or even multiple of π, the ellipse degenerates in to straight segment, coincident with the diagonal of the rectangle that lays in the first and third quadrant $Fig.2.2$. Indeed, in this case $Eq.(2.3.6)$ yield

$$\frac{E_y}{A_y} = \frac{E_z}{A_z} \Rightarrow E_y = \frac{A_y}{A_z} E_z \tag{2.3.10}$$

If y an odd multiple of π, Eq.(2.2.6) give

$$\frac{E_y}{A_y} = -\frac{E_z}{A_z} \Rightarrow E_y = -\frac{A_y}{A_z}E_z \tag{2.3.11}$$

2.3.3 Elliptically Polarized Light

The resultant wave is again linearly polarized, but now the optical disturbance is parallel to the other diagonal of the rectangle i.e. to that lying in the second and fourth quadrants Fig.2.3. If ψ is an odd multiple of $\frac{\pi}{2}$, Eq.(2.3.9) becomes

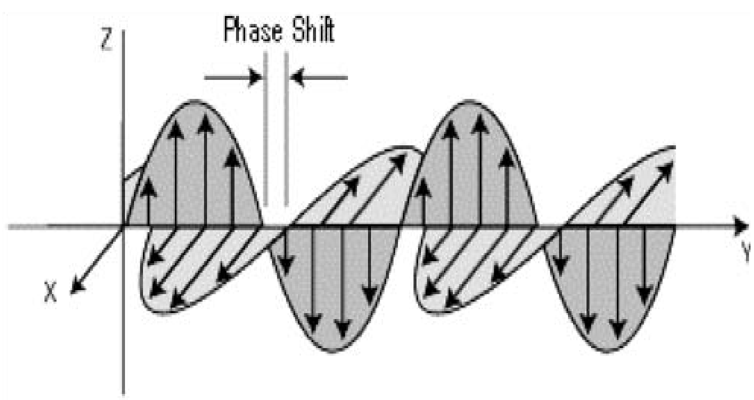

Figure 2.3: Elliptically Polarized Light

$$\frac{E_y^2}{A_y^2} + \frac{E_z^2}{A_z^2} = 1 \tag{2.3.12}$$

which is the equation of an ellipse having its axes in the y and z directions fig.2.3.

2.3.4 Circularly Polarized Light

If in particular $A_y = A_z$, the ellipse reduces to a circle and the wave is said to be circularly polarized. In this condition, the vector representing the optical disturbance at a given point in space rotates with uniform angular speed with out change of magnitude. Plane polarization is not the only way that a transverse wave can be polarized. In circular polarization the electric field vector at a point in space rotates

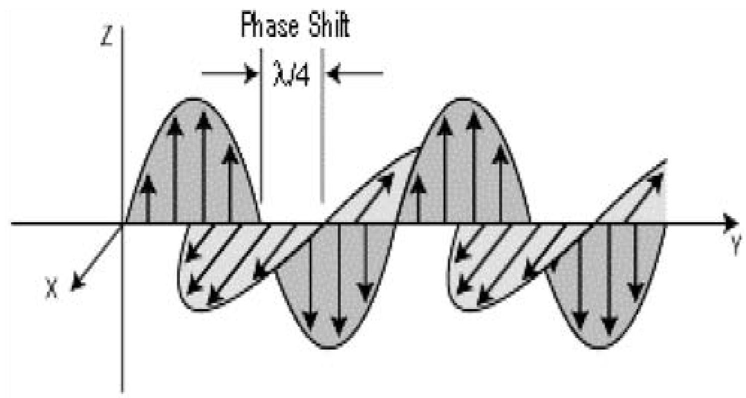

Figure 2.4: Circularly polarized light

in the plane perpendicular to the direction of propagation, instead of oscillating in a fixed orientation, and the magnitude of the electric field vector remains constant. Looking into the oncoming wave the electric field vector can rotate in one of two ways. If it rotates clockwise the wave is said to be right-circularly polarized and if it rotates anticlockwise the light is left-circularly polarized.

Actually circular polarization is not anything new. A circularly polarized elementary wave can be described as the superposition of two plane polarized waves with the same amplitude which are out of phase by a quarter of a cycle $\frac{\pi}{2}$ or three quarters of a cycle $\frac{3\pi}{2}$.

2.4 The Nature of Polarized Light

It has already been established that light may be treated as a transverse electromagnetic wave. Thus far we have linearly polarized or plane polarized light and circularly polarized light, that is, light for which orientation of the electric field is constant, although its magnitude and sign varies with time. In this case the electric field or optical disturbance resides in what is known as the plane of-vibration. That contains both $\vec{E}$ and $\vec{k}$, the electric field vector and the propagation vector in the direction of motion.

Imagine two harmonic, linearly polarized light waves of the same frequency, moving through the same region of space, in the same direction. If their electric field vectors are collinear, the superimposing disturbances will simply combine to form a resultant linearly polarized wave. If the two light waves are such that their respective electric-field directions are mutually perpendicular, the resultant wave may or may not be linearly polarized.

A monochromatic laser beam can be written as a plane wave in terms of two orthogonal components:

$$\vec{E} = [E_x\hat{x} + E_y\hat{y}]\exp[ikz - i\omega t] \tag{2.4.1}$$

where the beam is propagating in the z-direction. In general, the amplitudes E_x and E_y are complex in order to account for the phases of the components.

There are two cases of special interest.

Case(I): The first is when the phase angle between the complex amplitudes E_x and E_y is equal to 0 or π, in which case the total electric field always lies in a single plane resulting in a beam which is linearly polarized. The direction of the x-axis can be chosen to coincide with the plane of polarization, so the beam can be written as

$$\vec{E} = E_p\hat{x}\exp[ikz - i\omega t] \tag{2.4.2}$$

where E_p is the complex amplitude of the linearly polarized light.

Case(II): when the phase angle is $\pm\frac{\pi}{2}$, and $|E_x| = |E_y|$. In this case, $E_y = E_x \exp i\theta$, with $\theta = \pm\frac{\pi}{2}$. The total electric field has a constant magnitude, with the direction varying with the optical frequency ω so that the beam is circularly polarized. When $\theta = \frac{\pi}{2}$, the electric field has a positive, or right-handed, helicity. Such a beam is here called left circularly polarized. When $\theta = -\frac{\pi}{2}$, the beam has negative helicity and is called right circularly polarized.

A circularly polarized beam can always be written as

$$\vec{E} = [E_c\hat{x} \pm iE_c\hat{y}]\exp[ikz - i\omega t] \tag{2.4.3}$$

with the sign depending on whether the beam is left or right circularly polarized, and E_c is the complex amplitude. In general, however, the phase angle θ will have a value between these limiting values, or even if $\theta = \pm\frac{\pi}{2}$, $|E_x| \neq |E_y|$. In these cases, the beam is elliptically polarized, and the electric field vector E traces out an ellipse during each optical period.

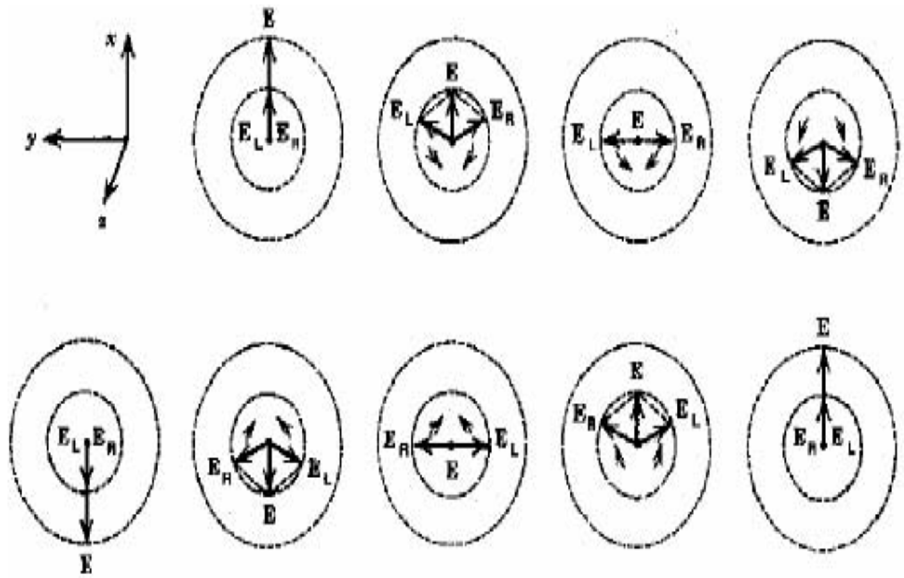

Figure 2.5: The linearly polarized light could be decomposed into two circularly polarized lights: right and left polarized

Recognizing that we can rewrite equation (2.4.3)for a circularly polarized beam as,

$$\vec{E} = E_c[\hat{x} + i\hat{y}]\exp[ikz - i\omega t] \tag{2.4.4}$$

We see that any beam can be represented as a sum of two circularly polarized components using the (complex) orthogonal basis vectors

$$\begin{aligned} \hat{e}_L &= \frac{1}{\sqrt{2}}[\hat{x} + i\hat{y}] \\ \hat{e}_R &= \frac{1}{\sqrt{2}}[\hat{x} - i\hat{y}] \end{aligned} \tag{2.4.5}$$

From this,

$$\begin{aligned} \hat{x} &= \frac{1}{\sqrt{2}}[\hat{e}_L + \hat{e}_R] \\ \hat{y} &= \frac{i}{\sqrt{2}}[\hat{e}_R - \hat{e}_L] \end{aligned} \tag{2.4.6}$$

The amplitudes of the left and right circular components can be found from the x and y amplitudes in the linear orthogonal representation (equation 2.2.1):

$$\begin{aligned} E_L &= \frac{1}{\sqrt{2}}[E_x - iE_y] \\ E_R &= \frac{1}{\sqrt{2}}[E_x + iE_y] \end{aligned} \tag{2.4.7}$$

From this

$$\begin{aligned} E_x &= \frac{1}{\sqrt{2}}[E_L + E_R] \\ E_y &= \frac{i}{\sqrt{2}}[E_L - E_R] \end{aligned} \tag{2.4.8}$$

When $|E_L| = |E_R|$, the beam is linearly polarized, with the plane of polarization given by the phase angle between the complex amplitudes E_L and E_R. If $E_L = 0$ (and $E_R \neq 0$), the beam is right circularly polarized, and left circularly polarized if $E_R = 0$.

2.5 The poynting vector and the Irradiance

An electromagnetic wave exists with in some region of space, and it is therefore natural to consider the radiant energy per unit volume, or energy density U. We suppose that the electric field itself can some how is store energy. This is a major logical step since it imparts to the field the attribute of physical reality. If the field has energy then the energy density of the electric field to be

$$U_E = \frac{\epsilon_0}{2}E^2 \tag{2.5.1}$$

Similarly the energy density of the $\vec{B}$ field is

$$U_B = \frac{1}{2\mu_0}B^2 \tag{2.5.2}$$

The relation ship $|\vec{E}| = c|\vec{B}|$ was derived specifically for plane waves; nonetheless, its quite general in its applicability. Using $c = \frac{1}{\sqrt{\epsilon_0\mu_0}}$, it follows that

$$U_E = U_B \tag{2.5.3}$$

The energy streaming through space in the form of an electromagnetic wave is shared equally between the constituent electric and magnetic fields, in as much as

$$U = U_E + U_B \tag{2.5.4}$$

From this

$$U = \epsilon_0 E^2 \tag{2.5.5}$$

And equivalently

$$U = \frac{1}{\mu_0} B^2 \tag{2.5.6}$$

To represent the flow of electromagnetic energy associated with a traveling wave, lets symbolize the transport of energy per unit time (the power) across unit area. In the SI system it has units of $\frac{W}{m^2}$. For a very small interval of time $\triangle t$, only the energy contained in the cylindrical volume, $U(c\triangle tA)$, will cross A. Thus

$$S = \frac{Uc\triangle tA}{\triangle tA} = Uc \tag{2.5.7}$$

Or using equation (2.3.5)

$$S = \frac{1}{\mu_0} EB \tag{2.5.8}$$

For isotropic media the energy flows in the direction of the propagation of the wave. The corresponding vector $\vec{S}$ is then

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \tag{2.5.9}$$

Or

$$\vec{S} = c^2 \epsilon_0 \vec{E} \times \vec{B} \tag{2.5.10}$$

The magnitude of $\vec{S}$ is the power per unit area crossing a surface whose normal is parallel to $\vec{S}$. Named John Henry Poynting (1852-1914), it has come to be known as the poynting vector.

Now lets apply these considerations to the case of a harmonic linearly polarized (the directions of the electric field $\vec{E}$ and the magnetic field $\vec{B}$ are fixed). Plane wave traveling through free space in the direction of $\vec{k}$:

$$\begin{aligned} \vec{E} &= \vec{E_0} \cos(\vec{k} \cdot \vec{r} - \omega t) \\ \vec{B} &= \vec{B_0} \cos(\vec{k} \cdot \vec{r} - \omega t) \end{aligned} \tag{2.5.11}$$

Using equation(2.3.10), we get

$$\vec{S} = c^2\epsilon_0\vec{E_0}\times\vec{B_0}\cos^2(\vec{k}\cdot\vec{r}-\omega t) \tag{2.5.12}$$

This is the instantaneous flow of energy per unit area per unit time.

In the past physicists generally used the word intensity to mean the flow of energy per unit area per unit time. By international if not universal, agreement, that term is slowly being replaced by the word Irradiance, denoted by I.[9]

The time-averaged value of the magnitude of the poynting vector, symbolized by $\langle S\rangle_T$, is a measure of irradiance (I). In the specific case of harmonic fields,

$$\langle S\rangle_T = c^2\epsilon_0|\vec{E_0}\times\vec{B_0}|\langle\cos^2(\vec{k}\cdot\vec{r}-\omega t)\rangle \tag{2.5.13}$$

For $T >> \tau$, then $\langle\cos^2(\vec{k}\cdot\vec{r}-\omega t)\rangle_T = \frac{1}{2}$

From this, the above equation becomes $\langle S\rangle_T = c^2\epsilon_0|\vec{E_0}\times\vec{B_0}|$ or

$$I \equiv \langle S\rangle_T = \frac{c\epsilon_0}{2}E_0^2 \tag{2.5.14}$$

The irradiance is proportional to the square of the amplitude of the electric field or the magnetic field

$$\begin{aligned} I &= \epsilon_0 c\langle E^2\rangle_T \\ I &= \frac{c}{\mu_0}\langle B^2\rangle_T \end{aligned} \tag{2.5.15}$$

with in a linear, homogeneous, isotropic dielectric medium, the expression for the irradiance becomes

$$I = \epsilon v\langle E^2\rangle_T \tag{2.5.16}$$

where ϵ-is permittivity of the medium v-is the speed of light in a medium then the time averaged irradiance is found as follows

$$I = \frac{c\epsilon_0}{2}\vec{E}\vec{E^*} \tag{2.5.17}$$

where

$$\begin{aligned} \vec{E} &= (E_L\hat{e}_L + E_R\hat{e}_R)\exp[ikz - i\omega t] \\ \vec{E^*} &= (E_L^*\hat{e}_L + E_R^*\hat{e}_R)\exp[ikz - i\omega t] \end{aligned} \tag{2.5.18}$$

Then it gives the irradiance is

$$I = \frac{c\epsilon_0 E_L^* E_L}{2} + \frac{c\epsilon_0 E_R^* E_R}{2} \tag{2.5.19}$$

And this gives us

$$I = I_L + I_R \tag{2.5.20}$$

Where- $I_L = \frac{c\epsilon_0 E_L^* E_L}{2}$ is the left circularly polarized light irradiance and $I_R = \frac{c\epsilon_0 E_R^* E_R}{2}$ is the right circularly polarized light irradiance.

Chapter 3

Optical Torque

3.1 Birefringence Crystals

In some materials light with different polarizations travels at different speeds. Since we can regard any wave as the superposition of two plane polarized waves, this is equivalent to saying that beam of light travels at different speeds in the material, that is the material has different refractive indices for light of the same frequency. Such materials are said to be doubly refracting or birefringent. Examples are crystals such as the minerals calcite (calcium carbonate) and quartz (silicon dioxide) or materials like Cellophane when it is placed under stress. The speed of light in a birefringent crystal depends, not only on the polarization, but also on the direction of travel of the light.

An optically isotropic material is one in which the index of refraction or, the phase velocity of a wave is the same in all direction. Generally, however, crystals are isotropic; the atomic binding forces on the electron clouds are different in different directions and as a result so are the refractive indices. We concern only with *uniaxial birefringent crystals* (which encompass the trigonal, hexagonal and tetragonal systems). Such crystal contains a single symmetry axis (actually a direction) known as the *optic axis* and displays two distinct principal indices of refraction. The latter corresponds to the light field oscillations parallel and perpendicular to the optic axis. Calcite,($CaCo_3$) is a good example. As usual we can regard any beam of light as a superposition of two linearly polarized components at right angles to each other. By choosing suitable directions for the polarization components it is found that one component wave, called the ordinary wave, travels at the same speed in all directions through the crystal,

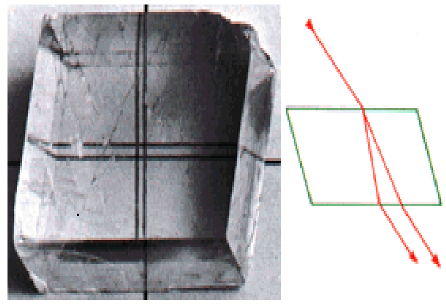

Figure 3.1: Birefringence crystal

but the speed of the other polarization component, called the extraordinary wave, depends on its direction of travel. There are some propagation directions in which all polarizations of light travel at the same speed and a line within the crystal parallel to one of those directions is called an optic axis. Some crystals, called uniaxial crystals have only one optic axis, while others, the biaxial crystals, have two. Figure

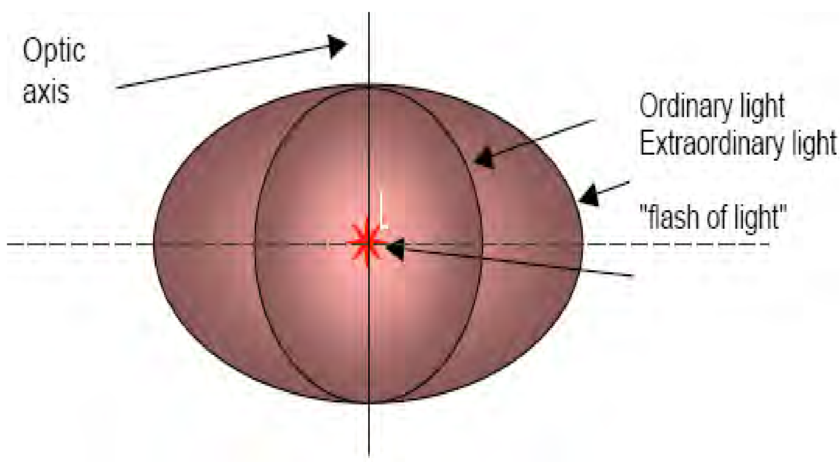

Figure 3.2: Ordinary and extraordinary waves

3.2. shows what would happen to light starting out from some point inside a calcite crystal. (This is not as silly as it may seem; Huygens' construction regards each point on a wavefront as a source of new waves. So the 'point source' considered here could be a point on a wavefront which originated outside the crystal). Calcite has one optic axis, along which the ordinary and extraordinary waves travel at the same speed, and the plane of the diagram has been chosen to include that axis. Two wavefronts are shown. Since the ordinary wave travels at the same speed (v_o) in all directions its wavefronts (for light coming from a point source) are spherical, and the section of the wavefront in the diagram is therefore circular. On the other hand, the speed (v_e) of the extraordinary wave depends on the direction of travel and the section of the wave front shown is elliptical. In calcite the speed of the extraordinary wave is always greater than or equal to the speed of the ordinary wave, so the extraordinary wavefront encloses the ordinary wavefront. In a crystal where the extraordinary wave is the slower of the two, its wavefront would stay inside the spherical wavefront of the ordinary wave. The ordinary wave is polarized perpendicular to the plane of the diagram and the polarization of the extraordinary wave is parallel to the plane of the diagram.

Light passing through a calcite crystal split into two rays. This process, first reported by Erasmus Bartholinus in 1669,[15] is called double refraction. The two rays of light are each plane polarized by the calcite such that the planes of polarization are mutually perpendicular. For normal incidence (a Snell's law angle of 0), the two planes of polarization are also perpendicular to the plane of incidence.

Table 3.1. Refractive indices of some uniaxial Birefringent crystals ($\lambda_0 = 589.3nm$)[9]

Crystal	n_o	n_e
Tourmaline	1.669	1.638
Calcite	1.6584	1.4864
Quartz	1.5443	1.5534
Sodium nitrate	1.5854	1.3369
Ice	1.309	1.313
Rutile $CTiO_2$	2.616	2.903

For normal incidence (a 0 degree angle of incidence), Snell's law predicts that the angle of refraction will be 0 . In the case of double refraction of a normally incident ray of light, at least one of the two rays must violate Snell's Law as we know it. For calcite, one of the two rays does indeed obey Snell's Law; this ray is called the ordinary ray (or O-ray). The other ray (and any ray that does not obey Snell's Law) is an extraordinary ray (or E-ray). For ordinary rays the vibration direction,

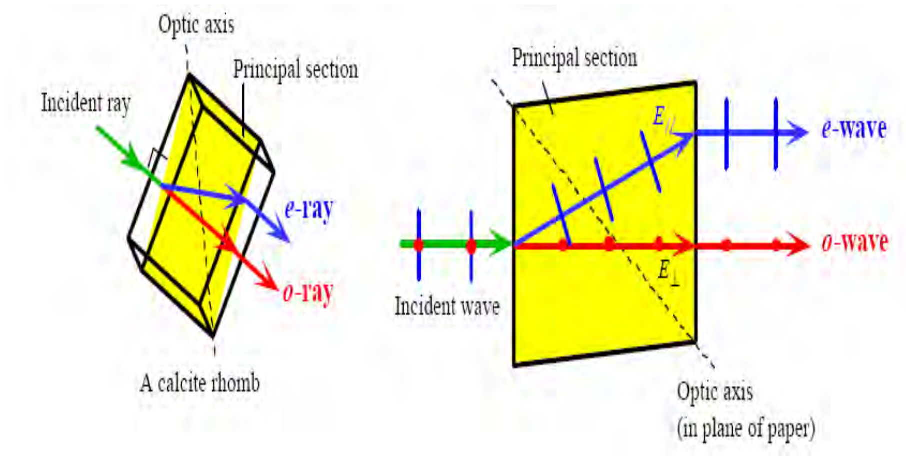

Figure 3.3: Ordinary and extraordinary waves in birefringence crystal

indicated by the electric vectors in our illustrations, is perpendicular to the ray path. For extraordinary rays, the vibration direction is not perpendicular to the ray path. The direction perpendicular to the vibration direction is called the wave normal.

All transparent crystals except those in the cubic system have the property of double refraction. For most crystals the image separation is not large enough to be visible.

However, we will observe other optical properties that result from the double refraction. For hexagonal and tetragonal crystals, there will be one O-ray and one E-ray. For orthorhombic, monoclinic, and triclinic crystals, there will be two E-rays. In general, the refractive indices for non-cubic crystals depend on vibration direction. Non-cubic crystals are, therefore, said to be optically anisotropic.

In most cases the refractive indices for the two rays produced by double refraction are not the same. One of the two rays will have a higher refractive index (and a lower velocity); this ray is called the slow ray. The other ray is then the fast ray.

Ordinary light is not polarized. Looking along a ray of light, the electric vectors make all angles with the vertical. Light that is plane polarized in the vertical plane has only vertical electric vectors. The plane of polarization is the plane that includes both the vibration direction and the ray path. Light may be polarized by crystals, by polarizing filters, and by reflection. Reflected light is partially polarized, favoring the vibration direction perpendicular to the plane of the ray path (including both the incident and reflected rays).

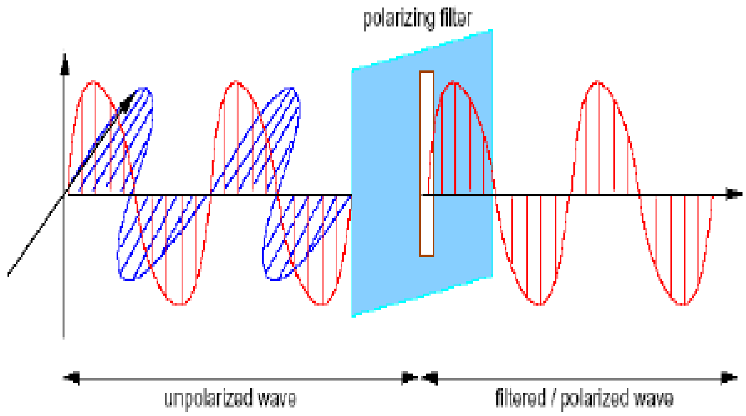

Figure 3.4: The electromagnetic waves filtered out by polarizing filters

Polarizing filters exclude all light not vibrating in the preferred direction of the filter. Polarizing sunglasses, by orienting their polarizing material vertically, selectively exclude the polarized portion of light reflected by horizontal surfaces. Transparent crystals do not exclude light, whatever its plane of polarization. Transparent anisotropic crystals resolve the electric vectors of incident light into two perpendicular electric vectors by the process of double refraction.

Upon emergence from the crystal, the two rays add together again according to the rules of vector addition. However, because the two rays have not traveled through the crystal with the same velocity, the combined emerging ray will not be identical to the incident ray.

3.2 Angular Momentum of the Light

Although we have only considered the beam as a classical electromagnetic wave so far, the fact that the angular momentum of left and right circularly polarized photons is $\pm\hbar$ can be used to simply find the angular momentum of the beam. Since the energy of a photon is $\hbar\omega$, the photon flux per unit area is

$$\Phi = \frac{I}{\hbar\omega} = \frac{I_L}{\hbar\omega} - \frac{I_R}{\hbar\omega} \tag{3.2.1}$$

giving an angular momentum flux per unit area of

$$L_z = \frac{(I_L - I_R)}{\omega} \tag{3.2.2}$$

Thus, the beam can be considered to have a net circularly polarized component with a power of $|I_L - I_R|$ which contributes to the angular momentum of the beam, and we can define a coefficient of circular polarization σ_z by

$$\sigma_z = \frac{(I_L - I_R)}{I} \tag{3.2.3}$$

and write the angular momentum flux density of the beam as

$$L_z = \frac{\sigma_z I}{\omega} \tag{3.2.4}$$

When the irradiance is integrated across the whole beam, the total power can be obtained and will be given by $P = P_L + P_R$

where $P_L = \int I_L dA$ and $P_R = \int I_R dA$ are the power of left circularly polarized and right polarized beam respectively.

A suitable average coefficient of circular polarization can be defined by

$$\sigma_z = \frac{(P_L - P_R)}{P} \tag{3.2.5}$$

with the resulting total angular momentum flux of the beam being

$$L_z = \frac{\sigma_z P}{\omega} \tag{3.2.6}$$

3.3 Optical Torque

It is well known that light can transport and transfer angular momentum and linear momentum. In recent years, this has been widely applied to rotate micro-particles in optical tweezers. Such rotational micromanipulation has been achieved using absorbing particles, birefringent particles, specially fabricated particles, and non-spherical particles using linearly polarized light or non-antisymmetric beams. Notably, particles rotated in rotationally symmetric beams spin at a constant speed, indicating that the optical torque driving the particle is balanced by viscous drag due to the surrounding medium. It has been noted that the optical torque can be measured optically, allowing direct independent measurement of the drag torque. For a pure

Gaussian beam that is circularly polarized, each photon also has an angular momentum of $\pm\hbar$ about the beam axis, equivalent to an angular momentum flux of $\pm\frac{P}{\omega}$ for the beam, where P is the power of an incident beam, and ω is the angular frequency of the optical field, which can be transferred to an object that absorbs the photon, or changes the polarization state. The reaction torque on a transparent object that changes the degree of circular polarization of the incident light is given by

$$\tau_R = \Delta\sigma \frac{P}{\omega} \tag{3.3.1}$$

where $\Delta\sigma$ is the change in the degree of circular polarization as the beam passes through the particle, P is the laser power and ω is the optical angular frequency.

If the beam passes through some birefringent material, the polarization will be affected. In general, σ_Z will change. The incident beam will have an initial coefficient of circular polarization σ_{Zin}, and will have an emergent polarization described by σ_{Zout}. Thus the angular momentum of the beam will change, and a reaction torque on the birefringent material will result. The reaction torque is equal to the change in the angular momentum flux:

$$\tau = [\sigma_{Zin} - \sigma_{Zout}] \frac{P}{\omega} \tag{3.3.2}$$

assuming that absorption and reflection can be ignored. Equation (3.3.2) is general, can be used to find the torque if the coefficients of circular polarization of the incident and outgoing beams are known or can be found. Although equation (3.3.2) applies in general, it is instructive to carry through a detailed calculation for a simple case: a uniform sheet of birefringent material, for example calcite.

A uniaxial birefringent material such as calcite can be described by two refractive indices: an ordinary refractive index n_o for electric fields normal to the optic axis, and an extraordinary refractive index n_e for electric fields parallel to the optic axis. For calcite, $n_o = 1.66$ and $n_e = 1.49$. Consider a thickness d of uniaxial birefringent material with the optic axis in the xy plane, at an angle of θ to the x-axis. The front face of the material is at $z = z_0$, and the rear face is at $z = z_0 + d$. If the electric field of the incident beam at the front surface of the material is given by equation (2.4.1), we can express it in terms of unit vectors $\hat{i}$ and $\hat{j}$ parallel to and normal to the optic axis respectively.

$$\vec{E} = [(E_x \cos\theta + E_y \sin\theta)\hat{i} + (-E_x sin\theta + E_y cos\theta)\hat{j}] \times exp(ikz - i\omega t) \tag{3.3.3}$$

In terms of circular components, this gives

$$\vec{E} = \frac{1}{\sqrt{2}}[(E_x - iE_y)exp(i\theta)\hat{e}_L^{'} + (E_x + iE_y)exp(-i\theta)\hat{e}_R^{'}]exp(ikz_0 - i\omega t) \qquad (3.3.4)$$

where $\hat{e}_L^{'} = \frac{1}{\sqrt{2}}[\hat{i} + i\hat{j}]$ and $\hat{e}_R^{'} = \frac{1}{\sqrt{2}}[\hat{i} - i\hat{j}]$
The coefficient of circular polarization is given by

$$\sigma_{Zin} = \frac{E_L^* E_L - E_R^* E_R}{E_L^* E_L + E_R^* E_R} \qquad (3.3.5)$$

or we can write this equation by substituting

$$\begin{aligned} E_L &= \frac{1}{\sqrt{2}}[E_x - iE_y] \qquad (3.3.6) \\ E_R &= \frac{1}{\sqrt{2}}[E_x + iE_y] \end{aligned}$$

And the complex conjugate of this is

$$\begin{aligned} E_L^* &= \frac{1}{\sqrt{2}}[E_x^* + iE_y^*] \qquad (3.3.7) \\ E_R^* &= \frac{1}{\sqrt{2}}[E_x^* - iE_y^*] \end{aligned}$$

By substituting these values in to the above equation (3.3.5) we can obtain

$$\sigma_{Zin} = i\frac{[E_y^* E_x - E_x^* E_y]}{E_x^* E_x + E_y^* E_y} \qquad (3.3.8)$$

After passing through the thickness d ,the field will be from equation (3.3.3)

$$\vec{E} = [(E_x \cos\theta + E_y \sin\theta)exp(ikdn_e)i + (-E_x sin\theta + E_y cos\theta)exp(ikdn_o)j] \times exp(ikz_o - i\omega t) \qquad (3.3.9)$$

Which we can express in terms of circular components

$$\begin{aligned} E_L &= \frac{1}{\sqrt{2}}[(E_x \cos\theta + E_y \sin\theta)exp(ikdn_e) - \qquad (3.3.10) \\ &\quad i(-E_x cos\theta + E_y sin\theta)exp(ikdn_o)] \\ E_R &= \frac{1}{\sqrt{2}}[(E_x \cos\theta + E_y \sin\theta)exp(ikdn_e) + \\ &\quad i(-E_x cos\theta + E_y sin\theta)exp(ikdn_o)] \end{aligned}$$

We define the convenient notation

$$\Delta = kd(n_o - n_e) \tag{3.3.11}$$

The coefficient of circular polarization of the emergent light is

$$\sigma_{Zout} = \frac{[icos\Delta(E_xE_y^* - E_x^*E_y) - sin\Delta\{(E_x^*E_x - E_y^*E_y)sin2\theta - (E_xE_y^* + E_x^*E_y)cos2\theta\}]}{[E_x^*E_x + E_y^*E_y]} \tag{3.3.12}$$

giving a torque per unit area

$$\tau = \frac{c\epsilon_o}{2\omega}[i(E_xE_y^* - E_x^*E_y)(1 - cos\Delta) + sin\Delta\{(E_x^*E_x + E_y^*E_y)sin2\theta - (E_xE_y^* + E_x^*E_y)cos2\theta\}] \tag{3.3.13}$$

If the incident light is linearly polarized ($E_y = 0$), the torque is

$$\tau = \frac{c\epsilon_0}{2\omega}sin\Delta E_0^*E_0 sin2\theta \tag{3.3.14}$$

which acts to align the slow axis of the particle with the plane of polarization if $n_o > n_e$. If the incident light is left circularly polarized ($E_y = iE_x$), the torque is

$$\tau = \frac{c\epsilon_0}{2\omega}E_0^*E_0(1 - cos\Delta) \tag{3.3.15}$$

which is independent of the orientation of the birefringent material.

If the birefringent material has a uniform thickness, the total torque can be simply calculated from this [6]. In general, a laser trapped birefringent particle will have a varying thickness, and direct calculation of the torque will not be feasible. Also, if the orientation of the birefringent particle is different, so the optic axis does not lie in the xy plane, the calculation will be further complicated. Equation (3.3.2), however, is general, and will still apply, and the torque acting on the particle can be deduced from the change in polarization of the light.

Chapter 4

Experimental Set Up, Measurement and Results

4.1 Introduction

The first experimental set up observation of the torque on a macroscopic object resulting from interaction of light was by Beth in 1936, who observed the deflection of a quartz wave plate suspended from a thin quartz fiber when circularly polarized light passed through it[8]. An experiment was proposed in 1957 to measure radiation torque using microwave radiation, and measurement of light torques were made in 1966. The experiments of Allen showed that the torque on a suspended dipole due to a circularly polarized radiation increased linearly with the intensity of the light[8].

An optical trap is based on the attraction of small particles in regions of high intensity in a tightly focused laser beam. This torque results from the transfer of momentum from the trapping beam to the particle. Optical torque will result if there is a transfer of angular momentum. Since the optical torques result from the change in angular momentum of the beam, it is in principle possible to measure the trapped torque by measuring the momentum of the scattered light[21].

Optical measurement torque is measured by trapping of a vibrating microscopic particles in a fluid. If we used a circularly polarized laser beam the particle rotates by the angular momentum of the beam as it transfer from the beam to the particle. Such microscopic particles will typically be confined within a laser trap. Strongly focussed laser light incident on a transparent particle, usually in a liquid medium, will produce a gradient force acting on the particle towards the region of highest irradiance. If this gradient force near the focus is stronger than scattering and absorption forces, the particle will be trapped at the beam focus, where the irradiance is highest. This technique of three-dimensional confinement and manipulation is called laser micro-manipulation, trapping, or optical tweezers. This method also used to determine the viscosity of the liquid. The viscosity of a liquid can be determined by measuring the torque required to rotate a sphere immersed in the liquid at a constant angular velocity and a concept we have implemented at the micrometer scale using a system based on optical tweezers[18].

The angular momentum carried by light can be characterized by the "spin" angular momentum associated with circular polarization and the "orbital" angular momentum associated with the spatial distribution of the wave [8]. The transfer of angular momentum from light to an absorbing particle can be understood using classical electromagnetic theory. The torque due to the polarization of the light on the particle that absorbs power P_{abs} is $\tau_{\sigma_z} = \frac{P_{abs}\sigma_z}{\omega}$, where σ_z is ± 1 for circularly polarized light and zero for plane polarized light and ω is the frequency of the light.

4.2 The Experimental set up

Our experimental set up was designed by using materials as described below. There are several materials used to measure the microscopic torques on the given birefringent crystal. The most important are:

- Helium-Neon laser beam with $633nm$ wavelength and power of $35mw$;
- Birefringent crystal(calcite);
- Quarter wave plates to produce circularly polarized beam;
- Analyzer to check whether the beam is circularly polarized beam or not;
- Filters to filter out the laser from entering the camera;
- Power meter to measure the power of the beam;
- Digital camera to capture video and image of the trapped particle;

First we set the laser beam about $63cm$ from the table and we fixed it from vibrating of any disturbance. The direction of the beam is along horizontal direction but the direction of the microscope is along vertical direction. To enter the beam in to the microscope eyepiece we must put the mirror at 45^0 from the beam direction. This alignment of the mirror bends the light at 90^0 and the beam entered through the microscope. The mirror we used for bending of light was coated in Polymer lab by using vacuum coating machine, this coating increases the reflectivity of the mirror.

The next step was the alignment of the beam from the eyepiece of the microscope to the camera. This step was the most difficult part of the experiment. First we checked the direction of the beam through all objectives and following this we adjust the $40X$ objective below the sample. This helps us to see the trapped particle from the experiment. After this step we put the filter, the purpose of the filter was to filter out the out going laser beam from trapped particle from entering to the camera. If it entered int to the camera it damages the camera and to block out only the laser is needed to enhance illumination at the sample. Before the camera there is an eyepiece, it helps to magnify the image of a trapped particle. Finally, we put the camera which is connected with a computer.

4.2.1 The materials

Trapping Laser

After we finished the set up of optical tweezers we started to check the optical trapping of particles. This is also too difficult like the experimental set up. Here we used a He-Ne gas laser of power $35mW$ and wave length $633nm$.

The basic requirement of a trapping laser is that it delivers a single mode output (typically, Gaussian TEM_{oo} mode) with excellent pointing stability and low power fluctuations. A Gaussian mode focuses to the smallest diameter beam waist and will therefore produce the most efficient, harmonic trap. Pointing instabilities lead to unwanted displacements of the optical trap position in the specimen plane, whereas power fluctuations lead to temporal variations in the optical trap stiffness. Pointing instability can be remedied by coupling the trapping laser to the optical trap via an optical fiber, or by imaging the effective pivot point of the laser pointing instability into the front focal plane of the objective. Thus, both power and pointing fluctuations introduce unwanted noise into any trapping system.

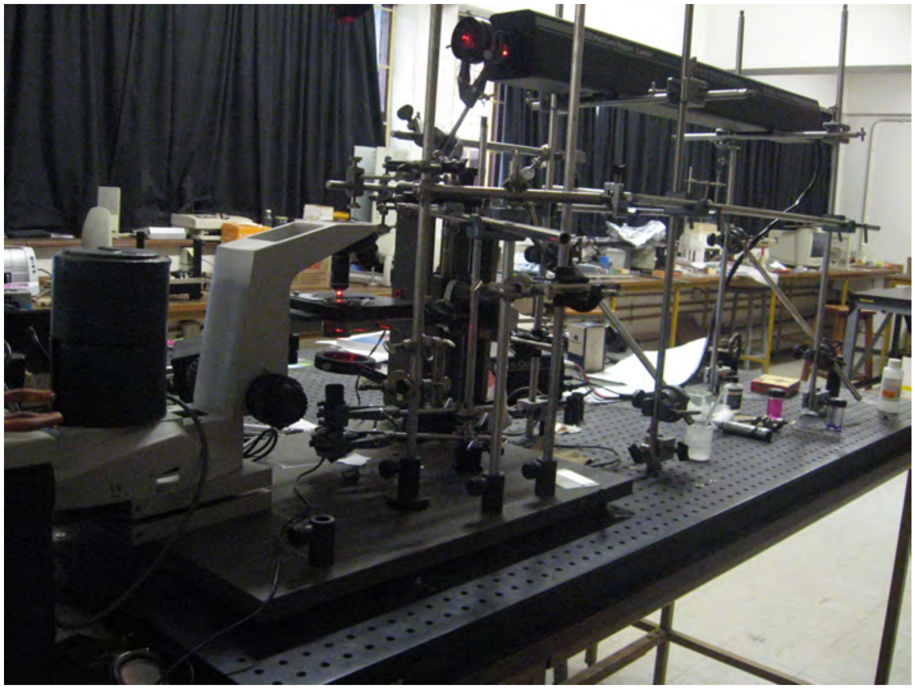

Figure 4.1: Experimental set up

The purpose of power-meter is to measure the power of the laser to trap the particle. We measured at different point the power i.e. RCP, LCP, and the power from the source to determine the coefficient of circular polarized beam.

Microscope

Most optical traps are built around a conventional light microscope, requiring only minor modifications. This approach reduces the construction of an optical trap to that of coupling the light from a suitable trapping laser into the optical path before the objective without compromising the original imaging capabilities of the microscope. In practice, this is most often achieved by inserting a dichroic mirror, which reflects the trapping laser light into the optical path of the microscope but transmits the light used for microscope illumination. Inverted, rather than upright, microscopes are often preferred for optical trapping because their stage is fixed and the objective moves, making it easier to couple the trapping light stably.

With more extensive modifications, a position detector can be incorporated into the trapping system. This involves adding a second dichroic mirror on the condenser side of the microscope, which reflects the laser light while transmitting the illuminating light. But for our case we used a bifocal light microscope for this experiment.

Objectives

The single most important element of an optical trap is the objective used to focus the trapping laser. The choice of objective determines the overall efficiency of the optical trapping system (stiffness versus input power), which is a function of both the NA and the transmittance of the objective. Additionally, the working distance and the immersion medium of the objective (oil) will set practical limits on the depth to which objects can be trapped. Spherical aberrations, which degrade trap performance, are proportional to the refractive index mismatch between the immersion medium and the aqueous trapping medium. The deleterious effect of these aberrations increases with focal depth. The working distance of most high NA oil immersion objectives is quite short (for $100X$ oil immersion is 0.1mm and for $40X$ is 0.6mm) and the large refractive index mismatch between the immersion oil ($n = 1.512$) and the aqueous trapping medium ($n \sim 1.32$) leads to significant spherical aberrations. In practice, this limits the maximum axial range of the optical trap to somewhere between 5 and $20\mu m$ from the cover glass surface of the trapping chamber[14].

Trapping deeper into solution can be achieved with water immersion objectives that minimize spherical aberration 10^5 and which are available with longer working distances. A high NA objective (typically, 1.2-1.4 NA) is required to produce an intensity gradient sufficient to overcome the scattering force and produce a stable optical trap for microscopic objects, such as calcite. The vast majority of high NA objectives are complex, multi-element optical assemblies specifically designed for imaging visible light, not for focusing an infrared laser beam. For this reason, the optical properties of different objectives can vary widely over the near infrared region. Generally speaking, objectives designed for general fluorescence microscopy display superior transmission over the near infrared compared to most general-purpose objectives, as do infrared-rated objectives specifically produced for use with visible and near infrared light. Given the wide variation in transmission characteristics for different objectives, an objective being considered for optical trapping should be characterized at the wavelength of the trapping light. It should be noted that the extremely steep focusing produced by high NA objectives can lead to specular reflection from surfaces at the specimen plane, so simply measuring the throughput of an objective by placing the probe of a power meter directly in front of the objective lens results in an underestimation of its transmission[13].

Quarter wave plate

An anisotropic plate for which the difference of optical thickness is a quarter of a wave length is called a *quarter wave plate*. Or circularly polarized light can be produced by introducing a phase shift of $\frac{\pi}{2}$ between two orthogonal components of linearly polarized light. The device for doing this is known as a quarter wave plate. This plates are made of doubly refracting transparent crystals, such as calcite or mica. Doubly refracting crystal in to slabs in such a way that an axis of maximum index n_1 (the slow axis) and an axis of minimum index n_2 (the fast axis) both lie at right angles to one an other in the plane of the slab. If the slab thickness is d, then the optical thickness is $n_1 d$ for light polarized in the direction of the slow axis and $n_2 d$ for light polarized in the direction of the fast axis. For a quarter wave plate, d is chosen to make the difference $n_1 d - n_2 d$ equal to *one-quarter wave length* , so that d is given by $d = \frac{\lambda_o}{4[n_1 - n_2]}$ in which λ_o is the vacuum wave length.

Quarter wave plates are used to turn-plane polarized light in to circularly polarized light and vice versa. To do this, we must orient the wave plate so that equal amounts of fast and slow waves are excited. We may do this by orienting an incident plane-polarized wave at 45^0 to the fast (slow) axis, as shown in fig.4.1.

When a plane-polarized light is incident up on an anisotropic plate the state of polarization in the emergent light depends up on (i) the difference of optical thickness (ii) the relation between the plane of polarization of the light and the privileged direction of the plate.

And when plane polarized light is incident up on a quarter-wave plate, the emergent light is in general elliptically polarized. The axis of the ellipse are parallel to the privileged direction in the plate. When the plane of polarization of the incident beam bisects the angle between the privileged directions, the light emerging from a quarter-wave plate is circularly polarized.

As we described in chapter two, there are two types of light, called polarized light and unpolarized light. Then, an arrangement which produces a beam of plane-polarized light from a beam of unpolarized light is called *polarizer*. And an arrangement detects plane polarized light is called an analyzer.

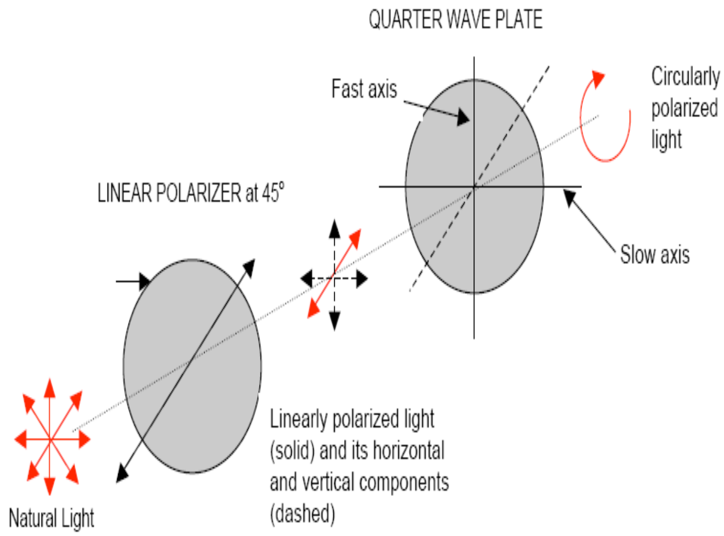

Figure 4.2: Producing circularly polarized beam

The physical arrangement for producing circularly polarized light is shown in fig.4.1. By choosing θ to be 45^0, the light entering the quarter wave plate can be resolved in to two orthogonal linearly polarized light components of equal amplitude and equal phase. On emerging from the quarter-wave plate, these two components are out of phase by $\frac{\pi}{2}$. Hence the emerging light is circularly polarized light. The sense of rotation the circularly polarized light depends on the value of θ and can be reversed by rotating the quarter-wave plate through an angle of 90^0 so that θ is 135 degrees[15,16].

For optical torque measurement the experimental set up was the same as for the trapping except entering the quarter wave plate. The He-Ne laser beam passes through the quarter wave plate and the analyzer in the form of left or right circular, depending on the direction or the angle of the quarter wave plate. Then this beam passes through the bifocal microscope to trap the vibrating particle. After trapping the beam pases through the objectives and go through the filter and it filtered out from entering the camera. We can see the trapping of particles with the help of the camera. The optical torque is determined by measuring of the power of the beam after trapping the particle.

4.3 Optical Trapping

Optical trapping, the trapping and manipulation of microscopic particles by a focussed laser beam, is a widely used and powerful tool. The most common optical trap, the single-beam gradient trap (optical tweezers) consists of a laser beam focussed by a lens, typically a high-numerical aperture microscope objective, with the same microscope being used to view the trapped particles (see Fig.4.2)[6]. The trapped particle is usually in a liquid medium, on a microscope slide.

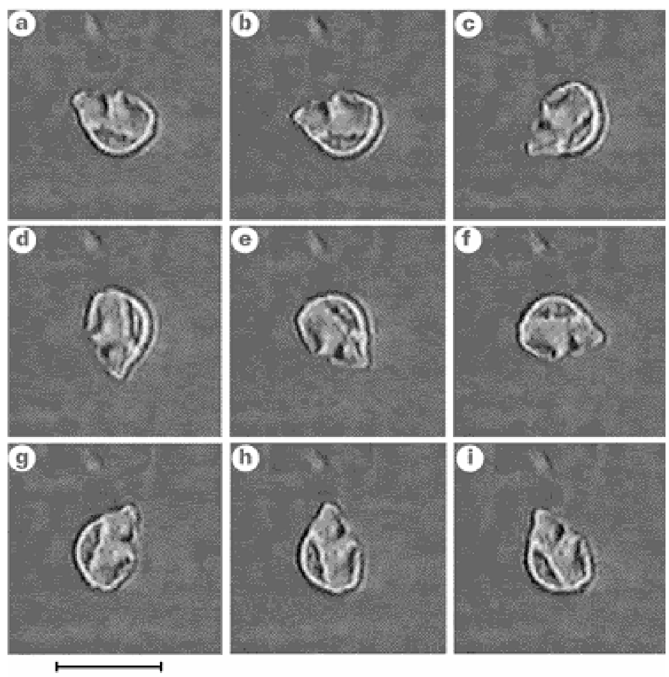

Figure 4.3: Nine Frames of a trapped calcite crystal showing free rotation due to an elliptically polarized trapping beam. Source: Friese et al. (1998).[6]

Although simple trapping and manipulation are sufficient for many applications, the use of optical trapping for quantitative research into physical, chemical, and biological processes, typically using a laser-trapped particle as a probe, requires an accurate quantitative theory of optical trapping. The minimization of damage to trapped biological specimens also indicates the desirability of using theoretical results to design traps in order to minimize the power absorbed by the trapped particle.

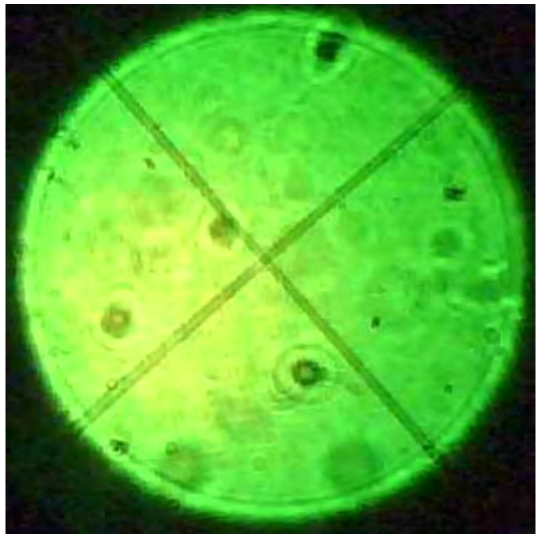

Figure 4.4: Trapped calcite crystal due to a circularly polarized trapping beam.

The concept of optical trapping is based on a gradient force causing small particles to be attracted to regions of high intensity in a tightly focussed laser beam. Other optical forces, due to absorption, reaction, and scattering are termed scattering forces. Both the gradient and scattering forces result from the transfer of momentum from the trapping beam to the particle. Optical torques can also be produced by the transfer of angular momentum from the beam. This can result from birefringence or scattering with a non-zero angular component relative to the particle center [4].

An accurate quantitative theory of optical trapping of wavelength-scale particles is highly desirable. Recent theoretical efforts have individually eliminated some of the deficiencies due to the various approximations usually used [5,6,7], but there still exists no general correct theory. The lack of suitable theory is even more acute when the trapping of non-spherical particles is considered. Non-spherical particles are of particular interest due to their suitability for use as microscopic probes, and the frequent natural occurrence of non-spherical biological and other structures. The possibility of rotating or controlling the orientation of non-spherical particles greatly extends the range of manipulation possible in an optical trap.

An optical trap is based on the attraction of small particles to regions of high intensity in a tightly focussed laser beam. This gradient force (and other forces due to absorption, reflection, etc - termed scattering forces) results from the transfer of momentum from the trapping beam to the particle. Optical torques will result if there is a transfer of angular momentum. Since the optical forces and torques result from the change in momentum and angular momentum of the beam, it is in principle possible to measure the applied force and torque by measuring the momentum of the scattered light. Direct optical determination of the force and torque gives an absolute measurement, immediately eliminating difficulties with calibration.

The degree of circular polarization remaining in the beam is determined by passing it through a $\frac{\lambda}{4}$ plate with the fast axis orientated at 45^0 to the axes of a polarizing beam splitter cube. Photodiodes monitoring the orthogonal outputs of the beam splitter provide a measure of the degree of circular polarization remaining in the beam, which allows the change in angular momentum and hence the torque applied to the particle to be known. The photodiodes are calibrated such that they provide a measure of the power at the trap focus. The sum of the two signals gives the total trapping power, and the difference between the signals gives the degree of circular polarization of the beam measured in units of power at the trap focus.

The torque applied to the particle is determined by measuring the difference between the initial degree of circular polarization of the beam in the absence of the particle and the final polarization, in accordance with equation (3.3.1). The frequency of rotation of the particle can be accurately determined by monitoring the transmitted light through a linear polarizer. A small amount of light is diverted from the main beam and is passed through a polarizing beam splitter cube, and the forward direction is monitored using a photodiode. The signal is modulated at twice the rotation rate of the particle, and the depth of the modulation is proportional to the amount of linearly polarized light in the beam. If the particle is thick enough to act as a $\frac{\lambda}{2}$ plate, then circularly polarized light passing through the particle is reversed in handedness and the frequency of the rotation cannot be measured as there is no linear component to the transmitted light. However, as the particles are spherical and are of similar diameter to the beam, the polarization at different radial distances from the rotation axis varies, and there will almost always be some linear component after transmission.

A typical optical-tweezers arrangement was used to trap microscopic calcite particles in three dimensions using $35mW$ of He-Ne laser light at a wavelength of $633nm$. The optical trap used a $100X$ oil-immersion, high numerical aperture ($NA = 1.25$) microscope objective. The trapping beam was initially linearly polarized, and the plane of polarization could be rotated using a half-wave plate. Alternatively, a quarter-wave plate allowed the ellipticity of polarization to be varied. They were dispersed in distilled water in a trapping cell consisting of a well in a microscope slide with a coverslip[6].

Because of their birefringent nature, calcite particles can act as wave-plates; a calcite particle $3\mu m$ thick is a $\frac{\lambda}{2}$ plate for $633nm$ light. On passage through a fragment of calcite, the ordinary and extraordinary components of the incident light will undergo different phase shifts. If this results in a change in the angular momentum carried by the light, there will be a corresponding torque on the material. Our results can be understood using a simple plane-wave picture; the interaction between an incident plane wave and a wave-plate is outlined below. We note that the calcite wave-plate is trapped at the focal point of the beam, where the wavefronts are nearly plane.

An incident laser beam can in general have both circularly polarized and plane polarized components; that is, it will be elliptically polarized. Elliptically polarized light can be described by

$$\vec{E} = E_o e^{i\omega t} cos\phi \hat{x} + i E_o e^{i\omega t} sin\phi \hat{y} \tag{4.3.1}$$

where ϕ describes the degree of ellipticity of the light ($\phi = 0$ or $\frac{\pi}{2}$ indicates plane-polarized light, $\phi = \frac{\pi}{4}$ circularly polarized light). To calculate the change in angular momentum of the light after passage through a birefringent material, the incident elliptically polarized light is first expressed in terms of components parallel and perpendicular to the optic axis of the material by equation (3.2.6).

The changes in the angular momentum of the light cause a reaction torque per unit area on the thickness d of material as described in Eq.(3.3.10). In general, the first term is the torque due to the plane-polarized component of elliptically polarized light while the second term is due to the change in polarization caused by passage through the medium. For plane-polarized light,$\phi = 0$ or $\frac{\pi}{2}$, so the torque on the particle is proportional to $sin2\theta$, so that a particle will experience torque so long as θ is non-zero, and will be at equilibrium when the fast axis of the crystal is aligned with the plane of polarization ($\theta = 0$). We found that calcite fragments trapped in plane-polarized light are aligned in a particular orientation, and a particular particle is always aligned in the same plane each time it is trapped. When the plane of polarization is rotated using a half-wave plate, a particles alignment exactly follows the rotation of the plane of polarization.

4.4 Methods of Measurement of Optical Torques

Consider a circularly polarized laser beam used to trap a microscopic particle composed of a uniaxial birefringent material such as calcite or a suitable polymer. If the optical torque is large enough to overcome forces holding the particle in place, the particle will rotate at a speed determined by the equilibrium between the optical torque and other forces such as viscous drag. In this way, a probe particle can be used to measure viscosity on a microscopic scale. If the particle does not rotate, the optical torque can be used to determine the torque due to static forces acting on the particle. The maximum torque and rotation rate will occur when the incident beam is completely circularly polarized ($i.e.\sigma_{Zin} = 1$). The torque in this case will also be constant as well as maximal [6], and we will only consider this case here. The torque τ acting on the trapped particle is given in equation (3.3.2) by the difference between the incident and outgoing angular momentum fluxes, and in this case, assuming no reflection or absorption, is

$$\tau = [1 - \sigma_{Zout}]\frac{P}{\omega} \tag{4.4.1}$$

Measurement of the outgoing polarization σ_{Zout} and beam power P gives an absolute measurement of the torque, which does not depend on the mechanical properties of the surrounding medium or the particular size or shape of the particle or laser beam.

We can also note that the plane of polarization of the linearly polarized component of the outgoing beam exiting a rotating birefringent particle will be rotating at the same rotation rate as the particle. If the outgoing beam is a (rotating) purely plane-polarized beam, as would occur if the particle acted as a quarter-wave plate, rotating at Ω, and of power P, and is passed through a linear polarizer, the measured power P_m will be $P_m = [1 + \cos 2\Omega t]\frac{P}{2}$ (with variation at a frequency of 2Ω since a rotation of 180^0 rotates the plane of polarization onto itself) [10]. By measuring this power, the rotation rate Ω of the trapped particle can be simply determined. This will still be the case for an elliptically polarized beam, as the same variation at a frequency of 2Ω will be observed. The angular momentum associated with this rotation of the plane of polarization will be negligible as $\Omega \ll \omega$.

The optical torque acting about the beam axis is always a result of the alteration of orbital and/or spin angular momentum of the incident beam by the trapped particle, by absorption or by scattering if there is either external (shape) or internal (birefringence) anisotropy [8]. Consequently, the torque can either originate from a beam where the incident light itself carries angular momentum that is transferred to the particle, or it can originate from a beam where the incident light carries zero angular momentum, but where the trapped particle induces angular momentum in the beam. In the general case, there will be an elliptically polarized outgoing beam, consisting of both plane and circularly polarized components. The measured power P_m after the outgoing beam passes through a linear polarizer acting as an analyzer will be

$$P_m = [1 + (1 - \sigma_{Zout}^2)^{\frac{1}{2}}) \cos 2\Omega t] \frac{P}{2} \tag{4.4.2}$$

Measurement of the variation of the transmitted power therefore allows the determination of the rotation period of the trapped particle, and the degree of (but not the direction of) circular polarization. The result of a measurement of this type will be as shown in fig.4.5. This measurement is an average over the beam, and it is not important whether or not the entire beam passes through the particle. In the case where the particle is not rotating, due to some restraining torque, the plane of polarization of the transmitted light will not be rotating. The degree of circular polarization can be measured in this case by rotating the linear polarizer, which will give the same result where Ω is the rotation rate of the polarizer relative to the particle. The orientation of the particle can also be determined from the position of the measured power maxima. The following result is by measuring the out going beam power of a $100mW$ and wavelength of $1064nm$ laser beam[17].
When the particle is insufficiently thick to change the direction of polarization (note that a calcite particle approximately $3\mu m$ thick is a $\frac{\lambda}{2}$ plate for $633nm$ light), or when the particle is small and the outgoing light is dominated by light that did not pass through the particle and has not changed in polarization. If necessary, the direction of circular polarization can be measured simply by placing a reversed circular polarizer (eg a quarter-wave plate followed by a linear polarizer appropriately oriented) in the beam path instead of a linear polarizer. In the case where the trapping beam has a righthanded helicity (left circularly polarized, with $\sigma_{Zin} = +1$, the light emergent from the particle can be described in terms of left and right circularly polarized components P_L and P_R, where $P_L = [1 + \sigma_{Zout}]\frac{P}{2}$ and $P_R = [1 - \sigma_{Zout}]\frac{P}{2}$. If the output beam is predominantly left circularly polarized, $\sigma_{Zout} > 0$, and $P_L > P_R$. A right circularly polarized beam has $\sigma_{Zout} < 0$, and $P_L < P_R$. It is only necessary to determine which

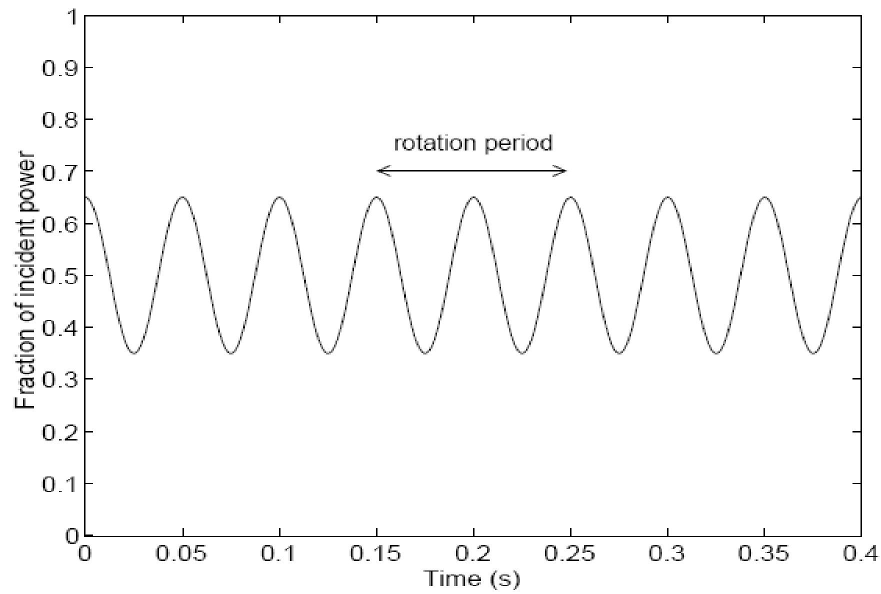

Figure 4.5: The power which would be measured through the plane polarizer after the beam passed through a birefringent particle.[17]

of these two components is larger, rather than to measure each one individually since $| \sigma_{Zout} |$is already known. In this way, the direction of circular polarization can be determined, and σ_{Zout} as opposed to merely $| \sigma_{Zout} |$ can be found. Once σ_{Zout} is known, the optical torque acting on the particle can be found using equation (3.3.2). A measurement of this type will be as shown in figure 4.6.

The two circularly polarized component of transmitted light can be measured to determine the direction of circular polarization. P_L is the power of left circularly polarized component, and P_R is the power of the right circularly polarized component.

Two cases are of particular interest. Firstly, a birefringent particle will have a uniform angle independent torque acting on it when trapped by a circularly polarized beam [8]. Secondly, non-spherical particles, and birefringent particles, will align in a particular orientation when trapped by a plane-polarized beam. If the particle can freely move to this orientation, there will be no torque acting. If, however, the particle cannot move freely, there will be an optical torque acting on the particle, which can be used to measure the restraining forces. The polarization of the scattered beam can

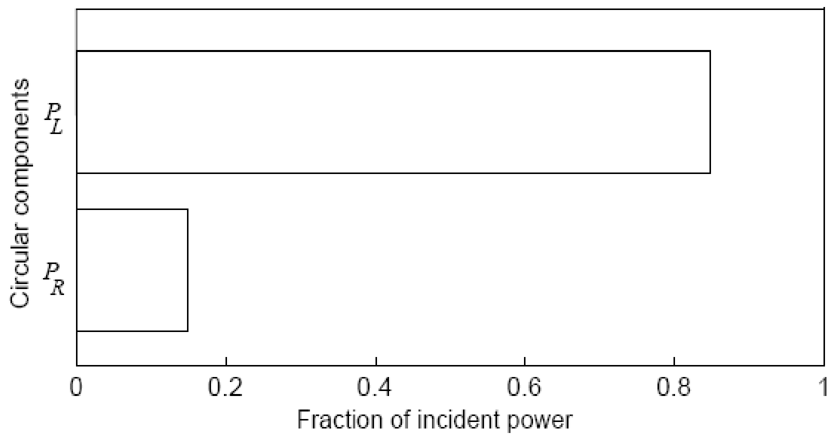

Figure 4.6: The two circularly polarized components of the transmitted light.

be measured by measuring the intensity through a plane-polarizer, which will result in a sinusoidal variation of the intensity as the particle rotates [9, 10]. This will provide simultaneous measurements of the optical torque and the rotation speed of the particle. The torque acting on a stationary particle can be measured by rotating the plane polarizer through which the intensity is measured. In general, the typical construction of an optical trap will make the measurement of torques other than that about the beam axis very difficult[17].

4.5 Measurement and result

Our measurement directly or indirectly depends on the power of the beam in different ways by rotating the quarter wave plates at different points. We measured the beam power before trapping and after trapping the birefringent crystals, calcite. From these measurements we can determine the optical torque acts on the particle and the rotating frequency of the trapped particle.

Here we measured first the power of LCP and RCP beam before entering to trap the particle just out going from the 100X,oil immersion objective, next we measured the out going beam power for both LCP and RCP cases and finally we measured the frequency of the out going beam for both LCP and RCP cases. From these measurements we can get the optical torque on the microscopic particles.

4.5.1 Measurement of Left and Right circularly polarized beam

From the experimental result the measurement of Left and Right circularly polarized of the beam is simply by measuring the power using power-meter by changing the $\frac{\lambda}{4}$ plate at 45^0 for right circularly polarized beam and at 135^0 for left circularly polarized beam. The left and right circularly polarized beam before trapping the particle is $P_L = 24.5mW$ and $P_R = 10.5mW$. And the angular frequency of the He-Ne laser beam of $35mW$ and $633nm$ from the source is $\omega = 2.98 \times 10^{15}\frac{rad}{sec}$. The two circularly

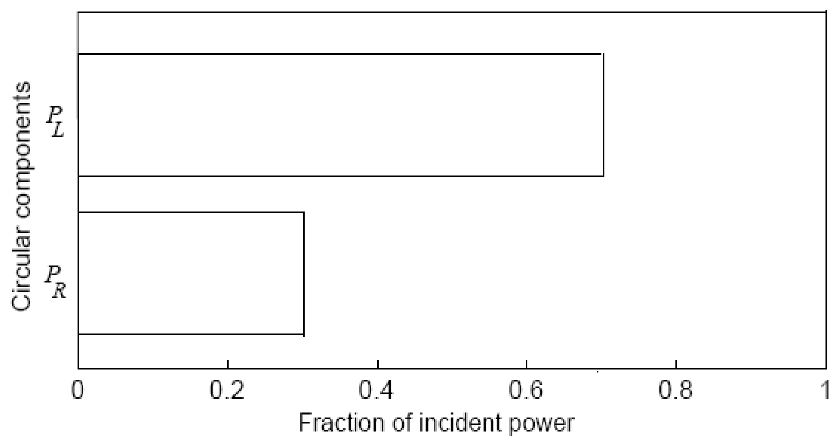

Figure 4.7: The two circularly polarized components of the transmitted light.

polarized component of transmitted light can be measured to determine the direction of circular polarization. P_L is the power of left circularly polarized component, and P_R is the power of the right circularly polarized component.

4.5.2 Measurement of the coefficient of circular polarization(σ_{zout})

The power which would be measured through the plane polarizer after the beam passed through a birefringent particle is shown in fig.4.7. The coefficient of circular polarization of the outgoing beam measured from the measurement of the left (LCP) and right (RCP) circularly polarized beam. We first measured the right circularly polarized beam power and next by rotating the quarter wave plate to determine

the LCP beam power. This measurement helps us to determine the coefficient of circularly polarized beam (σ_{zout}). The value of the power from the out going beam is $P_L = 6.6mW$ and $P_R = 1.4mW$, From this $\sigma_{zout} = +0.15$ and $\sigma_{Zin} = 1$ since our beam is completely circularly polarized beam [17]. There fore, the change in circular polarization is $\Delta\sigma = 0.85$.
After trapping the out going beam decreased due to some reflections from the surface of the cover slip and the slide. Measurement of the variation of the transmitted power therefore allows the determination of the rotation period of the trapped particle and the degree of circular polarization. The result of a measurement of this type will be as shown in fig.4.8. This measurement is an average over the beam, and it is not important whether or not the entire beam passes through the particle. In the case where the particle is not rotating, due to some restraining torque, the plane of polarization of the transmitted light will not be rotating [17]. The degree of circular polarization can be measured in this case by rotating the linear polarizer, which will give the same result where Ω is the rotation rate of the polarizer relative to the particle. The orientation of the particle can also be determined from the position of the measured power maxima [17].

4.5.3 Measurement of optical Torque

The optical torque measured by using equation (4.3.1) and by taking the frequency of the laser beam. The frequency of the laser beam is 2.98×10^{15} and the power of the incident beam is $P = 35mW$. But we get the value of the coefficient of circularly polarized out going beam $\sigma_{Zout} = +0.15$. Then, the optical torque acting on the vibrating particle is $9.983pN.\mu m$.
This method is simplest method to find the optical torque on the particle. And it does not depend on the shape and size of the particle. It should be noted that this technique is robust. It is not necessary to measure the power of the entire transmitted beam; it is sufficient to measure the portion of the beam that has passed through the trapped particle. Similarly, reflections are not likely to cause significant error. Some of the incident beam will be reflected from the trapped particle; the reflection will depend on the angle of incidence and the refractive indices of the particle and the surrounding medium. For calcite trapped in water, the Fresnel amplitude coefficients for reflection at normal incidence is

$$[r_{\shortparallel}]_{\theta i=0} = [-r_{\perp}]_{\theta i=0} = \frac{n_{water} - n_{calcite}}{n_{water} + n_{calcite}} \tag{4.5.1}$$

where $n_0 = 1.6584$ and $n_e = 1.4864$ for calcite and $n_{water} = 1.3333$ for water, then by substituting given values, gives reflected amplitudes of $-0.057E_x$ and $-0.11iE_x$ for linearly polarized components normal to and parallel to the optic axis respectively.In terms of circular components, this becomes $E_L = -0.02E_{L0}$ and $E_R = -0.08E_{L0}$, The torque produced by the reflected beam (from the calcite, water, slide cover and the glass slide) is $0.38pN.\mu m$ showing that the torque due to back reflected light will be less than 4 percent of the available torque. Therefore, the reflected light will not cause any significant error.

4.5.4 Measurement of the frequency of a rotating particle

The mean power measured through the polarizer is half of the power incident on the particle. The frequency of the variation is two times the rotation rate Ω of the particle. The optical torque can be found from the amplitude of the variation and the measured power once the direction of the transmitted polarization is known. In this case, for a $35mW$ trapping beam of wave length $633nm$, an optical torque of $9.983pN.\mu m$ is being exerted. This method gives us the frequency of the rotating particle. The result we measured by using an oscilloscope are for LCP is $136.94Hz$ and for RCP is $112.16Hz$. Here we used Fortran 90 to show the rotation frequency of the particle as shown in fig 4.8. From the figure the red color line graph indicates the LCP out going beam and the black color line is the RCP. This indicates the rotating frequency of the particle directly measured from the out going beam from the trapped particle for both RCP and LCP case.

The uncertainty of the frequency of the rotating particle can be determined by using $\triangle\Omega = \frac{|\Omega_L - \Omega_R|}{|\Omega_L + \Omega_R|}$, we obtained the $\triangle\Omega = 0.0995$.

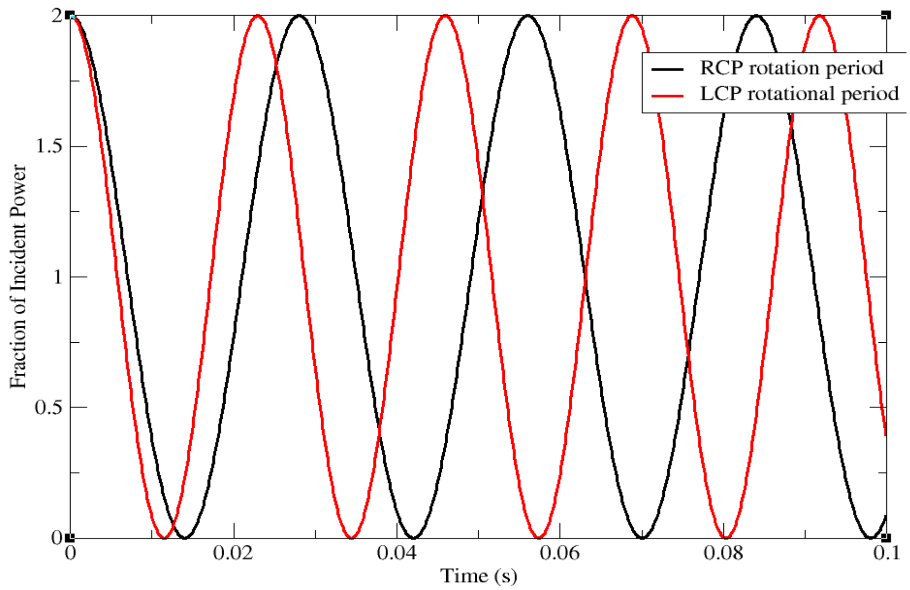

Figure 4.8: The power which would be measured through the plane polarizer after the beam passed through a birefringent particle

Chapter 5

Conclusion and Recommendation

5.1 Conclusion

A simple method of measuring the rotation speed and the optical torque applied to a laser trapped birefringent particle has been described. This method can be used even if the viscosity of the medium in which trapping is performed is unknown, and provides a means to measure this viscosity. Thus, this method is suitable for measuring an optical torque, which could be simply constructed by trapping a birefringent probe particle in the fluid of interest. A suitable test particle would be a small fragment of calcite, the exact shape not being critical at the very low Reynolds numbers encountered in these cases, or a more ideal shape could be fabricated from a birefringent polymer [11]. As the optical torque can be controlled by varying the power, the probe particle rotation speed can be varied, allowing, for example, the investigation of non-linear properties of the fluid. Wall effects, the size and the shape of the particles, and the property of a fluid doesn't affect the torque and the rotation speed of the particle. In this method the optical torque measurement depends on the power of the beam and its angular frequency. And this method helps to determine the viscosity of any liquid.

5.2 Recommendation

If one wants to determine the viscosity of any liquid with out any difficulty, one has to fulfil the following materials. This helps to eliminate the difficult part of the alignment.

- Stable set-up with appropriate parts is needed to minimize vibration and misalignment.
- Antireflection coated lenses are needed to reduce laser beam power by reflection.
- Better resolution and faster frame rate camera to see clearly the situation at the sample.
- A dichroic mirror to block out only the laser is needed to enhance illumination at the sample.
- A position sensitive detector (PSD) to calculate the speed and position of the trapped particles.
- Computer controlled moving stages [lab-microsecond precession] to have a better focus of the laser at the sample and for easier control of the experiment in general.

Appendix

```
c ****************************************************
program LCP beam frequency measurement
implicit none
c *****************************************************
c This program is used to determine the frequency of a rotating particle
c by applying a left circularly polarized beam
c ******************************************************
c Input Arguments:
c t - is the period of the oscillation of the out going beam
c a - is the the fraction of the power of the beam a=2Pm/P
c where - pm is the measured power of the out going beam
c Pm=[1+cos(2*om*t)]P/2
c -om is the frequency of the out going beam
c - P is the power of the laser beam
c ******************************************************
real::a,om,t,h
open(unit=1,file="ooo.dat",status="unknown")
h=0.0001
t=0
om=136.94
do
t=t+h
a=1+cos(2*om*t)
if(t >= 0.1)exit
!print*,t,a
write(1,*)t,a
```

```
end do
end program LCP frequency measurement
c ****************************************************************
program RCP frequency measurement
implicit none
c *************************************************************
c This program used to determine the frequency of a rotating particle
c by applying a right circularly polarized beam
c **************************************************************
c Input Arguments:
c t - is the period of the oscillation of the out going beam
c a - is the the fraction of the power of the beam a=2Pm/P
c where - pm is the measured power of the out going beam
c Pm=[1+cos(2*om*t)]P/2
c -om is the frequency of the out going beam
c - P is the power of the laser beam
c ************************************************************
real::a,om,t,h
open(unit=1,file="oo.dat",status="unknown")
h=0.0001
t=0
om=112.16
do
t=t+h
a=1+cos(2*om*t)
if(t >= 0.1)exit
!print*,t,a
write(1,*)t,a
end do
end program RCP beam frequency measurement
c **************************************************************
```

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DECLARATION

I here under signed declare that the thesis is my original work, has not been presented for a degree in any other university and that all sources of material used for the thesis have been dully acknowledged.

Name: Tamirat Abebe

Signature:——————————

This thesis has been submitted for examination with my approve as a university advisor.

Name: Dr. Araya Asfaw

Signature:——————————