STUDY ON THE BINDING ENERGY OF HYPERNUCLEI ${}^{9}_{\Lambda}Be$ AND RMS OF α-α CORE NUCLEI

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DECLARATION

I hereby declare that my thesis being entitled is my original work, had not been presented for degree in any other University. Sources of relevant materials taken from books and articles have been duly acknowledged.

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ABSTRACT

We present the results of variational Monte Carlo analysis of ${}^{9}_{\Lambda}Be$ binding energy. The hypernuclear system is treated as partially nine-body problem in the Λ-α-α cluster model. The potential parameters used are taken from the earlier work of s-shell hypernuclei. The binding energy of ${}^{9}_{\Lambda}Be$ is found to be sensitive to the choice of α-α potential. The root mean square separation between two-alpha is consistent with the assumption of α-cluster model for ${}^{9}_{\Lambda}Be$.

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1. Introduction

Hypernuclei are bound nuclear systems in which one or more nuclear constituents are hyperons apart from nucleons. The first hypernucleus was discovered by polish scientists Danysz and Pniewski [1] in 1953. The event observed is schematically shown in fig.1 and it was produced by interaction of high-energy cosmic rays with the emulsion. From analysis of range-energy relations it was found that thick track (AB in fig.1) tapering towards the end corresponds to a hypernucleus. The decay energy of the Λ° (lambda)- hyperon bound in the nucleus is transferred to the decay products of the hypernucleus. From the analysis of the parameters of tracks it was confirmed that the registered event is caused by the formation and decay of a system of boron nucleus in which one of neutrons is replaced by a Λ° -hyperon. Hypernucleus is denoted by symbol ${}^{A}_{\Lambda}Z$; A is total of (A-1) nucleons and a lambda particle and Z is charge of the nuclear core.

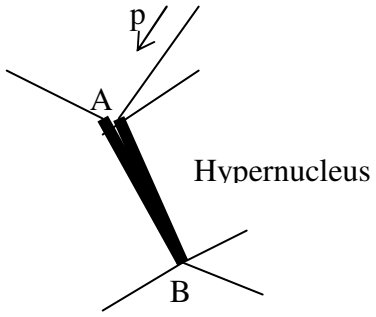

Figure 1: Formation and decay of a hypernucleus

The lightest amongst hyperons (Table 1) is the neutral Λ with strangeness S = -1 and spin = 1/2. It has a mass of 1115.6MeV, which is about 20 % larger than the mass of a nucleon.

Table 1:Some properties of hyperons along with their decay modes

Hyperon (mass: MeV)	Isospin (I,I_3)	Quark structure	Mean life time(sec)	Decay Mode(%)
$\Lambda(1115.6)$	(0,0)	sud	2.63×10^{-10}	$\rightarrow p\pi^-(64)$ $\rightarrow n\pi^0(36)$
$\Sigma^+(1189.4)$	(1,1)	suu	8.0×10^{-11}	$\rightarrow n\pi^+(48)$ $\rightarrow p\pi^0(52)$
$\Sigma^0(1192.5)$	(1,0)	sud	6.0×10^{-20}	$\rightarrow \Lambda\gamma(100)$
$\Sigma^-(1197.3)$	(1, -1)	sdd	1.48×10^{-10}	$\rightarrow n\pi^-(100)$
$\Xi^0(1318)$	(1/2, 1/2)	uss	2.9×10^{-10}	$\rightarrow \Lambda\pi^0(100)$
$\Xi^-(1315)$	(1/2, -1/2)	dss	1.64×10^{-10}	$\rightarrow \Lambda\pi^-(100)$
$\Omega^-(1672)$	(0,0)	sss	8.2×10^{-10}	$\rightarrow \Lambda K^-(67.8)$ $\rightarrow \Xi^-\pi^0(23.6)$

1.1 A Brief Review of the Literature

One identifies hypernuclei in emulsion by their decay modes, which occur predominantly in the ground state. In this way, ground state binding energies of light hypernuclear ($A \leq 15$) systems, together with decay modes bearing evidence on spins, have been established.

In the first two decades beginning 1953, emulsion experiments proved to be the only source of production and identification of hypernuclei. Due to the inherent limitation of the emulsion technique, the production and detection of Λ-hypernuclei and measurement of their ground and excited states energies for $A > 15$ could not be carried out. To understand the behavior of a Λ hyperon in a nucleus, systematic experimental data over the whole-mass range, especially ground and excited states of heavy hypernuclei are needed.

With the technological advancement in the K-meson accelerator and particles detection counter the above limitation of emulsion technique was over come. For the first time in 1973 at CERN experiments [2] with the K^- stopped in-flight have become a new promising tool of producing and measurement of the energies of ground and excited states of new hypernuclei. Basic mechanism is the strangeness exchange reaction (K^-,π^-). The

detection of outgoing pions with the help of spectrometer gives the information about the hypernucleus and its states. This led to renewed activities in producing new hypernuclei and setting of new facilities at BNL (USA), KEK (JAPAN) etc. to study many experimental properties, which earlier could not. The associated pair production (π^+, K^+) reaction also turned to be effective tool for spectroscopic study of heavy hypernuclei [3]. The usefulness of reaction was demonstrated experimentally at BNL and KEK where various excited states were resolved from ${}^{9}_{\Lambda}Be$ to ${}^{89}_{\Lambda}Y$. Combining these results with the previous Bubble Chamber experiments it has now become possible to clearly identify about 30 hypernuclei. Strangeness exchange process and associated pair production reaction responsible for productions of hypernuclei of not only Λ but also of other hyperons in ground and excited states are:

$$K^- n \rightarrow \pi^- \Lambda$$

$$K^- n \rightarrow \pi^- \Sigma^0$$

$$\pi^- p \rightarrow K^+ \Sigma^-$$

$$\pi^- n \rightarrow K^- \Lambda$$

$$\pi^+ p \rightarrow K^+ \Sigma^+.$$

The separation energy -B_Λ of Λ-particle in the Λ-hypernucleus is defined as energy required to separate Λ from the core nucleus provided it remains in the ground state. It is related to the masses M (${}^{A}_{\Lambda}Z$), M (${}^{A-1}Z$) and M (Λ) of the hypernucleus, core nucleus and Λ particle, respectively, through the following formula (c=1)

$$M\left({}^{A}_{\Lambda}Z\right) = M\left({}^{A-1}Z\right) + M(\Lambda) - B_\Lambda \ .$$

The Λ-hypernuclei with mass number A ≤ 5 are referred to as s-shell hypernuclei as their core nuclei belong to the s-shell. Those Λ-hypernuclear species whose baryon numbers lie in

the range 5<A<17 are known as p-shell hypernuclei since these consist of a Λ particle bound to p-shell core nuclei. Other hypernuclei (A>17) are called s-d shell.

Another quantity of interest related to hypernuclei is their lifetime (τ). It has been measured for a number of species. This lies in the interval $10^{-11} < \tau < 10^{-10}$ s. Lifetime is related to the decay modes of hypernuclei and are the ones induced by the following basic strangeness non-conserving weak decay:

$$\text{Mesonic decay: } \Lambda \rightarrow N\pi \qquad (1.1)$$

$$\text{Non- mesonic decay: } \Lambda \rightarrow NN \qquad (1.2)$$

Large fraction of decays of very light hypernuclei follows eq.(1.1) with the life times comparable to that of a free Λ-particle, namely 10^{-10}sec. In medium and heavy hypernuclei, the dominant mode is represented by the eq.(1.2). In these cases lifetime has the order of magnitude similar to that of light hypernuclei.

The hypernucleus lifetime indicates that on the time scale characteristics of strong interaction (10^{-23} sec), Λ-hypernuclei are considered stable in the sense of an ordinary nuclear system. During the average lifetime (~10^{-10}sec) of a hypernucleus, a Λ particle traverses a distance of about 10^{13} fm which is million million times nuclear diameter. Hence Λ may be considered to be trapped inside the hypernuclei. We can thus to a good approximation treat the ground and excited state levels of these systems as if these are bound.

The information about the Λ-particle and nucleons (ΛN) interaction comes mainly from studies of ground state binding (B_Λ) and excited states energies of Λ hypernuclei [Tables 2 and 3] many of which are now known with considerable accuracy over the almost whole periodic table. There are a few Λ-proton scattering [4] data points with large errors for total cross-section and forward to backward ratio with large uncertainties.

Table 2: B_Λ and spin values of available hypernuclei

Hypernuclear species(${}^{A}_{\Lambda}Z$)	${}^{5}_{\Lambda}He$	${}^{6}_{\Lambda}He$	${}^{7}_{\Lambda}Li$	${}^{7}_{\Lambda}Be$	${}^{8}_{\Lambda}He$	${}^{8}_{\Lambda}Li$	${}^{8}_{\Lambda}Be$
$B_\Lambda \pm \Delta B_\Lambda, MeV$	3.12± 0.02	4.25± 0.02	5.58± 0.03	5.16± 0.08	7.16± 0.70	6.80± 0.03	6.84± 0.05
spin(J)	1/2	-		1/2	-	1	-

Hypernuc-lear species $\left({}^{A}_{\Lambda}Z\right)$	${}^{9}_{\Lambda}Li$	${}^{9}_{\Lambda}Be$	${}^{9}_{\Lambda}B$	${}^{10}_{\Lambda}Be$	${}^{10}_{\Lambda}B$	${}^{11}_{\Lambda}B$	${}^{12}_{\Lambda}B$	${}^{13}_{\Lambda}C$	${}^{15}_{\Lambda}N$
$B_\Lambda \pm \Delta B_\Lambda$	8.53 ± 0.15	6.71 ± 0.04	7.88 ± 0.15	9.30 ± 0.26	8.89 ± 0.12	0.24 ± 0.05	11.7 ± 0.06	11.2 ± 0.08	13.9 ± 0.15
spin (J)	-	1/2	-	-	-	-	1	1/2	-

Table 3: Experimental values of single particle energy levels (in MeV) from various reactions. Notations A, B, and C stand for emulsion, (π^+, K^+) and (K^-,π^-) sources, respectively

Λ-hyper-nuclei	s-shell	p-shell	d-shell	f-shell
${}^{12}_{\Lambda}C$	10.2±0.7(A)			
${}^{12}_{\Lambda}C$	10.8±0.1(B)	0.1±0.2(C)		
${}^{13}_{\Lambda}C$	11.7±0.1(A)	0.8±0.5(C)		
${}^{16}_{\Lambda}O$	12.5±0.35(B)	2.5±0.5(C)		
${}^{28}_{\Lambda}Si$	16.0±0.3(B)	7.0±	-2.0	
${}^{32}_{\Lambda}S$	17.5±0.5(C)	8.1±0.6(C)	-1.0	
${}^{40}_{\Lambda}Ca$	18.7±1.1(B)	11.0±0.6(C)	1.0±0.5(C)	
${}^{51}_{\Lambda}V$	19.9±1.0(B)		4.0±0.5(C)	-5.0
${}^{89}_{\Lambda}Ye$	22.1±1.6(B)	16.0±1.0(B)	9.5±1.0(B)	2.5±1.0(B)

.

In a hypernucleus, the lambda is distinguishable from the nucleons, so it occupies lowest s-state in the Λ- nucleus potential without violating the Pauli principle. As the mass number of the hypernucleus increases the B_Λ grows monotonically reaching a saturation

value corresponding to very heavy hypernucleus. This is indication of the fact that the size of potential well increases and well depth reaches a saturation value which is a consequence of short-range behavior of ΛN forces.

The Λ-binding to an infinite nuclear matter (D_Λ) is another physical quantity of interest to theoretical physicists. Its value is determined from the extrapolation of a semi-empirical formula which fits the experimental B_Λ data of heavy hypernuclei. The radius $R(=r_0 A^{\frac{1}{3}})$ of the effective $\Lambda-$nucleus potential for a hypernucleus is very close to that of charge density of $\rho(r)$of the core. Using Uncertainty principle, kinetic energy of Λ-particle in the ground state of an infinite square well is given by

$$E_{kin} = < \frac{p^2}{2m_\Lambda} > \sim \left(\frac{\hbar\pi}{R}\right)^2 / 2m_\Lambda = \frac{\hbar^2\pi^2}{2m_\Lambda r_0^2} A^{-\frac{2}{3}},$$

which goes to zero with increasing A. In this simple picture the binding energy B_Λ and well depth D_Λ are related as

$$B_\Lambda \xrightarrow[A\to\infty]{} D_\Lambda - \frac{\hbar^2\pi^2}{2m_\Lambda r_0^2} A^{\frac{-2}{3}} \quad . \tag{1.3}$$

The formula [eq.(1.3)] is fitted to observed B_Λ values from around ${}^{13}_{\Lambda}C$ to heavy hypernuclei. The plot of B_Λ versus $A^{-\frac{2}{3}}$ is extrapolated to $A \to \infty$. The resulting intercept with the ordinate gives D_Λ which is regarded as empirical value of Λ-binding to infinite nuclear matter. The outcome of this fitting procedure gives the following value [5]

$$D_\Lambda \approx 30 \pm 3 MeV$$

with an upper limit of $D_\Lambda \leq 35 MeV$. Shoeb and Rahman Khan [6] has also developed a semi-empirical formula for B_Λ of hypernuclei which yields D_Λ=31.6MeV

Apart from hypernuclei of single Λ-particle, the hypernuclei [7,8,9] of double Λ and Ξ have been identified. Binding energies of these species are listed in Table 4 & 5. The existence of Σ hypernuclei has been subject of intense debate among experimentalist and theoretical physicists. At present there is a belief that Σ states [10] for A>5 will ever be seen. There is speculation that in future not only hypernuclei of Ω^-, charmed, beauty particles but also hypernucleui containing multi-strange ΞΞ, ΛΞ, $\Lambda\Lambda\Xi^-$…particles will also be discovered and their properties studied.

Table 4: Binding energies B_Ξ of Ξ-hypernuclei

Hypernuclear species ${}^{A}_{\Xi}Z$	${}^{8}_{\Xi}He$	${}^{11}_{\Xi}B$	${}^{13}_{\Xi}C$	${}^{15}_{\Xi}C$	${}^{17}_{\Xi}O$	${}^{28}_{\Xi}Al$
$B_\Xi \pm \Delta B_\Xi (MeV)$	5.9±1.2	9.2±2.2	18.1±3.2	16.00±4.7	16.00±5.5	23.2±6.8

Table 5: Binding energies $B_{\Lambda\Lambda}$ of ΛΛ-hypernuclei. Quantity with in round bracket in the second column is due to Prowse[8]

Hypernuclear species ${}^{A}_{\Lambda\Lambda}Z$	${}^{6}_{\Lambda\Lambda}He$	${}^{10}_{\Lambda\Lambda}Be$	${}^{13}_{\Lambda\Lambda}B$
$B_{\Lambda\Lambda} \pm \Delta B_{\Lambda\Lambda} (MeV)$	$7.25\pm0.19^{+0.18}_{-0.11}$ (10.8±0.6)	10.86±0.6	27.5±0.7

In the next pages we discuss the general methods that have been used to explore the nature of ΛN forces from the analysis of B_Λ data. A review article by Gal [5] has been quite helpful in this regard.

1.2 General methods of calculation of B_Λ

1.2.1 A rigid core approximation

In the rigid core approximation it is assumed that presence of Λ doesn't distort the nuclear core wave function ϕ_0. Consequently the total hypernuclear wave function for spin zero core is written as product of ϕ_0 and Λ-wave function $\phi(\vec{r})$, where $\vec{r}$ is the radius vector of the Λ with respect to center of mass the nuclear core. $\phi(\vec{r})$ is given by the solution of two-body $\Lambda-$nucleus Schrödinger equation

$$\left[-\left(\frac{\hbar^2}{2\mu_{\Lambda N}}\right)\Delta_r + V(r)\right]\varphi(r) = -B_\Lambda \varphi(r)$$

corresponding to the lowest eigenvalue for - B_Λ. The Λ-nucleus potential is obtained by folding two body ΛN potential $V_{\Lambda N}(\vec{r})$ with single particle nuclear density $\rho(r)$ as

$$V(r) = A\int \rho(r').V_{\Lambda N}(\vec{r}-\vec{r}').d^3\vec{r}',$$

where

$$\rho(r) = \int \left|\phi_0(\vec{r},\vec{r}_2,....,\vec{r}_A)\right|^2 d^3r_2...d^3r_A.$$

The rigid core model has mainly been applied for spinless-core hypernuclei, such as ${}^{5}_{\Lambda}He$, ${}^{7}_{\Lambda}Be$, ${}^{9}_{\Lambda}Be$ and ${}^{13}_{\Lambda}C$.

1.2.2 Variational Principle

A trial hypernuclear wave function Ψ is constructed which consists of variational parameters. The wavefunction must simulate both short-range correlations and the correct asymptotic behavior for each baryon in the system. The minimum energy $(\Psi, H\Psi)$ for the hypernucleus is obtained by varying the variational parameters. Likewise, the minimum of

$(\phi, H_N \phi)$ is found by varying parameters in the trial wave function ϕ of the core nucleus. The negative of the difference $(\Psi, H\Psi)$ - $(\phi, H_N \phi)$ give the binding energy B_Λ of the hypernucleus. A number of analyses [11,12,13] have been performed using this technique.

1.2.3 Cluster calculation

This method is applied to a hypernucleus under the condition that binding is small between two or more hypernuclear clusters compared to the separation energies involved with in each of these. By their nature these analyses [12,13] are confined to some isolated cases.

1.2.4 Hartree-Fock method

The self-consistent single-particle energies ε_i corresponding to the determinantal wavefunction ϕ are the observable energies of single nucleon in the nucleus. The solutions of the Hartree-Fock (HF) equations are not unique and, therefore, ϕ's have to be determined carefully. Different sets of solutions for the single-particle energies and wave functions are obtained. This leads to different HF wave functions ϕ corresponding to each a value of $<\phi \mid \mathrm{H} \mid \phi>$. The particular ϕ that leads to the lowest value of $<\phi \mid \mathrm{H} \mid \phi>$ is taken as the best HF solution for the ground state of the nucleus. The nuclear HF method has been extended to the hypernuclear systems. Here it is assumed that a set of single-particle orbitals for the nucleons and Λ can be found in a self-consistent way, namely that these orbitals for nucleons and Λ are eigenstates of a single-particle Hamiltonian [14] obtained by generating single-particle nucleon and Λ potentials from the same orbitals.

1.2.5 Shell Model Calculation

Shell model has been applied for p- shell hypernuclei. The core nuclei are described in terms of an intermediate coupling (between LS and jj coupling) model. Mainly two approaches have [5,15,16] been employed. In the first one [5,16] the Λ and nucleon wave functions are considered constant throughout the p-shell and hence the binding energy of all hypernuclei can be parameterized in terms of a relatively small number of reduced matrix elements of ΛN forces. The other one [15] is where Λ and nucleon wave functions vary from nucleus to nucleus throughout the p-shell.

In all the above mentioned methods attempts have been made to explore the nature of ΛN interaction from B_Λ data of Λ-hypernuclei. Unfortunately accurate scattering data are not available for the YN (Y stands for hyperon) system. Consequently reliable information cannot be extracted from these. Therefore, meson theoretical models developed by Nijmegen group [17,18] are very popular. These are based on one-meson exchange and take pseduoscalar, vector, and also scalar meson nonents into account. The models are augmented by pomeron and tensor meson exchange. The coupling constants with in these nonents are related by SU(3) symmetry and are mostly determined by fit to the NN and YN scattering data. Nijmegen group has provided a series of models SC 97 a-f, which give a very different spin-spin interaction.

As most of the YN models, Nijmegen group incorporates the strong conversion process of the ΛN to ΣN system explicitly. This is important because the conversion process is affected by the nuclear medium. The YN interaction is generally weaker than NN interaction leading to a core hyperon structure of the hypernuclei. In some cases $\Lambda - \Sigma$ conversion is suppressed if a change of the isospin of the core nucleus requires excitation.

The contribution of this process is much stronger than Δ-N conversion in ordinary nuclei; because it accounts for long-range part of the interaction, it is not suppressed in s- wave and because Λ-∑ mass difference is much smaller. As a result effective ΛN interaction requires strong three-baryon ΛNN interaction [11, 13, 19] and even higher order forces, which are quite unknown. On the other side an understanding of the medium dependence of the ΛN force provides insights into the interaction mechanism, which cannot be obtained from the study of the NN or even the YN system. Recently analyses [20] have been made to explain s-shell hypernuclear binding energy data, especially, the overbinding problem of ${}^{5}_{\Lambda}He$ including ΛN-ΣN channel explicitly. This is yet to be implemented for p-shell hypernuclei. Moreover, accurate Faddeeve-Yakubosky calculation for Λ separation energies of ground and exited states of ${}^{4}_{\Lambda}He$ and ${}^{4}_{\Lambda}H$ based on mesonic theoretic Nijmegen SC YN interactions shows that failure of these model in explaining the data.

The other approach is the interpretation of the binding energies of Λ hypernuclei in terms of reasonable phenomenological ΛN and ΛNN forces. By reasonable forces it is meant that forces are consistent with the general expectations of meson-exchange models. In this sprit Bodmer and Usmani [11,13] continued such efforts to obtain a consistent phenomenological description of a hypernuclear binding energies and low-energy Λp scattering in terms of reasonable phenomenological ΛN and ΛNN forces. Of course their potentials are to be considered as effective interactions. In particular, ΛNN forces are considered to be the result of eliminating ∑, Δ…degrees of freedom (from a coupled channel approach which includes these and which represents a more sophisticated level of phenomenology) to obtain a reduced description in terms of only Λ and nuclear degrees of freedom. They considered phenomenological ΛNN forces both of “dispersive” and two-pion-

exchange (TPE) type. A new feature of dispersive ΛNN forces was that they not only consider spin-independent forces but also spin dependent forces, depending on the nucleon spins, which are suggested by suppression mechanism due to ΛN-∑N coupling. Thus Bodmer and Usmani [11] were able to explain the B_Λ of the s-shell hypernuclei=3,4, 4^*, 5, where 4^* denotes the exited states of the A=4 hypernuclei p-shell hypernuclei, and also the Λ binding in nuclear matter (A=∞), i.e. the Λ well depth D_Λ using dispersive ΛNN force combined with two-body ΛN potential which fits scattering data. This method was extended with success to ΛΛ hypernuclei [13] in the cluster model approach.

1.3 Objective of the Analysis of the ${}^{9}_{\Lambda}$Be System

In this thesis we shall discuss the details of the analysis of B_Λ of ${}^{9}_{\Lambda}$Be system in the Λ-α-α model. A variety of α-α potentials available in the literature have been used to test the sensitivity of B_Λ on these. Detailed analysis of this system is essential as this will help us in extracting the information on ΛN spin-orbit force from the energy separation of 5/2-3/2 levels [21] observed in the spectra of ${}^{9}_{\Lambda}$Be. Moreover, information about ΛΛ interaction from the analysis of ${}^{10}_{\Lambda\Lambda}$Be will be influenced by the ${}^{9}_{\Lambda}$Be wave function.

We use variation Monte Carlo (VMC) methods to study the ${}^{9}_{\Lambda}$Be system. The separation energy of ${}^{9}_{\Lambda}Be$ (B_Λ=6.7±0.04) which has spinless p-shell core, is analyzed as a partially nine-body problem [12] in Λ-α-α model to a much more limited extent. Three types of α-α potential are used to explore the sensitivity of B_Λ data on these. The ΛN and ΛNN forces we use are central and so mostly are our NN forces for which we use Mafliet-Tjon (MT) potentials. Our wave functions include NN and ΛN correlation functions as three-body

ΛNN correlations are not found to be essential [19] to account B_Λ data of s-shell hypernuclei as emphasized earlier [11].

Organization of the present work is as follows. In the next chapter, which is divided into many sections, we discuss basics of Monte Carlo method and Metropolis algorithm. The Hamiltonian, two- and three-body potentials, two-and three-body correlations, the calculations and results for ${}^{9}_{\Lambda}Be$ are discussed in chapter 3. The conclusions of the analysis are presented in the last chapter.

2.The Monte Carlo Method

With the availability of fast computing machine and the advances made in the theoretical and computational aspects of the few-body systems it has become essential to develop numerical techniques to solve the equation of motion. One such technique is Monte Carlo. The technique of simulating many chances of experiment [22] and drawing inferences from the result of the simulated experiment is called the Monte Carlo method. In this chapter, we discuss some concepts basic to the Monte Carlo method and how this combined with Metropolis algorithm is used in the calculation of energy of a few-body system. An article by Gaurdiola [23] has been quite helpful in writing this chapter.

2.1 The Central Limit Theorem

The central limit theorem which is considered as corner stone of Monte Carlo method is stated as: calculate an average z of a large number of values $x_i (i = 1,2,3.....n)$ individually sampled from a probability distribution $f(x)$. Repeat this process many times and we get an ensemble $z_j (j = 1,2,3...n)$ of these averages. The values z_j so obtained have a Gaussian distribution. A non–rigorous proof of this theorem is given as below.

Let x is a random variable with probability distribution function $f(x)$. We construct a new variable z from random variables $x_1, x_2, ..., x_n$ as

$$z = \frac{(x_1 + x_2 + ... + x_n)}{n} \quad . \tag{2.1}$$

Probability distribution function in the new random variable z can be constructed using

$$g(z) = \int dx_1 dx_2 ... dx_n f(x_1) f(x_2) ... f(x_n) \delta\left(\frac{x_1 + x_2 + ... + x_n}{n} - z \right) \quad . \tag{2.2}$$

Making use of the integral representation of the delta function and changing the integration variables gives

$$g(z)=\frac{1}{2\pi}\int_{-.\infty}^{\infty}dt.e^{-.i(z-\mu).t}\left\{f(x).e^{\frac{it(x-\mu)}{n}}dx\right\}^{n} \quad . \tag{2.3}$$

If set of random variables n is very large, then we can write

$$\int f(x)\exp\left(\frac{it(x-\mu)}{n}\right).dx \approx \int f(x)dx\left\{1+\frac{it(x-\mu)}{n}-\frac{t^2(x-\mu)^2}{2n^2}...\right\} \tag{2.4}$$

$$\approx 1-\frac{t^2\sigma^2}{2n^2}. \tag{2.5}$$

Substituting this in equation (2.3) and integrating we get

$$g(z)=\frac{1}{\sqrt{2\pi}}\frac{1}{(\sigma/\sqrt{n})}\exp^{-\frac{(z-\mu)^2}{2(\sigma/\sqrt{n})^2}} \quad . \tag{2.6}$$

2.2 The Uniform Distribution $U(0,1)$

Usually it is a routine available in computer, which is based on a congruential relation

$$N_i=(aN_{i-1}+b)\,(\text{modulus M}), \tag{2.7a}$$

$$x_i=\frac{N_i}{M}\ . \tag{2.7b}$$

In the above relations N_i is an integer number, called a seed, and x_i, the random number in the interval (0,1). Note that 1 can never be reached, but 0 is included in the generation interval. The constants a, b and M are chosen so that the following three requirements are satisfied:

i. The resulting distribution of random number is uniform, and these are uncorrelated.

ii. The period of eq. (2.7) must be as long as possible, and

iii. The algorithm is fast.

A complete analysis of congruential methods may be found in the articles based on simulation. Currently the number M is determined by the number of bits of the integer word of the computer, i.e., the largest unsigned integer. In this form the modulus operation is automatically carried out by discarding the overflow digits in the previous integer operation. Analogously, the division N/M is never carried out, being just a shift of bits and the corresponding adjustment of the floating exponent.

The generation of uniformly distributed random numbers is important because it is used to generate random numbers corresponding to other probability distributions. In our problem Monte Carlo work can be carried out with the uniformly generated random numbers. We may remark that from x ε U(0,1) one can obtain z ε U(a, b) by the transformation

$$z = a + (b - a)x \tag{2.7c}$$

2.3 Crude Monte Carlo Quadrature

To begin with we will discuss as an example the computation of one-dimensional integral for any arbitrary function $h(x)$:

$$I = \int_a^b h(x)dx \tag{2.8}$$

using random numbers.

Let $(x_1, x_2, ..., x_n)$ be a set of random numbers generated from the relation (2.7c) and uniformly distributed in the interval (a,b). Using central limit theorem we may write integral (2.8) as

$$I = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} h(x_i) \ . \tag{2.9}$$

The variance in the integral is determined by

$$\sigma^2 = \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} h^2(x_i) - I^2 \ . \tag{2.10}$$

Several techniques have been developed to reduce the variance of crude Monte Carlo method and one of these is importance sampling.

2.4 Importance Sampling

The main idea of importance sampling technique is to modify the function to be sampled as well as the sampling algorithm, so as to significantly decrease the variance of the resulting function to be integrated. To achieve this objective we choose a probability distribution function $f(x)$ defined in the interval (a,b) of the quadrature. With this choice eq. (2.8) may be modified as

$$\mathrm{I} = \int_a^b (h(x)/f(x)) f(x) dx \ . \tag{2.11}$$

The Monte Carlo estimate of the integral can be done by drawing random numbers from the probability distribution $f(x)$ and computing the average

$$\mathrm{I} = \lim_{n\to\infty} \frac{1}{n} \sum (h(x_i)/f(x_i)) \ . \tag{2.12}$$

The new value of the variance is

$$\sigma^2 = \int_a^b (h(x)/f(x))^2 f(x) dx - I^2 \ . \tag{2.13}$$

A small variance may be achieved with a proper choice of $f(x)$. Hence the extra work required by the new formulae (2.12) may be compensated by the need of a smaller number of samples.

2.5 Markov Chains

The main aim of the importance sampling technique is to find and use the importance sampling function. The importance sampling function $f(x)$ is chosen in such a manner that it should be able to mimic the function $\mathrm{h}(x)$ to be integrated and thus lowering the variance. In many cases, the choice of importance sampling function is dictated by nature of the physical problem that we are interested in solving. For example in the case of the variational few-body problem, the importance sampling function is chosen as the trial wave function squared (to be discussed in chapter III). However, there still remains the problem of sampling from a normally very complicated distribution function. To solve it we use a stochastic algorithm, i.e., a random walk in the multivariate coordinate space, which generates asymptotically the desired set of random numbers. This algorithm is known with the name of Metropolis.

The random walk is known as Markov chain or Markov process. We consider a system, which can be in a discrete set of states $S_1, S_2, \ldots S_N$. The evolution of the system is governed by a clock, so that at every jiffy the system jumps from its current state to any other state of the set, itself included. This jump is characterized by a transition probability matrix p_{ij}, which represents the probability of jumping from the state i to any other state j. Note that p_{ii} is also included, and this matrix elements must fulfill the following conditions:

$$0 \leq p_{ij} \leq 1 \ , \tag{2.14}$$

$$\sum_j p_{ij} = 1 \ . \tag{2.15}$$

In addition it is required that the transition from state i to j is independent of the previous occupied states during the evolution. A matrix which satisfies both eq.(2.14) and (2.15) is called a stochastic matrix, and the process related to it is called a Markov chain.

The generalization from discrete to the case of the continuum states is called a Markov process. We use now x to label the states, and define a transition density $p(x,x')$ with the properties:

$$p(x,x') \geq 0 \tag{2.16}$$

$$\int p(x,x')dx' = 1 \quad . \tag{2.17}$$

The probability p_k of having passed to some state S_k, (assume we carry out n jumps, and let n_k be the number of stops to S_k) is defined as $p_k = \lim_{n\to\infty} \frac{n_k}{n}$,

provided the following conditions are fulfilled:

(i) The random walk must be endless.

(ii) The chain is irreducible.

The algebraic determination of the probability p_k is quite simple. The probability p_i of arriving to a state S_i is the sum of product p_k, the probability of being previously in another state S_k times the transition probablity, i.e.

$$p_i = \sum_k p_k p_{ki} \quad . \tag{2.18}$$

This is a set of homogeneous linear equations, which are not independent, as it can be checked by summing up over the free index i. It should be supplemented by the normalization condition $\sum p_i = 1$.

2.6 Metropolis Algorithm

Let q_{ij} be a stochastic and symmetric matrix, other wise arbitrary. Then our random walk matrix is given by

$$p_{ij} = q_{ij} \quad \text{, if } p_i > p_j \text{ ,} \quad \text{i≠j} \tag{2.19}$$

$$p_{ij} = q_{ij} \frac{p_j}{p_i} \text{, if } p_i \le p_j, \quad i \ne j \tag{2.20}$$

$$p_{ij} = q_{ii} + \sum_k {}' q_{ij} \left(1 - \frac{p_k}{p_i}\right), \quad \text{i=j} \tag{2.21}$$

The prime on the summation symbol means that only states, which satisfy $p_{k,} > p_i$ are considered. Metropolis algorithm can be generalized to the case of a continuous probability distribution $p(x)$. The move corresponding to the auxiliary matrix q_{ij} can now be carried out by means of a uniformly distributed displacement with in a segment line of length D centered at the current position x. The algorithm is as follows. Assume we are at point x, then

i) Try a move to x' = x + D (2z-1), Where z is from U(0,1).

ii) If $p(x') > p(x)$ then go to step (iv).

iii) If we arrive here is because $p(x')$ is smaller than p(x).

Then sample a new random number z from U(0,1) and do the following test:

If $\frac{p(x')}{p(x)}$ > z then go to (iv), or else

Put x'=x, i.e., remain at the old position.

iv) put x' in the list of random numbers, rename x' to x and go back to step (i).

The distance D is arbitrary, the only condition on the auxiliary matrix q being its symmetry. For very small D, the number of acceptances in steps ii) and iii) is large, but a large number of moves is necessary to cover the space and the successive random numbers are strongly correlated. On the contrary, when D is large the number of rejection is large, so that we may spend a long time at every position. It is usually admitted that the appropriate value of D should give a ratio of acceptances between 50% and 70%.

The most common problem in physics involves a probability distribution, which depends on the coordinates of A particles, i.e., 3A variables. The random walk is usually carried out in A steps, moving in turn each particle (not necessarily in the same order), and accepting or rejecting the individual motions with the usual test described above. After all particles have been moved, the resulting point is the new multidimensional random coordinates. It may happen that some of the new coordinates may have the same values. Metropolis algorithm is repeated a number of times before accepting the new point for sampling.

3. Variational Monte Carlo Analysis of B_Λ of $^{9}_{\Lambda}Be$

In a series of papers, Bodmer and Usmani [11,13,24] claimed to have solved overbinding problem of s-shell hypernuclei, explained the binding energies of α- clusters hypernuclei $^{9}_{\Lambda}Be$ and $^{13}_{\Lambda}C$, and Λ-binding to infinite nuclear matter. A phenomenological central spin and state-dependent Urbana type ΛN potential which fits Λp low energy scattering data and strongly repulsive ΛNN interaction of dispersive nature with or without Λ-spin dependence were used in the variational Monte Carlo (VMC) analyses. The variational wavefunction of the system were constructed as the product of appropriate two- and three-body correlations function. Two-body correlation functions (ΛN, NN and αα) were calculated using the procedure developed by the Urbana group [25]. The phenomenological dispersive ΛNN force was motivated from the NNN force used by Urbana group in the binding energy analyses [25] of s-shell nuclei and nuclear matter.

Recently Shoeb et al [19] re-examined the problem of overbinding of $^{5}_{\Lambda}He$ using a new form of dispersive, spin-dependent and non-central ΛNN force [26]. A detailed VMC analysis shows that freedom in adjusting the strength of the dispersive force can be used to resolve the overbinding problem with two-body correlations alone. Consequently, the ambiguity in the strength of dispersive ΛNN force masks the effect of 2π-exchange ΛNN interaction and three-body ΛNN correlation on the s-shell data. This work motivated Shoeb [27] to investigate the same old problem of overbinding using phenomenological dispersive spin-dependent ΛNN force used by Bodmer and Usmani [24]. VMC calculations were performed which included the B_Λ values of A=3,4,4^* (excited state) and 5 hypernuclei. The result supported the conclusions of the earlier work [19].

The conclusions summarized in the foregoing paragraph encouraged us to analyze the B_Λ data of p-shell hypernucleus ${}^{9}_{\Lambda}\mathrm{Be}$ to see whether the two-body correlations and dispersive ΛNN interactions as used in s-shell study [27] can be used to explain the B_Λ of this p-shell system as well. The forthcoming discussion is divided into many subsections where discussions on Hamiltonian, potentials, wave functions, energy calculations and result for ${}^{9}_{\Lambda}\mathrm{Be}$ will be presented.

3.1 General Hamiltonian for A-Baryon Hypernuclear System

Hypernucleus of mass number A consists of (A-1) nucleons and a Λ particle. The hypernuclear Hamiltonian H_H^A for A particles system, in general, can be written as the sum of H_N^{A-1}, Hamiltonian of the (A-1) nucleons of the core nucleus ground state and Λ particle Hamiltonian H_Λ:

$$H_H^A = H_N^{A-1} + H_\Lambda . \tag{3.1}$$

The nuclear Hamiltonian, H_N^{A-1} is given by

$$H_N^{A-1} = \sum_{i=1}^{A-1} T_N(i) + \sum_{i<j}^{A-1} V_{ij} + \sum_{i<j<k}^{A-1} V_{ijk}$$

$$= -\sum_{i=1}^{A-1} \frac{\hbar^2}{2m_i}\Delta + \sum_{i<j}^{A-1} V_{ij} + \sum_{i<j<k}^{A-1} V_{ijk} , \tag{3.2}$$

where V_{ij} and V_{ijk} are the two-nucleon NN and three-nucleon NNN potentials, respectively, and m_i is the mass of the nucleon.

The lambda Hamiltonian, H_Λ is given by

$$H_\Lambda = T_\Lambda + \sum_{i=1}^{A-1} V_{i\Lambda}(i\Lambda) + \sum_{i<j}^{A-1} V_{ij\Lambda}$$

$$= -\frac{\hbar^2}{2m_\Lambda}\Delta + \sum_{i=1}^{A-1} V_{i\Lambda}(i\Lambda) + \sum_{i<j}^{A-1} V_{ij\Lambda}\ , \qquad (3.3)$$

where $V_{i\Lambda}$ and $V_{ij\Lambda}$ are the two-body ΛN and three-body ΛNN potentials, respectively, m_Λ is the mass of the Λ particle. We now specialize to the system of interest and write the Hamiltonian for ${}^{9}_{\Lambda}Be$. It is treated as a partially nine-body problem with in the $\Lambda - \alpha - \alpha$ model as was done by Shoeb et al [12]. Its Hamiltonian is given by

$$H_H^{(A)} = H_N^{(A-1)} + T_\Lambda(r_\Lambda) + \left[\sum_{i=1}^{4} V_{\Lambda N}(r_{i\Lambda}) + \sum_{1=i<j}^{4} V_{\Lambda NN}(r_{ij\Lambda})\right] + \left[\sum_{i=5}^{A-1} V_{\Lambda N}(r_{i\Lambda}) + \sum_{5=i<j}^{A-1} V_{\Lambda NN}(r_{ij\Lambda})\right] + \sum_{\alpha_1}^{4}\sum_{\alpha_2}^{A-1} V_{\Lambda NN}(r_{\alpha_1\alpha_2\Lambda}) \quad , \qquad (3.4)$$

where square brackets represent the potential energy of Λ-α system and last term is the interaction energy of dispersive three-body ΛNN force, where in the triad a nucleon is taken from each alpha.

3.2Two-and Three-Body Potentials

For analyzing any nuclear many-or few-body systems we ought to know baryon-baryon interaction. The interactions chosen in this work, for computational ease, are simple and phenomenological and give good fits to the low-energy baryon-baryon scattering data. We below discuss the ΛN, NN, αα, and ΛNN potentials relevant for the present study.

3.2.1 Two-body ΛN, NN and αα Potentials

A) ΛN **potential**

The two-pion exchange (TPE) is the dominant part of the ΛN potential, which in turn is mainly determined by the strong tensor one-pion exchange (OPE) component acting twice. We use the central spin-dependent and space-exchange Urbana type ΛN potentials which have a theoretically reasonable attractive tail due to the TPE. It has the following form

$$V_{\Lambda N} = (1 - \varepsilon + \varepsilon P_X)\tilde{V}^0_{\Lambda N}, \tag{3.5}$$

where

$$\tilde{V}^0_{\Lambda N} = V_{2\pi} = V_0 - \left[\overline{V} - \frac{1}{4}V_\sigma(\vec{\sigma}_\Lambda.\vec{\sigma}_N)\right]T^2_\pi, \tag{3.6}$$

where $\overline{V}$ and V_σ are the spin-average and spin-dependent strengths, respectively. P_x is the Majorana space-exchange operator for Λ and nucleon. Parameter ε determines the importance of space-exchange potential. V_O is a fixed Woods-Saxon repulsive core, which is given

$$V_O(r) = W_o\left[1 + \exp\left[\frac{r-R}{d}\right]\right]^{-1} \tag{3.7}$$

with W_0 = 2137 MeV, R= 0.5fm, d = 0.2fm. T_π is the one-pion exchange tensor potential shape modified with a cut of,

$$T_\pi(r) = \left(1 + \frac{3}{\mu r} + \frac{3}{(\mu r)^2}\right)\frac{\exp(-\mu r)}{\mu r}\left(1 - \exp\left(-cr^2\right)\right)^2 \quad . \tag{3.8}$$

with μ=0.7fm^{-1} and the cut-off parameter c=2fm^{-2}. The exchange parameter (ε) is quite poorly determined from the Λp scattering data and is found to be ≈ 0.1-0.38. We take its (ε) mean value 0.25 as has been done in the past [19,24]. $V_{\Lambda N}$ parametrized in terms of $\overline{V}, V_\sigma$ which in turn are related to the singlet (V_s) and triplet (V_t) strengths of ΛN potential and are given by

$$\overline{V} = \frac{1}{4}V_S + \frac{3}{4}V_t \quad ; \quad V_\sigma = V_S - V_t. \tag{3.9}$$

The strength of the spin-average ΛN potential, which is consistent with Λp scattering, determined accurately [11, 24] and has a value

$$\overline{V} = (6.15 \pm 0.05) MeV \, .$$

For hypernuclei with zero-spin core nuclei, such as ${}^{9}_{\Lambda}Be$, the contribution to potential energy arises from the spin-average, $\overline{V}$; the spin component effectively contributes nothing.

B) N-N potential

For the N-N pair, we use the central, spin-isospin independent Mafliet-Tjon potential [24,25] that fits low energy NN data and has the form

$$V_{NN}(r) = \frac{\hbar c}{r}\left[7.39.e^{-3.11r} - 2.39e^{-1.55r}\right]. \qquad (3.10)$$

This potential, besides being simple, gives ground-state binding energy and rms radius of ^{4}He nucleus in reasonable agreement with experiment. The effect of the coulomb interaction is small and is neglected here as was done earlier.

C) $\alpha - \alpha$ Potentials

Since we shall not be considering the nucleon-nucleon interaction between the two-nucleon taken one from each alpha particle in ${}^{9}_{\Lambda}$Be in Λ-α-α model, we have to take α-α interaction into account to correctly reproduce energy of the core nucleus in the ${}^{9}_{\Lambda}Be$ system. We have chosen the following three types of phenomenological α-α potentials available in the literature [28-30] to test the sensitivity of B_Λ on these. For the ground state of ${}^{9}_{\Lambda}Be$, it is sufficient to use α-α potential only for orbital angular momentum l=0 because for all higher l contribution to B_Λ is found to be small [13].

1) Chien-Brown $\alpha - \alpha$ Potential [28].

For the Lth partial wave this potential.

$$V_{\alpha\alpha}^{(L)} = W_{\alpha\alpha}^{(L)} + V_{coul} \, , \qquad (3.11)$$

V_{coul} is the coulomb potential appropriate to Gaussian density and is given

$$Vcoul = \frac{4e^2}{r}\Phi\left[\left(\frac{2\alpha}{3}\right)^{\frac{1}{2}} r\right], \quad \text{with}$$

$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2} dt\,,$$

where $\alpha = 0.514$ fm^{-2}.The nuclear part of the potential is

$$W_{\alpha\alpha}^{(L)}(r) = V_L e^{-\mu_L r^2} + V_D(r)$$

in which V_L and μ_L of the repulsive part are L dependent, and V_L=0 for L $\geq$4.The direct part of α-α potential $V_D(r)$ (attractive part) is obtained by double-folding of a nucleon-nucleon Yukawa potential -(V_L / βr)exp$(-\beta r)$ with ρ_α as

$$V_D(r) = \int \rho_\alpha(r_1)\rho_\alpha(r_2)\left(\frac{-V_L}{\beta.r}\exp(-\beta r)\right)\delta(\vec{r}_2 - \vec{r}_1 - \vec{s} + \vec{r})d\vec{r}_1 d\vec{r}_2 d\vec{s}\,,$$

where $\vec{r}_1$ and $\vec{r}_2$ are measured from the center-of-mass of each α-particle and $\vec{s}$ connects the center-of-mass of two-alpha particle. The repulsive core potential parameters V_L and μ_L are dependent on the angular momentum L. The parameters of $W_{\alpha\alpha}^{(L)}$ were obtained from a fit to α-α scattering data and are given in Table 6.

Table 6: Potential parameters for Chien and Brown [28] obtained from a fit to α-α scattering data

L	V_0(MeV)	β(fm^{-1})	V_r(MeV)	μ_r(fm^{-1})
0	85	1.35	$287.5\pm^{30}_{26}$	0.635±0.015
2	85	1.35	$130.0\pm^{18}_{20}$	0.620±0.02

2) Ali-Bodmer $\alpha-\alpha$ potential[29]

The phenomenological potential V^L(r) to analyse α-α scattering data for L_{th} partial wave was chosen in the form

$$V^L(r) = V_R^{(L)} e^{-(\mu_R^L)^2 r^2} - V_A e^{-(\mu_A)^2 r^2} \tag{3.12}$$

where V_R^L and μ_R^L are parameters for the repulsive part and V_A and μ_A for the attractive part. The parameters which fits the scattering data are listed in Table 7.

Table 7:Potential parameters of Ali and Bodmer [29] from fit to α-α scattering data

L	$V_R^{(L)}(MeV)$	$\mu_R^L(fm^{-1})$	$V_A(MeV)$	$\mu_A(fm^{-1})$
0	475.0	0.7	130.0	0.475
2	320.0	0.7	130.0	0.475

3) Darriulat et al α -α potential[30]

α -α potential was chosen as combination of repulsive Saxon-Woods shape followed by another of attractive nature. It had six parameters and is given as

$$V^L(r) = U_R^{(l)} \left[1 + e^{\frac{(r - r_R^L)}{a_R^L}} \right]^{-1} - U_A^{(L)} \left[1 + e^{\frac{(r - r_A^{(L)})}{a_a^{(l)}}} \right]^{-1}, \tag{3.13}$$

where $U_R^L, U_A^L, r_R^L, r_A^L, a_R^L$ and a_a^l are parameters. The values of these parameters from a fit to experimental α-α scattering data is listed in Table 8

Table 8: Potential parameters for Darriulat et-al [30] obtained from a fit to the experimental data for $\alpha - \alpha$ scattering

L	$U_R^L(MeV)$	$r_R^L(fm)$	$a_R^L(fm)$	$U_A^L(MeV)$	$r_A^L(fm)$	$a_A^L(fm)$
0	150.0	1.65	0.10	9.2	3.72	0.40
2	150.0	1.63	0.05	16.0	3.55	0.30

3.2.2 Three-Body ΛNN Potentials

From the earlier studies on hypernuclei it was realized that two-body ΛN interactions which fits the Λp scattering data were not sufficient to explain the binding energies of s-shell hypernuclei and over binds the Λ in heavy hypernuclei and infinite nuclear matter. Dispersive type ΛNN potentials [13,19, 24, 26] have been introduced to resolve the over

binding. In the literature two types of ΛNN potentials have been used. First one arises from projecting out ∑, Δ, etc, degrees of freedom from coupled channel formalism. These are termed as dispersive potentials. The second type arises due to the exchange of 2π (TPE) between Λ and two-nucleon and is termed as genuine three-body ΛNN potential. Below we discuss these two types of potential. Although we shall be using central dispersive ΛNN force but for the sake of completeness we would like to briefly discuss the forces currently available in the literature.

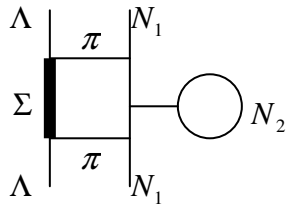

Figure 2: Dispersive ΛNN potential

a) Dispersive ΛNN forces.

Dispersive ΛNN forces, which have been used in the analysis of B_Λ data, are of two kinds. These forces are characterized by very different spatial, spin and isospin behaviors which we below:

i) Central dispersive ΛNN force

This force which we shall be using in our analysis of ${}^{9}_{\Lambda}Be$ is associated with suppression of the TPE ΛN potential arising from modification ("dispersion") of the intermediate $\sum, \mathrm{N}, \ldots$ by the medium (a " 2nd" nucleon N_2) as shown in the above figure. These types of phenomenological dispersive ΛNN potentials have the form [13,24]:

$$\text{Spin-independent: } V^{D}_{\Lambda NN} = W T^{2}_{\pi}(r_{\Lambda 1}) T^{2}_{\pi}(r_{\Lambda 2}) \tag{3.14}$$

and

$$\text{Spin-dependent: } V^{DS}_{\Lambda NN} = V^{D}_{\Lambda NN}\left[1 + \frac{1}{6}\vec{\sigma}_{\Lambda}.(\vec{\sigma}_1 + \vec{\sigma}_2)\right], \tag{3.15}$$

where $\vec{\sigma}_i$ (i=1,2) spin operator for i th nucleon and $\vec{\sigma}_\Lambda$ corresponding operator for Λ particle and for strength parameter W>0 the contribution of these forces by design is repulsive for all distances and ΛN correlations included in the wave function. $V^{D}_{\Lambda NN}$ and $V^{DS}_{\Lambda NN}$ are equivalent for spin-zero core nuclei (e.g. ${}^{5}_{\Lambda}He, {}^{9}_{\Lambda}Be$). The spin- dependent form $V^{DS}_{\Lambda NN}$ is obtained by assuming that dispersive (suppressive) modification act only for triplet ΛN_1 states, and then symmetrizing between N_1 and N_2. It is this later force (3.15), employed by Shoeb [27] in the analysis of s-shall hypernuclei, and shall be used in the present work.

ii) Non-central dispersive ΛNN force

Figure 3:(a) and (b) pion-exchange diagrams generating dispersive ΛNN interactions. Wavy lines denote the one-pion exchange.

Gal [26] has derived a dispersive spin-isospin dependent, and non-central ΛNN force This interaction represents the effect of the nuclear medium via third baryon on the two-body ΛN interaction [fig.3]. Propagation of ∑N pair in the nuclear medium generates this force. The potential when restricted to s-shell hypernuclei has the form

$$V^{DSN}_{\Lambda NN} = WY(r_{1\Lambda})Y(r_{2\Lambda})\{Y(r_{1\Lambda})T^2(r_{1\Lambda})[T(r_{2\Lambda})(3Cos^2\theta_{1\Lambda 2}-1)-1] + Y(r_{2\Lambda})T^2(r_{2\Lambda})[T(r_{1\Lambda})(3Cos^2\vartheta_{2\Lambda 1}-1)-1]\}\frac{1}{9}\vec{\tau}_1.\vec{\tau}_2(\vec{\sigma}_1.\vec{\sigma}_2+\vec{\sigma}_\Lambda.\vec{S}_{12}) \quad , \qquad (3.16)$$

where angle $\theta_{i\Lambda j}$ is angle between the radius vector joining the pair (iΛ) and (jΛ), the tensor radial shape T(r) and Yukawa function Y(r) modified with cut off are

$$T(r)=\left[1+\frac{3}{x}+\frac{3}{x^2}\right]\left(1-e^{-\hat{c}r^2}\right),$$

$$Y(r)=\frac{e^{-x}}{x}\left(1-e^{-\hat{c}r^2}\right),$$

with x = m_π r and $\vec{S}_{12}=(\vec{\sigma}_1+\vec{\sigma}_2)/2$.The expectation values of the spin-isospin factor

$$\frac{1}{9}\sum_{i<j}^{A-1}\left[\vec{\tau}_1.\vec{\tau}_2\left(\vec{\sigma}_1.\vec{\sigma}_2+\vec{\sigma}_\Lambda.\vec{S}_{12}\right)\right] \tag{3.17}$$

for a s-shell hypernuclei is known and is tabulated [26].

b) Two pion exchange ΛNN force

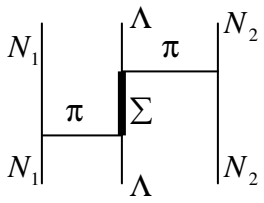

Figure 4:TPE ΛNN potential

This type of three-body ΛNN potential [31] arises from two-pion exchange, appropriate to a p-wave pion interaction of Λ with the i_{th} and j_{th} nucleons It has the form

$$V_{\Lambda NN}^{2\pi}=-\left(\frac{C_P}{6}\right)(\vec{\tau}_i.\vec{\tau}_j).\{X_{i\Lambda},X_{j\Lambda}\}.Y(r_{i\Lambda})Y(r_{j\Lambda}) \quad , \tag{3.18}$$

where $X_{K\Lambda}$, the one- pion–exchange operator for k th nucleon and Λ particle, is given by

$$X_{K\Lambda}=(\vec{\sigma}_K.\vec{\sigma}_\Lambda)+S_{K\Lambda}(r_{K\Lambda})T_\pi(r_{K\Lambda}) \tag{3.19}$$

with tensor operator for particles K and Λ

$$S_{K\Lambda}(r_{K\Lambda})=\frac{3(\vec{\sigma}_K.\vec{r}_{K\Lambda})(\vec{\sigma}_\Lambda.\vec{r}_{K\Lambda})}{r_{K\Lambda}^2}-\vec{\sigma}_K.\vec{\sigma}_\Lambda \quad , \tag{3.20}$$

and $\{A,B\} = AB + BA$,

where $\vec{\sigma}_K and \vec{\tau}_K$ are the spin and iso-spin Pauli operators, respectively, for K th particle.

3.3 Wave function of ${}^{9}_{\Lambda}Be$

Most of the effort in variational calculation goes into constructing a good trial wave function and it should contain the physics necessary to describe the ground state and be reasonably efficient to compute. Since we are using simple central ΛN [eq.3.5], NN [eq.3.10], and α-α [eqs.3.11-3.13] potentials, therefore, the wave function for ${}^{9}_{\Lambda}Be$ is the product of central two-body correlation functions alone. We are considering ${}^{9}_{\Lambda}Be$ as a partially nine-body problem in the $\Lambda - 2\alpha$ cluster model. A general trial wave function including three-body ΛNN correlation and has the following form

$$\Psi^{(9)} = \left\{\prod_{i=1}^{4} f_{\Lambda N}(i\Lambda)\prod_{i<j}^{4} f_{NN}(ij)\right\}\left\{\prod_{i=5}^{8} f_{\Lambda N}(i\Lambda)\prod_{i=5<j}^{8} f_{NN}(ij)\right\}\left\{\prod_{i<j}^{8} f_{\Lambda NN}(ij\Lambda) f_{\alpha_1\alpha_2}\right\}\chi^{(9)} \quad ,(3.21)$$

where $\chi^{(9)}$ is the appropriate spin functions. The two-body correlation functions $f_{\Lambda N}$, f_{NN} and $f_{\alpha_1\alpha_2}$ are spin-independent and are obtained with the procedures developed by the Urbana group. A Schroedinger type equation, which contain effective potentials through which the variational parameters enter is solved for each baryon-baryon pair. This equation has the form

$$\left[-\frac{\hbar^2}{2\mu_{BN}}\vec{\nabla}^2 + V_{BN} + \Lambda_{BN}\right] f_{BN} = 0, \qquad (3.22)$$

where B= Λ or N and BN=αα for two alphas and μ_{BN} is the reduced mass of the pair BN. For the hypernucles of mass number A we use for calculating the ΛN correlation $f_{\Lambda N}$, the potential

$$V_{\Lambda N} = V_C - sV_A T_\pi^2, \tag{3.23}$$

where V_A is the effective spin-average strength $V_{\Lambda N}$ for the mass number A given in terms of V_0 and V_σ. The variational parameter 's'(denoted by $s_{\Lambda N}$) is mostly used only when $V_{\Lambda NN}$ is included and is a convenient way of allowing for the effect of ΛNN forces on the two-body correlation functions $f_{\Lambda N}$; however sometimes it provides convenient additional flexibility to that provided by the parameters in Λ_{BN}. The auxiliary potential is given by

$$\Lambda_{BN} = -\frac{\hbar^2}{2\mu_{BN}}\left[\kappa_{BN}^2 - 2\frac{\kappa_{BN}(\nu_{BN}-1)}{r} - \frac{\nu_{BN}(\nu_{BN}-1)}{r^2}\right]\left(1 - e^{-\frac{r^2}{C_{BN}^2}}\right) +$$

$$\gamma_{BN}\left[1+\exp\{(r-R_{BN})/a_{BN}\}\right]^{-1},$$

where $\kappa_{BN}, C_{BN}, R_{BN}$ and a_{BN} are variational parameters. γ_{BN} is an eigenvalue parameter determined by matching logarithmic derivatives at some suitable distance 'r'. The form of Λ_{BN} is such that $f_{\Lambda N}, f_{NN}$ and $f_{\alpha\alpha}$ have the form of asymptotic behavior required by the full A-body Schroedinger equation, namely

$$f_{BN} \cong r^{-\nu_{BN}} \exp(-\kappa_{BN} r), \tag{3.24}$$

with the appropriate products of the f's then having the asymptotic behavior

$\sim r^{-1} \exp(-\kappa_B r)$, if the ν_{BN} are chosen appropriately ($\nu_{NN} = 0.54166$, $\nu_{\Lambda N} = 0.875$,and $\nu_{\alpha\alpha} = 0.51$ for ${}^{9}_{\Lambda}\mathrm{Be}$). Three-body ΛNN correlations $f_{\Lambda NN}$ of the forms [19,24,27,] have been used in the literature.

$$f_{\Lambda NN}^{D} = 1 - \alpha \tilde{Y}(r_{1\Lambda})\tilde{Y}(r_{2\Lambda}), \tag{3.25}$$

$$f_{\Lambda NN}^{NDS} = 1 - \alpha\left[\tilde{Y}(r_{1\Lambda}) + \tilde{Y}(r_{2\Lambda})\right]\left(3Cos^2\theta - 1\right)\tilde{Y}(r_{1\Lambda})\tilde{Y}(r_{2\Lambda}), \tag{3.26}$$

$$f_{\Lambda NN}^{2\pi} = 1 - \beta\left(3\cos^2\theta - 1\right)\tilde{Y}(r_{1\Lambda})\tilde{Y}(r_{2\Lambda}), \tag{3.27}$$

where $f^{D}_{\Lambda NN}$ is appropriate for $V^{D}_{\Lambda NN}$ and $V^{DS}_{\Lambda NN}$, $f^{NDS}_{\Lambda NN}$ for $V^{NDS}_{\Lambda NN}$ and $f^{2\pi}_{\Lambda NN}$ for $V^{2\pi}_{\Lambda NN}$. $\tilde{Y}(r)$ is the Yukawa functions with the range and cut- off parameters $\overline{\mu}$ and $\overline{c}$, respectively. The $\overline{\mu}, \overline{c}$ and the correlation strengths α and β are variational parameters. Since we are making calculation in the spirit of work [19,27] we shall assume that $f_{\Lambda NN}$ are not important and we set $f_{\Lambda NN}$ =1. For ${}^{9}_{\Lambda}Be$, wave function Ψ depends on a total of 13 variational parameters $\kappa_{\Lambda N}, C_{\Lambda N}, R_{\Lambda N}, a_{\Lambda N}, \kappa_{NN}, C_{NN}, R_{NN,} a_{NN}, \kappa_{\alpha\alpha}, C_{\alpha\alpha}, R_{\alpha\alpha}, a_{\alpha\alpha}, and \;\; s_{\Lambda N}$.

3.4 The Energy Calculation

The Monte Carlo method is a numerical technique of computing multidimensional integration appearing in the potential energy of ${}^{9}_{\Lambda}Be$ by means of random sampling. The variational Monte Carlo (VMC) is combination of Monte Carlo method and the Rayleigh-Ritz variational principle. It states that the expectation value of the Hamiltonian with an arbitrary normalized trial wave function Ψ_T is an upper bound E_V to the ground state energy E_0 i.e.

$$E_0 \le E_V = \frac{\int \Psi_T^*(R) H \Psi_T(R) dR}{\int \Psi_T^*(R) \Psi_T(R) dR}, \tag{3.28}$$

where R represents the set of 3A coordinates ($\vec{r}_1, \vec{r}_2, ..., \vec{r}_A$) of A spinless particles. The above equation, for the case when the Hamiltonian is a local operator, can be written in a form appropriate for Monte Carlo calculation. We define a local energy $E_L(R)$ as

$$E_L(R) = \frac{1}{\Psi_T(R)} H \Psi_T(R) \tag{3.29}$$

and introduce a multivariate probability distribution

$$p(R) = \frac{|\Psi_T(R)|^2}{\int |\Psi_T(R)|^2 dR} \quad . \tag{3.30}$$

Thus the variational energy is given by

$$E_V = \int p(R) E_L(R) dR \quad . \tag{3.31}$$

By drawing a set of random configuration in $R_1, R_2, \ldots, R_n$ and using Metropolis algorithm, [see sections 2.3, 2.4, and 2.6] we can obtain an estimate of the energy as

$$E_V = \lim_{n\to\infty} \frac{1}{n} \sum_i E_L(R_i), \tag{3.32}$$

with an statistical error

$$\Delta E_V = \left\{ \frac{1}{n} \sum_i E_L^2(R_i) - E_V^2 \right\}^{1/2} \quad . \tag{3.33}$$

The separation energy B_Λ of a hypernucleus is obtained from

$$-B_\Lambda = \frac{(\Psi^{(A)} | H^{(A)} | \Psi^{(A)})}{(\Psi^{(A)}, \Psi^{(A)})} - \frac{(\Psi^{(A-1)} | H^{(A-1)} | \Psi^{(A-1)})}{(\Psi^{(A-1)}, \Psi^{(A-1)})} \tag{3.34}$$

by minimizing separately both terms with respect to variational parameters which may be different for two wave functions. However, in our case the second term on the right hand side of (3.34) is known in terms of sum of energy of two-alpha and resonance energy of 0.1 MeV of $\alpha - \alpha$ system. The alpha particle energy [24,25] has already been calculated to be -31.2 MeV for Mafliet-Tjon potential using variational Monte Carlo method.

The general procedure for calculating energy in the VMC method is as follows:

From an initial configuration $R \equiv (\vec{r}_1, \ldots, \vec{r}_A)$ having weight w (i.e. $p(R)$) a trial configuration $R' \equiv (\vec{r}_1', \vec{r}_2', \ldots \vec{r}_A')$ is generated by adding to each component k of $\vec{r}_i$ a random shift,

$$r'_{i,k} = r_{i,k} + h(u - 0.5),$$

where h is a chosen step length(we took a step $h = 0.02$), and u a random number in the interval $(0,1)$. The new weight w' (i.e. $p(R')$) is calculated. Now we make use of Metropolis algorithm (see section 2.6). If w'>w the move is accepted and trial configuration R' is used as the initial configuration for the next move. If w' <w another random variable u is generated, and if w'/w > u the move is accepted, otherwise it is rejected and R is kept as the initial configuration for the next move. The calculations were started by setting $r_{i,k} = 0,$ and the system was allowed to move 100 times before energy calculation began. In order to reduce the correlation between two consecutive energy calculations, we move ten times the energy calculations.

The ΛN and NN potentials needed in the calculation of total energy of ${}^{9}_{\Lambda}$Be are given in eqs. (3.5) and (3.10), respectively. The parameters of ΛN and three-body ΛNN dispersive forces are the ones, which fit low energy Λp scattering data and B_Λ of s-shell hypernuclei and are taken from the analysis of Shoeb [27]. These are

$\bar{V}$ =-6.15MeV, ε=0.25, and W=0.02 MeV.

We select one of the three α- α potentials (eqs. (3.11) to (3.13)) one by one. There are no free potential parameters in the present analysis. For a given BN pair with the appropriate potential, the Schroedinger type eq.(3.22) is solved, for an initial guess of variational parameters(see end of section 3.3), to yield correlation functions $f_{\Lambda N}$, f_{NN}, and $f_{\alpha\alpha}$. These are used to construct the wave function. For a configuration which is accepted (see last paragraph on page 42) the energy is calculated using eq.(3.29) . Thus n=100,000 configurations are chosen to calculate the average energy using eq.(3.32) and statistical error eq.(3.33). The variational parameters corresponding to two-body correlations involved in the

wave function are varied taking one at a time to optimize the energy of hypernucleus ${}^{9}_{\Lambda}\mathrm{Be}$. From the past experience it is known that slight variation in the parameters c, R and a for all pairs does not alter the minima and we have also checked it. The $\kappa_{\Lambda N}$, κ_{NN} and $\kappa_{\alpha\alpha}$ and $s_{\Lambda N}$ are the parameters on which the energy depend sensitively. Subsequently these parameters were varied to optimize the energy.

3.5 Results and Discussion

The results of calculation of energy of ${}^{9}_{\Lambda}\mathrm{Be}$ for three-type of α-α potentials for optimum values of variational parameters (Table 9) are tabulated in Table 10. The ${}^{9}_{\Lambda}\mathrm{Be}$ system for Ali-Bodmer [29] and Darriult et-al [30] α-α potentials is over bound by about 2.0 Mev than that of the Chien and Brown [28]. The energy $-B_{\Lambda}$ of ${}^{9}_{\Lambda}\mathrm{Be}$ for the Chien and Brown potential after subtracting the energy (-62.3 MeV) of the core nucleus from the total energy (-70.04 MeV) of the system turn out to be –6.74 MeV which is equal to the experimental value listed in the Table 2. The r.m.s distance between two-alpha for the Chien and Brown case turns out to be slightly more than the Ali-Bodmer [29] and larger than the Darriult et-al [30] α-α potentials. Moreover, this separation is more than the twice the alpha particle r.m.s radius and thus justifying our assumption of alpha cluster model for the system under investigation. These results are at variance from those of Bodmer and Usmani [11] who have analyzed ${}^{9}_{\Lambda}Be$ in Λ-α-α cluster model where nucleonic degrees of freedom were frozen. Thus effect of dynamical NN and ΛN correlations were ignored which we have taken in to account.

Table 9: Variational parameters for minimum energy corresponding to different types of α- α potential
($C_{\Lambda N}$=2.0fm^{-1}, $a_{\Lambda N}$ =1.0fm, $R_{\Lambda N}$ =1.0fm, C_{NN}=1.0fm^{-1},a_{NN}=0.5fm, R_{NN} =1.0fm,
$C_{\alpha\alpha}$ =5.0fm^{-1}, $a_{\alpha\alpha}$=1.150fm, $R_{\alpha\alpha}$=0.0fm.)

Varational parameters	Chien andBrown[8]	Ali and Bodmer[9]	Darriult et al [10]
$\kappa_{\Lambda N}$ (fm^{-1})	0.105	0.085	0.105
κ_{NN} (fm^{-1})	0.260	0.260	0.260
$\kappa_{\alpha\alpha}$ (fm^{-1})	0.600	0.700	0.700
$s_{\Lambda N}$	0.900	0.900	0.600

Table 10:Variational results for optimum energy of the system ${}^{9}_{\Lambda}Be$ along with components energy for 100,000 configurations using various α-α potential

Components of energy And Total energy(MeV)	Chien and Brown[8]	Ali and Bodmer[9]	Darriult et al[10]
Kinetic energy of Λ particle $\langle K_{\Lambda} \rangle$	11.83	10.01	12.84
Total kinetic energy of all the nucleons $\langle K_{N} \rangle$	149.87	149.84	153.56
ΛN Potential energy - $\langle V_{\Lambda N} \rangle$	228.05	228.70	234.45
ΛN Space exchange energy - $\langle V_{\Lambda N}^{SE} \rangle$	6.42	5.75	6.94
Sum of dispersive three-body ΛNN potential energy - $\langle V_{\Lambda NN}^{DS} \rangle$ from each alpha	2.46	2.14	2.76
Dispersive three-body ΛNN potential energy from two-alpha $\langle V_{\Lambda\alpha\alpha}^{DS} \rangle$	0.16	0.12	0.20
Total energy –(E ± ΔE)	70.04 ± 0.15	72.32 ± 0.11	71.93 ± 0.07
Root mean square separation of α-α (in fm)	3.60	3.58	3.36

4. Conclusion

From the discussion given in the last section we find that central spin-dependent dispersive ΛNN force along with two-body ΛN Urbana type potential consistent with Λp scattering data and s-shell hypernuclear binding energies but without three-body correlation in the wave function is able to explain the binding energy of ${}^{9}_{\Lambda}Be$. The analysis further shows that binding energy data prefers the Chien and Brown potential to the others. The r.m.s distance 3.64 fm between two-alpha is twice the radius of alpha particle which justifies internal consistency of cluster model for ${}^{9}_{\Lambda}Be$ system. These conclusions are to be further strengthened using other available dispersive forces.

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