



THE COLLECTIVE PROPERTIES
OF DIPOLE PLASMA

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ABSTRACT

Usually the conventional theories consider the low temperature plasma as a system containing mobile charged particles and neutral atoms or molecules. In this thesis the theory of some collective properties of "dipole plasma" that consists of charged particles and polar molecules with constant dipole moment as the neutral component is developed.

In such a plasma the dipoles interact with the charged particles and the self-consistent fields that arise in the non-equilibrium state of the plasma. The interactions with the electrons and positive ions result in some correction terms in the thermodynamic functions of plasma; whereas, the dipole interactions with the self-consistent fields lead to some new physical effects such as the shifting of plasma frequencies, and appearance of additional damping plasma waves and a new branch of eigen-vibration which we call the "dipole-acoustic wave".

These phenomena in dipole plasma are investigated with the help of a collisionless kinetic equation derived for the rotational degrees of freedom of the dipoles.

The dielectric tensor is calculated from a perturbation analysis on the distribution function of the dipole plasma.

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CHAPTER I

INTRODUCTION

1. The Plasma State

A plasma is a quasineutral gas of charged and neutral particles which exhibits collective behaviour (2). By collective behaviour we mean motions that depend not only on local conditions but on the state of the plasma in remote regions as well.

Plasmas are composed of a very large number of particles, $N \sim 10^{23}$, all moving and interacting together. We can not ever hope to follow them all around their respective paths, even if we would obtain the exact equations to be solved. A statistical treatment is needed, for we can not predict the exact behaviour of every particle in the plasma.

The statistical method enable us to predict what happens to a collective of particles, on the average, subject to certain conditions.

The statistical method uses a kinetic description based on the velocity distribution function, $f(\vec{r}, \vec{v}, t)$, for each type of particles and the product $f(\vec{r}, \vec{v}, t) d^3r d^3v$ represents the number of particles in the phase volume $d^3r d^3v$. The number of particles in the elemental phase volume, when summed up for all possible velocities give the total number of particles (density number) in the entire

phase space.

That is,

$$n(\vec{r}, t) = \int_{-\infty}^{\infty} f(\vec{r}, \vec{v}, t) d^3v \quad (1.1.1)$$

In any event, it is important to know the form of the distribution function, both from the viewpoint of determining the plasma temperature and also for developing theories on the nature of plasma behaviour under non-equilibrium conditions.

For any physical quantity, $g(\vec{r}, \vec{v})$, we may determine its mean value to be

$$\langle g(\vec{r}, \vec{v}) \rangle = \frac{1}{n} \int g f(\vec{r}, \vec{v}, t) d^3v \quad (1.1.2)$$

where f is the distribution function.

The velocity distribution function satisfies an equation known as the Boltzmann equation:

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial r_i} + \frac{F_i}{m} \frac{\partial f}{\partial v_i} = \left(\frac{\partial f}{\partial t} \right)_{\text{coll}} \quad (1.1.3)$$

This equation is the law governing the variation of the distribution function with time.

The first term in eq. (1.1.3) is the local variation of the distribution function. The term $v_i \frac{\partial f}{\partial r_i}$ is the variation of the distribution function resulting from molecules streaming in and out of a given volume element. This term relates to the description of diffusion. The

third term is the variation of the distribution function resulting from the external forces, F_i , acting on the particles. The velocity distribution function will also vary as the result of collisions between the molecules of the gas. This variation is included in the collision term on the right-hand side of eq. (1.1.3).

A number of important plasma phenomena can be studied by treating the plasma as a whole or any of its constituents as a fluid characterized by a certain number of macroscopic variables such as the mass density, the average velocity, the kinetic pressure and the temperature. A set of moments of the Boltzmann equation appropriately truncated and suitably completed by an assumption on the highest velocity moment of the distribution function enables the determination of the equations governing these macroscopic variables. The electrons and ions form conducting fluids with the result there are also macroscopic electromagnetic variables such as the charge density, the current density, the electric field, and the magnetic flux density. These electromagnetic variables are governed by Maxwell's equations. The behaviour of the plasma can thus be analyzed in terms of the hydrodynamical and Maxwell's equations.

1.2 Basic Properties of Plasma.

A fundamental property of plasma is its tendency to preserve its macroscopic electrical neutrality. The plasma is said to be quasineutral; that is, neutral enough

that one can take the density of positive charges to be approximately equal to the density of negative charges (electrons), but not so neutral that all the interesting electromagnetic forces vanish.

When the electrical neutrality of plasma is disturbed, the electrons break into collective, high-frequency oscillations and the relatively heavy positive ions remain immobile and provide the necessary neutralizing background. On an average, over the time period of these high-frequency oscillations, the macroscopic electrical neutrality is maintained.

If an electric field due to a test charge is imposed on a plasma, the plasma particles redistribute themselves around the test charge in such a manner as to shield its influence from a major portion of the plasma. If the dimensions L of a plasma system are much larger than the Debye's length,

$$r_D = \left(\frac{T}{4\pi e^2 n} \right)^{\frac{1}{2}},$$

then whenever local concentrations of charge arise or external potentials are introduced into the system, these are screened out in a distance short compared with L , leaving the bulk of the plasma free of large electrostatic potentials or fields. Thus, a criterion for an ionized gas to be a plasma is that it be dense enough that r_D is much smaller than L . The picture of Debye shielding is valid only if there are enough particles in the charge cloud.

Clearly, if there are only one or two particles in the sheath region, Debye shielding would not be a statistically valid concept.

The number, N_D , of particles in a "Debye sphere" is:

$$N_D = n \frac{4}{3} \pi r_D^3 \quad (1.2.1)$$

where n is the plasma density.

Thus in addition to $r_D \ll L$, collective behaviour requires $N_D \gg 1$.

We have given two conditions that an ionized gas must satisfy to be called a plasma. A third condition has to do with collisions. If ω is the frequency of typical plasma oscillations and τ is the mean time between collisions with neutral particles, we require $\omega\tau > 1$ for the gas to behave like a plasma rather than a neutral gas.

From the fundamental length r_D and the basic speed, $v = \left(\frac{T}{m} \right)^{\frac{1}{2}}$ we construct a fundamental frequency,

$$\omega_p = \left(\frac{4\pi ne^2}{m} \right)^{\frac{1}{2}} \quad (1.2.2)$$

This is the so-called Langmuir plasma frequency.

The plasma frequency is such fundamental importance that the plasma density is frequently described by the quantity ω_p rather than by n itself.

We may also define the ion plasma frequency

$$\omega_{pi} = \left(\frac{4\pi n_i z^2 e^2}{m_i} \right)^{1/2} \quad (1.2.3)$$

this frequency is much lower than ω_{pe} since the ions have much greater mass (m_i).

Although plasma may be regarded as a special form of gaseous mixture, there are many important physical properties in which it differs from an ordinary gas containing only neutral particles. These differences are especially evident in the behaviour of plasma in electric and magnetic fields. An electric field - even a very weak one - gives rise to an electric current in the plasma. In a magnetic field, the plasma behaves as a diamagnetic material. It also exhibits a very strong interaction with electromagnetic waves.

The specific properties which are exhibited by plasma in its interaction with electric and magnetic fields serve as the basis for many scientific and technological applications of plasma.

1.3 The Dipole Plasma

The dipole plasma consists of electrons, positive ions, and polar molecules with constant dipole moment $\vec{d}$ as the neutral component. These dipoles rotate with angular frequency

$$\omega_T \sim \left(\frac{T}{I} \right)^{1/2} \quad (1.3.1)$$

where T is the temperature in energy units, and I is the

moment of inertia of the dipole molecule.

It is evident that the dipole molecules interact with the self-consistent fields that arise in non-equilibrium state and with the charged particles of the plasma. The latter interaction lead to the contribution of dipoles to the thermodynamic functions of the plasma. The interaction with the self-consistent fields result in new physical effects such as, displacement or shifting of plasma frequency, appearance of additional damping plasma waves, and new collective vibration which we may call the "Dipole-acoustic wave" - analogous to the ion-acoustic waves in the electron-ion plasma.

These phenomena in dipole plasma will be studied with the help of:

- i) thermodynamic description, which gives the thermodynamic functions of the dipole plasma; the heat of solution; and the equation of state of the dipole plasma in equilibrium state.
- ii) kinetic description using the set of Boltzmann equations for the electrons, positive ions, and for the rotational degrees of freedom of the dipoles in the collisionless case, with Maxwell's equations and the field sources permit the study of some collective properties of the dipole plasma.
- iii) perturbation analysis on the distribution function of dipole plasma, the dielectric tensor, additional damping coefficient and the "Dipole-acoustic wave" are obtained.

As indicated at the outset, the main concern will be the study of collective properties of the dipole plasma. In the subsequent chapters we will develop the theoretical method and use it to investigate the above mentioned physical phenomena.

CHAPTER II

THERMODYNAMIC DESCRIPTION OF DIPOLE PLASMA

2.1 The Electric Field of Dipole in plasma.

The method of calculation of thermodynamic functions for non-ideal gases of neutral particles is invalid for plasma - a gas characterized by the Coulomb interaction between particles, since due to the long-range nature of the coulomb forces, the group integrals in the virial expansion are divergent.

Physically, however, the fields of individual dipoles are effectively screened by the statistical distribution of other particles.

Let us consider a field in the neighbourhood of a dipole with dipole moment $\vec{d}$ in dipole plasma. For the sake of simplicity we assume the dipole to be located at the origin of coordinates.

The potential of the electric field satisfies the Poisson's equation

$$\Delta \psi = -4\pi\rho \quad (2.1.1)$$

where ψ is the potential of the electrostatic field, and ρ is the charge density given by,

$$\rho = e[zn_1 - n_e] - \text{div } \vec{p} \quad (2.1.2)$$

where e is the electronic charge, z is the atomic number, n_1 and n_e are concentrations of positive ions and electrons respectively, and $\text{div } \vec{p}$ describes the

induced charge associated with the polarized dipoles with the polarization,

$\vec{p} = n_d \langle \vec{d} \rangle$, n_d being the concentration of dipoles.

Substituting for ρ in eq. (2.1.1) and using the relation

$$\langle \vec{d} \rangle = \frac{\int \vec{d} \exp\left(\frac{\vec{d} \cdot \vec{E}}{T}\right) d\Omega}{\int \exp\left(\frac{\vec{d} \cdot \vec{E}}{T}\right) d\Omega} = \frac{d^2 \vec{E}}{3T} \quad (2.1.3)$$

for the case where

$$\frac{\vec{d} \cdot \vec{E}}{T} \ll 1, \text{ we obtain,}$$

$$\Delta \psi = -\frac{4\pi e}{\epsilon} (zn_i - n_e) \quad (2.1.4)$$

where $\epsilon = 1 + \frac{4\pi n_d d^2}{3T}$ is the Langevin's expression

for the dielectric constant,

According to the Boltzmann distribution, the densities of particles in plasma are given by

$$n = n_0 \exp\left(-\frac{e\psi}{T}\right)$$

where T is the temperature in energy units. By the quasineutrality condition the concentration when the potential ($e\psi$) is zero, is the same for the electrons and positive ions, that is

$$n_0 e = n_0 i = n_0$$

Thus,

$$\Delta \psi = - \frac{4\pi}{\epsilon} n_0 \left[(z \exp(-\frac{e\psi}{T}) - \exp(\frac{e\psi}{T})) \right] \quad (2.1.5)$$

we shall limit ourselves to the consideration of weakly ionized plasma in which the energy of the Coulomb interaction is small compared to the mean kinetic energy, that is, $e\psi/T \ll 1$.

Therefore, we can expand the exponentials in eq. (2.1.5) into a series in powers of $e\psi/T$ and limit ourselves to the first two terms of the expansion.

$$\text{Then } \Delta \psi = - \frac{\psi}{r_D^2} \quad (2.1.6)$$

Eq. (2.1.6) has solution of the form (4)

$$\psi(\vec{r}, \theta) = \frac{\vec{d} \cdot \vec{r}}{r^3} \left(1 + \frac{r}{r_D} \right) \exp \left(- \frac{r}{r_D} \right) \quad (2.1.7)$$

which satisfies the necessary boundary conditions,

that is $\psi = \frac{\vec{d} \cdot \vec{r}}{r^3}$ as $r \rightarrow 0$ and $\psi \rightarrow 0$ as $r \rightarrow \infty$.

In eq. (2.1.7),

$$r_D = \left\{ \frac{\epsilon T}{8\pi e^2 n_0} \right\}^{1/2}$$

which is the "Debye's length" in dipole plasma.

In the electron-ion plasma $\epsilon = 1$.

Expanding the exponential in eq. (2.1.7) into a power series in r , we get

$$\psi = \frac{\vec{d} \cdot \vec{r}}{r^3} \left\{ -\frac{1}{2} \left(\frac{r}{r_D} \right)^2 + \frac{1}{3} \left(\frac{r}{r_D} \right)^3 - \dots \right\}$$

from which the electric field

$$\vec{E} = -\frac{\vec{d}}{3r_D^3} + \frac{\vec{d}}{2rr_D^2} - \frac{(\vec{d} \cdot \vec{r}) \vec{r}}{2r^3 r_D^2} \quad (2.1.8)$$

2.2 Energy of Dipole in Plasma.

The energy of a dipole of constant molecular dipole moment $\vec{d}$ as a function of its orientation and position in the field $\vec{E}$ is given by, (8)

$$W = -\vec{d} \cdot \vec{E} \quad (2.2.1)$$

Using eq. (2.1.8),

$$W = \frac{d^2}{3r_D^3} + \frac{d^2 (\cos^2 \theta - 1)}{2 r r_D^2}$$

the energy of the dipole is obtained when $\theta \rightarrow 0, \pi$.

$$\text{Then, } W = \frac{d^2}{3r_D^3} \quad (2.2.2)$$

It is interesting to compare this expression with the energy of a test charge q in plasma, namely

$$W = -q^2 / r_D$$

The energy of dipole (2.2.2) gives a positive contribution to potential energy of the plasma, thereby increasing its energy as compared with the plasma without dipole component (electron - ion plasma).

2.3 Thermodynamic Functions of Dipole Plasma.

Compared with the expression for the internal energy of an ideal gas, the formula for the internal energy of dipole plasma contains additional terms associated with the existence of Coulomb interactions between the charged particles and the potential energy of the dipole component. Therefore, the internal energy of dipole plasma is given by

$$E = E_{\text{ideal}} + E_{\text{Coulomb}} + E_{\text{dipole}} \quad (2.3.1)$$

where

$$E_{\text{ideal}} = \frac{3}{2} (N_e + N_1 + N_d) T \quad (2.3.2)$$

N_e , N_1 , and N_d being the number of electrons, positive ions, and dipole molecules in dipole plasma.

The Coulomb interaction of the charged particles in plasma is (5)

$$E_{\text{Coul.}} = - \frac{1}{2} \frac{e^2}{\epsilon^{3/2} r_D} (n_e + z^2 n_1) V \quad (2.3.3)$$

where V is the volume occupied by plasma.

The dipole potential energy,

$$E_{\text{dip.}} = \frac{1}{2} \frac{d^2 n_d}{3r_D^3} V$$

since $\frac{d^2 n_d}{T} \ll 1$ in gas, one can expand $\epsilon^{-3/2}$ in eq. (2.3.3) using the Langevin expression for the dielectric constant and gets

$$E_{\text{Coul.}} = -e^3 \left(\frac{\pi}{VT}\right)^{1/2} (N_e + z^2 N_i)^{3/2} \left(1 - \frac{2\pi d^2 N_d}{VT}\right) \quad (2.3.4)$$

Using the expression for r_D we get,

$$E_{\text{dip.}} = \frac{4\pi d^2 N_d e^3}{3VT} \left(\frac{\pi}{VT}\right)^{1/2} (N_e + z^2 N_i)^{3/2} \quad (2.3.5)$$

Thus, the correction for the internal energy of dipole plasma becomes

$$E_{\text{corr.}} = -e^3 \left(\frac{\pi}{VT}\right)^{1/2} (N_e + z^2 N_i)^{3/2} \left(1 - \frac{10\pi}{3} \frac{N_d d^2}{VT}\right) \quad (2.3.6)$$

We now turn to the description of the thermodynamic properties of plasma with the aid of the thermodynamic function in variables T and V , namely, free energy.

Let us use for this purpose the Gibbs - Helmholtz equation and the relation following from it:

$$F = - T \int \frac{E}{T^2} dT + T f(V) \quad (2.3.7)$$

where $f(V)$ is an arbitrary function of volume.

Expression (2.3.1) including the internal energy of an ideal gas can not be extrapolated to cover the region $T \rightarrow 0$ and the Nernst theorem can not be used to determine $f(V)$.

Integration of eq. (2.3.7) leads to the expression

$$F = - C_v T \ln T - F_{\text{corr.}} + T f(V) \quad (2.3.8)$$

The function $f(V)$ can be defined with the aid of passing the limit as $V \rightarrow \infty$ at $T = \text{Constant}$, since for very rarefied plasma the expression $-C_v T \ln T + T f(V)$ must coincide with that of the free energy of an ideal gas. Consequently from eq. (2.3.8) we obtain

$$F = F_{\text{ideal}} - \frac{2}{3} e^3 \left(\frac{\pi}{VT} \right)^{\frac{1}{2}} (N_e + z^2 N_i)^{\frac{3}{2}} \left(1 - \frac{2\pi N_d d^2}{VT} \right) \quad (2.3.9)$$

as the free energy of dipole plasma.

Using the thermodynamic relation $P = - \left(\frac{\partial F}{\partial V} \right)_T$

we get the equation of state

$$P = P_{\text{ideal}} - \frac{e^3}{3V^{\frac{3}{2}}} \left(\frac{\pi}{T} \right)^{\frac{1}{2}} (N_e + z^2 N_i)^{\frac{3}{2}} \left(1 - \frac{2\pi N_d d^2}{VT} \right) \quad (2.3.10)$$

Formula (2.3.10) shows that the dipole component gives positive contribution to the pressure in dipole plasma which can be explained by the predominance of repulsive forces between the dipole molecules.

The entropy,

$$S = S_{ideal} - \frac{e^3}{3T} \left(\frac{\pi}{VT} \right)^{\frac{1}{2}} (N_e + z^2 N_i)^{\frac{3}{2}} \left(1 - \frac{6\pi N_d d^2}{VT} \right) \quad (2.3.11)$$

and differentiating this yields the heat capacity

$$C_v = (C_{ideal})_v + \frac{e^3}{2T} \left(\frac{\pi}{VT} \right)^{\frac{1}{2}} (N_e + z^2 N_i)^{\frac{3}{2}} \left(1 - \frac{10\pi N_d d^2}{VT} \right) \quad (2.3.12)$$

2.4 The Heat of Solution of Dipole Plasma.

We now consider the dipole plasma as a solution with a single solute and find expressions for its Gibb's free energy and the heat function whose change determines the heat liberated or absorbed depending on the type of molecules used as solvent.

In eq. (2.3.9) the correction term in the free energy associated with the Coulomb interaction between the charged particles and the energy of dipole molecules is small. Hence, one can simply use the equation of state for an ideal gas to get the Gibb's free energy from (2.3.9).

Thus,

$$G = G_{ideal} - \frac{2e^3}{3T} \left(\frac{\pi P}{N_e + N_i + N_d} \right)^{\frac{1}{2}} (N_e + z^2 N_i)^{\frac{3}{2}} \left\{ 1 - \frac{2\pi N_d P d^2}{T^2 (N_e + N_i + N_d)} \right\}$$

where P is the pressure. (2.4.1)

First we investigate the case when dipole molecules are used as solvent in the solution. That is, $N_d \gg N_e, N_i$ and the thermodynamic potential has the form

$$G \approx G_{ideal} - \frac{2e^3}{3T} \left(\frac{\pi P}{N_d} \right)^{\frac{1}{2}} (N_e + z^2 N_i)^{\frac{3}{2}} \left(1 - \frac{2\pi d^2 P}{T^2} - \frac{N_i + N_e}{2N_d} \right)$$

(2.4.2)

The heat function (enthalpy) of the solution is then given by

$$H = H_{ideal} - \frac{4e^3}{3T} \left(\frac{\pi P}{N_d} \right)^{\frac{1}{2}} (N_e + z^2 N_i)^{\frac{3}{2}} \left(1 - \frac{N_e + N_i}{N_d} - \frac{4\pi d^2 P}{T^2} \right)$$

(2.4.3)

From eq. (2.4.3) one can find the so-called heat of solution, that is released if a very large quantity of solvent is added (so that the concentration tends to zero). The heat released in this process is given by the change in the heat function.

Thus, from (2.4.3), the heat of solution,

$$Q = - \frac{4e^3}{3T} \left(\frac{\pi P}{N_d} \right)^{\frac{1}{2}} (N_e + z^2 N_i)^{\frac{3}{2}} \left(1 - \frac{N_i + N_e}{N_d} - \frac{4\pi d^2 P}{T^2} \right)$$

(2.4.4)

The condition for the applicability of the formule obtained

above is that the concentration be sufficiently small.

Now we can consider the opposite case in which $N_e, N_i \gg N_d$, that is, plasma as a solvent.

Then the Gibb's potential of the solution becomes

$$G = G_{ideal} - \frac{2e^3}{3T} \left(\frac{\pi P}{N_e + N_i} \right)^{\frac{1}{2}} (N_e + z^2 N_i)^{\frac{3}{2}} \left(1 - \frac{N_d}{2(N_e + N_i)} \right) \quad (2.4.5)$$

where G_{ideal} is the potential of ideal gas, and that of the pure solvent ($N_d \rightarrow 0$),

$$G_{solv.} = G_{ideal} - \frac{2e^3}{3T} \left(\frac{\pi P}{N_e + N_i} \right)^{\frac{1}{2}} (N_e + z^2 N_i)^{\frac{3}{2}} \quad (2.4.6)$$

The heat function in this case is

$$H = H_{ideal} - \frac{4e^3}{3T} \left(\frac{\pi P}{N_e + N_i} \right)^{\frac{1}{2}} (N_e + z^2 N_i)^{\frac{3}{2}} \left(1 - \frac{N_d}{2(N_e + N_i)} \right) \quad (2.4.7)$$

and that of the pure solvent,

$$H_{solv.} = H_{ideal} - \frac{4e^3}{3T} \left(\frac{\pi P}{N_e + N_i} \right)^{\frac{1}{2}} (N_e + z^2 N_i)^{\frac{3}{2}} \quad (2.4.8)$$

The change in the heat function gives the heat of solution,

$$Q = \frac{2e^3}{3T} \left(\frac{\pi P}{N_e + N_i} \right)^{\frac{1}{2}} (N_e + z^2 N_i)^{\frac{3}{2}} \left(\frac{N_d}{N_e + N_i} \right) \quad (2.4.9)$$

which is absorbed during the process.

The result obtained in this section, for the case $N_d \gg N_e, N_i$, justifies the positive contribution of dipole component to the energy of dipole plasma.

CHAPTER III

THE KINETIC DESCRIPTION OF DIPOLE PLASMA

(A Linear-Approximation)

3.1 Vlasov's Equation.

Let us consider the fully ionized plasma that consists of electrons of charge $(-e)$ and positive ions of charge (ze) , where z is the atomic number. The kinetic description is based on the velocity distribution function $f(\vec{r}, \vec{v}, t)$ for each species and it represents the number of particles in the phase volume $d^3r d^3v$. If the number of particles contained in the elemental phase volume is summed up for all possible velocities, the result is the total number of particles in the entire phase space. Therefore, it follows that

$$n(\vec{r}, t) = \int f(\vec{r}, \vec{v}, t) d^3v \quad (3.1.1)$$

The densities of positive ions and electrons are given by

$$n_i(\vec{r}, t) = \int f_i(\vec{r}, \vec{v}, t) d^3v \quad (3.1.2)$$

$$n_e(\vec{r}, t) = \int f_e(\vec{r}, \vec{v}, t) d^3v$$

where f_i and f_e are distribution functions of ions and electrons respectively.

The fundamental equation that the distribution function has to satisfy is the Boltzmann equation. Thus, for the two component plasma the Boltzmann equations

are:

$$\frac{\partial f_e}{\partial t} + \vec{V} \cdot \nabla f_e - \frac{e}{m} \left(\vec{E} + \frac{1}{C} \vec{V} \times \vec{B} \right) \cdot \frac{\partial f_e}{\partial \vec{V}} = I_{ee} + I_{ei}$$

and

$$\frac{\partial f_i}{\partial t} + \vec{V} \cdot \nabla f_i + \frac{e}{M} \left(\vec{E} + \frac{1}{C} \vec{V} \times \vec{B} \right) \cdot \frac{\partial f_i}{\partial \vec{V}} = I_{ii} + I_{ie}$$

(3.1.3)

where f_e and f_i are the distribution functions of electrons and ions respectively; $\vec{E}(\vec{r}, t)$ and $\vec{B}(\vec{r}, t)$ are self-consistent electric and magnetic fields; m and M are mass of electron and ions; I_{ee} , I_{ii} , and I_{ie} are electron-electron, ion-ion, and electron-ion collision terms respectively.

The self-consistent fields are described by Maxwell's equations:

$$\begin{aligned} \nabla \times \vec{E} &= -\frac{1}{C} \frac{\partial \vec{B}}{\partial t} ; & \nabla \cdot \vec{E} &= 4\pi\rho \\ \nabla \times \vec{H} &= \frac{4\pi}{C} \vec{j} + \frac{1}{C} \frac{\partial \vec{E}}{\partial t} ; & \nabla \cdot \vec{B} &= 0 \end{aligned}$$

(3.1.4)

where the field sources

$$\rho(\vec{r}, t) = \sum_{\alpha} e_{\alpha} \int f_{\alpha} d\vec{v}$$

and

$$\vec{j}(\vec{r}, t) = \sum_{\alpha} e_{\alpha} \int \vec{V}_{\alpha} f_{\alpha} d\vec{v}$$

(3.1.5)

are the charge and current densities. (α stands for the type of charge).

Collisions between plasma particles may be neglected if one considers physical processes for which the frequency $\omega \gg \nu$ (the collision frequency) and the Boltzmann equations assume the form

$$\frac{\partial f_e}{\partial t} + \vec{V} \cdot \nabla f_e - \frac{e}{m} (\vec{E} + \frac{1}{c} \vec{V} \times \vec{B}) \cdot \frac{\partial f_e}{\partial \vec{V}} = 0$$

and (3.1.6)

$$\frac{\partial f_i}{\partial t} + \vec{V} \cdot \nabla f_i + \frac{e}{M} (\vec{E} + \frac{1}{c} \vec{V} \times \vec{B}) \cdot \frac{\partial f_i}{\partial \vec{V}} = 0$$

for the electrons and ions respectively.

Eqs. (3.1.4), (3.1.5) and (3.1.6) represent the set of equations called Vlasov's equations which simultaneously determine the distribution functions f_e and f_i and the self-consistent fields $\vec{E}(\vec{r}, t)$ and $\vec{B}(\vec{r}, t)$.

3.2 Kinetic Equation for the Rotational Degrees of Freedom of the Dipole.

Now we develop the kinetic equation in the collisionless case for the rotational degrees of freedom of the dipoles (which are the neutral component of dipole plasma) in the self-consistent fields, which is analogous to Vlasov's equation in the electron-ion plasma.

The dipole molecules have angular frequency

$$\omega_T \sim \left(\frac{I}{I} \right)^{\frac{1}{2}} = \left(\frac{T}{mr_0^2} \right)^{\frac{1}{2}} \quad (3.2.1)$$

where T is the temperature, I the moment of inertia, and

m and r_0 , are the mass and length or size of the molecule respectively. The thermal or heat velocity of the dipole is

$$v_T \sim \left(\frac{T}{m} \right)^{\frac{1}{2}} \quad (3.2.2)$$

Thus, the average linear distance covered by the dipole is obtained from the above two equations

$$\bar{r} \sim \frac{v_T}{\omega_T} \sim r_0 \quad (3.2.3)$$

that means in the high-frequency region, that is $\omega \gg \omega_T$ one can neglect the translational motion of the dipole since $\bar{r} \ll r_0$. In the low-frequency region, that is, $\omega \ll \omega_T$ one can consider the mean dipole moment,

$$\langle \vec{d} \rangle \sim \frac{d^2}{3T} \vec{E} \quad (3.2.4)$$

and hence, the potential energy of the dipole

$$W \sim E^2 \quad (3.2.5)$$

Eq. (3.2.5) leads to a non-linear approximation. Therefore, from eqs. (3.2.3) and (3.2.5) it is legitimate to neglect the translational degrees of freedom of the dipole.

It is also possible to show that the vibrational degrees of freedom of dipoles can be neglected based on the following arguments.

Assume the dipole molecules as point spheres with dipole moment $\vec{d}$.

The classical Hamiltonian for such a dipole is given by

$$H = \frac{1}{2I} (p_\theta^2 + p_\psi^2) - \vec{d} \cdot \vec{E} \quad (3.2.9)$$

where the first term in (3.2.9) represents the rotational kinetic energy, and the second is the potential energy of the dipole.

The angular momenta p_θ and p_ψ are related to the moment of inertia,

$$p_\theta = I\dot{\theta} \quad \text{and} \quad p_\psi = I\dot{\psi} \quad (3.2.10)$$

and

$$\vec{d} \cdot \vec{E} = d (\sin\theta \cos\psi E_x + \sin\theta \sin\psi E_y + \cos\theta E_z)$$

Then the kinetic equation in collisionless case becomes:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \theta} \dot{\theta} + \frac{\partial f}{\partial \psi} \dot{\psi} + \frac{\partial f}{\partial p_\theta} \dot{p}_\theta + \frac{\partial f}{\partial p_\psi} \dot{p}_\psi = 0 \quad (3.2.11)$$

where the distribution function of dipoles

$$f = f(\theta, \psi, p_\theta, p_\psi; t)$$

Using Hamilton's equations, the kinetic equation takes

the non-equilibrium part of distribution function,
that is,

$$\vec{E}(\vec{r}, t) = \frac{1}{2\pi} \int \vec{E}(\omega) \exp(-i\omega t) d\omega$$

and

$$f_1(\theta, \psi, p_\theta, p_\psi; t) = \frac{1}{2\pi} \int d\omega \exp(-i\omega t) \sum_{m=-\infty}^{\infty} f_m(\theta, p_\theta, p_\psi; \omega) e^{im\psi} \quad (3.3.3)$$

the kinetic equation becomes:

$$\begin{aligned} \sum_{m=-\infty}^{\infty} (-i\omega f_m + im \frac{p_\psi}{I} f_m + \frac{p_\theta}{I} \frac{\partial f_m}{\partial \theta}) e^{im\psi} \\ = dE_z \sin\theta \frac{\partial f_0}{\partial p_\theta} + \frac{dE}{2} (i \sin\theta \frac{\partial f_0}{\partial p_\psi} - \cos\theta \frac{\partial f_0}{\partial p_\theta}) e^{-i\psi} \\ - \frac{dE}{2} (i \sin\theta \frac{\partial f_0}{\partial p_\psi} + \cos\theta \frac{\partial f_0}{\partial p_\theta}) e^{i\psi} \end{aligned} \quad (3.3.4)$$

Eq. (3.3.4) shows that the non-equilibrium
part of the distribution function can be written
as the sum of three parts:

$$f_1 = f_{1(m=0)} + f_{1(m=1)} e^{i\psi} + f_{1(m=-1)} e^{-i\psi} \quad (3.3.5)$$

which are obtained by solving eq. (3.3.4)
by the method of variation of parameters for
 $m = 0, \quad m = 1, \quad \text{and } m = -1 :$

$$f_{m=0} = \frac{dE_z}{2T} f_0 p_\theta \left[\frac{e^{-i\theta}}{p_\theta + i\omega} + \frac{e^{i\theta}}{p_\theta - i\omega} \right] \quad (3.3.6)$$

$$f_{m=1} = \frac{-idE_-}{4T} f_0 \left[\frac{(p_\theta + p_\psi)e^{i\theta}}{p_\theta + p_\psi - i\omega} - \frac{(p_\theta - p_\psi)e^{-i\theta}}{p_\theta - p_\psi + i\omega} \right] \quad (3.3.7)$$

$$f_{m=-1} = \frac{idE_+}{4T} f_0 \left[\frac{(p_\theta + p_\psi)e^{-i\theta}}{p_\theta + p_\psi + i\omega} - \frac{(p_\theta - p_\psi)e^{i\theta}}{p_\theta - p_\psi - i\omega} \right] \quad (3.3.8)$$

Together with eq. (3.2.14) the above three parts of the perturbation give the non-equilibrium distribution function of the dipole plasma.

The two Boltzmann equations (3.1.3) for the electron and ion components, the kinetic equation for the dipoles (3.2.12), Maxwell's equations with the field sources for the description of the self-consistent fields, and the two relations

$$\frac{\partial \vec{p}}{\partial t} = \vec{j} \quad \text{and} \quad \text{div} \vec{p} = -\rho \quad (3.3.9)$$

(where $\vec{p}$ is the polarization vector) are the complete set of equations for the kinetic description of dipole plasma.

3.4 Contribution of the Dipole Component to the Dielectric Tensor of the Dipole Plasma.

The dielectric tensor contains essentially all the information about the electromagnetic properties of plasma; it is determined from the calculation of the polarization vector p_i which is linearly related to the electric field E_j . That is,

$$p_i = \alpha_{ij} E_j \quad (3.4.1)$$

where α_{ij} is called the polarizability. The electric displacement,

$$D_i = E_i + 4\pi p_i$$

or

$$D_i = \epsilon_{ij} E_j$$

$$\text{where } \epsilon_{ij} = \delta_{ij} + 4\pi\alpha_{ij} \quad (3.4.2)$$

is called the dielectric tensor. Thus, knowledge of α_{ij} from (3.4.1) enables us to calculate the dielectric tensor using (3.4.2)

The z-component of the polarization vector

$$\langle p_z \rangle = \int d_z \int_{l(m=0)} d\theta d\psi \sin\theta dp_\theta dp_\psi \quad (3.4.3)$$

substitution for $f_{l(m=0)}$ from (3.3.6) and performing the integrations over θ , ψ , and p_ψ yield

$$\langle p_z \rangle = \frac{d^2 n_d E_z}{3T} \left[1 + \frac{\omega}{\sqrt{2\pi} \omega_T} \int_{-\infty}^{\infty} \frac{\exp(-p_\theta^2/2IT) dp_\theta}{p_\theta - I\omega} \right] \quad (3.4.4)$$

where ω_T is the rotational frequency of dipole, and E_z is the electric field component.

The change of variable

$$z^2 = \frac{p_\theta^2}{2IT} \quad \text{gives}$$

$$\langle p_z \rangle = \frac{d^2 n_d}{3T} \left[1 + F(x) \right] E_z \quad (3.4.5)$$

where

$$F(x) = \frac{x}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-z^2) dz}{z - x} \quad (3.4.6)$$

and

$$x = \frac{\omega}{\sqrt{2} \omega_T}$$

comparing eq. (3.4.5) with eq. (3.4.1), we get the polarizability

$$\alpha_{zz} = \frac{d^2 n_d}{3T} \left[1 + F(x) \right] \quad (3.4.7)$$

Similarly, introducing the relations

$$\langle p_{\pm} \rangle = \langle p_x \rangle \pm i \langle p_y \rangle$$

for the polarization vector components and

$$\langle d_{\pm} \rangle = \langle d_x \rangle \pm i \langle d_y \rangle \quad \text{for the components of the}$$

dipole moment, one can get α_{xx} and α_{yy} . That is,

$$\langle p_{\pm} \rangle = \int d_{\pm} f_{l(m=\pm 1)} d\theta d\psi \sin\theta dp_{\theta} dp_{\psi} \quad (3.4.8)$$

substituting the expressions for the non-equilibrium distribution functions and using the variable changes

$$p_{\theta} - p_{\psi} = p \quad \text{and} \quad p_{\theta} + p_{\psi} = 2\mathcal{P}$$

$$\text{and} \quad (3.4.9)$$

$$z = p / 2\sqrt{IT}$$

eq. (3.4.8) becomes

$$\langle p_{\pm} \rangle = \frac{d^2 n_d}{3T} \left[1 + \frac{\omega}{2\sqrt{\pi} \omega_T} F\left(\frac{\omega}{2\omega_T}\right) \right] E_{\pm} \quad (3.4.10)$$

or letting $y = \frac{\omega}{2\omega_T}$

$$\langle p_{\pm} \rangle = \frac{d^2 n_d}{3T} \left[1 + F(y) \right] E_{\pm} \quad (3.4.11)$$

Eq. (3.4.11) yields the polarizabilities

$$\alpha_{xx} = \alpha_{yy} = \frac{d^2 n_d}{3T} \left[1 + F(y) \right] \quad (3.4.12)$$

The expressions for the polarizabilities given above reduce to the Langevin's formula,

$$\alpha = \frac{d^2 n_d}{3T}$$

in the limit $\omega \rightarrow 0$.

CHAPTER IV
DIELECTRIC TENSOR AND COLLECTIVE PROPERTIES
OF COLLISIONLESS DIPOLE PLASMA

4.1 Longitudinal Waves ($\omega \gg kV_{Te}, \omega_T$)

The frequencies of interest are sufficiently high with the result that the relatively heavy positive ions are assumed to be immobile but they provide the necessary positive neutralizing background for the mobile electrons.

In the linear-approximation theory the dielectric tensor of collisionless dipole plasma can be represented as the sum that contains the contributions from the electrons, positive ions, and dipole molecules, (some properties of dipole media and dipole plasma have been treated in (1) and (6)). That is,

$$\epsilon_{ij}(k, \omega) = \epsilon_{ij}(k, \omega)_{\text{plasma}} + \epsilon_{ij}(\omega)_{\text{Dipole}} \quad (4.1.1)$$

where k is the wave number, and the dielectric tensor in general is given by

$$\epsilon_{ij}(k, \omega) = \epsilon_{\ell} \frac{k_i k_j}{k^2} + \epsilon_t \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) \quad (4.1.2)$$

ϵ_{ℓ} and ϵ_t being the longitudinal and transversal dielectric functions (3).

The longitudinal waves can propagate in collisionless plasma due to a spatial dispersion. The dispersion equation may be determined from the relation

$\epsilon_{\ell}(\omega, k) E(k, \omega) = 0$ and has a nontrivial solution if

$$\epsilon_{\ell}(k, \omega) = 0 \quad (4.1.3)$$

The longitudinal dielectric function of an electron-ion plasma is given by, (7),

$$\begin{aligned} \epsilon_{\ell}(\omega, k) = 1 + \frac{1}{(k r_{De})^2} \left[1 + F\left(\frac{\omega}{\sqrt{2} k V_{Te}}\right) \right] \\ + \frac{1}{(k r_{Di})^2} \left[1 + F\left(\frac{\omega}{\sqrt{2} k V_{Ti}}\right) \right] \end{aligned} \quad (4.1.4)$$

where k is the wave vector, r_{De} and r_{Di} are Debye's length for the electrons and ions, V_{Te} and V_{Ti} are thermal velocities of the electrons and ions respectively, and the function

$$F(x) = \frac{x}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-z^2) dz}{z - x} \quad (**) \quad (4.1.5)$$

where $x = \frac{\omega}{\sqrt{2} k V_{T\alpha}}$, $V_{T\alpha}$ being the thermal velocity of each component.

(**) see appendix for the evaluation of this integral.

In high-frequency limit, that is,

$\omega \gg \omega_p$, $\omega \gg k V_{Te}$ it is legitimate to ignore the motion of the positive ions and expanding (4.1.5) in powers of x ,

$$F(x) + 1 \approx -\frac{1}{2x^2} - \frac{3}{4x^4} + i\sqrt{\pi} x \exp(-x^2) \quad (x \gg 1) \quad (4.1.6)$$

This permits to write the limiting expression for the dielectric function

$$\begin{aligned} \epsilon_L(\omega, k) = 1 - \frac{\Omega_e^2}{\omega^2} \left(1 + \frac{3k^2 V_{Te}^2}{\omega^2} \right) \\ + i \left(\frac{\pi}{2} \right)^{\frac{1}{2}} \frac{\omega \Omega_e^2}{(k V_{Te})^3} \exp \left(-\frac{\omega^2}{2k^2 V_{Te}^2} \right) \end{aligned} \quad (4.1.7)$$

where $\Omega_e = \frac{(4\pi e^2 n_e)^{\frac{1}{2}}}{m_e}$ is the Langmuir frequency.

The system is isotropic; we may arbitrarily choose the z -axis in the direction of the wave vector and making use of the polarizability α_{zz} (3.4.7) the dipole contribution to the longitudinal dielectric function is obtained as

$$\frac{4\pi d^2 n_d}{3T} (1 + F(y)) \quad (4.1.8)$$

where

$$F(y) = \frac{y}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-z^2)}{z - y} dz \quad \text{and} \quad y = \frac{\omega}{\sqrt{2} \omega_T} \quad (4.1.9)$$

Expansion of (4.1.9), keeping only the leading term, in powers of y gives the result

$$\epsilon_{\ell}(\omega) = -\frac{\Omega_d^2}{\omega^2} + i \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{\omega}{\omega_T^2} \frac{\Omega_d^2}{\omega_T^2} \exp\left(-\frac{\omega^2}{2 \omega_T^2}\right) \quad (4.1.10)$$

$$(\omega \gg \omega_T)$$

$$\text{where} \quad \Omega_d^2 = \frac{4\pi d^2 n_d}{3I}$$

Thus, the longitudinal dielectric function of the collisionless dipole plasma becomes

$$\begin{aligned} \epsilon_{\ell}(\omega, k) = 1 - \frac{\Omega_e^2}{\omega^2} \left[1 + \frac{3k V_{Te}}{\omega^2} \right] - \frac{\Omega_d^2}{\omega^2} \\ + i \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \omega \left[\frac{\Omega_e^2}{(k V_{Te})^3} \exp\left(-\frac{\omega^2}{2k^2 V_{Te}^2}\right) \right. \\ \left. + \frac{\Omega_d^2}{\omega_T^3} \exp\left(-\frac{\omega^2}{2 \omega_T^2}\right) \right] \end{aligned} \quad (4.1.11)$$

Now, using the condition given by eq. (4.1.3) we can get the longitudinal waves of dipole plasma in the high-frequency limit.

In the limit $\omega \gg kV_{Te}$, ω_T the exponentials in eq. (4.1.11) could be neglected and we obtain the relation

$$\omega = \Omega_e \left[1 + \frac{1}{6} \frac{n_d}{n_e} \frac{m_e}{m_d} \right] \quad (4.1.12)$$

where n_e , m_e and n_d , m_d are concentration and mass of electron and dipole respectively.

Eq. (4.1.12) represents undamped high-frequency oscillation in dipole plasma for the case

$$\omega \approx \Omega_e \gg \Omega_d.$$

It is also independent of the propagation coefficient k , thus it follows that the oscillations are stationary in character. The particle velocity and the electric field associated with the plasma oscillations are in the same direction; there is also no magnetic field with this oscillation. Therefore, these oscillations are longitudinal, electrostatic and stationary.

We now seek a solution in the form of a complex frequency

$$\omega = \omega_0 + i\gamma \quad (4.1.13)$$

where ω_0 is the real part and γ , the imaginary part of the frequency, and

$$|\gamma/\omega_0| \ll 1$$

Introducing $\omega_0 = \Omega_e + \alpha$, where $\alpha \ll \Omega_e$ is a first-order small quantity, we compute the first order correction to ω .

That is,

$$\omega = \Omega_e \left[1 + \frac{3}{2} (kr_{De})^2 + \frac{1}{6} \frac{n_d}{n_e} \frac{m_e}{m_d} \right] \quad (4.1.14)$$

The first two terms represent the well-known result in the electron-ion plasma, whereas the third term gives the shifting in plasma frequency due to the interaction of the dipole component with the charged particles and the self-consistent fields.

The imaginary part of the frequency is obtained after substituting the assumed solution (4.1.13) into eq. (4.1.11) and equate it to zero. That is

$$\gamma = - \left(\frac{\pi}{8} \right)^{\frac{1}{2}} \frac{\omega^2}{\Omega_e} \left[\frac{1}{(kr_{De})^3} \exp \left(- \frac{\omega^2}{2k^2 v_{Te}^2} \right) + \frac{\Omega_e}{\Omega_d} \left(\frac{4\pi d^2 n_d}{3T} \right)^3 \exp \left(- \frac{\omega^2}{2\omega_T^2} \right) \right] \quad (4.1.15)$$

and describes the damping (where ω is as given by eq. (4.1.14)

The first term in eq. (4.1.15) is the well-known Landau damping coefficient in electron-ion plasma, whereas the second is the contribution of the dipoles. In the frequency limit $\omega \gg kv_{Te}$ and $\omega \sim \omega_T$,

$$\gamma = -\left(\frac{\pi}{8}\right)^{\frac{1}{2}} \frac{\omega^2}{\Omega_d} \left(\frac{4\pi d^2 n_d}{3T}\right)^{\frac{1}{2}} \exp\left(-\frac{\omega^2}{2\omega_T^2}\right) \quad (4.1.16)$$

that means, the damping is essentially due to the interaction of the dipole component.

The appearance of the negative imaginary part implies that the plasma oscillation can not live forever. This damping phenomena can be explained in terms of the resonant coupling between the waves and those particles which are moving with approximately the same velocities as the phase velocity, $V_p = \frac{\omega}{k}$, of the wave (3). If the amplitude of the wave is small but finite, particles moving slightly faster than the wave will decrease their average velocity to V_p through the resonance interaction, transferring their extra kinetic energy to the wave; particles moving slightly slower than the wave will be accelerated and absorb energy from the wave. Therefore, if the distribution function decreases with increasing velocity, as is the case with the Maxwellian, damping takes place, since there will be more slow particles to absorb energy than fast particles to transfer energy to the wave.

4.2 Longitudinal Waves ($kV_{Te} < \omega < \omega_T$)

In this frequency domain we may use the small argument expansions of $F(x)$ and $F(y)$ (see appendix) in eqs. (4.1.4) and (4.1.8) and neglect the small imaginary parts of the dielectric functions for the electrons and dipoles. The contribution of ions in this frequency limit can totally be ignored; thus the dielectric function of the dipole plasma becomes

$$\epsilon_{\ell}(\omega, k) \approx 1 - \frac{\Omega_e^2}{\omega^2} + \frac{4\pi n_d d^2}{3T} \quad (4.2.1)$$

Using the dispersion relation, $\epsilon_{\ell}(\omega, k) = 0$, we find

$$\omega = \left(\frac{4\pi e^2 n}{m_e \epsilon} \right)^{1/2} \quad (4.2.2)$$

where ϵ is the Langevin's dielectric constant.

Eq. (4.2.2) shows that the high-frequency plasma oscillation is modified. In the frequency domain under discussion the dipoles are playing the role of a medium for plasma with dielectric constant ϵ . It is also evident that, due to the presence of dipole component the self-consistent field decreases by a factor ϵ .

In concluding this section we shall consider the ion-acoustic waves in dipole plasma.

In the frequency domain, $kV_{Ti} < \omega < kV_{Te}$, the dispersion relation takes the form

$$\epsilon - \frac{\Omega_i^2}{\omega^2} + \frac{1}{(kr_{De})^2} = 0$$

or

$$\omega^2 = \frac{\Omega_i^2}{\epsilon + \frac{1}{(kr_{De})^2}} \quad (4.2.3)$$

In the long-wavelength limit, i.e., $kr_{De} \ll 1$

$$\omega \approx \Omega_i kr_{De} = k \left(\frac{T_e}{m_i} \right)^{1/2} \quad (4.2.4)$$

This shows that the dipoles have no contribution in this frequency domain.

4.3 The Dipole-Acoustic Wave

Debye screening and plasma oscillation are basic features associated with a single-system. The presence of the ions and dipoles, however, makes it possible to look for an additional frequency domain, in which the imaginary part of the dielectric function may take on small values. This possibility is offered especially because of the large mass ratios, m_i/m_e and m_d/m_e (where m_e , m_i and m_d are mass of electron, ion and dipole). We may thus investigate an intermediate-frequency domain such that

$$kV_{Ti} < \omega_T < \omega < kV_{Te} \quad (4.3.1)$$

The longitudinal dielectric function for the three-component system may be written as

$$\begin{aligned} \epsilon_{\ell}(\omega, k) = & 1 + \frac{1}{(kr_{De})^2} \left[1 + F\left(\frac{\omega}{\sqrt{2}kV_{Te}}\right) \right] \\ & + \frac{1}{(kr_{Di})^2} \left[1 + F\left(\frac{\omega}{\sqrt{2}kV_{Ti}}\right) \right] \\ & + \frac{4\pi n_d d^2}{3T} \left[1 + F\left(\frac{\omega}{\sqrt{2}\omega_T}\right) \right] \end{aligned} \quad (4.3.2)$$

where $F(x)$ in general is given by eq. (4.1.5).

In the frequency domain of (4.3.1) we may use the small argument expansion for the electrons and the large argument expansion for the ions and dipoles for the function $F(x)$ in eq. (4.3.2) (see appendix); we have

$$\epsilon_{\ell}(\omega, k) \approx 1 + \frac{1}{(kr_{De})^2} - \frac{\Omega_i^2}{\omega^2} - \frac{\Omega_d^2}{\omega^2} \quad (4.3.3)$$

The properties of the collective mode may be investigated through the dispersion relation (4.1.3); with the aid of (4.3.3), we have

$$\omega^2 = \frac{\Omega_d^2 + \Omega_i^2}{1 + \frac{1}{(kr_{De})^2}}$$

In the long-wavelength limit such that $kr_{De} \ll 1$, we get

$$\omega = k \left[\frac{T_e}{m_i} \left(1 + \frac{1}{3} \frac{n_d}{n} \frac{m_i}{m_d} \right) \right]^{\frac{1}{2}} \quad (4.3.4)$$

If one considers light dipoles with high density in plasma (4.3.4) reduces to

$$\omega = k \left(\frac{T_e n_d}{3n m_d} \right)^{\frac{1}{2}} \quad (4.3.5)$$

where n is the electron-ion plasma density.

Eq. (4.3.4) expressed in terms of the moment of inertia (I) and the dipole moment ($\vec{d}$) gives the collective mode which we may call the "Dipole-Acoustic Wave", i.e.,

$$\omega_d = k \left\{ \frac{T_e d^2 n_d}{3 I e^2 n} \right\}^{\frac{1}{2}} \quad (4.3.6)$$

The dipole-acoustic wave is essentially the collective mode associated with the dipoles.

These waves exist only in the dipole plasma due to the interaction of the dipole component with the self-consistent fields.

Below we shall prove that damping of these waves is comparatively small.

The damping associated with the dipole-acoustic wave is obtained by following the procedure of section (4.1) Eq. (4.3.2), keeping the exponential term for the electrons,

and using the dispersion relation becomes

$$1 - \frac{\Omega_d^2 + \Omega_i^2}{\omega^2} + \frac{1}{(k r_{De})^2} + i \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \frac{\omega}{(k v_{Te})^3} \Omega_e^2 = 0 \quad (4.3.7)$$

We seek a solution of this equation in the form of a complex frequency

$$\omega = \omega_0 + i\gamma \quad (4.3.8)$$

where ω_0 represents the real part given by (4.3.6).

Introducing one more assumption

$$|\gamma / \omega_0| \ll 1 \quad (4.3.9)$$

eq. (4.3.7) may be solved for the small imaginary part of the frequency as

$$\gamma = - \left(\frac{\pi}{8}\right)^{\frac{1}{2}} \left[\frac{m_e}{m_i} + \frac{1}{3} \frac{m_e n_d}{m_d n} \right] k v_{Te} \quad (4.3.10)$$

or for the case,

$$\frac{m_e}{m_i} \ll \frac{m_e}{m_d} \frac{n_d}{n} ,$$

$$\gamma = - \left(\frac{\pi}{8}\right)^{\frac{1}{2}} \frac{1}{3} \frac{m_e}{m_d} \frac{n_d}{n} k v_{Te} \quad (4.3.11)$$

Expressing γ in terms of ω_0 from eq. (4.3.6), gives the damping rate

$$\left| \gamma / \omega_d \right| = \left(\frac{m_e}{m_d} \frac{\pi}{24} \frac{n_d}{n} \right)^{\frac{1}{2}} ,$$

which proves the assumption (4.3.9).

4.4 Transverse Waves

The dielectric tensor $\epsilon(\vec{k}, \omega)$ for an isotropic medium contains only one element of directional dependence through that of the wave vector $\vec{k}$; the tensor character of $\epsilon(\vec{k}, \omega)$ must be determined from a general consideration so as to reflect this dependence.

The dielectric tensor can be expressed by eq. (4.1.2)

$$\epsilon(\vec{k}, \omega) = \epsilon_{\ell}(k, \omega) \frac{k_i k_j}{k^2} + \epsilon_t(k, \omega) \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right)$$

where the function $\epsilon_t(k, \omega)$ is calculated as

$$\epsilon_t(k, \omega) = \frac{1}{2} \left[\text{Tr } \epsilon(\vec{k}, \omega) - \epsilon_{\ell}(k, \omega) \right] \quad (4.4.1)$$

and Tr means the summation over the diagonal elements.

Using the expressions for $\epsilon_{\ell}(k, \omega)$ and $\text{Tr } \epsilon(\vec{k}, \omega)$ for the three plasma components, the transverse dielectric function

$$\begin{aligned} \epsilon_t(k, \omega) = 1 + \frac{\Omega_e^2}{\omega^2} F\left(\frac{\omega}{\sqrt{2}kV_{Te}}\right) + \frac{\Omega_i^2}{\omega^2} F\left(\frac{\omega}{\sqrt{2}kV_{Ti}}\right) \\ + \frac{4\pi n_d d^2}{3T} F\left(\frac{\omega}{2\omega_T}\right) \end{aligned} \quad (4.4.2)$$

where the function $F(x)$ in general is as given by eq. (3.4.6).

The propagation mode we are interested in should not have appreciable damping. We thus look for frequency domain in which the imaginary part of $F(x)$ is small; two such domains exist:

$$\omega \gg k V_{Te} , \omega_T \quad (4.4.3)$$

$$k V_{Te} \ll \omega \ll \omega_T \quad (4.4.4)$$

In the frequency domain (4.4.3) only electrons and dipoles give the main contribution, thus we may use the large argument expansion for $F(x)$; hence

$$\begin{aligned} \epsilon_t(k, \omega) = 1 - \frac{\Omega_e^2}{\omega^2} \left[1 + \frac{(k V_{Te})^2}{\omega^2} \right] + i \left(\frac{\pi}{2} \right)^{\frac{1}{2}} \frac{\Omega_e}{\omega k r_{De}} \exp \left(- \frac{\omega^2}{2 k^2 V_{Te}^2} \right) \\ - \frac{4 \pi d^2 n_d}{3 T} \left[1 + \frac{4 \omega^2 T}{\omega^2} \right] + \frac{4 \pi d^2 n_d}{3 T} i \sqrt{\pi} \frac{\omega}{2 \omega_T} \exp \left(- \frac{\omega^2}{4 \omega_T^2} \right) \end{aligned} \quad (4.4.5)$$

Clearly, $\text{Re } \epsilon_t < 1$; it follows from the dispersion relation

$$\epsilon_t(k, \omega) = (k^c / \omega)^2 \quad (4.4.6)$$

that $|\omega/k| > c$, the light velocity in vacuum.

This result is consistent with the original assumption (4.4.3).

The imaginary part of $\epsilon_t(k, \omega)$ in the expression (4.4.5) should then be set equal to zero, for it physically represents the effects of resonant coupling between wave and particles; no particles may exist with velocities greater than c (valid for nonrelativistic situation).

Then,

$$\epsilon_t(k, \omega) = 1 - \frac{\Omega_e^2}{\omega^2} - \frac{4\pi d^2 n_d}{3T}$$

and from (4.4.6) we get

$$\omega^2 = \frac{k^2 c^2 + \Omega_e^2}{1 - \left(\frac{4\pi d^2 n_d}{3T} \right)} \quad (4.4.7)$$

Since $\frac{4\pi d^2 n_d}{3T}$ is a small quantity this equation essentially describes the propagation of an electromagnetic wave in the plasma as in the case of the electron-ion plasma (3); i.e.,

$$\omega^2 \approx k^2 c^2 + \Omega_e^2 \quad (4.4.8)$$

We may similarly examine the dielectric function and the dispersion relation starting from (4.4.4). It will then be found that the results of the solution so obtained are the same as that given by (4.4.8)

The above two results show that the contribution of dipoles to the transverse dielectric function is comparatively small.

CHAPTER V

CONCLUSIONS.

In this work some collective properties of collisionless dipole plasmas-media that consist of electrons, positive ions and polar molecules have been studied. Such media occur in nature and could be produced under laboratory conditions, as for example, with the help of electrical breakdown in a polar gas.

Some important results have been obtained due to interactions of dipole with plasma. From the thermodynamic description, eqs. (2.2.2) and (2.3.10) show that the dipole component give positive contributions to the energy and pressure respectively, that means, the internal energy and pressure of the dipole plasma is increased as compared with that of the electron-ion plasma.

In the collisionless approximation, in the frequency domain $\omega < \omega_T$, the high-frequency electron oscillation is suppressed, for the dipoles act as a dielectric medium with

$$\epsilon = 1 + \frac{4\pi n_d d^2}{3T}$$

The collisionless approximation, that is, the case where the characteristic frequency ω must be larger than the collision frequency ν is valid if the following relation holds. In the high-frequency limit $\omega \approx \omega_p$,

APPENDIX
EVALUATION AND SERIES EXPANSIONS
OF THE FUNCTION $F(X)$

In Chapter III we have introduced a function

$$F(x) = \frac{x}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{\exp(-z^2)}{z-x} dz \quad (A.1)$$

If x lies on the contour of integration, a limiting process must be used, since the integral will "blow up" as it crosses the singularity.

We will define the Cauchy Principal Value of the integral to be

$$P \oint_C \frac{f(z)}{z-x} dz = \lim_{\delta \rightarrow 0} \left[\oint_C^{\nearrow} \frac{f(z)}{z-x} dz \right] \quad (A.2)$$

where $f(z) \equiv \exp(-z^2)$.

The symbol P appears in front of the integral to note that it is a principal value integral. The contour required for this integral is shown in the figure below in an expanded view.

δ is the radius of the path around the point x .

 is the original closed contour integral except for the small semicircular part of radius δ at the singularity

$$\begin{aligned} \frac{x}{\sqrt{\pi}} \mathcal{P} \int_{-\infty}^{\infty} \frac{\exp(-z^2)}{z-x} dz &= \frac{x \exp(-x^2)}{\sqrt{\pi}} \mathcal{P} \int_{-\infty}^{\infty} \exp(-u^2 - 2ux) \frac{du}{u} \\ &= \frac{x}{\sqrt{\pi}} \mathcal{P} \int_{-\infty}^{\infty} \exp(-u^2) \left(\frac{1}{u} - 2x \right) du \end{aligned}$$

Integrating, we get

$$F(x) \approx -2x^2 + \dots + i\sqrt{\pi} x, \quad x \ll 1 \quad (\text{A.5})$$

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