ADDIS ABABA UNIVERSITY
ADDIS ABABA INSTITUTE OF TECHNOLOGY
AFRICAN RAILWAY CENTER OF EXCELLENCE

ENERGY-EFFICIENT TRAIN OPERATION USING CONTROL PARAMETERISATION TECHNIQUE (CPT)

Case study: Addis Ababa-Light Rail Transit/East-West line (AALRT/E-W Line)

A Thesis in Railway Engineering (Traction and Train control)

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in Railway Engineering (Traction and Train control)

DECLARATION

I certify that the research work titled "**Energy-efficient train operation using control parameterisation technique (CPT)**" is my own work. The work has not been presented elsewhere for assessment. Where material has been used from other sources, it has been appropriately acknowledged/referenced.

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APPROVAL

The undersigned have examined the thesis entitled **'Energy-efficient train operation using control parameterization technique (CPT)' case of Addis Ababa Light Rail Transit/East-West line** presented by **INEZA Marie Aimee** of registration number **GSR/7568/11**, a candidate for the degree of Master of Science in Railway Engineering (Traction and Train Control) and hereby certify that it is worthy of acceptance.

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ABSTRACT

The energy consumed by the traction systems takes the most significant portion of the majority of total energy consumption in Addis Ababa Light Rail Transit (AALRT) system. Finding the energy-efficient train reference profile, also known as an optimal train control problem, is one of the most significant ways of saving energy. However, this optimal control problem belongs to complex computational problems which need robust and efficient solving techniques or algorithms.

In this thesis, an efficient numerical strategy known as the control parameterization technique (CPT) is deployed to solve this optimal train control problem. The core idea of this scheme is to approximate the control functions (i.e., the tractive and braking control forces) by piecewise-constant function. In this manner, the optimal train control problem is transformed into a non-linear constrained optimisation problem (NCOP), albeit one with one complex of the state constraints. To overcome this complexity, the constraints are introduced into the cost function via a penalty function in such a way that any violation of the constraints is penalised, transforming a constrained optimisation problem (COP) into an unconstrained optimisation problem (UOP) that can be easily solved with existing unconstrained solvers.

Various solutions for every section of the AALRT system/East-West line (i.e., from Ayat to Tor Hailoch station) were generated. The results showed that the speed profile meets the maximum speed limits between two consecutive stations. It has been noticed that the control force successfully satisfies the boundary control constraints between two consecutive station. Moreover, It is worth noting that the train energy consumption from Ayat to Tor Hailoch station has reduced about 9.709% when compared to the one consumed before optimisation.

Keywords: Energy-efficient train operation, control parameterization technique, constrained optimisation problem, unconstrained optimisation problem.

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LIST OF ABBREVIATIONS AND SYMBOLS

AALRT:	Addis Ababa Light Rail Transit
ERC:	Ethiopian Railway Corporation
LRT:	Light Rail Transit
E-W:	East-West
N-S:	North-South
CPT:	Control parameterization technique
$F_{t,max}$:	Maximum tractive effort
μ:	Adhesion coefficient
w:	Weight of a train
v:	Speed of a train
P:	Power of a locomotive
N:	Newton (unit)
g:	Acceleration of gravity
m:	Mass of a train
km/h:	Kilometre per hour (unit)
F_t:	Tractive effort
W:	Watts (unit)
η:	System coefficient
m/s:	Meter per second (unit)
J:	Joule (unit)
β:	Fraction
$F_{b,max}$:	Maximum braking force
R_b:	Basic resistance
n:	Number of axles
A:	The cross-sectional frontal area of the train
Ft:	Feet (unit)

DP:	Dynamic Programming
GA:	Genetic Algorithm
NLP:	Non-linear Programming
KN:	Kilo Newton (unit)
Mph:	Meter per hour (unit)
Kw:	Kilowatt (unit)
m:	Meter (Unit)
R_g:	Gradient resistance
α:	Gradient angle
Kg:	Kilogram (unit)
t:	Tonne (unit)
m/s^2:	Meter/second square (unit)
E:	Energy consumed by a train
F_b:	Braking force
ρ:	Mass rotating factor
KWh:	Kilowatt-hour (unit)
z_1:	Position of a train
z_2:	Instantaneous speed of a train
C:	Objective function
u(t):	Tractive control forces
N:	Number of subintervals
θ:	Constant control value
σ:	Penalty parameter
f_j:	State constraints
g_j:	Terminal constraints
h_j:	Control boundary constraints

CHAPTER ONE: INTRODUCTION

1.1 Background

The demand for a high capacity and speed transportation system in Addis-Ababa city has become a serious issue due to the gradually increasing population size and economy of the city. It has also been observed that the existing transportation services which are delivered by taxis and buses have highly increased the amount of environmental pollution in recent times. For these reasons, around 2014 government of Ethiopia decided to construct a two route Light Rail Transit (LRT) system. These routes are East-West (E-W) and North-South (N-S) lines with a total length of 34.25 km; E-W Line (line1) from Ayat to Tor Hailoch has 16.9 km in length with 22 stations and N-S Line (line2) from Kality to Menelik II square has17.4 km in length with 22 stations [1].

In general, metro-systems, such as Addis Ababa Light Rail Transit (AALRT), can provide frequent, comfortable, and safe journeys to a large number of passengers in a short period of time, making it the best mode of public transportation for relieving traffic congestion. Furthermore, AALRT can transport more passengers with less energy consumption and is a green transportation mode compared to buses and private car services. However, due to the large-scale operations of AALRT and high-frequency services, a significant amount of energy is consumed for daily operations [1].

According to the rail energy project [2], the majority of energy consumed in metro systems is consumed in traction, aeration, air conditioning, elevators, lighting, and drainage. As shown in Figure 1-1, traction energy takes the most considerable portion for the majority of total energy consumption in railway systems. Thus, minimising the traction energy can optimise the metro-system's energy efficiency as well as its operational costs. Hence, the energy consumption in traction has to be minimised by finding an optimal control sequence of traction and braking force under initial and terminal conditions, state and control constraints, known as an optimal train control problem. However, this optimal control problem belongs to complex computational problems which need robust and efficient solving techniques or algorithms [3], [4].

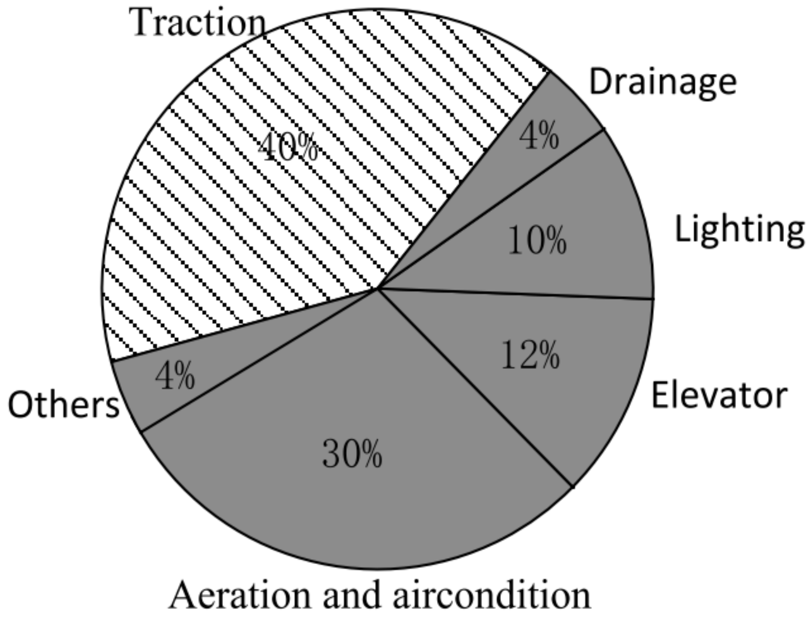


Figure 1-1:Energy consumption in metro-systems [2]

1.2 Problem Statement

Many companies today are concerned about reducing energy consumption in railway operators because they are among the largest consumers of electricity resources [5]. It has been demonstrated that traction energy accounts for the largest share of total energy consumption in railway systems [2]. The train for AALRT system also consumes the more energy that has to be minimised. Using an energy-efficient driving methodology is one of the essential suggestions for reducing train energy consumption. The energy-efficient driving methodology is an optimal control problem that aims to choose the optimal control sequence of tractive and braking forces such that the train energy consumption is minimised when a train moves from the initial point to the end-point of the road in a fixed journey time while adhering to control force constraints as well as safety speed restrictions [6]. Due to the complexity in solving this kind of optimal control problem, an efficient computational approach is required [3], [4].

1.3 Objectives of the Study

1.3.1 General Objective

The main objective of this thesis is to find the optimal train speed profile and tractive/braking control forces using Control Parameterization Technique (CPT) in order to minimise the tractive energy consumed by a train.

1.3.2 Specific Objectives

The specific objectives of this thesis work are:

i. To collect data and formulate a mathematical model of the optimal train control problem for the AALRT system.
ii. To Transform the original train control problem into a constrained non-linear optimization problem using Control Parameterization Technique (CPT).
iii. To develop an algorithm based on fminunc solver for solving a developed optimization problem.

1.4 Scope of the Study

In this work, an optimal train control problem is developed, which aims to find an optimal sequence of the tractive and braking forces such that the train energy consumption is minimised under a fixed journey time and safety speed restriction. However, due to the lack of information from the Ethiopia Railway Corporation (ERC), the gradient force and maximum allowable speed between two consecutive stations are assumed to be constant. The tunnel and curve resistances are not considered. Furthermore, this research merely focuses on the Est-West line (Line1) of the AARLT system.

1.5 Thesis Organisation

The dissertation is organised into five chapters, and they are arranged as follow:

- Chapter 1 gives the introductory part of the dissertation. It generally describes the background to the study, the problem statement, and presents the purposes of the study in terms of general and specific objectives, briefly talk about the scope of the study, and finally shows the work organisation.
- Chapter 2 reviews the theoretical background on the traction system, the basic concept of the control parameterization technique and summarises some of the previous works related to the optimal train control problem strategies.
- Chapter 3 describes the methodology and processes utilised to fulfil the objectives presented in Chapter 1. It first presents the data of the AALRT system/E-W line and then fit them in train model equations. It also contains the mathematical formulation of the optimal train control problem. Finally, it describes the procedures follow to solve the problem.

- Chapter 4 provides the different results simulated in MATLAB software. The first set of results present the train speed profile and control force for each station over the entire length of the E-W line. The last results show the minimum energy consumption between two consecutive stations and their riding modes.
- Chapter 5 provides the conclusion of the study, and the recommendations for future works.

CHAPTER TWO: LITERATURE REVIEW

This chapter reviews the theoretical background on the traction system, the basic concept of the control parameterization technique, and some of the previous essential works related to the optimal train control problem strategies to minimise train energy consumption.

2.1 Traction System

The traction system is the system that causes the propulsion of a vehicle in which the driving or tractive force is obtained from different devices like steam engine drives, electric motors, diesel engine drives, etc. It can be broadly divided into 2 types. The one which utilises electrical energy is known as electric traction systems, and non-electric traction systems, which do not utilise electrical energy for vehicle propulsion. These traction systems can be compared based on their efficiency and the weight-to-output-power relationship. Electric traction systems are now widely used in the railway industry due to their environmental friendliness, high starting torque, ease of speed controllability, and a variety of other factors [7] [8], [9].

2.2 Tractive Force and Adhesion

In railway engineering, the term tractive effort is also known as tractive force, which describes a locomotive's pushing or pulling capability. This force is needed to provide angular and linear acceleration to the train mass, as well as to overcome the gravity component of the train's weight, the frictional and wind resistance of the train, the gradient resistance, and the curve resistance.

The maximum tractive effort ($F_{t,max}$), a locomotive can produce to push or pull a stationary train is the starting tractive effort, which is derived by Equation (2.1) as a function of the wheel-rail adhesion factor(μ) and weight of a locomotive (w) but is independent to the speed (V) and power (P) of a locomotive. The adhesion factor is determined by the material of the rail and wheels, which is commonly 30 per cent for steel wheels on steel rail. The locomotive's maximum power is obtained at maximum torque. The power of the locomotive increases as it raises speed in this mode of operation [10].

$$F_{t,max} = \mu * w \tag{2.1}$$

where $F_{t,max}$ represents the maximum tractive effort of the locomotive in Newton (N)

μ denotes the adhesion coefficient

$w = m * g$, is the weight of the locomotive in Newton (N)

m stands the mass of a train in tonne

g is the acceleration of gravity.

The adhesion coefficient, μ, measures the percentage of locomotive's weight on the drivers that is available as effective tractive effort but is not constant because it varies with rail surface condition rail composition and wheel treads [11]. Since it is difficult to calculate the adhesion coefficient by using a theoretical method, so it can only be expressed through an experimental formula originated from a special test. The experimental formula shows the relationship between the computational adhesion coefficient and the operating speed of train under normal adhesive condition as expressed in Equation (2.2) [11], [12];

$$\mu = \frac{7.5}{3.6v+44} + 0.072 \tag{2.2}$$

where v is the speed of a train.

Thus, the maximum tractive effort becomes;

$$F_{t,max}(v) = \left[\frac{7.5}{3.6v+44} + 0.072\right] \times w \tag{2.3}$$

As the train gains speed, the locomotive's power increases until it reaches the rated horsepower, as shown in Figure 2-1.

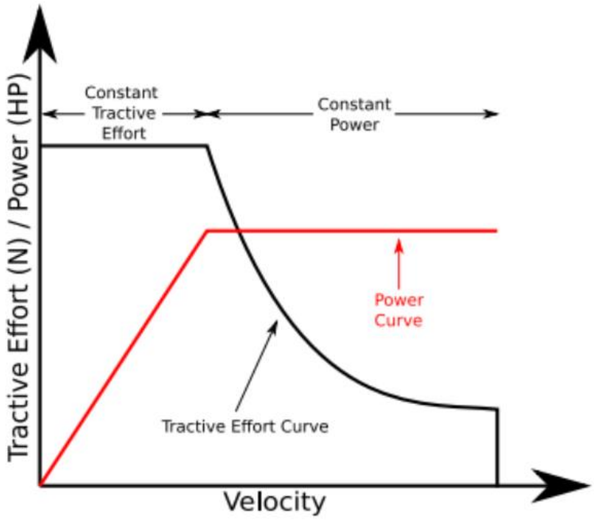


Figure 2-1: The relationship between the power on the train, tractive effort, and velocity [9].

This is known as the constant power region, and as the train accelerates, the tractive effort decreases. The relationship is given by Equation (2.4) [10]:

$$F_t = \frac{\eta * P}{V} \tag{2.4}$$

where P denotes the prime mover's power in watts (W), η denotes system efficiency, V denotes velocity in meter per second (m/s), and F_t denotes tractive force in Joule (J), respectively.

2.3 Braking Force

The braking force is the amount of force required by a train to decelerate, remain standing when parked, or control acceleration on a downhill slope [13].

The brake force available depends on two factors:

1. The adhesion between the wheels and rail being braked, and
2. The normal reaction of the rail on the wheels being braked (and hence on the weight per braked wheel).

In general, it is specified as a fraction (β) of the total weight of the train ($w = mg$), and its expression is given in Equation (2.5). A value of β is typically equal to 0.09 [14].

$$F_{b,max} = w\beta \tag{2.5}$$

2.4 Rolling or Basic Resistance

The rolling or basic resistance is friction-related resistance, and it includes resistance to rolling of the wheels, wind resistance, and friction in the axle bearings [15].

Generally, the unit basic resistance of a train, R_b (in lb), can be computed utilising an empirical expression given by Equation (2.6) [15]:

$$R_b(unit) = a + bv + cv^2 \tag{2.6}$$

where a, b, and c are coefficients describing the various resistive forces which are either dependent, independent, or affected by the square of the train speed, v.

Many formulas have been developed to calculate the coefficients a, b, and c based on the type of train. Davis' equation, which is used for various types of trains, is used to calculate these coefficients in this thesis work. The unit basic resistance formula for the train's rolling or basic resistance is given by Equation (2.7) [16].

$$R_b\ (unit) = \left(1.5 + \frac{18n}{w}\right) + bv + \frac{cA}{10000w}v^2 \tag{2.7}$$

Where

$R_b\ (unit)$ is the unit basic resistance of the vehicle in Ibs $\left(Ibs = 4.45\,N\right)$

n is the number of axles

m is the total weight in tonne of the train (t)

v is the velocity of a train in miles per hour (mph)

A is the cross-sectional frontal area of the train in feet square (ft^2).

Note that: For passenger train b=0.03 and c=0.00034 [16].

Therefore, the basic resistance in Newton (N) is given by Equation (2.8)

$$R_b = R_b\ (unit) * m * g \tag{2.8}$$

2.5 Track Resistance

The track resistance is related to the infrastructure. This resistance is caused by curves, gradients, and tunnels. In calculations, both tunnel and curve resistances are often neglected. The tunnel resistance is often neglected due to the lack of general formulae that expresses this resistance. While the curve resistance, which is caused by the flange of the wheel striking the rail and increasing resistance (see Figure 2-2), is relatively low in comparison to gradient resistance and thus neglected [17], [18].

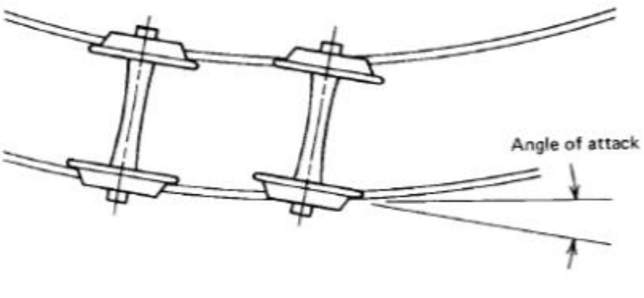


Figure 2-2: Curve resistance [16]

The gradient resistance is the force needed to overcome the gradient (see Figure 2-3). This force can be described by the Equation (2.9) [17].

$$R_g = m * g * sin(\alpha) \tag{2.9}$$

When the gradient is small, the expression in Equation (2.9) becomes;

$$R_g = m * g * tan(\alpha) \approx m * g * \frac{l}{1000} \tag{2.10}$$

where R_g is the gradient resistance in Newton (N)

m is the train mass in kilogram (kg)

g is the acceleration of gravity in meter per second square (m/s^2)

α is the gradient angle in radian or degrees (rad or deg)

l is the gradient in (${}^{o}/_{oo}$)

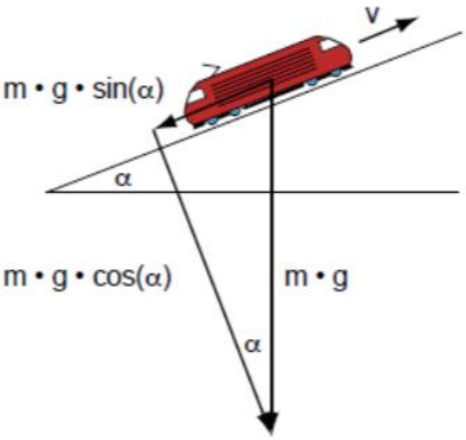


Figure 2-3: Gradient resistance [16]

2.6 Energy Consumption

The energy consumption in the traction system can be calculated in two different ways. The first is based on the total driving resistance and distance travelled by train, while the second is based on total power consumed and run time [19].

2.6.1 Energy Consumption Based on Resistance and Distance

The total resistance (which is tractive effort) is integrated over the total distance travelled by train to calculate energy.

$$E = \int_{s_1}^{s_2} F_t * dx \tag{2.11}$$

where:

F_t is the tractive effort which balances the total resistance for a distance between s_1 and s_2 and is dependent on the train's speed, E is the total energy consumed by the train in Joule (J).

2.6.2 Energy Consumption Based on Power and Time

Total power needed for a given trip at a given speed is expressed as follows:

$$P = F_t * v \tag{2.12}$$

where v is the speed of the train in meter per second (m/s)

The energy consumed by the train to complete the entire journey is calculated by integrating the required power over the total time duration [20].

$$E = \int_0^T Pdt \tag{2.13}$$

where P is the total power in Watt (W), T is the time duration of the journey, and the other parameters have already been specified. The time-based model is preferred for calculating energy consumption in this thesis work.

2.7 Train Model

Newton's second law is well-known for relating the acceleration that the train experiences when all of the different forces,F, are applied to it. Thus, the relationship is given in Equation (2.14)

$$\sum F = m * a \tag{2.14}$$

where $\sum F = F_t - F_b - R_b - R_g$

a is the acceleration in meter per second square (m/s^2)

F_t is the tractive force (N)

F_b is the braking force (N)

R_b is the basic or rolling resistance (N)

R_g is the gradient resistance (N)

Since the train is treated as a particle, and then its equations of the motion are expressed in the following form [21]:

$$\begin{cases} \frac{ds}{dt} = v, \\ \frac{dv}{dt} = \frac{1}{m\rho}\left(F_t - F_b - R_b - R_g\right). \end{cases} \tag{2.15}$$

where $\rho = 1.06$ is the rotating mass factor [22], and the other parameters have already been specified.

2.8 Control Parameterization Technique (CPT)

2.8.1 Introduction

The control parameterization technique (CPT) is well-known as a numerical technique used to solve optimal control problems. The fundamental idea behind CPT is to partition the control space by approximating the control function with a linear combination of basic functions. The optimal control problems are converted to a finite number of decision variables using this approximation strategy. Non - linear programming (NLP) methodologies can then be utilised to solve the approximate problem [23].

CPT is most typically implemented utilising a piecewise-constant approximation strategy, with control values serving as decision variables. The piecewise-constant approximation strategy is popular because it is simple to implement; applying a piecewise-constant control signal is relatively easier than applying a continuously-changing control signal. Furthermore, in several practical control problems (for example, bang-bang control problems), the accurate optimal control is a piecewise-constant function. Other benefits of the piecewise-constant approximation strategy are its strong convergence properties and versatility in dealing with non-standard optimal control problems [23].

2.8.2 Standard Optimal Control Problems

A dynamic model of the following form is involved in the canonical optimal control problem [23], [24]:

$$\begin{cases} \frac{dx}{dt} = f\big(t, x(t), u(t)\big), \qquad t \in [0, T], \\ x(0) = x_0. \end{cases} \tag{2.16}$$

where t represents time, $x(t)$ denotes the state variable at a time, $u(t)$ represents the control signal, x_0 is a predefined initial state, $T > 0$ is a predefined terminal time, and f is a predefined function. The system's time horizon is represented by the interval $[0, T]$.

The control signal in Equation (2.16) is an input variable that must be optimally selected, and it is subject to some bound constraints:

$$a_j \leq u_j(t) \leq b_j, \quad t \in [0, T], \quad j = 1, \dots, r \tag{2.17}$$

where a_j and $b_j, j = 1, \dots, r$, are predefined real number, $\ni a_j < b_j$.

The objective is to achieve a feasible control such that the following system cost function is minimised [23]:

$$g_0 \coloneqq \Phi_0\big(x(T, u)\big) + \int_0^T \mathcal{L}_0\,\big(t, x(t, u), u(t)\big)dt, \tag{2.18}$$

where Φ_0 and $\mathcal{L}_0$ define the terminal and running cost, respectively.

The standard optimal control problem is then described as follows:

Problem P: *Given the dynamic system in Equation (2.16), selected a feasible control u such that the cost function in Equation (2.18) is minimised.*

2.8.3 Piecewise-Constant Control Approximation

To solve the problem P, a piecewise-constant basis function is used to approximate the control signal u. Thus [23];

$$u(t) \approx u^p(t) = \xi^m, \quad t \in [\tau_{m-1}, \tau_m], \quad m = 1, \dots, p, \tag{2.19}$$

where $p \geq 1$ is the number of control sub-intervals, $[\tau_{m-1}, \tau_m]$ is the m^{th} control sub-interval, and ξ^m is the control value on the m^{th} sub-interval. Figure 2-4 depicts this approximation mechanism.

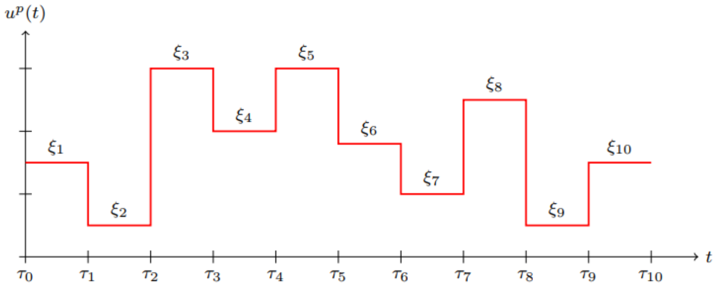


Figure 2-4: Approximation of control signal using piecewise-constant function [23].

In Equation (2.19), the control values, ξ^m, $m = 1, \dots, p$ are optimally selected decision variables, whereas $\tau_m, m = 1, \dots, p$ are prefixed knot points fulfilling

$$0 = \tau_0 < \tau_1 < \tau_2 .. < \tau_{m-1} < \tau_m = T. \tag{2.20}$$

Hence, the control signal can be approximated as piecewise-constant basis functions as expressed in the form below

$$u^p(t, \xi) := \sum_{m=1}^{p} \xi^p \chi_{[\tau_{m-1}, \tau_m)}(t), \quad t = [0, T] \tag{2.21}$$

where $\chi_{[\tau_{m-1}, \tau_m)}(t)$ is the characteristic function denoted as

$$\chi_{[\tau_{m-1}, \tau_m)}(t) = \begin{cases} 1 \ , t \in \ [\tau_{m-1}, \tau_m), \\ 0, t \notin [\tau_{m-1}, \tau_m). \end{cases} \tag{2.22}$$

Under this approximation, the control boundary constraints in Equation (2.17) becomes;

$$a_j \leq \ \xi_j^m \leq b_i, \quad m = 1, \dots, p, \quad j = 1, \dots, r \tag{2.23}$$

The same manner, the dynamic system in Equation (2.16) can be approximated as

$$\begin{cases} \frac{dx}{dt} = f(t, x(t), \xi^m), \qquad , t \in [\tau_{m-1}, \tau_m), \ m = 1, .., p \\ x(0) = x_0. \end{cases} \tag{2.24}$$

Finally, the cost function in Equation (2.18) becomes [23];

$$g_0^p(\xi) \coloneqq \Phi_0\big(x^p(T, \xi)\big) + \int_{m=1}^{p} \int_{\tau_{m-1}}^{\tau_m} \mathcal{L}_0 \left(t, x^p(T, \xi), \xi^m\right) dt, \tag{2.25}$$

Therefore, the approximation problem can be solved using the existing NLP algorithms.

2.9 Related Work

Since the 1960s, numerous researches on the optimal train control problem have been conducted to find energy-efficient train operations. Both numerical and analytical techniques were applied to deal with this problem.

In [25], Asnis et al. and Howlett [26] introduced the first comprehensive study of the optimal train control problem with continuous and bounded control input on a level track, in which the Pontryagin maximum principle was employed to optimize the optimal driving strategy. Khmelnitsky investigated an optimal train operation in [27], considering both an arbitrary variable grade profile and speed limitations. The study's aim was to develop a comprehensive scheme for traction and brake applications that gives the minimum energy consumption while moving the train along a specific route for a pre-determined time. In [28], Liu et al. described an analytical process for calculating the optimal operating sequences of a rail vehicle in order to reduce train energy consumption. The analytical solution that provides the equations and sequence of optimal controls to obtain the change points of control was the outcome of this specific research. Therefore, an energy-efficient train control computer program and calculation algorithm have been derived. Another analytical technique for train energy minimization was developed in [29] to determine optimal switching points while taking steep line gradients into account.

Due to the complex nature of the aforementioned analytical methods, some other scholars have adopted numerical techniques for solving the optimal train control strategy. In [30], Chang et al. and Han et al. [31] suggested a genetic algorithm (GA) for optimising train movements through suitable coast control based on riding comfort, punctuality, and energy consumption. However, in some cases, GA fails to produce good results [32]. Franke et al. [33], Ko et al. [34], and Vasak et al. [35] used dynamic programming (DP) methodologies to determine the optimal traction/braking force, hence the operating optimal train speed profile. However, DP typically necessitates a large number of computational resources. In [36], Wang et al. suggested a mixed-integer linear programming (MILP) method for calculating the optimal train operating speed profile. But, the MILP strategy does not perform well enough in terms of end-time violation.

In this thesis, an efficient numerical scheme known as the control parameterization technique (CPT) is deployed to find an optimal sequence of traction/braking force such that the train energy consumption is minimised while the travelling time is fixed under the constraints of safety speed and control force restrictions. In particular, CPT is a suitable scheme which has been well applied in many different practical problems [37], [38]. The core idea of this scheme is to approximate the control functions (i.e., the tractive and braking control forces) by piecewise-constant functions [23]. In this manner, the optimal train control problem is transformed into a non-linear constrained optimization problem, albeit one with one complex of the state constraints. To overcome this complexity, the constraints are introduced into the cost function via a penalty function so that any violation of the constraints is penalized, transforming a constrained optimization problem (COP) into an unconstrained optimization problem (UOP) that can be easily solved with existing unconstrained solvers.

CHAPTER 3: METHODOLOGY

This chapter describes the methodology and processes utilised to fulfil the objectives of the study. First of all, the track and train information for the AALRT system that are collected from ERC are presented. Then, the mathematical model for AALRT system is developed. Finally, it describes the steps follows to solve the developed mathematical model.

3.1 Data Collection

3.1.1 Description of the Study Area

The Addis Ababa Light Rail Transit (AALRT) Project comprises North-South and East-West lines, with a total length of 34.25 km (N-S line 16.9 km and E-W line 17.35 km). These two lines (N-S and E-W lines) utilise a common track of about 2.7 km. The entire line has 39 stations; the east-west line phase I project, which begins in Ayat and ends in Torhailoch, has 22 stations. While the south-north line phase II project begins at Menelik II Square and ends at Kality, it also contains 22 stations. The two lines utilise 5 common stations. This study focuses on the E-W Line (line1).

Figure 3-1: Map of AALRT [19]

[Source: Addis Ababa (E-W and N-S) route light rail transit project from ERC]

3.1.2 AALRT/E-W Track Information

Table 3-1:The distance, gradient angle and time horizon between two consecutive stations on AARLT/E-W line.

No	Station Name	Route Stations Symbol	Distance [m]	Gradient Angle	Time Horizon [sec]
1	Ayat ---------Meri	EW1---EW2	1300	0.905°	87.4881
2	Meri------CMC	EW2---EW3	1034	0.818°	73.7767
3	CMC----- St. Michael Church	EW3---EW4	896	0.917°	66.6633
4	St. Michael Church --- Civil Service University	EW4---EW5	865	2.767°	65.0654
5	Civil Service University -Management Institute	EW5---EW6	785	1.019°	60.9417
6	Management Institute -- Gurd Shola1	EW6---EW7	906	-0.79°	67.1788
7	Gurd Shola1----- Gurd Shola 2	EW7---EW8	922	0.127°	68.0036
8	Gurd-Shola 2----Megenagna	EW8---EW9	1004	1.031°	72.2304
9	Megenagna-----Lem Hotel	EW9--EW10	1035	-0.358°	73.8283
10	Lem Hotel--------Haya Hulet 1	EW10-EW11	838	-0.491°	63.6737
11	Haya Hulet 1--- Haya Hulet 2	EW11-EW12	694	-1.247°	56.251
12	Haya Hulet 2---St. Urael Church	EW12-EW13	875	-1.394°	65.5809
13	St. Urael Church---Bambis	EW13-EW14	570	-0.849°	49.8592
14	Bambis ----St.Estifanos	EW14-EW15	569	0.155°	49.8077
15	St.Estifanos -Stadium	EW15-EW16	700	0.093°	56.5603
16	Stadium---Laghar	EW16-EW17	865	0.458°	65.0654
17	Laghar-------- Mexico	EW17-EW18	555	0.102°	49.086
18	Mexico ----Tegbared	EW18-EW19	554	-1.591°	49.0345
19	Tegbared ----------St.Lideta	EW19-EW20	936	1.247°	68.7252
20	St.Lideta----------------Cocacola	EW20-EW21	848	1.302°	64.1891
21	Coca Cola---------Tor Hailoch	EW21-EW22	1149	0.504°	79.7046

[Source: Addis Ababa (E-W and N-S) route light rail transit project from ERC]

3.1.3 AALRT/E-W Train Specifications

Table 3-2: Main dimensions of the vehicle

No	Items	Dimension
1.	Axle load	11（1+3%）
2.	Wheelbase (power bogie)	1900 mm
3.	Wheelbase (unpowered bogie)	1800 mm
4.	Wheel diameter (new wheel)	660 mm
5.	The cross-sectional frontal area of the train	10m^2
6.	Number of train axles	6

[Source: Addis Ababa (E-W and N-S) route light rail transit project from ERC]

Table 3-3: Main technical indicators

No	Items	Dimensions
1.	Maximum operation speed	70 km/h
2.	Average travelling speed (average dwelling time of 30 seconds at each station)	20 km/h
3.	Base speed	41 km/h
4.	Maximum Acceleration	1 m/s^2
5.	Actual Running Acceleration	0.5 m/s^2
6.	Maximum service brake deceleration	1.1 m/s^2
7.	Emergency brake deceleration	2.0 m/s^2
8.	Maximum service acceleration	0.9 m/ s^2
9.	Rated speed of a motor	1800 rpm
10.	Rated operation frequency	71 HZ
11.	Poles of motor	6
12.	system efficiency	0.96
13.	Power per electric motor	130 KW

[Source: Addis Ababa (E-W and N-S) route light rail transit project from ERC]

Table 3-4: Vehicle weight (in Tonne)

Loads	**Carbody weight**	**Passenger weight**	**Total weight in tons**
Empty vehicle (t)	44	0	44
Seating capacity (t)	44	15.24	59.24
Overload capacity (t)	44	19.02	63.02
Axle load	11（1+3%）ton		

Note: Assuming that each passenger weighs 60kg on average.

[Source: Addis Ababa (E-W and N-S) route light rail transit project from ERC]

3.1.4 AALRT/E-W Kinematic Train Movement Models

Moving a vehicle or a train along a rail/route involves numerous forces, such as tractive and braking forces, basic resistance (rolling resistance), and line resistance (gradient resistance). These forces depend on the train and track parameters of a railway system. In this thesis work, they are computed based on AALRT system/E-W line.

i. Allowable Tractive force

This is related to vehicle characteristics and is primarily obtained from the manufacturer's data sheet. The locomotive of the AALRT system has a maximum speed of 4377rpm and a rated power of 130KW at a rated speed of 1800rpm. To determine maximum power, we must first determine maximum torque, assuming that maximum torque is achieved during maximum power. The motor's speed must be lower than the synchronous speed in order to produce maximum torque [39].

$$\begin{cases} T_{rated} = \frac{P_{rated}}{W_{rated}} \\ T_{max} = \frac{P_{max}}{W} \end{cases} \tag{3.1}$$

The margin of stability is defined as the ratio of maximum torque to rated torque. This ratio is between 2–3 range, but it is difficult to obtain a ratio greater than 2.3, and it is preferable to have a ratio greater than or equal to 1.6 [39].

$$\frac{T_{max}}{T_{rated}} \geq 1.6 \Longrightarrow T_{max} = 1.6 * T_{rated} \tag{3.2}$$

Alternatively;

$$\frac{P_{max}}{W} = 1.6\frac{P_{rated}}{W_{rated}} \tag{3.3}$$

Substituting the value of rated power and speed, yields;

$$\frac{P_{max}}{W} = \frac{1.6\times130\times10^3}{1800} \tag{3.4}$$

Then,

$$\frac{P_{max}}{W} = 116 \tag{3.5}$$

For AALRT system, motor of train used has six poles with rated operating frequency of 71 HZ. Thus,

$$W = (1-s)W_{sy} \tag{3.6}$$

where $W_{sy} = \frac{120f}{P} = \frac{120*71}{6} = 1420rpm$. As seen, the synchronous speed is less than the rated speed ($W_{rated} = 1800\,rpm$), this means that the motor always works in generator mode, which is not right, meaning the rolling stock specification given from ERC is not correct. In this study, we suppose that the motor of train has four poles which means $W_{sy} = 2130rpm$. Taking maximum torque at 25%. Thus,

$$W = 0.75\times2130 = 1597.5rpm \tag{3.7}$$

Substituting (3.7) into (3.5), gives;

$$P_{max} = 116\times1597.5 = 184.60\,KW \tag{3.8}$$

Now, using the manufacturer's data sheet and the calculation performed above, we can calculate the tractive force limit of the motor used by AALRT system.

- At low speed, the maximum tractive force times by base speed must be greater than or equal to the maximum power delivered by the four motors. Thus,

$$F_{t,max}\times V_{base} = 4\times184.60\,KW \tag{3.9}$$

Since , the base speed for the train of AALRT system is 41 km/h. Then;

$$F_{t,max} = 64.77KN \tag{3.10}$$

- At high speed, the maximu traction force decreases as the speed increases. In this case, it is called adhesion (A) and depends on the wheel-rail adhesion coefficient (μ). It is computed using Equation (2.1) and AALRT/E-W line data. Thus,

$$A = F_{t,max} = \mu * w \tag{3.11}$$

where $w = mg$ and $\mu = \frac{7.5}{3.6v+44} + 0.072$. Alternatively;

$$A = \left[\frac{7.5}{3.6v+44} + 0.072\right] \times mg \tag{3.12}$$

Then,

$$A = \left[\frac{7.5}{3.6v+44} + 0.072\right] \times (59.24 \times 9.81) = \left[\frac{4358.583}{3.6v+44} + 42.022\right] KN \tag{3.13}$$

Therefore, Equation (3.10) and Equation (3.13) can be combined to present the characteristics of the tractive force at low and high speeds. It gives;

$$F_{t,max} = \begin{cases} 64.77 & if \;\; 0 \le v \le 11.2 \\ \frac{4358.583}{3.6v+44} + 42.022 & if \; 11.2 < v \le 19.4 \end{cases} \tag{3.14}$$

where v is the train speed in m/s and F_t is the tractive force in KN

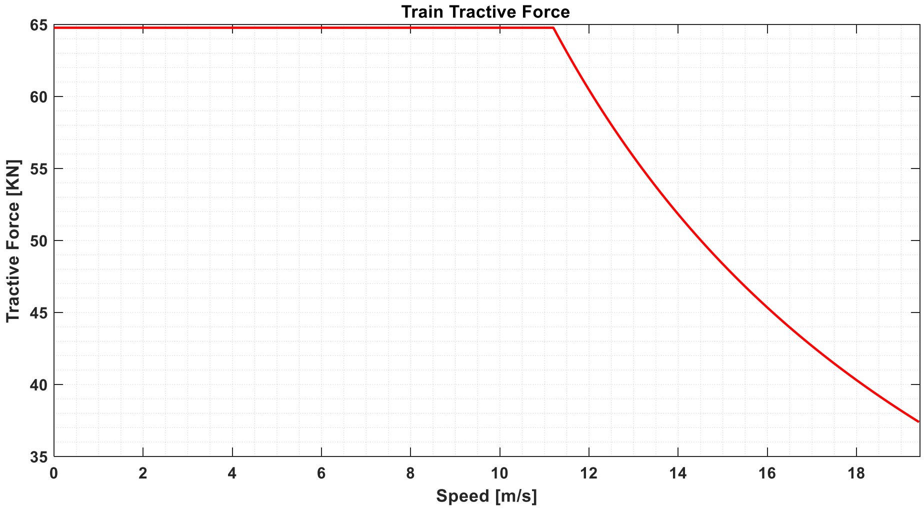


Figure 3-2: Characteristics of the Tractive force for AALRT/E-W line

ii. Allowable braking force

The allowable braking forces is calculated using Equation (2.5) and AALRT/E-W line data. It gives;

$$F_{b,max} = w\beta = 59.24 * 9.81 * 0.09 = 52.30KN \qquad (3.15)$$

iii. Basic resistance

Since the AALRT system has no specific formula for calculating the basic resistance, in this thesis, Davis's equation described in Equations (2.7-2.8) together with AALRT/E-W line data is used to compute this basic resistance. It yields;

$$R_b(v) = 1.9868 + 0.0415v + 0.0011v^2 \quad [KN]. \qquad (3.16)$$

iv. Gradient resistance

Gradient resistance is related to elevation changes and track alignment. Due to a lack of information from ERC, the gradient resistance between two consecutive stations is considered constant. Using the expression given in Equation (2.9) and AALRT/E-W line data. Thus, gradient resistance is equal to

$$R_g(s) = 618.2262\sin(\alpha(s))\, KN \qquad (3.17)$$

where $\alpha(s)$ is the line gradient angle between two consecutive station. For instance, the gradient angle for Lideta-Coca-Cola is $\alpha(s) = 1.302$. Then,

$$R_g = 14.047\, KN \qquad (3.18)$$

3.2 The Optimal Train Control Problem

3.2.1 Problem Formulation

The train's mathematical model is considered as the most crucial part of the train optimal control problem; usually, the mass-point model of the train described in Equation (2.15) is used. Let z_1 and z_2 be the position and instantaneous speed of a train in the route. Then, the equations (2.15),(3.16) and (3.17) can be rewritten as

$$(2.15) \Rightarrow \begin{cases} \frac{dz_1(\mathrm{t})}{dt} = z_2(\mathrm{t})\,, \\ \frac{dz_2(t)}{dt} = \frac{1}{m\rho}\big(u(t) - R_b(z_2) - R_g(z_1)\big). \end{cases} \qquad (3.19)$$

$$(3.16) \Rightarrow R_b(z_2) = 1.9868 + 0.0415z_2 + 0.0011{z_2}^2 \quad [KN]. \qquad (3.20)$$

$$(3.17) \Rightarrow \quad R_g(z_1) = 618.2262\sin(\alpha(z_1))\, KN \qquad (3.21)$$

where $u(t)$ is the traction control force bounded between maximum tractive force, $F_{t,max}$ and maximum braking force , $F_{b,max}$, t is the time taken by a train.

Alternatively, substituting Equations (3.20-3.21) into Equation (3.19) ,and replacing the value of $m = 59.24\ tonnes$ and $\rho = 1.06$,we get;

$$\begin{cases} \frac{dz_1}{dt} = z_2(t), \\ \frac{dz_2}{dt} = (0.149u(t) - 0.0297 - 0.0006z_2(t) - 0.00002z_2^2(t) - 9.255\sin(\alpha)). \end{cases} \tag{3.22}$$

Let *T* be the time taken by train between two stations determined by the timetable, and L is the distance between two consecutive stations. The speed and position at the starting point of the station must satisfy

$$z_2(0) = 0,\ \ z_1(0) = 0 \tag{3.23}$$

whilst the speed and position at the end-point of the station must satisfy

$$z_2(T) = 0,\ z_1(T) = L \tag{3.24}$$

The amount of energy consumed by the train during the journey is expressed as

$$C = \int_0^T u(t) * z_2(t)dt \tag{3.25}$$

In general, the behaviour of the optimal train control problem is that when a train moves from the starting point to the end-point of the road in a fixed travelling time, T, under the constraints of control force limitation and safety speed limit. Hence, the task is to obtain the sequence of the optimal tractive and braking forces such that the train energy consumption is minimised.

Thus, the optimal train control problem can be expressed as follows:

Problem Q: *Considering the train dynamics system in Equation (3.22) with the conditions at the starting point in Equation (3.23) and the end-point in Equation (3.24), find the optimal sequence of traction and braking forces of the train,u, such that the objective function, C, in Equation (3.25) is minimised subject to the control constraint conditions*

$$-F_{b,max} \leq u(t) \leq F_{t,max} \tag{3.26}$$

and state constraints

$$0 \leq z_2(t) \leq V_{max} \tag{3.27}$$

where

$F_{t,max} = \begin{cases} 64.77 & if\ 0 \leq v \leq 11.2 \\ \frac{4358.583}{3.6v+44} + 42.022 & if\ 11.2 < v \leq 19.4 \end{cases}$ *is the maximum allowa tractive force,*

$F_{b,max} = 52.30KN$ *is the maximum allowable braking force,*

$V_{max} = 19.4m/s$ *is the maximum acceptable speed between two consecutive stations.*

3.2.2 Problem Solving Procedure

A. Control Parameterization Technique

In this thesis, the optimal train control problem is solved using a general and effective numerical technique known as the control parameterization technique (CPT) [40]. This technique approximates the control signal as piecewise-constant functions, which results in an approximate optimization problem with a finite number of decision variables. The approximate problem can then be solved using existing NLP techniques [23].

Recall that the core concept of CPT is to split the time horizon T into N subintervals $[t_{p-1}, t_p]$, $p = 1, \dots, N,$ where t_p is a fixed knot point that satisfies

$$0 = t_0 \leq t_1 \leq \cdots \leq t_{p-1} \leq t_p = T \tag{3.28}$$

where $N \geq 1$ is a pre-determined integer. Referred to Equation (2.21), the control force functions can be approximated as piecewise-constant basis functions as expressed in the form below

$$u(t) \approx u^N(t) = \sum_{p=1}^{N} \theta^p \chi_{[t_{p-1}\ t_p)}(t), \tag{3.29}$$

where θ^p is the constant control value on the p^{th} subinterval and $\chi_{[t_{p-1}\ t_p)}(t)$ is the characteristic function denoted as

$$\chi_{[t_{p-1}\ t_p]}(t) = \begin{cases} 1\ , t \in\ [t_{p-1}\ t_p), \\ 0\ , t \notin [t_{p-1}\ t_p). \end{cases} \tag{3.30}$$

Thus, the train dynamics of the system in Equation (3.22) can be approximated as

$$\begin{cases} \dfrac{dz_1(t)}{dt} = z_2(t), \\ \dfrac{dz_2(t)}{dt} = \left[\sum_{p=1}^{N} 0.149\theta^p \chi_{[t_{p-1}\ t_p]}(t) - 0.0297 - 0.0006z_2(t) - 0.00002{z_2}^2(t) - 9.255\sin(\alpha)\right] \end{cases} \qquad (3.31)$$

the control boundary constraint becomes

$$-F_{b,max} \leq \theta^p \leq F_{t,max}, \quad p = 1, \dots, N. \qquad (3.32)$$

Let the solution of the dynamics system of the train in Equation (3.31) with the initial condition in Equation (3.23) be represented as $z^N(t) = [{z_1}^N(t), {z_2}^N(t)]^T$. Hence, the terminal constraint can be recast as

$$z^N(T) = [z_1^N(T), z_2^N(T)]^T \qquad (3.33)$$

the state constraint can be rewritten as

$$0 \leq z_2^N(t) \leq V_{max} \qquad (3.34)$$

and under this approximation, the objective function in Equation (3.25) can recast as

$$C^N = \int_{t_{p-1}}^{t_p} \sum_{p=1}^{N} \theta^p * z_2^N(t)\mathrm{dt}. \qquad (3.35)$$

Therefore, the train control problem Q can be reformulated as below:

Problem Q_N: *Select the constant control values of $\theta^p, p = 1, \dots, N$ such that the objective function in Equation (3.34) is minimised subjected to the system in Equation (3.31) with the initial conditions in Equation (3.23) and the constraints in Equations (3.32)-(3.34).*

Basically, the state vector could be deemed as an implicit function of θ, and this means that the solution of the dynamic system given in Equation (3.31) with the initial conditions in Equation (3.23) is represented as $z^N(t) = z^N(t/\theta)$. Therefore, the objective function in Equation (3.35) can also be considered as an implicit function of θ:

$$C^N(\theta) = \int_{t_{p-1}}^{t_p} \sum_{p=1}^{N} \theta^p * z_2^N(t/\theta)\mathrm{dt}. \qquad (3.36)$$

Similarly, the state constraint can be viewed as implicit functions of θ:

$$0 \leq z_2^N(t/\theta) \leq V_{max}, \qquad (3.37)$$

which can be recast as

$$\begin{cases} f_1(\theta) = -z_2^N(t/\theta) \leq 0, \\ f_2(\theta) = z_2^N(t/\theta) - V_{max} \leq 0. \end{cases} \tag{3.38}$$

Similar manner, the terminal constraint in Equation (3.33) can be recast as

$$\begin{cases} g_1(\theta) = z_1^N(T/\theta) - z_1(T) = 0, \\ g_2(\theta) = z_2^N(T/\theta) - z_2(T) = 0. \end{cases} \tag{3.39}$$

Again, the control boundary constraint in Equation can be rewritten in the following form;

$$\begin{cases} h_1(\theta) = -\theta^p - F_{b,max} \leq 0, \\ h_2(\theta) = \theta^p - F_{t,max} \leq 0. \end{cases} \qquad p = 1, \dots, N \tag{3.40}$$

As a matter of fact, the problem $\mathrm{Q_N}$ can be viewed as a non-linear optimization problem $\mathrm{Q_N}(\theta)$;

$$\begin{aligned} &\min C^N(\theta) \\ &subject\ to\ f_j(\theta) \leq 0, \quad j=1,2, \\ &\qquad g_j(\theta) = 0, \quad j=1,2, \\ &\qquad h_j(\theta) \leq 0 \quad j=1,2\ and\ \ p=1,2,\dots,N. \end{aligned} \tag{3.41}$$

B. Transformation of a Constrained Optimization Problem (COP) into an Unconstrained Optimization Problem (UOP)

$\mathrm{Q_N}(\theta)$ can be solved utilising the existing NLP optimization techniques. However, it includes the inequality constraint $f_j(\theta) \leq 0, j = 1,2$, which strictly limits the state variable $z_2(t)$ at an infinite number of time points in the interval $[0, T]$, as well as terminal condition equality and control bound inequality constraints, making it difficult to solve this non-linear optimization problem directly. By adopting a penalty function to penalise constraint violations, a constrained optimization problem (COP) is transformed into an unconstrained optimization problem (UOP) that can be solved efficiently using fminunc solver [3], [41].

The penalty function is defined as [3], [41];

$$penalty = \frac{1}{2}\sigma \left[\sum_{j=1}^{dimf}\left[max\left(0, f_j(\theta)\right)\right]^2 + \sum_{j=1}^{dimg}\left[g_j(\theta)\right]^2 + \sum_{j=1}^{dimh}\left[max\left(0, h_j(\theta)\right)\right]^2\right] \tag{3.42}$$

where $\sigma > 0$ is a penalty parameter and dim is the dimension of $f, g\ and\ h$ constraints

Then, the cost function becomes;

$$C_\sigma^N(\theta) = C^N(\theta) + penalty \tag{3.43}$$

As a result, the train control problem $\mathrm{Q_N}(\theta)$ can be reformulated as follows:

$\boldsymbol{Q_N^{\sigma}(\theta)}$: *Find the constant control values of* θ^p *such that the objective function in Equation (3.43) is minimised. Thus,*

$$\min_{\theta^p} C_{\sigma}^{N}(\theta) \tag{3.44}$$

where $p = 1, \dots, N$. $\mathrm{Q}_{\mathrm{N}}^{\sigma}(\theta)$ is then a non-linear optimization problem without constraints that can be directly solved using fminunc solver.

C. Algorithm for Solving Problem $\mathbf{Q}_{\mathbf{N}}^{\boldsymbol{\sigma}}(\boldsymbol{\theta})$

The steps used to solve the problem $\mathrm{Q}_{\mathrm{N}}^{\sigma}(\theta)$ is summarised as bellows:

Step 1: Initialisation

- ✓ Define the number of subintervals, N.
- ✓ Set the value of Penalty factor σ.
- ✓ Sets $\theta^p = 0$, $p = 1, \dots, N$.

Step 2: State trajectories

Generate SIMULINK model of train dynamic systems and use ode45 solver to determine the state variables.

Step 3: cost function and penalty

Utilise the z_1(t) and z_2(t) obtained in step 2 to evaluate the cost function and penalty function values.

Step 4: Optimization

Use MATLAB fminunc solver together with the information from steps 2-3 to find the optimal values of θ.

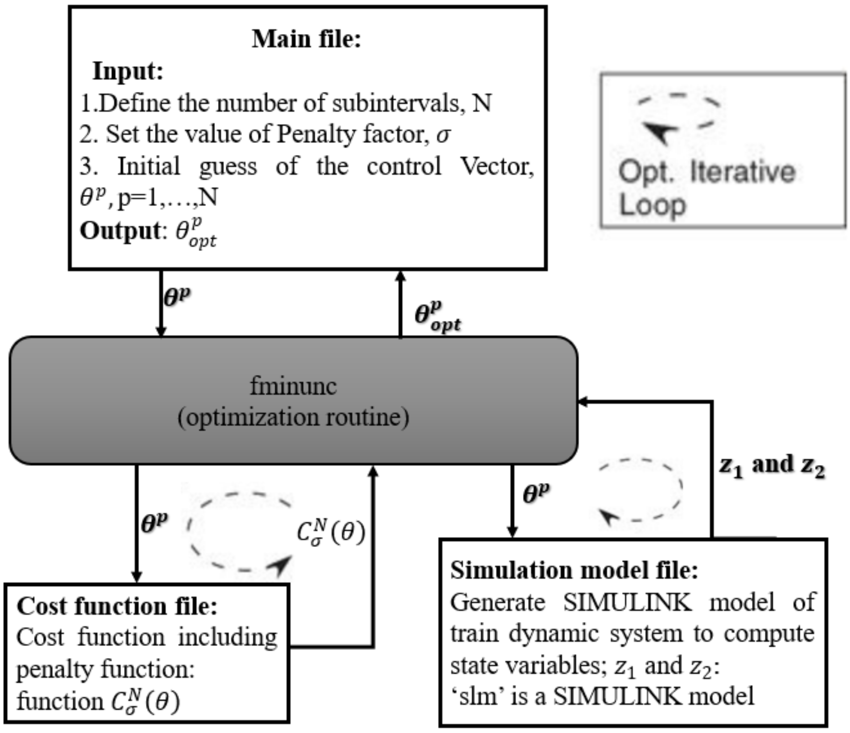


Figure 3-3: Optimization flowchart using fminunc solver

CHAPTER 4: SIMULATION RESULTS AND DISCUSSION

4.1 Introduction

The goal of this chapter is to test the computational method discussed in chapter three through simulation. To accomplish this, the AALRT system/East-West line (from Ayat station to Tor Hailoch station) is considered.

The implemented solutions are based on the MATLAB fminunc function, which is a part of optimization toolbox solvers. The SIMULINK model of the train dynamic system is generated to compute the state trajectories variables (i.e., the speed and position of a train at each instant of time) using the ode45 solver, as seen in Figure 4-1. The simulated state trajectories values are used to compute the cost function. Then, the unconstrained non-linear optimization problem is solved using a fminunc solver.

During simulation, the gradient force and speed limit between two consecutive stations are considered to be constant due to the lack of information from ERC. The number of subintervals for the time horizon, T, are chosen to be $N = 10$ and the penalty factor, $\sigma = 1000$, for the penalty function to penalise constraint violations.

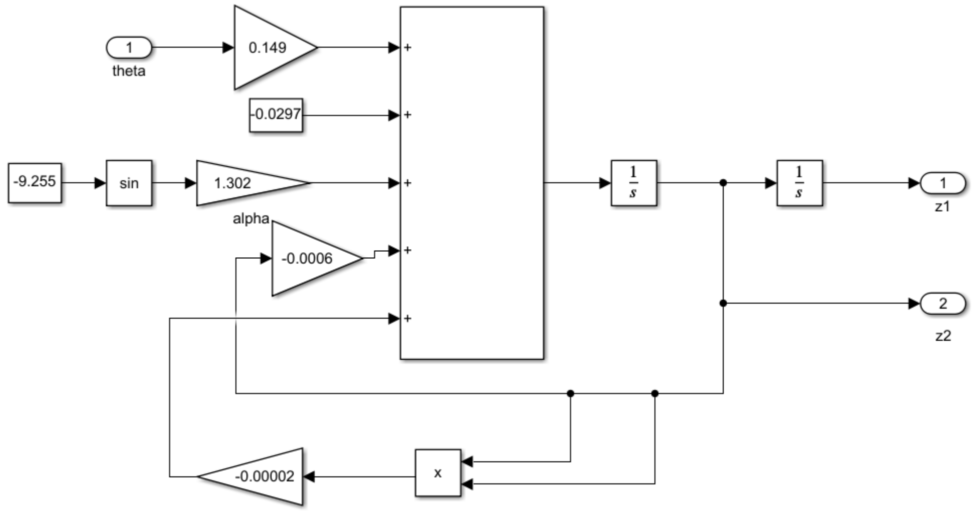
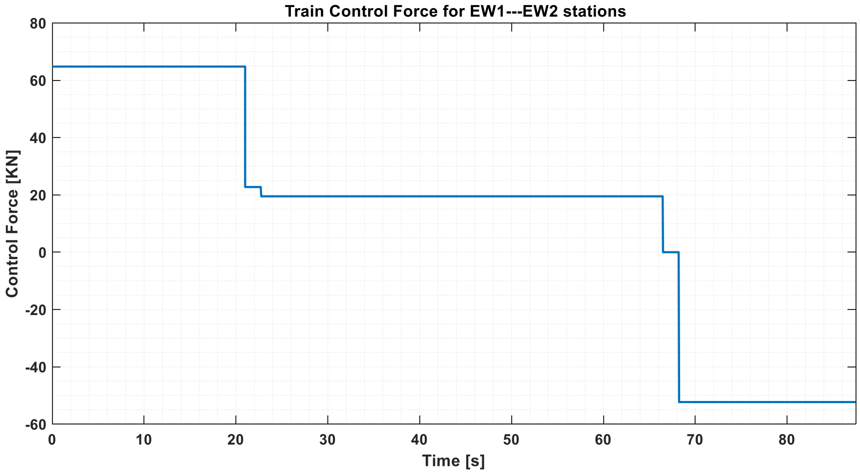


Figure 4-1: Simulink model of a train dynamic system’slm’

4.2 Optimal Train Speed Profile and Control Force

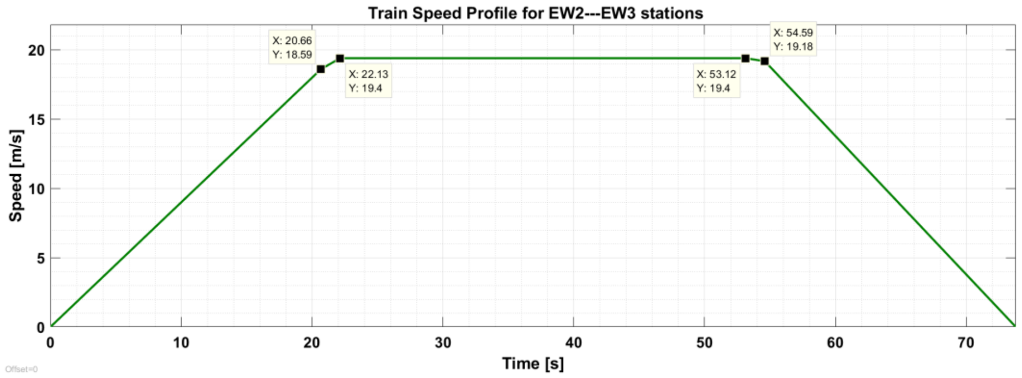
The optimal control force and speed trajectories for the AALRT/E-W line from Ayat station to Tor Hailoch station are presented in this section. Figures 4-2 to 4-22 show the train speed profile and control force solution for each station. The figures clearly demonstrate that the speed profile meets the maximum speed limits ($V_{max} = 19.4\ m/sec$) between two consecutive stations. It has also been observed that the control force successfully satisfies the boundary control constraints (i.e., $u_{max} = 64.77\ KN\ and\ u_{min} = -52.30\ KN$) between two consecutive stations. As can be seen from the Figures, the riding mode of a train for EW1-EW2,EW2-EW3,EW3-EW4,EW6-EW7,EW7-EW8,EW9-EW10,EW10-EW11,EW11-EW12,EW13-EW14,EW14-EW15,EW15-EW16,EW16-EW17,EW19-EW20,EW21-EW21,EW21-EW22 stations is a motor-cruise-coast-brake while for EW4-EW5,EW5-EW6,EW12-EW13,EW17-EW18,EW18-EW19 stations is a motor-cruise-brake. Thus, the stations which have the speed profile with coasting mode are energy saving as compared to those without a coasting regime.

(a)

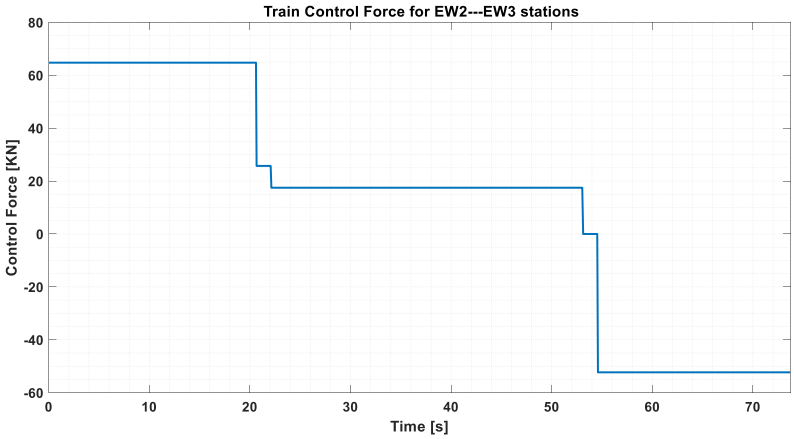


(b)

Figure 4-2: (a) Train Speed Profile, (b) Control Force from Ayat to Meri

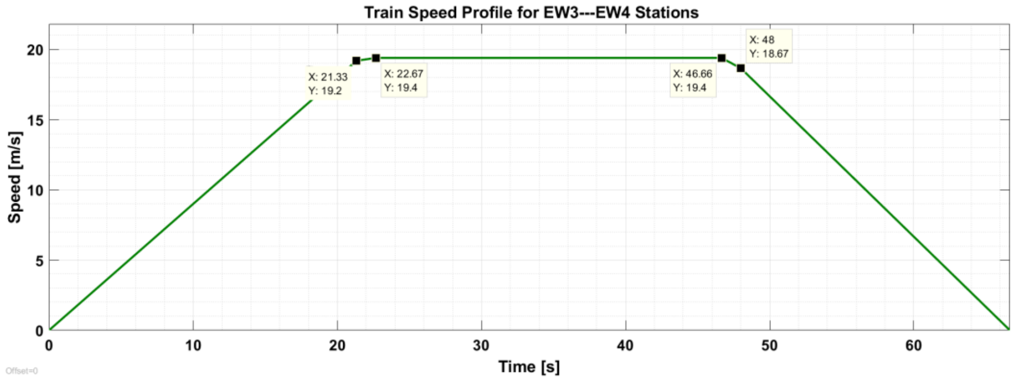


(a)

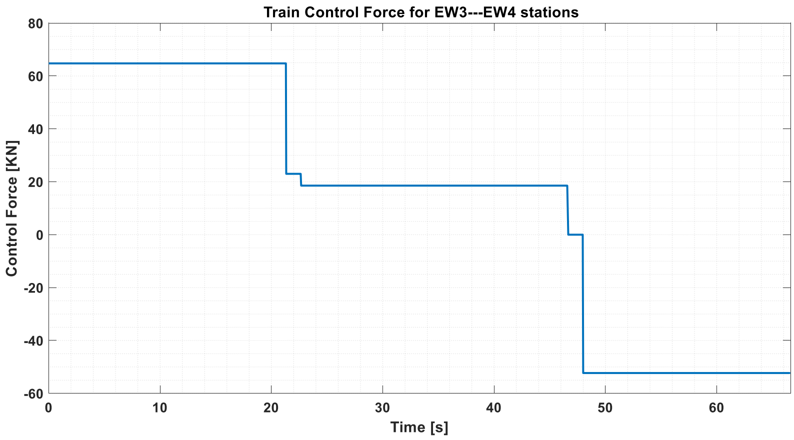


(b)

Figure 4-3: (a) Train Speed Profile, (b) Control Force from Meri to CMC

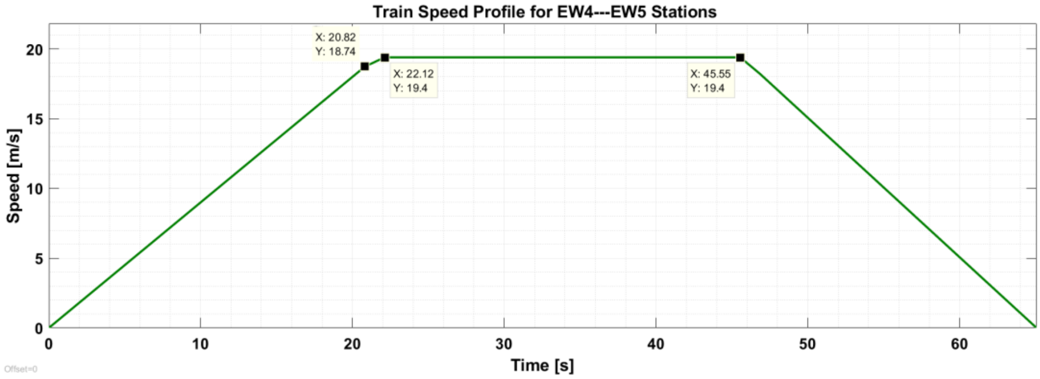


(a)

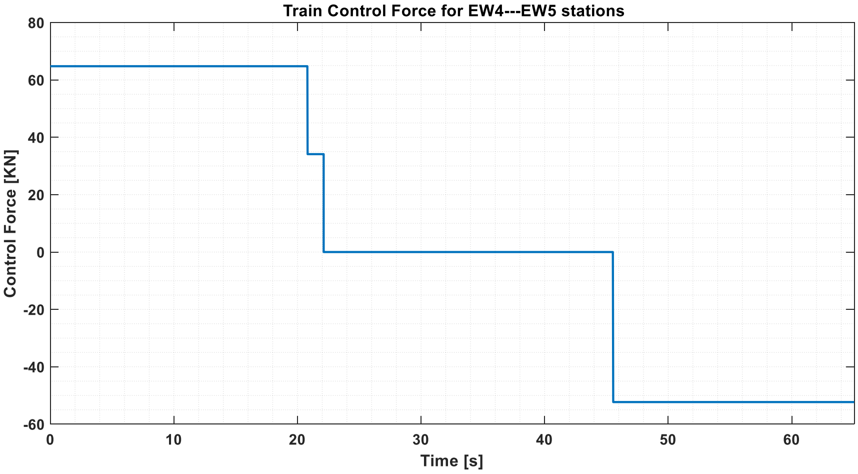


(b)

Figure 4-4: (a) Train Speed Profile, (b) Control force from CMC to St. Michael Church

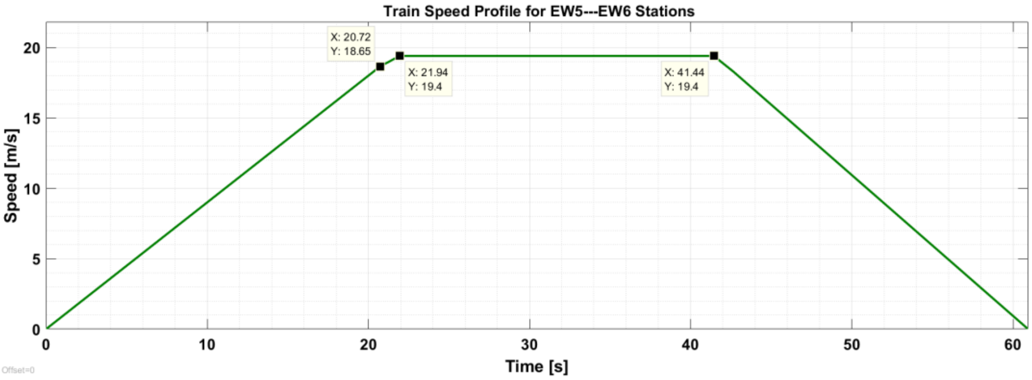


(a)

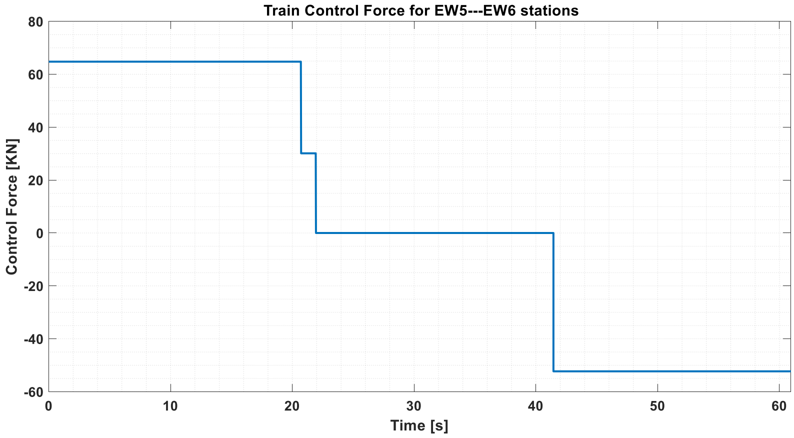


(b)

Figure 4-5: (a) Train Speed Profile, (b) Control Force from St. Michael Church to Civil Service University

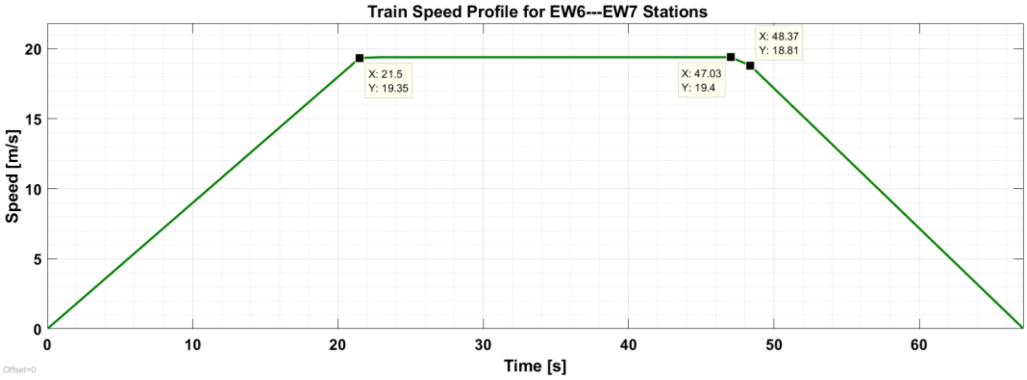


(a)

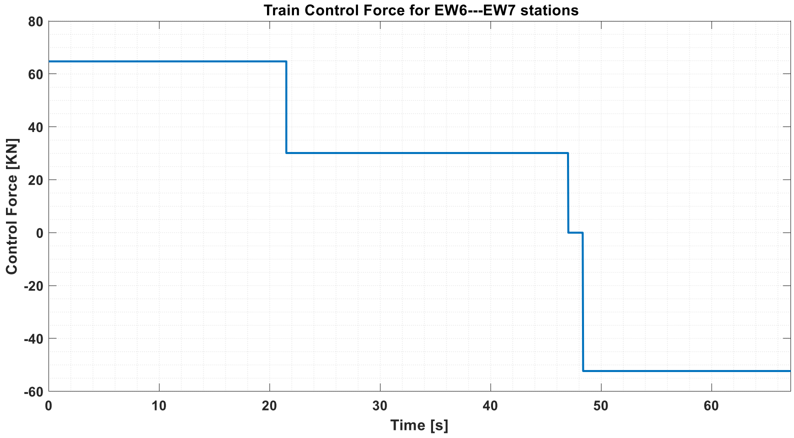


(b)

Figure 4-6: (a) Train Speed Profile, (b) Control Force from Civil Service University to Management Institute

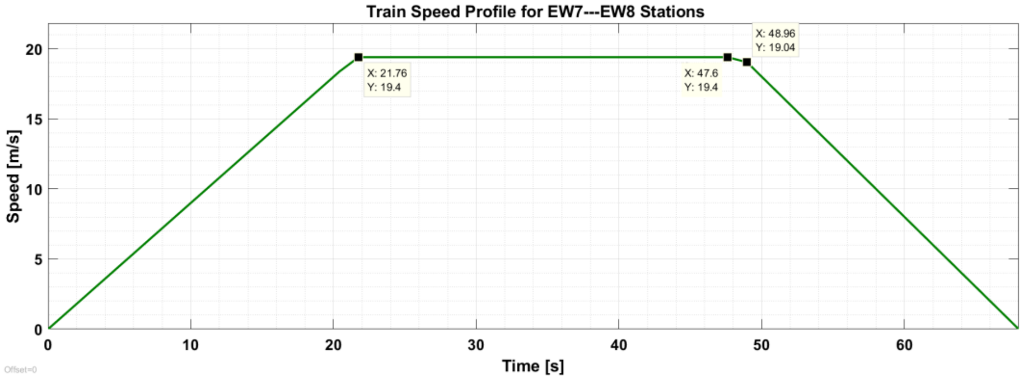


(a)

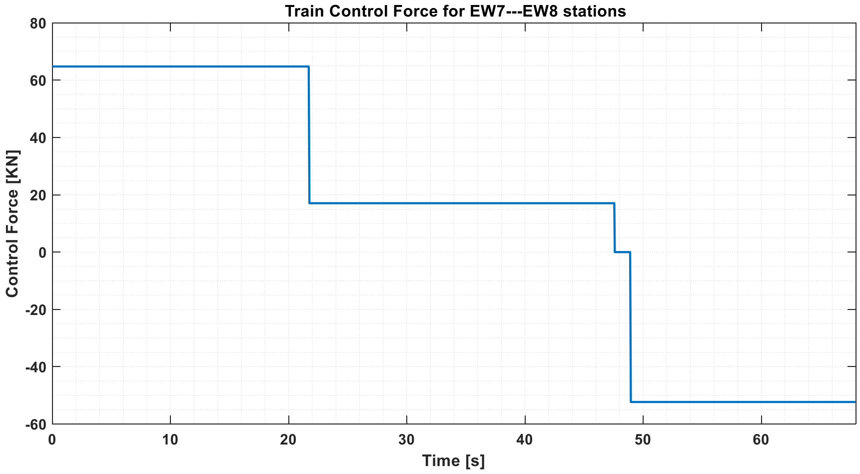


(b)

Figure 4-7: (a) Train Speed Profile, (b) Control Force from Management Institute to Gurd Shola1

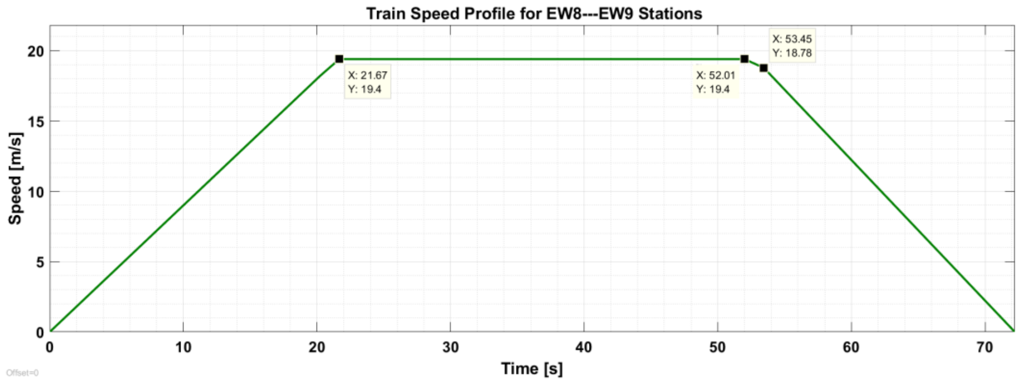


(a)

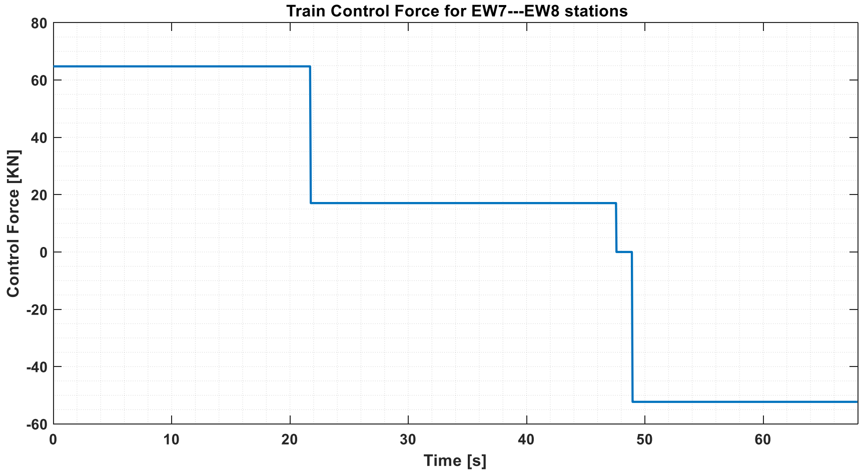


(b)

Figure 4-8: (a) Train Speed Profile, (b) Control Force from Gurd Shola1----- Gurd Shola 2

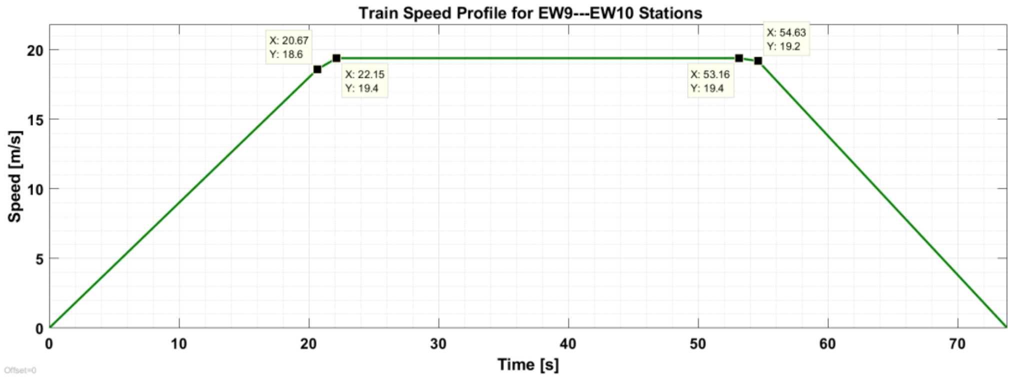


(a)

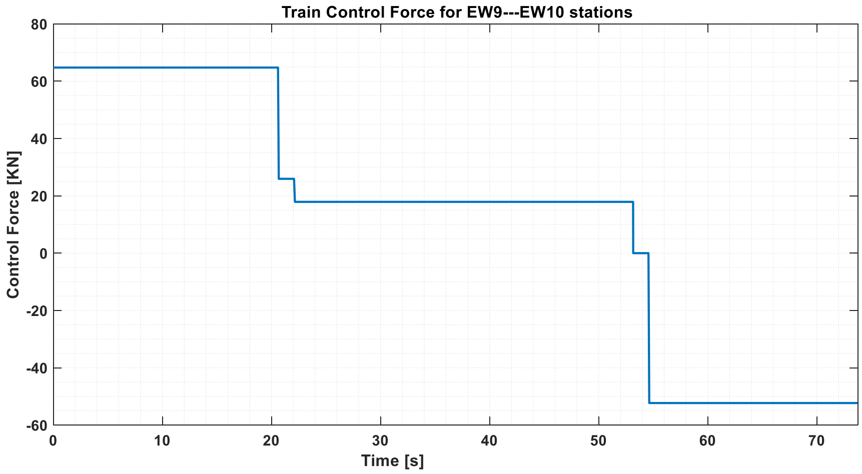


(b)

Figure 4-9: (a) Train Speed Profile, (b) control force from Gurd-Shola 2 to Megenagna

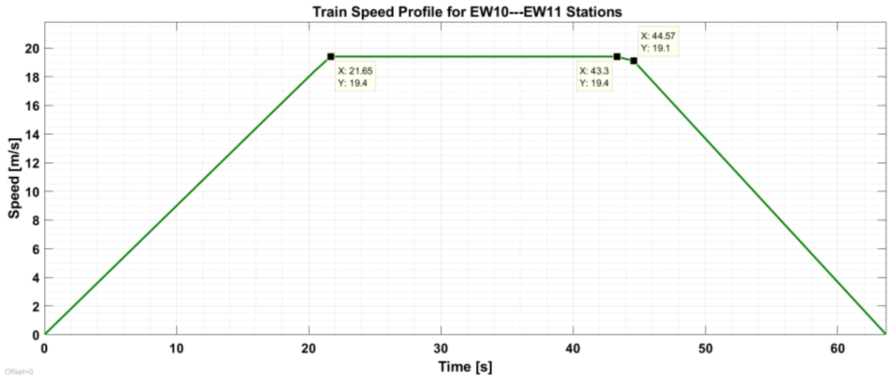


(a)

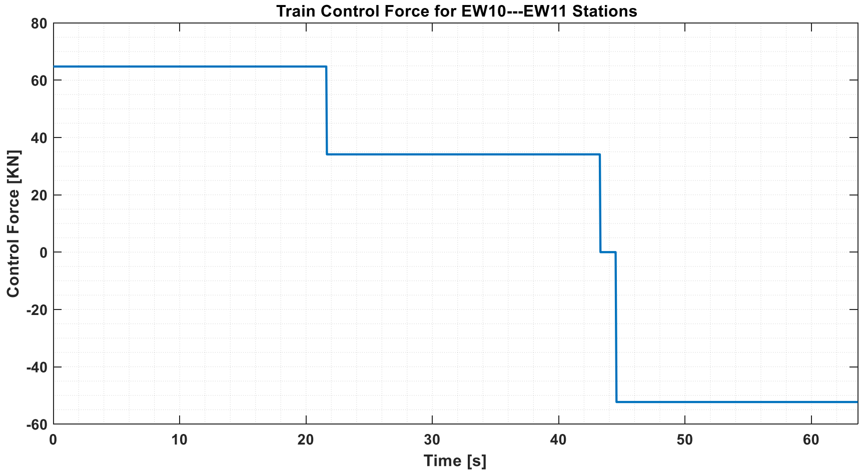


(b)

Figure 4-10: (a) Train Speed Profile, (b) Control Force from Megenagna to Lem Hotel

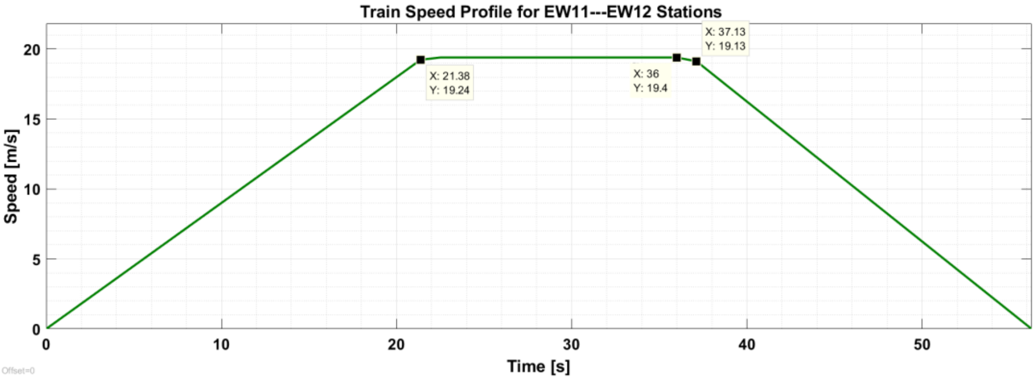


(a)

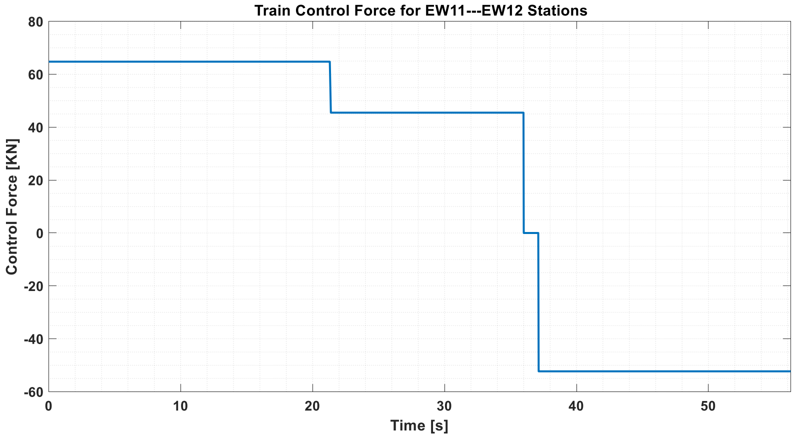


(b)

Figure 4-11: (a) Train Speed Profile, (b) Control Force from Lem Hotel to Haya Hulet 1

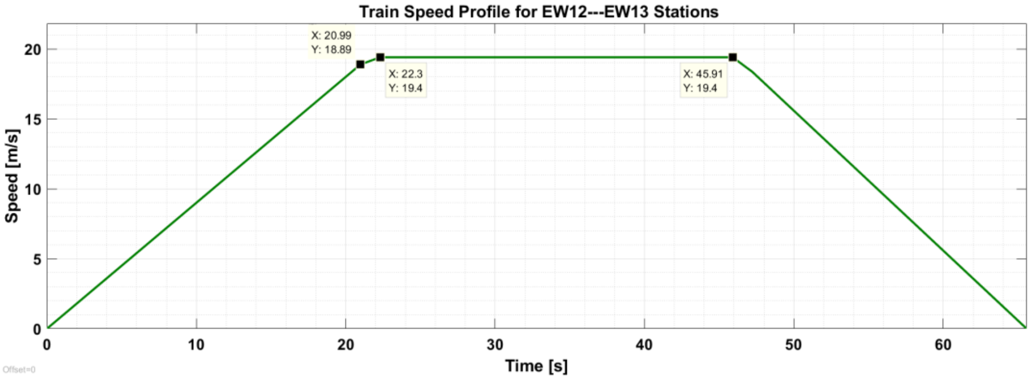


(a)

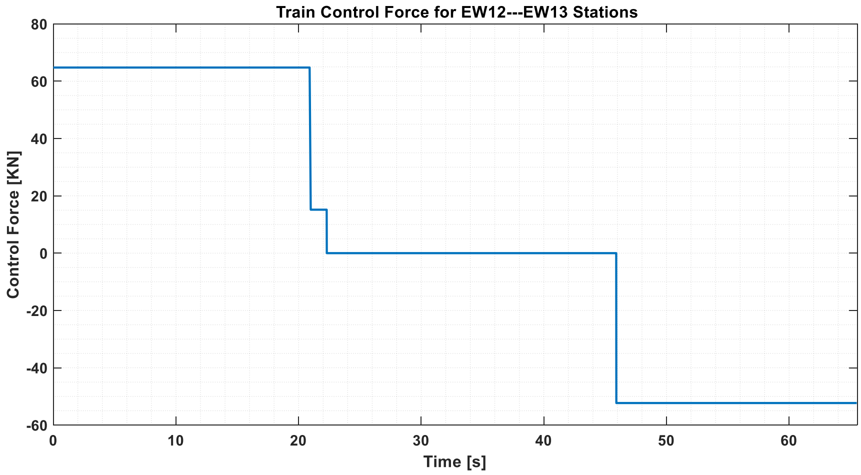


(b)

Figure 4-12: (a) Train Speed Profile, (b) Control Force from Haya Hulet 1 to Haya Hulet 2

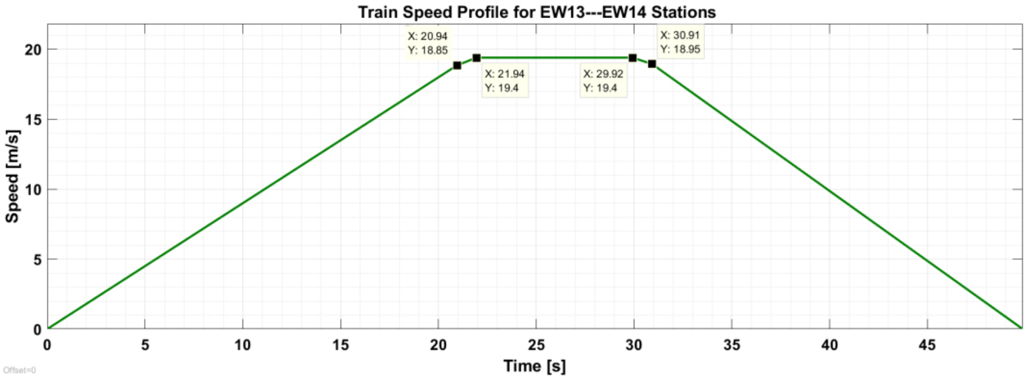


(a)

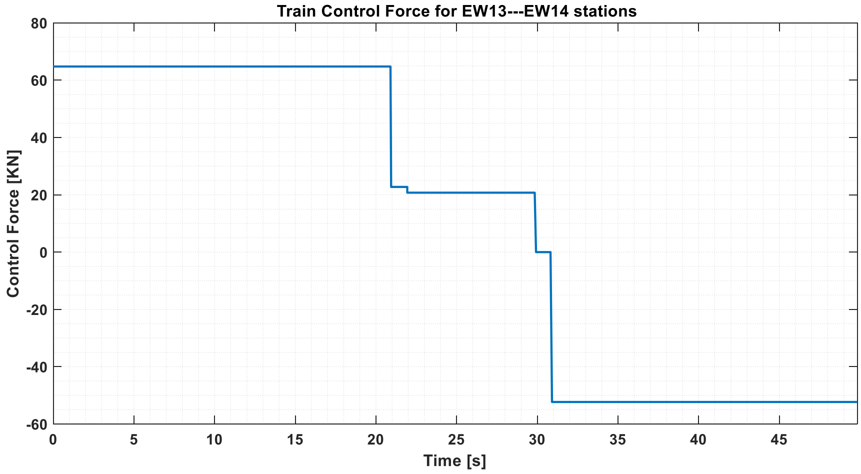


(b)

Figure 4-13: (a) Train Speed Profile, (b) Control Force from Haya Hulet 2 to St. Urael Church

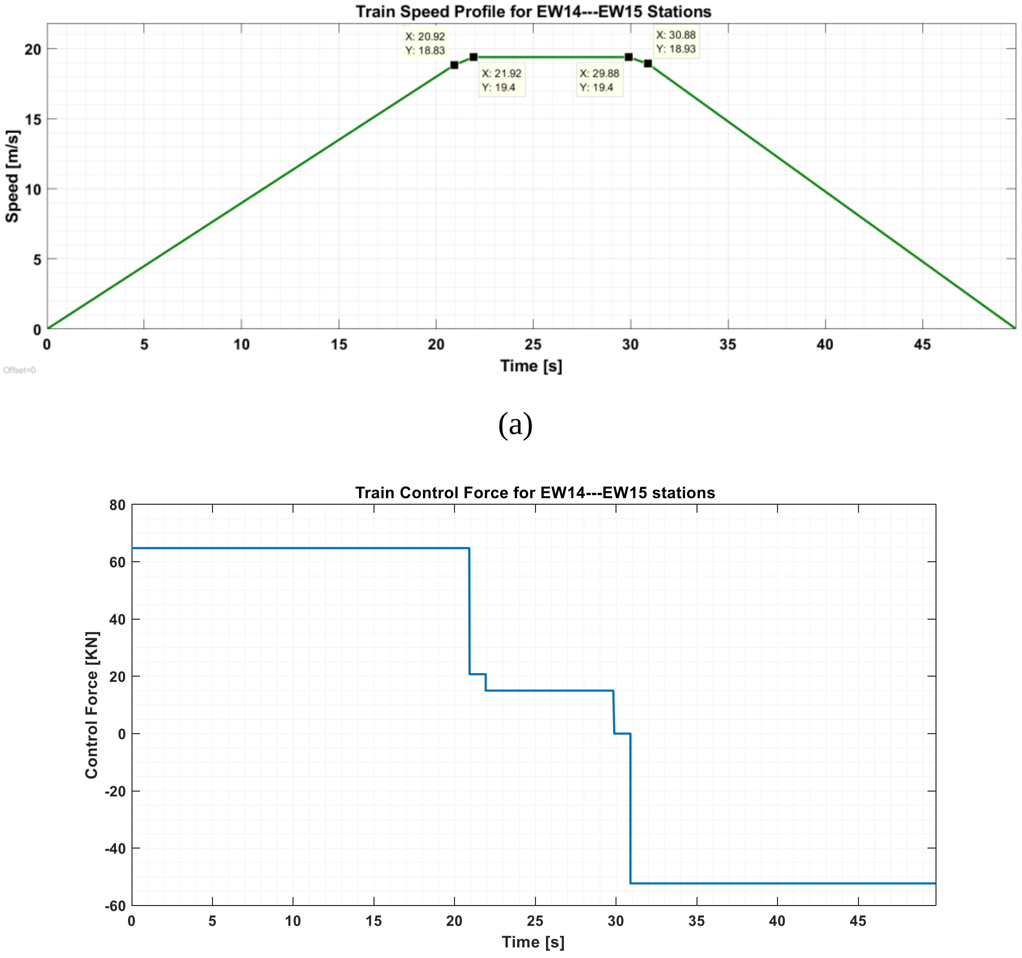


(a)



(b)

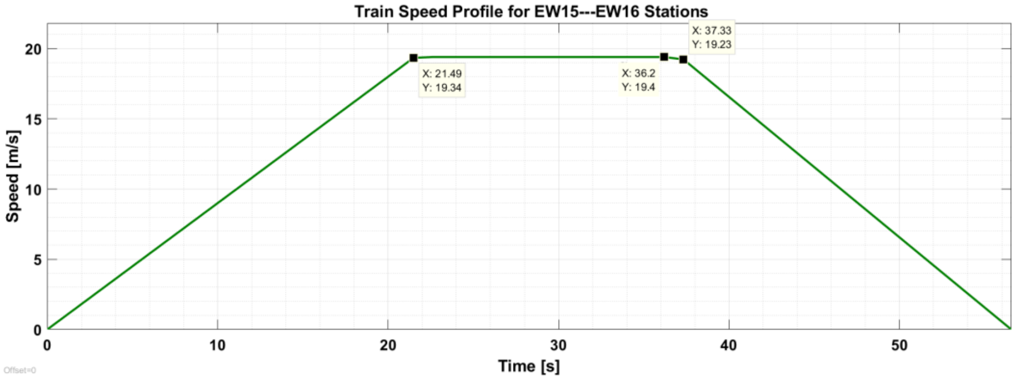
Figure 4-14: (a) Train Speed Profile, (b) Control Force from St. Urael Church to Bambis



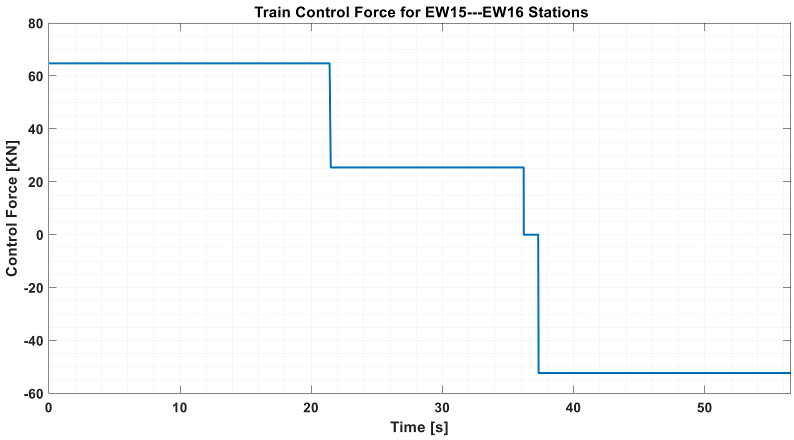
(a)

(b)

Figure 4-15: (a) Train Speed Profile, (b) Control Force from Bambis to St.Estifanos

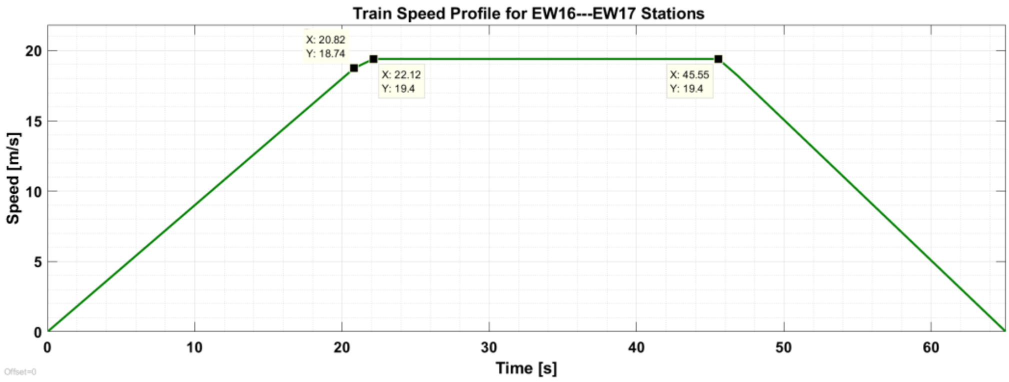


(a)

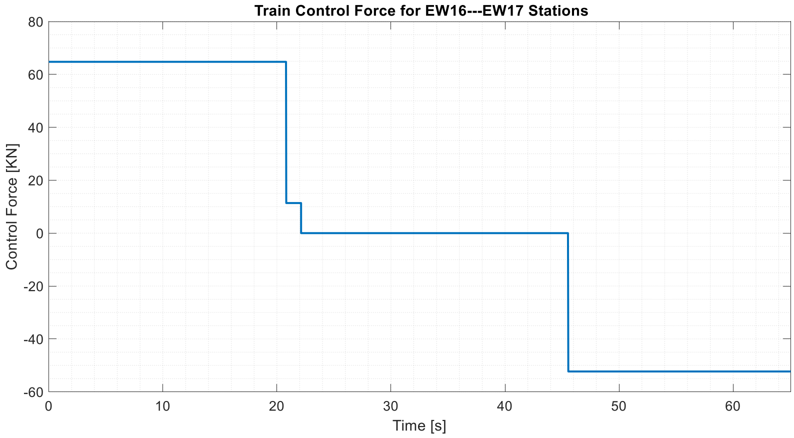


(b)

Figure 4-16: (a) Train Speed Profile, (b) Control Force from St.Estifanos to Stadium

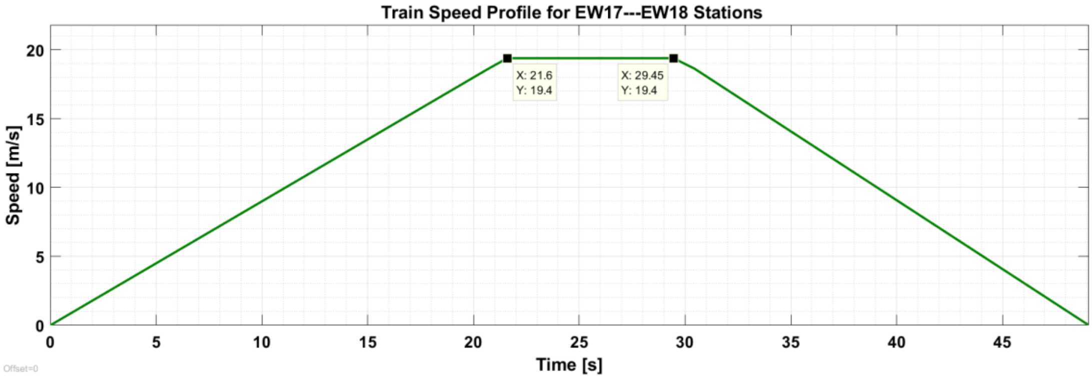


(a)

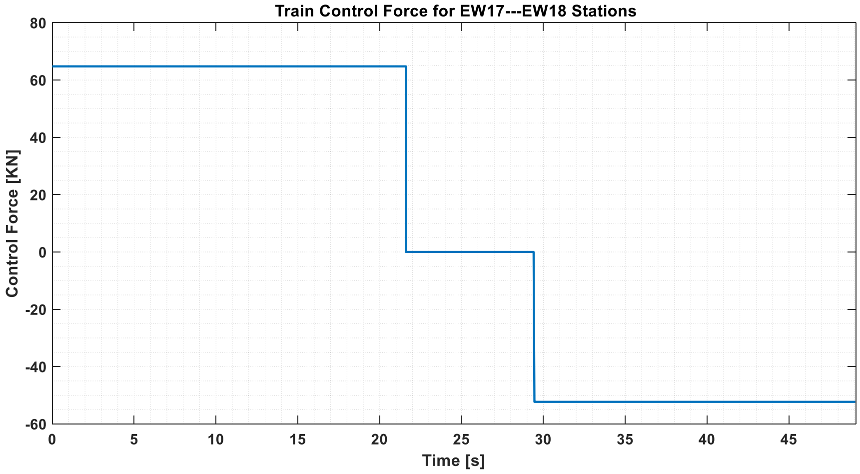


(b)

Figure 4-17: (a) Train Speed Profile, (b) Control Force from Stadium to Laghar

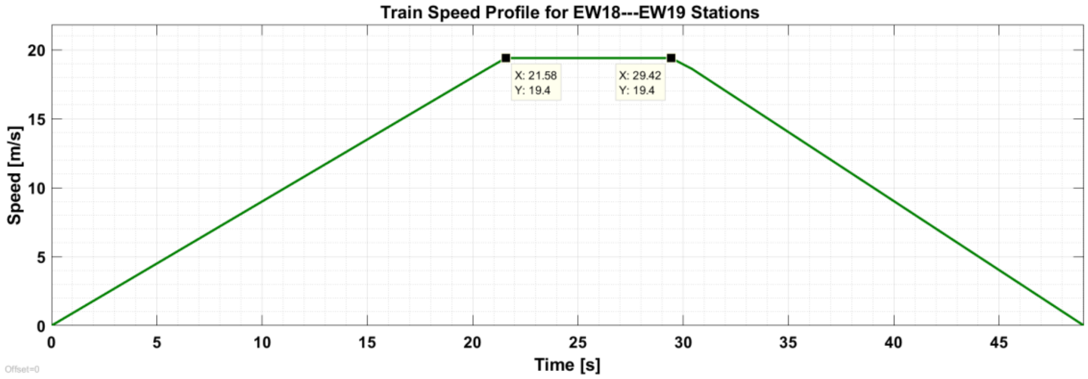


(a)

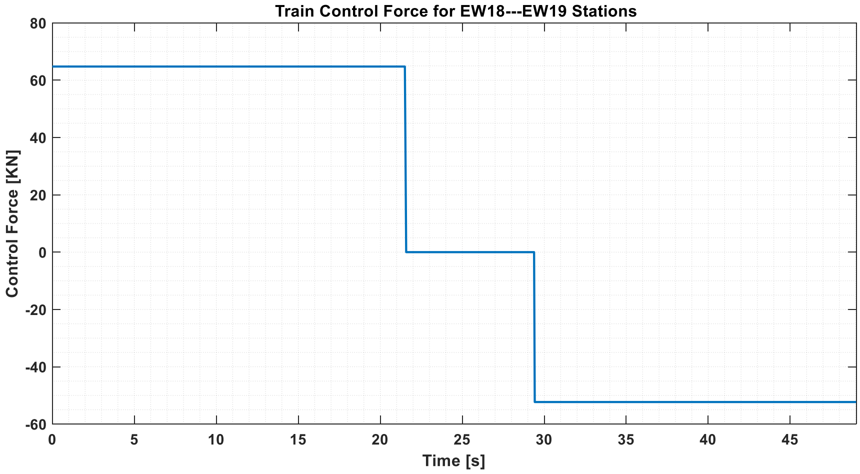


(b)

Figure 4-18: (a) Train Speed Profile, (b) Control Force from Laghar to Mexico

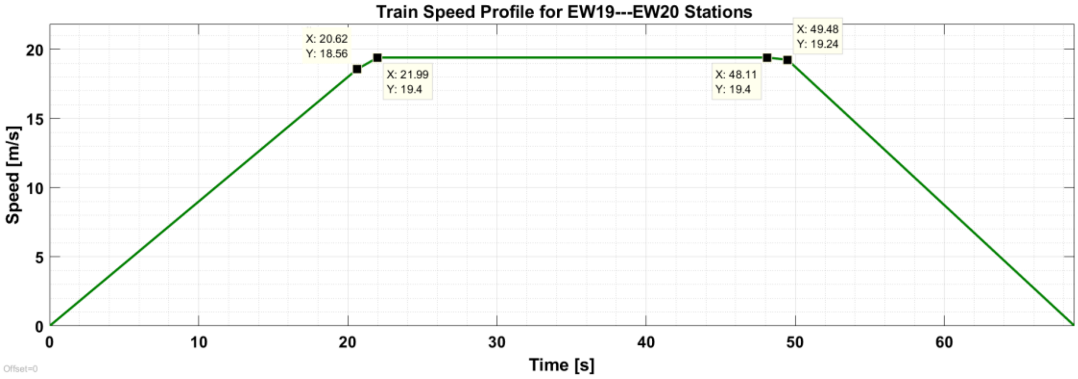


(a)

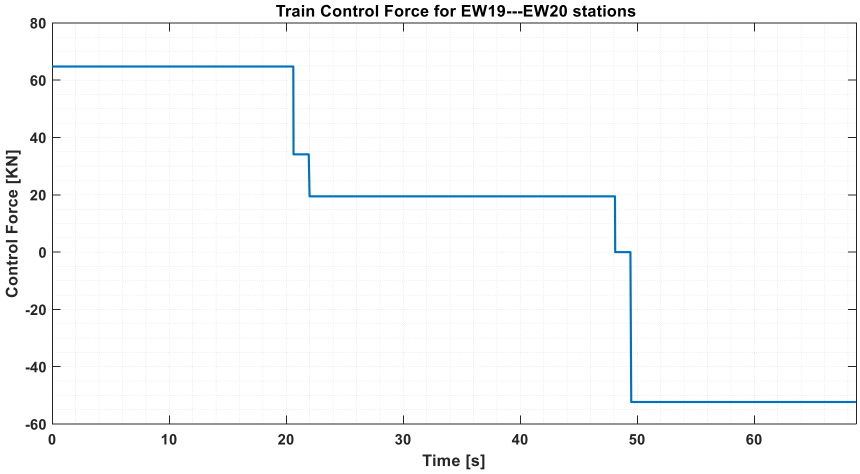


(b)

Figure 4-19: (a) Train Speed Profile, (b) Control Force from Mexico to Tegbared



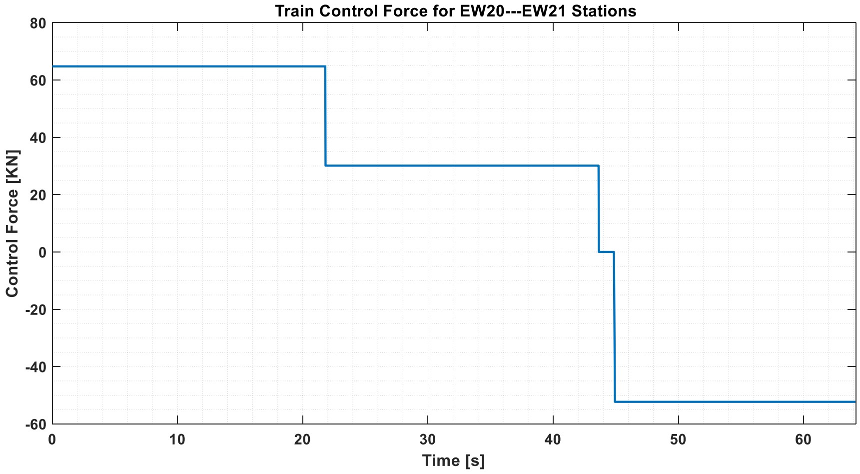
(a)



(b)

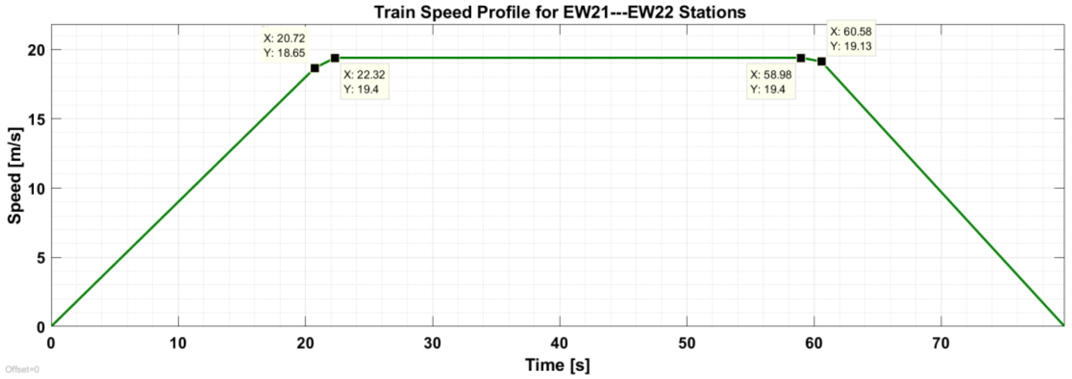
Figure 4-20: (a) Train Speed Profile, (b) Control Force from Tegbared to St.Lideta

(a)

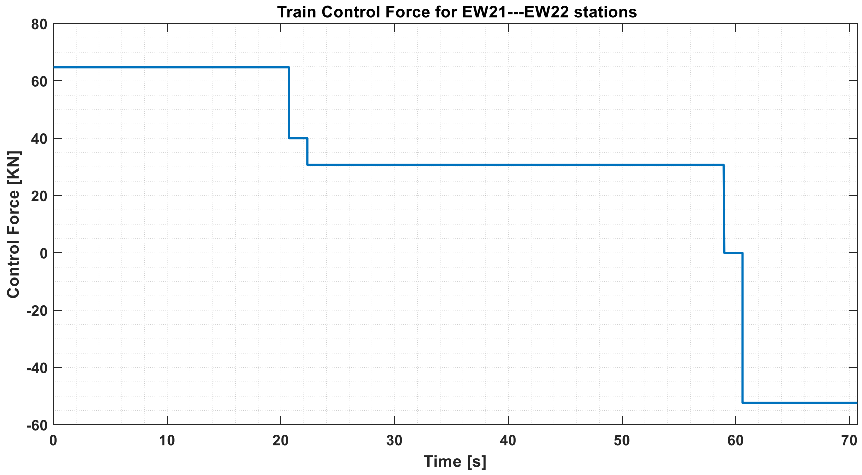


(b)

Figure 4-21: (a) Train Speed Profile, (b) Control Force from St.Lideta to Cocacola



(a)



(b)

Figure 4-22: (a) Train Speed Profile, (b) Control Force from Cocacola to Tor Hailoch

4.3 Minimum Train Energy Consumption

The minimum amounts of energy consumed by a train between two consecutive stations along the length of the E-W line for the AALRT system are presented in Table 4-1. As seen, more energy was consumed for the stations having a speed profile that has no coasting when compared to those with a coasting regime. Again, more energy was consumed for the stations having a longer section when compared to those having a shorter section.

$$Percentage\ of\ energy\ saving = \frac{(\sum E_{B0} - \sum E_{AO})}{\sum E_{BO}} \times 100$$

where

$\sum E_{BO}$ is the summation of energy consumption from Ayat to Tor Hailoch station before optimisation.

$\sum E_{AO}$ is the summation of energy consumption from Ayat to Tor Hailoch station after optimisation.

Then,

$$Percentage\ of\ energy\ saving = \frac{(129.673 - 117.082)}{129.673} \times 100 = 9.709\%$$

Table 4-1: Minimum Train Energy consumption between two stations and their driving strategies for E-W line.

Stations	**Distance [m]**	**Riding modes**				**Energy [MJ]**
EW1---EW2	1300	Motor	Cruise	Coast	Brake	7.384
EW2---EW3	1034	Motor	Cruise	Coast	Brake	5.642
EW3---EW4	896	Motor	Cruise	Coast	Brake	5.382
EW4---EW5	865	Motor	Cruise	-	Brake	8.743
EW5---EW6	785	Motor	Cruise	-	Brake	7.621
EW6---EW7	906	Motor	Cruise	Coast	Brake	5.391
EW7---EW8	922	Motor	Cruise	Coast	Brake	5.500
EW8---EW9	1004	Motor	Cruise	Coast	Brake	5.502
EW9--EW10	1035	Motor	Cruise	Coast	Brake	5.790
EW10-EW11	838	Motor	Cruise	Coast	Brake	5.298
EW11-EW12	694	Motor	Cruise	Coast	Brake	4.312
EW12-EW13	875	Motor	Cruise	-	Brake	10.325
EW13-EW14	570	Motor	Cruise	Coast	Brake	1.120
EW14-EW15	569	Motor	Cruise	Coast	Brake	0.811
EW15-EW16	700	Motor	Cruise	Coast	Brake	5.156
EW16-EW17	865	Motor	Cruise	Coast	Brake	5.373
EW17-EW18	555	Motor	Cruise	-	Brake	5.630
EW18-EW19	554	Motor	Cruise	-	Brake	5.510
EW19-EW20	936	Motor	Cruise	Coast	Brake	5.412
EW20-EW21	848	Motor	Cruise	Coast	Brake	5.328
EW21-EW22	1149	Motor	Cruise	Coast	Brake	5.852
Total Energy consumed from Ayat to Tor Hailoch						117.082

CHAPTER 5: CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

This thesis work sought to determine the energy-efficient train operation for the AALRT system, specifically the Est-West line (i.e., from Ayat station to Tor Hailoch station). The optimal train control problem is mathematically modelled using dynamic train systems, initial and terminal conditions, state and boundary control constraints, and train energy consumption as the objective function. The dynamic train operation is described by a set of kinematic equations that account for all forces acting on the train. Both tunnel and curve resistances are often neglected; tunnel resistance is often neglected due to the lack of general formulae that expresses this resistance. At the same time, the curve resistance is neglected because, compared to the gradient resistance is relatively low.

The optimal train control problem was reformulated as a non-linear constrained optimization problem by deploying the control parameterization technique (CPT), albeit with one complex of the state constraints. To address this complexity, the constraints were added to the objective function using a penalty function method, whereby a constrained optimization problem (COP) is transformed into an unconstrained optimization problem (UOP) that can be easily solved using the fminunc solver.

The train dynamic equations are modelled using Matlab/Simulink, and the ode45 solver is used to solve these systems of equations. The unconstrained optimization problem is then solved using the Matlab fminunc solver to determine the optimal tractive and braking control forces sequence and the corresponding cost function value.

Various simulations are carried out to find the optimal train speed profile, tractive and braking control forces sequence, and energy consumption between two consecutive stations from Ayat station to Tor Hailoch station for the AALRT system. The results revealed that the speed profile meets the maximum speed limits between two consecutive stations. It has been observed that the control force successfully satisfies the boundary control constraints between two consecutive stations. It has also been observed that the smallest and most significant amounts of energy consumed by train between two consecutive stations are 0.811 and 10.325 MJ for AALRT system/E-W line, respectively. An enormous amount of energy consumed by a train was associated with the segment between stations having a speed profile that has no coasting as well as a wider section.

5.2 Recommendations

- Due to insufficient information for the AALRT system for ERC, some approximations based on datasheets from railway manufacturing companies and international railway standards are taken during the mathematical modelling formulation of the optimal train control problem. For future works, the results obtained can be improved by incorporating the actual information for the AALRT system.
- The original optimal train control problem was transformed into a simple non-linear optimization problem employing CPT. Using this technique, the train control force was approximated by a piecewise-constant function with invariable switching time points. Under this approximation, only the piecewise-constant control force values on each subinterval were considered as decision variables. For further works, the variable switching time points would be taken as decision variables to achieve the train driving strategy for more energy efficiency.

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APPENDICES

Appendix A: Matlab Fminunc Solver

The fminunc solver is a Matlab Toolbox solver that is used to solve unconstrained optimization problems.

a. Purpose

Its purpose is to achieve the minimum values of a problem defined as [42]

$$\min_{x} f(x) \tag{A.1}$$

where x might be a scalar, vector, or matrix, and $f(x)$ is a function that takes a scalar.

b. Syntax

The following commands are used to compute a solution when fminunc solver is adopted;

$$\begin{cases} x = fminunc(fun, xo) \\ x = fminunc(fun, xo, options) \\ x = fminunc(fun, xo, options, P1, P2, \ldots) \\ [x, fval] = fminunc(\ldots) \\ [x, fval, exitflag] = fminunc(\ldots) \\ [x, fval, exitflag, output] = fminunc(\ldots) \\ [x, fval, exitflag, output, grad] = fminunc(\ldots) \\ [x, fval, exitflag, output, grad, hessian] = fminunc(\ldots) \end{cases} \tag{A.2}$$

c. Description

Beginning with an initial guess, fminunc obtains the minimum of a scalar function with multiple variables. This is commonly known as a nonlinear optimization problem without constrained. For instance, $x = fminunc(fun, xo)$ begins at the initial point, xo, finds a minimum value, x of the function defined in fun. xo might be a scalar, vector, or matrix.

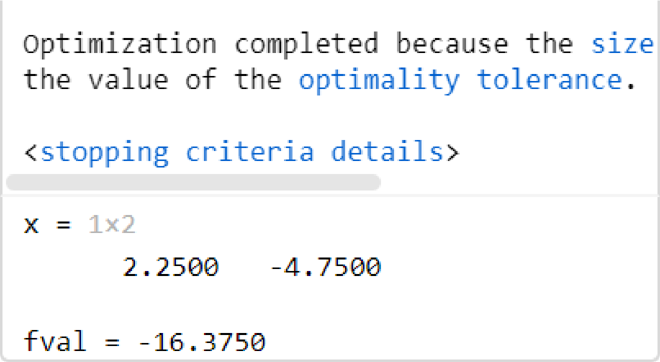
d. Example

Minimise the function $f(x) = 3x_1^2 + 2x_1x_2 + x_2^2 - 4x_1 + 5x_2$.

To do so,

```
%%% write an anonymous function fun that calculates the objective
fun = @(x)3*x(1)^2 + 2*x(1)*x(2) + x(2)^2 - 4*x(1) + 5*x(2);
%%% Then call fminunc to find a minimum of fun near [1,1].
x0 = [1,1];
[x, fval] = fminunc(fun, x0)
```

The solution, x, and the value of the function $fval$ at x are returned after some iterations.



```
Optimization completed because the size
the value of the optimality tolerance.

<stopping criteria details>

x = 1×2
      2.2500   -4.7500

fval = -16.3750
```

APPENDIX B: MATLAB CODES

APPENDIX B.1: Obj_pf.m for computing the cost function

```
%%%FUNCTION f THAT CALCULATES THE OBJECTIVE+PENALTY
FUNCTION
ftmax=64.77;
fbmax=52.30;
function f=obj_pf(X,h,Tfinal,sigma)
%% Defines the simulation options
opt=simset('solver','ode45','SrcWorkspace','Current','FixedSte
p',h);
%% Defines an input vector for the simulation.
T=[0:h:Tfinal-h]';
%% Defines an input vector for the simulation including time.
TU=[T X];
%% Carries out a simulation of the dynamic train system between
t=0 and Tfinal, given the current input trajectory defined in
TU.
[tout,xout,yout]=sim ('slm',[0 Tfinal],opt,TU);
%%  Calculates the objectives function value Excluding the
penalty function
f=trapz(t,yout(:,2).*X);
%%Computes the penalty terms the addition of three terms.
%%The first term considers the state constraints.
penalty=0.5*sigma*sum(max(0,0-yout(:,2)).^2);
penalty=penalty+0.5*sigma*sum(max(0,yout(:,2)-19.4).^2);
%% The second term considers the upper bound on the decision
variables.
penalty=penalty+0.5*sigma*sum(max(0,X-ftmax).^2);
%% The third term considers the lower bound on the decision
variables
penalty=penalty+0.5*sigma*sum(max(0,-X-fbmax).^2);
%% The fourth term considers the terminal constraints
penalty=penalty+0.5*sigma*sum(max(0,yout(:,1)-848).^2);
penalty=penalty+0.5*sigma*sum(max(0,yout(:,2)-0).^2);
%% Finally, the penalty function value is added to the objective
function value.
f=f+penalty;
```

APPENDIX B.2: main.m for calling fminunc solver and solve the problem

```
%%%% CALL FMINUNC SOLVER (OPTIMIZATION ROUTINE) AND SOLVE THE
PROBLEM
close('all')
clear;
clc
%% Defines the final time
Tfinal=64.1891;
%% Defines the number of steps in the simulation
N=10;
%%Define the penalty factor
sigma=1000;
%% Compute the integration step
h=Tfinal/N;
%% Defines the initial guess for the input trajectory
x0=zeros(N,1);
%% Call the 'fminunc' function to solve the optimization problem
and stores the optimization problem and stores the optimal
decision vector in X.
[X,costf]=fminunc('obj_pf',x0,h,Tfinal,sigma);
%% Defines a time vector T.
T=[0:h:Tfinal-h]';
%% Defines an input vector for the simulation, including time.
TU=[T X];
%% Defines the simulation options
opt=simset
('solver','ode45','SrcWorkspace','Current','FIxedStep',h);
%% Carries out a simulation of the optimal response of the
single-link manipulator between t=0 and Tfinal, given the
optimal input trajectory defined in TU.
[tout,xout,yout]=sim ('slm',[0 Tfinal],opt,TU);
```