Enhancing Secondary Students' Understanding of Descriptive Statistics Using Contextual and Socio-critical Modeling: The Case of West Shewa Zone, Ethiopia

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This is to certify that the dissertation research prepared by Mulugeta Woldemicheal Gebresenbet, entitled: *Enhancing Secondary Students' Understanding of Descriptive Statistics Using Contextual and Socio-critical Modeling: The Case of West Shewa Zone, Ethiopia*, and submitted in fulfillments of the requirements for the degree of Doctor of Philosophy in Science and Mathematics Education compiles with the regulations of Addis Ababa University and meets the accepted standard with respect to originality and quality.

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Declaration

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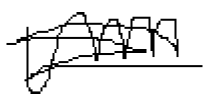

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Abstract

Enhancing Secondary Students' Understanding of Descriptive Statistics Using Contextual and Socio-critical Modeling: The Case of West Shewa Zone, Ethiopia

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The issue that this study addresses is the need for secondary students to work on relevant non-routine statistical problems to enhance their understanding of descriptive statistics. The purpose of this study is to explain how secondary students can enhance their understanding of descriptive statistics using Modeling Approach and to compare the amount of learning achieved using Modeling and Non-modeling Instructional Approaches. The Modeling Approach integrated the Contextual and Socio-critical Modeling Perspectives to teach a unit of descriptive statistics in secondary mathematics curriculum. The leading research question set forth is, to what extent and how do high school students enhance their understanding of descriptive statistics using a Modeling Approach?

This transformative embedded mixed method study looked into three facets of students' understanding of descriptive statistics: procedural, conceptual and critical. The study was conducted at two high schools in urban district in West Shewa Zone, Ethiopia. For comparisons, four Grade Nine sections with a total of 163 students were selected by purposive sampling technique. An instrument called Teaching for Robust Understanding in Mathematics (TRU Math) was used for observing Model-eliciting Activities (MEAs) to capture the Modeling Approach classroom practice (Schoenfeld, 2013). Quality Assurance Guide instrument was used to assess students' models on MEAs (Lesh et al.,

2000). Standard questions were used for achievement tests on procedural and conceptual understanding of descriptive statistics. An attitude survey instrument was administered to see students' change of attitude towards statistics using Modeling Approach. Teams of students did projects on their own themes for six weeks.

The quantitative data of the study was analyzed using descriptive statistics, t-tests and ANOVA. The qualitative data of the study was analyzed using thematic and content analyses. A statistically significant difference was found on conceptual understanding achievement test with medium effect size using Modeling Approach, but no statistical significant difference was found on procedural understanding achievement test except female comparison. Significant statistical effects were observed for attitude scale and subscales. Students enhanced their critical understanding of descriptive statistics doing projects through data modeling. The findings of this study suggested students more likely enhanced their understanding of descriptive statistics using Modeling Approach.

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List of Acronyms and Abbreviations

ARTIST	Assessment Resource Tools for Improving Statistical Thinking
BMI	Body Mass Index
CATALST	Change Agents for Teaching and Learning Statistics
CGERD	Common Guidelines for Educational Research and Development
ENEA	Ethiopian National Examination Agency
ESDP	Education Sector Development Program
GAISE	Guidelines for Assessment and Instruction in Statistics Education
MEA	Model-Eliciting Activity
MoE	Ministry of Education
OCED	Organization for Economic Co-operation and Development
STEM	Science, Technology, Engineering and Mathematics
TRU Math	Teaching for Robust Understanding in Mathematics
TGE	Transitional Government of Ethiopia
UNESCO	United Nations Educational, Scientific, and Cultural Organization
WASH	Water, Sanitation and Hygiene
WHO	World Health Organization

CHAPTER 1

INTRODUCTION

1.1 Introduction

Modeling, as an alternative for problem solving, is getting recognition in Mathematics Education for advancing students' understanding of a concept (Lesh & Doerr,2003; Sriraman & English, 2010). However, research studies are needed on how to incorporate modeling problems in an education system so as to enhance students' understanding of concepts and thereby improve students' achievements (Kaiser, 2010; Lesh, 2012); and little is known about Socio-critical Modeling that uses mathematics or statistics as a critical tool for analyzing and solving socio-cultural problems (Stillman, Kaiser, Blum & Brown, 2013). In this chapter, teaching statistics using Modeling Approach, which establishes the problem leading to the study, is discussed. The background of the study, the context of the study, statement of the problem, aims and purpose of the study, significance of the study, delimitations of the study, limitations of the study and definitions of terms are described. Finally, the structure of the dissertation is presented.

1.2 Background of the Study

In an education system, stakeholders agree students need to solve problems; however, there is little consensus on what entails problem-solving curriculum and instruction in mathematics education (Chamberlin, 2008, 2010). Teaching approaches that have some nature of problem solving in mathematics generally could be classified as teaching for, teaching about, and teaching through problem solving (Bostic, 2012; Cai, 2003; Schroeder, Thomas, & Lester, 1989).

Teaching through problem solving considers problem solving as integral to the development of an understanding of any given mathematical content and process (Doerr & Lesh, 2003; English, 2013a; Hamilton, 2007). It is different from the practice of teaching for problem solving which deals with solving problems using the already taught concepts. It also differs from teaching about problem solving which deals with problem solving heuristics and strategies. A Modeling Approach in the study integrates Contextual and Socio-critical Modeling Perspectives for teaching statistics through problem solving and, therefore; justifies the statistics rather than treating it as a means to an end.

Modeling problems could help enhancing students' critical understanding to deal with problems of socio-cultural and environmental complex systems (Barbosa, 2006; Kaiser & Schwarz, 2006; Sriraman & Lesh, 2006). How far problems in textbooks contribute for critical literacy aim and how teachers utilize such problems that could promote critical understanding is an under researched area in mathematics education (Ernest, 2008; Stillman, Kaiser, Blum & Brown, 2013). What remains to be explored is to study on mathematical or statistical modeling as a critical tool for enhancing students' critical understanding of mathematics or statistics (Kaiser, Blohomhoj & Sriraman, 2006).

The nature of problems beyond classroom demands educators to give students interdisciplinary problem solving experiences. Modeling needs to be integrated within all topic areas across the mathematics curriculum, and, there is a need, across disciplines (English, 2013a; Gouvea, Sawtelle, Geller & Turpen, 2013). There is, however, a limited research on how to integrate other disciplines within mathematics curriculum which can be done through modeling instruction (English, 2013a; Mousoulides & English, 2012).

Specifically, across all walks of life, statistics is essential for understanding the world and serves as a critical tool for solving socio-cultural problems. For example, the modeling ability to interpret, describe, and explain data and communicate results of data analysis is crucial in day to day life (Chamberlin & Coxbill, 2012). One way to promote statistical reasoning is through data modeling (Doerr & English, 2003; English, 2010, 2013b). However, there are very few studies that promote students' statistical thinking and reasoning at school using data modeling (English, 2010, 2013a; Kinnear, 2013; Lehrer & Schauble, 2007). According to Sriraman and English (2010), the issue of enhancing students' statistical reasoning through data modeling requires 'substantial research'.

If students gain the habit of working on statistical problems using models of representation systems, then models as thinking tools will help students to develop statistical reasoning and problem solving abilities. Cobb (2007) argues that, "Data sets are concrete, statistical models are abstracts, and every instant of modeling is an instance of abstraction-as a process" (p.339). However, surprisingly, there is little research on models and modeling in Statistics Education (Garfield & Ben-Zvi, 2008).

When looking at data, statistics is introduced into the modeling process in some way or other. However, there have been controversies in school curriculums on how to develop students' statistical reasoning based on pragmatic reasons whether or not to teach students with 'first order' emphasis on measure of central tendency at the expense of teaching variability as 'second order' emphasis (Gould, 2004; Konold & Pollatsek, 2002;

Shaughnessy, 2007). But, basically, both concepts of central tendency and variability could be taught using modeling instructional approach in a balanced way, because in essence they are founded on model-based reasoning.

More importantly, mathematics curricula must extend their goals to take account of key concepts and processes like statistical reasoning and mathematical modeling which optimize students' chance of being successful in the 21st century (English, 2002; Kuntze & Engel, 2011). Modeling would be appropriate for the nature of statistics which emphasize problem solving "in the wild" (Lesh & English, 2005; Roth, 2007) to describe and interpret meaningful socio-cultural real life situations. This kind of problem solving is needed in Ethiopian educational policy and curriculum as discussed in the next section.

1.3 Context of the Study

The contemporary educational policy of Ethiopia is rooted in the 1994 *Education and Training Policy* whose overarching aim was "strengthening the individual's and society's problem-solving capacity, ability and culture starting from basic education and at all levels" (TGE, 1994, p.1). In particular, a major goal of mathematics education in Ethiopia is to develop problem-solving abilities of students in socio-cultural context of their everyday life which is stated in different official documents (MoE, 2009a, 2009b, 2010a). Descriptive statistics is part of Ethiopian high school mathematics curriculum with the aim of teaching students for developing foundational concepts of statistics to solve problems related with 'every bit of students' everyday life' (MoE, 2010b).

Students need to study statistics for many reasons: to learn practical skills in daily life, to understand quantitative aspects of public policy critically, to pose and solve socio-cultural problems using statistics as a critical tool, and to prepare for careers that require quantitative reasoning. Students need to prepare for life by developing statistical problem solving abilities that are most needed for success beyond school in an age of information.

Here is an important question to ask: ‘How far do the problems which exist in textbooks help to achieve the above aims?’ An ongoing debate in mathematics education for several decades pertains to the extent to which low-level to high-level activities should be used (Chamberlin, 2008, 2010; Schoenfeld, 2004). Analysis model is suggested by Chamberlin (2010) for decision making for teachers to select problem solving tasks for low level and high level learners. Based on this model, examples are given as shown in Table 1 and content analysis is made on the current Ethiopian Grade Nine mathematics textbook on statistics unit which is published by Ethiopian Ministry of Education in 2010.

The percentages of exercises and word problems tasks in statistics unit in Ethiopian Grade Nine mathematics textbook are 68% and 27% respectively. Although Ethiopian education policy and documents on mathematical curriculum recommend that there should be relevant problem solving activities on socio-cultural issues, the percentage of statistical problem solving tasks (4%) in Grade Nine textbook is very low and non-routine statistical problem solving tasks are almost non-existent. Researchers have indicated that covering the text book is the most common teaching practice in Ethiopia instead of engaging students with rich problem-solving tasks (Asgedom, 2009; Micheal & O’Connell, 2014). This may lead teachers to use simple textbooks problems, and as a result students couldn’t develop cognitive abilities to solve problems in novel situations.

The best possible levels of cognitive tasks are required to prepare students for nature of understanding and abilities needed beyond schools (Lesh, 2000). The nature of problems which exists beyond schools call for considering what ranges of problems should be given to students at various achievement levels to promote equity. Significantly, students need to solve non-routine statistical problems like MEAs which are sometimes identified as 'rich tasks' (Bostic, 2012) so as to enhance students' foundational and critical understanding of statistics now and then (Chamberlin, 2012; Lesh & Doerr, 2003).

Table 1: *Levels for types of tasks*

Level	Tasks name	Example as task statement
Level 1	Exercises	*Find the mean of the following data 5, 7, 8, 9, 11
Level 2	Word or story problems	*In mathematics test the scores for boys were 6, 7, 8, 7, 5 and the scores for girls were 6, 3, 9, 8, 2, 2, 5, 7, 3. Find the mean and median for the boys and girls.
Level 3	Statistical problem solving tasks	*Mamo's brothers are 174cm, 180cm, 179cm, and 172cm tall. If Mamo and his brothers have an average height of 176.50 cm, then how tall is Mamo?
Level 4	Non-routine statistical problem solving tasks (like Model-Eliciting Activities in appendix A)	In your team, write a report that could be given to the news broadcast center by the director of the school using the provided students' data of three Grade Nine sections.

*The three examples are taken from Grade Nine mathematics textbook (MoE, 2010b)

1.4 Statement of the Problem

Traditionally, statistical curricula follow 'linear and hierarchical approach' to teach students foundational statistics concepts adopting from the formalist mathematics culture (Meletiou-Mavrotheris, 2007; Meletiou-Mavrotheris & Lee, 2002). For example, many students are taught about data distribution, measure of center, and variation sequentially on how to construct them rather than how to use them to think with data. The presupposition in the traditional approach is, students would learn statistics profoundly if they built one concept upon another 'basic' concept. However, teaching statistical contents step wisely based on an arranged sequence of curricular topics might attribute to divide knowledge haphazardly which may hinder students to see the interrelationships of the different statistical ideas (Meletiou-Mavrotheris, 2007).

The philosophical stance taken in this project is that statistical learning has to take place through problem solving which could motivate students as they solve relevant non-routine problem-solving tasks. As the problems arises from students real life, they may develop their intuitive knowledge moving from informal ideas to formal ideas and see the interconnection of statistical big ideas through modeling. For example, students could learn the concepts of variability, center and distribution concurrently from informal to formal solving MEAs, as researchers have found that students have informal intuition about variability and center that may serve as a good foundation for studying formal measures of these concepts (Garfield, delMas & Chance, 2007).

Though, it is difficult to motivate students to engage in reasoning about complex and challenging concepts like variability, it may be possible to engage them using a modeling approach as the problems start from meaningful real life situations. Educators may be able to develop students' foundational understanding of statistical concepts by providing modeling problems for students. The problems could in turn may help them for improving achievements in statistics and create positive attitude towards statistics.

Many developing and developed countries are giving emphasis on assessing their students at national and international levels on important learning outcomes like cognitive skills and problem solving abilities based on achievement tests (World Bank, 2006; Organization for Economic Co-operation and Development (OECD), 2014). According to Word Bank report (2006), the quality of learning outcomes in developing countries has been poor. Ethiopia, one of the developing countries, has conducted four round national learning assessments on key subjects that includes mathematics in 2010 and 2013 for grade 10 and 12 students. Both the two assessments have showed that secondary students' average achievement scores in mathematics were found to be low; and most students in secondary grade could not score at least 50% in mathematics (Ethiopian National Examination Agency (ENEA), 2010, 2013).

The results of the national assessments have shown the gaps are high in equitable learning outcomes based on gender and students achievement levels. Boys perform better than girls in both assessments. It is recommended that an intervention should be made revisiting the curriculum and the existing instruction practices (ENEA, 2010, 2013).

Program for International Students Assessment (PISA) for students at age 15 reveals that gender difference in students' attitude towards mathematics is dismaying and females have more negative attitude towards mathematics than males (OCED, 2014). These findings have serious implications not only for higher education, where females are already under-represented in the science, technology, engineering and mathematics fields of study, but also later on, when these females compete for different jobs (OCED, 2014). The same case seems to hold true for Ethiopia mathematics or statistics education (Semela, 2010). Educators have recommended that introducing modeling in schools would improve students' attitude and equity for learning mathematics or statistics in a school (Bostic, 2011; Kaiser, 2010).

Furthermore, students need to develop their critical understanding of statistics to solve socio-cultural problems. A study by Asgedom (2009) has found that the practice of Ethiopian Education mostly focuses on knowledge acquisition, or what UNESCO calls "learning to know," to the detriment of other useful purposes, learning to do, learning to be and learning to live together. A number of studies have shown that modeling develops system thinking using data modeling in Statistics Education (English, 2010, 2013a; Lehrer & Schauble, 2007). **What remains to be explored in school is how modeling could enhance students' sympathetic or critical understanding with the aim of living together in the world that is 'survival with dignity'** (Ernest, 2010; Stillman, Kaiser, Blum & Brown, 2013).

The more teachers incorporate and implement relevant non-routine real-world problems within the curriculum, the better their chances of enhancing secondary students' understanding of descriptive statistics by giving them an opportunity to develop their sense making and explain their reasoning iteratively. Using Modeling Approach, student could construct, revise and refine models; and they may enhance their understanding of descriptive statistics and apply statistics as a critical tool with positive attitude to solve problems in socio-cultural problems. The problems described in this section lead to consideration of Modeling Approach for enhancing students' understanding of descriptive statistics, one which is timely and important to research.

1.5 Aims and Purpose of the Research

The intent of this study is to explain how teaching statistics using Modeling Approach could enhance students' understanding of descriptive statistics and to compare the amount of learning achieved using Modeling and Non-modeling Instructional Approaches. The aim is to explain how to teach statistics through problem solving using Modeling Approach as a mechanism for enhancing students' understanding of descriptive statistics. The Modeling Approach integrates the Contextual and Socio-critical Modeling Perspectives to teach the descriptive statistics unit in secondary school mathematics curriculum. The participants in this study are 163 students in four Grade Nine sections in two high schools in West Shewa Zone, Ethiopia.

A transformative embedded mixed design is used. It involves quasi-experimental design collecting quantitative data using achievement tests on conceptual and procedural understanding of descriptive statistics. The purpose of the quasi experiment is to test the use of Modeling Approach using achievement tests on students' procedural and conceptual understanding of descriptive statistics. Also, means of students' achievement scores on procedural and conceptual understanding with respect to gender and achievement levels are compared. Pretest and posttest attitude surveys are administered to see students' change of attitude towards statistics using Modeling Approach.

Case study design is used for explaining the Modeling Approach based on data collection methods of classroom observation, content analysis of students' solutions on MEAs, and students' interviews. Moreover, qualitative case study design is used with Socio-critical Modeling Perspective as a transformative framework lens to explore students' critical understanding of statistics based on data sources of students' interviews, presentation videos, students' artifacts on MEAs, and students' project reports.

1.5.1 Research Questions

Leading Research Question: To what extent and how do high school students enhance their understanding of descriptive statistics using a Modeling Approach?

Research Questions

Question 1: How do students enhance their understanding of descriptive statistics using Modeling Instructional Approach?

Question 2: How do students enhance their critical understanding of descriptive statistics doing projects through data modeling?

Question 3: To what extent do students enhance their understanding of descriptive statistics using Modeling Instructional Approach?

Related with the third research question the following hypotheses in the next sub section are tested in the study.

1.5.2 Research Hypotheses

Comparison of Modeling and Non- modeling Groups by Schools and by Students

HO1: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on prior knowledge achievement test of descriptive statistics.

H02: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on post knowledge achievement test of descriptive statistics.

HO3: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on procedural understanding achievement posttest of descriptive statistics.

HO4: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on conceptual understanding achievement posttest of descriptive statistics.

HO5: There is no significant difference between students' mean scores on procedural and conceptual understanding achievement posttests of descriptive statistics.

Gender wise comparison between Modeling and Non-modeling groups

HO6: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on prior knowledge achievement test of descriptive statistics in gender wise.

H07: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on post knowledge achievement test of descriptive statistics in gender wise.

H08: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on procedural understanding achievement posttest of descriptive statistics in gender wise.

H09: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on conceptual understanding achievement posttest of descriptive statistics in gender wise.

H10: There is no significant difference between students' mean scores on procedural and conceptual understanding achievement posttests of descriptive statistics in gender wise.

Comparison by Gender using Modeling Approach

H11: There is no significant mean scores difference between students who have learnt using Modeling Approach on procedural understanding achievement posttest of descriptive statistics in gender wise.

H12: There is no significant mean scores difference between students who have learnt using Modeling Approach on conceptual understanding achievement posttest of descriptive statistics in gender wise.

Group comparison between Modeling and Non- modeling groups by achievement levels

H13: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on prior knowledge achievement test of descriptive statistics by achievement levels.

H14: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on post knowledge achievement test of descriptive statistics by achievement levels.

H15: There is no significant mean scores difference between students who have learnt using Modeling and Non-modeling Approach on procedural understanding achievement posttest of descriptive statistics by achievement levels.

H16: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on conceptual understanding achievement posttest of descriptive statistics by achievement levels.

H17: There is no significant difference between students' mean scores on procedural and conceptual understanding achievement posttests of descriptive statistics by achievement levels.

Comparison by achievement levels who have learned using Modeling

H18: There is no significant mean score difference among high, medium and low mathematics achievers who have learnt using Modeling Approach on procedural understanding achievement posttest of descriptive statistics.

H19: There is no significant mean score difference among high, medium and low mathematics achievers who have learnt using Modeling Approach on conceptual understanding achievement posttest of descriptive statistics.

Attitude towards Statistics using Modeling Approach

H20: There is no significance mean scores difference on students' attitude towards statistics survey before and after the intervention using Modeling Approach.

H21: There is no significant mean scores difference on students' pretest attitude towards statistics survey using Modeling Approach in gender wise.

H22: There is no significant mean scores difference on students' posttest attitude towards statistics survey using Modeling Approach in gender wise.

H23: There is no significant mean gain scores difference on students' pretest and posttest attitude towards statistics survey using Modeling Approach.

1.6 Significance of the Study

A study of the Modeling Approach to teach statistics is important for several reasons. First, it helps to lay the ground for further similar empirical studies across different school settings and populations on how to enhance students' understanding of descriptive statistics. Second, it serves as a basis for further effectiveness and large scale studies for teaching statistics using Modeling Approach. Third, the study will show research directions on how to design curriculum using transformative modeling framework in order to teach students with social justice aim using statistics as a critical tool. Fourth, it will help teachers as a heuristic devise on how to prepare and implement relevant non-routine problem solving tasks like MEAs. Fifth, the study will provide directions on aspects of professional development that have to be given for teachers for teaching statistics in secondary schools. Finally, it will serve as an available asset for policy decision makers, curriculum developer, and assessment builders on how to include relevant non-routine problem solving tasks in an education system.

1.7 Delimitations of the Study

This study is conducted in under-privileged urban secondary schools setting in West Shewa Zone in Ethiopia. It focuses on one statistics unit that contains descriptive statistics lessons. The descriptive statistics deals with basic ideas of data, distribution, center and variability. The study site is delimited to one district in two governmental secondary schools in non-technological school learning environment except the use of calculators, post it notes and plastic meters.

1.8 Limitations of the Study

This study focuses on secondary students' understanding of descriptive statistics. But, the impacts of a teacher on the effectiveness of an intervention need detailed research, since a teacher may affect the intervention even if one teacher taught the Modeling and Non-modeling Instructional Approaches in the two respective schools. Thus, the teacher variable could be the limitation of the study, since it is difficult to control for all aspects of teacher variables such as a teacher's belief, knowledge and orientations towards an instructional approach even if the researcher and the teachers discussed to synchronize goals before and after implementing instructional lessons (Schoenfeld, 2007, 2010).

Selecting the contexts for MEAs from socio-cultural topical issues and themes for projects are another important aspect of the study. However, teaching concepts using historical backgrounds are not included despite the history-culture dimension is one aspect of understanding concepts (Usiskin, 2012). For example, Bakker and Gravemiejer (2006) taught the concepts of mean and median using phenomenological history.

The limitation of the study on the methodology lies in that the study is conducted in natural school environment without assigning participants randomly to the teaching approaches. The methodology may limit the extent to generalize the findings across different school settings. But, the study will have ecological validity for potential and implied generality for students' populations, schools and districts with similar characteristics.

1.9 Definitions of Terms

Descriptive Statistics: Descriptive statistics is a branch of statistics that deals with basic statistical concepts of data, data distribution, center and variability without formal inferential statistics.

Equity: Equity is an equal opportunity for all students to learn statistics, to improve achievement, to engage in meaningful statistical experiences regardless of their gender and achievement levels.

Achievement: Achievement is students' educational outcome of an assessment after learning some lessons.

Model: "A model is a system for describing (or explaining or designing) another system(s) for some clearly specified purpose" (Lesh & Fennewald, 2010, p.7).

Model-Eliciting Activities: Model-Eliciting Activities are non-routine problems which require students to make interpretation and conclusion of meaningful real life situations by making models using system of representations.

1.10 Structure of the Dissertation

This research covers five chapters. Chapter one introduced the background of the study, the context of the study, the statement of the problem, aims and purpose of the study, the significance of the study, delimitation of the study, limitation of the study, and definitions of terms.

Chapter two presents an analysis of conceptual, theoretical and research literatures on students' understanding of descriptive statistics and modeling towards building a conceptual framework related to the research questions.

Chapter three establishes the study's methodological process and provides a rationale for using transformative embedded mixed method research. Data collection methods, data analysis and ethical considerations are presented.

Chapter four presents and discusses on Modeling Approach intervention based on classroom observation, students' reports on MEAs, and students' interview data. Then, this chapter presents and discusses on students' achievements based on quantitative students' scores data on procedural and conceptual understanding achievement tests of descriptive statistics. The chapter also presents and discusses on attitude towards statistics using Modeling Approach based on quantitative attitude survey data. It also presents and discusses on students' projects based on students' reports, video presentations and interviews data on the projects. Finally, it discusses on the general findings of qualitative and quantitative data towards answering the leading research question in the study.

Chapter five concludes by evaluating how far the research questions are answered, how far the theoretical and methodological aims are achieved. Then, it presents general discussion on the central findings to resolve the issue the study addresses. The chapter ends with conclusions, further implications and recommendations of the study for research and practice.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

This review aims to shed light on why it is not easy to develop high school students' understanding of descriptive statistics; and to consider how students' understanding of descriptive statistics might be enhanced using Modeling Approach. The review explores the published literature in relation to two review questions: (1) Why are statistics concepts difficult to learn with understanding for high school students in mathematics curriculum? (2) How could high school students' understanding of descriptive statistics be enhanced using Modeling Approach?

The review mainly draws on two areas of literature: Statistics Education and Modeling in Mathematics Education. Since it is not only in high school mathematics curriculum descriptive statistics is taught across the world, research on upper primary, middle schools mathematics curriculums and preparatory college introductory statistics with similar statistical contents and processes have been included. Though it is possible to include all research on tertiary education, only a few studies at college level are selected.

In this chapter, statistical literacy, reasoning and thinking are introduced and defined. These three learning outcomes in relation to mathematics proficiency strands are discussed. Then, what entails students' understanding of descriptive statistics is presented and research studies on students' understanding of descriptive statistics are examined. The two theoretical perspectives (Contextual Modeling Perspective and Socio-critical

Modeling Perspective) are pointed out and research studies on modeling are reviewed. Finally, conceptual framework of the study is presented and conclusions are drawn about to the extent to which the reviewed texts shed light on the two review questions.

2.2 Statistical Literacy, Reasoning and Thinking

Statistical literacy, reasoning and thinking are important learning outcomes in Statistics Education. Identifying what makes the learning outcomes different or the same with one another is crucial in Statistics Education; since they are essential in pointing directions "for formulating learning goals for students, designing instructional activities, and evaluating learning by using appropriate assessment instruments" (Ben-Zvi & Garfield, 2004, p.6). In this section, a selection of commonly cited definitions of statistical literacy, reasoning and thinking are presented to lay the ground for reviewing and discussing the literature on students' understanding of descriptive statistics.

Statistical Literacy

The two major contributors to the literature on statistics literacy were Idol Gal and Jane Watson. Gal (2002) purposed a model for statistical literacy which has six components that includes literacy skills, statistical knowledge, mathematical knowledge, contexts knowledge, and critical questions. With the six components, Gal also identified the affective elements like attitude and critical stance as part of statistical literacy. Statistical literacy refers to people's abilities to interpret, critically evaluate, and when relevant, express their opinion regarding statistical information and data related arguments (Gal, 2002). Statistical literacy also includes the ability to access, define, locate, extract and filter needed information in a complex array of information products (Gal, 2003).

Watson (1997) initially developed a view of statistical literacy based on media reports which focused on the data consumers. Watson and Callingham (2003) introduced a hierarchy of three tiers of statistical literacy that increase in sophistication: basic understanding of probabilistic and statistical terminologies; understanding of statistical language and concepts embedded in wider social discussion; and challenging claims in the media. Watson and her colleagues started to apply the three tires in their research studies to help them investigate the levels of students' statistical literacy abilities (Watson & Mortize, 2000). Watson and Callingham (2004) defined six levels of students' statistical literacy abilities: critical/mathematical, critical, consistent/non-critical, inconsistent, informal, and idiosyncratic. Watson (2006) also included the knowledge of how data were produced as an important component of statistical literacy.

Rumsey (2002) discussed several definitions of statistical literacy and argued that the term was too broad. As a consequence, she divided statistical literacy into two separate categories: "statistical competence" which corresponded to the basic knowledge of statistical reasoning and thinking and "statistical citizenship" which was the statistical ability to function in today's society. Works of Lindmeier, Kuntze, Sebastian and Reiss (2007) described statistical literacy encompasses competencies regarding the use of mathematical representations and the manipulation of data through reduction. Garfield and Ben-Zvi (2008) defined statistical literacy as basic and important skills that might be used in understanding statistical information or research results.

Models of statistical literacy help to identify critical statistical survival skills for both school students and adults. Models of statistical literacy usually have a 'prescriptive tone', implying that what students and life-long learning adults should be required to do so as to make evidence-based arguments and to live up to the expectation of modern citizens (Shaughnessy, 2007). Gal (2002) described that if critically evaluating other people's statistically based reports was not explicitly taught, then students would remain statistically illiterate. Statistical literacy is a construct which embraces critical thinking and evaluating reports other than working on curriculum-based formulas and definitions.

Statistical Reasoning

Statistical reasoning deals with the way people explain and make sense of statistical information (Ben-Zvi & Garfield, 2004). It involves making interpretations based on sets of data, representations of data, or statistical summaries of data. It also means connecting one concept to another (e.g., center and spread) (Garfield & Ben-Zvi, 2008). There has been positive growth in the past decades in statistics education towards using reasoning models to interpret students' responses (Shaughnessy, 2007). For example, Jones, Langrall, Mooney, and Thornton (2004) explained four process models: describing data, organizing and reducing data, representing data, and analyzing and interpreting data. Models of statistical reasoning are considered to be principally 'descriptive models' that explain how people think about statistics, what they seem to know and understand, and where they have difficulty (Shaughnessy, 2007).

Statistical Thinking

Statistical thinking is an understanding of why and how statistical investigations are conducted and the "big ideas" that underlie statistical investigations (Ben-Zvi & Garfield, 2004). Models of statistical thinking help both researchers and teachers to focus on the important concepts and processes in the teaching and learning of statistics (Chance, 2000).

In their landmark paper for Statistics Education, Wild and Pfannkuch (1999) have presented a four dimensional model of statistical thinking: an interrogative cycle, an investigative cycle, types of statistical thinking, and dispositions. The components of the model are considered to be interacting and dynamic when students work on statistical problems. Statistical thinking has been considered as understanding how and why; understanding when to describe, deal and explain variability; understanding the data; and being able to relate the data with the appropriate analysis to investigate a problem (Wild & Pfannkuch, 1999).

Although there are some commonalities among the definitions of the three learning outcomes, statistical thinking is considered to have high order cognitive statistical processes than the two; and statistical reasoning seems move beyond basic statistical literacy by making some connections of statistical concepts (Garfield & Ben-Zvi, 2008). Most statistics educators define students' statistical thinking as an ability to think like in a way a statistician thinks. Models of statistical thinking are, primarily, 'normative' models of statisticians' reasoning (Shaughnessy, 2007).

2.3 Statistics Learning Outcomes and Mathematics Proficiency Strands

Mathematics and Statistics Education

Despite there are no agreed up on definitions of statistical literacy, reasoning, and thinking; these constructs have been considered the most important learning outcomes in Statistics Education (Ben-Zvi & Garfield, 2004; Garfield & Ben-Zvi, 2008). In fact, research on students' learning outcomes in introductory statistics class depicted unsatisfactory statistical reasoning and thinking (delMas, Garfield, Ooms & Chance, 2007). What is needed is a radically different approach that is designed to help students to be more fruitful on these learning outcomes? There is a need to rethink how teachers could sow the basic seeds of statistical knowledge to students starting from lower grades in mathematics curriculum in schools in order to get the learning outcomes as fruits.

Statistics plays vital role in primary and secondary mathematics curriculum across many developed and developing countries in the 21st century (Batanero, Burrill, Reading, 2011). Statistics Education is established as a discipline form Statistics and Mathematics Education (Garfield & Ben-Zvi, 2008; Shaughnessy, 2007). According to Shaughnessy (2007), in the 1980s' statistics in the mathematics curriculum was considered as if it had been "a lost stepchild" (p.957). Though statistics is getting recognition as a distinct discipline and included well in curricula across the world, the relation between statistics and mathematics as disciplines in school curricula are not openly addressed.

Like a father and child relationship, in some mathematics school curricula, statistics is defined as part of mathematics; for instance, in Ethiopian Grade Nine mathematics textbook, statistics is defined as 'a branch of applied mathematics' (MoE, 2010b, p. 250). On the contrary, the *GAISE PER-K-12* curriculum framework stated that the focus on variability and the contextual nature of data separates statistics from mathematics (Franklin, et.al, 2007). According to Cobb and Moore (1997, p.803), in mathematics "context obscures structure…. In data analysis, context provides meaning." The GAISE report acknowledged that statistics necessarily uses mathematics, but it is stated that statistics should be taught as statistics in schools not as mathematics in order to promote learning outcomes such as statistical literacy (Franklin, et al., 2007).

The contributions of mathematics to statistics are well recognized in both disciplines, for example, the power of abstraction in looking at patterns in mathematics can be used in statistics (Cobb & Moore, 1997). But, what is missing in the literature and curriculums is the contribution of statistics to mathematics. In the last two decades, one of the shifts of mathematics education research has been the move not only to explore questions of how to improve mathematics teaching and learning, but also concerned with critical mathematics education on asking the question of 'why mathematics education' (Earnest, 2007, 2010; Stinson & Bullock, 2011).

Statistics educators have been teaching statistics in schools with the aim of social justice and equity (Frankenstein, 2010; Lesser, 2007, 2010). Gutstein (2006) taught mathematics using statistics in middle schools for empowering students to apply statistics to get sense of social agency. Thus, statistics can be taught in mathematics curriculum with the aim of promoting critical mathematics education.

Mathematics is necessarily important for statistics and the converse holds true. Both disciplines contribute to each other. In fact, statistics and mathematics both draw some knowledge from multiple disciplines and that educators should not ignore the contributions of the other fields to teach well both statistics and mathematics. Statistics educators argue that statistics education outcomes such as statistical literacy, reasoning and thinking somehow interrelate and in harmony with the goals of critical mathematics education (Campos, Wodewotzki, Jacobini, & Lambrado, 2010). Teaching statistics with social justice aim can be seen as a way of teaching statistics which includes conceptual and computational proficiency goals as recommended by mathematics and statistics educators (Kilpatrick, Swafford, & Findell, 2001; Lesser, 2007).

Statistics learning outcomes and mathematics proficiency strands

Reaching consensus at some levels on the definitions of what entails statistical literacy, reasoning and thinking is useful for decision making on what to teach, how to teach and how to assess statistics in schools (Ben-Zvi & Garfield, 2004). In most school mathematics curriculum, statistical literacy has been given more priority than statistical reasoning and thinking (Batanero, Burrill, Reading, 2011; Franklin, et al., 2007). However, this should be considered seriously, because it has implication that statistical reasoning and thinking outcomes reserved only for higher grades in tertiary education. The seeds of statistical reasoning and thinking, basic understanding of statistical concepts and processes, should be fertilized from lower grades to get the fruits of statistics education-the three statistics learning outcomes.

Mathematics proficiency has five components, or strands which is listed below as it is developed for K-8 school mathematics: (Kilpatrick, Swafford & Findell, 2001, p.116)

- *conceptual understanding*—comprehension of mathematical concepts, operations, and relations
- *procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*—ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification
- *productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

If you substitute mentally ‘statistics’ for ‘mathematics’ in the above description of mathematics proficiency strands, it makes sense that the strands would be useful for Statistics Education as suggested by mathematics educators as well (Kilpatrick, Swafford & Findell, 2001). The mathematical proficiency strands have some association with statistical literacy, reasoning and thinking. The statistics learning outcomes terminologies were used interchangeably even by the same statistics educator than the proficiency strands. For example, to critically evaluate statistical reports is referred as both as statistical literacy and thinking.

It is better to identify the unifying aspects of the statistical learning outcomes in order to lay basic understanding of concepts and processes of statistics in Statistics Education in mathematics curriculum at lower grades. Unfortunately, teachers of mathematics teach statistics at school across the world; and they may not have the knowledge of teaching statistics distinct from mathematics as they may not have special knowledge of teaching statistics (Batanero, Burrill, Reading, 2011). Hence, strands of mathematical proficiency need to be considered for better communication and practice in the two disciplines.

2.4 Understanding Basic Concepts of Descriptive Statistics

What does it mean to understand some basic concepts of descriptive statistics? In this section, what entails understanding of descriptive statistics is described relating the mathematics proficiency strands with statistical literacy, reasoning and thinking from the standpoint of the learner. This section presents discussion on concepts of statistics that deal with data, data distribution, center and variation in descriptive statistics.

Learning mathematics with understanding has increasingly get attention from educators and psychologists since the 20th century (Stylianides, A & Stylianides, G., 2007; Usiskin, 2012). Understanding may be expressed implicitly or explicitly based on one's view of understanding. According to Skemp (2006), instrumental and relational were the two types of mathematical understanding which correspondence with procedural and conceptual understanding of mathematics respectively. He described three advantages of instrumental mathematics, that is, rules without reasons. First, procedural understanding is easier to understand. Second, the rewards are straight forward. Third, one easily gets one exact answer with little cognitive thinking. Skemp argued that the instructional method should depend on one's value of desired learning outcomes, for instant, procedural or conceptual understanding of mathematics.

Usiskin (2012) disagreed with Skemp in that relational and instrumental understanding should refer two facets of a concept taking a bit of mathematics at atomistic level unlike taking them at instructional approach level. Further, he proposed five independent dimensions of understanding of a concept: the skill-algorithmic, the property-proof, the use-application, the representation-metaphor, and the history-culture dimensions. But, both holistic view of Skemp and atomistic view of Usiskin about understanding are important for the discussion of what entails understanding of descriptive statistics as shown in Figure 1.

In previous section, it is pointed out that statistics and mathematics are distinct disciplines. The learning outcomes in Statistics Education will help to frame what is statistical understanding of descriptive statistics from advance point of view in Statistics Education. Further, statistical literacy, reasoning and thinking in association with the five mathematical proficiency strands will give better orientation for discussing the knowledge, skills, abilities, and attitude that constitutes statistical proficiency at specific understanding levels of a concept.

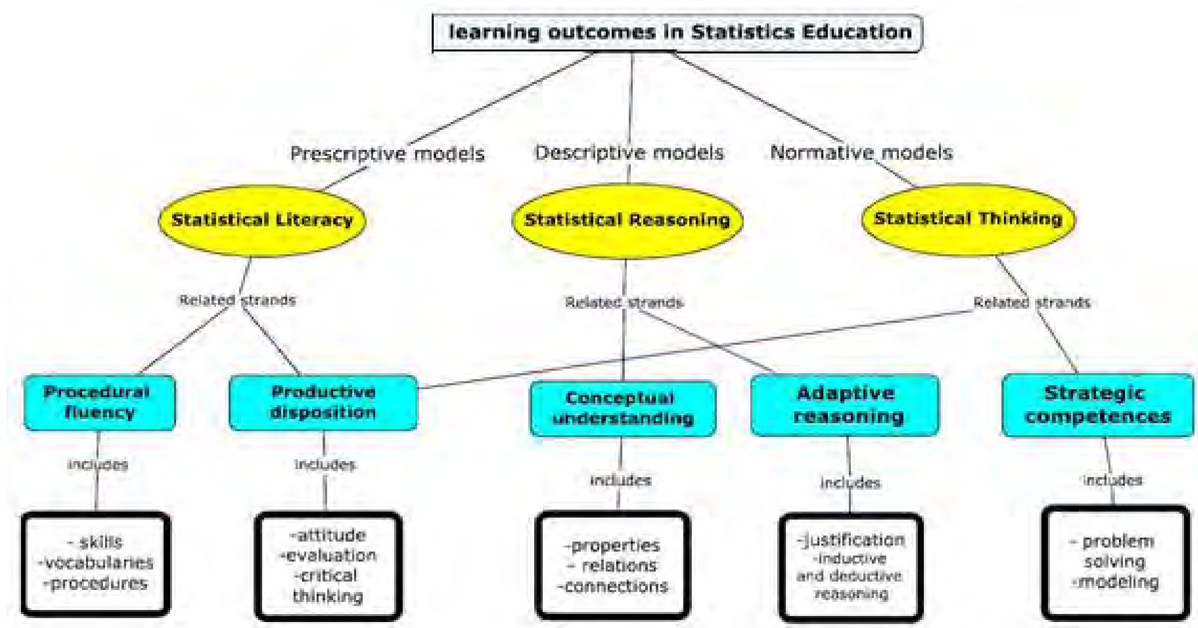


Figure 1: Understanding descriptive statistics: holistic and atomistic view.

Students' understanding of descriptive statistics from the advance point of statistics refer to students' knowledge of what are the big ideas of descriptive statistics which could develop students' statistical literacy, reasoning and thinking. A number of statistics educators have shown that understanding the concepts of variability and distribution in schools are important for developing students' statistical literacy, reasoning and thinking (Garfield & Ben-Zvi, 2008; Makar & Confrey, 2005; Shaughnessy, 2007).

The *Guidelines for Assessment and Instruction in Statistics Education* (GAISE) (PreK-12) states that in order to develop the ability to think statistically, students must learn that "statistical problem solving and decision-making depend on understanding, explaining, and quantifying the variability in the data" (Franklin et al., 2007, p. 6). However, the concept of variability has not been well addressed in statistics instruction at schools (Pfannkuch, 2005). Recognizing and interpreting variability in data lies at the heart of statistical reasoning and much needs to be learnt about how to cultivate and develop this type of reasoning in students (Garfield & Ben-Zvi, 2005). Understanding the "*omnipresence of variability*" as a component of statistical literacy has been considered a core concept to the very need for the discipline of statistics (Cobb & Moore, 1997, p. 801, italics in original).

Teaching about distribution is also taken as the major goal in Statistics Education (Garfield & Ben-Zvi, 2008). Shaughnessy (2007) stated that students' need to enhance an understanding of distribution starting from early grade by "reading, analyzing, critiquing, and making inference from distribution of data" (p.968). Bakker and Gravemiejer (2004) proposed a structure for analyzing the relationship between data and distribution as shown in Figure 2. Unfolding and predicting aggregate features of data sets that cannot be observed from individual cases makes the statistical analysis strong (Bakker & Gravemiejer, 2004).

The figure depicts both upwards (from particular data points to entire distributions of data) and downwards (from entire distributions back to particular data points) reasoning by students as they learn to reason about distributions of data. Bakker and Gravemiejer (2004) made clear that students as novices typically see individual values and use these to find values such as the median, range or standard deviation, but that this does not mean they are seeing the median, for example, as representative of a group. Thus, reasoning about data distribution is the other aspect of what entails basic understanding of descriptive statistics from the advance point of Statistics Education based on holistic view.

Distribution (Conceptual Entity)	
Center	**Spread**
Mean, median, midrange	**range, interquartile range, standard deviation**
Data (Individual Values)	

Figure 2: Relation between data and distribution (Bakker & Gravemiejer, 2004, p.148)

The three statistical learning outcomes in Statistics Education had relations with mathematics proficiency strands as shown in figure 1. Conceptual understanding refers to an integrated and functional grasp of statistical ideas. Students with conceptual understanding know more than isolated facts and methods as statistical reasoning. For example, suppose students are learning standard deviation in descriptive statistics. A student needs to co-ordinate several underlying statistical concepts from which the concept of standard deviation is formed. Students need to know the concept of standard deviation related with center and data distribution (delMas & Liu, 2007). Statistical literacy and thinking relate with productive dispositions, since they have aspects of students' attitude and inclinations towards statistics.

Adaptive reasoning refers to the capacity to think logically about relationships among concepts and situations. Statistical reasoning involves both inductive and deductive reasoning (Wild & Pfannkuch, 1999). One manifestation of adaptive reasoning is the ability to justify one work which is also related with conceptual understanding.

Procedural fluency relates with statistical literacy, since it refers to understanding procedures and facts (Gal, 2002; Kilpatrick, Swafford & Findell, 2001). Strategic competence refers to the ability to formulate statistical problems, plan, collect data, analyze the data and make conclusion which is related with statistical thinking. This strand is similar to modeling and problem solving in Statistics Education.

There are mutually supportive relations among the mathematics proficiency strands as there are relations among statistics education learning outcomes. For example, solving the statistical problem requires conceptual and procedural understanding of statistical concepts. Conversely, developing competence in solving non-routine problems helps to give a context and motivation to develop further procedural and conceptual understanding using routine problems (Kilpatrick, Swafford & Findell, 2001). A productive disposition develops when the other strands do and help each of them develop. For example, as students build strategic competence in solving non-routine problems, their attitude and beliefs as statistics learner becomes more positive. Adaptive reasoning interacts with other stands of proficiency during statistical problem solving activity, because students are expected to determine and show the legitimacy of a proposed a strategy with justification (Kilpatrick, Swafford & Findell, 2001).

2.5 Research on Students' Understanding of Descriptive Statistics

The intent of this section is to review studies on students' understanding of descriptive statistics in order to assess the challenge of enhancing students' understanding of descriptive statistics. Reviewing this would give directions to look for an instruction approach that could enhance students' understanding of descriptive statistics overcoming some of the challenges. In this section, the review explores the published literature on the review question: Why are statistics concepts difficult to learn with understanding for high school students in mathematics curriculum?

Research studies which focused on the concepts of center, variability, and data distribution for developing students' understanding of descriptive statistics have been included. First, the main texts under scrutiny are introduced and their main claims about students' understanding of descriptive statistics are compared. Then, the strength and the weakness of the main claims of the studies are discussed. Finally, conclusions are drawn about to the extent on which review texts shed light on the review question.

Mokros and Russels (1995) in Statistics Education studied students' conceptual understanding of average at schools. In order to develop students' statistical reasoning at school, Konold and Pollatsek (2002) argued that students should be taught with the focus on the signals in the noise processes starting from primary grades. Garfield, et. al. (2007) using Japanese lesson study claimed that students could develop their understanding of variability moving from informal to formal to develop their statistical thinking. Baker (2004) using teaching experiment had investigated that students understanding of descriptive statistics could be developed using diagrammatic and distributional reasoning.

On difficulties and various interpretations of average

Mokros and Russell (1995; Russell & Mokros, 1990) were the first educators who investigated on students' conceptual understanding of averages. They interviewed students and had found different students' conceptions of average: average as mode, average as algorithm, average as reasonable, average as the midpoint and average as a point of balance. The first two approaches were taken as non-representative, while the last three were taken as representative of a set of values.

They found that students faced challenge when their informal understanding of average was not related with formal algorithmic approach and students who focused on mode as an average found it difficult to work backward from the mean to construct a data set if they were not allowed to use the actual average value itself as an average. They also pointed out that modal thinking students did not have aggregate view of data.

Similarly, Cai (1993) had found that students' had difficulty working backward from the mean to find data values. According to Cai and Gorowara (2002), both students and teachers had difficult in understanding the average which includes procedural knowledge of the algorithm, a conceptual knowledge of the algorithm and a conceptual understanding of the concept to describe, compare and make sense of a data set.

In longitudinal study of students' concepts of average in grade 3 to 9, built upon the Markov and Russell's studies, Watson and Mortize (2000) suggested that it took many years for students to develop their concept of mean to the conception of mean as a representative of data sets. They identified the following levels of response in increasing order of sophistication: pre-average, single colloquial usage, multiple structures, representation and application in complex tasks. The last level was related to the concept of 'average as a balance' as recognized by Mokros and Russell.

Furthermore, Stack and Watson (2010) had done action research that instruction on the concept of average had many 'dilemmas'. They asserted students' informal understandings of average, media reports in everyday life using average and the technical terms in the curriculum (mean, median and mode) likely would create a challenge to make decision which meaning was intended to convey. The other challenge was to choose meaningful and motivating contexts that engage students to learn these concepts.

Finally, Zazkis (2013) added to statistics literature that the case of inference from a fixed total was another way of reasoning to understand mean, which referred to 'skipping the addition steps and performing only the division step' (p.206). And, this was related with Markov and Russell levels of 'average as reasonable', since one of students' thinking was it was not possible to compute average without each data value.

Average is a big idea in statistics and essential for students' understanding for summary of statistics and statistical tests (Shaughnessy, 2007). The previous review literature had showed that students' understanding of average was complex and it took years to develop the idea of average as a representative (considering data as an aggregate). Students had little conceptual understanding of averages; this may be because students in school usually learnt the procedural aspects of the concepts as drawn from the traditional formal approach of teaching mathematics (Meletiou-Mavrotheris & Lee, 2002; Shaughnessy, 2007). Almost, all the researchers (Cai, 1993; Cai & Gorowara, 2002; Mokros & Russell, 1995; Watson & Mortize, 2000; Zazkis, 2014) engaged with the intellectual project of understanding students' conception of average using qualitative interview. Only Stack and Watson (2010) had done action research to improve statistical literacy.

Stack and Watson had stated that choosing tasks to motivate students had been challenging. There is a need for further empirical studies on developing students' disposition towards statistics, adaptive reasoning and strategic competency in using average for gaining important learning outcomes. Though different interpretation and conceptual understanding were identified on measure of average, few intervention studies had been conducted to move from sequential linear and procedural approach to the one that grounded on students' intuitive and concrete understanding of average.

Moreover, traditional approach of teaching statistics in school had been criticized focusing only on averages with little emphasis on variation (Gould, 2004; Reading & Shaughnessy, 2004; Shaughnessy, Watson, Moritz, & Reading, 1999). Focusing only on measure of central tendency may lead students' to consider variation as an ad hoc to descriptive statistics, for example, a study showed that most students in school got neither procedural fluency nor conceptual understanding of standard deviation (Shaughnessy, 1997). There were studies that attempted to include the concept of variation in school instruction by a little or by large as discussed in the following sections.

Signals in noise processes

Konold and Pollatsek (2002) argued that students' instruction in schools primarily need to focus on center before variability or distribution as shape, since knowing average as central tendency (signals in noise processes) would develop statistical reasoning. They (2002) claimed that from a statistical point of view that the mean as "signal amid noise" was the most important and most useful conception of the mean, because they felt that this conception of the mean was the most helpful for comparing two data sets unlike average as data reducer and typical values.

They had reflected, theoretically, the mean could be interpreted as typical value, fair share, data reducer or signal amid noise drawn from the work of Mokros and Russell (1995). Research studies had showed that students had tendencies not to use averages to compare groups (Watson & Moritz, 2000). Konold and Pollatsek (2002) pointed the challenge of teaching the concept of average in school was students rarely interpreted average as signal from noise processes.

They recommended that statistics instruction in schools should teach early the concept of central tendency and should be the core concept in the curriculum. Despite they agreed that instruction should not merely emphasis measure of center at the expense of variability, they argued that the central idea should be searching for a signal, and that the idea of distribution came into focus when it was viewed as "the distribution around a signal". Similarly, they reflected measure of spread would capture group properties, and they measured an important property with good measure of center: the measures stabilized as more data is collected. They asserted that "nothing about analyzing data is model-free" and that the framework could also be adopted for non-standard processes.

Their intellectual project was theoretical reflection to improve students' reasoning on basic statistics education at school. They had also recommended that "suitable and potentially interesting contexts" should be considered to motivate students and real experiments should be done by students that involve a process of repeated measurement which could develop students' strategic competence (Konold & Pollatsek, 2002, p.191). Their category of mean as a fair share and mean as a typical are more related to data analysis perspective in statistics, while their category of mean as a data reducer and mean as a signal are more related to decision-making in statistics for developing their adaptive reasoning. It could have been better if they substantiated their work empirically to develop students' understanding of statistics.

They themselves acknowledged the challenge of teaching the signal in the noise, but claimed that it should be used before other conceptions of average. But, based on Mokros and Russell and of Mortize and Watson mean as a fair share, and subsequently mean as a typical value are better first introductions to the notion of measure of center, because they build on students' primary intuitions (Shaughnessy, 2007). The noise part of the signal would be complex for students to understand. Further, they gave emphasis on the signal than the noise which other statistics educators argued against them that emphasis should be given to the noise or to the variability of the data as discussed below (Gould, 2004).

Variability

Garfield, et.al (2007) argued variability was the most fundamental concepts students need to learn in descriptive statistics to develop statistical reasoning and thinking. They used constructivism approach in their study following collaborative work like Japanese Lesson Study on introductory statistics. Their main claim was students would develop understanding of formal variability building on students' informal notions of variability. Research had shown that even students could understand variation at grade 3 despite there were little curricula focus on it (Watson & Kelly, 2002).

Garfield and Ben-Zvi (2005) proposed the following seven dimensions of a theoretical framework representing key facets of understanding variation, or variability in data: (1) Developing intuitive ideas of variability (2) Describing and representing variability (3) Using variability to make comparisons (4) Recognizing variability in special types of distributions (5) Identifying patterns of variability in fitting models (6) Using variability to predict random samples or outcomes (7) Considering variability as part of statistical thinking. The framework proposed by Garfield and Ben-Zvi provides a comprehensive structure for looking at how people reason about variation, and incorporates multiple aspects of other researchers' models of conceptualizing variation. Within the framework of Garfield and Ben-Zvi, the focus of this research would be within the first three facets.

There is an interconnection of variability to concepts of distribution and center (Shaughnessy, 2007). Garfield, delMas, and Chance (2007) found that students had informal intuitions about variability that might serve as a good foundation for studying the formal measures. They found student understand better the concept of variability if they learn from informal to formal concepts of variability. Yet, they showed that it was the most difficult concept for students to understand and apply by making sense of its meaning, since enhancing understanding of variability as the "typical" spread from the center appears to be not very easy (Garfield et.al, 2007).

Part of the difficulty on learning variability might also originate from students' misunderstanding of how variability could be represented graphically. Cooper and Shore (2010) studied that being able to differentiate the underlying structure of different graphs types, including identifying the types of data and in which axis it was plotted was a necessary step to perceive variability graphically. For example, when presented with a histogram, some students judged the variability of the distribution on the basis of variation in the heights of bars, or the perceived "bumpiness" of the graph, rather than the relative density of the data around the mean (Garfield et.al, 2007). Histograms with thicker tails indicate greater variability, while clustering toward the middle indicates little variability (Cooper & Shore, 2010). Other researchers claimed that teaching students focusing on distributions would advance students' statistical reasoning and thinking as discussed in the following section.

Distribution

Baker (2004) conducted a design research using diagrammatic reasoning approach as the basis for developing middle school students' understanding of statistical concepts. The study had an aim of understanding students' concept development based on semiotic theory. In developing students' conception of statistics, Konold and Pollatsek (2002) argued that "the central idea should be that of searching for a signal and that the idea of distribution comes into better focus when it is viewed as the 'distribution around' a signal" (p. 262). Bakker (2004), on the other hand, argued that reasoning with shapes was basic for reasoning about distribution. By developing a lens of seeing the distribution as an entity, one can then look at statistical measures as characteristics of the distribution rather than as calculations from individual points (Bakker & Gravemiejer, 2004).

According to Bakker and Hoffmann (2005), to develop an understanding of statistical concepts students needed to learn to reason with diagrams as a sign. The thesis of their argument was that abstract concepts became visible in signs and in their use. An example of hypostatic abstraction was that of forming a notion of 'spread'. A similar design experiment research had been conducted on univariate and bivariate data analysis on descriptive statistics having distribution as a big idea by Cobb, McClain and Gravemiejer (2003). Measure of center, spread, and relative frequency were considered to be characteristics of the way the data were distributed. The instruction goal was therefore to support students' gradual development of a single, notion, that of a distribution, rather than a collection of topics to be taught as separate component of a curriculum unit.

Bakker, Derry and Konold (2006) conducted design experiments to advance students' understanding of center and variation. The main idea of the study was the meaning of statistical concepts such as mean and variation should be understood in their role in reasoning process and that this epistemological view had implication for pedagogy. The holistic view takes the stance concepts develop in ‘package’; but, in atomistic view, individual concepts can be defined and understood independently of other concepts.

A shift was noted from procedural knowledge to an argument based on data analysis to teach the informal aspect of distribution before introducing formal introduction of center and variation (Bakker & Gravemiejer, 2004; Cobb & McClain, 2003). The intellectual projects on distribution were for enhancing students’ statistical reasoning and they hadn’t addressed how to motivate students to do the hypostatic abstraction process if students were disengaged on the activities. Little attentions were also given in developing students’ strategic competence to solve statistical problems.

Section summary and research gaps

In this section, research studies were reviewed on the review question: Why are statistics concepts difficult to learn with understanding for high school students in mathematics curriculum? All the studies just described previously focused on one of big ideas of statistics (center, variability or distribution) and proposed that their focus would help to advance students’ understanding of descriptive statistics.

Though, Mokros and Russell (1995) and other similar studies identified different students’ interpretation of center, they found it was not easy to develop center as representative. According to Konold and Pollatsek (2000), even though they pointed out teaching about signal in noise processes should be central in school, they themselves acknowledged its complexity. Garfield et.al (2007) investigated on the concept of variability and they found it was challenging to develop the concept although it was helpful to teach from informal to formal ideas of variability. Similarly, Bakker (2004) did design research using distribution to enhance students' understanding and found students needed to go through iterative process to develop understanding of abstract concepts in statistics.

It seems that statistical concepts are not easy for students to understand because of their abstractness (Garfield & Ben-Zvi, 2008). A different approach to teaching statistics may be needed, one that will help students develop a deep understanding of the core statistical ideas, understand how concepts are connected, and build their statistical reasoning and thinking at the atomistic and holistic levels. Rather than present material in a logical fashion, as most textbooks and courses do, a mechanism will be required to create a new and innovative approach to teach students with understanding on descriptive statistics.

Konold and Pollatsek (2002) stated working on data cannot be 'model free' that we would need to enhance students reasoning with different statistical models with their limitations. Garfield, et.al (2007) recommended the teaching of variability using carefully designed MEAs would be appropriate for more insight how students' develop their reasoning on variability. Specifically, delMas and Liu (2007) pinpointed that MEAs may create conducive environment for promoting 'system-as-a-whole' thinking which is required for profound understanding of a concept.

The research studies reviewed in this section showed that they had emphasized one aspect of understanding strands over the other. Among the mathematical strands little attention were given for developing students' critical (positive) disposition towards statistics and for enhancing students' ability of strategic competence on statistics. The reason for this may be the intellectual projects were conducted on understanding students' conceptual development of statistics rather than developing students' attitudes and problem solving abilities. More intervention research studies are needed that could include all aspects of the intertwined understanding strands for improved learning outcomes. In the next section, Modeling Approach which combined two research traditions (concept development and problem solving research) that might solve some of the challenges is presented.

2.6 Theoretical Framework and Modeling Research

The purpose of this review section is to shed light on how teaching statistics using Modeling Approach could enhance students' understanding of descriptive statistics in relation to the challenges mentioned in the previous sections of this chapter. This section explores the published literature on the following review question: How could high school students' understanding of descriptive statistics be enhanced using Modeling Approach? The review focuses on Contextual and Social-critical Modeling Perspectives studies. First, the two perspectives, as a theoretical framework of the study, are presented. Then, research studies on modeling are discussed and conclusions are drawn about the extent to which the reviewed texts clarify on the review question.

2.6.1 Theoretical Framework

Contextual Modeling Perspective

It is for the purpose of motivating students in mathematics activities using real life problems and for developing their strategic competence to solve real world problems brought modeling come into existence in schools (Kaiser, 2010). Because modeling activities starts from meaningful real life situations, students could develop positive dispositions towards the subject. Contextual Modeling also known as Models and Modeling Perspective established as theoretical perspective from research studies that have been done for more than three decades by Lesh and his colleagues (Lesh & Doerr, 2003; Lesh & Fennewald, 2010; Lesh, Hamilton & Kaput, 2007). It blends two important, but separate research traditions in mathematics education: problem solving and conceptual development. That means, students would enhance their understanding of a concept as they solve modeling problems simultaneously. In this research the use of the term "mathematics" found in and referred to in Contextual and Socio-critical Modeling Perspectives literature includes statistics.

Theoretical perspectives for Contextual Modeling Perspective research begin their roots from the perspectives of Piaget, Vygotsky and American Pragmatists such as William James, Charles Sanders Peirce, Oliver Wendell Holmes, George Herbert Mead, and John Dewey (Kaiser & Sriraman, 2006; Lesh & Doerr, 2003). It is taken as a framework for developing models of students' modeling. A model in this perspective is defined as "a system of describing or (explaining, or designing) another system(s) for clearly specified purpose (Ferri & Lesh, 2013, p.58)."

Like Piaget, the perspective focuses on the development of elementary but deep cognitive structures with holistic characteristics that are not derived from characteristics of constituent parts (Lesh, 2012). It considers mathematics as the study of structures and the conceptual systems are with-in or between concept systems (Lesh, Yoon, & Zawojewski, 2007). In Models and Modeling Perspective, conceptual development is far more piecemeal and situated than Pigetians suggest. It considers the term 'decalage' that Piaget used for ladder like a sequence of development stages from concrete to abstract understanding as fuzzy, poorly differentiated which occurs along a variety of dimensions: concrete-abstract, simple-complex, situated-decontextualized, external-internal, specific-general, intuitive-formal, and unstable-stable (Ferri & Lesh,2013; Lesh & Doerr, 2003).

A Contextual Modeling perspective would consider students learning by putting them in situations where they express their current ways of thinking in forms that will be tested and revised in directions of increasing power without introducing the concepts as if they are new ideas following some other idea (Lesh & Doerr, 2003). For instance, to teach the interconnected concepts like center, variability and distributions in descriptive statistics, it considers most students could be at intermediate stage of development for such concepts that it may not be good to teach them sequentially like 'topic-topic-topic' approach. A central goal of the perspective is to facilitate students' development and refinement of their own abilities to interpret situations mathematically (statistically).

Statistical concepts in descriptive statistics are difficult to learn because they are abstract (Garfield & Ben-Zvi, 2008). So it would be possible to teach students by creating a learning situation through multiple dimensions of model development. For example, to learn about the statistical concept of mean, students could start from their own intuitive definitions, work on concrete activities like post-it note activities considering mean as a balance point, discussing its meaning on a media article and on textbooks and so on.

The other important point, in Contextual Modeling Perspective, is the important role that language plays in conceptual development and the usefulness of the notion of a zone of proximal development using Vygotsky ideas. Recently, statistics educators have argued that 'lexical words would hinder students' understanding and that we should start with students' prior conceptions to develop their understanding persistently (Kaplan, Rogness, & Fisher, 2014; Pfannkuch, Regan, & Wild, 2010). But, it emphasizes that language is only one among many different kinds of representational media such as complex mixes of spoken language, diagrams, and metaphors, as well as written symbols (Lesh & Doerr, 2003). In this perspective, zones of proximal development are multi-dimensional regions where interactions comprise not only teachers or other adults who are capable, but also learners interacting with peers with themselves and with powerful representational tools and media (Lesh & Doerr, 2003).

In Contextual Modeling Perspective models could be expressed in external representation systems and they could be outside the mind. This is the case where it obviously differs from constructivism (Glasersfeld, 2003, p.326). The perspective recognize that the only way that anybody can have access to another person's internal conceptual system is when they are expressed using some expressed media or representation systems (Lesh & Doerr, 2003).

According to Wild and Pfannkuch (1999), simple statistical thinking tools like statistical graphs are considered to be 'statistical models' which have vital role in statistical reasoning process. Cobb (2007) argues that, "Data sets are concrete, statistical models are abstracts, and every instant of modeling is an instance of abstraction-as a process" (p.339). Working with system of representations is at the heart of the Contextual Modeling perspective which is shown in Figure 3. The perspective has subject-related and psychological goals with the view that models could be inside students' mind implicitly or outside the mind explicitly (Kaiser & Sriraman, 2006; Lesh & Doerr, 2003).

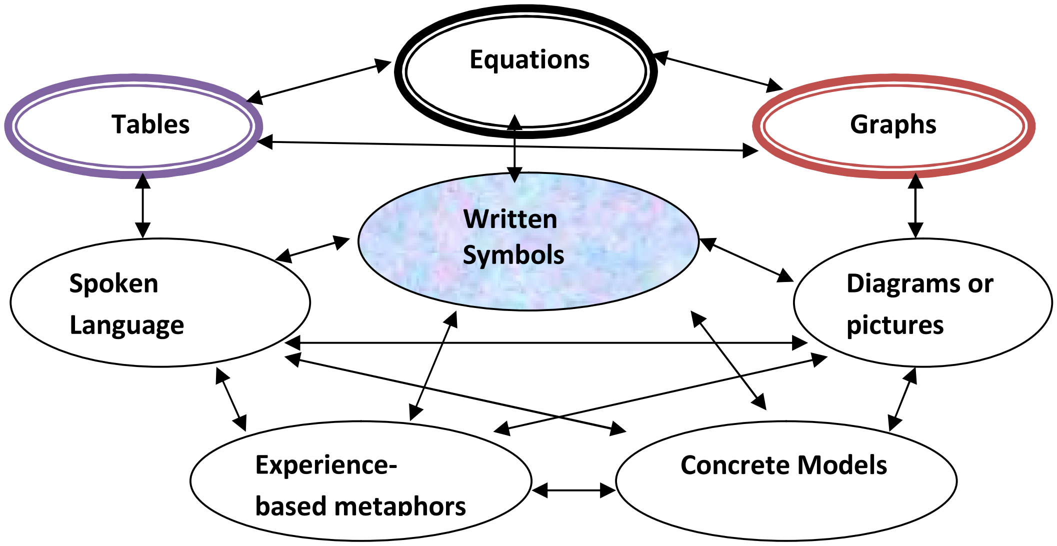


Figure 3: Representation systems (Lesh & Doerr, 2003, p. 449).

Model Eliciting Activities

At the core of the Contextual Modeling perspective is Model-Eliciting Activity (MEA) (Kaiser & Sriraman, 2006). Students are supposed to make sense of meaningful situations, and invent, extend, and refine their own mathematical constructs working on MEAs. It is a problem solving process which motivates and naturally allows students to make sense of real life situation. That means, it emphasizes promising aspects associated with both socio-cultural theories and theories of situated cognition (Kaiser & Sriraman, 2006). Yet, models need to be sharable and reusable by other people in other situation which represent generalizable way of thinking (Lesh & Doerr, 2003).

The following six principles are used to design the MEAs in the research from the works of Lesh and Doerr (2003) which are essential to help to determine what counts as MEA so that the task enable students to reveal their knowledge of relevant mathematical or statistical ideas as students make sense of real life situations:

1. **The reality principle-** Does the situation resemble some real life experience and warrant sense-making and extension of prior knowledge? It refers that problems must be "meaningful and relevant to the students" (Garfield, delMas & Zieffler, 2009).
2. **The model construction principle-** Does the situation create the need to test, modify or extend a mathematical significant construct? There are two parts to this component. The first component refers to the model requires to describe "patterns and rules governing the relations between numbers" (Garfield, delMas & Zieffler, 2009). The second component represents the model needs to be used in the immediate situation and for similar cases in the future.
3. **The self evaluation principle-** Does the situation require people to self asses? Students must be able to self assess or measure the usefulness of their solution. Students evaluate whether the client needs, purpose and conditions met with their products or results.
4. **The construct documentation principle-** Does the situation require to make visible their thinking about the situation? Students must be able to reveal and document their thinking process with their statistical and non-statistical interpretation of the problem situation which requires technical writing.
5. **The simple prototype principle-** Is the model adequately simple to use? The model should provide a useful prototype or metaphor for interpreting other situations (i.e., reusability).The model, which should represent "big ideas" in statistical thinking, should give an important learning prototype for interpreting other problems with similar underlying structure.

6. **The model generalization principle-** Does the solution serve as a useful model for interpreting other similar situations? This principle is also known as Model Share-Ability or Reusability principle refers solutions created by students should be generalizable or easily adopted to other situations or usable by others.

Socio-critical Modeling Perspective

The central aim of Socio-critical Modeling is developing students' critical understanding of the surrounding world with emancipator perspective (Kaiser & Sriraman, 2006). This perspective emphasizes the role of mathematics in society and claims the necessity to support critical thinking about the role of mathematics in society. It refers to socio-cultural dimensions of mathematics that could support students' critical understanding of their environment using mathematics/statistics as a critical tool. This emphasis is related to the idea that mathematics education must take part in effort to teach students be 'critical and engaged citizens' (Barbosa, 2006).

This perspective is similar with a librating pedagogy named as "pedagogy of the oppressed" by Freire (1970) for the purpose of social agency. Gutstein (2006, p.4) argued that "students need to be prepared through mathematics education to investigate and critique injustice, to challenge in words and action, oppressive structure and acts- that is to "read and write" the world with mathematics". A crucial aspect of teaching mathematics for social justice is what students do with the mathematics that they learn. Some mathematics educators stated that students at school learn mathematics better with social justice aim since it will show students the relevance of the subject they learned at school (Leonard & Moore, 2014).

Statistics educators argue that statistics education outcomes such as statistical literacy, reasoning and thinking somehow interrelate and in harmony with goals of critical mathematics education (Campos, Wodewotzki, Jacobini, & Lombardo, 2010). Socio-critical Modeling Perspective in statistics refers to students would develop a sense of empowerment to be able to use statistics critically (Lesser, 2007). In other words, statistics may not be considered simply as useful, but also as a tool to help effect social change in the world (Lesser, 2007). The perspective has pedagogical goals such as critical understanding of the surrounding world (Kaiser & Sriraman, 2006).

Originality of the research idea

MEAs are used to develop 'system at a whole' thinking to describe problem solving situations which involves complex systems (Lesh & English, 2005). The bad news is our world is not only advancing with complex system for a better life, but also the nature of problems that arises by nature or human is affecting negatively the welfare of people in the 21st century. For example, the mass killing of students in schools, the epidemic deadly disease like Ebola, weather variability, global warming, war crisis, immigrants' life at boarders, terrorist attack, poverty, violence, unemployment, threats of extremisms across the globe, lack of human basic needs and the like are rampant and happening across the world. One could say 'the world is in difficult time'. Educators need to instill in students systemic understanding as well sympathetic understanding to pose and solve problems in order to give chance for everyone to live safely in the world.

Because of much of a basic Socio-critical Modeling Perspective can be launched by problems arising in relevant contexts and by data sets students summarize and analyze, it is important to be able to identify 'investigative theme' as in the sense of Paulo Freire

(1970) for changing the world for better human living conditions which could be integrated in MEAs using modeling instruction. Educators need to enhance students' sympathetic understanding using Socio-critical Modeling Perspective with systemic thinking abilities using Contextual Modeling Perspective with the aim of living together in the world that is 'survival with dignity' (Ernest, 2010; Stillman, Kaiser, Blum & Brown, 2013).

The originality of this study lies in using MEAs for teaching statistics as a critical tool for transforming the life of students' as well as the society. MEAs are getting recognition: for developing concepts in mathematics and engineering as systematic thinking; for teaching mathematics in primary classes as case study for kids; for assessing students' thinking; for teaching mathematics for problem solving. There are hardly research studies that have been done in Statistics Education for the purpose of enhancing students' understanding using MEAs that arise from socio-cultural realities with personal meaningfulness.

Educational systems are heavily influenced by the social and cultural ideologies that characterize the particular society (Kim, 2005). According to Kim (2005), Eastern education in China, Japan and Korea is influenced by Confucius culture and it is believed that students need effort and hard work to be good at mathematics where as the western education is influenced by capitalist ideal of producing and consuming goods which value students excel at mathematics based on ability and creativity. Both systems, based on different ideologies, have strengths and weaknesses that are a function of their particular historical and cultural units. Teaching students by empowering students would be new perspective using mathematics/statistics as a critical tool in African socio-cultural contexts. The Modeling Approach in this study uses Socio-critical Modeling Perspective so that students would enhance their critical understanding of descriptive statistics.

2.6.2 Modeling Research

The intent of this section was to review research studies based on the review question: How could high school students' understanding of descriptive statistics be enhanced using Modeling Approach? There were only a few studies conducted on models and modeling processes in Statistics Education (Garfield & Ben-Zvi, 2008), but some studies using MEAs had been conducted and the relevant studies related to statistics education would be reviewed to answer the review question. The research background on the effectiveness of MEAs built on five areas: research on (1) the use of MEAs in Science, Technology, Engineering and mathematics (STEM) areas which includes statistics, (2) invention to learn activities and research on gifted education, (3) the role of prior knowledge and (4) research on problem-based learning (5) research on equity education.

MEAs possess specific qualities that ask students to engage in multiple iterations to solve the problem similar to the engineering design process (Chamberlin & Coxbill, 2012). The process of creating and refining multiple iterations of the models has been referred to as the process of, 'express, test, and revise' (Hamilton, Lesh, Lester & Yoon, 2007) in an attempt to seek a highly refined mathematical model. Magiera (2013) argued that MEAs have provided problem-solving experiences that help a wide range of mathematical expertise that creates problem solvers, innovators, inventors, self motivation and self reliance, logical thinkers, technological literacy which supports the goal of STEM. Researchers had found dramatic and positive results using MEAs in STEM education at different school levels (Diefes-Dux, Whittenberg & McKee, 2013; English, 2010, 2013a;

English & Mousoulides, 2011; Eric, 2008; Lehrer & Schauble, 2005; Moore, 2007; Mousoulides & English, 2012; Shuman et al., 2012).

Statistics is at the intersection of all these disciplines in order to provide “a coherent set of ideas and tools for dealing with data” (Cobb & Moore, 1997, p.801). English (2013b) had investigated that how first grade students learn to competently generate, test, revise and represent data through data modeling before formally being taught to do so. She had showed that through scaffolding using rich and motivating contexts, students at young age could construct their own representation, ways of understanding, and they learned to think about their data. A similar study conducted by Kinnear (2013) showed young students at age of 5 had competence and capacity to develop statistical reasoning.

Statistics Education research studies had indicated that MEAs promote statistical reasoning and thinking at upper primary and at introductory statistics courses. By considering a real world problem, a set of data, the need for a solution, students would experience the statistical enquiry cycle (Wild & Pfannkuch, 1999) as they solve MEAs problems. Doer and English (2003) investigated students at primary were able to create generalizable and reusable systems or models for selecting, ranking, and weighting data.

To offer students statistical problem solving based on real data Garfield, delMas and Zieffler (2011) showed how to develop MEAs funded by the CATALST (Change Agents for Teaching and Learning Statistics) project. Noll, Gebresenbet and Glover (2012), using CATALST curriculum that involve MEAs, explored various ways to support students’ thinking and to help them answer informal statistical inference questions in introductory statistics using dynamic statistical software. MEAs have recently been shown to be effective in statistics education for catalyzing group discourse during cooperative learning (Hjalmarson, Moore & delMas, 2011; Pearl et.al; 2012).

Chamberlin and Moon (2005), and Coxbill, Chamberlin and Weatherford (2012) argued traditional classroom method for identifying mathematical creative students were inadequate and conclude MEAs had the potential for both developing and identifying creatively gifted mathematicians. MEAs engaged students in an interdisciplinary, non-routine problem solving tasks and provided opportunity for students to develop the type of mathematical talent needed in different fields (Coxbill et.al, 2012).

Cognitive researcher had found that students' prior knowledge and intuition had conflict with new learning (Schwartz, Sears, Chang, 2007). Research suggested that the need for learning activities that help students work through inconsistencies in their prior knowledge and intuitions while at the same time building the scaffolding for future learning (Schwartz et.al, 2007). MEAs have been designed explicitly to reveal and test students' intuition and prior knowledge while at the same time providing for extensions, revision, integration of these ideas to develop a foundation for more abstract, formal ways of understanding (Lesh et.al, 2000).

MEAs are primarily used to investigate students' thinking and hold great potential in situation in which other assessment techniques fall short; a huge void is left in understanding why students come up with the answers that they do (Chamberlin & Coxbill, 2012). MEAs are typically used to 'plow the field' so that seeds of learning more formal content may be planted (Garfield, delMas & Zieffler, 2009). Researchers recommended that MEAs provides a richer set of assessment data (Chamberlin & Coxbill, 2012; Lesh & Doerr, 2003; Lesh & Fennewald, 2010).

An intervention research using experimental and control group on 403 students from eight elementary and secondary schools had showed students' models improved as they worked through sequence of six MEAs and also revealed a number of factors, such as students' grade, experiences with modeling activities, and modeling abilities that influenced their modeling process (Mousoulides, Christou & Sriraman, 2008). Three of the MEAs in the study involve statistical concepts such as average, ranking, weighting, aggregating. The range of models students constructed suggested teachers need to be aware MEAs produce various strategies and thinking in the part of the students (Mousoulides et.al, 2008)

Four levels of mathematical tasks were proposed by Chamberlin (2010) in increasing order of cognitive thinking: mathematical exercises, word or story problems, mathematical problems and non-routine mathematical problem solving task like MEA. He suggested that one-third of the task need to be low-level activities at level 1 and 2, and two-third of the tasks need to comprise high-level activities like MEAs. He argued that both activities are needed and the best possible percentage of each level should be reached to serve all students with equity. If teachers couldn't provide students relevant non-routine problems like MEAs, students would be tend to get bored, get negative dispositions, such as attitude, interest and value of mathematics; and get disengaged and less persistence with excessive exercise and word problems (Chamberlin, 2010).

Section summary and research gaps

In this section, research studies were reviewed on the review question: How could high school students' understanding of descriptive statistics be enhanced using Modeling Approach? Even though most of the studies are not large scale studies, as first-case and small scale studies, they had much to tell us how Modeling Approach that uses MEAs could enhance students' understanding.

Modeling Approach would help students' for developing concepts starting from their concrete real life experiences using multi-representation systems and their thinking abilities through iterative cycles along multiple dimensions (Lesh & Fennewald, 2010). The research studies reviewed had showed students' starting from lower grades could develop their statistical reasoning and thinking using data modeling. MEAs could be integrated in the curriculum, instruction and assessment at all grade levels so as to obtain important learning outcomes. But, studies are needed across different learning contexts, population and sites to outshine its effectiveness among other approaches.

According to English and Sriraman (2010), problem solving research is stagnant in the past decade and they claim that concept development through problem solving will be the next paradigm of problem solving research in the 21st century. They have pointed that there is a limited research on developing core concepts via problem solving and suggested modeling as an option could advance problem solving research. They also suggested that "one area in need of substantial research is the development of young children statistical reasoning" that uses data modeling (p.283).

Furthermore, little is known on how modeling approach would improve education equity in the classroom. The best possible ranges of low-level and high-level activities need to be used in one's curriculum and this could be explained only with empirical research. Relevant non-routine problem solving tasks need to be used to serve all students with equity and to enhance students' productive disposition; however, the literature on these aspects still shallow and needs further studies. MEAs are not known for teaching statistics with the aim of social agency to enhance students' critical understanding to solve socio-economic problems; and to this end, this research would investigate how to enhance secondary students' critical understanding of descriptive statistics using Modeling Approach to fill some of the void in statistics education literature.

2.7 Conceptual Framework and Conclusion

Conceptual Framework

The aim of this study was to teach statistics through problem solving using Modeling Approach as a means for enhancing students' understanding of descriptive statistics. As shown in Figure 4, the Modeling Approach integrates two modeling perspectives: Contextual Modeling and Socio-critical Modeling Perspectives. The study drives the research aim from problem solving policy and from the theoretical perspectives. Then, the perspectives would be substantiated in the curriculum design of MEAs and other follow up activities using representation systems to be enacted in the modeling classroom. The theoretical frameworks will also be used as a theoretical lens to observe the classroom practice of Modeling Approach intervention.

The designed activities would be implemented and observed using Schoenfeld (2013) five dimensions of classroom observation scheme. The following are the five dimensions to observe the Modeling Approach intervention on the Model-eliciting Activities: Statistics, Cognitive demand, Access, Authority and Identity, Agency, and Assessment.

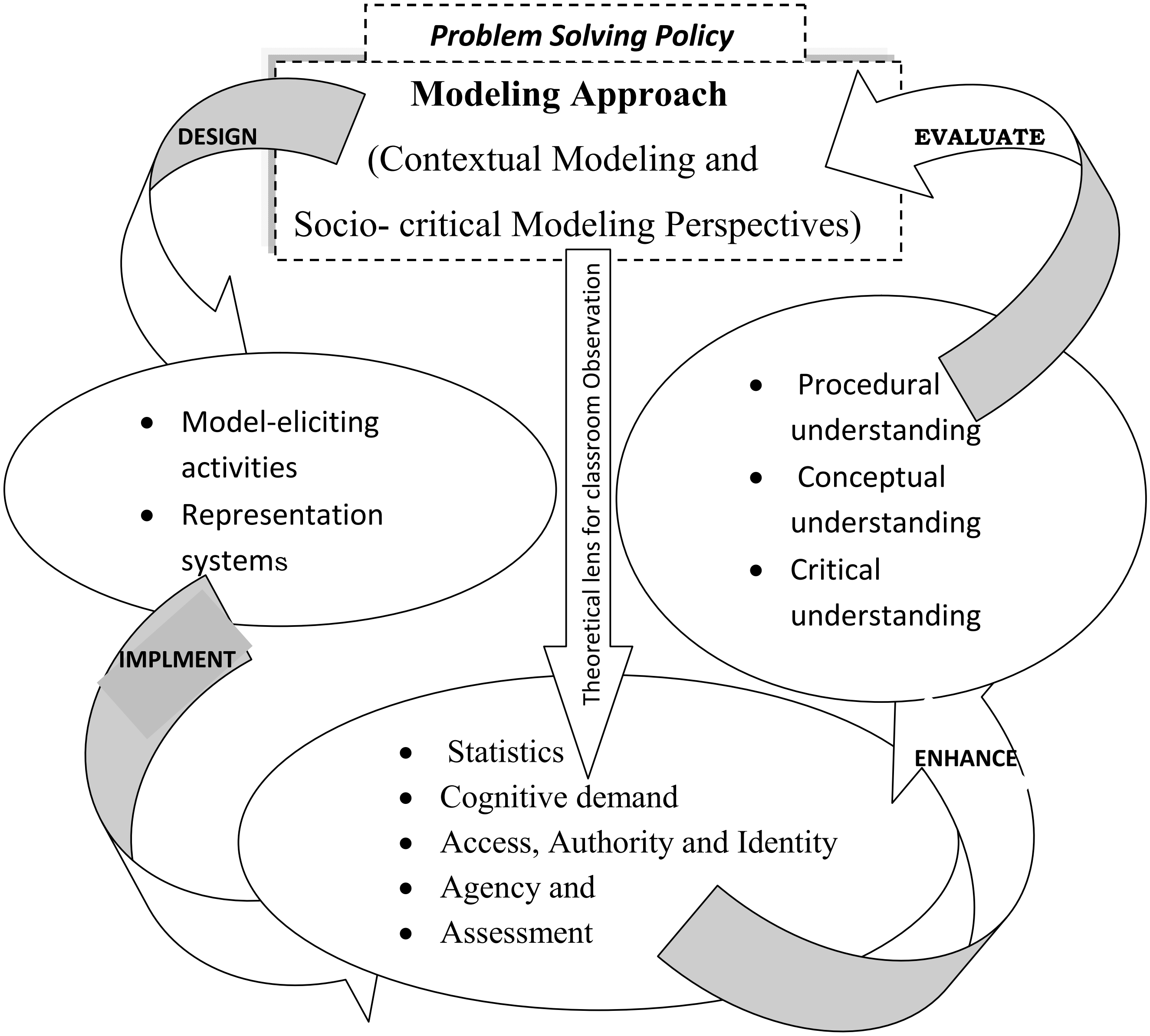


Figure 4: Conceptual Framework of the Study.

If educators are able to capture a classroom practice with better implementation of the five dimensions, then it would give us some insights how students' could enhance their understanding of descriptive statistics. The five dimensions schemes are originally created as a framework to look for 'equivalence classes' of essential elements of classroom activities, that is, with minimal overlap (Schoenfeld, 2013). If the Modeling Approach implemented well with respect to the five dimensions, then students' would enhance their conceptual, procedural and critical understanding of descriptive statistics.

Finally, the findings of the study would help to evaluate the extent to which Modeling Approach could enhance students' understanding of descriptive statistics.

Conclusion

In conclusion, this chapter provided a comprehensive answer on two review questions based on conceptual, theoretical and empirical research works in statistics and mathematics education that supported this research study. The two review questions were the following: (a) why are statistics concepts difficult to learn with understanding for high school students in mathematics curriculum? And, (b) how could high school students' understanding of descriptive statistics be enhanced using a modeling approach?

One of the challenges was students couldn't understand easily the concepts of descriptive statistics. An instructional approach which could move from intuitive to formal, concrete to abstract, specific to general, simple to complex is required to develop students' understanding of descriptive statistics, since statistical concepts need students' to think in abstract. For instance, the concept of mean which has been considered once as simple is difficult for students to develop as a representative of data distribution.

The intellectual projects reviewed focused on one of the big ideas of center, variation and distribution for developing students reasoning prioritizing one concept over the other. But, all the big ideas were interconnected and they were based on model-based reasoning. Students' understanding may be enhanced by teaching the concepts concurrently from informal to formal at the foundational level using Modeling Approach. The research studies reviewed also prioritized one understanding strands of the other and little research had been conducted that could enhance the intertwined aspects of understanding such as procedural fluency, conceptual understanding, strategic competence, adaptive reasoning and productive dispositions which has called for an alternative approach.

The alternative approach needed should promote students' understanding of descriptive statistics at holistic and at atomistic level. One instructional alternative proposed to enhance students' understanding of descriptive statistics is Modeling Approach. It involves teaching through problem solving; that is, as students solve modeling problems, they would naturally develop their conceptual understanding of a concept. Hence, it could be important to develop both strategic competencies and conceptual or procedural understanding of descriptive statistics. Modeling Approach requires students to justify their solution using model-based reasoning from real life situations which may promote their adaptive reasoning and productive dispositions.

One of the core aspects of the Modeling Approach is an MEA. An MEA is a non-routine problem solving task which could engage a team of students for high cognitive thinking and create condition for good classroom discourse. Studies showed the promising aspect of students' learning using MEAs in STEM disciplines and in particular in Statistics Education. But, much remains to be done across different contexts and populations; and little research has been done on equity education using MEAs for serving equal educational opportunity of all students. Lastly, there are barley studies using modeling approach to enhance students' critical understanding. The following chapter presents the methodology that supports the study on students' understanding of descriptive statistics.

CHAPTER 3

RESEARCH DESIGN AND METHODS

3.1 Introduction

This chapter presents the research approach and methods used to investigate secondary students' understanding of descriptive statistics using Modeling Approach. A transformative embedded mixed design in a pragmatism paradigm underpinned this study. To answer the research questions and to test the hypotheses (chapter 1), a transformative embedded mixed research design with lens of two modeling perspectives were used. First, this chapter presents theoretical considerations of the Modeling Approach Perspectives to pinpoint the rationale of the research approach and design of the study. Then, it presents the research methods describing the research site, the intervention and interview participants, the research instruments, the data collection methods and analysis. Finally, this chapter presents ethical issues of the study.

3.2 Theoretical Consideration

3.2.1 Modeling Approach Perspectives

This study was informed and shaped by the theoretical influences of Contextual Modeling and Socio-critical Modeling perspectives. The researcher blends the theoretical perspectives working like a bricoleur (Cobb, 2007). Given the purpose and context of the study, recognizing the theoretical research framework provides structure for explaining and justifying the usefulness and appropriateness of a study (Lester , 2010). Accordingly, two major theoretical influences on this study were Contextual Modeling and Socio-critical Modeling perspectives. They were used as a lens to investigate how secondary students enhance their understanding of descriptive statistics.

The guiding philosophy for this research was pragmatism based on Contextual Modeling, and Socio-critical Modeling perspectives which dictated the research questions and the whole research process at macro level as shown in Figure 5. Pragmatism philosophy considers truth is what works at the time and it is not based in a duality between reality independent of the mind and or within the mind (Lesh & Doerr, 2003). Pragmatists agree research occurs in social, historical, political and other contexts with theoretical lens of social justice and political aim (Creswell, 2014). The pragmatism philosophy allows the study to collect both quantitative and qualitative data (Creswell, 2014).

At meso level, research methodology must provide a rich source of students' understanding of statistics by letting them to construct, share, modify and document their conceptual constructs by externalizing their thinking through representation systems. In addition, it must reveal students' critical understanding and provide a mechanism to compare students' achievement scores on various assessment tests.

At micro level, modeling perspectives would help the researcher to speculate to what extent and how students enhance their understanding of descriptive statistics. The design and the method must give a means to compare the amount of learning achieved using different instructional approaches. To this end, the study used non-equivalent quasi-experimental pretest and posttest design on achievement tests. To see students' change of attitude towards statistics using Modeling Approach, pretest and posttest attitude surveys were administered. The study also used case study methods to answer the how question.

Macro Level: Pragmatism based on Contextual and Socio-critical Modeling perspectives:

- Ontology: mathematics (statistics) is the study of systemic properties of structurally interesting systems (models) and as well used as a critical tool to solve socio-cultural problems; Models exist both inside the minds of learner and outside mind in representational systems ((internal) conceptual system projected into the (external)world;
- Epistemology: learning occur being an extension or refinement of conceptual systems (models) that were already familiar; learner refine, test and modify models which exist at an intermediate levels in an individual or in a team to change the limit conditions
  - ✓ Substantiated in the design and student learning in the classroom
  - ✓ Drive goals for the modeling approach intervention

Implication 1: To design instructional materials and to investigate on students' procedural, conceptual and critical (sympathetic) understanding of descriptive statistics

- Research methodology must provide rich description of how students' could enhance their understanding of descriptive statistics using Modeling Approach

Implication 2: To compare students' mean scores on understanding tests of descriptive statistics using Modeling and Non-modeling instructional approaches

- Research methodology must provide a mechanism to compare the amount of learning achieved using Modeling and Non-modeling Approaches

Micro Level: Research on students' understanding of descriptive statistics

Implication 1: Use Qualitative case study methods such as interviews, students' artifacts, projects, video and observations data collection methods to understand how students' develop their understanding of descriptive statistics and do analysis by coding data, categorizing coded data into themes and finally make generalization on the findings.

Implication 2: Use comparison group to compare students' understanding of descriptive statistics and do analysis on students' score results using non-equivalent pre and post tests quasi-experimental design on achievement tests; and analyze students' pretest and posttest attitude survey data to see if there is change of attitude towards statistics using Modeling Approach.

Figure 5: Methodological Model of the Study.

3.2.2 Rationale for Research Approach and Design

Mixed methods research is an approach to inquiry that combines both quantitative and qualitative forms of research (Creswell, 2014). The research problems of this study led a transformative embedded mixed method approach to be utilized (Creswell, 2014; Niaz, 2008). The rationale for choosing the research approach was the nature of the mixed research question; that is, to what extent and how students' enhance their understanding of descriptive statistics using Modeling Approach. To investigate to what extent and how students' enhance their understanding of descriptive statistics using Modeling Approach may be complex at classroom level which involves intertwined understanding strands that may require simplification of discrete variables and more holistic qualitative approach.

Most importantly, Niaz (2008, p.298) make a strong claim that mixed method research in education help to supplement justifications for "hypotheses/theories/guiding assumptions/presuppositions to compete and provide alternatives". The rationale to utilize a transformative embedded mixed design was to understand experimental results by integrating views of individuals, that is, to give explanations on students' experiences within context of an experimental intervention; and to provide a clear picture of students' critical understanding with the aim of social agency that call for action (Creswell, 2014).

The transformative embedded mixed design in this study used quasi-experimental design for collecting quantitative data using achievement tests and attitude survey. A case study design was used on the Modeling Approach intervention and students' projects to investigate students' understanding of descriptive statistics as shown in Figure 6.

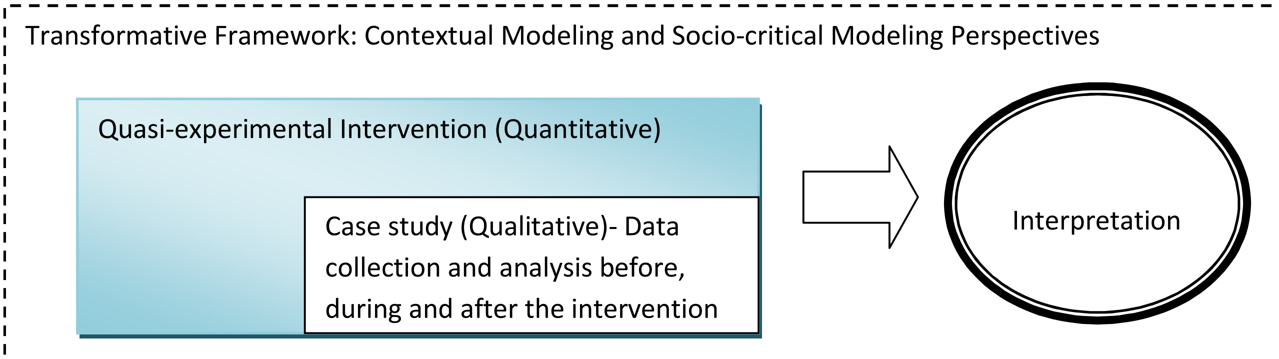


Figure 6: Transformative embedded mixed method.

The rationale using quasi-experimental design is to compare the effect of teaching approaches in the natural school setting, not by assigning students to random or equated groups but by following students in groups formed by their schools and using statistical methods to control for prior achievement (Boaler, 2009). To test the research hypothesis, the quasi-experimental design used in this study is known as non-equivalent control group pretest and posttest design as shown in Figure 7.

Experimental Group:	**NR**	$\boldsymbol{O_1}$	**X**	$\boldsymbol{0_2}$
Control Group:	**NR**	$\boldsymbol{O_1}$		$\boldsymbol{O_2}$

Figure 7: Quasi-experimental design of the Study.

In this design, NR represents non-randomization, O_1 represents pretest, X represent the treatment implemented, and O_2 represent the posttest (Campbell & Stanley, 1963). Besides, means of students' achievement scores on procedural and conceptual understanding with respect to gender and achievement levels were compared. Change of students' attitude before and after the intervention towards statistics in using Modeling Approach was investigated in this study. To test the hypothesis on change of attitude using Modeling Approach in statistics class a single pretest and posttest quasi-experimental design was used.

For explaining the intervention (the Modeling Approach), case study design was used based on the data that included students' solution artifacts on MEAs, observation data, and students' interviews. Furthermore, to explore students' critical understanding of statistics using Modeling Approach qualitative case study design was used based on students' interviews and project works.

The case study used was explanatory type to answer the how research questions on students' understanding of descriptive statistics using Modeling Approach (Yin, 2003). The case study design involved embedded multiple-case design. The unit of analysis in this study were classroom practices of Modeling Approach, teams of students and some individual students who participated on the intervention to investigate how students' enhance their understanding of descriptive statistics.

Threats to Internal Validity

There were several threats to validity that raised questions about an experimenters' ability to conclude that the intervention affects an outcome and not other factor. Internal validity threats are related with the participants (i.e., history, maturation, regression, selection, and mortality), those related to the use of an experimental treatment the researcher manipulates (i.e., diffusion, compensatory and resentful demoralization, and compensatory rivalry), and those involving procedures used in the experiment (i.e., testing and instrument) (Campbell & Stanley, 1963; Creswell, 2014). Potential threats to internal validity of this study were identified and designs created to minimize these threats were described in Table 2.

Table 2: *Threats to Internal Validity*

Types of threat	Description of threat	Action the researcher take
History	Time passes during experiment	Equal period allotment and same external events for both comparison groups are used
Maturation	Change during experiment	Similar age groups and the study took only for three months so there will be little change
Selection	One group outshine the other group	Select intact classes and use pretest scores to compare group
Mortality	Dropout during experiments	Consider the rate of dropout for data analysis
Diffusion of treatment	Communication of the comparison groups	Separate the two groups and keeps the instructional materials
Compensatory resentful/demoralization	Unequal benefits for comparison group	Both groups share their experiences during project presentations
Compensatory rivalry	One group feel they are being devalued	Take steps showing each group learned based on the curriculum
Testing	Familiarity with the outcome measures	Pretest and posttest are different items
Instrumentation	The instrument changes between pretest and posttest	Standard items are used for pretest and post tests

Threats to External Validity

External validity threats arise when the experimenters draw incorrect inference from the sample data to other persons, other settings, and past and future situations. These threats arise because of the characteristics of individuals selected for the sample, the uniqueness of the setting, and the timing of the experiment (Creswell, 2014). According to Schoenfeld (2007, p.89) though such type of study may have limited warranted generality, “it may offer an existence proof, bring important issues to the attention of the field, make theoretical contribution, or have the potential to catalyze productive new lines of enquiry”.

This research study took a position of research types of early efficacy study which engaged with some level of design and development research. The purpose of efficacy research was to determine whether an intervention or strategy can improve outcomes under what are sometimes called “ideal” conditions (Common Guidelines for Education Research and Development (CGERD), 2013). Some of the conditions in this research were the following: teachers were trained for a week and the study setting involved a homogenous sample of students, teachers and schools in a district. The purpose of Design and Development Research was to develop new or improved interventions or strategies to achieve well-specified goal or objectives, including making refinements on the basis of small-scale testing (CGERD, 2013). This early efficacy study was justified by its design and implementation using pilot study as a part of a design and development project.

The strength of this study was it happened in the natural school environment within the framework of the real world of students unlike research studies done within controlled laboratory environments which may lay strong claim to external validity but may lack ecological validity and make no sense in the real world (Design-Based Research Collective, 2003).

The claimed generality of this study would be limited to similar students' background, but it needed further efficacy studies. This study was conducted in two schools settings, but it needed further effectiveness studies across school districts. The external validity of a study that does not assign students into groups may be weaker than one that does, but this is compensated by the increased ecological validity of a study that examines the natural operating of a school. Thus, researcher using quasi-experimental design can study schools and students working in a ways that are realistic and achievable by other schools, rather than ways that has been artificially created by researchers (Boaler, 2009). The claimed generality of this study would be limited to similar students' background, but it needed further efficacy studies. This study may have potential and implied generality for students' populations, schools and district setting with similar characteristics.

3.3 Research Methods

3.3.1 Site and Participant Selection

Site Selection and its Rationale -the schools

This study was conducted at West Shewa Zone schools (School A and School B; pseudonyms) in Ethiopia based on grade 9 secondary mathematics school curriculum which had one unit of descriptive statistics with 22 period allotments (Micheal & O'Connell, 2014). There were 9 and 6 sections of grade students with average class size of 45 and 42 students per class in School A and B respectively. The dominant languages spoken by students were Amharic and Oromigna. According to Ethiopian National Examination Agency (2010, 2013), in Oromia region, students did not achieve the minimum requirement of at least 50% in the average of the five subjects including mathematics. In 2010, in this region, the mean percent scores of mathematics for secondary school girls and boys were respectively 37.7 and 31.0 on the national assessment.

The researcher chose this research site for a number of reasons. First, the researcher had familiarity of the study site living at the place for more than 15 years to investigate the nature of socio-cultural problems that could arise from their everyday life. Second, Ethiopian National Examination Agency (ENEA), (2010; 2013) had revealed that in secondary school 15.5% of the observed variation in students academic achievement attributed to differences among schools. The study selected the two schools to show direction how it was possible to work jointly to improve the quality of education. Third, the distance between the two schools is 0.5 km which could make the data collection process convenient.

Participants in the intervention

From the two high schools, 163 students participated in this study from four grade 9 sections (two modeling classes and two non modeling classes of students). The sampling method used to select the four classes for the quasi-experimental design was purposive sampling because the method would allow creating intact classroom group for the quasi experimental design. Lottery method was used to assign classes for Modeling and Non-modeling Approaches in the two schools. The demographics of the combined sections consisted of 56 % females (76 out of 163). The students' ages were within the range of 14-17 years old. Students at these ages were at the onset of youth age which could be the turning point for the rest of their life. Background information on the intervention participants based on their sex and mathematics achievements levels in the last semester were described as shown in Table 3.

Table 3: *Participants' background information*

Intervention Type	School / Sex / *Levels	School A Male	School A Female	School A Total	School B Male	School B Female	School B Total
Modeling	High	2	3	5	5	2	7
	Medium	7	6	12	5	5	10
	Low	12	11	23	11	12	23
Non Modeling	High	4	1	5	3	4	7
	Medium	11	3	14	7	5	12
	Low	7	14	21	5	18	23

*students achievement levels

Students score were transformed using standard scores and students whose mathematics score were below 50 taken as low achievers, if students score between 50 and 70 assumed to be medium achievers and lastly if a students' score were above 70 the students were considered as high achievers as set by the schools district officers.

Two teachers were selected one from school A, and the other from school B based on their willingness to participate in the research, and then two non-equivalent intact grade 9 classes from each school taught by the selected teacher were selected. In both schools, one class of students was taught using Modeling Instructional Approach and the other as a comparison class was taught using Non-Modeling Instructional Approach.

Participant in the interview

The sampling strategy to select the participant for the interview was purposive sampling. Fourteen and twelve interview participants were selected for interview on the Modeling Approach and on the students' projects respectively. Table 4 showed the description of the interview participants on the Modeling Intervention. Students were sent an informed consent to participate in the interview based on the following criteria. The first criterion was students' achievement levels based on the pervious semester mathematics result so that students at low, medium and high achievement levels would be included in the interview. The second criterion was sex of the student to capture some of female students' experiences on the Modeling Approach. The third criterion was on the place where the students live, rural or urban.

Table 4: *Descriptions of Interview Participants on the Modeling Approach*

Student*	Gender	Age	Rural or Urban	Achievement Level
Hanan	F	15	Urban	Medium
Saba	F	16	Rural	Low
Alemu	M	15	Urban	Medium
Helen	F	15	Rural	Medium
Chirenet	M	16	Urban	High
Mahari	F	15	Rural	High
Selam	F	16	Urban	Low
Fiker	M	15	Urban	High
Degenet	M	17	Urban	High
Andinet	M	16	Urban	Low
Tola	M	16	Urban	Low
Bereket	M	15	Urban	Medium
Lemessa	M	14	Rural	High
Ayne	F	16	Urban	Low

*All names of the students are pseudonyms to preserve their confidentiality and anonymity.

The criteria used for selecting students' for project interview were based on the project data collection sites and on the qualities of the project reports which were assessed using project rubric. Table 5 showed the description of the interview participants on students' projects.

Table 5: *Descriptions of Interview Participants on project works*

Student (s)*	Gender	Age	Topic	In or outside school
Netsanet & Helen	Females	15 and 16	Youth and Addictions	Outside
Ibrahim	Male	16	Amharic Language	Inside
Tsehay & Selam	Females	14 and 16	Students' Score	Inside
Robera & Alex	Males	Both are 15	Public Library	Outside
John	Male	16	Disability	Outside
Hanan	Female	15	Traffic Accident	Outside
Muyea	Female	14	Perceptions towards female	Inside
Chirenet and Degenet	Males	16 and 17	Body mass Index	Inside

*All names of the students are pseudonyms to preserve their confidentiality and anonymity.

3.3.2 Instruments of Data Collection

3.3.2.1 Instructional Design for the Modeling Approach

Four MEAs were designed for this study as described in Table 6 based on the six principles of constructing MEAs (chapter 2) with other statistical activities using representations systems. The MEAs had four main components: newspaper article, readiness or warm up questions, data table, and a problem statement (see appendix A). Each component serves a valid purpose and used to engage problem solvers in the task (Chamberlin & Moon, 2005).

Table 6: *Descriptions of the four Model Eliciting Activities*

Title of MEA	Problem Context	Statistical Concept	Objectives with Social Agency
Safe-Water	Students are asked to write a report based on 20 households data to give an awareness program for rural people how to drink safe water and keep their environment sanitation and hygiene.	Different graphs, distribution, average, little or a lot variability	To provide awareness on health and sanitation problems
Millennium Dam	Students are asked to write a report based on three grade 9 sections of students' data on the contribution of money to Ethiopian Millennium dam which is going to be sent by the director for the news agency.	Outliers, measure of centers (mean, median and mode) Range	Empower students on their contribution for developing their identity
Football	Students are asked to write a report to present on school min media to create awareness program and for consulting the Ethiopian football coach based on data on two football teams who plays against two other African football teams.	Center(mean, mode, and median), Distribution, Graphs	Helping others to control emotions and for creating awareness program about Ethiopian Football
Tourist	Students are asked to give reliable information based on the weather data of five tourist sites on what to eat, cloth and shoes to wear.	Distribution , measure of center, measure of variation	Use variability in everyday life like knowing variability of weather conditions

The purpose of the newspaper was to familiarize students with the context of the problem and to develop their statistical literacy. This article required 15 minutes or so to read and provide further information for the second part of MEAs. The second part of an MEA was readiness questions or warm-up questions. These questions were designed to evaluate their understanding of their reading and basic statistical literacy ability of the media article. The third part of MEA was a problem statement which required students pose and solve problems on the socio-cultural problem situations. The fourth part of an

MEA was usually a data table that may be used to solve the problem. A typical modeling approach lesson sequence was shown in Figure 8.

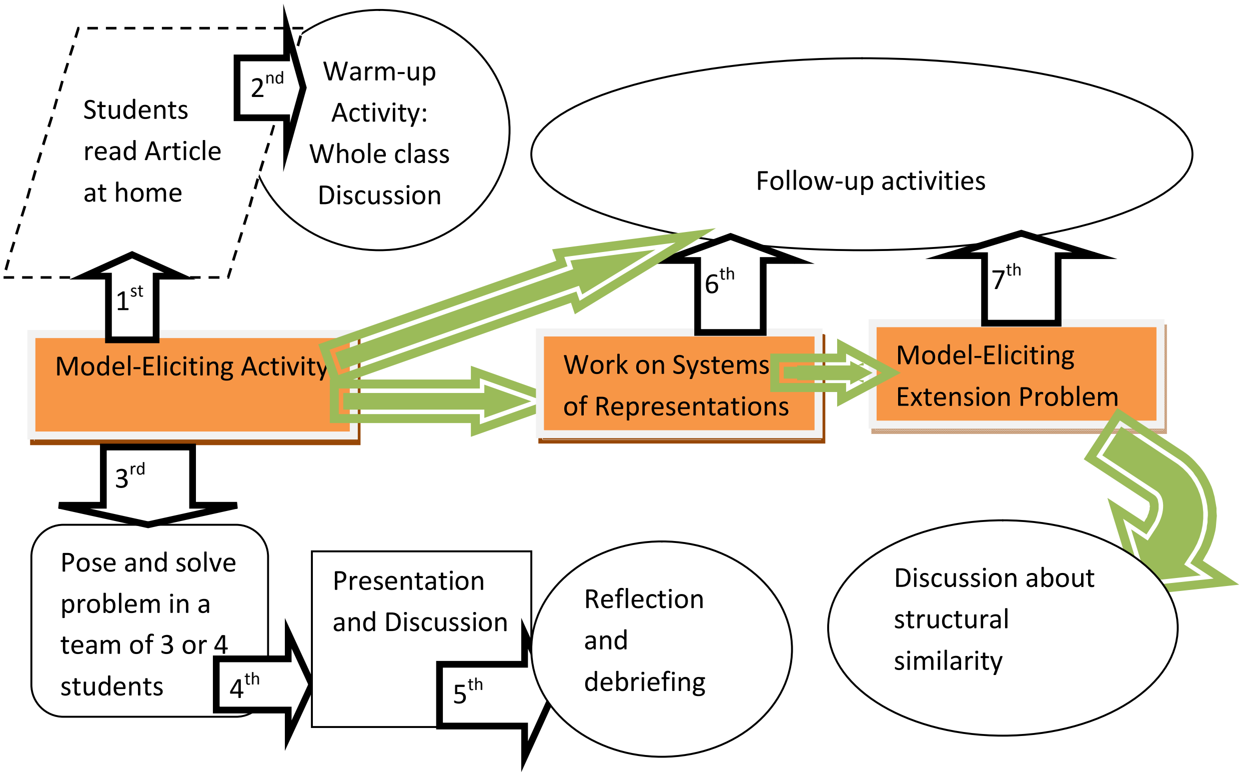


Figure 8: A typical Modeling Approach lesson sequence

After finishing the MEA, there would be other follow up activities that used system of representations and MEA extension problems. For example, in Safe-Water MEA students were introduce other graphs from simple graph type dot plot to histogram and how to interpret by comparing distribution of various graphs. Then, as assignment students wrote a report for Ministry of Water and Energy on Safe-Water MEA based on their own data and got feedback on the structural similarity of the pervious client report on Safe-Water MEA. Students were expected to use the newly introduced representation systems in the MEA extension problem.

Comparing and contrasting Modeling and Non-modeling Instructional Approaches

In this study, two instructional approaches were used in a statistics unit in secondary mathematics curriculum as shown in Table 7. The descriptive statistics concepts like data distribution, center and variability were covered by the same teachers within 4 weeks. The Modeling Approach included working on MEAs, systems of representations and a deliberate pedagogy for social justice. In this approach, students' were engaged in MEAs enhancing their reasoning by testing, revising and refining their ideas iteratively.

The control group used the Non-modeling Approach. This approach followed sequence of topic, topic and topic instructional approach and mainly used Ethiopian Grade 9 mathematics textbook to design lessons. The content analysis of the textbook (chapter 1) showed the percentages of exercises and word problems tasks in statistics topics were 68% and 27% respectively. There were barely non-routine problem solving tasks in Non-modeling Instructional Approach compared to Modeling Instructional Approach. The problems types in Non-modeling Approach often had one exact answer or solution.

Little reading and writing were being taken seriously as component of the activities in Non-modeling Approach, but reading an article and writing a letter or a report was an important part of MEA. The nature of data in the examples and exercise in Non-modeling Approach often used fake data and a single variable as compared to data in MEAs which used real data with multi-variables. Discourse in Non-modeling Approach was promoted mainly by answer and question session guided by teachers; but discourse in Modeling Approach was promoted having students critique and discuss on their different solutions of MEAs or other representational system activities with teachers as co-investigator of the limit-situations on the socio- cultural problems.

Table 7: *Comparison of Modeling and Non-modeling Instructional Approach*

Instructional aspects	Non-modeling Instruction Approach	Modeling Instruction Approach
Sequence of lessons	Topic-topic-topic approach	Informal to formal; concepts learned as one package
Main Instructional Materials	Lessons based on Ethiopian Grade 9 mathematics curriculum Textbook	Instructional lessons using MEAs, its extension problems and activities on Representation systems
Students-teacher focus	Students centered	Students centered
Discourse	Promote classroom discourse mainly through teachers questions and answers	Promotes classroom discourse by having students discuss and critique various solutions
Role of the teacher	Guides students by asking and explaining with examples and giving students feedbacks	Co-investigate with students on Model-Eliciting Activities with social-justice agenda and facilitate discussion
Data	Small data with univariate variable to illustrate and practice procedures	Real multivariate data sets to engage students in thinking and reasoning and making conjectures
Problem types	Exercises and word problems ; students either produce or learn the "correct" solution to a problem or question which could be done individually	MEAs and activities using representation systems; and different teams of students solutions are compared and expert solution provided at later time
Reading and writing	They are not parts of the tasks	They are parts of the tasks (MEAs)

3.3.3.2 Pretest and Posttest Assessments

To investigate the extent of prior statistical knowledge differences between Modeling and Non-modeling groups, an assessment was designed from Ethiopian National Primary Leaving Mathematics Examinations by selecting 15 questions related to basic statistics concepts from year 2008-2014. This assessment which was used as a pretest contained 15 items and all were closed ended multiple choice items. The items dealt with basic concepts of reading graphs like pie chart, bar graphs and calculating percentage, mean, median and mode.

Two posttests assessments were used to compare students' procedural and conceptual understanding of descriptive statistics after they learnt descriptive statistics unit. The posttest on procedural understanding had been collected from Ethiopian Secondary Leaving National Mathematics Examination from 2008 to 2014. The test contained 15 closed ended multiple choice items and the questions asked were procedural questions for calculating measure of center and measure of variation, identifying primary and secondary data, and reading information from bar graph.

The other posttest given at the same time with the procedural understanding test was on students' conceptual understanding of descriptive statistics. The items contained 15 closed-ended multiple choice items and selected from the ARTIST website which includes statistical reasoning assessment items on descriptive statistics. These items were released only to instructors, and were not freely accessible on the Web by students. According to Garfield & Ben-Zvi (2008), "with all of the excellent assessment resources available, statistics teacher need not try to write their own tests, and in fact, are advised to utilize high-quality assessment items such as those at the ARTIST Website" (p.86).

ARTIST assessment items were created for assessing statistical literacy, reasoning and thinking (Garfield & Ben-Zvi, 2008). The researcher adopted the ARTIST assessment items related with the conceptual understanding of descriptive statistics as described in Table 8.

Table 8: *Descriptions of achievement items on conceptual understanding test*

Item	Description of Concept Investigated
1	Statistically to describe and interpret the distribution of a variable
2	Comparing two histograms for the size of Standard Deviation (SD)
3	Explaining a given variable is quantitative or qualitative
4	Recognizing the non-negative nature of SD
5	Interpreting the meaning of average in a context
6	Reasoning with the typical meaning of an average
7	Choosing appropriate measures of center when there is an outlier
8	The relationships between the outliers and the measures of spread in terms of resistance to outliers
9	Justifying the nature of graph which describe the distribution of data
10	Comparing two histograms and interpreting using center and variability
11	Estimating (without calculating) the SD of a set of data points with a given mean
12	Interpreting the meaning of a zero standard deviation for a distribution of data points
13	Reasoning with the signal in noise process of measurement data
14	Reasoning by comparing two data sets if they had little or more variability
15	Reasoning based on data aggregate

3.3.3.3 Reflection Tools

The reflection tools helped students briefly record significant aspects of about what they had done on MEAs, for example changes that occurred in roles they played, strategies they used, feelings they had, or ways their team functioning. These tools were designed to help participants who are working individually or in teams on model MEAs which were translated in their mother tongue languages (Lesh, Lester, Yoon & Hamilton, 2007).

3.3.3.4 Quality Assurance Guide

The quality assurance Guide was designed to help teachers, researcher and students evaluate the products that were developed in response to the MEAs with the following characteristics: (a) the goal is to develop conceptual tools, (b) the client purposes are known and met, and (c) the tool must be sharable with other people and must be useful in situations where the data are different than those specified in the problem (appendix B).

The Quality Assurance Guide was used to quantitatively assess students' models. The levels were designed to categorize how well students' solution artifacts or reports satisfy the needs of the client and how well they explained their reports in general way (Lesh et.al., 2000). The range of response went from level 5, where the response satisfied the needs of the client for the current situation and for other similar situations as well, 1, where the response were going in the wrong direction and the team would need to rethink their reports completely.

3.3.3.5 Observation Tools

Observation tools were used to assess students' team functionality and observe students' presentation on MEAs. For observing classroom practice, *Teaching for Robust Understanding in Mathematics (TRU Math)* classroom analysis scheme was used with permission (Schoenfeld, et.al, 2014). TRU Math rubric as shown in the appendix E contains five general dimensions of classroom activities. Each of the five dimensions was coded separately during whole class discussion, small group work, and students' presentations. The TRU Math scheme comprised an analytic framework for characterizing essential dimensions of mathematics classroom activity and a scoring rubric for capturing their presence in instruction. One part of the scheme delineated a measurement scheme that focused on five minimally overlapping dimensions of mathematics classroom activity as shown in Table 9. Each of these five dimensions captured an essential aspect of productive mathematics classrooms to investigate how modeling approach enhance students' understanding.

Table 9: *TRU Math Dimensions* (Schoenfeld, et.al, 2014)

The Five Dimensions of Mathematically Powerful Classrooms:				
The Mathematics	**Cognitive Demand**	**Access to Mathematical Content**	**Agency, Authority, and Identity**	**Uses of Assessment**
The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.	*The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' mathematical development. There is a happy medium between spoon-feeding mathematics in bite-sized pieces and having the challenges so large that students are lost at sea.*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class. No matter how rich the mathematics being discussed, a classroom in which a small number of students get most of the "air time" is not equitable.*	*The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another's ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and authority (recognition for being mathematically solid), resulting in positive identities as doers of mathematics.*	*The extent to which the teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to move forward.*

Classrooms observation of the four MEAs was analyzed using the five minimal overlapping dimensions with three episodes for the MEA activities as shown in Figure 9. A typical MEA session involves the three distinct phases which is used as episodes to analyze the classroom observation data. If an episode was greater than 10 minute, the weighted average score of two or three episodes was assigned in each phase based on the TRU Math rubric. The rubric was used as a framework taking observation note because, according to Schoenfeld et.al, (2014, p.1) “it provides a straightforward and accessible language for discussing what happens (and should happen) in classrooms.”

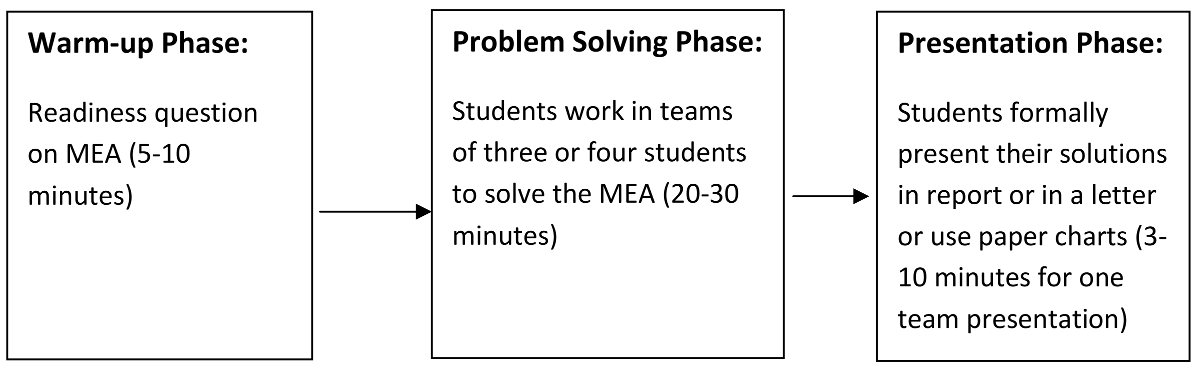


Figure 9: Phases of MEA to analyze episodes of classroom observation.

3.3.3.6 Interview Protocol

The interview protocol contained questions and tasks which were prepared for investigating students’ experience using Modeling Approach. The first two or three questions asked students about their general life and school experiences. The other questions were semi-structured in such way that they help to find out important themes that capture students’ experiences on MEAs and their follow up activities. The interview had been conducted within 40 and 60 minutes and took place in free classrooms in the schools during students’ free time without affecting their class sessions.

3.3.3.7 Students' Attitude towards Statistics Survey

The survey of attitude towards statistics was an instrument designed to assess students' change of attitude towards statistics using Modeling Approach. The survey consisted of four subscales that include usefulness, confidence, anxiety and interest as shown in Table 10. Respondents scored on a 5 point Likert-type scale corresponding to their level of agreement with each statement (Strongly disagree:1; Disagree:2; Neutral:3; Agree:4; Strongly agree:5).

Table 10: *Survey of attitude towards Statistics*

Subscale	No. of Items	Sample items
Usefulness	9	I feel that statistics is useful to me in my life.
Confidence	9	I lack confidence to analyze data.
Anxiety	9	I am under stress during statistics class.
Interest	9	I am not interested in solving statistical problems.

Of the attitude survey, 19 items were written in the positive and 17 items were written in the negative. Reversals of the negatively worded items were made before subscales scores items were determined so that a higher number response always indicated a more positive attitude. The survey instrument was prepared in Amharic and Oromigna language so that the students would take the survey in their mother tongue language (appendix H).

3.3.3.8 Students' Project Rubrics

The students wrote four homework assignments describing their work in each of the statistical investigative activities. The students consulted the teacher after completing each homework assignment to do the next stage of the subsequent homework. The homework assignments for the statistical investigation part were as follows: (1) formulating a question; (2) collecting data; (3) analyzing the data and interpreting the results; and (4) writing and presenting the final project report. These four components provided the students a scaffolding structure to do the project to develop statistical thinking (Franklin et al., 2007). These statistical investigations were conducted by team of three or four students for six weeks. Table 11 depicted the duration and summary of the required pages. Appendix F showed the rubrics in grading the assignment for final project.

Table 11: *Four phases of the investigative project*

Homework Assignment	Description of Students' writing Assignment
Formulating a Question (1-2 pages and 1 week)	Two to five research questions and a rationale for the project; students should select the topics by their own;
Collecting Data (1-2 pages and 2 weeks)	A description of how they collect data and the sample size. Students should use real data;
Analyzing the data and Interpreting the results (3-6 pages and 2 weeks)	Show 2 or more visual display of data and use measure of central tendency and variation;
Final Project (3-12 pages and 1 week)	Compilation of their previous assignment into the following: Introduction, Body and Conclusion and present their project report to students and teachers.

3.4 Pilot Study

Piloting the Intervention

As part of the design and development cycle of the research, the instructional design was piloted tested in two schools to see the feasibility of the design. First, the researcher designed the instructional design for the intervention based on MEA design principles (Chapter 2) and gave to experts for review and validation purpose. Second, the researcher selected six teachers for seminar on the Modeling Approach and there was discussion on the instructional design in a workshop and teachers' comments were taken. Third, the instructional design was pilot tested by two of the teacher from the seminar groups in their own classrooms. The researcher recorded some of the lesson videos and took observational notes. Then, after implementing the instructional design from the pilot test, the instructional design was revised and implemented in two Grade 9 classes in the selected school for the study. The school for a pilot test was assumed similar to the schools for the major study. There were major decisions made by the researcher in revising and improving the intervention in the pilot test. Students were not accustomed to modeling and the researcher, based on modeling processes cycle, developed a worksheet for each MEA that could help teacher to scaffold students' modeling of real world problems as shown in Figure 10.

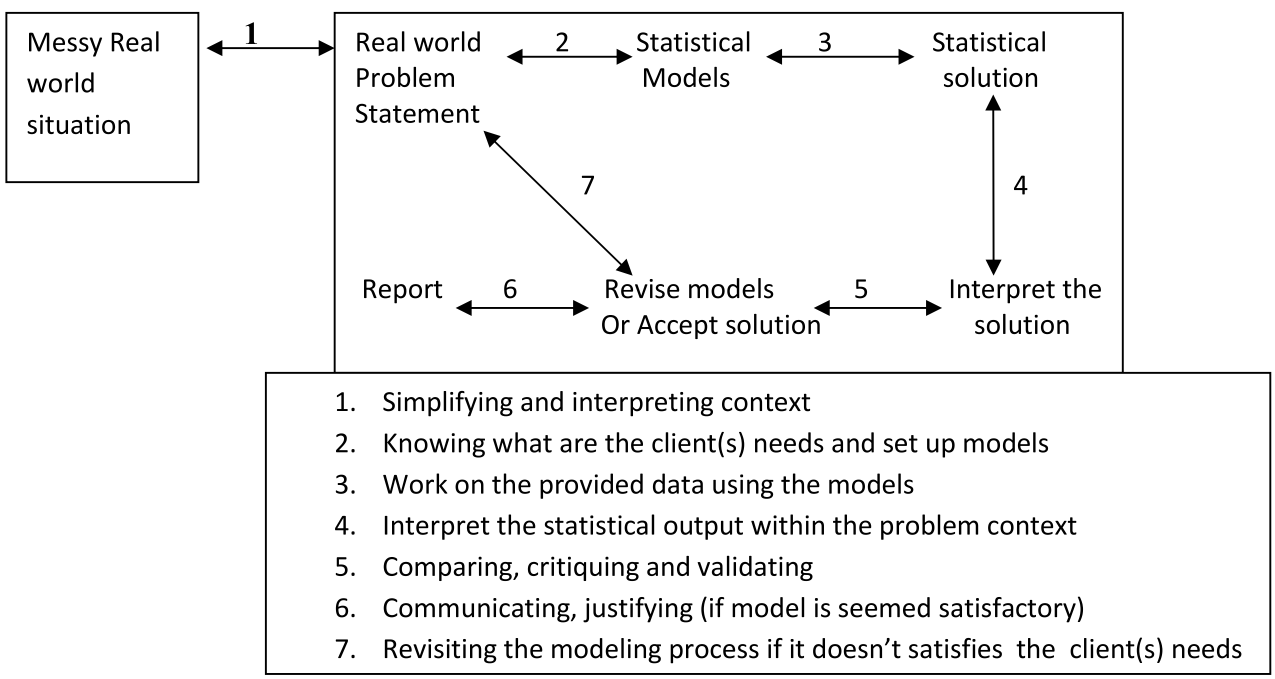


Figure 10: Modeling cycle used for scaffolding purpose adapted from Stillman, Galbraith and Edwards (2013).

Then, piloting the intervention helped to observe the feasibility of the study and to modify and improve some parts of the instructional design materials. Training was given on how to score on TRU Math rubric and on the Modeling Approach for a week on goals, orientation and resources of Modeling Approach to implement it in class. The design was pilot tested one year (in 2012/2013) before the final intervention (in 2014).

Piloting the Instruments

The purpose of piloting the instruments were for checking the reliability and validity of the instruments. The researcher used different validity tests in this study. Construct validity; face validity, expert reviews were used for checking the validity of the instruments. First, the construct validity and face validity was checked against the theoretical background and the literature review. Then, the instruments were provided for experts in the field to review the instrument and for language experts if there were any difficulty in the language use. Finally, 10 teachers at three schools who taught the lesson were asked to give comments on pretest and posttests of the achievement instruments. Test items which did not fit the purpose were dropped based on experts and teachers comments. The Cronbach Alpha of each subscale of survey of attitude towards statistics was shown in Table 12.

Table 12: *Cronbach Alpha for survey of attitude towards statistics*

Subscale	No. of Items	Cronbach Alpha
Usefulness	9	0.83
Confidence	9	0.89
Anxiety	9	0.80
Interest	9	0.85

For checking the reliability of instruments on the quasi-experimental intervention test-retest reliability was used. Test-retest reliability tests were used to check the reliability of achievement tests on the intervention. The students in the pilot study took the tests and one week later took the same test again. A test-retest reliability analysis yielded a reliability of 0.71 for posttest in procedural understanding achievement test and 0.75 for conceptual understanding achievement test. Triangulation, peer review and debriefing, negative case analysis, clarification of researcher bias, member checking, and external audit was used for the qualitative data.

3.5 Data collection Methods and Analysis

3.5.1 Data Collection Methods

As a corollary of the mixed research methodology, to answer the research questions, appropriate mixture of quantitative and qualitative data collection methods were used. Both quantitative and qualitative data collection methods administered during and after the implementation of the Modeling instruction. The source of data for the research drew on classes of data: quasi-experimental pretest and posttest assessments, student artifacts on MEAs, classroom observations, researcher's field notes, students' reports and presentations videos on the project and students' videotaped interviews.

The researcher gave paper and pen pretest achievement tests on descriptive statistics before the intervention and at the end of the intervention. Students took posttests using paper and pen achievements tests on procedural and conceptual understanding of descriptive statistics. During the implementation time the researcher and the one of the two teachers who was not teaching, as complete observer, observed the implementation of the Modeling Approach. A face-to-face – one on-one, in person semi-structured interview was conducted to investigate students' experiences of modeling approach during and just immediately after the intervention. The researcher collected documents of students' artifacts on students' models and took field notes on the classroom practice. Data collection of the study began on February 17, 2014 and ends on May 6, 2014.

3.5.2 Data Analysis

3.5.2.1 Statistical Analysis

Descriptive statistics such as means and standard deviations of pretests and posttests students' scores were reported. Data on achievement tests on procedural and conceptual understanding of descriptive statistics were analyzed using t-test and ANOVA based on the statistical assumptions. Responses for negatively worded items were reversed on attitude survey. Therefore, higher score on each item always represented a more usefulness, greater confidence and interest, and less anxiety. Repeated and independent sample t-tests were used to test the hypotheses on attitude. Data types such as classroom observations were analyzed quantitatively using weighted average. Inter-rater reliability coefficient was done using SPPS with crossed rating design of TRU Math rubric.

3.5.2.2 Case Analysis and Interview Coding

The qualitative data was analyzed using qualitatively categorization and interpretation of data in terms of common themes, and finally synthesized into overall conditions of the approach (Yin, 2003; Creswell, 2014). Interview data on students' Modeling Approach experiences were analyzed using qualitative methods of analysis. The case study analysis involved organization of details about the case, categorization of data, identifications of pattern, synthesis and generalization of the data.

3.5.2.3 ***Content Analysis***

Content Analysis was made on classroom observation data based on Schoenfeld et.al (2014) five analytic dimensions: Statistics, Cognitive demand, Access to Statistics, Agency, Authority and Identity and Assessment. The unit of analysis was episodes of MEAs. Content analysis of teams of students' models on the four MEAs was made using Quality Assurance Guide. The documents of students' reports on their projects were also analyzed using the rubric guide. The summary of data collection and analysis methods was shown in Table 13.

Table 13: *Data collection and analysis methods with primary aim*

Related research questions and hypothesis	**Data collection methods**	**Data analysis methods**	**Primary Aim**
Score comparisons on Achievement tests	Paper and pen pre and posttests on closed-ended multiple choice items	Independent t-tests and One way ANOVAs based on statistical assumptions	To test the hypothesis on the Modeling and Non-modeling Approaches
Attitude survey using Modeling Approach as pre and posttest	Likert scale Questionnaires before and after the intervention	Repeated independent sample t-test based on statistical assumptions	To test the hypothesis on change of attitude using Modeling Approach
How students' enhance their understanding	-Classroom observations -Students' artifacts on MEAs -Face to face one on one interview	-Inter-rater reliability -content analysis of students' artifacts - themes were found, classified, and organized	To capture the classroom practice and students' experiences on how students enhance understanding
How students enhance their critical understanding using projects	-Students' final project reports -face to face interview with single or pair of student(s)	- content analysis were made on the students' reports and on the video presentation and triangulated with the interview data	To answer the question on students' critical understanding of statistics and on the investigative cycle

3.6 Ethical Considerations

Prior to conducting the study the proposal was approved by the Science and Mathematics Education department; and then, the study got local permission from the school district officers and school directors. Ethical issues in the research were considered starting from topic selection of the research problem, to carrying research goals, to the interpretation and reporting of the research findings beyond informed consent.

At the beginning of the study, the benefits and the general purpose of the study were discussed with officers, school directors, teachers and students sequentially with respective orders. Even though classes of students were taught either using Modeling or Non-modeling approach based on active learning as recommend by Ministry of Education (MoE, 2010b), it was ethically challenging to assign a class of students to one approach since one group may feel less benefited than the other group. However, both groups were debriefed on the intervention on how they got benefit from the study. Besides, they shared their experiences and knowledge among themselves during students' project presentations. Among other benefits, teachers were supposed to gain or share knowledge on how to design and implement relevant non-routine problems like MEAs, since Grade 9 mathematics teacher's guide pointed prescriptively that teachers need to develop and implement 'high-level activities' in special cases (MoE, 2010b).

During data collection, the researcher respected the site and disrupt as little as possible. The researcher gave CD for each interview participants on the project which has video of their project presentation and teams of students were awarded mathematics books for their project participation. During data analysis, writing the dissertation and storing data, ethical considerations such as keeping anonymity and confidentiality of the participants, seeking permission to adopt work of others, writing honestly with unbiased language and sharing copies of the dissertation to stakeholders were indispensable parts of the study.

Summary. The theoretical perspectives, the nature of the problems, and the research questions led the study to use a transformative embedded mixed design. A non equivalent quasi-experimental pretest and posttest design was used for comparing the Modeling and the Non-modeling Approaches on procedural and conceptual understanding achievement tests of descriptive statistics. A pretest and posttest attitude survey was used to see the change of attitude using Modeling Approach. An embedded multiple case study design was used to investigate on the Modeling Approach intervention and students' projects.

The research methods were presented which described the rationale and the recruitment of the teachers and students for the intervention and interviews. Essential elements of the Modeling Approach lesson sequence were explained. Comparison also made between Modeling and Non-modeling Instructional Approaches. Instruments of data collections, issues of validity and reliability on the instruments, pilot study, method of data collection and analysis were presented. Threats to internal and external validity of the quasi experimental intervention and some of the researcher actions to reduce the threats were addressed. Overviews of the data collection methods and data analysis were provided. Finally, potential ethical issues of the study were identified. The following chapter presents results and discussions of the study.

CHAPTER 4

RESULTS AND DISCUSSIONS

4.1 Introduction

This chapter presents and discusses on Modeling Approach intervention based on classroom observation data, students' models on the MEAs with follow up activities, and interview data to answer the research question on how students enhance their understanding of descriptive statistics using Modeling Approach. Then, the chapter presents and discusses on the results of students' achievement and attitude data to test the hypotheses and to answer the research question which asks to what extent students enhance their understanding of descriptive statistics using Modeling Approach. The chapter also presents and discusses six cases of students' projects based on students' project reports, presentations and interview data to answer the research question on how students enhance their critical understanding of descriptive statistics through data modeling. Finally, the chapter discusses on the general findings of the study to answer the leading research question.

4.2 Results on Modeling Approach Intervention

4.2.1 Classroom Observation

In this section, classrooms observation of the four MEAs was analyzed using Schoenfeld (2013) TRU Math scheme and rubric (Chapter 3, p.81) on the three phases of MEA: Warm-up Phase, Problem solving Phase and Presentation Phase.

4.2.1.1 The Statistics

Warm-up Phase. At the end of the first day class, reading part of the Safe-Water MEA which was two pages and a half was given to each student as reading homework and they were told to come to class answering the readiness questions. On the second day of the modeling class, there was a whole class discussion on Safe-Water MEA readiness questions. Partly, the questions were comprehension questions on reading graphs and pie charts. Partly, the other questions were meant to introduce students the basic terminologies of statistical concepts like average and variability informally towards developing basic statistical literacy. For example, some of the questions were:

- How many glasses of water do you drink per day in average, explain?
- Do you think the number of glasses of water you drink per day over a week have a lot or little variability?

Similarly, students were given two or three pages media articles on the other MEAs as shown in appendix A as reading homework. All the MEAs lessons were started by asking students the readiness questions. Students explained their reasoning when they were elicited to express their thinking based on their informal and prior knowledge.

Problem-solving Phase. After discussing the readiness questions in each MEAs lesson, students in a team of four were asked to write a report or a letter for the modeling problems. Students were provided with multivariate data sets to write a report or a letter for each MEA considering the needs of a particular client in the problem. For example, the provided data table on Football MEA contained information on variables like players' numbers, position, players name, age, caps, goals, height, weight and the club name in which they were played. Similarly, the other MEAs have at least five variables and data sets at least with 12 cases. Teams of students interpreted the provided data by selecting variables and wrote reports using graphs, measure of center and variation.

Presentation Phase. In Safe-Water MEA, students presented report on different types of graphs like bar charts, pie charts and histograms based on the nature of data. Then, in Millennium dam MEA students present their report using wide range of statistical concepts like mean, median, mode, range, percent, aggregate sum, weighted mean, extreme values and graphs. Students presented their report on Football and Tourist MEAs using graphs, measure of center and variation making their own interpretation of the data.

4.2.1.2 Cognitive Demand

Warm-up Phase. The challenge in Safe-Water and Millennium MEA warm up phase was to develop informal reasoning of variability and average to lay the foundations for formal measures of center and variability by eliciting students' thinking. Rather than introducing directly the formal definitions of the three averages (mean, mode and median) and variation to the students, they were elicited in the two MEAs to express their ideas informally from their experiences. The challenge was to come to agreement on the definitions of the concepts among the colloquial meaning in the students, in the media articles and in the formal definitions of the curriculum. There was little cognitive demand on warm-up phases of Football and Tourist MEAs.

Problem-solving phase. For guiding students on the modeling cycle (Chapter 3, p.85), worksheets which had guiding questions on the modeling cycle were distributed for each team. Teams were provided the scaffolding through the worksheet guiding questions;

however, they were challenged to satisfy the clients' needs and no direct answers or techniques of solving the problems were given or explained to them. As they were not used to data modeling processes, they were guided initially to figure out the general modeling cycle; however, they were never told about how to choose variables and use statistics concepts to write the report or the letter.

Presentation Phase. The challenges in this phase for students were presenting a report or a letter in the shoe of the clients that could satisfy the clients' needs based on the problem statements of MEAs. Students tried to create an awareness program on Safe-Water MEA. Students made presentation on Millennium Dam MEA as if they presented the news program in Ethiopia radio or television. Student wrote letters on Football MEA for creating an awareness program which could be presented in school mini media. Students presented reports in Tourist MEA as if they were tour guides.

4.2.1.3 Access to Statistical Content

Warm-up Phase. Students took part on the whole class discussion answering MEAs readiness questions with a lot of opportunities to participate in class. Both girls and boys participated in class discussion. They were encouraged to participate without fear of making any mistakes. Colloquial meaning of concepts of average and variability were discussed. The formal definitions of the concept of average and variation came later after discussing on their experiential knowledge of averages in their everyday life.

Problem-solving Phase. In the four MEAs, the students were required to prepare report, to describe, explain, justify and communicate their ideas and strategies within their team members. Students were told that making mistakes were normal and they would create opportunities for learning. They were motivated to solve the problem and they were able to keep their preservation to finish writing the letter or the report with good engagement.

Presentation Phase. In almost all presentation except the first MEA, students presented the news in pairs or more. Of the two pairs, one of the students introduced the news and the other student acted as a reporter. Students were getting used to overcome the fears to present reports in front of students from one MEA lesson to the next MEA lesson.

4.2.1.4 Agency, Authority and Identity

Warm-up Phase. Students gave their own informal definitions on the concepts of center and variability on the readiness questions of the four MEAs. Students' ideas were not explored in detail with the amount of time given for students' responses in warm-up phase, but the authority of students' ideas were not neglected on MEAs warm-up phases.

Problem-solving Phase. Teams of students wrote reports on MEAs in various ways using the provided data and chose variables independently to satisfy the clients' needs. Students selected statistical concepts for analyzing variables and wrote reports by their own style. They produced different solutions for the stated problem in the MEAs. Students had ownership of ideas and shared ideas among each other in their team rather than directing every question to teachers or getting answer from textbook.

Presentation Phase. Presenters had the opportunity to demonstrate their proficiency without being tightly constrained by text or teacher. The multiple solutions of students on the MEAs showed students had authorities of ideas. Besides, students had been presenting reports with the aim of creating social justice and trying to create fun presenting jokes with different strategies to develop their authorships of ideas.

4.2.1.5 Assessment

Warm-up Phase. Individual students were asked in warm up phase of each MEA to feel My Learning Goal Assessment (see appendix C). Students' prior understanding of types of graphs, students' informal concepts of average and variability concepts were elicited based on readiness questions, then effort was made to compare the similarity and the difference between students' informal definitions and textbook formal definitions of measures of central tendency and variation.

Problem-solving Phase. Each teams of students were given Quality Assurance Guide (see appendix D) to assess their work on each MEA problem. In each MEA, students were asked the following leading questions more than twice to get sense of what they were doing: (1) who is the client on the MEA problem statement? (2)What kind of report the client needs in the problem statement? (3)What does the client need to be able to do with the report? Students assigned roles by themselves on problem-solving phase like chairperson, secretary, reporter and facilitator interchangeably on the four MEAs. Students assessed their individual roles in the team work and their team functioning based on group reflection tools on transition points on the course of solving MEAs such as brainstorming ideas, selecting strategies, communicating and documenting as shown in appendix D.

Presentation Phase. Students reasoning or thinking was not surfaced or pursued while they were presenting the report on Safe-Water MEA. There were only a few peer feedbacks on the students' presentations on Safe-Water MEA, but in other MEAs' presentation phase, teams of students gave feedbacks to one another. Students got expert solution feedbacks after teams of students had finished their presentations.

Aspects of Modeling Approach classroom practice were observed with minimum non-overlapping dimensions based on TRU Math rubric. The maximum score on TRU Math rubric is 3 and the minimum is 1. The scores of the four MEAs using weighted average was shown in table 14 on the five dimensions of TRU Math rubric (appendix E). The crossed inter-rater reliabilities for the four MEAs for the two respective schools were the following: MEA-1, 0.90, 0.87; MEA-2, 0.81, 0.82; MEA-3, 0.83, 0.77; MEA-4, 0.89, 0.80. Table 14 showed the Modeling Approach score lies most of the time between 2 and 3 in weighted average across the four MEAs on the five dimensions of TRU Math rubric.

Table 14: *Weighted average of TRU Math scores across the four MEAs*

Episodes / Dimensions	Warm-up Phase		Problem solving Phase		Presentation Phase	
	*Sch A	Sch B	Sch A	Sch B	Sch A	Sch B
Statistics	1.62	2.13	2.88	3	2.88	2.75
Cognitive Demand	1.5	1.5	3	2.5	2.38	2.25
Access to statistics	2.5	3	2	2	2.5	3
Agency, authority, identity	2.75	2.25	3	2.62	3	3
Assessment	2.38	2.25	2.75	2.5	2.5	2.62

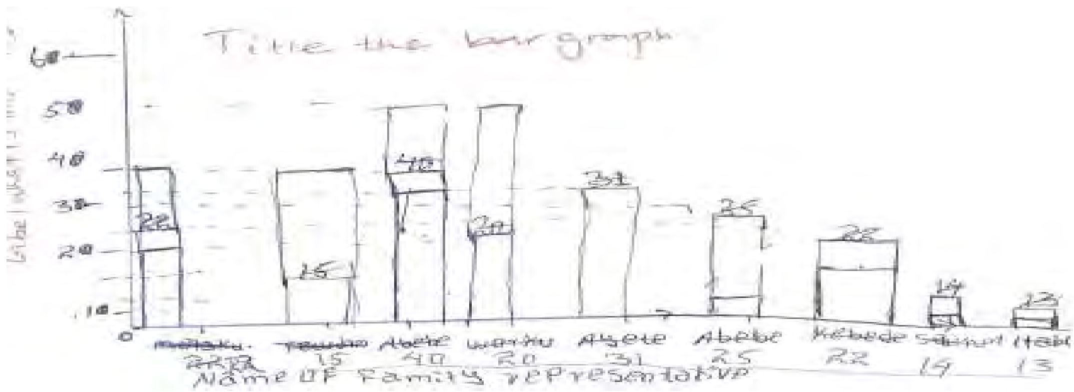
*Sch represents school

4.2.2 Students' Models for MEAs

4.2.2.1 Students' Models for Safe-Water MEA

Students' Models at Level 1: Safe-Water MEA

Two teams of students were at level 1on Safe-Water MEA. The reports were unrelated to the clients' needs and did not fit the purpose of the task. For example, one team of students drew the bar graph as shown in Figure 11. First, they drew the bar graph putting on the x-axis the name of the representative of the 20 householders. Then, they drew the annual income of householders in thousands on the y-axis.



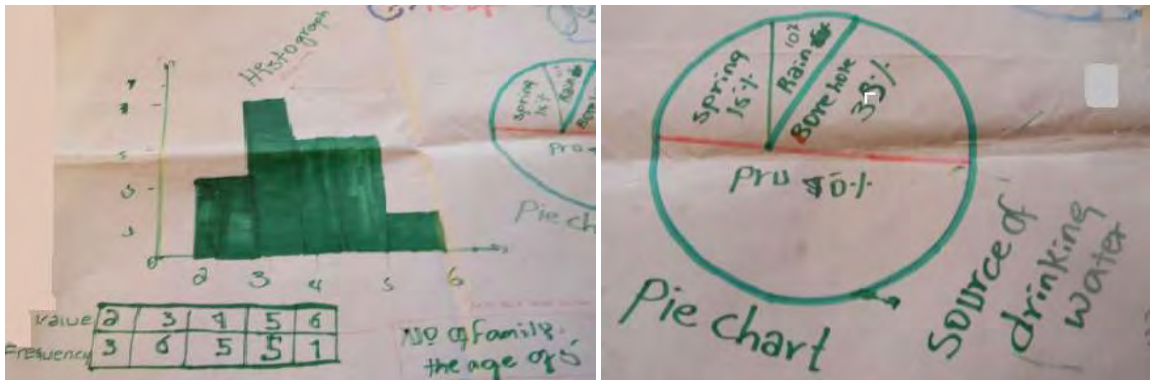
Report: Dear Ladies and Gentlemen, today we will give you an awareness program on keeping the sanitation of latrine houses. We have drawn a bar graph with the income of family in thousand birr and the number of children whose age are greater than or equal to the age of 5 years old. From the graph, we see that a family with 22 thousand birr income has 4 children, a family with 15 thousand birr income has again 4 children and a family who has 40 thousand birr income has 5 children and so on. Each family has better income and at least has 2 children at home. Thus, a family should keep the latrine clean together and should not use open deification and wash hands after going to latrine house. We thank you for your attention!

Figure 11: Team of students' model on Safe-Water MEA at Level 1.

They tried to modify the graph putting the income of householders on the x-axis and the number of children on the y-axis. But, they couldn't justify why they drew the graph. In addition, the report merely included little data from the provided data table and partial data was used for the bar graph. The report needs a complete redirection.

Students' Models at Level 2: Safe-Water MEA

Seven teams' of students were at this level. One team of students' drew pie chart on drinking water sources and histogram on the number of families whose age was greater or equal to 5 as shown in Figure 12. The team drew a histogram with the bars overlapping, but the variable on the x-axis was a discrete variable. The team showed meaningful interpretation, though they selected only two variables. The work was on the right direction, but it needs adjustment to satisfy the clients' needs. The team had to redraw the histogram by labeling the axis and making the bars non-overlapping.

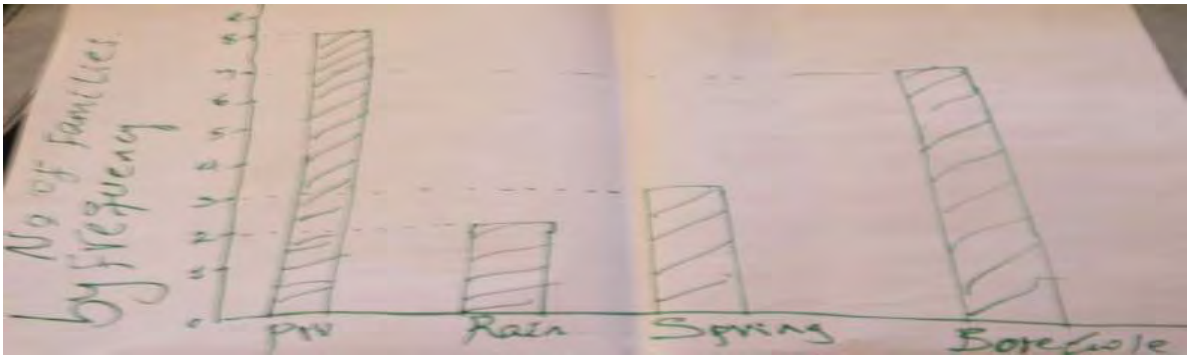


Report: Good morning! Today, we will see how to treat drinking water in a family. Based on our data as shown in the pie chart, for 20 householder families 40% of drinking water source is protected well, 35% -borehole, 15% -spring and 10% comes from rain water. The histogram has also showed the number of family members whose age is greater or equal to 5. There are 3 householders who have 2 family members, 6 householders who have 3 family members, 5 house holders have 4 family members and another 5 householders have 5 members and one householder have the maximum number of family members which is 6. Dear families, the water you drink is not well treated which causes many water born diseases like typhoid and typhus. We should first boil the water and then cool it for drinking purpose. Every family member should feel responsible for keeping the water safe. We thank you for listening!

Figure 12: Team of students' model on Safe-Water MEA at Level 2.

Students' Models at Level 3: Safe-Water MEA

Nine teams of students were at level 3. Teams of students' reports at this level were closely ready to give an awareness program on safe drinking water, but the reports needs some modifications. Among these teams of students, one team of students prepared a report on safe drinking water on how to wash hands after using a latrine as shown in Figure 13. The two bar graphs and the pie charts were appropriate and visible except they need titles. The interpretations on the graphs were correct and used more than three variables unlike the students' models at Level 2.



Report: Welcome! How are you? We are going to discuss with you on keeping our environment sanitation and on how to give our children safe drinking water based on data and graphs. As the graph shows, out of 20 householders' in our rural community, there are 8 householders who have used protected well for drinking water source, 2 householders use rain water, 3 from spring and 7 get from Borehole water source. There are family members who do not wash their hands. For example, out of 9 people 6 people do not wash hands taking 9 householders sample from the 20 householders. This is a serious matter because 50% of householders in the community use open defecation. Hence based on the data we need a solution to keep our children safe because every family have at least one child except two householders. Children are most affected by unsafe water. We recommend the following solution: (1) All people have to wash their hands, and (2) All people have to save the children. Next time we will see how to filter and treat the water.

Figure 13: Team of students' model on Safe-Water MEA at Level 3

Students' Models at Level 4: Safe-Water MEA

One team of students' was at level 4. The team wrote the report to give an awareness program for the society. The team drew histogram for the income of the householders for 20 families and the income ranges from 13 thousand birr to 90 thousand birr. They also drew bar graphs and pie charts. Then, they made association with the area each householders had in its compounds. They could see the paradox in that the area the 20 householders own was large and they had high incomes (Figure 14). But, half of the householders with their family practiced open deification. This was interesting, because they had created relations among the variables looking at some data pattern. But, the report might not be sharable or reusable if the data provided or the context was different.

Report: Dear Ladies and Gentlemen, Good morning! We have called you to solve the problems the community have for building the latrine and keeping our environment safe and wash our hands. Based on our data we found that 20 householders had better income and enough area. The people income ranges from 13 thousand birr to 90 thousand birr yearly. It surprises us because half of them (50%) practice open defecation and lack no latrine houses. Most of the people use water sources like protected well (40%), borehole (35%), spring water (15%), and rain water (10%). All of this water sources may contain impurities and should be treated. But, we need also to build pump water or ask the government to get tap water service. So we need to solve the problem together. How could we create awareness for every member of our family young and old to wash hands? How we build latrines in our compound? And how do we get safe drinking water? Please, give us your comments if you have any better options!

Figure 14: Team of students' model on Safe-Water MEA at Level 4

4.2.2.2 Students' Models for Millennium Dam MEA

Students' Model at Level 1: Millennium Dam MEA

Two teams of students were at this level. One team wrote the report without meeting the client's needs as shown in Figure 15, because the director wanted a news report to be given for a news agency based on three Grade 9 sections money contribution for the Millennium dam. The models needed to be done again with teacher's feedbacks.

NEWS: Secondary Students in School A contribute money for the Millennium Ethiopian dam by buying a bond of 2415 birr. The school director *Teshome Bikila told the media that it is not the first time for students to buy bonds for the Millennium dam. He explained that the students' strong contribution will continue in the future. Some students stated that besides their education they are ready to give support for the country's continuing development and be there to give the necessary help. They said that this bond buying is one part of ensuring the country's development. We present the details as we get more on the NEWS!

Figure 15: Team of students' model on Millennium dam at Level 1.

*The name is pseudonym to preserve the confidentiality and anonymity of the director.

Students' Model at Level 2: Millennium Dam MEA

Four teams of students were at this level. One team of students' report included the number of the participation of girls and boys as shown in Figure 16. The report did not include other data like what were sources of money the students got to contribute for the Millennium dam and the age of students who made contribution.

NEWS: Here are the breaking NEWS! First the headlines! Secondary students in school A have contributed money for the Millennium dam. I am *Alexander Fikadu, the presenter. Stay with us! In Oromia town, students in secondary school have supported by donating money to Ethiopia Millennium dam. From the three sections a total of 2364 birr has been donated and separately grade 9 section A students has contributed 771 birr and each students in average contributed 38.55 birr. In this section equal number of girls and boys were participated. In grade 9 in section B, students had contributed 837 birr and each students in average contributed 41.5. In section B, 7 girls and 13 boys participated. In section C, students had contributed 576 and each students in average contributed 37.80 birr. In section C, 11 girls and 9 boys participated.

Figure 16 : Team of students' model on Millennium dam MEA at Level 2.

* The name is pseudonym to preserve the confidentiality and anonymity of the student.

Students' Model at Level 3: Millennium Dam MEA

Twelve teams of students were at this level. They were similar to teams at level 2, but they used more than three models like sum, mean and mode or median and mode. They worked on more than three variables on the provided data. For example, one team of students wrote a report working on four variables: sex of the students, amount of money contribution, sources of money contribution and sections of a student as shown in Figure 17. The students had used mean and mode to prepare the news; however, they could not identify the mean was affected by extreme values. They did not use the age of the students from the provided data to satisfy the client's need. In their report, they used figures like 39.9834 birr which may confuse listeners, since it is not common way to describe currency with four decimal places.

> **NEWS:** First the Headlines! Students in School A have contributed money for the Millennium dam. Students in three grade 9 sections contribute 2399 birr and 32 girls and 28 boys take part in the contribution. All students in average contributed 39.9834 birr. The sources from where students get the money for the dam contribution include families, friends, uncle, brother and the like. Out of these sources most students contributed from their pocket and they are 24 students. To finalize the news, students' contribution for the Millennium dam is encouraging.

Figure 17: Team of students' model on Millennium dam MEA at Level 3.

Students' Model at Level 4: Millennium dam MEA

One team of students was at this level. The team identified that one student in section B had contributed 200 birr which attributed section B students' contribution to be the highest among the three sections as shown in Figure 18. The team had used models like sum, extreme value, median and range. They had used variables such as students' age, sex, students' contribution of money, and students sections. They used the provided data and variables except money source to prepare the report. They recognized the highest value which could affect the mean of section B contribution. This created an opportunity to teach the median is more appropriate than mean, whenever data contain extreme values or outliers.

> **NEWS:** Here is the news! First the headlines! From three grades 9 sections, 28 girls and 32 boys had contributed money for the Millennium dam. The girls' participation is encouraging. Students' ages in the three sections are between 13 and 22. From the three sections, students in section B contribute the highest amount, because one student has contributed 200 birr. The students contributed 2399 birr. In average, a student contributes close to 35 birr. We ask, your excuse, not showing you the video. We have finished the news and invited you to watch the rest programs!

Figure 18: Team of students' model on Millennium dam MEA at Level 4

4.2.2.3 Students' Models for Tourist MEA

Students' Models at Level 1: Tourist MEA

Two teams of students were at this level. For example, a team of students gave general information on what clothes and shoes to wear, what food to eat and what drinks to drink, but they barely used data in their report to meet the client needs as shown in Figure 23.

> Dear Tourist, it is advisable that a tourist visit a country in September because the weather condition is conducive. When the tourist wants to visit a country, he should not bring clothes for cold weather, that is, he/she should bring light clothes. The shoes should be Sandals as there is no snow in the tourist sites. The tourist needs to bring an umbrella in case it rains. Besides, the tourist should bring fast foods and cold drinks. M.D = 28+23+35+23/4 =23 M.D= /-5/+/-7/+/-5/ +/0/ divided by 4 equals 7 *And M.D= 1.25*

Figure 19: Team of students' model on Tourist MEA at Level 1**.**

Students' Models at Level 2: Tourist MEA

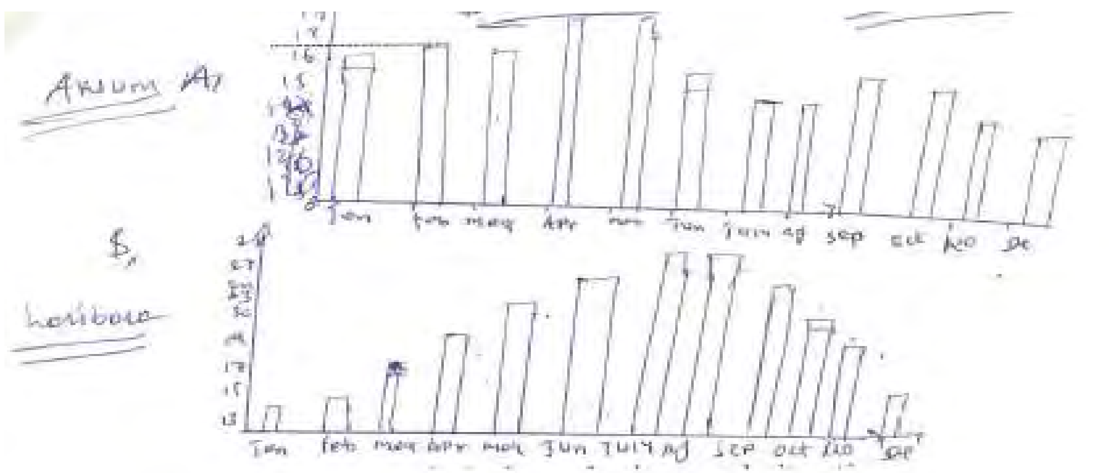
Eight teams of students were at level 2. They used only one model which was range to describe the average annual temperature of two tourist attraction sites out of the four sites. They used partial data of the provided data and they tried to give information on two tourist attractions sites. The reports needed further refinements using more variables and data to furnish the tourist with good information for the four tourist sites.

> Dear Tourist, Welcome to the attractive tourist sites of Ethiopia!
>
> We would like you to introduce two historic tourist attraction sites in Ethiopia. The two sites are known as Harar and Aksum. Harer is a city protected by stone wall and it is recognized by UNESCO. The people of Harer are well known for their hospitality and love. Both Christian and Muslims have lived in Harmony for centuries. Harer is found at an altitude of 55m above sea level. The range of average temperature and rain days for a year in Harer is 5 degree centigrade and 9 respectively. Axum is a city well known for its obelisks for example one obelisk has a height of 33 meters. Aksum is found at an altitude of 2355 m which is at higher altitude than Harar. The range of average temperature and rain days for a year in Harer is 3 degree centigrade and 9 respectively. Dear tourist, Harar is hotter than Axum. You need to visit both places, because both of them are historical places and their social life and culture are interesting. Good Luck!

Figure 20: Team of students' Model on Tourist MEA at Level 2.

Students' Models at Level 3: Tourist MEA

Eight teams of students were at level 3 on Tourist MEA. For example, a team of students at this level used two models (range and bar graphs) to give information to the tourists using the provided data. They tried to present the data using bar graphs and range on average annual temperature on the two tourist attraction sites (Figure 21).

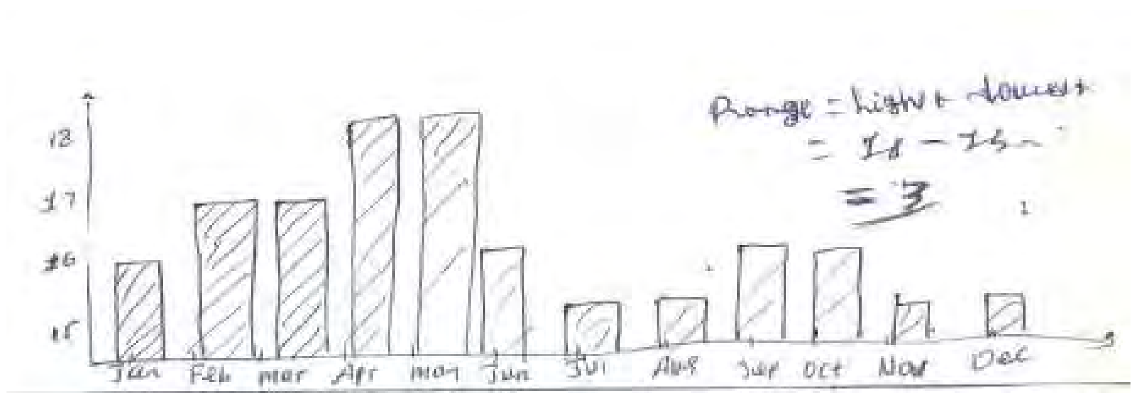


> Dear Tourist, We would like to give you reliable information about health requirement, customs, transport, time, currency, topography, etc. We would like to introduce two tourist attraction sites which are known as Axum and Lalibla. The range of the annual average temperature of Axum is 3^0c and the range of the annual temperature of Lalibla is 15^0c. So the Lalibla temperature is hotter than the Axum temperature. So if you go to Lalibla you must wear white or light clothes, since it will be hot there. You can see and compare the temperature difference using the pair of bar graphs as shown for the two sites. Come and visit us we will give you further information!

Figure 21: Team of students' Model on Tourist MEA at Level 3.

Students' Models at Level 4: Tourist MEA

One team of students was at level 4. Similar to team of students at level 3, the team had used range and bar graphs. But the team of students gave description on the rainfall amount of the four places in addition to using the average annual temperature.



Things to do for Tourist! There are many things that we do for tourists; they may come from a country far from Ethiopia. Thus, they may not know our local languages that we have to translate the local language for them. We can also help them by carrying their goods, food, clothes and other necessary materials. We can also use range to show the tourist sites climate variations to give information for tourists.

- Axum- has low range b/c 18-15= 3^0c   and rain-high =11mm
- Lalibela-has high variation b/c 28-13=15^0c and rain-low=1mm
- Gonder-has high variation b/c 35-23= 12^0c and rain-low=1mm
- Harar-low variation b/c 28-23=5^0c and rain high=9mm

Dear tourists, welcome to the attractive sites of Ethiopia. Ethiopia is a country abundant with varied tourist sites which are attractive and you will have memories of these sites in your mind. We will say, welcome again! Now I am going to tell you about Ethiopian tourist sites. Ethiopia has many innumerable tourist sites that it is difficult to count in short period of time. Among the well known sites by tourism sector, we take today Axum, Lalibla, Gonder and Harar. We can see different amazing things at these places. Our dear tourists, if you want to come to Axum, you have to wear sweater, normal trousers and you need to have tea because there will be rain days. Again if you want to come to Lalibla, you have to wear t-shirt and need to have cold water and you have to have vegetable food because this place is very hot. As we notice from the graphs most months have high variation by temperature. At the end, please try to come to visit Ethiopian tourism sector if you need our help!

Figure 22:Team of students' model on Tourist MEA at Level 4.

Students' Models at Level 5: Tourist MEA

One team of students was at level 5. The team's report assumed to be sharable and reusable as the students used all the variables from the provided data and different models to describe the four tourist sites. They interpreted the data correctly within cultural contexts of the tourist sites like wearing style, social life of the people. They presented the information as if it was given in FM radio transmission.

This is Ethiopian FM RADIO!

Dear tourists, first welcome to Ethiopia! We are happy to announce you that you will be happy for visiting Ethiopia, the country which has several historic, cultural and wildlife tourist attraction sites. Among the cities for tourist attractions I will give you important information on Harer and Gonder. And Helen will give you information about the attractive tourist sites of Lalibla and Axum. Please be with us!

Based on data, Harer is located at an altitude of 55m above sea levels. It is known for its people kindness and the city is called a ‘love country’. And thus, this culture is closer to Brazilian culture and that many Brazilian come to visit Harar. The range of the average temperature for Harar for a year is 5 degree centigrade with medium temperature. Thus, we need to wear light clothes like traditional Harar clothes called ‘dereya’. When we look at the rainfall amount it has a standard deviation of $\sqrt{6.24}$. When we go to Gonder, it is located at an altitude of 380m above sea levels and it is a city that we found several historic and cultural places to visit. The range of the average temperature for Gonder is 12 degree centigrade for a year with hot temperature. The standard deviation of the rainfall amount of Gonder for the year is $\sqrt{2.39}$. Now, Helen will present you information on other two cities.

Thank you Hanan! I will present you information on the great Ethiopian cultural Heritage placed called Aksum and Lalibla. Axum is a place where it attracts many tourists in the world and located at an altitude of 2355m. The range of average temperature for Axum is 3 degree centigrade which means it is not a hot place. The standard deviation of the rainfall amount is $\sqrt{14.85}$ and you can wear whatever cloths you like. When we see Lalibla, it has range of average temperature of 15 degree centigrade. The people who lived in Lalibla often wear white clothes to reflect the sun light radiation. You could also stay there wearing light clothes suitable for the weather condition. Lalibla is located at an altitude of 74 m from sea level and the standard deviation of the rainfall amount is $\sqrt{1.34}$. Thank you for staying with us! We will meet in another program.

Figure 23: Team of students’ model on Tourist MEA at Level 5.

Sample of teams of students' models at each level for Football MEA was shown in Appendix A. Agreement was reached among the two teachers and the researcher using Quality Assurance Guide for the four MEAs that the majority of teams of students were at level 2 and 3 as shown in Table 15. The percentage of number of teams of students' solution at level 1 and 2 were 10% and 38.8% respectively. The percentage of number of teams of students' solution at level 3 and 4 were 45% and 5% respectively. Only one team of students' solution was considered at level 5 across the four MEAs.

Table 15: *Number of teams' at the five performance levels for the four MEAs*

Performance Level	MEA-1	MEA-2	MEA-3	MEA-4
Level one: Requires Redirection	3	2	1	2
Level two: Requires major extension or revision	7	8	8	8
Level three: Requires editing and revision	9	9	10	8
Level four: Useful for the specific data given	1	1	1	1
Level five: Shareable and reusable	0	0	0	1

4.2.2.4 Follow up Activities

Follow up Activities for Safe-Water MEA. Students did the following follow up activities on Safe-Water MEA: distinguishing distribution, little or a lot variability and Safe-Water MEA extension problem. Students in a team of three or four compared different distributions of dots plots and explained possible reason why students' score in mathematics could differ using center and variation informally as shown in Figure 24.

Students were also introduced the concept of variability informally by asking them to describe whether a variable had a little or a lot of variability that could measure about themselves on a daily or weekly basis and examine over a period of week or month. Finally, revisiting Safe-Water MEA as an extension problem, student was given an individual assignment to write a report on safe drinking water in their community.

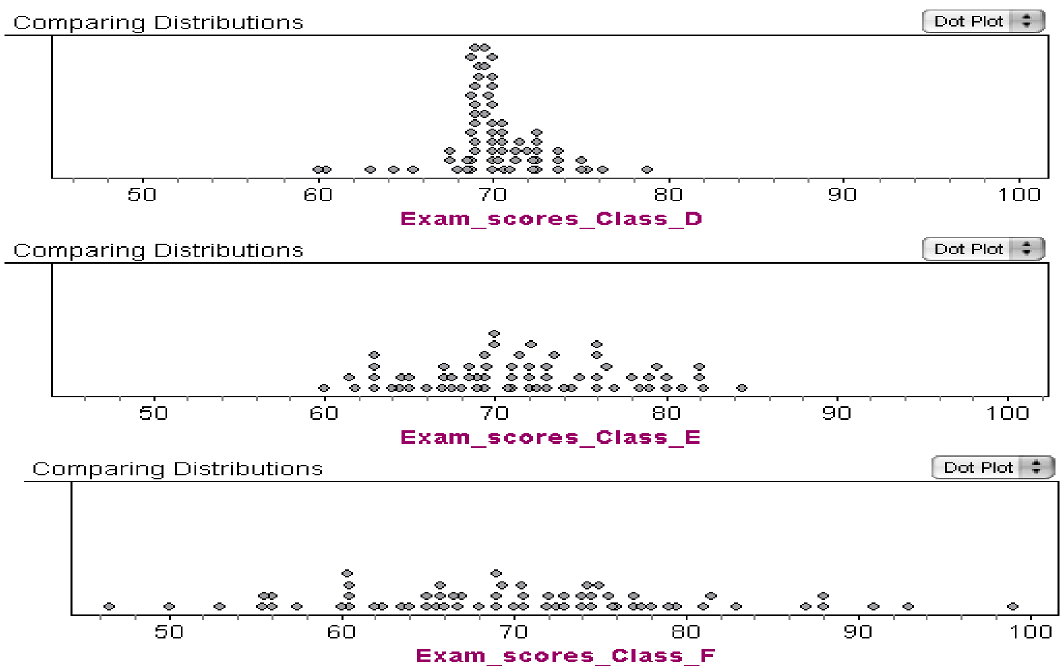


Figure 24: Comparing data distribution using dot plots.

Follow up Activities for Millennium Dam MEA. Students did the following Millennium Dam MEA follow up activities: choosing appropriate measure of center and Post-it Note activities. On choosing appropriate measure of center, students used the students' survey data by using graphs to decide which 'typical' value (mean or median) is appropriate examining a graph of distribution of variable. Students in their team did Post-it Notes activities to get sense of the mean as the 'balance' point as a representative of data and to show a median is not affected by extreme values. Further, students were introduced the concept of mean deviations on the Post-it Notes activity investigating the distance of each data points from the mean. For example, they showed the mean age of 10 students was 21 by explaining that the mean might not be the age of one of the students in the data sets. That is, they were able to move all post-it notes, yet the mean age was still 21 years as shown in Figure 25.

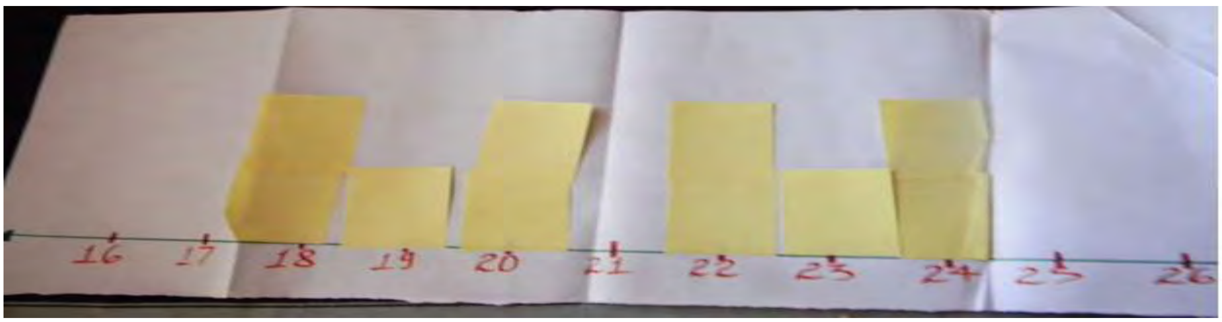


Figure 25: Team showing none of students age is 21 but mean is 21.

Follow up Activities for Football MEA. Students were given Millennium dam MEA extension problems as an assignment on properties of mean weather their report would change if each student added the same amount of money, doubled and tripled their contribution. Then, they got feedbacks on the properties of mean using chart.

Follow up Activities for Tourist MEA. One extension problem on Football MEA was to write a report or a news using statistical concepts on top ten provided data statistics for European and Ethiopian football clubs (English-Premier League, Spain-La Liga, Italy-Seria A, Germany- Buendeseliga and Ethiopia-Ethio League). In ‘How big is your head?’ follow up activity, each team of students’ was given a plastic meters and collected data on head circumference for each member of the team. One student from each team wrote the team data on the blackboard and one student was selected to measure the head circumference of every student in the class.

Students were asked what the typical head circumference was looking at the distribution of the two data sets using dot plots. Students compared the two dot plots with respect to center and spread. Then, the students were asked what could be the possible reason for the variability of the data in the measurements of neck circumferences. Students were elicited why the variation existed and discussed the sources of data variability such as natural and measurement variability.

As extension to Tourist MEA, students described variation of weather data for three Towns for five consecutive days from TV, Radio or newspaper. Lastly, teams of students had compared the size of standard deviation in pair of histogram visually without calculating the standard deviation as shown in Figure 26.

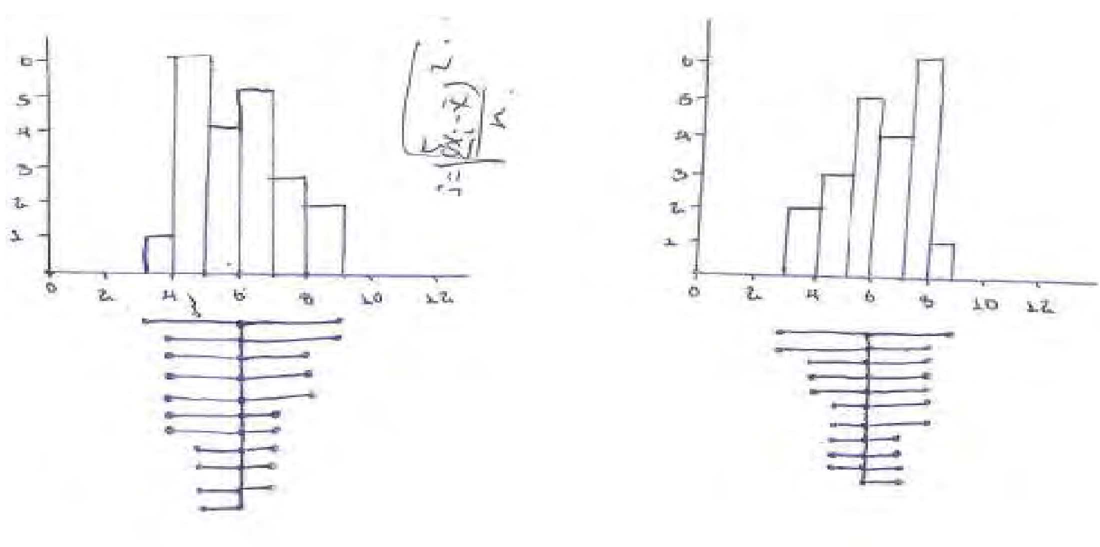


Figure 26: Students' solutions comparing standard deviation using histogram

4.2.3 Students' Experiences of Modeling Approach

The researcher conducted interviews with 14 students to capture their experiences of Modeling Approach. Students' were interviewed with semi-structured questions on the four MEAs: Safe water, Millennium Dam, Football and Tourist and follow up activities. Based on students' interview data the following were the emerging themes: Relevance of MEAs, Power of Representation systems, Choice of models, Team functioning and Dispositions towards statistics.

Relevance of MEAs

Students had expressed their views that all MEAs were relevant to the society and to themselves. Since the problems in MEAs arose from their familiar situations, students were able to learn meaningfully and engage in challenging tasks with motivation. In the following, how students experienced each MEA in the Modeling Approach is discussed.

Relevance of Safe-Water MEA

In Safe-Water MEA, students who lived both in urban and rural had found the activity relevant to their life. The problem was real for both students who came from the rural or from the town, because the problems of safe drinking water and sanitation existed in both rural and urban communities. When asked how it was related with their life, students in the interview responded that:

> Safe-Water MEA has much relation with our life. I was born in rural and the problem still exists there (Alemu: 17/03/2014).
>
> Normally, though it has slight difference in the use of the water, it is related with our life and with the society (Bereket: 19/03/2014).
>
> When we solve Safe-Water MEA, there were several rural people who had big incomes but they lack safe drinking water and practice open deification. This

> surprise me and I infer they are illiterate and I would like to help them and for us we learned how to drink safe water (Chirenet: 8/04/2014).

Students developed their sense of social agency; in that, similar problems exist in the society. They hoped to solve some of the problems by creating awareness program using statistics as a critical tool. They expressed their great concern that a person should not use unsafe drinking water with sympathetic understanding:

> I know the water quality in the community is very bad. I still have many worries that I have seen the water getting spoiled and in the town the most common diseases are typhus and typhoid. I would like to present statistical report on this problem (Selam: 15/04/2014).
>
> In our community we use tap water, but I see dirty particles moving inside the water. I wanted to find out why this happened and what kind of drinking water parents give for their children and there are scarcity of drinking water and we use pump water at times so I want to collect data and write a report on keeping safe drinking water to solve the problems in the society (Hanan: 27/03/2014).

Relevance of Millennium Dam MEA

In Millennium Dam MEA, all interviewed students had been contributing money and they said they would continue to do so in the future. They stated that the activity was part of their life and motivated to write their report as if they were taking part in the country development plan towards fighting poverty. Students had said that the Millennium Dam MEA was relevant and expressed their concern with sense of social agency:

> Everywhere electricity is necessary. I have to take part in this contribution as a responsible citizen (Bereket: 19/03/2014).
>
> Yes, the Millennium dam is not only for Ethiopia to avoid poverty but will be sold for the whole Africans, for example for Egypt and Sudan (Andinet: 14/04/2014).
>
> It would be good for rural areas, because my family lived in rural where there is no electricity (Alemu: 17/03/2014).

Relevance of Football MEA

Except two respondents, students in the interview said that they liked football the most among other sports. But, the two students who didn't like football said that football was part of their life because they had families and friends who strongly supported European football clubs. Students expressed their view that they were motivated to solve the Football MEA in their teams:

> What surprises me most is that the emotional aspects our team members expressed as supporters as if they own the clubs; and, I wonder why there was less support for Ethiopians clubs (Lemessa: 20/03/2014).
>
> All of us enjoyed the Football MEA in my team. One of the students in our team is a coach of little boys (Mahari: 3/04/2014).

Relevance of Tourist MEA

Students responded that Tourist MEA had relevance with their everyday life. Students had found that knowing weather and climate conditions using statistics was an important part of everyday life. Students noted that statistics can be applied in everyday life. They had also indicated that they had been highly motivated during tourist MEA because telling information about the tourist sites would give them some happiness. Even the activity motivates two teams of students to do projects in relation with weather variability. One member of team of student who did project related to weather and attendance problem responded in the interview on the relevance of the MEA:

> The Tourist MEA helped me to understand how to measure average and variability of weather conditions. Besides, it motivated me to choose the project in student attendance over the school year. The school calendar should be based on the weather variability so that we students could attend class properly and teachers could cover all topics. It is hard for teachers to finish the topics because during harvest time there was a break for two weeks (Tola: 21/03/2014).

All students in the interview had showed that all MEAs were relevant with their everyday life and had got good experiences of Modeling Approach:

> Yes, every person will prefer if you start teaching him from what he or she is very familiar with (Mahari: 3/04/2014).
>
> I know nothing about statistics before, but now I can see statistics everywhere. It is good to decide things in real life using statistical data (Andinet: 14/04/2014).

In summary, students had showed that learning statistics with relevant real life problems helped them to see statistics very useful in their everyday life and would create a sense of social agency to maneuver their room for the betterment of their lives and others in the community.

Power of Representation systems

Students were asked if there was little or more variability in their use of water in different contexts which would help them to get sense of the concept of average and variability. Students' responded on simulated recall interview on variability of number of glass of water they drink per day over a week and the time that took them to have shower:

> A lot variability since today I drink 6 glasses of water in 24 hours, on the next day I will drink 2 glasses of water and on the other next day I will drink 4 glasses (Helen: 24/03/2014).
>
> There is only one safety tank. Since we do not get water every day, I need to take bath quickly without wasting much water. The durations to take shower are similar that there is less variability of durations for taking shower (Fiker: 5/04/2014).

Students in the interview were asked to compare data distribution using the concept of average and variation informally on pair of histograms by making some estimation visually. Students were able to compare the distribution of the two histograms correctly as described in table 16 by estimating the average and the variation looking at the bars.

Table 16: *Comparing histograms using average and variation correctly*

	Average only	Variation only	Both average and variation	Looking at the heights of the bars
Number of students	1	1	10	2 (Incorrect)

Students commented in the interview how powerful was to learn with representation systems such as: verbal representation, visual graphic representations, concrete objects, using metaphors, formulas and tables:

> A picture speaks more than thousand words', graphs have more descriptive power (Bereket: 19/3/2014).
>
> We can see and visualize using graphs. If a person has disability problems like if he can't hear but see or if he can see and can't hear, we can present the report easily. When we present the news, if he can only hear then he can hear our report, and if he can only see, then he can visually see the graphs (Helen: 24/03/2014).
>
> The methods we learned sealed in our mind that we will not forget it. It is really good. Above all, the activity what we did using post-it notes were unforgettable (Bereket: 19/03/2014).
>
> We haven't learnt measuring things practically. I like the way we learned on body measurement data (Ayne: 6/04/2014).

All students in the interview correctly describe the procedural aspect of average and standard deviations. They all gave response that standard deviation cannot be negative. Students interpreted what does zero standard deviation and larger standard means with a context:

> Zero doesn't mean there is no rain in the region in that week. It means that it has uniform rainfall amount. The larger the standard deviation, the larger the variation of the rainfall amount of a region will be in the week days (Lemessa: 20/03/2014).

One student in the interview responded 0 standard deviation amount of rainfall amount in a region means there was no rainfall in that region in that week. Possibly, if there was no rainfall amount in that region, then standard deviation would be 0. But, when the student was probed if there was uniform amount of rainfall in that region in the particular week, the student couldn't respond the standard deviation would be 0. Thus, the student interpretation is considered partially correct as shown in Table 17.

A student interpretation is correct if he/she responded there was uniform amount of rainfall in that particular week. A student response is incorrect interpretation if he/she responded the rainfall amounts of some days of the weeks are zero. If students interpreted larger standard deviation of rainfall amount means larger variations of rainfall amount, then it was taken as correct interpretation. If for larger standard deviation the respondent interpreted it as large rainfall amount of a region for the week, then the response will marked incorrect interpretation. That is, larger standard deviation doesn't mean necessarily a lot of or a small rainfall amount in the days of the week, but how the data is distributed. In brief, students expressed that they got experience of working on representation systems like verbal, concrete, metaphors, formula, and graphic representation systems to study the formal measure of center and variation and aspects of distribution.

Table 17: *Interpretation of zero and larger standard deviation*

	Incorrect Interpretation	Partially Correct Interpretation	Correct Interpretation
Zero Standard deviation	1	1	12
Larger standard deviation	1	2	11

Choice of Models

Students in the interview were asked what they would do if they were asked by Ministry of Health and Water to write a report on safe drinking water. Out of the 14 interviewed students, 12 were able to explain the whole investigative cycle that is, identifying the problem, prepare plan, collect data, and make conclusion. The rest two students had explained partial investigative cycle like they responded that 'I will collect data on how they use drinking water and present the report using graphs.' A student who lived in a rural explained the complete investigative cycle on the problem:

> First, using statistical data I would ask them what kind of report they are looking for. Then, after understanding their need I will prepare the method to collect data. Then, I will collect the data. Finally, I will organize the data and analyze the data using various graphs and methods that involve calculations (Bereket: 17/03/2014).

To see how students chose models for comparing two data distributions similar to Millennium Dam MEA task-based interview was made and students responded the following to compare two teams' contributions using measure of center and variation:

> I could use graphs. We can add, and then divide the data values to compare the teams using mean. You can also use mode and median (Fiker: 5/04/2014).
>
> I will compare using graphs. I will make graphs for both, and then I could see and compare which has more variation to write the report (Andinet: 14/04/2014).

Similar to the football MEA, students were interviewed how they would choose statistical models for making decision in order to see how they go about data modeling comparing scores of two basketball teams. Students were probed further on how they would compare two data distribution if they had different choice of models. They responded that they would use different models based on their preference and correct reasoning using measure of centers, measure of variation, extreme values and visual graphs.

Some students made further enquires that they need adequate data to make decision based on data to select the better team. They said the data given for the two basketball teams' scores were not rich enough to decide which team was better to play with, unless they got other data on variables like on the player age, height, weight, jump height and additional qualitative data. They claimed sufficient data were needed to make reliable decisions.

Team functioning

Another feature of Modeling Approach experiences was the roles students played such as facilitator, chairperson, secretary, and presenter. The premise behind this idea is that all students have important work to do in their team, without which the team cannot function. Students had expressed that they experienced multiple roles while they were solving MEAs:

> We share ideas and I played multiple roles (Alemu: 17/03/2014).
>
> I was assigned as chairperson, but I had been also secretary, strategy designer. Though each team members assigned a role, we moved from one role to the other interchangeably to solve the MEAs by helping each other (Bereket: 19/03/2014).
>
> I played multi-roles. I have been chairperson, secretary, and presenter (Helen: 24/03/2014).

Students had responded that presenting the report in teams helped them to reduce anxiety during presentation and at the same time it helped them to gain some confidence and engagement:

> When students present in front of class in teams, they will develop confident (Degenet: 21/03/2014).
>
> First, we do not know each other, but we could communicate well and present ideas in the best ways. Now I have confidence because we work as one, but I had never presented like this before. I feel at ease to present the report in teams (Selam: 15/04/2014).

> We haven't participated like this before, but we have a lot of engagement and motivation in this method. We present report making comparisons, we discuss what we feel and we argue and interpret data with good reasoning (Lemessa: 20/03/2014).

Again, students expressed their views on the importance of working in teams to share and exchange ideas. They expressed their experience that they got the habit of working and living together with love and sympathetic understanding helping one another by sharing ideas within a team:

> Those who had knowledge should share for others working in groups. We should try to help other citizens to use statistics (Saba: 18/03/2014).
>
> We developed how to work together. If others could learn like this, they could develop their confidence to use statistics (Ayne: 6/04/2014).
>
> It was good that working in a team helped us to communicate well. Before, we haven't got acquainted with each other, but now we intimately know and love each other very well (Fiker: 5/04/2014).

Dispositions towards Statistics using Modeling

Students had found Modeling Approach helped them to learn statistics with sense making and it increases their motivation and interest to learn mathematics and statistics:

> Till now we had not learned by doing; the teachers often tell us something and then go out. But, we present report in this approach, we worked deeply and our attitude towards statistics changed (Chirenet: 8/04/2014).
>
> Before I learn statistics using the Modeling Approach, I have little interest in mathematics. But, after I learn using this approach mathematics becomes easier and it is now one of my favorite subjects. So, the statistics that I learned helped me to love mathematics (Selam: 15/04/2014).
>
> I like mathematics more than any school subjects and I didn't like statistics before I learned this. When I learned using this approach, I get used to love it more and more (Mahari: 3/04/2014).

Similarly, students in the interview responded that the anxieties related to statistics went down due to they work together in a team of students in modeling problems:

> When they present reports their anxiety will be lost so it would be good if students learned using this approach from lower grades (Degenet: 21/03/2014).
>
> It helped us to develop our reading skills and it reduced our anxiety to present reports since we were afraid of presenting reports previously (Lemessa: 20/03/2014).

In general, students found their modeling experience were beneficial and they expressed in the interview that they had learnt deeply, gained confidence, and appreciate how statistics come across in their everyday and future life. Finally, they wish if other students got the similar experiences, because they found the experience valuable as they reflected on their experience:

> Now, I know a lot about statistics. It will not be a challenge for me to collect data when I join university as well as after graduation from university (Chirenet: 8/04/2014).
>
> Before, we didn't know deeply about statistics and we couldn't understand and see graphs in different offices. Now, we identified the four kinds of graphs deeply. Now, we can solve problems driving formula, but before we haven't learned solving problems. We have learnt measure of mean deviation and standard deviation along with measures of location (Alemu: 17/03/2014).
>
> For me, it is very good! It is related with our everyday life. I would like to tell other students that the approach relates statistics to our life and we could develop skills on how to use statistics in everyday life. On top of this, we develop a habit of working in a team and in future if students learned this way, they will develop confidence (Degenet: 21/03/2014).

Traditionally, there has been a myth that statistics and mathematics considered as male dominated subjects. Girls worked against such myth in Modeling Approach and got empowerment in using statistics as a critical thinking tool:

> It would be good for females. This is because in the past we afraid to present in front of class, now we have learned in this approach to present reports without any fear. When students presented in front of other people, they would develop confidence and this approach could be used in other subjects too. There is a common saying that mathematics is not for girls, but we learned statistics is connected and using this approach it would be easier for girls (Ayne: 6/04/2014).

> I have a message that every student should develop statistical knowledge and teach the community by using statistics to solve problems rather than simply saying I come to school learning some statistics (Helen: 24/03/2014).

4.2.4 Discussion on Modeling Approach Intervention Results

This section discusses on Modeling Approach intervention results in relation to the relevant literature review based on classroom observation data, students' models on MEA with follow up activities and interview data to answer the research question: How do students enhance their understanding of descriptive statistics using Modeling Approach?

The statistics

Using Modeling Approach, students could develop reasoning on aspects of data distribution using graphic representations, center and variability. Three challenges have been noticed to develop students' reasoning on features of data distribution that could be improved using Modeling Approach. The first challenge was how to motivate students to learn the abstract statistical concepts on the aspects of data distributions. The second challenge was how to help students to see the concept of variability as spread from the center (Garfield, et.al, 2007). The third challenge was how to teach students to see data sets as entity or aggregate values not as individual values (Ben-Zvi, 2004).

On the first challenge, the MEAs supported meaningful connections between statistical concepts, procedures and contexts. Modeling Approach started form situated knowledge that could arise from students' meaningful experience to develop abstract concepts (Lesh & Doerr, 2003). Reading part of MEAs give the background context for data modeling problems (Chamberlin & Coxbill, 2012; Kinnear,2013). Knowing the context is critical aspect of statistical investigation (Pfannkuch & Wild, 1999) and students have been shuffling back and forth between the statistical and the context world in the Modeling Approach class while solving the MEAs.

According to Garfield, delMas and Chance (2007), the context has important effect on students reasoning and thinking. If students understand and care about the data, they will engage in statistical reasoning processes. Students used real data throughout the Modeling Approach. The challenge of statistics education has been to motivate and engage students to learn abstract statistical concepts like the concept of mean and variability as a representative of data distribution (Garfield.et.al, 2007; Mokros & Russell, 1995). Students will be on task in statistical class if the problems are relevant (Garfield & Ben-Zvi, 2008). Students responded in the interview, MEAs are relevant problems which suggested the MEAs in this study satisfied the 'reality principle' to help them learn statistical concepts with sense making which is supported by other empirical studies (Lehrer & Schauble, 200; Libman, 2010).

Moreover, the contexts of MEAs were originated from the topical issues of the society where the study was conducted. The first investigative theme on Safe-Water MEA was identified as one of priority theme by the Ministry of education on problems of water, sanitation and hygiene (WASH program) (MoE, 2010a). The Tourist MEA was related with investigative theme of topical issue of the world on weather variability which is also found in students Geography curriculum at the grade level (Dalelo, 2012). The MEAs would provide students an opportunity to think outside of ‘subject boxes’ on multi-disciplinary areas (English, 2013a; Gouvea, Sawtelle, Geller, & Turpen, 2013).

Before students were introduced the formal aspect of statistical concepts, they described the concepts informally in the Modeling Approach. Students discussed the definitions of statistical terms such as average, variability, typical value, range with the meanings derived from students’ everyday experiences, media articles and formal statistical definitions. Building vocabularies on statistical terminologies is part of statistical literacy and the study had found that they could build their statistical literacy from informal to formal using Modeling Approach.

According Pfannkuch, Regan and Wild (2010, p.3), “intuition is such an important factor in building conceptual understanding that it makes sense to operate as closely as possible to natural language but to some compromises around shared meanings.” Language plays an important role as a notion of Zone of Proximal Development in Contextual Modeling Perspective as part of representation systems. In the Modeling Approach, students had been discussing ‘lexical words’ like average, center and variability in the warm-up activity which could have hindered students’ motivation and understanding to improve from their prior conceptions (Kaplan, 2014).

Students during MEAs were building statistical terminologies and basic statistical concepts based on their prior and informal knowledge towards enhancing their statistical literacy. It was found that students could enhance their statistical literacy by reading different types of graphs and statistical terminologies answering readiness questions of

MEAs similar with others findings that students had intuitive, informal and prior knowledge of statistical concepts (Garfield et.al, 2007; Schwartz, Sears & Chang, 2007).

On the second challenge, developing the concept of variability, students learned variability from informal to formal revisiting the concept on MEAs and representation systems iteratively. To develop students' statistical reasoning, the Modeling Approach focused on the three facets of variability based on theoretical framework of Garfield and Ben-Zvi (2005): (1) developing intuitive ideas of variability, (2) describing and representing variability and (3) using variability to make comparison.

Students were discussing on how to ask statistics questions which need data that vary on the first day class. Then, they discussed on the definition of average and variability informally like whether the water they drink had little or lot variability on Safe-Water MEA. Within a team of students, they further work on the deviation from the mean using post-it notes, because standard deviations involve a co-ordination of distribution and deviation from the mean (delMas & Liu, 2007). Then, students used formula such as range, standard deviation and mean deviation to compare data distribution. Further, they estimate the size of standard deviation without calculations and they were able to compare pair of histograms visually drawing deviation lines from the mean. The finding of students' interview data confirmed students enhanced their informal and formal understanding of variability using Modeling Approach.

On the third challenge, on developing aggregate view of data, it was observed that it was not easy to develop aggregate view of data as noted by other researchers (Ben-Zvi & Arcavi, 2001). However, making comparison of data distribution through representation systems was found to be good experience for developing aggregate views of data distribution (Ben-Zvi, 2004). Initially, student used bar graphs, dot plots, pie chart and histogram, but found it difficult to see aggregate view of data and describe the data sets as an individual values. Then, students did follow up activities in comparing data distribution in dot plots and histograms to enhance aggregate views of data distribution using center and variability informally.

Furthermore, to reinforce the aggregate view, students revisited the concept in different tasks by using representation systems. Finally, students were making comparison using the concept of measure of center and variability formally. Students need to learn procedural knowledge and skill development on constructing graphs. But, more importantly, students need to be encouraged to think with data using statistical models through external representations as is the case using the Modeling Approach. This approach is also supported by other studies; that is, to develop distributional reasoning from informal to formal using representation systems (Bakker & Hoffmann, 2005; Garfield et.al, 2007; Lem, Onghena, Verschaffel, & Dooren, 2013).

For example, in Hydro Dam MEA majority of students compare the distribution of data on students' contribution of money for the Millennium dam writing a report using measure of center, range and graphs as the students report in the MEA showed. To build further on the concept of mean or median as representative of data sets, students worked on Football MEA on deciding which team should be played by comparing data distribution of two football teams. Then, in the follow up activity students did 'post-it notes' activities to study mean and median as a balance point to represent the whole data distribution. Then, they used graphs to determine whether the mean or median was the appropriate measure of center for the given data distribution on students' survey. Finally, students compared data distributions on Tourist MEA using formal measure of center and variation.

Traditionally, statistical curricula follow the linear and hierarchical approach to teach students foundational statistics concepts adopting from the formalist mathematics culture (Meletiou-Mavrotheris & Lee, 2002). It is assumed that this simplifies the process of learning by gradually leading students from basic to more complex connections. However, it is not true that the development of a second concept necessarily follows the development of the first concept on a predictable manner. For example, students may not necessarily learn center before variability. Research had shown that students at the age of 3 could develop intuitive ideas of variability before they know any idea about center (Watson & Kelly, 2002).

A Contextual Modeling Perspective assumes that students are at intermediate stage of development for nearly any construct or conceptual system that teachers want their students to develop further (Lesh & Doerr, 2003). For example, a teacher wanted to teach about measure of variation and students may have a wide variety of concrete, intuitive, or situated understanding which already existed. So, the challenge of teachers is not simply to introduce new ideas sequentially. The goal is to put students in situations where students express their current ways of thinking iteratively to develop the concepts further.

For related concepts such as center, variability and distribution, it is not necessary that one concept precede the other (Watson, 2009). There has been a debate in Statistics Education for pragmatic reason whether teachers should teach the signal in the distribution with first order concern or the distribution around a signal (Bakker, 2004; Konold & Pollatsek, 2002). There was also argument whether the focus should be on measure of center tendency at the expense of measure of variation or not (Gould, 2005; Konold & Pollatsek, 2002; Reading & Shaughnessy, 2004; Shaughnessy, 2007).

Even though center, variability and distribution are important concepts students should learn in descriptive statistics, teachers may not need teach students one concept followed the other as if it is new concept that build on the pervious, because it may not help see the interconnectedness between the concepts and a student may have better intuitive understanding for one concept to the other. Traditionally, students learn measure of central tendency at the expense of measure of variation which could limit their conceptual understanding of the big ideas of variability in statistics education (Shaughnessy, 2007). Putting much emphasis on central tendency in turn may not help enhancing students' statistical reasoning (Garfield, et.al, 2007).

However, students could learn statistical concepts concurrently for enhancing their conceptual understanding as it was the case in this study using Modeling Approach. For example, students learned the concept of variability and center for describing and comparing about distribution of data as one package unlike 'topic-topic-topic approach'. In Contextual Modeling Perspective, it is considered that students develop concepts by testing, revising and refining concepts iteratively from informal to formal, from concrete to abstract, from particular to general in multiple dimensions.

The whole is greater than the parts in Modeling Approach to develop the 'system as a whole' thinking; that is, it may help to enhance students' knowledge of the intertwined statistical concepts (Lesh & Doerr, 2003). Concepts came as package and thus the design of the Modeling Approach involved iterative process of developing concepts based on model-based reasoning rather than following a linear approach to develop students' conceptual understanding. By testing, constructing and revising students could develop concepts as they solve problems.

Hence, the common threads students were doing in Modeling Approach was to enhance statistical concepts as a 'package' on features of data distribution as an aggregate working iteratively (Bakker, Derry & Konold, 2006). To see aggregate features of data distribution, first they developed intuitive ideas of center and variability on aspect of distribution informally, second they described data distribution using measure of center and variation and finally they compared data distributions using the concept of center and variation both informally and formally. The MEAs were useful to elicit students thinking to build the concept of data distribution as an aggregate by testing, constructing and revising their prior knowledge. Students were able to reason and think with data, because the context in the MEA tasks gave them meaningful experiences to build on their prior and intuitive knowledge (English, 2013b; Kinnear, 2013).

Thus, one could see that Modeling Approach more likely balances the big ideas of descriptive statistics: center, variation and distribution in secondary mathematics curriculum for students' to develop their statistical reasoning and conceptual understanding. The statistical concepts were related and interconnected that Modeling Approach may be suitable for developing their statistical reasoning, because students could see the relations between and within concepts working on MEAs. The findings suggested that student could enhance their conceptual understanding seeing the interconnection between and with-in concepts using systemic thinking as a whole working in MEAs and through representation systems making justification and interpretation (delMas & Liu, 2007; Lesh & Kaput, 2007).

The cognitive Demand

In data modeling, students had their own preferences to select any model for describing and comparing data distribution. Teachers have two choices in undertaking data analysis with the students: teachers can lead them to organize, represent and compare their data in a way that makes sense to the teacher, or teacher can support them as they organize and represent their data in a way which makes sense to them (English, 2013b).

In the first case, they learn some rules and then they try to follow the rules to do the task. In the second case, they learn to think about their data even with common sense. Students need to construct their own representations and ways of understanding, even when their decision do not seem correct to adults. The important finding in the Modeling Approach class was students started to draw conclusion based on data in context prematurely without being expert in modeling as they did MEAs.

Many mathematics educators argued that modeling was considered a practice reserved for tertiary education. But, recently, many mathematics educators have recommended students need to learn data modeling even from elementary classes. Empirical research had showed that even students in lower grades learn to competently to generate, test, revise and represent data through data modeling (English, 2010, 2013b; Kinnear, 2013). Though there was high cognitive demand in the problem solving phases of MEAs as observed using Schoenfeld, et.al (2014) observation scheme and rubrics , the class observation and students' models on MEAs would suggest that student can undergo data modeling with proper scaffolding without team of students' being expert at modeling.

Initially, students' may not have experience working in modeling problems and the cognitive demand on MEAs would be high as the tasks considered to have high cognitive processes from other exercises and word problems (Chamberlin, 2010). The MEAs were cognitively demanding tasks for students in Modeling Approach based on TRU math rubric as weighted average score of cognitive demand was close to 3 across the four MEAs. This can be taken as a continuum in which at one end students had a chance to learn just by spoon feeding or at another end students would be at sea and lack any direction what to do in a given statistical problem. There is a need to strike the balance and develop our students' cognitive thinking by giving them challenging problems; yet they need proper scaffolding on the modeling cycles (English, 2013b). Through scaffolding, students' were able to work on data modeling tasks and produce conceptual artifacts as the findings of students' report and classroom observation showed on MEAs.

One objection not to use Modeling in school curriculum was to think of students could not be expert modelers at lower grade levels and should first develop concepts in a discipline. The MEAs were helpful to see students thinking which fill the void in assessing students' prior knowledge even if students could not produce solutions like experts (Hjalmarson, Moore & delMas, 2011). But, students' models were like 'windows' to see students thinking to enhance their understanding (Lesh & Fennewald, 2010).

Even though, students did not collect data they were able to pose problem, select variables and models, analyze data and make conclusions which is the common statistical investigative cycle (Wild & Pfannkuch, 1999).When students were asked on similar problem on MEA using interview, out of the 14 interviewed students, 12 were able to explain the investigative cycle, that is, identifying the problem, prepare plan, collect data, and make conclusion.

In Contextual Modeling Perspective, students are at some intermediate stage of modeling cycle that the intent of the MEAs are to put students in a situation where they externalize their models for improving their conceptual system (Lesh & Doerr, 2003) For example, in MEAs students prepared a report at different levels as they were assessed using Quality Assurance Guide. Even though students were not expert modelers on the initial stage, they were able to develop their concepts working on MEAs by testing, refining and revising their models working on MEAs extensions and follow-up activities. The finding on MEAs report suggests, students could enhance their adaptive reasoning and procedural understanding of statistical concepts, because the statistics in Modeling Approach involve making interpretation and justification as well as computation.

Students may have implicit models, but the models are accessible when they externalize their models through external representations (Lesh & Doerr, 2003). Representational fluency is found to be important, because it allows students to enhance their thinking of 'system as a whole' by switching from one representation to another (Lem, Onghena, Verschaffel, & Dooren, 2013). Students had been working on representation systems to develop their conceptual understanding, because conceptual understanding involves making connections between concepts. Representation systems depict different aspects of relations and connection between and within concepts. Students worked from one representation system to another representation system along a variety of dimensions. The zone of proximal development (ZPD) of an individual is not only teachers and or peer members in their team, but also working in different powerful representation systems. The finding of the study suggests team of students working on representation systems could work on cognitively challenging problems to enhance their understanding.

Access to Statistics

Students could learn statistics with equity using Modeling Approach, because students got a chance to enhance their statistical reasoning. According to Boaler (2008) equity could refer to relational equity in which students could get enough opportunity to participate in class regardless of student's sex or achievement level, unlike considering equity only in terms of students' achievements. Equity in mathematics education may include helping students 'to participate in significant out of-school practices in relatively substantial ways' (Burner, 1986 cited in Cobb & Hodge, 2007, p. 179).

MEAs are interdisciplinary problems which would need students to draw knowledge from several disciplines and the modeling classroom was 'multidimensional'. According to Boaler (2008, p.4), "a multidimensional mathematics classroom could reward students for using different methods, asking questions, representing ideas and having good discussions in addition to the execution of procedures." The theory is that as classrooms become more multidimensional more students have access to ideas and may be regarded as contributing in important ways.

Each team members need to contribute and actively participate in MEAs, because MEAs do not have one exact answer. Each team members were playing roles in their teams as students responded in the interview. The premise behind this idea is that all students have important work to do in their teams, without which the team cannot function. The team interdependency would promote equity education. Students could get relational equity with treating each other with respect, commitment to the learning of others (Boaler, 2008). In the Modeling Approach, the teams of students were working with the sense of learning to be together as they solve problems and present reports in a team.

Modeling Approach may promote equity for students at different achievement levels. An ongoing debate in mathematics education for several decades pertains to the extent to which low-level to high-level activities should be used (Chamberlin, 2010). Best possible levels of tasks are needed to serve all students with equity at different achievement levels (Chamberlin, 2010). If students worked only on low level activities on simple exercises and word problems, then it would be a problem to enhance students' productive dispositions and statistical thinking. MEAs are relevant non-routine tasks that could enhance students' higher cognitive thinking. Modeling Approach had MEAs and representational systems to serve all students with equity at different achievement levels, since MEAs are non-routine problem solving tasks which could not be solved by using facts and skills automatically with little cognitive processes.

Students at different achievement levels were engaged in the modeling class and got access as observed using TRU math rubric with weighted average of 2.5. Most teams in MEAs present reports in pairs or in three that students were learning with equity, since they gained access to statistical concepts as they present their statistical report with justification. Besides, students develop sense of good relational equity by helping one another in their team and in the class as whole. Students can develop team interdependency (Moore, 2005) and living togetherness as they solved problems in team.

Contextual Modeling Perspective considers statistics is taught to students because it is useful in everyday life (Lesh & Doerr, 2003). Students need to develop systemic thinking to deal with the kind of problem that arises in their everyday lives that involve multi-variables and complex systems. The findings of the Modeling Approach intervention offer the chance to students to enhance their understanding working in teams and in multidisciplinary areas (Lesh, 2000). It is likely that educators could develop students' systemic understanding using data modeling for creating equity education for the nature of problems students face beyond school.

Agency, identity and authority

Modeling Approach prompts students' empowerment to solve their own and the community problems, because the problems in the modeling approach started from the concrete situations of students' life. Students' had been reading and writing with social agency agenda to transform 'limiting conditions' in their life and in their society using statistics. Similarly, Gustine (2006) had showed students develop their sense of social agency through reading and writing the world with mathematics. Students used statistics in the modeling class as a critical tool to pose problems by reading media articles on the socio-cultural problems. The findings of the study suggests, students used statistics as a critical tool using Modeling Approach by developing a sense of social agency through writing and presenting reports and letters using statistical data and concepts.

Teachers may quit teaching statistics with social justice agenda for the reason that it takes more lessons to teach with social agency agenda than traditional lessons. But, it was possible to teach statistics with social agency agenda using Modeling Approach within the provided period allotment, since modeling problems started from real life and we could make the reality principle from the concrete situations of socio-cultural problems which could promote students' critical understanding.

According to Ministry of Education (2010a), in ESDP IV students are expected to be change agents on School, Water, Sanitation and Hygiene (WASH) which was one of the priority programs in the document. To this end, students were presenting awareness program on safe drinking water and sanitation. Students could act like social agents as they were observed and expressed their views in the interviews. Students had also developed sympathetic understanding for the death of children under the age of 5 and prepared a report that children under the age of 5 at least should drink boiled water. Students both in rural and urban developed a sense of agency to help their families and community towards keeping safe drinking water and sanitation.

Teaching students using the Modeling Approach cultivates consciousness and the development of students' identity, since MEAs problems in the Modeling Approach mainly deals with the current socio-political contexts. Reading and writing the world with mathematics could improve 'the awakening of political- consciousness' (Gustine, 2006). In Hydro-dam MEA students were reading and writing a report with statistics on students' money contribution for Millennium dam. Students are part of the country's development plan for fighting against poverty by contributing money. Students also reflected in the interview that doing the Hydro-dam MEA they were able to create identities as actors of the socio-political conditions of their country.

A crucial aspect of teaching statistics for social justice is what students do with the statistics that they learn (Gustine, 2006). In Football MEA activity students were preparing a report using statistics to give an awareness program on Ethiopian football. There is nothing wrong to support European football, but the youth got so emotional that could affect one's life and may get identity crisis which would force him to follow cultural models contradictory with the local culture. Students were developing critical understanding of statistics informing and helping others like presenting statistical report for creating awareness on Ethiopian football how to support their national team and to stop being very emotional not to affect one's wellbeing.

Students could develop sympathy helping others by giving information using statistics. In Tourist MEA students learned creating their sense of social agency giving information on four tourist sites using statistics as critical tool on weather conditions. Students were also learning to live together as they responded in the interview that doing in a team of students help them to know each other very well and to develop sympathetic understanding helping each other. Based on TRU classroom observation, students were developing agency, authority and identity of their voice using statistics as a critical tool.

Assessment

Assessment was an integral part of Modeling Approach to enhance students' understanding of descriptive statistics; students develop concepts iteratively by testing, revising and refining their models through continuous feedbacks. The classroom observation showed students were assessed during warm-up, problem solving and presentation phases. The Modeling Approach had assessment as learning (self assessment) and assessment for learning (giving feedback) in addition to the common assessment of learning (for evaluation). In assessment as learning, students assess themselves using My Learning Goal assessment and each MEA was designed with self assessment principle which could give students sense of direction and purpose as noted in modeling classroom observation. Students also assessed their solution processes on MEA using Quality Assurance Guide.

Making mistakes were normal part of modeling class taken as an opportunity for important learning to occur. Though statistical concepts are counter intuitive and difficult to develop, students could iteratively develop good understanding of statistical concepts through continuous feedbacks. MEAs were used primarily to assess students' prior knowledge by eliciting students thinking without being expert in modeling and the instruction was built on students' models.

Teams of students were at different levels of performance on MEAs. Students MEAs solutions serve as an assessment tool for giving feedback on their solutions. MEAs feel a void for a need for pre assessment before introducing formal concepts to students. In Modeling Approach, teachers develop concepts by building on students' models looking at the MEAs solutions. Assessment is an integral part of Modeling Approach. Teacher scaffolds students with continuous assessment to make students models shareable and reusable in other context as they worked on MEAs follow up activities. Students also assessed their individual roles and their team functioning on the MEAs processes using reflection tools.

Productive Dispositions

Students were solving relevant non-routine problem solving tasks in Modeling Approach. Their productive dispositions towards statistics were more likely promoted, because modeling problems started from real life problems related to students' life. Students developed interest, since they can see the usefulness of statistics in their immediate and future life. Students in the interview responded that they came to see sensibly the usefulness of statistics now and in their future lives.

Students could also enhance productive dispositions by developing confidence and reducing anxiety towards statistics working in a team of students. Students, working in a team of students in data modeling, could reduce their anxiety as they share ideas and help each other in their team. Students responded in the interview that they reduced anxiety, because their team functioned interdependently and they had played multiple roles. As students developed their data modeling abilities, at the same time they could develop their confidence by solving different problems using statistics. As students present statistical report on MEAs, they had no anxiety in presenting their report since they shared responsibility for their team. Students presented reports on MEAs in pairs which might help them to work cooperatively and gain positive dispositions.

Students were able to create fun and different styles when they wrote reports and news on the MEAs. This could create positive dispositions towards statistics, because students were having fun in the class despite the fact that MEAs are cognitively challenging tasks. A study had showed that students would engage in statistics class if they are motivated and got some fun in the class (Lesser, et.al, 2013). Students found that working in MEAs were 'hard fun' to develop their dispositions towards statistics. Students were reporting their solution using news and letters in funny ways which had created students motivations to work on MEAs. Though data modeling was challenging, students were able to engage and preserve in completing the modeling cycle, because students got a chance to present their report in their own styles as observed in modeling class which is supported by similar research findings (Boaler, 2001).

4.3 Results on Achievement and Attitude Data

4.3.1 Results on Achievement Data

A descriptive analysis including sample means, standard deviation, independent sample t-test and ANOVA were conducted to test hypotheses based on Skewness-Kurtosis normality test at $\alpha = 0.05$ significance level to compare scores of students' using achievement tests on prior knowledge, procedural and conceptual understanding between students who learned descriptive statistics using Modeling and Non-modeling Approach. List wise deletion was made for students who didn't take the posttests and 3 cases were deleted. The prior knowledge test was taken as a pretest and achievement tests on procedural and conceptual understanding of descriptive statistics was taken as posttests.

Comparison between modeling versus non-modeling students by schools

HO1: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on prior knowledge achievement test.

HO2: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on post knowledge achievement test.

HO3: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on procedural understanding achievement posttest.

HO4: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on conceptual understanding achievement posttest.

HO5: There is no significant scores difference between students on procedural and conceptual achievement posttests on descriptive statistics

Result of group comparison in School A and B

Inspection of the two groups means indicated that the average conceptual understanding achievement test for Non-modeling students (8.08 and 7.88 for school A and B respectively) is significantly lower than the score (9.50 and 9.10 for school A and B respectively) for students who have learnt using Modeling Approach. Table 18 shows there was statistically significant difference between students who have learnt using Modeling Approach and Non-modeling Approach on conceptual understanding of descriptive statistics achievement test, (p=.004 at α =.05) in School A and (p=.005 at α =.05) in School B. The effect sizes (Cohen's d) were approximately .67 and .64 for school A and B respectively.

Students who learned using Modeling Approach did not differ significantly from students who learned with Non-modeling Approach on achievement test on procedural understanding of descriptive statistics (p= .424 and p=.099). There was no significant difference between the two groups in the pretest prior knowledge in school A and B.

Table 18: *Comparison of students' scores on a descriptive statistics achievement tests in school A and B (n=40 Modeling and 40 Non-modeling for each school)*

Achievement test	*School A (n=80)*				*School B (n=80)*			
	M(SD)	*t*	*df*	*p*	*M(SD)*	*t*	*df*	*P*
Prior Knowledge		1.451	78	.151		.528	78	.599
Modeling	7.93 (2.94)				7.68(2.97)			
Non-modeling	7.08 (2.26)				7.38(2.02)			
Post Knowledge		2.140	78	.035		3.955	78	.000
Modeling	17.53(4.39)				16.98(2.89)			
Non-modeling	15.63 (3.49)				14.60 (2.47)			
Procedural		.804	78	.424		1.67	78	.099
Modeling	8.03 (2.89)				7.88 (1.54)			
Non-modeling	7.55 (2.37)				7.18 (2.16)			
Conceptual		3.005	78	.004		3.269	78	.005
Modeling	9.50 (2.31)				9.10 (2.08)			
Non-modeling	8.08 (1.91)				7.88 (1.70)			

Furthermore, Table 19 showed that students' procedural and conceptual understanding of descriptive statistics were significantly different on the achievement tests for modeling group (p=.014 and p=.004) unlike the Non-modeling group (p=.280 and p=.111) in school A and B. Looking at the two dependent variables score means, the average score conceptual understanding achievement test is (8.03 for school A and 7.88 for school B) was lower than the score (9.50 and 9.10 for school A and B respectively) for students who had learnt using Modeling Approach.

Table 19: *Comparison between procedural and conceptual understanding scores for Modeling and Non-modeling students*

	School A				School B			
Achievement test	*M(SD)*	*T*	*df*	*P*	*M(SD)*	*t*	*df*	*P*
Modeling		2.523	78	.014		2.989	78	.004
Procedural	8.03(2.89)				7.88(1.54)			
Conceptual	9.50(2.31)				9.10(2.09)			
Non-modeling		1.089	78	.280		1.612	78	.111
Procedural	7.55(2.37)				7.18(2.16)			
Conceptual	8.08(1.91)				7.88(1.70)			

Result of comparison across groups

Combing two sections of students who had learned using Modeling Approach, comparison was made across groups by combining another two sections of students who had learned using Non-modeling Approach. Checking the normality using Skewness-Kurtosis tests, an independent t-test was conducted and the following result was obtained.

Table 20 showed that students who had learnt using Modeling Approach were significantly different from students who had learnt using Non-modeling on both conceptual understanding (p=.000) but not on procedural understanding (p=.105) of descriptive statistics achievement test across the groups in both schools. Comparing the two groups means showed that the average conceptual understanding achievement test for Non-modeling students (7.98) was significantly lower than the score (9.30) for students who had learned using modeling approach with effect size 0.66.

Table 20: *Comparison of modeling and non modeling class of students' scores in two Schools (n=80 modeling and 80 non-modeling students)*

Achievement test	*M*	*SD*	*t*	*df*	*P*
Prior knowledge			1.416	158	.159
Modeling	7.80	2.94			
Non modeling	7.23	2.14			
Procedural			1.629	158	.105
Modeling	7.95	2.30			
Non-modeling	7.36	2.26			
Conceptual			4.174	158	.000
Modeling	9.30	2.20			
Non-modeling	7.98	1.80			

Similarly, Table 21 indicated that students' procedural and conceptual understanding of descriptive statistics were significantly different on the achievement tests for modeling group (p=.000) unlike the Non-modeling group (p=.060) in both schools. Examining the two dependent variables score means, the average score on procedural understanding achievement test (7.95) was lower than the score (9.30) on conceptual understanding achievement test for students who had learnt using Modeling approach. The difference between the means was 1.35 points on a 15-point test.

Table 21: *Comparison of students' scores on procedural and conceptual understanding of descriptive statistics for Modeling and Non-modeling students*

Achievement test	*M*	*SD*	*t*	*df*	*P*
Modeling			3.798	158	.000
Procedural	7.95	1.54			
Conceptual	9.30	2.30			
Non modeling			1.895	158	.060
Procedural	7.36	2.26			
Conceptual	7.98	1.80			

Gender wise comparison across and between groups

HO6: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on prior knowledge achievement test of descriptive statistics in gender wise.

H07: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on post knowledge achievement test of descriptive statistics in gender wise.

H08: There is no significant mean scores difference between students who have learnt using Modeling and Non-modeling Approach on procedural understanding achievement posttest of descriptive statistics in gender wise.

HO9: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling approach on conceptual understanding achievement posttest of descriptive statistics in gender wise.

H10: There is no significant mean scores difference between students on procedural and conceptual understanding of descriptive statistics achievement posttests in gender wise.

H11: There is no significant mean scores difference on procedural understanding achievement posttest in gender wise for students who have learnt using Modeling Approach.

H12: There is no significant mean scores difference on conceptual understanding achievement posttest in gender wise for students who have learnt using Modeling Approach.

Result of comparison between Modeling and Non-modeling in gender wise

Table 22 showed that male (female) students who had learnt using Modeling approach were significantly different from male (female) students who had learnt using Non-modeling on conceptual understanding test, (p=.049 for male and .005 for female), but there was no significance difference for male students in procedural understanding achievement test (p=.851) unlike female students (p=.016) in the two Schools. Comparison of the two groups means showed that the average conceptual understanding achievement test for Non-modeling male students (8.14) was significantly lower than the score (9.02) for Modeling male students. The difference between the means was 0.88 points on a 15-point test. The effect size d was approximately 0.46.

Comparison of the two groups means showed that the average conceptual understanding achievement test for Non-modeling female students (7.85) was significantly lower than the score (9.59) for Modeling female students. The difference between the means was 2.45 points on a 15-point test. The effect size d was approximately 0.84. Female students who learned using Modeling Approach also differed significantly with Non-modeling female students on procedural understanding of descriptive statistics achievement test with effect size 0.54.

Table 22: *Comparison of students' scores by gender between Modeling and Non-modeling groups in both schools (n=41 males for modeling and 36 males for non modeling group)*

	Male (n=41 modeling and 36 non-modeling)				*Female (n=39 modeling and 44 non-modeling)*			
Achievement test	*M(SD)*	*t*	*df*	*p*	*M(SD)*	*t*	*df*	*P*
Prior Knowledge		1.451	78	.609		.187	81	.852
Modeling	8.32 (3.07)				7.26 (2.73)			
Non-modeling	7.31 (2.34)				7.16 (1.98)			
Post Knowledge		2.140	78	.035		3.955	78	.000
Modeling	17.53(4.39)				16.98(2.89)			
Non-modeling	15.63 (3.49)				14.60 (2.47)			
Procedural		.189	75	.851		3.002	81	.016
Modeling	7.49 (2.20)				8.44 (2.33)			
Non-modeling	7.58 (2.23)				7.18 (2.30)			
Conceptual		2.002	75	.049		3.002	81	.005
Modeling	9.02 (1.90)				9.59 (2.46)			
Non-modeling	8.14 (1.97)				7.85 (1.67)			

Result of comparison between procedural and conceptual understanding for Modeling and Non-modeling students in gender wise.

Table 23 showed male students' procedural and conceptual understanding of modeling class were significantly different on the achievement tests, (*p*=.001) unlike the Non-modeling male students in the two high schools. Looking at the two dependent variables score means, the average score on procedural understanding achievement test (7.49) was lower than average score on conceptual understanding achievement test (9.02) for male students who have learnt using Modeling Approach. The difference between the means was 1.53 points on a 15-point test.

Similarly, female students' procedural and conceptual understanding of Modeling group were significantly different on the achievement tests, (*p*=.036), unlike the Non-modeling group. Looking at the two dependent variables score means, the average score on conceptual understanding achievement test (9.59) was greater than the average score on procedural understanding achievement test (8.44) for female students who had learnt using Modeling Approach. The difference between the means was 1.15 points on a 15-point test.

Table 23: *Comparison of students' scores with respect to their procedural and conceptual understanding of descriptive statistics on achievement tests in the two schools*

	Male				Female			
Achievement test	*M (SD)*	*t*	*df*	*P*	*M (SD)*	*t*	*df*	*P*
Modeling		3.379	80	.001		2.130	76	.036
Procedural	7.58(1.97)				8.44(2.46)			
Conceptual	8.14(2.23)				9.10(2.09)			
Non-modeling		1.118	70	.267		1.545	86	.126
Procedural	7.58(1.97)				7.18(1.67)			
Conceptual	8.14(2.23)				7.84(2.30)			

Result of gender wise comparison of students' score in using Modeling Approach

Table 24 showed that males neither differed significantly from females on procedural understanding achievement test (p=.065) nor on conceptual understanding achievement test (p=.252). However, female students scored more than male students in average both on procedural and conceptual understanding achievement tests of descriptive statistics. The differences of the mean scores are 0.95 and 0.57 for procedural and conceptual understanding achievement tests respectively.

Table 24: *Comparison of students' scores by gender in the modeling group (n=39 males and 41 females)*

Achievement tests	*M*	*SD*	*t*	*df*	*P*
Prior knowledge			-1.630	78	.107
Male	8.32	3.07			
Female	7.26	2.73			
Procedural			1.872	78	.065
Male	7.49	2.20			
Female	8.44	2.33			
Conceptual			1.154	78	.252
Male	9.02	1.90			
Female	9.59	2.46			

Comparison by achievement levels

H13: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on prior knowledge achievement test of descriptive statistics by achievement levels.

H14: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on post knowledge achievement test of descriptive statistics by achievement levels.

H15: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on procedural understanding achievement posttest of descriptive statistics by achievement levels.

H16: There is no significant mean scores difference between students who have learnt using Modeling Approach and Non-modeling Approach on conceptual understanding achievement posttest of descriptive statistics by achievement levels.

H17: There is no significant mean scores difference between students on procedural and conceptual understanding achievement posttests by achievement levels.

Result of Comparisons between achiever levels across groups

Table 25 showed high achiever Modeling students mean scores were significantly different from high achiever Non-modeling students on conceptual understanding of descriptive statistics achievement tests, (p=.029) in the two high schools. Comparison of the two groups means showed that the average conceptual understanding achievement test for high achiever Non modeling students (8.50) was significantly lower than the score (10.92) for high achiever Modeling students. The difference between the means was 2.42 points on a 15-point test. The effect size d was approximately 0.95.

Medium achiever Modeling students mean scores were significantly different from medium achiever Non-modeling students on conceptual understanding of descriptive statistics achievement test, (p=.022) in the two high schools. Comparison of the two groups means showed that the average conceptual understanding achievement test for medium achiever Non-modeling students (7.50) was significantly lower than the score (8.73) for medium achiever Modeling students. The difference between the means was 1.23 points on a 15-point test. The effect size d was approximately 0.74. Medium achiever Modeling students did not differ significantly with medium achiever Non-modeling students on procedural understanding of descriptive statistics achievement test (p=.143).

Low achiever Modeling students mean scores were significantly different from low achiever Non-modeling students on conceptual understanding of descriptive statistics achievement test, (p=.014) in the two schools. Comparison of the two groups means showed that the average conceptual understanding achievement test for low achiever Non-modeling students (8.12) was significantly lower than the score (9.15) for low achiever Modeling students. The difference between the means was 1.03 points on a 15-point test. The effect size d was approximately .53. Medium achiever Modeling students did not differ significantly with medium achiever Non-modeling students on procedural understanding of descriptive statistics achievement test (p=.269).

Table 25: Comparison of high achiever students' scores between Modeling and Non-modeling groups (n=12, 22 and 46 (n= 12, 26 and 42) for high, medium and low achievers in modeling (non modeling) group respectively)

Achievement	High			Medium			Low		
Test	*M (SD)*	*T*	*P*	*M (SD)*	*t*	*p*	*M(SD)*	*t*	*P*
Prior		.098	.845		.530	.599		1.551	.125
Modeling	8.58(3.63)			7.50 (2.91)			7.74(2.79)		
Non-modeling	8.33(2.43)			7.12 (2.09)			6.93(2.01)		
Post		1.089	.280		1.612	.111		2.835	.006
Modeling	7.55(2.37)			7.18 (2.16)			16.59(3.5)		
Non-modeling	8.08(1.91)			7.88 (1.70)			14.67(2.7)		
Procedural		.189	.286		1.492	.143		.486	.628
Modeling	9.67(1.78)			8.09(2.33)			7.43(2.22)		
Non-modeling	8.58(2.94)			7.12(2.20)			7.21(2.02)		
Conceptual		2.331	.029		2.431	.022		2.508	.014
Modeling	10.92(2.02			8.73(2.19)			9.15(2.09)		
Non-modeling	8.14(2.97)			7.50(.99)			8.12(1.74)		

Comparison by achievement levels

H18: There is no significant mean score difference among high, medium and low mathematics achievers who have learnt using Modeling Approach on procedural understanding achievement test of descriptive statistics.

H19: There is no significant mean score difference among high, medium and low mathematics achievers who have learnt using Modeling Approach in students' on conceptual understanding achievement test of descriptive statistics.

Result of comparison by achievement levels

Comparison of mean scores of prior knowledge achievement test for low, medium and high achiever of Modeling students in table 26 showed that high achievers' score (9.25) is larger than the medium achievers (7.77) and low achievers (7.17).

Table 26: *Comparison of students' scores by achievement levels in modeling groups*

	High achievers n=12	Medium achievers n=22	Low achievers n=46		
	M (SD)	*M (SD)*	*M (SD)*	*F*	*p*
Prior knowledge	9.25 (3.25)	7.77 (2.45)	7.17 (2.32)	3.294	.042
Procedural	9.67 (1.78)	8.09(2.33)	7.43 (2.22)	4.999	.009
Conceptual	10.92 (2.02)	8.73 (2.19)	9.15 (2.09)	4.466	.015

Similar trend was found for high achievers' compared to medium and low achievers in the posttests procedural and conceptual understanding achievement tests, however the mean score of low achiever students (9.15) was larger than the mean score of medium achievers (8.73) on achievement test of conceptual understanding.

Result of post hoc analysis on achievement levels comparisons

As the post hoc analysis in table 27 showed high achiever students significantly differ on procedural understanding achievement test both with medium achiever (p=.049) and low achiever students (p=.002). But, there was no significance difference on procedural understanding achievement test between medium and low achiever (p= .252).

Table 27: Nested Post Hoc tests of multiple comparisons of students' scores by achievement levels for Modeling Group

Achievement Levels		*Achievement test*	*Mean difference*	*P*
		Prior Knowledge		
High achiever	Medium achiever		1.477	.105
	Low achiever		*2.076*	*.013*
Medium achiever	Low achiever		. 519	.360
		Procedural		
High achiever	Medium achiever		1.576	.049
	Low achiever		2.232	.002
Medium achiever	Low achiever		.656	.252
		Conceptual		
High achiever	Medium achiever		2.189	.005
	Low achiever		1.764	.012
Medium achiever	Low achiever		-.425	.439

4.3.2 Results on Attitude Data

H20: There is no significance mean scores difference in students' attitude towards statistics using Modeling Approach before and after the intervention.

H21: There is no significant mean scores difference between male and female students in students' pretest attitude towards statistics.

H22: There is no significant mean scores difference between students in students' posttest attitude towards statistics using Modeling Approach in gender wise.

H23: There is no significant mean gain scores difference between male and female students in students' posttest attitude towards statistics using Modeling Approach.

Based on Skewness-Kurtosis normality test repeated measure sample t-test was conducted to compare mean of scores from pretest to posttest using attitude survey to see changes of attitude towards statistics using Modeling Approach at α=0.05 significance. The result in Table 28 showed that there was a significant difference on attitude towards statistics using Modeling Approach before and after the intervention (p=.000).

Similarly, based on Skewness-Kurtosis normality test repeated measure sample test t-test was conducted to compare mean of scores from pretest to posttest using attitude survey subscale which are usefulness, confidence, anxiety and interest to see changes of attitude towards statistics using Modeling Approach at α=0.05 significance level . As shown in Table 28 students' attitude towards statistics using Modeling Approach changes significantly with respect to subscale dimensions of usefulness, anxiety, confidence, anxiety and interest.

Table 28: *Comparison of students' pretest and posttest score on attitude towards statistics using Modeling Approach*

		Pretest	Posttest			
	N	*M (SD)*	*M (SD)*	t	df	*P*
Attitude	75	118.8(24.85)	141.20(15.78)	7.698	74	.000
Male	35	117.8(25.50)	139.31(16.53)	2.620	34	.012
Female	40	119.7(24.56)	142.85(15.11)	2.620	39	.007
Usefulness	75	31.21 (7.53)	36.49 (4.26)	5.977	74	.000
Male	35	30.54 (7.97)	36.46 (4.67)	4.466	34	.000
Female	40	31.80 (7.17)	36.53 (3.94)	3.963	39	.000
Confidence	75	30.75 (7.93)	36.05 (5.16)	5.568	74	.000
Male	35	30.77 (7.18)	35.89 (4.85)	3.707	34	.001
Female	40	30.73 (8.63)	36.20 (5.50)	4.106	39	.000
Anxiety	75	24.36 (6.44)	32.68 (6.19)	7.658	74	.000
Male	35	24.34 (6.73)	31.66 (6.70)	5.033	34	.001
Female	40	25.78 (6.70)	33.58 (5.63)	5.767	39	.000
Interest	75	32.48 (9.97)	35.95 (5.96)	3.602	74	.001
Male	35	32.14 (8.45)	35.31 (6.03)	2.368	34	.024
Female	40	32.78 (9.16)	36.55 (5.85)	2.687	39	.011

Results of comparison of pretest Attitude towards statistics in Gender wise

Based on Skewness-Kurtosis tests for normality an independent sample test t-test was conducted to compare mean of scores on pretest attitude towards statistics survey based on gender difference to see whether there was a difference on scores of attitude towards statistics before the intervention. The finding in Table 29 showed that students did not differ significantly on the attitude scale before the intervention in gender wise comparison and similarly there were no significance differences on subscales of the attitude survey: usefulness, confidence, anxiety and interest in gender wise comparison.

Table 29: *Comparison of male and female students' pretest score on attitude towards statistics using modeling (n=35 males and n=40 males)*

Variable	*M*	*SD*	*t*	*df*	*P*
Attitude			-0 .324	73	.747
Male	117.80	25.50			
Female	119.68	24.56			
Usefulness			-.719	73	.475
Male	30.54	7.97			
Female	31.80	7.17			
Confidence			-0.025	73	.980
Male	30.73	8.63			
Female	30.77	7.18			
Anxiety			-0.021	73	.983
Male	24.34	6.73			
Female	24.38	6.26			
Interest			-.309	73	.758
Male	33.20	9.32			
Female	33.83	9.64			

Comparison of posttest Attitude towards statistics in Gender wise

Similarly, based on Skewness-Kurtosis tests for normality test an independent sample test t-test was conducted to compare mean of scores on posttest attitude towards statistics survey to see whether there was a significant difference on scores of attitude towards statistics after the intervention in gender wise. The result in Table 30 showed a comparison between male and female students' attitude towards statistics using Modeling Approach and they did not differ significantly on the attitude scale. Similarly, there were no significance differences on subscales of the attitude survey: usefulness, confidence, anxiety and interest in gender wise comparison.

Table 30: *Comparison of male and female students' posttest score on attitude towards statistics using Modeling Approach (n=35 males and n=40 males)*

Variable	*M*	*SD*	*t*	*df*	*P*
Attitude			-.968	73	.336
Male	139.31	16.53			
Female	142.85	15.11			
Usefulness			-.068	73	.946
Male	36.46	4.66			
Female	36.53	3.94			
Confidence			-0.261	73	.795
Male	35.89	4.85			
Female	36.20	5.50			
Anxiety			-1.346	73	.182
Male	31.66	6.70			
Female	33.58	5.63			
Interest			-.900	73	.371
Male	35.31	6.03			
Female	36.55	5.85			

Result of Comparison on mean gain difference scores on attitude survey by gender

Based on Skewness-Kurtosis tests for normality test an independent sample test t-test was conducted to compare mean gain difference scores from posttest to pretest students' scores on attitude towards statistics survey based on gender difference to see whether there was a significant difference on mean gain scores before and after the intervention. Table 31 showed that male and female students' mean gain difference from posttest to pretest attitude towards statistics did not differ significantly (p=.951). Comparing males and females mean gain differences shows that the average attitude score for male students (8.00) was not significantly (p=.506) larger than the score (7.75) for female students who have learned using Modeling Approach.

Table 31: *Comparison of male and female students' mean gain difference score on attitude towards statistics using modeling (n=35 males and n=40 males)*

Variable	*M*	*SD*	*t*	*df*	*P*
Attitude			-.283	73	.778
Male	21.51	24.89			
Female	23.18	25.76			
Usefulness			.669	73	.506
Male	5.91	7.83			
Female	4.73	7.54			
Confidence			-0.188	73	.852
Male	5.11	8.16			
Female	5.48	8.43			
Anxiety			-0.864	73	.390
Male	7.31	8.60			
Female	9.20	10.1			
Interest			-.309	73	.759
Male	3.17	8.27			
Female	3.78	9.08			

Similarly in the sub components of the attitude, the mean gain difference usefulness score for male (5.91) was not significantly larger than the mean gain difference (4.73) for female students. Even though, females mean gain difference scores did not differ significantly with their male counterpart on attitudinal components of confidence and interest (p=.852 and *p*=.761 respectively), female students mean gain difference scores in confidence (5.48) and interest (2.73) was better than male students mean gain difference scores by small margin in confidence (5.11) and interest (2.11). Finally, male students anxiety mean gain difference score (-5.14) was a little bit larger than female students anxiety mean gain difference score (-5.18), but they did not differ significantly (*p*=.987).

4.3.3 Discussion on Results of Achievement and Attitude Data

This section discusses on achievement and attitude data results in relation to the relevant literature review to answer the research question: To what extent do students enhance their understanding of descriptive statistics using Modeling Approach?

Achievement

A comparison was made between and across groups of students who learned using Modeling and Non-modeling Instruction Approaches on procedural and conceptual understanding achievement tests of descriptive statistics. The findings showed that there was a significance difference between groups on conceptual understanding achievement test between students who learned Modeling and Non-modeling Approaches. The achievement tests contained items on interpretation of statistical concepts and reasoning items. Study showed that if students learned making connections within and between concepts, they would have better conceptual understanding (Boaler, 2006). In Modeling Approach, students iteratively developed concepts as a package. For example, students had learned the concept of variability and center to describe aspect of data distribution from informal to formal to see the interconnected concepts using representation systems.

According to Garfield, et.al (2007) students could develop their conceptual understanding well if students started working building on informal knowledge. Students had some kind of intuitive knowledge to build on a robust understanding. Students had been challenged to make interpretation and justification while solving MEAs. Modeling could develop reasoning ability, because MEAs and other follow up activity challenges students to enhance their cognitive thinking (Chamberlin, 2010).

A study conducted by Hjalmarson, Moore and delMass (2011) showed that students should be elicited on their prior knowledge without creating cognitive obstacles. Using Modeling instruction, students could develop their conceptual understanding through problem solving. First, as a sequence of model development, they tested, revised and refined their models by using different representation systems. The approach begins from meaningful and concrete situations which more likely help to develop formal notions of the concepts, because model development is local conceptual development (Lesh & Doerr, 2003).

The other point that could help students to develop conceptual understanding would be students had been learning statistics as a system to develop understanding on features of data distribution. Students often looked at individual values rather than the aggregate values (Ben-Zvi & Arcavi, 2005), but through continuous feedback using representation systems, it seems that it is possible to help students see aggregate values using Modeling Approach. Students had been working on representation systems to describe the distributional aspect of data sets. In the Modeling Approach, students worked on multivariate data to write a report which support their systemic thinking (delMass & Liu, 2005, 2007).

There was no statistical significance difference on achievement test of procedural understanding within and across groups between Modeling and Non-modeling Instructional Approach. Procedural understanding is helpful to develop the conceptual understanding of concepts in the Modeling Approach, since it reduce the cognitive challenge as it automates the process of solving cognitively challenging problems (Chamberlin , 2010). But, mean score of students who learned using Modeling Approach was greater than mean scores of students who learned using Non-modeling approach. This suggests that Modeling Approach would enhance conceptual understanding without compromising their procedural understanding of descriptive statistics.

In the previous section, equity was discussed holistically in terms of relational equity and on the nature of problems students would experience on the subject they learnt at school and beyond school in their real life. Equity could also be expressed in terms of students' achievement using different outcome measures. There was a statistical significance difference among students by their achievement levels in the modeling class on conceptual understanding achievement test of descriptive statistics.

Research showed advance students may lack engagement and interest, because the students often did low level cognitive tasks which will not challenge them (Chamberlin, 2010). MEAs would be helpful to identify creative individuals and could serve as relevant non-routine problem solving tasks to serve all students with equity (Bostic, 2012). The Study found that there was statistical significance difference between high achievers in Modeling and Non-modeling groups, which may suggest that the modeling group were favored by cognitively demanding MEAs tasks. But, there was no statistical significance difference between students who achieve low and medium in Modeling Approach. This might for the reason that both low and medium students engaged actively in modeling class to solve problems which in turn attribute for both groups to enhance their understanding of descriptive statistics (Doerr & Lesh, 2003).

The findings of the study had showed that there was statistical significance result across groups between students in gender wise in both achievement tests. In mathematics education, there was a stereotype that mathematics is male domain subject (Forgasz & Rivera, 2012; Oise, 2002). According to Boaler and Sengputa-Irving (2006), there is nothing wrong with the girls, but rather the way teacher teaches affects girls' attitude and achievement in mathematics. Their study showed girls were more interested in knowing why the procedure works and they liked to learn procedure with sense making. Modeling approach provided a chance for students to develop systemic thinking with rich connection using representation systems that could offer a chance for girls to enhance their understanding of descriptive statistics.

Attitude

The findings of the attitude data showed that there was significance mean gain difference from the pretest to the posttest students' scores on attitude towards statistics survey .The attitude survey instrument included affective issues of usefulness, anxiety, confidence and interest on descriptive statistics. The study found that students likely develop their positive attitude using Modeling Approach. Research had showed that modeling would help to develop sense of how statistics is useful and motivate students, because the problems start from real life situations (Curran, Carlson, & Celotta, 2013; Falsetti & Rodriguez, 2012).

The study found that students could easily see the usefulness of statistic using Modeling Approach, because the approach allows students to learn concepts of statistics meaningfully (Shaughnessy, Garfield, & Geer, 1996). One of the principles of designing MEAs is reality principle and that students may look into relevance of the problem in their everyday life. In particular, in this study students were working with socio-cultural problems that arose from the concrete limiting conditions of the society which likely help them to see the usefulness of statistics. They even further used statistics as a critical tool towards social justice for giving awareness on topical issues for themselves and for others. Hence, it seems that Modeling Approach could more vividly show the usefulness of statistics in the present and future life of the students.

More probably students also enhance their confidence using Modeling Approach. Students had been working on data modeling solving MEAs. Remember that MEAs was observed to be cognitively demanding tasks as observed in this study and mathematics educators identified MEAs as cognitively challenging tasks (Chamberlin, 2010). Teachers scaffold students in the modeling cycle and they likely develop habit of mind on the statistical investigative process (Wild & Pfannkuch, 1999).

The big idea in statistics is to draw conclusion from data with in context. Students were able to make decision and conclusion working on relevant non-routine problem solving tasks which may add their confidence dealing with statistical problems. The other point is, students work in teams in MEAs that they could develop confidence because each member of the team could share responsibility. Students were playing multiple roles in MEAs, wrote and presented in a team that could perticipitate theire lack of confidence as they made contributions in their team (Moore, 2005).

Students' anxiety could also be reduced using Modeling Approach as it was found in this study. More importantly, students' fear of asking teacher would likely be reduced, because students were most of the time engaged in the MEAs and the teacher walk around much of the time to scaffold their data modeling. Students' anxiety seems decreased using Modeling Approach, since they share ideas in their teams and making mistakes were encouraged as an opportunity for learning. Moreover, students were presenting reports in pairs that may decrease student anxiety to present reports statistically.

The other finding is students could enhance their interest using Modeling Approach. Though the finding of the study on interest subscale on attitude survey did not differ significantly before and after the intervention in gender wise, the finding showed students score higher on interest subscales other than other subscale like usefulness and confidence. However, classroom observation suggests students likely gain interest towards statistics using Modeling Approach. Students found MEAs 'hard fun' activities. They made their report funny using news, jokes and comics incorporating it in their statistical reports initiated by students themselves unlike the fun created by teachers to teach statistics (Lesser, et al., 2013)

4.4 Students' Project

This section presents results on students' projects to answer the second research question: How do students enhance their critical understanding of descriptive statistics through data modeling using projects? First, students' project phases are identified and summarized using percentage and frequencies. Second, six cases of students projects are presented based on their final project reports, project interviews and students' presentation data.

4.4.1 Phases of Students' Project

The four phases of students' project were the following: (1) formulating a question; (2) collecting data; (3) analyzing data and interpreting the results and (4) final report and presentation of the project. Table 32 showed the distribution of number of phases completed by 160 students in a team of three or four students. Except team of students' reports who had completed the whole phases of the project were excluded for analysis.

Table 32: *Number of teams of students on the phases of the projects*

Modeling Completed	**Frequency**	**Percentage**	**Non-modeling completed**	**Frequency**	**Percentage**
All 4 phases	14	70%	All 4 phases	13	65%
Three phases	1	5%	Three phases	1	5%
Two phases	2	10%	Two phases	1	5%
One phase	1	5%	One phase	2	10%
None	2	10%	None	3	15%

Question Formulation Phase

The students were required to submit two to five questions for the project. The average number of research questions was 5 (taken the median of all questions on the 27 projects). Nine teams of students (33.3%) submitted more than 5 research questions for the project. Sixteen teams of students (66.7 %) submitted up to 5 questions more than the minimum requirement of two questions. The minimum number of questions submitted by one team of students was 3 and the maximum number of questions was 12.

It was of some interest to see the quality of the questions that the students posed for the project. From 27 teams of students' project topics there were 169 statistical questions were prepared for data collection and four major types of questions were identified that the students proposed. These are (1) questions that asked discrete quantitative data (30%); (2) questions that asked continuous quantitative data (25%); (3) questions that asked for binary data (18%); and (4) questions that asked categorical data (27%).

Investigative Themes

Teams of students selected the investigative themes as shown in Table 33. They chose the themes by their own to go through data modeling process to solve social-cultural problems on issues of health, education, culture, economic, gender inequities. Some of the themes focused on giving an awareness program to students to take some preventive cares related to health issues like dental care, body mass index, sleep, and first aid.

Investigative themes in education includes problems of learning Amharic language, motivation towards mathematics, problems of school attendance, problems of students achievements in last semester, problems of keeping balance between study and entertainment. Teams of students did projects on economic and transport problems which could solve existing problems in the community with social justice agenda.

Some of the themes dealt with traffic accidents and transportation problems in the town. One of the students on traffic project put her concern in the interview that she was responsible for creating awareness about traffic accidents for students with a sense of social justice:

> We know there were several traffic accidents in the town. Recently, a girl we know died in car accident in grade 9 and this is a great concern, because it will take away the precious life of many people. We plan to do this project with the aim of giving awareness program using statistics. Students often go in groups which increased the risks, because they may lack concentration and we need to give more awareness on the traffic laws (Hanan: 1/5/2014).

The Ethiopia education system acknowledge there was a need to include sensitive issues about gender in the curriculum and pointed out that community attitude towards girls education has been low and there has been still a gap on gender based violence in and around schools (MoE, 2010).

Table 33: *Investigative Themes in students' projects*

Investigative themes	**Aim of the investigation**	**Number of questions**
Disability and their working conditions	To create positive awareness on attitude towards disabled people	12
Problems in Amharic language	To solve problems of learning Amharic as a second language	8
Motivation in Maths	How to improve girls motivation in mathematics	12
School Attendance	To solve problems of absenteeism	4
Types of students shoes	Types of shoe students need to wear	4
Students' scores	How to improve achievement scores	8
Youth and addiction	To avoid bad addictions habits like drinking alcohol, smoking cigarette and chewing chat	4
Entertainment	To help balancing study and entertainment	10
Library (2 teams)	Give an awareness program on how to use library	5 and 4
Traffic Accident	Give an awareness program on traffic	8
Recreation and sports	To create a balance between fun and study	10
School Café	Give an awareness program on the use of café	5
Gender and house chores	To support girls to have enough study time	4
Sleeping	To address how to avoid sleeping problems	5
Body Mass Index (2 teams)	To avoid over weight and under weight	4 and 4
Religion	To develop tolerance with different Religions	10
Patients in clinic	To improve clinic service in a community	3
Cultural clothes	Keeping the identity of one's culture	5
Red Cross and Ambulance	Show how to give first aid in case of emergency	4
Dental Care	To give an awareness program on dental care	4
Global warming	To give an awareness program on global warming	4
Marketing	To give an awareness on how to buy cereals	4
Gender equality (2 teams)	To change negative attitude towards females	10 and10
Transport problems	To address transportation problems in the town	4

Three investigative themes deal with socio-cultural problems of gender inequities both in perceptions, attitude, and division of jobs at home. A student who participated on 'gender inequality' responded in the interview that women can be leaders of a country like German Chancellor, Angela Merkel:

> In the country or in our society there are still negative perceptions that females are inferior to males. The males may not think they have affected females in some way that we need to give an awareness program to create positive attitude. If females are leader at home, they can also be a leader for a country (Muyea: 2/5/2014).

One team of students did project on religion with the aim of promoting positive attitude to live together peacefully with people believing in different religions by developing tolerance against religion extremism. One team of students worked on how to keep cultural identity of different ethnic group on wearing style traditions. According to MoE (2010a), substance abuse like ‘chat’, alcohol, tobacco and cigar has been a significant risk for many youngsters and little attention has been given to address this issue. One team of students had investigated theme regarding how to avoid bad addictions in youth which includes chewing chat, drinking alcohol and smoking cigarettes.

Data Collection Phase

In the second phase, the teams of students were required to finalize their survey questionnaire and measurement protocol. The students prepared the survey, after making some revision on their questions discussing with their teachers. Teams of students had collected data with the minimum sample size of at least 30. Teams of students used primary data, but three teams of students used both primary and secondary data.

Data Analysis and Interpretation Phase

Table 34 showed the different ways how teams of students in Modeling and Non-modeling classes presented the data. If a team of students used histogram as a model and displayed three histograms to represent the data, then the three histograms was counted as one model. The content analyses for the 27 projects were done on the choice of models for 14 and 13 teams’ project reports of Modeling and Non-modeling groups respectively.

Table 34: *Choice of models to display data*

	Frequency for Modeling teams	Frequency for Non-Modeling teams
Bar or line graph	13	9
Pie Chart	13	4
Dot plot	7	2
Histogram	8	6
Table of frequency	13	6
Measure of center		
Mean	10	7
Mode	11	6
Median	11	6
Measure of variation		
Range	10	6
Mean deviation	4	3
Standard deviation	7	3

Final Report and Presentation Phase

Teams of students wrote a project report within 3 to 18 pages. Every team of students prepared a colorful poster displaying various graphs and use descriptive statistics to present reports on their projects. Teams of students had prepared from outstanding to inadequate reports as shown in Table 35 in both Modeling and Non-modeling groups. Six case studies of the final reports will be discussed on the next section.

The teams were trying to present their project using different styles. Three teams used dramas to present their projects and one of the dramas showed by teams of students was on giving awareness on avoiding bad addictions habits. Others presented news, poems and comedy to present their reports attractively. One team of students who worked on cultural clothes had a fashion show. Three teams of students included interviews on their presentations, for example, on BMI project the students interviewed one of their team members as medical doctor to give advice on how to keep normal BMI.

Table 35: *Students' final project reports using project rubrics*

	Number of teams in Modeling group (%)	**Number of teams in Non-modeling group (%)**
Outstanding report	5 (35.7 %)	2 (15.4 %)
Good report	3 (21.4 %)	3 (23.1 %)
Adequate report	4 (28.6 %)	5 (38.4 %)
Inadequate report	2 (13.3 %)	3 (23.1 %)
Total	14 (100.0 %)	13 (100.0%)

4.4.2 Six Case Studies

Table 36 is a summary of students' background who participated in interviews with their statistical investigation topics of the projects. The six case studies of the projects were selected based on the quality of their final project reports and sites of their data collection. Six projects were included as cases. Three teams of students were selected from the Modeling group and another three teams of students were selected from Non-modeling group from the two schools.

Table 36: *Summary of the 6 case studies participants of the projects*

	Modeling Project		**Non-modeling Project**	
Quality of the report	**Team member(s)**	**Topic (site of data collection)**	**Team member(s)**	**Topic (site of data collection)**
Outstanding Report	Selam	Scores of students (inside school)	Ibrahim	Amharic language (inside school)
Good report	Chirenet & Degenet	Body Mass Index (inside school)	Netsanet & Helen	Youth and Addiction (outside school)
Adequate Report	Robera & Alex	Public Library (outside school)	Leoul	Disability (outside school)

Projects in the Schools Compound by the Modeling Groups

Case #1 Selam's team project was about students' scores in the first semester and her team wrote in their final project report that the vision of their investigation was to see students' scores to be much better in the second semester than the previous semester. The team stated that their objective was to help students achieve high scores by initiating students to study hard. Selam said the following in the interview on how they selected the project:

> When we select the theme of our project, we noticed that students' scores in the first semester are very low and we discussed how to improve the scores. When we asked some students they said, 'We wish there was no first semester and we wish if the first semester pass over unknowingly….' In our section, a female student whom we do not know stood first. We are doing this project to initiate students to score better in the second semester than the first semester (Selam: 29/4/2014).

The method they used to collect data on this project was oral survey by asking students from class to class. The team asked six statistical questions which helped them to collect continuous and categorical data. Their data analysis used all measure of dispersion. They used visual displays such as frequency distribution tables, dot plot, bar graph, pie chart and histogram. The graphs were appropriately labeled on the axis and contain titles as shown in Figure 27 and the calculations on measure of center and dispersion of the data were accurate with correct interpretations. Selam's team wrote the following result in their report from the oral survey of 30 students:

> Even if, one of the reasons for low students score was not studying hard, there are other reasons for students' low scores like work burden on female students at home, the living conditions of students, and lack of students' hope in their education. As a result, students lack any interest to learn something. In general, students, who had a lot of work to do at home, are forced not to have study time; since there is no one to help their family. Especially, this problem is common for rural female students. For urban students, the major problem is lack of interest

and hope in their education, because they see college and university students at home without getting any jobs after graduation.

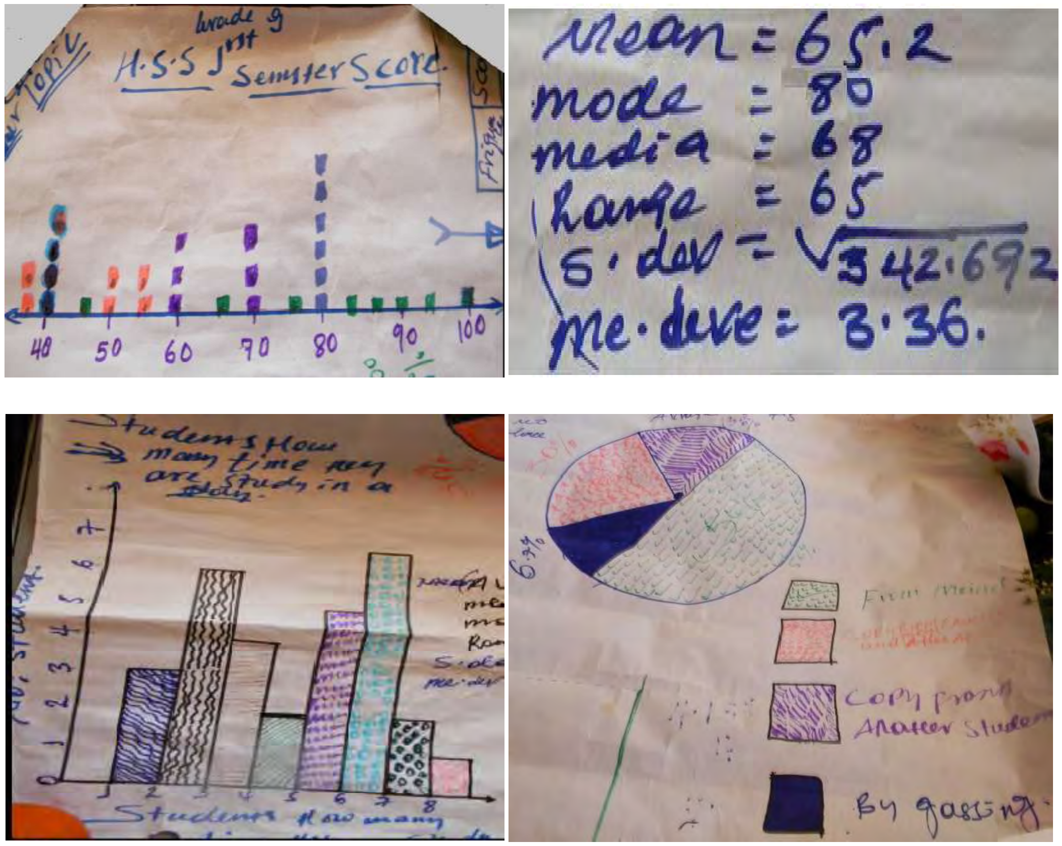


Figure 27: Students' scores project visual displays of data.

After pointing some of remedies for the above problems, the team started to summarize the data on last semester average using dot plot and wrote:

> As the saying goes, 'If you do not sow the seed well, you can't get the harvest.' Students result for few are good and for the majority it is unsatisfactory. The range for 30 students' data score is 65 with mean score 65.2 and students' data scores standard deviation is $\sqrt{342.692}$. The standard deviation shows that there is a big variation among students' scores in the last semester.

The other main concern of the project was the problem of copying during examinations. They reported using pie chart that 50% of the students did independently by themselves, 30% copy from exercise books and notes, 13.3% copy from other students and 6.7% did just by guessing while taking tests and examinations. Overall, all components of the project report were clearly present and of excellent quality. Among the project phases, Selam had expressed her views that data organization and analysis phase was difficult.

Case #2 Chirenet and Degenet were members of a team who did a project on Body Mass Index (BMI). Chirenet responded in the interview that the team chose the project theme for the following reason:

> First, we could not estimate whether a student is normal, overweight or underweight without any measurement looking at students' body. For example looking at a student's body, we can't say he is overweight since his weight could be distributed over his height. If he is tall and if we divide the weight by the square of his height, he may have normal BMI. Most people do not know their height, weight and BMI; that, they will be exposed for many health problems. Thus, we think the project is important for us and for many other students (Chirenet: 28/04/2014).

The team found that 50% of the students were underweight and the team worked on the project with a social agency aim to alleviate the problems giving an awareness program:

> If we do not weigh their weight and measure their height, then we could not know the BMI. Since we know how to use statistics, we are able to give an awareness program for others and for ourselves on how to keep our health (Chirenet: 28/04/2014).

Chirenet and Degenet team made a pie chart, dot plot, histogram and a bar graph as shown in Figure 28. The team used dot plots to show the age of students, bar graph to show the sex of students and a histogram to show the distribution of students' weight. They used also measure of center and variation accurately with correct interpretation of the data. But, they had not used standard deviation as measure of variation.

Chirenet and Degenet expressed their views that among the project phases, data collection phase was difficult. Overall, their final project report was good and all components were clearly presented. In their final presentation, Degenet made an interview with Chirenet as if he was a doctor to advice students on the need to check one's BMI.

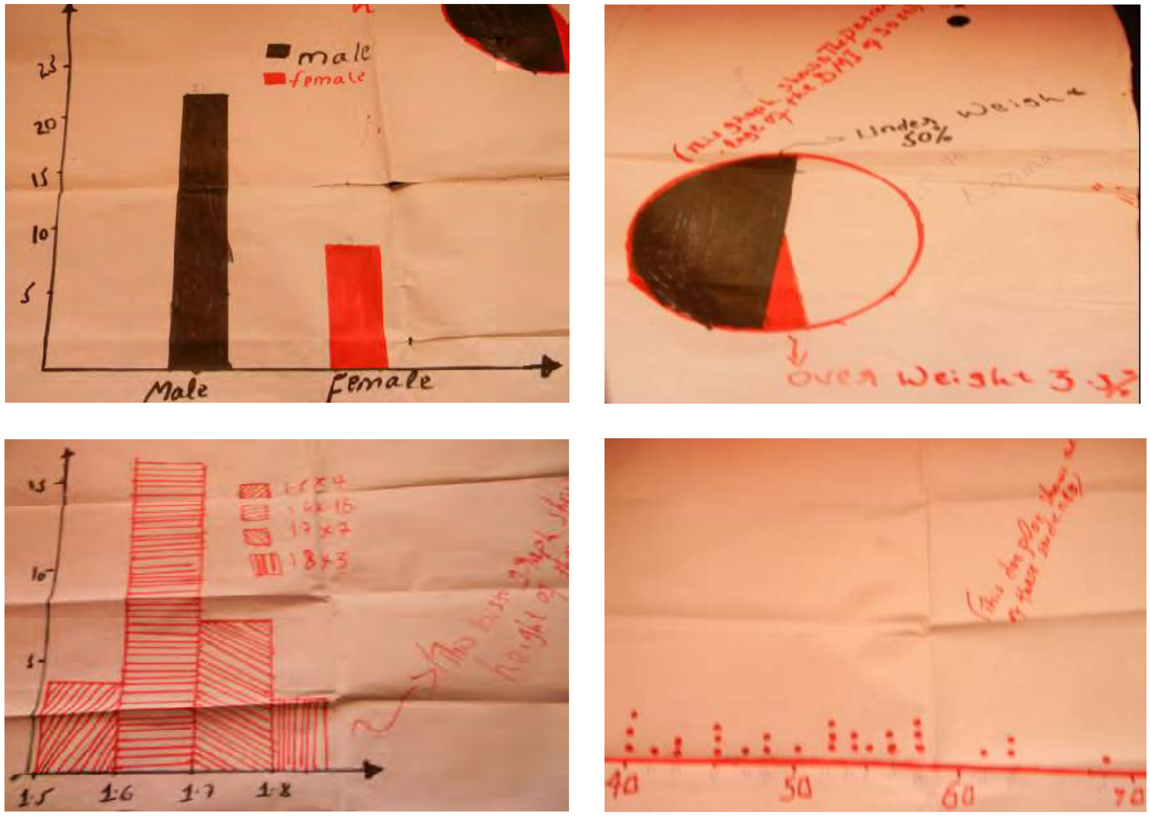


Figure 28: BMI project visual displays of data

Projects outside the school compound by the Modeling group

Case #3 Robera and Alex with other team members did on public library project. The school had no library that their project raised an important issue. They responded in the interview that they selected the theme for the following reason:

> We are students and books are necessary to support our learning. There are students who do not know about library and thus we choose the project for helping students to use library. We ourselves know little about the use of library (Robera: 30/04/2014).

> Besides what he says, we learned in the modeling class that when we do a project it has to solve some problem. Hence, doing a project on library will solve a lot of problems for students (Alex: 30/04/2014).

The team had collected primary data from students on the use of library and used secondary data on the number of books the public library had. They asked questions that allowed them to collect both quantitative and qualitative data. The team used bar graphs and histogram to present their data as shown in Figure 29. All the graphs had labels and titles. But, they incorrectly make histogram for the number of days students visit the library in the last weeks; because, it is a discrete quantitative variable. The team had also used measure of central tendency, but they had not used measure of variation like range and standard deviation.

Overall, the project report on library was adequate and all components were present. They display their data with appropriate graphs, but their analysis was limited to describe the data using only measures of center. Among the project phases, Robera and Alex responded that organizing the data and displaying the data with appropriate graphs were challenging. They made good presentation on their project for giving awareness program on the importance of using library and they were supporting their arguments with evidences from their data. They expressed their great concern on problems of library use in gender wise as the school had no library:

> Most students went to library for getting information, but there are few individuals who went for entertainment. Besides, we found in our data that the majority of students who used library were male students more than female students. We conjecture that females would be busy doing house work most of the time (Alex: 30/04/2014).

When they were probed further on how they created an awareness program to use the public library, they responded the following with social agency aim:

> Yes, we know girls had a lot of works to do at home. But, boys are not busy at home and if one of their friends showed them the library, they easily used to it. On the contrary, most girls stay at home and no one will show them the library. If somebody helped them to start using the library, there would not be any problem. Yes, we need to give them some awareness (Alex: 30/04/2014).

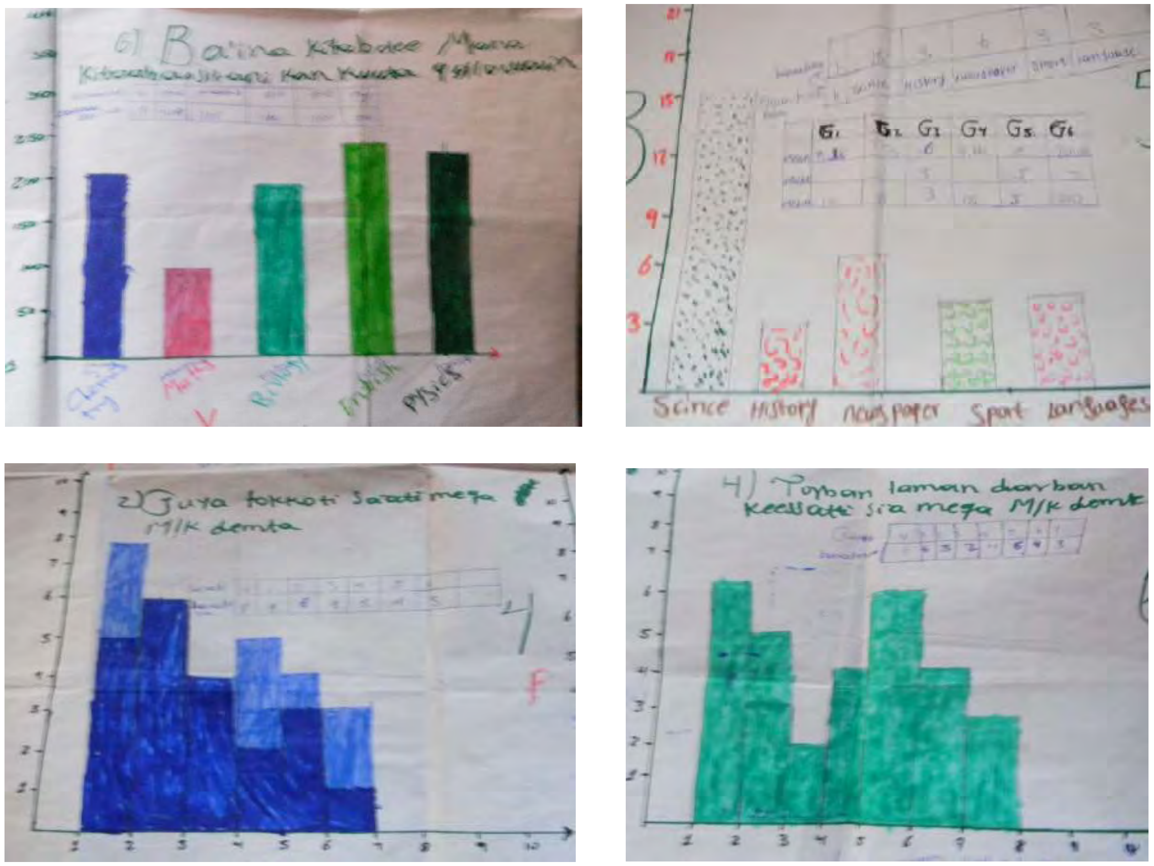


Figure 29: Library project visual display of data.

Projects in the School Compound by Non-modeling Groups

Case #4 Ibrahim team had done a project on students' problems of learning Amharic language. Ibrahim said in the interview that they selected the title because in all six sections of Grade Nine in their school, most students had problems to learn and communicate well in Amharic. They wrote in their final project report that they collected data from 10 students whose mother tongue language was Amharic and from 20 students whose mother tongue language was Oromigna.

Ibrahim team collected data from 15 girls and 15 boys in the school. In the interview Ibrahim responded this on how they had collected data from 30 students' participants:

> First, we simply distribute questionnaires on the project to students, but we found students do not give us candid responses. Therefore, we collected the data using face to face answer and response type of questioning on one-one basis method with the selected 30 students for the project (Ibrahim: 25/04/2014).

After collecting their data they displayed the graphs as shown in Figure 30 and wrote the following on their project report:

> The data from 30 students showed that no students failed in Amharic language, but their achievement was not satisfactory. Boys scored in the range of 51-89 and girls scored in the range 55-87. In both sexes, students had scored better result in the range of 80-89, and some students achieved unsatisfactory result that were in the range of 51-79. So, we need to ask how to improve this result?

The team drew all the graphs with appropriate labels and titles. Their interpretations were correct and accurate. Ibrahim's team wrote in the final project that they found a paradox from their data. The paradox was, eight students who achieved higher scores in the last semester had responded themselves that they had communication problems in speaking, reading, writing and listening in Amharic language. Ibrahim expressed in the interview that this problem existed in the school and it was critical which needed attentions to be solved:

> We had noticed some students like to speak in Amharic, but couldn't listen well. We know some students could write well, but I know some other students who couldn't write their names in Amharic alphabets. During Amharic test or exam, other students who write well helped those students who couldn't write their names on their on the answer sheet. This is a big problem (Ibrahim: 25/04/2014).

Finding controversies in the data they collected, they made interview with their teacher:

> The teacher told us in the interview that most students need support in Amharic language, because most students' mother tongue language was Oromigna. Students who didn't knew the language copied at exam not to get a score of less than 50. So, this was the reason we asked the Amharic language teacher to give us some explanation why students' who have less ability got higher scores. Because in the first semester, we can see coping at exam was the norm in most class. We had also seen students who couldn't write their names get all the answers correct at exam. We asked on how it was possible students got better result without reading ability and our teacher in the interview explained this for us (Ibrahim: 25/04/2014).

Overall Ibrahim team's project report was outstanding. All components of the statistical investigation processes from problem formulation to conclusion were clearly presented and were of excellent quality. The teams were working together and Ibrahim expressed his view in the interview that in his team they had found data analysis was the difficult part from the project phases. All the team members were actively participated in the presentation of their project and they had displayed video of their interview with the Amharic teacher using flattop computer for the audience during their presentation.

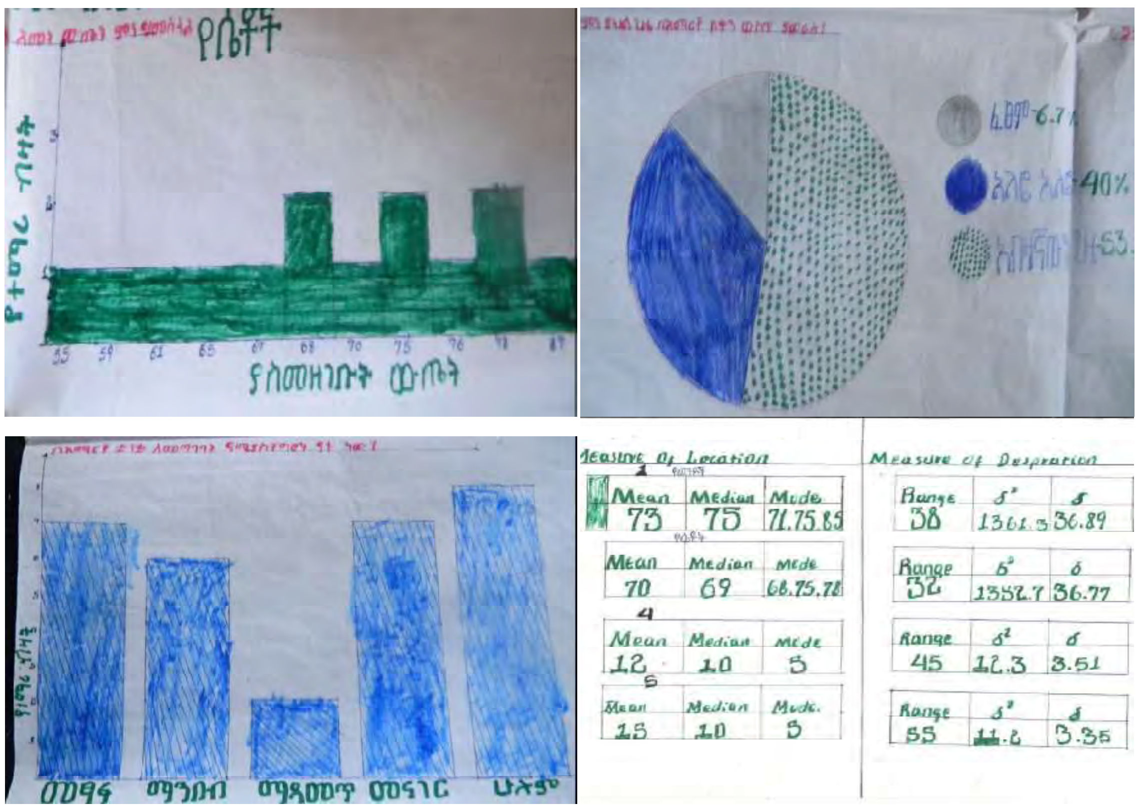


Figure 30: Amharic Language Project visual displays of data.

Projects outside the School Compound by Non-modeling Group

Case #5 Netsanet and Helen responded in the interview that they did not enjoy mathematics much and selected the project as a social agent for youth not to be addicted by bad habits like drinking alcohol, smoking cigar and chewing chat:

> Yes, we are at youth age and it is critical period because the youth wanted to try anything new even if it is bad. For this reason, we choose the project on bad addiction because it concerns all of us (Helen: 6/05/2014).
>
> What I could add on what she said is, when we choose this title the majority of the participants were unemployed. Before they started to get used to bad addictions, we need to reach for them and give support not to be victims of bad addictions. We were interested in finding remedies by giving awareness program (Netsanet: 6/0/2014).

They wrote the following point in their final project report:

> Our project title is about the youth. To do this project, we collected data from 30 people. Among the 30 people, 15 are males and 15 are females. When we do this project, we noticed the youth has many problems. At this age, the youth must scarify everything to live a happy life in the future. If we do good things at this age, then we will be happy in the rest of our life and if we do bad things, then we will regret in our future or we may lose hope forever in our life.

Then, they wrote in the report that how bad addictions would affect one's health and life economically. They set five quantitative and four qualitative statistical questions. This team made appropriate display of data as shown in Figure 31. They used both measure of variation and measure of center for describing the data. But, they used the highest frequency as mode instead of finding the value of the data with the highest frequency.

Overall, the team of the students demonstrated a good understanding of the project problem and made good analysis and interpretations of the data. Netsanet and Helen reflected that data analysis phase of the project was difficult for the team. They also described that they faced challenge on collecting data, because their topic was very sensitive. Netsanet said, "We know some of the youth participants in our study were addicted and liked to conceal their personal information". They collected data using questionnaires and the participants were not asked to reveal their names to protect the participant privacy. They made organized and proactive presentation. The team presented drama and poem for 30 minutes, and then presented their statistical report by explaining each statistical question on the project with graph. They summarized the data with measure of center and dispersion.

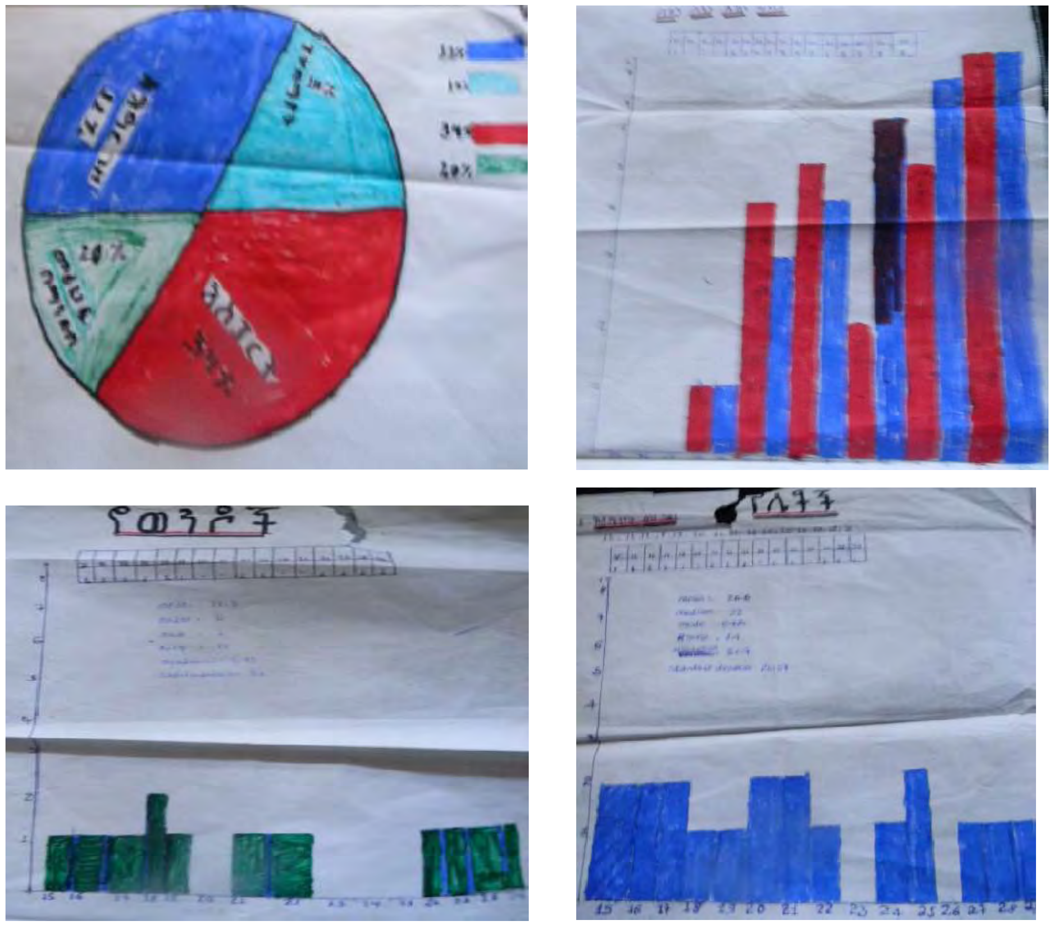


Figure 31: Youth and addiction Project Visual displays of data.

Case #6 John was an average student in mathematics class and his mathematics teacher described him as having fixed mathematics ability; even if, he put much effort in mathematics class. John responded in the interview that his team selected the project entitled ‘disability’ because they wanted to give an awareness program on disability because the team believed the society did not have positive attitude:

> The community does not have good perceptions for people who have some disability caused by leprosy, because there is a myth that leprosy is incurable and transmitted by family background or bloodline (John: 24/04/2014).

The team wrote the following in their final project report:

> The disable people are like other members of the community and they have the right
>
> - To live by working
> - To change their living conditions
> - To contribute significantly for the country development
>
> In different time the disabled people had been exemplary without being dependent on others and helping to solve their family problems. If the society respects the disabled people, then the disabled ones will develop a sense of equality and patriotism for their country. Therefore, it is a duty of every citizen to respect the right of the disabled people.

The team prepared 12 qualitative and quantitative statistical questions on the disability project. They collected data from 6 and 24 people in and outside the town respectively. The team went more than 30km to collect data from the disabled people. John responded that two of the participants were lepers. In the final project, the team described that among the participants 37% and 63% were females and males respectively. They had used connivance sampling and reported that 30% of the participants were daily workers, 36.67% were beggars, 6.67% works in office and 26.67% were merchants. Though there were beggars, they wrote in their report that the majority of the participants believed that they were productive human power. The team was strongly worked towards social agency agenda using statistics as a critical tool:

> I think, if we go to foreign country, you will see several disability people. Most people get disabled by traffic accident and different disease. If we looked at China, most people are older and there is less productive human power. If we go Europe, we found very less youth population who are productive human power. They are in danger after 35 or 36 years, because they may lack productive human power. In our case, we do not have many intellectuals in our country. We have

disabled youth who could be part of human productive power. To change the society perceptions, we need to give awareness program through social media and other means. I don't have such understanding and knowledge before, data gives you evidence than just doing things based on opinions (John: 24/04/2014).

The team drew 17 visual displays. The team used pie chart, line graph, bar graphs and frequency distribution table as shown in Figure 32. The team had not used histogram, despite they collected continuous data. The graphs had labels and titles and the team selected the appropriate graphs with correct interpretation. They used measure of center and variation correctly but they made little interpretation on the summary. Overall, the team final project report on was adequate.

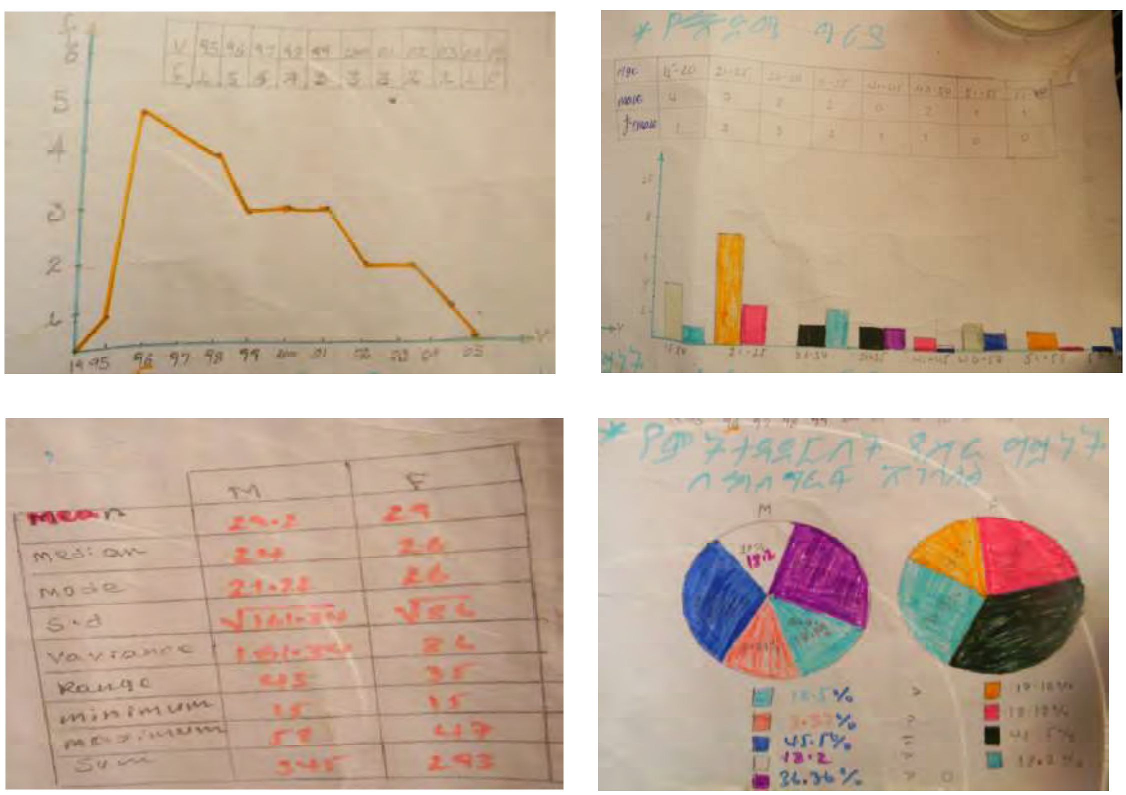


Figure 32: Disability project visual displays of data.

4.4.3 Discussion on the Findings of Students' Projects

This section discusses on the results of students' projects in relation to the relevant literature review to answer the research question: How do students enhance their critical understanding of descriptive statistics doing projects through data modeling?

The project for Modeling and Non-modeling group

Teams of students in both Modeling and Non-modeling projects had found different parts of the phases difficult. The project involved four phases of statistical investigation processes. It was not a kind of 'let them students do the project' as pointed by Wild and Pfannkuch (1999), but teachers helped in the meantime if students faced with any problems during the four phases of the project: question formulation, data collection, data analysis and interpretation and final report writing. The teacher gave feedbacks identifying students' challenges during the investigative processes of the project (Pfannkuch, Regan, & Wild, 2010).

Teachers need to give equal attention for all the phases of the project and give feedback to complete the project successfully. Among the project phases, teams of students may even found question formulation phase difficult. In both Modeling and Non-modeling groups, four teams of students found question formulation phase difficult. The teams of students set up more general questions or problems. Students in modeling group had learned in the first class how to set up statistical questions that need data that vary. Hence, this suggested students need to learn on how to prepare statistical questions that allow them to collect both quantitative and qualitative data.

Doing project is one aspect of Modeling Approach which could promote independent learning. Though both Modeling and Non-modeling groups had been doing the project, it seemed that modeling group could easily undergo the investigative process unlike the Non-modeling group, because they were get used to the modeling cycle in the Modeling Approach. Both, the project and the Modeling Approach involved data modeling.

Some of the difference between the MEAs and the project were whether students choose the investigative themes and collect data by themselves and the extent to which students do data modeling independently. Thus, students' learning of data modeling could help them to reduce the challenge doing the project independently, because they may already develop the ability to work together in a team and to do the data modeling working in MEAs. Both teachers in the school had been reflecting that students in Non-modeling group found the projects more difficult than the Modeling group. The reason for this may be students in Non-modeling group may not have the habit of mind to go through the data modeling. The completion rate of the project of the Modeling group is better than the Non-modeling group. Modeling Approach likely enhances students' representational fluency. Students' in the modeling approach showed more choice of models than the non-modeling groups of students'. The study found that teams in modeling group had used more choice of models than the non-modeling teams.

Students' empowerment

According to Freire (1970), it is better if students select the investigative themes by themselves than investigative themes assigned by a teacher. Students had developed their critical understanding of descriptive statistics using statistics as critical tool towards solving the socio-cultural problems. Students were able to pose and solve their own and others problems using projects by creating awareness programs. Students were working on themes that arose from educational to economic to social life and culture problems. Students were presenting awareness program on avoiding copying during examination, avoiding bad addiction habits, keeping dental care and body weight regularly, resolving problems in gender and religion inequities.

Gustine (2006) used both statistics and mathematics with social agency, but there were a number of subtle similarity and differences in this study from his approach. Gustine chose the investigative themes for students, but in the Modeling Approach, students first worked on MEAs problems with investigative themes having relevance with socio-cultural problems. Then, students selected the themes by themselves towards helping others with sympathetic understanding. In the Modeling Approach, students learned according to the time plan of the lessons on descriptive statistics set by Ministry of Education on Grade Nine curriculum syllabuses. One of the advantage of MEA was it took not more than two or three periods unlike other non-routine problem solving tasks that requires an extended period of time.

Students had been doing the projects having some kind of team sprits and the finding of the study suggest they likely develop abilities to work in teams. In the 21st century working in multi-disciplinary areas with a team is one of the abilities students need to develop (English, 2010; Lesh, 2000). Students were working in teams with interdependency and helped each other towards finishing their project with preservation. They developed team sprite and learning to live together working on the projects.

There is a myth that students had fixed abilities in doing mathematics. However, research had showed that ability and intelligence grow with effort and practice; when teachers believe that everybody's ability can grow and they give students chances to achieve at high levels (Boaler, 2013). Pedagogy with social agency could promote students empowerment beyond fixed abilities with open-ended projects. Students could struggle to develop their empowerment through critical pedagogy by trusting students to do their own project (Freire, 1970). The teacher considered a student in disability project had a fixed mind, but on the contrary, he was the pioneer of the project to work on significantly on the statistics concepts and to present the project. The school director, after students presented their final project report, commented that "How could students from highly underprivileged background whom we consider incapable do projects like these?"

Students' were able to develop their critical productive disposition towards statistics. Students' had used different entertainment and fun using statistics as a critical tool to give an awareness program on their project themes. Students had used drama, live interview, fashion show, poems which could add value to their productive dispositions towards statistics as a tool that could be used in their everyday life to solve their own and society's problems. Students could enhance the statistical thinking which basically emphasis making conclusion based on data in a context. More importantly, the finding of the students' project showed that students at their youth age could pose and solve significant problems as social agents.

4.5 General Discussions on the Results

This section discusses on the findings of the study towards answering the leading research question: To what extent and how do high school students enhance their understanding of descriptive statistics using a Modeling Approach?

To answer the how research question on the modeling approach intervention the units of analysis were the classroom practice, team of students and individual students. Like doctors observe a patient body by checking respiratory system, circulatory system, digestive system, muscular system and so on, Schoenfeld (2013) TRU Math scheme with five non-overlapping dimensions serve as a tool to observe the Modeling Approach classroom practice. The findings of the observation scheme and rubric suggested MEAs were cognitively demanding tasks and promoted learning equity for all students at different achievement levels. Students could also enhance their conceptual understanding learning the interrelated statistical concepts as a package from informal to formal. Further, students advance their understanding working on follow up activities and on MEAs extension problems organizing their original perceptions of the modeling tasks.

To continue with the medical metaphor, as the doctor interview the patient what are his/her interpretations of the case, MEAs serve as an interview tool to see students' statistical interpretation of the modeling tasks. The unit of analysis in this case was team of students' models on MEAs. Students worked on multi data sets and on multi variables to write reports and letters. Learning statistics is not only doing, but also involves describing and interpreting situations statistically (Lesh, 2000, 2012). The finding of the study suggests students could draw conclusion informally from data in a context on modeling tasks. Students working in a team by constructing, testing, revising their implicit models and expressing their thinking through representation systems enhanced their understanding of descriptive statistics (Lesh & Doerr, Lesh & Fennewald, 2010). The multiplicity of students' solution on MEAs showed students could enhance authorships of ideas than relaying simple on textbooks and teachers. The finding also suggests assessment, as learning and for learning, was an integral part of the Approach.

Furthermore, the unit of analysis on the Modeling Approach experiences interview was individual students. The finding suggested students found the MEAs relevant to their life and they were 'hard fun' activities. Students were able to engage and enhance their productive dispositions, because they could learn meaningfully using Modeling Approach (Garfield et. al, 2007). They also responded that they think 'outside of the box' in multidisciplinary areas using representation systems. Finally, they also develop sense of social agency using statistics as a critical tool working on the modeling tasks. Students also reflected they 'learn to be together' using Modeling Approach (Asgedom, 2009)

As a doctor inquires further tests to understand fully the patient case, the study administered achievement tests and attitude survey to investigate to what extent students understand descriptive statistics using Modeling Approach. The core finding of the study suggested students could enhance their conceptual understanding of descriptive statistics with medium effect size without compromising their procedural understanding of descriptive statistics using Modeling Approach. This finding is similar with other studies using problem-driven curriculum (Schoenfeld, 2004). The quantitative achievement data results also suggest Modeling Approach could promote equity with respect to gender and achievement levels similar to other research works (Boaler, 2006; Chamberlin, 2010).

Modeling can also be enacted in teaching statistics as a final project after they learned the descriptive statistics through data modeling. But, the findings of the study suggested it is better if students work on relevant non-routine problem solving tasks like MEAs before doing projects in a team for three reasons. First, students develop habit of working in collaboration with in a team, since MEAs enhance their team functioning habit. Second, students get used to the statistical inquiry cycle doing MEAs (Wild & Pfannkuch, 1999) before they do projects. By working on a real world problem, a set of data, the need for a solution, students would experience the statistical enquiry cycle (Wild & Pfannkuch, 1999) as they solve MEAs problems. Third, students could select sensitive issues for themselves as they see statistics serve as a critical tool to solve societal problems. The core finding of the students' projects was students could draw conclusion from data in context informally using statistics as a critical tool.

To recap the section, based on qualitative and quantitative data using observation, interpretation and cognitive assessments an attempt was made to answer the leading research question: To what extent and how students' enhance their understanding of descriptive statistics using Modeling Approach? Teaching is more complex than the medical diagnosis if one considers the analogy, since it involves teacher's dynamic decisions on the fly and has interrelated webs of interactions among the teacher, team of students, and individual students (Schoenfeld, 2010). But, the medical diagnosis of a patient analogy is a good one to illustrate a synergy of Modeling Instructional Approach with an assessment triangle of observation, interpretation and cognition more likely enhance students understanding of descriptive statistics. The next chapter winds up the dissertation study by presenting summary, conclusion, and recommendations.

CHAPTER 5

SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

The chapter presents a summative claim on what has been found related to the study purpose, aim and design. It addresses on how far the findings have answered the research questions. Then, the chapter discusses the implication of the study for practice and research supported by the findings. Lastly, the chapter presents concluding remarks on the degree of certainty with which the findings from the specific context could be generalized in contributing to answering the central question, and then it points out recommendations and suggestions for future research.

5.2 Summary of Study Purpose, Aim and Design

The purpose of this study was to explain how secondary students could enhance their understanding of descriptive statistics using Modeling Approach and to compare the amount of learning achieved using Modeling and Non-modeling Instructional Approaches. The leading research question was this: To what extent and how do high school students enhance their understanding of descriptive statistics using Modeling Approach? Students' understanding of descriptive statistics entails students' procedural, conceptual and critical understanding of descriptive statistics. Specifically, the research questions were the following:

1. How do students enhance their understanding of descriptive statistics using Modeling Approach?

2. How do students enhance their critical understanding of descriptive statistics doing projects through data modeling?
3. To what extent do students enhance their understanding of descriptive statistics using Modeling Approach?

The research questions were answered so as to (1) capture holistically the modeling classroom practice which may enhance students' understanding of descriptive statistics, (2) to explain how modeling approach might improve students' critical understanding , and (3) to look into to what extent modeling approach improve students' achievement by comparing students' scores between and across Modeling and Non-modeling groups on achievement tests of procedural and conceptual understanding of descriptive statistics.

The study used a transformative embedded mixed design. Using Non-equivalent pretest and posttest quasi-experimental design, a comparison was conducted on achievement tests of procedural and conceptual understanding to test hypothesis on Modeling and Non-modeling Approaches. A qualitative case study was used to investigate how Modeling Approach could enhance students' understanding underpinned by two theoretical perspectives, Contextual and Socio-critical Modeling perspectives.

The study's methodology and method enable to observe how students could develop their models iteratively working on MEAs and representation systems. Lesson designs in which students worked was prepared by the two theoretical modeling perspectives. Different quantitative and qualitative data collection methods were utilized as the study was an embedded mixed method to answer the leading mixed research question.

Quantitative achievement tests, observation rubrics, and attitude survey were used. Qualitative interviews, observation, content analysis and video tape were used.

5.3 Answering the Research Questions and Central Findings

Chapter 4 presented and discussed the findings of the study to address the research questions and hypotheses on students' understanding of descriptive statistics. The study found students solving relevant non-routine problems more likely can enhance their understanding of descriptive statistics. Using TRU Math rubric, important aspects of Modeling Approach classroom practice were observed to investigate the mechanism on how it is possible to enhance students' understanding of descriptive statistics using Modeling Approach. Students did MEAs which arose from relevant socio-cultural problems and through solving the problems they developed models which are local conceptual developments; since problem solving abilities and concept development evolved at the same time during MEAs (Lesh & Doerr, 2003).

Chapter 4 presented and discussed the findings of the study using students' achievement tests on procedural and conceptual understanding of descriptive statistics comparing students' score who had learned by Modeling and Non-modeling Approaches. It also presented and discussed the findings of the study on students' attitude survey. Finally, chapter 4 presented and discussed on the findings of the study on students' project on the phases of the projects and on six projects cases to explain how students could enhance their critical understanding of descriptive statistics.

The findings from this chapter give a spectrum of students' understanding to explain the mechanism how Modeling Approach enhances students' understanding. The findings underscore Modeling Approach enhances students understanding of descriptive statistics using non-routine problem solving tasks and representation systems. The research questions in relation to the findings are briefly summarized in the next sub sections.

5.3.1 Research Question 1.

How do students enhance their understanding of descriptive statistics using Modeling Approach?

The central findings of this study revealed that Modeling Approach were likely to enhance their understanding of descriptive statistics in statistics education at holistic level. Understanding statistical information from media article was one aspect of enhancing students' statistical literacy. Students were asked interpreting graphic displays in media article and presented reports using statistics in MEAs. Students were also developed their vocabularies and statistical terminologies discussing on the definitions of statistical concepts from colloquial intuitive meaning from informal to formal as part of statistical literacy. Students were able to read and understand the big ideas of distribution expressing the concept of center and variability intuitively and by using concepts for describing and comparing data distribution.

In the Modeling Approach, students were making interpretation and justification for the choice of models in MEAs and were using representation systems which enhanced their statistical reasoning. MEAs were not problems with exact answer; and thus students had different solutions which opened the door for students to convince others the viability of their solutions. Statistics using Modeling Approach is not only about computing, but also making interpretation and description of meaningful situations using statistics. Shuttling between the context and statistical sphere with justifying why they used a particular statistical concept, students learned how concepts in descriptive statistics were connected using representation systems.

Students worked in a team of students to enhance their statistical thinking through data modeling. The findings revealed that students through teacher scaffolding could undergo modeling cycle and develop habit of mind to think statistically selecting models and variables by their own in MEAs that had multi-variables and complex data sets. The finding showed that students found MEAs challenging in terms of their cognitive thinking level from simple exercise and word problems, but they were interested to engage in data modeling processes. More importantly, they were able to draw conclusion based on data in a context; even if they were dealt with descriptive statistics without making formal inferences.

The other core finding was students more likely could enhance intertwined strands of understanding which includes procedural fluency, conceptual understanding, adaptive reasoning, and productive dispositions of descriptive statistics using Modeling Approach. The Modeling Approach was teaching through problem solving that used MEAs. The students solving MEAs through data modeling could develop their strategic competence. In Modeling Approach, problem solving and concept development co-evolve together; that is, when students solve MEAs, they developed models as a local conceptual development (Lesh & Doerr, 2003). Doing follow up activities working on MEA extension problems and representation systems, students could enhance their conceptual understanding making connections within or between concept systems by developing models iteratively.

Similarly, the other interrelated strands could be developed in feedback loops because in the Modeling Approach the whole seemed to be considered greater than the parts. Using the reality principle, MEAs in this study were designed from the concrete life situations of socio-cultural problems. The reality principle assumed personal meaningfulness, but in this study MEAs were designed from socio-cultural problems with the assumption that the students would find the problems relevant as the problems were 'limiting conditions' of the society and the students were part of their community. It was found that all students were interested in MEAs and see statistics as sensible to solve problems in their everyday life; and since students could solve the MEAs they also likely developed confidence in dealing with statistical situations.

Furthermore, students in a team of students were writing and presenting reports which could reduce their anxiety towards statistics. Representing reports in news and letters created fun in statistics class which also helped them to have more interest in the statistics they learned. Hence, Modeling Approach may enhance productive dispositions and which in turn could help students to preserve and engage in problem solving tasks to enhance other understanding strands.

The students could enhance their representational fluency working in the Modeling Approach, because they were working in MEAs and follow up activities by externalizing their thinking by translating one form of representation to another representation system. In the age of information, from the traditional 3Rs skills which are 'Reading, wRitings and aRithmetic' there is a shift to 4Rs which includes Representational fluency, (Scientific) Reasoning, Reflection and Responsibility that students need to develop (Lesh, 2000). Students who had learnt using Modeling Approach promisingly may narrow the gap between the understanding they developed in the school and the understanding they need for life beyond school in the age of information, because in the Modeling Approach one of aims was to develop abilities and understanding needed beyond school such as working in a team of specialists, having representational fluency, and designing projects in collaborations.

An important finding of this study was students more likely get equity education using Modeling Approach. The first finding in relation to equity is in terms of engagement and access to get statistical knowledge for students. In Modeling Approach, students were found to be engaged. If educators are able to give relevant non-routine tasks for students, they would less likely to be bored and develop negative dispositions towards statistics. Students had got more motivation and engagement, since the problems arose from the concrete real life situations of the students.

The high level cognitive demand of MEAs also helped advance students be engaged and more focused from getting disinterested practicing only simple exercise and word problems. Low achievers could also produced complex models in MEAs that enhance their understanding, because using the Modeling Approach they more likely learned statistics meaningfully and with sense making since their knowledge could organized around experience at least as much as abstractions which could students access in multi-disciplinary areas (Lesh, 2012).

Working in a team, students could develop their sense of social agency by giving awareness program by solving MEAs. Students could develop systemic and sympathetic understanding on how to live together with 'survival with dignity' and help other people and themselves for the betterment of human life in the community working on MEAs.

5.3.2 Research Question 2

How do students enhance their critical understanding of descriptive statistics through data modeling working on project works?

The other major finding of the study was the finding on students' project that teams of students were able to draw conclusion based on data informally, despite they worked on basic descriptive statistics. The findings showed that students could develop their abilities on statistical investigative process and they could also develop their critical understanding working on projects. The majority of teams of students' in Modeling and Non-modeling group could undergo the investigative processes in the students' project selecting their own project themes.

Teams of students in both groups found different phases of the project difficult for them; and teachers need to scaffold students to complete the project phases successfully. It was also found that team of students in the Modeling group had multiple model choices than the Non-modeling group which may suggest that student working in representation systems could develop their representational fluency better; and this in turn, could help them to go easily through the investigative processes as they get habit of mind in data modeling in MEAs.

Students' more likely could enhance their critical understanding of descriptive statistics doing projects using statistics as a critical tool. Team of students could pose problems and give an awareness program that could solve limiting conditions of the socio-cultural problems inside school or beyond school. The findings showed that students at their youth age could develop a sense of social agency to transform the society for the betterment of human life in our world.

5.3.3. Research Question 3

To what extent do students enhance their understanding of descriptive statistics using Modeling Approach?

The core finding of this study was students who learned using Modeling Approach had enhanced their conceptual understanding with medium effect size than the Non-modeling students. This may be for the reason that students learned statistics by making connections between concepts and within concepts using Modeling Approach. The second reason could be students were using statistical concepts with interpretation that could help them to enhance their conceptual understanding. The third reason might be as the other intertwined strands develop, their conceptual understanding also develops. The students could have enhanced their conceptual understand along with the other strands of understanding statistics; because the strands of understanding co-evolve together one strand affecting the other.

The findings could not show whether Modeling Approach enhance students' procedural understanding of descriptive statistics using achievement test. Except female students, there is no significant difference between students on achievement tests on procedural understanding of descriptive statistics for comparisons between across groups of students in Modeling and Non-modeling groups with respect to achievement levels, gender or the instructional approach used.

The finding of the achievement tests seems to suggest that female students enhance both procedural and conceptual understanding using Modeling Approach. The Modeling Approach emphasized sense making and learning the concepts of statistics relating it with their everyday life. Research had showed that girls prefer more to learn with knowing why than simply accepting procedures without sense making and making connection than boys who could learn mathematics with only following procedures (Boaler, 2006). There is a myth that statistics or mathematics is not for girls. But, research had showed that girls can do better if we change the way we teach students, because Modeling Approach was a cooperative learning as others got similar results (Boaler, 2002; Fennema & Leder, 1990). The finding of this Study showed that female students in Modeling scored higher in both procedural and conceptual understanding than Non-modeling female students and there was a significance difference in achievement tests on both posttests.

The other findings of this chapter was, students who had learned using Modeling Approach outperformed students in Non-modeling students on achievement test of conceptual understanding in both sexes and at different achievement levels. But, the research found that there was no significant difference on both posttests between male and female students who had learned using Modeling Approach which seems to show modeling promotes equity with respect to gender. There was significant difference on procedural and conceptual understanding achievement tests among students within modeling groups by comparing students based on achievement levels. The study found that in the post hoc analysis high achiever students outperform in both procedural and conceptual understanding achievement tests. But, there was no significance difference between low and medium achiever on procedural and conceptual understanding within the Modeling group.

It seemed that high achiever got cognitively challenging tasks to enhance their procedural and conceptual understanding, because as they solve relevant non-routine problem solving tasks they would enhance their cognitive thinking which may enhance other aspects of understanding. Research had showed that better problem solvers would develop rich connections of concepts and co-ordination of procedures and vice versa (Lesh, Hamilton, & Kaput, 2007). Low achievers likely get meaningful learning working with real life problems to develop their productive dispositions to preserve and engage in multi-representation systems to cope with the medium achievers.

5.4 General Discussion on the overall Findings

The issue that this study addressed was the need for secondary students to work on relevant non-routine problem solving tasks to enhance their understanding of descriptive statistics. Problem solving using modeling made significant progress towards the goal of enhancing students' understanding in Mathematics Education (Doerr & Lesh, 2003; Sriraman & English, 2010). However, in school mathematics curriculum, instruction and assessment modeling problems were barely incorporated (Kaiser, 2010; Lesh, 2012). An ongoing debate in mathematics education for several decades pertains to weather students should learn basics concepts or students should learn using problems solving (Chamberlin, 2008, 2010; Schoenfeld, 2004).

Teaching methods that involve problem solving can be categorized into three types: teaching for problem solving, teaching about problem solving, and teaching through problem solving (Bostic, 2012; Cai, 2003; Schroeder, Thomas, & Lester, 1989). The first approach, teaching for problem solving, refers to teaching the application of some concepts that students have already learned. The usual practice involving routine word problems, which Hamilton (2007) points to as the "concept-then-word problem" approach (p. 4), engages students in a single or two step process of mapping problem information onto simple operations. In most cases, the problem information has already been cautiously mathematized for the students. Their goal is to unmask the statistics by mapping the problem information in such a way that students produce an answer using familiar quantities and basic operations. If the majourity of students' classroom statistics experience are of teaching for problem solving, then their ability to solve problems in the real world will be compromised. This may attribute statistics content and process objectives to be separated.

The second approach, teaching about problem solving, deals with the practice of teaching problem-solving heuristics and strategies. However, more than a decade of mathematics education research studies have shown that teaching about problem solving heuristics and strategies couldn't help students to develop their cognitive thinking in mathematics (Lester & Kehle, 2003; Schoenfeld, 1992). According to English and Sriraman (2010), teaching about heuristics like Polya's (1945) problem solving steps doesn't work, because they do not help students where, when, why and how to use heuristics and strategies while they are solving problems. This approach also may disregard the big ideas of statistics, since the focus is how to go about the processes rather than the contents or the concepts.

The third approach, teaching through problem solving, typically starts with relevant non-routine problem-solving tasks like MEAs. This approach considers problem solving as integral to the development of any given statistical contents and process (Doerr & Lesh, 2003; English, 2013a). The Modeling Approach in this study used MEAs for teaching statistical concepts and processes through problem solving. The position that this study takes on is the need for students to learn statistics using Modeling Approach to enhance their understanding of descriptive statistics.

There are a number of reasons for purposing this approach as a better option for enhancing students' understanding of descriptive statistics. First, modeling problems like MEAs are cognitively demanding tasks that could help to enhance students' statistical reasoning and thinking. Although it is agreeable that students need to solve problems, the level of cognition needed to solve the problems will depend on nature of the tasks. Simple exercises and word problems may not be challenging enough to meet cognitive process objectives; that is, they just need recalling simple facts and following few procedures. If students learn statistics only with exercises and word problems that can be solved automatically, then students may not enhance statistical reasoning and thinking that needs cognitive processes. The findings of this study suggest MEAs enhances their understanding going through data modeling processes.

Second, teaching statistics using Modeling Approach could help students to see the usefulness of statistics that could help them to engage in statistics tasks with sense making to enhance their understanding of descriptive statistics. Though teaching abstract statistical concepts is common practice, study showed that students need to learn from concrete to abstract, from informal to formal (Garfield et.al, 2007). Teaching statistics formally may not motivate students to develop students' productive dispositions, since the concepts may not give them any sense. There is an intertwined relationship among productive dispositions and other understanding strands. Studies showed that affect had significant effect to enhance cognitive thinking (Roth, 2007). The findings of the study suggest that MEAs were relevant tasks that develop students' productive dispositions, because the problems deal with concrete problematic situations of students' everyday life and thus students could easily see statistics is useful to solve problems in everyday life.

Third, it is morally important to teach statistics using Modeling Approach to enhance students' critical understanding of descriptive statistics. It is good that people live in the world with sympathetic understanding, that is, 'survival with dignity'. The youth can be misinformed at the age of information if they did not discern the good from the bad. Even though the world is not a safe place to live for everyone in the world nowadays, it is valuable to work against dehumanization using data based evidences. Statistics may not be considered simply as useful, but also as a tool to help solve social problem. Students need to learn how to live together working in a team using statistics as a critical tool by solving real social modeling problems in class and through project works. The findings of this study suggest students could enhance their critical understanding using Modeling Approach, since it offers a chance to work on real social problems to transform the life of disadvantaged society.

Fourth, modeling problems could offer integrated knowledge to enhance students' understanding of descriptive statistics. The nature of problems beyond school demands educators to give students interdisciplinary problems. Statistics is about making (and making sense of) patterns and regularities in complex systems at least as much as it is about calculations with pieces of data (Lesh, 2000; Lesh & Doerr, 2003). Modeling Approach draws from many disciplines to help students to think ‘out of subject box’. Statistics is at the intersection of many disciplines to serve students’ need to work on real data and on multiple variables in complex systems. Students could enhance their understanding of concepts organically, because real life problems need to draw integrated knowledge from multi-disciplinary areas for the reason that the whole is greater than the parts. Modeling Approach would enhance students’ understanding with rich connections, since an MEA could draw from multi-disciplinary areas during data modeling.

Fifth, Modeling Approach could serve to offer equity Statistics Education for all students regardless of their gender and achievement levels to enhance their understanding of descriptive statistics. According to Chamberlin (2010), advance students need to work on non-routine tasks that could challenge them cognitively, otherwise they may get bored and disinterested with routine task that need only a little cognitive thinking. MEAs serve as assessment tool to elicit students intuitive and prior knowledge, thus students at any achievement level can get a chance to build the abstract concepts on their informal concepts. Study showed girls could enhance their understanding if they learn meaningfully with sense making (Boaler, 2006). The findings of this study suggests, girls could enhance their understanding using Modeling Approach, since girls may enhance their understanding making connection from informal to formal meaningfully.

Defenders claim that if teachers teach statistics using Modeling Approach, students may not get the basic building blocks of knowledge. They assume that students need to learn first the basics, then apply what they learned later on in another statistics lessons or in application problems. Of course students need to learn a healthy dose of basic concepts and a focus only on problem solving process without skills will deter their procedural fluency; however it is possible to use Modeling Approach to enhance basic skills as well as the cognitive thinking and reasoning at the same time, since Modeling Approach enhance students' basic knowledge while they solve modeling problems (Lesh, 2012). A decade of mathematics education research showed that students could enhance high cognitive thinking on 'rich tasks' without compromising the basic skills and knowledge (Schoenfeld, 2004, 2007). The findings of the study suggest that it is possible to enhance students' conceptual understanding without affecting their procedural understanding.

Defenders also may claim that modeling problems are challenging that students could not attempt to solve the problems. Neither a teacher wants to teach statistics by giving students challenging problems which frustrate them to attempt, nor does he/she want to teach students by spoon feeding. However, teachers need to strike a balance to teach students with some sense of challenges so that students get a chance to enhance their cognitive thinking. Even a child from the early age naturally requires some challenge; the challenge for six month baby may be walking.

Teachers need not avoid modeling problems in the pretext of students are young that they do not need to work on modeling problems for the reason that they are challenging. But, learning theory supports that students learn well through perturbations, since human minds disallow the disequilibrium created by the challenge. Through scaffolding, students could solve the modeling problems through iterative cycles of assimilation and accommodation. The findings of the study suggest, students could undergo data modeling processes; even if, they were not experts at modeling. Social interactions in teams of students were the cause of accommodations as observed in MEAs lessons.

Many defenders raises practical issues by claiming that it is not possible to teach statistics using Modeling Approach due to time constraints and content coverage in the school programs. Of course, some problem-driven curriculum may take longer period of time than the allotted time in schools (Chamberlin & Coxbill, 2008). However, as this study among others showed, MEAs did not take longer more than one or two periods that time constraints may not be a problem (Chamberlin & Coxbill, 2008, Lesh, 2012). Students need to develop both content and process objectives for learning descriptive statistics. Most importantly, the best possible ranges of problem types that involve routine and non-routine problems need to be used. Students could learn on big ideas of descriptive statistics using MEAs based on few design principles as this study suggests. Further, students could benefited more if they do open-ended project to experience statistical investigative processes that could save enough time to meet the process objectives as this study findings suggested.

Other defenders simply leave the matter of using non-routine problem solving tasks like MEAs for teachers who teach statistics in schools. Even though the teacher is the ultimate decision maker in the classroom, it is not an honest recommendation since using non-routine needs a coordinated effort from all stakeholders. First the teacher may have little time to design MEAs, because MEAs requires selecting topical themes and need to follow design principles to construct them. Second, using Modeling Approach the teacher needs to use diagnostic teaching method to attend students' thinking (Schoenfeld, 2010).

It is not just the lesson goes according the plan way of teaching is used for Modeling Approach. But, among other studies, the study showed MEAs primary purpose is to elicit students thinking by externalizing their thinking using external representation systems or models (Doerr & English, 2003). Thus, students' models serve as windows so that teacher made interpretation on their models to build the instruction (Lesh & Fennewald, 2010). In the Modeling Approach, teaching is not only doing; it also involves observation and interpretation and making inference based on cognitive tasks as this study suggests. Hence, to use modeling problems in an education system needs serious attention beyond leaving the matter simply to teachers on what they liked to do with non-routine problems.

Finally, many defenders of the modeling practice claim that it is simply a tradition to teach statistics using a formal approach and teachers ought to continue with the usual practice. However, following traditions may have negative side to enhance the kinds of understanding students' need in the 21st century in an age of information. Statistics being in mathematics curriculum may be taught in traditional formal approach, despite statistics as discipline originates to solve practical problems.

In fact, to reveal a particular practice as having a status of a tradition couldn't shed light on whether it is a good one or a bad one. Statistics is at the intersection of many disciplines that stakeholders need to take part for enhancing students' understanding for the benefit of all not just for mathematicians or mathematics educator. If Modeling Approach enhances students' understanding of descriptive statistics within the interest of all stakeholders, then it needs to be considered seriously to incorporate modeling problems in the statistics curriculum, instruction and assessment in an education system.

5.5 Conclusion

Students worked on relevant non-routine problem solving tasks called MEAs and representation systems to lay the basic foundations for important learning outcome in statistics education such as statistical literacy, reasoning and thinking. In an age of information, students at their youth age need to solve relevant non-routine problem solving tasks to enhance their critical understanding which could enable them to cope up with the nature of problems in daily life. Students could get a sense of social agency using statistics as a critical tool solving socio-cultural problems using data modeling.

The study revealed that students who learned using Modeling approach could enhance their understanding of descriptive statistics. Students' could elicit their models by externalizing thinking through representation systems working on MEAs and develop their models by testing, revising, and refining iteratively with follow up activities. The findings of this study are reminders that Modeling Approach more likely enhances students' understanding of descriptive statistics with similar school settings, but needs further efficacy study at different school settings and populations; it also needs large scale effectiveness study to meet the interests of stakeholders in an education system.

5.6 Recommendations

Implication for Practice

Curriculum and instruction draws educational aim from educational policy. Problem-driven curriculum and instruction would draw aim from problem solving policy. The contemporary educational policy of Ethiopia is rooted in the 1994 *Education and Training Policy* whose overarching aim was to enable students capable of solving their own and societal problems similar with other countries' educational policy (TGE, 1994).

Problem solving is one of the pillars of educational policy that the goals of mathematics education curriculum could be derived. But, neither mathematics teacher educators (mathematicians), nor a teacher could set up the objectives alone to design problem-driven curriculum for statistics teaching. For the mathematics educators, the objectives of the curriculum could be to prepare students for the next mathematics or statistics with formalistic approach without using any modeling problems from the interdisciplinary subject areas. This may create conflict of interest in educational stakeholders, since at least all students will not become mathematics educators or mathematicians.

Moreover, statistics lies at the intersection of several disciplines in STEM education and in social science education to deal with data modeling. Thus, the best possible ranges of problem solving tasks need to be used to serve the interest of stakeholders. Problem-driven curriculum using Modeling Approach may be the better option to enhance students understanding of descriptive statistics under the umbrella of problem solving policy as purposed by the researcher as shown in Figure 33. Using Modeling Approach students would need to solve both non-routine and routine tasks using representation systems. Many mathematics educators leave the matter of preparing high cognitive tasks to the teachers alone; however, teachers may have little training in statistics teaching and they may often rely on textbooks which has simple exercise and words problems which could not develop students' cognitive thinking (Asgedom, 2009; Micheal & O'Connell, 2014).

The researcher prepared the themes of MEAs selecting topical issues from the society. To set non routine interdisciplinary problems within the interest of all stakeholders, key stakeholders such as teachers, students, STEM educators, and social-agents should co-ordinate being evolving experts. The study was conducted in non-technological environment, but technology will further strength the use of Modeling Approach making the representation system dynamic. Statistics Educators and researchers will play a major role informing what big ideas and cognitive process need to include in the curriculum. Social-agents also identify investigative themes that could create sense of social agency solving social problems. Modeling Approach curriculum drives its epistemology of constructing, testing and revising models from engineering as they are heavy users of mathematics and the nature of problems beyond school are of these types (Lesh, 2012).

After the problem-driven curriculum is set, then it is operationally implemented in the Modeling Instructional Approach. The main elements in interaction in Modeling Instructional Approach are the modeling tasks, students and the teacher. The tasks involve MEAs which are non-routine problem solving tasks. The study suggests that students find them relevant. If the tasks are not relevant, the teacher and other stakeholders need to find sensitive and timely issues going back to design the curriculum. Thus, the curriculum and the instruction would inform each other back and forth. Problem-driven curriculum using Modeling Approach would give a framework to see what goals drive every action in the instruction.

The third aspect of the model, which is crucial together with curriculum and instruction to enhance students' understanding of descriptive statistics in Modeling Approach, is assessment. As this study suggest, teaching is not only doing. But, it involves making observation and interpretation. Teachers make assessment of Modeling Approach using TRU Math rubric, which has five dimensions: Statistics, Cognitive Demand, Access,

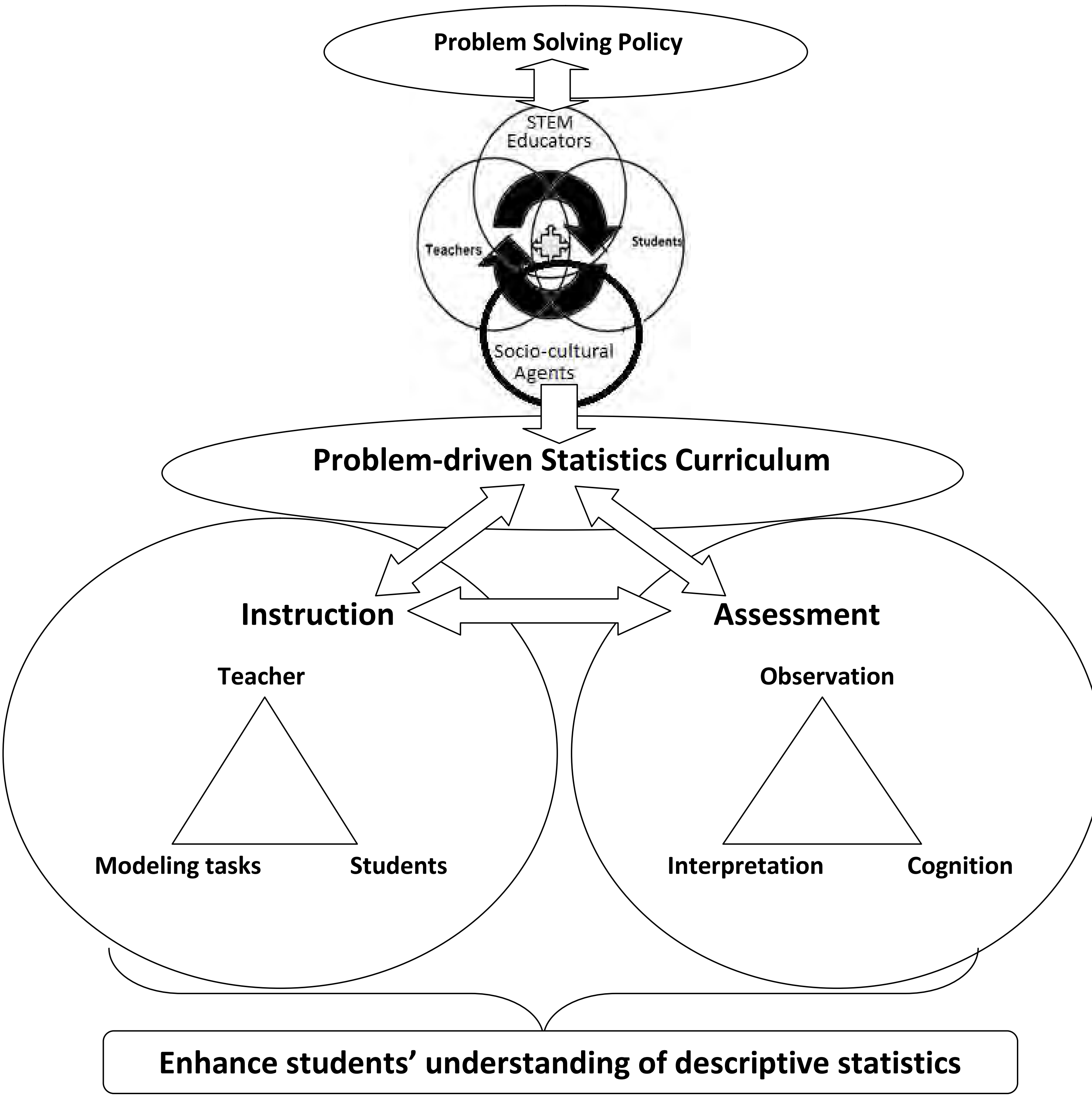


Figure 33: Researcher's Purposed Model of the Modeling Approach

Agency, identity and authority, and Assessment (Schoenfeld, 2013). These dimensions give accessible language for teachers and researcher to observe the implementation of modeling classroom practice to enhance students' understanding of descriptive statistics.

Then, teacher also makes interpretation of students' models on MEAs. Statistics involves not only doing analysis on data sets, but also it involves interpreting realistic complex situations (Lesh, 2000). Finally, teachers could assess students' individual cognition using assessment tool that asks statistical reasoning. Project work could also be used as an open-ended assessment tool to see students' thinking. The study triangulates observation, interpretation and cognition assessments to explain how Modeling Approach enhances students understanding of descriptive statistics.

The finding of this study suggests that if there is a synergy of curriculum, instruction and assessment using Modeling Approach in an education system, students likely could enhance their understanding of descriptive statistics. This is preliminary research which could inform problem solving policy in secondary mathematics curriculum, but further research is needed to inform the policy better as discussed below.

Implication for Research

The best possible levels of tasks will be required to promote achievement and learning equity. Students need not work only on simple exercise and word problems. Students need to work on relevant non-routine problem solving tasks; not only to develop their cognitive thinking, but also to solve problems in socio-cultural problems. To what extent non-routine tasks and routine tasks like low level exercises and word problems should be used needs further research across different topic areas, populations and settings.

The study was conducted in a statistic unit in mathematics curriculum on multidisciplinary areas in few specific subjects like Geography, environmental and sport education using Modeling Approach. There is a need to widen the disciplinary boundaries in STEM education. Though, modeling getting recognition in STEM education, there is barley research on critical modeling that includes aspect of sympathetic understanding. Ethical issues ought to be an important aspect of STEM education. Thus, further study is needed on critical understanding aligning it with STEM educational objectives.

Statistics is part of mathematics curriculum in secondary school in many countries. However, as statistics is more useful for the youth, there is a need to use it in an age of information as a critical tool in several disciplines. Further studies are required on how statistics could be taught across curriculums and enhance disciplinary knowledge domains. The finding showed that statistics can contribute a lot for critical mathematics education in showing direction how to use mathematics as a critical tool, which needs further research.

This research was at alpha level that involved design development and efficacy study. There is a need to do longitudinal study to see students' retention on their understanding using Modeling Approach. In addition to, there is a need for further efficacy study with different school setting in rural and urban. There is a need for further scale up and effectiveness study on using Modeling Approach to enhance students' understanding.

Suggestions for Future Research

One component of Modeling Approach was working on representation systems. Working on technological software like tinker plots or computer program will make working on representation systems more dynamic and easy to work on representational system for data transnumeration (Shaughnessy, 2007). There may be a need for empirical research to see the tension how critical understanding could be enhanced under this learning environment. This study is conducted in under privileged school setting; there is a need to do further research for other school setting. Critical understanding is not only important for secondary students for developing nations at the onset of their youth in the age of information, it may be more important for developed nation too. The developed nation had flux of information, which the youth can't discern good or bad. Hence, there seems to be research needed on how to develop sympathetic understanding for developed nation for 'survival with dignity' using statistics as a critical tool.

This study is conducted on students' understanding of descriptive statistics. A future study is needed on professional development setting at classroom, school, district and regional levels on how to enhance students' understanding using Modeling Approach. The professional development could be on how to prepare the MEAs and assessment tools. More importantly, the professional development based on TRU Math Rubric on Modeling Approach need further research in order to improve the Modeling Approach using Schoenfeld, et.al (2014) observation scheme and rubric, because "it provides a straight forward and accessible language for discussing what happens and (should happen) in classrooms" (Schoenfeld, 2015).

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Appendix A: MODEL-ELICITING ACTIVITIES

Safe Water Model-Eliciting Activity (MEA-1)

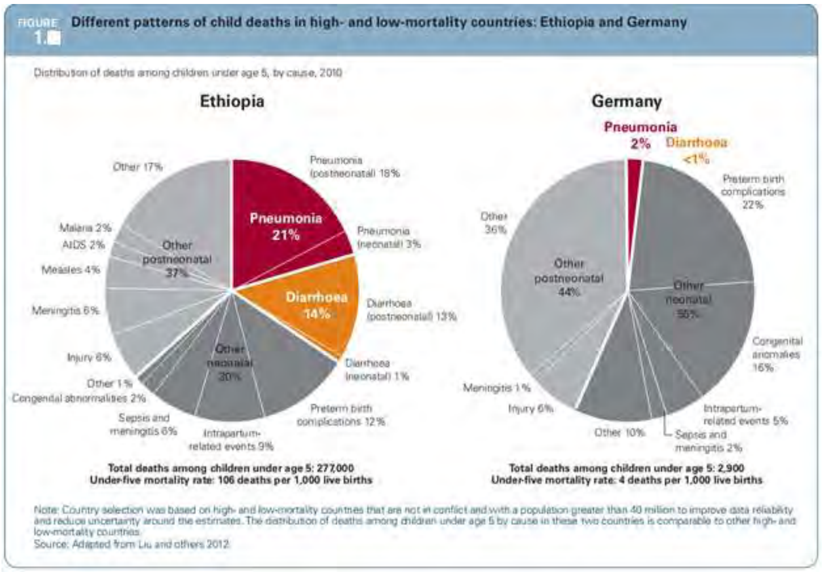
PART I: READING HOMEWORK

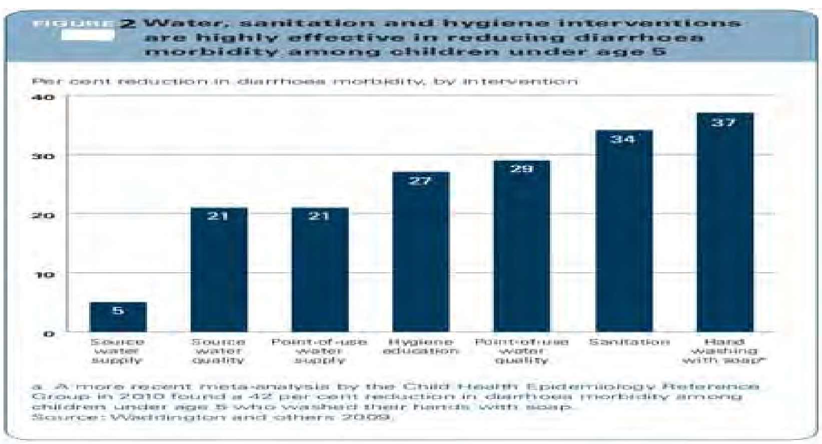
ETHIOPIA: Safe water - a glass half full

ADDIS ABABA, 27 June 2012 (IRIN) - More than half of all Ethiopians have access to an improved source of drinking water, but the country still has much work to do if it hopes to achieve its goal of providing access to safe water and sanitation for its 83 million people by 2015, experts say. "Despite an increase in coverage, the number of people that require access to sanitation and hygiene, for instance, are still the highest in Africa, if not the world," said Kebede Faris, water and sanitation expert for the World Bank's Ethiopia office. "As a result, a significant number of Ethiopians are still facing WASH [water, sanitation and hygiene]-related health problems and also losing their lives."

A recently released study by the UN Children's Fund (UNICEF) shows that some 271,000 Ethiopian children under the age of five died in 2010 alone, with pneumonia and diarrhoea causing more than one-third of those deaths. The authors said "basic steps" such as hand-washing with soap, expanding access to safe drinking water and sanitation, along with providing other medical services, could have saved their lives. The problem remains: "An overwhelming majority, nine households in every 10, does not treat their drinking water," leaving them susceptible to various health problems. Figure 1 below shows the different patterns of child deaths in high and low-mortality countries: Ethiopia and Germany. Ethiopia is seeking to meet the UN Millennium Development Goals' targets on water, sanitation and hygiene through its Universal Access Plan II, which seeks to provide 98.5 percent of the population with access to safe water along with 100 percent access to sanitation by 2015.

Nationally, the proportion of Ethiopian households with access to an improved source of drinking water - categorized as a public tap or stand pipe, borehole, a protected well, spring water and rainwater - has reached 54 percent, according to the *Demographic Health Survey 2011*. As shown in the Figure 2 below, at the international level, it is possible to reduce the death of children under age 5 using intervention programs on water, sanitation and hygiene. However, there are significant disparities between urban households, where 95 percent of people have access to an improved source of drinking water, and rural areas where just 42 percent access safe drinking water.



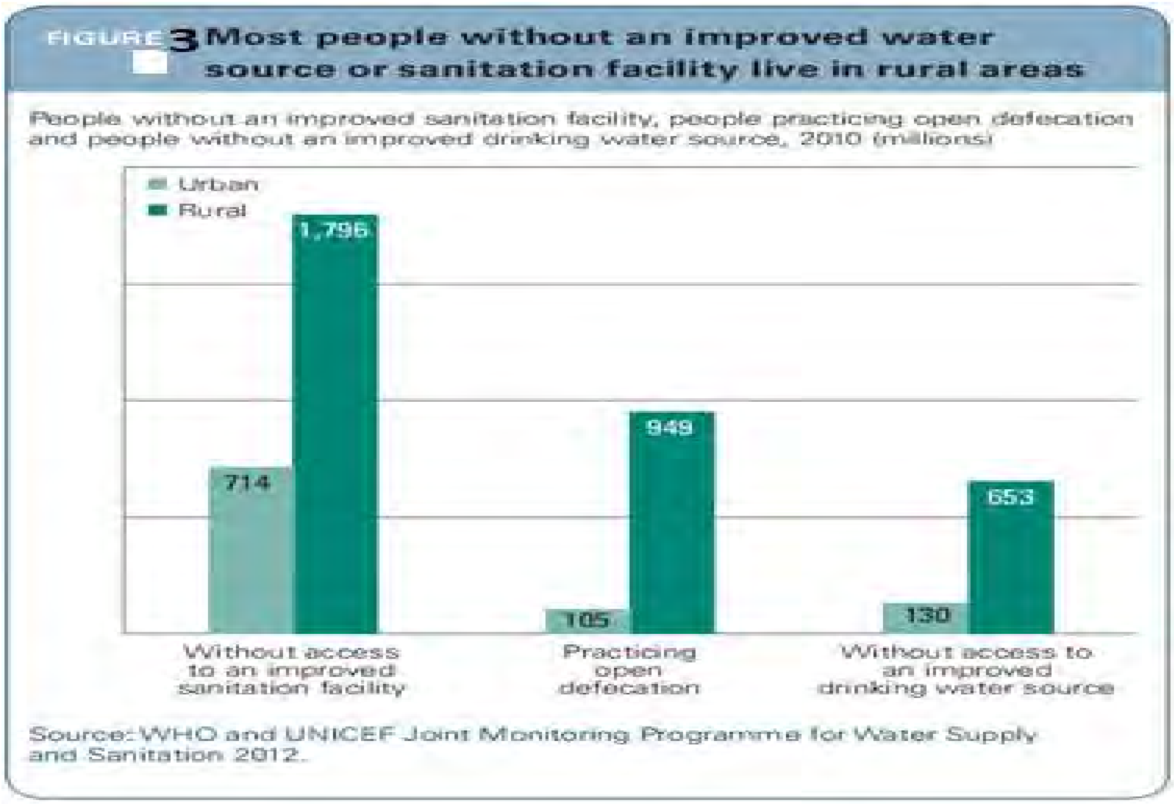


Rural-urban divide

Like access to water services, government data shows a wide gap in access to sanitation between urban and rural households. But, this is also a global trend as shown in figure 3 for the disparity between urban and rural on water source and sanitation facilities. According to the country's Growth and Transformation Plan 2010, the national coverage of sanitation stands at 60 percent, with rural coverage at 56 percent, compared to 88 percent for urban households.

"Building latrines is not enough. A systematic approach that focuses on quality or building to minimum standards, maintenance and use are equally important," said the World Bank's Kebede, adding that there was a need "to invest now to save more future lives and impairments of many kinds". Other water-borne diseases are also common as a result of poor water and sanitation. "With more than 65 million people living in the trachoma endemic parts of rural

Ethiopia, we need to reach out to more people and fast, with proper sanitation and hygiene practices, as most of the cases are happening in areas where water supply and sanitary conditions are poor," said Menebere Alemu, country representative for NGO International Trachoma Initiative.



PART II: READINESS QUESTION

Section A

1) How is water treated in your family? Do you drink tap water, boiled water, spring water or rain water?

2) How many glasses of water do you drink per day in average? Explain

3) Have you ever seen or heard about how Ethiopian rural people fetch and treat drinking water?

4) What do you think children under the age of 5 are much more vulnerable for water born diseases?

Section B (Use figure 1, 2 and 3 to answer the questions below)

1) What is the number of children death under the age of 5 in post neonatal pneumonia and diarrhea in Ethiopia?

2) In which distribution of deaths in the pie charts among children under age 5, Ethiopia and Germany have the same percentage?

3) Which intervention has the highest percentage of cause of death in diarrhea?

4) Which intervention has the same percentage of death reduction in diarrhea?

5) In which particular case is the urban showing a big difference than the rural? Explain.

PART III: PROBLEM STATEMENT

Team work 6.2

Safe Water Mode-eliciting Activity 1

Problem Statement

Helen and Alemu are active members of Environment Protection and Sanitation Club in the first Ethiopian established school, Menlik II. During the vacation, the club members go to the nearby rural area to give an awareness program on health and sanitation. Helen and Alemu are assigned together to give an awareness program on water sanitation and hygiene for 16 families households. They collect the following demography and background data on the 16 families as shown in table 1 below. They wanted to present and summarize the data for the ten families using different graphs and to create an awareness program on one of the weekends. Helen and Alemu badly need help on how to present the data to create awareness on water treatment and they asked the senior club members, who are experienced in presenting set of data, to write a good presentation report with posters for the 16 families' households at the rural district. Now, as the senior experienced club members in your team of 3 or 4, write the report and prepare the posters to give for Alemu and Helen.

PART IV: DATA TABLE

Demography and background information for 20 families' households

Name of family representative	Age of the family Representative	No. of Children under the age of 5	No. of family ≥ the age of 5	Income in thousand birr	Area of leaving compound	Source of drinking water	Practicing open defecation	No. of family ≥ the age of 5 washing hands
Melaku	36	3	4	22	250	PW	No	3
Tewdros	31	2	4	15	550	Rain	Yes	2
Abebe	44	0	5	40	650	Spring	No	1
Worku	38	1	5	20	250	PW	Yes	0
Ayele	34	2	3	31	360	Borehole	Yes	2
Abebe	35	2	3	25	250	PW	No	1
Kebede	35	3	2	22	270	Borehole	Yes	2
Sewnet	39	2	5	14	360	PW	No	0
Habte	43	1	5	13	250	Borehole	Yes	0
Kider	38	2	2	90	250	Rain	No	1
Worku	37	2	3	17	280	PW	No	0
Bikila	45	2	6	13	300	Borehole	Yes	0
Adem	37	1	4	24	245	Borehole	No	0
Nuru	46	1	5	15	500	PW	No	0
Bahru	49	1	4	16	350	Borehole	Yes	1
Belay	38	4	3	22	520	PW	Yes	1
Berhan	34	0	2	45	420	Spring	No	2
Mulugeta		5	4	32	340	Pw	Yes	0
Chirenet	27	2	3	44	250	Spring	No	1
Seyome	35	4	3	40	250	Borehole	No	1

Millennium DAM Model-Eliciting Activity (MEA-2)

PART I: READING HOMEWORK

Dams and hydropower in Ethiopia

Ethiopia considers itself the powerhouse of Africa due to its high hydropower potential. Only a fraction of this potential has been harnessed so far. In 2009 less than 10% of Ethiopians had access to electricity and the country was plagued by power outages. In order to overcome this situation, the government has embarked on an ambitious dam building program. Three hydropower plants with a combined capacity of 1.18 GW were commissioned in 2009 and 2010 alone, more than doubling the previous installed capacity of the country. The largest hydroelectric plant in Ethiopia, Beles, began initial operation in May 2010. Contracts for five more large dams have been signed. Once completed, which is expected to be around 2015, these dams would increase the installed capacity by more than 11 GW from less than 1 GW in 2008. The construction of more large dams is foreseen in a Master Plan that aims to bring capacity to 15 GW. While this increase is enormous, it is barely equivalent to the capacity of all power plants in Portugal. Power demand in Ethiopia is constrained by poverty, and the country thus plans to export power to Sudan, Kenya, Djibouti and even Yemen or Egypt. The benefits of the dams are not only limited to hydropower. Many dams are multi-purpose dams that are also designed to provide water for irrigation and flood control. However, hydropower is expected to be the main benefit of the dams.

The construction of large dams entails many tangible and intangible costs. The financial cost itself is already substantial. Resettlement adds to the social costs of the dams. Sedimentation from unchecked erosion in the upper watershed of rivers reduces the lifespan of reservoirs.

Environmental costs are imposed on communities living downstream of the dams in Ethiopia. And neighboring countries, in particularly Egypt, see their historical water rights affected and threaten to take action against the dams. The almost exclusive reliance on hydropower makes electricity generation vulnerable to droughts, which may be exacerbated by climate change. Earthquakes can also endanger the dams and associated tunnels. There were a total of 16 recorded earthquakes of magnitude 6.5 and higher in Ethiopia's seismic active areas in the 20th century.

Name	Installed capacity	Commissioning	Basin	Contractor	Financing	Cost	Remarks
Fincha	134 MW	1973	Fincha	NA	NA	NA	also for irrigation
Gilgel Gibe I	180 MW	2004	Gilgel Gibe	Salini	World Bank	$331m	Gilgel Gibe I
Tekezé	300 MW	2009	Tekeze (Atbara	Sinohydro Corporation	Chinese	$365m	producing 300 MW
Beles	460 MW	2010	Lake Tana	Salini	Gov.	NA	Irrigation of 140,000 ha
Gilgel Gibe II	420 MW	2010	Omo River	Salini	Italy and EIB	Euro 370m	Gilgel Gibe II
Gilgel Gibe III	1,870 MW	2012-13	Omo River	Salini	Italy	Euro 1.55bn	faces stiff environmental criticism but work in progress
Fincha Amerti Nesse (FAN)	100 MW	Planned	Fincha (Blue Nile)	China Gezhouba Group Co.	Exim Bank of China	$276m	Fincha Amerti Nesse (FAN)
Halele Worabese	440 MW	2014	Omo River	Sinohydro Corporation	Fair Fund?	Euro 470m	Halele Worabese
Gilgel Gibe IV	2,000 MW	2014	Omo River	Sinohydro Corporation	Chinese	$1.9bn	Gilgel Gibe IV
Chemoga Yeda	278 MW	2013	Blue Nile	Sinohydro Corporation	Chinese	$555m	Will consist of 5 interconnected dams
Genale Dawa III	256 MW	awarded in 2009	Oro	Chinese CGGC	Chinese	$408m	Feasibility study by Lahmeyer
Grand Ethiopian Renaissance	5,250 MW	2014	Blue Nile River	Salini	Gov.	Euro 4.8 billion	Awarded without competitive bid

PART II: READINESS QUESTIONS

1) What is the average capacity of hydropower plants commissioning before 2012?
2) What will be the average capacity of hydropower commissioning in between 2012 to 2015? Compare your answer with the first question.
3) Which contractor has done the most hydropower plants in Ethiopia?
4) Which country or organization supports Ethiopia most in constructing hydropower plants in terms of cost?
5) "Poverty is our number one enemy!" Explain the pros and cons of using hydropower plants in Ethiopia in relation to our poverty?

PART III: STATEMENT OF THE PROBLEM

Millennium Model -eliciting Activity 2

Problem Statement

Indeed, whilst continuing its efforts to secure international cooperation to build the Millennium dam, the Ethiopian governments is ready and determined to complete the project- with or without-foreign grant or loans. The estimated total construction of the dam is around 70 to 80 billion birr. Given the possibility of additional cost, it may appear to be an enormous challenge to mobilize and invest such a huge amount of resource in so short a time- but it is not impossible. Through the collaboration of all our citizens, the private sector, including those who live abroad, it is possible to do even more. With this in mind, the Ethiopian government has availed bonds for sale. The government calls on all Ethiopians to leave their mark on Ethiopia's transformative development by purchasing the government bonds to enable the country and people make full use of their natural resources. The data below describes the contribution made by Karamara secondary High school students in three grade 9 sections to build the Millennium dam which is collected by the school record office. The school director wants to report summary of these data for ERTV news broadcast. In your team, write a report that could be given to the news broadcast center by the director using the given data for the three sections.

PART IV: DATA TABLE

Sec A-Student Name	Sex	Age	Birr	Money Source	Sec B-Student Name	Sex	Age	Birr	Money Source	Sec C-Student Name	Sex	Age	Birr	Money Source
Melat	F	16	33	Pocket	Serawit	M	15	30	Parents	Abel	M	17	30	Pocket
Hana	F	14	21	Parents	Worku	M	16	60	Friends	Fikadu	M	15	50	Parents
Ayne	F	15	44	Pocket	Cherota	M	14	200	Part-time	Helen	F	16	21	Pocket
Kedir	M	16	35	Uncle	Negest	F	15	40	Parents	Gebru	M	17	15	Uncle
Hayat	F	13	40	Pocket	Helen	F	15	35	Grand ma	Bekila	M	15	26	Parents
Fana	F	14	50	Uncle	Melaku	M	16	19	Uncle	Froal	M	18	59	Part time
Yalew	M	14	100	Friend	Shewit	F	14	32	Parents	Tigest	F	19	16	Pocket
Mitiku	M	13	35	Parents	Getenet	M	14	44	Pocket	Mhiret	F	12	24	Parents
Chiru	M	14	49	Parents	Habte	M	15	25	Frinds	Chala	M	14	55	Uncle
Lenssa	F	15	25	Pocket	Girma	M	15	36	Poket	Tigest	F	15	60	Pocket
Mike	M	13	16	Friend	Tayitu	F	15	21	Friends	Kidest	F	14	15	Parents
Sara	F	15	38	Pocket	Gemechu	M	15	48	Parents	Rediet	F	16	14	Brother
Weda	M	13	60	Pocket	Addis	F	15	24	Pocket	Ayle	M	18	24	Grandma
Helen	F	13	17	Parents	Tewdros	M	16	37	Friends	Bethe	F	14	35	Parents
Abebe	M	14	28	Friend	Ashenafi	M	15	39	Parents	Fikru	M	13	45	Pocket
Sitotaw	M	14	33	Pocket	Mohe	M	15	22	Pocket	Getenet	M	15	40	Sister
Aman	M	15	44	Pocket	Worku	M	16	44	Pocket	Nesanet	F	18	70	Parents
Tigest	F	13	23	Parents	Hana	F	15	32	Pocket	Mahlet	F	15	85	Grandpa
Addis	M	22	50	Parents	Shenkute	M	13	44	Grandpa	Helen	F	16	44	Pocket
Fetsame	F	18	30	Pocket	Sara	F	14	40	Parents	Fatuma	F	17	28	Parents

Football Model-eliciting Activity (MEA-3)

Part I: READING HOMEWORK

Are We Getting Obsessed with European Football?

By Seble Teweldebirhan

Addis Ababa, March 4, 2012 (Ezega.com) - I just do not get the obsession my generation has with European football. Do not get me wrong, as many of my friends do, I tried to understand and fall in love with one of the teams. I did go to a house with DSTV (The pay TV in Ethiopia), that shows the matches live and tried to engage with the game emotionally. I also showed an effort to understand about the characters of the coaches, salaries of the players, the brand of their cars and shampoo and their love life, which I think has many models in it. However, any of it never worked for me. Therefore, I started to go to the DSTV houses not to see the games but watch the supporters of the game. I became more impressed about their wisdom on football, the life of players, and many other things, which I think the Ethiopian Football Federation is missing out.

Near to where I live, a DSTV house shows every European game live with a big screen and LCD TV. The entrance fee is 20 birr and in some very important games 30 birr. I have never seen anyone mind paying the entrance and the house is always full of football fans. There are even places reserved for a certain team supporters. Especially the supporters of Manchester United and Arsenal reserve the opposite front raw and the supporters do not mix. Even when one side is full and there are extra spaces on the other side, supporters prefer to stand instead sharing a chair with team supporters. The case is serious for them, and 'take it easy, it is just a game' does not work in this situation.

Whenever there is a game, the discussion before the games begins is my favorite part. Everyone has a suggestion and opinion on which player should be in or out. The coaches are criticized a lot as if the team of experts is evaluating them. Though I always have a hard time to understand and remember which team is up or down, I am guessing the season is not working for Arsenal's coach Arsen Venger. I have heard a lot of cursing on his performance and I even noticed some fantasize by writing a resignation letter for him.

The discussions and opinions from random football lovers make you ask yourself how a country with this kind of expertise in football has a hard time winning any game. The supporters use many technical terms when talking about the games. They explain why a certain team won or lost and what could have happened if that or this player was not injured. They predict the results of the game with some sound arguments based on previous and current performances of the teams. In some cases, they suggest what the coach should or should not do to win the game. Then the discussion will turn to the players and their personal lives. "He was in Spain to visit his grandmother" or "he and his wife took a trip to Hawaii, so I don't think he will be in a best shape today," are some of the most common opinions you might hear in the fantasyland. When someone says, "he had a minor accident on Tuesday" you might hear a surprising question, as "which one of his cars was he driving" and the funniest part will be someone answering saying "oh he was driving his brand new Chevrolet, you know the one he bought last week".

These football lovers know the players, their coach, assistants, the field, and many other things inside out. Some will quote sayings of the coaches from decades back and others remember how Chelsea lost the game to whatever team whenever with details. I even heard some talking about school friends of one player's son.

The most surprising thing is not only their detailed oriented knowledge but also the way they talk about the players and the team. I have never seen a sense of ownership on people that much. They always use the word "ours" and seem to feel like they share both the successes and

the failures. As much as they are happy when they win, it looks like it is their fault when the team loses.
Once the game begins, no one will notice if you start a fire there. The focus is too much and they do not miss any detail in it. They shout “offsite” before the commentator or the judge for that matter. Knowing whether fouls result a yellow or a red card, is probably the simplest thing for them. They seem to know the rules of the field well and are not afraid to criticize the judge or the players. If their team scores, they will shout and jump as if it is for their personal credit. If the team misses, then the player will be insulted and criticized in a language they will never understand or know it exists for that matter. For the first half of the game all eyes are on the screen. The arguments on what could and could not have been will start during the 15 minutes break. Then again, another 45 minutes of absolute focus will follow.

European football today is not just an entertainment. For many Ethiopians who are more drawn to it everyday, it is part of life. It is inside families, determining the relationship between brothers, couples, and friends. In some cases, the fans will hold it against you if you say bad things about their team or the players. They will shout, argue the whole day, and cry if that is what it takes to convince their team is the best. Probably the love and commitment they show is no like other. I personally know people who refuse to work or spend time with their family if their team, or in some cases any other team, is playing. In the houses where the football is transmitted live, fight between opposite team supporters is inevitable. For a reason no one understands except them, people get so emotional that they say and do things they obviously regret. In one of my watching the event, I hardly remember who was playing, but one of the supporters insulted a player from the opposite team. The next thing I remember was the fight that made the house looks like a bad gangster movie. For that, most houses hire a guard and in some cases identify those with potential to get into fighting and refuse to let them in.
Showing games live is also a booming business all over the country. Today bars and restaurants understood they would not have customers in most days of the week if they do not show European football.

Public transportations, cafes, and workplaces now have a topic unrelated to any of the things going on in the country. European football is taking over all the conversation you might have with especially the youth. The first thing most people ask the minute they get to their office is “have you seen the game yesterday”. Especially those whose team won, they will talk about it a lot. Therefore, opposite team supporters might avoid contacts the whole day. I have two friends who work for the same company. Unfortunately, they support different teams. In most cases, they tell me one of them did not see the other the whole day. The reason for that is one team had won and the other lost. The supporter for the team that lost will have his lunch and coffee in a place the other one does not know.
The media is also in love with European football. Most FM radios talk about it the whole day and ETV sport news is always dominated by it. Since in Ethiopia, the media loves to revolve on agenda that is already set, it might not be surprising to see it follow the people’s interest.
It is funny to witness romantic relationships torn up because of English Premier League. These relationships especially are going to the ground because the obsession of men with football annoys many women. Many women consider it as immature and sometimes irresponsible to abandon work and family for European football. Especially because most games are at night, the fact that men stay out late drives most women crazy. For those who have children, it is imaginable how frustrating this could be. Men are claiming women should understand and leave

them alone when it comes to football. Since most people cannot afford to have a DSTV and watch the games from their house, keeping relationships and marriages together is having one more challenge in its plate.
However, that is not the end of the story for European football and relationships. Previously, mostly men were obsessed with foreign football. Nowadays, women are also joining in greater numbers. Therefore, dating obviously is being affected by football games. However, men who love European football love to date women with the same interests. Of course if she is supporting a different team, most nights will probably go unromantic, if not a fight. The other most serious problem related to this obsession is the fact that the games are now turning in to gambling. For some, there is a lot of money at stake and it is not surprising to see them act like crazy during the games. There are stories about people gambling with all they have including their car and houses, finally torn their lives and families apart.

Why is all of this going on for something that is hardly related to what actually matters in people's lives? It is obvious that people love football. What is complex is the extent of it. How can a game happening millions of miles away steal the hearts and minds of our youth in a way anything in the country can? Many European football lovers told me they do not know what they would do without the games. For them there is nothing to do and nowhere to go and the football gave them the opportunity of spending their spare time with pleasure. Some say that Ethiopians always loved football and since the country's football is nothing but a big disappointment, they have to find the pleasure somewhere. Ethiopia never had it good with home grown football in a long time. The move is always one-step forward and two steps backward. For that, many in the country have given up hoping things might turn around for the better. Still, as someone who goes to the European live games to observe the fans, I say the love is more than we can handle. If we are having friends fighting, couples breaking up and families disappointed for something that is not ours, it is time to evaluate how far we are willing to go for it.

PART II: READINESS QUESTIONS

1.) Have you ever supported any one of the European football clubs?
2.) Is there anyone in your family that you know who has strong emotional feeling about European football? Could you describe him/her?
3.) Why do you think Ethiopian football is so unfamiliar to Ethiopian supporters?
4.) Ethiopian is well known for athletics in the world, when do you think our football will be well known even in Africa?

PART III: PROBLEM STATEMENT

The Ethiopian Football Federation investigates how to attract more Ethiopian football supporters in Ethio-league football competitions. One of the greatest problems for the federation has been lack of enough media coverage. The federation committee decides to broadcast everyday Ethio-league football news through government and private Medias on TV, radio and newspapers. Additionally, the federation has wanted a team of students who could work in school media and prepare weekly football sport news for the school. The federation has planned to give pocket money for the team of students going to be selected. Ethiopian National male football team (Walia) has participated after 30 years (one generation) in African Football Cup this year 2013. Now, the team is competing to qualify for 2014 world foot ball cup in Brazil. The Walia has a match against the national football team of Botswana and South Africa in June, 7 and 14, 2013 respectively. It is said that the honorable Walia coach, Sewnet Bishaw has two teams within the Walia football team based on some similarities of football skills (Team A and Team B as shown below) and sometimes it is a challenge for him to decide which team should play against the opposite team. Now, in your team, write a report for the coach to decide which team is better (team A or team B) to play against Botswana and South Africa football team using the data below which could also be presented in school media. Notice that the Congo football team players are longer and massive, and play long balls, while South African players are physical similar with Ethiopian and they have medium height, and play short balls.

PART IV: DATA TABLE Team A

Number	Position	Player	Age	Caps	Goals	Height	Weight	Club
1	GK	Sisay Bacha	23	6	0	1.60m	55kg	Dedebit
4	DF	Abebaw Butako	25	27	0	1.65m	65kg	Ethiopian Coffee
5	DF	Aynalem Hailu	26	17	0	1.70m	68kg	Dedebit
12	DF	Biadgelegn Elias	24	10	0	1.75m	72kg	Saint George
3	MF	Yared Zinabu	23	24	0	1.78m	75kg	Saint George
15	MF	Dawit Estifanos	25	3	0	1.66m	59kg	Ethiopian Coffee
14	MF	Minyahil Teshome	27	11	0	1.65m	67kg	Dedebit
8	MF	Tesfay Alebachew	24	15	1	1.78m	75kg	Saint George
21	FW	Addis Hintsa	25	10	0	1.80	70kg	Dedebit
7	FW	Saladin Said	24	17	10	1.72m	65kg	Leirse
9	FW	Getanhe Kebede	20	16	5	1.74m	68kg	Dedebit

Team B

Number	Position	Player	Age	Caps	Goals	Height	Weight	Club
22	**GK**	Jemal Tasew	**23**	**7**	**0**	**1.74m**	**66kg**	**Ethiopian Coffee**
2	**DF**	Degu Debebe	**29**	**46**	**0**	**1.69m**	**70kg**	**Saint George**
6	**DF**	Alula Girma	**19**	**24**	**0**	**1.70m**	**69kg**	**Saint George**
10	**DF**	Birhanu Bogale	**27**	**22**	**1**	**1.78m**	**76kg**	**Dedebit**
3	**MF**	Yared Zinabu	**23**	**24**	**0**	**1.78m**	**75kg**	**Saint George**
8	**MF**	Asrat Megressa	**25**	**13**	**0**	**1.70m**	**62kg**	**EEPCO**
18	**MF**	Shimelis Bekele	**22**	**23**	**7**	**1.70m**	**65kg**	**Saint George**
16	**MF**	Yesuf Saleh	**28**	**7**	**0**	**1.80m**	**77kg**	**Tobol**
19	**FW**	Adane Girma	**27**	**36**	**8**	**1.72m**	**72kg**	**Saint George**
9	**FW**	Oumed Oukri	**22**	**23**	**9**	**1.75m**	**74kg**	**Saint George**
13	**FW**	Fuad Ibrhaim	**21**	**4**	**1**	**1.73m**	**70kg**	Minnesota Stars

Tourist Model-Eliciting Activity (MEA-4)

PART I: READING HOMEWORK

Ethiopia's Historical & Natural Tourist Attractions

The natural beauty of Ethiopia amazes the first-time visitor. Ethiopia is a land of rugged mountains (some 25 are over 4000 meters high) broad savannah, lakes and rivers. The unique Rift Valley is a remarkable region of volcanic lakes, with their famous collections of birdlife, great escarpments and stunning vistas. Tisisat, the blue Nile falls, must rank as one of the greatest natural spectacles in Africa today. With 14 major wildlife reserves, Ethiopia provides a microcosm of the entire sub-saharan ecosystem. Birdlife abounds, and indigenous animals from the rare Walia ibex to the shy wild ass, roam free just as nature intended. Ethiopia, after the rains, is a land decked with flowers and with many more native plants than most countries in Africa.

Ethiopia, the oldest independent nation in Africa, has a heritage dating back to first century AD. Traders from Greece, Rome, Persia and Egypt knew of the riches of what is now Ethiopia, and by the first century AD, Axum was the capital of a great Empire. This realm became one of the first Christian lands of Africa. Late in the 10^{th} Century, Axum declined and a new Zagwe dynasty, centred what is now Lalibela, ruled the land. Axum, Lalibela and Gonder now provide our greatest historical legacy. It was in the 16^{th} Century that the son of the great explorer Vasco Da Gama came to Ethiopia, but then found a land of many kingdoms and provinces beset by feuds and War.

Legend has it that Emperor Menelik I, the son of the Queen of Sheba and king Solomon, brought the Ark of the Covenant from Jerusalem to Axum, where he settled and established one of the world's longest known, uninterrupted monarchical dynasties. This is only one example of Ethiopia's magnificent history, which encompasses legend and tradition, mystery and fact, from a powerful and religious ancient civilization. The well -trodden path through Ethiopia's famous and fascinating historic places takes you through a scenically magnificent world of fairy - tale names, such as Lalibela, Gondar, Deber Damo and Bahar Dar.

Several of Ethiopia's more remote areas are excellent for walking safaris, which are offered by several good tour operators in the country. Walking tours, best planned for the dry season, offer the traveler the opportunity for awe-inspiring vantage points from which to view many of Ethiopia's natural wonders, cultural riches and architectural heritage. In Gondar, there are fairy tale castles dating back to the 17th century. In Harar, the visitor can enjoy the incense-flavored mysteries of narrow alleyways and towering minarets. Wide, tree-lined streets, fine architecture, glorious weather and the incongruity of donkey trains along the boulevards make Addis Ababa a city of surprises and a delightful place to explore. The clear mountain air gives the city the bracing atmosphere of a summer highland resort. It enjoys a mild climate, with an average temperature of 61 degrees Fahrenheit.

PART II: READINESS QUESTIONS

a) What does it tells you about the temperature in Addis when it says in an average the temperature is 61 degrees Fahrenheit?
b) What do you think the temperature in Addis Abeba might be for 6 different days in the year? -------, --------, ----------, ---------,--------,---------.
c) Why do you make these choices?

PART III: STATEMENT OF THE PROBLEM

Problem Statement
Smokeless industry- tourism is one of the prominent industries in Ethiopia. Tour and travel agencies are expected to give reliable information about health requirements, customs, transport, time, currency, topography, language, climate and etc. Write a report for the tourist describing and comparing the climate data given below for the four tourist attraction sites telling them what type of clothes they should wear and what type of food(drinks) they should take or have.

PART IV: DATA TABLE

Temperature by: centigrade

AXUM 8 98 N, 38 80 E, 7726 feet (2355 meters) above sea level.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Avg. Temperature	16	17	17	18	18	16	15	15	16	16	15	15
Avg. Max Temperature	22	24	23	24	24	22	20	20	21	21	22	22
Avg. Min Temperature	7	8	11	11	11	11	11	11	10	8	7	6
Avg. Rain Days	1	1	3	4	3	8	11	11	7	2	0	0
Avg. Snow Days	0	0	0	0	0	0	0	0	0	0	0	0

LALIBLA 30 13 N, 31 40 E, 242 feet (74 meters) above sea level.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Avg. Temperature	13	15	17	20	24	27	28	28	27	23	19	15
Avg. Max Temperature	18	20	22	27	31	33	34	34	33	29	24	20
Avg. Min Temperature	8	10	11	14	18	21	22	23	21	18	14	10
Avg. Rain Days	1	1	1	0	0	0	0	0	0	0	1	0
Avg. Snow Days	0	0	0	0	0	0	0	0	0	0	0	0

GONDER 15 60 N, 32 54 E, 1246 feet (380 meters) above sea level.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Avg. Temperature	23	25	27	32	35	35	31	30	32	32	28	24
Avg. Max Temperature	31	33	36	40	42	42	38	37	38	38	35	31
Avg. Min Temperature	16	17	19	24	27	27	26	25	26	25	21	17
Avg. Rain Days	0	0	0	0	0	0	1	2	1	0	0	0
Avg. Snow Days	0	0	0	0	0	0	0	0	0	0	0	0

Harar 4 3 S, 39 61 E, 180 feet (55 meters) above sea level.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Avg. Temperature	26	27	28	27	25	24	23	23	24	25	26	26
Avg. Max Temperature	31	32	32	30	28	28	27	27	28	28	30	31
Avg. Min Temperature	22	22	23	22	21	20	18	19	20	21	21	22
Avg. Rain Days	2	1	3	7	10	6	8	7	6	6	7	4
Avg. Snow Days	0	0	0	0	0	0	0	0	0	0	0	0

Students' Models for Football MEA

Students' Models at Level 1: Football MEA

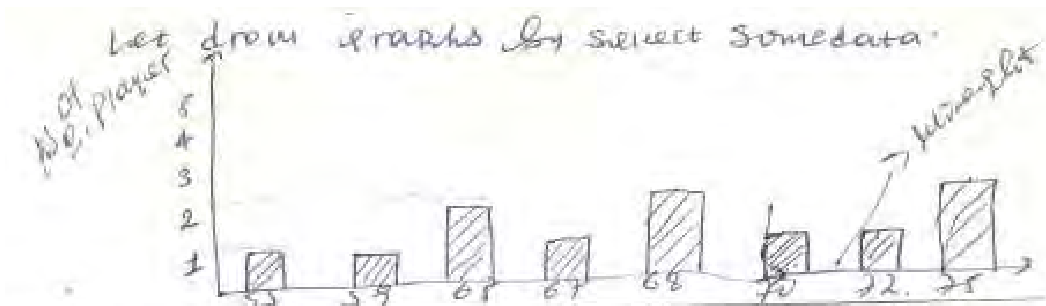
NEWS: A letter to Mr. Sewnet Bishaw: Dear Coach,

In our team, we discuss together and decide the following: Team A should play with South Africa and Team B should play with Congo. We have decided using the data interpreting it with mode, the value with the highest frequency. We think that if they play with these teams, we hope they would develop their confidence in the future!

Students' Models at Level 2: Football MEA

NEWS: Now is the time for Sport NEWS! According to the Ethiopian football Coach, the Congo team football players are longer and massive. Team B has more height (1.73m in average) and weight (70.54 in average) than the height (1.71m in average) and weight (67.1 in averages) in team A. He told the media that, team A will play against South Africa because their height and weight is similar. Team B will play against Congo because they have more height and weight and most of them are from one club (Saint George) and will communicate well. Therefore, Team B has to play against Congo football team. We have finished the sport news! Good bye!

Students' Models at Level 3: Football MEA



Dear Mr. Sewnet Bishaw, We found that most players in team B played for Saint George using mode. Besides, team B has a median height of 1.73m, but the median height of team A is 1.72m. If we see the weight, the median weight for team B is 70 kg but for team A is 68kg as shown in the bar graph. Hence, we conclude that Team B is better to play against Congo because they could score a goal with head as they are taller. Team A is better to play against South Africa because they could defend well.

Students' Models at Level 4: Football MEA

Dear respected Coach: Sewnet Bishaw, While I am giving this letter with my hands to you, I feel like I am talking with you and consider it as if you put your honored signature to me. I like to start first by admiring you about your effort in different occasions to make Ethiopian football team always to be winners and pioneer in African football. And, this attributes to consider you as a person who has a special place in my life. When you speak your face often shines as I think it is the white meat you ate in your childhood. This is fascinating! I wish my face shine like you and I would like to put some ointment in my face if it shines just like you. This is, because; oftentimes we do not eat white meat and butter in our time, but we ate a piece of bread and oil. Having said this let me talk on the report! From the provided teams, team A and team B, I will first compare the two teams before we decide which team should play with South Africa or Congo. With respect to Age both teams are equivalent, in average their age is 24.18. When we compare the weight we find team A has 67.18 kg in average and team B has a weight of 70.54 kg. When we go to the height, team A has 1.71 m and team B has 1.73 m in average. In general, it is better decision if team B and A play against Congo and South Africa respectively. The highest goal scorer from both teams is Saldine Sied with 10 goals. Thank you! I hope better time will come in the future and that we eat white meat and butter together*!*

Appendix B: QUALITY ASSURANCE GUIDE

Performance Level	How useful is the product?	What might the client say?
Level one: Requires Redirection	The product is on the wrong track. Working longer or harder won't work. The students may require some additional feedback from the teacher.	*"Start over. This won't work. Th about it differently. Use different or procedures."*
Level two: Requires Major Extensions or Refinements	The product is a good start toward meeting the client's needs, but a lot more work is needed to respond to all of the issues.	*"You're on the right track, but th needs a lot more work before it'll a form that's useful."*
Level three: Requires Only Minor Editing	The product is nearly ready to be used. It still needs a few small modifications, additions, or refinements.	*"Hmmm, this is close to what I n You just need to add or change a small things."*
Level four: Useful for this Specific Data Given	No changes will be needed to meets the immediate needs of the client.	*"Ahhh, this will work well as it i won't even need to do any editing*
Level five: Sharable or Reusable	The tool not only works for the immediate situation, but it also would be easy for others to modify and use it in similar situations.	*"Excellent, this tool will be easy f to modify or use in other similar s ations— when the data are sligh different."*

Appendix C: MY LEARNING GOAL

My Learning Goal on Distribution and Histogram

My current knowledge about Distributions and Histograms

☐ Excellent ☐ Above average ☐ Average ☐ A bit weak

My goal is to reach the following level of understanding Distribution and Histograms

☐ Excellent ☐ Above average ☐ Average

Can I reach this goal?

☐ Absolutely confident ☐ Strong confident

☐ Rather confident ☐ Absolutely not confident

My learning goal in Distribution and Histograms:

☐ Understand different data types could be represented in different graphs

- [ ] Understand looking at the center, shape and spread will give lots of information
- [ ] Understand how to construct Histogram using frequency distribution table
- [ ] Understand how to compare Histograms using center, shape and spread aspects of data
- [ ] Understand about my classmate using student survey data in Histograms
- [ ] Understand the difference between different types of graphs

Appendix D: Individual Reflection on MEA

INDIVIDUAL ROLES REFLECTION TOOL

Group Problem-solving Self-Reflection

Your name: ____________________ Name of activity/problem: ____________________________

1. ***Group Transition Points*** Use the following timeline to show the various transitions your group made over the course of the activity. Please label each transition, explaining what happened during each phase.

Start -- Finish

2. ***Individual Roles*** Recreate the lines from the timeline you created above to make as many columns on the chart as your group had transitions. Title each column using the same labels you gave in the timeline. Using the scale below, please rank the roles that you played during the problem-solving activity.

4 – Mostly 3- Somewhat 2- A little 1- Hardly at all 0-None

Role	
Facilitating/ Leading	
Recording	
Monitoring	
Strategizing	
Other (specify)	

3. ***Dominant Roles*** What was the primary role each group member took during the problem-solving activity?

Appendix E: TRU Math RUBRIC

Whole Class Activities: Launch, Teacher Exposition, Whole Class Discussion

On the score sheet, Circle one of L / E / D if the episode is primarily of that type. If a Launch is primarily logistical, some dimensions may be labeled N/A.

	The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
	How accurate, coherent, and well justified is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent does the teacher support access to the content of the lesson for all students?*	*To what extent are students the source of ideas and discussion of them? How are student contributions framed?*	*To what extent is students' mathematical thinking surfaced; to what extent does instruction build on student ideas when potentially valuable or address misunderstandings when they arise?*
1	Classroom activities are unfocused or skills-oriented, lacking opportunities for engagement in key practices such as reasoning and problem solving.	Classroom activities are structured so that students mostly apply memorized procedures and/or work routine exercises.	There is differential access to or participation in the mathematical content, and no apparent effort to address this issue.	The teacher initiates conversations. Students' speech turns are short (one sentence or less), and constrained by what the teacher says or does.	Student reasoning is not actively surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	Activities are primarily skills-oriented, with cursory connections between procedures, concepts and contexts (where appropriate) and minimal attention to key practices.	Classroom activities offer possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" the challenges, removing opportunities for productive struggle.	There is uneven access or participation, but the teacher makes some efforts to provide mathematical access to a wide range of students.	Students have a chance to explain some of their thinking, but "the student proposes; the teacher disposes": in class discussions, student ideas are not explored or built upon.	The teacher refers to student thinking, perhaps even to common mistakes, but specific students' ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	Classroom activities support meaningful connections between procedures, concepts and contexts (where appropriate) and provide opportunities for engagement in key practices.	The teacher's hints or scaffolds support students in productive struggle in building understandings and engaging in mathematical practices.	The teacher actively supports and to some degree achieves broad and meaningful mathematical participation; OR what appear to be established participation structures result in such engagement.	Students explain their ideas and reasoning. The teacher may ascribe ownership for students' ideas in exposition, AND/OR students respond to and build on each other's ideas.	The teacher solicits student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings.

Small Group Work

If students are engaged in early brainstorming, the role of the teacher is to support students in exploring and justifying. This is the reason for "ORs" in the scoring.

	The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
	How accurate, coherent, and well justified is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent do teacher support and/or group dynamics provide access to meaningful participation for all students?*	*To what extent do teacher support and/or group dynamics provide access to "voice" for students?*	*To what extent does the teacher monitor and help students refine their thinking within small groups?*
1	Students are aimed at "answer getting" without addressing underlying reasoning.	Activities or teacher intervention constrain students to activities such as applying straightforward or memorized procedures.	Some students are disengaged or marginalized, and differential access is not addressed.	Teacher interventions, if any, either constrain students to producing short responses to the teacher **OR** do not address clear imbalances in group discussions.	The teacher does not engage students in discussion, or actions are simply corrective.
2	Student explanations, if they appear, are largely procedural; students are not expected to explain their ideas or supported in doing so.	Activities offer possibilities of conceptual richness or problem solving challenge, **BUT:** students are either left unsupported when lost, **OR,** the teacher's interventions remove the challenge.	All team members appear to be engaged, **OR** the teacher makes moves to engage team members who are not participating.	At least one student has a chance to talk about the mathematical content, but "the student proposes, the teacher disposes": students are not encouraged to build on each other's ideas.	Teacher points to student errors or responds to questions, but does not help students build on nascent ideas or encourage group discussion of potentially problematic issues.
3	Explanation of and justification for the central mathematical ideas is accurate; **OR** teacher supports students in focusing on central mathematical ideas and explaining and justifying them.	Activities offer possibilities of conceptual richness or problem solving challenge, **AND** students are provided scaffolding (if they are lost) that enables them to grapple with the tasks at hand, but for which the challenge has not been removed.	Everyone in the team contributes to the mathematical discussion, **OR** teacher makes moves to try to give all team members access to making contributions.	At least one student puts forth and defends his/her ideas/reasoning. The teacher may ascribe ownership for students' ideas in exposition, **AND/OR** students respond to and build on each other's ideas.	The teacher solicits student thinking, **AND** subsequent discussion responds to those ideas, by building on productive beginnings or addressing possible misunderstandings.

Student Presentations

Some episodes are in essence a conversation between teacher and student presenter(s); some conversations that involves the whole class. Scoring in the rubrics corresponds to the presence of these two different participation structures: ***C*** *for a teacher-presenter conversation, and* ***W*** *for whole-class involvement.*

	The Mathematics	Cognitive Demand	Access to Mathematical Content	Agency, Authority, and Identity	Uses of Assessment
	How accurate, coherent, and well justified is the mathematical content?	*To what extent are students supported in grappling with and making sense of mathematical concepts?*	*To what extent does the teacher support presenters or class in engaging with the mathematics?*	*To what extent are students the source of presented ideas and response to presented ideas?*	*To what extent is students' mathematical thinking surfaced and serve as grounds for conversation?*
1	Presentation is aimed at "answer getting" without addressing underlying reasoning.	Presentation and classroom discussion focus on straightforward or familiar facts and procedures.	**(C):** Presenter(s) need support/encouragement but do not receive it; **OR (W):** A significant number of students appear disengaged.	Presenter role is structured by teacher/text and student is narrowly constrained in response to teacher questions.	Student reasoning is not surfaced or pursued. Teacher actions are limited to corrective feedback or encouragement.
2	The mathematics presented is largely procedural; presenter(s) are not expected to explain their ideas or supported in doing so.	Presentation offers possibilities of conceptual richness or problem solving challenge, but teaching interactions tend to "scaffold away" these possibilities, resulting in a focus on straightforward or familiar facts and procedures.	**(C):** Teacher encourages presenters but does not provide effective scaffolding; **OR (W):** The presentation evolves into whole class activity. There is uneven participation and the teacher does not provide structured support for many students to participate in meaningful ways.	Presenters have the opportunity to demonstrate individual proficiency, without being tightly constrained by text or teacher. **BUT**, the discussions **do not build on students' ideas.** (*To qualify as an *idea*, what is referred to must extend beyond the tasks, diagrams, etc., that students were given.)	In presentation and discussion the teacher refers to student thinking, perhaps even to common mistakes, but specific student ideas are not built on (when potentially valuable) or used to address challenges (when problematic).
3	The mathematics presented is relatively clear and correct, **AND** either includes justifications or explanations **OR** the teacher encourages students to focus on central mathematical ideas and explaining and justifying them.	The teacher's hints or scaffolds support presenters and/or class in "productive struggle" in building understandings and engaging in mathematical practices.	**(C):** Teacher supports presenters (if needed) in engaging, **OR (W):** The presentation evolves into whole class activity in which the teacher actively supports broad participation and/or what appear to be established participation structures result in such participation.	Student presentations result in further discussion of relevant mathematics, **OR** students make meaningful reference to other students'/groups' ideas in their presentations. (*To qualify as an idea, what is referred to must extend beyond the tasks, diagrams, etc., that students were given.)	In presentation and discussion the teacher solicits student thinking and responds to student ideas by building on productive beginnings or addressing emerging misunderstandings.

Appendix F: PROJECT RUBRIC

Rubric for assessing the final project report

(4) Outstanding Report. All components are clearly present and are of excellent quality. The team of students demonstrates solid conceptual understanding of the nature of the task, as well as completes an insightful analysis of the problem chosen by the team. The report is excellent quality; demonstrate clearly the data collection, organization, presentation, analysis and interpretation describing the variation of data. The team used real data and make clear introduction to the problem. Report is concise, very well written, convey coherent thoughts and utilizes appropriate tables, graphs, measure of center and dispersion.

(3) Good Report. All components are clearly presented. The team of students demonstrates a good understanding of the nature of the task, as well as completes a good analysis of the problem chosen by the team. The report is good quality; demonstrate the data collection, organization, presentation, analysis and interpretation describing the variation of data. Report is well written, coherent thoughts and utilizes appropriate tables, graphs, measure of center and dispersion.

(2) Adequate Report. All components are present. The team of students demonstrates adequate understanding of the nature of the task, as well as an adequate analysis of the questions, but may miss important but subtle features of the presentation. Not enough demonstration of data collection, organization, presentation, analysis and interpretation describing the variation of data. Report generally shows little reflective thought. The report demonstrates statistical skills, but may contain several incorrect choices of tables, graphs, measure of center and dispersion.

(1) Inadequate Report. Team of students demonstrates little understanding of the nature of the task, or does an incomplete analysis of the problem chosen by the team. The report indicates little thought making it difficult to understand. Tables, graphs, measure of center and dispersion are inappropriate and used incorrectly.

(0)Very Poor Report. Team of students demonstrates virtually no understanding of the nature of the task and does not complete an adequate analysis of the questions. The assignment may not be completed, not turned in, or is incoherent.

Appendix G: Researcher’s Philosophy of Statistic Education

It is very distressing that students at their youth age have little retention of statistical understanding after leaving school in an age of information where various displays of data and related reports can easily mislead the young in the wrong directions. The fruit of statistics education such as statistical literacy, reasoning, and thinking could be obtained if students develop the seed of basic statistics concept starting from lower grades. Statistics becomes an indispensible tool for everyone in the world in the 21st century and thus students need to be taught on how to use statistics critically in their everyday life.

Metaphorically speaking, as the seed has a potential to be a big tree and could produce flowers and fruits, students have potential to enhance their understanding to achieve desired learning outcomes in statistics education. Further, the fruit of their work could be realized prematurely at their youth age through empowering themselves as social agents solving their own and socio-cultural problems.

A seed needs optimal levels of soil, minerals, water, air and sunlight to germinate and produce fruits. With similar figure of speech, procedural fluency, conceptual understanding, adaptive reasoning, strategic competency and productive dispositions would be indispensable intertwined aspects of an instructional approach for enhancing students’ understanding towards desired learning outcomes in Statistics Education.

Moreover, to continue the metaphor, as healthy soil is very essential for the seed to develop with its potential by driving water, air and essential nutrients from the soil, students need to work on authentic problem solving tasks to develop their understanding using their adaptive reasoning, conceptual and procedural knowledge as they solve problems. The problems need to be authentic that could arise from the concrete situations of students’ life to engage them in the tasks and enhance their understanding. For example, teaching statistics as dry and cold using formalist tradition of teaching mathematics may inhabit students’ conceptual understanding from growing as it would be difficult for a seed to germinate on a dry soil.

Analogically, to plant the seed of statistics knowledge we need to assess and develop students’ prior and informal knowledge as we make the soil fertile in order the seed to grow well. Students may need to solve problems that could mimic their life or socio-cultural problems so that they could develop their knowledge organically form informal to formal and which could engage and motivate them in high statistical discourses to lay the foundation for high cognitive thinking.

Moreover, authentic problem solving tasks could help us to elicit and assess students’ thinking before formal instruction to serve all students’ with equity statistics education as they may be at different knowledge levels for different concepts like we plant seeds at different depth depending on the seed’s size and its requirement for light. We need to identify the level of students' thinking since learning occurs being an extension or refinement of conceptual systems (models) that are already familiar.

Students could learn the concepts well at their grade level, nonetheless they may not develop the kind of dispositions to use statistics in their present life or future life as the seed start to germinate but get dried by sunlight because the roots do not develop well. Students need to learn statistics sensibly and meaningfully with optimal levels of the intertwined strands integrated. As a seed requires sunlight; students need to solve interesting problems to develop their productive dispositions and maneuver their room to act on limiting situations or solve the existing problems in their own life or in the society.

Furthermore, we may not need to teach statistics as if it was given by some authorities. Students would require taking ownership of ideas by externalizing their thinking using representation systems. The traditional linear approach of statistic curricular need to be revisited so as to allow students invent, revise, construct and refine ideas iteratively. Seeds often are sown close together in case some of them do not germinate successfully, concepts could be taught concurrently to enhance understanding of statistics.

Though students could have a gut to learn statistics well, they might have incorrect conceptions of statistical contents and processes which hinder their concept development like weeds compete to stop the growth of young plants. We have to provide students continuous feedbacks iteratively in order to improve their reasoning and data modeling abilities. Co-operative learning needs to be promoted for students' to share ideas flexibly and develop concepts meaningfully in the ways of statistical thinking in the discipline.

Finally, education aim need to include work against 'dehumanization' in our world by developing their critical understanding using statistics as a critical tool to promote the well being of human being as we avoid thorns, insects and diseases that inhibit the plant to be fruitful. Students could be empowered to make statistical reports posing problems using complex realistic data sets to develop a sense of social agency and do projects by their own on socio-cultural real life problems. And, this probably could be done teaching statistics using Modeling Instructional Approach.

Appendix H: Survey of Attitude towards Statistics

ትዕዛዛት፡ ለሚከተሉት አርፈተ ነገሮች የ ”x” ምልዕክትን በተሰጠው ባዶ ሳጥን ውስጥ በመፃፍ ከተሰጡት አማራጭ ውስጥ የምትስማማበትን መረጥ/ጭ፡፡ እባኳዎን ሁሉንም የመጠይቅ አረፍተ ነገሮች በማንበብ ከተሰጡት አማራጮች ውስጥ አመለካከቶን በ ”x” ምልዕክትን በመጠቀም ይምረጡ ፡፡

ጠቀሜታ

	በጣም አልሳማማም	አልስማማም	አስተያየት የለኝም	እስማማለሁ	በጣም እስማማለሁ
1.እስታቲስቲክስ በህይወቴ ጠቃሚ እንደሆነ ይሰማኛል፡፡	☐	☐	☐	☐	☐
2.እንደማንበብና መፃፍ ሁሉ እስታቲስቲክስን መገንዘብ ብቁ ዜጋ ለማፍራት ያስፈልጋል፡፡	☐	☐	☐	☐	☐
3.ለአብዛኛው ሰው የእስታቲሲቲክስ ትምህርት ጠቃሚ ነው፡፡	☐	☐	☐	☐	☐
4. እንደሚሰማኝ እስታቲስቲክስ ለእያንዳንዱ ሰው ከታች ክፍል ጀምሮ አያስፈልግም፡፡	☐	☐	☐	☐	☐
5. በዕለትተዕለት ኑሮ ስታቲስቲካል ዳታ ለመወያያ አያሰፈልግም፡፡	☐	☐	☐	☐	☐
6.እስታቲስቲክስ ማጥናት ጥቅመ ቢስ ነው፡፡	☐	☐	☐	☐	☐
7. እስታቲስቲክስ ለሌሎች ትምህርቶች ጠቃሚ ነው፡፡	☐	☐	☐	☐	☐
8.እስታቲስቲክስ መረጃን ለማወቅ አያስፈልግም፡፡	☐	☐	☐	☐	☐
9. እስታቲስቲክስ ኮሌጅ ገብቼ ለማጠናው ፊልድ ጠቃሚ ነው፡፡	☐	☐	☐	☐	☐

በራስ መተማመን

	በጣም አልሳማማም	አልስማማም	አስተያየት የለኝም	እስማማለሁ	በጣም እስማማለሁ
10. እስታቲስቲክስ በአግባቡ ለመጠቀም ለእኔ ከባድ ነው፡፡	☐	☐	☐	☐	☐
11. ተገቢውን የእስታቲስቲክስ ፎርሙላ ለመምረጥ በራስ መተማመን የለኝም፡፡	☐	☐	☐	☐	☐

	በጣም አልሳማማም	አልስማማም	አስተያየት የለኝም	እስማማለሁ	በጣም
12. ዳታን ለመተንተን በራስ መተማመን የለኝም፡፡	☐	☐	☐	☐	☐
13. ችግሮችን ለመፍታት እስታቲስቲክስ ሞዴሎችን መጠቀም ለእኔ ቀላል ነው፡፡	☐	☐	☐	☐	☐
14. እስታቲስቲክስ ያለውን መረጃ ለመረዳት በራስ መተማመን የለኝም፡፡	☐	☐	☐	☐	☐
15. ዳታን ለመሰብሰብ በራስ መተማመን አለኝ፡፡	☐	☐	☐	☐	☐
16. በእስታቲስቲካል አክቲቪቲ መሥራት ለእኔ ቀላል ነው፡፡	☐	☐	☐	☐	☐
17. የእስታቲስቲካል ችግሮችን ለመፍታት በራስ መታመማን አለኝ፡፡	☐	☐	☐	☐	☐
18. እስታቲስቲክስ እንደ መረጃ ለመጠቀም በራስ መታመማን የለኝም፡፡	☐	☐	☐	☐	☐
ጭንቀት					
19. እስታቲስቲክስ ያለው ሪፖርት ለማቅረብ ጭንቀት አይሰማኝም፡፡	☐	☐	☐	☐	☐
20. በስታቲስቲካል ክፍል ተማሪ መሆኔን ሳስበው ጭንቀት ይሰማኛል፡፡	☐	☐	☐	☐	☐
21.በእስታስቲክስ ዳታ መስራት አያስጨንቀኝም ፡፡	☐	☐	☐	☐	☐
22.ዳታን መተርጐም አያስጨንቀኝም፡፡	☐	☐	☐	☐	☐
23.የእስታቲስትክስ ፈተና መወሰድ ያስጨንቀኛል፡፡	☐	☐	☐	☐	☐
24. በቡድን መሥራት ያስጨንቀኛል፡፡	☐	☐	☐	☐	☐
25. አስተማሪዬን የእስታቲስትክስ ጥያቄ ለመጠየቅ ያስጨንቀኛል፡፡	☐	☐	☐	☐	☐
26. ዳታን መሰብሰብ አያስጨንቀኝም፡፡	☐	☐	☐	☐	☐
27. ዳታን መተንተን አያስጨንቀኝም፡፡	☐	☐	☐	☐	☐

ፍላጐት					
	<u>በጣም አልሳማማም</u>	<u>አልስማማም</u>	<u>አስተያየት የለኝም</u>	<u>እስማማለሁ</u>	<u>በጣም</u>
28. እስታስቲክስ ለመማር ፍላጐት አለኝ፡፡	☐	☐	☐	☐	☐
29. እስታቲስቲካል ፕሮብሌሞችን ለመፍታት ፍላጐት የለኝም፡፡	☐	☐	☐	☐	☐
30. የእስታቲስቲክስ መረጃ ከሌሎች ተማሪዎች ጋር አብሮ ለመወያየት ፍላጐት አለኝ፡፡	☐	☐	☐	☐	☐
31. የእስታቲስቲክስ መረጃን ለመረዳት ፍላጐት አለኝ፡፡	☐	☐	☐	☐	☐
32. ዳታን መሰረት ያደረገ የህይወት ችግሮችን ለመፍታት ፍላጐት የለኝም፡፡	☐	☐	☐	☐	☐
33.ተማሪዎች ዳታን መሰረት ያደረገ መረጃ በመያዝ የሚድሩጉትን ክርክር ለመገምገም ፍላጐት የለኝም፡፡	☐	☐	☐	☐	☐
34.ከዜና አውታሮች የሚቀርቡትን የእስታቲስቲክስ መረጃን ለመረዳት ፍላጐት የለኝም፡፡	☐	☐	☐	☐	☐
35.በእለት ትእለት ኑሮ ወሰጥ እስታቲስትክስን ለመጠቀም ፍላጐት አለኝ፡፡	☐	☐	☐	☐	☐
36. በተለያዮ የእስታቲስቲክስ ሞዴሎች ለመስራት ፍላጐት አለኝ፡፡	☐	☐	☐	☐	☐

ፆታ፡ ☐ ወንድ ☐ ሴት ዕድሜ (በዓመት)------------