INVESTIGATION OF DAMPING EFFECTS ON POWER LOSS IN AN OPTICAL FIBER DUE TO HIGHER ORDER NONLINEAR TERM

By

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Abstract

Many properties of optical fibers namely power loss, noise, distortion, attenuation, absorption, damping etc are guided by nonlinearities. They must be checked and calculated their effects before put into applications. We used a developed an-harmonic oscillator model (by Sunita Sharma, S. K. Ghoshal and Devendra Mohan) in which two oxygen atoms are connected to the silicon atom by springs which undergoes into an-harmonic vibration. This formulation is targated for nanosecod pulses in long haul optical communication. For such laser signal a nonlinearity is quite prominent. It is this an-harmonic motion that leads to nonlinear effects. The equations for second-order, third-order, and fifth-order linear and nonlinear susceptibilities are derived from this an-harmonic model. The equations for the dielectric constants and for the index of refraction are also derived. The power loss due to imaginary part of the higher-order nonlinear refractive index for Pure Silica Core Fiber, Dispersion Shifted Fiber and Dispersion Compensating Fiber is explicitly calculated. The variation of power loss with damping constant is also calculated at 2mW. The power loss by the Kerr and the electrostrictive nonlinear refractive index and also the total power radiated is theoretically calculated from the model. Our results demonstrate that the electrostriction, the Kerr the damping effects are significant in optical fibers. These results confirm some of the recent theoretical and experimental observations. The model is quite general and is suitable for calculating many other nonlinear properties of fiber materials.

Acknowledgements

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Introduction

In this chapter we will describe the geometry of the optical fiber and define some of the fiber parameters that are used to characterize its physical shape. We begin by describing the different type of fibers based on the refractive index profile and number of modes propagating though the fiber. We will see that a narrow pulse that originates at the optical source will spread out as it propagates down the fiber. This spread in the pulse width (called pulse dispersion) can limit the data rate. Multimode fibers are especially susceptible to this pulse spread; single mode fibers have inherent advantages over multimode fibers in reducing dispersion. Now that we have fairly powerful laser sources (i.e.,greaterthan 1mw) and small fiber cores, we are able to generate high power densities ($mw/\mu m$) in the fiber. Nonlinear effects that are negligible at lower densities or at shorter propagation lengths can become important. These nonlinearities can limit the data rate (or link distance) that we can achieve. In this chapter we will also see that the contribution of the Kerr and the electrostrictive effects to the total power loss. In chapter two we will see that electromagnetic waves that encounter materials create a complex of interactions with the charged particles of the medium. Forces are exerted on the charges by the electric field of the waves and, because of the motions of the charges, also by the magnetic field of the waves. In responding to these oscillating fields, the charges themselves oscillate

and act as radiators of secondary electromagnetic waves. Thus, in determining the net field at some point, the field of both the source waves and the waves emitted by the charged oscillators must be taken into account. In the case of ordinary fields, smaller than those now attainable with high-energy lasers, the net fields are assumed to be a linear superposition of the constituent fields. The complicated effects of all the microscopic contributions to the resultant field by the charges in the material can, for certain purposes, be simply described by macroscopic material parameters, the optical constants of the material. In this chapter, we will show that in particular how the refractive index for the nonconducting (insulators or dielectrics) materials as a function of damping factor and an operating frequency can be understood. In order to do this we use Maxwells equations and the mathematical techniques of vector calculus. Our main objective is that to derive the fifth order nonlinear susceptibility, by using anharmonic microscopic model which was derived earlier by Sharma etc., and estimate its nonlinearities. At the same time we are also re-driving the lower order nonlinearity terms and cheak their analytical results with other experimental results.

Chapter 1

Optical Fiber

1.1 Structures And Types

To guide light propagation, a basic optical fiber has a circular cross section as depicated in Fig. 1.1. Although a practical fiber has many layers, only the core and cladding are important to light propagation. Both the core and cladding are typically made of silica glass. However, the core has a higher refractive index to confine light inside. As light propagates inside a fiber, most of its power is confined in the core region, which is surrounded by the cladding. The cladding has a slightly lower optical density (or refraction index), typically between 1 percent and a few percent. Most fibers have the cladding diameter around $125\mu m$. Its size is generally not important to light propagations. Outside the cladding are several layers of protection jackets. The jackets prevent the fiber surface from being scratched or cut by mechanical forces. We know that the light or optical signals are guided through the silica glass fibers by total internal reflection. A typical glass fiber consists of a central core glass ($\cong 50\mu m$) surrounded by a cladding made of a glass of slightly lower refractive index than the cores refractive index. The overall diameter of the fiber is about 125 to $200\mu m$. Cladding is necessary to provide proper light guidance i.e. to retain the

light energy within the core as well as to provide high mechanical strength and safety to the core from scratches.

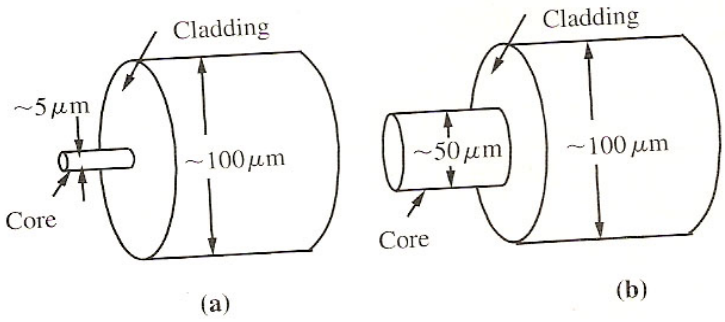

Figure 1.1: Basic structures of (a) single-mode and (b) multimode optical fibers1.

1.1.1 Step Index Fiber

In the step index fiber, the refractive index of the core is uniform throughout and undergoes an abrupt or step change at the core cladding boundary. The light rays propagating through the fiber are in the form of meridional rays which will cross the fiber axis during every reflection at the core cladding boundary and are propagating in a zig-zag manner as shown in figure 1.3a.

1.1.2 Graded Index Fiber

In the graded index fiber, the refractive index of the core is made to vary in the parabolic manner such that the maximum value of refractive index is at the center of the core. The light rays propagating through it are in the form of skew rays or helical rays which will not cross the fiber axis at any time and are propagating around

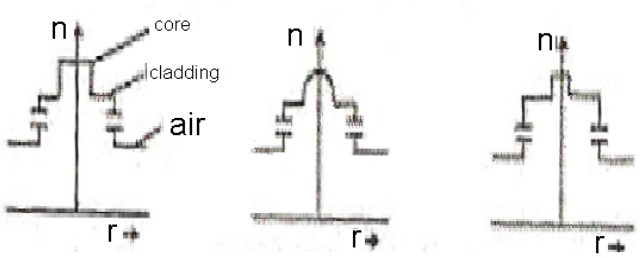

Figure 1.2: Refractive index profile

the fiber axis in a helical (or) spiral manner as shown in figure 1.1b. Based on the number of modes propagating the fiber, there are multimode fibers and single mode fibers. Mode is the mathematical concepts of describing the nature of propagation of electromagnetic waves in a waveguide. Mode means the nature of the electromagnetic field pattern (or) configuration along the light path inside the fiber. In metallic waveguides there are transverse electric (TE) modes for which Ez = 0 but Hz $\neq$ 0 and the transverse magnetic (TM) modes fore which Hz = 0 but $Ez \neq 0$ when the propagation of microwaves is along the z-axis. In optical fibers, along with TE and TM modes, there are also hybrid modes which have both axial electric and magnetic fields Ez and Hz. The hybrid modes are further classified in to EH and HE modes. In EH modes, the axial magnetic field Hz is relatively strong where as in HE modes the actual electric field Ez is relatively strong. Based on the linearly polarized nature of light, today these modes are designated as linearly polarized (LP) modes. For example LP01 mode corresponds to HE11 mode. LP11 mode is the combination of

HE21, TE01 and TM01 modes.

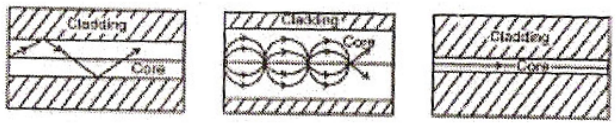

Figure 1.3: Nature of light ray propagation(a)Multi mode step index fiber(b)Multi mode Graded index fiber (c) Single mode step Index fiber

1.1.3 Single mode fibers

In single mode fiber, only one mode (LP01 mode) can propagate through the fiber (figure 1.2c). Normally the number of modes propagating through the fiber is proportional to its V-number where

$$V - number = \frac{2\Pi}{\lambda} n_1 a \sqrt{2\Delta} \tag{1.1.1}$$

Here a=radius of the core of the fiber,n_1=refractive index of the core,λ=wave length of light propagating through the fiber,Δ relative index=$\frac{n^2-n_2}{2n_1^2} \approx n_1 - n_2$, where n_2= refractive index of cladding. In the case of single mode fiber, V-number$\leq$2.405. The single mode fiber has a smaller core diameter ($10\mu m$) and the difference between the refractive indices of the core and the cladding is very small. Fabrication of single mode fibers is very difficult and so the fiber is expensive. Further the launching of

light in to single mode fibers is also difficult. Generally in single mode fibers, the transmission loss and dispersion or degradation of the signal are very small. So the single mode fibers are very useful in long distance communication.

1.1.4 Multi mode fibers

Multimode fibers allow a large number of modes for the light rays traveling through it. Here the V-number is greater than 2.405. Total number of modes M propagating through a given multi mode step index fiber is given by

$$M = \frac{v^2}{2} = 4.9(\frac{dn_1\sqrt{2\triangle}}{\lambda})^2 \tag{1.1.2}$$

Where d is the diameter of the core of the fiber. For a multimode graded index fiber having parabolic refractive index profile core,

$$M = \frac{v^2}{4} \tag{1.1.3}$$

Which is half the number supported by a multimode step index fiber. Generally in multimode fibers, the core diameter and the relative refractive index difference are larger than in the single mode fiber. In the case of multimode graded index fiber, signal distortion is very low because of self-focusing effects. Here the light rays travel at different speeds in different paths of the fiber because of the parabolic variation of refractive index of the core. As a result, light rays near the outer edge travel faster than the light rays near the center of the core. In fact, light rays are continuously refocused as they travel down the fiber and almost all the rays reach the exit end of the fiber at the same time due to the helical path of the light propagation. Launching of light into the fiber and fabrication of the fiber are easy. These fibers are generally used in local area networks.

1.1.5 Dispersion and losses in fibers

Dispersion in the fiber means the broadening of the signal pulse width due to dependence of the refractive index of the material of the fiber on the wavelength of the carrier. If we send digitized signal pulses in the form of square pulses, they are converted in to broadened Gaussian pulses due to dispersion. The dispersion leads to the distortion or degradation of the signal quality at the output end due to overlapping of the pulses. There are two kinds of dispersion mechanisms in the fiber: (i) intramodal dispersion and (ii) intermodal dispersion. The dispersion effect can be explained on the basis of behavior of group velocities of the guided mode in the optical fiber. Group velocity is the velocity at which the energy in a particular mode travels along the fiber. The propagation constant therefore $\beta = n_1\frac{2\pi}{\lambda} = \frac{n_1\omega}{c}$ group velocity $v_g = \frac{d\omega}{d\beta} = \frac{d\lambda}{d\beta\frac{d\omega}{d\lambda}}$ since $\beta = n_1\frac{2\pi}{\lambda}$, $\frac{d\beta}{d\lambda} = \frac{2}{\lambda}\frac{dn_1}{d\lambda} - n_1\frac{2\pi}{\lambda^2}$ using $\omega = \frac{2\pi}{\lambda}$, $\frac{d\omega}{d\lambda} = \frac{2\pi}{\lambda}$
Therefore

$$v_g = \frac{d\lambda}{d\beta}\frac{d\omega}{d\lambda} = \frac{\frac{2\pi}{\lambda^2}}{(}\frac{2\pi}{\lambda}\frac{dn_1}{d\lambda} - n_1\frac{2\pi}{\lambda_2}) = \frac{c}{(n_1-\lambda)}\frac{dn_1}{d\lambda} = \frac{c}{N_g} \tag{1.1.4}$$

Where$N_g = n_1 - \lambda\frac{dn_1}{d\lambda}$is called the group index of the fiber. Thus the group velocity and phase velocity ($v_p = \frac{c}{n_1}$) are different in the optical fiber. Otherwise an optical fiber is a dispersive medium.Intramodal dispersion arises due to the dependence of group velocity in the wave length. Further it increases with the increase in spectral width of the optical sources. This spectral width is the range of wavelengths emitted by the optical sources. For example in the case of LED, it has a large spectral width about 40nm since it emits wavelengths from 830-870nm with the peak emission wavelength at 850nm. In the case of laser diode which has a very narrow spectral width is about 1 or 2nm only. Thus the intramodal dispersion can be reduced in an optical

fiber using single mode laser diode as an optical source. Intramodal dispersion arises due to the dispersive properties of the optical fiber material (material dispersion) and the guidance effects of the optical fiber (wave guide dispersion).

1.1.6 Material dispersion or chromatic dispersion

This dispersion arises due to the variation of the refractive index of the core material with the wavelength or frequency of light. It is directly proportional to the frequency bandwidth of the transmitted pulse. A material exhibits material dispersion when $d2n1/d2n2 \neq 0$. For pure silica, the material dispersion tends to zero at the wavelength of $1.3\mu m$. further by using an optical source with a narrow spectral width, the material dispersion can be reduced. For shorter wavelengths around $0.6\mu mto0.8\mu m$, the material dispersion exponentially rises to a higher value.

1.1.7 Waveguidedispersion

This dispersion arises due to the finite frequency bandwidth and the dependence of the mode group velocity on the frequency of light. Higher frequency band width of the transmitted pulse, higher will be the waveguide dispersion. The amount of waveguide dispersion depends on the fiber design like core radius, since the propagation constant β is a function of a/λ. In the case of single mode fibers, waveguide dispersion arises when$d^2\beta/d\lambda^2 \neq 0$. In the case of multimode fibers, most of the modes propagate far from the cutoff value. Therefore then all are almost free from waveguide dispersion.

1.1.8 Intermodal dispersion or multimode dispersion

Intermodal dispersion or multimode dispersion arises due to the variation of group velocity for each mode at a single frequency. Different modes arrive at the exit end

of the fiber at different times. So there is multimode dispersion and hence there is broadening of the signal pulses. Dispersion in different fibers: among the three dispersions, Multimode dispersion > material dispersion > waveguide dispersion. Based on the dispersion effects, one can get the following results. i.The multimode step index fibers exhibit a large value of dispersion due to the enormous amount of multimode dispersion which gives the greatest pulse broadening. ii.In the case of single mode step index fibers, they have only intramodal dispersion. Further among the intramodal dispersions, the waveguide dispersion is the dominant one. The material dispersion in them is almost negligible due to axial ray propagation and small core radius. When we compare it with the dispersion in the dispersion in the multimode graded index fiber, the dispersion in the single mode fiber is negligible. That is why single mode fibbers are highly useful in long distance communication systems.

1.1.9 Dispersion-shifted single mode fibers

Generally in single mode fibers, zero dispersion is obtained at a wavelength of about $1.3\mu m$. since there is a finite loss in the silica fiber at $1.3\mu m$, to day the fibers are designed such that there is zero dispersion at 1.5mu m with a minimum loss. At $1.55\mu m$, the material dispersion in single mode fiber is positive and large, while the waveguide dispersion is negative and small. So to increase the waveguide dispersion equal to that of material dispersion, the relative refractive index difference Δ may be slightly increased by adding more GeO2 in the core (which increases the refractive index of the core) or adding more fluorine in the cladding (which decreases the refractive index of the cladding) or instead of the parabolic refractive index profile, a triangular refractive index profile can be designed. Thus the dispersion-shifted fibers have minimum loss and zero dispersion at $1.5\mu m$.

1.1.10 Dispersion-compensating fibers

At present the installed fiber optic links are operating at the wavelength of $1.3\mu m$ using conventional single mode fibers. Instead of $1.3\mu m$ wavelength if one wants to use $1.5\mu m$ wavelength to reduce the transmission loss, then the whole fiber optic link should be replaced with the new dispersion-shifted fibers. This will require an enormous expenditure. To avoid this huge expenditure and to use the old fiber optic links dispersion compensating fibers were evolved. These fibers have a large negative dispersion at $1.5\mu m$, while the conventional single mode fibers operating at $1.3\mu m$ have positive dispersion at $1.5\mu m$. By suitably replacing 1km length of conventional single mode fiber in the fiber optic link with the dispersion compensating fiber fore every 100km length of conventional single mode fiber optic link, one can achieve minimum loss and zero dispersion also.

1.1.11 The Electrostriction Effects

All dielectric materials undergo a strain when subjected to an applied electric field. It results in a slight change in shape. This change in length is termed electrostriction. It causes density changes in the core of a cylindrical fiber that excite the transverse acoustic vibrational eigenmodes of the fiber, generating refractive-index variations by means of the elasto-optic effect.3 The electrostrictive contribution (n_{2e}) to the refractive index is due the intensity dependence of the variation of material density associated with intense electric field from the source.

1.1.12 The Kerr Effects

When the optical medium is isotropic, as in the case of liquids and glasses, the Pockels effect is absent and the polarization is modified by the third-order [in the

expansion of the polarization of the medium, see eq.(3) electro-optic effect, better known as the Kerr effect. This effect, like all third-order effects, occurs whether or not a material possesses inversion symmetry.). The Kerr nonlinearity (n_{2k}) originates from third-order non-resonant electronic susceptibility and Raman susceptibility. The Kerr and electrostrictive effects are strongly interrelated to each other, as both are associated with the change in material density with high electromagnetic field interaction. However, the electrostrictive effect is observed with long pulses and if the higher-order susceptibilities are significant then the Kerr effect also contributes to the change of nonlinear refractive index. Electrostriction has the unique characteristic of being explicitly dependent on the transverse optical intensity profile. Quadratic electrostriction is considered to be one of the effects that can influence the power damage threshold of materials used with high-power laser applications.

1.1.13 Power loss due to electrostriction and Kerr effects

Electrostrictive[3] effect contributes significantly to self action effects in optical fibers, and is 19fields that vary slowly compared with 1 ns as suggested by Smolorz et al. The total nonlinear contribution in the refractive index is given by

$$n_2 = n_{2k} + n_{2e}. \tag{1.1.5}$$

The total power loss (PTLN) due to Kerr non-linearity (P2k) and electrostriction (P2e) in the optical fiber is

$$P_{TLN} = P_{2k} + P_{2e} \tag{1.1.6}$$

The power loss due to Kerr non-linearity is3

$$p_{2k} = \frac{1}{2} n_{2k} A c \varepsilon_o |E|^2 \tag{1.1.7}$$

where c is the velocity of light in free space and A is the area of the core of the optical fiber. Similarly, the power loss due to electrostriction is calculated.

Chapter 2

Optical Properties of Materials

2.1 Polarization of a Dielectric Medium

We take as our model a simple dielectric, that is, a nonconducting material whose properties are isotropic. By nonconducting we mean that the medium, unlike a metal, contains no free charges. Positive charges are associated with the constituent nuclei and negative charges with the electrons bound to such nuclei. By isotropic we mean that the relevant physical properties we consider are independent of direction in the medium, so that we may treat the physical constants as scalar quantities. Application of an electric field to such a medium causes charge displacement, in which the negative charge distribution bound to the nuclei shifts in a direction opposite to the electric field. The shift may occur in a polar molecule, like H2o, because the molecule has a permanent electric dipole, that is, the effective centers of its positive and negative charge distributions do not coincide. In this case, application of the field produces some reorientation of the molecules so that, on the average, the positive end of the dipole is in the direction of the field. The tendency toward alignment is counteracted by the thermal motions of the molecules. The shift in charge distribution may also occur in nonpolar molecules, such as O_2, in which positive and negative

charge distributions normally have the same effective center. Application of the field results in a slight shift of the electron could relative to its nucleus, producing an induced dipole. In either case, the dipole moment due to each atom or molecule is given by the product of the magnitude of the displaced charge and the vector that locates the effective negative charge center relative to the effective positive charge center in the dipole, or

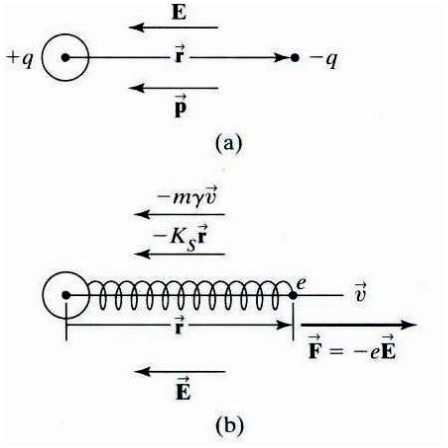

Figure 2.1: The elementary electric dipole.(a)Alignment with the field(b)Forces acting on a dipole when the electric field has the direction indicated.

$$\vec{P} = -q\vec{r} \tag{2.1.1}$$

As indicated in figure 2.1. The direction of the dipole moment is from the negative toward the positive charge. The magnitude of the dipole moment for a given material depends on how easily charge is displaced under the influence of a given electric field. The polarization $\vec{P}$ of the medium is then said to be the collective dipole moment per unit volume, the sum of dipole moments given by

$$\vec{P} = -Ne\vec{r} \tag{2.1.2}$$

where N is the number of elementary dipoles per unit volume and e is the magnitude of the electronic charge. Electrons behave as though the forces binding them to the nuclei are elastic forces given by Hookes law, where the restoring force is proportional to the displacement and oppositely directed. The more massive nuclei can be considered stationary since they are unable to respond to the rapid changes in the field representing an electromagnetic wave in the optical region of the spectrum. A simple model in which electrons are held by springlike forces to a fixed nucleus is therefore applicable. In an alternating electric field, however, forced oscillations of electrons remove a certain amount of energy from the incident radiation, the energy that the electrons radiate in turn and the energy of interaction with neighboring atoms that shows up as thermal energy. The model of the oscillating electron is therefore that of a damped, harmonic oscillator, with a frictional force proportional to the velocity. Newtons second law, applied to the electron in the model of Figure 2.1b, then leads to the equation of motion,

$$-k_s\vec{r} - m\gamma\frac{d\vec{r}}{dx} - \vec{E} = m\frac{d^2\vec{r}}{dt^2} \tag{2.1.3}$$

In Eq. (2.1.3), k_s is the force constant of the effective spring, m is the electronic mass, and γ is a frictional constant with dimensions of reciprocal time. Notice that the force $(e\vec{v} \times \vec{B})$ on the electron due to the magnetic field of the radiation is omitted; it is, in fact, negligible compared with the force $(e\vec{E})$ due to the electric field. When the applied $\vec{E}$-field is static; there is no oscillation of the dipoles, so that both velocity and acceleration of the electron vanishes. In this special case, Eq.(2.1.3) reduces to

$$k_s\vec{r} = e\vec{E} \tag{2.1.4}$$

or, eliminating $\vec{r}$ with the help of Eq. (2.1.2), the static polarization is given by

$$\vec{P} = \frac{Ne^2\vec{E}}{k_s} \tag{2.1.5}$$

Suppose now that $\vec{E}$ is a harmonic field with time dependence given by $\vec{E} = \vec{E}_o \exp -i\omega t$ and that the oscillations respond with a similar dependence,$\vec{r} = r_o \exp -i\omega t$. Note that we are representing the electric field $\vec{E}$ and the dipole displacement vector $\vec{r}$ by complex vectors.

2.2 Propagation of Light Waves in a Dielectric

The four Maxwell equations may be written in the general form

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_o} \tag{2.2.1}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2.2.2}$$

$$\nabla \cdot \vec{B} = 0 \tag{2.2.3}$$

$$\nabla \times \vec{B} = \frac{1}{c^2}(\frac{\partial \vec{E}}{\partial t} + \frac{\vec{J}}{\varepsilon_o}) \tag{2.2.4}$$

In this equation ρ is the charge density, which in general includes both the free charge density ρ_f and the bound charge density ρ_b , so that $\rho = \rho_f + \rho$ .In a dielectric, however, $\rho_f = 0$. It is standard practice in a course in electricity and magnesium to show that the bound-charge density is related to the polarization by

$$\rho_b = - \bigtriangledown \cdot \vec{P} \tag{2.2.5}$$

The quantity $\vec{J}$ similarly represents the current density and can arise from both free and bound charge, as indicated by $\vec{J} = \vec{J}_b + \vec{J}_f$ In a dielectric where $\rho_f = \vec{J}_f = 0$

also. Furthermore, it can be shown that

$$\vec{J_b} = \frac{\partial \vec{P}}{\partial t} \tag{2.2.6}$$

With these constraints, the four Maxwell equations for a dielectric can be written

$$\bigtriangledown \cdot \vec{E} = \frac{-\bigtriangledown \cdot \vec{P}}{\varepsilon_o} \tag{2.2.7}$$

$$\bigtriangledown \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2.2.8}$$

$$\nabla \cdot \vec{B} = 0 \tag{2.2.9}$$

$$c^2 \bigtriangledown \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \frac{1}{\varepsilon_o}\frac{\partial \vec{P}}{\partial t} \tag{2.2.10}$$

Now we take the curl of both sides of Eq. (2.1.13), giving

$$\bigtriangledown \times (\bigtriangledown \times \vec{E}) = \bigtriangledown \times -\frac{\partial \vec{B}}{\partial t} \tag{2.2.11}$$

where we have interchanged the order of differentiation with respect to space and time in the last step. The left member of Eq. (2.1.15) can be reexpressed by the identity

$$\bigtriangledown \times (\bigtriangledown \times \vec{E}) = \nabla(\bigtriangledown \cdot \vec{E}) - \bigtriangledown^2 \vec{E} \tag{2.2.12}$$

In a homogeneous dielectric, the effect of polarization is to produce a net surface charge density, while leaving the internal charge density $\rho_b = 0$ unchanged. The internal charge density is zero because, in any internal closed surface, every bit of charge that moves into the enclosed volume in response to a polarizing field is balanced by an equal bit of charge that moves out. The surface charge density appears because such balancing is not possible there. Thus by Eqs. (17) and (19) we conclude that $\bigtriangledown \cdot \vec{E} = 0$ and substitute the remainder of Eq. (24) into Eq. (28), giving

$$\bigtriangledown^2 \vec{E} = \frac{\partial(\bigtriangledown \times \vec{B})}{\partial t} \tag{2.2.13}$$

For the right member we may make use of Maxwells equation (27) and write

$$c^2 \bigtriangledown^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial t^2} + \varepsilon_o \frac{\partial^2 \vec{P}}{\partial t^2} \tag{2.2.14}$$

The last term is expressed as

$$c^2 \bigtriangledown^2 \vec{E} = [1 + \frac{Ne^2}{m\varepsilon_o}(\omega_o^2 - \omega^2 - i\omega\gamma)] \frac{\partial^2 \vec{E}}{\partial t^2} \tag{2.2.15}$$

For a harmonic wave expressed as $\vec{E} = \vec{E}_o \exp ikz - \omega t$, in which case $\bigtriangledown^2 \vec{E} = -k^2 \vec{E}$ and $\frac{\partial^2 \vec{E}}{\partial t^2}$, Eq. (32) solved for k^2 becomes

$$k^2 = \frac{\omega^2}{c^2}[1 + \frac{Ne^2}{m\varepsilon_o(\omega_o^2 - \omega^2 - i\omega\gamma)}] \tag{2.2.16}$$

We conclude that the analysis of plane waves propagating in a homogeneous dielectric requires in general that the propagation constant be a complex number. Consequently, we write

$$k = k_R + ik_I \tag{2.2.17}$$

Inserting this form into the expression for a harmonic wave, we have

$$\vec{E} = \vec{E}_o \exp i(k_R z + k_I z) - \omega t = \exp k_I z \exp ik_R z - \omega t \tag{2.2.18}$$

The exponential factor in k_I represents a depth-dependent absorption of an otherwise harmonic wave, and it measures the amplitude attenuation of the wave. By taking the square of the magnitude of both sides of Eq. (35), the result describes instead the energy flux density, giving

$$I = I_o \exp -\alpha z \tag{2.2.19}$$

where $\alpha = 2k_I$ is the absorption coefficient of the medium. If the propagation constant is complex, so must be the refractive index, since

$$k \;\; = \;\; \frac{2\pi}{\lambda} = \frac{2\pi\nu}{\upsilon} = (\frac{\omega}{c})n \tag{2.2.20}$$

If we identify the real and imaginary parts of the complex refractive index by

$$n = n_R + in_I \tag{2.2.21}$$

where n_R is the usual refractive index and n_I is called the extinction coefficient, it follows from Eqs. (34) and (37) that

$$k_R + ik_I = (\frac{\omega}{c})n_R + in_I \tag{2.2.22}$$

yielding the relations

$$k_R = (\frac{\omega}{c})n_R \tag{2.2.23}$$

and

$$k_I = (\frac{\omega}{c})n_I \tag{2.2.24}$$

Writing n^2 as

$$n^2 = (n_R + in_I)^2 = (\frac{ck}{\omega})^2 \tag{2.2.25}$$

and relating this equation to Eq. (33) gives

$$n^2 = (n_R + in_I)^2 \tag{2.2.26}$$

$$n^2 = 1 + (\frac{Ne^2}{m\varepsilon_o})(\frac{1}{\omega_o^2 - \omega^2 - i\omega\gamma}) \tag{2.2.27}$$

Expressions for the real and the imaginary parts of the refractive index can be found by equating real and imaginary parts in Eq. (42). The left member is

$$(n_R + in_I)^2 = (n_R^2 + n_I^2) + i2n_R n_I \tag{2.2.28}$$

The right member can also be written as the sum of a real and imaginary part. The complex term is first rewritten by multiplying numerator and denominator by the complex conjugate of the denominator. The result, after simplification, is

$$(n_R + in_I)^2 = 1 + \frac{Ne^2}{m\varepsilon_o}(\frac{\omega_o^2 - \omega^2}{\omega_o^2 - \omega^{2^2} + \omega^2\gamma^2} + i\frac{\omega\gamma}{\omega_o^2 - \omega^{2^2} + \omega^2\gamma^2}) \tag{2.2.29}$$

Now by comparing the right members of Eqs. (43) and (44),

$$n_R^2 - n_I^2 = 1 + \frac{Ne^2}{m\varepsilon_o}\left(\frac{\omega_o^2 - \omega^{2\,2}}{\omega_o^2 - \omega^2} + \omega^2\gamma^2\right) \tag{2.2.30}$$

and

$$2n_R n_I = \frac{Ne^2}{m\varepsilon_o}\left[\frac{\omega\gamma}{\omega_o^2 - \omega^{2\,2} + \omega^2\gamma^2}\right] \tag{2.2.31}$$

The equation can be solved simultaneously for n_R and n_I. The appearance of the mass m in the denominator of this equations shows that electronic oscillations are more important than ionic oscillations in determining the index of refraction. Ionic polarization may be significant in the region of resonance, however, where the large-bracketed terms in Eqs. (45) and (46) balance the small prefactors containing the mass. Figure 5 shows both n_R and n_I calculated from Eqs. (45) and (46) as a function of driving frequency ω. The absorption described by the extinction coefficient n_I is seen to peak at the resonant frequency ω_o. The real refractive index experiences a sharp rise and then a fall as ω increases toward and then passes through resonance, after which it increases again, approaching the value $n_R = 1$ at high frequencies. The narrow region where n_R decrease with frequency is contrary to the usual dispersion of transparent media and is called the region of anomalous dispersion. Figure 4.Angular frequency dependence of the refractive index n_R and the extinction coefficient n1 for a dielectric5.Assumed values are $\omega_o = 1x10^{16}$rad/sec,$\gamma = 10^{14}s^{-1}$and $N = 10^{28}m^{-3}$. A resonance frequency such as ω_o for the dielectric means that, for incident photons of frequency, there is a high probability of absorption. Absorption of such a photon corresponds to a transition between states differing in energy by $\hbar$=$h\nu_o$ in the energy-band structure of the material. As ω is varied, there will be series of resonance frequencies characteristic of the material. If such a resonance occurs in the visible range of frequencies, for example, the material absorbs a portion of the spectrum

and appears colored, while transmitting the reminder. Transparent materials like glass have resonance frequencies in the infrared and ultraviolet regions but not in the visible. In terms of our simplified model of a dielectric, we interpret the existence of a number of resonance frequencies to mean that electrons experience different degrees of freedom in response to the applied field. To take this in to account formally, Eq. (42) is usually generalized to include a number of terms summed over the resonant frequencies ω_j,giving by

$$n^2 = 1 + (\frac{Ne^2}{m\varepsilon_o}) \sum_j \frac{f_j}{\omega_j^2 - \omega^2 - \gamma_j \omega} \tag{2.2.32}$$

where f_j called the oscillator strength for the resonance ω_j, represents the fraction of dipoles having this resonant frequency. The determination of the oscillatory strength for a given resonant transition requires the application of quantum theory.

Chapter 3

Formulation of Model

3.0.1 Second order non-linear effect

In anharmonic oscillator model, the equation of motion of the displaced electron as a forced harmonic oscillator is represented by:

$$\ddot{x} + \gamma\dot{x} + \omega_o^2 x + Dx^2 = (\frac{\varepsilon_O \vec{E_O}}{2m})(\exp(i\omega t) + \exp(-i\omega t)) \tag{3.0.1}$$

Where γ the damping factor, e is electronic charge, m is mass of the electron, ω is the frequency of the laser, $\omega_o$is oscillating frequency of the atom, D second order anharmonic term, We assume a solution of the form

$$x = \frac{1}{2}[q_1 \exp(i\omega t) + q_2 \exp(2i\omega t) + q_3 \exp(3i\omega t) + c.c] \tag{3.0.2}$$

$$\dot{x} = \frac{1}{2}[i\omega q_1 \exp(i\omega t) + 2i\omega q_2 \exp(2i\omega t) + 3i\omega q_3 \exp(3i\omega t) + c.c] \tag{3.0.3}$$

$$\ddot{x} = \frac{1}{2}[-\omega^2 q_1 \exp(i\omega t) - 4\omega^2 q_2 \exp(2i\omega t) - 9\omega^2 q_3 \exp(3i\omega t) + c.c] \tag{3.0.4}$$

Substituting the value of x,$\dot{x} and \ddot{x}$ in equation (3.0.1), we get;

$$-\frac{\omega^2}{2}q_1\exp(i\omega t) - 2\omega^2 q_2\exp(2i\omega t) - \frac{9}{2}\omega^2 q_3\exp(3i\omega t)$$

$$+\frac{\gamma}{2}[i\omega q_1\exp(i\omega t) + 2i\omega q_2\exp(2i\omega t)$$

$$+3i\omega q_3\exp(3i\omega t)] + \frac{\omega_o^2}{2}[q_1\exp(i\omega t) + q_2\exp(2i\omega t) + q_3\exp(3i\omega t)]$$

$$+\frac{D}{4}[q_1^2\exp(2i\omega t) + q_2^2\exp(4i\omega t) + q_3^2\exp(6i\omega t) + 2q_1q_2\exp(3i\omega t)$$

$$+2q_2q_3\exp(5i\omega t) + 2q_1q_3\exp(4i\omega t)] = \frac{eE_o}{2m}(\exp(i\omega t) + \exp(-i\omega t))$$

By equating the coefficients of $\exp(i\omega t)$ on both sides in equation (3.0.5),and then after some rearrangement the nonlinear susceptibility becomes

$$\chi_{NL}^{\omega} = \frac{Ne^2}{m\varepsilon_o(\omega_o^2 - \omega^2 + i\omega\gamma)} \tag{3.0.5}$$

By equating the coefficient of $\exp(2i\omega t)$ on both sides in equation (3.0.5),and by making some rearrangement the second order nonlinear susceptibility becomes

$$\chi_{NL}^{2\omega} = \frac{-DNe^3}{2m^2\varepsilon_o}[\frac{1}{[(\omega_o^2 - \omega^2) + i\omega t]^2}\frac{1}{[(\omega_o^2 - \omega^2) + 2i\gamma\omega]}] \tag{3.0.6}$$

By rationalizing and splitting of eq. (3.0.7) into real and imaginary parts, we get;

$$RE[\chi^{2\omega}] = \frac{-DNe^3}{2m^2\varepsilon_o}[\frac{(\omega_o^2 - 4\omega^2)[(\omega_o^2 - \omega^2)^2 - \omega^2\gamma^2] - 4\omega^2\gamma^2(\omega_o^2 - \omega^2)}{[(\omega_o^2 - \omega^2)^2 + \omega^2\gamma^2]^2[(\omega_o^2 - 4\omega^2)^2 + 4\omega^2\gamma^2]}] \tag{3.0.7}$$

$$IM[\chi^{2\omega}] = \frac{DNe^3}{2m^2\varepsilon_o}[\frac{2\omega\gamma[(\omega_o^2 - \omega^2)[(\omega_o^2 - \omega^2) + (\omega_o^2 - \omega^2)] - \omega^2\gamma^2]}{[(\omega_o^2 - \omega^2)^2 + \omega^2\gamma^2]^2[(\omega_o^2 - 4\omega^2)^2 + 4\omega^2\gamma^2]}] \tag{3.0.8}$$

3.0.2 Third-order non-linear effect

In anharmaonic oscillator model, the equation of motion of the displaced electron as a forced harmonic oscillator is represented by

$$\ddot{x} + \gamma\dot{x} + \omega_o^2 x + Dx^2 + Gx^3 = (\frac{\varepsilon_O \vec{E_O}}{2m})(\exp(i\omega t) + \exp(-i\omega t)) \tag{3.0.9}$$

Where γ is the damping factor, e is electronic charge, m is mass of the electron, ω is the frequency of the laser,ω_o is oscillating frequency of the atom, D second order anharmonic term,G is third order anharmonic term or (3rd order nonlinear coefficient) We assume a solution of the form

$$x = \frac{1}{2}[q_1 \exp(i\omega t) + q_2 \exp(2i\omega t) + q_3 \exp(3i\omega t) + c.c] \tag{3.0.10}$$

$$\dot{x} = \frac{1}{2}[i\omega q_1 \exp(i\omega t) + 2i\omega q_2 \exp(2i\omega t) + 3i\omega q_3 \exp(3i\omega t) + c.c] \tag{3.0.11}$$

$$\ddot{x} = \frac{1}{2}[-\omega^2 q_1 \exp(i\omega t) - 4\omega^2 q_2 \exp(2i\omega t) - 9\omega^2 q_3 \exp(3i\omega t) + c.c] \tag{3.0.12}$$

Substituting the values of x,$\dot{x} and \ddot{x}$ in eq. (3.0.10) we get By rearranging and equating the coefficients of $\exp(i\omega t)$ on both side of equation (3.0.14), we get

$$q_1 = \frac{eE_o}{m}\frac{1}{(\omega_o^2 - \omega^2 + i\omega\gamma)} \tag{3.0.13}$$

By equating the coefficients of $\exp(2i\omega t)$ on both sides in eq. (3.0.10) we get;

$$q_2 = \frac{-De^2E_o^2}{2m^2}\frac{1}{[(\omega_o^2 - \omega^2) + i\omega\gamma]^2[(\omega_o^2 - 2\omega^2) + 2i\omega\gamma]} \tag{3.0.14}$$

By equating the coefficients of $\exp(3i\omega t)$ on both sides in eq. (3.0.10) we get;

$$q_3 = \frac{e^3E_o^3}{2m^3}\frac{1}{[(\omega_o^2 - \omega^2) + i\omega\gamma]^3[(\omega_o^2 - 9\omega^2) + 3i\omega\gamma]}[\frac{D^2}{[(\omega_o^2 - 4\omega^2) + 2i\omega\gamma]} - \frac{G}{2}] \tag{3.0.15}$$

The third-order nonlinearity susceptibility becomes

$$\chi^{3\omega} = \frac{Ne^4}{2m^2\varepsilon_o[[\omega_o^2 - \omega^2 + i\omega\gamma]^3][\omega_o^2 - 9\omega^2 + 3i\omega]}[\frac{D^2}{[\omega_o^2 - 4\omega^2 + 2i\omega\gamma]} - \frac{G}{2}] \tag{3.0.16}$$

By rationalizing and splitting of eq. (3.0.18) in to real and imaginary part we gate the imaginary part of the third order susceptibility

$$IM[\chi^{3\omega}] = \frac{Ne^4}{2\varepsilon_o m^3}[\frac{-D^2[\omega\gamma[(\omega_o^2-\omega^2)^3-3\omega^2\gamma^2](5\omega_o^2-30\omega^2)+[3\omega\gamma(\omega_o^2-\omega^2)^2-\omega^3\gamma^3][(\omega_o^2-\omega^2)^2-}{[(\omega_o^2-\omega^2)^2+\omega^2\gamma^2]^3[(\omega_o^2-9\omega^2)^2+9\omega^2\gamma^2][(\omega_o^2-4\omega^2)^2+4\omega^2\gamma^2]} \tag{3.0.17}$$

3.0.3 Fifth Order nonlinear effect

In anharmonic oscillator model, the equation of motion of the displaced electron as a forced harmonic oscillator is represented by:

$$\ddot{x}+\gamma\dot{x}+\omega_o^2 x+Dx^2+Fx^5=(\frac{\varepsilon_O\vec{E_O}}{2m})(\exp(i\omega t)+\exp(-i\omega t)) \tag{3.0.18}$$

Where γ the damping factor, e is electronic charge, m is mass of the electron,ω is the frequency of the laser, $\omega_o$is oscillating frequency of the atom, D second order anharmonic term,G is third order anharmonic term or (3rd order nonlinear coefficient), F is the fifth-order anharmonic term. We assume a solution of the form

$$\ddot{x}+\gamma\dot{x}+\omega_o^2 x+Dx^2=(\frac{\varepsilon_O\vec{E_O}}{2m})(\exp(i\omega t)+\exp(-i\omega t)) \tag{3.0.19}$$

Where γ the damping factor, e is electronic charge, m is mass of the electron, ω is the frequency of the laser, $\omega_o$is oscillating frequency of the atom, D second order anharmonic term, We assume a solution of the form

$$x=\frac{1}{2}[q_1\exp(i\omega t)+q_2\exp(2i\omega t)+q_3\exp(3i\omega t)+c.c] \tag{3.0.20}$$

$$\dot{x}=\frac{1}{2}[i\omega q_1\exp(i\omega t)+2i\omega q_2\exp(2i\omega t)+3i\omega q_3\exp(3i\omega t)+c.c] \tag{3.0.21}$$

$$\ddot{x}=\frac{1}{2}[-\omega^2 q_1\exp(i\omega t)-4\omega^2 q_2\exp(2i\omega t)-9\omega^2 q_3\exp(3i\omega t)+c.c] \tag{3.0.22}$$

Substituting the values of x,$\dot{x} and \ddot{x}$ in equation (3.0.21) we get

$$\frac{-\omega^2}{2}q_1\exp(i\omega t) - 2\omega^2 q_2\exp(2i\omega t) - \frac{9}{2}\omega^2 q_3\exp(3i\omega t) - \frac{25}{2}\omega^2 q_5\exp(5i\omega t)$$

$$+\frac{\gamma}{2}[i\omega q_1\exp(i\omega t) + 2i\omega q_2\exp(2i\omega t) + 3i\omega q_3\exp(3i\omega t) + q_5\exp(5i\omega t)]$$

$$+\frac{\omega^2}{2}[q_1\exp(i\omega t) + q_2\exp(2i\omega t) + q_3\exp(3i\omega t) + q_5\exp(5i\omega t)] + \frac{D}{4}(q_1\exp(i\omega t)$$

$$+q_2\exp(2i\omega t) + q_3\exp(3i\omega t) + q_5\exp(5i\omega t))^2 + \frac{G}{8}(q_1\exp(i\omega t)$$

$$+q_2\exp(2i\omega t) + q_3\exp(3i\omega t) + q_5\exp(5i\omega t))^2 + \frac{G}{8}(q_1\exp(i\omega t)$$

$$+q_2\exp(2i\omega t) + q_3\exp(3i\omega t) + q_5\exp(5i\omega t))^3\frac{F}{32}(q_1\exp(i\omega t)$$

$$+q_2\exp(2i\omega t) + q_3\exp(3i\omega t) + q_5\exp(5i\omega t))^5 = \frac{eE_o}{2m}(\exp(i\omega t) + \exp(-i\omega t))$$

Equating the coefficients of $\exp(i\omega t)$ on both sides in eq. (3.0.25) we get

$$q_1 = \frac{eE_0}{m}[\frac{1}{\omega_o^2 - \omega^2 i\omega\gamma}] \tag{3.0.23}$$

Equating the coefficients of $\exp(2i\omega t)$ on both sides in eq. (3.0.25) we get

$$q_2 = \frac{-De^2E_0^2}{2m^2}[\frac{1}{[\omega_o^2 - \omega^2 + i\omega\gamma]^2[(\omega_o^2 - 4\omega^2) + 2i\omega\gamma]}] \tag{3.0.24}$$

Equating the coefficients of $\exp(3i\omega t)$ on both sides in eq. (3.0.25) we get

$$q_3 = \frac{-D^3e^3E_0^3}{2m^3}[\frac{1}{[(\omega_o^2-\omega^2)][\frac{1}{[(\omega_o^2-\omega^2)+i\omega\gamma]^2}}][\frac{1}{[(\omega_o^2-4\omega^2)+2i\omega\gamma]}[\frac{1}{(\omega_o^2-9\omega^2)+3i\omega\gamma}] \tag{3.0.25}$$

Equating the coefficients of $\exp(5i\omega t)$ on both sides of eq. (3.0.25) we get;

$$q_5 = \frac{e^5E_o^5}{4m^5}\frac{1}{[(\omega_o^2-\omega^2)+i\omega\gamma]^5[(\omega_o^2-25\omega^2)+5i\omega\gamma]}$$

$$[\frac{D^2}{[(\omega_o^2-4\omega^2)+2i\omega\gamma][(\omega_o^2-9\omega^2)+3i\omega\gamma]}$$

$$(\frac{D^2}{(\omega_o^2-4\omega^2)+2i\omega\gamma}-\frac{G}{2})-\frac{3G}{4}\frac{D^2}{[(_o^2-4\omega^2)+2i\omega\gamma}-\frac{G}{2})$$

$$-\frac{3G}{4}\frac{D^2}{[(_o^2-4\omega^2)+2i\omega\gamma]^2}-\frac{3G}{2[(\omega_o^2-9\omega^2)+3i\omega\gamma]}$$

$$[\frac{D^2}{[(\omega_o^2-4\omega^2)+2i\gamma][(\omega_o^2-9\omega^2)+3i\omega\gamma]}-\frac{G}{2}]-\frac{F}{4}]$$

The imaginary parts fifth order nonlinear susceptibility is given by;

(3.0.26)

$$IM[\chi^{5\omega}] = \frac{Ne^6}{4m^5\varepsilon_o}[\frac{D^4[e((rv+su)+f(ru-sv)]}{A1A2A3^2A4} \tag{3.0.27}$$

(3.0.28)

$$-\frac{GD^2}{2}\frac{(rv+su)}{A1A2A3A4} - \frac{3GD^2}{4}\frac{(xs+yr)}{A1A2A3^2} \tag{3.0.29}$$

(3.0.30)

$$\frac{3GD^2}{4}\frac{j(gs+hr)+k(gr-hs)}{A1A2A3A4} + \frac{3G^2}{4}\frac{gs+hr}{A1A2A4} - \frac{Fs}{4A1A2}] \tag{3.0.31}$$

(3.0.32)

Constants used in eq.(3.0.19)are given below.

$$e = (\omega_o^2 - 4\omega^2) \quad f = 2\omega\gamma$$

$$\begin{aligned} r = [(\omega_o^2 - 25\omega^2)[(\omega_o^2 - \omega^2)^5 - 4\omega^2\gamma^2(\omega_o^2 - \omega^2)^3 \\ -3\omega^4\gamma^4(\omega_o^2 - \omega^2) - 2\omega^4\gamma^4(\omega_o^2 - \omega^2) + 6\omega^2\gamma^2(\omega_o^2 - \omega^2)^3] + \\ +5\omega\gamma[(\omega_o^2 - \omega^2)^3\omega^3\gamma^3 - 3\omega\gamma(\omega_o^2 - \omega^2)^4 - \omega^5\gamma^5 - 3\omega^3\gamma^3(\omega_o^2 - \omega)^2 - \\ 2\omega\gamma(\omega_o^2 - \omega^2)^4 + 6\omega^3\gamma^3(\omega_o^2 - \omega^2)^2]] \end{aligned}$$

$$\begin{aligned} s = [-5\omega\gamma[(\omega_o^2 - \omega^2)^5 + 3\omega^2\gamma^2(\omega_o^2 - \omega^2)^3 + \omega^2\gamma^2(\omega_o^2 - \omega^2)^3 - 3\omega^4\gamma^4(\omega_o^2 - \omega^2) - \\ 2\omega^4\gamma^4(\omega_o^2 - \omega^2) + 6\omega^2\gamma^2(\omega_o^2 - \omega^2)^3] + (\omega_o^2 - 25\omega^2)[\omega^3\gamma^3(\omega_o^2 \\ -\omega^2)^3 - \omega^5\gamma^5 + 3\omega^3\gamma^3(\omega_o^2 - \omega^2)^2 - 2\omega\gamma(\omega_o^2 - \omega^2)^4 + 6\omega^3\gamma^3(\omega_o^2 - \omega^2)^2]] \end{aligned}$$

$$u = [(\omega_o^2 - 4\omega^2)(\omega_o^2 - 9\omega^2) - 6\omega^2\gamma^2] \quad v = [3\omega\gamma(\omega_o^2 - 4\omega^2) + 2\omega\gamma(\omega_o^2 - 9\omega^2)]$$

$$x = [(\omega_o^2 - 4\omega^2)^2 - 4\omega^2\gamma^2] \qquad y = 4\omega\gamma(\omega_o^2 - 4\omega^2)$$

$$g = [\omega_o^2 - 9\omega^2] \qquad j = [\omega_o^2 - 4\omega^2]$$

$$A1 = [(\omega_o^2 - \omega^2)^2 + \omega^2\gamma^2]^5 \qquad A2 = [(\omega_o^2 - 25\omega^2)^2 + 25\omega^2\gamma^2]$$

$$A3 = [(\omega_o^2 - 4\omega^2)^2 + 4\omega^2\gamma^2] \qquad A4 = [(\omega_o^2 - 9\omega^2)^2 + 9\omega^2\gamma^2]$$

$$A6 = (A3)^2 \qquad A7 = A4$$

$$A8 = A3 \qquad h = 2\omega\gamma$$

The value of the second, third, and fifth order anharmonic term (i.e. the value of D,G, and F. G = (2%)D , and F=(1%)G) can be calculated as:

$$D = \frac{-39.9e^2}{4\pi\varepsilon_o m r_o^2} = -2.5x10^{23} \tag{3.0.33}$$

$$\gamma = \frac{\mu_o e^2 \omega_o^2}{6\pi mc} \cong 0.0625 \tag{3.0.34}$$

We used all the above expreessions to generate data for nonlinear index of refraction, susceptibilities, and power loss using matlab in the next chapter.

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Chapter 4

Results and Discussions

4.1 Results

Results for second-, third-, and fifth-order nonlinear susceptibilities. Using $\gamma_{1,2} = (\pm x\%)\gamma$. (Where x = 1,2,5.).

The results which are shown in the next pages are obtained by using matlab. in the matlab we used expressions which are drived from the microscopic model we used. The model says that, two oxygen atoms are connected with the silicon atom with an ideal spring, which oscillates anharmonically. This type of oscillation makes the system to have nonluinearity property. In our calculation we used some constants which are very important to generate data for nonlinear index of refraction, susceptibilities, and power loss as a function of both damping factor and frequency.

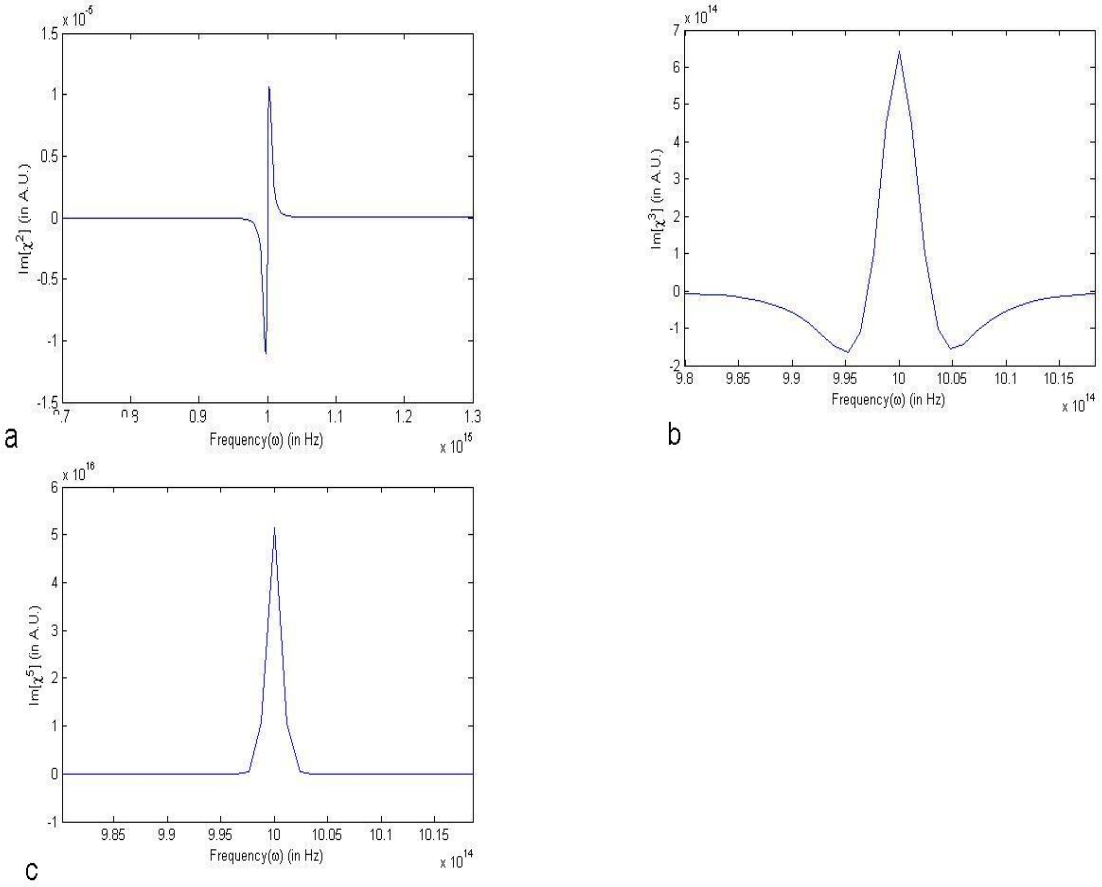

Figure 4.1: Variation of imaginary part of nonlinear susceptibility versus frequency.By using $\gamma 1 = (-1\%)\gamma$. (a)second-order (b)third-order (c) fifth-order

From the graph we observed that, $Im[\chi^{2\omega}]$, $Im[\chi^{3\omega}]$, and $Im[\chi^{5\omega}]$ increases dramatically as ω_o, so that ω_o represents a resonance frequency for the dipoles of the medium. As the driving frequency approaches the response frequency ω_o of the oscillator, the amplitude of the vibrations becomes very large and subsides again as the frequency increases beyond ω_o.

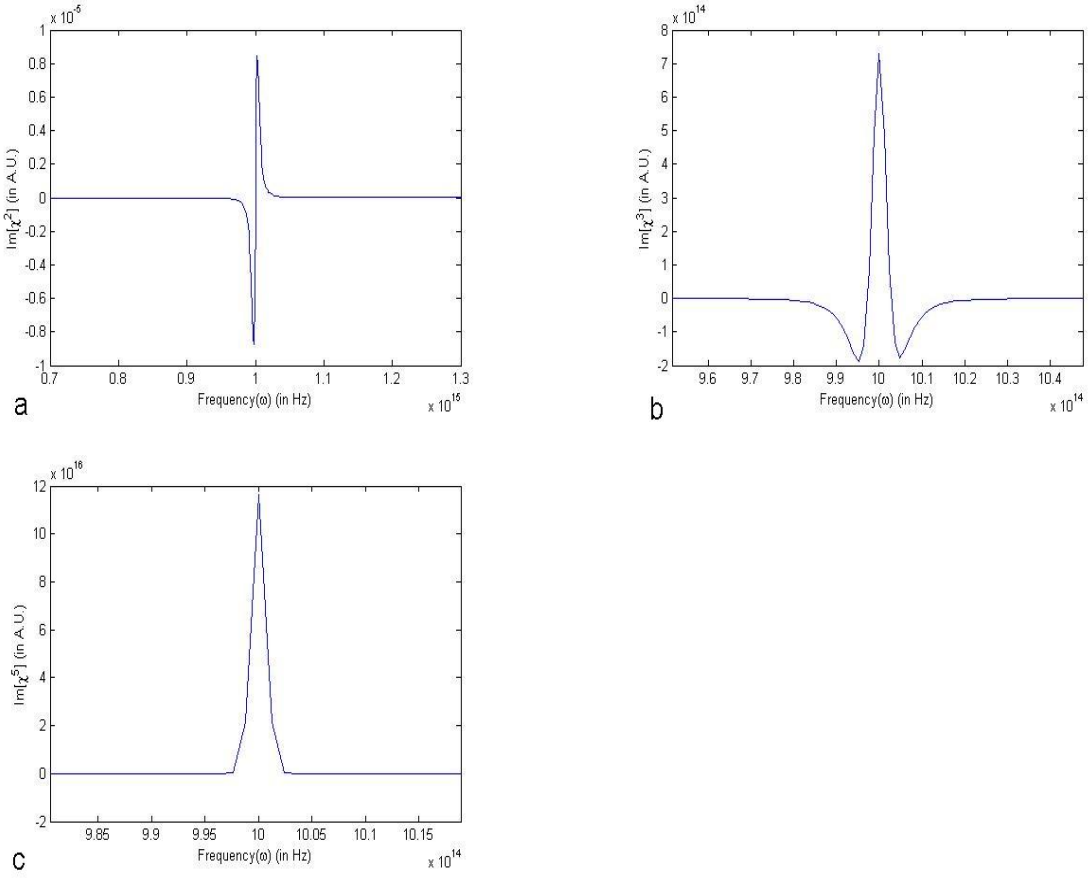

Figure 4.2: Variation of imaginary part of nonlinear susceptibility versus frequency.By using $\gamma 1 = (-2\%)\gamma$. (a)second-order (b)third-order (c) fifth-order
It shows that, the response of the imaginary part of the nonlinear susceptibility becomes very large as the driving frequency approaches the resonant frequency of the oscillator. Results for second-, third-, and fifth-order nonlinear refractive index.

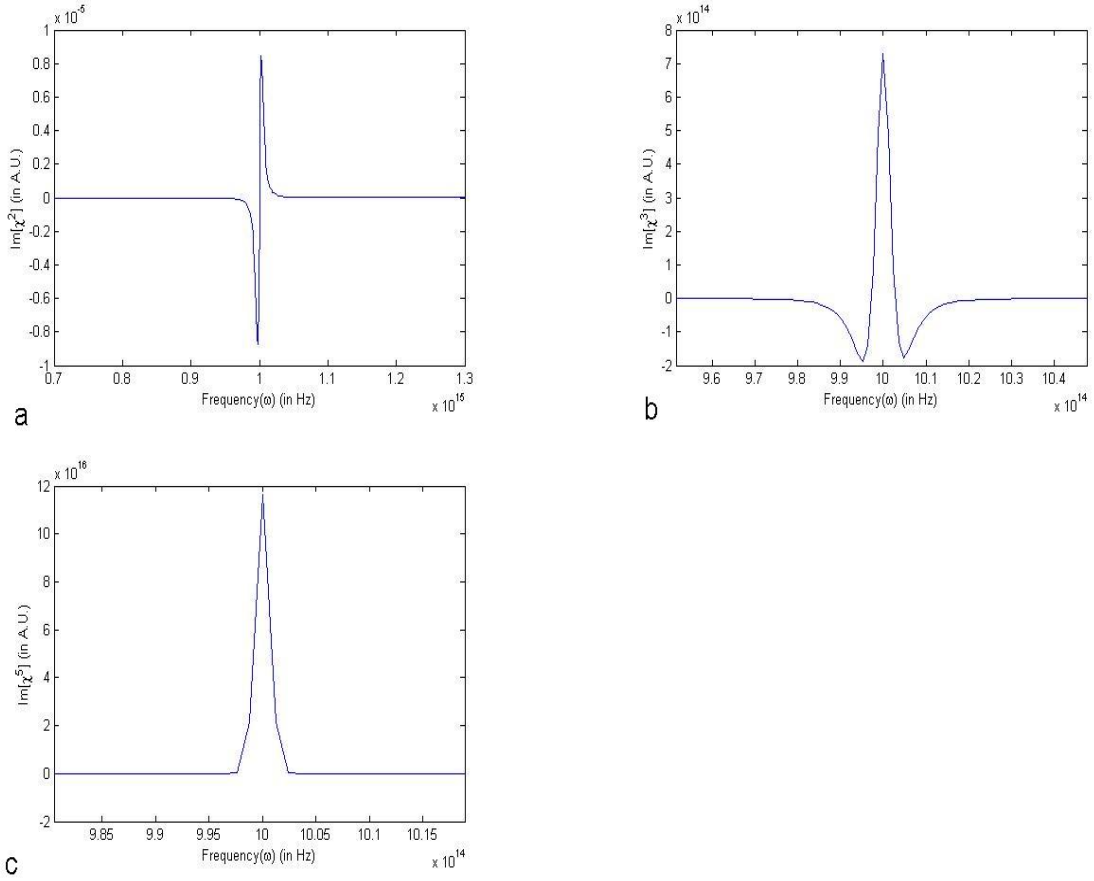

Figure 4.3: Variation of imaginary part of nonlinear susceptibility versus frequency.By using $\gamma 1 = (-5\%)\gamma$. (a)second-order (b)third-order (c) fifth-order
It shows that, the response of the imaginary part of the nonlinear susceptibility becomes very large as the driving frequency approaches the resonant frequency of the oscillator. Results for second-, third-, and fifth-order nonlinear refractive index.

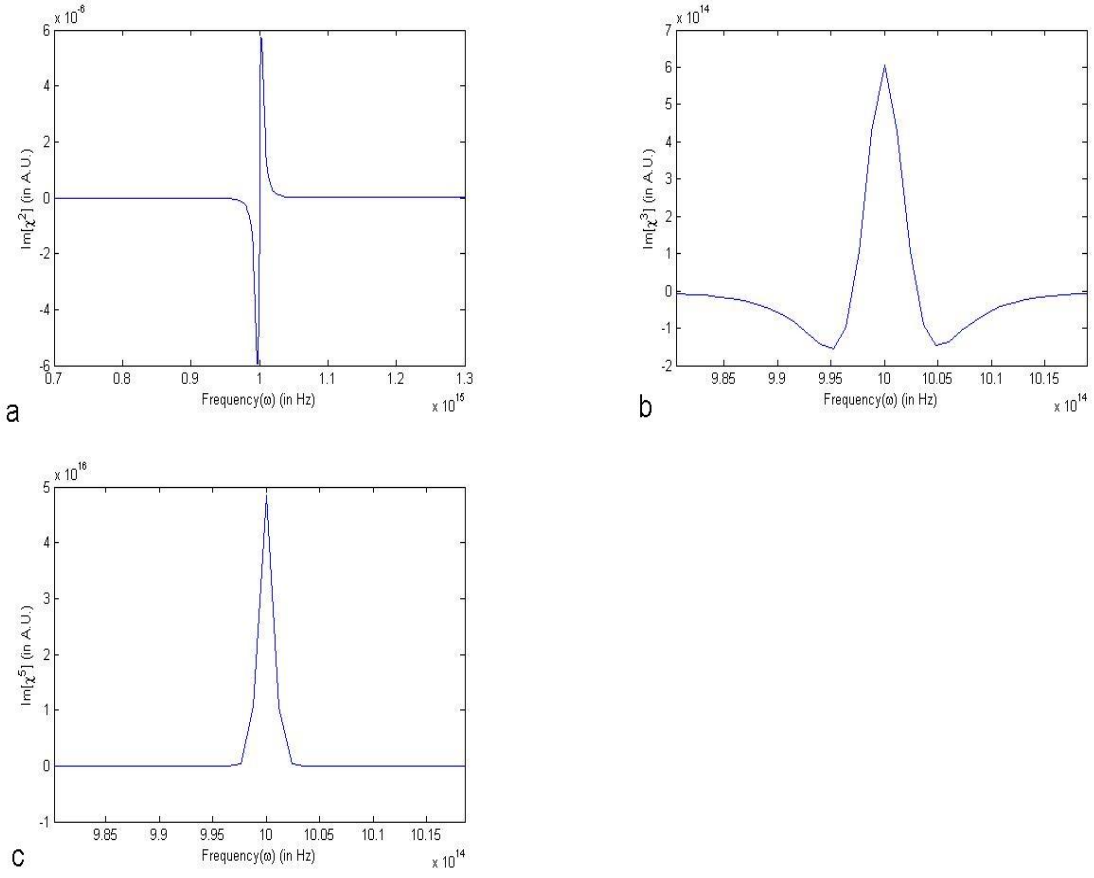

Figure 4.4: Variation of imaginary part of nonlinear susceptibility versus frequency.By using $\gamma 2 = (+1\%)\gamma$.By using $\gamma 1 = (+1)\gamma$. (a)second-order (b)third-order (c) fifth-order

From the graph we observed that, $Im[\chi^{2\omega}]$, $Im[\chi^{3\omega}]$, and $Im[\chi^{5\omega}]$ increases dramatically as ω_o, so that ω_o represents a resonance frequency for the dipoles of the medium. As the driving frequency approaches the response frequency ω_o of the oscillator, the amplitude of the vibrations becomes very large and subsides again as the frequency increases beyond ω_o.

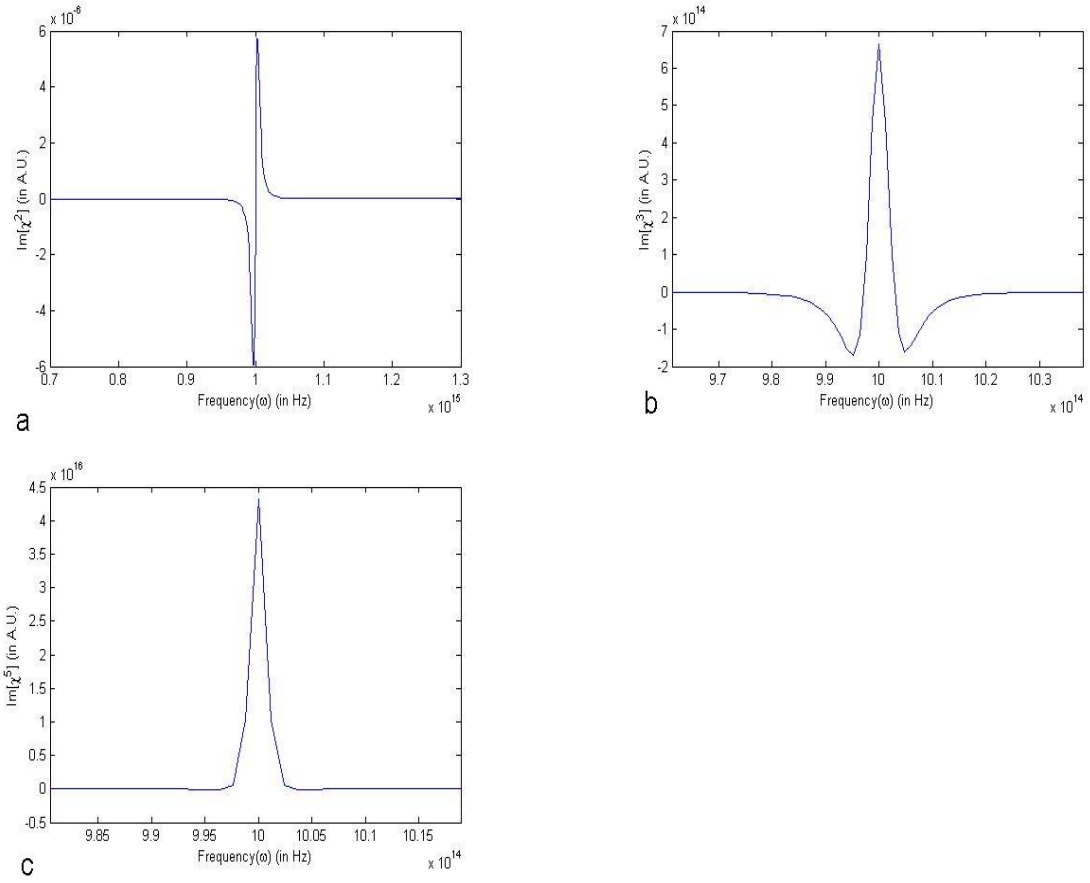

Figure 4.5: Variation of imaginary part of nonlinear susceptibility versus frequency.By using $\gamma_2 = (+2\%)\gamma$. (a)second-order (b)third-order (c) fifth-order
The figure shows that, the response of the imaginary part of the nonlinear susceptibility becomes very large as the driving frequency approaches the resonant frequency of the oscillator.

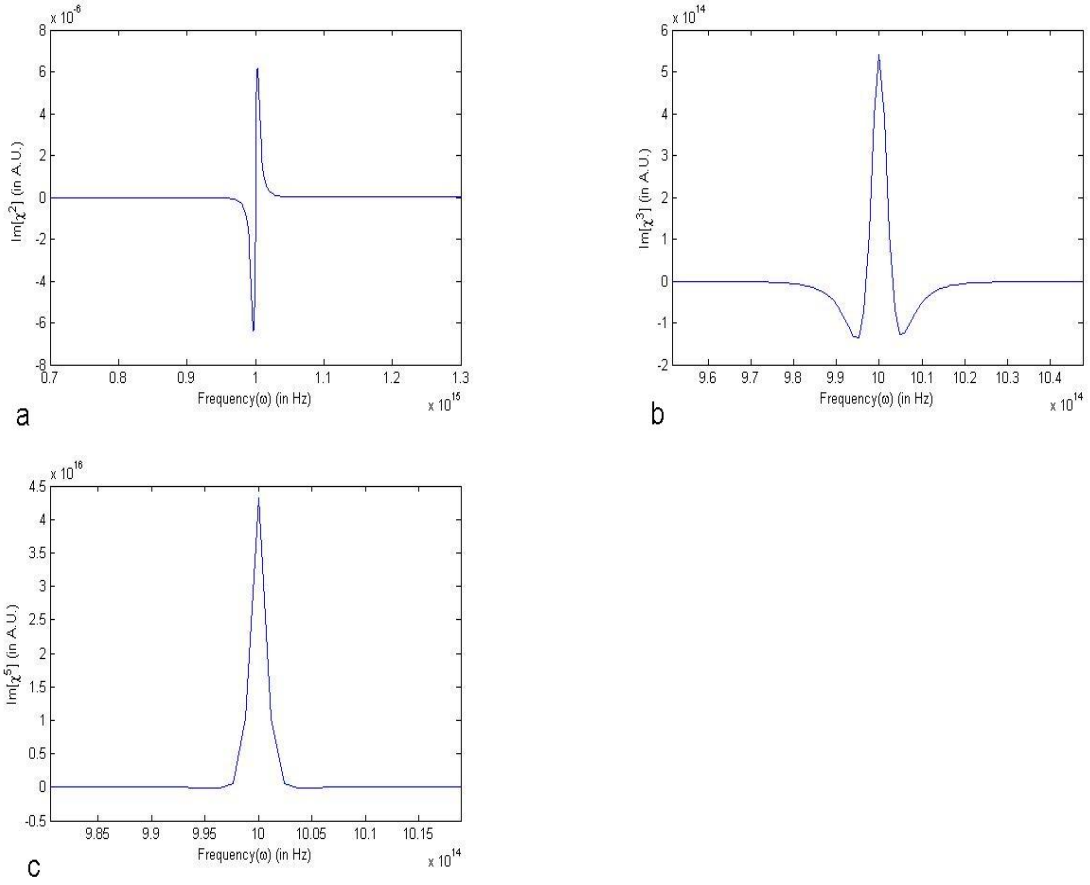

Figure 4.6: Variation of imaginary part of nonlinear susceptibility versus frequency.By using $\gamma_2 = (+5\%)\gamma$. (a)second-order (b)third-order (c) fifth-order
From the graph we observed that, $Im[\chi^{2\omega}]$, $Im[\chi^{3\omega}]$, and $Im[\chi^{5\omega}]$ increases dramatically as ω_o, so that ω_o represents a resonance frequency for the dipoles of the medium. As the driving frequency approaches the response frequency ω_o of the oscillator, the amplitude of the vibrations becomes very large and subsides again as the frequency increases beyond ω_o.

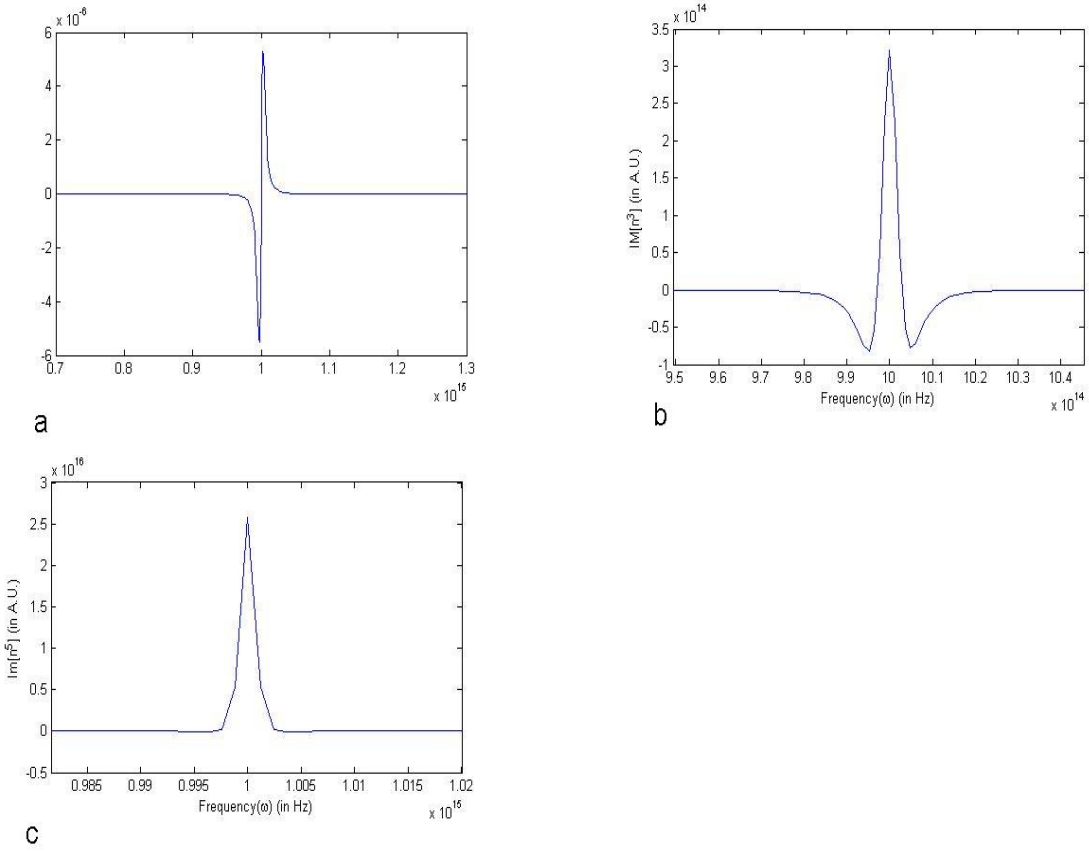

Figure 4.7: Variation of imaginary part of nonlinear index of refraction versus frequency.By using $\gamma 1 = (-1\%)\gamma$. (a)second-order (b)third-order (c) fifth-order
From the figure we understand that, the imaginary part of the index of refraction is seen to peak at the resonant frequency ω_o.

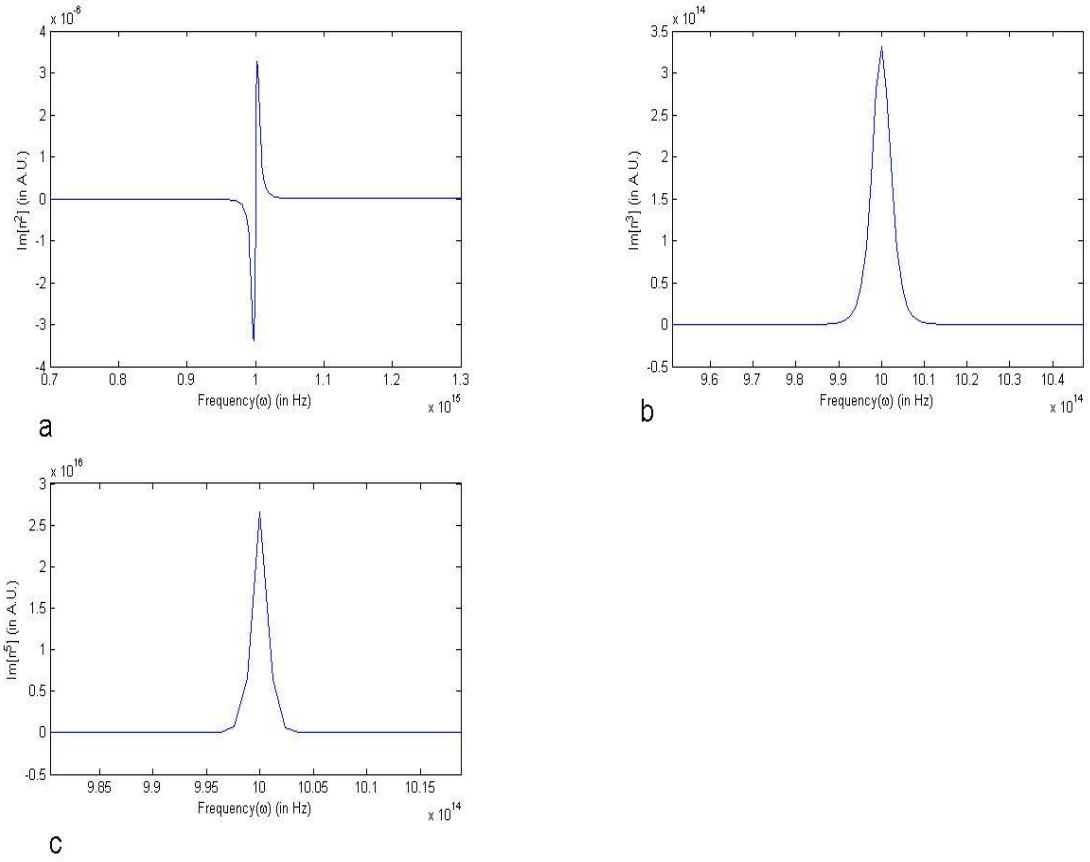

Figure 4.8: Variation of the imaginary part of the nonlinear index of refraction versus frequency.By using $\gamma 1 = (-2\%)\gamma$. (a)second-order (b)third-order (c) fifth-order .The figure shows that, the response of the imaginary part of the nonlinear refractive index becomes very large as the driving frequency approaches the resonant frequency of the oscillator. so that ω_o represents a resonance frequency for the dipoles of the medium.

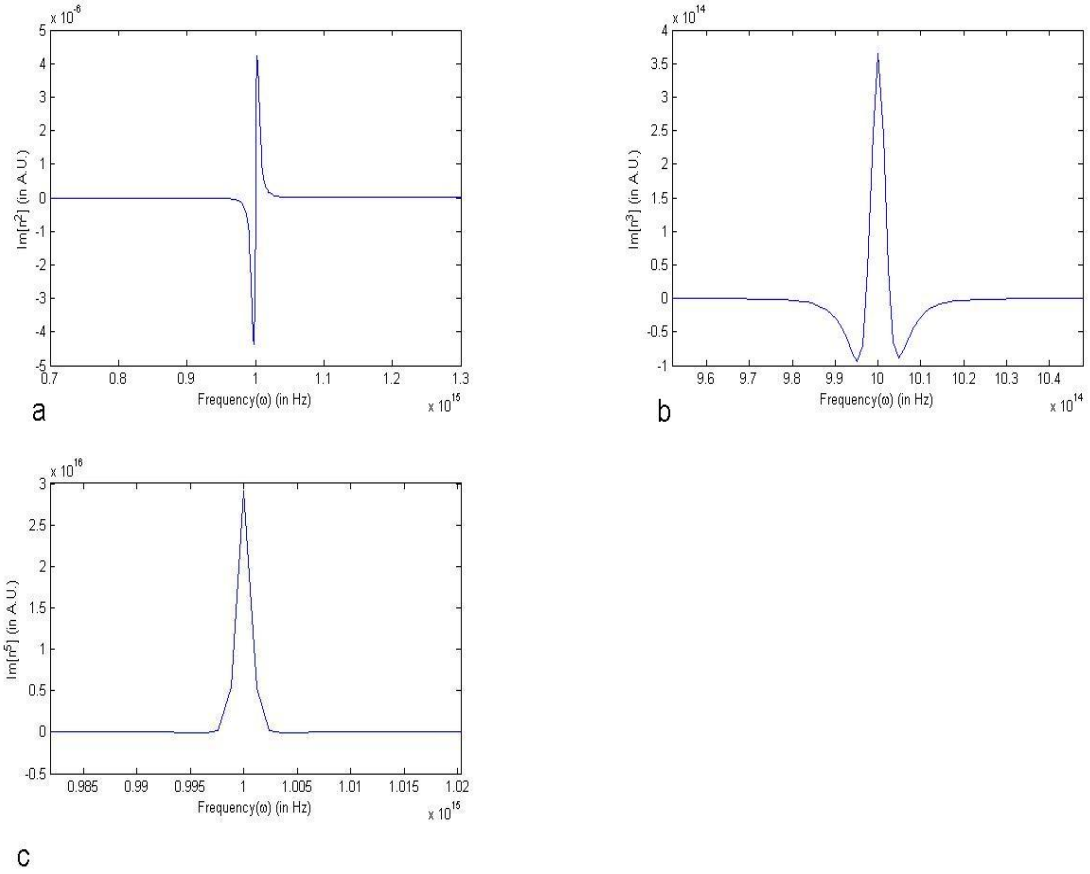

Figure 4.9: Variation of the imaginary part of the nonlinear index of refraction versus frequency. By using $\gamma 1 = (-5\%)\gamma$. (a)second-order (b)third-order (c) fifth-order .The figure tell us that, the fifth order nonlinear index of refraction becomes maximum as the field frequency nearer to the resonance frequency of the oscillator.

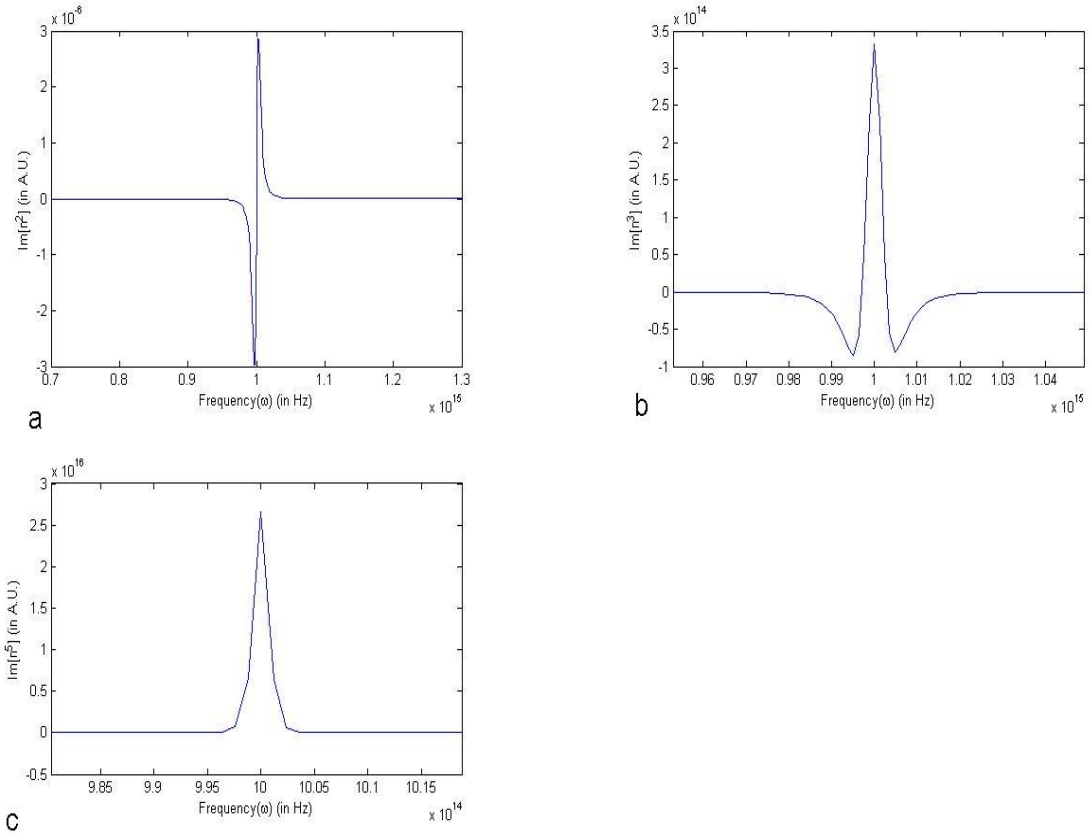

Figure 4.10: Variation of the imaginary part of the nonlinear index of refraction versus frequency. By using $\gamma 2 = (+5\%)\gamma$. (a)second-order (b)third-order (c) fifth-order
The figure shows that, the response of the imaginary part of the nonlinear refractive index becomes very large as the driving frequency approaches the resonant frequency of the oscillator. so that ω_o represents a resonance frequency for the dipoles of the medium.

Fiber type	Damping factor γ=	Nonlinear refractive Index (in A.U.)		Power loss (in A.U.)	
PSCF (1)	γ_{1}= 0.061875	n_{2e}^{2}	1.045	p_{2e}^{2}	27.44
		n_{2k}^{2}	4.455	p_{2k}^{2}	116.99
	γ_{2} = 0.062625	n_{2e}^{2}	0.95	p_{2e}^{2}	4.95
		n_{2k}^{2}	4.05	p_{2k}^{2}	106.35
DSF (2)	γ_{1}= 0.06125	n_{2e}^{2}	0.665	p_{2e}^{2}	17.46
		n_{2k}^{2}	2.835	p_{2k}^{2}	74.45
	γ_{2} = 0.06325	n_{2e}^{2}	0.57	p_{2e}^{2}	14.97
		n_{2k}^{2}	2.43	p_{2k}^{2}	63.81
DCF (3)	γ_{1}= 0.059375	n_{2e}^{2}	0.855	p_{2e}^{2}	22.45
		n_{2k}^{2}	3.645	p_{2k}^{2}	95.72
	γ_{2} = 0.065625	n_{2e}^{2}	0.627	p_{2e}^{2}	16.47
		n_{2k}^{2}	2.673	p_{2k}^{2}	70.19

Table 1. Power loss due to Kerr and Electrostrictive effects while considering second-order nonlinearity.

It is shown that the power loss due to the imaginary part of the second order nonlinear index of refraction by DCF is largerthan the loss by the DSF which in turn has greater power loss tha the PSCF.

Fiber type	Damping factor	Nonlinear refractyive Index (in A.U.)		Power loss (in A.U.)	
PSCF (1)	γ_{1}= 0.061875	n_{2e}^{2}	0.627	p_{2e}^{2}	16.47
		n_{2k}^{2}	2.673	p_{2k}^{2}	70.19
	γ_{2} = 0.062625	n_{2e}^{2}	0.570	p_{2e}^{2}	14.97
		n_{2k}^{2}	2.430	p_{2k}^{2}	63.81
DSF (2)	γ_{1}= 0.06125	n_{2e}^{2}	0.646	p_{2e}^{2}	16.96
		n_{2k}^{2}	2.754	p_{2k}^{2}	72.30
	γ_{2} = 0.06325	n_{2e}^{2}	0.646	p_{2e}^{2}	16.96
		n_{2k}^{2}	2.754	p_{2k}^{2}	72.32
DCF (3)	γ_{1}= 0.059375	n_{2e}^{2}	0.667	p_{2e}^{2}	17.50
		n_{2k}^{2}	2.835	p_{2k}^{2}	74.45
	γ_{2} = 0.065625	n_{2e}^{2}	0.513	p_{2e}^{2}	13.47
		n_{2k}^{2}	2.187	p_{2k}^{2}	21.27

Table 2. Power loss due to Kerr and Electrostrictive effects while considering third-order nonlinearity.

It is shown that the power loss due to the imaginary part of the third order nonlinear index of refraction by DCF is larger than the loss by the DSF which in turn has greater power loss that the PSCF. But these loss is smaller than the loss produced by the second order nonlinear case. The table also indicates that, the loss by the Kerr nonlinearity effects is larger than that of the loss due to the electrostrictive effects.

Fiber type	Damping factor	Nonliniear refractive Index (in A.U.)		Power loss (in A.U.)	
PSCF (1)	$\gamma_1 = 0.061875$	n_{2e}^2	0.243	p_{2e}^2	6.39
		n_{2k}^2	1.037	p_{2k}^2	27.23
	$\gamma_2 = 0.062625$	n_{2e}^2	0.230	p_{2e}^2	6.04
		n_{2k}^2	0.980	p_{2k}^2	25.74
DSF (2)	$\gamma_1 = 0.06125$	n_{2e}^2	0.253	p_{2e}^2	6.64
		n_{2k}^2	1.077	p_{2k}^2	28.29
	$\gamma_2 = 0.06325$	n_{2e}^2	0.251	p_{2e}^2	6.59
		n_{2k}^2	1.069	p_{2k}^2	28.08
DCF (3)	$\gamma_1 = 0.059375$	n_{2e}^2	0.277	p_{2e}^2	7.28
		n_{2k}^2	1.183	p_{2k}^2	31.06
	$\gamma_2 = 0.065625$	n_{2e}^2	0.243	p_{2e}^2	6.39
		n_{2k}^2	1.037	p_{2k}^2	21.27

Table 3. Power loss due to Kerr and Electrostrictive effects while considering fifth-order nonlinearity

It is shown that the power loss due to the imaginary part of the fifth order nonlinear index of refraction by DCF is largerthan the loss by the DSF which in turn has greater power loss tha the PSCF. But these loss is smaller than the loss by both the third order and the second order nonlinear case.The table also indicates that, the loss by the Kerr nonlinearity effects is largerthan that of the loss due to the electrostrictive effects.

Fiber type	Average refractive index (in A.U.)			Average total power loss (in A.U.)		
	2^{nd}-order	3^{rd}-order	5^{th}-order	2^{nd}-order	3^{rd}-order	5^{th}-order
PSCF (1)	3.120	2.205	1.245	81.93	57.90	32.69
DSF (2)	3.200	2.235	1.325	84.03	58.69	34.79
DCF (3)	3.305	2.265	1.370	86.79	59.48	35.98

Table 4. The average refractive index and the average total power loss for PSCF, DSF, and DCF fiber.

In all ordered terms, we observed that the imaginary part of the refractive index of DCF is larger than that of DSF which in turn has larger refractive index than PSCF

Fiber type	Power loss (in A.U.)	
	P_{2e}	P_{2k}
PSCF	15.57	66.36
DSF	15.97	68.06
DCF	16.49	70.30

Table 5. Total power loss due toKerr (P_{2k}) and electrostrictive effects (P_{2e}) in diferent type of fibers.

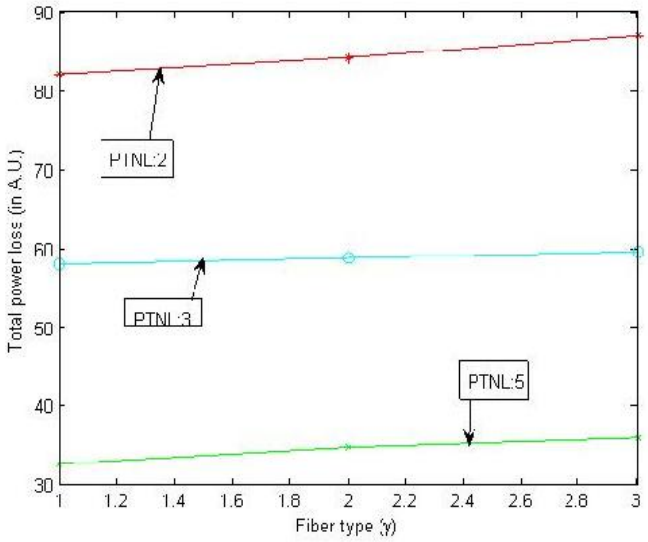

Figure 4.11: Total power loss versus different types of fibbers. The figure shows that, the power loss due to second order nonlinearity is larger than that of due to the third order nonlinearity which in turn greater than that of due to the fifth order nonlinearity.

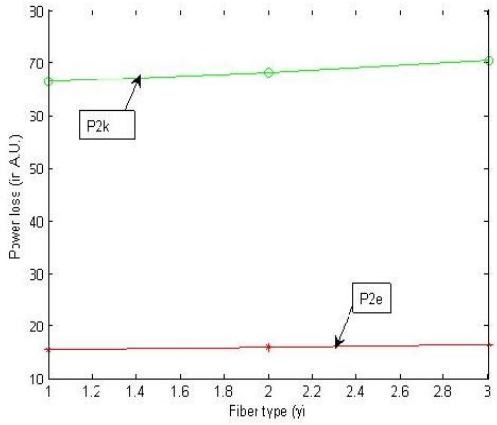

Figure 4.12: Total power loss due to Kerr (P_{2k}) and electrostrictive effects (P_{2e}) in PSCF, DSF, and DCF.The figure depicts that, the power loss by the Kerr nonlinearity is larger than the power loss by the electrostrictive effects.

Fiber type	Refractive index (in A.U.)		Percentile Diference (in %)
	2^{nd} -order	3^{rd} -order	
PSCF	3.120	2.205	34.37
DSF	3.200	2.235	35.51
DCF	3.305	2.265	38.42

Table 6. Percentile comparison of the 2nd and the 3rd order refractive index for PSCF, DSF, and DCF fibers.

Fiber type	Refractive index (in A.U.)		Percentile Diference (in %)
	2^{nd} -order	5^{th} -order	
PSCF	3.120	1.245	85.91
DSF	3.200	1.325	41.44
DCF	3.305	1.370	82.78

Table 7, shows that the percentile change between 2nd and 5th order nonlinear refractive index of DCF is larger than that of DSF which in turn has larger percentile change of refractive index than PSCF.

Fiber type	Refractive index (in A.U.)		Percentile Diference (in %)
	3^{rd} -order	5^{th} -order	
PSCF	2.205	1.245	55.65
DSF	2.235	1.325	51.12
DCF	2.265	1.370	49.24

Table 8. Percentile comparison of the 3rd and the 5th order refractive index for PSCF, DSF, and DCF fibers.

Table 8, shows that the percentile change between 2nd and 5th order nonlinear refractive index of PSCF is larger than that of DSF which in turn has larger percentile change of refractive index than DCF.

4.2 Discussion

Nonlinearities in optical fibers and materials play crucial role in a long haul and high band with communication. It has many complication like damping loss, distortion, in signal propagation. As the size of pulse become narrow and laser power increases, nonlinearities become more predominant. In order to understand the origin and mechanism of such effects we used a simple microscopic model by taking Silica corer fiber as mass and spring in which the springs vibration can be anharmonic. It is this anharmonicity that lead to Kerr and electrostrictive loss. In terms of model microscopic parameter we calculate explicitly the higher order susceptibilities dielectric function, refractive index as a function of damping for different kinds of commercial fibres. We see that our calculation gives quite nice quantitative understanding of nonlinear effects and its impact on power loss as well as damping. The imaginary part of the refractive index of DCF is larger than that of DSF which in turn has larger refractive index than PSCF. With the increase in refractive index the coupling power also increases. Figure 4.11, shows that the total power loss due to 2nd order is larger than that of the 3rd order which in turn larger than the 5th order. Figures 4.12, shows that the power loss due to Kerr effects is more than that due to electrostrictive effects in higher order nonlinearity. Kerr nonlinearity can be enhanced with increasing the coupling power . We have been calculated that the power loss due to Kerr effect increases with increase in refractive index (Table, 5) and hence with coupling power which is in confirmation with the contribution of power loss due to imaginary part of refractive index is significant and hence absorption, scattering and dispersion phenomena also play an important role in power loss in materials of

optical fibers. From figure, 4.11 one can concluded that with increase in order of nonlinearity, the power loss decreases. So the power loss due to 7th order nonlinearity will be less than 5th order and hence the contribution of higher order nonlinearity is neglected. Therefore, one needs to pay careful attention to the role of the damping factor while interpreting measurement of the higher-order nonlinear optical susceptibility. Hence, more careful experimental and theoretical studies are necessary to quantify these issues. The imaginary part of refractive index (n_2) is a convenient way of expressing the loss term. This loss is due to γ (damping factor) which originates due to complex interaction of Si and O atoms with external electro magnetic field. Therefor, inorder to estimate the effect of damping on power loss, we have used a developed model in which then two oxygen atoms are connected to the silicon atom via springs that undergoes into anharmonic vibration. The problem now reduces to solve the equation of motion under anharmonic potential and find out response of the system under external intense electric field. This model now used to look at the nonlinear response in Pure Silica Core Fiber (PSCF), Dispersion Shifted Fiber (DSF) and Dispersion Compensating Fiber (DCF). We present some analytical results of the damping factor's contribution to the power loss in optical fibers due to higher order term based on model calculation. Using the anharmonic model the contribution of the Kerr and the electrostrictive effects to the total power loss in PSCF, DSF, and DCF is examined.

Chapter 5

Conclusions and Future outlooks

Study of nonlinearities in optical fibers have received lot of attention in recent years due to its applicability and control in soliton communication. However, the origin and the mechanism is not clearly understood yet. There are different theoretical calculation, modeling and experiments to estimate the nonlinearities. Most of the observations are limited to the 3rd order effects and no study exists on 5th order nonlinearities. In pure silica fiber as well as doped fiber materials there are two kinds of nonlinearities involved. They are Kerr and electrostrictive nonlinearities.Depending on the nature of pulses they respond differently and manifest effects on signal propagation. There are some reports that suggests that 3rd and 5th order nonlinearities can dis-stabilize the soliton and make the signal noisy. It is therefore, important to examine higher order nonlinearities (even if they are very small) and make an estimate for higher order refractive indices and susceptibilities. To understand the origin and mechanism of such higher order nonlinearities we extend an earlier developed microscopic model. Our main focus of attention was to see the 5th order nonlinearities from this anharmonic oscillator model. We observe that 5th order effect is very small

as expected but not negligible. It is thus important take into consideration the 5th order contribution in studying the signal distortion and unstable. We further calculate the power loss due to dfferent order of nonlinearities at $\lambda = 1.55\mu m$. Our main aim was to estimate qualitatively the Kerr and electrostrictive contribution to the power loss and compare with earlier observation. We indeed find agreement. Further the calculation is made on different kinds of commercial fibers Pure silica, DCF, and DSF to observe the nature of loss. It is found from the model and Pure silica fiber has low loss and is good for communication. Our results on refractive indices for different fiber also agrees with other investigations. The effects of damping on nonlinear power loss and susceptibilities of different order is investigated in depth. As we found that damping has greet role on nonlinear behavior.

Appendix A
IMPORTANT PHYSICAL AND MATHEMATICAL CONSTANTS

Constants	Descriptions	Values
c	Speed of light	2.99792×10^{8} m/s
m	Stationary electron mass	9.11×10^{-31} Kg
q	Unit electron charge	1.602×10^{-19} C
$\varepsilon_o = 1/\mu_o c^2$	Vacuum permittivity	8.85418×10^{-12} F/m
μ_o	Vacuum permeability	$4\pi \times 10^{-7} = 1.25663 \times 10^{-6}$ H/m
π	Area of unit circle	3.1415926

Appendix B
INTERNATIONAL SYSTEM OF UNITS (SI)
Base Units

Base Units

Quantity	unit	symbol
Length	meter	m
Mass	kilogra	Kg
time	second	s
current	ampere	A

Secondary Units

Quantity	unit	symbol	Dimension
Force	newton	N	$Kg.m/s^2$
Frequency	hertz	Hz	1/s
Energy	joule	J	N.m
Power	watt	W	J/s
Electric charge	coulomb	C	A.s
Potential	volt	V	J/C=W/A
Capacitance	farad	F	C/V
Magnetic flux	weber	Wb	V.s
Magnetic induction (B)	tesla	T	Wb/m^2
Inductance	henry	H	Wb/A

Appendix c

IMPORTANT VARIABLE DEFINITIONS

parameter	unit	description
a	m	core radius
$\vec{B}$	A/m	magnetic field strength
D		second order anharmonic term
e	C	magnitude of the electric charge
$\vec{E}$	V/m	electric field
$\vec{E_o}$	V/m	amplitude of Electric field
EH		hybrid mode
Ez	V/m	z components of an electricfield
F		third order anharmonic term
fi		oscillatory strength
G		fifth order anharmonic term
HE		hybrid mode
Hz	A/m	z components of a magnetic field
J	A/m^2	current density
k	1/m	propagation constant
K_R,k_I	1/m	real and imaginary part of k.
k_s	N/m	force constant of effective spring
LP	V/F	linearly polarized
M		mode number
N		number of dipoles per unit volume
N_g		group index of refraction
n		complex refractive index
n_R,n_I		refractive index and extinction coefficient
n_1,n_2		core and cladding refractive index
$\vec{p}$	C.m	induced dipole moment
$\vec{P}$	V/F	induced polarization
q_1,q_2,q_3,q_5	m	displacement of charges in 1st,2nd, 3rd & 5th order term
$\vec{r}$	m	distance b/n negative & positive charge
χ^{ω}_{L}		linear susceptibility
$\chi^{2\omega}_{NL}$		second order nonlinear susceptibility
$RE\left[\chi^{\omega}_{NL}\right], IM\left[\chi^{2\omega}_{NL}\right]$		real and imaginary part of $\chi^{2\omega}_{NL}$
$\chi^{3\omega}_{NL}$		third order nonlinear susceptibility

(Appendix c)continuous...

$RE\left[\chi_{NL}^{3\omega}\right], IM\left[\chi_{NL}^{3\omega}\right]$		real and imaginary part of $\chi_{NL}^{3\omega}$
$\chi_{NL}^{5\omega}$		fifth order nonlinear susceptibility
$RE\left[\chi_{NL}^{5\omega}\right], IM\left[\chi_{NL}^{5}\right]$		real and imaginary part of $\chi_{NL}^{5\omega}$
TE	V/m	transverse electric
TM	A/m	transverse magnetic
V		normalized frequency
vg	m/sec	group velocity in a dielectric medium
x	m	displacement of charge
$\dot{x}, \ddot{x}$	m/s,m/s^2	First and second derivatives
Υ	$\frac{H \cdot C^2 \cdot S^{-1}}{Kg \cdot m^2}$	damping factor
ω	rad/sec	laser circular frequency
ω_o	rad/sec	resonant frequency
ρ	C/m^3	charge density
α	1/m	absorption coefficient
εo	F/m	Vacuum electric permittivity
Δ		refractive index change
λ	m	wavelength
β	1/m	propagation constant

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