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This project entitled "**Applications of Banach Fixed Point Theorem to Differential Equations**" is prepared by Shewaseged H/giorgis in partial fulfillment of the requirements for the degree of Master of Science in Mathematics.

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Abstract

Banach fixed point theorem states sufficient conditions for the *existence* and *uniqueness* of a fixed point (point that is mapped onto itself). The applications of Banach fixed point theorem to differential equations arise in connection with certain class functions called *contractions*. The theorem also provides a constructive procedure (called *iteration*) by which we can obtain better and better approximation to the solution of a problem and error bounds. In this project we apply the *existence, uniqueness* and *iterative* properties of the theorem to Differential Equations.

Preface

This project contains the following three chapters each of them is briefed below.

Chapter One discusses some basic mathematical concepts such as Metric Space, Cauchy Sequence, Normed Space, Banach Space, and Lipschitz Conditions as background elements.

Chapter Two states and proves The Banach Fixed Point Theorem (BFPT), beginning with the definitions of a fixed point and Contraction.

Chapter Three considers the Applications of Banach Fixed Point Theorem to Differential Equations in detail. In this chapter, in fact, the Banach Fixed Theorem has been applied mainly to prove the famous Picard's Existence and Uniqueness theorem which plays a very important role in the theory of ordinary differential equations (ODEs).After detail consideration of Picard's Existence and Uniqueness theorem proof, we concentrate on questions regarding the *sufficient* and *necessary* conditions of Picard's Theorem, and then we apply Picard's Iteration Method to find solution for an initial value problem of ODE

Notations

$\mathcal{R}$	Real line or the field of real numbers
N	The set of natural numbers
$\mathcal{R}^n$	Euclidian n-space
$\mathbb{C}$	Complex plane or the field of Complex numbers
$\mathbb{C}^n$	Unitary n-space
F	Field
$d: X \to X$	Metric
$d(x, y)$	Distance from x to y
$\|.\|: X \to X$	Norm
$\| x \|$	Norm of x
(X, d)	Metric space
$(X, \|.\|)$	Normed space
$C(\mathcal{R})$	The space of continuous functions on $\mathbb{R}$
$C[a, b]$	The space of all continuous functions on $I = [a, b] \subseteq \mathcal{R}$
Tx	A notation that represents $T(x)$

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Abbreviations

IVP	Initial Value Problem
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
BFPT	Banach Fixed Point Theorem

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