THERMODYNAMIC PROPERTIES OF STRONGLY DEGENERATE FERMI GAS

By

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Abstract

In this project we have studied thermodynamic properties of strongly degenerate Fermi gases. Using the basic law of thermodynamic we derived Grand potential (Ω), number of fermions (N), Heat capacity (Cv), pressure (p), Energy (E), Entropy (S), Chemical potential (μ), Magnetization (M) as a functions of temperature (T) and magnetic field (H). We have also the concept of pauli paramagnetism and Landau diamagnetism in degenerate fermi gas (Electron gas). Generally, we can get the result from the derived equation the plot of these parameters as temperature increase the grand potential is large negative. As temperature increases the chemical potential decreases, the pressure increases as the temperature increases, the pressure decreases as magnetic field increases at low temperature. The total energy versus magnetic field graph shows that the energy increases rapidly when magnetic field is less than 1500 T. The density of state has discontinuous behavior and has a real and positive value. We also observed that as the total energy, grand potential and pressure depend on density of states and thus show similar behavior.

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Chapter 1

Introduction

Thermodynamics properties of strongly degenerate fermi gas may be regarded as a relatively new research area. It study the physical properties (mechanical, thermal, magnetic), etc. It studies the properties of bodies, without going into the mechanism of phenomena, i.e., not taking into consideration the relation between the internal structure of substance and phenomena, it generalizes experimental results[1]. As a result of such a generalization, postulates and laws of thermodynamics made their appearance. These laws make it possible to find general relations between the different properties of macroscopic systems and the physical events occurring in them. As a law of nature, identical fundamental particles are indistinguishable[2]. In quantum mechanics, this law causes the wave function of identical particles to fall into two classes of symmetry symmetric or antisymmetric[3]. If two identical particles have a symmetric wave function, they are named as bosons and obey Bose-Einstein statistics. In contrast, the particles with an antisymmetric wave function are called fermions and obey Fermi-Dirac statistics. For atoms, the intrinsic spin decides whether an atom is a boson or fermion atoms with integer spin (0, 1, and so on) are bosons, while fermions have half-integer spin (1/2, 3/2, and so on). At very low temperature, these two classes of atoms show quite different behavior bosons "like each other," and occupy the same quantum state to form a condensate, whereas the Pauli-exclusion principle makes the fermions "avoid each other" by filling the energy levels from the lowest state up to the highest state labeled as Fermi energy. Such Fermi gases have a tower structure in the energy domain and are called as degenerate Fermi gases[4]. From the perspective of modern physicists, Fermi gases are more important sources of new physics than Bose gases, because all the material elementary particles are fermions, such as quarks, electrons, muons, taus and neutrinos. The intriguing properties of many-body quantum physics are usually related to complex interactions between fermionic particles[5, 6, 7]. One of the most compelling problems is to study strong interacting fermions. In an interacting system, strong interaction is defined by the condition that the scattering length of the inter-acting particles is much larger than the average inter particle spacing. Strong interactions between fermions dominate behavior of a wide scale of matter in the universe, which appears in terms of all four fundamental forces. The first law of thermodynamics was discovered by the German physiologist Julius Robert von Mayer (1842) and the English physicist James Prescott Joule (1843)[8, 9]. The first law of thermodynamics is a law of conservation of energy for closed processes. In 1847, the German physicist and physiologist Hermann von Helmholtz generalized this law for any non-closed thermodynamic processes[10, 11]. The second law of thermodynamics was discovered independently by both the German physicist Rudolf Clausius (1850) and the English physicist William Thomson (Lord Kelvin). They introduced in the theory a new function of state-entropy, in the statistical sense, and discovered the law of increasing entropy. The third law of thermodynamics was discovered in 1906 by the German physicistchemist Walther Nernst. According to this law, entropy of all systems independently of external parameters tends to the identical value (zero) as temperature approaches the absolute zero. Note that the first law of thermodynamics is a law about energy, and the second and the third ones are about entropy[12]. The founders of thermodynamics are J.R.von Mayer, J.P.Joule, H.vonHelmholtz, R.Clausius, W.Kelvin, and W.Nernst. Statistical physics received its development only in the last quarter of the nineteenth century. The founders of classical statistical physics are R.Clausius, J.C. Maxwell, L Boltzmann, and

J.W.Gibbs. The height of development of classical statistical physics is the method of Josiah Willard Gibbs (1902). The application of classical statistics to many problems provided results, though not coinciding with the experimental facts of that time. Black radiation (thermodynamics of a photon gas), heat capacity of metals, Pauli paramagnetism, etc. These difficulties of classical statistics were circumvented only after the rise of quantum mechanics (L.deBroglie, W.Heisenberg, E.Schrodinger, and P.Dirac) and quantum statistics, created on its basis (E.Fermi, P.Dirac, S.N.Bose, A.Einstein) during 19241926 of the book, occupy an important place[13]. It is shown that of all the thermodynamic functions, the most important are the function of free energy and grand thermodynamic potential, which are determined from the Gibbs canonical distribution. Understanding free energy and grand thermodynamic potential, it is easy to determine entropy, thermal and caloric equations of state. To do this in the case of classical systems, it is sufficient to know the Hamilton function energy as a function of coordinates and impulses of particles of the system, forming it, and for quantum systems, it is the energy spectrum, i.e the dependence of energy on quantum numbers. It is also an essence of the Gibbs method which is applied to ideal gas[15]. It is shown how the difficulties of classical statistics, associated with its application to an electron gas in metals, are circumvented. The statistics of the electron gases are considered in detail in this paper. A separate topic is devoted to study thermodynamic properties of strongly degenerate fermi gas in a quantizing magnetic field.

Chapter 2

Completely Degenerate Fermi Gas

2.1 Equations of States of Fermi Gases

. The thermal equation of the state of Fermi gases can be found from the basic law of thermodynamic. Consider temperature (T), volume (V) and chemical potential (μ) are independent variables of the function, called the grand thermodynamic potential or Ω potential that is $\Omega = \Omega(T, V, \mu)$. Using this basic law we can calculate number of particles (N), pressure (P), heat capacity (C_V), energy (E), entropy (S) and chemical potential (μ). From First law of thermodynamic given by

$$d\Omega = -SdT - PdV - Nd\mu \tag{2.1}$$

Then system of equation is given by

$$\begin{aligned} P &= -\left(\frac{\partial\Omega(T, V, \mu)}{\partial V}\right)_{T,\mu} \\ N &= -\left(\frac{\partial\Omega(T, V, \mu)}{\partial \mu}\right)_{V,T} \end{aligned} \tag{2.2}$$

Where $\Omega(T, V, \mu)$ is the grand thermodynamic potential of the system.

To finding the explicit form of the grand thermodynamic potential $\Omega = \Omega(T, V, \mu)$. Assume that an ideal quantum gas of volume V consists of N fermions or bosons. The expression of the grand thermodynamic potential of such gases has the form.

$$\Omega = -k_B T \sum_k \ln\left(1 + e^{\frac{(\mu - \varepsilon_k)}{k_B T}}\right) \tag{2.3}$$

To calculate the sum (2.3), it is necessary to pass from the summation with respect to k to the integration.

$$\Omega = -\frac{k_B T V g_0}{(2\pi)^3} \int \ln\left(1 + e^{\frac{(\mu-\varepsilon_k)}{k_B T}}\right) d^3k \tag{2.4}$$

where $g_o = (2s+1)$ is the multiplicity of degeneracy with respect to the spin s, in (2.4), we move to the spherical coordinate system given by $d^3k = \sin\Theta k^2 d\theta d\varphi k^2 dk$ and take into account that the integral over angles equals 4π. Then, we get

$$\Omega = -\frac{k_B T V g_0}{2\pi^2} \int_0^\infty \ln\left(1 + e^{\frac{(\mu-\varepsilon_k)}{k_B T}}\right) k^2 dk \tag{2.5}$$

On the strength of the model it is more favorable to pass from the integral over dk to the integral over $d\varepsilon$. Then, the expression (2.5), takes the following final form

$$\Omega = -\frac{k_B T V g_0}{(2\pi)^2} \frac{(2m)^{3/2}}{\hbar^3} \int_0^\infty ln\left(1 + e^{(\mu-\varepsilon)/k_B T}\right) \varepsilon^{1/2} d\varepsilon \tag{2.6}$$

Here and henceforth, we will follow the notation $\varepsilon_k \equiv \varepsilon$. If we take into account the expression of the grand thermodynamic potential (2.6) in (2.2), we get the following expression for pressure

$$P = -\frac{k_B T g_0}{(2\pi)^2} \frac{(2m)^{3/2}}{\hbar^3} \int_0^\infty ln\left(1 + e^{(\mu-\varepsilon)/k_B T}\right) \varepsilon^{1/2} d\varepsilon \tag{2.7}$$

This expression can be integrated by parts once thereupon take the derivatives from (2.6), with respect to μ and substitute it in the second equation of (2.2). As a result, the system of equations (2.2), acquires the form

$$\begin{aligned} P &= \frac{2}{3} \frac{g_0 (2m)^{\frac{3}{2}}}{(2\pi)^2 \hbar^3} \int_0^\infty \frac{\varepsilon^{\frac{3}{2}}}{e^{\frac{(\varepsilon-\mu)}{k_B T}} + 1} d\varepsilon, \\ N &= \frac{V g_0 (2m)^{\frac{3}{2}}}{(2\pi)^2 \hbar^3} \int_0^\infty \frac{\varepsilon^{\frac{1}{2}}}{e^{\frac{(\varepsilon-\mu)}{k_B T}} + 1} d\varepsilon \end{aligned} \tag{2.8}$$

we can find the function of the density of states $g(\varepsilon)$, i.e. the number of quantum states accounting for the unit range of energy per unit volume is;

$$g(\varepsilon) = \frac{g_0 (2m)^{\frac{3}{2}}}{(2\pi)^2 \hbar^3} \varepsilon^{\frac{1}{2}} \tag{2.9}$$

The energy density can be presented in the form

$$\frac{E}{V} = \int_0^\infty g(\varepsilon)\varepsilon f(\varepsilon) d\varepsilon \tag{2.10}$$

where $f(\varepsilon) = \frac{1}{e^{\frac{(\varepsilon-\mu)}{k_B T}}+1}$ is the fermi function. By substituting equation (2.9) in to (2.10), the mean value of the total internal energy takes the form

$$E = \frac{V g_0 (2m)^{\frac{3}{2}}}{(2\pi)^2 \hbar^3} \int_0^\infty \frac{\varepsilon^{\frac{3}{2}}}{e^{\frac{(\varepsilon-\mu)}{k_B T}}+1} d\varepsilon \tag{2.11}$$

Comparing (2.8) with (2.11), the simple relationship between the energy density and pressure

$$P = \frac{2}{3}\frac{E}{V} \tag{2.12}$$

2.2 Completely Degenerate Fermi Gas

If the gas satisfies condition described by the Fermi distribution function $f(\varepsilon)$. Assume that energy of fermions is given by a simple parabolic dispersion law, equation of state for a Fermi gas can be rewritten in the form

$$\begin{aligned} P &= \frac{2}{3}\frac{g_0 (2m)^{\frac{3}{2}}}{(2\pi)^2 \hbar^3} \int_0^\infty \varepsilon^{\frac{3}{2}} f(\varepsilon) d\varepsilon, \\ N &= \frac{V g_0 (2m)^{\frac{3}{2}}}{(2\pi)^2 \hbar^3} \int_0^\infty \varepsilon^{\frac{1}{2}} f(\varepsilon) d\varepsilon, \end{aligned} \tag{2.13}$$

At $(T = 0)$, (2.8) by integrating by parts once then, this system of equations takes the form

$$\begin{aligned} P &= \frac{4}{15}\frac{g_0 (2m)^{\frac{3}{2}}}{(2\pi)^2 \hbar^3} \int_0^\infty \left(-\frac{\partial f}{\partial \varepsilon}\right) \varepsilon^{\frac{5}{2}} d\varepsilon, \\ N &= \frac{2}{3}\frac{V g_0 (2m)^{\frac{3}{2}}}{(2\pi)^2 \hbar^3} \int_0^\infty \left(-\frac{\partial f}{\partial \varepsilon}\right) \varepsilon^{\frac{3}{2}} d\varepsilon, \end{aligned} \tag{2.14}$$

At T=0, the energy spectrum is completely filled up to the Fermi energy μ_F, and the levels above are empty. The distribution function in this case has a step-like form

$$\lim_{T \longrightarrow 0} f(\varepsilon) = 1, \quad if \quad \varepsilon \leq \mu_F \quad and \quad 0, \quad if \quad \varepsilon > \mu_F \tag{2.15}$$

The derivative of the distribution function with respect to ε behaves like the δ-function

$$\lim_{T\longrightarrow 0}\left(-\frac{\partial f}{\partial \varepsilon}\right)=\delta(\varepsilon-\mu_F) \tag{2.16}$$

A gas described by such a distribution is called a completely degenerate Fermi gas. If we take into account this property of the distribution function then the integration in the system of equations (2.13), is easily evaluated. As a result, we get

$$\begin{aligned} P_0 &= \frac{4}{15}\frac{g_0(2m)^{\frac{3}{2}}}{(2\pi)^2\hbar^3}\mu_F^{\frac{5}{2}}, \\ N &= \frac{2}{3}\frac{Vg_0(2m)^{\frac{3}{2}}}{(2\pi)^2\hbar^3}\mu_F^{\frac{3}{2}}. \end{aligned} \tag{2.17}$$

Hence, for the considered gas we can find the following Fermi energy

$$\mu_F=\frac{\hbar^2}{2m}\left(\frac{6\pi^2}{g_0}\frac{N}{V}\right)^{\frac{2}{3}}, \tag{2.18}$$

Fermi impulse

$$p_0=\hbar k_F=(2m\mu_F)^{\frac{1}{2}}=\hbar\left(\frac{6\pi^2}{g_0}\frac{N}{V}\right)^{\frac{1}{3}}, \tag{2.19}$$

Zero pressure of a Fermi gas using (2.18) in to (2.17), becomes

$$P_0=\frac{2}{5}\frac{\hbar^2}{2m}\left(\frac{6\pi^2}{g_0}\right)^{\frac{2}{3}}\left(\frac{N}{V}\right)^{\frac{5}{3}} \tag{2.20}$$

Zero energy using (2.20) into (2.12), the zero energy is given by

$$E_0=\frac{3}{5}\frac{\hbar^2}{2m}\left(\frac{6\pi^2}{g_0}\frac{N}{V}\right)^{\frac{2}{3}}N. \tag{2.21}$$

2.3 Thermodynamic Properties of Strongly Degenerate Fermi Gas

When we speak of a strongly degenerate Fermi gas (electron gas) we simply a gas that is found in the statistical state schematically presented in fig (2.1) and fig (2.2). This state is characterized by the fact that under the action of finite

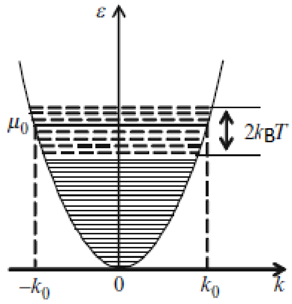

Figure 2.1: The filling of the energy spectrum by fermions at temperature distinct from zero

but small ($T \ll T_o$) temperatures, a small part of fermions found in quantum states below the Fermi energy pass to higher levels. As a result, the Fermi energy acquires a finite width on the order of $2k_BT$. However, note that this width is much smaller than the Fermi energy $2k_BT \ll \mu_F$.

Here, we solve the system of equations (2.14), in the first approximation with respect to the dimensionless small parameter $k_BT/\mu_F = T/T_0 \ll 1$ and consider the influence of strong degeneracy on the thermodynamic properties of the gas. Here, $T_0 = \mu_F/k_B$ is temperature of degeneracy.

From the system of equations (2.14), it is seen that to find the equation of state it is necessary to calculate integrals of the type

$$I = \int_0^\infty \left(-\frac{\partial f}{\partial \varepsilon}\right) \varphi(\varepsilon) d\varepsilon \tag{2.22}$$

In the first approximation with respect to the parameter $k_BT/\mu_F \ll 1$. In our case, $\varphi(\varepsilon) = \varepsilon^{5/2}$ and $\varphi(\varepsilon) = \varepsilon^{3/2}$. The distribution function of a strongly degenerate gas and its derivative are presented in (2.2) from the figure (2.2), it is seen that at finite temperature the derivative $(-\partial f/\partial\varepsilon)$ for the value of energy $\varepsilon = \mu_F(T)$ is maximum, where μ_F is the Fermi level at temperature T Therefore, to calculate the integral (2.22) the function $\varphi(\varepsilon)$ can be expanded into a series around μ_F in powers of $(\varepsilon - \mu_F)$. Then,

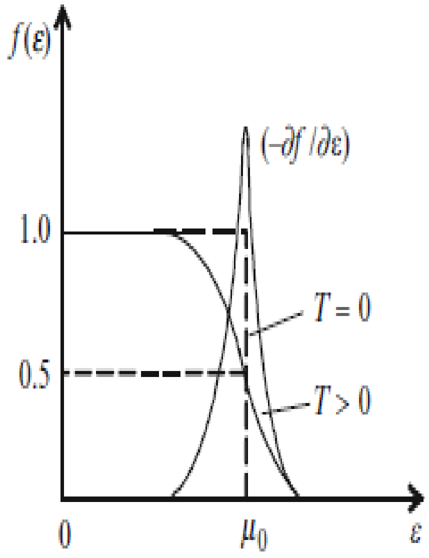

Figure 2.2: The plots of the distribution function fermi and its derivative distinct from zero

integral (2.22), takes the form

$$I = \varphi(\mu_F) + I_1 \left(\frac{d\varphi}{d\varepsilon}\right)_{\varepsilon=\mu_F} + \frac{1}{2} I_2 \left(\frac{d^2\varphi}{d\varepsilon^2}\right)_{\varepsilon=\mu_F} + ... \tag{2.23}$$

Here, we used the property of the distribution function (2.22) and employed the following notations

$$I_1 = \int_0^\infty (\varepsilon - \mu_F)\left(-\frac{\partial f}{\partial \varepsilon}\right) d\varepsilon; I_2 = \int_0^\infty (\varepsilon - \mu_F)^2 \left(-\frac{\partial f}{\partial \varepsilon}\right) d\varepsilon \tag{2.24}$$

We introduce the new dimensionless variable $x = (\varepsilon - \mu_F)/k_B T$. At temperature $T \ll T0$, when $\mu/k_B T \gg 1$, the lower boundary of integrals (2.23), with respect to x can be replaced with $-\infty$. Simultaneously, we take into account that the function $(-\partial f/\partial x) = e^x/(e^x + 1)^2$ is even that is $e^x/(e^x + 1)^2 = e^{-x}/(e^{-x} + 1)^2$. Then, because the function $x(-\partial f/\partial x)$ under the first integral sign is odd, the first integral is equal to zero, i.e.

$$I_1 = (k_B T)^2 \int_{-\infty}^{\infty} x \left(-\frac{\partial f}{\partial x}\right) dx = 0 \tag{2.25}$$

and the second one

$$I_2 = (k_B T)^2 \int_{-\infty}^{\infty} x^2 \left(-\frac{\partial f}{\partial x}\right) dx = 2(k_B T)^2 \int_{0}^{\infty} x^2 \left(-\frac{\partial f}{\partial x}\right) dx. \tag{2.26}$$

on integrating (2.23), once by parts, we get

$$I_2 = (2k_B T)^2 \int_{0}^{\infty} \frac{x}{e^x + 1} dx \tag{2.27}$$

If we take into account

$$\int_{0}^{\infty} \frac{x}{e^x + 1} dx = \frac{\pi^2}{12}, \tag{2.28}$$

then integral (2.22), takes the form

$$I = \int_{0}^{\infty} \left(-\frac{\partial f}{\partial \varepsilon}\right) \varphi(\varepsilon) d\varepsilon = \varphi(\mu_F) + \frac{\pi^2}{6}(k_B T)^2 \left(\frac{d^2\varphi}{d\varepsilon^2}\right)_{\varepsilon=\mu_F} + ... \tag{2.29}$$

On applying this formula if we preliminarily integrate the expression (2.6), twice by parts, we get

$$\Omega = -\frac{4}{15}\frac{V g_0}{(2\pi)^2}\frac{(2m)^{\frac{3}{2}}}{\hbar^3} \int_{0}^{\infty} \varepsilon^{\frac{5}{2}} \left(-\frac{\partial f}{\partial \varepsilon}\right) d\varepsilon \tag{2.30}$$

Note that from comparison of expressions (2.14) and (2.29) follows the known thermodynamic relationship $\Omega = PV$. On applying $\varphi(\varepsilon) = \varepsilon^{\frac{5}{2}}$ the approximation (2.29), to the expression of the grand thermodynamic potential (2.30), with the necessary accuracy we get

$$\Omega = -\frac{4}{15}\frac{V g_0}{(2\pi)^2}\frac{(2m)^{\frac{3}{2}}}{\hbar^3}\mu_F^{\frac{5}{2}} \left[1 + \frac{5\pi^2}{8}\left(\frac{k_B T}{\mu_0}\right)^2\right]; T \ll T_0 \tag{2.31}$$

by substituting this in equation (2.2), we get P and N as follows

$$P = \frac{4}{15} \frac{g_0 (2m)^{\frac{3}{2}}}{(2\pi)^2 \hbar^3} \mu_F^{\frac{5}{2}} \left[1 + \frac{5\pi^2}{24} \left(\frac{k_B T}{\mu_o} \right)^2 \right]; T \ll T_0, \tag{2.32}$$

$$N = \frac{2}{3} \frac{V g_0 (2m)^{\frac{3}{2}}}{(2\pi)^2 \hbar^3} \mu_F^{\frac{3}{2}} \left[1 + \frac{\pi^2}{8} \left(\frac{k_B T}{\mu_0} \right)^2 \right]; T \ll T_0, \tag{2.33}$$

From the second equation of this system in the first approximation with respect to degeneracy, we can find the temperature dependence of the Fermi level given by

$$\mu(T) = \mu_F \left[1 - \frac{\pi^2}{12} \left(\frac{k_B T}{\mu_0} \right)^2 \right]; T \ll T_0, \tag{2.34}$$

Here, μ_F is the Fermi energy at $T = 0$, derived in (2.18). It is seen that at finite but small temperatures the Fermi energy decreases. The reason for this lies in the fact that the density of quantum states increases as $g(\varepsilon) \sim \sqrt{\varepsilon}$ the density of quantum states above the Fermi energy μ_F is somewhat more than that below the Fermi energy. Therefore, a specified amount of fermions (electrons) passing the Fermi energy upwards occupies a narrower strip of energy. As a result, on average the Fermi level decreases. If we substitute the first equation (2.17) in to (2.32), at very low temperatures ($T \ll T_0$) the equation of the state of a Fermi gas takes the form

$$P(T) = P_0 \left[1 + \frac{5(\pi)^2}{12} \left(\frac{k_B T}{\mu_0} \right)^2 \right]; T \ll T_0, \tag{2.35}$$

The mean energy of a degenerate Fermi gas in the first approximation can be found using the relationship $E = 3PV/2$ and also expressions (2.32) and (2.33)

$$E = E_0 + \frac{\pi^2}{4} \mu_F N \left(\frac{k_B T}{\mu_0} \right)^2; T \ll T_0 \tag{2.36}$$

In the same approximation, the heat capacity of a Fermi gas equals

$$Cv = \frac{\pi^2}{2} k_B N \left(\frac{k_B T}{\mu_0} \right); T \ll T_0 \tag{2.37}$$

If in (2.31), we take the derivative with respect to T at constant volume V and the chemical potential μ_F, the entropy of a Fermi gas takes the very simple form

$$S = \frac{\pi^2}{2} k_B N \left(\frac{k_B T}{\mu_0} \right) ; T \ll T_0 \tag{2.38}$$

Chapter 3

Pauli Paramagnetism

3.1 Second Difficulty of Classical Statistics

The second difficulty of the Boltzmann classical statistics is associated with calculating the paramagnetic susceptibility(χ) of a free electron gas in metals. The paramagnetic susceptibility calculated on the basis of the Boltzmann statistics and its temperature dependence do not coincide with values from experiment. Thus, for instance, the experimental value of the paramagnetic susceptibility is lower by two orders of magnitude and does not depend on temperature, where as according to the classical statistics $\chi << 1/T$. This difficulty was circumvented by Pauli in 1927, after applying the new Fermi statistics to compute the paramagnetic susceptibility. He came to the conclusion that the cause of the divergence is the fact that **an electron gas is not classical but a strongly degenerate quantum gas.**

Assume that in a metal of volume V there is a gas consisting of N free electrons. Each electron possesses an intrinsic magnetic moment associated with the spin, which is equal to the Bohr magneton $\mu_B = e\hbar/2mc$. In the absence of an external magnetic field, the intrinsic magnetic moments compensate each other (since, according to the Pauli principle in one quantum level there exist two electrons with opposite spins) and the magnetic moment of the metal as a whole equals zero.

On placing the metal in an external uniform magnetic field H, the number of electrons

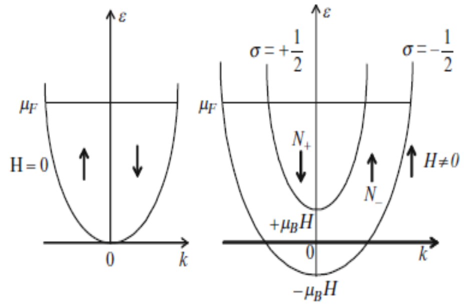

Figure 3.1: The energy spectra of an electron gas in the absence and in the presence of a magnetic field

with spins directed along the magnetic field is more than that with the opposite spins, and therefore the electron gas possesses a paramagnetic moment. Using the Fermi statistics, we can calculate the magnetization of an electron gas in metals. Energy of an electron in an external magnetic field H with regard to the spin has the form

$$\varepsilon = \frac{h^2k^2}{2m} + \sigma g_0 \mu_B H, \tag{3.1}$$

where m is the electron mass, $\mu_B = e\hbar/2mc = 0.93 \times 10^-20$ erg/G is the Bohr magneton, $\sigma = \mp 1/2$ (the sign "-" refers to electrons whose intrinsic magnetic moments are directed along the magnetic field, and "+" refers to electrons whose intrinsic magnetic moments are directed opposite to the field) and $g_0 = (2s + 1)$ is the degree of the spin splitting degeneracy for a free electron $s = 1/2$ and $g_0 = 2$. Energy spectra of an electron gas in the absence and in the presence of a magnetic field are schematically shown in fig (3.1) the total number of electrons can get

$$N = \sum_{k,\sigma} f(k,\sigma) = \frac{V}{(2\pi)^3} \sum_{\sigma} \int f(k,\sigma) d^3k. \tag{3.2}$$

Here $f(k,\sigma)$ is the Fermi-Dirac distribution function and, we have changed from the summation with respect to k to the integration gives

$$N = \frac{V}{(2\pi)^3} \frac{(2m)^{\frac{3}{2}}}{\hbar^3} \sum_{\sigma} \int_{-\sigma g_0 \mu_B H}^{\infty} f(\varepsilon)(\varepsilon + \sigma g_0 \mu_B H)^{\frac{1}{2}} d\varepsilon \tag{3.3}$$

Up on integrating once by parts, we get

$$N = \frac{2V}{3(2\pi)^2} \frac{(2m)^{\frac{3}{2}}}{\hbar^3} \sum_{\sigma} \int_{-\sigma g_0 \mu_B H}^{\infty} \left(-\frac{\partial f}{\partial \varepsilon}\right) (\varepsilon + \sigma g_0 \mu_B H)^{\frac{3}{2}} d\varepsilon \tag{3.4}$$

If we add up with respect to $\sigma = 1/2$ and take $g_0 = 2$, (3.4) takes the form,

$$N = N_+ + N_- \tag{3.5}$$

where

$$N_- = \frac{2V}{3(2\pi)^2} \frac{(2m)^{\frac{3}{2}}}{\hbar^3} \sum_{\sigma} \int_{-\mu_B H}^{\infty} \left(-\frac{\partial f}{\partial \varepsilon}\right) (\varepsilon + \mu_B H)^{\frac{3}{2}} d\varepsilon \tag{3.6}$$

is the number of electrons whose spins are parallel to the magnetic field H, and

$$N_+ = \frac{2V}{3(2\pi)^2} \frac{(2m)^{\frac{3}{2}}}{\hbar^3} \sum_{\sigma} \int_{+\mu_B H}^{\infty} \left(-\frac{\partial f}{\partial \varepsilon}\right) (\varepsilon - \mu_B H)^{\frac{3}{2}} d\varepsilon \tag{3.7}$$

is the number of electrons whose spins are antiparallel to the magnetic field H. It is evident that as a whole the paramagnetic moment of an electron gas in metal can be defined as

$$M = \mu_B (N_- - N_+) \tag{3.8}$$

If we take into account that for a completely degenerate electron gas $(-\partial f/\partial \varepsilon) = \delta(\varepsilon - \mu_F)$, from (3.6) to (3.8), we get

$$M = \frac{\mu_B 2V}{3(2\pi)^2} \frac{(2m)^{\frac{3}{2}}}{\hbar^3} \mu_F^{\frac{3}{2}} \left[\left(1 + \frac{\mu_B H}{\mu_F}\right)^{\frac{3}{2}} - \left(1 - \frac{\mu_B H}{\mu_F}\right)^{\frac{3}{2}} \right] \tag{3.9}$$

in a weak magnetic field using Taylor's series one can set

$$M = \frac{\mu_B^2 V}{2\pi^2} \frac{(2m)^{\frac{3}{2}}}{\hbar^3} \mu_F^{\frac{1}{2}} H \tag{3.10}$$

then the fermi energy at T=0, is given by

$$\mu_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{\frac{2}{3}} \tag{3.11}$$

For the paramagnetic susceptibility $\chi_0 = M/VH$ of a completely degenerate electron gas, from (3.10), we get the simple expression

$$\chi_0 = \mu_B^2 g(\mu_F) \tag{3.12}$$

where

$$g(\mu_F) = \frac{(2m)^{\frac{3}{2}}}{2\pi^2 \hbar^3} \mu_F^{\frac{1}{2}} \tag{3.13}$$

If at finite values of temperature aid of we calculate integrals (3.6), and (3.7), for the paramagnetic susceptibility we get

Magnetization at finite Temprature

$$\int_0^\infty \frac{\varphi(x)}{d} x e^{x-y} + 1 = \int_0^y \varphi(x) dx + \frac{\pi^2}{6}\left(\frac{d\varphi}{dx}\right)_{x=y} + \frac{7\pi^4}{360}\left(\frac{d^3\varphi}{dx^3}\right)_{x=y} + \frac{31\pi^6}{15720}\left(\frac{d^5}{dx^5}\right)_{x=y} + \dots\dots\dots \tag{3.14}$$

but

$$\varphi(\varepsilon) = (\varepsilon \pm \mu_B H)^{\frac{3}{2}} d\varepsilon \tag{3.15}$$

$$I_- = \int_{\mu_B H}^\infty \frac{(\varepsilon + \mu_B H)^{\frac{1}{2}}}{1 + e^{\beta(\varepsilon - \mu_F)}} d\varepsilon \tag{3.16}$$

Let u=$\beta(\varepsilon + \mu_B H)$

then

$$I_+ = (k_B T)^{\frac{3}{2}} \int_0^\infty \frac{x^{\frac{1}{2}}}{1 + e^{x-y}} dx \tag{3.17}$$

$$I_\pm = (k_B T)^{\frac{3}{2}} \int_0^\infty \frac{x^{\frac{1}{2} dx}}{1 + e^{x-y}} \tag{3.18}$$

let y is given by

$$y = (\mu_F + \mu_B H)/k_B T; \varphi = X^{\frac{1}{2}} \tag{3.19}$$

$$I_- = (k_B T)^{\frac{3}{2}} \left[\frac{2}{3} y^{\frac{3}{2}} + \frac{\pi^2}{6} \frac{1}{2\sqrt{y}} + \frac{21\pi^4}{360} \frac{1}{8y^{\frac{5}{2}}} \right] ... \tag{3.20}$$

$$I_- = \frac{2}{3}(\mu_F + \mu_B H)^{\frac{3}{2}} + \frac{(\pi k_B T)^2}{12(\mu_F + \mu_B H)^{\frac{3}{2}}} + \frac{21}{2888} \frac{(\pi k_B T)^4}{(\mu_F + \mu_B H)^{\frac{5}{2}}} + \frac{3255}{503040} \frac{(\pi k_B T)^6}{(\mu_F + \mu_B H)^{\frac{9}{2}}} \tag{3.21}$$

similarly

$$I_+ = \int_{\mu_B H}^{\infty} \frac{(\varepsilon - \mu_B H)^{\frac{1}{2}}}{1 + e^{\beta(\varepsilon - \mu_F)}} d\varepsilon \tag{3.22}$$

$$I_+ = (k_B T)^{\frac{3}{2}} \int_0^{\infty} \frac{x^{\frac{1}{2}}}{1 + e^{x-y}} dx \tag{3.23}$$

$$y = (\mu_F - \mu_B H)/k_B T; x = \beta\varepsilon \tag{3.24}$$

$$I_+ = \frac{2}{3}(\mu_F - \mu_B H)^{\frac{3}{2}} + \frac{(\pi k_B T)^2}{12(\mu_F - \mu_B H)^{\frac{3}{2}}} + \frac{21}{2888} \frac{(\pi k_B T)^4}{(\mu_F - \mu_B H)^{\frac{5}{2}}} + \frac{3255}{503040} \frac{(\pi k_B T)^6}{(\mu_F - \mu_B H)^{\frac{9}{2}}} \tag{3.25}$$

then the magnetization is given by

$$M = \mu_B(N_- - N_+) \tag{3.26}$$

$$M = \frac{v\mu_B^2 (2m)^{\frac{3}{2}}}{2\pi^2 \hbar^3} \mu_F^{\frac{1}{2}} \left(1 - \frac{(k_B T)^2}{8\mu_F^3} - \frac{105}{5760} \frac{(k_B T)^4}{\mu_F^4} \right) \tag{3.27}$$

$$M = V\chi H \tag{3.28}$$

then $\chi(T)$ is given by

$$\chi(T) = \frac{\mu_B^2 (2m)^{\frac{3}{2}}}{2\pi^2\hbar^3\mu_F}\mu_F^{\frac{3}{2}}\left(1 - \frac{(k_BT)^2}{8\mu_F^3} - \frac{105}{5760}\frac{(k_BT)^4}{\mu_F^4} - \frac{3255}{503040}\frac{(\pi_BT)^6}{\mu_F}\right) \tag{3.29}$$

let $y = k_BT/\mu_F$

$$\chi(T) = \frac{\mu_B^2 (2m)^{\frac{3}{2}(k_BT)^{\frac{1}{2}}}}{2\pi^2\hbar^3 y^{\frac{1}{2}}}\left[1 - \frac{\pi^2 y^2}{8\mu_F} - \frac{105\pi^4 y^4}{5760} - \frac{3255\pi^6 y^6}{503040}\right] \tag{3.30}$$

$$\chi(T) = \mu_B^2 g(\mu_F)\left(1 - \frac{\pi^2}{12}\frac{k_BT}{\mu_F}\right). \tag{3.31}$$

Hence, it is seen that at temperatures $k_BT \ll \mu_F$ the paramagnetic susceptibility χ_0 depends on temperature T very weakly.

3.2 Thermodynamic Properties of Electron Gas in Quantizing Magnetic Field

To determine the criterion of degeneracy of an electron gas in a quantizing magnetic field, it is necessary to find the relation of its chemical potential to concentration and temperature. To do this, it is needed to know the explicit form of the grand thermodynamic potential Ω_e as a function of volume, temperature, chemical potential and magnetic field $\Omega_e = \Omega_e(T, V, \mu, H)$. The thermodynamic relationship for Ω_e in a magnetic field is given by

$$d\Omega_e = -SdT - PdV - N_e d\mu - VMdH \tag{3.32}$$

where Ne is the number of free electrons, V and μ are volume and chemical potential of an electron gas, respectively, M is the magnetization, and the rest of the notations are generally accepted. The influence of quantization of the motion in a magnetic field on the energy spectrum of an electron is schematically shown in fig (3.2a). For comparison the same is found in fig (3.2b), where the dependence of energy on the wave vector $\varepsilon(k)$ in the absence of a magnetic field is adduced. It is seen that in the presence of a

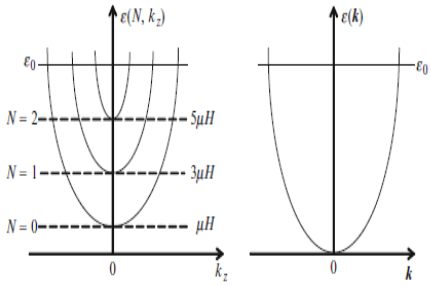

Figure 3.2: Energy as a function of wave vector: a) in the presence of magnetic field,discrete Landau levels and one-dimensional bands appear, b) in the absence of magnetic field.

magnetic field, in the spectrum discrete levels separated from each other by the energy distance $2\mu H$ appear. These are called the Landau levels. For the given level, energy continuously depends only on k_z, i.e. one dimensional parabolic energy bands appear.

Energy of circular motion in x-y plane is quantized and localized by magnetic field is

$$\varepsilon(xy_{plane}) = (j + 1/2)\hbar\omega_c \tag{3.33}$$

The total energy is given by

$$\varepsilon(j, k_z) = (j + 1/2)\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m} \tag{3.34}$$

with respect to quasi-continuous quantum numbers k_y and k_z to integrals, according to the known rule also we have

$$Z = \frac{2}{V}\sum_{j,k_z} \to \frac{2L_x L_y}{V(2\pi)^2}\sum_{j}\int dk_z \tag{3.35}$$

The grand partition function is given by

$$\Omega = -k_B T \ln Z \tag{3.36}$$

$$\ln Z = 2\sum_{j,k_z} \ln\left[1 + e^{-\beta(\varepsilon-\mu)}\right] \tag{3.37}$$

it is seen that knowing the explicit form of $\Omega_e = \Omega_e(T, V, \mu, H)$. Grand thermodynamic potential for fermions (electrons) is given by

$$\Omega_e = -2k_BT\sum_{j,k_z} \ln\left[1 + exp\left(\frac{\mu - \varepsilon(j, K_z)}{k_BT}\right)\right] \tag{3.38}$$

since one quantum state of an electron is determined by three quantum numbers $k \to (j, kz); \varepsilon(j, kz)$ and the factor 2 takes into account degeneracy with respect to the spin. Then from the integral over dk_z we pass to the integral over energy $d\varepsilon$ takes the form

$$\Omega_e = \frac{4k_BTV}{(2\pi R)^2}\sum_j \int_{\varepsilon_j}^{\infty} \frac{dk_z(\varepsilon, j)}{d\varepsilon} d\varepsilon \ln\left[1 + exp\left(\frac{\mu - \varepsilon}{k_BT}\right) d\varepsilon\right] \tag{3.39}$$

where the lower boundary of the integral $\varepsilon_j = (j + 1/2)\hbar\omega_c$, and R is the magnetic length given by $R = \left(\frac{\hbar c}{eH}\right)^{\frac{1}{2}}$

Up on integrating by parts, we get

$$\Omega_e = \frac{4V}{(2\pi R)^2}\sum_j \int_{\varepsilon_j}^{\infty} k_z(\varepsilon, j) f(\varepsilon) d\varepsilon \tag{3.40}$$

Then pressure can be determined as follows

$$P = -\left(\frac{\partial\Omega_e}{\partial V}\right)\mu, H, T; \tag{3.41}$$

$$P = \frac{4}{(2\pi R)^2}\sum_j \int_{\varepsilon_j}^{\infty} k_z(\varepsilon, j) f(\varepsilon) d\varepsilon \tag{3.42}$$

If we use $k_z = (\varepsilon, j) = \frac{\sqrt{2m}}{\hbar}(\varepsilon - \varepsilon_j)^{\frac{1}{2}}$ and, (3.35) gives

$$P = \frac{4(2m)^{\frac{1}{2}}}{\hbar(2\pi R)^2}\sum_j \int_{\varepsilon_j}^{\infty} (\varepsilon - \varepsilon_j)^{\frac{1}{2}} f(\varepsilon) d\varepsilon \tag{3.43}$$

Up on integrating by parts, we get

$$P = \frac{8(2m)^{\frac{1}{2}}}{3\hbar(2\pi R)^2} \sum_j \int_{\varepsilon_j}^{\infty} (\varepsilon - \varepsilon_j)^{\frac{3}{2}} \left(-\frac{\partial f}{\partial \varepsilon}\right) d\varepsilon \qquad (3.44)$$

For an arbitrary degree of degeneracy of an electron gas the integral in (3.44), cannot be analytically calculated.

For Degenerate electron gas; if in the zeroth approximation with respect to temperature we take into account that $(-\frac{\partial f}{\partial \varepsilon}) = \delta(\varepsilon - \mu_F)$, from (3.44), for the zero pressure at(T=0) we get

$$P_0 = \frac{8(2m)^{\frac{1}{2}}}{3\hbar(2\pi R)^2} \sum_{j=0}^{j_0} [\mu_F - (2j+1)\mu_B H]^{\frac{3}{2}} \qquad (3.45)$$

where

$$j_0 = \frac{(\varepsilon - \mu_B H)}{2\mu_B H} \qquad (3.46)$$

In the quantum limit; when all electrons are found at the zero Landau level, i.e. when $\mu_B H < \mu_F < 3\mu_B H$, in sum it can be restricted to the term with j=0 and from form as follows that

$$P_0 = \frac{8(2m)^{\frac{1}{2}}}{3\hbar(2\pi R)^2} [\mu_F - \mu_B H]^{\frac{3}{2}} \qquad (3.47)$$

Then for the thermal equation of the state of a degenerate electron gas in the quantum then we get

$$P_0 = \frac{\pi^2}{3} \frac{\hbar^4 C^3}{me^2} \frac{n^3}{H^2} \qquad (3.48)$$

where

$$n = \frac{(2m)^{\frac{3}{2}}}{\pi\hbar^3} \mu_B H \sum [\mu(H) - (2j+1)\mu_B H]^{\frac{1}{2}} \qquad (3.49)$$

the zero pressure of a degenerate electron gas in the quantum limit strongly depends on the concentration n and magnetic field $P_0 \sim n^3/H^2$.

The caloric equation of state or mean energy; An electron gas is defined as follows

$$E = \frac{6V}{(2\pi R)^2} \sum_j \int (\varepsilon - \varepsilon_j) f(\varepsilon) dk_z \tag{3.50}$$

From the integration over dk_z we pass to the integration over $d\varepsilon$ and multiply by 2, taking into account that two values k_z correspond to one value of ε. As a result, we have

$$E = \frac{12V}{(2\pi R)^2} \sum_j \int_{\varepsilon_j}^{\infty} (\varepsilon - \varepsilon j) f(\varepsilon) \left(\frac{dk_z}{d\varepsilon} \right) d\varepsilon \tag{3.51}$$

where

$$\frac{dk_z}{d\varepsilon} = \frac{(2m)^{\frac{1}{2}}}{2\hbar} \frac{1}{\sqrt{\varepsilon - \varepsilon_j}} \tag{3.52}$$

for the caloric equation of state (the mean energy) we get

$$E = \frac{6V}{\hbar(2\pi R)^2} \sum_j \int_{\varepsilon_j}^{\infty} (\varepsilon - \varepsilon j)^{\frac{1}{2}} f(\varepsilon) d\varepsilon \tag{3.53}$$

Up on integrating by parts, we get

$$E = \frac{4V(2m)^{\frac{1}{2}}}{\hbar(2\pi R)^2} \sum_j \int_{\varepsilon_j}^{\infty} [\varepsilon - (2j+1)\mu_B H]^{\frac{3}{2}} \left(\frac{-df}{d\varepsilon} \right) d\varepsilon \tag{3.54}$$

$$E = E_0 + \frac{\pi^2}{2} (k_B T)^2 \frac{V(2m)^{\frac{1}{2}}}{\hbar(2\pi R)^2} \sum_j [\mu_F - (2j+1)\mu_B H]^{\frac{-1}{2}} \tag{3.55}$$

where $E_0 = 3P_0V/2$ is the zero energy at $T = 0$, and P_0 is determind above

Entropy; According to (3.32), entropy of an electron gas is defined as follows:

$$S = -\left(\frac{\partial \Omega_e}{\partial T} \right)_{V,\mu,H} \tag{3.56}$$

We started with the expression (3.40), for the grand thermodynamic potential Ω_e. From this expression it is seen that only the Fermi distribution function $f(\varepsilon)$ depends on temperature. If we take into account that

$$\left(\frac{\partial f}{\partial T} \right)_{\mu_F,H} = \frac{(\varepsilon - \mu_F)}{T} \left(\frac{\partial f}{\partial \varepsilon} \right) \tag{3.57}$$

from (3.40), for entropy of an electron gas in a quantizing magnetic field we get

$$S = \frac{4V}{(2\pi R)^2 T} \sum_j \int_{\varepsilon_j}^{\infty} k_z(\varepsilon, \varepsilon_j)(\varepsilon - \mu_F) \left(-\frac{\partial f}{\partial \varepsilon}\right) d\varepsilon \tag{3.58}$$

or, we have

$$S = \frac{4V}{(2\pi R)^2 T} \frac{(2m)^{\frac{1}{2}}}{\hbar} \sum_j \int_{\varepsilon_j}^{\infty} (\varepsilon - \varepsilon_j)^{\frac{1}{2}} (\varepsilon - \varepsilon_j)^{\frac{1}{2}} (\varepsilon - \mu_F) \left(-\frac{\partial f}{\partial \varepsilon}\right) d\varepsilon \tag{3.59}$$

At high temperature; Using equation (3.38), the entropy gives

$$S = \frac{4V}{(2\pi R)^2 T} \frac{(2m)^{\frac{1}{2}}}{\hbar} \sum_j \left[\varphi(\varepsilon_F) + \frac{\pi^2 (k_B T)^2}{6} \frac{d^2\varphi}{d\varepsilon^2}\right] \tag{3.60}$$

The heat capacity; An electron gas in a quantizing magnetic field can be calculated, using the expression for energy (3.59), as $C_V = (\partial E/\partial T)V$.

To calculate heat capacity of a degenerate electron gas integrate up

$$E = \frac{4V(2m)^{\frac{1}{2}}}{\hbar(2\pi R)^2} \sum_j \int_{\varepsilon_j}^{\infty} [\varepsilon - (2j+1)\mu_B H]^{\frac{3}{2}} \left(\frac{-df}{d\varepsilon}\right) d\varepsilon \tag{3.61}$$

$$E = E_0 + \frac{\pi^2}{2}(k_B T)^2 \frac{V(2m)^{\frac{1}{2}}}{\hbar(2\pi R)^2} \sum_j [\mu_F - (2j+1)\mu_B H]^{\frac{-1}{2}} \tag{3.62}$$

From (3.62), for the heat capacity of a degenerate electron gas in a quantizing magnetic field we get the expression

$$C_V = \pi^2 k_B^2 T \frac{V(2m)^{\frac{1}{2}}}{\hbar(2\pi R)^2} \sum [\mu_F - (2j+1)\mu_B H]^{\frac{-1}{2}} \tag{3.63}$$

which can be presented as

$$C_V = \frac{\pi^2}{4} k_B^2 T V g_H(\mu_F) \tag{3.64}$$

where g_H (μ_F) is the density of quantum states at the Fermi level is given by

$$g_H(\varepsilon) = \frac{4}{(2\pi R)^2} \sum_j \frac{dk_z(\varepsilon, j)}{d\varepsilon} \tag{3.65}$$

In order to find the explicit form of $gH(\varepsilon)$, it is necessary to begin with a concrete form of the dispersion law $\varepsilon(kz, N,)$, where the spin splitting is disregarded. Then the summation with respect to spin is reduced to the factor 2 and by virtue of (3.34), density of states (3.65), takes the form

$$g_H(\varepsilon) = \frac{4}{(2\pi R)^2}\frac{(2m)^{\frac{1}{2}}}{\hbar}\sum_N[\varepsilon - (N + \frac{1}{2})\hbar\omega_c]^{\frac{1}{2}} \tag{3.66}$$

The summation in (3.66), is carried out with respect to all integer values of N, for which the radicand expression is positive. Note that if in weak magnetic fields $\hbar\omega_c << \varepsilon$ in (3.66), from the summation with respect to N pass to the integral in the limits from 0 to $(\varepsilon - \frac{1}{2}\hbar\omega_c/\hbar\omega_c) = N_{max}$, we get the known result (2.9), for the density of states without a magnetic field. From (3.66), it is seen that the density of states has a certain peculiarity: every time, when energy coincides with one of the Landau levels, it is converted to infinity. The behaviour of $g_H(\varepsilon)$ is schematically shown in fig.3.3. Continuously distributed quantum states in the k-space in the presence of a magnetic field basically group at Landau levels, but because of that the total number of states is conserved. The fact that many quantum states account for one Landau level is associated with the fact that each level in the magnetic field is degenerate with respect to the quantum number k_y.

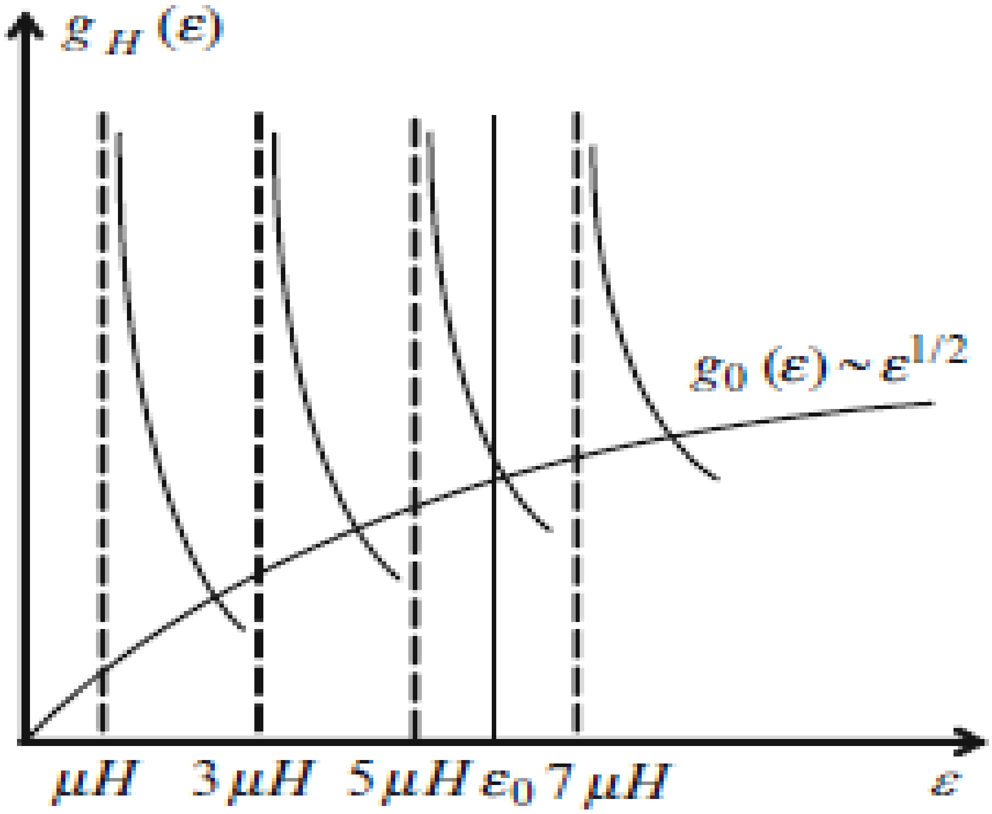

Figure 3.3: Quantum state densities in magnetic field, $g_H(\varepsilon)$.

In the quantum limit ($\mu_F < 3\mu_B H$) in (3.64), it can be restricted to the term with j=0, and the heat capacity takes the form

$$C_V = \pi^2 k_B T \frac{V(2m)^{\frac{1}{2}}}{\hbar(2\pi R)^2} \left[\mu_F - \mu_B H\right]^{\frac{-1}{2}} \tag{3.67}$$

For the heat capacity of a degenerate electron gas in the quantum limit

$$\mu(n, H) = \mu_B H \left[1 + 3\left(\frac{\mu_F}{3\mu_B H}\right)^3\right] \tag{3.68}$$

we get

$$C_V = \frac{k_B^2 T V (2m)^3}{(2\pi)^2 \hbar^2 n} (\mu_B H)^2 \tag{3.69}$$

It is seen that, the heat capacity of a degenerate electron gas in the quantum limit strongly depends on the magnetic field.

3.3 Landau Diamagnetism

According to classical mechanics, free electrons in metals placed in a uniform magnetic field move in spiral trajectories, and in the plane perpendicular to the magnetic field follow a cyclotron orbit. In1930 L.D.Landau showed that if the motion of an electron is considered on the basis of quantum mechanics and quantization of energy of free electrons in a magnetic field[1, 2, 3]. The diamagnetic susceptibility of an electron gas does not equal zero. Here we expound the theory of Landau diamagnetism on the basis of energy spectrum From (3.32), it follows that the magnetization of an electron gas M can be found originating from the explicit form of the grand thermodynamic potential $\Omega_e = \Omega_e(T, V, \mu, H)$ as follows

$$M = -\frac{1}{V}\left(\frac{\partial \Omega_e}{\partial H}\right) T, V, \mu \tag{3.70}$$

once integrate up by parts, for the grand thermodynamic potential we get

$$\Omega_e = -\frac{8V(2m)^{\frac{1}{2}}}{3\hbar(2\pi R)^2}\sum_j \int_{\varepsilon j}^{\infty} (\varepsilon - \varepsilon j)^{\frac{3}{2}} \left(\frac{-\partial f}{\partial \varepsilon}\right) d\varepsilon \tag{3.71}$$

For any degree of degeneracy of an electron gas it is impossible to find the analytical form of Ω_e therefore, the classical and the quantum statistics are considered separately. **Degenerate electron gas**; In this case in the zeroth approximation with respect to temperature $(-\partial f/\partial\varepsilon) = \delta(\varepsilon - \mu_F)$ and from (3.71), for the grand thermodynamic potential of a degenerate electron gas Ω_e we get

$$\Omega_e = \frac{-8V(2m)^{\frac{1}{2}}}{3(2\pi R)^2}\sum_{j=0}^{j} [(\mu_F - (2j+1)\mu_B H)]^{\frac{3}{2}} \tag{3.72}$$

where $j_0 = (\mu_F - \mu_B H)/2\mu_B H$.

In the general form for an arbitrary value of the magnetic field it is impossible to conduct the summation in (3.72). Therefore we consider different limiting cases. In weak magnetic fields, when $\mu_B H \ll \mu_F$ i.e. at large j_0, in the zeroth quasi-classical approximation in (3.72), the summation with respect to j. Then in the zeroth approximation for the grand thermodynamic potential of a completely degenerate electron gas we get the following result

$$\Omega_e = -\frac{8V(2m)^{\frac{3}{2}}}{15(2\pi)^2\hbar^3}\mu_F^{\frac{5}{2}} \tag{3.73}$$

In order to find the quantum correction to the grand thermodynamic potential, we calculate the sum in (3.72), with the aid of the Euler summation formula everywhere neglect $\mu_B H$ compared with μ_F. As a result,

$$\Omega_e = \Omega_e^{\prime(0)} + \frac{V(2m)^{\frac{3}{2}}}{3\hbar^3(2\pi)^2}\mu_F^{\frac{5}{2}}(\mu_B H)^2 \tag{3.74}$$

From (3.70) and (3.74), for the diamagnetic magnetization we have

$$M_{dia} = \frac{-(2m)^{\frac{3}{2}}}{6\pi^2\hbar^3}\mu_B^2 H \mu_F^{\frac{5}{2}} \tag{3.75}$$

the ratioM_{dia}/M_{para}= χ_{dia}/χ_{para} we get the same result as for a non-degenerate electron gas in a weak magnetic field. In the quantum limit, when $\mu_F < 3\mu_B H$, in sum (3.72), can be restricted only to one term (N=0). Then we have

$$\Omega_e = \frac{-8V(2m)^{\frac{3}{2}}}{3\hbar^3(2\pi)^2}\frac{eH}{\hbar c}(\mu_F - \mu_B H)^{\frac{3}{2}} \tag{3.76}$$

Hence, according to (3.70), find the magnetization M, where for $(\mu_F - \mu_B H)$. As a result, we get

$$M = \frac{-8(2m)^{\frac{3}{2}}}{3\hbar(2\pi)^2}\frac{e}{\hbar c}(\mu_F)^{\frac{3}{2}}\left[\left(\frac{\mu_F}{3\mu_B H}\right)^3 - \frac{1}{2}\right] \tag{3.77}$$

Chapter 4

Results and Discussion

In this section, we have studied about results and discussion graphically expiration of relationship between different parameters of thermodynamic properties of strongly degenerate fermi gas with regards to the derived equation from the above section.

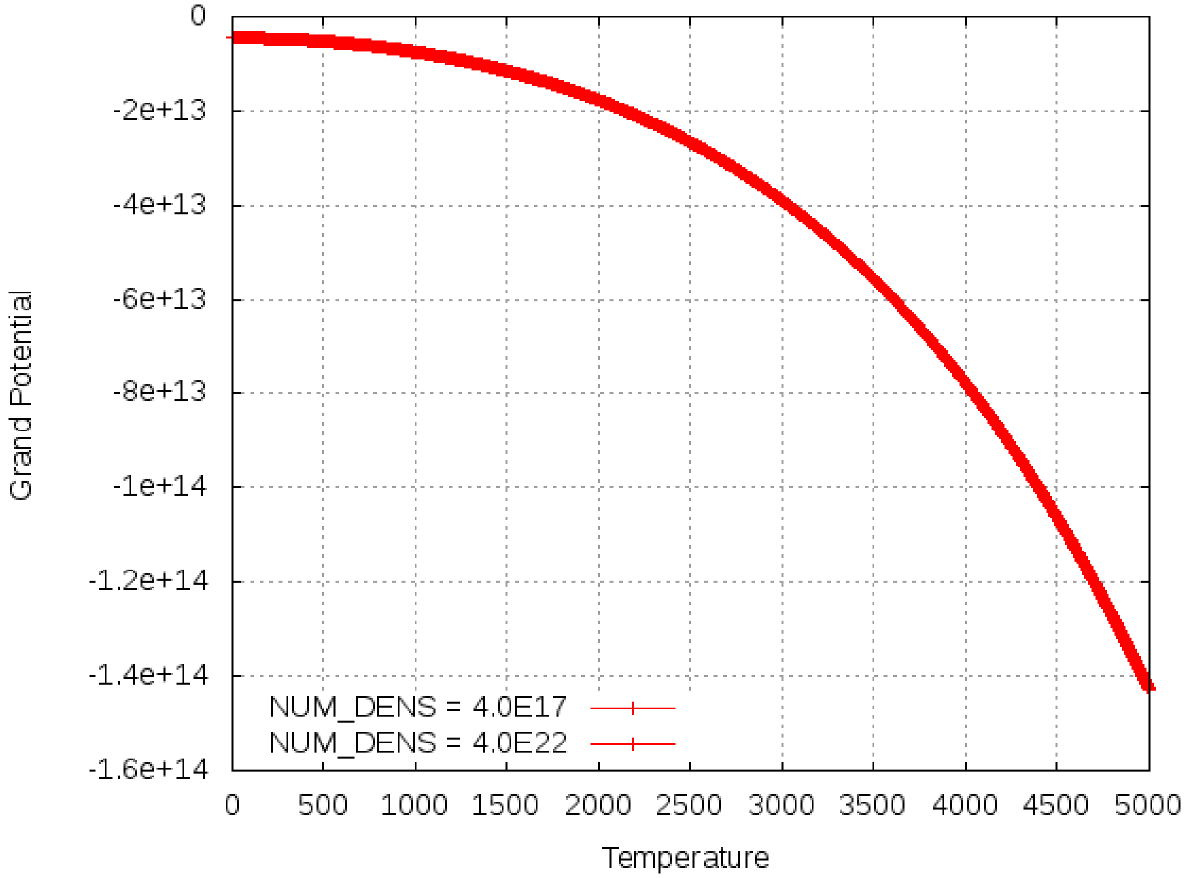

Figure 4.1: The plots of graph grand potential as a function of temperature

The above graph shows the relationship between grand potential and temperature is half parabolic concave downward from equation (2.31). When the temperature increase the grand potential becomes large negative.

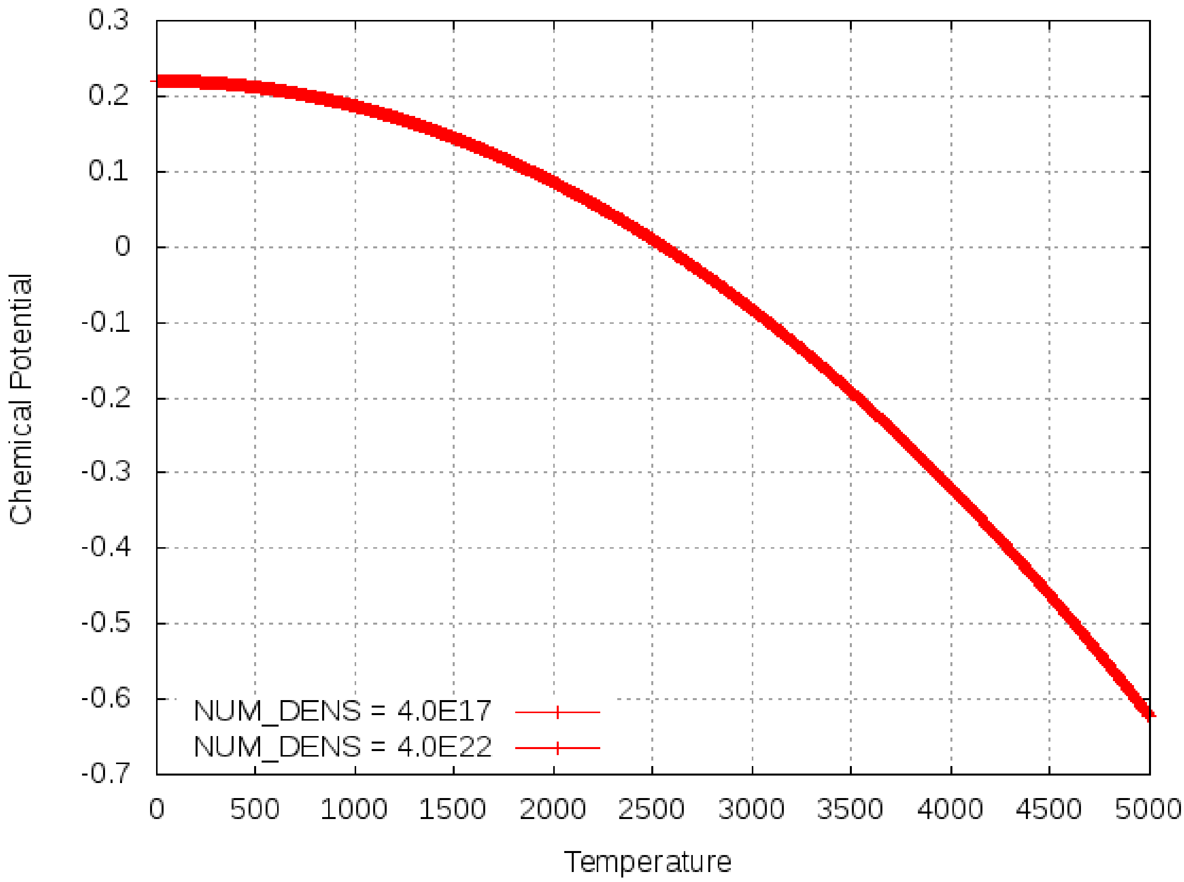

Figure 4.2: The plots of graph chemical potential as a function of temperature

From equation (2.34), the plots of the graph of chemical potential as a function of temperature show. As can be seen from the graph at finite but small temperatures the Fermi energy decreases and has positive value and become negative for large value of temperature. As shown from the graph for temperature greater than 2500K the chemical potential is positive, zero for T=2500K and negative for temperature greater than 2500K.

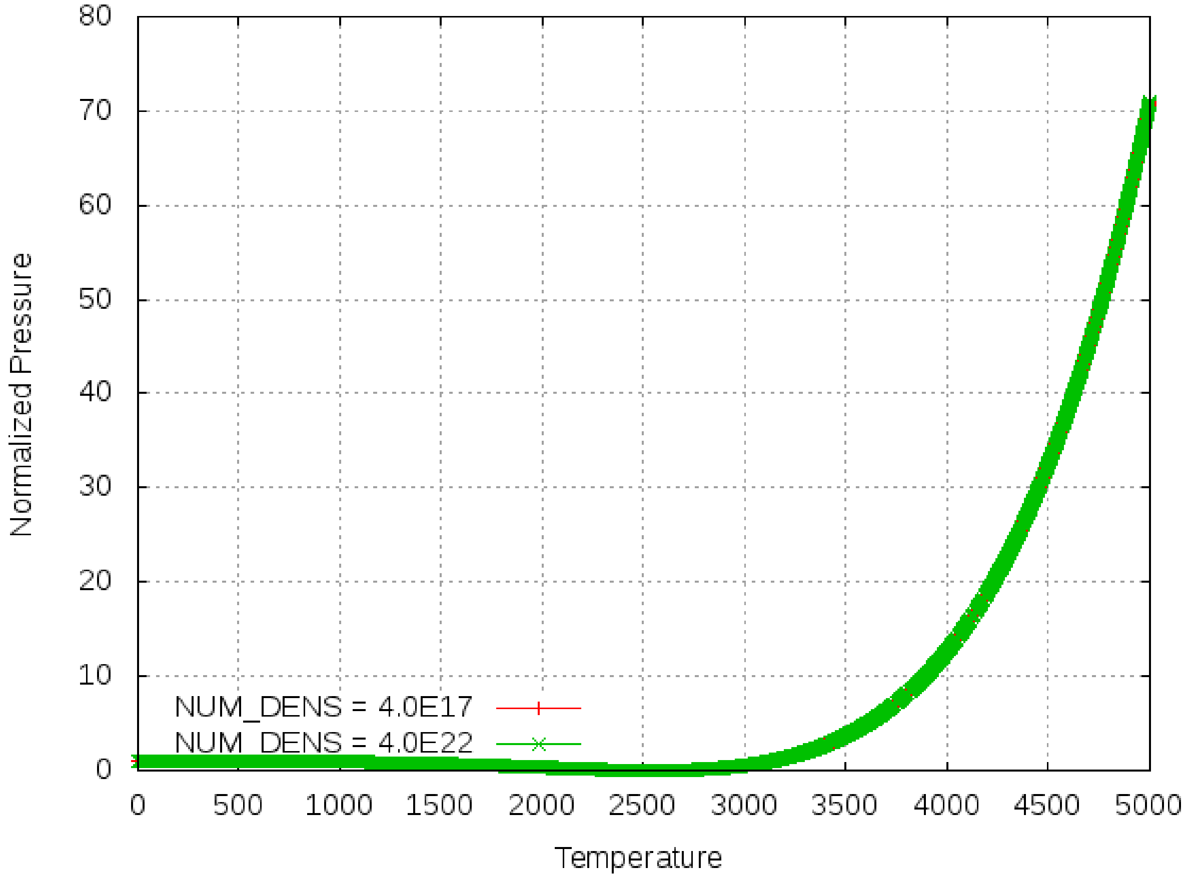

Figure 4.3: The plots of graph normalized pressure as a function of temperature

From equation (2.35), the normalized pressure as a function of temperature shows fig 4.3, the pressure is almost zero for temperature less than 3000K and then increase rapidly.

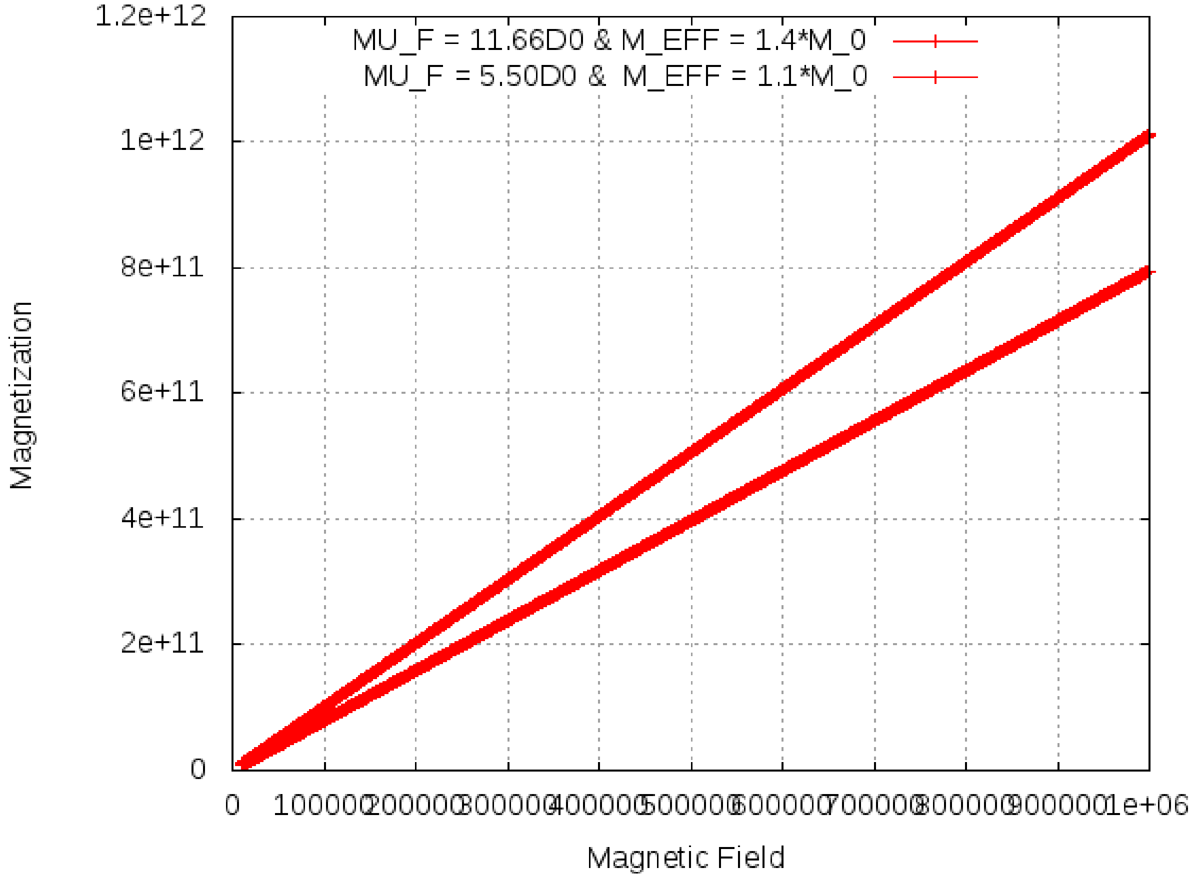

Figure 4.4: The plots of graph magnetization as a function of magnetic field

The above graph is the plot of the magnetization as a function of magnetic field shown in equation (3.9). As the graph shows, magnetization increase linearly as magnetic field increase.

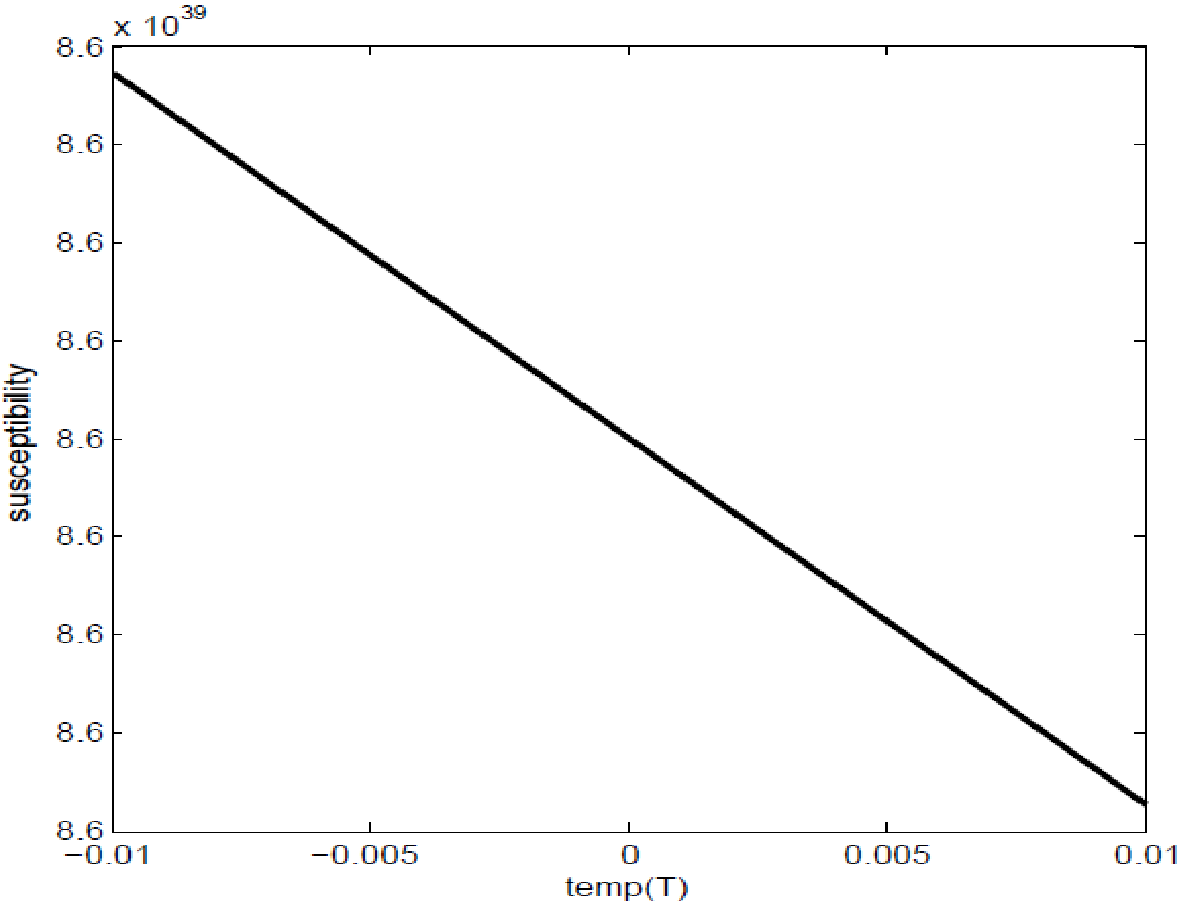

Figure 4.5: The plots of susceptibility as a function of temperature

The plots of graph susceptibility versus temperature from equation(3.31), shown in fig 4.5, it is seen that at temperatures $k_B T \ll \mu_F$ the paramagnetic susceptibility χ_0 depends on temperature T very weakly. Then the graph shows decrease susceptibility as temperature increase.

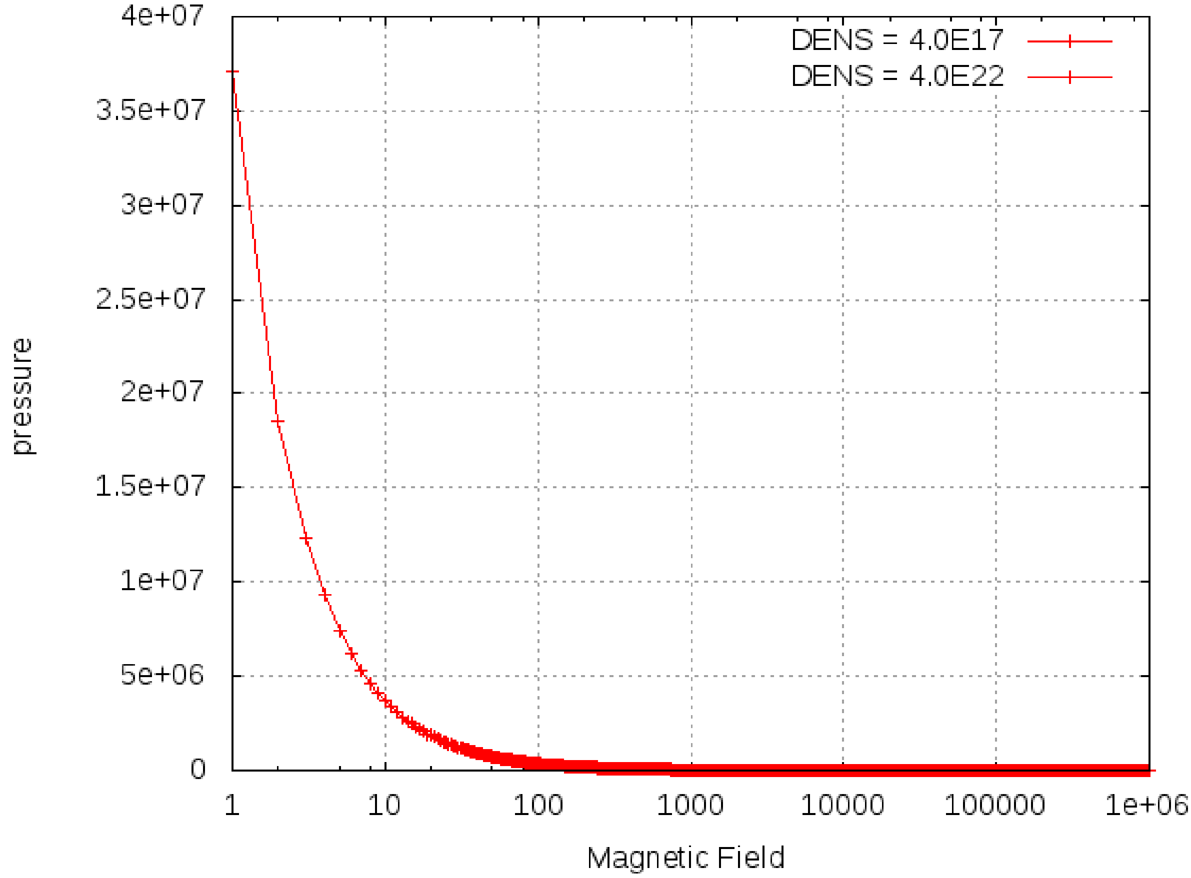

Figure 4.6: The plots of graph pressure as a function of magnetic field

From equation (3.47), the graph of pressure as a function of Magnetic field is shown in fig 4.6, as the magnetic field increase from 1 to 100T the pressure decrease parabolically and become zero grater than 100T. Because as magnetic field increase, magnetic moment become more aligned and become rigid thus mobility decrease becomes like a solid. Thus the pressure decrease.

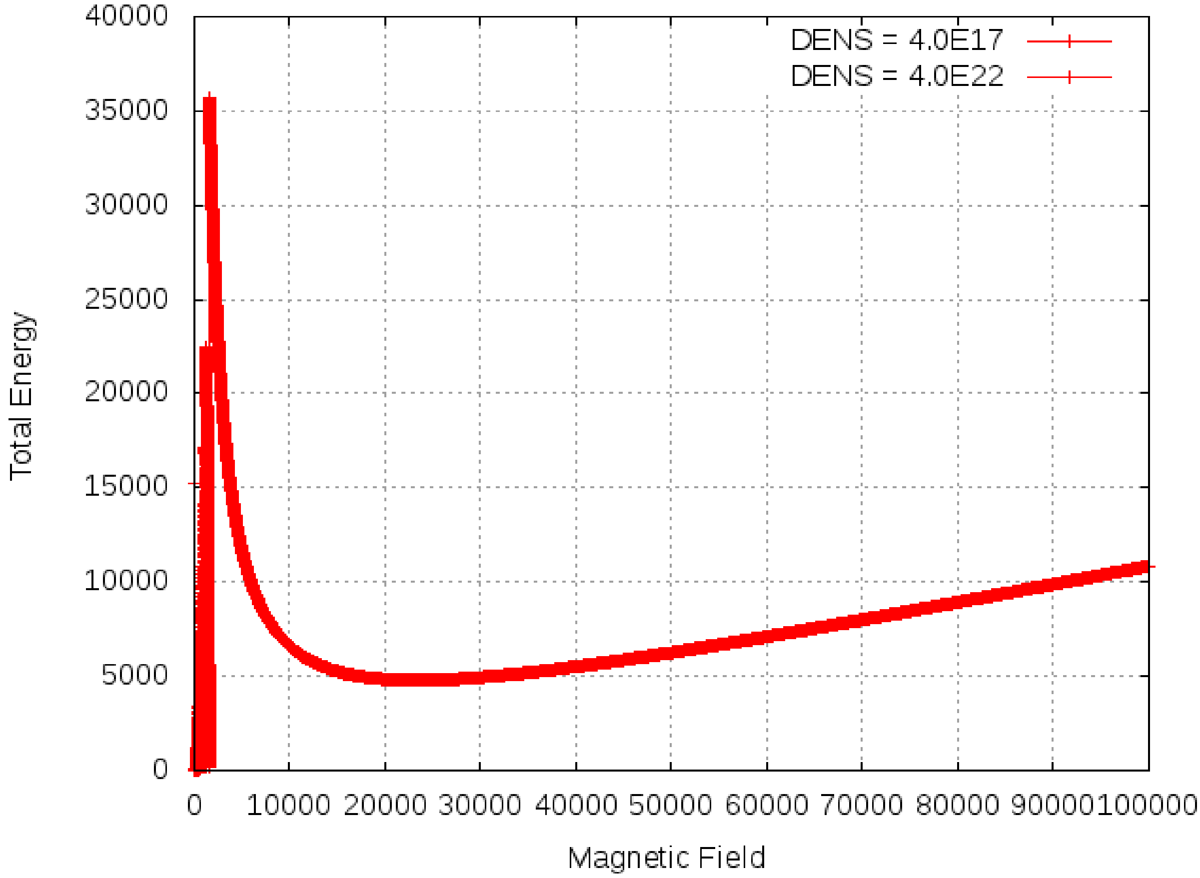

Figure 4.7: The plots of graph total energy as a function of magnetic field

From equation (3.55), the total energy as a function of magnetic field is plotted in fig 4.7, for magnetic field less than 1500T, energy increase rapidly. The density of state has the discontinuous behavior. Density of state is real and positive. As N goes to j_{max}, Density of state becomes infinite and then it falls down. It shows, discontinuity at j_{max}.

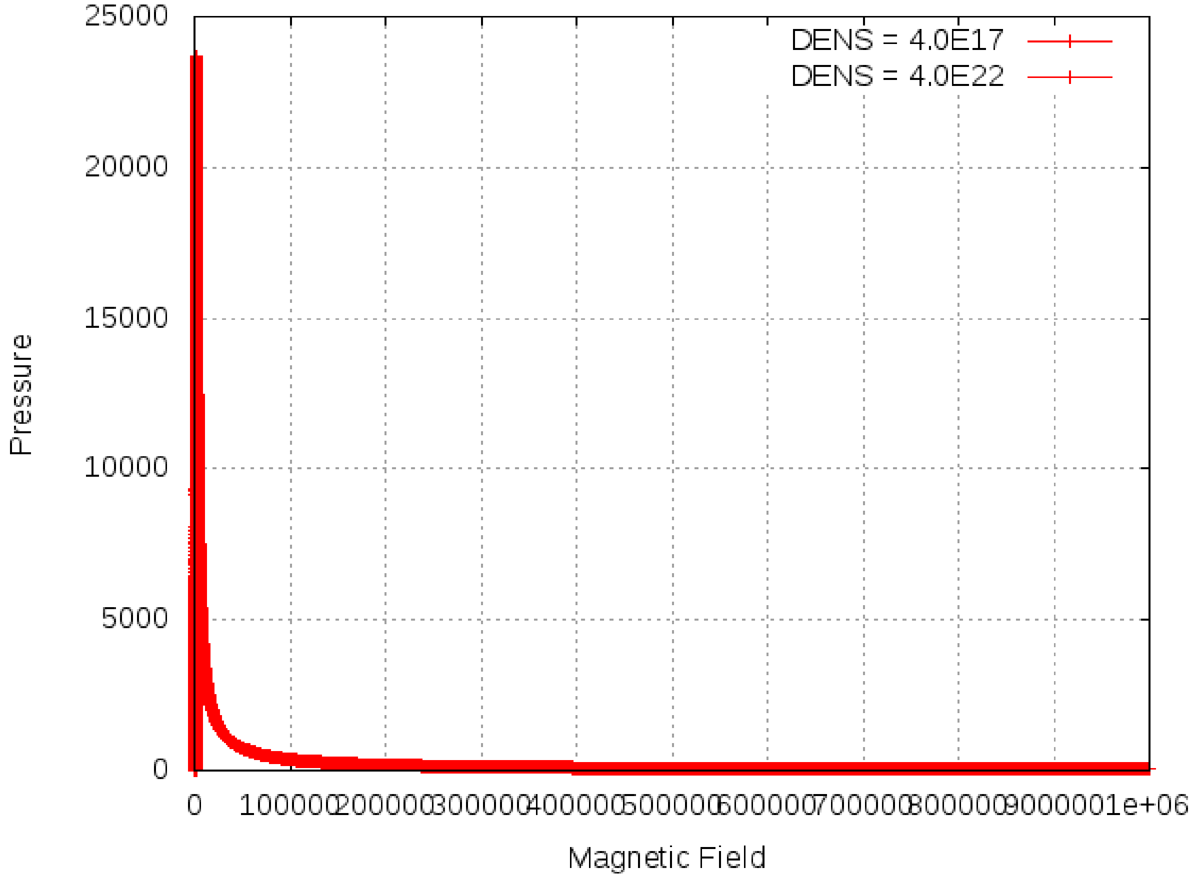

Figure 4.8: The plots of graph pressure as a function of magnetic field

As shown from the above fig 4.8, in equation (3.47), it is more general to show the relationship between the graph of pressure as a function of Magnetic field, total energy as a function of Magnetic and grand potential as a function of Magnetic show analogous behavior.

Chapter 5

Summery and Conclusion

►Thermodynamics properties of strongly degenerate fermi gas is defined as a gas be found in the strongly degenerate state, it is necessary that its *concentration n be large, the mass of fermions small and the temperature low.*

► The Second Difficulty of Classical Statistics was circumvented by Pauli in 1927, after applying the new Fermi statistics to compute the paramagnetic susceptibility. He came to the conclusion that the cause of the divergence is the fact that **an electron gas is not classical but a strongly degenerate quantum gas.**

►To determine the criterion of degeneracy of an electron gas in a quantizing magnetic field, it is needed to know the explicit form of the grand thermodynamic potential Ω_e as a function of volume, temperature, chemical potential and magnetic field $\Omega_e = \Omega_e(V, T, \zeta, H)$. The thermodynamic relationship for Ω_e in a magnetic field is given by $d\Omega_e = -SdT - PdV - N_e d\zeta - VMdH$.

►Generally, we can get the result from the derived equation the plot of these parameters as temperature increase the grand potential is large negative. As temperature increases the chemical potential decreases, the pressure increases as the temperature increases, the pressure decreases as magnetic field increases at low temperature. The total energy versus magnetic field graph shows that the energy increases rapidly when magnetic field is less than 1500 T. The density of state has discontinuous behavior and has a real and positive value.

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