Quantum Hall Effect in 2DES and Graphene

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By

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DEPARTMENT OF PHYSICS

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Table of Contents

List of Figures

Abstract

We consider the behavior of 2D systems in the strong magnetic fields and at low temperature. The main attention is devoted to the quantum Hall effect in two dimension electron gas and in one layered graphene . The original part of the work is the analysis of Landau level of mono layer graphene in cylinderical cooridinate.

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Introduction

When electric current carrying conductor is placed in a perpendicular magnetic field, there will be a production of potential difference (Hall voltage) across a conductor this effect is called Hall effect. The Hall effect was discovered in 1879 by Edwin Herbert Hall while he was working his doctoral degree at Johns Hopkins University. The Hall effect is due to the nature of electric current in a conductor. Current is the movement of electrons, holes or ions. When a magnetic field is applied that is perpendicular to a moving charges this charges experience a force called Lorentz force. Due to the existence of the magnetic field the electrons or charges move to a curved path between collisions. But for absent of magnetic field charge follow straight path between collision. Because of magnetic field the electrons move to the left, mostly accumulate up on left edge, leaving uncompensated to the right as we see in (Fig.1). The separation of positive and negative charge produces an electric field $\vec{E}$ within strip as we see in (Fig.1). This field exerts an electric force F_E on electrons directly opposite to magnetic force. So an equilibrium develops in which electric force on each electrons build up until just cancel magnetic force that made electrons to follow curved path [13].

The Hall voltage V_H is associate with induced electric field across strip.

$$V_H = Ed \tag{0.0.1}$$

where V_H is Hall voltage, E electric field and d is the width of conductor. After the equilibrium attains no more accumulation of charge on strip so that the Hall voltage

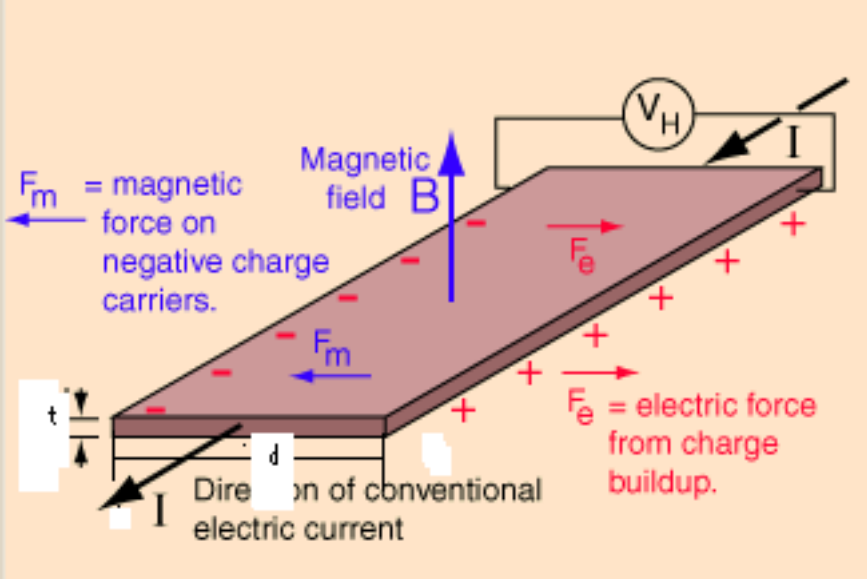

Figure 1: Hall effect in a conductor

and induced electric field kept constant as (Fig.1). As accumulation of charges increase the induced electric field also increase. Finally, the force on electron due to this induced electric field able to cancel the Lorentz force. So the electrons follow straight path due to the equilibrium. Therefore, there is no more accumulation of charges in strip so this will be peak voltage, Hall voltage.

$$\vec{F}_E = \vec{F}_m \tag{0.0.2}$$

where $\vec{F}_E$ is electric force and $\vec{F}_m$ is magnetic force.

$$eE = ev_dB, \tag{0.0.3}$$

$$v_d = \frac{E}{B}. \tag{0.0.4}$$

where E is induced electric field, v_d is drift velocity, e is charge of electron and B is magnetic field.

Current density in conductor is given by

$$J = nev_d \Rightarrow v_d = \frac{-J}{ne} \tag{0.0.5}$$

The negative value taken from electron charge where J is current density, n is charge density. The cross section area of conductor is given by,

$$A = t \times d \tag{0.0.6}$$

A is cross section area, t is thickness of conductor and d is the width of strip. From equation (0.0.1) and (0.0.4)

$$V_H = Bv_d d \tag{0.0.7}$$

here V_H is Hall voltage, d is width of the strip, B is applied magnetic field and v_d is drift velocity of electron.

Now substitute equation (0.0.5) into (0.0.6)

$$V_H = -B\frac{J}{ne}d \Rightarrow V_H = -\frac{BdJ}{ne} \tag{0.0.8}$$

Next let multiply right hand side by $\frac{t}{t}$ where t is thickness of the wire. As we write based equation (0.0.6) the equation will be

$$V_H = -\frac{BJA}{net} \tag{0.0.9}$$

But it is known that $JA = I$

$$V_H = \frac{-BI}{ne} \tag{0.0.10}$$

The Hall constant is given by

$$R_H = \frac{E_y}{j_x B} \tag{0.0.11}$$

where E_y is induced electric field due to Hall effect, R_H is Hall resistivity, j_x is current density and B is magnetic field.

$$E_y = \frac{V_H}{d} \tag{0.0.12}$$

Now substitute equation (0.0.12) in to (0.0.11) and multiply by thickness $\frac{t}{t}$.

$$R_H = \frac{V_H t}{j_x Btd} \tag{0.0.13}$$

$$R_H = \frac{V_H t}{j_x AB} \tag{0.0.14}$$

And current is equal to $j_x A$

$$R_H = \frac{V_H t}{IB} \tag{0.0.15}$$

Finally by putting equation (0.0.10) into equation (0.0.15) we can get:

$$R_H = -\frac{\frac{IB}{net} \times t}{IB} = -\frac{IB}{neIB} \tag{0.0.16}$$

$$R_H = -\frac{1}{ne} \tag{0.0.17}$$

n is charge density, e is electron charge and R_H is Hall coefficient measured in m^3/C. Hall effect is a very useful as means of measuring the carrier density and sign of charge whether a body is negative or positive.

Hall probe is one of the application of Hall effect. It is often used to as magneto meter. Hall effect device produced quite low signal level but instead must be amplified by transistor based amplifier. Hall effect device used in motion sensing and motion limit switches. Hall effect sensor also used for automotive ignition timing device in different distributor and fuel injection electric motor sensing and space craft propulsion.

Chapter 1

Classical Hall Effect

Let now look into classical calculation of the equation of electron motion in crossed magnetic and electric field.

$$F = m\frac{dv}{dt} = -e[\vec{E} + \frac{1}{c}v \times \vec{B}] \tag{1.0.1}$$

where m is mass of electron, v is speed of electron, F is force, c is speed of light, $\vec{E}$ is electric field and $\vec{B}$ is magnetic field.

In presence of friction or collision in conductor (1.0.1) can be written as [1]

$$m(\frac{d}{dt} + \frac{1}{\tau})v = -e\vec{E} - \frac{e}{c}v \times \vec{B} \tag{1.0.2}$$

here τ is relaxation time between collision .

Each component in (1.0.2) can be given as:

$$m(\frac{d}{dt} + \frac{1}{\tau})v_x = -e\vec{E_x} - (\frac{e}{c}v \times \vec{B})_x, \tag{1.0.3}$$

$$m(\frac{d}{dt} + \frac{1}{\tau})v_y = -e\vec{E_y} - (\frac{e}{c}v \times \vec{B})_y, \tag{1.0.4}$$

$$m(\frac{d}{dt} + \frac{1}{\tau})v_z = -e\vec{E_z} - (\frac{e}{c}v \times \vec{B})_z. \tag{1.0.5}$$

where v_x,v_y and v_z are speed of electron; E_x,E_y and E_z are electric field in different component.

$$v \times \vec{B} = \begin{pmatrix} i & j & k \\ v_x & v_y & v_z \\ 0 & 0 & B \end{pmatrix} \tag{1.0.6}$$

Since the magnetic field is in z direction and by considering steady flow electric current, $\frac{dv}{dt} = 0$ (1.0.3), (1.0.4) and (1.0.5) will look like:

$$v_x = -\frac{e}{m}\tau E_x - \frac{Be}{mc}\tau v_y, \tag{1.0.7}$$

$$v_y = -\frac{e}{m}\tau E_y + \frac{Be}{mc}\tau v_x. \tag{1.0.8}$$

Now, by substituting (1.0.8) into (1.0.7) and vice versa we can obtain:

$$v_x = \frac{-e}{m}\tau E_x - \frac{Be}{mc}\tau(-\frac{e}{m}\tau E_y + \frac{Be}{mc}\tau v_x), \tag{1.0.9}$$

$$v_x = \frac{-\tau e}{m(1+\tau\omega_c^2)}E_x + \frac{\tau^2 e\omega_c}{m(1+\tau\omega_c^2)}E_y, \tag{1.0.10}$$

$$v_y = \frac{-e\tau}{m(1+\tau^2\omega_c^2)}E_y - \frac{\tau^2 e\omega_c}{m(1+\tau^2\omega_c^2)}E_x, \tag{1.0.11}$$

$$v_z = -\frac{e}{m}\tau E_z. \tag{1.0.12}$$

where ω_c is cyclotron frequency which is $\omega_c = \frac{eB}{mc}$.

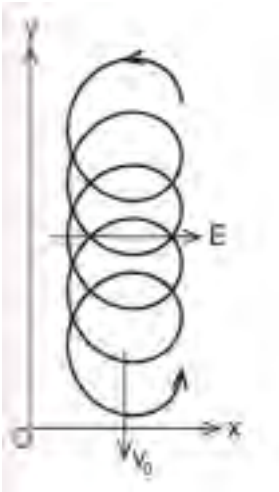

Figure 1.1: Motion of electron in crossed electric and magnetic field

As we see in (Fig.1.1) the electron move simultaneously rotate in plane. If there was only magnetic field the electron only rotate by cyclotron frequency not move through a conductor. But when the case is in cross electric and magnetic field electron will

make both moving through the conductor and rotates by cyclotron frequency.

The current density given by

$$j = nqv \Longrightarrow j = nev \tag{1.0.13}$$

where j current density, n number of charge, q charge, e electron charge and v speed of electron.

Using (1.0.10), (1.0.11) and (1.0.12) with equation (1.0.13) one can obtain components of current density.

$$j_x = nev_x = ne[-\frac{\tau e}{m(1+\tau\omega_c^2)}E_x + \frac{\tau^2 e\omega_c}{m(1+\tau\omega_c^2)}E_y], \tag{1.0.14}$$

$$j_x = -\frac{n\tau e^2}{m(1+\tau\omega_c^2)}E_x + \frac{n\tau^2 e^2\omega_c}{m(1+\tau\omega_c^2)}E_y, \tag{1.0.15}$$

$$j_y = nev_y = ne[\frac{-e\tau}{m(1+\tau^2\omega_c^2)}E_y - \frac{\tau^2 e\omega_c}{m(1+\tau^2\omega_c^2)}E_x], \tag{1.0.16}$$

$$j_y = \frac{-ne^2\tau}{m(1+\tau^2\omega_c^2)}E_y - \frac{n\tau^2 e^2\omega_c}{m(1+\tau^2\omega_c^2)}E_x, \tag{1.0.17}$$

$$j_z = nev_z = ne[-\frac{e}{m}\tau E_z], \tag{1.0.18}$$

$$j_z = -\frac{ne^2}{m}\tau E_z. \tag{1.0.19}$$

where j_x ,j_y and j_z are current density.

Current density also given in this form

$$j_i = \sigma_{ij}E_j \tag{1.0.20}$$

where j_i is current density, σ_{ij} is conductivity tensor and E_i is electric field.

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} \tag{1.0.21}$$

By plugging (1.0.15), (1.0.17) and (1.0.19) with (1.0.21) one can obtain conductivity tensor as

$$\frac{\sigma_o}{(1+\omega_c\tau)^2} \begin{pmatrix} 1 & -\omega_c\tau & 0 \\ \omega_c\tau & 1 & 0 \\ 0 & 0 & (1+\omega_c\tau)^2 \end{pmatrix} \tag{1.0.22}$$

here $\sigma_o = \frac{ne^2\tau}{m}$. The most important result we are in for two dimensional system for strong magnetic field $\omega_c\tau >> 1$ led to

$$\sigma_{xy} = -\frac{nec}{B} \tag{1.0.23}$$

And the $\omega_c\tau >> 1$ vanish

$$\sigma_{xx} = o \tag{1.0.24}$$

The diagonal component of conductivity (σ_{xx}) go to zero while the Hall conductivity (σ_{xy}) has a finite value as for strong magnetic field. We are going to discuss this case in both 2DES and graphene.

Chapter 2

Two Dimensional Electron System

A two dimensional electron gas can be seen at the interface between semiconductor. Two dimensional electron gas can be realized in different material: like in Si MOS-FET, HMT(high mobility transistor) like GaAs and AlGaAs, in quantum well and liquid helium. For first time Si MOS is fabricated in 1960. Si MOS made by combining Si, Sio_2, insulator, and metal. When a potential difference is applied between metal and Si, semiconductor, electrons appear between semiconductor and insulator and positive charges collected in between metal and insulator. High quality of two dimensional electron system realized by using trivalent (Ga) and pentavalent (As). GaAs is one of compound that are produced by method of molecular beam epitaxy (MBE). Quantum well is produced by sandwich type structure where a thin GaAs layer is placed inside bulk AlGaAs. Finally, two dimensional electron system can be realized by putting electrons on free surface of liquid helium. Helium liquefies at 4.2 k under 1 atm, free surface is quite flat.

Electron approaching the surface from above attracted to surface but the electrons cannot dive into Helium due to it has 1s orbital that are completely filled, and 2s orbitals energy is so high. So electrons pushes aside and form the bubble in surface of helium [4] and [2].

2.1 Landau level and Two Dimensional Electron System

In chapter one we have seen classical treatment electrons in constant magnetic field. Now, let us consider the solution of Schrodinger for an electron in constant magnetic field. Hamiltonian operator of electron given by

$$\hat{H} = \frac{1}{2m^*}(\hat{p} - \frac{e}{c}\overrightarrow{A})^2 \tag{2.1.1}$$

where $\hat{H}$ is Hamiltonian operator, $\hat{p}$ is momentum operator , $\vec{A}$ is magnetic vector potential, m* is effective mass of electron, e is charge of electron and c is speed of light.

Landau gauge given by:

$$A_x = -B_o y, A_y = 0, A_z = 0 \tag{2.1.2}$$

here A_x, A_y and A_z are vector potentials, B_o is applied magnetic field and y is displacement.

$$\overrightarrow{B} = \nabla \times \overrightarrow{A} = \begin{pmatrix} i & j & k \\ \frac{\partial}{\partial_x} & \frac{\partial}{\partial_y} & \frac{\partial}{\partial_z} \\ -B_o y & 0 & 0 \end{pmatrix} \tag{2.1.3}$$

where $\vec{B}$ is magnetic field, $\vec{A}$ is vector potential, B_o is applied magnetic field and y is displacement. Two dimensional electron gas in the x, y plane and magnetic field is in z direction.

$$\frac{1}{2m^*}\{(\hat{p}_x + \frac{eB_o}{c}y)^2 + \hat{p}_y^2\}\psi(x,y) = E\psi(x,y) \tag{2.1.4}$$

where $\psi(x,y)$ is wave function, E is energy eigenvalue value and $\hat{p}_x$ and $\hat{p}_y$ are momentum operators.

We seek a solution for (2.1.4) in form of

$$\psi(x,y) = \varphi(y)e^{ikx} \tag{2.1.5}$$

$\varphi(y)$ and $\psi(x,y)$ are wave functions, k is wave vector and y is displacement.

Substitute equation (2.1.5) in equation (2.1.4):

$$\frac{1}{2m^*}\{(\hbar k + \frac{eB_o}{c}y)^2 + \hat{p}_y^2\}\varphi(y) = E\varphi(y) \tag{2.1.6}$$

where $\hbar$ is Plank constant Now we insert $2m$ to bracket multiply by m of the first term in equation

$$\{\frac{e^2B_o^2m^*}{2c^2m^{*2}}(y-y_o)^2 - \frac{\hbar^2d^2}{2m^*dy^2}\}\varphi(y) = E\varphi(y) \tag{2.1.7}$$

where $y_o = -\frac{\hbar kc}{eB_o}$. Next let we substitute $y = \alpha\xi$ where α has length dimension and ξ is dimensionless and putting cyclotron frequency $\omega_c = \frac{eB_o}{mc}$ we can get:

$$-\frac{\hbar^2}{2m^*\alpha^2}\frac{d^2}{d\xi^2}\varphi + \frac{m^*\omega_c^2\alpha^2}{2}(\xi-\xi_o)^2\varphi = E\varphi \tag{2.1.8}$$

here α has length dimension.

Let us rewrite equation (2.1.8):

$$\frac{d^2\varphi}{d\xi^2} + [\frac{2m^*E}{\hbar^2}\alpha^2 - \frac{m^{*2}\omega_c^2\alpha^4}{\hbar^2}(\xi-\xi_0)^2]\varphi = 0 \tag{2.1.9}$$

Here we put

$$\frac{m^{*2}\omega_c^2\alpha^4}{\hbar^2} = 1 \tag{2.1.10}$$

From (2.1.10)

$$\alpha = \sqrt{\frac{\hbar}{m^*\omega_c}} \tag{2.1.11}$$

Substitute equation (2.1.10) and (2.1.11) into equation (2.1.9)

$$\frac{d^2\varphi}{d\xi^2} + [\frac{2m^*E}{\hbar^2}\frac{\hbar}{m^*\omega_c} - (\xi-\xi_0)^2]\varphi = 0 \tag{2.1.12}$$

This led to Schrodinger equation of the harmonic oscillator motion:

$$\frac{d^2\varphi}{d\xi^2} + [\lambda - (\xi-\xi_0)^2]\varphi = 0 \tag{2.1.13}$$

where λ

$$\lambda = \frac{2E}{\hbar\omega_c} \tag{2.1.14}$$

$$\lambda = 2n + 1 \Longrightarrow \frac{2E}{\hbar\omega_c} = 2n + 1 \tag{2.1.15}$$

$$E_n = \hbar\omega_c(n + \frac{1}{2}) \tag{2.1.16}$$

here n is integer. Since it has form simple harmonic oscillator its wave function look like

$$\varphi_n(y) = N_n e^{-\frac{m\omega}{2\hbar}(y-y_o)^2} H_n([y - y_o]\sqrt{\frac{m\omega}{\hbar}}) \tag{2.1.17}$$

One can substitute equation (2.1.17) in to equation (2.1.5) to get

$$\psi_{nk}(x, y) = Ae^{ikx} e^{-\frac{m\omega}{2\hbar}(y-y_o)^2} H_n([y - y_o]\sqrt{\frac{m\omega}{\hbar}}) \tag{2.1.18}$$

A and N_n are the normalization coefficients. When we look in equation (2.1.16) energy depends on n level where as in equation (2.1.18) the wave function is depend on both n and k, means for a given value n there can be different k that is$(\psi_{nk1}, \psi_{nk2}, \psi_{nk3}...)$, this thing shows us for given eigenvalue there are number of wave functions this what we call degeneracy. Let take in to account (Fig. 2.1) to calculate degeneracy. We point out that magnetic field is applied in z direction.

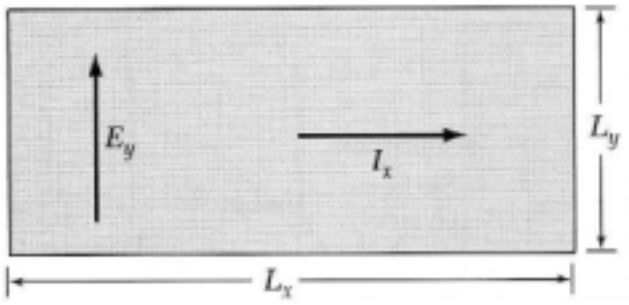

Figure 2.1: Hall effect in 2DES

$$y_o = \frac{-\hbar kc}{eB_o} \tag{2.1.19}$$

where $\hbar$ is Plank constant, k is wave vector, e is electron charge and c is speed of light. Now we find the degeneracy of the Landau levels.

$$k_{max} = \frac{2\pi}{L_x} p_{max} \tag{2.1.20}$$

here p is integer, k is wave vector and L_x is width of 2DEG.

From (2.1.19) maximum k_{max} can calculated.

$$k_{max} = \frac{eB_oL_y}{hc} \tag{2.1.21}$$

By equating (2.1.20) and (2.1.21) one can get p_{max}

$$p_{max} = g = \frac{L_xL_yeB}{ch} \tag{2.1.22}$$

$$g = \frac{eAB_o}{hc} = \frac{AB_o}{\frac{hc}{e}} \tag{2.1.23}$$

here is degeneracy of Landau levels.

$$g = \frac{\Phi}{\Phi_o} \tag{2.1.24}$$

where $\Phi_o = \frac{hc}{e}$, $A = L_xL_y$ is the area of a conductor, B_o is applied magnetic field, The degeneracy depend on the are of 2DES and magnetic filed application as the magnetic field applied increases the degeneracy also increase linearly. In (Fig.2.2) when the magnetic field $B = 0$ the energy level look like continuous but $B_1 \neq 0$ Landau level produced.

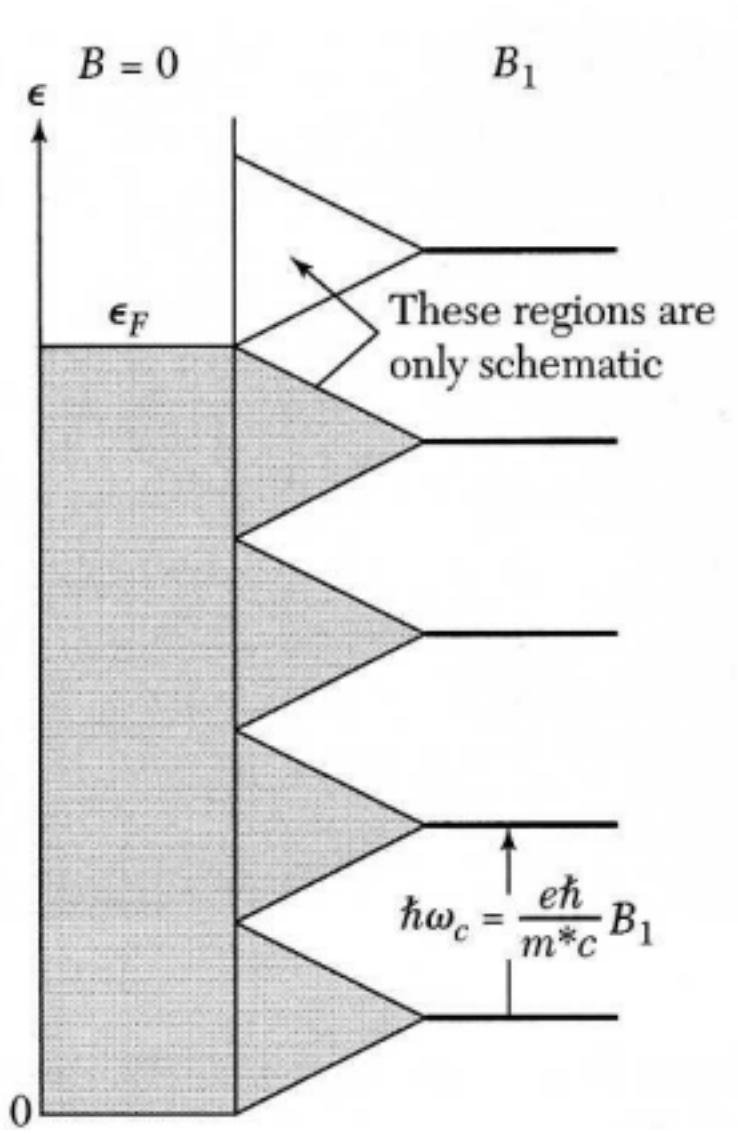

Figure 2.2: Landau level

Chapter 3

Hall Effect In Two Dimensional Electron System (2DES)

Quantum Hall effect is observed in 2DES which is the quantization of Hall voltage. When a strong magnetic field is applied at lower temperature in 2DES all electrons drop to lowest Landau level (LLL). Filling factor given by

$$\nu = \frac{n}{g} \tag{3.0.1}$$

Where n is number of electrons in a level, g is degeneracy. Filling factor also depend on magnetic field

$$\nu = \frac{nh}{eB} \tag{3.0.2}$$

Where n is electrons number density, In quantum Hall effect: there is the presence of quantized plateaus of the Hall resistance ρ_{xy}, and vanishing of the diagonal resistivity, ρ_{xx} in 2DES, (eg GaAs-GaAlAs) at integer ν and fractional occupation $\nu = \frac{p}{q}$ of Landau level where p is integer and q is odd integer,1,3,5,7.. The Hall conductivity and Hall resistivity given by [6], [7] and [8]

$$\sigma_{xy} = -\nu\frac{e^2}{h} \Rightarrow \rho_{xy} = -\frac{h}{\nu e^2} \tag{3.0.3}$$

where ν is filling factor and h is Plank constant.

When ν has an integer value it is called an Integer Quantum Hall effect (IQHE) while ν has a fractional occupation $\nu = \frac{p}{q}$ of Landau level we call Fractional Quantum Hall effect (FQHE).

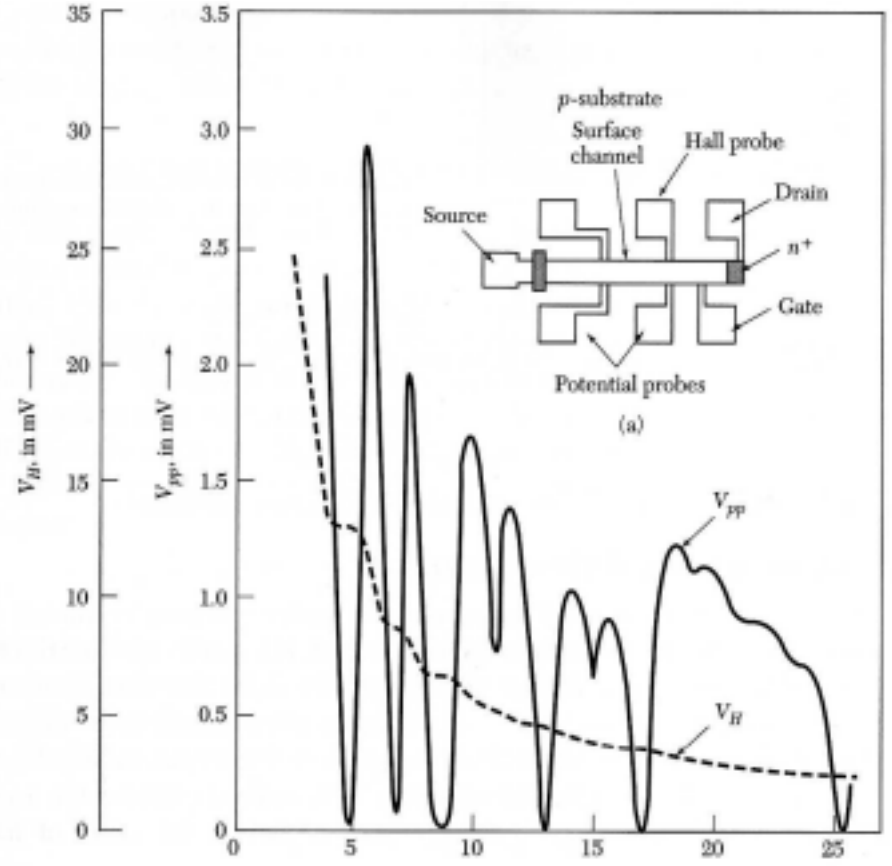

Figure 3.1: V_{pp}, V_H versus the gate voltage,V_g [1]

3.1 Integer Quantum Hall Effect(IQHE)

As we see in figure (Fig.3.1) of gate voltage the voltage drop in direction of current flow goes to zero, as the effective conductance were infinite. Second, there are plateaus of Hall resistivity V_H/I_x, (Fig.2.1) at these plateaus are accurately quantized at 25,813/s ohms which is value of $\frac{h}{e^2}$. The number of electrons surface concentration is given by

$$n_s = \frac{seB}{hc} \tag{3.1.1}$$

The quantization result follow

$$n_s = \frac{h}{se^2} = \frac{2\pi}{sc\alpha} \tag{3.1.2}$$

n_s number of electrons surface concentration, s is integer and α is the fine structure constant which is $\alpha = \frac{1}{137}$. Hall resistivity is quantized at 25,813/s independent of semiconductor high purity and perfection. The density of state in electron gas in strong magnetic field. In Ideal 2D crystal the Landau level is sharp. In real crystal the Landau level is broadened. But this does not affect the Hall resistivity as we see in (Fig.3.1).

In real crystal the Landau level is broadened in contrast with ideal crystals as we see in (Fig.3.2a) and (Fig.3.2b). We see in (Fig.3.1) the occurrence of plateaus in the Hall resistance curve. But this not expected in ideal system. Laughlin interpreted the result for real system as the expression of general principle of gauge invariance. Laughlin made experiment which is called Laughlin's thought -experiment. In this experiment the 2D system, which exactly in (Fig.2.1), is bent to form a cylinder (Fig.3.3) whose surface is pierced every where by a strong magnetic field B normal to surface. The current I, I_x in (Fig.2.1), circle the loop. The magnetic field B acts on charge carrier to produce a Hall voltage V_H. Circulating current I produce a small magnetic flux. The primary goal of thought- experiment is to find the relation

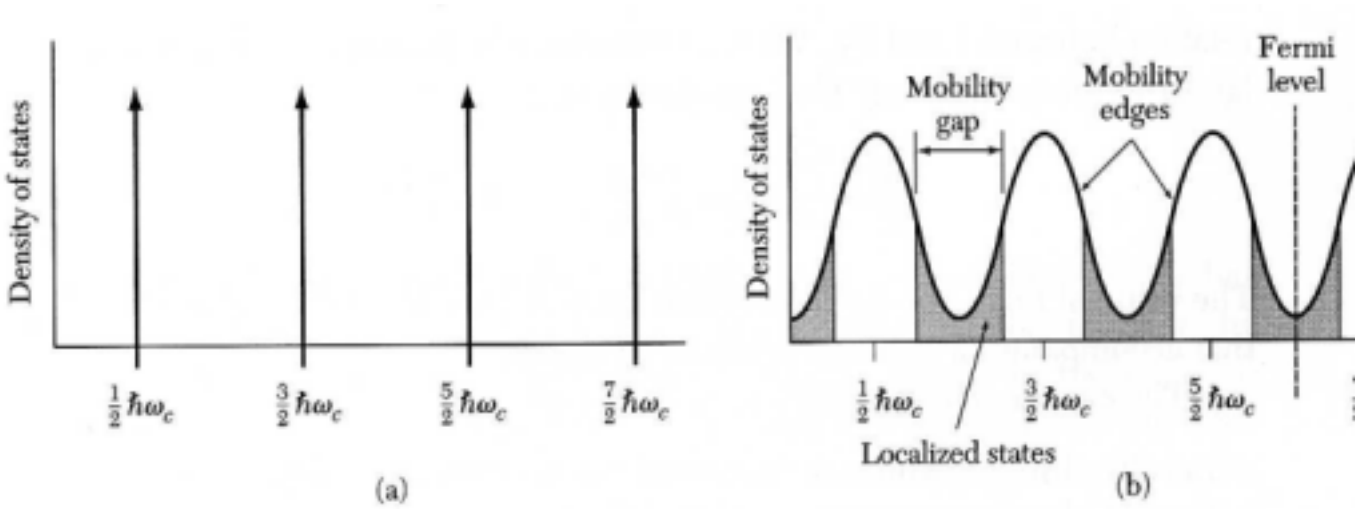

Figure 3.2: Density of state in a 2D electron gas in strong magnetic field .(a) Ideal 2D crystal. (b) Real 2D crystal.

between I and V_H. I and U are related can be related by

$$\frac{\partial U}{\partial t} = -V_x I_x = \frac{I}{c}\frac{\partial \varphi}{\partial t} \tag{3.1.3}$$

where φ is magnetic flux, U is total energy, I is current and c is speed of light [1]. From equation (3.1.1)

$$I = c\frac{\delta U}{\delta \varphi} \tag{3.1.4}$$

There are two carriers which are:

Localized state, which are not continuous around the loop.

Extended state, continuous around the loop.

The two class of states respond differently to the application of the flux. For localized state change in flux consider as gauge transformation, which cannot affect energy. But for extended state, their energy may be changed. However, if the flux varied by quantum flux $\delta\varphi = \frac{hc}{e}$, all extended orbit state are identical to those before the flux quantum was added. If the Fermi level fall within localized state, all extended state (Landau level), below the the Fermi level will be filled with electrons and after the flux change, $\delta\varphi = \frac{hc}{e}$. However, during the change an integral number state, general one per Landau level, enter the cylinder in one edge and leave it at

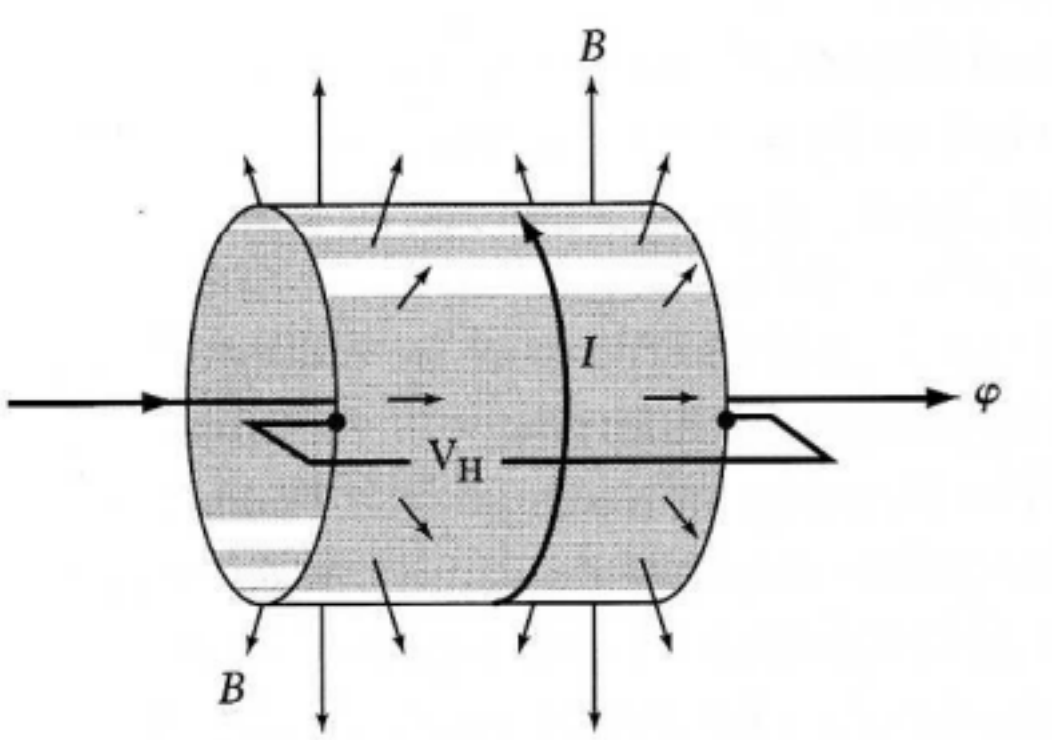

Figure 3.3: Laughlins thought- experiment Loop

the opposite edge. In order to the system be identical before and after the flux change the number must be integral. If the transferred state is transfered while occupied by one electron, it contribute an energy change eV_H. If N occupied state transferred , the energy change is NeV_H. Landau gauge for vector potential is given by $A = -By\hat{x}$. An increase δA that corresponds to the flux increase $\delta\varphi$ is equivalent to a displacement of extended state by $\delta A/B$ in y direction. We have $\delta\varphi = L_x\delta A$ from Stokes theorem and definition of vector potential. So $\delta\varphi$ causes a motion of the entire electron gas in y direction. As the current and the total energy U of a resistance less system.

$$\delta U = NeV_H \tag{3.1.5}$$

$$\delta\varphi = \frac{hc}{e} \tag{3.1.6}$$

By substituting equation (3.1.2) and (3.1.3) in equation (3.1.1) Laughlin obtained.

$$I = \frac{Ne^2}{h} V_H \tag{3.1.7}$$

From $V_H = IR_H$ one can get Hall resistance.

$$R_H = \frac{h}{Ne^2} \tag{3.1.8}$$

In large magnetic field diagonal resistivity tensor ρ_{xx} approach to zero , and the

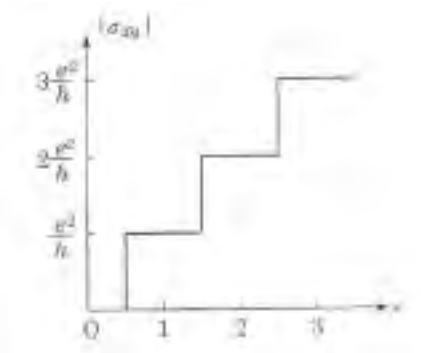

Figure 3.4: Conductivity and filling factor [4]

creation of plateaus at integral value ρ_{xy}. This agree partially with what we got in (1.0.23) and (1.0.24). In classical treatment we got only ρ_{xy} is finet value not quantized but in both case, in classical and quantum, $\rho_{xx} = \frac{1}{\sigma_{xx}}$ is vanish.

3.2 Fractional Quantum Hall Effect(FQHE)

There are only integer Hall plateaus, and no fractional plateaus is exist up to some magnetic field limit (Fig 3.4), but fractional Hall effect happen when electron-electron interaction make localized states delocalized and make (FQHE) possible. To obtain this kind of delocalization strong magnetic field is applied. There are two energies that have to be considered impurity potential and Coulomb interaction potential, potential between two separated electrons. If the Coulomb potential is negligible, the motion of electron is along an equipotential line of impurity potential. Since this is a closed line, the electron is localized. If the Coulomb potential

is larger than impurity potential, the static closed orbit along equipotential lines can not survive, the electronic state are delocalized. And this delocalization led to FQHE. The distance between two electrons is given by r_o, the average distance can be calculated in 2DEG [4]

$$r_o = \frac{1}{\sqrt{n_e}} = \frac{\sqrt{2\pi}l}{\sqrt{\nu}} \tag{3.2.1}$$

$$U_o = \frac{e^2}{4\pi\epsilon r_o} = \frac{\sqrt{\nu}}{\sqrt{2\pi}}\frac{e^2}{4\pi\epsilon l} \tag{3.2.2}$$

where ν is filling factor electron density, is permittivity and l is magnetic length which is equal to

$$l = \sqrt{\frac{\hbar c}{|e|B}} \tag{3.2.3}$$

here e is electron charge, n_e is density of electrons, B is magnetic field and $\hbar$ is Plank constant and U_o is Coulomb potential between electrons. From (3.2.2) and (3.2.3) one can get $U_o \propto \sqrt{B}$ where as separation between Landau levels is $\hbar\omega_c \propto$ B. So at strong magnetic field $\hbar\omega_c \gg U_o$. In extreme quantum limit, of higher magnetic field and at lower temperature the lowest Landau level is partially occupied. Power full method used by Laughlin using variational method to calculate expectation of Hamiltonian ground state by applying appropriate wave function $\Psi(\alpha)$ that contain a parameter α a candidate for ground state and calculate the expectation value of Hamiltonian $< \Psi(\alpha)|H|\Psi(\alpha) > / < \Psi(\alpha)|\Psi(\alpha) >$. This value always grater than or equal to true ground state [4].

$$< H >= \frac{< \Psi(\alpha)|H|\Psi(\alpha) >}{< \Psi(\alpha)|\Psi(\alpha) >} \tag{3.2.4}$$

where $\Psi(\alpha)$ is Laughlin's wave function.

Laughlin reached a trial wave function. First for single electron in Hilbert space to the lowest Landau level. And the wave function of single electron look like. $\Phi(r)$

$$\Phi(r) = f(z)e^{\frac{-|z|^2}{4}} \tag{3.2.5}$$

here $z = x - iy/l$ is the complex number representation of coordinates, and $f(z)$ is a polynomial in z. Now for N_e electrons can be with linear combination of wave function and applying this for all state, exited state Laughlin trial function appear to be

$$\Phi(r_1, r_2, r_3, ..., r_N) = \prod_{i>j}(z_i - z_j)^q e^{\sum_i |z|^2/4} \tag{3.2.6}$$

This wave function is called Laughlin's wave function. This wave function is depend on parameter q. The filling factor depend on this parameter q.

$$\nu = \frac{p}{q} \tag{3.2.7}$$

The fractional charged excitation is introduced by Laughlin it remarkable because it has fractional of elementary electron charge and neutralized by electron. Quark has this kinds of fractional charge [4].

$$e^* = -\frac{e}{q} \tag{3.2.8}$$

As we see in (Fig 3.5) at integral value there are plateaus at filling factor 4,3,2 and 1. In extreme quantum limit, of higher magnetic field and at lower temperature the lowest Landau level is partially occupied. Hall resistance R_H is quantized in units of $3h/e^2$ at filling factor 1/3 and 2/3. Other fraction are reported at 2/5, 2/7, 3/5 and 3/7. [6], [7] and [8].

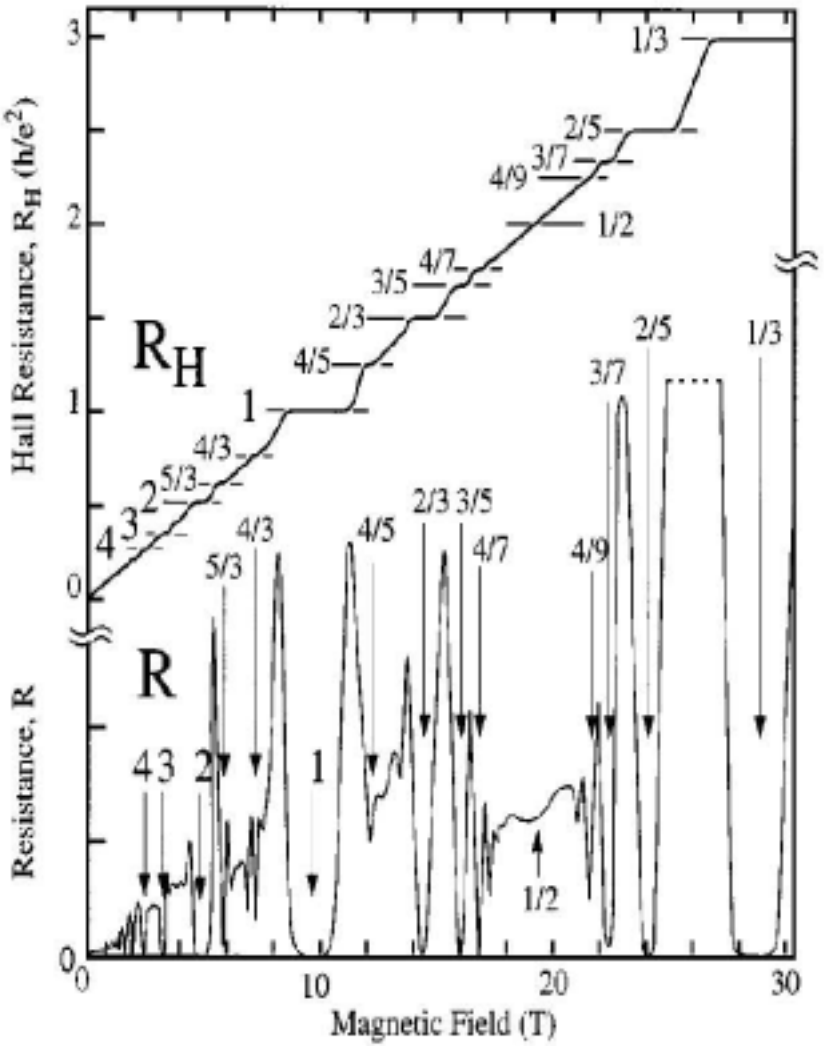

Figure 3.5: Integer and Fractional Quantum Hall effect [4]

Chapter 4

Graphene

4.1 Discovery Of Graphene

In 1916 the structure of graphite was solved by power diffraction method and it was studied in detail by V.Kohlschuter and P. Haenni in 1918. Its structure was determined from single-crystal diffraction in 1924. The first theory of graphene is explored in 1947 by P.R Wallace. In 1970 single layer graphite were grown by epitaxial on the top of other material. For first time, in 2002, the production of graphene was announced, titled 'Nano-scaled Graphene Plates'. In 2004 Ander Geim and Kostya Novoselov at university of Manchester extracted single-atom-thick crystallites from bulk crystallites from bulk graphite. They pulled graphene layer from graphite and transfered them into thin SiO_2 on silicon warfer in the process Mic cro mechanical cleavage. Even though graphene on nickel and on silicon carbide have both existed in laboratory [10]

4.2 Physical Properties Of Graphene

Graphene is the thinnest known material a sheet of carbon atoms in honey comb like structure has a single atom thick, yet stronger than diamond. Graphene was known experimentally in 2004, it has significant application is nanotechnology, solid state

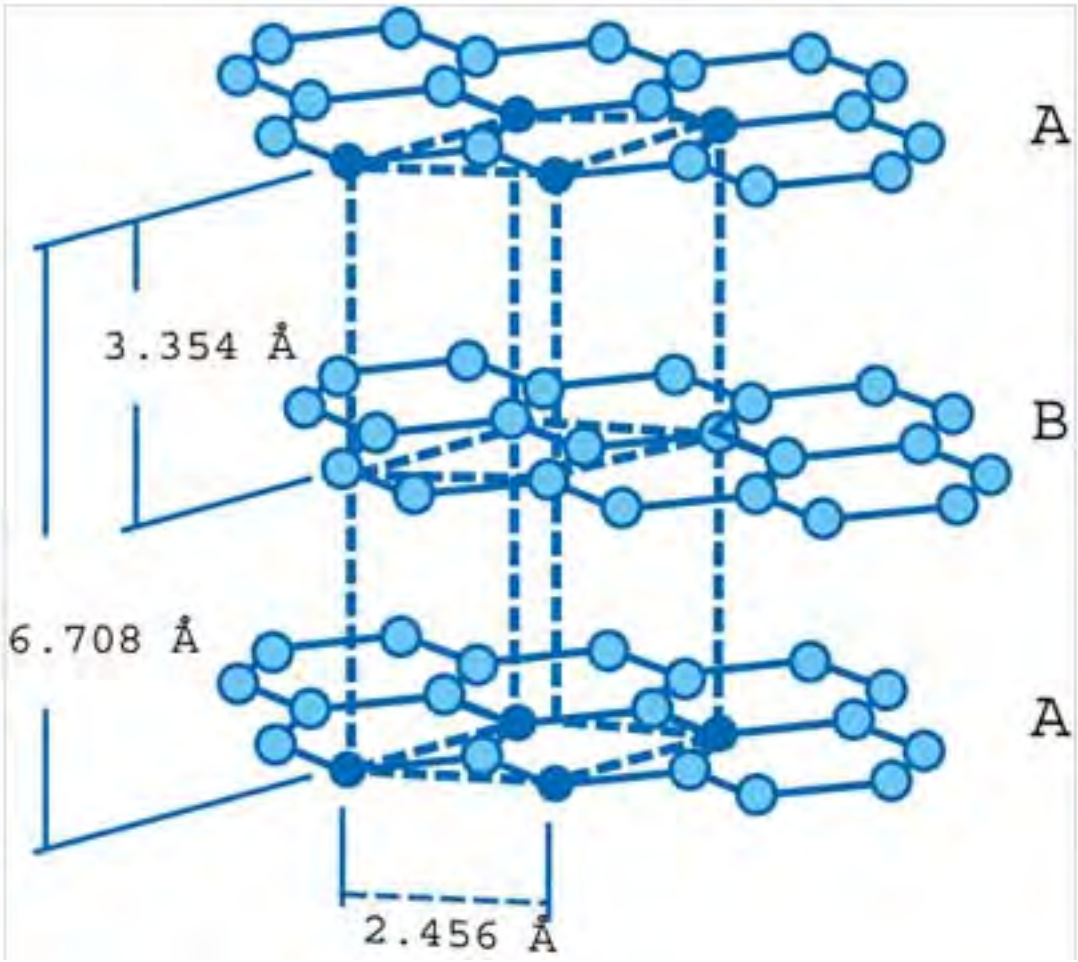

Figure 4.1: The structure of graphite

realization of high-energy physics phenomena and host important in condensed matter in revolutionized soft-matter and two-dimensional physics. Graphene is made only from carbon atoms arranged on a honey comb structure made out of hexagons. Physical properties graphene structure its low energy excitation mass less and Dirac fermions. Graphene extracted from graphite structure as we see in (Fig.4.1). Mono layer graphene is just a layer of this graphite (Fig.4.1). The speed of electrons in graphene is $10^6 m/s$. Dirac fermions behave in very unusual ways compared to ordinary electrons that subject to magnetic field. This led to unconventional kind of Quantum Hall effect.Second graphene has a property of insensitive to external electro static potential due to the so-called Klein-paradox which is Dirac fermions

can be transmitted with probability 1 through classically forbidden region. Third it has been identified experimentally that electrons can propagate with out scattering over large distance of order of micro meter in graphene. The fourth property of graphene is its softness is related with the fact it has plane vibration modes (phonon) that cannot found in 3D solids. Fifth properties graphene made from only carbon atoms and carbon-based atoms show an unlimited number of different structure. Some of the structure are carbon nanotube and fullerenes. Carbon nanotube can be made by rolling graphene along and reconnecting the carbon bonds. Since, carbon nanotubes have only hexagons and can be thought as one dimensional objects as we see in (Fig.4.2) left. Fullerenes are molecules where carbon atoms are arranged spherically, therefore physical point of view are zero dimensional object. Fullerenes can be obtained from graphene with the introduction of pentagon, that create positive curvature defect as we see in (Fig.4.2) right [10]. The flexibility of graphene

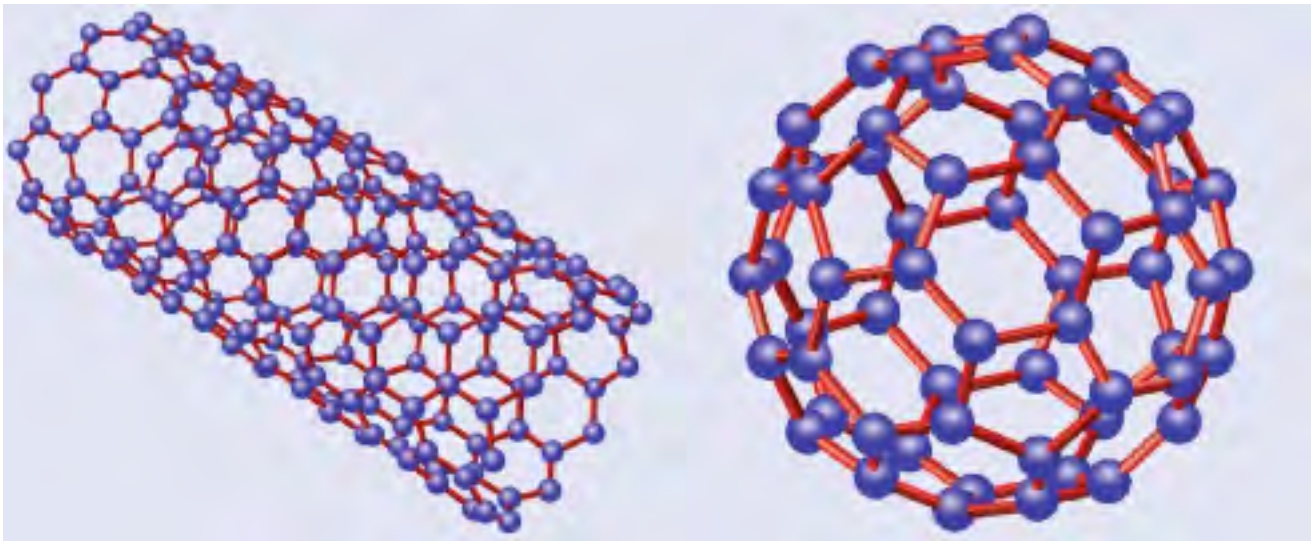

Figure 4.2: Nanotube and Fullerene

structure is due to sp^2 hybridization between one s-orbital and two p-orbitals leads to a triangular planar structure with the formation of δ-bond. And this δ-bond is responsible for the robustness of the lattice structure in all allotropes. Graphene grown epitaxial on metal surface by using catalytic decomposition of hydro carbon

or carbon oxides. When these surface are heated, oxygen or hydrogen desorbs, and the carbon form a graphene mono layer. Graphene also be formed on surface of SiC. During heating, the silicon from top layer desorbs. And a few of layers of graphene are left on surface. The resulting graphene could be reaches up to a micrometer. The number of layers can be controlled by limiting time and temperature of heating treatment. The quality and number of layers depend on the SiC face used for their growth. Angle resolved photo-emission experiment (ARPES) shows that epitaxial graphene grown on SiC has linearly dispersing quasi particles (Dirac fermions) agreement with theoretical result. And graphene grown this way is heavily doped due to the charge transfer from substrate to graphene layer. There is also evidence for strong interaction between a substrate and the graphene layer leading to the appearance of gaps at the Dirac point. Graphene is honey comb like structure and

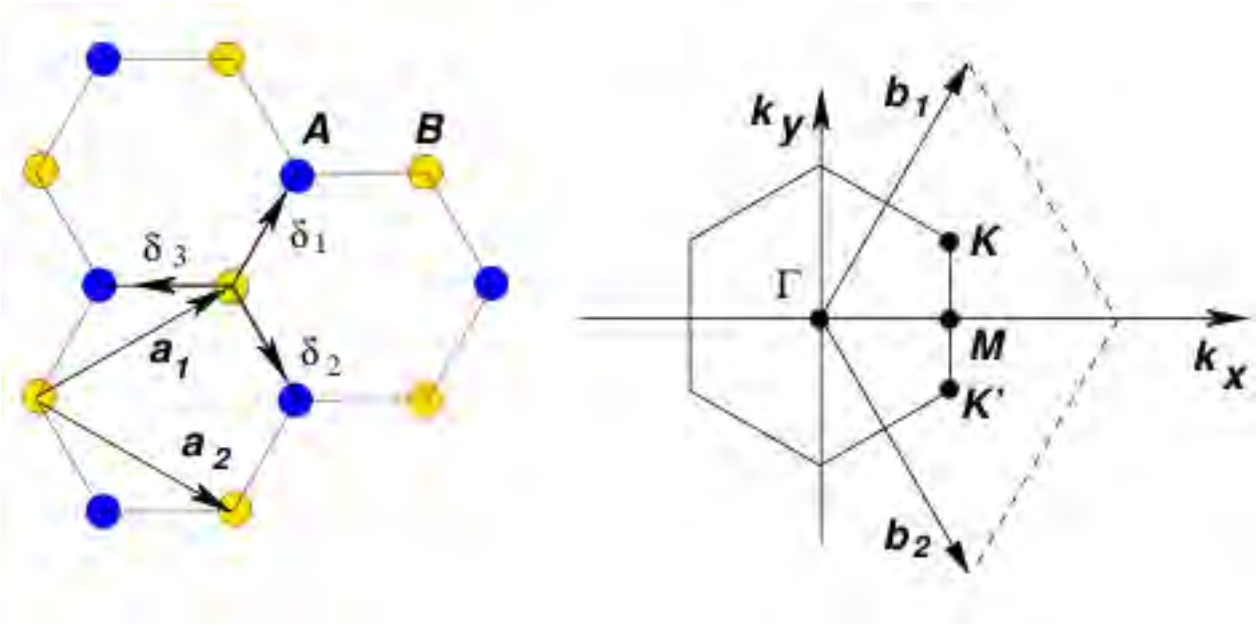

Figure 4.3: Graphene structure in primitive and reciprocal lattice respectively

densely packed in regular Sp^2 bonded. It is a gap less semiconductor. And it also has a chirle effect. Graphene contains two atoms per cell. As we see in (Fig.4.3) left

the primitive cell has two sub lattice in real space which given by

$$a_1 = \frac{a}{2}(3, \sqrt{3}) \tag{4.2.1}$$

$$a_2 = \frac{a}{2}(3, -\sqrt{3}) \tag{4.2.2}$$

where a is carbon-carbon distance $a = 1.42A^o$, a_1 and a_2 are primitive lattices. The reciprocal lattice vectors are given by :

$$b_1 = \frac{2\pi}{3a}(1, \sqrt{3}) \tag{4.2.3}$$

$$b_2 = \frac{2\pi}{3a}(1, -\sqrt{3}) \tag{4.2.4}$$

where b_1 and b_2 are reciprocal lattices as we see (Fig.4.3) right. And very important point in graphene in Brillouin zone called Dirac points and their position in momentum space given by:

$$K = (\frac{2\pi}{3a}, \frac{2\pi}{3\sqrt{3}a}) \tag{4.2.5}$$

$$K' = (\frac{2\pi}{3a}, -\frac{2\pi}{3\sqrt{3}a}) \tag{4.2.6}$$

where K and K' are Dirac points. And the three nearest neighbors vectors in real space are given by

$$\delta_1 = \frac{a}{2}(1, \sqrt{3}), \delta_2 = \frac{a}{2}(1, \sqrt{3}), \delta_3 = -a(1, 0) \tag{4.2.7}$$

Graphene contains two atoms per cell:

Sub lattice A and sub lattice B are equivalent. Each atom of the sub lattice A (blue) is surrounded by 3 atoms of sub lattice B (yellow) as we see in (Fig.4.3) left.

The quantum equation which describe the free electron in graphene take the following form.

$$v_F \sigma.\hat{p} \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \end{pmatrix} = E \begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \end{pmatrix} \tag{4.2.8}$$

$$v_F\{\sigma_x\hat{p}_x + \sigma_y\hat{p}_y\}\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} = E\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} \tag{4.2.9}$$

where σ_x and σ_y are Pauli spin matrix given by

$$\sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \tag{4.2.10}$$

We see that (4.2.9) looks like 2D Dirac equation for masslesse electrons.

Now let us substitute (4.2.10) into (4.2.9)

$$v_F[\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\hat{p}_x + \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}\hat{p}_y]\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} = E\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} \tag{4.2.11}$$

$$v_F[\begin{pmatrix}0 & p_x\\ \hat{p}_x & 0\end{pmatrix} + \begin{pmatrix}0 & -i\hat{p}_y\\ i\hat{p}_y & 0\end{pmatrix}]\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} = E\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} \tag{4.2.12}$$

$$v_F[\begin{pmatrix}0 & \hat{p}_x - i\hat{p}_y\\ \hat{p}_x + i\hat{p}_y & 0\end{pmatrix}\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} = E\begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix} \tag{4.2.13}$$

$$(\hat{p}_x - i\hat{p}_y)\Psi_2 = \frac{E}{v_F}\Psi_1 \tag{4.2.14}$$

$$(\hat{p}_x - i\hat{p}_y)\Psi_1 = \frac{E}{v_F}\Psi_2 \tag{4.2.15}$$

We seek the solution of the form

$$\Psi_1 = Ae^{i(k_x x + k_y y)} \tag{4.2.16}$$

$$\Psi_2 = Be^{i(k_x x + k_y y)} \tag{4.2.17}$$

where A and B are the normalization coefficients, k_x and k_y are wave vector.

Next let substitute (4.2.16) and (4.2.17) into (4.2.14) and (4.2.15) respectively led to

$$(\hbar k_x - i\hbar k_y)B = A\frac{E}{v_F} \tag{4.2.18}$$

$$(\hbar k_x + i\hbar k_y)A = B\frac{E}{v_F} \tag{4.2.19}$$

where $\hbar$ is Plank constant, k_x and k_y are wave vectors, v_F is Fermi velocity and E is energy eigenvalue.

Let multiply (4.2.18) by (4.2.19)

$$(\hbar k_x - i\hbar k_y)(\hbar k_x + i\hbar k_y)AB = AB\frac{E^2}{v_F^2} \tag{4.2.20}$$

The constant AB in both side cancel out

$$\hbar^2 k_x^2 + \hbar^2 k_y^2 = \frac{E^2}{v_{F^2}} \tag{4.2.21}$$

By putting $k_x^2 + k_y^2 = k^2$ equation (4.2.21) look like

$$k^2\hbar^2 v_F^2 = E^2 \tag{4.2.22}$$

So the value of energy from equation (4.2.22) in graphene is equal to

$$E = \pm v_F \hbar k \tag{4.2.23}$$

where v_F is Fermi velocity. The energy of graphene what we get in (4.3.23) has

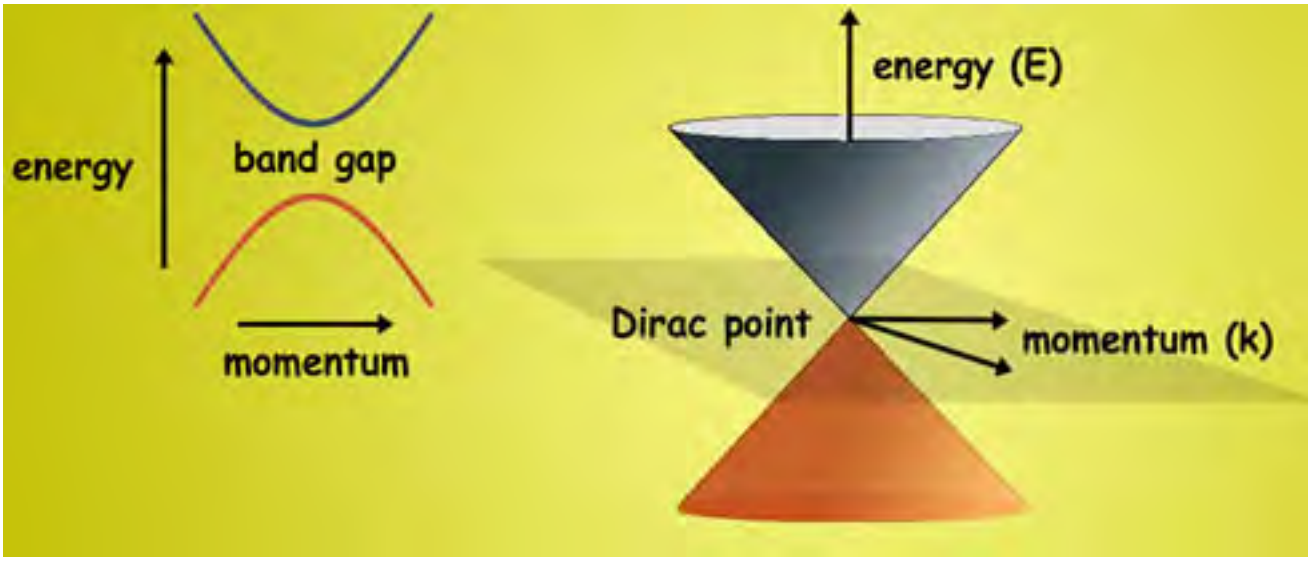

Figure 4.4: Energy dispersion of semiconductor (left) and graphene (right)

linear dispersion with wave vector while the energy in semiconductor $E = \frac{\hbar^2 k^2}{2m*}$ this

make the graphene the gap less semiconductor. Moreover, as we see from (Fig.4.4) left there is an energy gap between conduction band and valance band. But in case of graphene conduction band and valance band contact each other (Fig.4.4) right.

4.3 Landau Level In Graphene

The Hamiltonian of mass less electron in graphene in presences of constant magnetic field is given by Dirac like equation which is similar with Dirac equation of neutrino.

$$\hat{H}_D = v_F\hat{\sigma}(\hat{p} - \frac{e}{c}A)^2 \tag{4.3.1}$$

$$\hat{H}\begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \end{pmatrix} = E\begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \end{pmatrix} \tag{4.3.2}$$

The equation of electron in graphene look like:

$$v_F\hat{\sigma}.(\hat{p} - \frac{e}{c}\vec{A}))\begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \end{pmatrix} = E\begin{pmatrix} \Psi_1 \\ \Psi_2 \\ \end{pmatrix} \tag{4.3.3}$$

In section 4.2 there is no the application of magnetic field but in this section we consider magnetic field.

$$\hat{H} = v_F\sigma_x(\hat{p}_x - \frac{e}{c}\vec{A}_x) + V_F\sigma_y(\hat{p}_y - \frac{e}{c}\vec{A}_y) \tag{4.3.4}$$

where σ_x and σ_y are the Pauli spin matrix given by (4.2.10)

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \\ \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \\ \end{pmatrix} \tag{4.3.5}$$

Plugging equation (4.3.4) and (4.3.5) into (4.3.3) one can obtain

$$\hat{H} = v_F\begin{pmatrix} 0 & 1 \\ 1 & 0 \\ \end{pmatrix}(\hat{p}_x - \frac{e}{c}\vec{A}_x) + v_F\begin{pmatrix} 0 & -i \\ i & 0 \\ \end{pmatrix}(\hat{p}_y - \frac{e}{c}\vec{A}_y) \tag{4.3.6}$$

Now it is convenient to use Landau gauge which is

$$A_x = -\frac{e}{c}B_o y, A_y = 0, A_z = 0 \tag{4.3.7}$$

Substituting (4.3.7) into (4.3.6)

$$\hat{H} = v_F \begin{pmatrix} 0 & \hat{p}_x + \frac{e}{c}B_o y \\ \hat{p}_x + \frac{e}{c}B_o y & 0 \end{pmatrix} + v_F \begin{pmatrix} 0 & -i\hat{p}_y \\ i\hat{p}_y & 0 \end{pmatrix} \tag{4.3.8}$$

$$\hat{H} = v_F \begin{pmatrix} 0 & \hat{p}_x - i\hat{p}_y + \frac{e}{c}B_o y \\ \hat{p}_x + i\hat{p}_y + \frac{e}{c}B_o y & 0 \end{pmatrix} \tag{4.3.9}$$

$\hat{p}_-$ and $\hat{p}_+$ can be defined

$$\hat{p}_- = \hat{p}_x + i\hat{p}_y, \hat{p}_+ = \hat{p}_x - i\hat{p}_y \tag{4.3.10}$$

$\hat{p}_-$, $\hat{p}_+$, $\hat{p}_x$ and $\hat{p}_x$ are momentum operator and i is complex number. By substituting equation (4.3.10) to (4.3.9) and again into equation (4.3.2) one can obtains.

$$v_F \begin{pmatrix} 0 & \hat{p}_- + \frac{e}{c}B_o y \\ \hat{p}_+ + \frac{e}{c}B_o y & 0 \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = E \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} \tag{4.3.11}$$

$$(\hat{p}_- + \frac{e}{c}B_o y)\psi_2 = \frac{E}{v_F}\psi_1 \tag{4.3.12}$$

$$(\hat{p}_+ + \frac{e}{c}B_o y)\psi_1 = \frac{E}{v_F}\psi_2 \tag{4.3.13}$$

By finding ψ_2 from equation (4.3.13) and substituting in equation (4.3.12) and for ψ_1 obtain from (4.3.12) and substituting in (4.3.13) on can obtain:

$$(\hat{p}_- + \frac{e}{c}B_o y)(\hat{p}_+ + \frac{e}{c}B_o y)\psi_1 = (\frac{E}{v_F})^2\psi_1 \tag{4.3.14}$$

$$(\hat{p}_+ + \frac{e}{c}B_o y)(\hat{p}_- + \frac{e}{c}B_o y)\psi_2 = (\frac{E}{v_F})^2\psi_2 \tag{4.3.15}$$

We seek solution

$$\psi_1(x, y) = \varphi_1(y)e^{ikx} \tag{4.3.16}$$

$$\psi_2(x,y) = \varphi_2(y)e^{ikx} \tag{4.3.17}$$

Let us find for φ_1 by substituting (4.3.16) into (4.3.14)

$$[(\hbar k - i\hat{p}_y) + \frac{eB_o}{c}y][(\hbar k + i\hat{p}_y) + \frac{eB_o}{c}y]\varphi_1(y) = (\frac{E}{v_F})^2\varphi_1(y), \tag{4.3.18}$$

where y is displacement.

$$[(\hbar k + \frac{eB_o}{c}y) - i\hat{p}_y][(\hbar k + \frac{eB_o}{c}y) + i\hat{p}_y)]\varphi_1(y) = (\frac{E}{v_F})^2\varphi_1(y), \tag{4.3.19}$$

$$\hbar^2[\frac{eB_o}{\hbar c}(y + \frac{\hbar kc}{eB_o}) - i\frac{\hbar}{i\hbar}\frac{d}{dy}][\frac{eB_o}{\hbar c}(y + \frac{\hbar kc}{eB_o}) + i\frac{\hbar}{i\hbar}\frac{d}{dy}] = (\frac{E}{v_F})^2\varphi_1(y). \tag{4.3.20}$$

We define a_H

$$a_H = \sqrt{\frac{\hbar c}{eB_o}} \tag{4.3.21}$$

Now let substituting equation (4.3.20) into equation (4.3.19) and rearranging it.

$$[\frac{1}{a_H^2}(y - y_o) - \frac{d}{dy}][\frac{1}{a_H^2}(y - y_o) + \frac{d}{dy}]\varphi_1(y) = [\frac{E}{v_F\hbar}]^2\varphi_1(y) \tag{4.3.22}$$

Again let us substitute $y = \alpha\xi$ in equation (4.3.21) where α has length dimension and ξ is dimensionless, this what we did in chapter 2.

$$[\frac{1}{a_H^2}\alpha(\xi - \xi_o) - \frac{1}{\alpha}\frac{d}{d\xi}][\frac{1}{a_H^2}\alpha(\xi - \xi_o) - \frac{1}{\alpha}\frac{d}{d\xi}] = [\frac{E}{v_F\hbar}]^2\varphi_1(y) \tag{4.3.23}$$

Factor out $\frac{1}{\alpha}$ from both brackets

$$\frac{1}{\alpha^2}[\frac{\alpha^2}{a_H^2}(\xi - \xi_o - \frac{d}{d\xi})][\frac{\alpha^2}{a_H^2}(\xi - \xi_o + \frac{d}{d\xi})]\varphi_1(y) = [\frac{E}{v_F\hbar}]^2\varphi_1(y) \tag{4.3.24}$$

But from (2.1.11) and (4.3.20) both a_H and α have the same value and rearranging equation

$$[(\xi - \xi_o) - \frac{d}{d\xi}][(\xi - \xi_o) + \frac{d}{d\xi}]\varphi_1(y) = [\frac{E\alpha}{v_F\hbar}]^2\varphi_1(y) \tag{4.3.25}$$

$$[(\xi - \xi_o)^2 - (\frac{d}{d\xi})^2 + (\xi - \xi_o)\frac{d}{d\xi} - \frac{d}{d\xi}(\xi - \xi_o)]\varphi_1(y) = [\frac{E\alpha}{v_F\hbar}]^2\varphi_1(y), \tag{4.3.26}$$

$$[-\frac{d}{d\xi}^2 + (\xi - \xi_o)^2]\varphi_1(y) + (\xi - \xi_o)\frac{d}{d\xi}\varphi_1(y) - \varphi_1(y) - (\xi - \xi_o)\frac{d}{d\xi}\varphi_1(y) = [\frac{E\alpha}{v_F\hbar}]^2\varphi_1(y), \tag{4.3.27}$$

$$[-\frac{d}{d\xi}^2 + (\xi-\xi_o)^2 - 1]\varphi_1(y) = [\frac{E\alpha}{v_F\hbar}]^2\varphi_1(y), \tag{4.3.28}$$

$$\frac{d}{d\xi}^2 \varphi_1(y) + [[\frac{E\alpha}{v_F\hbar}]^2 + 1]\varphi_1(y) - (\xi-\xi_o)^2\varphi_1(y) = 0. \tag{4.3.29}$$

$$\lambda = 2n+1 = [\frac{E\alpha}{v_F\hbar}]^2 + 1 \tag{4.3.30}$$

$$E_n^2 = \frac{2n\hbar^2 v_F^2}{\alpha^2} \tag{4.3.31}$$

By substituting the value of α

$$E_{n1} = \pm\hbar v_F\sqrt{\frac{2eB_o}{\hbar c}}\sqrt{n}, = \pm\hbar\omega_D\sqrt{n} \tag{4.3.32}$$

where $\omega_D = \sqrt{\frac{2eB_o v_F^2}{\hbar c}}$. Wave function can written us

$$\varphi_{n1}(y) = N_n e^{-\frac{eB}{2\hbar c}(y-y_o)^2} H_n([y-y_o]\sqrt{\frac{eB}{\hbar c}}), \tag{4.3.33}$$

H_n is Hermite polynomial.

Using equation (4.3.15)

$$\psi_{nk1}(x,y) = A_1 e^{ikx} e^{-\frac{eB}{2\hbar c}(y-y_o)^2} H_n([y-y_o]\sqrt{\frac{eB}{\hbar c}}). \tag{4.3.34}$$

where N_n and A_1 are the normalization coefficients.

Next let substitute (4.3.17) into (4.3.15) following the same as above finally we reach to:

$$[(\xi-\xi_o) + \frac{d}{d\xi}][(\xi-\xi_o) - \frac{d}{d\xi}]\varphi_2(y) = [\frac{E\alpha}{v_F\hbar}]^2\varphi_2(y) \tag{4.3.35}$$

$$[(\xi-\xi_o)^2 - (\frac{d}{d\xi})^2 - (\xi-\xi_o)\frac{d}{d\xi} + \frac{d}{d\xi}(\xi-\xi_o)]\varphi_2(y) = [\frac{E\alpha}{v_F\hbar}]^2\varphi_2(y), \tag{4.3.36}$$

$$[-\frac{d}{d\xi}^2 + (\xi-\xi_o)^2]\varphi_2(y) - (\xi-\xi_o)\frac{d}{d\xi}\varphi_2(y) + \varphi_2(y) + (\xi-\xi_o)\frac{d}{d\xi}\varphi_2(y) = [\frac{E\alpha}{v_F\hbar}]^2\varphi_2(y) \tag{4.3.37}$$

$$[-\frac{d}{d\xi}^2 + (\xi-\xi_o)^2 + 1]\varphi_2(y) = [\frac{E\alpha}{v_F\hbar}]^2\varphi_2(y), \tag{4.3.38}$$

$$\frac{d}{d\xi}^2 \varphi_2(y) + [[\frac{E\alpha}{v_F\hbar}]^2 - 1]\varphi_2(y) - (\xi-\xi_o)^2\varphi_2(y) = 0. \tag{4.3.39}$$

$$\lambda = 2n + 1 = [\frac{E\alpha}{v_F\hbar}]^2 - 1 \tag{4.3.40}$$

$$E_n^2 = \frac{2n\hbar^2 v_F^2}{\alpha^2} + 2 \tag{4.3.41}$$

By substituting the value of α

$$E_{n2} = \pm\hbar v_F\sqrt{\frac{2eB_o}{hc}}\sqrt{n+1} = \pm\hbar\omega_D\sqrt{n+1} \tag{4.3.42}$$

where $\omega_D = \sqrt{\frac{2eB_o v_F^2}{\hbar c}}$. Wave function can written us

$$\varphi_{n2}(y) = N_n e^{-\frac{eB}{2\hbar c}(y-y_o)^2} H_n([y-y_o]\sqrt{\frac{eB}{\hbar c}}) \tag{4.3.43}$$

Using equation (4.3.16)

$$\psi_{nk2}(x,y) = A_2 e^{ikx} e^{-\frac{eB}{2\hbar c}(y-y_o)^2} H_n([y-y_o]\sqrt{\frac{eB}{\hbar c}}) \tag{4.3.44}$$

where N_n and A_2 are the normalization coefficients.

Now let compare our result of graphene to 2DEG. For graphene, we have two wave function and each wave function depend on n_1 or n_2 and k while energy eigenvalue depend only n_1 and n_2 because of these two wave functions have led more degeneracy than 2DEG. Second point is energy eigenvalue have linear dependence with B since $E \propto \omega_c \propto B$ from (2.1.16) where in case of graphene it has $E \propto \sqrt{B}$. The last important thing is energy eigenvalue of 2DEG has linear dependence with n, quantum number, this led to equal energy gap between Landau levels which is $\hbar\omega_c$. Whereas the energy eigenvalue $E \propto \sqrt{n}$ this led unequal energy gap between Landau levels in graphene. The ground state of, E_{n1} has zero value this impossible for 2DEG.

4.4 Landau level of Graphene in Cylindrical Coordinate

Using a Cartesian coordinate we got the solution of graphene in constant magnetic field. With a wave solution and energy eigenvalue. Now let solve this problem in

cylindrical coordinate. Equation (4.3.6) can be written as

$$\hat{H} = v_F \begin{pmatrix} 0 & \hat{p}_x - \frac{e}{c}A_x \\ \hat{p}_x - \frac{e}{c}A_x & 0 \end{pmatrix} + v_F \begin{pmatrix} 0 & -i\hat{p}_y - i\frac{e}{c}A_y \\ i\hat{p}_y - i\frac{e}{c}A_y & 0 \end{pmatrix} \tag{4.4.1}$$

We can define other operator.

$$\hat{\Pi}_x = \hat{p}_x - \frac{e}{c}A_x, \hat{\Pi}_y = \hat{p}_y - \frac{e}{c}A_y \tag{4.4.2}$$

And

$$\hat{\Pi}_+ = \hat{\Pi}_x + i\hat{\Pi}_y, \hat{\Pi}_- = \hat{\Pi}_x - i\hat{\Pi}_y \tag{4.4.3}$$

Using equation (4.4.2) and (4.4.3) equation (4.4.1) look like

$$\hat{H} = v_F \begin{pmatrix} 0 & \hat{\Pi}_x \\ \hat{\Pi}_x & 0 \end{pmatrix} + v_F \begin{pmatrix} 0 & -i\hat{\Pi}_y \\ i\hat{\Pi}_y & 0 \end{pmatrix} = v_F \begin{pmatrix} 0 & \hat{\Pi}_- \\ \hat{\Pi}_+ & 0 \end{pmatrix} \tag{4.4.4}$$

The Dirac equation look like:

$$\hat{H} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} = E \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} \tag{4.4.5}$$

Using (4.4.4) and (4.4.5) one can obtain

$$\hat{\Pi}_+\hat{\Pi}_-\psi_1 = (\frac{E}{v_F})^2\psi_1 \tag{4.4.6}$$

If we substitute (4.4.2) in (4.4.4)

$$\hat{\Pi}_-\hat{\Pi}_+\psi_2 = (\frac{E}{v_F})^2\psi_2 \tag{4.4.7}$$

But

$$\hat{\Pi}_+\hat{\Pi}_- = \hat{p}^2 + \frac{e^2}{c^2}\vec{A}^2 - \frac{e}{c}[2\vec{A}.\hat{p} + \hbar B] \tag{4.4.8}$$

$$\hat{\Pi}_-\hat{\Pi}_+ = \hat{p}^2 + \frac{e^2}{c^2}\vec{A}^2 - \frac{e}{c}[2\vec{A}.\hat{p} - \hbar B] \tag{4.4.9}$$

Adding (4.4.6) and (4.4. 7)

$$(\hat{\Pi}_+\hat{\Pi}_- + \hat{\Pi}_-\hat{\Pi}_+)\psi_{1,2} = 2(\frac{E}{v_F})^2\psi_{1,2} \tag{4.4.10}$$

where $\psi_{1,2}$ is coupled wave function ψ_1 and ψ_2.

Substitute (4.4.8) and (4.4.9) into (4.4.10)

$$(\hat{p}^2 + \frac{e^2}{c^2}\vec{A}^2 - \frac{e}{c}[2\vec{A}.\hat{p} \pm \hbar B])\psi_{1,2} = (\frac{E}{v_F})^2\psi_{1,2} \tag{4.4.11}$$

Let rearrange in convenient way (4.4.11)

$$(\hat{p}^2 - (\frac{E}{v_F})^2)\psi_{1,2} = (\frac{e^2}{c^2}\vec{A}^2 - \frac{e}{c}[2\vec{A}.\hat{p} \pm \hbar B])\psi_{1,2} \tag{4.4.12}$$

But $\hat{p}^2 = -\hbar^2\nabla^2$ let substitute this in (4.4.12)

$$(-\hbar^2\nabla^2 - (\frac{E}{v_F})^2)\psi_{1,2} = (\frac{e^2}{c^2}\vec{A}^2 - \frac{e}{c}[2\vec{A}.\hat{p} \pm \hbar B])\psi_{1,2} \tag{4.4.13}$$

This led to

$$(\nabla^2 + k^2)\psi_{1,2} = \frac{1}{\hbar^2}\{\frac{e^2}{c^2}\vec{A}^2 - \frac{e}{c}[2\vec{A}.\hat{p} \pm \hbar B]\}\psi_{1,2} \tag{4.4.14}$$

where

$$k^2 = (\frac{E}{\hbar v_F})^2 \tag{4.4.15}$$

$$(\nabla^2 + k^2)\psi_{1,2} = \hat{V}_{1,2}\psi_{1,2} \tag{4.4.16}$$

$$\hat{V}_{1,2} = \frac{1}{\hbar^2}\{\frac{e^2}{c^2}\vec{A}^2 - \frac{e}{c}[2\vec{A}.\hat{p} \pm \hbar B]\} \tag{4.4.17}$$

here $\hat{V}_{1,2}$ is potential. This equation was obtained in [5].

In cylindrical coordinate

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} \tag{4.4.18}$$

Using Luttinger gauge

$$A_x = -\frac{1}{2}By, A_y = \frac{1}{2}Bx \tag{4.4.19}$$

Substitute (4.4.19) into (4.4.17) led to

$$\hat{V}_{1,2} = \frac{1}{\hbar^2}\{\frac{e^2}{4c^2}\vec{B}^2(x^2+y^2) - \frac{e}{c}[2(\vec{A}_x\hat{p}_x + \vec{A}_y\hat{p}_y) \pm \hbar B]\} \tag{4.4.20}$$

The bracket term can be written as

$$\vec{A}_x\hat{p}_x + \vec{A}_y\hat{p}_y = -\frac{1}{2}By\frac{\hbar}{i}\frac{\partial}{\partial x} + \frac{1}{2}Bx\frac{\hbar}{i}\frac{\partial}{\partial y} \tag{4.4.21}$$

$$\vec{A}_x\hat{p}_x + \vec{A}_y\hat{p}_y = -\frac{1}{2}B\frac{\hbar}{i}[x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}] = \frac{1}{2}B\hat{L}_z \tag{4.4.22}$$

Using final result (4.4.22) and $x^2+y^2=r^2$ result equation on can obtain equation (4.4.20) as

$$\hat{V}_{1,2} = \frac{1}{\hbar^2}\{\frac{e^2}{4c^2}\vec{B}^2r^2 - \frac{e}{c}[B\hat{L}_z \pm \hbar B]\} \tag{4.4.23}$$

Using equation (4.4.16), (4.4.18) and (4.4.23) one can get

$$(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2} + k^2)\psi_{1,2} = \frac{1}{\hbar^2}\{\frac{e^2}{4c^2}\vec{B}^2r^2 - \frac{e}{c}[B\hat{L}_z \pm \hbar B]\}\psi_{1,2} \tag{4.4.24}$$

The solution for $\psi_{1,2}$ will be

$$\psi_{1,2}(r,\varphi) = R_{1,2}(r)e^{im\varphi} \tag{4.4.25}$$

Now substitute equation (4.4.26) into equation (4.4.16)

$$(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{m^2}{r^2} + k^2)R_{1,2}(r) = \frac{1}{\hbar^2}\{\frac{e^2B}{4c^2}r^2 - \frac{e}{c}[B\hbar m \pm \hbar B]\}R_{1,2}(r) \tag{4.4.26}$$

$$R''_{1,2}(r) + \frac{1}{r}R'_{1,2}(r) + (k^2 - \frac{m^2}{r^2})R_{1,2}(r) = [\frac{e^2B^2}{4c^2\hbar^2}r^2 - \frac{eB\hbar}{c\hbar^2}(m\pm1)]R_{1,2}(r) \tag{4.4.27}$$

Rearranging one can get

$$R''_{1,2}(r) + \frac{1}{r}R'_{1,2}(r) + [k^2 - \frac{m^2}{r^2} - (\frac{eB}{2c\hbar})^2r^2 + 2\frac{eB}{2c\hbar}(m\pm1)]R_{1,2}(r) = 0 \tag{4.4.28}$$

Now let substitute r with $r=\alpha\rho$, ρ is dimensionless α has the dimension of length.

$$\frac{1}{\alpha^2}R''_{1,2}(\rho) + \frac{1}{\alpha^2}\frac{1}{\rho}R'_{1,2}(\rho) + [k^2 - \frac{m^2}{\alpha^2\rho^2} - (\frac{eB}{2c\hbar})^2\alpha^2\rho^2 + \frac{eB}{c\hbar}(m\pm1)]R_{1,2}(\rho) = 0 \tag{4.4.29}$$

$$R''_{1,2}(\rho) + \frac{1}{\rho}R'_{1,2}(\rho) + [k^2\alpha^2 - \frac{m^2}{\rho^2} - (\frac{eB}{2c\hbar})^2\alpha^4\rho^2 + \frac{eB}{c\hbar}\alpha^2(m\pm1)]R_{1,2}(\rho) = 0 \tag{4.4.30}$$

The most convenient to choose is

$$\alpha^4(\frac{eB}{2c\hbar})^2 = 1 \tag{4.4.31}$$

In consequence

$$\alpha^2 = \frac{2c\hbar}{eB} \tag{4.4.32}$$

$$R''_{1,2}(\rho) + \frac{1}{\rho}R'_{1,2}(\rho) + [\frac{2E^2c}{B\hbar v_F^2 e} - \frac{m^2}{\rho^2} - \rho^2 + 2(m \pm 1)]R_{1,2}(\rho) \tag{4.4.33}$$

$$[\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} - \frac{m^2}{\rho^2} + k^2\alpha^2 - \rho^2 + 2(m \pm 1)]R_{1,2}(\rho) = 0 \tag{4.4.34}$$

where $k = \frac{E}{\hbar v_F}$ again let introduce other parameter

$$\xi = \rho^2 \tag{4.4.35}$$

so

$$\frac{d}{d\rho} = \frac{d}{d\xi}\frac{d\xi}{d\rho} = 2\rho\frac{d}{d\xi} \tag{4.4.36}$$

And

$$\frac{d^2}{d\rho^2} = 2\frac{d}{d\xi} + 2\rho\frac{d^2}{d\xi}\frac{d\xi}{d\rho} = 2\frac{d}{d\xi} + 4\rho^2\frac{d^2}{d\xi^2} \tag{4.4.37}$$

Let put equation (4.4.34) in terms of ξ by substituting (4.4.36) and (4.4.37) and (4.4.35) in it

$$[2\frac{d}{d\xi} + 4\rho^2\frac{d^2}{d\xi^2} + 2\frac{d}{d\xi} - \frac{m^2}{\rho^2} + k^2\alpha^2 - \rho^2 + 2(m \pm 1)]R_{1,2}(\xi) = 0 \tag{4.4.38}$$

Rearranging and substituting equation (4.4.35)

$$[4\frac{d}{d\xi} + 4\xi\frac{d^2}{d\xi^2} - \frac{m^2}{\xi} + k^2\alpha^2 - \xi + 2(m \pm 1)]R_{1,2}(\xi) = 0 \tag{4.4.39}$$

This led to by dividing by 4 ξ

$$[\frac{d^2}{d\xi^2} + \frac{d}{\xi d\xi} - \frac{m^2}{4\xi^2} + \frac{k^2\alpha^2}{4\xi} - \frac{1}{4} + \frac{(m \pm 1)}{2\xi}]R_{1,2}(\xi) = 0 \tag{4.4.40}$$

Let take a case as $\xi \to 0$ the last three terms can be ignore in equation (4.4.40) and so

$$\xi\frac{d^2}{d\xi^2}R_{1,2}(\xi) + \frac{d}{d\xi}R_{1,2}(\xi) - \frac{m^2}{4\xi}R_{1,2}(\xi) = 0 \tag{4.4.41}$$

We seek equation for

$$R_{1,2}(\xi) = \xi^s \tag{4.4.42}$$

Substitute (4.4.42) into (4.4.41) we can get

$$s(s-1)\xi^{s-1} - \frac{m^2\xi^{s-1}}{4} + s\xi^{s-1} = 0 \tag{4.4.43}$$

$$s^2 - s - \frac{m^2}{4} + s = 0 \Rightarrow s^2 = \frac{m^2}{4} \Rightarrow s = \frac{|m|}{2} \tag{4.4.44}$$

Now by substituting equation (4.4.44) into (4.4.42)

$$R_{1,2}(\xi) = \xi^{\frac{|m|}{2}} \tag{4.4.45}$$

We seek equation in form

$$R_{1,2}(\xi) = \xi^{\frac{|m|}{2}} Q_{1,2} \tag{4.4.46}$$

Substitute (4.4.46) into (4.4.40)

$$R'_{1,2}(\xi) = \frac{|m|}{2}\xi^{\frac{|m|}{2}-1}Q_{1,2} + \xi^{\frac{|m|}{2}}Q'_{1,2} \tag{4.4.47}$$

$$R''_{1,2}(\xi) = \frac{|m|}{2}(\frac{|m|}{2} - 1)\xi^{\frac{|m|}{2}-2}Q_{1,2} + 2\frac{|m|}{2}\xi^{\frac{|m|}{2}-1}Q'_{1,2} + \xi^{\frac{|m|}{2}}Q''_{1,2} \tag{4.4.48}$$

Substitute equation (4.4.47) and (4.4.48) into equation (4.4.40)

$$\xi Q''_{1,2} + Q''_{1,2}[|m| + 1] + Q_{1,2}[\frac{k^2}{4} - \frac{1}{4}\xi + \frac{m \pm 1}{2}] = 0 \tag{4.4.49}$$

Now let another asymptotic case $\xi \to \infty$ for equation (4.4.49) only two terms can remain we ignore the rest which are

$$\xi Q''_{1,2} - \frac{1}{4}\xi Q_{1,2} \tag{4.4.50}$$

The solution is substitute equation (4.4.47) and (4.4.48) into equation (4.4.40)

$$\xi Q''_{1,2} + Q''_{1,2}[|m| + 1] + Q_{1,2}[\frac{k^2\alpha^2}{4} - \frac{1}{4}\xi + \frac{m \pm 1}{2}] = 0 \tag{4.4.51}$$

Now take asymptotic case $\xi \to \infty$ for equation (4.4.49) only two terms can remain we ignore the rest which are

$$\xi Q''_{1,2} - \frac{1}{4}\xi Q_{1,2} = 0 \tag{4.4.52}$$

The solution for (4.4.52)is

$$Q_{1,2} = e^{-\frac{\xi}{2}} w(\xi)_{1,2} \tag{4.4.53}$$

where some unknown function

$$Q'_{1,2} = \frac{1}{2} e^{-\frac{\xi}{2}} w + e^{-\frac{\xi}{2}} w' \tag{4.4.54}$$

$$Q''_{1,2} = \frac{1}{4} e^{-\frac{\xi}{2}} + e^{-\frac{\xi}{2}} w'' \tag{4.4.55}$$

Now substitute equation (4.4.52) and (4.4.53) into equation (4.4.49) led to

$$\xi w''_{1,2} + [|m| + 1] w'_{1,2} + [\frac{k^2\alpha^2}{4}] - \frac{m \pm 1 - |m| - 1}{2}] w_{1,2} = 0 \tag{4.4.56}$$

This equation led to two equation from $\pm$ which are

$$\xi w''_1 + [|m| + 1] w'_1 + [\frac{k^2\alpha^2}{4}] - \frac{m - |m|}{2}] w_1 = 0 \tag{4.4.57}$$

and when it is minus

$$\xi w''_2 + [|m| + 1_\xi] w'_2 + [\frac{k^2\alpha^2}{4}] - \frac{m - |m| - 2}{2}] w_2 = 0 \tag{4.4.58}$$

We define w_{12} as power series

$$w_{12} = \sum_p a_p \xi^p \tag{4.4.59}$$

By substituting equation (4.4.59) into (4.4.57) and (4.4.58) we get

$$\sum_p \{p(p-1) a_p^1 \xi^{p-1} + p a_p^1 \xi^{p-1} [|m| + 1 - \xi] + [\frac{k^2\alpha^2}{4} - p - \frac{|m| - m}{2}] a_p^1 \xi^p\} = 0 \tag{4.4.60}$$

Rearranging and increasing by 1 step the first power series

$$\sum_p \{p(p+1) a_{p+1}^1 + p + 1 a_{p+1}^1 [|m| + 1 - \xi] + [\frac{k^2\alpha^2}{4} - p - \frac{|m| - m}{2}] a_p^1\} \xi^p = 0 \tag{4.4.61}$$

$$a_{p+1}^1 = \frac{\frac{|m|-m}{2} + p - \frac{k^2\alpha^2}{4}}{p(p+1)^2} a_p^1 \tag{4.4.62}$$

For cutting power series the numerator of equation (4.4.60) has to be zero

$$\frac{|m| - m}{2} + p - \frac{k^2\alpha^2}{4} = 0 \tag{4.4.63}$$

We introduce a quantum number n

$$n = \frac{|m| - m}{2} + p \tag{4.4.64}$$

By substituting (4.4.15), (4.4.32) and (4.4.64) into (4.4.63) we can obtain

$$E_{n1}^2 = \frac{eB}{c\hbar}\frac{4\hbar^2 v_F^2}{2} \tag{4.4.65}$$

Where n is integer and finally

$$E_{n1} = \pm\hbar v_F\sqrt{\frac{2eB_o}{\hbar c}}\sqrt{n}, = \pm\hbar\omega_D\sqrt{n} \tag{4.4.66}$$

where $\omega_D = \sqrt{\frac{2eB_o v_F^2}{\hbar c}}$. In the same way for equation (4.4.60) one can obtain

$$E_{n2} = \pm\hbar v_F\sqrt{\frac{2eB_o}{\hbar c}}\sqrt{n+1}, = \pm\hbar\omega_D\sqrt{n+1} \tag{4.4.67}$$

For positive m $|m| - m = 0$ using (4.4.64)

$$n = p \tag{4.4.68}$$

For negative $|m| - m = 2m$ using (4.4.64)

$$n = p + m \tag{4.4.69}$$

We again obtain energy level, (4.4.66) and (4.4.67),which coincide with equation what we obtained in (4.3.32) and (4.3.42) respectively. The result in (4.3.32) and (4.4.66) have dependence with quantum number of $\sqrt{n}$. While for (4.3.42) and (4.4.67) have dependence with quantum number by $\sqrt{n+1}$. Further, the results for positive m quantum number n is equal to integer p while for negative m quantum number n is equal to p plus m.

Chapter 5

Hall effect in Graphene

In chapter 4 we have seen the eigenvalue and eigenfunction are obtained for steady magnetic field. Because of high cyclotron gap, it is expected that QHE in this material can observed for much higher temperature and lower magnetic field than semiconductor because of high electron velocity in around Dirac point. A plateaus in Hall resistance followed by the vanishing of diagonal resistance. Filing factors has its own series

$$\upsilon = \pm 2(2n+1) = \pm 2 \pm 6 \pm 10... \tag{5.0.1}$$

where n is integer n=0,1,...

The filling factor steps in units of four between successive plateaus because of four folded spin- valley degeneracy. There are some graphene result in an unconventional kinds of Hall quantization called Anomalous Quantum Hall effect .

$$\sigma_{xy} = -\frac{2e^2}{h}(2n+1), \tag{5.0.2}$$

There high field have been observed at

$$\upsilon = \pm 1, \pm 4 \tag{5.0.3}$$

which are not belong to series in equation (5.0.1) this is because of electron -electron interaction as a consequance of maximally polarized quantum Hall ferromagnetic states.

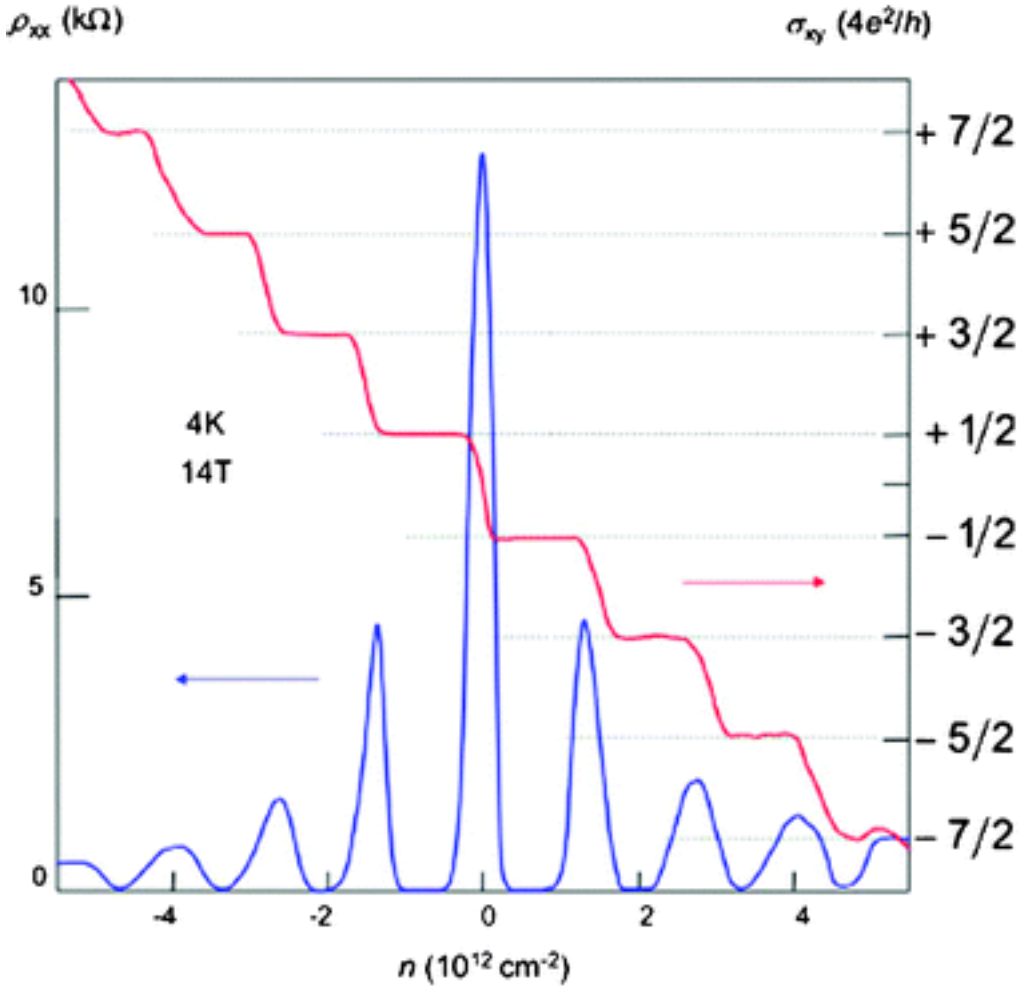

Figure 5.1: Quantum Hall effect in graphene [14] and [15]

5.1 Fractional Quantum Hall Effect in Graphene

We have seen in section 3.1 the cause for FQHE is delocalization and also we review the work Laughlin and Laughlin's wave function. For graphene case this wave function, Laugline wave function, can be extended to include Su(4) degree of freedom. This additional symmetry can be obtained by Halperin wave function. This set of wave function is readily generalized to the four components.

$$\psi^{SU(4)}_{m_1..m_4} = \phi^{L}_{m_1..m_4}\phi^{inter}_{n_{ij}} \tag{5.1.1}$$

where

$$\phi^{L}_{m_1..m_4} = \prod_{j=1}^{4} \prod_{k_j<l_j}^{N_j} (z^{j}_{k_j} - z^{j}_{l_j})^{m_j} e^{-\sum_{j=1}^{4}\sum_{k_j=1}^{N_j} |z^{(j)}_{k_j}|^2/4} \tag{5.1.2}$$

And inter-component correlations.

$$\phi^{inter}_{n_{ij}} = \prod_{i<j}^{4} \prod_{k_i}^{N_i} \prod_{k_j}^{N_j} (z^{(i)}_{k_i} - z^{(j)}_{k_j})^{n_{ij}} \tag{5.1.3}$$

here, $z^{(j)}_{k_j}$ is the complex coordinate of a particle in componenet j and N_j is the total number of j -type particles, m_j and n_{ij} relate to the componenet of filling factor. As we see in (Fig 5.1) the FQHE graph is done by applying 14T magnetic field and at 4k temperature. This figure shows fractional quantization [14] and [15].

Chapter 6

Conclusion and Summary

The Hall effect shows so many kinds of phenomena when it is applied to ordinary metal like copper by creating Hall voltage. When it is applied to 2DEG it exibts the quantization phenomena at high magnetic field and low temperature. With the advent of graphene the Hall effect quantization able to be observed at normal temperature because of graphene has high cyclotron energy than its thermal energy at this temperature. Finally, in our work we review the Dirac equation of graphene and got the eigenvalue and eigenfunction in rectangular coordinate in section 4.3 . We solve the same problem, Dirac equation, in cylindrical coordinate that has the same eigenvalue with what is solved in section 4.3. Beside for different value of m it exhibit different value. For positive m quantum number n is equal to integer p while for negative m quantum number n is equal to p plus m.

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Declaration

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

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