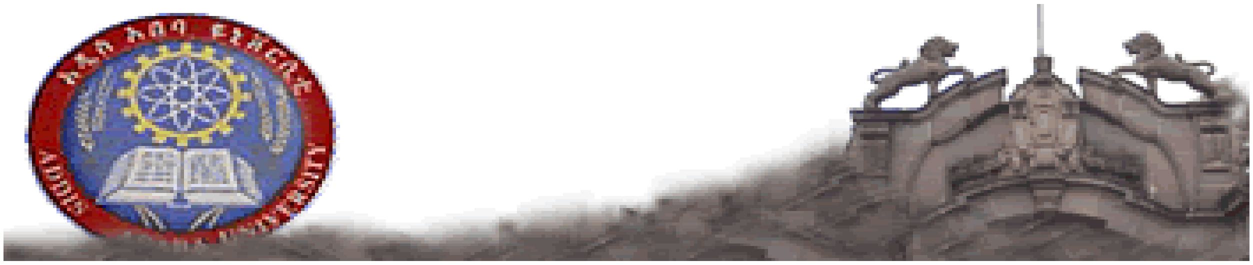

ADDIS ABABA UNIVERSITY

GRADUATE STUDIES PROGRAMME

COLLEGE OF NATURAL SCIENCES

DEPARTMENT OF STATISTICS

The Macroeconomic Determinants of Volatility in Precious Metals Prices in Ethiopia Using GARCH and RiskMetrics Models

By

Amare Wubishet

A Thesis submitted to the School of Graduate Studies of Addis Ababa University in partial fulfillment of the requirements for the Degree of Master of Science in Statistics

Addis Ababa, Ethiopia
JUNE, 2014

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This is to certify that the thesis prepared by Amare Wubishet, entitled: **The Macroeconomic Determinants of Volatility in Precious Metals Prices in Ethiopia Using GARCH and RiskMetrics Models** and submitted in partial fulfillment of the requirements for the Degree of Master of Science in Statistics, complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

Signed by the Examining Committee:

Examiner _____________________ Signature ______________Date __________

Examiner ____________________ Signature ______________Date __________

Advisor _____________________ Signature ______________Date __________

Chair of Department or Graduate Program Coordinator

DECLARATION

I, the undersigned, declare that this thesis is my original work and has not been presented for a degree in any other university, and that all sources of materials used for the thesis have been duly acknowledged.

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Date: ________________

This thesis has been submitted for examinations with my approval as an advisor.

Advisor Name: Emmanuel G/Yohannes (Dr.) Signature____________ Date ____________

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Table of Contents

List of Acronyms

ACF	Autocorrelation Function
ADF	Augmented Dickey Fuller Test
AIC	Akaike Information Criteria
AR	Autoregressive
ARCH	Autoregressive Conditionally Heteroskedastic
ARMA	Autoregressive Moving Average
CCC	Constant Conditional Correlations
CLRM	Classical Linear Regression Model
EGARCH	Exponential Generalized Autoregressive Conditionally Heteroscedastic
EWMA	Exponentially Weighted Moving Average
GARCH	Generalized Autoregressive Conditionally Heteroskedastic
GARCH-M	Generalized Autoregressive Conditionally Heteroskedastic in Mean
GED	Generalized Error Distribution
IID	Independent and Identical Distribution
JB	Jarque-Bera Test
L	Log likelihood function
LM	Lagrange Multiplier
MA	Moving Average
MAD	Mean Absolute Deviation
MAE	Mean Absolute Error
MAPE	Mean Absolute Percent Error
MGARCH	Multivariate Generalized Autoregressive Conditionally Heteroskedastic
ML	Maximum Likelihood
OLS	Ordinary Least Square Estimation
PACF	Partial Autocorrelation Function
PP	Phillips and Perron Test
RMSE	Root Mean Square Error
RSS	Residual Sum of Squares
SBIC	Schwarz Bayesian Information Criteria
TGARCH	Threshold Generalized Autoregressive Conditionally Heteroskedastic
TU	Theils- U Inequality

ABSTRACT

Modelling and forecasting volatility for the price of precious metals has become a fertile field of empirical research in financial markets. Since volatility is considered as an important concept in many economic and financial applications. The objective of this study was to model and forecast the volatility dynamics in precious metals prices in the Ethiopian market using GARCH family and RiskMetrics models using data from January1998 to January 2014.

The price return series of gold and silver show the characteristics of financial time series such as leptokurtic distributions and thus, can suitably be modeled using EWMA and GARCH family models. Empirical investigation was conducted in order to model price volatility using EWMA and GARCH family models. Among the GARCH family models considered in this study, ARMA (0, 1)-GARCH-M (2, 2) model with Student's t-distributional assumption of residuals and ARMA (1, 3)-EGARCH (3, 2) model with normal distributional assumption of residuals were found to be better fit for price volatility of gold and silver, respectively.

Saving interest rate, exchange rate and price of crude oil were found to have statistically significant effect on monthly price volatility of gold. On the other hand, saving interest rate and general inflation rate have statistically significant effect on monthly price volatility of silver. The risk premium effect for GARCH-M (2, 2) model was positive and statistically significant. This implies that an increase in volatility would increase the mean return. The asymmetric term was found to be positive and significant in EGARCH (3, 2) volatility model for sliver. This is an indication that unanticipated increase in price had larger impact on price volatility than unanticipated decrease in the price of silver.

A comparison was made between GARCH family models and exponentially weighted moving average (EWMA) model. The study suggests that GARCH class of models appear to be better in volatility forecasting than EWMA model as judged by RMSE and MAE criteria.

Key words: EWMA model, GARCH model, Precious metals and Volatility.

CHAPTER ONE

INTRODUCTION

1.1. Background of the Study

Precious metals are rare and naturally occurring metallic chemical elements that have high economic value. Precious metals such as gold, platinum and silver are interesting and receiving much attention for a number of reasons. They have served as monetary media and media of international exchange. They are used for savings, personal investment, fashion, art, medical and industrial purposes. Futures contracts on these precious metals are traded on different exchanges around the world along with other derivatives (such as call and put options).

Historically, precious metals have commanded much higher prices than common metals. Prices in precious metals, such as gold and silver, show dramatically increasing pattern over the period 1991 to 2011. Such dramatic increases are due to a number of factors, such as inflation expectations, the recent economic crisis, and higher demand by emerging markets (Lee and Lin, 2010). As a result, such dramatic changes in both markets have attracted the attention of investors since these precious metals serve as important stores of value and play a role in diversifying risk (Adrangi *et al*., 2003; Lucey and Tully, 2006).

Volatility is the relative rate at which the price of a security moves up and down. It shows how the prices fluctuate in a given period of time. The more the price moves up and down, the more volatility it is considered to have. Ross (1989) argued that volatility is a measure of information flow; the analysis can be viewed as an investigation of the extent to which the rate of information flows from one market to another. Over the last few years, modelling and forecasting volatility of financial time series (asset returns) has become a fertile area of research in finance, and has been receiving considerable attention from academics and practitioners. This is because volatility is an important concept for many economic and financial applications, like portfolio optimization, risk management and asset pricing. A special feature of volatility, which according to Tsay (2010) is "the conditional variance of the underlying asset returns", is that it is not directly observable. Consequently, financial analysts are especially keen to obtain good estimates of this conditional variance in order to improve portfolio allocation, risk management or valuation of financial derivatives. Since the 1980s a number of models have been developed that are especially suited to estimate the conditional volatility of financial assets.

Financial and commodity markets have been highly volatile in recent years. Volatility brings risk and opportunity to traders and investors, and should therefore be examined. There are many reasons for volatility to occur in commodity markets. Political unrest or extreme weather conditions in commodity producing countries can cause supply disruptions which can create volatility in commodity prices. Introduction of new financial innovations, such as futures, options, exchange-traded funds (ETFs), and use of precious metal as collateral for trading can affect volatility. Changes in demand for the product of an industry that uses commodities as an input may lead to fluctuations in prices of commodities. Market participants form different expectations of profitable opportunities, perform cross-market hedging across different asset classes, process information at different speeds, and build and draw inventories at different levels. These factors contribute to volatility of commodities over time and across markets (Hammoudeh *et al*, 2011).

Price discovery is one of the key functions of futures markets, which provides a market place for industry participants and investors to manage the price risk of commodities. Metal prices are volatile, reflecting the global nature of the changing markets. In the 43 years since President Richard Nixon effectively ended the use of gold and silver as the money standard for the U.S. dollar in 1971, prices of precious metals have been extremely volatile, reacting to the interactions of global factors such as inflation, interest rates and various economic and political events (Xu and Fung, 2004). The objective of investors is to obtain a return as high as possible from their investments assuming a risk as low as possible. When investors accept a higher risk, they are expecting a higher return. The measure typically associated to risk is the volatility. In particular, volatility and risk have a direct relationship (the higher the volatility, the higher the risk). One of the main characteristics of volatility is the asymmetry of its behaviour. The asymmetric behavior, also known as leverage effect, is of crucial importance. There is leverage effect in case where volatility is higher when the return in the previous period has been negative than when such a return has been positive (Centeno *et al.*, 2010).

There is a common belief that the price of commodities tends to move in unison. The reason is that they are influenced by common macroeconomic factors such as interest rate, exchange rate and inflation (Hammoudeh *et al*, 2008). Precious metals, among others, are the strategic commodities which have received much attention recently, partly due to the surges in their prices and the increases in their economic uses like industrial use in jewelry, medicine, electronic, auto catalytic industries and investment.

Appropriate processes for modelling volatility need to accurately capture the properties of financial time series. These properties have been identified as fat tails, sharp peaks and volatility clustering. Also, if there is correlation between lagged returns, a model that accounts for this should be chosen. From theory, the lagged correlation for any measure of volatility quantifies the volatility's memory shape and magnitude. Furthermore, the memory shape of lagged correlation in empirical data is considered to be well described by a slow power law or a logarithmic decay. The generalized autoregressive conditional heteroscedasticity (GARCH) and exponential weighted moving average (EWMA) processes on the other hand presume that the lagged correlations decay as an exponential. This implies that the memory in GARCH and RiskMetrics EWMA processes decay too fast and so not making use of all available information (Gustafsson and Soderstom, 2010).

A number of techniques have been used to model volatility. The autoregressive conditional heteroscedasticity (ARCH) model developed by Engle (1982), and later generalized by Bollerslev (1986), is the most popular method used for analyzing high frequency financial time series data. The estimates of volatility persistence for each return series provide information about the extent to which post shocks and volatility matter in the construction of forecast of future conditional variance. The greater the persistence, the more weight should be given to recent observations of volatility in explaining future behavior of the variance. The volatility of the series will return to its conditional variance faster than would be the case when there is greater persistence.

Standard GARCH models assume that positive and negative shocks have a symmetric effect on the volatility, that is, good and bad news have the same effect on volatility. In practice this assumption is frequently violated, in particular by stock returns, in that volatility increases more after bad news than after good news. From an empirical point of view, volatility reacts asymmetrically to the sign of the shocks and, therefore, a number of parameterized extensions of the standard GARCH model have been suggested recently. Some of the most important ones are the exponential GARCH (EGARCH) and the threshold GARCH (TGARCH) models. The other model is the EWMA model which uses historical observations to capture the dynamic features of the volatility. It assigns the highest weight to the latest observations and the least to the oldest observations in the volatility estimate. The assignment of these weights enables volatility to react to large return (jump) in the market, and following a jump, the volatility declines exponentially as the weight of the jump falls.

1.2. Statement of Problem

In recent years, the variance (volatility) in prices of precious metals has increased relative to other commodities. The volatile precious metal price environment requires market risk quantification (Hammoudeh *et al*, 2011). Even though there is a significant prior work on the macroeconomic determinants of volatility in the commodity markets, there is limited evidence that explains the relationship in precious metals when treated as an asset of class. Volatility plays a central role in empirical finance and financial risk management and lies at the heart of any model for pricing derivative securities (commodity). Research on changing volatility (conditional variance) using time series models has been active since the creation of the original ARCH model in 1982. From there, ARCH models grew rapidly into a rich family of empirical models for volatility forecasting during the last twenty years and they are now widespread and essential tools in financial econometrics (David, 2008).

Various studies have been conducted to analyze the volatility of the price return of precious metals. However, as to the knowledge of the researcher, no information (study) is available in Ethiopia in the dynamics of price volatility of precious metals. Most of the studies in other countries have not incorporated the determinants of price return volatility of precious metals and have not considered the RiskMetrics volatility models. This paper concerns on the investigation of the key macroeconomic variables that affect the price return volatility of precious metals and on formulation of various ARCH family models and RiskMetrics volatility models to assess the volatility of price return of precious metals, namely gold and silver, in Ethiopia.

1.3. Objective of the Study

1.3.1. Main objective of the study

- The main objective of this study is to identify the macroeconomic determinants of volatility in precious metals prices in the Ethiopian market by using GARCH and RiskMetrics models.

1.3.2. Specific objectives of the study

The specific objectives of this study are

- To find out whether GARCH family models are an improvement over the RiskMetrics type models in terms of modeling and forecasting precious metal price volatility.
- To investigate and assess the key macroeconomic factors that have a significant impact on the price volatility of precious metals in the Ethiopian market.

1.4. Significance of the Study

This study is very important to investors, researchers, governmental and nongovernmental organizations for policy formulation and planning in precious metals marketing. It can also be used as a supportive tool for price volatility management of precious metals. As to the knowledge of the researcher, there are no studies that have been designed to identify empirically the relative impact of significant factors contributing to the volatility of precious metals price in Ethiopia. The result of this paper could help to understand factors that affect volatility of precious metals price in the study area. Furthermore, the result of this study will be used as a basis for further study in this area.

CHAPTER TWO

LITERATURE REVIEW

2.1. General Overview of Volatility

The main characteristic of any financial asset is its return which is typically considered to be a random variable. The spread of outcomes of this variable, known as assets volatility, plays an important role in numerous financial applications. Its primary usage is to estimate the value of market risk. Volatility is also a key parameter for pricing financial derivatives. All modern option pricing techniques rely on a volatility parameter for price evaluation. Volatility is also used for risk management assessment and in general portfolio management. It is crucial for financial institutions not only to know the current value of the volatility of the managed assets, but also to be able to predict their future values. Volatility forecasting is especially important for institutions involved in options trading and portfolio management.

Accurate estimation of the future behavior of the values of financial indicators is obscured by complex interconnections between these indicators, which are often convoluted and not intuitive. This makes forecasting the behavior of volatility a challenging task even for experts in this field. Mathematical modeling can assist in establishing the relationship between current values of the financial indicators and their future expected values. Model-based quantitative forecasts can provide financial institutions with a valuable estimate of a future market trend. Although some experts believe that future events are unpredictable, some empirical evidence to the contrary exists. For example, financial volatility has a tendency to cluster and exhibits considerable autocorrelation (i.e. the dependency of future values on past values). These features provide the justification for formalizing the concept of volatility and creating mathematical techniques for volatility forecasting. Starting from the late 70's a number of models for volatility forecasting have been introduced (Sergiy, 2009).

Volatility refers to the spread of all outcomes of an uncertain variable. In finance, we are interested in the outcomes of assets returns. Volatility is associated with the sample standard deviation of returns over some period of time. It is a quantified measure of market risk. Volatility is related to risk, but it is not exactly the same. Risk is the uncertainty of a negative outcome of some event (e.g. stock returns); volatility measures the spread of outcomes. This includes positive as well as negative outcomes.

2.2. Characteristics of Financial Time Series

Modeling financial time series is a complex problem. This complexity is not only due to the variety of the series in use (stocks, exchange rates, interest rates, etc.), to the importance of the frequency of observation (second, minute, hour, day, etc) or to the availability of very large data sets. It is mainly due to the existence of statistical regularities (stylized facts) which are common to a large number of financial series and are difficult to reproduce artificially using stochastic models (Francq and Zakoian, 2010). Financial time series data sets have several characteristics which are crucial to note for the purposes of modeling and forecasting.

Nonstationarity of price series: Samples paths of prices are generally close to a random walk without intercept. On the other hand, sample paths of returns are generally compatible with the second-order stationarity assumption.

Fat tail: When the empirical distribution of returns is drawn, one can generally observe that it does not resemble a Gaussian distribution, which means market returns have distributions with fatter tails than the normal distribution. Classical tests typically lead to rejection of the normality assumption at any reasonable level. More precisely, the densities have fat tails (decreasing to zero more slowly than normal and are sharply peaked at zero), and they are called leptokurtic. A measure of the leptokurticity is the kurtosis coefficient, defined as the ratio of the sample fourth-order moment to the squared sample variance. Asymptotically equal to three for Gaussian iid observations, this coefficient is much greater than three for returns series. When the time interval over which the returns are computed increases, leptokurticity tends to vanish and the empirical distributions get closer to a Gaussian. Monthly returns, for instance, defined as the sum of daily returns over the month, have a distribution that is much closer to the normal than daily returns.

Volatility clustering: refers to the phenomenon that large changes tend to be followed by large changes of either sign, and small changes tend to be followed by small changes. A quantitative manifestation of this fact is that, while returns themselves are uncorrelated, absolute returns $|Y_t|$ or their squares display a positive, significant and slowly decaying autocorrelation function.

Leverage effect: It was noted by Black (1976), and refers to an asymmetry of the impact of past positive and negative values on the current volatility. Negative returns (corresponding to price decreases) tend to increase volatility by a larger amount than positive returns (price increases) of the same magnitude.

2.4. Theoretical Literature Review

Within the last three decades various approaches to volatility modelling have been suggested in the econometrics and financial literature. In the following we will provide a brief overview of developments in the literature starting with the autoregressive conditional heteroscedasticity (ARCH) models (Engle, 1982).

One of the assumptions of the classical linear regression model (CLRM) is that the variance of the error terms is constant, which is known as homoscedasticity. If the variance of the errors is not constant, the situation is known as heteroscedasticity. If the errors are heteroscedastic, and yet we assume that they are homoscedastic, an implication would be that standard error of the estimators could be biased. It is unlikely in the context of financial time series that the variance of the errors will be constant over time, and hence it makes sense to consider a model that does not assume that the variance is constant, and which describes how the variance of the errors evolves. Due to this several models have been suggested for capturing special features of financial data, and most of these models have the property that the conditional variance depends on the past. One of the best known and most often used is the autoregressive conditionally heteroscedastic (ARCH) process introduced by Engle (1982). This is the first model that provides a systematic framework for volatility modeling. The basic idea of ARCH models is that the shocks of an asset return are serially uncorrelated, but are dependent, and the dependence of ε_t can be described by a simple quadratic function of its lagged values. ARCH models are simple and easy to handle, and take care of clustered errors, as well as nonlinearities. Even if ARCH model has been used in volatility modeling, it has its own weakness. According to Tsay (2005) some of the weaknesses are:

1. The model assumes that positive and negative shocks have the same effects on volatility since they depend on the square of the previous shocks. In practice, it is well known that the price of financial assets responds differently to positive and negative shocks.

2. The ARCH model is rather restrictive. For instance, α_1^2 of an ARCH (1) model must be in the interval $[0, \frac{1}{3}]$ if the series has a finite fourth moment. The constraint becomes complicated for higher order ARCH models. In practice, it limits the ability of ARCH models with Gaussian innovations to capture excess kurtosis.

3. ARCH models are likely to over predict the volatility because they respond slowly to large isolated shocks to the return series.

Although ARCH model has been used by researchers and practitioners, one of its characteristics is that it requires many parameters to describe appropriately the volatility process of an asset return. Thus, alternative models must be further searched, one of them being the one developed by Bollerslev (1986) who propose a useful extension known as the Generalized ARCH.

Bollerslev (1986, 1987) introduced the generalized ARCH (GARCH) model. This model is a weighted average of past squared residuals and variances, but has declining weights that never go completely to zero. It gives parsimonious models that are easy to estimate and, even in its simplest form, has proven surprisingly successful in predicting conditional variances. The most widely used GARCH specification asserts that the best predictor of the variance in the next period is a weighted average of the long-run average variance, the variance predicted for this period, and the new information in this period that is captured by the most recent squared residual. This model is also consistent with the volatility clustering often seen in financial returns data, where large changes in returns are likely to be followed by further large changes.

As compared to the ARCH model, GARCH model requires less lags to capture the property of time-varying variance (Wang, 2009). This feature allows GARCH model to be more parsimonious than the ARCH model. Moreover, ARCH incorporates the feature of autocorrelation observed in return volatility of most financial assets; GARCH improves ARCH by adding a more general feature of conditional heteroscedasticity.

Since the introduction of these models, they have been widely used in volatility modelling and forecasting. Researchers such as French *et al.* (1987) and Akgiray (1989) utilised GARCH models to capture the behaviour of stock market price volatilities. Akgiray (1989) compared the GARCH (1, 1) model to other historical estimation methods and found that the GARCH (1,1) model outperformed its competitors. Many extensions of the GARCH model have been introduced in the literature since then: e.g. GARCH-in-mean (GARCH-M) models (Engle *et al.*, 1987), EGARCH models (Nelson, 1991), Threshold ARCH (TARCH) and Threshold GARCH (TGARCH) (Glosten, Jaganathan, and Runkle, 1993; Zakoïan, 1994) and Power ARCH (PARCH) models (Ding *et al*., 1993). A number of studies have focused on optimal model specification and the performance of various GARCH models in financial markets providing no clear-cut results. Hansen and Lunde (2005) carried out comprehensive testing of 330 variants of ARCH type models on their performance in estimating volatility in exchange rates and stock returns. The study found that the GARCH (1,1) model outperforms other models in estimating exchange rate volatilities but underperforms in estimating stock returns. McMillan *et al*. (2000) tested a set of ten volatility estimation models including random walk, moving average and GARCH models in forecasting UK stock market returns at different frequencies. They found that the performance of each model varied depending on the length of frequencies, the series as well as the type of loss function being applied. The random walk model outperformed others at the monthly frequency, while GARCH and moving average models were superior using daily forecasts.

Brooks and Persand (2002, 2003) examine various ARCH and GARCH type models with respect to volatility forecasting. They report that, while the forecasting performance of the models depended on the considered data series and time horizon, the overall most preferred model is a simple GARCH (1,1). This is also consistent with many other studies such as e.g. Bollerslev *et al*. (1992). On the other hand, Braisfold and Faff (1996) evaluate volatility models in forecasting stock returns, and found that none of the models significantly outperforms the others.

Exponentially weighted moving average (EWMA) model is also a widely used technique for modeling and forecasting volatility of equity returns in financial markets, and the well-known RiskMetrics approach is virtually the variation of EWMA. A great deal of existing studies using EWMA model on various markets demonstrated that EWMA model has different performance (Kun, 2011).

Akgiray (1989) examined the forecasting performance of EWMA technique on volatility forecasting on stocks returns of New York stock exchange (NYSE). The study also examines the predictive power of ARCH and GARCH models. The forecasting performance is evaluated and compared through a number of statistics: mean error (ME), root mean square error (RMSE), mean absolute error (MAE) and mean absolute percent error (MAPE). The findings indicated that EWMA model is useful for forecasting time series, but based on the relative values of these statistics, the GARCH model performs better for forecasting volatility of stock returns.

Tse and Tung (1992) investigated monthly volatility movements in Singapore stock market using three different volatility forecasting models: the naïve method based on historical sample variance, EWMA and GARCH models. The study result suggests that EWMA model is better for predicting volatility of monthly returns for Singapore market using the RMSE and MAE as comparison criteria.

Walsh and Tsou (1998) investigated the volatility of Australian index from January 1, 1993 to December 31, 1995 using a variety forecasting techniques: an improved extreme-value method, the ARCH/GARCH class of models and EWMA model. Hourly, daily and weekly data were used for analysis. Their finding indicates that the EWMA model outperforms other volatility forecasting technique within the sample period using forecasting accuracy measures.

2.5. Empirical Literature Review

Price volatility in commodity markets has been studied extensively in the academic literature. Many researchers have examined commodity price volatility from different perspectives. Some studies have examined price volatility from the view point of price efficiency. For example, Aggarwal and Sundararaghavan (1987) reported that the silver market was not efficient in the weak form[1]. But, Solt and Swanson (1981) found that futures market for gold and silver were weak form efficient and that investors cannot earn abnormal profits. Moreover, Ciner (2001) examined the long run trend in prices of gold and silver futures contracts listed on the Tokyo Commodity Exchange. Using daily closing prices from 1992 to 1998 with Johansen's (1991) cointegration analysis, the results indicated that the long run stable relationship between gold and silver future prices had disappeared.

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[1] Weak form efficient hypothesis assumes that the rates of return on the market should be independent; past rates of return have no effect on future return rates.

Batten *et al*. (2010) employ a monthly VAR framework to assess macroeconomic determinants of volatility in the precious metal market. Their findings suggest that precious metals are "too distinct to be considered a single asset class" because volatilities do not appear impacted jointly by the same key factors. While both monetary and financial variables proved significant for gold, neither of them was found significant for silver.

To study the relations between precious metal and FX markets, Capie *et al*. (2005) used a combination of regression and GARCH models for weekly returns on gold prices in USD with the Sterling (Yen) to USD exchange rate as covariate in the regression term. The authors conclude that during the span of thirty years from the early 1970s onward, gold has served as a hedge against USD exchange rate fluctuations, "but that it has done so to a degree that seems highly dependent on somewhat unpredictable political attitudes and events". Pukthuanthong and Roll (2011), however, found no empirical evidence for a special relation between the USD price of gold and USD weakness that would allow to imply a positive impact from foreign currencies' appreciation. Using daily data from the preceding forty years, and applying a conditional correlation GARCH model, they show that a higher price of gold can be associated with depreciation in every currency (USD, Euro, Pound, and Yen) over the same time period. Sari *et al*. (2010) investigate co-movements and information spillovers among the precious metal, oil and FX markets on the basis of a VAR model. They found evidence of close linkage in the short run after shocks.

Morales and Andreosso (2011) use the normal GARCH (1, 1) model and the ICSSGARCH extended model which incorporates the volatility breakpoints identified by the iterated cumulative sum of squares (ICSS) algorithm to study the break points or sudden changes in the variance of precious metals returns in US-Dow Jones Industrial, Japan-Nikkei 225 and UK-FTSE 100 markets using the standardized residuals obtained through the GARCH (1,1) mean equation. Their findings show that there is a significant positive relationship between precious metals market returns and the Brent returns; both the GARCH (1, 1) and the GARCH (1, 1) with dummies show a positive significant relationship between precious metal returns and oil returns. This situation reflects that, in general when the oil markets are appreciating, there is a trend of increasing returns for the gold market. In relation to the equity markets, most of the coefficients appear to be insignificant. This means that shocks in equity markets do not tend to generate major effects in precious metals markets.

Jonathan *et.al* (2008) model the monthly price volatilities of four precious metals (gold, silver, platinum and palladium) and investigate the macroeconomic determinants (business cycle, monetary environment and financial market sentiment) of these volatilities. To conduct the empirical tests and to establish the volatility linkages between the various macroeconomic variables and the precious metals market, they use the estimated conditional standard deviations (GARCH) model and vector autoregressive (VAR) framework to calculate block exogeneity causality tests. Gold volatility is shown to be explained by monetary variables, but this is not true for silver. Overall, there is limited evidence that the same macroeconomic factors jointly influence the volatility processes of the four precious metal price series, although there is evidence of volatility feedback between the precious metals. These results are consistent with the view that precious metals are too distinct to be considered a single asset class, or represented by a single index.

Toraman *et.al* (2011) studied the determinant factors affecting the price of gold with a multivariate GARCH (MGARCH) model using data from 1992 and 2010. Oil price and US exchange, inflation and real interest rates were included in the model as covariates. First differences of US exchange rate and real interest rates are taken and return series of the other variables are used. An MGARCH model is estimated by Constant Conditional Correlations (CCC) model for the reason that it gives the best results for the estimation of the coefficients. According to the empirical findings, the high negative correlation is found between gold prices and US exchange rate. Secondly, a positive correlation is found between gold prices and oil prices. Moreover, a significant negative relationship is found between the return of gold and the return of US Dollar, while the remaining variables have no significant relationship with the return of gold. This result is valid both for the conditional and unconditional correlation relationship. On the contrary, Topcu (2010) found that there is no significant relationship between oil prices and return of gold and inflation rate and the return of gold.

Harper (2008) examines the price volatility in the silver spot market. His results provide some evidence that both good and bad news have no significant effect on silver price volatility. Both the GARCH (1, 1) and EGARCH (1, 1) models were well fit in respect to silver price volatility. The results also have implications for the various agents that use silver. The volatility in the silver spot market could impact the futures market. Therefore, the various agents that use silver should observe the futures markets in order to determine if hedging silver price volatility is an appropriate risk management tool.

Ghosh *et al*. (2002) using monthly data between 1976 and 1999 investigated the effects of worldwide inflation level, US inflation level, worldwide income, value of US Dollar and random shocks on gold prices with a VAR model. It was concluded that gold prices are related with US Inflation level, interest rates and dollar exchange rate. Furthermore, a long run relationship was found between gold prices and US Consumer Price Index from cointegration analysis.

Xu and Fung (2004) study the cross-linkage between U.S and Japanese markets on precious metals futures trading. In estimating bivariate GARCH model for gold, platinum and silver, they first compared the models with one lag, two lags and three lags by using the Akaikian information criterion (AIC). For all three precious metals futures, the two-lag model has the smallest AIC; hence it was selected as the appropriate model. They estimate the model by maximizing the log-likelihood function following the Berndt *et al*. (1974) algorithm. The results indicate that the pricing transmission across both markets is strong and significant. The error-correction terms, which reflect long-term relationships between the U.S. and the Japanese markets, are highly significant, suggesting that both markets are moving closer together over time. The result of the volatility analyses indicate that GARCH (2, 2) model well fits with different spillover coefficients for all precious metals considered.

Kiohos and Sariannidis (2010) examined the role of financial factors in the gold market using a GJR-GARCH model. More specifically, they examined the influence of the Crude Oil Light Sweet index, S&P 500 Stock index, the U.S. dollar/yen exchange rate and the TNX 10- Year Treasury Note on the gold future price. The empirical results show that the first determinant factor, the crude oil, reflects a positive transmission effect from the leading energy market to the gold market. The importance of these spillover effects reflects, to a large extent, the world economic activity. On the other hand, the S&P 500 Stock index, the U.S. dollar/yen exchange rate and the TNX 10-Year Treasury Note influence negatively the gold market not only because gold is a hedge against economic or political turmoil, but also because it offers alternative approaches in portfolio management.

The structural analysis of volatility showed that the impact of old news on conditional volatility was higher than that of the current news. In addition, the volatility of the U.S. dollar/yen exchange rate returns exerts significant influence on the conditional variance of the gold series. Finally, the results have shown that the volatility of the gold market tends to overact in response to positive shock contrary to the equity markets. In times of market stress or turmoil the increased volatility from the other markets is transmitted to the gold market which acts as a safe haven. Their empirical evidence of the study suggests that the role of gold in a portfolio investment is beneficial since the increased volatility of gold enhances the useful negative correlation of gold with other markets. The increased price and positive shocks of gold as a financial instrument protects against declining movements in the price of other assets (Kiohos and Sariannidis, 2010).

Korkmaz and Aydin (2002) compared EWMA and GARCH models in their study of ISE-30 index return volatility. RiskMetrics method has used EWMA model and various data ranges (number of days) have been used in the analysis. It is determined that the most recent data have asserted more influence on future volatility than past data. ARCH effects have been recorded for all the stocks in ISE 30 Index in the study and for all the stocks' returns, the optimal lag length was found to be one using AIC and SIC. Afterwards, GARCH model is tested and the constraints of all GARCH parameters are satisfied. Their finding suggests that GARCH model provides more accurate analysis than EWMA model. On the contrary, Tse (1991) studied volatility of stock returns of the Japanese market during the period of 1986 to 1989 using ARCH, GARCH and EWMA models. The study results revealed that the EWMA model outperforms ARCH and GARCH models for volatility forecasting of stock returns in Tokyo stock exchange during the sample period.

Ekvall (2012) compared EWMA, implied volatilities and three GARCH specifications- GARCH (1,1) and EGARCH(1,1) with student's errors and a GARCH(1,1) with a Hansen's skewed t errors- on his study of variance forecasting in bull and bear markets on three asset indices: the DAX, the standard & poor's 500 and the currency shares Euro trust. He computes 30 days ahead variance forecast using daily data and the true latent variance approximated by the intra-month realized variance in the two markets. The performance of the models was compared by R^2 (obtained from a regression of the realized variance on the estimated past variance), quasi likelihood (QLIKE) statistic and the mean square error (MSE). His result shows that the implied volatility forecast was better in bull markets and the GARCH and EGARCH forecast was better in bear markets than EWMA.

CHAPTER THREE

DATA AND STATISTICAL METHODOLOGY

3.1. Data Sources

In this study, the data are secondary data on monthly price of precious metals (gold and silver), exchange rate, saving interest rate, inflation rate and price of crude oil which were obtained from the National Bank of Ethiopia (NBE) over the period from January 1998 to January 2014.

3.2. Definition and Variables of the Study

The response variable in this the study is the monthly price returns of precious metals in the Ethiopian market**.** The explanatory (predictors) variables considered are those that are assumed to affect price volatility of precious metals. These are:

- Exchange rate (of Birr against the US dollar)
- General Inflation rate (the rate at which the general level of prices for goods and services rise and fall)
    - Inflation rate of food items
    - Inflation rate of non-food items
- Saving interest rate (the rate at which interest is paid by a borrower (debtor) for the use of money for a lender (creditor)).
- Price of crude oil (Us dollar per barrel)

3.3. Statistical Methodology

Risk is often measured in terms of price changes. These changes can take a variety of forms such as absolute price change, relative price change, and log price change. When the price change is defined relative to some initial price, it is known as a return. If we denote the price of an item (commodity) at time t by P_t, then the absolute price change on the commodity between time t and $t-1$ is defined as:

$$d_t = P_t - P_{t-1} \qquad (3.1)$$

The relative price change or percent return for the same period is given by:

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{3.2}$$

If the gross return on an item is just $(1 + r_t)$, then the log price changes (or continuously-compounded return), Y_t of an item is defined to be the natural logarithm of its gross return. That is:

$$Y_t = \ln(1 + r_t) = \ln\left(\frac{P_t}{P_{t-1}}\right) \tag{3.3}$$

In practice, the main reason for working with returns rather than prices is that, the return of the series gives a complete and scale-free summary of the series and return series are easier to handle than price series because they have more attractive statistical properties like stationarity. Further, returns (relative and log price changes) are often preferred to absolute price changes because the latter do not measure change in terms of the given price level (Morgan and Reuters, 1996). But the log-return is usually more preferable than absolute and relative returns. The reason of typically considering log-returns instead of other returns is to use the additive property of log-returns, which is not shared by absolute and relative returns. The log-return series display many of the typical facts in financial series such as leptokurtosis and volatility clustering.

3.3.1. Stochastic Process and stationarity

A stochastic process is a series of time indexed random variables $Y(\omega, t)$, where ω belongs to a sample space and t to an index set. For a fixed set t, $Y(\omega, t)$ is a random variable; for a given ω, $Y(\omega, t)$ as a function of t is called a realization. The population that consists of all possible realizations is called the ensemble in stochastic process and time series analysis. Thus, time series is a realization or sample function from a certain stochastic process (William, 2006). A time series Y_t is a collection of observations on a variable indexed sequentially over several time points t = 1, 2... T. Time series observations $Y_1, Y_2, \dots\dots, Y_T$ are inherently dependent. From a statistical modeling perspective, this means it is inappropriate to treat a time series as a random sample of independent observations.

A stationary process has the property that the mean, variance and autocorrelation structure do not change over time. It is an essential property to define a time series process. Stationarity may be weak or strong.

3.3.1.1 Weak Stationarity

A stochastic process $\{Y_t\}$ is said to be weakly stationary if both the mean of Y_t and the covariance between Y_t and Y_{t+h} are time invariant, where h is an arbitrary integer. More specifically, $\{Y_t\}$ is weakly stationary if:

- $E(Y_t) = \mu$ (Constant).
- $var(Y_t) = \sigma^2 < \infty$ (Finite variance).
- For any *t*, $cov(Y_t, Y_{t+h}) = \gamma_X(h)$, which depends only on h and not on *t*.

3.3.1.2 Strong (Strict) Stationarity

A time series is said to be strictly stationary if its properties are not affected by a change in the time origin. That is, if the joint probability distribution associated with m observations $\{Y_{t_1}, Y_{t_2}, \ldots, Y_{t_m}\}$ made at any set of times $t_1, t_2, \ldots, t_m$ is exactly the same as the joint probability distribution of the observations $\{Y_{t_{1+k}}, Y_{t_{2+k}}, \ldots, Y_{t_{m+k}}\}$ made at times $t_{1+k}, t_{2+k}, \ldots, t_{m+k}$. Mathematically it can be expressed as:

$$\mathrm{F}(Y_{t_1} \le \tau_1, Y_{t_2} \le \tau_2, \ldots, Y_{t_m} \le \tau_m) = \mathrm{F}\left(Y_{t_{1+k}} \le \tau_1, Y_{t_{2+k}} \le \tau_2 \ldots, Y_{t_{m+k}} \le \tau_m\right) \qquad (3.4)$$

Many financial time series appear to be non-stationary. Different statistical issues arise when analyzing non-stationary process. When we deal with non-stationary series, the effect of a shock never dies away and it leads to spurious regressions, that is, one can regress completely unrelated series and find high R^2 (indicating how good one term is at predicting another) and the standard tests are not valid. The first step for an appropriate analysis is to determine whether the time series is stationary or not. Due to non-stationarity, regressions with time series data are very likely to result in spurious results.

In order to ensure the condition of stationarity, a non-stationary series must to be integrated. A non-stationary time series is called integrated if it can be transformed into a stationary process by differencing once or a very few times. The order of integration is the minimum number of times the series needs to be differenced to yield a stationary series. In order to detect the presence and form of non-stationarity, unit root tests are used.

3.3.2 Testing Stationarity: Unit Roots Tests

Many economic and financial time series exhibit a trending behavior or non stationarity in the mean. Unit root tests can be used to determine if trending data should be first differenced or regressed on deterministic functions of time to render the series stationary (Granger and Newbold ,1974; Zivot and Wang, 2006). Consider an AR (1) process:

$$Y_t = \mu + \rho Y_{t-1} + \varepsilon_t \tag{3.5}$$

where ε_t is a white noise process. Unit root tests are based on testing the null hypothesis that $H_0: \rho = 1$ against the alternative $H_1: \rho < 1$. They are called unit root tests because, under the null hypothesis, the characteristic polynomial has a root equal to unity. There are several procedures which have been developed to test the stationarity of time series. The most popular ones are Augmented Dickey- Fuller (ADF) test due to Dickey and Fuller (1979), and the Phillip-Perron (PP) test due to Phillips (1986) and Phillips and Perron (1988).

3.3.2.1. Augmented Dickey-fuller (ADF) Unit Root Test

The ADF test tests whether a unit root is present in an autoregressive model. It is developed by the statisticians David Dickey and Wayne Fuller in 1979. In its simplest form it considers an autoregressive order one process given by:

$$Y_t = \mu + \rho Y_{t-1} + \varepsilon_t \tag{3.6}$$

where Y_t is the variable of interest and ε_t is the error term which is assumed white noise. If a unit root is present (that is, if ρ = 1), then the model would be non-stationary. In this case the regression model can be written as:

$$\Delta Y_t = \mu + (\rho - 1)Y_{t-1} + \varepsilon_t = \mu + \varphi Y_{t-1} + \varepsilon_t \tag{3.7}$$

Testing for a unit root in the above model is equivalent to testing:

$\mathrm{H_0}: \varphi = 0$ (the series is nonstationary) versus $\mathrm{H_1}: \varphi < 0$ (the series is stationary)

The test statistic is given by:

$$t_\varphi = \frac{\hat{\varphi}}{se(\hat{\varphi})} \tag{3.8}$$

Due to the non-stationary of Y_t under the null hypothesis, the test statistic given by (3.8) does not follow the conventional student's t-distribution. Dickey and Fuller (1979) derived asymptotic results and simulated critical values for various tests and sample sizes. MacKinnon (1991, 1996) implemented a much larger set of simulations than those tabulated by Dickey and Fuller. In addition, MacKinnon estimated response surfaces from simulation results, permitting the calculation of Dickey-Fuller critical values and p-values for arbitrary sample sizes.

The simple Dickey-Fuller unit root tests described above is valid only if the series is an AR (1) process. If the series is correlated at higher order lags, the assumption of white noise disturbances of ε_t is violated. The ADF test constructs a parametric correction for higher-order correlation by assuming that the series follows an AR (*p*) process and adding lagged difference terms of the dependent variable. If the series are serially correlated, then ε_t can be expressed as:

$$\varepsilon_t = \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \cdots + \theta_p\varepsilon_{t-p} + u_t \tag{3.9}$$

Assuming u_t are identically and independently distributed (iid) random variables, then equation (3.7) can be expressed as:

$$\Delta Y_t = \mu + \varphi Y_{t-1} + \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \cdots + \theta_p\varepsilon_{t-p} + u_t \tag{3.10}$$

From equation (3.6), when $\rho = 1$ and imposing $\mu = 0$, we have $\varepsilon_t = Y_t - Y_{t-1} = \Delta Y_t$, so that equation (3.10) can be rewritten as:

$$Y_t = \beta_0 + \beta_1 t + \varphi Y_{t-1} + \theta_1\Delta Y_{t-1} + \theta_2\Delta Y_{t-2} + \cdots + \theta_p\Delta Y_{t-p} + u_t$$

$$Y_t = \beta_0 + \beta_1 t + \varphi Y_{t-1} + \textstyle\sum_{j=1}^{p}\theta_j\,\Delta Y_{t-j} + u_t \tag{3.11}$$

This augmented specification is then used to test for unit root using the t-ratio in equation (3.8). An important result obtained by Dickey and Fuller (1979, 1981) is that the asymptotic distribution of the t -ratio for φ is independent of the number of lagged first differences included in the ADF regression. Moreover, while the assumption that Y_t follows an AR process may seem restrictive, Said and Dickey (1984) demonstrate that the ADF test is asymptotically valid in the presence of a moving average component provided that sufficient lagged difference terms are included in the test regression.

The practical issue for the implementation of the ADF test is the specification of the lag length p. If p is too small then the remaining serial correlation in the errors will bias the test, and if p is too large then the power of the test will suffer. The number of augmenting lags (p) is determined by minimizing the Schwartz Bayesian information criterion and Akaike information criterion or lags are dropped until the last lag is statistically significant.

3.3.2.2. Phillips and Perron (PP) Unit Root Tests

The statistic proposed by Phillips and Perron (1988) arise from their considerations of the limiting distributions of the various Dickey-Fuller statistics when the assumption that ε_t is an iid process is relaxed. The test regression in the Phillips-Perron test is given by:

$$\Delta Y_t = \varphi Y_{t-1} + \alpha + \beta t + \varepsilon_t \tag{3.12}$$

where ε_t is a stationary process (which may be heteroscedastic). The PP test correct for any serial correlation and heteroscedasticity in the errors ε_t of the test regression by directly modifying the Dickey-Fuller test statistic. The modified statistics, denoted by $Z_t \; and \; Z_\varphi$, are given by:

$$Z_t = \left(\frac{\hat{\sigma}^2}{\hat{\lambda}^2}\right)^{\frac{1}{2}} t_{\varphi=0} - \frac{1}{2}\left(\frac{\hat{\lambda}^2 - \hat{\sigma}^2}{\hat{\lambda}^2}\right)\left(\frac{Tse(\hat{\varphi})}{\hat{\sigma}^2}\right)$$

$$Z_\varphi = T\varphi - \frac{1}{2}\left(\frac{T^2 se(\hat{\varphi})}{\hat{\sigma}^2}\right)\left(\hat{\lambda}^2 - \hat{\sigma}^2\right) \tag{3.13}$$

The terms $\hat{\sigma}^2$ and $\hat{\lambda}^2$ are consistent estimates of the variance parameters

$$\hat{\sigma}^2 = \lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} E(\varepsilon_t^2)$$

$$\hat{\lambda}^2 = \lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} E(S_T^2) \quad where \; S_T = \sum_{t=1}^{T} \varepsilon_t$$

Under the null hypothesis that $H_0 : \varphi = 0$, the Z_t and Z_φ statistics have the same asymptotic distributions as the ADF t-statistic. In the Dickey-Fuller specification, we can use the critical values given by Dickey and Fuller for the various statistics if ε_t is an iid, and we should use Phillips-Perron's counter parts if it is not iid. One advantage of the PP tests over the ADF tests is that the PP tests are robust to general forms of heteroscedasticity in the error term ε_t, and the user does not have to specify a lag length for the test regression.

3.4. Model Specification of the Study

There are two distinct equations or specifications in this study, the first for the conditional mean and the second for the conditional variance. We first employ an autoregressive moving average (ARMA) model for the conditional mean in the return series as an initial regression. Then a test of the null hypothesis that there is no ARCH effect in the residual series is conducted. Exponentially weighted moving average (EWMA), autoregressive conditional heteroscedasticity (ARCH) and its generalization (GARCH) models represent the main methodologies that have been applied in modelling and forecasting the volatility of precious metals price. In this paper different univariate extension of GARCH specifications are employed to model monthly precious metal price volatility in Ethiopia. These include GARCH-M, EGARCH and TGARCH.

3.5. Family of ARMA and ARIMA Models

Univariate time series models are a class of specifications where one attempts to model and predict time series variables using only information contained in their own past values and possibly current and past values of the error term. Time series models are usually theoretical, implying that their construction and use is not based upon any underlying theoretical model of the behaviour of a variable. Instead, time series models are an attempt to capture empirically relevant features of the observed data that may have arisen from a variety of different (but unspecified) structural models. An important class of time series models is the family of Autoregressive Integrated Moving Average (ARIMA) models, usually associated with Box and Jenkins (1976).

The aim of time series analysis is to construct a model for the underlying stochastic process. This model is then used for analyzing the causal structure of the process or to obtain optimal predictions. In statistical analysis of time series, autoregressive–moving average (ARMA) model provides a parsimonious description of a weakly stationary process in terms of two polynomials, one for the auto-regression and the second for the moving average. The model is usually represented by $\mathrm{ARMA}(\mathrm{p}, \mathrm{q})$ model, where p is the order of the autoregressive part and q is the order of the moving average part.

3.5.1 Autoregressive (AR) Process

An autoregressive (AR) model is one where the current value of a variable Y_t depends upon on the values of the p past values of the variable plus an error term. Specifically an AR model of order p, denoted as AR (p), can be expressed as:

$$Y_t = \mu + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-2} + \alpha_3 Y_{t-3} \ldots + \alpha_p Y_{t-p} + \varepsilon_t = \mu + \sum_{i=1}^{p} \alpha_i Y_{t-i} + \varepsilon_t \qquad (3.14)$$

where ε_t is a white noise disturbance term and we assume that it is independent of the past values of the response variable. This expression can be written more compactly using the backward shift operator L as:

$$(1 - \alpha_1 L^1 - \alpha_2 L^2 - \cdots - \alpha_P L^P) Y_t = \alpha(L) Y_t = \varepsilon_t \qquad (3.15)$$

The characteristic equation for AR (p) process is given by: $\alpha(L) = 1 - \alpha_1 L^1 - \alpha_2 L^2 - \cdots - \alpha_P L^P = 0$. The autoregressive process of order p is stationary if the roots of the characteristic equation lie outside the unit circle (or, if complex, have modulus greater than one) (Box and Jenkins, 1976, Chatfield, 1996).

A useful property of an AR (p) process is that the partial ACF is zero at all lags greater than p. This means that the sample partial ACF can be used to help determine the order of an AR process (assuming the order is unknown as is usually the case) by looking for the lag value at which the sample partial ACF "cuts off" (meaning that it should be approximately zero, or at least not significantly different from zero, for higher lags) (Box and Jenkins, 1976; Chatfield, 1996).

3.5.2 Moving average (MA) process

A time series Y_t is said to be a moving average process of order q if it is a weighted linear sum of the last q random shocks /errors. In general, the moving average process of order q, denoted as MA (q), can be expressed as:

$$Y_t = \mu + \varepsilon_t - \beta_1\varepsilon_{t-1} - \beta_2\varepsilon_{t-2} - \beta_3\varepsilon_{t-3} - \cdots - \beta_q\varepsilon_{t-q} = \mu - \sum_{i=1}^{q}\beta_i\varepsilon_{t-i} + \varepsilon_t \tag{3.16}$$

Using the backward shift operator, equation (3.16) can be expressed as:

$$Y_t = \left(1 - \beta_1 L - \beta_2 L^2 - \beta_3 L^2 - \cdots - \beta_q L^2\right)\varepsilon_t = \beta(L)\varepsilon_t \tag{3.17}$$

where q is the number of past innovations included in the moving average, $\beta_1, \beta_2, \dots, \beta_q$ are the MA parameters (coefficients) which describe the effect of the past innovations on Y_t and ε_t is the error term which is assumed as a white noise process.

Unlike the autoregressive process, a moving average process is always stationary since it is a finite linear combination of a white noise sequence for which the first two moments are time-invariant. The moving average process of order q has the characteristic equation given by $\beta(L) = 1 - \beta_1 L - \beta_2 L^2 - \beta_3 L^3 - \cdots - \beta_q L^q = 0$. The MA *(q)* is invertible if all the roots of the characteristic equation $\beta(L) = 0$ lie outside the unit circle (or, if complex, have modulus greater than one) (Box and Jenkins, 1976; Green, 1993).

The order of the moving average model can be determined by analysis of the autocorrelation function (ACF) which cuts off after q lags and partial ACF that decays exponentially fast.

3.5.3. Autoregressive Moving average (ARMA) Process

In some applications, the AR or MA models become cumbersome because one may need a higher-order model with many parameters to adequately describe the dynamic structure of a process. To overcome this difficulty, the autoregressive moving-average (ARMA) models are introduced in 1960's by Box-Jenkins (see Box *et al.*, 1994). The ARMA model states that the current value of the series Y_t depends linearly on its own previous values plus a combination of current and previous values of a white noise error term. A stationary process Y_t is called an

ARMA (p, q) process, where p and q are integers, if there exist real coefficients $\alpha_o, \alpha_{1\,,} \dots . \alpha_p$; $\beta_1 \dots \dots . \beta_q$ such that,

$$Y_t = \mu + \sum_{i=1}^{p} \alpha_i Y_{t-i} + \varepsilon_t - \sum_{j=1}^{q} \beta_j \varepsilon_{t-j} \qquad \forall t \in \boldsymbol{Z} \tag{3.18}$$

Using the backward shift operator, equation (3.18) can be expressed as:

$$\boldsymbol{\alpha(L)} Y_t = \mu + \boldsymbol{\beta(L)} \varepsilon_t \tag{3.19}$$

where ε_t is the error term which is assumed to be a white noise process; and $\alpha(L)$ and $\beta(L)$ are polynomials in L of finite order p and q, respectively, defined by:

$$\alpha(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \cdots - \alpha_P L^P \text{ and } \beta(L) = 1 - \beta_1 L - \beta_2 L^2 - \dots - \beta_q L^q$$

The response series is stable if the roots of the homogeneous characteristic equation $\alpha(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \alpha_3 L^3 - \cdots - \alpha_P L^P = 0$ lie outside the unit circle.

3.5.4. Autoregressive Integrated Moving Average (ARIMA) Process

In practice many time series are non-stationary and we cannot apply stationary AR, MA or ARMA processes directly. A series which is stationary after being differenced once is said to be integrated of order 1 (denoted by I (1)). Differencing techniques are normally used to transform a non-stationary time series into stationary by subtracting each datum in a series from its predecessor. The set of observations that correspond to the initial time period (t) when the measurement was taken is described as the series in level. If a non-stationary time series has to be differenced *d* times to make it stationary, that time series is said to be integrated of order *d* and denoted as I (d) (Pole, 1994; Weigend, 1993).

If the original the series is differenced *d* times before fitting an ARMA (p, q) process, then the model for the original undifferenced series is said to be an ARIMA(p, d, q) process. The general form of ARIMA (p, d, q) process is given by:

$$\alpha(L)(1 - L)^d Y_t = \alpha_o + \beta(L)\varepsilon_t \tag{3.20}$$

where $\alpha(L)$ and $\beta(L)$ are polynomials of order p and q as defined above.

3.6 Building ARMA Models: The Box–Jenkins Approach

Box and Jenkins (1976) introduced the first approach for estimating an ARMA models in a systematic manner. Their approach uses an iterative three-stage modeling approach.

3.6.1. Model identification

This is the way of making sure that the variables under the study are stationary, identifying seasonality in the dependent series (seasonal differencing if necessary), and using plots of the autocorrelation and partial autocorrelation functions of the time series variable to decide the tentative (if any) autoregressive or moving average components in the model.

Model selection criteria

In time series analysis there may be several adequate models that can fit a given data set. Thus, various criteria for model selection have been introduced in the literature. When there are multiple adequate models, the selection criterion is normally based on the likelihood function and the number of free parameters from the fitted model or on forecast errors calculated from out-of sample forecast.

Akaike's Information Criterion (AIC)

Akaike (1973) suggested measuring the goodness of fit for ARMA (*p, q*) by balancing the error of the fit against the number of parameters in the model. It is defined by:

$$AIC = -2\ln(L) + 2k \tag{21}$$

where L is the maximized value of the likelihood function and k is the number of (free) parameters in the model (i.e. $k = p + q + 1$). Given a set of candidate models, the preferred model is the one with the minimum AIC value. However, AIC may not result in the selection of a parsimonious model, that is, it may favor over-parameterized models. In order to remove this deficiency, Hurvich and Tsai (1989) introduced a corrected version, *AICc*, defined by:

$$AICc = -2\ln(L) + \frac{2nk}{n-k}$$

Bayesian Information Criterion (BIC)

Bayesian information criterion (Schwarz, 1978) is another criterion for model selection among a set of competing models. It is based, in part, on the likelihood function and it is closely related to the Akaike information criterion (AIC). It is defined by:

$$BIC = -2ln(L) + kln(T) \tag{3.22}$$

BIC is also called the Schwarz Information Criterion (SBIC). Various simulation studies have tended to verify that BIC does well at getting the correct number of parameters in large samples, whereas bias corrected AIC (AICc) tends to be superior in smaller samples where the relative number of parameters is large (McQuarrie and Tsai, 1998).

3.6.2. Parameter estimation

Having identified the appropriate p, d and q values, the next step is to estimate the parameters of the autoregressive and moving average terms included in the model. We can estimate the unknown parameters by ordinary least squares or by maximum likelihood (ML) methods. Both estimation procedures are based on the innovation series ε_t. The least-squares methods minimize the sum of squares of the innovation terms, while the ML method maximizes the joint probability density function of the innovation terms $\varepsilon_1, \varepsilon_2, \ldots., \varepsilon_T$.

3.6.3. Model diagnostic checking

Once we have identified and estimated the candidate ARMA models, we want to assess the adequacy of the selected models. Tests related to serial correlation and normality of residuals should be performed at this stage.

3.6.3.1. Test of serial correlation of residuals

Two types of tests of serial correlation of the residual series are applied in this work: Breusch-Godfrey LM and Ljung-Box tests.

Lagrange Multiplier (LM) Test for Serial Correlation

This test was developed by Breusch and Godfrey in 1978, and is used to test for serial correlation in the error terms. Suppose that the residuals ε_t follow an autoregressive scheme given by:

$$\varepsilon_t = \theta_1\varepsilon_{t-1} + \theta_2\varepsilon_{t-2} + \cdots + \theta_p\varepsilon_{t-p} + u_t \qquad (3.23)$$

where u_t is a white noise process. The Lagrange Multiplier (LM) test for p^{th} order serial correlation is computed first by estimating the sample residuals $\hat{\varepsilon}_t$ by ordinary least squares (OLS) and regress the current residual $\hat{\varepsilon}_t$ on the p lagged residuals. The auxiliary regression model of residuals is given by:

$$\hat{\varepsilon}_t = \theta_1\hat{\varepsilon}_{t-1} + \theta_2\hat{\varepsilon}_{t-2} + \cdots + \theta_p\hat{\varepsilon}_{t-p} + u_t \qquad (3.\ 24)$$

The null hypothesis of absence of serial correlation up to lag p is $H_0\colon \theta_1 = \theta_2 = \cdots = \theta_p = 0$.

If the usual R^2 statisistic is computed for this model, then the following asymptotic approximation can be used for the distribution of the test statistic:

$$TR^2 \sim \chi^2(p) \qquad (3.25)$$

Ljung-Box for serial correlation Test

The Ljung- Box (LB) test is a statistical test which is used to test whether there is serial correlation in the residuals or not. The null hypothesis of absence of serial correlation up to lag m is $H_0\colon \rho_1 = \rho_2 = \cdots = \rho_m = 0$. The test statistic is given by:

$$Q_{LB} = T(T+2)\sum_{k=1}^{m}\left(\frac{\hat{\rho}_k{}^2}{T-K}\right) \qquad (3.26)$$

where $\hat{\rho}_k$ is the lag *k-th* autocorrelation of the residuals of the fitted ARMA(*p, q*) model and *T* is the number of observations. Under the null hypothesis, Q_{LB} is asymptotically distributed as chi-square with degrees of freedom equal to $m-(p+q)$.

3.6.3.2. Testing Normality of the Residual

Normality tests are used to ascertain whether the residuals of the regression are normally distributed or not. The null hypothesis is that the residuals are normally distributed. Several tests for normality are available but the most commonly used test for normality of regression disturbances is due to Jarque and Bera (1981). The Jarque-Bera test statistic is given by:

$$JB = T\left[\frac{s^2}{6} + \frac{(k-3)^2}{24}\right] \qquad (3.27)$$

where T is the sample size, s and k are the sample skewness and kurtosis coefficients, respectively, which are defined by:

$$s = \frac{\hat{\mu}_3}{(\hat{\sigma}^2)^{\frac{3}{2}}} \qquad and \qquad k = \frac{\hat{\mu}_4}{(\hat{\sigma}^2)^2}$$

where $\hat{\mu}_3$ and $\hat{\mu}_4$ are the third and fourth central moments of the residuals, respectively. Under the null hypothesis of a normal distribution, this test statistic is asymptotically distributed as $\chi^2(2)$. Thus, large values of this test statistic relative to the quantiles from the $\chi^2(2)$ distribution lead to rejection of the null hypothesis.

3.7 Random Variance Models

Time series models such as autoregressive moving average models cannot capture the main stylized facts of financial series like leptokurticity, the unpredictability of returns (i.e. large/ small volatility in returns followed by large/small volatility in returns, and the existence of positive autocorrelations in the squared returns. The fact that large absolute returns tend to be followed by large absolute returns (whatever the sign of the price variations) is hardly compatible with the assumption of constant conditional variance. This phenomenon is called conditional heteroscedasticity.

The models for specific nature of financial time series (price series or log-returns, interest rates, inflation rates, etc.) are generally written in the multiplicative form:

$$\varepsilon_t = \sigma_t \quad {}_t \tag{3.28}$$

where ${}_t$ and σ_t are real processes such that σ_t is measurable with respect to a sigma-field denoted by ψ_{t-1} , η_t is an iid process with mean zero and unit variance and independent of ψ_{t-1} and ε_u for $u < t$, and σ_t is non-negative. This formulation implies that the sign of the current price variation (that is, the sign of ε_t) is that of ${}_t$ and is independent of past price variations. Moreover, if the first two conditional moments of ε_t exist, they are given by:

$$E(\varepsilon_t \quad \psi_{t-1}) = 0 \qquad \text{and} \qquad E(\varepsilon_t^2 \quad \psi_{t-1}) = \sigma_t^2 \tag{3.29}$$

The random variable σ_t is called the volatility of ε_t which is not directly observable. Different classes of volatility models can be distinguished depending on the specification adopted for σ_t :

1) Conditionally heteroscedastic (or GARCH-type) processes for which $\psi_{t-1} =$ (ε_s ; s < t) is the -field generated by the past values of ε_t. The volatility here is a deterministic function of the past values of ε_t. Processes of this class differ by the choice of a specification for this function. The standard GARCH models are characterized by a volatility specified as a linear function of the past values of ε_t^2.

2) Stochastic volatility processes for which $_{t-1}$ is the σ-field generated by $\{v_t, v_{t-1}, v_{t-2}, v_{t-3}, \ldots\}$, where ($v_t$) is a strong white noise and is independent of $_t$. In these models, volatility is a latent process. The most popular model in this class assumes that $log\ \sigma_t$ follows an autoregressive process of order one of the form:

$$log\ \sigma_t = \omega + \varphi\ log\ \sigma_{t-1} + v_t \tag{3.30}$$

where the noises v_t and $_t$ are independent of each other.

3) Switching-regime models for which $\sigma_t = \sigma$ (Δ_t , ψ_{t-1}), where (Δ_t) is a latent (unobservable) integer-valued process, independent of ($_t$). The state of the variable Δ_t is interpreted as a regime and, conditionally on this state, the volatility of $_t$ has a GARCH specification. The process (Δ_t) is generally supposed to be a finite-state Markov chain. Such models are called Markov-switching models (Christian and Zakola, 2010).

In this paper we consider the specification for σ_t given by specification (1) by considering the price return series of precious metals as a response variable and incorporating a set of explanatory variables in the conditional variance equation.

3.8. 1. RiskMetrics Models

One of the most popular volatility models in risk management framework is the RiskMetrics model which is introduced by Morgan in 1995. RiskMetrics is a set of methodologies for measuring market risk and volatility. Market risk means the potential for changes in the value of a position resulting from changes in market prices. This approach was developed so as to enable other financial institutions, corporate treasuries, and investors to estimate their market risks in a consistent and reasonable fashion (Morgan and Reuters, 1996).

RiskMetrics measures the volatility by using exponentially weighted moving average (EWMA) model that gives the heaviest weight on the immediate past observation. EWMA model gives immediate reaction to market crashes or huge changes. If the same weight is given to every data, it is hard to capture extraordinary events and effects. Therefore, EWMA is considered to be a good model to solve the problem. EWMA model assumes that the weight of the immediate past is more than old time and it assumes that assets price changes through time.

EWMA responds to volatility changes and it does assume that volatility is not constant through time. Using EWMA for modelling volatility, the equation is given by:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)y_{t-1}^2 \tag{3.31}$$

where λ (0< $\lambda < 1$) denotes the decay factor which determines how much weight is given to recent versus older observations, y_t is the return series at time t, and σ_t^2 denotes the variance at time t. This model emphasizes that the volatility at a given time $(t-1)$ is actually used as a predictor for the volatility of the next time t.

EWMA model applies weighting factors which decrease exponentially. The weighting for each older data point decreases exponentially, giving much more importance to recent observations while still not discarding older observations entirely. To understand how equation (3.31) corresponds to weights that decrease exponentially, we proceed as follows:

$$\sigma_t^2 = \lambda[\lambda\sigma_{t-2}^2 + (1-\lambda)y_{t-2}^2] + (1-\lambda)y_{t-1}^2$$

$$= \lambda^2\sigma_{t-2}^2 + \lambda(1-\lambda)y_{t-2}^2 + (1-\lambda)y_{t-1}^2$$

$$\sigma_t^2 = \lambda^2[\lambda\sigma_{t-3}^2 + (1-\lambda)y_{t-3}^2] + \lambda(1-\lambda)y_{t-2}^2 + (1-\lambda)y_{t-1}^2$$

$$= \lambda^3\sigma_{t-3}^2 + \lambda^2(1-\lambda)y_{t-3}^2 + \lambda(1-\lambda)y_{t-2}^2 + (1-\lambda)y_{t-1}^2$$

$$\vdots$$

$$\sigma_t^2 = \lambda^m\sigma_{t-m}^2 + (1-\lambda)[y_{t-1}^2 + \lambda\, y_{t-2}^2 + \lambda^2\, y_{t-3}^2 + \cdots + \lambda^{m-1}\, y_{t-m}^2]$$

$$= \lambda^m \sigma_{t-m}^2 + (1-\lambda) \sum_{j=1}^{m} \lambda^{j-1} y_{t-j}^2$$

For large m, the term $\lambda^m \sigma_{t-m}^2$ is sufficiently small to be ignored so that equation (3.31) is the same as:

$$\sigma_t^2 = (1-\lambda) \textstyle\sum_{j=1}^{m} \lambda^{j-1} y_{t-j}^2 \tag{3.32}$$

Here the weights for the y_{t-j}^2 decline at a rate of λ as we move back through time.

The EWMA approach has the attractive feature that relatively little data need to be stored. At any given time we need to remember only the current estimate of the variance rate and the most recent observation on the value of the market variable. When we get a new observation on the value of the market variable, we calculate a new time percentage change and use equation (3.31) to update our estimate of the variance rate the old estimate of the variance rate and the old value of the market variable can then be discarded.

Morgan (1996) stated that the most appropriate value for the decay factor is $\lambda = 0.94$ for one day continuously compounded returns and $\lambda = 0.97$ for one month continuously compounded returns. But in this study we estimate the decay factor by using maximum likelihood estimation.

3.8.1.1. Methods for Determining the Optimal Decay Factor

Consider the forecast of the variance of the return at time $(t+1)$. Suppose we are interested in forecasting the one-step-ahead conditional variance which is given by:

$$E(y_{t+1}^2 | \psi_t) = \sigma_{t+1|t}^2 \tag{3.34}$$

where ψ_t is the information set at time t. Then the forecast error of the variance is defined as:

$$\varepsilon_{t+1|t}(\lambda) = y_{t+1}^2 - \sigma_{t+1|t}^2(\lambda) \tag{3.35}$$

with expected value of zero, i.e. $E\left(\varepsilon_{t+1|t}(\lambda)\right) = E\left((y_{t+1}^2 | \psi_t)\right) - \sigma_{t+1|t}^2(\lambda) = 0$.

Under the assumption that the errors of the variance are conditionally normal, the objective here is to specify the joint probability density of errors given a value of the decay factor λ. The density function for the error term at time t is given by:

$$f(\varepsilon_t|\lambda) = \left(\frac{1}{\sigma_{t|t-1}(\lambda)\sqrt{2\pi}}\right) exp\left[-\left(\frac{\varepsilon_t^2}{2\sigma_{t|t-1}^2(\lambda)}\right)\right] \qquad (3.36)$$

By combining all the conditional distributions for the time history under consideration, we obtain the likelihood function given as:

$$f(\lambda|\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots, \varepsilon_T) = L = \prod_{t=1}^{T}\left(\frac{1}{\sigma_{t|t-1}(\lambda)\sqrt{2\pi}}\right) exp\left[-\left(\frac{\varepsilon_t^2}{2\sigma_{t|t-1}^2(\lambda)}\right)\right] \qquad (3.37)$$

Equation (3.37) is known as the normal likelihood function and its value depends on λ. In practice, it is often easier to work with the log-likelihood function, which is simply the natural logarithm of the likelihood function. The maximum likelihood (ML) principle stipulates that the optimal value of the decay factor λ is one which maximizes the likelihood function. This is equivalent to finding the value of λ that maximizes the log-likelihood function given by:

$$l = \sum_{t=1}^{T}\left[-\ln\,(\sigma_{t|t-1}(\lambda) - \frac{1}{2}\left(\frac{\varepsilon_t^2}{\sigma_{t|t-1}^2(\lambda)}\right)\right] \qquad (3.38)$$

Another way of choosing the optimal decay factor λ is by finding the value of λ that minimizes the Mean Square Error. Mathematically the MSE is expressed as:

$$MSE = \frac{1}{k}\sum_{i=1}^{k-1}\left[y_{t-i+1}^2 - \hat{\sigma}_{t-i+1|t-i}^2(\lambda)\right]^2 \qquad (3.39)$$

where k is the forecasting horizon.

3.8.2. Autoregressive Conditionally Heteroscedastic (ARCH) Models

Autoregressive conditionally heteroscedastic (ARCH) models were introduced by Engle (1982) and they are specifically designed to model and forecast conditional variances. Let Y_t denote a stationary time series with conditional mean of ARMA family models. The error terms $_t$ are split into a stochastic component η_t and a time dependent standard deviation σ_t characterizing a typical size of the term given by:

$$\varepsilon_t = \sigma_t \eta_t \qquad (3.40)$$

where the random variable η_t is a white noise process and ${}_t$ is iid with mean zero. To allow conditional heteroscedasticity, we assume that $\mathrm{var}({}_t|\psi_{t-1}) = \mathrm{E}(\varepsilon_t^2|\psi_{t-1}) = {}_t^2$ since $E({}_t|\psi_{t-1}) = 0$ where ψ_{t-1} denotes the information set at time $t-1$. Here ${}_t^2$ denotes the variance conditional on information at time $\mathrm{t}-1$, and is modeled in the following way:

$$\sigma_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \alpha_3\varepsilon_{t-3}^2 + \cdots + \alpha_p\varepsilon_{t-p}^2 \tag{3.41}$$

Here ${}_{t-1}^2, \dots, {}_{t-p}^2$ are the lagged squared residuals from the conditional mean equation and we impose the non-negativity restrictions $\alpha_0 > 0$, $\alpha_i \geq 0$, $i: 1,2,\dots,p$. Equation (3.41) can be rewritten as an AR representation of squared residuals as:

$$\varepsilon_t^2 = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \alpha_3\varepsilon_{t-3}^2 + \cdots + \alpha_p\varepsilon_{t-p}^2 + \delta_t \tag{3.42}$$

where ${}_t = \varepsilon_t^2 - \mathrm{E}(\varepsilon_t^2|\psi_{t-1})$ is a white noise process. The model given by equation (3.41) is known as the autoregressive conditional Heteroscedasticity (ARCH) model of Engle (1982), which is usually referred to as the ARCH (*p*) model.

The conditional variance equation of an ARCH (*p*) model with explanatory variables is given by:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}\alpha_i\varepsilon_{t-i}^2 + \boldsymbol{X_t\phi} \tag{3.43}$$

where $\boldsymbol{X_t} = (X_{1t}, X_{2t}, \dots, X_{rt})$ is a vector of explanatory variables at time t, $\boldsymbol{\phi} = (\phi_1, \phi_2, \dots, \phi_r)$ is a vector of regression coefficients that show the effect of the explanatory variables on the volatility of the price return series under consideration, ${}_0$ shows long term volatility and $\alpha_1, \alpha_2, \dots, \alpha_p$ indicate the effect of past shocks irrespective of their sign.

Before estimating a full ARCH model for a financial time series, it is usually a good practice to test for the presence of ARCH effects in the residuals. To test for the presence of heteroscedasticity in the residuals, the Lagrange Multiplier (LM) test and the Ljung-Box test can be applied. If the there are no ARCH effects in the residuals, then the ARCH model is unnecessary and misspecified.

3.8.2.1 Testing for ARCH Effect

a) Lagrange Multiplier Test

In order to test the presence of ARCH effects in the residuals we can use the AR representation of squared residuals in the following way. Based on equation (3.42), we construct an auxiliary regression:

$$\hat{\boldsymbol{\varepsilon}}_t^2 = \alpha_0 + \alpha_1\hat{\varepsilon}_{t-1}^2 + \alpha_2\hat{\varepsilon}_{t-2}^2 + \alpha_3\hat{\varepsilon}_{t-3}^2 + \cdots + \alpha_p\hat{\varepsilon}_{t-p}^2 + \delta_t \qquad (3.44)$$

where $\hat{\varepsilon}_t$ are the residuals from the mean equation. The significance of the parameters α_i would indicate the presence of conditional volatility. Under the null hypothesis that there is no ARCH effect, i.e. $\mathrm{H}_0\colon \alpha_1 = \alpha_2 = \cdots . \alpha_p = 0$, the test statistic:

$$LM = TR^2 \qquad (3.45)$$

is distributed as χ^2 with p degrees of freedom , where T is the sample size and R^2 is the coefficient of determination computed from the regression equation (3.44).

b) Ljung-Box Test for the Squared Residuals

This test was developed by Box and Pierce (1970) and modified by Ljung and Box (1978) and tests the joint significances of serial correlation in the standardized and squared standardized residuals for the first *m* lags instead of testing individual significance. To test for ARCH effect, we apply the Ljung-Box test to the squared residuals of the model. The null corresponds to insignificance of correlation coefficients with lags up to *m*, and the test statistic given by:

$$LB = T(T+2)\sum_{i=1}^{m}\frac{\hat{\rho}_i{}^2}{T-i} \qquad (3.46)$$

where T denotes the number of observations under consideration, and $\hat{\rho}_i$ is the sample autocorrelation for the squared residuals at lag i. The test statistic given by equation (3.46) is distributed as $\chi_{(m)}^2$ under the null hypothesis.

If the LM and LB test statistics for ARCH effects are significant, one could proceed to estimate an ARCH model and obtain estimates of the time varying volatility σ_t based on past history. However, in practice it is often found that a large number of lags m , and thus a large number of parameters, are required to obtain a good model fit. Moreover, ARCH model assumes that positive and negative shocks have the same effects on volatility because it depends on the square of the previous shocks. In practice, it is well known that the price of a financial asset responds differently to positive and negative shocks. Due to these reasons a more general framework was proposed by Bollerslev (1986) and Taylor (1986).

3.8.3. Generalized Autoregressive Conditionally Heteroscedastic (GARCH) Models

GARCH is an extension of an ARCH model of Engle (1982) by Bollerslev (1986). GARCH is a mechanism that includes past variances in the explanation of future variances. More specifically, GARCH is a time series modeling technique that uses past errors and past variance forecasts to forecast future variances (Akgiray, 1989; Bollerslev *et al.*, 1992).

GARCH modeling takes into account excess kurtosis (fat tail behavior) and volatility clustering. It provides accurate forecasts of variances and covariance of asset returns through its ability to model time-varying conditional variances. As a consequence, we can apply GARCH models to option pricing, inflation rate, and foreign exchange rate among others.

The assumptions of the GARCH family models are:

1. $E(\ _{t}|\ _{t-1})$=0.
2. Heteroscedasticity of the errors terms, i.e. $\mathrm{var}(\ _{t}|\ _{t-1}) = \sigma_t^2$.
3. No severe multicollinearity among the explanatory variables.
4. The error terms $_{t}$ are assumed to follow- normal, student – t or generalized error distribution (GED) with mean zero and variance $^{2}_{t}$.
5. The error terms $_{t}$ are serially uncorrelated but not independent.

The symmetric GARCH (*p, q*) model is given by:

$$Y_t = \mu + \varepsilon_t \ ,$$

$$\varepsilon_t = \sigma_t \eta_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \, \sigma_{t-j}^2 \quad , \quad t \epsilon Z \qquad (3.47)$$

where $\mathrm{var}(\ _{t}|\ _{t-1}) = \mathrm{E}(\ ^{2}_{t}|\ _{t-1}) = \ ^{2}_{t}$ and $E(\ _{t}|\ _{t-1}) = 0$. We impose the restrictions $\alpha_0 > 0$, $\alpha_i \geq 0$ and $\beta_j \geq 0$ for $i = 1,2,..,p$ and $j = 1,2,\ldots,q$ to ensure that the conditional variance is non-negative, and $\left(\sum_{i=1}^{p}\alpha_i + \sum_{j=1}^{q}\beta_j < 1\right)$ is necessary and sufficient condition for the stability of the conditional variance equation.

Equation (3.47) can be written in a more compact way using the backward shift (or lag) operator L as:

$$\beta(L)\sigma_t^2 = \alpha_0 + \alpha(L)\varepsilon_t^2 \tag{3.48}$$

where (L) and (L) are polynomials in L of finite order p and q, respectively, given by:

$$\alpha(L) = \alpha_1\mathrm{L} + \alpha_2\mathrm{L}^2 + \cdots + \alpha_\mathrm{p}\mathrm{L}^\mathrm{p} = \sum_{i=1}^{p}\alpha_i L^i$$

$$\beta(L) = 1 - \beta_1\mathrm{L} - \beta_2\mathrm{L}^2 - \cdots - \beta_\mathrm{q}\mathrm{L}^\mathrm{q} = \left(1 - \sum_{j=1}^{q}\beta_j L^j\right)$$

By definition, the innovation of the process in (3.47) is the variable which is defined by $\delta_t^2 = \varepsilon_t^2 - \sigma_t^2$. Replacing σ_{t-j}^2 by $\varepsilon_{t-j}^2 - \delta_{t-j}^2$ in equation (3.47), we get the representation:

$$\varepsilon_t^2 = \alpha_0 + \sum_{i=1}^{r}\varpi_i\,\varepsilon_{t-i}^2 + \delta_t^2 - \sum_{j=1}^{p}\beta_j\delta_{t-j}^2 \tag{3.49}$$

where $\varpi_i = (\alpha_i + \beta_\mathrm{i})$, $r = max(p, q)$, with the convention $\alpha_i = 0$ $(\beta_j = 0)$ if $i > q$ $(j > p)$. This equation has the linear structure of an ARMA model, allowing for simple computation of the linear predictions. Under additional assumptions (implying the second-order stationarity of ε_t^2), we can state that if (ε_t) is GARCH (p, q), then $(\varepsilon_\mathrm{t}^2)$ is an ARMA(r, p) process. The ARMA representation will be useful for the estimation and identification of GARCH processes (Christian and Zakola, 2010). One can identify the orders of the GARCH model using the correlogram of the squared residuals. They will coincide with ARMA orders of the squared residuals of the time series (Akgiray, 1989; Green, 1993).

The conditional variance equation of a GARCH (p, q) model with explanatory variables is given by:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}\alpha_i\varepsilon_{t-i}^2 + \sum_{j=1}^{q}\beta_j\,\sigma_{t-j}^2 + \mathbf{X_t\phi} \qquad , t\epsilon Z \tag{3.50}$$

where $_0$ shows long term volatility; $_1$, $_2$, …, $_\mathrm{p}$ indicate the effect of past shocks and $_1$, $_2$, …, $_\mathrm{q}$ show the influence of past volatility on the current volatility.

3.7.4 GARCH-in-Mean (GARCH-M) Model

In financial investment, high risk is often expected to lead to high returns. Most models used in finance suppose that investors should be rewarded for taking additional risk by obtaining a higher return, which means the return of a security may be determined by its risk. Engle, Lilien and Robins (1987) suggested an ARCH-M specification, where the conditional variance of asset returns enters into the conditional mean equation. Since GARCH models are now considerably more popular than ARCH, it is more common to estimate a GARCH-M model. An example of a GARCH-M model is given by the specification:

$$\left.\begin{array}{c} Y_t = \mu + \delta\sigma_{t-1} + \varepsilon_t \qquad \varepsilon_t|\psi_{t-1} \sim D(0, \sigma_t^2) \\ \sigma_t^2 = \alpha_0 + \alpha_1\, \varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \end{array}\right\} \tag{3.51}$$

If δ is positive and statistically significant then increased risk, given by an increase in the conditional variance, leads to a rise in the mean return, and δ can be interpreted as a risk premium. In some empirical applications, the conditional variance term, σ_{t-1}^2 appears directly in the conditional mean equation, rather than in square root form σ_{t-1}.

A more general form for the GARCH-M model by incorporating exogenous explanatory variable is given by:

$$\left.\begin{array}{c} Y_t = \mu + \delta\sigma_{t-1} + \varepsilon_t \\ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i\, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^2 + \mathbf{X}_\mathbf{t}\boldsymbol{\phi} \end{array}\right\} \tag{3.52}$$

3.7.5 Asymmetric GARCH Models

One weakness of ARCH and GARCH models is that they account for the volatility reactions for positive and negative changes (shocks) in a symmetric way. A solution was given by the asymmetric models which are capable of capturing the asymmetric features of the series (Green, 1993; Kiohos and Sariannidis, 2010).

An interesting feature of asset price is that bad news seems to have a more pronounced effect on volatility than does good news. For many stocks, there is strong negative correlation between the current return and the future volatility. The tendency for volatility to decline when returns rise and to rise when returns fall is often called the leverage effect (Enders, 2004). The main drawback of symmetric GARCH models is that the conditional variance is unable to respond asymmetrically to rises and falls in ε_t. In the GARCH (p, q) model, the conditional variance is a function of past conditional variances and squared innovations; therefore, sign of returns cannot affect the volatilities (Knight and Satchell, 2007). The symmetric GARCH models described above cannot account for the leverage effects observed in returns, and consequently, a number of models have been introduced to deal with this phenomenon. These models are called asymmetric models. This paper uses EGAECH and TGARCH models for capturing the asymmetric phenomena.

3.7.5.1 The Exponential GARCH (EGARCH) Model

This model captures asymmetric responses of the time-varying variance to shocks and, at the same time, ensures that the variance is always positive. To overcome some weaknesses of the GARCH model in handling financial time series, Nelson (1991) proposes the exponential GARCH (EGARCH) model. EGARCH (1, 1) is specified as:

$$\boldsymbol{ln(\sigma_t^2) = \alpha_0 + \alpha_1 \left\{ \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right\} + \beta_1 ln(\sigma_{t-1}^2)} \quad (3.53)$$

where $\boldsymbol{\gamma}$ is the asymmetric response parameter or leverage parameter. The sign of $\boldsymbol{\gamma}$ is expected to be positive in most empirical cases so that a negative shock increases future volatility or uncertainty while a positive shock eases the effect on future uncertainty. In macroeconomic analysis, financial markets and corporate finance, a negative shock usually implies bad news, leading to a more uncertain future. Higher order EGARCH models can be specified in a similar way. EGARCH (p, q) model is defined as:

$$\boldsymbol{ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left\{ \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| - \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right\} + \sum_{j=1}^{q} \beta_j ln(\sigma_{t-j}^2)} \quad (3.54)$$

The parameter γ_i thus signifies the leverage effect of ε_{t-i}. Bad news can have a larger impact on volatility; again we expect γ_i to be negative in real applications. An EGARCH (p, q) variance equation with explanatory variables is given by:

$$\boldsymbol{ln(\sigma_t^2) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \left\{ \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right| - \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}} \right\} + \sum_{j=1}^{q} \beta_j ln(\sigma_{t-j}^2) + X_t \phi} \quad (3.55)$$

3.7.5.2 The Threshold GARCH (TGARCH) Model

Another volatility model commonly used to handle leverage effects is the threshold GARCH (TGARCH) model which was proposed by Glosten, Jagannathan, Runkle (1993) and Zakoian (1994). The general specification of the conditional variance equation using the TGARCH (*p, q*) model is given by:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \gamma_i s_{t-i} \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{3.56}$$

where s_{t-i} is a dummy variable (indicator function) defined by:

$$\mathrm{s_{t-i}} = \begin{cases} 1 \ \text{ if } \ {}_{\mathrm{t-i}} < 0, \mathit{bad} \ \text{ news} \\ 0 \ \text{ if } \ {}_{\mathrm{t-i}} \geq 0, \text{good news} \end{cases}$$

α_i, γ_i and β_j are parameters satisfying the conditions for non-negativity of σ_t^2, that is, $\alpha_0 > 0$, $\alpha_i > 0$, and $(\alpha_i + \gamma_i) \geq 0$. Depending upon whether ε_{t-i} is above or below the threshold value, ε_{t-i}^2 has a different effect on the conditional variance σ_t^2. In case of negative shocks, the impact of the shocks on σ_t^2 is $(\gamma_i + \alpha_i)\varepsilon_{t-i}^2$, whereas this quantity is $\alpha_i \varepsilon_{t-i}^2$ for positive shocks, that is, negative shocks have higher impact on the conditional variance than positive shocks. The model uses zero as its threshold to separate the impacts of past shocks. The conditional variance equation of the TGARCH (p, q) model in the presence of explanatory variables is given by:

$$\sigma_t^2 = \alpha_0 + \textstyle\sum_{i=1}^{p} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \gamma_i s_{t-i} \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \mathbf{X_t \phi} \tag{3.57}$$

3.8 Parameter Estimation of ARCH/GARCH models

Since GARCH model is no longer of the usual linear form, OLS cannot be used for model estimation. There are a variety of reasons for this, but the simplest and most fundamental is that ordinary least square (OLS) minimizes the residual sum of squares (RSS). The RSS depends only on the parameters in the conditional mean equation, and not on the conditional variance, and hence RSS minimization is no longer an appropriate objective (Brook, 2008).

In order to estimate the unknown parameters of the GARCH family models, the maximum likelihood (ML) method is employed with various distributional assumptions for the error terms.

Essentially, this method works by finding the most likely values of the parameters given the actual data. More specifically, a log-likelihood function (L) is formed and the values of the parameters that maximize it are sought. Maximum likelihood estimation can be employed to find parameter values for both linear and non-linear models.

In this study, three distributions are considered: namely the normal, student-t and generalized error distribution. The log-likelihood functions under different distributional assumptions are given below.

i. If $_t \sim N(0, \sigma_t^2)$, the log-likelihood function is given as:

$$\log(L) = \sum_{t=1}^{T}\left(-\frac{1}{2}\log(2\pi) - \frac{1}{2}\log(\sigma_t^2) - \frac{\varepsilon_t^2}{2\sigma_t^2}\right) \tag{3.58}$$

ii. If $\varepsilon_t \sim t(0, \sigma_t^2, \tau)$, where τ is the number of degrees of freedom , then the log-likelihood function is given as:

$$\log(L) = \sum_{t=1}^{T}\left[\log\left(\frac{\Gamma\left(\frac{+1}{2}\right)}{\Gamma\left(\frac{}{2}\right)}\right) - \frac{1}{2}\log\left(\frac{\pi(\tau-2)}{\sigma_t^2}\right) - \left(\frac{\tau+1}{2}\right)\log\left(1 + \frac{\varepsilon_t^2}{(\ -2)\sigma_t^2}\right)\right] \tag{3.59}$$

where $\Gamma(.)$ is the gamma function, and $\tau > 2$.

iii. Under the assumption that the errors follow independent GED with mean zero, variance $\sigma_t^2 > 0$, degree of freedom (shape parameter) $v > 0$ and skewness parameter $\omega, -1 < \omega < 1$, that is, $_t \sim GED(0, \sigma_t^2, \omega)$, then the log-likelihood function given as:

$$\log(L) = \sum_{t=1}^{T}\left[\log\left(\frac{v}{\omega}\right) - \log\left(\Gamma\left(\frac{1}{v}\right)\right) - \left(\frac{1}{2}\right)\log(\sigma_t^2) - \left(1 + \frac{1}{v}\right)\log(2) - \frac{1}{2}\left|\frac{t}{\omega\sigma_t}\right|^{v}\right] \tag{3.60}$$

For more description of the above log likelihood functions, suppose that values of $\{\varepsilon_t\}$ are drawn from a normal distribution having a mean of zero and conditional variance σ_t^2 . From a standard distribution theory, the likelihood function using T independent observations is given by:

$$l = \prod_{t=1}^{T}\frac{1}{\sqrt{2\pi\sigma_t^2}}e^{\left(-\frac{\varepsilon_t^2}{2\sigma_t^2}\right)} = \prod_{t=1}^{T}\frac{1}{\sqrt{2\pi\sigma_t^2}}e^{\left(-\frac{\left(Y_t - \mu - \sum_{i=1}^{p}\alpha_i Y_{t-i} - \sum_{j=1}^{q}\beta_j\varepsilon_{t-j}\right)^2}{2\sigma_t^2}\right)} \tag{3. 61}$$

The log likelihood function becomes:

$$lnL = -\frac{T}{2} ln2\pi - \frac{1}{2}\sum_{t=1}^{T} \sigma_t^2 - \frac{1}{2}\sum_{t=1}^{T} \frac{\left(Y_t - \mu - \sum_{i=1}^{p} \alpha_i Y_{t-i} - \sum_{j=1}^{q} \beta_j \varepsilon_{t-j}\right)^2}{\sigma_t^2} \quad (3.62)$$

Unfortunately, maximizing the log likelihood function for a model with time-varying variances is trickier than in the homoscedastic case. Analytical derivatives of the log likelihood function in (3.62) with respect to the parameters have been developed, but only in the context of the simplest of GARCH specifications. Moreover, the resulting formulae are complex, and thus, a numerical procedure is often used instead to maximize the log-likelihood function. Essentially, all methods work by ‘searching’ over the parameter-space until the values of the parameters that maximize the log-likelihood function are found (Brook, 2008).

3.9 Model Adequacy Checking

For a properly specified GARCH model, the standardized residuals:

$$\omega_t = \frac{\varepsilon_t}{\sigma_t} \quad , t = 1,2,3 \ldots\ldots, T \quad (3.63)$$

form a sequence of identically and independently distributed (iid) random variables. Therefore, one can check the adequacy of a fitted GARCH model by examining the series $\{\omega_t\}$. The following methods are used to check the adequacy of the fitted model in this study.

a) The ACF and PACF of the standardized residuals: If the model is adequate, the ACF and PACF of squared standardized residuals should be indicative of a white noise process.
b) The standardized residuals should be identically and independently distributed as standard normal even if student-t and generalized error distribution (GED) are assumed (Tsay, 2010). This can be checked through the Jarque-Bera test.
c) The Ljung-Box test is one of the most widely used lack-of-fit tests, that is, a test for the appropriateness of the fitted model. The Ljung-Box test statistic uses the Q-statistic to test whether there is a group of significant *k* autocorrelations, to test whether the mean model is appropriately specified and to test for remaining ARCH effects under the null hypothesis that there is no autocorrelation among *m* lags of standardized residuals and squared standardized residuals for mean and GARCH specification, respectively. Thus, if the statistic *Q* at all lags is insignificant, it indicates the absence of autocorrelation in the residuals, and this is evidence that the model selected fits the data well.

3.10 Forecasting using ARCH/GARCH Family models

Once an ARCH/GARCH model is fitted and satisfies all the diagnostic tests, then we can use it to forecast the volatility of the series.

Forecasting Volatility Using ARCH Model

The main use of the ARCH model is to predict the future conditional variances. Consider an ARCH (*p*) model given by equation (3.41). The 1-step ahead forecast of σ_t^2 is given by:

$$\hat{\sigma}_t^2(1) = \hat{\alpha}_0 + \hat{\alpha}_1\varepsilon_t^2 + \hat{\alpha}_2\varepsilon_{t-1}^2 + \cdots + \hat{\alpha}_p\varepsilon_{t+1-p}^2 \tag{3.64}$$

The $2-step$ ahead forecast is given by:

$$\hat{\sigma}_t^2(2) = \hat{\alpha}_0 + \hat{\alpha}_1\hat{\sigma}_t^2(1) + \hat{\alpha}_2\varepsilon_t^2 + \cdots + \hat{\alpha}_p\varepsilon_{t+2-p}^2 \tag{3.65}$$

and the $k-step$ ahead forecast for σ_t^2 is given by:

$$\hat{\sigma}_t^2(k) = \hat{\alpha}_0 + \sum_{i=1}^{p}\hat{\alpha}_i\hat{\sigma}_t^2(k-i) \tag{3.66}$$

where $\hat{\sigma}_t^2(k-i) = \varepsilon_{t+k-i}^2$ if $(k-i) \leq 0$.

Forecasting Volatility Using GARCH Model

Forecasts using GARCH model can be obtained using the same recursive procedures similar to that of an ARCH model by including lagged values of σ_t^2. Consider GARCH (1, 1) model given by:

$$\sigma_{t+1}^2 = \alpha_0 + \alpha_1\varepsilon_t^2 + \beta_1\sigma_t^2 \tag{3.67}$$

where ε_t and σ_t^2 are known at the time $(t+1)$. Assuming the forecast origin is t, the $1-step$ ahead forecast is given by:

$$\hat{\sigma}_t^2(1) = \hat{\alpha}_0 + \hat{\alpha}_1\varepsilon_t^2 + \hat{\beta}_1\sigma_t^2 \tag{3.68}$$

For multistep ahead forecasts, consider $\varepsilon_t^2 = \sigma_t^2\eta_t^2$ from equation (3.40), and rewrite equation (3.67) as:

$$\hat{\sigma}_{t+1}^2 = \hat{\alpha}_0 + (\hat{\alpha}_1 + \hat{\beta}_1)\sigma_t^2 + \hat{\alpha}_1\sigma_t^2(\eta_t^2 - 1) \tag{3.69}$$

Since $E(\eta_{t+1}^2 - 1) = 0$, the 2-step ahead volatility forecast at the forecast origin t satisfies the equation:

$$\hat{\sigma}_t^2(2) = \hat{\alpha}_0 + (\hat{\alpha}_1 + \hat{\beta}_1)\hat{\sigma}_t^2(1) \tag{3.70}$$

Therefore, by considering the forecasting horizon k, we have the general forecast equation given by:

$$\hat{\sigma}_t^2(k) = \hat{\alpha}_0 + (\hat{\alpha}_1 + \hat{\beta}_1)\hat{\sigma}_t^2(k-1), \quad k > 1 \tag{3.71}$$

The same recursive procedures like GARCH (1, 1) models are applied for GARCH models of higher orders allowing us to compute multistep ahead forecasts.

Forecasting evaluation and accuracy criteria

Evaluating the performance of different forecasting models plays a very important role in choosing the most accurate models. There are several criteria for assessing the predictive accuracy of an ARCH-GARCH family model. The most widely used statistical evaluation measures are Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE) and Theil's inequality Coefficient (U). These are applied to measure the forecasting accuracy of the fitted model in this study. Their formal expressions are given below:

$$MAE(k) = \frac{1}{T}\sum_{t=1}^{T}\left|\sigma_{t+k} - \sqrt{\hat{\sigma}_t^2(k)}\right| \tag{3.72}$$

$$RMSE(k) = \sqrt{\frac{\sum_{t=1}^{T}\left(\sigma_{t+K} - \hat{\sigma}_t(k)\right)^2}{T}} \tag{3.73}$$

$$MAPE(k) = \frac{100}{T}\sum_{t=1}^{T}\left|\left(\frac{\sigma_{t+k} - \hat{\sigma}_t(k)}{\sigma_{t+k}}\right)\right| \tag{3.74}$$

$$U(k) = \frac{\sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\sigma_{t+k} - \hat{\sigma}_t(k)\right)^2}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T}\hat{\sigma}_t^2(k)}\ \sqrt{\frac{1}{T}\sum_{t=1}^{T}\sigma_{t+k}^2}} \tag{3.75}$$

where σ_t is the conditional standard deviation obtained from the fitted model, $\hat{\sigma}_t$ is the forecast standard deviation and k is forecast horizon. U is scale invariant, and lies between zero and one. If U = 0, then $\sigma_t = \hat{\sigma}_t$ indicating a perfect fit; if U = 1, the prediction performance is not good. In general, the smaller the error statistic is, the better the forecasting ability of that model under consideration.

CHAPTER 4

RESULTS AND DISCUSSIONS

The objective of this study was to model and forecast the volatility dynamics in precious metals price in the Ethiopian market by using GARCH and RiskMetrics volatility models. To undertake these tasks, R 3.1.0 software for EWMA analysis and Eviews 5 software for symmetric and asymmetric GARCH analysis have been employed.

4.1 Descriptive Statistics

The data used in this study were the monthly prices of gold and silver (in birr per gram) in the Ethiopian market for the period from January 1998 through January 2014. The logarithmic return series $Y_t = \ln(p_t/p_{t-1})$ were computed from the monthly price series p_t to measure price volatility. Summary statistics are displayed in Table 4.1 below.

Table 4.1: Summary statistics for monthly prices of gold and silver (price per gram in birr) and their return series in the Ethiopian market during the study period.

Statistic	Gold price	Silver price	Return series for gold	Return series for silver
Mean	309.9607	35.57731	0.014453	0.011810
Median	151.1511	29.13362	0.013842	0.003608
Maximum	1018.100	109.1157	0.230761	0.385227
Minimum	47.14667	6.086868	-0.170060	-0.436659
Std. Dev.	313.3813	25.52546	0.057612	0.112231
Skewness	1.119337	1.209819	**0.376187**	**-0.293167**
Kurtosis	2.682689	3.437361	**5.627439**	**5.326049**
Jarque-Bera	41.11180	48.61935	59.75602	46.03432
Probability	0.00000	0.00000	0.00000	0.00000
Observations	193	193	192	192

As can be seen in Table 4.1 the monthly average price of gold and silver were about 309.96 and 35.57 birr with minimum prices of 47.14 and 6.08 birr and maximum prices of 1018.10 and 109.11 birr, respectively. The return series display positive skewness of about 0.3761 for gold and negative skewness of about -0.293167 for silver. The kurtosis coefficient of gold and silver price return series were about 5.627 and 5.32604, respectively, meaning that the return series are highly leptokurtic. This phenomenon has been widely known in the literature of return series. Likewise, the Jarque-Bera (JB) test also confirms that the null hypothesis of normality for the monthly return series should be rejected at the 1% level of significance. The rejection of the hypothesis of normality of gold and silver return series might be due to the existence of excess kurtosis.

Figure 1 shows the plot of the monthly price trend of precious metals under consideration. It can be observed that monthly prices of gold and silver show an increasing trend over the study period.

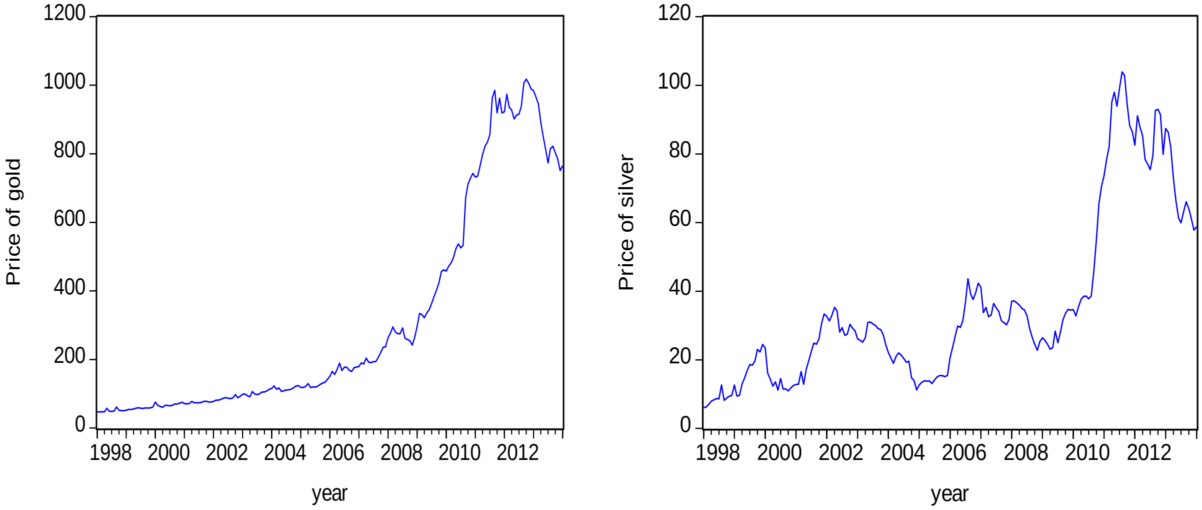

Figure 1: The mothly price trend of gold and silver over the study period.

The absolute return series were constructed for each of the prices in order to examine the presence of volatility in the series. As can be seen from Figure 2, periods of high volatility are observed for both of the series under consideration.

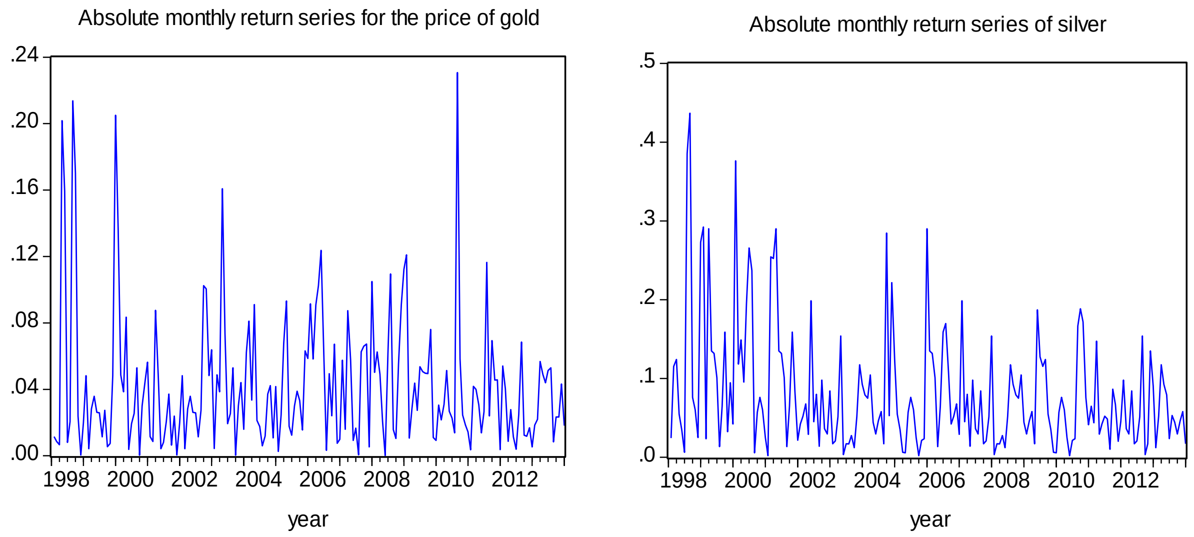

Figure 2: The absolute monthly return series for the price of gold and silver.

4.2. Unit root test for non-stationarity

The time series under consideration should be checked for stationarity before one can attempt to fit a suitable model. That is, variables have to be tested for the presence of unit root(s) and the order of integration for each series should be determined. The unit root tests first impose the null hypothesis that the series has a unit root problem, versus the alternative hypothesis that the series is stationary. In this study, Augmented Dickey-Fuller test (ADF) and Phillip Perron (PP) tests are used to check the stationarity of the monthly natural log return series of gold and silver prices and explanatory variables.

Table 4.3 summarizes the unit root test results for the price return series and explanatory variables under consideration. As we can see from the table, the null hypothesis of unit root would not be rejected for all explanatory variables in both the ADF and PP tests. On the other hand, for the price return series of gold and silver, the null hypothesis of unit root is rejected at the 1% level of significance indicating that both of the price return series are stationary.

Since both of unit root tests reject the stationarity of the explanatory variables in levels, we take first differences of the series and test if the unit root problem still presents. As can be seen from Table 4.4, the null hypothesis of unit root is rejected at the 1% level of significance for all explanatory variables by both ADF and PP tests. Thus, all explanatory variables are integrated of order one (I (1)).

Table 4.3: Unit root tests for the series (at level).

Variables	ADF Test		PP Test		Critical values		
	t -Statistic	P -value	t- Statistic	P - value	1%	5%	10%
Price of crude oil	-1.512	0.525	-1.282	0.637	-3.464	-2.876	-2.575
Exchange rate	1.93213	0.999	1.7948	0.999			
general inflation rate	1.599841	0.999	2.1324	0.999			
Inflation rate of food items	0.66696	0.991	1.2221	0.998			
Inflation rate of nonfood items	-0.26015	0.9913	-0.3478	0.988			
Saving interest rate	-1.83399	0.363	-1.837	0.361			
Price return series of gold	-15.8167	**0.000**	-15.92	**0.000**			
Price return series of silver	-8.30578	**0.000**	-13.99	**0.000**			

Table 4.4: The ADF and PP unit root tests for the first differenced explanatory variables

Variable	**ADF test**		**PP test**	
	t-statistic	P- value	t-statistic	P- value
Price of crude oil	-9.425757	0.0000	-9.408736	0.0000
Exchange rate	-12.01914	0.0000	-12.01914	0.0000
general inflation rate	-5.120955	0.0000	-8.648473	0.0000
Inflation rate of food items	-6.107370	0.0000	-9.921000	0.0000
Inflation rate of nonfood items	-3.727101	0.0044	-10.86181	0.0000
Saving interest rate	-13.76724	0.0000	-13.76724	0.0000

4.3. Specification of the Conditional Mean Equation

In order to model the volatility of the return series, we need first to specify their conditional mean equation. The return for current time will depend on returns in previous periods (autoregressive component) and the error terms in current and previous periods (moving average component). To specify the conditional mean equation for the series, comparison of various AR (p), MA (q) and ARMA (p, q) models are performed and the one with smallest information criteria is selected.

4.3.1 Model identification

In the specification of the mean equation, the sample ACF and PACF plots of the stationary series can be used to tentatively identify the order of autoregressive terms and/or moving average terms (Green, 1993). In most applications, lower order ARMA models are often considered. In this study, the fifteen combinations of AR (0-3) and MA (0-3) were considered, since the return series show insignificant spikes for most of the lags (see Figures B2 and B3 in Annex B).

Table A3 (in Annex A) presents AIC and SBIC statistics for optimal order selection for each combination of the conditional mean models across different lag specifications. Models with no serial correlation in the residuals are presented in Table 4.5.

4.3.2 Parameter Estimation for the ARMA Model

The parameter estimates for Box-Jenkins models are usually obtained by maximum likelihood method, which is asymptotically correct for any time series (Brockwell and Davis, 1996). Hence, we use maximum likelihood estimation method for the monthly return series of precious metals price to estimate the parameters. The results are summarized in Table 4.5 below.

The optimal lag length was then selected based on AIC and BIC. As can be seen in Table 4.5, the ARMA (0, 1) has the smallest SBIC and ARMA (2, 2) has the smallest AIC. Here ARMA (0, 1) model is selected as the mean equation for the price return series of gold since AIC may favor over-parameterized models than SBIC. On the other hand, ARMA (1, 3) model is selected as the mean equation for the price return series of silver since it has the smallest AIC and SBIC.

Table 4.5: Parameter estimates of competing ARMA models with information criteria.

	Model	Parameter	Coefficients	Std. error	t-statistic	P - value	Information criteria	
							AIC	SBIC
Gold	ARMA(2,2)	μ	0.014645	0.004048	3.617973	0.0004	-2.88829	-2.80285
		α_1	0.672981	0.013788	48.80785	0.0000		
		α_2	-0.968801	0.014650	-66.12835	0.0000		
		β_1	-0.694506	0.008410	-82.58130	0.0000		
		β_2	0.981544	0.012771	76.85848	0.0000		
	ARMA(0,2)	μ	0.014518	0.003231	4.493371	0.0000	-2.872846	-2.82195
		β_1	-0.156251	0.072616	-2.151750	0.0327		
		β_2	-0.061174	0.072961	-0.838444	0.4028		
	ARMA(1, 0)	μ	0.014570	0.003642	4.000165	0.0001	-2.86978	-2.83572
		α_1	-0.138758	0.071997	-1.927267	0.0554		
	ARMA(0,1)	μ	0.014487	0.003425	4.229788	0.0000	**-2.878408**	**-2.84447**
		β_1	-0.169516	0.071516	-2.370318	0.0188		
Silver	ARMA(1,1)	μ	0.011834	0.008006	1.478138	0.1410	-1.516185	-1.46051
		α_1	-0.893660	0.170867	-5.230149	0.0000		
		β_1	0.863399	0.194319	4.443214	0.0000		
	ARMA(0,3)	μ	0.011818	0.009060	1.304482	0.1937	-1.529673	-1.46108
		β_1	-0.010978	0.072843	-0.150702	0.8804		
		β_2	0.187898	0.071573	2.625254	0.0094		
		β_3	-0.049687	0.072936	-0.681248	0.4966		
	ARMA(1,3)	μ	0.007302	0.003281	2.225434	0.0273	**-1.546252**	**-1.46111**
		α_1	0.949876	0.029223	32.50395	0.0000		
		β_1	-0.969116	0.076887	-12.60441	0.0000		
		β_2	0.158534	0.099622	1.591355	0.1132		
		β_3	-0.186329	0.072526	-2.569123	0.0110		

4.3.3 Model adequacy checking

Before we consider the fitted model as a better fit and interpret its findings, it is essential to check whether the model is correctly specified, that is, whether the model assumptions are supported by the data. If some key model assumptions seem to be violated, then a new model should be specified until it provides an adequate fit to the data.

4.3.3.1 Test of serial correlation in the residuals

The presence of serial correlation in the residuals was tested using the Lagrange Multiplier (LM) and Ljung-Box tests for each of the tentatively selected ARMA models: ARMA (0, 1) and ARMA (1, 3) models for the conditional mean in the return series of gold and silver, respectively. The null hypothesis asserts that there is no serial correlation in the residual series up to lag 3. The results of this examination are summarized in Table 4.6.

Table **4.6:** Summary result for Breusch-Godfrey Serial Correlation LM Test of fitted models.

		Gold price return series			Silver price return series		
Statistic	lag	1	2	3	1	2	3
F-statistic		1.267203 (0.261719)	0.631646 (0.53284)	0.539018 (0.539018)	0.946195 (0.331960)	0.476227 (0.621886)	2.481865 (0.062425)
χ^2 statistic		1.278699 (0.258141)	1.281513 (0.526894)	1.646010 (0.649003)	0.851700 (0.356072)	0.863393 (0.649406)	7.351168 (0.061508)

Note: values inside the bracket are p-values.

The Breusch–Godfrey serial correlation LM test results in Table 4.6 provide evidence that there is no serial correlation in the residuals of the mean equation. Moreover, the Ljung-Box test (Tables A4 and A5 of Annex A) indicates that there is no significant serial correlation for lags up to 36 for both of the series under consideration. Hence, there is no significant serial correlation in the residuals.

4.3.3.2 Test of normality of the residuals

To investigate whether the residuals of the fitted model (mean equation) are normally distributed, the Jarque-Bera test has been applied. The results are reported in Table 4.7 and Figures B7 and B8 (Annex B).

We can see from Table 4.7 that the Jarque-Bera statistic is not significant, and hence, there is no significant evidence to reject the null hypothesis of normality. This indicates that the residuals of the fitted model are normally distributed for both of the series under consideration.

Table 4.7: Normality test of residuals from the mean equation.

Variable	Skweness	Kurtosis	Jarque-Bera Statistic	p-value
Price of gold return series	0.202164	3.641932	4.556498	0.102463
Price of sliver return series	0.139068	3.479716	2.447089	0.294186

4.4 Test for ARCH Effects

One of the most important issues before applying the exponential weighted moving average (EWMA) and generalized autoregressive conditional heteroscedasticity (GARCH) models is to first examine the residuals of the price return series of precious metals for evidence of heteroscedasticity. There are different approaches for testing heteroscedasticity (ARCH effect): the Lagrange Multiplier (LM) test proposed by Engle (1982) and Ljung-Box (1978) test for squared residuals.

Table 4.8: ARCH effect test using LM test for squared residuals of the fitted models.

	Gold price return series			Silver price return series		
	F-statistic	Chi-squared statistic (χ^2)	SBIC	F-statistic	Chi-squared statistic (χ^2)	SBIC
ARCH(1)	11.91151 (0.000688)	11.32388 (0.000765)	-7.22852	12.31028 (0.000564)	11.67665 (0.000633)	-4.50069
ARCH(2)	6.583388 (0.001726)	12.49802 (0.001932)	-7.203554	6.083447 (0.002759)	11.60407 (0.003021)	-4.46758
ARCH(3)	4.406010 (0.005086)	12.60333 (0.005578)	-7.172079	4.228577 (0.006421)	12.12552 (0.006965)	-4.43852

Note: Values in parenthesis are p-values.

The results of Lagrange Multiplier (LM) test for ARCH effects in the residuals are presented in Table 4.8. As can be seen from the table, the null hypothesis of no ARCH effect in the first three lags of residuals from the mean equations for both monthly price return series is rejected. This implies that the conditional variance of the monthly price return series of gold and silver are non-constant.

To test for ARCH effects, we can also apply the Ljung-Box test for the squared residuals of the fitted model. The null corresponds to insignificance of correlation coefficients of the squared residuals up to lag q. Table A12 (Annex A) shows the results of ARCH effect test using the Ljung-Box test up to lag 18 for the squared residuals of the mean equation. The Q-statistics confirmed the presence of significant correlation in the squared residuals.

Thus, both tests provide strong evidence for the presence of ARCH effects in the residuals. These results indicate that the respective return series under consideration have a non-constant variance (heteroscedasticity) and need to be modeled using EWMA and (G)ARCH family models.

4.5 Specification of Volatility Models

Here we consider exponentially weighted moving average (EWMA) model and GARCH family models.

4.5.1 Exponentially Weighted Moving Average (EWMA) Model

In exponentially weighted moving average model, the weights assigned to the squared return series y_t^2 decline exponentially as we move back through time. This is given by:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)y_{t-1}^2$$

In this model we need only remember the current estimate of the variance rate and the most recent observation on the market variable once we estimate the decay factor parameter λ.

Determining the Optimal Decay Factor

Maximum likelihood (ML) principle works under the assumption that the return follows a conditional normal distribution. In this study both of the observed financial return series have tails that are "fatter" than those implied by conditional normality. The Jarque-Bera (JB) test confirms that the null hypothesis of normality for the return series should be rejected at the 1% level of significance. Therefore, it is important to look for another way to estimate the optimal decay factor λ. The MSE criterion does not impose any distributional assumptions in the determination of the optimal value of decay factor (Morgan and Reuters, 1996) and Liu *et al.*

(2004). Therefore, in this study we consider the MSE criterion in order to estimate the optimal decay factor. The optimal decay factor is estimated using a grid search over the parameter space in the interval of $\lambda \in (0,1)$ so that the mean square error is minimum. The estimates of the optimal decay factor for both of the return series are presented in Table 4.10.

Table 4.10: Estimated decay factor for the series under consideration.

Estimated decay factor		Minimum RMSE	
gold	silver	gold	sliver
0.9499764	0.8907354	0.13649	0.8146

The forecast of the volatilities for both of the series are presented in Figures 3 and 4 and Tables A6 and A7 (Annex A). It is observed from the figures that there are periods of high and low forecasted volatilities across the study period.

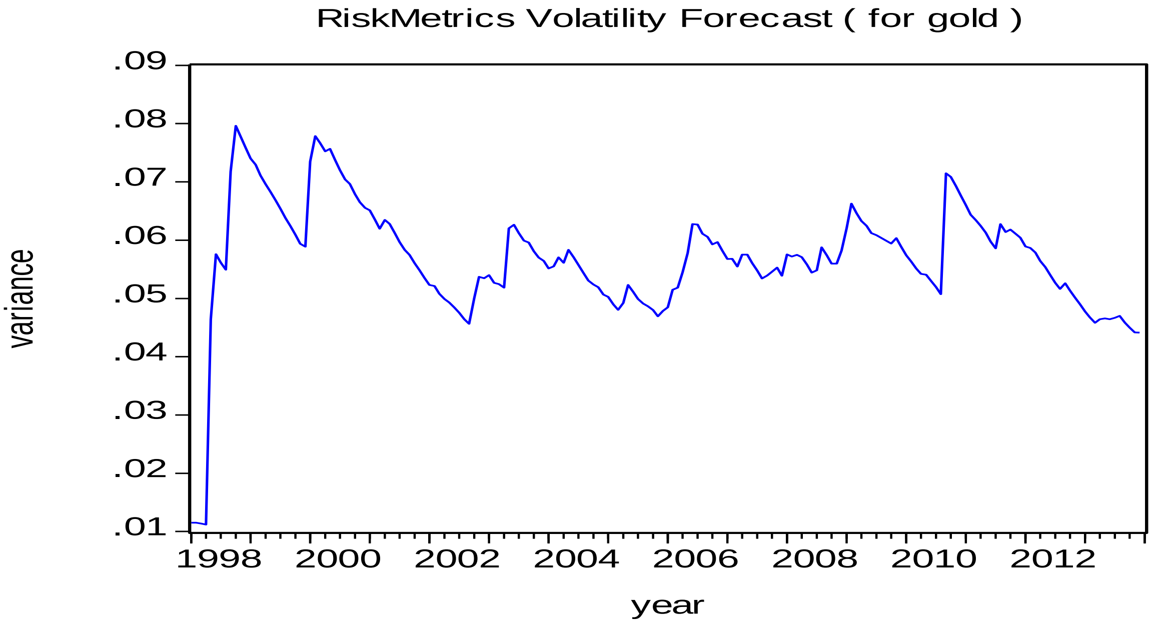

Figure 3: In-sample monthly forecast of conditional volatility using EWMA model with estimated decay factor $\hat{\lambda} = 0.9499764$ (for gold).

We can see from Figure 3 that low price volatility of gold was observed around the years 1999, 2002, 2005 and 2010. On the other hand, high price volatility of gold was observed around the years 1998, 2000 and 2011.

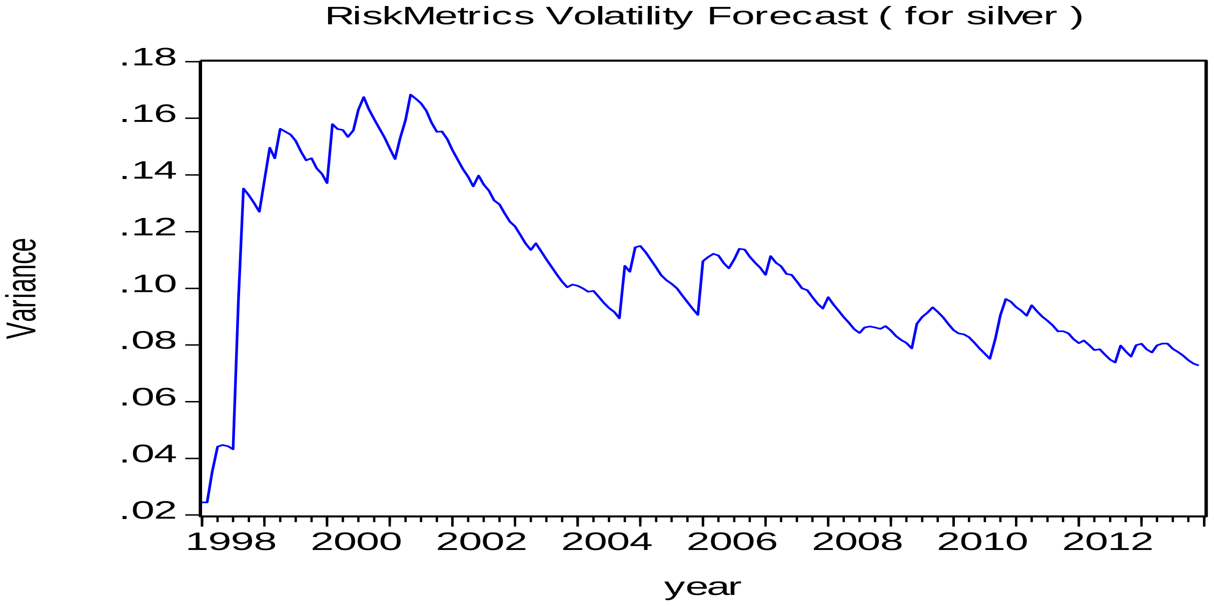

Figure 4: In-sample monthly forecast of conditional volatility using EWMA model with estimated decay factor $\hat{\lambda} = 0.8907354$ (for silver).

We can see from Figure 4 that high price volatility of silver was observed between 1999 and 2001. On the other hand, low price volatility of sliver was observed after 2006. Moreover, it can be seen that the price of silver shows low volatility around the year 2010 over the study period.

4.5.2 G(ARCH) Family Models

Once the presence of ARCH effects is confirmed, then the optimal lag for GARCH family models has to be determined prior to the construction of the final model. Various symmetric (GARCH, GARCH-M) and asymmetric (EGARCH, TGARCH) models for gold and silver price return series are displayed in Tables 8-11 of Annex A. The parameters of the models are estimated using maximum likelihood method under the assumption of different error distributions.

4.5.2.1 Model selection of GARCH Family Model

One practical problem in model selection is the determination of the ARCH order p and the GARCH order q for a particular series. Since GARCH models can be treated as ARMA models for squared residuals, traditional model selection criteria such as the Akaikian information criterion (AIC) and the Bayesian information criterion (SBIC) can be used for selecting models.

Low order GARCH (p, q) models are generally preferred to a high order ARCH (p) for reasons of parsimony and better numerical stability of estimation (Andersen, 2009).

In our model selection procedure we first fit different symmetric and asymmetric GARCH models of different orders of p and q. GARCH (2, 0), GARCH-M(2, 2) and EGARCH(1, 0) models under Student's t-distributional assumption for residuals, EGARCH(3, 3) model under normal distributional assumption for residuals, and TGARCH(1, 0) model under Student's t-distributional assumption for residuals were selected as candidate models for the price volatility of gold since they possess minimum AIC and /or SBIC. The summary results are displayed in Table 4.11 below.

Table 4.11: Optimal lag selected based on AIC and /or SBIC under different error distributional assumptions for gold price return series.

Model Specification	Error distribution	AIC	SBIC	Asymmetric effect
ARMA(0,1)-GARCH(2,0)	Student's t-distribution	-3.043	-2.9412	*
ARMA(0,1)-GARCH-M(2,2)	Student's t-distribution	-3.1142	-2.9615	*
ARMA(0,1)-EGARCH(1,0)	Student's t-distribution	-3.0490	-2.9642	Significant
ARMA(0,1)-EGARCH(3,3)	Normal distribution	-3.1107	-2.9580	Significant
ARMA(0,1)-TGARCH(1,0)	Student's t-distribution	-3.6862	-2.9844	Non-Significant

Additionally, GARCH(2, 0) and GARCH(3, 3) models under generalized error distributional (GED) assumption for residuals, EGARCH(3,2) model under normal distributional assumption for residuals, and GARCH-M(1,2), TGARCH(0,1) and TGARCH(3,3) models under GED assumption for residual were selected as candidate models for the price volatility of silver since they possess minimum AIC and / or SBIC. The summary results are displayed in Table 4.12.

Table 4.12: Optimal lag selected based on AIC and/or SBIC under different error distributional assumptions for silver price return series.

Model Specification	Error Distribution	AIC	SBIC	Asymmetric effect
ARMA(1, 3)-GARCH(2, 0)	Generalized error distribution	-1.7479	-1. 5946	*
ARMA(1, 3)-GARCH(3, 3)	Generalized error distribution	-1.7655	-1.5441	*
ARMA(1, 3)-GARCH-M(1,2)	Generalized error distribution	-1.8016	-1.6143	*
ARMA(1, 3)-EGARCH(3,2)	Normal distribution	-1.8494	-1.6620	Significant
ARMA(1, 3)-TGARCH(0,1)	Generalized error distribution	-1.7344	-1.5812	Non-Significant
ARMA(1, 3)-TGARCH(3,3)	Generalized error distribution	-1.7546	-1.5162	Significant

Moreover, to select the appropriate conditional volatility model, we consider the forecasting performance of the selected symmetric and asymmetric GARCH models given in Tables 4.11 and 4.12. The forecast performance of fitted GARCH family models are evaluated by four conventional error measurements (forecast accuracy statistics): RMSE, MAE, MAPE and Theil inequality coefficient. The models with the smallest statistics are considered to be better fit for modeling the conditional volatility of the price of gold and silver. The summary results are displayed in Tables 4.13 and 4.14.

Table 4.13: Forecast accuracy statistics for residuals from the mean equation of gold.

Model	Error distribution	Forecast accuracy measure			
		RMSE	MAE	MAPE	Theil
GARCH(2,0)	Student's t-distribution	0.0580	0.0410	236.58	0.7808
GARCH-M(2,2)	Student's t-distribution	0.0552	0.0391	242.42	0.6768
EGARCH(1,0)	Student's t-distribution	0.0578	0.0410	226.84	0.7850
EGARCH(3,3)	Normal distribution	0.0578	0.0410	210.37	0.8070
TGARCH(1,0)	Student's t-distribution	0.0593	0.0411	253.89	0.7620

Table 4.14: Forecast accuracy statistics for residuals from the mean equation of silver.

Model	Error distribution	Forecast accuracy measure			
		RMSE	MAE	MAPE	Theil
GARCH(2,0)	Generalized error distribution	0.1229	0.0803	148.82	0.7392
GARCH(3,3)	Generalized error distribution	0.1206	0.0802	151.49	0.7587
GARCH-M(1,2)	Generalized error distribution	0.1172	0.0768	140.51	0.7223
EGARCH(3,2)	Normal distribution	0.1157	0.0766	146.63	0.7145
TGARCH(0,1)	Generalized error distribution	0.1162	0.0786	143.34	0.7614
TGARCH(3, 3)	Generalized error distribution	0.1248	0.0777	149.40	0.7619

The forecast accuracy measures indicate that ARMA (0, 1)-GARCH-M (2, 2) model with Student's t-distributional assumption for residuals, and ARMA (1, 3)-EGARCH (3, 2) model with Normal distributional assumption for residuals perform better to describe volatility since they possess the smallest forecast error measures in the majority of the statistics considered for price return series of gold and silver, respectively.

4.5.2.2 Parameter Estimation

Once the ARMA (0, 1)-GARCH-M (2, 2) model with Student's t-distributional assumption for residuals and ARMA (1, 3)-EGARCH (3, 2) model with Normal distributional assumption for residuals were selected as better fit based on AIC and / or SBIC and forecast accuracy measures, then the next step is to perform analysis of the determinants of monthly price volatility of gold and silver. The parameters in the mean and variance equations are estimated using the maximum likelihood (ML) method. The results are shown in Tables 4.16 and 4.17.

Before going to the analysis of the determinants of monthly price return volatility of gold and silver, it is crucial to check for the presence of a multicollinearity problem. Multicollinearity is a statistical phenomenon in which two or more predictor variables in a multiple regression model are highly correlated, meaning that one can be linearly predicted from the others with a non-trivial degree of accuracy. In order to check the presence of multicollinearity problem we use the variance inflation factor (VIF) which quantifies the severity of multicollinearity. The results (displayed in Table A13 of Annex A) show that there is no multicollinearity problem.

Table 4.16: ML parameter estimates of ARMA (0, 1) -GARCH-M (2, 2) volatility model under Student's t-distributional assumption of residual for the price return series of gold.

Parameter	Variables	Coefficients	Std.error	Statistic	P-value
Mean equation	Sqrt(GARCH)	2.450141	0.027318	89.68928	0.0000*
	Constant	0.170967	0.002493	68.57100	0.0000*
	MA(1)	-0.632888	0.007252	-87.27090	0.0000*
Variance equation	Constant	0.002483	1.15E-05	215.9308	0.0000*
	ARCH(-1)	0.101341	0.013398	7.563999	0.0000*
	ARCH(-2)	0.046473	0.024321	1.910846	0.0460**
	GARCH(-1)	0.543957	0.051308	10.60179	0.0000*
	GARCH(-2)	0.162452	0.052548	3.091484	0.0020*
	General inflation rate	0.000424	0.000558	0.761155	0.4466
	Saving interest rate	-0.006344	6.18E-06	-1027.198	0.0000*
	Price of crude oil	-0.000335	9.97E-05	-3.358358	0.0008*
	Exchange rate	-0.022376	0.001549	-14.44758	0.0000*

Note: * indicates significance at 1% level and ** indicates significance at 5% leve1.

We can see from Table 4.16 that the coefficient estimate of price of crude oil is negative and statistically significant at the 1% level. Thus, an increase in price of crude oil leads to a decrease in the current month price volatility of gold. The result suggests that the energy market influences the gold market. This result was consistent with findings by Kiohos and Sariannidis (2010) and Morales and Andreosso (2011).

The coefficient of exchange rate (birr against US dollar) was negative and statistically significant at the 1% level, that is, exchange rate has a significant influence on gold price movements. This result concurred with the findings of Kiohos and Sariannidis (2010) and Toraman *et.al* (2011). The coefficient of saving interest rate was negative and statistically significant at the 5% level. The opportunity cost of holding gold increases with a real interest rate decreases and decreases with increase in real interest rates. This result is consistent with the findings of Ghosh *et al.* (2002) which asserted that gold prices are related with saving interest rate. On the contrary, this result is not in line with the findings of Toraman *et al.* (2011) who asserted that saving interest rate has no relationship with the price of gold. Specifically, our result shows that gold volatility responds to monetary variables. This seems to be consistent with the argument that gold can be regarded as surrogate money.

Among the explanatory variables which are considered in this study, general inflation rate show a non-significant effect on the current month price volatility of gold. This result is consistent with the findings of Topcu (2010) which assert that there is no relationship between inflation rate and the price of gold. On the contrary, this result is not in line with the findings of Toraman *et al.* (2011).

The result also indicates that the first two month lagged shocks (i.e. ARCH (-1) and ARCH (-2)) of the monthly price of gold are statistically significant at the 1% level. This indicates that the current month price volatility of gold was affected by its 1-month and 2-months lagged shocks. This may be an indication that current price volatility is sensitive to price movements in the past. Similarly, GARCH (-1) and GARCH (-2) terms are statistically significant at the 1% level. This indicates that current month price volatility of gold was also affected by its 1-month and 2-months lagged price volatility.

Additionally, the estimated risk premium coefficient in the mean equation is positive and statistically significant at the 1% level. This implies that the mean of return series considerably depends on past innovations and past conditional standard deviation. Here the conditional standard deviation is used as proxy for risk of return. This result indicates that higher risk is often expected to lead to high returns.

As can be seen in Table 4.17, the coefficient estimate of saving interest rate is statistically significant at the 5% level. Moreover, the coefficient estimate of general inflation rate is statistically significant at the 1% level. This indicates that saving interest rate and general inflation rate have statistically significant effect on the current month price volatility of silver. This result was consistent with findings by XU and Fung (2004) which asserted that the price of silver is extremely volatile reacting to the interactions of global factors such as inflation, saving interest rate and various economic and political events. On the contrary, this result is not in line with the findings of Batten *et al.* (2010) which asserted that both monetary and financial variables show non-significant effect on the price of silver.

Table 4.17: ML parameter estimates of ARMA (1, 3)-EGARCH (3, 2) volatility model under Normal distributional assumption of residual for price return series of silver.

Parameter	Variables	Coefficients	Std.error	Statistic	P-value
Mean equation	Constant	-0.000686	0.005747	-0.119411	0.9049
	AR(1)	-0.423396	0.195479	-2.165938	0.0303**
	MA(1)	0.675648	0.154133	4.383531	0.0000*
	MA(2)	0.374537	0.028299	13.23510	0.0000*
	MA(3)	0.211123	0.031042	6.801143	0.0000*
Variance equation	Constant	-13.35269	0.648682	-20.58433	0.0000*
	ARCH(-1)	1.021688	0.239459	4.266642	0.0000*
	ARCH(-2)	0.959286	0.164583	5.828567	0.0000*
	ARCH(-3)	1.205999	0.260989	4.620888	0.0000*
	Asymmetric(-1)	0.545054	0.182957	2.979136	0.0029*
	EGARCH(-1)	-0.432781	0.016068	-26.93501	0.0000*
	EGARCH(-2)	-0.878201	0.030126	-29.15064	0.0000*
	General inflation rate	-0.774831	0.046881	-16.52760	0.0000*
	Saving interest rate	-0.791274	0.298948	-2.646862	0.0081*
	Exchange rate	0.422090	0.225022	1.875771	0.0607
	Price of crude oil	0.024219	0.013810	1.753759	0.0795

Note: * indicates significance at 1% level and ** indicates significance at 5% level.

Among the explanatory variables which are considered in this study, exchange rate and price of crude oil show a non-significant effect on the current month price volatility of silver.

The results also indicate that lagged shocks (i.e. ARCH (-1), ARCH (-2) and ARCH (-3) terms) of the monthly price of silver have statistically significant effect on the current month price volatility of silver. Similarly, EGARCH (-1) and EGARCH (-2) terms are statistically significant at the 1% level. These indicate that the current month price volatility of silver was affected by its 1- and 2-month lagged price volatility.

Additionally, the coefficient of the asymmetric term was positive and statistically significant at the 1% level - an indication that bad news (unexpected increase in monthly price) had larger impact on the price volatility of silver than good news (unexpected decrease in monthly price).

4.6 Checking the Adequacy of the Fitted Models

In order to check whether the fitted models are good fit to the data or not different diagnostic tests were performed. The Ljung-Box test indicates that the autocorrelations in the standardized residuals are not significantly different from zero for the first 36 lags (Tables A16 and A19 in Annex A). As can be seen in Table 4.18, the Breusch–Godfrey Serial Correlation LM test indicates that the standardized residuals of the fitted model did not exhibit any additional ARCH effect. Moreover, the Ljung-Box test for the squared standardized residuals of the fitted model also indicates insignificance ARCH effects (Tables A17 and A20 in Annex A).

Table 4.18: ARCH-LM test for standardized residuals of the fitted volatility models.

ARCH order	χ^2 statistic(G)	F statistic(G)	χ^2 statistic(S)	F statistic(S)
ARCH(1)	1.715760	1.713183	0.056136	0.055561
	(0.192162)	(0.19216)	(0.08127)	(0.8139)
ARCH(2)	1.703363	0.845817	0.088513	0.04357
	(0.426697)	(0.430842)	(0.9567)	(0.9574)
ARCH(3)	1.779908	0.586267	0.142597	0.04655
	(0.619317)	(0.62473)	(0.9863)	(0.9866)

Note: Values in parenthesis are p-values and G: for gold and S: for sliver.

Figures 7 and 8 (Annex B) show the skewness and kurtosis coefficients of the residuals of the fitted models. The results reveal that the coefficients of skewness were -0.022641 and 0.078173 and the coefficients of kurtosis were 3.134983 and 3.7824 for gold and silver, respectively. The Jarque-Bera test statistic was also insignificant and hence, there is no evidence to reject the null hypothesis of normality. This indicates that the residuals of the fitted model were approximately normally distributed. Therefore, the selection of GARCH-M (2, 2) model with Student's t-distributional assumption of residuals and EGARCH (3, 2) model with normal error distributional assumption to investigate the determinants of the price volatility of gold and silver, respectively, was well justified.

4.7 Forecasting price volatility

One of the fundamental uses of developing GARCH model is forecasting. In this section we examine the forecasting accuracy of the fitted models and then we make in-sample forecasts.

Evaluation of forecast accuracy

The root mean square error (RMSE), mean absolute error (MAE) and Theil U statistics were used to assess the forecasting performance of the fitted models. Table 4.10 reports the forecasting accuracy measures of the fitted models.

Table 4.10: Forecasting accuracy measures of the fitted models.

Accuracy measure	Price of gold return series	Price of sliver return series
Root Mean Squared Error	0.02362	0.112239
Mean Absolute Error	0.03218	0.06321
Mean Absolute percent error	146.231	112.3571
Theil Inequality Coefficient	0.63217	0.7803

The graph of the forecast values of the price volatility of the series under consideration are displayed below.

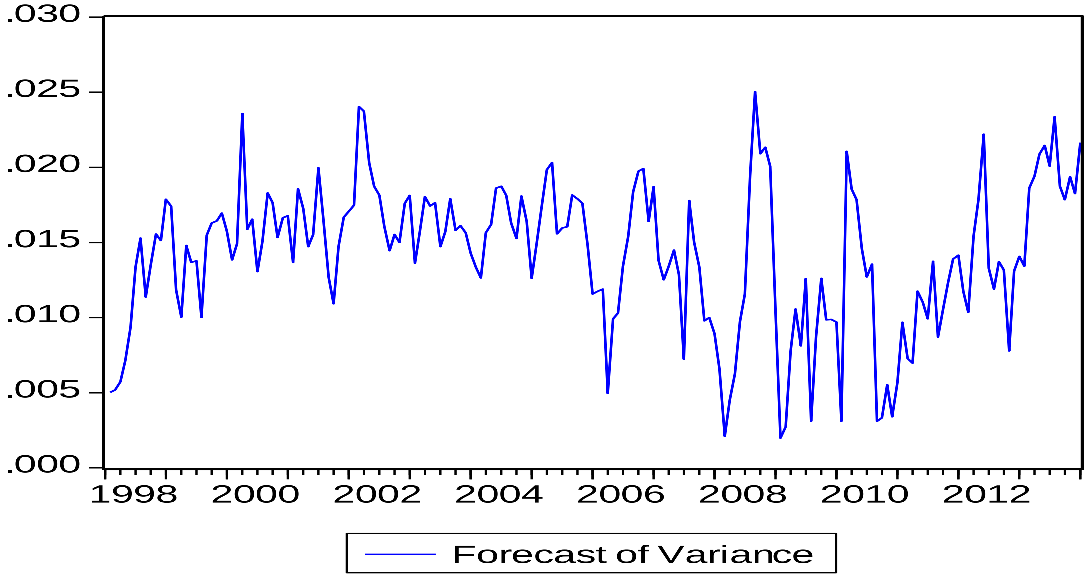

Figure 5: In-sample forecast of monthly price volatility of gold using GARCH-M (2, 2) volatility model.

We can see from Figure 5 that high price volatility of gold was observed around the years 2000, 2002 and 2008. On the other hand, low price volatility was observed around the years 2006, 2007 and 2010.

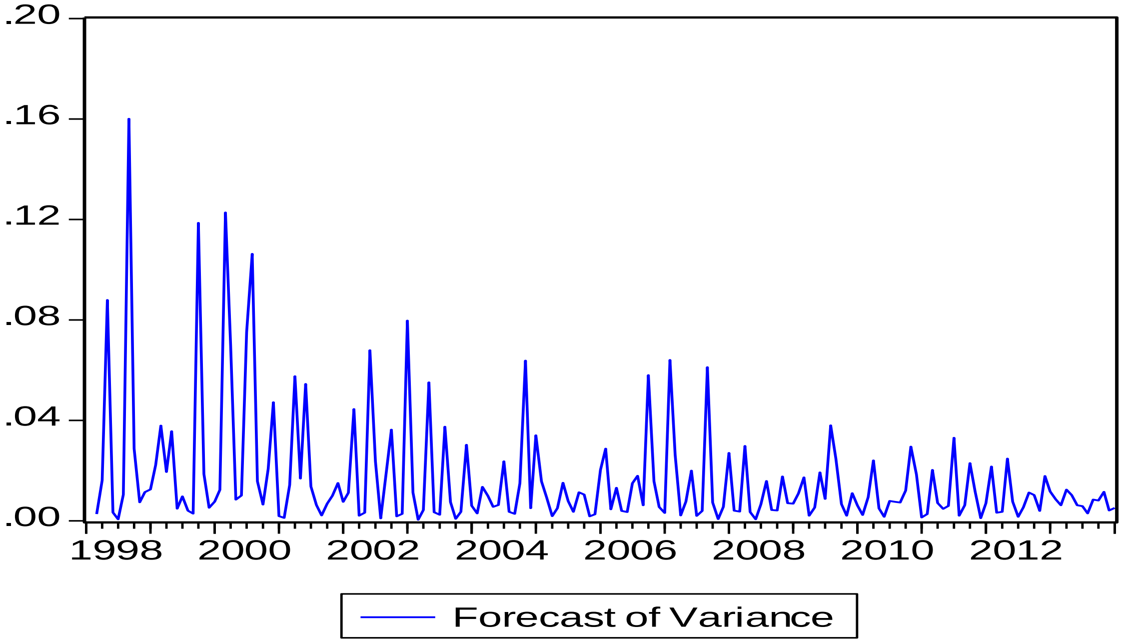

Figure 6: In-sample forecast of monthly price volatility of silver using EGARCH (3, 2) volatility model.

We can see from Figure 6 that high price volatility of sliver was observed around the years 1998, 1999 and 2000. On the other hand, low price volatility of sliver was observed around the years 2001, 2005 and after 2008.

4.7 Comparing the Volatility Models

In this study, the RiskMetrics EWMA and GARCH models were used to model and forecast the price volatility dynamics of precious metals. Figures 3 and 4 show the in-sample forecast using RiskMetrics EWMA model, and Figures 5 and 6 show the in-sample forecast using GARCH family models. In this study the forecasting performance of volatility models are evaluated and compared through the in-sample root mean squared forecast errors (RMSE) and mean absolute forecast errors (MAE).

Table 4.11: Summary on the comparison of EWAM and GARCH family models.

Series	**Model**	**RMSE**	**MAE**
Gold	GARCH Family (GARCH-M(2,2))	0.02362	0.012736
	EWMA ($\hat{\lambda} = 0.9499$)	0.13649	0.05750323
Sliver	GARCH Family (EGARCH(3,2))	0.11224	0.06321
	EWMA ($\hat{\lambda} = 0.8907$)	0.8146	0.09043705

When we compare the RMSE and MAE statistics for the models (i.e. EWMA and GARCH), GARCH family models show the lowest RMSE and MAE in both of the series under consideration than RiskMetrics EWMA model. Therefore, GARCH family models perform better for forecasting the price volatility of gold and silver in the Ethiopian market. This result was consistent with findings by Akgiray (1989), Korkmaz and Aydin (2002), and Ekvall (2012). On the contrary, this result is not consistent with Tse (1991), Tse and Tung (1992), and Walsh and Tsou (1998).

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1. Conclusion

The objective of this study was to model and forecast the volatility dynamics in precious metals prices in the Ethiopian market using GARCH and RiskMetrics models over the period from January 1998 to January 2014. From the preliminary analysis over the time period considered, both of the price series show an increasing pattern. Additionally, the price return series of gold and silver show the characteristics of financial time series such as leptokurtic distributions. This provides an adequate ground for the use of EWMA and GARCH family models.

Before going to the specification of volatility models, we specify the conditional mean equation using ARMA models. Here ARMA (0, 1) and ARMA (1, 3) were selected as the mean equations of the price return series of gold and silver, respectively, using AIC and/ or SBIC criteria. Moreover, the ARCH-LM and Ljung-Box tests support the presence ARCH effects in the residuals of the conditional mean equations.

Exponentially weighted moving average (EWMA) and Generalized Autoregressive Conditionally Heteroscedastic (GARCH) models including both symmetric (GARCH (p, q), GARCH-M (p, q)) and asymmetric (EGARCH (p, q) and TGARCH (p, q)) models were considered in order to model the price volatility of gold and silver in the Ethiopian market.

Symmetric GARCH-M (2, 2) model with Student's t-distributional assumption of residuals and asymmetric EGARCH (3, 2) model with normal distributional assumption for residuals were found to be better fit for the price volatility of gold and silver, respectively. The forecast performances of the models were evaluated using the MAE, MAPE, RMSE and Theil inequality coefficient.

Saving interest rate, exchange rate and price of crude oil have statistically significant effect on monthly price volatility of gold. On the other hand, saving interest rate and general inflation rate were found to have a statistically significant effect on monthly price volatility of silver.

The risk premium effect for GARCH-M (2, 2) model is statistically significant at the 1% level, and the sign of the risk premium is positive for gold. This implies that increase in volatility would increase returns, which is an expected result. The asymmetric term was found to be positive and significant in EGARCH (3, 2) volatility model for sliver. This is an indication that an unanticipated increase in prices had larger impact on price volatility than an unanticipated decrease in price of silver.

At last, a comparison was made between volatility forecasting techniques: GARCH class of models and exponentially weighted moving average (EWMA) model. The study suggests that GARCH family models appear to be better in volatility forecasting than EWMA model as judged by RMSE and MAE criteria.

5.2. Recommendations

From the empirical findings of this study, the following recommendations are drawn:

- The price volatility of gold was influenced by macroeconomic factors such as exchange rate, saving interest rate and price of crude oil, and the price volatility of silver was influenced by macroeconomic factors such as inflation rate and saving interest rate. Therefore, concerned bodies should give due attention to these factors in policy formulation.
- GARCH family models that incorporate exogenous inputs perform better in forecasting price volatility of precious metals than EWMA model. Thus, the inclusion of exogenous factors in volatility forecasting is recommended.

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ANNEX

Annex A: Tables

Table A1: Mean equation specification for the natural log return series for the price of gold.

Dependent Variable: GRETURN
Method: Least Squares
Date: 05/12/14 Time: 12:11
Sample (adjusted): 1998M02 2014M01
Included observations: 192 after adjustments
Convergence achieved after 7 iterations
Backcast: 1998M01

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.014487	0.003425	4.229788	0.0000
MA(1)	-0.169516	0.071516	-2.370318	0.0188

R-squared	0.023549	Mean dependent var	0.014453
Adjusted R-squared	0.018409	S.D. dependent var	0.057612
S.E. of regression	0.057079	Akaike info criterion	-2.87840
Sum squared resid	0.619017	Schwarz criterion	-2.84447
Log likelihood	278.3272	F-statistic	4.582148
Durbin-Watson stat	1.971273	Prob(F-statistic)	0.033581

Table A2: Mean equation specification for the natural log return series for the price of silver.

Dependent Variable: SRETURN
Method: Least Squares
Date: 05/12/14 Time: 12:09
Sample (adjusted): 1998M03 2014M01
Included observations: 191 after adjustments
Convergence achieved after 29 iterations
Backcast: 1997M12 1998M02

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.007302	0.003281	2.225414	0.0273
AR(1)	0.949876	0.029224	32.50280	0.0000
MA(1)	-0.969116	0.076888	-12.60431	0.0000
MA(2)	0.158534	0.099622	1.591355	0.1132
MA(3)	-0.186329	0.072526	-2.569126	0.0110

R-squared	0.060145	Mean dependent var	0.011744
Adjusted R-squared	0.039933	S.D. dependent var	0.112522
S.E. of regression	0.110253	Akaike info criterion	-1.546252
Sum squared resid	2.260958	Schwarz criterion	-1.461114

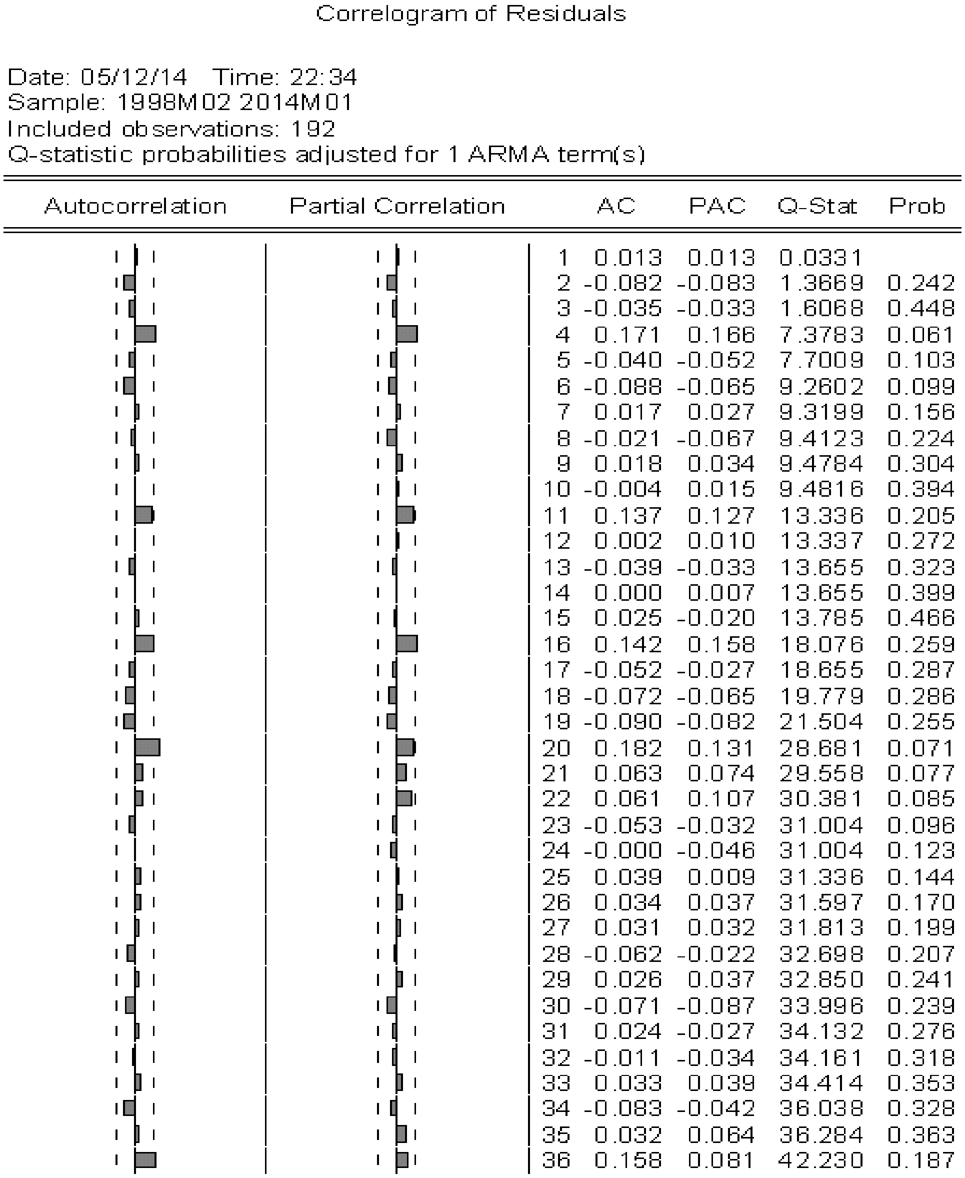
Log likelihood	152.6671	F-statistic	2.975701
Durbin-Watson stat	2.046530	Prob(F-statistic)	0.020596

Inverted AR Roots	.95		
Inverted MA Roots	1.00	-.01+.43i	-.01-.43i

Table A3: Summary on the mean equation specification and order selection criteria for return series of gold and silver.

	Gold		Silver	
Mean equation	AIC	BIC	AIC	BIC
ARMA(1 ,0)	-2.869780	-2.835724	-1.51584	-1.481784
ARMA(2, 0)	-2.863283	-2.812014	-1.530557	-1.530557
ARMA(3 , 0)	-2.854811	-2.786203	1.52034	-1.451731
ARMA(0 , 1)	-2.878408	-2.844476	-1.521054	-1.487122
ARMA(1, 1)	-2.867028	-2.815945	-1.516185	-1.460510
ARMA(2, 1)	-2.854216	-2.785857	-1.526858	-1.458500
ARMA(3, 1)	-2.948846	-2.863085	-1.513589	-1.427829
ARMA(0, 2)	-2.872846	-2.821948	-1.537698	-1.486799
ARMA(1, 2)	-2.858068	-2.789957	-1.524068	-1.455957
ARMA(2, 2)	-2.888299	-2.802851	-1.517541	-1.432093
ARMA(3, 2)	-2.941150	-2.838238	-1.503361	-1.400448
ARMA(0, 3)	-2.862483	-2.794618	-1.529673	-1.461081
ARMA(1, 3)	-2.921572	-2.836434	-1.546252	-1.461114
ARMA(2, 3)	-2.966027	-2.863489	-1.509853	-1.407315
ARMA(3, 3)	-2.942992	-2.822927	-1.618875	-1.498810

Table A4: Ljung-Box test for the presence of serial correlation in the residuals of the mean equation (gold).

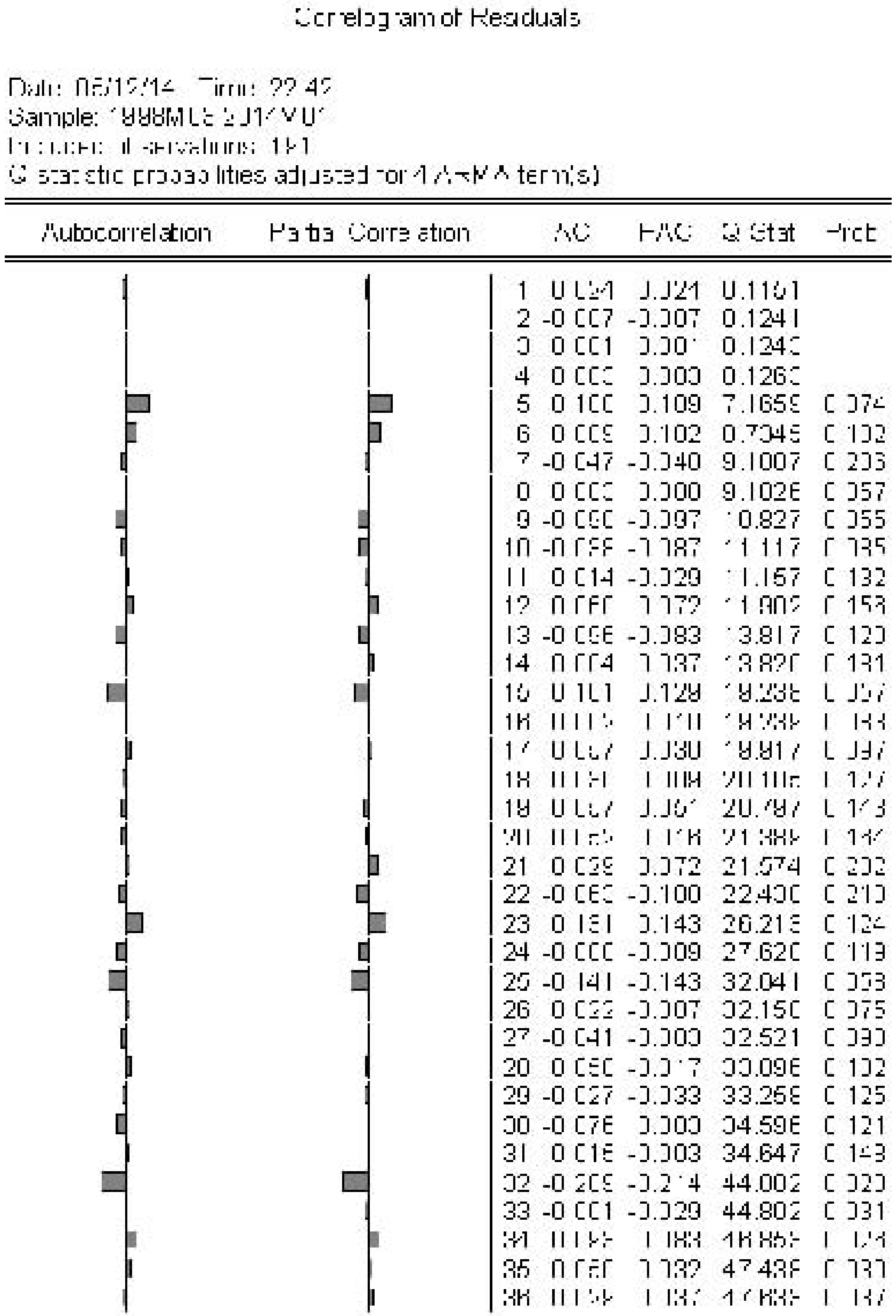


Correlogram of Residuals

Date: 05/12/14 Time: 22:34
Sample: 1998M02 2014M01
Included observations: 192
Q-statistic probabilities adjusted for 1 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.013	0.013	0.0331	
		2	-0.082	-0.083	1.3669	0.242
		3	-0.035	-0.033	1.6068	0.448
		4	0.171	0.166	7.3783	0.061
		5	-0.040	-0.052	7.7009	0.103
		6	-0.088	-0.065	9.2602	0.099
		7	0.017	0.027	9.3199	0.156
		8	-0.021	-0.067	9.4123	0.224
		9	0.018	0.034	9.4784	0.304
		10	-0.004	0.015	9.4816	0.394
		11	0.137	0.127	13.336	0.205
		12	0.002	0.010	13.337	0.272
		13	-0.039	-0.033	13.655	0.323
		14	0.000	0.007	13.655	0.399
		15	0.025	-0.020	13.785	0.466
		16	0.142	0.158	18.076	0.259
		17	-0.052	-0.027	18.655	0.287
		18	-0.072	-0.065	19.779	0.286
		19	-0.090	-0.082	21.504	0.255
		20	0.182	0.131	28.681	0.071
		21	0.063	0.074	29.558	0.077
		22	0.061	0.107	30.381	0.085
		23	-0.053	-0.032	31.004	0.096
		24	-0.000	-0.046	31.004	0.123
		25	0.039	0.009	31.336	0.144
		26	0.034	0.037	31.597	0.170
		27	0.031	0.032	31.813	0.199
		28	-0.062	-0.022	32.698	0.207
		29	0.026	0.037	32.850	0.241
		30	-0.071	-0.087	33.996	0.239
		31	0.024	-0.027	34.132	0.276
		32	-0.011	-0.034	34.161	0.318
		33	0.033	0.039	34.414	0.353
		34	-0.083	-0.042	36.038	0.328
		35	0.032	0.064	36.284	0.363
		36	0.158	0.081	42.230	0.187

Table A5: Ljung-Box test for the presence of serial correlation in the residuals of the mean equation (silver).



Correlogram of Residuals

Date: 05/12/14 Time: 22:42
Sample: 1988M03 2014M01
Included observations: [illegible]
Q-statistic probabilities adjusted for 4 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	0.024	0.024	0.1161	
2	-0.007	-0.007	0.1241	
3	0.001	0.001	0.1243	
4	0.003	0.003	0.1263	
5	0.100	0.109	7.1659	0.074
6	0.009	0.102	8.7045	0.102
7	-0.047	-0.040	9.1007	0.203
8	0.003	0.000	9.1028	0.057
9	-0.090	-0.097	10.827	0.055
10	-0.055	-0.087	11.117	0.085
11	0.014	-0.029	11.157	0.132
12	0.060	0.072	11.902	0.153
13	-0.098	-0.083	13.817	0.120
14	0.004	0.037	13.820	0.131
15	0.101	0.128	18.236	0.057
16	[illegible]	[illegible]	[illegible]	[illegible]
17	0.067	0.030	18.817	0.097
18	[illegible]	[illegible]	[illegible]	[illegible]
19	0.067	0.067	20.787	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]
21	0.029	0.072	21.574	0.202
22	-0.063	-0.100	22.430	0.210
23	0.131	0.143	26.213	0.124
24	-0.000	-0.009	27.620	0.113
25	-0.141	-0.143	32.041	0.033
26	0.022	-0.007	32.150	0.075
27	-0.041	-0.003	32.521	0.030
28	0.050	-0.017	33.098	0.102
29	-0.027	-0.033	33.258	0.125
30	-0.076	0.000	34.598	0.121
31	0.015	-0.003	34.647	0.143
32	-0.205	-0.214	44.002	0.020
33	-0.001	-0.029	44.802	0.031
34	[illegible]	[illegible]	[illegible]	[illegible]
35	0.050	0.032	47.438	0.030
36	[illegible]	[illegible]	[illegible]	[illegible]

Table A6: In- sample forecast of conditional volatility using RiskMetrics EWMA model (For Gold).

```
  [1] 0.0115298 0.0115291 0.01139943 0.01120919 0.04642396 0.05759173
  [7] 0.05616200 0.05493119 0.07176642 0.07962080 0.07778806 0.07581756
 [13] 0.07403278 0.07295728 0.07111557 0.06960067 0.06830867 0.06683633
 [19] 0.06539901 0.06379380 0.06247803 0.06090760 0.05938797 0.05888174
 [25] 0.07346495 0.07786790 0.07667342 0.07522673 0.07565799 0.07374646
 [31] 0.07200884 0.07041444 0.06964506 0.06788082 0.06650358 0.06556560
 [37] 0.06513566 0.06353795 0.06195802 0.06348583 0.06278159 0.06119876
 [43] 0.05967593 0.05834484 0.05746990 0.05603282 0.05487488 0.05348486
 [49] 0.05232240 0.05212265 0.05081133 0.04992455 0.04931458 0.04842219
 [55] 0.04754801 0.04641432 0.04565039 0.05004755 0.05370799 0.05344914
 [61] 0.05401615 0.05265703 0.05246691 0.05185942 0.06202636 0.06264660
 [67] 0.06121325 0.05993269 0.05960297 0.05809313 0.05702114 0.05644578
 [73] 0.05513136 0.05550374 0.05705317 0.05611253 0.05835655 0.05707850
 [79] 0.05577126 0.05437501 0.05306414 0.05238239 0.05192262 0.05066500
 [85] 0.05025577 0.04898632 0.04806028 0.04922572 0.05231099 0.05114231
 [91] 0.04992444 0.04911858 0.04865903 0.04800681 0.04692032 0.04786666
 [97] 0.04846063 0.05147615 0.05183989 0.05445718 0.05781031 0.06276937
[103] 0.06266946 0.06108633 0.06055496 0.05926618 0.05968588 0.05819831
[109] 0.05676695 0.05680782 0.05548396 0.05749940 0.05751123 0.05609243
[115] 0.05479843 0.05341043 0.05391205 0.05457767 0.05528313 0.05389631
[121] 0.05753910 0.05719775 0.05747881 0.05708785 0.05583475 0.05442032
[127] 0.05481783 0.05877138 0.05739177 0.05598689 0.05596198 0.05821007
[133] 0.06205175 0.06625705 0.06462258 0.06330154 0.06247031 0.06119483
```

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[139] 0.06083993 0.06037539 0.05988631 0.05940921 0.06035010 0.05887301
[145] 0.05741903 0.05637810 0.05516309 0.05420859 0.05406860 0.05304111
[151] 0.05194567 0.05072413 0.07147045 0.07087207 0.06929498 0.06766774
[157] 0.06603134 0.06436366 0.06342577 0.06246331 0.06126515 0.05979348
[163] 0.05857886 0.06274980 0.06139639 0.06181649 0.06110690 0.06043161
[169] 0.05890664 0.05867287 0.05789055 0.05645764 0.05537668 0.05403473
[175] 0.05267338 0.05163887 0.05261153 0.05135433 0.05012241 0.04899761
[181] 0.04777176 0.04674450 0.04582525 0.04643173 0.04656311 0.04643487
[187] 0.04669038 0.04702246 0.04587003 0.04501334 0.04418242 0.04413423
```

Table A7: In- sample forecast of conditional volatility using RiskMetrics EWMA model (For silver).

```
 [1] 0.02446916 0.02446915 0.03514351 0.04411845 0.04472320 0.04429198
 [7] 0.04319420 0.09589518 0.13518116 0.13285339 0.13019176 0.12701754
 [13] 0.13803597 0.14958466 0.14589024 0.15628845 0.15529726 0.15420579
 [19] 0.15198306 0.14816448 0.14513816 0.14585865 0.14235035 0.14034975
 [25] 0.13712027 0.15793182 0.15619083 0.15584165 0.15338744 0.15572620
 [31] 0.16300245 0.16748891 0.16325116 0.15963005 0.15651603 0.15314900
 [37] 0.14937466 0.14559137 0.15289502 0.15936050 0.16832171 0.16681715
 [43] 0.16524081 0.16262721 0.15853693 0.15520049 0.15538879 0.15260784
 [49] 0.14882138 0.14535878 0.14215567 0.13938141 0.13600784 0.13980784
 [55] 0.13664213 0.13437785 0.13101112 0.12956249 0.12654467 0.12351624
 [61] 0.12185012 0.11882684 0.11591072 0.11356094 0.11592608 0.11299225
 [67] 0.11019677 0.10747310 0.10493345 0.10231177 0.10039753 0.10131707
 [73] 0.10090415 0.09993607 0.09884091 0.09913087 0.09713102 0.09490255
 [79] 0.09307955 0.09165059 0.08941269 0.10791524 0.10584362 0.11445344
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 [85] 0.11496719 0.11272695 0.11015157 0.10737089 0.10465900 0.10280836
 [91] 0.10164170 0.09998515 0.09761351 0.09514189 0.09285601 0.09065660
 [97] 0.10960982 0.11102412 0.11215330 0.11161587 0.10883132 0.10706209
[103] 0.11023755 0.11396570 0.11366660 0.11118935 0.10899836 0.10731338
[109] 0.10479911 0.11138624 0.10903609 0.10777047 0.10508702 0.10474723
[115] 0.10242049 0.10004486 0.09931089 0.09687299 0.09453432 0.09285776
[121] 0.09684581 0.09439589 0.09208469 0.08983319 0.08777625 0.08559635
[127] 0.08423671 0.08620076 0.08653823 0.08619244 0.08567035 0.08670857
[133] 0.08509620 0.08320533 0.08175973 0.08074457 0.07879427 0.08746427
[139] 0.08990167 0.09134703 0.09327392 0.09173828 0.08975850 0.08749664
[145] 0.08529006 0.08410999 0.08373054 0.08272175 0.08082102 0.07877504
[151] 0.07692956 0.07516519 0.08218542 0.09052312 0.09622687 0.09531182
[157] 0.09335813 0.09214269 0.09034463 0.09403699 0.09189173 0.09006093
[163] 0.08855082 0.08699252 0.08481927 0.08490532 0.08409255 0.08208949
[169] 0.08065722 0.08161666 0.07996806 0.07822250 0.07853050 0.07663962
[175] 0.07484392 0.07385305 0.07980796 0.07779039 0.07591693 0.07992258
[181] 0.08047316 0.07848223 0.07737532 0.07985711 0.08054904 0.08048900
[187] 0.07862898 0.07755263 0.07624061 0.07460473 0.07345262 0.07276550
```

Symmetric GARCH Model Specification

Table A8: GARCH model specification and order selection criteria for price returns of gold and silver.

GARCH(p, q)	Error Distribution Specification	Information Criteria			
		AIC^G	$SBIC^G$	AIC^S	$SBIC^S$
GARCH(0,1)	Normal	-2.8687	-2.8008	-1.4933	-1.3741
	t	*	*	-1.5498	-1.4136
	GED	-2.9532	-2.8684	-1.6584	-1.5222
GARCH(0,2)	Normal	-2.8583	-2.7735	-1.6486	-1.5124
	t	-2.9349	-2.8331	-1.5329	-1.3797
	GED	-2.9743	-2.8725	-1.7233	-1.5701
GARCH(0,3)	Normal	-2.9176	-2.8158	-1.6885	-1.5352
	t	-2.9224	-2.8037	-1.6895	-1.5193
	GED	-2.9644	-2.8456	-1.6699	-1.4996
GARCH(1,0)	Normal	-2.9094	-2.8415	-1.6086	-1.4894
	t	-3.0332	-2.9484	-1.6579	-1.5217
	GED	-3.0069	-2.9221	-1.7273	-1.5911
GARCH(1,1)	Normal	-2.9237	-2.8388	-1.5812	-1.4450
	t	*	*	-1.6517	-1.4984
	GED	-3.0334	-2.9316	-1.7297	-1.5765
GARCH(1,2)	Normal	-2.9031	-2.8013	-1.6094	-1.4561
	t	-3.0354	-2.9167	-1.7277	-1.5574
	GED	-3.0091	-2.8903	-1.7086	-1.5383
GARCH(1,3)	Normal	-2.9549	-2.8362	-1.7508	-1.5805
	t	-3.0357	-2.900	*	*
	GED	-3.0175	-2.8817	-1.7224	-1.5351
GARCH(2,0)	Normal	-2.9241	-2.8393	-1.5816	-1.4454
	t	-3.0430	-2.9412	-1.6422	-1.4889
	GED	-3.0169	-2.9151	-1.7479	-1.5946
GARCH(2,1)	Normal	-3.0215	-2.9197	-1.7323	-1.5790
	t	-3.0334	-2.9146	-1.6356	-1.4654
	GED	-3.0020	-2.8832	-1.6268	-1.4566
GARCH(2,2)	Normal	-3.0404	-2.9216	-1.5700	-1.3990
	t	-2.9304	-2.7946	-1.7340	-1.5467
	GED	-2.9963	-2.8606	-1.7148	-1.5275
GARCH(2,3)	Normal	-2.9282	-2.7925	-1.6040	-1.4167
	t	*	*	-1.6958	-1.4914
	GED	-2.9989	-2.8462	-1.7296	-1.5253
GARCH(3,0)	Normal	-2.9141	-2.8123	-1.5921	-1.4389
	t	-2.9360	-2.8172	-1.6398	-1.4695
	GED	-3.0066	-2.8878	-1.6464	-1.4761
GARCH(3,1)	Normal	-3.0436	-2.9248	-1.5641	-1.3938
	t	-3.0231	-2.8874	-1.6091	-1.4218
	GED	-2.9952	-2.8595	-1.7299	-1.5426
GARCH(3,2)	Normal	-2.9506	-2.8149	-1.5460	-1.3587
	t	-3.0401	-2.8874	-1.5838	-1.3795
	GED	-3.0204	-2.8677	-1.7512	-1.5469
GARCH(3,3)	Normal	-2.9455	-2.7920	-1.5663	-1.3620
	t	-3.0256	-2.8560	-1.5742	-1.3528
	GED	-2.9963	-2.8267	-1.7655	-1.5441

Table A9: GARCH-M model specification and order selection criteria for price returns of gold and silver.

GARCH-M (p, q)	Error distribution specification	Information criteria			
		AIC^G	$SBIC^G$	AIC^S	$SBIC^S$
GARCH-M (0,1)	Normal	-2.8968	-2.8120	-1.5374	-1.4012
	t	*	*	-1.7262	-1.5729
	GED	-2.9609	-2.8591	-1.7094	-1.5561
GARCH-M(0,2)	Normal	-2.9463	-2.8445	-1.5707	-1.4175
	t	-3.0256	-2.9068	-1.7146	-1.5443
	GED	-2.9496	-2.8308	-1.6944	-1.5241
GARCH-M (0,3)	Normal	-2.8464	-2.7276	-1.5592	-1.3889
	t	-3.0124	-2.8767	-1.7066	-1.5193
	GED	-2.9871	-2.8514	-1.7397	-1.5524
GARCH-M(1,0)	Normal	-2.9466	-2.8617	-1.5878	-1.4516
	t	*	*	-1.6558	-1.5026
	GED	-3.0457	-2.9439	-1.7279	-1.5747
GARCH-M(1,1)	Normal	-2.9396	-2.8378	-1.6107	-1.4575
	t	-2.9650	-2.8462	-1.7131	-1.5428
	GED	*	*	-1.7100	-1.5398
GARCH-M(1,2)	Normal	-2.9293	-2.8105	-1.5819	-1.4116
	t	-3.0958	-2.9601	-1.6665	-1.4792
	GED	-2.5500	-2.4143	-1.8016	-1.6143
GARCH-M(1,3)	Normal	-2.9638	-2.8280	-1.6140	-1.4267
	t	*	*	-1.7059	-1.5015
	GED	-3.0638	-2.9111	-1.7945	-1.5902
GARCH-M(2,0)	Normal	-2.9386	-2.8368	-1.6186	-1.4654
	t	-3.0742	-2.9555	*	*
	GED	-3.0607	-2.9419	-1.7398	-1.5695
GARCH-M(2,1)	Normal	-2.9292	-2.8105	-1.6215	-1.4512
	t	-3.0706	-2.9349	-1.6944	-1.5071
	GED	-3.0497	-2.9140	-1.7138	-1.5264
GARCH-M(2,2)	Normal	-2.9584	-2.8227	-1.6284	-1.4411
	t	-3.1142	-2.9615	-1.6588	-1.4545
	GED	-3.0841	-2.9314	-1.7121	-1.5078
GARCH-M(2,3)	Normal	-2.9874	-2.8347	-1.5753	-1.3710
	t	-3.1045	-2.9348	-1.6959	-1.4745
	GED	-3.0548	2.8852	-1.7284	-1.5071
GARCH-M(3,0)	Normal	-2.9300	-2.8112	-1.5752	-1.4049
	t	-3.0648	-2.9290	-1.7431	-1.5558
	GED	-3.0503	-2.9145	-1.6935	-1.5062
GARCH-M(3,1)	Normal	-2.9196	-2.7838	-1.5915	-1.4042
	t	-3.0545	-2.9018	-1.5696	-1.3653
	GED	-3.0647	-2.9120	-1.7385	-1.5342
GARCH-M(3,2)	Normal	-3.0534	-2.9007	-1.5342	-1.3298
	t	-3.1042	-2.9345	-1.5738	-1.3525
	GED	-3.0078	-2.8382	-1.7657	-1.5443
GARCH-M(3,3)	Normal	-2.9536	-2.7840	-1.5438	-1.3225
	t	-3.0480	-2.8613	-1.6828	-1.4440
	GED	-3.0153	-2.8287	-1.7160	-1.4777

Asymmetric GARCH Model Specification

Table A10: EGARCH model specification and order selection criteria for price returns of gold and silver.

EGARCH(p, q)	Error distribution specification	Information criteria			
		AIC^G	$SBIC^G$	AIC^S	$SBIC^S$
EGARCH(0,1)	Normal	-2.8843	-2.8165	-1.4934	-1.3742
	t	-2.8930	-2.8081	-1.6581	-1.5219
	GED	-2.9535	-2.8686	-1.6478	-1.5117
EGARCH(0,2)	Normal	-2.9774	-2.8756	-1.5146	-1.3784
	t	-2.9671	-2.8483	-1.6507	-1.4975
	GED	-3.0273	-2.9086	-1.6471	-1.4939
EGARCH(0,3)	Normal	-2.9928	-2.8910	-1.5900	-1.4368
	t	-2.8718	-2.7530	-1.6790	-1.5088
	GED	-3.0359	-2.9172	-1.7080	-1.5377
EGARCH(1,0)	Normal	-2.9038	-2.8359	-1.5947	-1.4755
	t	-3.0490	-2.9642	-1.7367	-1.6005
	GED	-3.0191	-2.9343	-1.7213	-1.5851
EGARCH(1,1)	Normal	-2.8943	-2.8095	-1.5871	-1.4509
	t	-3.0390	-2.9372	-1.7319	-1.5786
	GED	-3.0104	-2.9086	-1.7105	-1.5573
EGARCH(1,2)	Normal	-2.9405	-2.8387	-1.5878	-1.4348
	t	-3.0495	-2.9308	-1.7468	-1.5766
	GED	-3.0300	-2.9113	-1.7104	-1.5401
EGARCH(1,3)	Normal	-2.9605	-2.8418	-1.6005	-1.4297
	t	-3.0420	-2.9063	-1.8021	-1.6148
	GED	-3.0238	-2.8880	-1.8046	-1.6173
EGARCH(2,0)	Normal	-2.8951	-2.8102	-1.5893	-1.4530
	t	-3.0391	-2.9373	-1.7312	-1.5779
	GED	-3.0121	-2.9103	-1.7117	-1.5585
EGARCH(2,1)	Normal	-2.8933	-2.7915	-1.7597	-1.6064
	t	-3.0421	-2.9234	-1.7226	-1.5523
	GED	-3.0278	-2.9090	-1.7794	-1.6091
EGARCH(2,2)	Normal	-2.9367	-2.8179	-1.7325	-1.5622
	t	-3.0401	-2.9043	-1.7399	-1.5526
	GED	-3.0196	-2.8839	-1.7648	-1.5775
GARCH(2,3)	Normal	-2.9611	-2.8254	-1.6843	-1.4970
	t	-3.0317	-2.8790	-1.8124	-1.6081
	GED	-3.0149	-2.8622	-1.7735	-1.5691
EGARCH(3,0)	Normal	-2.9402	-2.8384	-1.5866	-1.4334
	t	-3.0411	-2.9224	-1.7248	-1.5545
	GED	-3.0189	-2.9001	-1.7019	-1.5317
EGARCH(3,1)	Normal	-2.9467	-2.8280	-1.5761	-1.4058
	t	-3.0353	-2.8996	-1.7154	-1.5281
	GED	-3.0203	-2.8846	-1.6928	-1.5055
EGARCH(3,2)	Normal	-3.0651	-2.9294	-1.8494	-1.6620
	t	-3.0299	-2.8772	-1.7462	-1.5419
	GED	-3.0050	-2.8523	-1.7989	-1.5946
EGARCH(3,3)	Normal	-3.1107	-2.9580	-1.6152	-1.4109
	t	-3.0213	-2.8517	-1.7396	-1.5182
	GED	-3.0902	-2.9205	-1.7441	-1.5227

Table A11: TGARCH model specification and order selection criteria for price returns of gold and silver.

TGARCH(p, q)	Error distribution specification	Information criteria			
		AIC^G	$SBIC^G$	AIC^S	$SBIC^S$
TGARCH(0,1)	Normal	-2.9352	-2.8503	-1.5154	-1.3792
	t	*	*	*	*
	GED	-2.9818	-2.8800	-1.7344	-1.5812
TGARCH(0,2)	Normal	-2.9249	-2.8231	-1.4008	-1.2475
	t	-2.9934	-2.8746	-1.5280	1.3577
	GED	-2.9762	-2.8574	-1.7156	-1.5454
TGARCH(0,3)	Normal	-2.9170	-2.7987	-1.6653	-1.4950
	t	-2.9370	-2.8013	-1.5169	-1.3296
	GED	-2.9759	-2.8401	-1.6550	-1.4677
TGARCH(1,0)	Normal	-2.9630	-2.8782	-1.5856	-1.4494
	t	-3.6862	-2.9844	-1.6402	-1.4870
	GED	-3.0548	-2.9530	-1.7050	-1.5514
TGARCH(1,1)	Normal	-2.9659	-2.8641	-1.7198	-1.5665
	t	-3.0804	-2.9616	-1.7294	-1.5592
	GED	-3.0571	-2.9384	-1.7191	-1.5488
TGARCH(1,2)	Normal	-2.9963	-2.8775	-1.5844	-1.4141
	t	-3.0654	-2.9296	-1.6771	-1.4898
	GED	-3.0529	-2.9172	-1.7090	-1.5217
TGARCH(1,3)	Normal	-2.9525	-2.8168	-1.6131	-1.4258
	t	-3.0823	-2.9296	-1.6854	-1.4811
	GED	-3.0741	-2.9215	-1.6733	-1.4690
TGARCH(2,0)	Normal	-2.9753	-2.8735	-1.5779	-1.4247
	t	-3.0876	-2.9637	-1.6244	-1.4541
	GED	-3.0635	-2.9448	-1.7317	-1.5614
TGARCH(2,1)	Normal	-2.9672	-2.8484	-1.5985	-1.4282
	t	-3.0733	-2.9376	-1.7364	-1.5491
	GED	-3.0507	-2.9150	-1.7075	1.5202
TGARCH(2,2)	Normal	-2.9798	-2.8440	-1.5970	-1.4097
	t	-3.0699	-2.9172	-1.6534	-1.4491
	GED	-3.0568	-2.9041	-1.6882	-1.4839
TGARCH(2,3)	Normal	-2.9831	-2.8304	-1.6750	-1.4706
	t	-3.0664	-2.8907	-1.6379	-1.4165
	GED	-3.0584	-2.9227	-1.7222	-1.5009
TGARCH(3,0)	Normal	-2.9736	-2.8548	-1.5959	-1.4256
	t	*	*	-1.6254	-1.4381
	GED	-3.0584	-2.9227	-1.6779	-1.4906
TGARCH(3,1)	Normal	-2.9647	-2.8290	-1.5721	-1.3848
	t	-3.0580	-2.9053	-1.7104	-1.5061
	GED	-3.0470	-2.8943	-1.7008	-1.4964
TGARCH(3,2)	Normal	-2.9281	-2.7754	-1.5503	-1.3460
	t	-3.0708	-2.9012	-1.6110	-1.3896
	GED	-3.0524	-2.8827	-1.6789	-1.4575
TGARCH(3,3)	Normal	-2.9469	-2.7773	-1.5353	1.3139
	t	-3.01831	-2.8317	*	*
	GED	-3.0471	-2.8605	-1.7546	-1.5162

Table A12: ARCH effect test using Ljung-Box test for squared residuals of the fitted models.

	Gold price return series				Silver price return series			
Lags	Acf	Pacf	Q-statistic	p-value	Acf	Pacf	Q-statistic	p-value
1	0.244	0.244	11.577		0.247	0.247	11.827	
2	-0.021	-0.086	11.665	0.001	0.053	-0.009	12.366	
3	0.006	0.035	11.673	0.003	-0.046	-0.061	12.781	
4	0.130	0.126	15.014	0.002	0.024	0.054	12.897	
5	0.003	-0.066	15.016	0.005	0.155	0.149	17.633	0.000
6	-0.064	-0.039	15.844	0.007	0.079	-0.000	18.879	0.000
7	-0.101	-0.083	17.890	0.007	0.133	0.116	22.425	0.000
8	-0.099	-0.084	19.880	0.006	0.095	0.059	24.228	0.000
9	-0.083	-0.043	21.285	0.006	-0.013	-0.064	24.261	0.000
10	-0.039	-0.005	21.597	0.010	0.080	0.096	25.576	0.000
11	0.000	0.026	21.597	0.017	-0.054	-0.106	26.184	0.000
12	-0.025	-0.019	21.726	0.027	0.041	0.034	26.526	0.001
12	-0.034	-0.019	21.968	0.038	0.041	0.034	26.526	0.001
13	-0.064	-0.071	22.817	0.044	0.135	0.124	30.289	0.000
14	0.056	0.069	23.483	0.053	0.070	-0.016	31.296	0.001
15	0.141	0.103	27.684	0.024	0.093	0.049	33.113	0.001
16	-0.009	-0.078	27.700	0.034	0.009	0.021	33.132	0.001
17	-0.045	0.000	28.137	0.043	0.236	0.239	44.909	0.000
18	0.244	0.244	11.577	0.001	0.150	0.019	49.718	0.000

Table A13: Test for multicollinearity using variance inflation factor (VIF).

Regressed variable	$\boldsymbol{R_j^2}$	$\boldsymbol{VIF = 1/1 - R_j^2}$
D(general inflation rate)	0.8928	9.3283
D(food inflation rate)	0.8690	7.6335
D(non food inflation rate)	0.8594	7.1123
D(exchange rate)	0.0611	1.0650
D(price of crude oil)	0.0927	1.1021
D(interest rate)	0.00061	1.0006

Note: R_j^2 is the coefficient of determination from the regression X_j on the remaining $(r-1)$ predictor variables.

Table A14: The fitted ARMA (0, 1)-GARCH-M (2, 2) volatility model for price return series for gold.

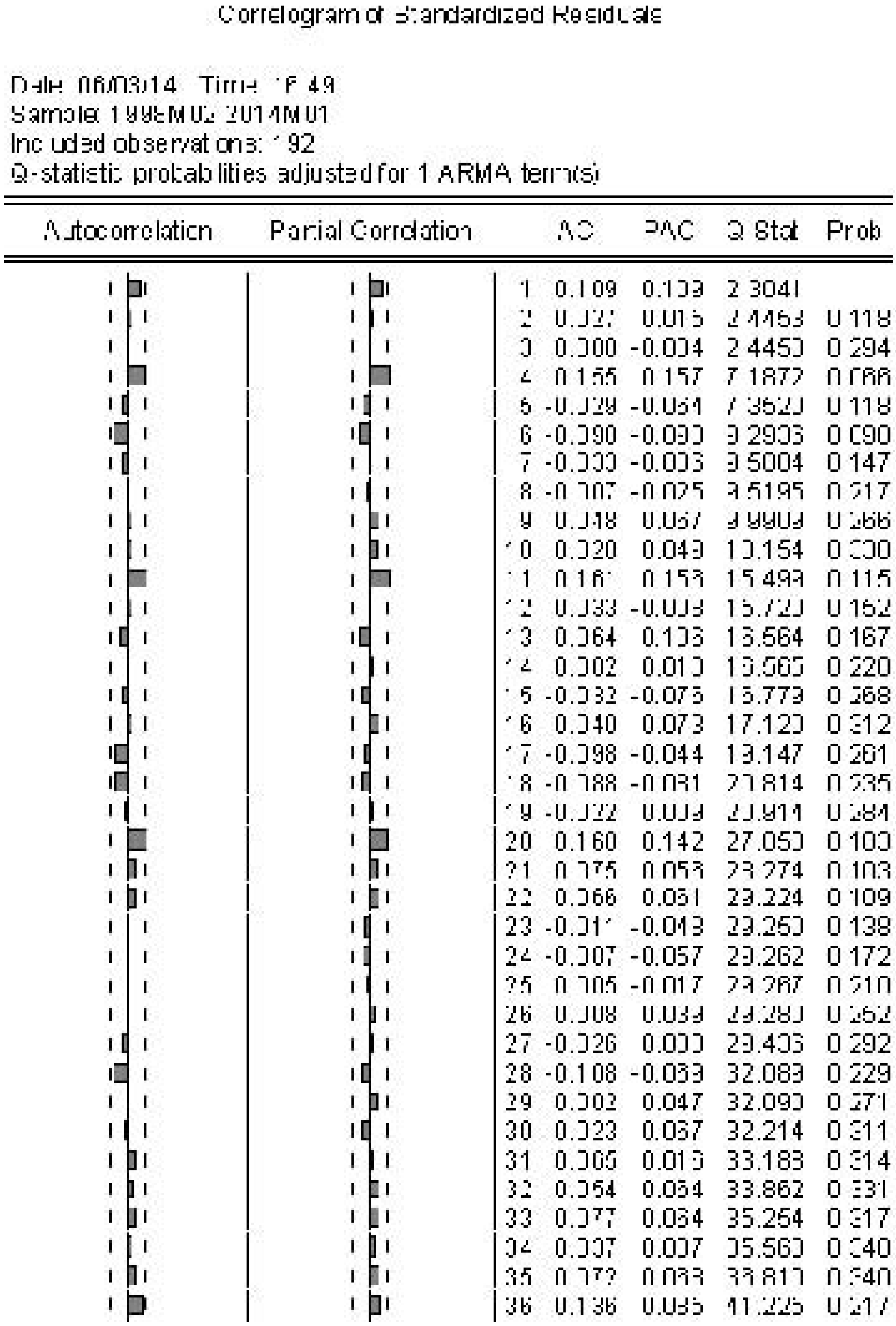
```
Dependent Variable: GRETURN
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 06/09/14   Time: 15:18
Sample (adjusted): 1998M02 2014M01
Included observations: 192 after adjustments
Convergence achieved after 17 iterations
MA backcast: OFF, Variance backcast: OFF
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*RESID(-2)^2 + C(7)
        *GARCH(-1) + C(8)*GARCH(-2) + C(9)*D(GENERALINFLATION
        RATE) + C(10)*D(INTERESTRATE) + C(11)*D(PRICEOFCRUDE
        OIL) + C(12)*D(EXCHANGERATE)
==========================================================================
                              Coefficient   Std. Error   z-Statistic   Prob.
==========================================================================
        @SQRT(GARCH)            2.450141     0.027318     89.68928     0.0000
             C                  0.170967     0.002493     68.57100     0.0000
           MA(1)               -0.632888     0.007252    -87.27090     0.0000
==========================================================================
                              Variance Equation
==========================================================================
             C                  0.002483     1.15E-05     215.9308     0.0000
        RESID(-1)^2             0.101341     0.013398     7.563999     0.0000
        RESID(-2)^2             0.046473     0.024321     1.910846     0.0460
         GARCH(-1)              0.543957     0.051308     10.60179     0.0000
         GARCH(-2)              0.162452     0.052548     3.091484     0.0020
 D(GENERALINFLATIONRATE)        0.000424     0.000558     0.761155     0.4466
     D(INTERESTRATE)           -0.006344     6.18E-06    -1027.198     0.0000
    D(PRICEOFCRUDEOIL)         -0.000335     9.97E-05    -3.358358     0.0008
     D(EXCHANGERATE)           -0.022376     0.001549    -14.44758     0.0000
==========================================================================
        T-DIST. DOF             18.52262     22.89405     0.809058     0.4185
==========================================================================
R-squared                      0.0326381    Mean dependent var        0.014453
Adjusted R-squared             0.0021914    S.D. dependent var        0.057612
S.E. of regression              139.5991    Akaike info criterion     2.760139
Sum squared resid               3488334.    Schwarz criterion         2.980699
Log likelihood                 -251.9734    Durbin-Watson stat        0.029243
==========================================================================
Inverted MA Roots                  .63
==========================================================================
```

Table A15: The fitted ARMA (1, 3)-EGARCH (3, 2) model for price return series for sliver.

```
Dependent Variable: SRETURN
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/04/14   Time: 21:44
Sample (adjusted): 1998M03 2014M01
Included observations: 191 after adjustments
Convergence achieved after 63 iterations
LOG(GARCH) = C(6) + C(7)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(8)
     *ABS(RESID(-2)/@SQRT(GARCH(-2))) + C(9)*ABS(RESID(-3)
     /@SQRT(GARCH(-3))) + C(10)*RESID(-1)/@SQRT(GARCH(-1)) +
     C(11)*LOG(GARCH(-1)) + C(12)*LOG(GARCH(-2)) + C(13)
     *D(GENERALINFLATIONRATE) + C(14)*D(INTERESTRATE) + C(15)
     *D(EXCHANGERATE) + C(16)*D(OIL)
==============================================================================
      Variable          Coefficient    Std. Error    z-Statistic      Prob.
==============================================================================
         C               -0.000686      0.005747      -0.119411      0.9049
       AR(1)             -0.423396      0.195479      -2.165938      0.0303
       MA(1)              0.675648      0.154133       4.383531      0.0000
       MA(2)              0.374537      0.028299       13.23510      0.0000
       MA(3)              0.211123      0.031042       6.801143      0.0000
==============================================================================
                             Variance Equation
==============================================================================
        C(6)             -13.35269      0.648682      -20.58433      0.0000
        C(7)              1.021688      0.239459       4.266642      0.0000
        C(8)              0.959286      0.164583       5.828567      0.0000
        C(9)              1.205999      0.260989       4.620888      0.0000
        C(10)             0.545054      0.182957       2.979136      0.0029
        C(11)            -0.432781      0.016068      -26.93501      0.0000
        C(12)            -0.878201      0.030126      -29.15064      0.0000
        C(13)            -0.774831      0.046881      -16.52760      0.0000
        C(14)            -0.791274      0.298948      -2.646862      0.0081
        C(15)             0.422090      0.225022       1.875771      0.0607
        C(16)             0.024219      0.013810       1.753759      0.0795
==============================================================================
R-squared                -0.041469    Mean dependent var             0.011744
Adjusted R-squared       -0.063866    S.D. dependent var             0.112522
S.E. of regression        0.116060    Akaike info criterion         -1.804896
Sum squared resid         2.505403    Schwarz criterion             -1.532454
Log likelihood            188.3675    Hannan-Quinn criter.          -1.694544
Durbin-Watson stat        2.450440
==============================================================================
Inverted AR Roots           -.42
Inverted MA Roots        -.03-.58i       -.03+.58i        -.62
==============================================================================
```

Model Adequacy checking of the fitted model

Table A16: Ljung-Box test for Standardized Residuals from Volatility Model for gold Price returns.



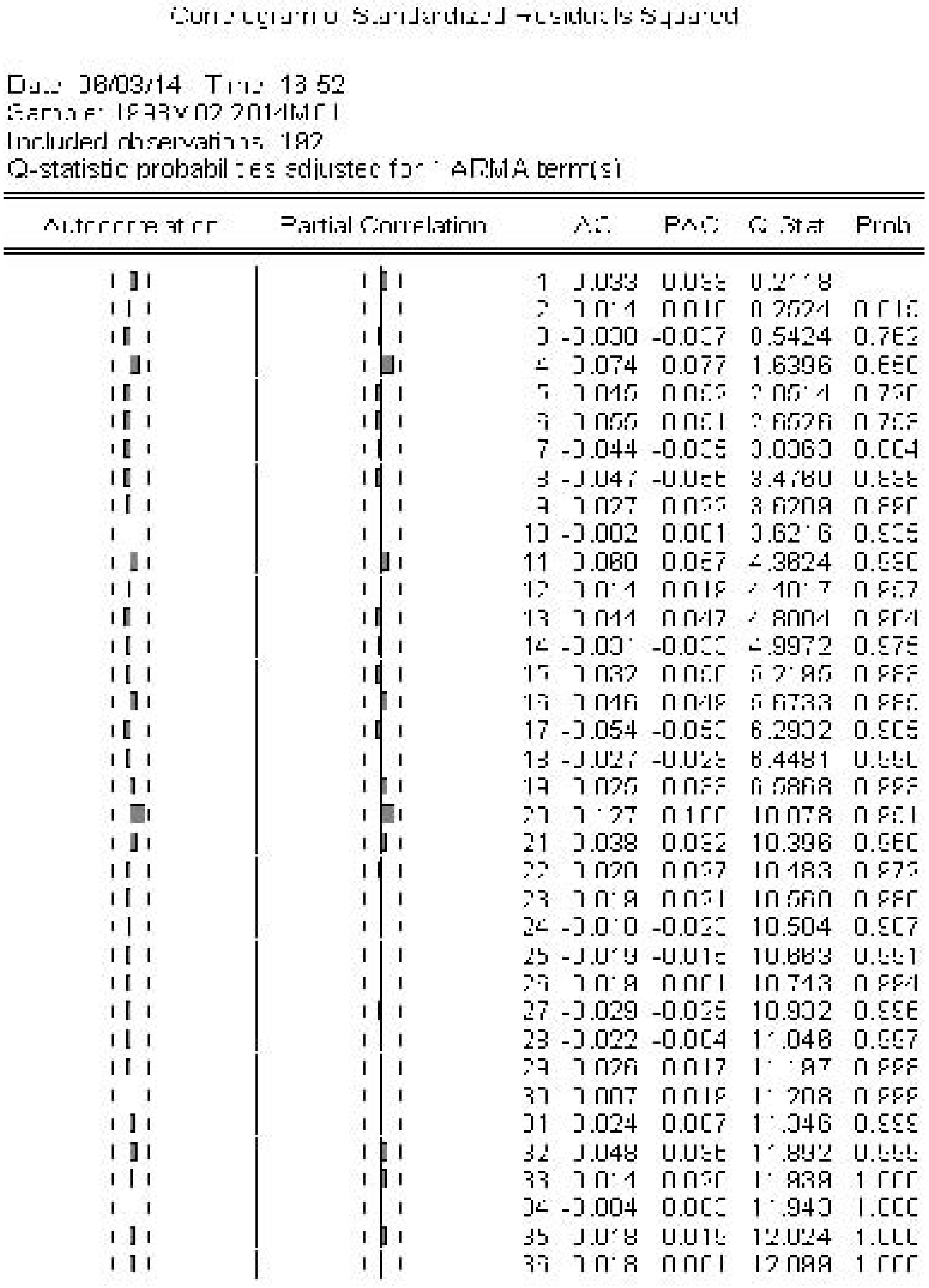
```
                  Correlogram of Standardized Residuals

Date: 06/03/14   Time: 16:49
Sample: 1998M02 2014M01
Included observations: 192
Q-statistic probabilities adjusted for 1 ARMA term(s)

 Autocorrelation   Partial Correlation      AC     PAC   Q-Stat   Prob

                                       1   0.109   0.109  2.3041
                                       2   0.027   0.015  2.4463  0.118
                                       3   0.000  -0.004  2.4450  0.294
                                       4   0.155   0.157  7.1872  0.066
                                       5  -0.029  -0.051  7.3620  0.118
                                       6  -0.090  -0.090  9.2903  0.090
                                       7  -0.030  -0.003  9.5004  0.147
                                       8  -0.007  -0.025  9.5195  0.217
                                       9   0.048   0.057  9.9903  0.266
                                      10   0.020   0.043  10.154  0.330
                                      11   0.161   0.153  15.493  0.115
                                      12   0.033  -0.008  15.720  0.152
                                      13   0.064   0.103  16.564  0.167
                                      14   0.002   0.010  16.565  0.220
                                      15  -0.032  -0.075  16.773  0.268
                                      16   0.040   0.073  17.120  0.312
                                      17  -0.098  -0.044  19.147  0.261
                                      18  -0.088  -0.081  20.814  0.235
                                      19  -0.022   0.009  20.911  0.284
                                      20   0.160   0.142  27.050  0.103
                                      21   0.075   0.053  28.274  0.103
                                      22   0.066   0.051  29.224  0.109
                                      23  -0.011  -0.013  29.250  0.138
                                      24  -0.007  -0.057  29.262  0.172
                                      25   0.005  -0.017  29.267  0.210
                                      26   0.008   0.039  29.280  0.252
                                      27  -0.026   0.000  29.406  0.292
                                      28  -0.108  -0.039  32.083  0.229
                                      29   0.002   0.047  32.090  0.271
                                      30   0.023   0.037  32.214  0.311
                                      31   0.065   0.015  33.183  0.314
                                      32   0.054   0.054  33.862  0.331
                                      33   0.077   0.064  35.254  0.317
                                      34   0.037   0.037  35.563  0.340
                                      35   0.072   0.033  36.817  0.340
                                      36   0.136   0.085  41.225  0.217
```

TableA17: ARCH effect test using Ljung-Box test for squared residuals of the fitted volatility model for gold.



Correlogram of Standardized Residuals Squared

Date: 06/03/14 Time: 13:52
Sample: 1983M02 2014M01
Included observations: 182
Q-statistic probabilities adjusted for 1 ARMA term(s)

	AC	PAC	Q-Stat	Prob
1	0.033	0.033	0.2118	
2	0.014	0.010	0.2524	0.615
3	-0.030	-0.027	0.5424	0.762
4	0.074	0.077	1.6396	0.650
5	0.046	0.053	2.0614	0.725
6	0.056	0.061	2.6626	0.752
7	-0.044	-0.035	3.0363	0.804
8	-0.047	-0.056	3.4780	0.838
9	0.027	0.033	3.6208	0.890
10	-0.002	0.001	3.6216	0.935
11	0.060	0.067	4.3824	0.929
12	0.014	0.019	4.4017	0.957
13	0.044	0.047	4.8004	0.964
14	-0.031	-0.033	4.9972	0.975
15	0.032	0.030	5.2185	0.983
16	0.046	0.049	5.6733	0.985
17	-0.054	-0.050	6.2902	0.985
18	-0.027	-0.026	6.4481	0.989
19	0.026	0.033	6.5868	0.993
20	0.127	0.100	10.078	0.931
21	0.038	0.032	10.396	0.943
22	0.020	0.027	10.483	0.959
23	0.018	0.031	10.500	0.970
24	-0.010	-0.023	10.504	0.977
25	-0.019	-0.015	10.669	0.991
26	0.018	0.001	10.743	0.994
27	-0.029	-0.035	10.932	0.995
28	-0.022	-0.004	11.048	0.997
29	0.026	0.017	11.187	0.998
30	0.007	0.019	11.208	0.999
31	0.024	0.007	11.346	0.999
32	0.048	0.056	11.892	0.999
33	0.014	0.030	11.939	1.000
34	-0.004	0.003	11.943	1.000
35	0.018	0.015	12.024	1.000
36	0.018	0.001	12.088	1.000

Table A18: ARCH effect test using LM test for squared residuals of the fitted volatility model for gold.

ARCH Test:

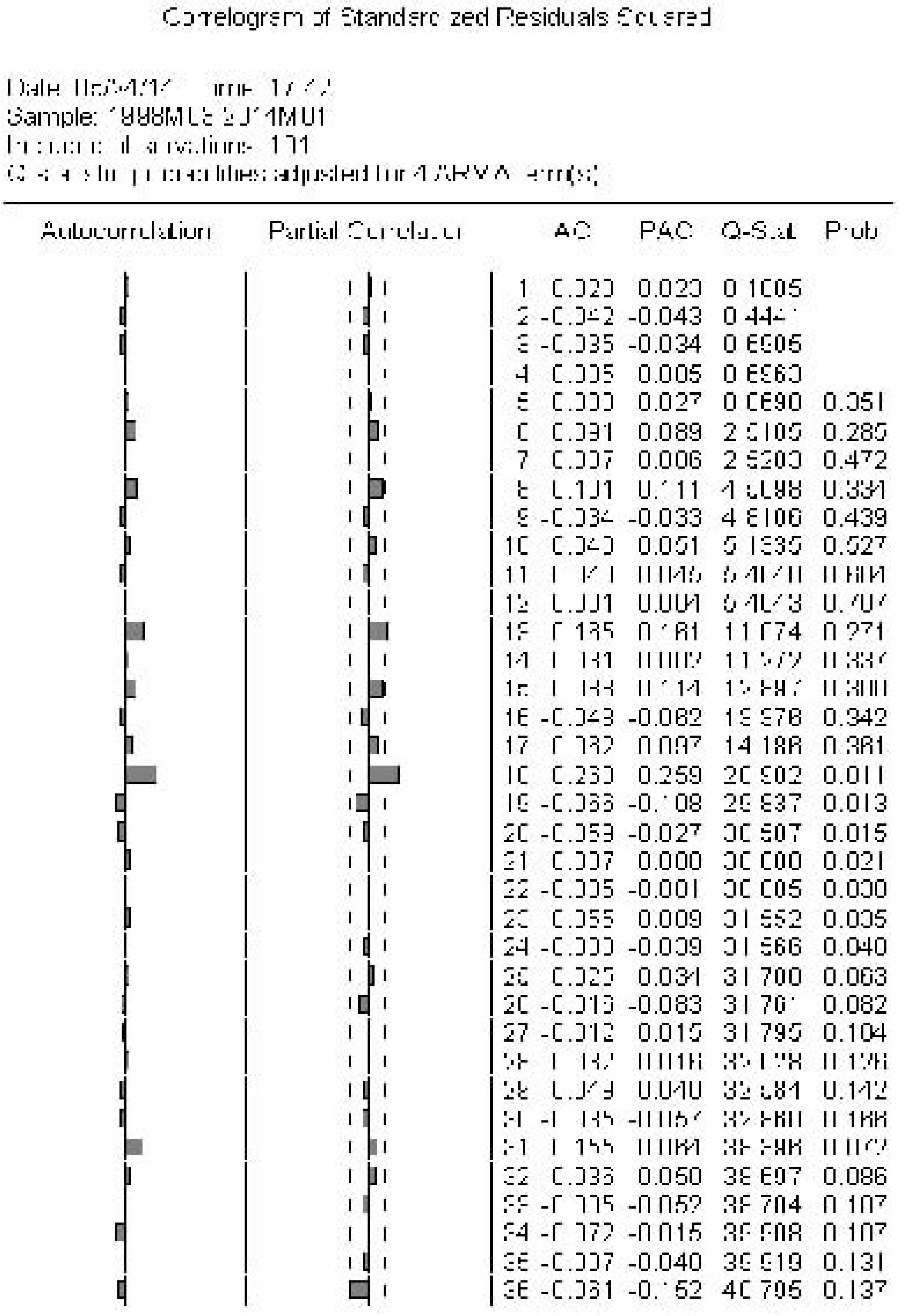
F-statistic	1.713183	Probability	0.192162
Obs*R-squared	1.715760	Probability	0.190240

Test Equation:
Dependent Variable: STD_RESID^2
Method: Least Squares
Date: 06/09/14 Time: 15:34
Sample (adjusted): 1998M03 2014M01
Included observations: 191 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	2.283416	0.419341	5.445248	0.0000
STD_RESID^2(-1)	0.094749	0.072389	1.308886	0.1922

R-squared	0.008983	Mean dependent var	2.521989
Adjusted R-squared	0.003740	S.D. dependent var	5.229094
S.E. of regression	5.219307	Akaike info criterion	6.153023
Sum squared resid	5148.581	Schwarz criterion	6.187078
Log likelihood	-585.6136	F-statistic	1.713183
Durbin-Watson stat	2.000569	Prob(F-statistic)	0.192162

Table A19: Ljung-Box test for Standardized Residuals from Volatility Model for sliver Price Return series.



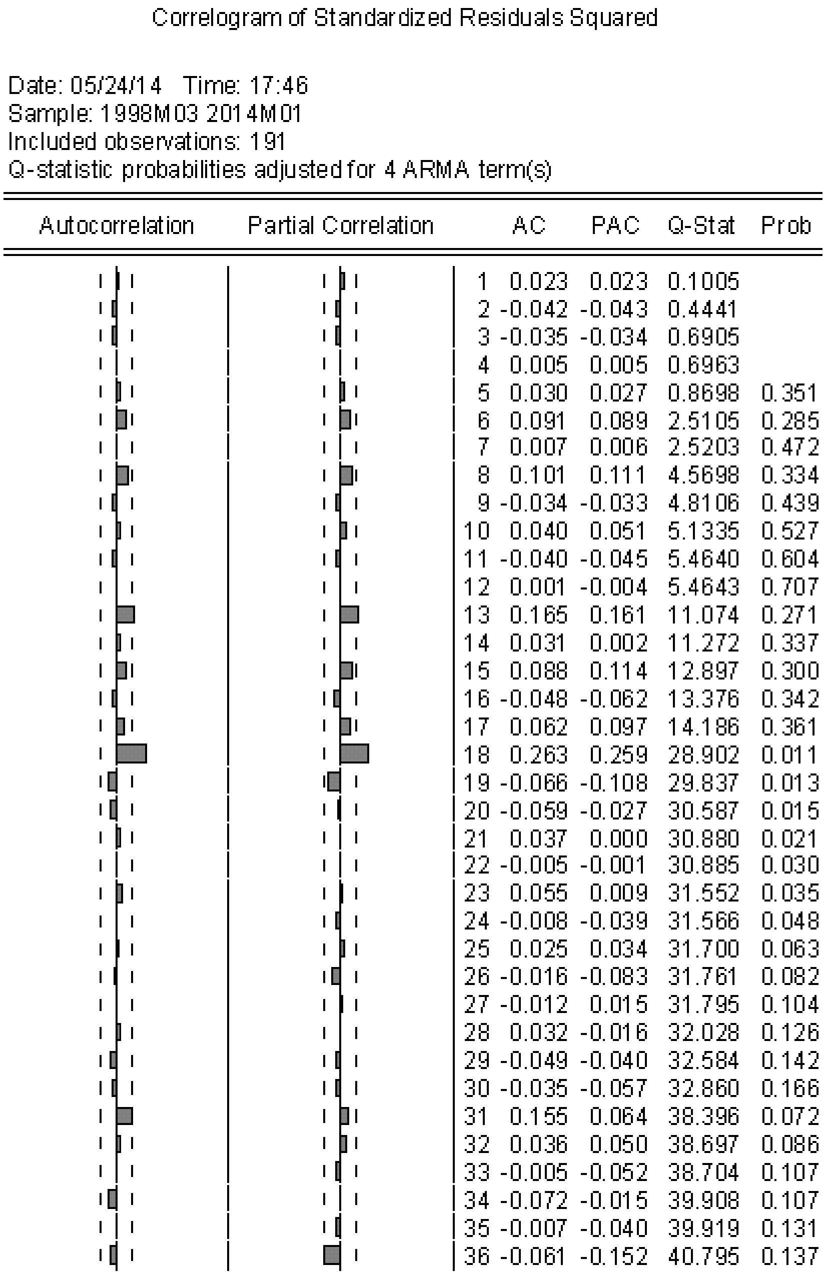
```
Correlogram of Standardized Residuals Squared

Date: [illegible]   Time: [illegible]
Sample: 1988M03 2014M01
Included observations: [illegible]
Q-statistic probabilities adjusted for 4 ARMA term(s)

Autocorrelation   Partial Correlation        AC      PAC      Q-Stat   Prob

                                       1   0.020   0.020   0.1005
                                       2  -0.042  -0.043   0.4441
                                       3  -0.035  -0.034   0.8506
                                       4   0.005   0.005   0.8563
                                       5   0.000   0.027   0.8690   0.351
                                       6   0.091   0.089   2.5105   0.285
                                       7   0.007   0.006   2.5203   0.472
                                       8   0.101   0.111   4.5898   0.331
                                       9  -0.034  -0.033   4.8106   0.439
                                      10   0.040   0.051   5.1335   0.527
                                      11 [illegible] 0.046 [illegible] [illegible]
                                      12   0.001   0.004 [illegible] [illegible]
                                      13   0.135 [illegible] 11.074  0.271
                                      14 [illegible] [illegible] [illegible] [illegible]
                                      15 [illegible] [illegible] [illegible] [illegible]
                                      16  -0.043  -0.062   13.378   0.342
                                      17   0.032   0.097   14.188   0.361
                                      18   0.230   0.259   29.502   0.011
                                      19  -0.033  -0.108   29.837   0.013
                                      20  -0.053  -0.027   30.507   0.015
                                      21   0.007   0.000   30.600   0.021
                                      22  -0.005  -0.001   30.605   0.030
                                      23   0.055   0.009   31.552   0.035
                                      24  -0.000  -0.039   31.566   0.040
                                      25   0.020   0.034   31.700   0.063
                                      26  -0.010  -0.083   31.701   0.082
                                      27  -0.012   0.015   31.795   0.104
                                      28 [illegible] 0.018 [illegible] [illegible]
                                      29   0.043   0.040   32.681   0.142
                                      30 [illegible] [illegible] [illegible] [illegible]
                                      31 [illegible] [illegible] [illegible] [illegible]
                                      32   0.033   0.050   36.807   0.086
                                      33  -0.055  -0.052   37.704   0.107
                                      34  -0.072  -0.015   39.508   0.107
                                      35  -0.007  -0.040   39.519   0.131
                                      36  -0.051  -0.152   40.705   0.137
```

Table A20: ARCH effect test using Ljung-Box test for squared residuals of the fitted volatility model for silver.



```
Correlogram of Standardized Residuals Squared

Date: 05/24/14   Time: 17:46
Sample: 1998M03 2014M01
Included observations: 191
Q-statistic probabilities adjusted for 4 ARMA term(s)

Autocorrelation   Partial Correlation      AC     PAC   Q-Stat   Prob

                                      1  0.023  0.023  0.1005
                                      2 -0.042 -0.043  0.4441
                                      3 -0.035 -0.034  0.6905
                                      4  0.005  0.005  0.6963
                                      5  0.030  0.027  0.8698  0.351
                                      6  0.091  0.089  2.5105  0.285
                                      7  0.007  0.006  2.5203  0.472
                                      8  0.101  0.111  4.5698  0.334
                                      9 -0.034 -0.033  4.8106  0.439
                                     10  0.040  0.051  5.1335  0.527
                                     11 -0.040 -0.045  5.4640  0.604
                                     12  0.001 -0.004  5.4643  0.707
                                     13  0.165  0.161  11.074  0.271
                                     14  0.031  0.002  11.272  0.337
                                     15  0.088  0.114  12.897  0.300
                                     16 -0.048 -0.062  13.376  0.342
                                     17  0.062  0.097  14.186  0.361
                                     18  0.263  0.259  28.902  0.011
                                     19 -0.066 -0.108  29.837  0.013
                                     20 -0.059 -0.027  30.587  0.015
                                     21  0.037  0.000  30.880  0.021
                                     22 -0.005 -0.001  30.885  0.030
                                     23  0.055  0.009  31.552  0.035
                                     24 -0.008 -0.039  31.566  0.048
                                     25  0.025  0.034  31.700  0.063
                                     26 -0.016 -0.083  31.761  0.082
                                     27 -0.012  0.015  31.795  0.104
                                     28  0.032 -0.016  32.028  0.126
                                     29 -0.049 -0.040  32.584  0.142
                                     30 -0.035 -0.057  32.860  0.166
                                     31  0.155  0.064  38.396  0.072
                                     32  0.036  0.050  38.697  0.086
                                     33 -0.005 -0.052  38.704  0.107
                                     34 -0.072 -0.015  39.908  0.107
                                     35 -0.007 -0.040  39.919  0.131
                                     36 -0.061 -0.152  40.795  0.137
```

Table A21: ARCH effect test using LM test for squared residuals of the fitted volatility model for silver.

```
ARCH Test:
==========================================================================
F-statistic               0.055561    Prob. F(1,188)                0.8139
Obs*R-squared             0.056136    Prob. Chi-Square(1)           0.8127
==========================================================================
Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 06/05/14   Time: 10:15
Sample (adjusted): 1998M04 2014M01
Included observations: 190 after adjustments
==========================================================================
        Variable          Coefficient    Std. Error    t-Statistic   Prob.
==========================================================================
           C               1.018282      0.147140      6.920517     0.0000
     WGT_RESID^2(-1)       0.017192      0.072935      0.235715     0.8139
==========================================================================
R-squared                 0.000295    Mean dependent var          1.036106
Adjusted R-squared       -0.005022    S.D. dependent var          1.735488
S.E. of regression        1.739841    Akaike info criterion       3.955935
Sum squared resid         569.0847    Schwarz criterion           3.990114
Log likelihood           -373.8138    Hannan-Quinn criter.        3.969780
F-statistic               0.055561    Durbin-Watson stat          1.999145
Prob(F-statistic)         0.813911
==========================================================================
```

Annex B: Figures

Figure B1: Time series plot of monthly price of gold and silver data in Ethiopia market over the sample period.

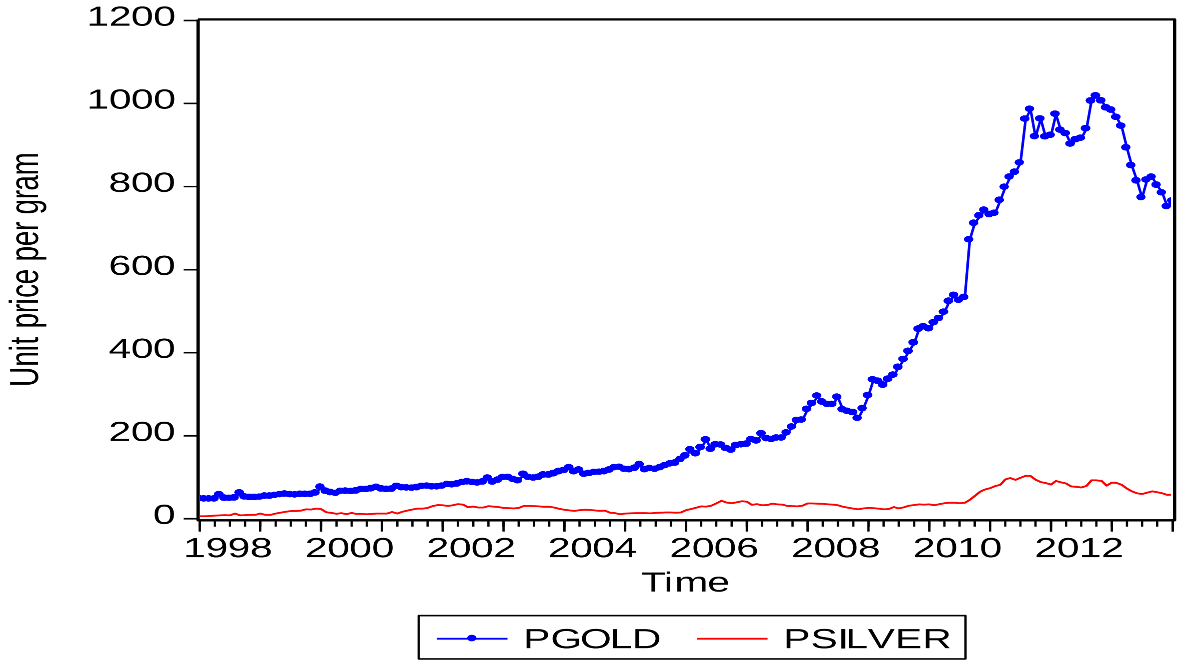


Figure B2: Sample ACF and PACF of natural log returns for price of gold in Ethiopian market.

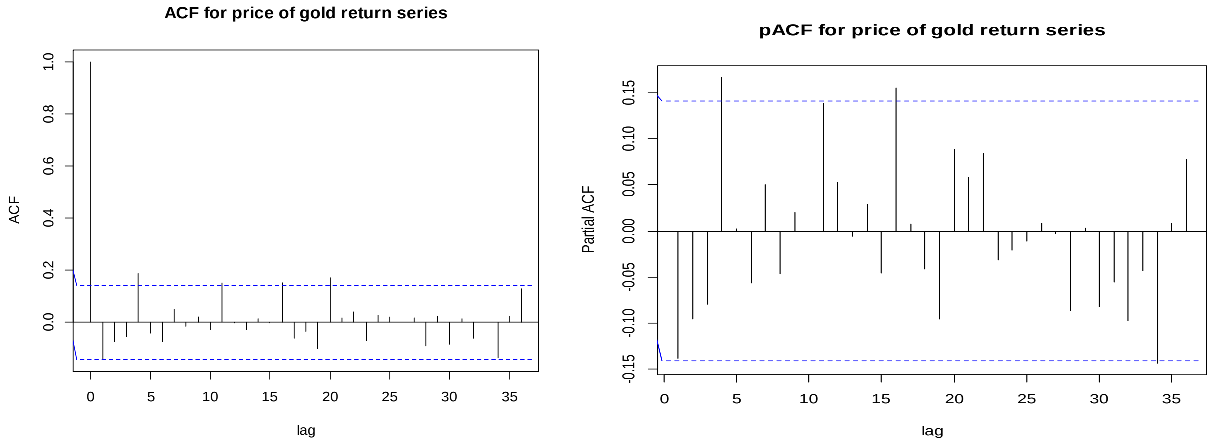


Figure B3: Sample ACF and PACF of natural log returns for price of silver in Ethiopian market.

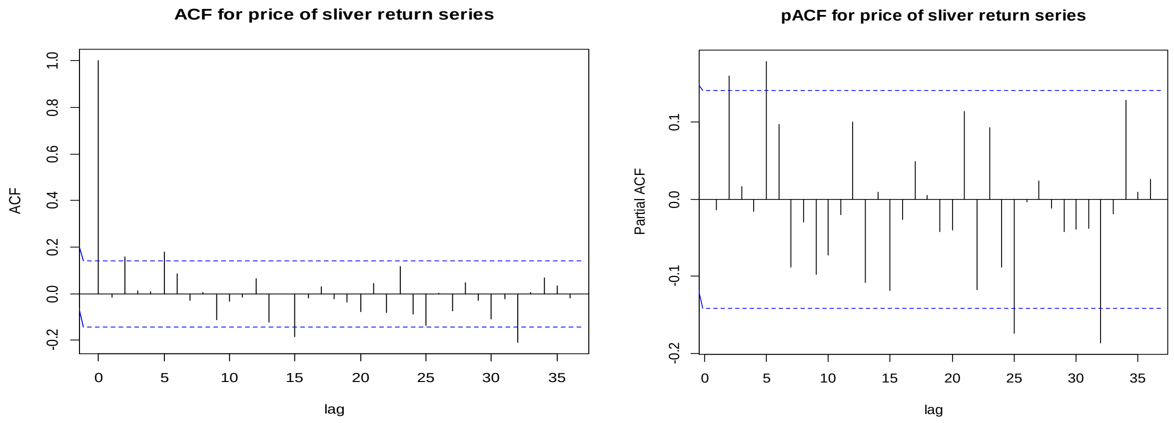
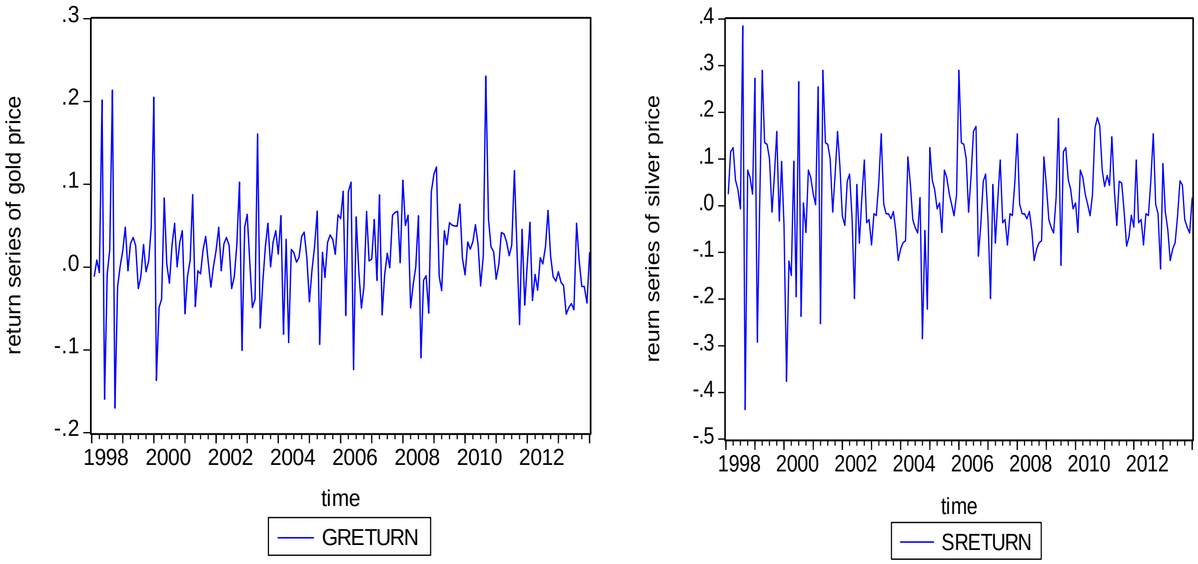
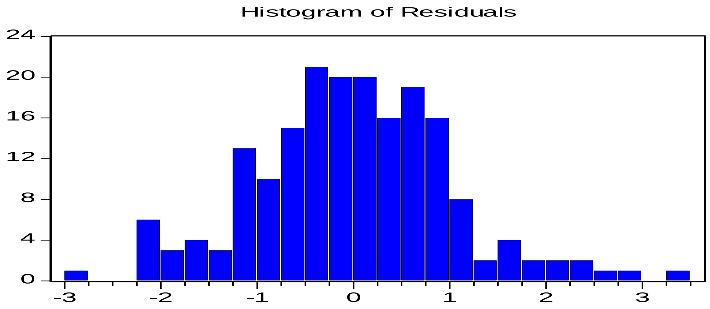


Figure B4: Plot of the natural log return series under consideration.



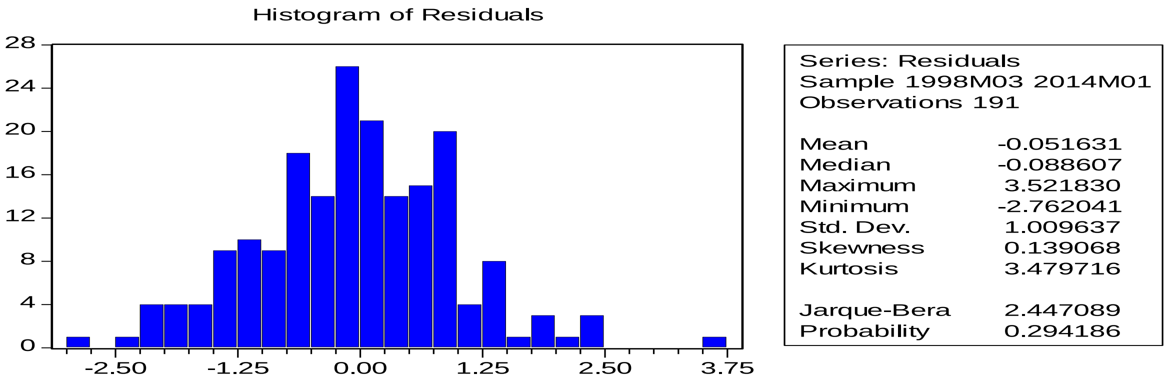
FigureB5: Histogram of Residuals and Diagnostic test for normality of Residuals from ARMA (0, 1) model for gold price return series.



Series: Residuals
Sample 1998M03 2014M01
Observations 191

Mean	0.004336
Median	-0.002145
Maximum	3.349814
Minimum	-2.760034
Std. Dev.	1.010462
Skewness	0.202164
Kurtosis	3.641932
Jarque-Bera	4.556498
Probability	0.102463

Figure B6: Histogram of Residuals and Diagnostic test for normality of Residuals from ARMA (1, 3) model for sliver price log return series.

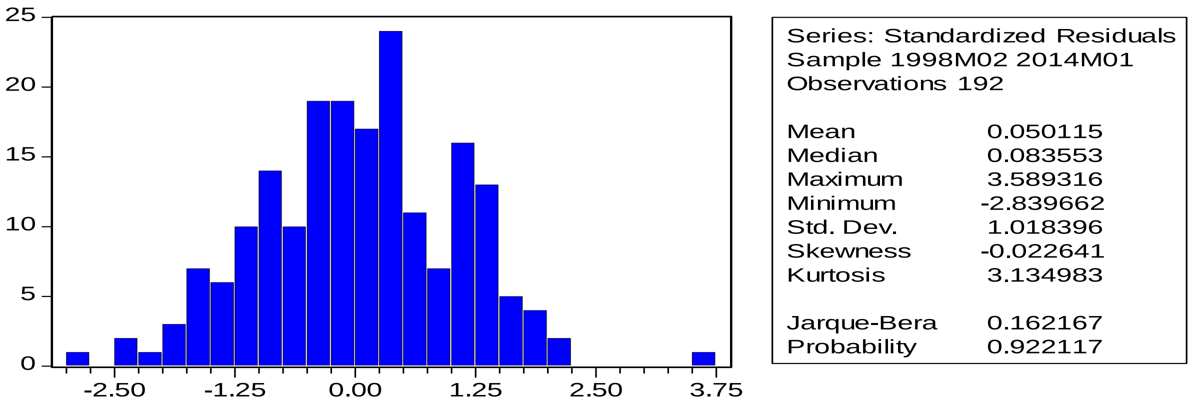


```
Series: Residuals
Sample 1998M03 2014M01
Observations 191

Mean          -0.051631
Median        -0.088607
Maximum        3.521830
Minimum       -2.762041
Std. Dev.      1.009637
Skewness       0.139068
Kurtosis       3.479716

Jarque-Bera    2.447089
Probability    0.294186
```

FigureB7: Histogram of residuals standardized residuals for test of normality of Residuals from the fitted model for price of gold natural log return series.

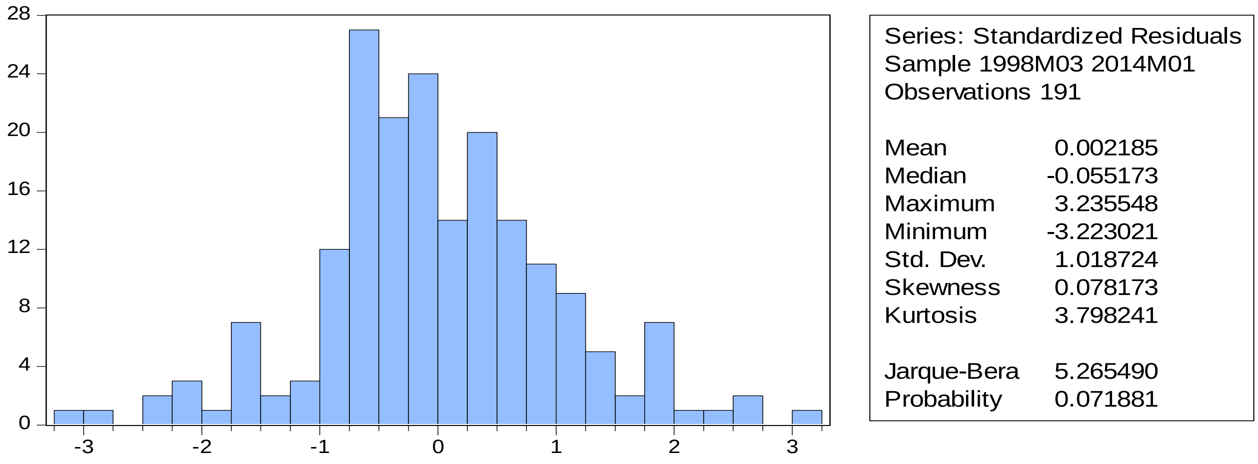


```
Series: Standardized Residuals
Sample 1998M02 2014M01
Observations 192

Mean           0.050115
Median         0.083553
Maximum        3.589316
Minimum       -2.839662
Std. Dev.      1.018396
Skewness      -0.022641
Kurtosis       3.134983

Jarque-Bera    0.162167
Probability    0.922117
```

FigureB8: Histogram of residuals standardized residuals for test of normality of Residuals from the fitted model for price of sliver natural log return series.



```
Series: Standardized Residuals
Sample 1998M03 2014M01
Observations 191

Mean           0.002185
Median        -0.055173
Maximum        3.235548
Minimum       -3.223021
Std. Dev.      1.018724
Skewness       0.078173
Kurtosis       3.798241

Jarque-Bera    5.265490
Probability    0.071881
```