

SIMULATION OF THE EFFECT OF RANDOM
GEOPOTENTIAL HEIGHT DISTURBANCE ON AIR
FLOW ON A BAROTROPIC ATMOSPHERE USING
COARSE RESOLUTION GRID

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By
Abera Debebe

Addis Ababa, Ethiopia

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ADDIS ABABA UNIVERSITY
FACULTY OF SCIENCE
DEPARTMENT OF PHYSICS

The undersigned hereby certify that they have read and recommended to the Faculty of Science School of Graduate Studies for acceptance a thesis entitled “**Simulation of the effect of random geopotential height disturbance on air flow on a barotropic atmosphere using coarse resolution grid** ” by **Abera Debebe** in partial fulfillment of the requirements for the degree of **Master of Science in Physics**.

Name

Signature

Dr. Gizaw Mengistu, Advisor

Dr. Esayas Belay, Examiner

Dr. Tilahun Tesfaye, Examiner

Dedicated

to

my parents.

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Addis Ababa, Ethiopia

Abera Debebe

Abstract

The atmosphere is chaotic in nature and full of random processes. This random process affects the atmospheric field variables directly or indirectly. In this thesis, to identify the effect of random height disturbance, linearized shallow water equation is used as a model. The equation is solved numerically using finite difference method on nonstaggered Arakwan grid A . When the geopotential height is randomly disturbed, the initial distribution of the velocity field creates a wave of a standing type that consists of an enhanced and diminished pattern depending on the magnitude of the perturbation. However in the presence of mean flow, the perturbed system and the initial distribution of the field variables are carried away based on the direction of the mean flow.

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Introduction

The earth's atmosphere is a natural laboratory in which a wide variety of physical and chemical processes take place and hence it is too complex to explain not only in process but also in composition and property.

Unlike laboratory physicist, atmospheric researchers can not perform controlled experiments on the large scale atmosphere. The standard scientific method cannot be applied directly . In this case models are developed following atmospheric phenomena by including significant processes. Most of the time, these models are formulated in terms of mathematical equations and the experiments are performed by solving the model equations under various conditions and interpreting the solution in terms of physical behavior. However, the model does not represent all the process that takes place in the atmosphere, it depends on the temporal and spatial scale [1].

The atmosphere is not uniform from the surface of the earth on the way to the outer space; rather it is divided into various layers based on physical properties like temperature, composition, and pressure. The non-homogeneity of the atmosphere leads to the existence of various processes at different places and forbids to use a single model for the whole of the atmosphere. This shows that various models are required for variuos locations depending on the temporal and spatial scales.

The behavior of the atmosphere is governed by a set of conservative equations. These conservative equations are derived from a number of interrelated field variables like temperature, density, pressure, and velocity using physical principles.

One of the motion type observed in the atmosphere is oscillatory (wave). Those waves are created either by compressibility of the air, or topography of the earth, or convection of the air, or thunderstorm, or energy difference based on the magnitude of the causes. In general, all those obstacles together create a spectrum of atmospheric wave from the small scale sound wave to the large scale planetary wave. Various parts of the atmosphere communicate with each other due to the presence of these waves.

The wave type depends on the type of perturbation that creates it. The paper published in 1965 by Akira and Kasahara by using a circular obstacle in the laboratory, identify the nature of the wave as a function of time [2]. They identified that the air moving over the obstacle from the west create oscillation but not the one from the east [2]. The interaction of moving air over the stationary "swell" of the surface of water was also studied and it is identified that the swell create a moving wave pattern [3].

Apart from the above and other study on the atmospheric wave formation by disturbances, the purpose of our study is to identify the effect of random geopotential height disturbance on the atmospheric air flow on large scale with time. The study aims to answer the question "How does a random geopotential height disturbance affect atmospheric motion with time?"

To this end, first the atmospheric governing equations are developed from the conservation of matter (mass), conservation of momentum, and conservation of energy

for barotropic inviscid atmosphere. With the help of scale analysis, unwanted type of motions are filtered and discarded to find the shallow water equation. The derivation of the equation is available in any standard atmospheric textbooks and has not been dealt in depth. These are given in Chapter 1.

In order to analyze the effect of the disturbance, atmospheric equations have to be solved. The model equations are solved numerically using finite difference method. The stability condition of the numerical solution, the type of grid used, and the error that exist in the numerical solution are discussed in the second Chapter.

The effect of disturbance on the atmospheric motion is investigated through the presence of stochastic process in the dynamics. This part focuses on the solution of the shallow water equation initialized with the random disturbance. In general, in this section attention is given to study the role of the stochastic initial condition on the shallow water equation. This is done in such a way that the random process is introduced into the system as the initial values of the system stochastically from which the motion of the perturbed system evolves. This is covered in the third chapter of this thesis. The Results of the study of the numerical analysis is presented and discussed in Chapter 4.

Finally, summary and conclusion along with outlook on implication of this work for future direction are given in chapter 5.

Chapter 1

Dynamics of the atmosphere

1.1 Introduction

The mathematical models of the atmospheric motions are derived based on conservation principles. The conservation principles that are considered in this thesis are conservation of mass, momentum, and energy. Large scale motions of the atmosphere are influenced by the distribution of land mass and water bodies [4].

To understand the dynamical influence of mountain barriers, a number of laboratory experiments were carried out in Chicago University by a number of researchers [2]. They investigate nearly two-dimensional motions of a homogeneous fluid around an obstacle on rotating hemispherical shells with the obstacle and the fluid in between. In 1952 it was stated that when a westerly current flows past the obstacle, the fluid goes into a series of oscillations extending around a whole globe. On the contrary, an easterly current flows past the obstacle does not oscillate and is little disturbed by the obstacle [2]. To understand and analyze this fact computationally, the model equations have to be derived and solved to crosscheck with the laboratory results. Because of this reason, this chapter gives due attention to the discussion of the governing

equations for all types of atmospheric motion.

The dynamical equations derived based on the three conservation principles contain all information from the simplest acoustic wave to the planetary wave. This demands, scale analysis to be applied to the equations to filter out and to remove the unnecessary wave from the meteorological important large scale waves.

The derivations are available in different meteorology and atmospheric textbooks and therefore we have taken the final equations [1,4].

1.2 Basic conservation laws in the atmosphere

As stated earlier atmosphere is a fluid or continuum in which a wide variety of flows and processes take place. These differences come from either energy content, or material content, or momentum content. Since the atmosphere is too wide and complex, it is impossible to study as a whole. However, one can consider a small element of the atmosphere (parcel of air) in the same way as point particle in classical mechanics to develop the governing equation.

The motion of the atmosphere can be modelled by deriving mass, energy and momentum equations. From these three principles, there are five basic variables namely: three component of velocity (i.e. zonal, meridional and vertical velocities), and any two of thermodynamic variables (i.e. temperature, pressure, density, entropy etc). For these five variables, there are five corresponding equations; three component of momentum, continuity, and energy (first law of thermodynamics) equations. For the turbulent-free atmosphere, it is quite possible to derive equal number of equations

with that of the unknown so that the equations become closed. On the contrary, if the atmospheric turbulence is taken into consideration, especially in the planetary boundary layer or over the equator, or when a small scale is considered, then the number of unknown exceeds the number of equation [5].

For the mid latitude free troposphere synoptic scale, tackling the problem by neglecting the turbulent effect is an appropriate approximation. All the subsequent equations in this thesis are taken based on this assumption [6, 4].

1.2.1 Conservation of mass

One of the three fundamental conservation principles, used to derive equation for the atmospheric motion, is the conservation of matter (mass). The equation developed based on this principle is continuity equation. By using the Eulerian frame of reference, conservation of mass is applied using the principle "mass is neither created nor destroyed" for classical systems. At this stage there is no difficulty whether or not the atmosphere is compressible.

Considering flow of matter along the three directions with velocity components u (eastward), v (northward), and w (upward), one can arrive at the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \quad (1.2.1)$$

where $\vec{V} = ui + vj + wk$.

This equation shows that the sum of the local rate of change of density and mass divergence is zero. Using the chain rule the above equation can be written as

$$\frac{\partial \rho}{\partial t} + \vec{V} \cdot \nabla \rho + \rho \nabla \cdot \vec{V} = 0 \quad (1.2.2)$$

The second term is the advection term which is negative for air flow from higher to lower density and tells that the addition or removal of property to or from region of interest. Rearranging further equation (1.2.2) gives the continuity equation in the form of velocity divergence as

$$\frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \vec{V} = 0 \quad (1.2.3)$$

where $\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$.

This equation states that the fractional rate of increase of density following the motion (Lagrangian frame) of the air parcel is equal to velocity convergence. The equation holds true for conditions in which there are no chemical reaction, molecular diffusion as a sink and source of species [7]. However, for the large scale non-planetary boundary layer and below almost 100 *km* altitude the equation is quite sufficient description [4].

1.2.2 Conservation of momentum

Atmospheric motions are governed by Newton's second law of motion which is stated as "the rate of change of momentum of an object equals the sum of all the forces acting on it". Even if Newton's laws of motion are applicable in all inertial reference frame, they need some modification to make them applicable in non-inertial reference frame. This modification evolves the addition of some fictitious forces. The equation developed based on this principle is termed as Navier-Stoke's equation. The above

statement of Newton's law for the air parcel per unit mass in non-inertial earth's reference frame can be summarized as

$$\frac{D\vec{V}}{Dt} = \sum F \quad (1.2.4)$$

From this equation, the left hand side is the rate of change of absolute velocity following the motion as viewed in an inertial reference frame whereas the right hand side of the equation is the sum of all real and virtual forces acting on the air parcel. The forces can be categorized as body and surface forces. Using those body and surface forces and by taking the angular velocity of the earth as constant, the momentum equation for the neutral atmosphere in vector form becomes

$$\frac{D\vec{V}}{Dt} = -2\vec{\Omega} \times \vec{V} - \Omega^2 \vec{r} - \vec{g} - \frac{1}{\rho} \nabla P + \nu \nabla^2 \vec{V} \quad (1.2.5)$$

where ρ is density, ν is kinetic coefficient of friction, and Ω is angular velocity of the earth.

The term $(-2\vec{\Omega} \times \vec{V})$ is the deflecting Coriolis force due to the rotation of the earth that the atmosphere experiences. The second term $(\Omega^2 \vec{r})$ is the centrifugal force which arises due to the spherical shape of the earth and acts out ward from the axis of rotation of the earth. The fourth term $(-\frac{1}{\rho} \nabla P)$ is the pressure gradient force acting on the surface which arises as a result of pressure difference. The last term $(\nu \nabla^2 \vec{V})$ is the frictional force because of the normal and tangential stress on the surface. The pressure gradient and frictional forces are surface forces and the rest are body forces. Finally the term from the left hand side $(\frac{D\vec{V}}{Dt})$ is the sum of the local rate of change of velocity and the advection term.

1.2.3 Conservation of energy

The third conservation principle that we are going to discuss in this chapter is the conservation of energy described in terms of first law of thermodynamics. The transformation of heat energy into various forms of mechanical energy is the process that determine atmospheric motion.

From the first law of thermodynamics, conservation of energy for a system at rest in thermodynamic equilibrium is stated as "the change in internal energy of the system (the air parcel in this case) is equal to the difference between the heat added to the system and the work done by the system". The internal energy of the air is the energy due to the random motion, rotation and internal vibration of the molecules. In equation form, since temperature is the measure of the internal energy of a system, and hence the expression for dry air for an infinitesimal change at rest is given as

$$dU = C_v dT \quad (1.2.6)$$

where C_v is specific heat capacity of air at constant volume, U is internal energy and T is temperature.

Even if the first law of thermodynamics is stated for a system at rest, it also applies to a system in motion like the atmosphere. Considering the Lagrangian frame, the air parcel can not be in thermodynamic equilibrium with the environment but still the first law of thermodynamics is applicable in such a way that the thermodynamic energy is given as the sum of the internal energy and the kinetic energy

$$E_{th} = U + K_E \quad (1.2.7)$$

Here, the kinetic energy is due to the macroscopic motion of the air parcel.

However, as long as there is an exchange of energy among the air parcel and the environment, for long time scale, the rate of change of the total thermodynamic energy is equal to the sum of the rate of the diabatic heating (due to radiation, conduction, and latent heat release) and the rate at which work is done by all the forces. In equation form, it can be expressed as

$$\frac{dE_{th}}{dt} = \frac{dq}{dt} + \frac{dW}{dt} \quad (1.2.8)$$

where the first term from the right hand side is rate of diabatic heating and the second term on the same side is rate of work done. All the forces that do work, as stated under conservation of momentum, are the body and surface forces.

By using the air parcel as a system and based on the above equations for dry air, one can arrive at the equation of the first law of thermodynamics in differential form

$$C_p \frac{DT}{Dt} - \alpha \frac{DP}{Dt} = J \quad (1.2.9)$$

where C_p is the specific heat capacity at constant pressure, α is specific volume, and J is the diabatic heating

The main physical processes contributing to the diabatic heating in the lower and middle atmosphere are the latent heating from condensation, and latent cooling from evaporation of water vapor, and radiation heating from absorption and radiation cooling from emission of electromagnetic radiation. Rearranging equation (1.2.9) further gives

$$\begin{aligned} C_p \frac{D \ln T}{Dt} - R \frac{D \ln P}{Dt} &= \frac{J}{T} \\ \frac{D}{Dt} (C_p \ln T - R \ln P) &= \frac{J}{T} \end{aligned} \quad (1.2.10)$$

where R is the universal gas constant.

For an ideal gas undergoing an adiabatic process (*i.e* when the time scale of the motion is much less than the time required for heat transfer), the thermodynamic energy equation becomes

$$C_p D \ln T - R D \ln P = 0 \quad (1.2.11)$$

For such an adiabatic process it is not temperature that is conserved but potential temperature(θ). From equation (1.2.11), potential temperature is given as

$$\theta = T \left(\frac{P_o}{P} \right)^{\frac{R}{C_p}} \quad (1.2.12)$$

where P_o is pressure on the surface.

Differentiating equation (1.2.12) with respect to time and equating with equation (1.2.10) gives the final expression of the energy equation in terms of potential temperature (θ), temperature (T), and diabatic heating rate (J) as

$$C_p \frac{D \ln \theta}{Dt} = \frac{J}{T} \quad (1.2.13)$$

1.3 Momentum equation in spherical coordinate

Since the large scale motion of the atmosphere follows the shape of the earth, it is quit logical to express the equation in spherical coordinate. In addition to this the vector form of the conservative equations, are not handy for theoretical and numerical analysis, and hence it is necessary to expand into scalar components on a spherical coordinate. Even though the earth is not a perfect sphere, considering it as a perfect one is an acceptable approximation.

To expand the momentum equation at a point on the surface of the earth, it can be identified by the coordinates (r, ϕ, λ) where r is the distance from the center of the earth given as the sum of the radius of the earth (a) and the height (z), ϕ is latitude, and λ is longitude as given in the schematic diagram in Fig. 1.1.

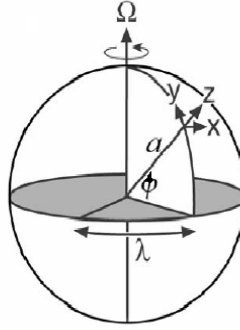


Figure 1.1: Spherical coordinate

Since the Earth is in rotational motion, unit vectors i , j and k pointing northward, eastward, and vertically upward respectively, are not constant with respect to time. With this information we can arrive at the momentum equation on a spherical coordinate system given below:

$$\frac{Du}{Dt} = \frac{uv \tan \phi}{r} - \frac{wu}{r} - 2\Omega(w \cos \phi - v \sin \phi) - \frac{1}{\rho r \cos \phi} \frac{\partial p}{\partial \lambda} + \nu \nabla^2 u \quad (1.3.1)$$

$$\frac{Dv}{Dt} = \frac{-u^2 \tan \phi}{r} - \frac{wv}{r} - 2\Omega u \sin \phi - \frac{1}{\rho r} \frac{\partial P}{\partial \phi} + \nu \nabla^2 v \quad (1.3.2)$$

$$\frac{Dw}{Dt} = \frac{u^2 + v^2}{r} + 2u\Omega \cos \phi - \frac{1}{\rho} \frac{\partial P}{\partial z} - g + \nu \nabla^2 w \quad (1.3.3)$$

Those equations are the eastward, northward, and upward momentum equations respectively. The total derivative in spherical coordinate is given as

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{u}{r \cos \phi} \frac{\partial}{\partial \lambda} + \frac{v}{r} \frac{\partial}{\partial \phi} + w \frac{\partial}{\partial z}$$

1.4 Scale analysis of the atmospheric governing equations

Scaling is important for estimating the magnitude of various terms in the governing equations according to a particular type of motion. This concerns whether the scaling is for small scale motion, or medium scale motion, or large scale motion. It is performed at least based on the following key points:

- magnitude of the field variables
- amplitude of fluctuation in the field variables
- characteristics of length, depth, and time scales on which the fluctuation occurs

Scaling is applied not only to simplify the mathematics but also to filter the meteorological important waves. The equation given so far in this thesis describe all types of motions from the small scale sound wave to large scale planetary wave. To filter out this unwanted type of motion, we can scale these three equations based on the universally accepted scale values. Some of the scale values, and constants for the mid latitude synoptic scale (motions on a scales of hundreds to thousands of kilometers) are given in Table 1.1.

Scale	Symbol	Typical magnitude
Horizontal scale	L	1000 <i>km</i>
vertical scale	H	10 <i>km</i>
horizontal velocity(u,v)	U	10 <i>m/s</i>
vertical velocity (w)	W	1 <i>cm/s</i>
time scale(<i>L/U</i>)	T	10 ⁵ sec
surface density	ρ	1 <i>kg/m³</i>
acceleration due to gravity	g	10 <i>m/s²</i>
coriolis parameter(2Ω)	f	10 ⁻⁴ 1/s
earth's radius	a	6400 <i>km</i>

Table 1.1: Standard values of some constants and scales

In addition to those given values, the molecular friction is so small except near the surface of the earth within few centimeter and above 100 *km* altitude and hence it is neglected [8,1,4]. In the following analysis we use the above values and some other assumptions as the situation requires.

1.4.1 Parametrization of continuity equation

In addition to the values given, we can analyze based on the fact that any field variables can be expressed as the sum of the basic part (the average value) and the deviation part (undetermined value). Therefore the density can be expressed as

$$\rho(x, y, z, t) = \rho_o(z) + \rho'(x, y, z, t) \quad (1.4.1)$$

As can be seen from the equation, the average value depends only on the vertical coordinate and the deviation part depends on all space coordinate and time. Substituting this in the equation (1.2.3) gives

$$\frac{1}{(\rho_o + \rho')} \frac{D}{Dt} (\rho_o(z) + \rho'(x, y, z, t)) + \nabla \cdot \vec{V} = 0$$

$$\frac{1}{\rho_o(1 + \frac{\rho'}{\rho_o})} \left(\frac{D\rho'}{Dt} + \frac{D\rho_o}{Dt} \right) + \nabla \cdot \vec{V} = 0$$

This reduces to

$$\frac{1}{\rho_o} \left(\frac{\partial \rho'}{\partial t} + \vec{V} \cdot \nabla \rho' \right) + \frac{w}{\rho_o} \frac{D\rho_o}{Dz} + \nabla \cdot \vec{V} = 0$$

Since the deviation is too small compared to the basic state and the ratio is on the order of

$$\begin{aligned} \frac{\rho'}{\rho_o} &\sim 10^{-2} \ll 1 \\ \frac{\rho' U}{\rho_o L} &\approx 10^{-7} \\ \frac{W}{H} &\approx 10^{-6} \end{aligned}$$

and

$$\frac{U}{L} \approx 10^{-5}$$

the analysis leads to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + w \frac{D \ln \rho_o}{Dz} = 0 \quad (1.4.2)$$

In vector form (1.4.2) can be written as

$$\nabla \cdot (\rho_o \vec{V}) = 0$$

This shows that for the synoptic scale motion, the mass flux computed using the basic state density is not divergent. In addition to this, for purely horizontal motion, the atmosphere is incompressible (*i.e.* when $w = 0$), the last term of equation (1.4.2) becomes zero. In general for any situation, if the atmosphere is considered as incompressible which is especially true for the synoptic and mesoscales (scales of tens to hundreds of kilometers), then the divergence of velocity is zero.

1.4.2 Scale analysis of momentum equation

The procedure that we follow here is similar to the scale analysis of the continuity equation. Using the value of constants and scale values given in Table 1.1, the horizontal and vertical equation of motion are given in Table 1.2.

$\frac{Du}{dt} =$	$2\Omega v \sin \phi$	$-2\Omega w \cos \phi$	$\frac{-uw}{r}$	$\frac{uv \tan \phi}{r}$	$\frac{-1}{\rho \cos \phi r} \frac{\partial p}{\partial \lambda}$	$\nu \nabla^2 \vec{u}$
$\frac{Dv}{dt} =$	$-2\Omega u \sin \phi$	0	$\frac{-vw}{r}$	$-\frac{u^2 \tan \phi}{r}$	$\frac{-1}{\rho r} \frac{\partial p}{\partial \phi}$	$\nu \nabla^2 \vec{v}$
$\frac{u^2}{L} =$	$f_o U$	$f_o w$	$\frac{Uw}{a}$	$\frac{U^2}{a}$	$\frac{\delta P}{\rho L}$	$\frac{\nu U}{H^2}$
$10^{-4} =$	10^{-3}	10^{-6}	10^{-8}	10^{-5}	10^{-3}	10^{-12}

Table 1.2: Scale analysis of the horizontal momentum equation

As shown from the analysis, the value of frictional force is too small compared with the other terms which indicates that neglecting the frictional force in all air motion except the boundary layer is a good approximation. Neglecting terms smaller than the total acceleration gives the scaled momentum equation suitable for the mesoscale and synoptic scale as

$$\frac{Du}{Dt} = 2v \sin \phi - \frac{1}{r \rho \cos \phi} \frac{\partial P}{\partial \lambda} \quad (1.4.3)$$

$$\frac{Dv}{Dt} = -2u \sin \phi - \frac{1}{\rho r} \frac{\partial P}{\partial \phi} \quad (1.4.4)$$

The vertical component of momentum equation can also be filtered in a similar fashion as shown in Table 1.3.

$\frac{Dw}{Dt} =$	$2\Omega u \cos \phi$	$\frac{u^2+v^2}{r}$	$\frac{-1}{\rho} \frac{\partial P}{\partial z}$	$-g$	$\nu \nabla^2 w$
$\frac{UW}{L}$	$f_o U$	$\frac{U^2}{a}$	$\frac{P_o}{\rho H}$	g	$\frac{\nu W}{H^2}$
10^{-7}	10^{-3}	10^{-5}	10	10	10^{-15}

Table 1.3: Scale analysis of the vertical momentum equation

From Table 1.3, except the pressure gradient force and the effective gravity force (the sum of gravity and centrifugal forces), all are negligible. This leads to hydrostatic balance equation

$$\frac{\partial P}{\partial z} = -\rho g \quad (1.4.5)$$

1.4.3 Scale analysis of energy equation

The potential temperature can be written as the sum of the basic part and the deviation part

$$\theta(x, y, z, t) = \theta_o(z) + \theta'(x, y, z, t) \quad (1.4.6)$$

Substituting this in equation (1.2.13) and using the fact that the ratio of the deviation to the basic part is much much less than unity gives

$$\frac{1}{\theta_o} \left(\frac{\partial \theta'}{\partial t} + \vec{V} \cdot \nabla \theta' \right) + w \frac{d \ln \theta_o}{dz} = \frac{J}{C_p T}$$

For large scale, when there is no active precipitation of water, the part that contributes for the diabatic heating is the net radiative heating. This fact gives

$$\frac{J}{C_p} \leq 1$$

Above the boundary layer, the horizontal potential temperature fluctuation is approximated as

$$\vec{V} \cdot \nabla \theta' \approx 40^{\circ}C$$

Similarly the vertical advection of the basic state (adiabatic cooling) when the vertical velocity has a value of around 1 *cm/s* in 1 *km* is

$$w \left(T \frac{d \ln \theta_o}{dz} \right) \approx 4 \frac{^{\circ}C}{day}$$

As the scaling indicates, in the absence of strong diabatic heating, the rate of change of perturbation of potential temperature following the motion is equal to the diabatic heating or cooling because of the vertical motion of the basic state of potential temperature [4]. This leads to the equation

$$\frac{D\theta}{Dt} + w \frac{d\theta_o}{dz} = 0 \quad (1.4.7)$$

1.4.4 Shallow water equation

In general the scale analysis of the equations derived in the preceding sections constitute the so called primitive equations given as

$$\frac{Du}{Dt} = -\frac{1}{r\rho \cos \phi} \frac{\partial P}{\partial \lambda} + 2\Omega \sin \phi v \quad (1.4.8)$$

$$\frac{Dv}{Dt} = -\frac{1}{\rho r} \frac{\partial P}{\partial \phi} - 2\Omega \sin \phi u \quad (1.4.9)$$

$$\frac{\partial P}{\partial z} = -\rho g \quad (1.4.10)$$

$$\nabla(\rho_o \vec{V}) = 0 \quad (1.4.11)$$

$$\frac{D\theta}{Dt} + w \frac{\partial \theta_o}{\partial z} = 0 \quad (1.4.12)$$

When the motion is horizontal, the atmosphere can be considered as an incompressible which allows decoupling of the energy equation from the primitive equation to be treated alone. This assumption along with additional mathematical simplification give the linearized shallow water equation about a resting basic state:

$$\frac{\partial u}{\partial t} = -\frac{g}{r \cos \phi} \frac{\partial h}{\partial \lambda} + 2v\Omega \sin \phi \quad (1.4.13)$$

$$\frac{\partial v}{\partial t} = -\frac{g}{r} \frac{\partial h}{\partial \phi} - 2u\Omega \sin \phi \quad (1.4.14)$$

$$\frac{\partial h}{\partial t} = -\frac{H}{r \cos \phi} \left[\frac{\partial u}{\partial \lambda} + \frac{\partial}{\partial \phi} (v \cos \phi) \right] \quad (1.4.15)$$

where $H=h_{final}-h_{initial}$ is the free height and h is the perturbation

This shallow water equation is a prototype for three dimensional atmospheric wave motion. The solution of this equation gives a good clue about the solution of the general wave equation and that is why we focus on the shallow water equation to simulate the effect of the random disturbance of the geometrical height on meteorological important wave motion.

1.5 Atmospheric Motion

Air in the atmosphere on large scale on a free troposphere is either in straight line motion or in wave motion. The wave motions are oscillations in the field variables such as pressure, temperature, velocity, and density that propagate in space. The oscillation can be either horizontal or vertical or oblique depending on the type of wave. Waves are initiated by various mechanisms like gravity of the earth, rotation of the earth, compressibility of the atmosphere, and energy difference of parts of the earth [9].

From the general circulation of the atmosphere, the motion of the air over the globe is categorized into three cells namely the Haddly cell, Ferrel cell, and polar cell. The Haddly cell is situated around the equator where as the polar cell is found around the poles and Ferrel cell lies between the two (around mid-latitude) [6]. Each of them are bounded with a high and low pressure belts.

On the earth, since it is in rotational motion, the motion of the air on large scale is

governed by the pressure gradient and Coriolis forces. The Coriolis force acts on the direction of flow and the pressure gradient force on the magnitude of the flow. These forces constraint the air to flow parallel to the isobar. In the northern hemisphere the pressure of air moving to the south deflects to the right and viceversa in the southern hemisphere. Even if the Coriolis force is too small (*i.e* 1.5^{-5}s^{-1} at the poles and 0 at the equator), it is this force that imparts an east/west component to meridional atmospheric motion [6].

From the above division of the general circulation of the atmosphere, the air flow in the Frell cell (in mid-latitude) is westerlies. The dominant wave in this region is the Rossby wave.

A given propagating wave can be identified by its amplitude, period, and phase. The frequency of oscillation depends on the wave number and the property of the medium. Unless the frequency of the wave is directly proportional to the wave number or if the phase speed varies with the wave number, the wave become dispersive (a wave group doesn't preserve its shape). Those wave are used to transport energy from one region to the other with the speed of group velocity. To put the equation mathematically, if κ is a wave number vector, and ω is the frequency, then the phase speed can be given as

$$C_p = \frac{\omega}{\kappa} \quad (1.5.1)$$

and the group velocity is also given as

$$C_g = \frac{\partial\omega}{\partial\kappa} \quad (1.5.2)$$

Some of the waves that exist in the neutral atmosphere ranging from small scale wave to large scale wave are: acoustic (sound) wave, shallow water gravity wave, gravity

wave, and planetary (Rossby) wave. Among those waves gravity wave range from small scale to large scale that feels the rotation of the Earth.

1.5.1 Rossby waves

One of the most dominant and important large scale atmospheric waves that exist in mid-latitude are the Rossby waves. These waves were first identified in the atmosphere in 1939 by Carl-Gustaf Arvid Rossby who went on to explain their motion. Rossby waves are a subset of inertial waves [1,6]. They absorbed on thousands of spatial scale and a period of several days. Since they occur on large horizontal wave length and with low frequency, they have low Rossby number (R_o) determined from the ratio of advection term to the coriolis term:

$$R_o = \frac{u \frac{du}{dx}}{fu} \quad (1.5.3)$$

If the Rossby number is much less than unity, the coriolis term exceeds the advection term and the geostrophic balance holds true and fails otherwise. In general for large scale weather system at mid latitude, the momentum equation due to scaling is reduced to the two hydrostatic and geostrophic balance equations. The geostrophic balance equation is

$$fv_g = \frac{1}{\rho} \frac{\partial p}{\partial x} \quad (1.5.4)$$

$$fu_g = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (1.5.5)$$

where v_g and u_g are the meridional and zonal geostrophic wind

The origin of Rossby wave is the shape and rotation of the Earth . The restoring force for the parcel oscillation is from the meridional gradient of the planetary vorticity. In

an invicid barotropic atmosphere of constant depth, if the horizontal divergence of velocity is zero, then absolute vorticity is conserved due to the variation of Coriolis force along latitude. The absolute vorticity (η) is the sum of relative vorticity (ξ) and planetary vorticity (f)

$$\eta = \xi + f \quad (1.5.6)$$

where $\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the vertical component of relative vorticity on rectangular coordinate system.

In general, knowing the distribution of vorticity gives information about this meteorological important wave (large scale wave). Concerning the equation, one can develop it from the atmospheric governing equation following scale analysis or directly derive from the geostrophic relation.

In general for the barotropic atmosphere, the vertical component of vorticity is conserved following the motion ($w = 0$) in mid latitude as

$$\frac{D}{Dt}(\xi + f) = 0 \quad (1.5.7)$$

Rossby wave can be created due to longitudinal temperature difference, barriers (obstacles), these in general can be termed as forced waves. The wave pattern of this wave also can be identified from the parcel oscillation.

To identify the dispersion relation of this wave, let's return back to the above equation:

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\xi + \beta v = 0 \quad (1.5.8)$$

Since in mid latitude the wind is almost zonal, considering the motion of air consisting of the mean zonal velocity \bar{u} and the perturbation of each field variables, linearizing

equation (1.5.8) gives

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\xi' + \beta v' = 0 \quad (1.5.9)$$

where $\beta = \frac{df}{dy}$

Now, from the geostrophic relation a stream function (ψ) is defined by using the equation

$$\psi = \frac{P}{f\rho} \quad (1.5.10)$$

where P is pressure and ρ is density

Velocity components and stream function are related by

$$u = -\frac{\partial\psi}{\partial y} \quad (1.5.11)$$

and

$$v = \frac{\partial\psi}{\partial x} \quad (1.5.12)$$

This implies that

$$\xi' = \nabla^2\psi' \quad (1.5.13)$$

Now the linearized equation becomes

$$\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\nabla^2\psi' + \beta\frac{\partial\psi'}{\partial x} = 0 \quad (1.5.14)$$

Since the above equation is a plane wave type, like the Maxwell equation, look for a solution of the form [1]

$$\psi = \text{Re}(\psi_0 e^{i(kx+ly-\nu t)}) \quad (1.5.15)$$

Using this solution in equation (1.5.14) gives the following dispersion relation

$$\nu = k\bar{u} - \frac{\beta k}{(k^2 + l^2)} \quad (1.5.16)$$

where k , l , and ν are zonal wave number, meridional wave number and the relative phase frequency respectively. This dispersion relation is used to find the phase velocity and the group velocity by using equation (1.5.1) and (1.5.2).

Equation (1.5.16) clearly shows that the presence of β which represents coriolis force variation along latitude is necessary for the existence of Rossby wave. Basically the synoptic scale Rossby wave move towards east as the mean flow but at a phase speed less than the mean wind flow and the zonal phase propagation relative to the mean flow is always west ward. It is one peculiar characteristics of Rossby wave. The phase speed increases as the wave length increases and the west ward propagating Rossby wave may be large enough to balance the mean flow. This condition forces the wave to be stationary relative to the ground. Unlike the phase speed, the group velocity may be either eastward or westward depending on the ratio of zonal and meridional wave number. It moves westward for zonal long wave length ($\frac{k}{l} < 1$) and vice versa. For the stationary case, it propagates eastward.

Qualitatively using the conservation of absolute vorticity on a barotropic atmosphere owing to the variation of planetary and relative vorticity can be described with help of Fig.(1.2).

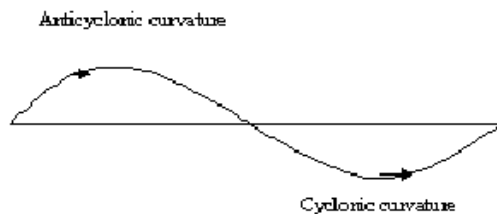


Figure 1.2: Propagation of Rossby waves due to the conservation of absolute vorticity for the northern hemisphere

Since planetary vorticity decreases as the air parcel move towards south, as a response due to the conservation of absolute vorticity, relative vorticity increases, and hence for southward displacement the relative vorticity is positive (Cyclonic) where as for the northward displacement relative vorticity is negative (for anticyclonic). These relative vorticity negative maximum (for clock wise rotation) and positive maximum (anti clock wise rotation) pattern moves towards the west (if the mean westerly flow is small) and this pattern constitute the Rossby wave.

1.6 Atmospheric disturbances

Disturbances in the atmosphere are initiated by different causes. Some of them are;

- differential heating of different parts of the atmosphere
- topography of the earth; and
- advection of momentum, energy, and matter.

Since the large scale motion depends on the horizontal position and time, considering barotropic atmosphere, following the motion absolute vorticity is conserved. In this case when the air flows over a certain obstacle, the effect of the disturbance is treated based on the conservation of absolute vorticity assuming gradient of potential temperature with respect to pressure ($\frac{\partial\theta}{\partial P}$) or geometrical height(δz) between two isentropic surfaces remain constant. Using this conservation principle, one can identify the flow of air after interacting with the obstacle.

In general for those different forcing, the effect of the disturbance are studied using the atmospheric governing equation. For the large scale, the adjustment (linearized

shallow water equation) about a resting state is used to develop the vorticity equation:

$$\frac{D(\xi + f)}{Dt} = f \frac{\partial w}{\partial z} \quad (1.6.1)$$

The term at the right hand side of the equation (1.6.1) is a stretching term due to the divergence from the adjustment equation.

For longitudinally uniform surface, the flow averaged over time (season) should be completely characterized by the zonally symmetrical component of the circulation. However, on real earth the above listed facts affect the flow. These produce longitudinally asymmetric planetary waves.

When a Rossby waves are superimposed on the zonal mean circulation, such waves produces local regions of enhanced and diminished time mean westerly winds which strongly influence the development and propagation of transient weather disturbances [6].

Chapter 2

Numerical solution of linearized shallow water equation

2.1 Introduction

The mathematical methods of most problems in science involve rates of change with respect to one or more independent variables like time and space which lead to partial differential equations. Those equations are either elliptic or parabolic or hyperbolic type. The equations are used to express the evolution of field variables with respect to time or the gradients with respect to the space coordinates [10].

Most of those partial differential equations do not have analytical solution. This difficulty forces researchers to look for another means for solution. The technique that is applied to solve equations that are impossible to solve analytically is the numerical method. For the first time, it is Richardson (1922) who have shown the numerical method to predict weather even if the result was poor due to initial condition and the approach of the problem [4]. However after three decades, the numerical prediction method revived and Charny developed a model based on Richardson's equation from the primitive atmospheric model equations using the geostrophic and hydrostatic

balance. Since then the numerical method becomes the only means to solve a number of complex equation using computers.

There are a number of numerical methods to solve different types of differential and integral equations. The most frequently applicable numerical methods are the spectral method, the finite element method and the finite difference method. The spectral and finite element methods are used to solve equations whether or not the domain is regularly shaped. The finite difference method is the simplest method compared to the two and effective for the regular shaped domain of integration and in this thesis we are going to use the finite difference method [11].

This chapter focuses with the property of numerical solution of differential equation using finite difference method. The convergence criterion, grid type, and the type of errors associated are also part of the chapter.

2.2 Finite difference method

Finite difference method is the simplest type of numerical technique to integrate differential equation using grid point values determined recursively. It is highly suitable for a regular domain or region of integration. There are different types of finite difference formulation for a given differential equation. For the differential equation that involves time, the finite difference method can be expressed as centered (3-time level), backward (2-time level) and forward (2-time level) schemes [12].

The finite difference method is formulated using the Taylor series expansion. It uses a number of values on a grid points in such a way that the domain is divided into

a number of rectangles. The distribution of the field variables on the mesh points depend on the type of grid used. In a broad sense, based on the assignment of field variables on mesh points, the grid can be categorized as nonstaggered and staggered.

2.2.1 Unstaggered Arakwan "A" Grid

The choice of the above finite difference type has to be followed with the appropriate grid type. The grid types are the Arakawan *A*, *B*, *C*, *D*, and *E* grids. Except Arakawan *A* grid the rest are staggered one. In a nonstaggered grid field variables are placed at the same grid points whereas in the staggered grid, field variables are partially or totally distribute among grid points. The choice of the Arakwan grid type depends on the problem at hand [11]. For medium-range weather prediction and longer time integration, the nonstaggered regular latitude-longitude grid is used [12].

The nonstaggered grid *A* is the simplest one compared to the other since all the field variables are found on each of the mesh points. The scheme is given in Fig. (2.1) based on the geopotential height (h), zonal velocity (u), and meridional velocity (v) on a longitude-latitude grid point.

step has a rounding error. In general it contribute its part to the deviation of the solution from the exact analytical solution [10].

The truncation error exist due to the formulation of the finite difference equation from the continuous differential equation. Since finite difference equation for a certain differential equation is developed by using Taylor series expansion by neglecting higher order terms, the difference between the continuous equation and the truncated difference equation gives the truncation error. If $\frac{\partial f}{\partial x}$ is required to be solved numerically, and δf is the finite difference counter part, then the truncation error is

$$T_{er} = \frac{\partial f}{\partial x} - \delta f \quad (2.2.2)$$

where T_{er} is the truncation error

On the other hand the discretization error is a difference between the exact solution of the continuous differential equation and the corresponding finite difference equation. If U is the exact solution of the differential equation at the grid point i , and u is the exact solution of the finite difference equation at the same grid point i , then in equation form it is given as

$$d_{er} = U_i - u_i \quad (2.2.3)$$

where d_{er} is discretization error

The discretization error can be identified by substituting the exact solution of the continuous equation in the finite difference equation.

2.2.3 Convergence of numerical solution

A solution of the dicretized differential equation is said to be convergent if it approaches the solution of the continuous equation (the differential equation) when the

discretization becomes finer and finer.

In general, to solve differential equation using numerical method, discretizing the equation and calculating the value is not enough for it may not be the approximation of the true solution. It needs a check up whether or not it is convergent. Checking the convergency of the numerical solution is not as such an easy task. Even if it is difficult to check it directly, it is possible through consistency and stability. One of them alone doesn't guarantee convergence [13].

Consistency is achieved when the truncation error of the discretization equation tends to zero as the discretization (grid separation) becomes smaller and smaller or when the discretized equation is similar to the continuous equation. If the truncation error tends to zero as the grid separation becomes smaller and smaller, then the method is said to be consistent [14].

On the other hand the discretization scheme is said to be stable if the solution is uniformly bounded function of initial state for every value of time step. Stability is violated due to discretization and rounding error. The scheme is stable if the error remains bounded. It can be tested by various method, among which Von Neumann's method (Fourier series method) is the one. In this method a solution of the following type is used

$$\Psi(x, y, z, t) = \Psi_o \lambda^n e^{i(kd+ld+md)} \quad (2.2.4)$$

where λ is the amplification factor for the mode, and n is exponential.

This equation is substituted in the numerical equation and after a certain mathematical manipulation, if the modulus of the amplification factor is less than or equal to unity, then the solution is stable [12,13].

Stability further can be categorized as conditionally stable and unconditionally stable. Unconditionally stability exists whenever the scheme is implicit where as conditionally stability happens when the scheme is explicit. In conditionally stable case, the time step is restricted. The need for this restriction was noted by Courant, Friedrich, and Lewy condition. The condition for this restriction is abbreviated as *CFL* condition from the first letter of their names. Once stability and consistency are fulfilled they are sufficient for convergence [14].

Chapter 3

Role of stochastic process on the atmospheric air motion

3.1 Introduction

Atmosphere is an intricate dynamical system with many degrees of freedom and the state is described by the spacial distribution of field variables. The processes that takes place through time are chaotic.

Stochastic process is the probabilistic, or completely random process in which chance plays the role. It is free of any biases. This stochastic or chaotic process directly or indirectly affects the atmospheric field variables for they are interrelated. The word "chaotic" is a completely undeterminate process that is susceptible to the initial condition [8].

Stochastic processes are initiated by a number of causes. Irregular distribution of land masses and water bodies, ridges and valleys, and cold and hot regions are some of the causes [6]. This chapter concerns the description of random disturbance especially targeted to random height perturbation on the linearized shallow water equation. We

are going to identify the influence on the adjustment processes on a mid latitude atmosphere where the Rossby wave is the dominant of the air circulation.

3.2 Stochastic process of field variables in the atmosphere

The atmospheric field variables are randomly perturbed or disturbed by different forcings. The spectrum of atmospheric wave from the small scale to the large scale are the result of those perturbations. As the waves are different from each other, the causes are also different. In this chaotic atmosphere, processes that have a small difference at earlier time are completely different at any latter time. This is expressed with the chaotic nature of the atmosphere.

The air motion are disturbed differently due to the random distribution of rural and urban areas, land masses and water bodies, mountain ridges and valleys, warm and cold areas. Those are some of the main causes that bring change in the atmospheric field variables stochastically [6]. The shape and rotation of the earth also disturb the air motion.

During the rainy day the droplets of water or ice strike the surface of water and create a surface wave with out any pattern is an example of a disturbed wave. Similarly the ice strikes the land with out any predictable pattern is a common experience. These all are a random processes that we observe in our day to day activities. In general the atmosphere especially in the small scale is a full of random processes that put its effect on the regional and global scale.

3.3 Finite difference discretization of the model equation

The region of study lies in mid latitude to have the hydrostatic and geostrophic balance. This region is characterized by the domination of the Rossby wave from perspective view of the atmospheric circulation . The wind flow is westerlies bounded by the high pressure and low pressure belts [6].

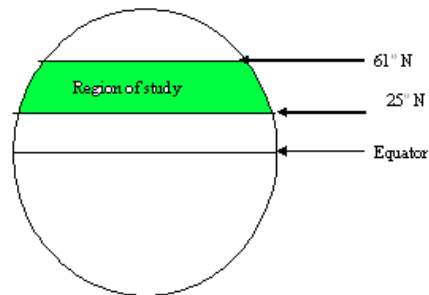


Figure 3.1: This figure shows the region of study situated in mid latitude

Here the model equation is the linearized shallow water equation on a spherical coordinate. The numerical method that we apply to solve the equation and to identify the effect of random height disturbance is the finite difference method in which all the field variables are put at the same mesh point (Arakwan grid A).

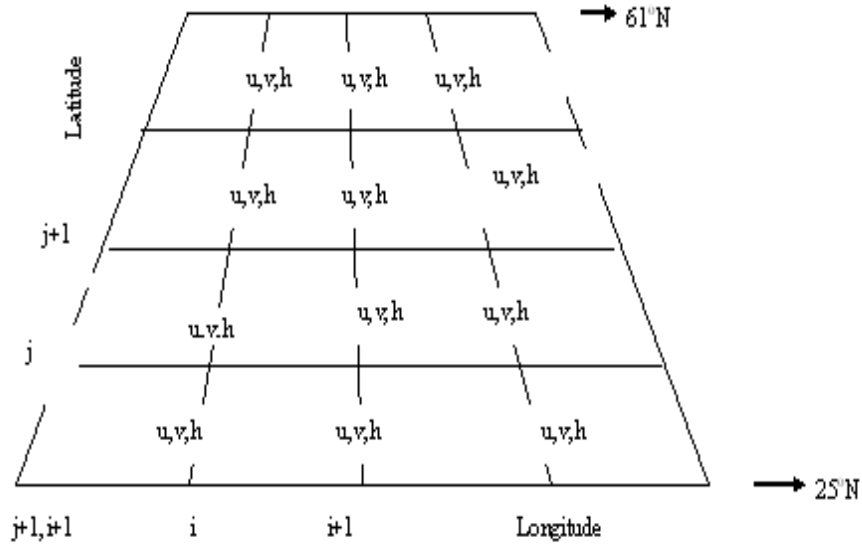


Figure 3.2: Domain of integration on a spherical coordinate

Since the circumference of the latitude circle becomes shorter and shorter as one advances to the pole, the *west–east* grid length decreases. This different length brings noise to the solution due to the reason that the minimum wavelength is determined from the longest grid length [15]. To identify this pattern, the circumference of the latitude circle at the i^{th} point is given as

$$C_i = 2\pi r_i = 2\pi a \cos \phi_i$$

where a is radius of the earth, ϕ is the latitude angle, and r_i is the radius of the circle at point i .

If n is the number of the grid separation along the latitude circle (equal in number in all circles), then the length of the grid in the i^{th} circle is

$$d_i = \frac{2\pi r_i}{n} = \frac{2\pi a \cos \phi_i}{n}$$

This implies that, the grid length at $\phi = 28^\circ$ is

$$d_1 = \frac{2\pi a \cos(28)}{n}$$

and at $\phi = 61^\circ$ is

$$d_2 = \frac{2\pi a \cos(61)}{n}$$

For our case the grid separation is 3 degrees in both directions. The number of the grid separation becomes 120 (*i.e.* $n = 120$). From this, one can easily identify how the grid separation decreases towards the poles. In this regular longitude-latitude grid case, the south-north grid separation is constant.

From this calculation the longest grid separation is around 294 *km*, and the shortest grid length is around 162 *km*. Using this, the shortest wavelength resolved by this grid system becomes 588 *km*. Any wave having wavelength less than 588 *km* cannot be resolved.

Time step stable for the whole of the domain of integration is taken based on the shortest grid length [16,15]. In this study the time step (Δt) is 1 minute. The CFL condition is stated as information cannot move a distance greater than a grid separation in one time step. The fastest phase speed in the governing equations is the sound wave which is around $C = 300$ *m/s* and the other type of wave have below this value [4]. Using the shortest east-west grid separation, the following inequality gives guarantee for the validity of CFL condition for one-dimension.

$$c\Delta t \leq \Delta\lambda$$

For two-dimensional linearized shallow water equation about a resting state, the time step stable for the integration on a rectangular coordinate system is constrained by the general inequality

$$\Delta t \leq \frac{1}{\sqrt{c^2 \left(\frac{\sin^2 k \Delta x}{\Delta x^2} + \frac{\sin^2 l \Delta y}{\Delta y^2} \right) + f^2}}$$

where f is the planetary vorticity. k and l are zonal and meridional wave number respectively. Δx represent west-east grid length where as Δy stands for north-south grid length.

The variation of the coefficients along latitude is included during the development of the program. Using this method, the model equations are discretized. Here averaging of the space derivatives of the variables are required to remove the grid noise. From now onwards we are going to use the following averaging and difference operator through out the discretization. For a certain variable f which is a function of time and space coordinates, the operators become

$$\delta_x f = \frac{f_{x+\frac{\Delta x}{2}} - f_{x-\frac{\Delta x}{2}}}{\Delta x}$$

$$\bar{f}^x = \frac{f_{x+\frac{\Delta x}{2}} + f_{x-\frac{\Delta x}{2}}}{2}$$

Starting from the divergence equation

$$\frac{\partial h}{\partial t} = \frac{-u_o}{r \cos \phi} \frac{\partial h}{\partial \lambda} - \frac{h}{r \cos \phi} \left[\frac{\partial u}{\partial \lambda} + \frac{\partial (v \cos \phi)}{\partial \phi} \right]$$

using the above operators, the equation will reduce to

$$\frac{\partial h}{\partial t} = \frac{-u_o}{r \cos \phi} \delta_\lambda \bar{h}^\lambda - \frac{h}{r \cos \phi} \delta_\lambda \bar{u}^\lambda - \frac{h}{r \cos \phi} \delta_\phi (v \bar{\cos} \phi)^\phi$$

by making use of leapfrog scheme, we obtain

$$\frac{h_{i,j}^{n+1} - h_{i,j}^{n-1}}{2\Delta t} = \frac{-u_o}{r \Delta \lambda \cos \phi_j} (h_{i+\frac{1}{2},j}^n - h_{i-\frac{1}{2},j}^n) - \frac{h_{i,j}^n}{r \Delta \lambda \cos \phi_j} (u_{i+\frac{1}{2},j}^n - u_{i-\frac{1}{2},j}^n)$$

$$-\frac{h_{i,j}^n}{r\Delta\phi\cos\phi_j}(v_{i,j+\frac{1}{2}}\cos\phi_{j+\frac{1}{2}}-v_{i,j-\frac{1}{2}}\cos\phi_{j-\frac{1}{2}})^n$$

which reduces to

$$h_{i,j}^{n+1} = h_{i,j}^{n-1} - z_1(h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j})^n - z_2 h_{i,j}^n (u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j})^n - z_3 h_{i,j}^n v_{i,j+\frac{1}{2}}^n + z_4 h_{i,j}^n v_{i,j-\frac{1}{2}}^n \quad (3.3.1)$$

after rearranging terms and defining new variables

$$\begin{aligned} z_1 &= \frac{2u_o\Delta t}{r\cos\phi\Delta\lambda} \\ z_2 &= \frac{2\Delta t}{r\Delta\lambda\cos\phi_j} \\ z_3 &= \frac{2\Delta t\cos\phi_{j+\frac{1}{2}}}{r\Delta\phi\cos\phi_j} \\ z_4 &= \frac{2\Delta t\cos\phi_{j-\frac{1}{2}}}{r\Delta\phi\cos\phi_j} \end{aligned}$$

where n , i , and j indicates time, longitude, and latitude indices respectively. Since

the coefficients are constant for any time step, using matrix form, equation(3.3.1)

becomes

$$h^{n+1} = h^{n-1} + A_3 h_h^n + C u_h^n + D v_h^n + D3 \quad (3.3.2)$$

where

$$\begin{aligned} h_h &= (h_{i-\frac{1}{2},j} - h_{i+\frac{1}{2},j}) \\ u_h &= h_{i,j}(u_{i-\frac{1}{2},j} - u_{i+\frac{1}{2},j}) \\ v_h &= h_{i,j}(v_{i,j-\frac{1}{2}} - v_{i,j+\frac{1}{2}}) \end{aligned}$$

are column matrices. The coefficients A_3 , C , and D are also column matrices.

Following the same steps for the equation of zonal velocity

$$\frac{\partial u}{\partial t} + \frac{u}{r\cos\phi}\frac{\partial u}{\partial\lambda} = \frac{-g}{r\cos\phi}\frac{\partial h}{\partial\lambda} + 2\Omega v\sin\phi$$

as the geopotential height, we arrive at

$$\frac{\partial u}{\partial t} = \frac{-u_o}{r \cos \phi} \delta_\lambda \bar{u}^\lambda - \frac{g}{r \cos \phi} \delta_\lambda \bar{h}^\lambda + 2\Omega v \sin \phi$$

$$\frac{u_{i,j}^{n+1} - u_{i,j}^{n-1}}{2\Delta t} = \frac{-u_o}{r \Delta \lambda \cos \phi_j} (u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j})^n - \frac{g}{r \cos \phi_j \Delta \lambda} (h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j})^n + 2\Omega v_{i,j}^n \sin \phi_j$$

defining coefficients in compact form

$$x_1 = \frac{2u_o \Delta t}{r \Delta \lambda \cos \phi_j} = z_1$$

$$x_2 = \frac{2g \Delta t}{r \Delta \lambda \cos \phi_j}$$

$$x_3 = 4\Omega \Delta t \sin \phi_j$$

$$u_{i,j}^{n+1} = u_{i,j}^{n-1} - x_1 (u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j})^n - x_2 (h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j})^n + x_3 v_{i,j}^n \quad (3.3.3)$$

Equation(3.3.3) can be written in a compact matrix notation as

$$u^{n+1} = u^{n-1} + A_3 u_u^n + A h_u^n + S v_u^n + D1 \quad (3.3.4)$$

where

$$u_u = (u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j})$$

$$h_u = (h_{i+\frac{1}{2},j} - h_{i-\frac{1}{2},j})$$

$$v_u = v_{i,j}$$

are column matrices and the coefficients A , and S are also column matrices.

Similar analysis for the meridional velocity equation

$$\frac{\partial v}{\partial t} + \frac{u_o}{r \cos \phi} \frac{\partial v}{\partial \lambda} = \frac{-g}{r} \frac{\partial h}{\partial \phi} - 2\Omega u \sin \phi$$

yields

$$\frac{\partial v}{\partial t} = \frac{-u_o}{r \cos \phi} \delta_\lambda \bar{v}^\lambda - \frac{g}{r} \delta_\phi \bar{h}^\phi - 2\Omega u \sin \phi$$

after discretizing, it becomes

$$\frac{v_{i,j}^{n+1} - v_{i,j}^{n-1}}{2\Delta t} = \frac{-u_o}{r\Delta\lambda \cos \phi_j} (v_{i+\frac{1}{2},j} - v_{i-\frac{1}{2},j})^n - \frac{g}{r\Delta\phi} (h_{i,j+\frac{1}{2}} - h_{i,j-\frac{1}{2}})^n - 2\Omega u_{i,j}^n \sin \phi_j$$

as before defining coefficients

$$w_1 = \frac{2u_o\Delta t}{r\Delta\lambda \cos \phi_j} = x_1$$

$$w_2 = \frac{2g\Delta t}{r\Delta\phi}$$

$$w_3 = 4\Delta t\Omega \sin \phi_j = x_3$$

$$v_{i,j}^{n+1} = v_{i,j}^{n-1} - w_1(v_{i+\frac{1}{2},j} - v_{i-\frac{1}{2},j})^n - w_2(h_{i,j+\frac{1}{2}} - h_{i,j-\frac{1}{2}})^n - w_3 u_{i,j}^n \quad (3.3.5)$$

Using matrix form, equation (3.3.5) becomes

$$v^{n+1} = v^{n-1} + A_3 v_v^n + B h_v^n - S u_v^n + D_2 \quad (3.3.6)$$

where

$$v_v = (v_{i+\frac{1}{2},j} - v_{i-\frac{1}{2},j})$$

$$h_v = (h_{i,j+\frac{1}{2}} - h_{i,j-\frac{1}{2}})$$

$$u_v = u_{i,j}$$

are column matrices and the coefficient B is also a column matrix. In all equations D_1 , D_2 , and D_3 are a boundary value column matrices.

In this case the truncation error can be estimated from the Taylor series expansion. Since the scheme that we apply is the leapfrog scheme (both center in space and time), the truncation error in both cases are second order.

3.4 Introducing random height perturbation in the solution of linearized shallow water equation

The linearized shallow water equation, being the prototype of a three dimensional atmospheric equation of motion, helps to identify the medium scale and large scale phenomena based on the Rossby number. In this case the perturbation aims on the geopotential or pressure field to identify the effect on the speed of air at the extra tropical regions where the Rossby wave is dominant. This is done by introducing (adding) a random number on the height field.

The random numbers are produced using a random number generator between 0 and 1. If the domain of integration contains k internal number of mesh points, then the random numbers produced are equal to k in number. Those k random numbers are distributed among the grid points. To make some of the grid points undisturbed, zero is assigned to the corresponding random numbers. The disturbance in equation form can be expressed as

$$h_k = Ph' \tag{3.4.1}$$

where

P is the column matrix containing the random numbers

h' is unit geopotential height at each grid point

h_k is the perturbation of height whose effect is required.

The magnitude of the disturbance is going to be modified by multiplying with a

certain number n greater than 1. This shows that the maximum disturbance is less than the multiplier number n .

$$h_k = n(Ph') \quad (3.4.2)$$

Assuming the air motion is horizontal(usually true in mid latitude), the perturbation can be added to the model equations given by Eqns (3.3.2, 3.3.4, 3.3.6) of the system at any time when perturbation is required.

Chapter 4

Result and discussion

This chapter focuses on the discussion and analysis of random disturbance on atmospheric motion especially in mid-latitude where the Rossby wave is the dominant. The result is interpreted based on the analysis from the simulation of the shallow water equation. The response of zonal velocity and meridional velocity for the disturbance is the same and therefore the zonal velocity has been selected for the present discussion. In all cases the simulation covers $\frac{1}{10}$ of the globe along latitude circle which lies between 25° and 61° along a given longitude in the northern hemisphere. The grid separation along latitude and longitude are equal which is 3 degrees.

4.1 Atmospheric air motion disturbance along latitude circle

The results of simulation of undisturbed motion is compared with that of disturbed atmospheric flow along a given latitude circle. The outcome of the comparison is interpreted in view of established and current understanding of atmospheric dynamics in the following sections.

4.1.1 Motion of the atmosphere with out disturbance along a given latitude circle

From the atmospheric general circulation point of view, in mid-latitude, air moves west to east. If the earth's topography is even, air motion would be longitudinally symmetric. With out introducing the height disturbance into the solution of the governing equations used in our study, air motion exhibits zonally symmetric flow. Fig. (4.1) shows the evolution of the zonal velocity under this condition.

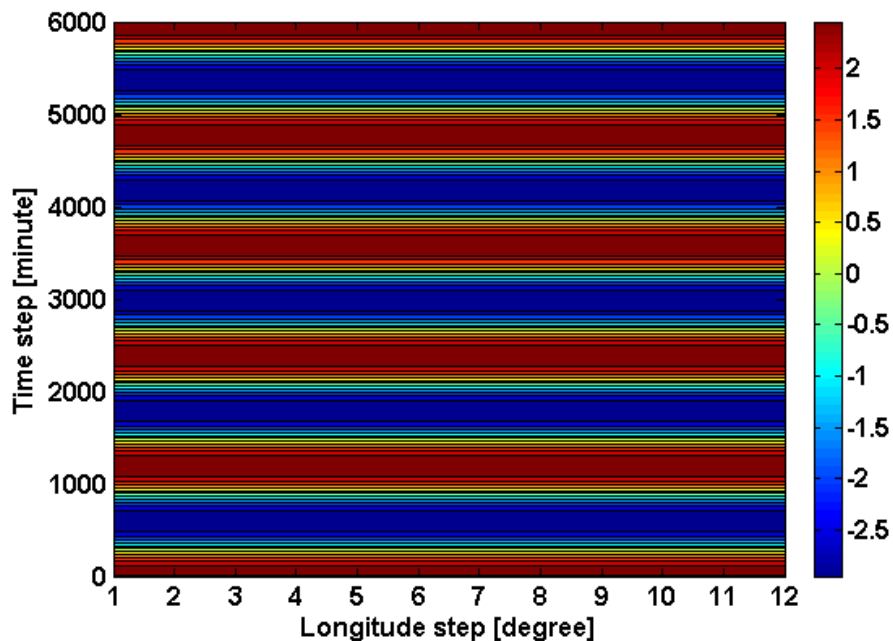


Figure 4.1: Zonal velocity for a given latitude circle. Simulation of atmospheric motion in the absence of disturbance. Vertical axis represent time step in minute and the horizontal axis longitude step (each 3 degrees)

Similar results as in Fig. (4.1) have been obtained for other latitude circles. This shows that even if the Coriolis force has its own deflecting effect on the motion of

the air, it is the same along the same latitude circle. In the absence of disturbance of the motion, the air propagate forward with time with out change of shape. In this case the initial distribution of velocity repeats itself with time keeping kinetic energy constant.

The constant phase of the shallow water equation, being a wave equation, repeats itself with time with out change of shape. The parcel oscillation follows the corrugated sheet. Fig. 4.2 shows the same results as in Fig. (4.1) but in a form of plane wave.

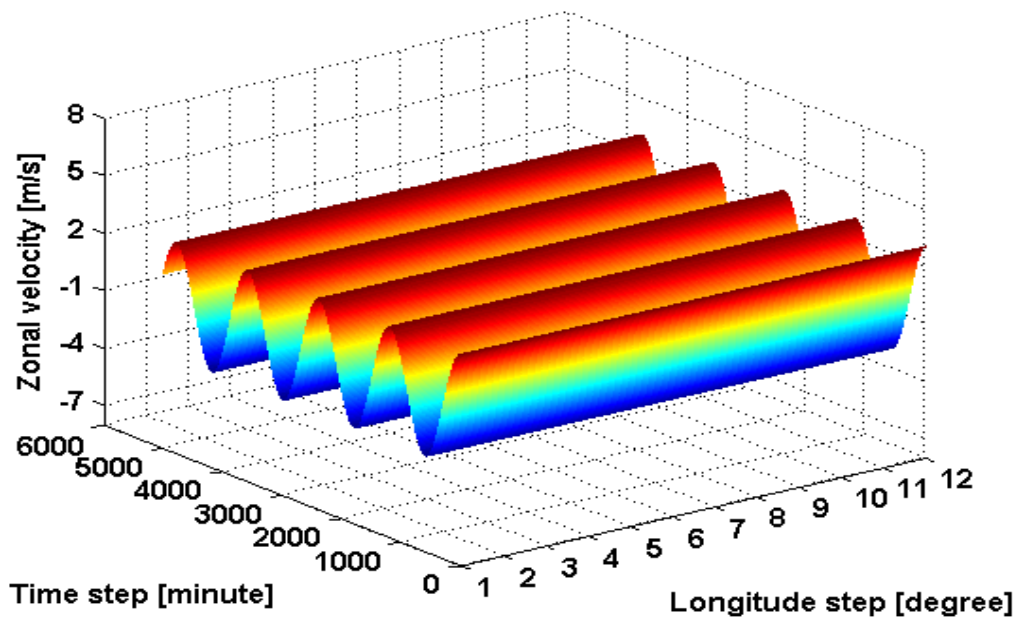


Figure 4.2: Zonal velocity for a given latitude circle. The constant of phase repeats itself with time. The horizontal axis represents time step in minute and longitude step (each 3 degrees). The vertical axis is zonal velocity

Similar to that of Fig. (4.1), the effect of the Coriolis force along a given latitude circle is the same with time. This shows that the motion along the latitude circle is

symmetric and the initial situation repeats itself with time. For a barotropic atmosphere, since the horizontal variation of temperature vanishes, the situation shown in both figures remain the same with altitude.

4.1.2 Motion of the atmosphere with disturbance along a given latitude circle

When the random disturbance is introduced into the system through the geopotential height, the pattern of the evolved system is different from the one given in Fig. (4.1). Fig. 4.3 shows the evolution of zonal wind speed along a given latitude circle.

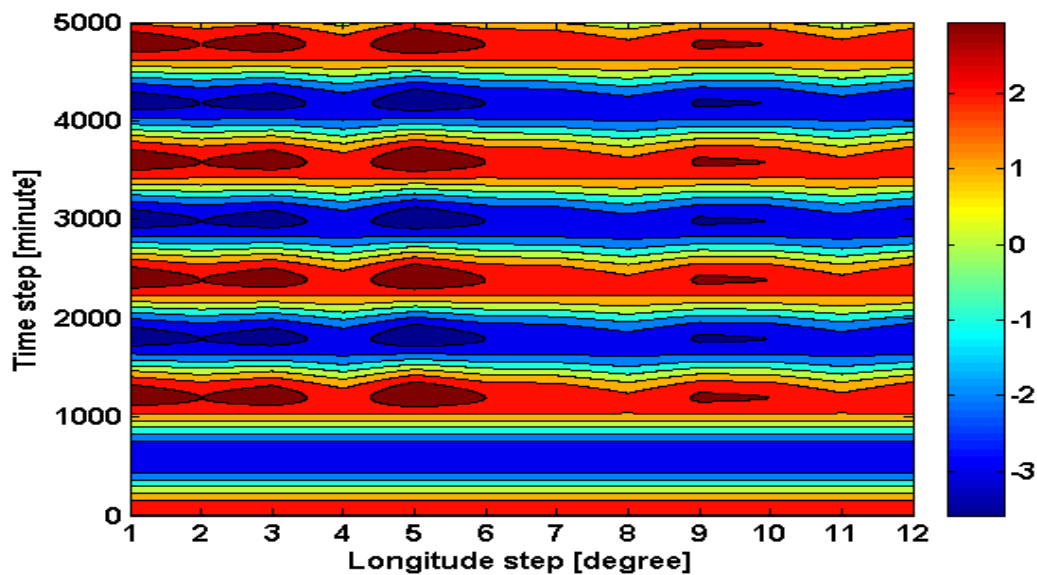


Figure 4.3: Zonal velocity for a given latitude circle with random height disturbance showing the evolved system. The vertical axis is time step in minute and the horizontal axis is longitude step (each 3 degrees)

In Fig. (4.3), there is a wavy pattern that indicates the effect of the disturbance on

the motion of air. The wavy structure repeats itself one after the other with time indicating that it is the same as stationary wave created by mountains. The wavy pattern produced in this situation is a result of a maximum of 20 *m* height perturbation. The minimum separation between two consecutive disturbance is around 162 *km*. The magnitude of the system evolved from the perturbation depends on the magnitude of the disturbance.

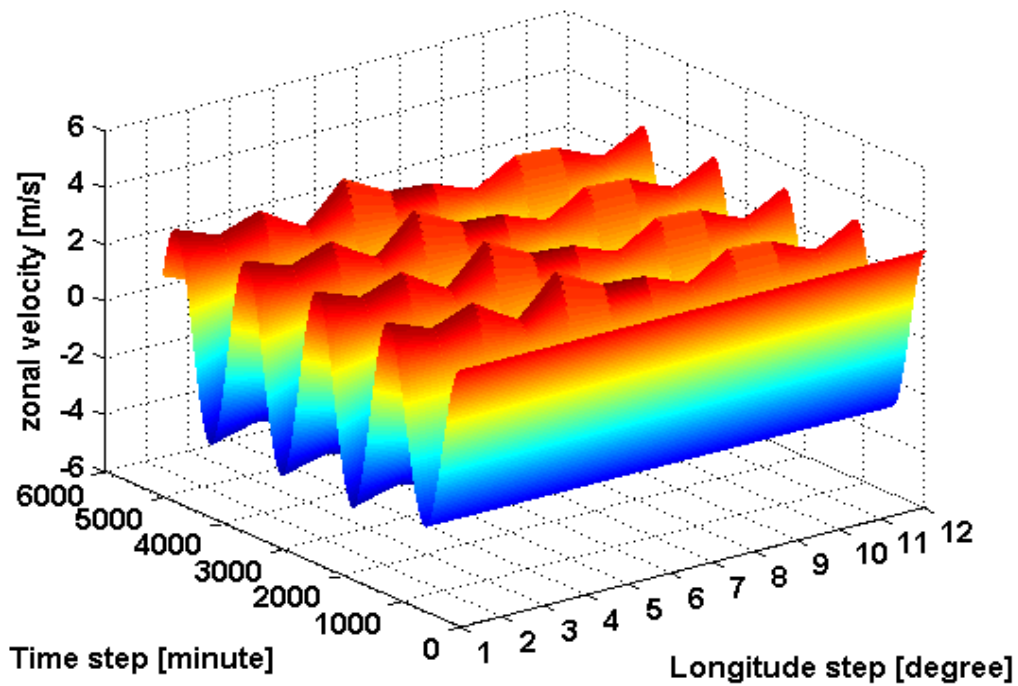


Figure 4.4: Zonal velocity for a given latitude circle with random height disturbance showing the evolved system on a time longitude position plane. The horizontal axis represent the time step in minute and the longitude step (each 3 degrees). The vertical axis stands for zonal velocity.

The corrugated sheet shown in Fig. (4.4) is not similar to patterns indicated in Fig. (4.2) for undisturbed flow. In the present case the troughs and the ridges are deformed

in response to the disturbance. The system was perturbed at time $t=1000$ minute for a duration of 60 minute and the perturbation was switched off afterwards. However, the effect continues to persist. The amplitude (speed) of the atmospheric motion is not constant. In some region it increases while in the other region it decreases. The effect of the disturbance on a given latitude circle is similar except the period.

4.2 Atmospheric air motion disturbance along a given longitude

We are going to investigate what happens on the atmospheric motion meridionally when one observes the effect of disturbance along a given longitude. Here the latitudinal position varies and the Coriolis force comes into play on the motion of air. As in latitude circle, the evolution of motion of the disturbed system has been analyzed based on the theoretical background.

4.2.1 Atmospheric motion on a given longitude with out disturbance

On the same longitudinal position, the simulation of the zonal velocity of air motion of air for the undisturbed case is depicted in Fig. (4.5).

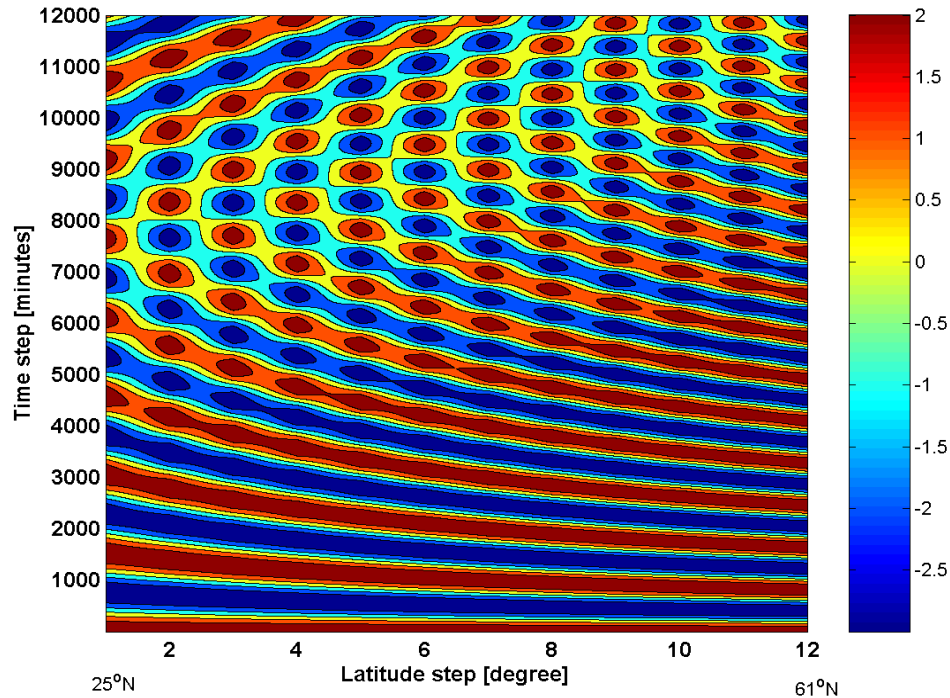


Figure 4.5: Zonal velocity for a given longitude with out random height disturbance showing the evolved system. The vertical axis is time step in minute and the horizontal axis is latitude step (each 3 degrees).

Since the coriolis force increases towards the poles, the deflecting effect increases. This forces the air parcel found on the same longitude to undergo different oscillation. From Fig. (4.5) the pattern of the contour lines below $t=8000$ minutes is the same. Beyond this time to $t=16000$ minutes the pattern is reversed. Starting from $t=16000$ to $t=24000$ minutes is the same as the pattern from $t=0$ to $t=16000$ minutes. Each one is a complete cycle. The period for each cycle is about 16000 minutes which is around 11 days. Wave with this period is characterized as large scale planetary wave.

Fig (4.5) can further be analyzed using 4 different points on the same longitude as

given in Fig. (4.6).

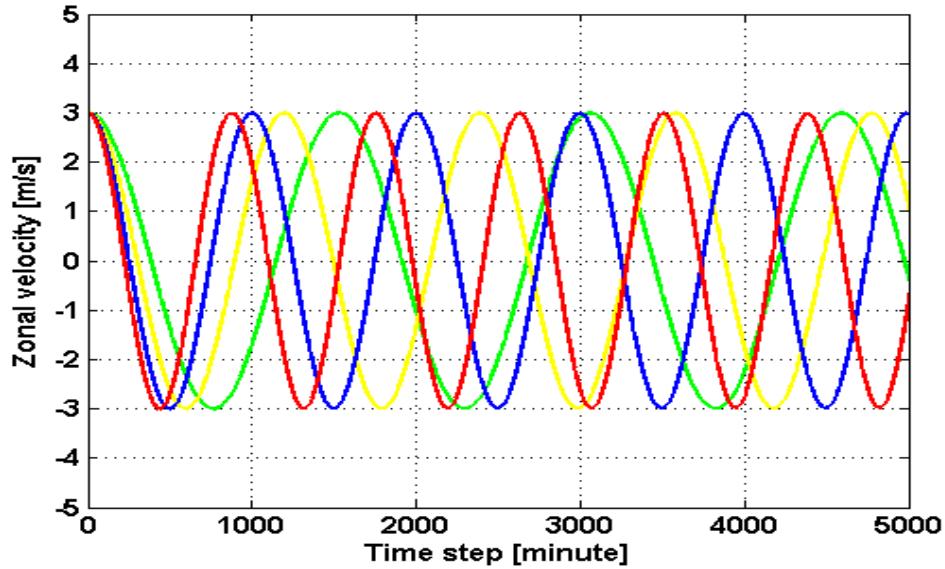


Figure 4.6: Zonal velocity for a given longitude with out random height disturbance showing oscillation of parcel of air on different latitude circle. The horizontal axis is time step in minute and the vertical axis in zonal velocity

The different lines in Fig. (4.6) represent different latitude positions such that the green is for $\phi = 28^\circ$, yellow for $\phi = 37^\circ$, blue for $\phi = 46^\circ$, and red for $\phi = 55^\circ$ latitudes. All points are in the Northern Hemisphere. The period of blue (28°) is 1000 minutes which is around 17 hours, and period of green (28°) is 1500 minutes which is 25 hours. This shows that period of parcel oscillation increases towards the equator. This difference is related to the variation of Coriolis force due to the spherical shape of the earth. If we consider the average coriolis force, there will not be any difference in frequency and all the lines will be the same. The amplitude of the velocity is the same with time.

4.2.2 Atmospheric motion on a given longitude with height disturbance

The disturbed system in this section is analyzed to identify the effect of the disturbance on different latitude circle on a given longitude. For this situation the simulation reveals features shown in Fig. (4.7).

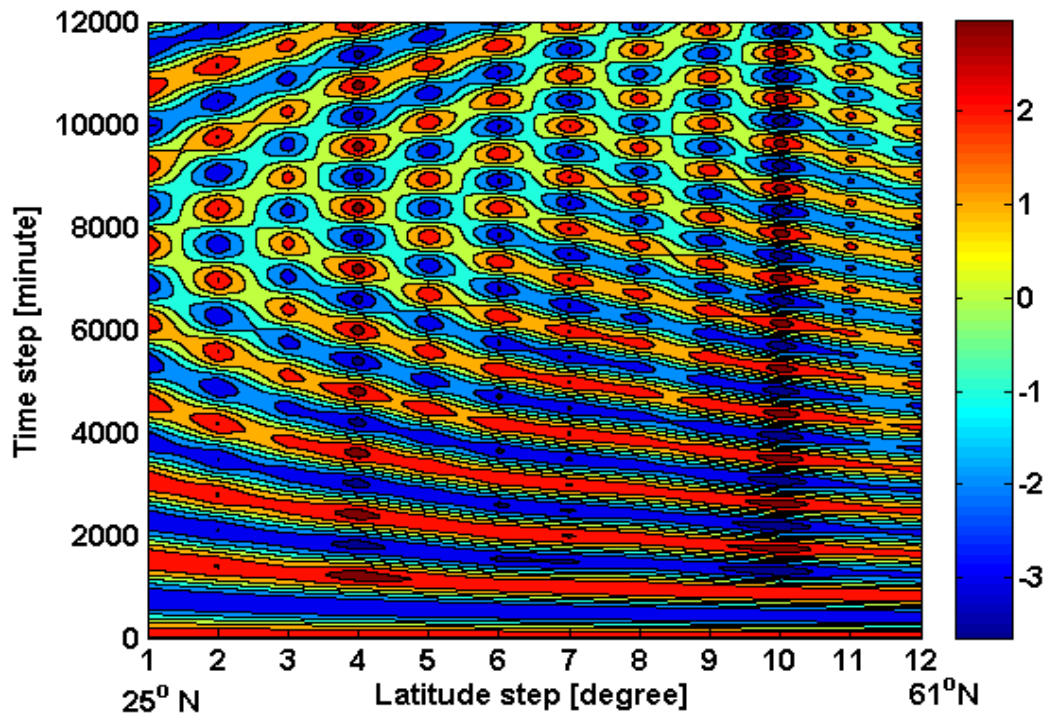


Figure 4.7: Zonal velocity for a given longitude with random height disturbance showing the evolved system. The horizontal axis is latitude step (each 3 degrees) and the vertical is time step in minute.

Compared to Fig. (4.5) which represent quiet atmosphere, this one is highly diffused showing that the motion is disturbed. The large scale planetary wave is still observed

in the same manner as in Fig. (4.5). However, one can notice the presence of diminished and enhanced regions of the atmospheric motion. The effect on the amplitude can be illustrated using Fig. (4.8).

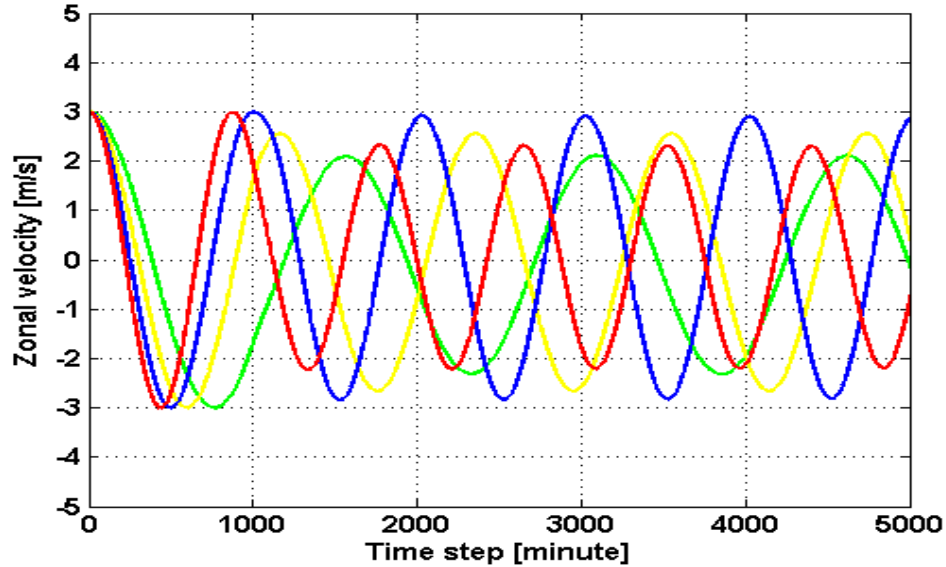


Figure 4.8: Zonal velocity for a given longitude with random height disturbance showing oscillation of parcel of air on different latitude circle. The vertical axis represent zonal velocity and the horizontal axis represents time step in minute.

The period is the same with that of the undisturbed case seen in Fig. (4.6). However, the amplitude is different. In some points, the amplitude is magnified while in the others, it is diminished. This shows that the disturbance does not affect the period but the amplitude which is related with the energy. The disturbance forces the motion to change its speed which is directly related with the kinetic energy. Change in amplitude (speed) brings change in kinetic energy of the system.

4.3 The effect of advection on atmospheric field variables

So far we have seen the effect of random height disturbances using the linearized shallow water equation with a resting mean state. In that situation there is no additional or removal of field variables. In the present discussion, we consider the mean zonal flow. This mean flow may have either a removal or addition effect on the local field properties. For the present case we use a mean zonal flow of 15 m/s . For the undisturbed case, the effect of the mean flow along a given latitude circle is illustrated in Fig. (4.9).

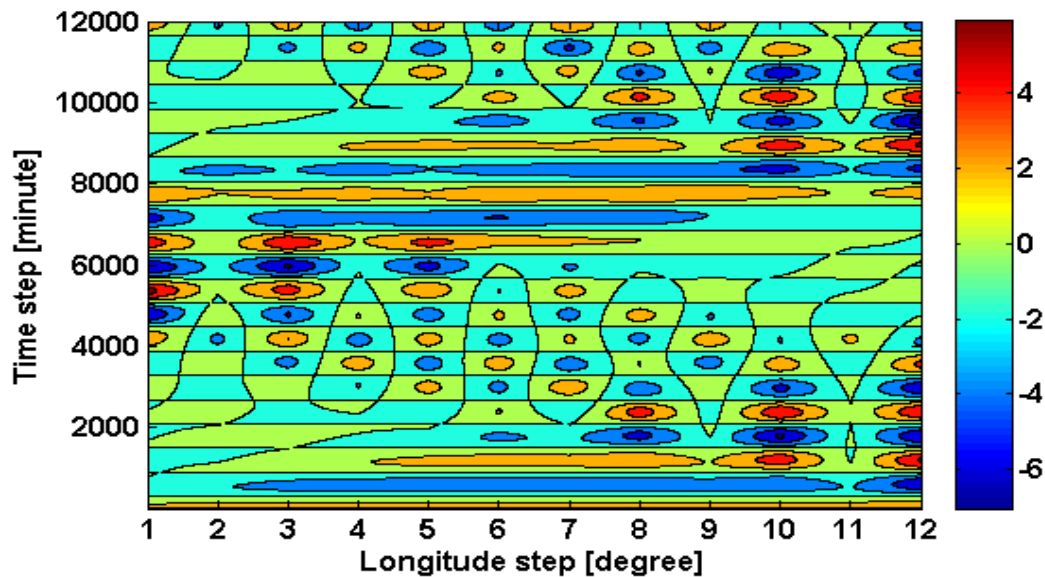


Figure 4.9: Zonal velocity with mean flow from west to east on a given latitude circle. The horizontal axis in longitude step (each 3 degrees) and the vertical axis is time step in minute.

Fig. (4.9) shows that the mean wind field which blows into the system advects away

initial value of the system with the wind. This mean wind disturbs the system from west to east following the direction of the mean wind flow and the magnitude of the effect depends on the magnitude of the mean wind field.

4.4 The effect of advection on a disturbed atmospheric field variables

In this situation the mean flow carries not only the initial distribution of field variables but also the disturbance introduced.

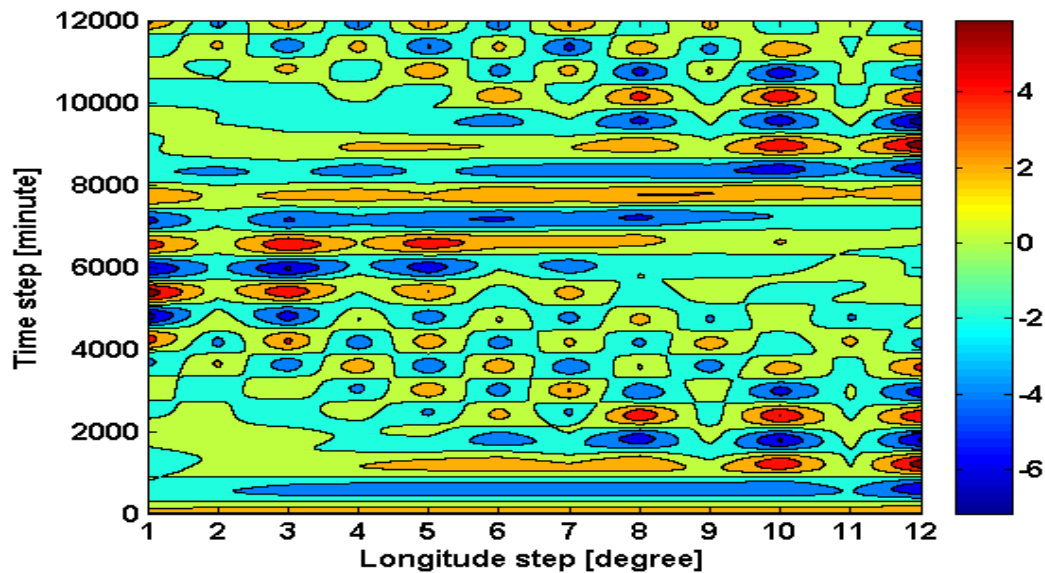


Figure 4.10: Zonal velocity for a given latitude circle with random height disturbance showing the effect of mean zonal flow. The vertical axis is time step in minute and the horizontal axis is longitude step (each 3 degrees).

Fig. (4.10) shows that the disturbance is carried by the mean flow from one region to the other region of simulation. Since we have assumed the flow is zonally symmetric,

the mean wind can advect the field variables into and away from the region of study. This illustrates that disturbance happening at some place may influence the other place.

As we have seen earlier, random disturbance creates the enhanced and the diminished wave pattern which may bring shift in kinetic energy. In the case of a resting system, kinetic energy shifts somewhere in the region of integration. However, in this case the enhanced and diminished wave pattern are carried and taken away from the region depending on the direction of the mean flow.

Chapter 5

Summary, conclusion, and future direction

5.1 Summary and conclusion

The atmospheric motion is governed by a number of conservation principles. Among them, conservation of mass, momentum, and energy are important at all scales of motion. The mathematically formulated equations of these principles are used to model the atmospheric motion. The solution of the model equation based on the appropriate boundary and/or initial condition is crucial to interpret the state of the atmosphere.

The atmosphere being chaotic, is full of a natural random process, affects one or more of field variables. This study has tried to determine the effects of the random height disturbance on the evolution of other atmospheric state variables using a linearized shallow water equation.

In the absence of the mean flow (advection term) and without the disturbance the initial distribution of field variables repeats itself with time with out any change of

shape showing that energy is conserved with time. The period of the oscillation is typical to latitude circle. This coincides with the effect of the variation of the coriolis force with latitude.

However, when the system is disturbed randomly, an enhanced and diminished pattern is created. This enhanced and diminished patterns form a wavy structure which is similar to a standing wave created by a mountain ridges.

On the other hand, when there is a mean flow, both the initial distribution and the disturbed systems (enhanced and diminished) are carried by the mean flow in the direction of the flow. This may bring a favorable or unfavorable atmospheric condition somewhere out of the region.

In general, a random disturbance of a geopotential height affects the wind velocity field and carried out of the region with the advection term and influence the atmospheric condition away from the origin of the disturbance.

5.2 Future direction

So far what is done is simulation of random disturbance using linearized shallow water equation around mid-latitude. In future, based on this result, the simulation of primitive equations for the whole globe for the same problem for different field variables will be done as part of effort to develop indigenous numerical weather prediction models at regional scale in Ethiopia.

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