ADDIS ABABA UNIVERSITY
LIBRARIES

INVESTIGATION OF ACADEMIC STAFF STRUCTURE IN THE COLLEGE OF SOCIAL SCIENCES USING MARKOV MODELS AND ADAPTATION OF A COMPUTER PROGRAM FOR ANALYSIS AND SUBSEQUENT APPLICATIONS

By

Negussie Gorfe

A THESIS SUBMITTED IN PARTIAL FULFILLMENT FOR THE DEGREE OF MASTER OF SCIENCE IN STATISTICS IN THE ADDIS ABABA UNIVERSITY

Addis Ababa

June 1990

ACKNOWLEDGEMENT

I would like to express my deep gratitude to my Advisor, Professor Ayenew Mekonnou, who has suggested the methods employed in this research and has also spent his valuable time in guiding and advising at every stage of this study.

I am also indented to Ato Gebre-Egziabher Kiros, a fellow graduate student and programmer of the NUPI Computing Center.

Finally, my thanks also go to W/t Zenebech W.Tsadik, Secretary of the Department of Statistics, for typing the manuscript quickly and efficiently.

ABSTRACT

A partially stochastic approach is employed to predict the size of academic staff in the College of Social Sciences (CSS) of the Addis Ababa University (AAU). Two Markov models, namely; Markov model with given input and Markov model with given total size are used to predict the size of academic staff in the CSS from 1989/90 to 1993/94.

To estimate parameter values, the data of the CSS on academic staff size by rank, the number of promotions to each of the ranks, the number of recruits to each of the ranks and the number of losses from each of the ranks, from 1982/83 to 1986/87, have been utilized. The method of maximum likelihood was applied on the past data of the College in order to estimate transition probabilities between the ranks, recruitment probabilities to each of the ranks and probabilities of loss from each of the ranks. The average number of annual recruits was also estimated.

The future likely structure of academic staff, based on the data of the past five years, was investigated. In addition, three different recruitment options were considered in the prediction of the size of academic staff by rank in the CSS. The predicted sizes of academic staff which were obtained by using the three recruitment options were compared with the target academic staff structure of the College envisaged by the Academic Vice-President's

Office (1988). To attain the envisaged target structure different likely promotion probabilities were also sought. Besides, we checked the attainability and maintainability of the envisaged target structure under prevailing recruitment and promotion conditions.

To facilitate computations, a computer program was adapted from Bartholomew and Forbes (1979).

TABLE OF CONTENTS

		PAGE
1.	INTRODUCTION	1
2.	ACADEMIC STAFF RECRUITMENT, LOSS AND PROMOTION IN THE CSS FROM 1982/83-1987/88.....	5
	2.1 Introduction	5
	2.2 Size of academic staff in the CSS.....	6
	2.3 Recruitment and loss of academic staff	7
	2.4 Academic staff promotion	9
3.	MODELS FOR THE INVESTIGATION OF ACADEMIC STAFF STRUCTURE	11
	3.1 The basic Markov model	11
	3.2 Markov model for a system with given input.....	12
	3.3 Markov model for a system with given size	14
	3.4 Control of structure	15
	3.5 Estimation of parameters.....	21
4.	INVESTIGATION OF ACADEMIC STAFF STRUCTURE OF THE CSS	26
	4.1 Validation of the Markov model.....	26
	4.1.1 Point estimation of parameter values	26
	4.1.2 Model validation	29
	4.2 Prediction of academic staff structure using average CSS inputs over 1982/83 to 1986/87.....	31
	4.3 Reality and AAU planned rank structure	35
	4.3.1 Planned structure and growth patterns	35
	4.3.2 Prediction of structure using different sizes.....	37

PAGE

4.4 Control of structure 44

4.4.1 Control by recruitment 44

4.4.2 Control by promotion 45

5. DISCUSSION AND CONCLUSION 52

REFERENCES 55

APPENDIX: A. Computer Program 57

B. Academic staff structure based on average input..... 62

C. Predicted number of academic staff using Q and $M_1(T)$..... 63

D. Predicted number of academic staff using Q and $M_2(T)$..... 64

E. Predicted number of academic staff using Q and $M_3(T)$..... 65

F. Predicted number of academic staff using Q_1 and $M_1(T)$..... 65

G. Predicted number of academic staff using Q_1 and $M_2(T)$..... 66

H. Predicted number of academic staff using Q_1 and $M_3(T)$..... 66

1. INTRODUCTION

Higher education is a sub-system within the educational system. For education, particularly higher education, to play its proper role in national development through appropriate development of human potential, the issues of excellence and efficiency assume great importance. To realize these concerns higher education institutions require academic staff, facilities and finance, among others.

Higher education is a very expensive venture and the resources available to it are mostly limited. Therefore, it is mandatory that the resources available be managed efficiently and effectively. This suggests that students and faculty need to be properly matched in order to achieve institutional objectives.

As the quality of higher education is largely affected by the quality of its academic staff, it is important to give serious attention to staff recruitment, promotion, and development. This is central to the future growth of any higher educational institution. To train qualified technicians, to reduce the attrition rate of students and to conduct successful research, the quality of academic staff must have a reasonable standard and an appropriate structure. The objectives of this research are, therefore, to:

i) project staff composition, for given promotion and withdrawal conditions, as recruitment rates vary over a reasonable range;

ii) find limiting structures for some reasonable recruitment rates;

iii) seek recruitment rates suitable for achieving the academic staff structure envisaged by the Academic Vice-President's (AVP) Office;

iv) seek promotion probabilities suitable for achieving the academic staff structure envisaged by AVP's Office, and

v) adapt a computer program that may be useful for future use in manpower planning.

The academic staff categories that are to be studied are those that are currently employed in AAU and these include the five ranks of Graduate Assistant, Assistant Lecturer, Lecturer, Assistant Professor and Associate Professor. The rank of Professor will not be considered since there is very little promotion experience to this rank.

To realize these objectives this research will employ a method that makes use of a partially stochastic approach based on Markov models as described in Bartholomew and Forbes (1979) and Bartholomew (1982).

Gani (1963) had applied Markov models in order to project student enrollment at each grade and the degrees to be awarded in Australian Universities. Armitage et.al. (1970) had also used the models to estimate high school graduates in the United Kingdom. Our investigation will be the first of its kind in relation to our University which has a fairly limited planning experience.

The lattest AAU planning exercise is that of the AVP's Office (1988) and the Planning Office (1988 a) which set the target of academic staff composition to be ultimately attained. AAU also plans to expand its graduate programs both vertically and horizontally. In addition, the University is planning to put more emphasis on improving the quality of education and research output. The target is, therefore, set in order to cope with the planned graduate program expansion, and to meet the desired quality of education and research which require an increase in the percentage of senior academic staff. To arrive at the target academic staff structure, the types of programs (diploma, undergraduate degree, graduate degree) offered by a college are taken into account. A college which offers both undergraduate and graduate degree programs is designed to have a higher percentage of senior staff than one which offers undergraduate degree programs only.

In this research, we will first estimate the promotion rates, withdrawal rates, and recruitment rates for the College of Social Sciences (CSS) using the 1982/83 - 1986/87 experience of the college. The data on the size of academic staff and its promotion, recruitment, and loss come from the Academic Vice-President's Office, the Academic Program's Office, the Personnel Department, the Planning Office, and the Office of the Secretariat of the University Senate.

The data for 1988/89 will be used for validation of the methods employed. We will then project the academic staff composition of the CSS for the years 1989/90 - 1993/94 and compare the results obtained with that of the AVP's Office. Different likely input structures will also be considered in the Markov models and the resulting limiting structures will be sought. An attempt will also be made to figure out possible recruitment rates and promotion probabilities that will enable the attainment of CSS's staff structure as planned by the AVP's Office.

A computer program will be adapted from Bartholomew and Forbes (1979) in order to facilitate the analysis. This program may find useful applications in AAU's Planning Office and other offices which may be concerned with manpower planning.

2. ACADEMIC STAFF RECRUITMENT, LOSS AND PROMOTION IN THE COLLEGE OF SOCIAL SCIENCES (CSS) FROM 1982/83-1987/88

2.1 Introduction

Modern higher education started in Ethiopia in March 1950 with the founding of the University College of Addis Ababa which evolved into what is now called Addis Ababa University (AAU). Presently, AAU has nine campuses which house twelve faculties and colleges as well as the School of Graduate Studies and the Continuing Education Division. Out of the total of nine campuses, four are located in four different administrative regions. The remaining five campuses are located within Addis Ababa (Planning Office, 1989).

The College of Social Sciences, which is located within Addis Ababa at the Sidist Kilo Campus, is one of the largest units of AAU, having resulted from the merger of the former Faculty of Arts, the College of Business Administration and the School of Social Work (AAU, 1981).

The College administers four-year degree programs in eight fields, namely: Accounting, Economics, Geography, History, Management and Public Administration, Philosophy, Political Science and International Relations and Sociology and Social Administration. The College also has graduate programs in the fields of Economic Planning and Development, Geography and History.

2.2 Size of academic staff in the CSS

The size of the academic staff in the CSS at the beginning of each academic year, as given by AMU's Planning Office (1988b) for 1982/83 - 1988/89, is presented in Table 1. It was indicated earlier that the CSS [illegible] undergraduate and graduate degree programs. The rank-mix to be attained by the CSS, according to AVP's Office (1988), is 9% for Graduate Assistants, 7% for Assistant Lecturers, 13% for Lecturers, 37% for Assistant Professors, 24% for Associate Professors and 10% for Professors so that the percentage of Associate Professors and full Professors adds up to 34%.

Table 1: Size of academic staff in the CSS by rank and target structure

Rank	Target Structure	Academic Year*						
		1982/83	1983/84	1984/85	1985/86	1986/87	1987/88	1988/89
Graduate Assistant	14	14	10	5	9	3	11	12
Assistant Lecturer	12	28	26	27	17	16	13	15
Lecturer	21	23	25	35	34	41	40	42
Assistant Professor	61	28	31	23	19	20	20	22
Associate Professor	39	11	12	15	15	22	24	18
Professor	17	2	2	2	5	2	1	2
Total	164	106	106	107	99	104	109	111

In general, no definite pattern can be observed in the total size of academic staff. The average rank-mix of academic staff over the years 1982/83 - 1986/87 is 7% for Graduate Assistants, 19% for

*Each academic year starts in September.

Assistant Lecturers, 33% for Lecturers, 22% for Assistant Professors, 17% for Associate Professors and 2% for Professors. There is an increase in the number of Lecturers and Associate Professors, and a decrease in almost all other ranks. This may be due to a decrease in the recruitment of Graduate Assistants, due to an increase in the number of academic staff who may have left the University for various reasons, including retirement at senior ranks, or because of problems related to promotion to the next rank.

2.3 Recruitment and Loss of academic staff

Recruitment refers to those Ethiopian staff returning to work upon completion of their studies and sabbaticals, new Graduate Assistants and those local and expatriate staff newly recruited by the College. On the other hand, staff loss includes those Ethiopian staff who went on study leave and on sabbaticals, those who left the University for various reasons, including expatriate staff whose contract had been terminated. The academic staff recruitment and loss for a given year is assessed in September of every year so that at the beginning of each academic year, the size of the faculty represents the size a year ago plus the net gain over the preceding 12-month period.

The average number of staff newly recruited per year by the College during the period 1982/83 - 1986/87, was 8, while the average staff loss per year during the same period was 7.

The number of staff newly recruited and those who left the College for various reasons, according to the various annual reports of the Personnel Department of AAU, are given in Table 2 for different ranks.

Table 2: Staff recruitment and loss in the CSS

Rank	1982/83		1983/84		1984/85		1985/86		1986/87		1987/88	
	Recruitment	Loss	Recruitment	Loss	Recruitment	Loss	Recruitment	Loss	Recruitment	Loss	Recruitment	Loss
Graduate Assistant	2	-	1	-	7	-	-	-	10	-	1	-
Assistant Lecturer	-	4	2	-	-	8	-	2	-	-	2	-
Lecturer	-	1	3	-	-	5	4	-	-	5	2	-
Assistant Professor	2	-	-	5	-	4	1	-	-	-	2	-
Associate Professor	2	-	-	-	-	1	5	-	1	-	-	6
Professor	-	1	-	-	3	-	-	3	-	1	1	-
Total	6	6	6	5	10	18	10	5	11	6	8	6

Yearly academic staff losses in the years under consideration were between 5 and 18. On the other hand, staff recruitment reached its maximum in 1986/87 while the lowest was attained in 1982/83 and 1983/84. In almost all the years under consideration, recruitment of staff was more than staff loss.

2.4 Academic staff promotion

The number of academic staff who were promoted to the various ranks, according to the AVP's Office, the Office of the Secretariat of the University Senate and the Personnel Department, for 1982/83 - 1986/87 are given in Table 3.

Table 3: Academic staff flows* in the CSS

1982/83

Rank	Rank						Loss	Total
	1	2	3	4	5	6		
1. Graduate Assistant	8	6	0	0	0	0	0	14
2. Assistant Lecturer	0	20	4	0	0	0	4	28
3. Lecturer	0	0	21	1	0	0	1	23
4. Assistant Professor	0	0	0	28	0	0	0	28
5. Associate Professor	0	0	0	0	10	1	0	11
6. Professor	0	0	0	0	0	1	1	2

1983/84

Rank	Rank						Loss	Total
	1	2	3	4	5	6		
1. Graduate Assistant	4	6	0	0	0	0	0	10
2. Assistant Lecturer	0	19	7	0	0	0	0	26
3. Lecturer	0	0	25	0	0	0	0	25
4. Assistant Professor	0	0	0	23	3	0	5	31
5. Associate Professor	0	0	0	0	12	0	0	12
6. Professor	0	0	0	0	0	2	0	2

*Promotion for a given year is that cumulated over 12-month period, starting in September.

1984/85

Rank	Rank 1	2	3	4	5	6	Loss	Total
1. Graduate Assistant	2	3	0	0	0	0	0	5
2. Assistant Lecturer	0	14	5	0	0	0	8	27
3. Lecturer	0	0	29	1	0	0	5	35
4. Assistant Professor	0	0	0	18	1	0	4	23
5. Associate Professor	0	0	0	0	14	0	1	15
6. Professor	0	0	0	0	0	2	0	2

1985/86

Rank	Rank 1	2	3	4	5	6	Loss	Total
1. Graduate Assistant	3	6	0	0	0	0	0	9
2. Assistant Lecturer	0	10	5	0	0	0	2	17
3. Lecturer	0	0	32	2	0	0	0	34
4. Assistant Professor	0	0	0	17	2	0	0	19
5. Associate Professor	0	0	0	0	15	0	0	15
6. Professor	0	0	0	0	0	2	3	5

1986/87

Rank	Rank 1	2	3	4	5	6	Loss	Total
1. Graduate Assistant	1	2	0	0	0	0	0	3
2. Assistant Lecturer	0	11	5	0	0	0	0	16
3. Lecturer	0	0	35	1	0	0	5	41
4. Assistant Professor	0	0	0	19	1	0	0	20
5. Associate Professor	0	0	0	0	22	0	0	22
6. Professor	0	0	0	0	0	1	1	2

The promotion of academic staff was susnended in AAU owing to budgetary constraints from July 1987 to Sept.1988.

3. MODELS FOR THE INVESTIGATION OF ACADEMIC STAFF STRUCTURE

The objective of this research is to investigate the academic staff structure of the CSS. To make this objective a reality we will first estimate, from past data, the promotion probability of academic staff to the next higher rank, the probability of loss of academic staff by rank and the probability of recruitment to one of the ranks. To achieve this end, we will use Markov models for a system with a given input and with a given size. The general description of the Markov models, according to Bartholomew and Forbes (1979) and Bartholomew (1982) is given below.

3.1 The basic Markov model

The assumptions for the Markov model are that individuals move independently and with identical probabilities which do not vary over time.

We will consider a population whose members are divided into k grades. Let p_{ij} be the probability that an individual in grade i at the start of a given time interval or year is in grade j at the end. The matrix $P = \{p_{ij}\}$ is called a transition matrix, where i=1,2,...,k; j=1,2,...,k; its elements are invariant with time, as assumed above. Further, let w_i be the probability that an individual of grade i at the start has left by the end of the year. The row vector

$W=(w_1,w_2,\ldots,w_k)$ is called a wastage or loss vector.

The elements of P and W will be estimated from past data. Since each member of the academic staff must either stay where he is, move to another grade, or leave, the row sums

$$\sum_{j=1}^{k} p_{ij}+w_i=1, \text{ for all } i \qquad (3.1)$$

Suppose that the total number of recruits between points of time (T-1) and T, denoted by $R(T)$, is given. Let these recruits be allocated to the grades with probabilities $r_1,r_2,\ldots,r_k$, where $\sum_{i=1}^{k} r_i=1$. $r=(r_i)$ is called a recruitment vector. Then, the transition probabilities between each of the grades may be set out in an array as follows:

p_{11}	p_{12}	p_{13}	$\cdots$	p_{1k}	w_1
p_{21}	p_{22}	p_{23}	$\cdots$	p_{2k}	w_2
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
p_{k1}	p_{k2}	p_{k3}	$\cdots$	p_{kk}	w_k

3.2 Markov model for a system with given input

We next consider a population whose members are divided into k grades. Let $n_j(T)$ denote the number of individuals in grade j at time $T(T=0,1,2,\ldots;\ j=1,2,3,\ldots,k)$; $n_j(0)$ is the initial number of people in grade j and it is assumed to be given. For $T\geq 1$, the grade sizes are random variables and we will, therefore, work with their expected values.

Let $\bar{n}_{ij}(T-1)$ be the expected number of people moving from category i to category j in the period (T-1) to T. Thus, the expected number of people in category j at time T is given by

$$\bar{n}_j(T)=\sum_{i=1}^{k}\bar{n}_{ij}(T-1)+\bar{n}_{oj}(T) \qquad (3.2)$$

where $\bar{n}_{oj}(T)$ is the expected number of recruits allocated to grade j at time T.

If the expected size in category i at time (T-1) is denoted by $\bar{n}_i(T-1)$ and the total recruitment by $\bar{R}(T)$, then the expected flows are

$$\left.\begin{aligned}\bar{n}_{oj}(T)&=\bar{R}(T)r_j\\ \bar{n}_{ij}(T-1)&=\bar{n}_i(T-1)p_{ij}\end{aligned}\right\} \qquad (3.3)$$

By combining (3.2) and (3.3), the expected grade size at time T can be expressed as follows:

$$\bar{n}_j(T)=\sum_{i=1}^{k}\bar{n}_i(T-1)p_{ij}+\bar{R}(T)r_j \qquad (3.4)$$

or, in matrix notation,

$$\bar{n}(T)=\bar{n}(T-1)P+\bar{R}(T)r \qquad (3.5)$$

It follows from (3.5) that if there is a limiting vector, n say, then it must satisfy

$$n = nP + Rr \qquad (3.6)$$

where R is the limit of the sequence $\{\bar{R}(T)\}$ which must be assumed to exist. Then, Equation (3.6) gives

$$n = Pr(I-P)^{-1} \qquad (3.7)$$

$(I-P)^{-1}$ always exists (Bartholomew and Forbes, 1979, p. 90) for P matrices with row sums strictly less than one. Relation (3.7) then gives the limiting structure for given constant r, R and P.

3.3 Markov model for a system with given size

This model differs from the one described in the preceding section only in that the total size rather than the input is fixed. Here we will have a sequence of total sizes $\{N(T)\}$ which may be a sequence of given numbers.

Let $M(T)$ be the increase in size which takes place between periods (T-1) and T; thus

$$M(T) = N(T) - N(T-1) \qquad (3.8)$$

Under this model $\{R(T)\}$ is unknown but the number of recruits must be sufficient to achieve the desired expansion and to replace the losses from the system. The expected number of recruits required at period T is, therefore, given as

$$\bar{R}(T) = M(T) + \sum_{i=1}^{k} \bar{n}_i(T-1)W_i \qquad (3.9)$$

substituting (3.9) into (3.5) we have

$$\bar{n}(T) = \bar{n}(T-1)Q + M(T)r \qquad (3.10)$$

where Q is the matrix $\{q_{ij}\}$ and $Q = P + W'r$ so that

$$\sum_{j=1}^{k} p_{ij} + W_{ij} \sum_{j=1}^{k} r_j = 1$$

Hence, Q is also a stochastic matrix since $\sum_{j=1}^{k} r_{ij} = 1$.

To facilitate the computations of equations (3.5) and (3.10), a computer program in Basic is adapted from Bartholomew and Forbes (1979).

The terms in Equation (3.10) can be identified in the following way:

$\bar{n}(T-1)P$ represents normal internal movements;

$\bar{n}(T-1)W'r$ represents recruits who replace leavers,

and $M(T)r$ represents recruits filling new or created vacancies.

3.4 Control of structure

A manpower system with reasonable looking promotion rates may lead to the higher grades growing at the expense of the lower. This feature is often observed in practice where it commonly occurs at the end of a period of expansion (Bartholomew and Forbes, 1979).

Measures designed to control a desired structure may affect the prospects of individuals whose aspirations must be taken seriously if the system is to function harmoniously and efficiently. We will, therefore, investigate what can be achieved under the constraints which the requirements of good management impose. In practice, only some flows can be controlled and even then there may be limits to the degree of control which can be exercised. Control can be exercised

through the flows of people who may be classified into one of the three categories:

i) Wastage (the vector W);

ii) Promotion (the matrix P); and

iii) Recruitment. This has two parts; (the total number $\{R(T)\}$ and their allocation to the ranks, r.)

The wastage flow can be controlled to some extent. It can be increased by dismissing people or by offering them financial and other inducements to leave, and decreased by improved conditions or inducements to stay. But these methods of control are uncertain, undesirable and often expensive.

Another method of control can be exercised over the promotion flows through direct management decisions, such as those introduced by Addis Ababa University in 1987/88. This method also has drawbacks. For example, an increase in promotion rate may involve the promotion of inadequately qualified people and make it difficult to return to the original standards should this prove necessary later. Equally, a decrease in promotion rates is likely to create problems among people who may see their expectation for advancement being eroded.

The recruitment flow offers the most attractive means of control since decisions to recruit more or fewer people at a given level can be taken without the same immediate

impact on those already serving. This method too has drawbacks:

i) There may be practical difficulties in finding enough recruits at the required levels and

ii) High recruitment near the top may affect the promotion prospects of existing academic staff members. It may also be unduely expensive.

Nevertheless, of the three methods of control the adjustment of the recruitment vector seems to offer, perhaps, the least painful means of control.

Control has two aspects which are called attainability and maintainability. Attainability is concerned with whether or not a goal, such as that of a faculty structure, can be reached and, if so, by what means. Maintainability, on the other hand, has to do with remaining at the goal structure once it has been attained.

Suppose n^* is a steady-state structure which is to be maintained. Then there must exist values of P, W and r such that we have, from (3.10),

$$n^* = n^*P + n^*W + r \qquad (3.11)$$

If recruitment is the only flow subject to control, then P and W are fixed and r is to be determined. This leads to what is known as recruitment control.

It is to be noted that every structure may not be maintainable. Thus, it is important to delineate those

structures which can be maintained from those which cannot.

Let $q_i(T)$ be the relative size at time T. Therefore,

$$q_i(T) = \frac{\bar{n}_i(T)}{\bar{N}(T)} \quad (i=1,2,\ldots,k) \tag{3.12}$$

where $\bar{N}(T)$ is the total fixed size of the system at time T. The relative size at time T is then given by:

$$q(T)=q(T-1)P+q(T-1)W'r \tag{3.13}$$

A structure $q(T)$ can be maintained if we can find values of the control parameters such that for a stable structure,

$$q(T) = q(T-1) = q \tag{3.14}$$

Equations (3.13) and (3.14) will then give rise to:

$$r = q(I-P)/qW' \tag{3.15}$$

If the values of the elements of r so obtained add up to 1, and they are positive, Equation (3.15) gives a valid structure; if not, the structure is not maintainable.

With a given P and W, it would be useful to be able to characterize the set of structures which can be maintained. For Equation (3.15) to be positive we have to have a set of $\bar{q}$'s for which

$$q \geq qP \tag{3.16}$$

One may also discuss recruitment control in the context of growth as follows.

Hence, suppose that the total size of the syster is growing or contracting; expansion or contraction may be looked upon as a further control process.

Since rate of expansion is often a critical factor in manpower planning, we shall generalize the problem by incorporating an expansion rate and deal with the quasi-stationary behaviour of the stock proportions. To do this let $M(T)$ be as defined in Equation (3.8). If α is the rate of expansion, we have,

$$M(T) = \bar{n}(T-1)1'\alpha \tag{3.17}$$

where $1_{1xk}=(1,1,\ldots,1)$. If the system is contracting, α will be negative.

Under the above conditions we can write

$$\bar{n}(T)1'=(1+\alpha)\bar{n}(T-1)1' \tag{3.18}$$

Equation (3.12) can be rewritten as

$$q(T)=\bar{n}(T)/\bar{n}(T)1' \tag{3.19}$$

substituting Equations (3.18) and (3.19) in (3.10), we have

$$(1+\alpha)q(T)=q(T-1)\{P+(W'+1'\alpha)r\} \tag{3.20}$$

which has a stationary structure q satisfying

$$(1+\alpha)q=q\{P+(W'+1'\alpha)r\} \tag{3.21}$$

If such an r exists it is given by

$$r=\{q(I-P)+\alpha q\}(qW'+\alpha)^{-1} \tag{3.22}$$

the elements of this vector should sum to 1; they will be non-negative if

$$(1+\alpha)q \geq qP \tag{3.23}$$

Thus, for given α and P Equation (3.23) provides an easy arithmetical check on whether a particular structure a is maintainable.

Still another method of control is that effected by introducing changes in P when all others are fixed and such a procedure is referred to as <u>control by promotion.</u> When control has to be exercised by promotion, W and r are fixed and the problem is then to find a matrix P satisfying Equation (3.21).

In the case of recruitment control, the unknown was a vector in an equation which had a unique solution. With promotion control, the unknown in the Equation (3.21) is the matrix P and, in general, there will be infinitely many solutions. This will be an advantage from the planning point of view because it allows a measure of choice (Bartholomew and Forbes, 1979). To find whether there is an admissible solution for a P matrix, with non-negative elements and row sums not exceeding one, we can rewrite Equation (3.21) in the form:

$$aP=(1+\alpha)a-(aW'+\alpha)r \tag{3.24}$$

Since, for any admissible P, the vector aP must have non-negative elements, we have

$$aP=(1+\alpha)a-(aW'+\alpha)r\geq 0 \tag{3.25}$$

Thus

$$(1+\alpha)a \geq (aW'+\alpha)r \tag{3.26}$$

The structure c is, therefore, maintainable if

$$c > \{(q^{W'}+\alpha)/(J+\alpha)\}r \qquad (3.27)$$

3.5 Estimation of parameters

In any practical application, the parameter values have to be assigned either by making hypothetical assumptions or by estimating their values from historical data.

To obtain point estimates of the transition probabilities from historical data, we will apply the method of maximum likelihood; in order to apply this method we need to have the total stock and flow data. We will follow Bhat's (1972) procedure of estimating the transition probabilities of an individual who was in grade i at time (T-1) and is in grade j at time T.

Suppose there is a finite Markov model with k grades and total stock n. That is:

	1	2	3	...	J	Total
1	n_{11}	n_{12}	n_{13}	...	n_{1J}	n_1
2	n_{21}	n_{22}	n_{23}	...	n_{2J}	n_2
3	n_{31}	n_{32}	n_{33}	...	n_{3J}	n_3
⋮	⋮	⋮	⋮		⋮	⋮
K	n_{k1}	n_{k2}	n_{k3}	...	n_{kJ}	n_k
						n

(3.28)

Let n_{ij} be the number of transitions from grade i to grade j and let $\sum_{j=1}^{J} n_{ij} = n_i$ $(i=1,2,\ldots,k;\ j=1,2,\ldots,J)$.

We are interested in the estimates of the transition probabilities and let these estimates be denoted by $\hat{P}_{ij}$ $(i=1,2,\ldots,k;\ j=1,2,\ldots,J)$.

For a given initial state i and a number of trials n_i, the sample transition counts $(n_{i1},n_{i2},\ldots,n_{iJ})$ can be considered as a sample of size n_i from the multinomial distribution with probabilities $(P_{i1},P_{i2},\ldots,P_{iJ})$ such that $\sum_{j=1}^{J} P_{ij}=1$. The probability of this outcome can, therefore, be given as

$$\frac{n!}{n_{i1}!n_{i2}!\ldots n_{iJ}!} P_{i1}^{n_{i1}} \cdot P_{i2}^{n_{i2}} \ldots P_{iJ}^{n_{iJ}} \qquad (3.29)$$

Extending the argument for the k initial states $(1,2,\ldots,k)$, when the breakdown of the total number of trials n into $(n_1,n_2\ldots,n_k)$ is given, the probability of the realization of the transition counts is given by:

$$\prod_{i=1}^{k} \frac{n_i!}{n_{i1}!n_{i2}\ldots n_{iJ}!} P_{i1}^{n_{i1}} P_{i2}\ldots P_{iJ}^{n_{iJ}} \qquad (3.30)$$

In Equation (3.28), the row sums $(n_1,n_2,\ldots,n_k)$ are also random variables and the likelihood function, $L(P_{ij})$, of the sample observation consists of another factor giving the joint distribution of these random variables. This factor is independent of the probability elements (Whittle, 1955), P_{ij} and let it be denoted by $A(n_{ij})$. The likelihood function can then be expressed as

$$L(P_{ij}) = A(n_{ij}) \cdot \prod_{i=1}^{k} \frac{n_i!}{n_{i1}! \cdots n_{iJ}!} p_{i1}^{n_{i1}} \cdots p_{iJ}^{n_{iJ}}$$

Taking logarithms, we find

$$\ell n\, L(p_{ij}) = \ell n\, B(n_{ij}) + \sum_{i=1}^{k} \sum_{j=1}^{J} n_{ij} \ell n\, p_{ij} \qquad (3.31)$$

where $\ell n\, B(n_{ij})$ contains all terms independent of p_{ij}'s.

To derive maximum likelihood estimates, we maximize (3.31) under the condition $\sum_{j=1}^{J} p_{ij} = 1$, $(i=1,2,\ldots,k)$. Then, we can rewrite (3.31) as:

$$\ell = \ell n\, L(p_{ij}) - \ell n\, B(n_{ij}) = \sum_{i=1}^{k} \sum_{j=1}^{J=1} n_{ij} \ell n\, p_{ij} + \sum_{i=1}^{k} n_{iJ} \ell n (1 - p_{i1} - p_{i2} - \ldots - p_{i,J-1}) \qquad (3.32)$$

From the structure of the log likelihood function, estimates can be obtained separately. For a specific value of (i,j) we have:

$$\ell_{ij} = n_{ij} \ell n p_{ij} + n_{iJ} \ell n (1 - p_{i1} - p_{i2} - \ldots - p_{i,J-1}) \qquad (3.33)$$

where $\ell_{ij} = (i,j)^{th}$ component of ℓ, $i=1,2,\ldots,k$; $j=1,2,\ldots,J$

Differentiating (3.33) with respect to p_{ij} $(j=1,2,\ldots,J-1)$ and setting it equal to zero, we get:

$$\frac{n_{i1}}{p_{i1}} - \frac{n_{iJ}}{1-p_{i1}-p_{i2}-\cdots-p_{i,J-1}} = 0$$

$$\frac{n_{i2}}{p_{i2}} - \frac{n_{iJ}}{1-p_{i1}-p_{i2}-\cdots-p_{i,J-1}} = 0$$

$$\vdots$$

$$\frac{n_{i,J-1}}{p_{i,J-1}} - \frac{n_{iJ}}{1-p_{i1}-p_{i2}-\cdots-p_{i,J-1}} = 0$$

Combining the above equations, we can write

$$\frac{n_{i1}}{p_{i1}} = \frac{n_{i2}}{p_{i2}} = \cdots = \frac{n_{i,J-1}}{p_{i,J-1}} = \frac{n_{iJ}}{1-p_{i1}-p_{i2}-\cdots-p_{i,J-1}}$$

In all the equations above, p_{ij} should strictly be replaced by $\hat{p}_{ij}$, the estimate.

Hence,

$$n_{ij} = Cp_{ij} \quad , \quad i=1,2,\ldots,k; \; j=1,2,\ldots,J \qquad (3.34)$$

where C is a proportionality constant. But

$$n_i = \sum_{j=1}^{J} n_{ij} = C \sum_{j=1}^{J} p_{ij} = C$$

Using this in (3.30), we obtain the estimators: so that

$$\hat{p}_{ij} = \frac{n_{ij}}{n_i} \quad i=1,2,\ldots,k; \; j=1,2,\ldots,J \qquad (3.35)$$

where n_i is the total stock in grade i and n_{ij} is the observed number of flows from grade i to grade j during the period (T-1,T).

If the total stock and the flow data are available over several years for each of which the transition proba-

bilities can be assumed to be the same, then

$$\hat{r}_{ij} = \frac{\sum_T n_{ij}(T)}{\sum_T n_i(T)} \qquad (3.36)$$

These relationship assume constant transition probabilities over time, but, in practice this condition is unlikely to hold, but the Markov model has still been found (Bartholomew and Forbes, 1979; p. 106) to be useful in such manpower studies. The estimates of r and w will clearly follow from (3.35) as well.

4. INVESTIGATION OF ACADEMIC STAFF STRUCTURE OF THE CSS

4.1 Validation of the Markov model

4.1.1 Point estimation of parameter values

The method of maximum likelihood was used to estimate the promotion probabilities to the next higher rank, the probability of loss from one of the ranks and the probability of allocating a new recruit into one of the ranks. To estimate these probabilities we have used Equation (3.36) of Section 3.5.

It should also be noted that, as indicated earlier, the rank of Professor could not be considered since there was almost no promotion experience to that rank in the College in the years under consideration. The estimation of transition probabilities was based on data for the period, 1982/83 - 1986/87, the estimates were used to forecast the number of promotions and the size of academic staff in each rank from 1988/89-1993/94.

It would also be expected that predictions for 1988/89 and possibly those of 1989/90 would be lower than actual values on account of the accumulation of unpromoted cases over the preceding year. These promotionswould be expected to consist partly of those that were also due in 1987/88 when promotions were suspended in the University as a whole.

The estimated values of these transition probabilities are denoted by P, W and r for this period, 1982/83 - 1986/87, and they are given below:

$$P = \begin{bmatrix} .439 & .561 & 0 & 0 & 0 \\ 0 & .649 & .228 & 0 & 0 \\ 0 & 0 & .899 & .032 & 0 \\ 0 & 0 & 0 & .868 & .058 \\ 0 & 0 & 0 & 0 & .987 \end{bmatrix} \quad (4.1)$$

$W = [.000, .123, .069, .074, .013]$

$r = [.500, .050, .175, .075, .200]$

The estimates indicate that the probability that a Graduate Assistant will get promoted to the rank of Assistant Lecturer within a given year in the period is 0.561 and the promotion probability to the rank of Lecturer is 0.228. The promotion probabilities to the rank of Assistant Professor and Associate Professor are 0.032 and 0.058, respectively.

The highest probability of loss is observed in the rank of Assistant Lecturer followed by those of Assistant Professor and Lecturer, respectively. These may be mainly due to study leave. The withdrawal probability for the most senior rank is likely to be an understimate, especially for the years to come.

The probabilities of new recruits in the five ranks are found to be 0.500 for Graduate Assistant, 0.050 for Assistant Lecturer, 0.175 for Lecturer, 0.075 for Assistant Professor and 0.200 for Associate Professor.

The average number of new annual recruits, $\bar{p}(T)$, was 8 for the period 1982/83 - 1986/87.

Using the results given above, we will check the validity of the model by first predicting the number of promotions and the size of academic staff by rank and then comparing the predicted numbers with observed values.

The objectives of fitting a model to the data of the CSS are to provide some insight into the dynamics of the system and to make projections if the model is appropriate. Our model will be judged on how successfully it achieves these objectives. It may not matter very much whether there is heterogeneity in the data or lack of independence between persons' behaviour so long as the effect on predictions is negligible. An inadequate model may well be useful for the purpose in hand and in such a case, it is not reasonable to reject it. "It is this philosophy which underlies the widespread use of Markov models in many fields of application where the assumptions, strictly interpreted,

In general, after allowing for the fact that promotions were suspended during 1987/88, there appears to be no large difference between the observed and the predicted size of academic staff in 1988/89. Therefore, the use of P, W and r in the prediction of the size of academic staff through the years 1989/90 to 1993/94 may not be unreasonable.

4.2 Prediction of academic staff structure using average CSS inputs over 1982/83 - 1986/87

A Markov model with given average input will be used to predict the structure of academic staff in the CSS in the five years, 1989/90 - 1993/94. Use will be made of the promotion, loss and recruitment experience of the College during the five years, 1982/83 - 1986/87. An answer to the question "is it possible to achieve the desired rank structure envisaged by AVP's Office (1988), if we follow past trends?" will also be sought using this model.

The Ten-Year Prespective Plan (ONCCP, 1984) covers the period 1984/85 - 1993/94. The Ten-Year Plan of AAU is part of this long-term national plan. It is, therefore, useful to predict the academic staff size of the CSS up to 1993/94, which is the end of the long-term plan period.

The average number of new annual recruits from 1982/83 to 1986/87 was found to be 8 while the average number of

Table 5: Predicted number of academic staff in the CSS

	1989/90	1990/91	1991/92	1992/93	1993/94
Graduate Assistant	9(8%)	8(7%)	7(7%)	7(6%)	7(6%)
Assistant Lecturer	17(15%)	17(15%)	16(14%)	15(13%)	14(12%)
Lecturer	43(39%)	44(39%)	44(39%)	45(39%)	45(39%)
Assistant Professor	21(19%)	20(18%)	20(17%)	19(17%)	19(17%)
Associate Professor	21(19%)	23(21%)	26(23%)	28(25%)	30(26%)
Total	111(100%)	112(100%)	113(100%)	114(100%)	115(100%)

The number of Assistant Lecturers will decrease to 14, and that for Assistant Professors to 19. On the other hand, the number of Lecturers and Associate Professors will show an increase during the same period. The number of Lecturers will grow to 45, while the number of Associate Professors will increase to 30 - an increase of 43% over the five year period.

It is also important to find the limiting structure of the CSS academic staff, if it exists, for the average recruitment of 8 per year. If this limiting structure exists it is given by Equation (3.7) of Section 3.2. This equation is given as

$$n = Rr(I-P)^{-1}$$

where I is the 5 x 5 identity matrix.

$(I-P)^{-1}$ always exists for P matrices with row sums strictly less than one. The row sums of the matrix P are 1, .877, .931, .926 and .987 for rows 1,2,3,4 and 5 respectively.

The values of the matrices (I-P) and $(I-P)^{-1}$ and the vector Rr are found to be as follows:

$$(I-P) = \begin{bmatrix} .561 & -.561 & 0 & 0 & 0 \\ 0 & .351 & -.228 & -.033 & 0 \\ 0 & 0 & .101 & -.032 & 0 \\ 0 & 0 & 0 & .132 & -.053 \\ 0 & 0 & 0 & 0 & .013 \end{bmatrix}$$

$$(I-P)^{-1} = \begin{bmatrix} 1.783 & 2.849 & 6.431 & 1.559 & 6.956 \\ 0 & 2.349 & 6.431 & 1.559 & 6.956 \\ 0 & 0 & 9.901 & 2.400 & 10.709 \\ 0 & 0 & 0 & 7.576 & 33.300 \\ 0 & 0 & 0 & 0 & 76.923 \end{bmatrix}$$

$$Rr = [4, 0.4, 1.4, 0.6, 1.6]$$

Substituting these values in Equation (3.7) of Section 3.2, we obtained the limiting structure to be

$$n = n[\infty], [13, 42, 15, 189]$$

After 451 years, the academic staff structure to a steady state and the total size of academic staff in the CSS is then expected to reach 266. The computer outputs is given as Appendix B. The ultimate academic staff structure to be attained, if we follow the experience of the past five years, is 3% for Graduate Assistants, 5% for Assistant Lecturers, 16% for Lecturers, 5% for Assistant

Professors and the remaining 71% for Associate Professors. This apparently top-heavy structure is quite unlike the target structure envisaged by AVP's Office (1988). Hence, with current rates of promotion, withdrawal and recruitment, the target structure planned for AAU appears to be unattainable.

4.3 Reality and AAU planned rank structure

4.3.1 Planned structure and growth patterns

A comarison of the predicted academic staff structure for 1993/94, which was obtained in Section 4.2 by using P, W, r, n(0), and the average annual inputs over the years 1982/83-1986/87, and the target academic staff structure (AVP's Office, 1988) for the year 1993/94 is provided in Table 6.

Table 6: The target and predicted academic staff structure

Rank	Predicted (%)	Target (%)
Graduate Assistant	6	9
Assistant Lecturer	12	7
Lecturer	39	13
Assistant Professor	17	37
Associate Professor*	26	34
Total	100	100

*Includes Professors.

To attain the target rank structure of the CSS, we need to increase the percentage share of senior staff. On the otherhand, the percentage shares of Assistant Lecturers and Lecturers need to be reduced in order to achieve the rank structure envisaged for the College by the year, 1993/94.

The size of academic staff in 1988/89 was 109 and it is planned, by AVC's Office (1988), to increase the total academic staff size to 164 by the year, 1993/94. This means that we need to increase the size of academic staff by 55 during the period 1989/90-1993/94. This comes to an average increase of 11 per year. To attain this planned academic staff expansion, three different options are considered.

The first option assumes a constant increase of 11 academic staff every year from 1989/90-1993/94.

The second option considers a linear or arithmetic growth rate. The net increase is assumed to be 7 in 1989/90 and it increases linearly by 2 every year so that by the year 1993/94, the number of recruits will reach 15.

The third option, on the other hand, assumes a geometric growth pattern. Here the number of recruits starts at 5 in 1989/90 and increases multiplicatively by a factor of 1.4. The size of the net increase, according to this option, by the end of the plan-period, will be 19.

The yearly increments for the three different options, denoted by $M_1(T)$, $M_2(T)$ and $M_3(T)$ are provided below, for the year 1989/90 - 1993/94.

Option	1989/90	1990/91	1991/92	1992/93	1993/94	Total
$M_1(T)$	11	11	11	11	11	55
$M_2(T)$	7	9	11	13	15	55
$M_3(T)$	5	7	10	14	19	55

(4.2)

The values of $M_1((T)$, $M_2(T)$ and $M_3(T)$, which are given in (4.2) above, will be used to predict academic staff sizes by rank in the next sections.

4.3.2 Prediction of structure using different sizes

A method of forecasting the size of academic staff by rank is by using the Markov model with given total size. In this model we need to know about the variation of the total size of academic staff in the CSS in the years to come.

The CSS, according to the AVP's Office (1988), plans to increase its academic staff in terms of quality and quantity. The quality of academic staff is to be realized by increasing the percentage share of senior academic staff members, which, according to the classi-

fication of AAU, include those within the ranks of Assistant Professor, Associate Professor and Professor.

The size of academic staff in the CSS by the year 1993/94 is planned to be 164. The distribution of academic staff among the six ranks is expected to be 14 Graduate Assistants, 12 Assistant Lecturers, 21 Lecturers, 61 Assistant Professors, 39 Associate Professors and 17 Professors. Since the rank of Professor will not be considered here for reasons given earlier, the target for the number of faculty at the rank of Associate Professor will be taken as 56. To attain the desired academic staff size by 1993/94, we will provide for an expansion in the size of the staff every year.

To predict the structure of academic staff, we use Equation (3.10) of Section 3.3. This equation requires the values of P, W, r, W'r, O, M(T) and n(0), the size of academic staff at the beginning of 1988/89. The values of these matrices and vectors are given below:

$$P = \begin{bmatrix} .439 & .561 & 0 & 0 & 0 \\ 0 & .649 & .228 & 0 & 0 \\ 0 & 0 & .899 & .032 & 0 \\ 0 & 0 & 0 & .868 & .053 \\ 0 & 0 & 0 & 0 & .987 \end{bmatrix}$$

$$W = [.000,\ .123,\ .069,\ .074,\ .013]$$

$$r = [.500,\ .050,\ .175,\ .075,\ .200]$$

$$W'r = \begin{bmatrix} .000 & .000 & .000 & .000 & .000 \\ .061 & .006 & .022 & .009 & .025 \\ .035 & .003 & .012 & .005 & .014 \\ .037 & .004 & .013 & .005 & .015 \\ .006 & .001 & .002 & .001 & .003 \end{bmatrix}$$

$$Q = \begin{bmatrix} .439 & .561 & .000 & .000 & .000 \\ .061 & .655 & .250 & .009 & .025 \\ .035 & .003 & .911 & .037 & .014 \\ .037 & .004 & .013 & .873 & .073 \\ .006 & .001 & .002 & .001 & .990 \end{bmatrix}$$

and $\quad n(0) = [12,\ 15,\ 42,\ 22,\ 18]$

The values of P, W and r are those that have already been given in (4.1).

The three different recruitment options, which were given in (4.2), have been used in predicting academic staff size by rank.

The predicted numbers of academic staff in the CFS which are obtained by substituting the values given above and in (4.2) in Equation (3.10)are presented in Tables 7, 8 and 9.

To facilitate the computation of Equation (3.10) we have again used the adapted Computer Program from Bartholomew and Forbes (1979). Table 7 corresponds to $M_1(T)$, the constant yearly increment and the computer output is in Appendix C.

Table 7: Predicted number of academic staff using $M_1(T)$

	1989/90	1990/91	1991/92	1992/93	1993/94
Graduate Assistant	14(12%)	15(11%)	16(11%)	17(11%)	17(10%)
Assistant Lecturer	17(14%)	20(15%)	22(15%)	24(16%)	26(16%)
Lecturer	44(37%)	47(36%)	50(35%)	53(35%)	57(35%)
Assistant Professor	22(18%)	22(17%)	22(16%)	22(14%)	22(13%)
Associate Professor*	23(19%)	27(21%)	32(23%)	37(24%)	42(26%)
Total	120(100%)	131(100%)	142(100%)	153(100%)	164(100%)

*Includes Professors.

With the exception of Assistant Professors, the predicted number of academic staff showed an increase in all the remaining four ranks. The number of Assistant Professors remained constant during the plan period. This is

may be due to the number of promotions and recruits to the rank of Assistant Professor being equal to the number of promotions to the rank of Associate Professor and losses from the rank of Assistant Professor, every year. The highest increase in the size of academic staff is observed at the rank of Associate Professor, followed by Lecturer and Assistant Lecturer, respectively. During the plan period under consideration, the total number of Graduate Assistants will have increased by 3.

On the other hand, the percentage share of Graduate Assistants decreased by 2%, Lecturers by 2% and Assistant Professors by 5%. Meanwhile, the percentage share of Assistant Lecturers and Associate Professors increased by 2% and 7%, respectively. Table 8 arises from M_2 (T), which represents linear growth in which the increment varies linearly with time. The computer output is in Appendix D.

Table 8: Predicted number of academic staff using M_2(T)

Rank	1989/90	1990/91	1991/92	1992/93	1993/94
Graduate Assistant	12(10%)	13(10%)	15(11%)	17(11%)	19(12%)
Assistant Lecturer	17(15%)	19(15%)	20(15%)	23(16%)	26(16%)
Lecturer	44(38%)	46(37%)	49(36%)	52(35%)	56(16%)
Assistant Professor	21(18%)	21(17%)	21(16%)	21(14%)	22(13%)
Associate Professor*	22(19%)	26(21%)	31(22%)	36(24%)	41(25%)
Total	116(100%)	125(100%)	136(100%)	149(100%)	164(100%)

*Includes Professors.

Table 9 results from $M_3(T)$ whereby the yearly increments exhibit geometric growth. The computer output is in Appendix E.

Table 9: Predicted number of Academic staff using $M_3(T)$

Rank	1989/90	1990/91	1991/92	1992/93	1993/94
Graduate Assistant	11(10%)	12(10%)	14(10%)	17(12%)	21(13%)
Assistant Lecturer	17(15%)	18(15%)	19(15%)	21(14%)	25(15%)
Lecturer	43(38%)	45(37%)	48(36%)	51(35%)	55(34%)
Assistant Professor	21(19%)	21(17%)	21(16%)	21(15%)	22(13%)
Associate Professor*	21(19%)	25(21%)	29(23%)	35(24%)	41(25%)
Total	113(100%)	121(100%)	131(100%)	145(100%)	164(100%)

*Includes Professors.

Although three different growth patterns were used in the prediction of academic staff, there appears to be little difference between the predicted values. In almost all cases, the percentage shares of Graduate Assistant and Assistant Lecturer showed a slight increase; the percentage shares of Lecturer and Assistant Professor showed a decrease while that of Associate Professor has increased.

The predicted structures of academic staff which are obtained using $M_1(T)$, $M_2(T)$ and $M_3(T)$ are compared with the target academic staff structure (AVP's Office, 1988) for 1993/94 in Table 10 below.

Table 10: Target and predicted academic staff structures in the CSS in 1993/4

Rank	Predicted Structure (%)			Target Structure (%)
	$M_1(T)$	$M_2(T)$	$M_3(T)$	
Graduate Assistant	10	12	13	9
Assistant Lecturer	16	16	15	7
Lecturer	35	34	34	13
Assistant Professor	13	13	13	37
Associate Professor*	26	25	25	34
Total	100	100	100	100

*Includes Professor.

The predicted staff structures which are obtained using different size options are very close to each other in each of the five academic ranks, but there is a big difference between the target and the predicted staff structures in all the ranks. In the ranks of Assistant Lecturer and Lecturer, the predicted percentages are larger than those in the target. On the other hand, in the ranks of Assistant Professor and Associate Professor the target's percentages are larger than the predicted ones. Hence, the overall picture is that even though one may attain the same total size with various growth patterns, existing promotion, withdrawal and recruitment rates do not appear to lead to the planned staff structure of AAU.

4.4 Control of Structure

4.4.1 Control by Recruitment

Suppose we assume that the target structure is some how attained. It is then important to check whether or not this target structure is maintainable. To do this first we have to compute the relative sizes of academic staff in the target structure. Let these relative sizes be denoted by the vector, a, where

$$a(T) = n_i(T)/N(T)$$

where $N(T) = n(T)1'$, $1'$ being a column vector each element of which is unity.

Using the above formula we have obtained the relative sizes of the target structure to be $a=[.085, .073, .128, .372, .342]$. To check the maintainability of the given structure we need to find out if we can find valid values of the vector, r, using Equation (3.22) of Section 3.4. Hence we have

$$r = \{a(I-P)+\alpha c\} / (aW'+\alpha)$$

where $W = [.000, .123, .069, .074, .013]$, I_{5x5} is the identity matrix of order 5, P is the matrix which was given earlier.

In order to estimate α we have applied the geometric growth rate method [Shryock and Siegel],1971), which follows from

$$\frac{A_n}{A_o} = (1 + \alpha)^n$$

where

A_n = the size of academic staff in 1993/94 which is 164.

A_o = the size of academic staff in 1988/89, which is 109.

n = number of years, which is 5 here.

using $A_5 = 164$, $A_o = 109$, α comes to be 0.085.

If, upon substituting the values of Q, P, α, and W in the above equation, the computed values of the vector r are all positive, then the structure is said to be maintainable; otherwise it is not.

Here, the value of the vector r obtained by using the above formula are

$$r = [.407, -.119, .052, .571, .089].$$

The values of the vector r are not all positive which implies that even if the target structure envisaged by AVP's Office (1988) is attained, it is not maintainable when the promotion matrix P, α and W are those based on the experience of 1982/83 - 1986/87.

4.4.2 Control by Promotion

When control has to be exercised by promotion, W and r are fixed and the problem is then to find a

matrix P satisfying Equation (3.21) of Section 3.4. Under promotion control, the structure q is maintainable if the condition given by Equation (3.27) of Section 3.4 is satisfied, that is

$$\alpha > \{(\alpha W' + \alpha)/(1+\alpha)\}\, r$$

where r, α, W and α are given.

Using these values of q, W, α and r, we find that $\{(\alpha W' + \alpha)/(1+\alpha)\}\, r = [.062, .006, .022, .009, .025]$.

Since the inequality given by Equation (3.27) is satisfied for all elements of the vector α, the structure is maintainable under promotion control. Hence, a stuitable matrix, P, can be found such that the desired structure can be maintained for fixed α, W and r.

Hence, in order to attain the target structure envisaged by AVP's Office (1988), one alternative is to change the promotion probabilities in the matrix P, which was given earlier, within a reasonable range. The promotion probabilities to the various ranks need to be changed in such a way that the predicted academic staff number is closer to the target staff number by the year 1993/94.

To attain the target staff structure, it is necessary to increase the number of Assistant Professors, and to decrease the number of Lecturers and

Assistant Lecturers. This is done by increasing the promotion probability to rank of Assistant Professor.

To realize the above concerns a matrix P_1, which is given below, is selected. When compared with matrix P, which was given earlier, the promotion probabilities to all the ranks are increased in P_1:

$$P_1 = \begin{bmatrix} .350 & .650 & 0 & 0 & 0 \\ 0 & .377 & .500 & 0 & 0 \\ 0 & 0 & .500 & .431 & 0 \\ 0 & 0 & 0 & .841 & .085 \\ 0 & 0 & 0 & 0 & .937 \end{bmatrix}$$

P_1 is one of the many promotion matrices which will lead to the attainment of the target structure. A general solution is obtained in Rao and Mitra (1971).

In order to predict academic staff, we have to find a matrix Q_1, where

$$Q_1 = P_1 + W'r$$

and

$$Q_1 = \begin{bmatrix} .350 & .650 & .000 & .000 & .000 \\ .061 & .383 & .522 & .009 & .025 \\ .035 & .003 & .512 & .436 & .014 \\ .037 & .004 & .013 & .846 & .100 \\ .006 & .001 & .002 & .001 & .990 \end{bmatrix}$$

Equation (3.10) of Section 3.3 is then used to predict the number of academic staff for 1989/90 - 1993/94. The vectors and the matrix required for the prediction are W and r, which were given in (4.1), and Q_1, which is given above. n(0)=[12, 15, 42, 22, 18], the size of academic staff at the beginning of 1988/89.

The three different recruitment options, which were given in (4.2) are again used in predicting academic staff size by rank.

The predicted numbers of academic staff in the CSS which are obtained by substituting the above values in Equation (3.10)are given in Tables 11, 12,and 13 for $M_1(T)$, $M_2(T)$ and $M_3(T)$, respectively. The computer outputs are in Appendices F, G and H, respectively.

Table 11: Predicted number of academic staff using $M_1(T)$

Rank	1989/90	1990/91	1991/92	1992/93	1993/94
Graduate Assistant	13(11%)	14(10%)	14(10%)	14(9%)	15(9%)
Assistant Lecturer	14(12%)	15(11%)	15(10%)	16(11%)	16(10%)
Lecturer	32(26%)	26(20%)	24(17%)	23(15%)	22(14%)
Assistant Professor	38(32%)	47(36%)	52(37%)	55(36%)	58(35%)
Associate Professor*	23(19%)	30(23%)	37(26%)	45(29%)	53(32%)
Total	120(100%)	132(100%)	142(100%)	153(100%)	164(100%)

*Includes Professors.

Table 12: Predicted number of academic staff using $M_2(T)$

Rank	1989/90	1990/91	1991/92	1992/93	1993/94
Graduate Assistant	11(9%)	12(9%)	13(10%)	15(10%)	17(10%)
Assistant Lecturer	14(12%)	13(11%)	14(10%)	15(10%)	16(10%)
Lecturer	31(27%)	25(20%)	22(16%)	22(15%)	22(14%)
Assistant Professor	38(33%)	46(37%)	51(38%)	54(36%)	56(34%)
Associate Professor*	22(19%)	29(23%)	36(26%)	44(29%)	53(32%)
Total	116(100%)	125(100%)	136(100%)	150(100%)	164(100%)

*Includes Professors.

Table 13: Predicted number of academic staff using $M_3(T)$

Rank	1989/90	1990/91	1991/92	1992/93	1993/94
Graduate Assistant	10(9%)	10(9%)	12(9%)	15(10%)	18(11%)
Assistant Lecturer	14(12%)	12(10%)	12(9%)	14(10%)	16(10%)
Lecturer	31(27%)	25(20%)	22(17%)	21(14%)	22(13%)
Assistant Professor	37(33%)	46(38%)	50(38%)	53(36%)	56(34%)
Associate Professor*	22(19%)	29(23%)	35(27%)	43(30%)	52(32%)
Total	114(100%)	121(100%)	131(100%)	146(100%)	164(100%)

*Includes Professors

There is little difference among the predicted numbers of academic staff by rank which are obtained using $M_1(T)$, $M_2(T)$ and $M_3(T)$. Besides, the predicted numbers of academic

staff which are obtained using the three different growth options are quite close to the target structure envisaged by the AVP's Office (1988). Table 14 summarizes the results for 1993/94.

Table 14: Target and predicted academic staff structures in the CSS in 1993/94

Rank	Predicted staff structure (%) $M_1(T)$	$M_2(T)$	$M_3(T)$	Target Structure (%)
Graduate Assistant	9	10	11	9
Assistant Lecturer	10	10	10	7
Lecturer	14	14	13	13
Assistant Professor	35	34	34	37
Associate Professor*	32	32	32	34
Total	100	100	100	100

*Includes Professors.

As is evident in Table 14, it is possible to attain the target structure envisaged by AVP's Office (1988) by changing the promotion probabilities. So, if it is desired to attain the target rank structure in the CSS, it is necessary that the current promotion policy of the University be relaxed considerably.

Promotions to the various ranks result from direct management control. It is important to note, however, that an increase in the promotion probabilities must be weighed against the danger of the promotion of inadequately qualified staff, which, in turn, may lead to a further lowering of the quality of education at AAU.

5. DISCUSSION AND CONCLUSION

A partially stochastic approach has been used to predict the number of academic staff by rank in the CSS.

In order to estimate parameter values we have applied maximum likelihood method on the data of the CSS from 1982/83 to 1986/87. Two models, namely: Markov model with given input and Markov model with given total size have been used to predict the size of academic staff by rank from 1989/90-1993/94.

We have predicted the number of academic staff by rank in the CSS based on the past promotion, recruitment and loss experiences of the College. In addition, three growth options of which one exhibits a constant increment per year and the rest embody arithmetic and geometric growths have been considered under the two Markov models.

The predicted numbers of academic staff which were obtained by using the three growth options and the two Markov models, were then compared with the target staff structure envisaged by AVP's Office (1988).

In general, the results obtained have indicated that the numbers of Graduate Assistants were nearly the same. On the other hand, there were more Assistant Lecturers and Lecturers in the prediction while the number of Assistant Professors and Associate Professors were far below the desired target.

The three different growth options have been unable to attain the target structure envisaged by the AVP's Office (1988). We have also seen that the target structure was not attainable under prevailing conditions.

To attain the envisaged target structure the only alternative was to change the promotion probabilities in matrix P. So a matrix P_1, which has relatively higher promotion probabilities to each of the ranks when compared with P_0 was used. Matrix P_1 is one of the many promotion matrices which lead to the attainment of the target structure.

The predicted number of academic staff in 1993/94, which were based on the matrix Q_1 (a stochastic matrix based on P_1, W and r), n(0) and the three different growth options were found to be very close to the target structure. The target staff structure was thus found to be maintainable under promotion control.

In summary, we conclude that:

i) So far, no statistical models have been applied to predict academic staff structure in any higher education institution in Ethiopia. The models applied here may be of considerable help in manpower planning, especially in academic staff prediction by rank;

ii) The models have also been found to be useful tools in projecting student enrollments at each grade (Cani, 1963;

Armitage, et.al, 1970). Therefore, Markov models may also have useful applications in planning student enrollment and graduation in Ethiopia.

iii) The adapted computer program in Basic, which is attached as Appendix A, may be useful in facilitating the computations using the Markov models.

iv) Markov models may also be useful in assessing whether or not a certain target structure is attainable under existing promotion, recruitment and loss experiences. The models will also be useful in checking the maintainability of a target structure.

The study involved only the CSS. The results obtained are, therefore, concerned only with this College. But it is the belief of this investigator that such Markov models can be useful tools in University-wide planning.

REFERENCES

1. Academic Vice-President's Office (1988). Optimal Number of Academic Staff Required in AAU. Addis Ababa University, Addis Ababa.

2. Addis Ababa University (1981). General Information Handbook for a Faculty. Addis Ababa.

3. Addis Ababa University (1985). Long-Term Plan for Strengthening and Expanding the Graduate Programmes of AAU (1987/88-1997/98). Addis Ababa.

4. Armitage, P.H., C.M. Phillips and J. Davies (1970). "Towards a Model of the Upper Secondary School System." Journal of the Royal Statistical Society, Vol. 133, Series A, Part 2, pp. 166-205.

5. Bartholomew, D.J. (1982). Stochastic Models for Social Processes, 3rd Edition. John Wiley and Sons, New York.

6. Bartholomew, D.J. and A.F. Forbes (1979). Statistical Techniques for Manpower Planning. John Wiley and Sons, New York.

7. Bhat, U.N. (1972). Elements of Applied Stochastic Processes. John Wiley and Sons, New York.

8. Gani, J. (1963). "Formulae for Projecting Enrollments and Degrees Awarded in Universities." Journal of the Royal Statistical Society, Vol. 126, Series A, pp. 400-409.

9. Planning Office (1989). Facts and Figures. Addis Ababa University, Addis Ababa.

10. Planning Office (1988a). Long-term Plan of the Undergraduate Programs of AAU (1986/87-1997/98.) Addis Ababa University, Addis Ababa.

11. Planning Office (1988b). A Statistical Digest. Addis Ababa University, Addis Ababa.

12. Office of the National Committee for Central Planning 1984). Ten-year Perspective Plan 1984/85-1993/94. Addis Ababa.

13. Rao, C.R. and S.K. Mitra (1971). Generalized Inverse of Matrices and its Applications. John Wiley and Sons, New York.

14. Shryock, H.S. and J.S. Siegel (1971). The Methods and Materials of Demography, Vol. 2. U.S. Department of Commerce, Bureau of the Census.

15. Whittle, P. (1955). "The Outcome of a Stochastic Epidemic, a note on Bailey's Paper." Biometrica, 42, pp. 116-122.

```
10 DIM N(1,10),P(10,10),R(1,10),M(10),X(1,10),Q(1,10),W(10)
20 PRINT "BASEQN EVALUATES N(T+1)=N(T)*P - R"
30 ON ERROR GOTO 980
40 LET A=0
50 REM
60 REM DATA INPUT SEGMENT
70 PRINT
80 PRINT " K=";
85 LPRINT " K= ";
90 INPUT K
95 LPRINT K
100 REM
105 PRINT " N= ";
106 LPRINT " N= ";
110 FOR I=1 TO K
120 PRINT " N(1";",";I;") =";
130  INPUT N(1,I)
135 LPRINT N(1,I),
140 NEXT I
145 LPRINT
150 IF A=1 THEN 1010
160 D0=.0000001
170 FOR J=1 TO K
180 M(J)=N(1,J)
190 D0=D0+N(1,J)
200 NEXT J
210 REM
220 PRINT "  P= ";
225 LPRINT " P= "
230 FOR I=1 TO K
240 FOR J=1 TO K
245 PRINT "P(";I;",";J;")=";
250 INPUT P(I,J)
255 LPRINT P(I,J),
260 NEXT J
263 LPRINT
265 NEXT I
270 IF A=1 THEN 1010
275 REM
280 PRINT " ENTER DATA   R = ";
285 LPRINT " R =";
290 FOR I=1 TO K
295 PRINT "Q(1";",";I;")=";
300 INPUT Q(1,I)
305 LPRINT Q(1,I),
310 NEXT I
312 LPRINT
315 PRINT "ENTER DATA FOR R,R$,E"
320 INPUT R,R$,E
330 IF R=-1 THEN 2050
340 IF C$="*R=" THEN 375
350 IF A=1 THEN 1010
360 REM
370 PRINT "  T,% =      ":PRINT "---------- -------"
375 PRINT "ENTER DATA FOR   S,B$ ";:INPUT S,B$
390 IF C$="*R=" THEN 410
400 GOTO 980
410 S=S-1
420 GOTO 620
```

```
 30 REM
 40 REM CALC AND OUTPUT SEGMENT
 50 PRINT
 60 T1=-1
 70 T2=0
 80 REM TABLE HEADING
 90 LPRINT " T ";
100 FOR J=1 TO K
110 IF B$="YES" THEN 550
120 IF B$="YESO" THEN 570
130 LPRINT USING "  ###";J;
140 GOTO 580
150 LPRINT USING "      ###";J;
160 GOTO 580
170 LPRINT USING "       ###";J;
180 NEXT J
190 LPRINT "       TOTAL";
200 LPRINT  "      R"
210 LPRINT
220 FOR T=0 TO S
230 IF T1=-1 THEN 720
240 REM TO SET MAT R
250 GOSUB 1620
260 FOR I=1 TO K
265 TENT=0
270 FOR J=1 TO K
280 TENT=TENT + N(1,J)*P(J,I)
290 NEXT J
300 X(1,I)=TENT
310 NEXT I
313 FOR I=1 TO K
314 N(1,I)=X(1,I)+R(1,I)
315 NEXT I
320 REM
330 D=1E-08
340 FOR J=1 TO K
350 D=D+N(1,J)
360 NEXT J
370 T1=T1+1
380 IF T1=T2 THEN 800
390 GOTO 960
800 T2=T2+1
810 LPRINT USING "##";T1;
820 LPRINT " ";
830 FOR J=1 TO K
840 IF B$="YES" THEN 880
850 IF B$="YESO" THEN 900
860 LPRINT USING "  ###";N(1,J);
870 GOTO 910
880 LPRINT USING "####(##%)";N(1,J),100*N(1,J)/D;
890 GOTO 910
900 LPRINT USING "####(###%)";N(1,J),100*N(1,J)/M(J);
910 NEXT J
920 LPRINT USING "######(###%)";D,100*D/D0;
930 IF T1=0 THEN 950
940 LPRINT USING " ####";R1;
950 LPRINT
960 NEXT T
```

```
1550 GOTO 340
1560 REM N IS RESET TO ORIGIONAL N(0)
1570 FOR J=1 TO K
1580 N(1,J)=M(J)
1590 NEXT J
1600 GOTO 1810
1610 REM
1620 REM SUB TO SET MAT R
1630 IF R=-1 THEN 1660
1640 IF R$="0" THEN 1950
1650 GOTO 1810
1660 REM TOTAL SIZE SYSTEM FIXED
1670 R1=0
1680 FOR J=1 TO K
1690 R1=R1+N(1,J)*W(J)
1700 NEXT J
1710 IF R$="+" THEN 1740
1720 IF R$="*" THEN 1760
1730 PRINT "ERROR IN R$ AT LINE 1730"
1740 R1=R1+E
1750 GOTO 1770
1760 R1=R1+D*(E-1)
1770 FOR I=1 TO K
1780 R(1,I)=R1*Q(1,I)
1790 NEXT I
1800 GOTO 2030
1810 REM THIS SEG FOR R NE 1 AND R$ NE 0
1820 REM R1=R
1830 FOR I=1 TO K
1840 R(1,I)=R1*Q(1,I)
1850 NEXT I
1870 REM CALC SCAL R READY FOR FOLLOWING YEAR
1880 IF R$="+" THEN 1910
1890 IF R$="*" THEN 1930
1900 PRINT "ERROR IN RS AT LINE 1900"
1910 R=R+E
1920 GOTO 2030
1930 R=R*E
1940 GOTO 2030
1950 REM THIS SEG FOR R$ =  0
1960 FOR I=1 TO K
1970 R(1,I)=Q(1,1)
1980 NEXT I
1990 R1=0
2000 FOR J=1 TO K
2010 R1=R1 +R(1,J)
2020 NEXT J
2030 RETURN
2040 REM
2050 REM CALC WAST RATES IF R=-1
2060 FOR I=1 TO K
2070 W(I)=1
2080 FOR J=1 TO K
2090 W(I)=W(I)-P(I,J)
2100 NEXT J
2110 NEXT I
2120 GOTO 340
2130 REM
2140 REM CHANGE INDIVIDUAL ELEMENTS OF P
```

```
970 REM
980 A=1
990 REM
1000 REM CONTROL SEGMENT
1010 REM
1020 PRINT "(*) ENTER YOUR CONTROL SEGMENT ";
1030 INPUT C$
1040 IF C$="RUN" THEN 650
1050 IF C$="N" THEN 100
1060 IF C$="N=" THEN 110
1070 IF C$="P" THEN 210
1080 IF C$="P=" THEN 330
1090 IF C$="R" THEN 275
1100 IF C$="R=" THEN 290
1110 IF C$="*R=" THEN 290
1120 IF C$="T%" THEN 360
1130 IF C$="T%=" THEN 375
1140 IF C$="T=" THEN 1520
1150 IF C$="+T=" THEN 1480
1160 IF C$="START" THEN 40
1170 IF C$="RESTART" THEN 40
1180 IF C$="BEGIN" THEN 40
1190 IF C$="A" THEN 40
1200 IF C$="END" THEN 2270
1210 IF C$="FINISH" THEN 2270
1220 IF C$="Z" THEN 2270
1230 IF C$="PIJ" THEN 2140
1240 IF C$="PIJ=" THEN 2160
1250 IF C$="ROWP" THEN 2200
1260 IF C$="ROWP=" THEN 2220
1270 IF C$="N=N(O)" THEN 1560
1280 IF C$="PARAMS" THEN 1310
1290 PRINT "NO SUCH CONTROL";
1300 GOTO 1010
1310 REM PARAM PRINT-CHECK SEGMENT
1320 PRINT
1325 PRINT "  N = "
1330 FOR I=1 TO K
1340 PRINT "N(";I;")=";N(J,I)
1350 NEXT I
1360 PRINT "P="
1370 FOR I=1 TO K
1380 FOR J=1 TO K
1390 PRINT "P(";I;",";J;")=";P(I,J)
1400 NEXT J
1410 NEXT I
1420 PRINT "R="
1430 FOR I=1 TO K
1440 PRINT "Q(1,";I;")=";Q(1,I)
1450 NEXT I
1460 PRINT R,R$,E
1470 GOTO 1010
1480 REM FURTHER YEARS SAME TABLE
1490 INPUT "ENTER FURTHER YEARS SAME TABLE";S
1500 S=S-1
1510 GOTO 620
1520 REM PRINT ONLY SPECIFIED YEAR
1525 PRINT "Enter only specified year".
1530 INPUT T2
1540 S=T2-T1-1
```

```
2150 PRINT "I;J;P(I,J)= ";
2160 INPUT "ENTER VALUES FOR I AND J RESP.";I,J
2170 INPUT "P(";I;",";J;")= ";P(I,J)
2180 GOTO 1010
2190 REM
3200 REM CHANGE WHOLE ROW OF P
2210 PRINT "I;P(I,J),J=1,K; ";
2220 INPUT "ENTER # FOR I ";I
2230 FOR J=1 TO K
2240 INPUT "ENTER DATA FOR P(";I;",";J;")= ";P(I,J)
2250 NEXT J
2260 GOTO 1010
2270 END
```

Appendix B: Academic staff structure based on average input

K = 5					
N = 12	15	42	22	18	
P = .139	.561	0	0	0	
0	.649	.228	0	0	
0	0	.899	.032	0	
0	0	0	.968	.058	
0	0	0	0	.987	
R = 4	.4	1.0	.6	1.6	

T	1	2	3	4	5	TOTAL	R
0	12(11%)	15(14%)	42(39%)	22(20%)	18(17%)	109(100%)	
1	9(8%)	17(15%)	43(39%)	21(19%)	21(19%)	111(101%)	8
2	8(7%)	17(15%)	44(39%)	20(18%)	23(21%)	112(102%)	8
3	8(7%)	16(14%)	44(39%)	20(17%)	26(23%)	113(103%)	8
4	7(6%)	15(13%)	45(39%)	19(17%)	28(25%)	114(105%)	8
5	7(6%)	14(12%)	45(39%)	19(16%)	30(26%)	115(106%)	8
160	7(3%)	13(5%)	42(17%)	15(6%)	168(69%)	245(225%)	8
170	7(3%)	13(5%)	42(17%)	15(6%)	171(69%)	248(227%)	8
175	7(3%)	13(5%)	42(17%)	15(6%)	172(69%)	249(228%)	8
180	7(3%)	13(5%)	42(17%)	15(6%)	173(69%)	250(229%)	8
190	7(3%)	13(5%)	42(17%)	15(6%)	175(70%)	252(231%)	8
200	7(3%)	13(5%)	42(17%)	15(6%)	177(70%)	253(232%)	8
228	7(3%)	13(5%)	42(16%)	15(6%)	180(70%)	257(236%)	8
240	7(3%)	13(5%)	42(16%)	15(6%)	182(70%)	258(237%)	8
250	7(3%)	13(5%)	42(16%)	15(6%)	183(70%)	259(238%)	8
270	7(3%)	13(5%)	42(16%)	15(6%)	184(71%)	261(239%)	8
300	7(3%)	13(5%)	42(16%)	15(6%)	186(71%)	262(241%)	8
320	7(3%)	13(5%)	42(16%)	15(6%)	186(71%)	263(241%)	8
330	7(3%)	13(5%)	42(16%)	15(6%)	187(71%)	263(242%)	8
340	7(3%)	13(5%)	42(16%)	15(6%)	187(71%)	264(242%)	8
350	7(3%)	13(5%)	42(16%)	15(6%)	187(71%)	264(242%)	8
360	7(3%)	13(5%)	42(16%)	15(6%)	187(71%)	264(242)	8
370	7(3%)	13(5%)	42(16%)	15(6%)	188(71%)	264(242%)	8
380	7(3%)	13(5%)	42(16%)	15(6%)	188(71%)	264(243%)	8
400	7(3%)	13(5%)	42(16%)	15(6%)	188(71%)	265(243%)	8

410	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	265(243%)	8
420	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	265(243%)	8
430	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	265(243%)	8
440	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	265(243%)	8
450	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	265(243%)	8
451	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8
452	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8
453	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8
454	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8
455	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8
456	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8
457	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8
458	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8
459	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8
460	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8
461	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8
462	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8
500	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8
550	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8
600	7(3%)	13(5%)	42(16%)	15(6%)	189(71%)	266(243%)	8

Appendix C: Predicted number of academic staff using Q and $M_1(T)$

K = 5

N =	12	15	42	22	18
P =	.439	.561	0	0	0
	.061	.655	.25	.009	.025
	.035	.003	.911	.037	.014
	.037	.004	.013	.873	.073
	.006	.001	.002	.000	.99
R =	5.5	.55	1.925	.825	2.2

T	1	2	3	4	5	TOTAL	R
0	12(11%)	15(14%)	42(39%)	22(20%)	18(17%)	109(100%)	11
1	14(12%)	17(14%)	44(37%)	22(18%)	23(19%)	120(110%)	11
2	15(12%)	20(15%)	47(36%)	22(17%)	27(21%)	131(120%)	11
3	16(11%)	22(16%)	50(35%)	22(15%)	32(22%)	142(130%)	11
4	17(11%)	24(16%)	53(35%)	22(14%)	37(24%)	153(140%)	11
5	17(10%)	26(16%)	57(35%)	22(13%)	42(26%)	164(150%)	11

Appendix D: Predicted number of academic staff using Q and $M_2(T)$

K = 5

N = 12 15 42 22 22 18 1.4

R = 3.5 .35 1.225 .585 1.4

T	1	2	3	4	5	TOTAL	R
0	12(11%)	15(14%)	42(39%)	22(20%)	18(17%)	109(100%)	R
1	12(10%)	17(15%)	44(38%)	21(18%)	22(19%)	116(106%)	7
2	13(11%)	19(15%)	46(37%)	21(17%)	26(21%)	125(115%)	9
3	15(11%)	20(15%)	49(36%)	21(16%)	31(22%)	136(125%)	11
4	17(11%)	23(15%)	52(35%)	21(14%)	36(24%)	149(137%)	13
5	19(12%)	26(16%)	56(34%)	22(13%)	41(25%)	164(150%)	15

Appendix E: Predicted number of academic staff using Q and $M_3(T)$

K = 5

N = 12 15 42 22 18

R = 2.5 .25 .875 .375 1

T	1	2	3	4	5	TOTAL	R
0	12(11%)	15(14%)	42(39%)	22(20%)	18(17%)	109(100%)	
1	11(10%)	17(15%)	43(38%)	21(19%)	21(19%)	113(105%)	5
2	12(10%)	18(15%)	45(37%)	21(17%)	25(21%)	131(111%)	7
3	14(10%)	19(15%)	48(36%)	21(16%)	29(23%)	131(120%)	10
4	17(12%)	21(15%)	51(35%)	21(15%)	35(24%)	145(133%)	14
5	21(13%)	25(15%)	55(34%)	22(13%)	41(25%)	164(150%)	19

Appendix B: Predicted number of academic staff using Q_1 and $M_1(T)$

K = 5

N = 12	15	42	22	18
P = .35	.65	0	0	0
.061	.383	.522	.009	.025
.035	.003	.512	.436	.014
.037	.004	.013	.846	.1
.006	.001	.002	.001	.990
R = 5.5	.55	1.925	.825	2.2

T	1	2	3	4	5	TOTAL	R
0	12(11%)	15(14%)	42(39%)	22(20%)	18(17%)	109(100%)	
1	13(11%)	14(12%)	32(26%)	38(32%)	23(19%)	120(110%)	11
2	14(10%)	15(11%)	26(20%)	47(36%)	30(23%)	132(120%)	11
3	14(10%)	15(11%)	24(17%)	52(37%)	37(26%)	142(130%)	11
4	14(9%)	16(10%)	23(15%)	55(36%)	45(29%)	153(140%)	11
5	15(9%)	16(10%)	22(14%)	58(35%)	53(32%)	164(150%)	11

Appendix G: Predicted number of academic staff using Q_1 and $M_2(T)$

K = 5

N = 12	15	42	22	18
R = 3.5	.35	1.225	.525	1.4

T	1	2	3	4	5	TOTAL	R
0	12(11%)	15(14%)	42(39%)	22(20%)	18(17%)	109(100%)	
1	11(9%)	14(12%)	31(27%)	38(32%)	22(19%)	116(106%)	7
2	12(9%)	13(11%)	25(20%)	46(37%)	29(23%)	125(115%)	9
3	13(10%)	14(10%)	22(17%)	51(38%)	36(26%)	136(125%)	11
4	15(10%)	15(10%)	22(15%)	54(36%)	44(29%)	149(137%)	13
5	17(10%)	16(10%)	22(14%)	56(34%)	53(32%)	164(150%)	15

Appendix H: Predicted number of academic staff using Q_1 and $M_3(T)$

K = 5

N =	12	15	42	22	18
R =	2.5	.25	.875	.375	1

T	1	2	3	4	5	TOTAL	R
0	12(11%)	15(14%)	42(39%)	22(20%)	18(17%)	109(100%)	=
1	10(9%)	14(12%)	31(27%)	37(33%)	22(19%)	114(105%)	5
2	10(9%)	12(10%)	25(20%)	46(38%)	28(23%)	121(111%)	7
3	12(9%)	12(9%)	22(16%)	50(38%)	35(26%)	131(120%)	10
4	15(10%)	14(9%)	21(14%)	53(37%)	43(29%)	145(133%)	14
5	18(11%)	16(10%)	22(13%)	56(34%)	52(32%)	164(150%)	19