Two-Mode Coherent and Squeezed Vacuum States

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The undersigned hereby certify that they have read and recommend to the School of Graduate Studies for acceptance a thesis entitled "**Two-Mode Coherent and Squeezed Vacuum States**" by **Alelign Worku** in partial fulfillment of the requirements for the degree of **Master of Science in Physics**.

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Abstract

Using the antinormally-ordered characteristic function, we have determined the Q function for a two-mode coherent state as well as a two-mode squeezed vacuum state. Using the pertinent Q function, we have calculated the quadrature variance, the mean and the normally-ordered variance of the photon number sum and difference. Furthermore, we have obtained the Q function for the superposition of two-mode coherent and squeezed vacuum states. Employing the resulting Q function, we have calculated the quadrature variance, the mean and the normally-ordered variance of the photon number sum and difference.

We have found that the mean of the photon number sum for the superposition of two-mode coherent and squeezed vacuum states is the sum of the mean of the photon number sums for the individual states. And the mean of the photon number difference for the superposition of two-mode coherent and squeezed vacuum states is the same as that of the two-mode coherent state. The quadrature variance for the superposition of two-mode coherent and squeezed vacuum states is the same as that of a two-mode squeezed vacuum state.

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Chapter 1

Introduction

The quantum description of radiation is one of the central topics in quantum optics. This description requires the quantization of the radiation field. The quantization of the radiation field leads to the introduction of various possible quantum sates of light such as the number states, the coherent states, and the squeezed states [1-12].

Of all states of the radiation field, the coherent states are the most important and arise frequently in quantum optics. Not only can they be an accurate representation of the radiation field produced by a stabilized laser operating well above threshold, but also many of the techniques for studying the properties of the radiation fields rely on the properties of the coherent states [1-7]. On the other hand, squeezed light has potential applications in the detection of weak signals and in low-noise communications [1,3,7,8,13,14].

Single-mode coherent states are minimum uncertainty states with equal noise in both quadratures and they have Poissonian photon statistics. On the other hand, in a single-mode squeezed vacuum state the fluctuations in the plus quadrature are below the coherent state level with enhanced fluctuations in the minus quadrature [1,2,3,4,5,6]. The mean photon number for the superposition of single-mode coherent and squeezed vacuum states is the sum of the individual state mean photon numbers [15].

In this thesis, we seek to study the quantum properties of the superposed two-mode coherent and squeezed vacuum states. To this end, we first determine the Q functions

for the individual states. Using the pertinent Q function, we calculate the quadrature variance, the mean and the normally-ordered variance of the photon number sum and difference. Furthermore, we obtain the Q function for the superposition of two-mode coherent and squeezed vacuum states. Applying the resulting Q function, we calculate the quadrature variance, the mean and the normally-ordered variance of the photon number sum and difference.

Chapter 2

Two-Mode Coherent States

Coherent states can easily be generated using the unitary displacement operator [1-7]. The two-mode coherent state $\big|\alpha, \beta\big\rangle$ may be defined in terms of the displacement operator

$$\hat{D}(\alpha, \beta) = e^{(\alpha\hat{a}^\dagger - \alpha^*\hat{a})} e^{(\beta\hat{b}^\dagger - \beta^*\hat{b})}, \tag{2.0.1}$$

as

$$\big|\alpha, \beta\big\rangle = \hat{D}(\alpha, \beta)\big|0_a, 0_b\big\rangle, \tag{2.0.2}$$

where α and β are complex numbers.

2.1 The Q function

The Q function for a two-mode light is defined by

$$Q(\alpha, \beta) = \frac{\langle\beta, \alpha\big|\hat{\rho}\big|\alpha, \beta\big\rangle}{\pi^2}. \tag{2.1.1}$$

This function is expressible in terms of the antinormally-ordered characteristic function defined by [1]

$$\phi_{a,b}(z, \eta) = Tr\big(\hat{\rho} e^{-z^*\hat{a}} e^{z\hat{a}^\dagger} e^{-\eta^*\hat{b}} e^{\eta\hat{b}^\dagger}\big). \tag{2.1.2}$$

Applying the completeness relation for coherent states and using fact that operators for separate modes are commute with each other, we can write the above equation as

$$\phi_{a,b}(z,\eta) = Tr\Big(\int \frac{d^2\alpha\; d^2\beta}{\pi^2}\hat{\rho} e^{-z^*\hat{a}}\; e^{-\eta^*\hat{b}}\big|\alpha,\beta\big\rangle\big\langle\beta,\alpha\big|e^{z\hat{a}^\dagger}e^{\eta\hat{b}^\dagger}\Big). \tag{2.1.3}$$

From the property of the trace operation and on account of (2.1.1), we have

$$\phi_{a,b}(z,\eta) = \int d^2\alpha\; d^2\beta\; Q(\alpha,\beta)e^{(z^*\alpha - z\alpha^* + \eta^*\beta - \eta\beta^*)}. \tag{2.1.4}$$

Since the Q function is the inverse Fourier transform of this characteristic function, we see that

$$Q(\alpha,\beta) = \frac{1}{\pi^4}\int d^2z\; d^2\eta\; \phi_{a,b}(z,\eta)e^{(z^*\alpha - z\alpha^* + \eta^*\beta - \eta\beta^*)}. \tag{2.1.5}$$

Next we seek to obtain the Q function for the two-mode coherent state $\big|\ell,\lambda\big\rangle$. The antinormally-ordered characteristic function for this state can be written as

$$\phi_{a,b}(z,\eta) = Tr\big(\big|\ell,\lambda\big\rangle\big\langle\lambda,\ell\big|e^{-z^*\hat{a}}e^{z\hat{a}^\dagger}e^{-\eta^*\hat{b}}e^{\eta\hat{b}^\dagger}\big). \tag{2.1.6}$$

With the aid of the identity [1-6]

$$e^{\hat{A}}e^{\hat{B}} = e^{\hat{B}}e^{\hat{A}}e^{[\hat{A},\hat{B}]}, \tag{2.1.7}$$

we get

$$e^{-z^*\hat{a}}e^{z\hat{a}^\dagger}e^{-\eta^*\hat{b}}e^{\eta\hat{b}^\dagger} = e^{-(z^*z+\eta^*\eta)}e^{z\hat{a}^\dagger}e^{-z^*\hat{a}}e^{\eta\hat{b}^\dagger}e^{-\eta^*\hat{b}}. \tag{2.1.8}$$

In view of this result, expression (2.1.6) can be put in the form

$$\phi_{a,b}(z,\eta) = e^{-(z^*z+\eta^*\eta)+z\ell^*-z^*\ell+\eta\lambda^*-\eta^*\lambda}. \tag{2.1.9}$$

On account of this relation, Eq. (2.1.5) can be written as

$$Q(\alpha,\beta) = \frac{1}{\pi^4}\int d^2z d^2\eta e^{-(z^*z+\eta^*\eta)+z\ell^*-z^*\ell+\eta\lambda^*-\eta^*\lambda+z^*\alpha-z\alpha^*+\eta^*\beta-\eta\beta^*}. \tag{2.1.10}$$

This can be rewritten as

$$\begin{aligned} Q(\alpha,\beta) = \quad & \frac{1}{\pi^2}\int\frac{d^2z}{\pi}e^{-z^*z+(\ell^*-\alpha^*)z+(\alpha-\ell)z^*} \\ & \times\int\frac{d^2\eta}{\pi}e^{-\eta^*\eta+(\lambda^*-\beta^*)\eta+(\beta-\lambda)\eta^*}. \end{aligned} \tag{2.1.11}$$

Integrating over η and z using the relation [1]

$$\int \frac{d^2z}{\pi} exp\big(- azz^* + bz + cz^* + Az^2 + Bz^{*2}\big)$$
$$= \left[\frac{1}{a^2 - 4AB}\right]^{1/2} exp\left[\frac{abc + Ac^2 + Bb^2}{a^2 - 4AB}\right], a > 0 \tag{2.1.12}$$

we find

$$Q(\alpha, \beta) = \frac{1}{\pi^2} exp\big[- (\ell\ell^* + \lambda\lambda^* + \alpha^*\alpha + \beta^*\beta) + \ell^*\alpha + \ell\alpha^* + \lambda^*\beta + \beta^*\lambda\big], \tag{2.1.13}$$

which is the Q function for a two-mode coherent state.

2.2 The mean of the photon number sum and difference

The photon number sum and difference are defined by [1]

$$\hat{n}_+ = \hat{n}_a + \hat{n}_b \tag{2.2.1}$$

and

$$\hat{n}_- = \hat{n}_a - \hat{n}_b, \tag{2.2.2}$$

with

$$\hat{n}_a = \hat{a}^\dagger \hat{a} \tag{2.2.3}$$

and

$$\hat{n}_b = \hat{b}^\dagger \hat{b}. \tag{2.2.4}$$

Where $\hat{n}_a$ and $\hat{n}_b$ are the photon number operators for the separate modes. The mean of the photon number sum and difference are given by

$$\bar{n}_+ = \bar{n}_a + \bar{n}_b \tag{2.2.5}$$

and

$$\bar{n}_{-} = \bar{n}_a - \bar{n}_b. \tag{2.2.6}$$

The mean photon number of mode a is expressible as

$$\bar{n}_a = \int d^2\alpha \; d^2\beta Q(\alpha,\beta)\alpha\alpha^* - 1. \tag{2.2.7}$$

Upon inserting (2.1.13) into (2.2.7), we get

$$\begin{aligned}\bar{n}_a = \quad & \frac{1}{\pi^2}\int d^2\alpha \; d^2\beta exp\big[-(\ell\ell^* + \lambda\lambda^* + \alpha^*\alpha + \beta^*\beta) \\ & +\ell^*\alpha + \ell\alpha^* + \lambda^*\beta + \beta^*\lambda\big]\alpha\alpha^* - 1. \end{aligned} \tag{2.2.8}$$

This can be rewritten as

$$\begin{aligned}\bar{n}_a = \quad & exp\big[-\ell\ell^* - \lambda\lambda^*\big] \\ & \times\frac{d}{d\eta}\frac{d}{d\gamma}\Big[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \eta\alpha + \ell^*\alpha + \ell\alpha^* + \gamma\alpha^*\big] \\ & \times\int \frac{d^2\beta}{\pi} exp\big[-\beta\beta^* + \lambda^*\beta + \lambda\beta^*\big]\Big]_{\eta=\gamma=0} - 1. \end{aligned} \tag{2.2.9}$$

On performing the integration over β, we find

$$\begin{aligned}\bar{n}_a = \quad & exp\big[-\ell\ell^*\big] \\ & \times\frac{d}{d\eta}\frac{d}{d\gamma}\Big[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \eta\alpha + \ell^*\alpha + \ell\alpha^* + \gamma\alpha^*\big]\Big]_{\eta=\gamma=0} - 1, \end{aligned} \tag{2.2.10}$$

and on carrying out the integration over α, we have

$$\bar{n}_a = \frac{d}{d\eta}\frac{d}{d\gamma}\Big[exp\big[\eta\ell + \eta\gamma + \gamma\ell^*\big]\Big]_{\eta=\gamma=0} - 1. \tag{2.2.11}$$

After differentiating and applying the condition $\eta = \gamma = 0$, we obtain

$$\bar{n}_a = \ell\ell^*. \tag{2.2.12}$$

In a similar manner, one can also show that

$$\bar{n}_b = \lambda\lambda^*. \tag{2.2.13}$$

Upon substituting (2.2.12) and (2.2.13) into (2.2.5) and (2.2.6), the mean of the photon number sum and difference for a two-mode coherent state are given by

$$\bar{n}_+ = \ell\ell^* + \lambda\lambda^*, \tag{2.2.14}$$

and

$$\bar{n}_- = \ell\ell^* - \lambda\lambda^*. \tag{2.2.15}$$

2.3 The normally-ordered variance of the photon number sum and difference

The normally-ordered variance of the photon number sum is given by

$$:(\Delta n_+)^2:= \langle :\hat{n}_+^2: \rangle - \langle :\hat{n}_+: \rangle^2. \tag{2.3.1}$$

Upon substituting (2.2.1) along with (2.2.3) and (2.2.4) into this, we have

$$:(\Delta n_+)^2:= \langle :(\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b})^2: \rangle - \langle \hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b} \rangle^2. \tag{2.3.2}$$

This is the same as

$$:(\Delta n_+)^2:= \langle \hat{a}^{\dagger 2}\hat{a}^2 \rangle + 2\langle \hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b} \rangle + \langle \hat{b}^{\dagger 2}\hat{b}^2 \rangle - (\langle \hat{a}^\dagger\hat{a} \rangle + \langle \hat{b}^\dagger\hat{b} \rangle)^2. \tag{2.3.3}$$

Using the commutation relations $[\hat{a}, \hat{a}^\dagger] = 1$ and $[\hat{b}, \hat{b}^\dagger] = 1$, this can be rewritten in the antinormal order as

$$:(\Delta n_+)^2:= \langle \hat{a}^2\hat{a}^{\dagger 2} \rangle + 2\langle \hat{a}\hat{a}^\dagger\hat{b}\hat{b}^\dagger \rangle + \langle \hat{b}^2\hat{b}^{\dagger 2} \rangle - 6\big[\bar{n}_a + \bar{n}_b + 1\big] - \big[\bar{n}_a + \bar{n}_b\big]^2. \tag{2.3.4}$$

Similarly

$$:(\Delta n_-)^2 := \langle \hat{a}^2\hat{a}^{\dagger 2}\rangle - 2\langle \hat{a}\hat{a}^\dagger\hat{b}\hat{b}^\dagger\rangle + \langle \hat{b}^2\hat{b}^{\dagger 2}\rangle - 2\big[\bar{n}_a + \bar{n}_b + 1\big] - \big[\bar{n}_a - \bar{n}_b\big]^2. \qquad (2.3.5)$$

The expectation value of $\hat{a}^2\hat{a}^{\dagger 2}$ in terms of the Q function for a two-mode coherent state is expressible as

$$\langle \hat{a}^2\hat{a}^{\dagger 2}\rangle = \int d^2\alpha\ d^2\beta Q(\alpha,\beta)\alpha^2\alpha^{*2}. \qquad (2.3.6)$$

Upon inserting (2.1.13) into this, we get

$$\begin{aligned}\langle \hat{a}^2\hat{a}^{\dagger 2}\rangle = \quad & \frac{1}{\pi^2}\int d^2\alpha\ d^2\beta exp\big[-(\ell\ell^* + \lambda\lambda^* + \alpha^*\alpha + \beta^*\beta) \\ & +\ell^*\alpha + \ell\alpha^* + \lambda^*\beta + \beta^*\lambda\big]\alpha^2\alpha^{*2}. \end{aligned} \qquad (2.3.7)$$

This can be rewritten as

$$\begin{aligned}\langle \hat{a}^2\hat{a}^{\dagger 2}\rangle = \quad & exp\big[-\ell\ell^* - \lambda\lambda^*\big] \\ & \times\frac{d^2}{d\eta^2}\frac{d^2}{d\gamma^2}\Big[\int\frac{d^2\alpha}{\pi}exp\big[-\alpha\alpha^* + \eta\alpha + \ell^*\alpha + \ell\alpha^* + \gamma\alpha^*\big] \\ & \times\int\frac{d^2\beta}{\pi}exp\big[-\beta\beta^* + \lambda^*\beta + \lambda\beta^*\big]\Big]_{\eta=\gamma=0}. \end{aligned} \qquad (2.3.8)$$

On performing the integration over β, we find

$$\begin{aligned}\langle \hat{a}^2\hat{a}^{\dagger 2}\rangle = \quad & exp\big[-\ell\ell^*\big] \\ & \times\frac{d^2}{d\eta^2}\frac{d^2}{d\gamma^2}\Big[\int\frac{d^2\alpha}{\pi}exp\big[-\alpha\alpha^* + \eta\alpha + \ell^*\alpha + \ell\alpha^* + \gamma\alpha^*\big]\Big]_{\eta=\gamma=0}, \end{aligned} \qquad (2.3.9)$$

and on carrying out the integration over α, we have

$$\langle \hat{a}^2\hat{a}^{\dagger 2}\rangle = \frac{d^2}{d\eta^2}\frac{d^2}{d\gamma^2}\Big[exp\big[\eta\ell + \eta\gamma + \gamma\ell^*\big]\Big]_{\eta=\gamma=0}. \qquad (2.3.10)$$

After differentiating and applying the condition $\eta = \gamma = 0$, we obtain

$$\langle \hat{a}^2\hat{a}^{\dagger 2}\rangle = 2 + 4\ell\ell^* + \ell^2\ell^{*2}. \qquad (2.3.11)$$

Following a similar procedure, we arrive at

$$\langle \hat{b}^2 \hat{b}^{\dagger 2} \rangle = 2 + 4\lambda\lambda^* + \lambda^2\lambda^{*2}. \tag{2.3.12}$$

The expectation value of $\hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger$ in terms of the Q function for a two-mode coherent state is expressible as

$$\langle \hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger \rangle = \int d^2\alpha \; d^2\beta Q(\alpha,\beta)\alpha\alpha^*\beta\beta^*. \tag{2.3.13}$$

Upon inserting (2.1.13) into this, we get

$$\begin{aligned} \langle \hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger \rangle = \quad & \frac{1}{\pi^2}\int d^2\alpha \; d^2\beta exp\big[-(\ell\ell^* + \lambda\lambda^* + \alpha^*\alpha + \beta^*\beta) \\ & +\ell^*\alpha + \ell\alpha^* + \lambda^*\beta + \beta^*\lambda\big]\alpha\alpha^*\beta\beta^*. \end{aligned} \tag{2.3.14}$$

This can be rewritten as

$$\begin{aligned} \langle \hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger \rangle = \quad & exp\big[-\ell\ell^* - \lambda\lambda^*\big] \\ & \times \frac{d}{d\eta}\frac{d}{d\gamma}\frac{d}{d\mu}\frac{d}{d\nu}\Bigg[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \eta\alpha + \ell^*\alpha + \ell\alpha^* + \gamma\alpha^*\big] \\ & \times \int \frac{d^2\beta}{\pi} exp\big[-\beta\beta^* + \lambda^*\beta + \mu\beta + \nu\beta^* + \lambda\beta^*\big]\Bigg]_{\eta=\gamma=\mu=\nu=0}. \end{aligned} \tag{2.3.15}$$

On performing the integration over β, we get

$$\begin{aligned} \langle \hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger \rangle = \quad & exp\big[-\ell\ell^*\big] \\ & \times \frac{d}{d\eta}\frac{d}{d\gamma}\frac{d}{d\mu}\frac{d}{d\nu}\Bigg[exp\big[\nu\lambda^* + \mu\nu + \mu\lambda\big] \\ & \times \int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \eta\alpha + \ell^*\alpha + \ell\alpha^* + \gamma\alpha^*\big]\Bigg]_{\eta=\gamma=\mu=\nu=0}, \end{aligned} \tag{2.3.16}$$

and on carrying out the integration over α, we have

$$\langle \hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger \rangle = \frac{d}{d\eta}\frac{d}{d\gamma}\frac{d}{d\mu}\frac{d}{d\nu}\Bigg[exp\big[\nu\lambda^* + \mu\nu + \mu\lambda + \eta\ell + \eta\gamma + \gamma\ell^*\big]\Bigg]_{\eta=\gamma=\mu=\nu=0}. \tag{2.3.17}$$

After differentiating and applying the condition $\mu = \nu = \eta = \gamma = 0$, we get

$$\langle \hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger \rangle = 1 + \ell\ell^* + \lambda\lambda^* + \ell\ell^*\lambda\lambda^*. \tag{2.3.18}$$

Upon inserting (2.2.12), (2.2.13), (2.3.11), 2.3.12), and (2.3.18) into (2.3.4) and (2.3.5), the normally-ordered variance of the photon number sum and difference are given by

$$: (\Delta n_+)^2 := 0, \tag{2.3.19}$$

and

$$: (\Delta n_-)^2 := 0. \tag{2.3.20}$$

Since the normally-ordered variance of the photon number sum and difference are zero, the photon statistics of a two-mode coherent state is Poissonian.

2.4 The quadrature variance

The squeezing properties of a two-mode light are described by two quadrature operators defined by [1]

$$\hat{c}_+ = \frac{1}{\sqrt{2}}(\hat{a}_+ + \hat{b}_+) \tag{2.4.1}$$

$$\hat{c}_- = \frac{1}{\sqrt{2}}(\hat{a}_- + \hat{b}_-), \tag{2.4.2}$$

where

$$\hat{a}_+ = \hat{a}^\dagger + \hat{a} \tag{2.4.3}$$

$$\hat{a}_- = i(\hat{a}^\dagger - \hat{a}), \tag{2.4.4}$$

and

$$\hat{b}_+ = \hat{b}^\dagger + \hat{b}, \tag{2.4.5}$$

$$\hat{b}_- = i(\hat{b}^\dagger - \hat{b}). \tag{2.4.6}$$

The operators $\hat{c}_+$ and $\hat{c}_-$ are Hermitian and satisfy the commutation relation

$$[\hat{c}_+, \hat{c}_-] = 2i. \tag{2.4.7}$$

A two mode light is said to be in a squeezed state if either $\Delta c_+ < 1$ or $\Delta c_- < 1$ such that $\Delta c_+ \Delta c_- \geq 1$.

The variance for the plus and the minus quadratures are defined by

$$(\Delta c_+)^2 = \langle \hat{c}_+^2 \rangle - \langle \hat{c}_+ \rangle^2 \tag{2.4.8}$$

and

$$(\Delta c_-)^2 = \langle \hat{c}_-^2 \rangle - \langle \hat{c}_- \rangle^2. \tag{2.4.9}$$

On account of (2.4.1) and (2.4.2) along with (2.4.3), (2.4.4), (2.4.5), and (2.4.6), the variance for the plus and the minus quadratures are given by

$$\begin{aligned}(\Delta c_+)^2 = \quad & \frac{1}{2}\big[(\Delta a_+)^2 + (\Delta b_+)^2\big] + \langle \hat{a}\hat{b} \rangle + \langle \hat{a}\hat{b}^\dagger \rangle + \langle \hat{a}^\dagger \hat{b} \rangle + \langle \hat{a}^\dagger \hat{b}^\dagger \rangle \\ & -\langle \hat{a} \rangle\langle \hat{b} \rangle - \langle \hat{a} \rangle\langle \hat{b}^\dagger \rangle - \langle \hat{a}^\dagger \rangle\langle \hat{b} \rangle - \langle \hat{a}^\dagger \rangle\langle \hat{b}^\dagger \rangle\end{aligned} \tag{2.4.10}$$

and

$$\begin{aligned}(\Delta c_-)^2 = \quad & \frac{1}{2}\big[(\Delta a_-)^2 + (\Delta b_-)^2\big] - \langle \hat{a}\hat{b} \rangle + \langle \hat{a}\hat{b}^\dagger \rangle + \langle \hat{a}^\dagger \hat{b} \rangle - \langle \hat{a}^\dagger \hat{b}^\dagger \rangle \\ & +\langle \hat{a} \rangle\langle \hat{b} \rangle - \langle \hat{a} \rangle\langle \hat{b}^\dagger \rangle - \langle \hat{a}^\dagger \rangle\langle \hat{b} \rangle + \langle \hat{a}^\dagger \rangle\langle \hat{b}^\dagger \rangle,\end{aligned} \tag{2.4.11}$$

Where

$$(\Delta a_+)^2 = 1 + 2\langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{a}^{\dagger 2} \rangle + \langle \hat{a}^2 \rangle - \langle \hat{a} \rangle^2 - \langle \hat{a}^\dagger \rangle^2 - 2\langle \hat{a}^\dagger \rangle\langle \hat{a} \rangle \tag{2.4.12}$$

and

$$(\Delta a_-)^2 = 1 + 2\langle \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^{\dagger 2} \rangle - \langle \hat{a}^2 \rangle + \langle \hat{a} \rangle^2 + \langle \hat{a}^\dagger \rangle^2 - 2\langle \hat{a}^\dagger \rangle\langle \hat{a} \rangle, \tag{2.4.13}$$

with a similar definitions for $(\Delta b_+)^2$ and $(\Delta b_-)^2$.

The expectation value of $\hat{a}$ in terms of the Q function for a two-mode coherent state is expressible as

$$\langle \hat{a} \rangle = \int d^2\alpha \ d^2\beta Q(\alpha, \beta)\alpha. \tag{2.4.14}$$

On account of (2.1.13), we have

$$\langle \hat{a} \rangle = \frac{1}{\pi^2} \int d^2\alpha \; d^2\beta exp\big[-(\ell\ell^* + \lambda\lambda^* + \alpha^*\alpha + \beta^*\beta) + \ell^*\alpha + \ell\alpha^* + \lambda^*\beta + \beta^*\lambda \big] \alpha. \tag{2.4.15}$$

This can be rewritten as

$$\langle \hat{a} \rangle = exp\big[-\ell\ell^* - \lambda\lambda^* \big] \times \frac{d}{d\gamma} \Big[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \gamma\alpha + \ell^*\alpha + \ell\alpha^* \big] \times \int \frac{d^2\beta}{\pi} exp\big[-\beta\beta^* + \lambda^*\beta + \lambda\beta^* \big] \Big]_{\gamma=0}. \tag{2.4.16}$$

On performing the integration over β, we find

$$\langle \hat{a} \rangle = exp\big[-\ell\ell^* \big] \frac{d}{d\gamma} \Big[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \gamma\alpha + \ell^*\alpha + \ell\alpha^* \big] \Big]_{\gamma=0}, \tag{2.4.17}$$

and on carrying out the integration over α, we have

$$\langle \hat{a} \rangle = \frac{d}{d\gamma} \Big[exp\big[\gamma\ell\big] \Big]_{\gamma=0}. \tag{2.4.18}$$

After differentiating and applying the condition $\gamma = 0$, we get

$$\langle \hat{a} \rangle = \ell. \tag{2.4.19}$$

Similarly the expectation value of $\hat{b}^\dagger$ is given by

$$\langle \hat{b} \rangle = \lambda. \tag{2.4.20}$$

The expectation value of $\hat{a}^2$ in terms of the Q function for a two-mode coherent state is expressible as

$$\langle \hat{a}^2 \rangle = \int d^2\alpha \; d^2\beta Q(\alpha, \beta) \alpha^2. \tag{2.4.21}$$

On account of (2.1.13), we have

$$\langle \hat{a}^2 \rangle = \frac{1}{\pi^2}\int d^2\alpha\ d^2\beta exp\big[-(\ell\ell^* + \lambda\lambda^* + \alpha^*\alpha + \beta^*\beta) + \ell^*\alpha + \ell\alpha^* + \lambda^*\beta + \beta^*\lambda\big]\alpha^2. \tag{2.4.22}$$

This can be rewritten as

$$\langle \hat{a}^2 \rangle = exp\big[-\ell\ell^* - \lambda\lambda^*\big] \times \frac{d^2}{d\gamma^2}\Big[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \gamma\alpha + \ell^*\alpha + \ell\alpha^*\big] \times \int \frac{d^2\beta}{\pi} exp\big[-\beta\beta^* + \lambda^*\beta + \lambda\beta^*\big]\Big]_{\gamma=0}. \tag{2.4.23}$$

On performing the integration over β, we find

$$\langle \hat{a}^2 \rangle = exp\big[-\ell\ell^*\big]\frac{d^2}{d\gamma^2}\Big[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \gamma\alpha + \ell^*\alpha + \ell\alpha^*\big]\Big]_{\gamma=0}, \tag{2.4.24}$$

and on carrying out the integration over α, we have

$$\langle \hat{a}^2 \rangle = \frac{d^2}{d\gamma^2}\Big[exp\big[\gamma\ell\big]\Big]_{\gamma=0}. \tag{2.4.25}$$

After differentiating and applying the condition $\gamma = 0$, we get

$$\langle \hat{a}^2 \rangle = \ell^2. \tag{2.4.26}$$

Similarly the expectation value of $\hat{b}^2$ is given by

$$\langle \hat{b}^2 \rangle = \lambda^2. \tag{2.4.27}$$

The expectation value of $\hat{a}\hat{b}$ in terms of the Q function for a two-mode coherent state is expressible as

$$\langle \hat{a}\hat{b} \rangle = \int d^2\alpha\ d^2\beta Q(\alpha, \beta)\alpha\beta. \tag{2.4.28}$$

Upon inserting (2.1.13) into this, we get

$$\langle\hat{a}\hat{b}\rangle = \frac{1}{\pi^2}\int d^2\alpha\ d^2\beta exp\big[-(\ell\ell^* + \lambda\lambda^* + \alpha^*\alpha + \beta^*\beta) + \ell^*\alpha + \ell\alpha^* + \lambda^*\beta + \beta^*\lambda\big]\alpha\beta. \tag{2.4.29}$$

This can be rewritten as

$$\langle\hat{a}\hat{b}\rangle = exp\big[-\ell\ell^* - \lambda\lambda^*\big] \times\frac{d}{d\eta}\frac{d}{d\gamma}\Big[\int\frac{d^2\alpha}{\pi}exp\big[-\alpha\alpha^* + \ell^*\alpha + \eta\alpha + \ell\alpha^*\big] \times\int\frac{d^2\beta}{\pi}exp\big[-\beta\beta^* + \gamma\beta + \lambda^*\beta + \lambda\beta^*\big]\Big]_{\eta=\gamma=0}. \tag{2.4.30}$$

On performing the integration over β, we find

$$\langle\hat{a}\hat{b}\rangle = exp\big[-\ell\ell^*\big] \times\frac{d}{d\eta}\frac{d}{d\gamma}\Big[exp\big[\gamma\lambda\big]\int\frac{d^2\alpha}{\pi}exp\big[-\alpha\alpha^* + \ell^*\alpha + \eta\alpha + \ell\alpha^*\big]\Big]_{\eta=\gamma=0}, \tag{2.4.31}$$

and on carrying out the integration over α, we have

$$\langle\hat{a}\hat{b}\rangle = \frac{d}{d\eta}\frac{d}{d\gamma}\Big[exp\big[\gamma\lambda + \ell\eta\big]\Big]_{\eta=\gamma=0}. \tag{2.4.32}$$

After differentiating and applying the condition $\eta = \gamma = 0$, we obtain

$$\langle\hat{a}\hat{b}\rangle = \ell\lambda. \tag{2.4.33}$$

The expectation value of $\hat{a}\hat{b}^\dagger$ in terms of the Q function for a two-mode coherent state is expressible as

$$\langle\hat{a}\hat{b}^\dagger\rangle = \int d^2\alpha\ d^2\beta Q(\alpha,\beta)\alpha\beta^*. \tag{2.4.34}$$

Upon inserting (2.1.13) into this, we get

$$\langle\hat{a}\hat{b}^\dagger\rangle = \frac{1}{\pi^2}\int d^2\alpha\ d^2\beta exp\big[-(\ell\ell^* + \lambda\lambda^* + \alpha^*\alpha + \beta^*\beta) + \ell^*\alpha + \ell\alpha^* + \lambda^*\beta + \beta^*\lambda\big]\alpha\beta^*. \tag{2.4.35}$$

This can be rewritten as

$$\begin{aligned}\langle\hat{a}\hat{b}^{\dagger}\rangle = \quad & exp\big[-\ell\ell^{*}-\lambda\lambda^{*}\big] \\ & \times\frac{d}{d\eta}\frac{d}{d\gamma}\bigg[\int\frac{d^{2}\alpha}{\pi}exp\big[-\alpha\alpha^{*}+\eta\alpha+\ell^{*}\alpha+\ell\alpha^{*}\big] \\ & \times\int\frac{d^{2}\beta}{\pi}exp\big[-\beta\beta^{*}+\lambda^{*}\beta+\lambda\beta^{*}+\gamma\beta^{*}\big]\bigg]_{\eta=\gamma=0}. \end{aligned} \tag{2.4.36}$$

On performing the integration over β, we have

$$\begin{aligned}\langle\hat{a}\hat{b}^{\dagger}\rangle = \quad & exp\big[-\ell\ell^{*}\big] \\ & \times\frac{d}{d\eta}\frac{d}{d\gamma}\bigg[exp\big[\gamma\lambda^{*}\big]\int\frac{d^{2}\alpha}{\pi}exp\big[-\alpha\alpha^{*}+\eta\alpha+\ell^{*}\alpha+\ell\alpha^{*}\big]\bigg]_{\eta=\gamma=0}, \end{aligned} \tag{2.4.37}$$

and on carrying out the integration over α, we have

$$\langle\hat{a}\hat{b}^{\dagger}\rangle = \frac{d}{d\eta}\frac{d}{d\gamma}\bigg[exp\big[\gamma\lambda^{*}+\ell\eta\big]\bigg]_{\eta=\gamma=0}. \tag{2.4.38}$$

After differentiating and applying the condition $\eta=\gamma=0$, we obtain

$$\langle\hat{a}\hat{b}^{\dagger}\rangle = \ell\lambda^{*}. \tag{2.4.39}$$

On account of (2.4.19), (2.4.20), (2.4.26), (2.4.27), (2.4.33), and (2.4.39) along with their complex conjugate, and in view of (2.2.12), (2.2.13), (2.4.10), (2.4.11), (2.4.12), and (2.4.13), the variance for the plus and the minus quadratures have the form

$$(\Delta c_{+})^{2} = 1 \tag{2.4.40}$$

and

$$(\Delta c_{-})^{2} = 1. \tag{2.4.41}$$

From these results we observe that two-mode coherent states are minimum uncertainty states with equal noise in both quadratures.

Chapter 3

Two-Mode Squeezed Vacuum States

A two-mode squeezed vacuum state is defined by [1]

$$\big|0_a, 0_b, r\big\rangle = \hat{S}(r)\big|0_a, 0_b\big\rangle, \tag{3.0.1}$$

in which

$$\hat{S}(r) = e^{r(\hat{a}\hat{b}-\hat{a}^\dagger\hat{b}^\dagger)}, \tag{3.0.2}$$

is a two-mode squeeze operator.

3.1 The Q function

From Eq. (2.1.5), we know that the Q function for a two-mode light is expressible as

$$Q(\alpha, \beta, r) = \frac{1}{\pi^4}\int d^2z d^2\eta \phi_{a,b}(z, \eta, r) e^{(z^*\alpha - z\alpha^* + \eta^*\beta - \eta\beta^*)}, \tag{3.1.1}$$

where the anti-normally ordered characteristic function is defined by

$$\phi_{a,b}(z, \eta, r) = Tr\bigg(\hat{\rho} e^{-z^*\hat{a}(r)} e^{z\hat{a}^\dagger(r)} e^{-\eta^*\hat{b}(r)} e^{\eta\hat{b}^\dagger(r)}\bigg). \tag{3.1.2}$$

Applying the Baker-Hausdorff identity [1-7]

$$e^{\hat{A}}e^{\hat{B}} = e^{\hat{A}+\hat{B}+\frac{1}{2}[\hat{A},\hat{B}]}, \tag{3.1.3}$$

we have

$$\phi_{a,b}(z,\eta,r) = exp\big[-\frac{1}{2}(z^*z+\eta^*\eta)\big]Tr\bigg(\hat{\rho}e^{z\hat{a}^\dagger(r)-z^*\hat{a}(r)+\eta\hat{b}^\dagger(r)-\eta^*\hat{b}(r)}\bigg). \tag{3.1.4}$$

This can rewritten as

$$\begin{aligned}\phi_{a,b}(z,\eta,r) = \quad & exp\big[-\frac{1}{2}(z^*z+\eta^*\eta)\big] \\ & \times Tr\bigg(\hat{\rho}\ exp\big[(z\cosh r+\eta^*\sinh r)\hat{a}^\dagger-(z^*\cosh r+\eta\sinh r)\hat{a}\big] \\ & \times exp\big[(\eta\cosh r+z^*\sinh r)\hat{b}^\dagger-(\eta^*\cosh r+z\sinh r)\hat{b}\big]\bigg),\end{aligned} \tag{3.1.5}$$

where [1]

$$\hat{a}(r) = \hat{a}\cosh r - \hat{b}^\dagger\sinh r \tag{3.1.6}$$

and

$$\hat{b}(r) = \hat{b}\cosh r - \hat{a}^\dagger\sinh r. \tag{3.1.7}$$

Applying the Baker-Hausdorff identity once more, we have

$$\begin{aligned}\phi_{a,b}(z,\eta,r) = \quad & exp\big[-\frac{1}{2}(z^*z+\eta^*\eta)\cosh^2 r-(z\eta+z^*\eta^*)\cosh r\sinh r\big] \\ & \times Tr\bigg(\hat{\rho}\ e^{(z\cosh r+\eta^*\sinh r)\hat{a}^\dagger}e^{(\eta\cosh r+z^*\sinh r)\hat{b}^\dagger} \\ & \times e^{-(z^*\cosh r+\eta\sinh r)\hat{a}}e^{-(\eta^*\cosh r+z\sinh r)\hat{b}}\bigg).\end{aligned} \tag{3.1.8}$$

Since mode a and b are initially in a two-mode vacuum state $\big|0_a,0_b\big\rangle$, we have

$$\begin{aligned}\phi_{a,b}(z,\eta,r) = \quad & exp\big[-\frac{1}{2}(z^*z+\eta^*\eta)\cosh^2 r-(z\eta+z^*\eta^*)\cosh r\sinh r\big] \\ & \times\big\langle 0_a,0_b\big|e^{(z\cosh r+\eta^*\sinh r)\hat{a}^\dagger}e^{(\eta\cosh r+z^*\sinh r)\hat{b}^\dagger} \\ & \times e^{-(z^*\cosh r+\eta\sinh r)\hat{a}}e^{-(\eta^*\cosh r+z\sinh r)\hat{b}}\big|0_a,0_b\big\rangle.\end{aligned} \tag{3.1.9}$$

It then follow that

$$\phi_{a,b}(z,\eta,r) = exp\Big[-\frac{1}{2}(z^*z+\eta^*\eta)\cosh^2 r - (z\eta + z^*\eta^*)\cosh r \sinh r\Big]. \qquad (3.1.10)$$

Substitution of (3.1.10) into (3.1.1) leads to

$$\begin{aligned} Q(\alpha,\beta,r) = \quad & \frac{1}{\pi^4}\int d^2zd^2\eta \; exp\Big[-\frac{1}{2}(z^*z+\eta^*\eta)\cosh^2 r - (z\eta+z^*\eta^*)\cosh r\sinh r\Big] \\ & \times exp\big[z^*\alpha - z\alpha^* + \eta^*\beta - \eta\beta^*\big]. \end{aligned} \qquad (3.1.11)$$

This can be rewritten as

$$\begin{aligned} Q(\alpha,\beta,r) = \quad & \frac{1}{\pi^2}\int \frac{d^2z}{\pi} \; exp\Big[-\frac{1}{2}z^*z\cosh^2 r + z^*\alpha - z\alpha^*\Big] \\ & \times \int \frac{d^2\eta}{\pi} \; exp\Big[-\frac{1}{2}\eta^*\eta\cosh^2 r \\ & -(z\eta+z^*\eta^*)\cosh r\sinh r + \eta^*\beta - \eta\beta^*\Big]. \end{aligned} \qquad (3.1.12)$$

Integrating over η and z, we have

$$Q(\alpha,\beta,r) = \frac{sech^2 r}{\pi^2} exp\Big[-\alpha\alpha^* - \beta\beta^* - \alpha\beta\tanh r - \alpha^*\beta^*\tanh r\Big], \qquad (3.1.13)$$

which is the Q function for a two-mode squeezed vacuum state.

3.2 The mean of the photon number sum and difference

In view of (2.2.5) and (2.2.6) the mean of the photon number sum and difference are expressed as

$$\bar{n}_+ = \bar{n}_a + \bar{n}_b \qquad (3.2.1)$$

and

$$\bar{n}_- = \bar{n}_a - \bar{n}_b. \qquad (3.2.2)$$

The mean photon number of mode a in terms of the Q function for a two-mode squeezed vacuum state is given by

$$\bar{n}_a = \int d^2\alpha\ d^2\beta Q(\alpha,\beta,r)\alpha\alpha^* - 1. \tag{3.2.3}$$

Upon inserting (3.1.13) into (3.2.3), we get

$$\bar{n}_a = \frac{sech^2 r}{\pi^2}\int d^2\alpha\ d^2\beta\ exp\big[-\alpha\alpha^* - \beta\beta^* - \alpha\beta\tanh r - \alpha^*\beta^*\tanh r\big]\alpha\alpha^* - 1. \tag{3.2.4}$$

This can be rewritten as

$$\begin{aligned}\bar{n}_a = \quad & sech^2 r\ \frac{d}{d\eta}\frac{d}{d\gamma}\bigg[\int\frac{d^2\alpha}{\pi}exp\big[-\alpha\alpha^* + \eta\alpha + \gamma\alpha^*\big] \\ & \times\int\frac{d^2\beta}{\pi}exp\big[-\beta\beta^* - \alpha\beta\tanh r - \alpha^*\beta^*\tanh r\big]\bigg]_{\eta=\gamma=0} - 1.\end{aligned} \tag{3.2.5}$$

Integrating with respect to β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we find

$$\bar{n}_a = sech^2 r\ \frac{d}{d\eta}\frac{d}{d\gamma}\bigg[\int\frac{d^2\alpha}{\pi}exp\big[-\alpha\alpha^* sech^2 r + \eta\alpha + \gamma\alpha^*\big]\bigg]_{\eta=\gamma=0} - 1, \tag{3.2.6}$$

and integrating with respect to α, we have

$$\bar{n}_a = \frac{d}{d\eta}\frac{d}{d\gamma}\bigg[exp\bigg(\frac{\eta\gamma}{sech^2 r}\bigg)\bigg]_{\eta=\gamma=0} - 1. \tag{3.2.7}$$

After differentiating and applying the condition $\eta = \gamma = 0$, we obtain

$$\bar{n}_a = \frac{1}{sech^2 r} - 1 = \sinh^2 r. \tag{3.2.8}$$

In a similar manner, one can also show that

$$\bar{n}_b = \sinh^2 r. \tag{3.2.9}$$

On account of (3.2.1) and (3.2.2) along with (3.2.8) and (3.2.9), the mean of the photon number sum and difference are given by

$$\bar{n}_+ = 2\sinh^2 r \tag{3.2.10}$$

and

$$\bar{n}_{-} = 0. \tag{3.2.11}$$

3.3 The normally-ordered variance of the photon number sum and difference

In view of (2.3.4) and (2.3.5) the normally-ordered variance of the photon number sum and difference are given by

$$:(\Delta n_{+})^2 := \langle \hat{a}^2 \hat{a}^{\dagger 2}\rangle + 2\langle \hat{a}\hat{a}^{\dagger}\hat{b}\hat{b}^{\dagger}\rangle + \langle \hat{b}^2\hat{b}^{\dagger 2}\rangle - 6\big[\bar{n}_a + \bar{n}_b + 1\big] - \big[\bar{n}_a + \bar{n}_b\big]^2 \tag{3.3.1}$$

and

$$:(\Delta n_{-})^2 := \langle \hat{a}^2 \hat{a}^{\dagger 2}\rangle - 2\langle \hat{a}\hat{a}^{\dagger}\hat{b}\hat{b}^{\dagger}\rangle + \langle \hat{b}^2\hat{b}^{\dagger 2}\rangle - 2\big[\bar{n}_a + \bar{n}_b + 1\big] - \big[\bar{n}_a - \bar{n}_b\big]^2. \tag{3.3.2}$$

The expectation value of $\hat{a}^2\hat{a}^{\dagger 2}$ in terms of the Q function for a two-mode squeezed state is expressible as

$$\langle \hat{a}^2 \hat{a}^{\dagger 2}\rangle = \int d^2\alpha\ d^2\beta Q(\alpha,\beta,r)\alpha^2\alpha^{*2}. \tag{3.3.3}$$

Upon inserting (3.1.13) into (3.3.3) , we get

$$\langle \hat{a}^2 \hat{a}^{\dagger 2}\rangle = \frac{sech^2 r}{\pi^2}\int d^2\alpha\ d^2\beta\ exp\big[-\alpha\alpha^* - \beta\beta^* - \alpha\beta\tanh r - \alpha^*\beta^*\tanh r\big]\alpha^2\alpha^{*2}. \tag{3.3.4}$$

This can be rewritten as

$$\begin{aligned}\langle \hat{a}^2 \hat{a}^{\dagger 2}\rangle = \quad & sech^2 r\,\frac{d^2}{d\eta^2}\frac{d^2}{d\gamma^2}\bigg[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \eta\alpha + \gamma\alpha^*\big] \\ & \times \int \frac{d^2\beta}{\pi} exp\big[-\beta\beta^* - \alpha\beta\tanh r - \alpha^*\beta^*\tanh r\big]\bigg]_{\eta=\gamma=0}.\end{aligned} \tag{3.3.5}$$

Integrating with respect to β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we find

$$\langle \hat{a}^2 \hat{a}^{\dagger 2}\rangle = sech^2 r\,\frac{d^2}{d\eta^2}\frac{d^2}{d\gamma^2}\bigg[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* sech^2 r + \eta\alpha + \gamma\alpha^*\big]\bigg]_{\eta=\gamma=0} \tag{3.3.6}$$

and integrating with respect to α, we have

$$\langle \hat{a}^2 \hat{a}^{\dagger 2} \rangle = \frac{d^2}{d\eta^2} \frac{d^2}{d\gamma^2} \left[exp\left(\frac{\eta\gamma}{sech^2 r} \right) \right]_{\eta=\gamma=0}. \quad (3.3.7)$$

After differentiating and applying the condition $\eta = \gamma = 0$, we obtain

$$\langle \hat{a}^2 \hat{a}^{\dagger 2} \rangle = \frac{2}{sech^4 r} = 2\cosh^4 r. \quad (3.3.8)$$

Following a similar procedure, we arrive at

$$\langle \hat{b}^2 \hat{b}^{\dagger 2} \rangle = 2\cosh^4 r. \quad (3.3.9)$$

The expectation value of $\hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger$ in terms of the Q function for a two-mode squeezed state is expressible as

$$\langle \hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger \rangle = \int d^2\alpha \; d^2\beta Q(\alpha, \beta, r)\alpha\alpha^* \beta\beta^*. \quad (3.3.10)$$

Upon inserting (3.1.13) into (3.3.10) , we get

$$\langle \hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger \rangle = \frac{sech^2 r}{\pi^2} \int d^2\alpha \; d^2\beta \; exp\big[-\alpha\alpha^* - \beta\beta^* - \alpha\beta \tanh r - \alpha^*\beta^* \tanh r \big] \alpha\alpha^* \beta\beta^*. \quad (3.3.11)$$

This can be rewritten as

$$\begin{aligned} \langle \hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger \rangle = \quad & sech^2 r \frac{d}{d\eta} \frac{d}{d\gamma} \frac{d}{d\mu} \frac{d}{d\nu} \left[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \eta\alpha + \gamma\alpha^* \big] \right. \\ & \times \int \frac{d^2\beta}{\pi} exp\big[-\beta\beta^* - \alpha\beta \tanh r \\ & \left. + \mu\beta + \nu\beta^* - \alpha^*\beta^* \tanh r \big] \right]_{\eta=\gamma=\mu=\nu=0}. \end{aligned} \quad (3.3.12)$$

Integrating with respect to β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we find

$$\begin{aligned} \langle \hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger \rangle = \quad & sech^2 r \frac{d}{d\eta} \frac{d}{d\gamma} \frac{d}{d\mu} \frac{d}{d\nu} \left[exp\big[\mu\nu \big] \right. \\ & \times \int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* sech^2 r + \eta\alpha - \nu\alpha \tanh r \\ & \left. + \gamma\alpha^* - \mu\alpha^* \tanh r \big] \right]_{\eta=\gamma=\mu=\nu=0}, \end{aligned} \quad (3.3.13)$$

and integrating with respect to α, we have

$$\langle \hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger \rangle = \frac{d}{d\eta}\frac{d}{d\gamma}\frac{d}{d\mu}\frac{d}{d\nu}\left[exp\left(\frac{1}{sech^2 r}\left[\mu\nu + \eta\gamma - \eta\mu \tanh r - \gamma\nu \tanh r\right]\right)\right]_{\eta=\gamma=\mu=\nu=0}. \quad (3.3.14)$$

After differentiating and applying the condition $\eta = \gamma = 0$, we obtain

$$\langle \hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger \rangle = \cosh^4 r + \sinh^2 r \cosh^2 r. \quad (3.3.15)$$

Upon substitution of (3.2.8), (3.2.9), (3.3.8), (3.3.9), (3.3.15) into (3.3.1) and (3.3.2), the normally-ordered variance of the photon number sum and difference take the form

$$:(\Delta n_+)^2: = 4\sinh^4 r + 2\sinh^2 r \quad (3.3.16)$$

and

$$:(\Delta n_-)^2: = -2\sinh^2 r. \quad (3.3.17)$$

3.4 The quadrature variance

On account of Eqs. (2.4.10) and (2.4.11) the variance for the plus and the minus quadratures for a two-mode state are given by

$$\begin{aligned}(\Delta c_+)^2 = \quad & \frac{1}{2}\left[(\Delta a_+)^2 + (\Delta b_+)^2\right] + \langle \hat{a}\hat{b} \rangle + \langle \hat{a}\hat{b}^\dagger \rangle + \langle \hat{a}^\dagger \hat{b} \rangle + \langle \hat{a}^\dagger \hat{b}^\dagger \rangle \\ & -\langle \hat{a} \rangle\langle \hat{b} \rangle - \langle \hat{a} \rangle\langle \hat{b}^\dagger \rangle - \langle \hat{a}^\dagger \rangle\langle \hat{b} \rangle - \langle \hat{a}^\dagger \rangle\langle \hat{b}^\dagger \rangle \quad (3.4.1)\end{aligned}$$

and

$$\begin{aligned}(\Delta c_-)^2 = \quad & \frac{1}{2}\left[(\Delta a_-)^2 + (\Delta b_-)^2\right] - \langle \hat{a}\hat{b} \rangle + \langle \hat{a}\hat{b}^\dagger \rangle + \langle \hat{a}^\dagger \hat{b} \rangle - \langle \hat{a}^\dagger \hat{b}^\dagger \rangle \\ & +\langle \hat{a} \rangle\langle \hat{b} \rangle - \langle \hat{a} \rangle\langle \hat{b}^\dagger \rangle - \langle \hat{a}^\dagger \rangle\langle \hat{b} \rangle + \langle \hat{a}^\dagger \rangle\langle \hat{b}^\dagger \rangle, \quad (3.4.2)\end{aligned}$$

where

$$(\Delta a_+)^2 = 1 + 2\langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{a}^{\dagger 2} \rangle + \langle \hat{a}^2 \rangle - \langle \hat{a} \rangle^2 - \langle \hat{a}^\dagger \rangle^2 - 2\langle \hat{a}^\dagger \rangle\langle \hat{a} \rangle \quad (3.4.3)$$

and

$$(\Delta a_-)^2 = 1 + 2\langle \hat{a}^\dagger \hat{a}\rangle - \langle \hat{a}^{\dagger 2}\rangle - \langle \hat{a}^2\rangle + \langle \hat{a}\rangle^2 + \langle \hat{a}^\dagger\rangle^2 - 2\langle \hat{a}^\dagger\rangle\langle \hat{a}\rangle. \tag{3.4.4}$$

with a similar definition for $(\Delta b_+)^2$ and $(\Delta b_-)^2$.

The expectation value of $\hat{a}$ in terms of the Q function for a squeezed vacuum state is expressible as

$$\langle \hat{a}\rangle = \int d^2\alpha\ d^2\beta Q(\alpha,\beta,r)\alpha. \tag{3.4.5}$$

On account of (3.1.13), we have

$$\langle \hat{a}\rangle = \frac{sech^2 r}{\pi^2}\int d^2\alpha\ d^2\beta\ exp\big[-\alpha\alpha^* - \beta\beta^* - \alpha\beta\tanh r - \alpha^*\beta^*\tanh r\big]\alpha. \tag{3.4.6}$$

This can be rewritten as

$$\begin{aligned}\langle \hat{a}\rangle = \quad & sech^2 r\ \frac{d}{d\gamma}\bigg[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \gamma\alpha\big] \\ & \times \int \frac{d^2\beta}{\pi} exp\big[-\beta\beta^* - \alpha\beta\tanh r - \alpha^*\beta^*\tanh r\big]\bigg]_{\gamma=0}.\end{aligned} \tag{3.4.7}$$

Integrating with respect to β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we find

$$\langle \hat{a}\rangle = sech^2 r\ \frac{d}{d\gamma}\bigg[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* sech^2 r + \gamma\alpha\big]\bigg]_{\gamma=0}, \tag{3.4.8}$$

and integrating with respect to α, we have

$$\langle \hat{a}\rangle = \frac{d}{d\gamma}\bigg[1\bigg]_{\gamma=0}. \tag{3.4.9}$$

After differentiating and applying the condition $\gamma = 0$, we get

$$\langle \hat{a}\rangle = 0. \tag{3.4.10}$$

Following similar procedure, we arrive at

$$\langle \hat{b} \rangle = 0. \tag{3.4.11}$$

The expectation value of $\langle \hat{a}^2 \rangle$ in terms of the Q function for a squeezed vacuum state is expressible as

$$\langle \hat{a}^2 \rangle = \int d^2\alpha \; d^2\beta Q(\alpha, \beta, r)\alpha^2. \tag{3.4.12}$$

On account of (3.1.13), we have

$$\langle \hat{a}^2 \rangle = \frac{sech^2 r}{\pi^2} \int d^2\alpha \; d^2\beta \; exp\big[-\alpha\alpha^* - \beta\beta^* - \alpha\beta \tanh r - \alpha^*\beta^* \tanh r \big]\alpha^2. \tag{3.4.13}$$

This can be rewritten as

$$\begin{aligned} \langle \hat{a}^2 \rangle = \quad & sech^2 r \, \frac{d^2}{d\gamma^2} \bigg[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \gamma\alpha \big] \\ & \times \int \frac{d^2\beta}{\pi} exp\big[-\beta\beta^* - \alpha\beta \tanh r - \alpha^*\beta^* \tanh r \big] \bigg]_{\gamma=0}. \end{aligned} \tag{3.4.14}$$

Integrating with respect to β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we find

$$\langle \hat{a}^2 \rangle = sech^2 r \, \frac{d^2}{d\gamma^2} \bigg[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* sech^2 r + \gamma\alpha \big] \bigg]_{\gamma=0}, \tag{3.4.15}$$

and integrating with respect to α, we have

$$\langle \hat{a}^2 \rangle = \frac{d^2}{d\gamma^2} \bigg[1 \bigg]_{\gamma=0}. \tag{3.4.16}$$

After differentiating and applying the condition $\gamma = 0$, we get

$$\langle \hat{a}^2 \rangle = 0. \tag{3.4.17}$$

Similarly one can easily obtain the expectation value of $\hat{b}^2$ as

$$\langle \hat{b}^2 \rangle = 0. \tag{3.4.18}$$

The expectation value of $\hat{a}\hat{b}$ in terms of the Q function for a squeezed vacuum state is given by

$$\langle \hat{a}\hat{b} \rangle = \int d^2\alpha \ d^2\beta Q(\alpha, \beta, r)\alpha\beta. \tag{3.4.19}$$

Upon inserting (3.1.13) into (3.4.19), we get

$$\langle \hat{a}\hat{b} \rangle = \frac{sech^2 r}{\pi^2} \int d^2\alpha \ d^2\beta \ exp\big[-\alpha\alpha^* - \beta\beta^* - \alpha\beta \tanh r - \alpha^*\beta^* \tanh r \big]\alpha\beta. \tag{3.4.20}$$

This can be rewritten as

$$\begin{aligned} \langle \hat{a}\hat{b} \rangle = \quad & sech^2 r \ \frac{d}{d\eta}\frac{d}{d\gamma}\Bigg[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \eta\alpha \big] \\ & \times \int \frac{d^2\beta}{\pi} exp\big[-\beta\beta^* - \alpha\beta \tanh r + \gamma\beta - \alpha^*\beta^* \tanh r \big] \Bigg]_{\eta=\gamma=0}. \end{aligned} \tag{3.4.21}$$

Integrating with respect to β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we find

$$\langle \hat{a}\hat{b} \rangle = sech^2 r \ \frac{d}{d\eta}\frac{d}{d\gamma}\Bigg[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* sech^2 r + \eta\alpha - \gamma\alpha^* \tanh r \big] \Bigg]_{\eta=\gamma=0}, \tag{3.4.22}$$

and integrating with respect to α, we have

$$\langle \hat{a}\hat{b} \rangle = \frac{d}{d\eta}\frac{d}{d\gamma}\Bigg[exp\bigg(\frac{-\eta\gamma \tanh r}{sech^2 r} \bigg) \Bigg]_{\eta=\gamma=0}. \tag{3.4.23}$$

After differentiating and applying the condition $\eta = \gamma = 0$, we obtain

$$\langle \hat{a}\hat{b} \rangle = \frac{-\tanh r}{sech^2 r} = -\sinh r \cosh r. \tag{3.4.24}$$

The expectation value of $\hat{a}\hat{b}^\dagger$ in terms of the Q function for a squeezed vacuum state is expressible as

$$\langle \hat{a}\hat{b}^\dagger \rangle = \int d^2\alpha \ d^2\beta Q(\alpha, \beta, r)\alpha\beta^*. \tag{3.4.25}$$

Upon inserting (3.1.13) into (3.4.25), we get

$$\langle \hat{a}\hat{b}^{\dagger}\rangle = \frac{sech^2 r}{\pi^2}\int d^2\alpha\ d^2\beta\ exp\big[-\alpha\alpha^* - \beta\beta^* - \alpha\beta\tanh r - \alpha^*\beta^*\tanh r\big]\alpha\beta^*. \tag{3.4.26}$$

This can be rewritten as

$$\begin{aligned}\langle \hat{a}\hat{b}^{\dagger}\rangle = \quad & sech^2 r\ \frac{d}{d\eta}\frac{d}{d\gamma}\bigg[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \eta\alpha\big] \\ & \times \int \frac{d^2\beta}{\pi} exp\big[-\beta\beta^* - \alpha\beta\tanh r + \gamma\beta^* - \alpha^*\beta^*\tanh r\big]\bigg]_{\eta=\gamma=0}.\end{aligned} \tag{3.4.27}$$

Integrating with respect to β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we find

$$\langle \hat{a}\hat{b}^{\dagger}\rangle = sech^2 r\ \frac{d}{d\eta}\frac{d}{d\gamma}\bigg[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* sech^2 r + \eta\alpha - \gamma\alpha\tanh r\big]\bigg]_{\eta=\gamma=0}, \tag{3.4.28}$$

and integrating with respect to α, we have

$$\langle \hat{a}\hat{b}^{\dagger}\rangle = \frac{d}{d\eta}\frac{d}{d\gamma}\bigg[1\bigg]_{\eta=\gamma=0}. \tag{3.4.29}$$

After differentiating and applying the condition $\eta = \gamma = 0$, we obtain

$$\langle \hat{a}\hat{b}^{\dagger}\rangle = 0. \tag{3.4.30}$$

On account of (3.4.10), (3.4.11), (3.4.17), (3.4.18), (3.4.24) and (3.4.30) along with their complex conjugate, and in view of (3.2.8), (3.2.9), (3.4.1), (3.4.2), (3.4.3), and (3.4.4) the variance for the plus and the minus quadratures are given by

$$(\Delta c_+)^2 = 1 + 2\sinh^2 r - 2\sinh r\cosh r \tag{3.4.31}$$

and

$$(\Delta c_-)^2 = 1 + 2\sinh^2 r + 2\sinh r\cosh r. \tag{3.4.32}$$

From the definition of hyperbolic sine and cosine, we see that

$$\sinh^2 r = \frac{e^{2r} - 2 + e^{-2r}}{4}, \tag{3.4.33}$$

$$\cosh r \sinh r = \frac{e^{2r} - e^{-2r}}{4}. \tag{3.4.34}$$

In view of theses relations, expressions (3.4.31) and (3.4.32) finally take the form

$$(\Delta c_+)^2 = e^{-2r} \tag{3.4.35}$$

and

$$(\Delta c_-)^2 = e^{2r}. \tag{3.4.36}$$

For $r > 0$, we easily observe that the fluctuations in the plus quadrature are below the coherent state level with enhanced flactuations in the minus quadrature.

Chapter 4

The Superposition of Two-Mode Coherent and Squeezed Vacuum States

4.1 The Q function

Suppose a two-mode light beam is injected into a lossless cavity initially having no photons.

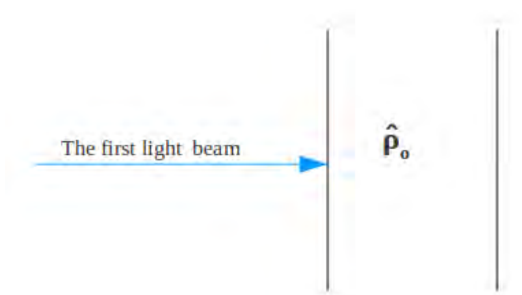

Figure 4.1: The injection of the first light beam into a lossless cavity initially having no photons

The density operator for this light is given by

$$\hat{\rho}_1 = \hat{\rho}_1(\hat{a}^\dagger, \hat{b}^\dagger, \hat{a}, \hat{b}) = \sum_{klmn} C_{klmn} \hat{a}^{\dagger k} \hat{b}^{\dagger l} \hat{a}^m \hat{b}^n. \tag{4.1.1}$$

The completeness relation for a two-mode light is given by

$$\frac{1}{\pi^2} \int d^2\eta_a d^2\eta_b |\eta_a, \eta_b\rangle\langle\eta_b, \eta_a| = \hat{I}. \tag{4.1.2}$$

Substituting this into (4.1.1), we have

$$\hat{\rho}_1 = \int d^2\eta_a d^2\eta_b \frac{1}{\pi^2} \sum_{klmn} C_{klmn} \big|\eta_a, \eta_b\big\rangle\big\langle\eta_b, \eta_a\big|\hat{a}^{\dagger k}\hat{b}^{\dagger l}\hat{a}^m\hat{b}^n, \tag{4.1.3}$$

then it follows

$$\hat{\rho}_1 = \int d^2\eta_a d^2\eta_b \frac{1}{\pi^2} \sum_{klmn} C_{klmn} \quad \eta_a^{*k}\big(\eta_a + \frac{\partial}{\partial\eta_a^*}\big)^m \quad \eta_b^{*l}\big(\eta_b + \frac{\partial}{\partial\eta_b^*}\big)^n \big|\eta_a, \eta_b\big\rangle\big\langle\eta_b, \eta_a\big|, \tag{4.1.4}$$

where

$$\big|\eta_a, \eta_b\big\rangle\big\langle\eta_b, \eta_a\big|\hat{a}^{\dagger k} = \big|\eta_a, \eta_b\big\rangle\big\langle\eta_b, \eta_a\big|\eta_a^{*k} \tag{4.1.5}$$

$$\big|\eta_a, \eta_b\big\rangle\big\langle\eta_b, \eta_a\big|\hat{b}^{\dagger l} = \big|\eta_a, \eta_b\big\rangle\big\langle\eta_b, \eta_a\big|\eta_b^{*l} \tag{4.1.6}$$

$$\big|\eta_a, \eta_b\big\rangle\big\langle\eta_b, \eta_a\big|\hat{a}^m = \big(\eta_a + \frac{\partial}{\partial\eta_a^*}\big)^m \big|\eta_a, \eta_b\big\rangle\big\langle\eta_b, \eta_a\big| \tag{4.1.7}$$

$$\big|\eta_a, \eta_b\big\rangle\big\langle\eta_b, \eta_a\big|\hat{b}^n = \big(\eta_b + \frac{\partial}{\partial\eta_b^*}\big)^n \big|\eta_a, \eta_b\big\rangle\big\langle\eta_b, \eta_a\big| \tag{4.1.8}$$

Eq. (4.1.4) can be further written as

$$\begin{aligned} \hat{\rho}_1 = \quad & \int d^2\eta_a d^2\eta_b \Big[\frac{1}{\pi^2} \sum_{klmn} C_{klmn} \quad \eta_a^{*k}\big(\eta_a + \frac{\partial}{\partial\eta_a^*}\big)^m \\ & \times\eta_b^{*l}\big(\eta_b + \frac{\partial}{\partial\eta_b^*}\big)^n \hat{D}(\eta_a)\hat{D}(\eta_b)\ \hat{\rho}_0\ \hat{D}(-\eta_a)\hat{D}(-\eta_b)\Big], \end{aligned} \tag{4.1.9}$$

where

$$\big|\eta_a, \eta_b\big\rangle\big\langle\eta_b, \eta_a\big| = \hat{D}(\eta_a)\hat{D}(\eta_b)\big|0_a, 0_b\big\rangle\big\langle 0_b, 0_a\big|\hat{D}(-\eta_a)\hat{D}(-\eta_b), \tag{4.1.10}$$

with

$$\hat{\rho}_0 = \big|0_a, 0_b\big\rangle\big\langle 0_b, 0_a\big|, \tag{4.1.11}$$

represents the density operator for the light initially in the cavity.

Now if we inject another light beam into the cavity, then the density operator for the superposition of the light beams can be written as

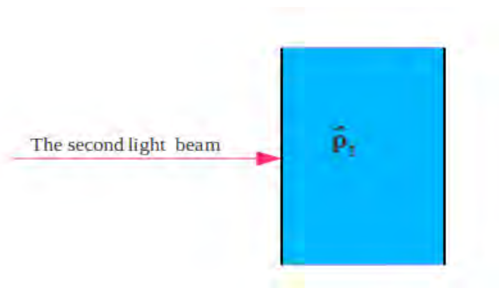

Figure 4.2: The injection of the second light beam into a lossless cavity initially having some photons with the density opeator $\hat{\rho}_1$

$$\hat{\rho} = \int d^2\lambda_a d^2\lambda_b \Big[\frac{1}{\pi^2}\sum_{k'l'm'n'} C_{k'l'm'n'} \ \lambda_a^{*k'}\big(\lambda_a + \frac{\partial}{\partial\lambda_a^*}\big)^{m'} \times\lambda_b^{*l'}\big(\lambda_b + \frac{\partial}{\partial\lambda_b^*}\big)^{n'}\hat{D}(\lambda_a)\hat{D}(\lambda_b)\ \hat{\rho}_1\ \hat{D}(-\lambda_a)\hat{D}(-\lambda_b)\Big]. \tag{4.1.12}$$

Introducing Eq. (4.1.4) into this, we have

$$\hat{\rho} = \int \frac{d^2\lambda_a d^2\lambda_b d^2\eta_a d^2\eta_b}{\pi^4}\Big[\sum_{k'l'm'n'} C_{k'l'm'n'} \ \lambda_a^{*k'}\big(\lambda_a + \frac{\partial}{\partial\lambda_a^*}\big)^{m'}\lambda_b^{*l'}\big(\lambda_b + \frac{\partial}{\partial\lambda_b^*}\big)^{n'} \times\sum_{klmn} C_{klmn}\ \eta_a^{*k}\big(\eta_a + \frac{\partial}{\partial\eta_a^*}\big)^{m}\ \eta_b^{*l}\big(\eta_b + \frac{\partial}{\partial\eta_b^*}\big)^{n} \times\hat{D}(\lambda_a)\hat{D}(\lambda_b)\big|\eta_a,\eta_b\big\rangle\big\langle\eta_b,\eta_a\big|\hat{D}(-\lambda_a)\hat{D}(-\lambda_b)\Big]. \tag{4.1.13}$$

This is equal to

$$\hat{\rho} = \int \frac{d^2\lambda_a d^2\lambda_b d^2\eta_a d^2\eta_b}{\pi^4}\Big[\sum_{k'l'm'n'} C_{k'l'm'n'} \ \lambda_a^{*k'}\big(\lambda_a + \frac{\partial}{\partial\lambda_a^*}\big)^{m'}\lambda_b^{*l'}\big(\lambda_b + \frac{\partial}{\partial\lambda_b^*}\big)^{n'} \times\sum_{klmn} C_{klmn}\ \eta_a^{*k}\big(\eta_a + \frac{\partial}{\partial\eta_a^*}\big)^{m}\ \eta_b^{*l}\big(\eta_b + \frac{\partial}{\partial\eta_b^*}\big)^{n} \times\big|\eta_a+\lambda_a,\eta_b+\lambda_b\big\rangle\big\langle\eta_b+\lambda_b,\eta_a+\lambda_a\big|, \tag{4.1.14}$$

where

$$\hat{D}(\lambda_a)\hat{D}(\lambda_b)\big|\eta_a,\eta_b\big\rangle\big\langle\eta_b,\eta_a\big|\hat{D}(-\lambda_a)\hat{D}(-\lambda_b) = \big|\eta_a+\lambda_a,\eta_b+\lambda_b\big\rangle\big\langle\eta_b+\lambda_b,\eta_a+\lambda_a\big|. \tag{4.1.15}$$

In view of (2.1.1) the Q function for a two-mode light beam is given by

$$Q(\alpha,\beta) = \frac{\langle\beta,\alpha\big|\hat{\rho}\big|\alpha,\beta\rangle}{\pi^2}. \tag{4.1.16}$$

Upon inserting (4.1.14) in to this, we get

$$\begin{aligned} Q(\alpha,\beta) = \quad & \int \frac{d^2\lambda_a d^2\lambda_b d^2\eta_a d^2\eta_b}{\pi^4}\Big[\sum_{k'l'm'n'} C_{k'l'm'n'} \;\; \lambda_a^{*k'}\big(\lambda_a + \frac{\partial}{\partial\lambda_a^*}\big)^{m'} \\ & \times\lambda_b^{*l'}\big(\lambda_b + \frac{\partial}{\partial\lambda_b^*}\big)^{n'} \sum_{klmn} C_{klmn} \;\; \eta_a^{*k}\big(\eta_a + \frac{\partial}{\partial\eta_a^*}\big)^{m} \;\; \eta_b^{*l}\big(\eta_b + \frac{\partial}{\partial\eta_b^*}\big)^{n} \\ & \times\big\langle\beta,\alpha\big|\eta_a+\lambda_a,\eta_b+\lambda_b\big\rangle\big\langle\eta_b+\lambda_b,\eta_a+\lambda_a\big|\alpha,\beta\big\rangle. \end{aligned} \tag{4.1.17}$$

From the fact that

$$\big\langle\beta,\alpha\big|\eta_a+\lambda_a,\eta_b+\lambda_b\big\rangle = \big\langle\alpha\big|\eta_a+\lambda_a\big\rangle\big\langle\beta\big|\eta_b+\lambda_b\big\rangle, \tag{4.1.18}$$

with

$$\big\langle\alpha\big|\eta_a+\lambda_a\big\rangle = exp\Big[\alpha^*(\eta_a+\lambda_a) - \frac{1}{2}|\alpha|^2 - \frac{1}{2}|\eta_a+\lambda_a|^2\Big] \quad and \tag{4.1.19}$$

$$\big\langle\beta\big|\eta_b+\lambda_b\big\rangle = exp\Big[\beta^*(\eta_b+\lambda_b) - \frac{1}{2}|\beta|^2 - \frac{1}{2}|\eta_b+\lambda_b|^2\Big] \tag{4.1.20}$$

and

$$\big\langle\eta_b+\lambda_b,\eta_a+\lambda_a\big|\alpha,\beta\big\rangle = \big\langle\eta_a+\lambda_a\big|\alpha\big\rangle\big\langle\eta_b+\lambda_b\big|\beta\big\rangle. \tag{4.1.21}$$

with

$$\big\langle\eta_a+\lambda_a\big|\alpha\big\rangle = exp\Big[\alpha({\eta_a}^*+{\lambda_a}^*) - \frac{1}{2}|\eta_a+\lambda_a|^2 - \frac{1}{2}|\alpha|^2\Big] \quad and \tag{4.1.22}$$

$$\big\langle\eta_b+\lambda_b\big|\beta\big\rangle = exp\Big[\beta({\eta_b}^*+{\lambda_a}^*) - \frac{1}{2}|\eta_b+\lambda_b|^2 - \frac{1}{2}|\beta|^2\Big] \tag{4.1.23}$$

Eq. (4.1.17) can be put in the form

$$\begin{aligned} Q(\alpha,\beta) = \quad & f_1\int \frac{d^2\lambda_a d^2\lambda_b d^2\eta_a d^2\eta_b}{\pi^6}\Big[f_2\sum_{k'l'm'n'} C_{k'l'm'n'} \;\; \lambda_a^{*k'} \\ & \times\big(\lambda_a + \frac{\partial}{\partial\lambda_a^*}\big)^{m'} f_3 \;\; \lambda_b^{*l'}\big(\lambda_b + \frac{\partial}{\partial\lambda_b^*}\big)^{n'} f_4 \\ & \times\sum_{klmn} C_{klmn} \;\; \eta_a^{*k}\big(\eta_a + \frac{\partial}{\partial\eta_a^*}\big)^{m} f_5 \;\; \eta_b^{*l}\big(\eta_b + \frac{\partial}{\partial\eta_b^*}\big)^{n} f_6\Big], \end{aligned} \tag{4.1.24}$$

where

$$f_1 = exp\big[-\alpha\alpha^* - \beta\beta^*\big] \tag{4.1.25}$$

$$f_2 = exp\big[\eta_a\alpha^* + \lambda_a\alpha^* + \lambda_b\beta^* + \eta_b\beta^*\big] \tag{4.1.26}$$

$$f_3 = exp\big[\lambda_a^*(\alpha - \eta_a - \lambda_a)\big] \tag{4.1.27}$$

$$f_4 = exp\big[\lambda_b^*(\beta - \eta_b - \lambda_b)\big] \tag{4.1.28}$$

$$f_5 = exp\big[\eta_a^*(\alpha - \eta_a - \lambda_a)\big] \tag{4.1.29}$$

$$f_6 = exp\big[\eta_b^*(\beta - \eta_b - \lambda_b)\big]. \tag{4.1.30}$$

From binomial expansion, we have

$$\begin{aligned}\big(\eta_b + \frac{\partial}{\partial\eta_b^*}\big)^n f_6 \quad &= \big(\eta_b + \frac{\partial}{\partial\eta_b^*}\big)^n exp\big[\eta_b^*(\beta - \eta_b - \lambda_b)\big], \\ &= \sum_{k=0}^{n} \frac{n!}{k!(n-k)!}\eta_b^{n-k}(\frac{\partial}{\partial\eta_b^*})^k exp\big[\eta_b^*(\beta - \eta_b - \lambda_b)\big]. \end{aligned} \tag{4.1.31}$$

After differentiation and applying binomial theorem back, we get

$$\begin{aligned}\big(\eta_b + \frac{\partial}{\partial\eta_b^*}\big)^n f_6 \quad &= (\beta - \lambda_b)^n exp\big[\eta_b^*(\beta - \eta_b - \lambda_b)\big], \\ &= (\beta - \lambda_b)^n f_6. \end{aligned} \tag{4.1.32}$$

Similarly

$$\big(\eta_a + \frac{\partial}{\partial\eta_a^*}\big)^m f_5 = (\alpha - \lambda_a)^m f_5, \tag{4.1.33}$$

$$\big(\lambda_b + \frac{\partial}{\partial\lambda_b^*}\big)^{n'} f_4 = (\beta - \eta_b)^{n'} f_4, \tag{4.1.34}$$

$$\big(\lambda_a + \frac{\partial}{\partial\lambda_a^*}\big)^{m'} f_3 = (\alpha - \eta_a)^{m'} f_3. \tag{4.1.35}$$

Substituting (4.1.32), (4.1.33), (4.1.33), (4.1.34), and (4.1.35) into (4.1.24), we have

$$\begin{aligned}Q\big(\alpha, \beta\big) = \quad & \frac{1}{\pi^2}\int d^2\lambda_a\ d^2\lambda_b\ d^2\eta_a\ d^2\eta_b\Big[Q\big(\lambda_a^*,\ \lambda_b^*,\ \alpha - \eta_a,\ \beta - \eta_b\big) \\ & \times Q\big(\eta_a^*,\ \eta_b^*,\ \alpha - \lambda_a,\ \beta - \lambda_b\big) \\ & \times exp\big(-|\alpha - \lambda_a - \eta_a|^2 - |\beta - \lambda_b - \eta_b|^2\big)\Big]. \end{aligned} \tag{4.1.36}$$

This is the Q function for the superposition of a pair of two-mode light beams in general.

Now we seek to obtain the Q function for the superposition of two-mode coherent and squeezed vacuum states. With the aid of (2.1.13) and (3.1.13), one can write

$$Q\big(\lambda_a^*,\ \lambda_b^*,\ \alpha-\eta_a,\ \beta-\eta_b\big) = \frac{1}{\pi^2}exp\bigg[-\big[\ell\ell^*+\lambda\lambda^*+\lambda_a^*(\alpha-\eta_a)+\lambda_b^*(\beta-\eta_b)\big] +\ell^*(\alpha-\eta_a)+\ell\lambda_a^*+\lambda^*(\beta-\eta_b)+\lambda_b^*\lambda\bigg] \tag{4.1.37}$$

and

$$Q\big(\eta_a^*,\ \eta_b^*,\ \alpha-\lambda_a,\ \beta-\lambda_b\big) = \frac{sech^2 r}{\pi^2}exp\bigg[-\eta_a^*(\alpha-\lambda_a)-\eta_b^*(\beta-\lambda_b) -((\beta-\lambda_b)(\alpha-\lambda_a)+\eta_b^*\eta_a^*)\tanh r\bigg]. \tag{4.1.38}$$

Upon inserting (4.1.37) and (4.1.38) into (4.1.36), we find

$$\begin{aligned} Q\big(\alpha,\beta\big) = \quad & \frac{sech^2 r}{\pi^6}\int d^2\lambda_a\ d^2\lambda_b\ d^2\eta_a\ d^2\eta_b\ exp\bigg[-\alpha\alpha^*-\beta\beta^*-\ell\ell^* \\ & -\lambda\lambda^*+\ell^*\alpha-\ell^*\eta_a+\ell\lambda_a^*+\lambda^*\beta-\lambda^*\eta_b+\lambda_b^*\lambda+\lambda_a\alpha^* \\ & -\lambda_a\lambda_a^*+\eta_a\alpha^*-\eta_a\eta_a^*+\lambda_b\beta^*-\lambda_b\lambda_b^*+\eta_b\beta^*-\eta_b\eta_b^* \\ & +(-\alpha\beta+\lambda_a\beta+\alpha\lambda_b-\lambda_a\lambda_b-\eta_b^*\eta_a^*)\tanh r\bigg]. \end{aligned} \tag{4.1.39}$$

This can be rewritten as

$$\begin{aligned} Q\big(\alpha,\beta\big) = \quad & \frac{sech^2 r}{\pi^2}\ exp\bigg[-\alpha\alpha^*-\beta\beta^*-\ell\ell^*-\lambda\lambda^*+\ell^*\alpha+\lambda^*\beta-\alpha\beta\tanh r\bigg] \\ & \times\int\frac{d^2\lambda_a}{\pi}exp\bigg[-\lambda_a\lambda_a^*+\lambda_a\beta\tanh r+\lambda_a\alpha^*+\ell\lambda_a^*\bigg] \\ & \times\int\frac{d^2\lambda_b}{\pi}exp\bigg[-\lambda_b\lambda_b^*+\lambda_b\beta^*+\alpha\lambda_b\tanh r-\lambda_a\lambda_b\tanh r+\lambda_b^*\lambda\bigg] \\ & \times\int\frac{d^2\eta_a}{\pi}exp\bigg[-\eta_a\eta_a^*+\eta_a\alpha^*-\ell^*\eta_a\bigg] \\ & \times\int\frac{d^2\eta_b}{\pi}exp\bigg[-\eta_b\eta_b^*-\lambda^*\eta_b+\eta_b\beta^*-\eta_b^*\eta_a^*\tanh r\bigg]. \end{aligned} \tag{4.1.40}$$

Integrating over the variable η_b using the relation given in Eq. (2.1.12), we get

$$\begin{aligned} Q(\alpha,\beta) = \quad & \frac{sech^2 r}{\pi^2}\ exp\Big[-\alpha\alpha^* - \beta\beta^* - \ell\ell^* - \lambda\lambda^* + \ell^*\alpha + \lambda^*\beta - \alpha\beta\tanh r\Big] \\ & \times \int \frac{d^2\lambda_a}{\pi} exp\Big[-\lambda_a\lambda_a^* + \lambda_a\beta\tanh r + \lambda_a\alpha^* + \ell\lambda_a^*\Big] \\ & \times \int \frac{d^2\lambda_b}{\pi} exp\Big[-\lambda_b\lambda_b^* + \lambda_b\beta^* + \alpha\lambda_b\tanh r - \lambda_a\lambda_b\tanh r + \lambda_b^*\lambda\Big] \\ & \times \int \frac{d^2\eta_a}{\pi} exp\Big[-\eta_a\eta_a^* + \eta_a\alpha^* - \ell^*\eta_a \\ & +\lambda^*\eta_a^*\tanh r - \beta^*\eta_a^*\tanh r\Big]. \end{aligned} \tag{4.1.41}$$

Similarly integrating with respect to the variable η_a, we find

$$\begin{aligned} Q(\alpha,\beta) = \quad & \frac{sech^2 r}{\pi^2}\ exp\Big[-\alpha\alpha^* - \beta\beta^* - \ell\ell^* - \lambda\lambda^* + \ell^*\alpha + \lambda^*\beta - \alpha\beta\tanh r \\ & +\alpha^*\lambda^*\tanh r - \alpha^*\beta^*\tanh r - \ell^*\lambda^*\tanh r + \ell^*\beta^*\tanh r\Big] \\ & \times \int \frac{d^2\lambda_a}{\pi} exp\Big[-\lambda_a\lambda_a^* + \lambda_a\beta\tanh r + \lambda_a\alpha^* + \ell\lambda_a^*\Big] \\ & \times \int \frac{d^2\lambda_b}{\pi} exp\Big[-\lambda_b\lambda_b^* + \lambda_b\beta^* \\ & +\alpha\lambda_b\tanh r - \lambda_a\lambda_b\tanh r + \lambda_b^*\lambda\Big], \end{aligned} \tag{4.1.42}$$

and integrating over the variable λ_b, we obtain

$$\begin{aligned} Q(\alpha,\beta) = \quad & \frac{sech^2 r}{\pi^2}\ exp\Big[-\alpha\alpha^* - \beta\beta^* - \ell\ell^* - \lambda\lambda^* + \ell^*\alpha + \lambda^*\beta + \lambda\beta^* - \alpha\beta\tanh r \\ & +\alpha^*\lambda^*\tanh r - \alpha^*\beta^*\tanh r - \ell^*\lambda^*\tanh r + \ell^*\beta^*\tanh r + \lambda\alpha\tanh r\Big] \\ & \times \int \frac{d^2\lambda_a}{\pi} exp\Big[-\lambda_a\lambda_a^* + \lambda_a\beta\tanh r \\ & +\lambda_a\alpha^* - \lambda\lambda_a\tanh r + \ell\lambda_a^*\Big]. \end{aligned} \tag{4.1.43}$$

Finally integrating over the variable λ_a, we have

$$\begin{aligned} Q(\alpha,\beta) = \quad & \frac{sech^2 r}{\pi^2}\ exp\Big[-\alpha\alpha^* - \beta\beta^* - \ell\ell^* - \lambda\lambda^* + \ell^*\alpha + \lambda^*\beta + \lambda\beta^* - \alpha\beta\tanh r \\ & +\alpha^*\lambda^*\tanh r - \alpha^*\beta^*\tanh r - \ell^*\lambda^*\tanh r + \ell^*\beta^*\tanh r + \lambda\alpha\tanh r \\ & +\ell\beta\tanh r + \ell\alpha^* - \ell\lambda\tanh r\Big]. \end{aligned} \tag{4.1.44}$$

This is the Q function for the superposition of two-mode coherent and squeezed vacuum states.

Now let us integrate this function over the variables α and β

$$\begin{aligned}\int d^2\alpha d^2\beta Q(\alpha,\beta) = \quad & sech^2 r\ exp\Big[-\ell\ell^* - \lambda\lambda^* - \ell^*\lambda^*\tanh r - \ell\lambda\tanh r\Big] \\ & \times\int\frac{d^2\alpha}{\pi}exp\Big[-\alpha\alpha^* + \ell^*\alpha + \lambda\alpha\tanh r + \ell\alpha^* + \alpha^*\lambda^*\tanh r\Big] \\ & \times\int\frac{d^2\beta}{\pi}exp\Big[-\beta\beta^* + \lambda^*\beta - \alpha\beta\tanh r + \ell\beta\tanh r \\ & +\lambda\beta^* - \alpha^*\beta^*\tanh r + \ell^*\beta^*\tanh r\Big]. \end{aligned} \tag{4.1.45}$$

Integrating over the variable β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we get

$$\begin{aligned}\int d^2\alpha d^2\beta Q(\alpha,\beta) = \quad & sech^2 r\ exp\Big[-sech^2 r\ell\ell^*\Big] \\ & \times\int\frac{d^2\alpha}{\pi}exp\Big[sech^2 r(-\alpha\alpha^* + \ell^*\alpha + \ell\alpha^*)\Big], \end{aligned} \tag{4.1.46}$$

and integrating over the variable α, we have

$$\int d^2\alpha d^2\beta Q(\alpha,\beta) = 1. \tag{4.1.47}$$

This shows that the Q function for the superposition of two-mode coherent and squeezed vacuum states is normalized.

4.2 The mean of the photon number sum and difference

On account of (2.2.5) and (2.2.6) the mean of the photon number sum and difference are expressed as

$$\bar{n}_+ = \bar{n}_a + \bar{n}_b \tag{4.2.1}$$

and

$$\bar{n}_- = \bar{n}_a - \bar{n}_b. \tag{4.2.2}$$

The mean photon number of mode a in terms of the Q function for the superposition of two-mode coherent and squeezed vacuum states is given by

$$\bar{n}_a = \int d^2\alpha\ d^2\beta Q(\alpha,\beta)\alpha\alpha^* - 1. \tag{4.2.3}$$

In view of (4.1.44), we have

$$\begin{aligned}\bar{n}_a = \quad & \frac{sech^2 r}{\pi^2}\int d^2\alpha\ d^2\beta\ exp\Big[-\alpha\alpha^* - \beta\beta^* - \ell\ell^* - \lambda\lambda^* + \ell^*\alpha + \lambda^*\beta + \lambda\beta^* \\ & -\alpha\beta\tanh r + \alpha^*\lambda^*\tanh r - \alpha^*\beta^*\tanh r - \ell^*\lambda^*\tanh r + \ell^*\beta^*\tanh r \\ & +\lambda\alpha\tanh r + \ell\beta\tanh r + \ell\alpha^* - \ell\lambda\tanh r\Big]\alpha\alpha^* - 1. \end{aligned} \tag{4.2.4}$$

This can be rewritten as

$$\begin{aligned}\bar{n}_a = \quad & sech^2 r\ exp\big[-\ell\ell^* - \lambda\lambda^* - \ell^*\lambda^*\tanh r - \ell\lambda\tanh r\big] \\ & \times\frac{d}{d\eta}\frac{d}{d\gamma}\bigg[\int\frac{d^2\alpha}{\pi}exp\big[-\alpha\alpha^* + \eta\alpha + \ell^*\alpha + \lambda\alpha\tanh r + \ell\alpha^* + \gamma\alpha^* + \alpha^*\lambda^*\tanh r\big] \\ & \times\int\frac{d^2\beta}{\pi}exp\big[-\beta\beta^* + \lambda^*\beta - \alpha\beta\tanh r + \ell\beta\tanh r \\ & +\lambda\beta^* - \alpha^*\beta^*\tanh r + \ell^*\beta^*\tanh r\big]\bigg]_{\eta=\gamma=0} - 1. \end{aligned} \tag{4.2.5}$$

Integrating with respect to β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we find

$$\begin{aligned}\bar{n}_a = \quad & sech^2 r\ exp\big[-\ell\ell^*\ sech^2 r\big] \\ & \times\frac{d}{d\eta}\frac{d}{d\gamma}\bigg[\int\frac{d^2\alpha}{\pi}exp\big[-\alpha\alpha^*\ sech^2 r + \eta\alpha + \ell^*\alpha\ sech^2 r \\ & +\ell\alpha^*\ sech^2 r + \gamma\alpha^*\big]\bigg]_{\eta=\gamma=0} - 1, \end{aligned} \tag{4.2.6}$$

and integrating with respect to α, we have

$$\bar{n}_a = \frac{d}{d\eta}\frac{d}{d\gamma}\left[exp\left(\frac{1}{sech^2 r}\left[\eta\ell\ sech^2 r + \eta\gamma + \gamma\ell^*\ sech^2 r\right]\right)\right]_{\eta=\gamma=0} - 1. \tag{4.2.7}$$

After differentiating and applying the condition $\eta = \gamma = 0$, we obtain

$$\bar{n}_a = \frac{1}{sech^2 r} + \ell\ell^* - 1 = \sinh^2 r + \ell\ell^*. \tag{4.2.8}$$

In a similar manner, one can also show that

$$\bar{n}_b = \sinh^2 r + \lambda\lambda^*. \tag{4.2.9}$$

Upon substituting (4.2.8) and (4.2.9) into (4.2.1) and (4.2.2), the mean of the photon number sum and difference for the superposition of two-mode coherent and squeezed vacuum states are given by

$$\bar{n}_+ = \ell\ell^* + \lambda\lambda^* + 2\sinh^2 r \tag{4.2.10}$$

and

$$\bar{n}_- = \ell\ell^* - \lambda\lambda^*. \tag{4.2.11}$$

From these results we have understand that the mean of the photon number sum for the superposition of two-mode coherent and squeezed vacuum states is the sum of the mean of the photon number sums for the individual states. And the mean of the photon number difference for the superposition of two-mode coherent and squeezed vacuum states is the same as that of the two-mode coherent state.

4.3 The normally-ordered variance of the photon number sum and difference

In view of (2.3.4) and (2.3.5) the normally-ordered variance of the photon number sum and difference are given by

$$:(\Delta n_+)^2 := \langle \hat{a}^2\hat{a}^{\dagger 2}\rangle + 2\langle \hat{a}\hat{a}^{\dagger}\hat{b}\hat{b}^{\dagger}\rangle + \langle \hat{b}^2\hat{b}^{\dagger 2}\rangle - 6\big[\bar{n}_a + \bar{n}_b + 1\big] - \big[\bar{n}_a + \bar{n}_b\big]^2 \tag{4.3.1}$$

and

$$:(\Delta n_-)^2 := \langle \hat{a}^2\hat{a}^{\dagger 2}\rangle - 2\langle \hat{a}\hat{a}^{\dagger}\hat{b}\hat{b}^{\dagger}\rangle + \langle \hat{b}^2\hat{b}^{\dagger 2}\rangle - 2\big[\bar{n}_a + \bar{n}_b + 1\big] - \big[\bar{n}_a - \bar{n}_b\big]^2. \tag{4.3.2}$$

The expectation value of $\hat{a}^2\hat{a}^{\dagger 2}$ in terms of the Q function for the superposition of two-mode coherent and squeezed vacuum states is given by

$$\langle \hat{a}^2\hat{a}^{\dagger 2}\rangle = \int d^2\alpha\ d^2\beta Q(\alpha,\beta)\alpha^2\alpha^{*2}. \tag{4.3.3}$$

In view of (4.1.44), we have

$$\begin{aligned}\langle \hat{a}^2\hat{a}^{\dagger 2}\rangle = \quad & \frac{sech^2 r}{\pi^2}\int d^2\alpha\ d^2\beta\ exp\Big[-\alpha\alpha^* - \beta\beta^* - \ell\ell^* - \lambda\lambda^* + \ell^*\alpha + \lambda^*\beta + \lambda\beta^* \\ & -\alpha\beta\tanh r + \alpha^*\lambda^*\tanh r - \alpha^*\beta^*\tanh r - \ell^*\lambda^*\tanh r + \ell^*\beta^*\tanh r \\ & +\lambda\alpha\tanh r + \ell\beta\tanh r + \ell\alpha^* - \ell\lambda\tanh r\Big]\alpha^2\alpha^{*2}. \end{aligned} \tag{4.3.4}$$

This can be rewritten as

$$\begin{aligned}\langle \hat{a}^2\hat{a}^{\dagger 2}\rangle = \quad & sech^2 r\ exp\big[-\ell\ell^* - \lambda\lambda^* - \ell^*\lambda^*\tanh r - \ell\lambda\tanh r\big] \\ & \times\frac{d^2}{d\eta^2}\frac{d^2}{d\gamma^2}\Big[\int\frac{d^2\alpha}{\pi}exp\big[-\alpha\alpha^* + \eta\alpha + \ell^*\alpha + \lambda\alpha\tanh r + \ell\alpha^* + \gamma\alpha^* + \alpha^*\lambda^*\tanh r\big] \\ & \times\int\frac{d^2\beta}{\pi}exp\big[-\beta\beta^* + \lambda^*\beta - \alpha\beta\tanh r + \ell\beta\tanh r \\ & +\lambda\beta^* - \alpha^*\beta^*\tanh r + \ell^*\beta^*\tanh r\big]\Big]_{\eta=\gamma=0}. \end{aligned} \tag{4.3.5}$$

Integrating with respect to β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we obtain

$$\begin{aligned}\langle \hat{a}^2 \hat{a}^{\dagger 2} \rangle = \quad & sech^2 r \; exp\big[- \ell\ell^* \; sech^2 r\big] \\ & \times \frac{d^2}{d\eta^2}\frac{d^2}{d\gamma^2}\bigg[\int \frac{d^2\alpha}{\pi} exp\big[- \alpha\alpha^* \; sech^2 r + \eta\alpha + \ell^*\alpha \; sech^2 r \\ & + \ell\alpha^* \; sech^2 r + \gamma\alpha^*\big]\bigg]_{\eta=\gamma=0}, \end{aligned} \tag{4.3.6}$$

and integrating with respect to α, we have

$$\langle \hat{a}^2 \hat{a}^{\dagger 2} \rangle = \frac{d^2}{d\eta^2}\frac{d^2}{d\gamma^2}\bigg[exp\bigg(\frac{1}{sech^2 r}\big[\eta\ell \; sech^2 r + \eta\gamma + \gamma\ell^* \; sech^2 r\big]\bigg)\bigg]_{\eta=\gamma=0}, \tag{4.3.7}$$

After differentiating and applying the condition $\eta = \gamma = 0$, we obtain

$$\langle \hat{a}^2 \hat{a}^{\dagger 2} \rangle = \frac{2}{sech^4 r} + \frac{4\ell\ell^*}{sech^2 r} + \ell^2\ell^{*2} = 2\cosh^4 r + 4\ell\ell^* \cosh^2 r + \ell^2\ell^{*2}. \tag{4.3.8}$$

Following a similar procedure, we arrive at

$$\langle \hat{b}^2 \hat{b}^{\dagger 2} \rangle = 2\cosh^4 r + 4\lambda\lambda^* \cosh^2 r + \lambda^2\lambda^{*2}. \tag{4.3.9}$$

The expectation value of $\hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger$ in terms of the Q function for the superposition of two-mode coherent and squeezed vacuum states is given by

$$\langle \hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger \rangle = \int d^2\alpha \; d^2\beta Q(\alpha,\beta)\alpha\alpha^*\beta\beta^*. \tag{4.3.10}$$

In view of (4.1.44), we have

$$\begin{aligned}\langle \hat{a}\hat{a}^\dagger \hat{b}\hat{b}^\dagger \rangle = \quad & \frac{sech^2 r}{\pi^2}\int d^2\alpha \; d^2\beta \; exp\bigg[- \alpha\alpha^* - \beta\beta^* - \ell\ell^* - \lambda\lambda^* + \ell^*\alpha + \lambda^*\beta + \lambda\beta^* \\ & -\alpha\beta\tanh r + \alpha^*\lambda^*\tanh r - \alpha^*\beta^*\tanh r - \ell^*\lambda^*\tanh r + \ell^*\beta^*\tanh r \\ & +\lambda\alpha\tanh r + \ell\beta\tanh r + \ell\alpha^* - \ell\lambda\tanh r\bigg]\alpha\alpha^*\beta\beta^*. \end{aligned} \tag{4.3.11}$$

This can be rewritten as

$$\begin{aligned}\langle \hat{a}\hat{a}^\dagger\hat{b}\hat{b}^\dagger\rangle = \quad & sech^2 r\ exp\big[-\ell\ell^* - \lambda\lambda^* - \ell^*\lambda^*\tanh r - \ell\lambda\tanh r\big] \\ & \times\frac{d}{d\eta}\frac{d}{d\gamma}\frac{d}{d\mu}\frac{d}{d\nu}\Big[\int\frac{d^2\alpha}{\pi}exp\big[-\alpha\alpha^* + \eta\alpha + \ell^*\alpha + \lambda\alpha\tanh r \\ & +\ell\alpha^* + \gamma\alpha^* + \alpha^*\lambda^*\tanh r\big] \\ & \times\int\frac{d^2\beta}{\pi}exp\big[-\beta\beta^* + \lambda^*\beta - \alpha\beta\tanh r + \ell\beta\tanh r + \mu\beta \\ & +\nu\beta^* + \lambda\beta^* - \alpha^*\beta^*\tanh r + \ell^*\beta^*\tanh r\big]\Big]_{\eta=\gamma=\mu=\nu=0}. \qquad (4.3.12)\end{aligned}$$

Integrating with respect to β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we have

$$\begin{aligned}\langle \hat{a}\hat{a}^\dagger\hat{b}\hat{b}^\dagger\rangle = \quad & sech^2 r\ exp\big[-\ell\ell^*\ sech^2 r\big] \\ & \times\frac{d}{d\eta}\frac{d}{d\gamma}\frac{d}{d\mu}\frac{d}{d\nu}\Big[exp\big[+\nu\lambda^* + \nu\ell\tanh r + \mu\nu + \mu\lambda + \mu\ell^*\tanh r\big] \\ & \times\int\frac{d^2\alpha}{\pi}exp\big[-\alpha\alpha^*\ sech^2 r + \eta\alpha + \ell^*\alpha\ sech^2 r - \nu\alpha\tanh r \\ & +\ell\alpha^*\ sech^2 r + \gamma\alpha^* - \mu\alpha^*\tanh r\big]\Big]_{\eta=\gamma=\mu=\nu=0}, \qquad (4.3.13)\end{aligned}$$

and integrating with respect to α, we have

$$\begin{aligned}\langle \hat{a}\hat{a}^\dagger\hat{b}\hat{b}^\dagger\rangle = \quad & \frac{d}{d\eta}\frac{d}{d\gamma}\frac{d}{d\mu}\frac{d}{d\nu}\Big[exp\Big(\frac{1}{sech^2 r}\big[\nu\lambda^* sech^2 r + \mu\nu + \mu\lambda sech^2 r + \gamma\ell^* sech^2 r \\ & +\eta\ell sech^2 r + \eta\gamma - \eta\mu\tanh r - \gamma\nu\tanh r\big]\Big)\Big]_{\eta=\gamma=\mu=\nu=0}. \qquad (4.3.14)\end{aligned}$$

After differentiating and applying the condition $\eta = \gamma = \mu = \nu = 0$, we obtain

$$\begin{aligned}\langle \hat{a}\hat{a}^\dagger\hat{b}\hat{b}^\dagger\rangle = \quad & \cosh^4 r + \sinh^2 r\cosh^2 r + \ell\ell^*\cosh^2 r + \lambda\lambda^*\cosh^2 r \\ & -\ell\lambda\sinh r\cosh r - \ell^*\lambda^*\sinh r\cosh r + \ell\ell^*\lambda\lambda^*. \qquad (4.3.15)\end{aligned}$$

Upon substitution of (4.2.8), (4.2.9), (4.3.8), (4.3.9) and (4.3.15) into (4.3.1) and (4.3.2), the normally-ordered variance of the photon number sum and difference take the form

$$\begin{aligned}: (\Delta n_+)^2 := \quad & 4\sinh^4 r + 2\sinh^2 r + 2\ell\ell^*\sinh^2 r + 2\lambda\lambda^*\sinh^2 r \\ & -2\ell\lambda\sinh r\cosh r - 2\ell^*\lambda^*\sinh r\cosh r \qquad (4.3.16)\end{aligned}$$

and

$$: (\Delta n_-)^2 := \quad -2\sinh^2 r + 2\ell\ell^* \sinh^2 r + 2\lambda\lambda^* \sinh^2 r$$
$$+2\ell\lambda \sinh r \cosh r + 2\ell^*\lambda^* \sinh r \cosh r. \tag{4.3.17}$$

4.4 The Quadrature Variances

On account of Eqs. (2.4.10) and (2.4.11) the variance for the plus and the minus quadratures for a two-mode state are given by

$$(\Delta c_+)^2 = \quad \frac{1}{2}\left[(\Delta a_+)^2 + (\Delta b_+)^2\right] + \langle \hat{a}\hat{b}\rangle + \langle \hat{a}\hat{b}^\dagger\rangle + \langle \hat{a}^\dagger\hat{b}\rangle + \langle \hat{a}^\dagger\hat{b}^\dagger\rangle$$
$$-\langle \hat{a}\rangle\langle \hat{b}\rangle - \langle \hat{a}\rangle\langle \hat{b}^\dagger\rangle - \langle \hat{a}^\dagger\rangle\langle \hat{b}\rangle - \langle \hat{a}^\dagger\rangle\langle \hat{b}^\dagger\rangle \tag{4.4.1}$$

and

$$(\Delta c_-)^2 = \quad \frac{1}{2}\left[(\Delta a_-)^2 + (\Delta b_-)^2\right] - \langle \hat{a}\hat{b}\rangle + \langle \hat{a}\hat{b}^\dagger\rangle + \langle \hat{a}^\dagger\hat{b}\rangle - \langle \hat{a}^\dagger\hat{b}^\dagger\rangle$$
$$+\langle \hat{a}\rangle\langle \hat{b}\rangle - \langle \hat{a}\rangle\langle \hat{b}^\dagger\rangle - \langle \hat{a}^\dagger\rangle\langle \hat{b}\rangle + \langle \hat{a}^\dagger\rangle\langle \hat{b}^\dagger\rangle, \tag{4.4.2}$$

where

$$(\Delta a_+)^2 = 1 + 2\langle \hat{a}^\dagger\hat{a}\rangle + \langle \hat{a}^{\dagger 2}\rangle + \langle \hat{a}^2\rangle - \langle \hat{a}\rangle^2 - \langle \hat{a}^\dagger\rangle^2 - 2\langle \hat{a}^\dagger\rangle\langle \hat{a}\rangle \tag{4.4.3}$$

and

$$(\Delta a_-)^2 = 1 + 2\langle \hat{a}^\dagger\hat{a}\rangle - \langle \hat{a}^{\dagger 2}\rangle - \langle \hat{a}^2\rangle + \langle \hat{a}\rangle^2 + \langle \hat{a}^\dagger\rangle^2 - 2\langle \hat{a}^\dagger\rangle\langle \hat{a}\rangle. \tag{4.4.4}$$

with a similar definition for $(\Delta b_+)^2$ and $(\Delta b_-)^2$.

The expectation value of $\langle \hat{a}^\dagger\rangle$ in terms of the Q function for the superposition of two-mode coherent and squeezed vacuum states is expressible as

$$\langle \hat{a}\rangle = \int d^2\alpha \; d^2\beta Q(\alpha,\beta)\alpha. \tag{4.4.5}$$

On account of (4.1.44), we have

$$\langle \hat{a} \rangle = \frac{sech^2 r}{\pi^2} \int d^2\alpha \; d^2\beta \; exp\Big[- \alpha\alpha^* - \beta\beta^* - \ell\ell^* - \lambda\lambda^* + \ell^*\alpha + \lambda^*\beta + \lambda\beta^*$$
$$-\alpha\beta \tanh r + \alpha^*\lambda^* \tanh r - \alpha^*\beta^* \tanh r - \ell^*\lambda^* \tanh r + \ell^*\beta^* \tanh r$$
$$+\lambda\alpha \tanh r + \ell\beta \tanh r + \ell\alpha^* - \ell\lambda \tanh r \Big] \alpha. \tag{4.4.6}$$

This can be rewritten as

$$\langle \hat{a} \rangle = sech^2 r \; exp\big[- \ell\ell^* - \lambda\lambda^* - \ell^*\lambda^* \tanh r - \ell\lambda \tanh r \big]$$
$$\times \frac{d}{d\gamma} \Big[\int \frac{d^2\alpha}{\pi} exp\big[- \alpha\alpha^* + \gamma\alpha + \ell^*\alpha + \lambda\alpha \tanh r + \ell\alpha^* + \alpha^*\lambda^* \tanh r \big]$$
$$\times \int \frac{d^2\beta}{\pi} exp\big[- \beta\beta^* + \lambda^*\beta - \alpha\beta \tanh r + \ell\beta \tanh r$$
$$+\lambda\beta^* - \alpha^*\beta^* \tanh r + \ell^*\beta^* \tanh r \big] \Big]_{\gamma=0}. \tag{4.4.7}$$

Integrating with respect to β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we get

$$\langle \hat{a} \rangle = sech^2 r \; exp\big[- \ell\ell^* \; sech^2 r \big]$$
$$\times \frac{d}{d\gamma} \Big[\int \frac{d^2\alpha}{\pi} exp\big[- \alpha\alpha^* \; sech^2 r + \gamma\alpha + \ell^*\alpha \; sech^2 r$$
$$+\ell\alpha^* \; sech^2 r \big] \Big]_{\gamma=0}, \tag{4.4.8}$$

and integrating with respect to α, we have

$$\langle \hat{a} \rangle = \frac{d}{d\gamma} \Big[exp\Big(\frac{1}{sech^2 r} \big[\gamma\ell \; sech^2 r \big] \Big) \Big]_{\gamma=0}. \tag{4.4.9}$$

After differentiating and applying the condition $\gamma = 0$ once more, we get

$$\langle \hat{a} \rangle = \ell. \tag{4.4.10}$$

Similarly one can easily obtain the expectation value of $\hat{b}$ as

$$\langle \hat{b} \rangle = \lambda. \tag{4.4.11}$$

The expectation value of $\langle \hat{a}^2 \rangle$ in terms of the Q function for the superposition of two-mode coherent and squeezed vacuum states is expressible as

$$\langle \hat{a}^2 \rangle = \int d^2\alpha \; d^2\beta Q\big(\alpha, \beta\big)\alpha^2. \tag{4.4.12}$$

On account of (4.1.44), we have

$$\begin{aligned}\langle \hat{a}^2 \rangle = \quad & \frac{sech^2 r}{\pi^2} \int d^2\alpha \; d^2\beta \; exp\Big[-\alpha\alpha^* - \beta\beta^* - \ell\ell^* - \lambda\lambda^* + \ell^*\alpha + \lambda^*\beta + \lambda\beta^* \\ & -\alpha\beta \tanh r + \alpha^*\lambda^* \tanh r - \alpha^*\beta^* \tanh r - \ell^*\lambda^* \tanh r + \ell^*\beta^* \tanh r \\ & +\lambda\alpha \tanh r + \ell\beta \tanh r + \ell\alpha^* - \ell\lambda \tanh r \Big] \alpha^2.\end{aligned} \tag{4.4.13}$$

This can be rewritten as

$$\begin{aligned}\langle \hat{a}^2 \rangle = \quad & sech^2 r \; exp\big[-\ell\ell^* - \lambda\lambda^* - \ell^*\lambda^* \tanh r - \ell\lambda \tanh r \big] \\ & \times \frac{d^2}{d\gamma^2} \Big[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* + \gamma\alpha + \ell^*\alpha + \lambda\alpha \tanh r + \ell\alpha^* + \alpha^*\lambda^* \tanh r \big] \\ & \times \int \frac{d^2\beta}{\pi} exp\big[-\beta\beta^* + \lambda^*\beta - \alpha\beta \tanh r + \ell\beta \tanh r \\ & +\lambda\beta^* - \alpha^*\beta^* \tanh r + \ell^*\beta^* \tanh r \big] \Big]_{\gamma=0}.\end{aligned} \tag{4.4.14}$$

Integrating with respect to β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we have

$$\begin{aligned}\langle \hat{a}^2 \rangle = \quad & sech^2 r \; exp\big[-\ell\ell^* \; sech^2 r \big] \\ & \times \frac{d^2}{d\gamma^2} \Big[\int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* \; sech^2 r + \gamma\alpha + \ell^*\alpha \; sech^2 r \\ & +\ell\alpha^* \; sech^2 r \big] \Big]_{\gamma=0},\end{aligned} \tag{4.4.15}$$

and integrating with respect to α, we have

$$\langle \hat{a}^2 \rangle = \frac{d^2}{d\gamma^2} \left[exp\left(\frac{1}{sech^2 r} \big[\gamma\ell \; sech^2 r \big] \right) \right]_{\gamma=0}. \tag{4.4.16}$$

After differentiating and applying the condition $\gamma = 0$ once more, we get

$$\langle \hat{a}^2 \rangle = \ell^2. \tag{4.4.17}$$

Similarly one can easily obtain the expectation value of $\hat{b}^2$ as

$$\langle \hat{b}^2 \rangle = \lambda^2. \tag{4.4.18}$$

The expectation value of $\hat{a}\hat{b}$ in terms of the Q function for the superposition of two-mode coherent and squeezed vacuum states is given by

$$\langle \hat{a}\hat{b} \rangle = \int d^2\alpha \ d^2\beta Q(\alpha,\beta)\alpha\beta. \tag{4.4.19}$$

In view of (4.1.44), we have

$$\begin{aligned} \langle \hat{a}\hat{b} \rangle = \quad & \frac{sech^2 r}{\pi^2} \int d^2\alpha \ d^2\beta \ exp\Big[- \alpha\alpha^* - \beta\beta^* - \ell\ell^* - \lambda\lambda^* + \ell^*\alpha + \lambda^*\beta + \lambda\beta^* \\ & -\alpha\beta \tanh r + \alpha^*\lambda^* \tanh r - \alpha^*\beta^* \tanh r - \ell^*\lambda^* \tanh r + \ell^*\beta^* \tanh r \\ & +\lambda\alpha \tanh r + \ell\beta \tanh r + \ell\alpha^* - \ell\lambda \tanh r \Big] \alpha\beta. \end{aligned} \tag{4.4.20}$$

This can be rewritten as

$$\begin{aligned} \langle \hat{a}\hat{b} \rangle = \quad & sech^2 r \ exp\big[- \ell\ell^* - \lambda\lambda^* - \ell^*\lambda^* \tanh r - \ell\lambda \tanh r \big] \\ & \times \frac{d}{d\eta}\frac{d}{d\gamma}\Big[\int \frac{d^2\alpha}{\pi} exp\big[- \alpha\alpha^* + \eta\alpha + \ell^*\alpha + \lambda\alpha \tanh r + \ell\alpha^* + \alpha^*\lambda^* \tanh r \big] \\ & \times \int \frac{d^2\beta}{\pi} exp\big[- \beta\beta^* + \gamma\beta + \lambda^*\beta - \alpha\beta \tanh r + \ell\beta \tanh r \\ & +\lambda\beta^* - \alpha^*\beta^* \tanh r + \ell^*\beta^* \tanh r \big] \Big]_{\eta=\gamma=0}. \end{aligned} \tag{4.4.21}$$

Integrating with respect to β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we obtain

$$\begin{aligned}\langle \hat{a}\hat{b}\rangle = \quad & sech^2 r \ exp\big[-\ell\ell^* \ sech^2 r\big] \\ & \times \frac{d}{d\eta}\frac{d}{d\gamma}\Bigg[exp\big[\lambda\gamma + \gamma\ell^* \tanh r\big] \\ & \times \int \frac{d^2\alpha}{\pi} exp\big[-\alpha\alpha^* \ sech^2 r + \eta\alpha + \ell^*\alpha \ sech^2 r \\ & -\gamma\alpha^* \tanh r + \ell\alpha^* \ sech^2 r\big]\Bigg]_{\eta=\gamma=0}, \end{aligned} \tag{4.4.22}$$

and integrating with respect to α, we have

$$\langle \hat{a}\hat{b}\rangle = \frac{d}{d\eta}\frac{d}{d\gamma}\Bigg[exp\bigg(\frac{1}{sech^2 r}\big[\lambda\gamma \ sech^2 r - \eta\gamma \tanh r + \ell\eta \ sech^2 r\big]\bigg)\Bigg]_{\eta=\gamma=0}. \tag{4.4.23}$$

After differentiating and applying the condition $\eta = \gamma = 0$, we obtain

$$\langle \hat{a}\hat{b}\rangle = \ell\lambda - \sinh r \cosh r. \tag{4.4.24}$$

The expectation value of $\hat{a}\hat{b}^\dagger$ in terms of the Q function for the superposition of two-mode coherent and squeezed vacuum states is given by

$$\langle \hat{a}\hat{b}^\dagger\rangle = \int d^2\alpha \ d^2\beta Q(\alpha,\beta)\alpha\beta^*. \tag{4.4.25}$$

In view of (4.1.44), we have

$$\begin{aligned}\langle \hat{a}\hat{b}^\dagger\rangle = \quad & \frac{sech^2 r}{\pi^2}\int d^2\alpha \ d^2\beta \ exp\Big[-\alpha\alpha^* - \beta\beta^* - \ell\ell^* - \lambda\lambda^* + \ell^*\alpha + \lambda^*\beta + \lambda\beta^* \\ & -\alpha\beta \tanh r + \alpha^*\lambda^* \tanh r - \alpha^*\beta^* \tanh r - \ell^*\lambda^* \tanh r + \ell^*\beta^* \tanh r \\ & +\lambda\alpha \tanh r + \ell\beta \tanh r + \ell\alpha^* - \ell\lambda \tanh r\Big]\alpha\beta^*. \end{aligned} \tag{4.4.26}$$

This can be rewritten as

$$\begin{aligned}\langle\hat{a}\hat{b}^{\dagger}\rangle = \quad & sech^2 r\ exp\big[-\ell\ell^* - \lambda\lambda^* - \ell^*\lambda^*\tanh r - \ell\lambda\tanh r\big] \\ & \times\frac{d}{d\eta}\frac{d}{d\gamma}\bigg[\int\frac{d^2\alpha}{\pi}exp\big[-\alpha\alpha^* + \eta\alpha + \ell^*\alpha + \lambda\alpha\tanh r + \ell\alpha^* + \alpha^*\lambda^*\tanh r\big] \\ & \times\int\frac{d^2\beta}{\pi}exp\big[-\beta\beta^* + \lambda^*\beta - \alpha\beta\tanh r + \ell\beta\tanh r \\ & +\gamma\beta^* + \lambda\beta^* - \alpha^*\beta^*\tanh r + \ell^*\beta^*\tanh r\big]\bigg]_{\eta=\gamma=0}. \end{aligned} \tag{4.4.27}$$

Integrating with respect to β and applying the relation $sech^2 r = 1 - \tanh^2 r$, we get

$$\begin{aligned}\langle\hat{a}\hat{b}^{\dagger}\rangle = \quad & sech^2 r\ exp\big[-\ell\ell^*\ sech^2 r\big] \\ & \times\frac{d}{d\eta}\frac{d}{d\gamma}\bigg[exp\big[\gamma\lambda^* + \gamma\ell\tanh r\big] \\ & \times\int\frac{d^2\alpha}{\pi}exp\big[-\alpha\alpha^*\ sech^2 r + \eta\alpha + \ell^*\alpha\ sech^2 r \\ & -\gamma\alpha\tanh r + \ell\alpha^*\ sech^2 r\big]\bigg]_{\eta=\gamma=0}, \end{aligned} \tag{4.4.28}$$

and integrating with respect to α, we have

$$\langle\hat{a}\hat{b}^{\dagger}\rangle = \frac{d}{d\eta}\frac{d}{d\gamma}\bigg[exp\big[\gamma\lambda^* + \eta\ell\big]\bigg]_{\eta=\gamma=0}. \tag{4.4.29}$$

After differentiating and applying the condition $\eta = \gamma = 0$, we obtain

$$\langle\hat{a}\hat{b}^{\dagger}\rangle = \ell\lambda^*. \tag{4.4.30}$$

On account of (4.4.10), (4.4.11), (4.4.17), (4.4.18), (4.4.24), and (4.4.30) along with their complex conjugate, and in view of (4.2.8), (4.2.9), (4.4.1), (4.4.2), (4.4.3) and (4.4.4) the variance for the plus and the minus quadratures are given by

$$(\Delta c_+)^2 = 1 + 2\sinh^2 r - 2\sinh r\cosh r \tag{4.4.31}$$

and

$$(\Delta c_-)^2 = 1 + 2\sinh^2 r + 2\sinh r\cosh r. \tag{4.4.32}$$

From the definition of hyperbolic sine and cosine, we see that

$$\sinh^2 r = \frac{e^{2r} - 2 + e^{-2r}}{4}, \tag{4.4.33}$$

$$\cosh r \sinh r = \frac{e^{2r} - e^{-2r}}{4}. \tag{4.4.34}$$

In view of these relations, expressions (4.4.31) and (4.4.32) finally take the form

$$(\Delta c_+)^2 = e^{-2r} \tag{4.4.35}$$

and

$$(\Delta c_-)^2 = e^{2r}. \tag{4.4.36}$$

For $r > 0$, we easily observe that the fluctuations in the plus quadrature are below the coherent state level with enhanced flactuations in the minus quadrature. It then immediately follows that the superposition of two-mode coherent and squeezed vacuum state is in a squeezed state. We also note that, the variance of the quadrature operators for the superposition are the same as that of the two-mode squeezed vacuum state. This implies that, the superposition of a two-mode coherent state with a two-mode squeezed vacuum state does not affect the squeezing properties of the two-mode squeezed vacuum state.

Chapter 5

Conclusion

In this thesis we have obtained the Q functions for a two-mode coherent state, a two-mode squeezed vacuum state, and the superposition of these states. And with aid of these Q functions, we have studied the statistical and squeezing properties of these states.

We have observed that two-mode coherent states are the minimum uncertainty states with equal noise in both quadratures. Since the normally-ordered variance of the photon number sum and difference for coherent sates are zero, the photon statistics of a two-mode coherent state is Poissonian. From the quadrature variance of a two-mode squeezed vacuum state, we have seen that the fluctuations in the plus quadrature are below the coherent state level with enhanced fluctuations in the minus quadrature.

The mean of the photon number sum for the superposition of two-mode coherent and squeezed vacuum states is the sum of the mean of the photon number sums for the individual states. The mean of the photon number difference for the superposition of two-mode coherent and squeezed vacuum states is the same as that of the two-mode coherent state. And its quadrature variance shows that the fluctuations in the plus quadrature are below the coherent state level with enhanced fluctuations in the minus quadrature. It then immediately follows that the superposition of two-mode coherent and squeezed vacuum state is in a squeezed state.

We have also observed that, the variance of the quadrature operators for the superposition of two-mode coherent and squeezed vacuum states is the same as that of the two-mode squeezed vacuum state. This implies that the superposition of a two-mode coherent state with a two-mode squeezed vacuum state does not affect the squeezing properties of the two-mode squeezed vacuum state.

References

[1] Fesseha Kassahun, Fundamentals of Quantum Optics (Lulu, North Carolina, 2008).

[2] D. F. Walls and G. J. Milburn, Quantum Optics (Springer-Verlege, Berlin, 1994).

[3] M. O. Scully and M. S. Zubairy, Quantum Optics (Cambridge University press, Cambridge, 1997).

[4] S. M. Barnett and P. M. Radmore, Methods in Theoretical Quantum Optics (Clarendon press, Oxfored, 1997).

[5] C. C. Gerry and P. L. Knight, Introductory Quantum Optics (Cambridge University press, Cambridge, 2004).

[6] L. Manedel and E.Wolf, Optical Coherence and Quantum Optics (Cambridge University press, Cambridge, 1995).

[7] H. A. Bachor, A Guide to Experiments in Quantum Optics (Wiley-VCH, New York, 1998).

[8] A. M. Fox, Quantum Optics: *An Introduction* (Oxford University Press, New York, 2006).

[9] P. Meystre and M.Sargent III, Elements of Quantum Optics (Springer-Verlag, Berlin, 1991).

[10] R. R. Puri, Mathematical Methods of Quantum Optics (Springer, Berlin, 2001).

[11] W. Vogel and D. G Welsch, Quantum Optics (Wiley-VCH, New York, 2006).

[12] U. Leonhardt, Measuring the Quantum State of Light (Cambridge University Press, Cambridge, 1997).

[13] C. M. Caves, Phys. Rev. D **23**, 1693 (1981).

[14] M. Xiao, L. A. Wu and H. J. Kimble, Phys. Rev. Lett. **59**, 278 (1987).

[15] Limenew Alemu, MSc Thesis, Addis Ababa University, 2010.

Declaration

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

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This thesis has been submitted for examination with my approval as University advisor.

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