Addis Ababa University

Addis Ababa Institute of Technology

School of Electrical and Computer Engineering

VOLTAGE CONTROL OF A DC-DC BUCK CONVERTER USING SECOND ORDER SLIDING MODE CONTROL

A Thesis Submitted to Addis Ababa Institute of Technology, School of Graduate Studies, Addis Ababa University

in Partial Fulfillment of the Requirement for the Degree of Master of Science in Electrical Engineering (Control Engineering)

By

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DECLARATION

I hereby declare that this thesis work is my original work, has not been presented for a degree in this or any other universities, and all sources of materials used for the thesis work have been fully acknowledged.

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Place: Addis Ababa Institute of Technology, Addis Ababa University, Addis Ababa

Date of Submission: February, 2017

This thesis has been submitted for examination with my approval as a university advisor.

Dr. Dereje Shiferaw ______________________

Advisor's Name Signature

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List of Symbols

C ……………………………........ Capacitor

C_{min} ……………………….….…... Minimum capacitor

D ……………………………..…... Average duty cycle

d ………………………………..… Duty cycle

$\hat{d}$ ………………………………..… Duty cycle variation

f ……………………………….….… Switching frequency

i_c ……………………………….… Actual capacitor current

I_L ………………………………… Average inductor current

i_L ………………………………… Actual inductor current

$\hat{i}_L$ ………………………………… Small signal variation of the inductor current

i_{Lmax} …………………………... Maximum inductor current

i_{Lmin} …………………………... Minimum inductor current

I_o …………………………….... Average output current

i_o …………………………….... Actual output current

Is …………………………….... Average source current

i_s ………………………….……… Actual source current

$\hat{i}_s$ ……………………………….… Small signal variation of the source current

K_d ………………………….……... Derivative gain

K_i ………………………………… Integral gain

K_p ………………………….....…. Proportional gain

L ………………………………..… Inductor

L_{min} ………………………….…… Minimum inductor

R ………..…………………….….... Load resistor

S …………………………….……… Switch

T ………………………….………… Switching period

t_p ……………………………….... Peak time

t_r ………………………………..…. Rise time

t_s ………………………………..…. settling time

u …………………….……....….... Control signal

V_c ……………………..….…. Average capacitor voltage

v_c …………………………...…. Actual capacitor voltage

$\hat{v}_c$ …………………………….... Small signal variation of the capacitor voltage

V_{in} …………………………..…. Average input voltage

v_{in} …………………………..…. Actual input voltage

$\hat{v}_{in}$ …………………………...…. Small signal variation of the input voltage

v_L …………………………...…. Actual inductor voltage

V_o …………………………...…. Average output voltage

v_o………………………………. Actual output voltage

$\hat{v}_o$ ………………………………. Small signal variation of the output voltage

V_{ref} …………………………...…. Reference voltage

V_s ………………………………. Average source voltage

w_n ………………………………. Natural frequency

ξ ………………………………… Damping ratio

ΔQ ………………………………. Change of charge

Δi_L ………………………………..…. peak-to-peak inductor current

ΔV_o ……………………………… Ripple output voltage

τ ………………………………..… Sampling time

List of Acronyms

CCM …………………………..…… Continuous Conduction Mode

DC ………………………….……… Direct Current

DCM ……………………….……… Discontinuous Conduction Mode

GTO ……………………….……… Gate Turn off Thyristor

HOSM ……………………..….… Higher Order Sliding Mode

IGBT ………………………..….... Insulated Gate Bipolar Transistor

MOSFET……………………….... Metal oxide Semiconductor Field Effect Transistor

PID …………..……………..…… Proportional Integral Derivative

PI …………..…….….……….….. Proportional Integral

SCR …………..…….….……... Silicon Controlled Rectifier

SMC …………..…….…..….…. Sliding Mode Control

SM …………..…………..…… Sliding Mode

SOSMC ……………….…..…... Second Order Sliding Mode Control

SMPS ……………….……..…… Switch Mode Power Supply

1-SMC ……………………..… First Order Sliding Mode Control

VSS …………..…………..….. Variable Structure System

Abstract

DC-DC converters are non-linear and the most widely used circuits in power electronics. Generally they are used in all situations where there is need of stabilizing a given DC voltage to a desired value. DC-DC buck converter is used in applications for voltage step-down.

The output voltage of this converter alone is usually unstable, oscillates, it has large overshoot, and long settling time. Also it is unable to give the desired output voltage under input voltage and load variations. To overcome this problem and obtain constant stable output voltage and fast response various controllers are required. PID controllers have been usually applied to the converters to obtain the desired output voltage because of their simplicity. But application of PID controller is not reliable and satisfactory in the case of non-linear systems. Therefore, non-liner controllers are required to improve system performance.

In this thesis second order sliding mode controller based on the prescribed convergence algorithm has been designed to achieve fast and stable performance of buck converter. The proposed controller performance is compared with PID controller based on dynamic response of the system in terms of overshoot, settling time, rise time, and voltage deviation from desired value using MATLAB/Simulink. In order to test the performance of proposed controllers the load resistance increased and decreased by 62.5% from operating point while input voltage decreased by 20.83% and increased up to 41.67% from operating point. Also to test effectiveness of SOSM control the input voltage is varied from operating point (i.e. 24V) up to 200V.

Simulation results show that, using SOSM controller the rise and settling time is improved by 5.228% and 46.39% respectively as compared to that obtained using PID controller. The overshoot is reduced from 51.3% to 9.455% using PID controller while SOSM controller totally removes it. Both controllers overcame the effect of load resistance variations. The overshoot is increased from 9.455% to 17.5% for input voltage increased by 41.67% using PID controller is eliminated using SOSM controller. The actual output voltage is not deviated from desired value even for large input voltage variation using SOSM controller. Generally from the result it is possible to conclude that the performance of SOSM controller is better than PID controller.

Key words: DC-DC Converter, Buck Converter, PID Control, SOSM Control.

CHAPTER ONE

INTRODUCTION

1.1. Background

DC-DC converters are an electronic circuit which converts a source of direct current (DC) from one voltage level to another with a desired voltage level. They are the most widely used circuits in power electronics. They can be found in almost every electronic device nowadays, since all semiconductor components are powered by DC sources. They are basically used in all situations where there is the need of stabilizing a given DC voltage to a desired value [1].

DC-DC converters are widely used in switch-mode power supplies. These switch mode DC-DC converters have received an increasing deal of interest in many areas. This is due to their wide applications like power supplies for personal computers, office equipments, appliance control, telecommunication equipments, DC motor drives, automotive, aircraft, etc [2]. There are different types of DC-DC converters. The most commonly known are usually buck converter, boost converter and buck-Boost converter. The buck converter is used for voltage step-down/reduction, while the boost converter is used for voltage step-up and the buck-boost converter can be used for either step-down or step-up.

DC-DC converters consist of power semiconductor devices which are operated as electronic switches. Operation of the switching devices causes the inherently nonlinear characteristic of the DC-DC converters including the buck converter. The output voltage of buck converter alone is usually unstable. In order to obtain constant stable output voltage, linear controllers have been usually applied to this converter because of their simplicity but unable to produce the best performance when applied to nonlinear system. So that, nonlinear control method that gives best performance under any condition is always in demand. In this thesis second order sliding mode controller has been designed for buck converter in order to obtain constant stable output voltage and fast response.

1.2. Problem Statement

DC-DC converters consist of power semiconductor devices which are operated as electronic switches. Operation of the switching devices causes the inherently nonlinear characteristic of the DC-DC converters including the buck converter. The output voltage of buck converter alone is usually unstable, oscillates, it has large overshoot and long settling time. Also it is unable to give the desired voltage under input voltage and load variations. Consequently, this converter requires a controller with a high degree of dynamic response. Proportional-Integral- Differential (PID) controllers have been usually applied to the converters because of their simplicity to obtain the desired voltage. However, applications of PID controller does not reliable and satisfactorily in case of non-linear systems. It is difficult to account the variation of system parameters and also it produce longer rise time and settling time which in turn influence the voltage regulation of buck converter. Therefore, in this thesis second order sliding mode controller has been designed for buck converter to improve the converter performance.

1.3. Objectives

1.3.1. General Objective

The thesis general objective is to improve the buck converter robustness against load and input voltage variations.

1.3.2. Specific Objectives

- To model and analyse the buck converter without controller and simulate using MATLAB/Simulink.
- To design second order sliding mode controller for voltage regulation of the buck converter.
- To investigate the effectiveness of the proposed second order sliding mode controller as compared to conventional PID controller at operating point and under input voltage and load variations.

1.4. Significance of Thesis

- It gives stable output voltage from the buck converter.
- It gives understanding how second order sliding mode control law has been designed for the buck converter.
- It gives understanding how regulated voltage obtained from buck converter.
- It verifies the importance of second order sliding mode controller compared to conventional PID controller.

1.5. Scope of Thesis

- All of the models are implemented in simulation method only using MATLAB/ Simulink software.
- The effectiveness between second order sliding mode controller and conventional PID controller is studied in terms of maximum overshoot, rise time (t_r), settling time (t_s) and voltage deviations from desired value under input voltage and load variations.
- Analyses of the converter have been carried out for continuous conduction mode (CCM) only.

1.6. Methodology

The following methodologies have been used for the accomplishment of this thesis:

- Different literatures which are related to this thesis work are studied.
- Mathematical model of DC-DC buck converter is developed and second order sliding mode controller is designed for voltage regulation of DC-DC buck converter.
- The proposed system model is developed and analyzed using MATLAB/simulink.
- Performance is investigated by varying the input voltage and load resistance.
- Finally conclusion is given from simulation result.

1.7. Thesis Report Layout

This thesis is organized as follows;

Chapter 1 briefs the overall background of the study. The main part of study such as problem statement, thesis objective, thesis significances, thesis scope, methodology used to accomplish this thesis, and thesis report layout is presented well through this chapter.

Chapter 2 covers the literature review of previous study and general information about buck converter and theoretical review on PID control, sliding mode control and second order sliding mode control system are also described in this chapter.

Chapter 3 presents the methodology used to derive the open loop buck converter and used to design PID controller and second order sliding mode controllers described well in this chapter.

Chapter 4 presents results of the buck converter obtained with and without the PID controller and second order sliding mode controller. Also the effectiveness of PID controller and second order sliding mode controller are compared.

Chapter 5 presents conclusion and recommendation for future study. References cited and supporting appendices are given at the end of this thesis report.

CHAPTER TWO
LITERATURE REVIEW

2.1. Related Work

DC-DC converters are widely used in switch-mode power supplies. These switch mode DC-DC converters have received an increasing deal of interest in many areas due to their wide applications. Therefore analysis, control and stabilization of switching converters are the main factors that need to be considered. Many control methods are used for control of switch mode DC-DC converters and the simple and low cost controller structure is always in demand for most industrial and high performance applications. Every control method has some advantages and drawbacks due to which that particular control method consider as a suitable control method under specific conditions, compared to other control methods. The control method that gives the best performances under any conditions is always in demand.

Guo Liping (2006) has found that to achieve a stable steady-state response and fast transient response under varying operating points, nonlinear controllers need to be used for DC-DC converters. Fuzzy controller has many advantages such as: exact mathematical models are not required for the design of fuzzy controllers, complexities associated with nonlinear mathematical analysis are relatively low, and fuzzy controllers are able to adapt to changes in operating points. However, fuzzy controllers are usually designed based on expert knowledge of converters, and extensive tuning is required based on a trial and error method. The tuning can be quite time consuming. In addition, the response is not easy to predict [3].

Md. Shamim-Ul-Alam, Muhammad Quamruzzaman, and K. M. Rahman (2010) proposed design of a sliding mode controller based on fuzzy logic for a DC-DC boost converter. Sliding mode controller ensures robustness against all variations and fuzzy logic helps to reduce chattering phenomenon introduced by sliding controller, thereby increasing efficiency and reducing error, voltage and current ripples. The proposed system is simulated using MATLAB/SIMULINK. This model is tested against variation of input and reference voltages and found to perform better than conventional sliding mode controller. The challenges here may also the tuning can be quite time consuming [4].

S. S. Muley and R. M. Nagarale **(**2013**)** compare the performance and properties of sliding mode controller with Proportional Integral Derivative (PID) controller and Proportional Integral (PI) controller. The results shows that the sliding mode control scheme provides good voltage regulation and is suitable for boost DC-DC conversion purposes. The derived controller/converter system is suitable for any changes on line voltage and parameters at input keeping load as a constant [5].

Betcy Mariam David and Sreeja K.K. **(**2015) have been presented DC-DC converters showed a high potential for improving the dynamic performance by applying Sliding Mode Control. This nonlinear control scheme is especially well suited for Variable Structure System (VSS) like DC-DC converters. The main advantage of SMC over common linear control schemes is its high robustness against line, load and parameter variations [6].

Siew-Chong Tan, Y. M. Lai, and Chi K. Tse (2008) compared the performance of sliding mode control with that of conventional linear control for the control of DC-DC converters in terms of transient characteristics. It has been shown that the use of sliding mode control can lead to an improved robustness in providing consistent transient responses over a wide range of operating conditions. Its major advantages are the guaranteed stability and the robustness against parameter, line and load uncertainties [7].

Sumita Dhali, P.Nageshwara Rao, Praveen Mande, and K.Venkateswara Rao (2012) compared the general aspects of the performances and properties of the sliding mode controller with the Proportional Integral Derivative (PID) controller and Proportional Integral (PI) controller for pulse width modulation based sliding mode (SM) controller for DC-DC boost converter operating in continuous conduction mode. Pulse width modulation based sliding mode controller shows acceptable performance than PID and PI controller having lowest deviation from reference voltage under internal losses and input voltages changes. Using the sliding mode controller, the non-linearity and un-stability of power converters can be improved which is applicable in many engineering applications [8].

Mohammad Khalid Khan (2003) proposed that second order sliding mode control keeps the main advantages of standard sliding modes control and have the additional advantage that it can be used to remove chattering effect, providing smooth or at least piecewise smooth control. The method also provides better accuracy with respect to switching delays [9].

The main drawback of sliding mode control is that they exhibit chattering. Therefore in order to overcome the problem in this thesis second order sliding mode controller has been designed for buck converter in order to obtain constant stable output voltage and fast response. This controller preserves the advantage of SM control, provides better accuracy with respect to switching delays and in addition used for reduction of chattering problem.

2.2. DC-DC Converter

DC-DC converters are electronic devices that are used to change DC electrical power efficiently from one voltage level to another. They are an electronic circuit which converts a DC signal from one voltage level to another level by storing the input energy and realizing that energy to the output at different voltage level. They use an inductor and a capacitor as energy storage elements so that energy can be transferred from the input to the output. DC- DC converters are widely used in switched-mode power supplies (SMPS) and have a wide range of uses today and are becoming increasingly more important in everyday use. There are different types of DC-DC converters. The most commonly known are:

- Buck converter
- Boost converter
- Buck-boost converter

2.3. Buck Converter

Buck converter is used to convert unregulated DC input voltage to a controlled DC output voltage with a desired voltage level. It is sometimes called step down power stage. The topology gets its name from producing an output voltage that is lower in magnitude than the input voltage.

The circuit diagram of buck converter is as shown below by Figure 2.1.

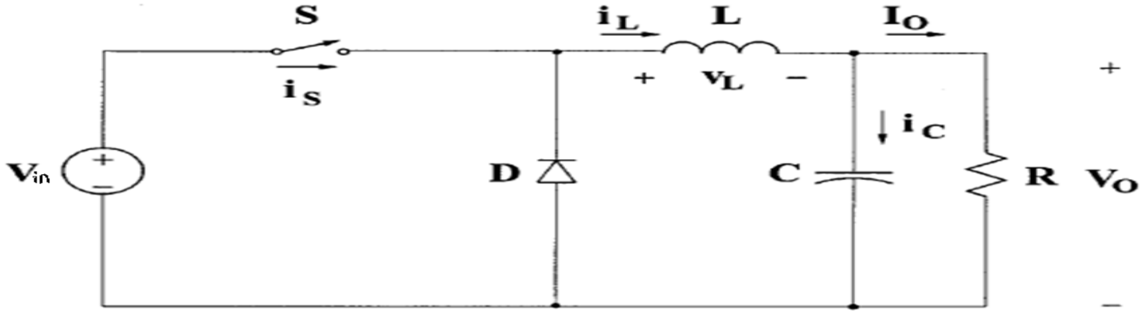

Figure 2.1: Buck converter circuit [10].

The waveforms of buck converter for continuous conduction mode of operation are shown in the Figure 2.2 below.

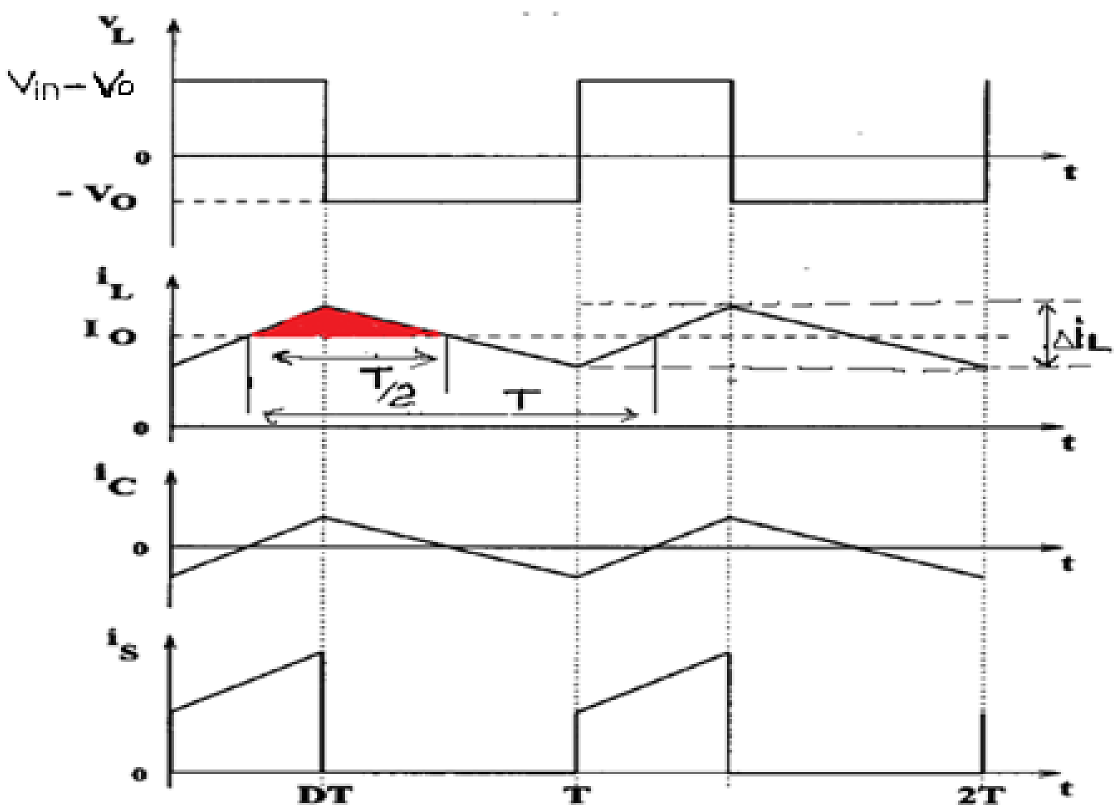

Figure 2.2: Waveforms of buck converter for continuous conduction mode operation [10].

2.3.1. Buck Converter Formula

Based on the buck converter analysis for the switch closed and open and using Kirchhoff's voltage law and current law it is possible to calculate the following.

Calculation for Duty Ratio

For calculation of the duty ratio, first of all assume that the converter is in steady state. The switches are treated as being ideal, and the losses in the inductive and the capacitive elements are neglected. Also it is important to point out that the following analysis does not include any parasitic resistances (all are ideal case). The analysis also has the assumption that the converter is operating in continuous conduction mode only i.e. inductor current is greater than zero. To obtain the duty ratio the analysis of the two state of buck converter is required.

2.3.1.1. Buck Converter Analysis for the Switch Closed (ON)

The equivalent circuit of buck converter when the switch closed is as shown below in Figure 2.3.

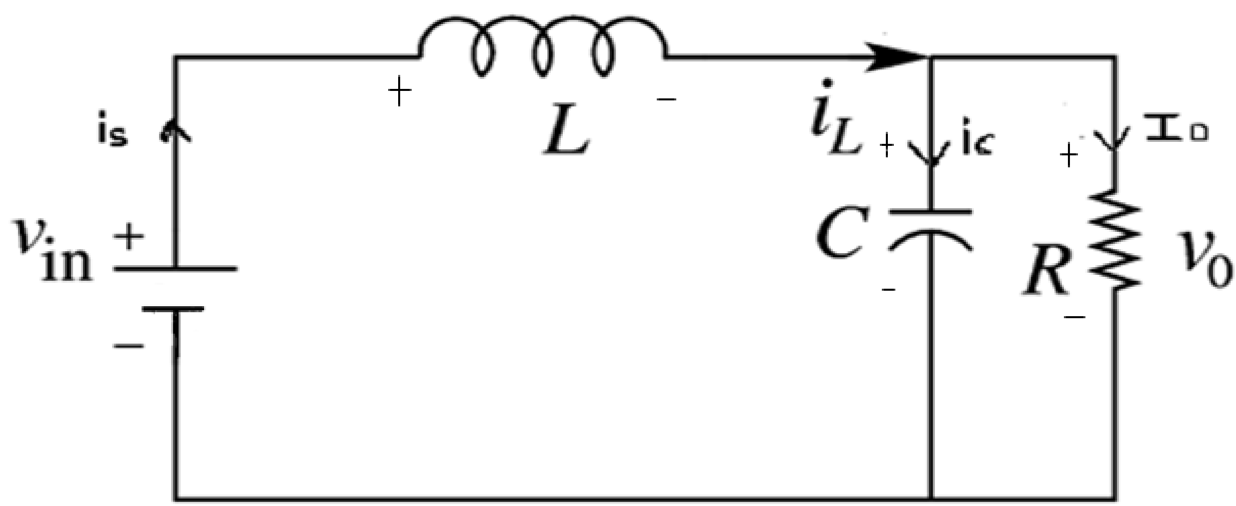

Figure 2.3: Buck converter circuit when the switch is closed.

When the switch is closed (ON) for time duration t_{on}, the switch conducts the inductor current and the diode becomes reverse biased. This results in a positive voltage across the inductor.

$$v_{in} = L\frac{di_L}{dt} + v_o \tag{2.1}$$

Rearranging Equation (2.1) gives,

$$L\frac{di_L}{dt} = v_{in} - v_o \tag{2.2}$$

Equation (2.2) can be written as,

$$\frac{\Delta i_L}{\Delta t} = \frac{v_{in} - v_o}{L} \tag{2.3}$$

Since the duration of time when the witch ON is given as $\Delta t = DT$, Equation (2.3) is given as,

$$\Delta i_{L(closed)} = \left(\frac{v_{in} - v_o}{L}\right) DT \tag{2.4}$$

Where, D= duty ratio.

2.3.1.2. Buck Converter Analysis for the Switch Open (OFF)

The equivalent circuit of buck converter is shown below when the switch is open.

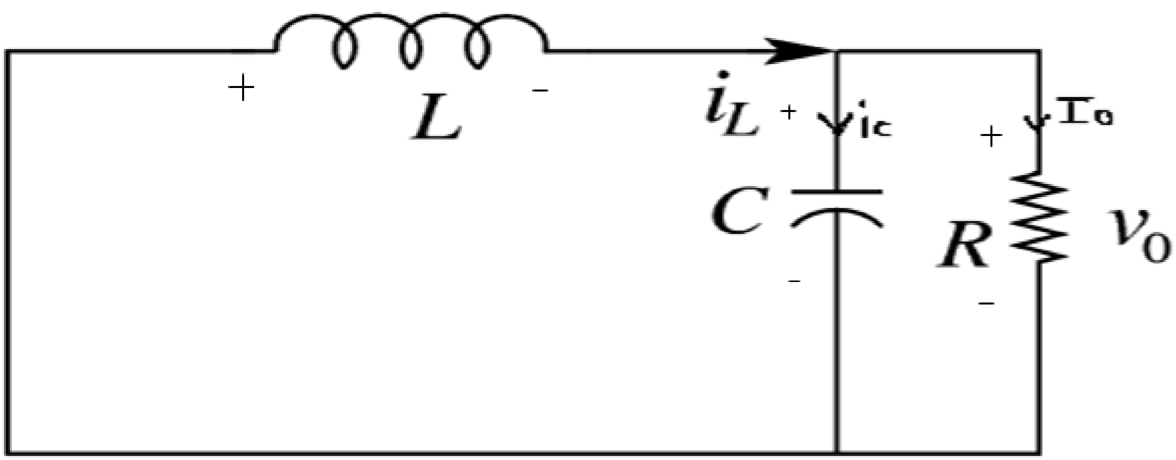

Figure 2.4: Buck converter circuit when the switch is open.

When the switch is turned off, because of the inductive energy storage, inductor current continues to flow. During this time diode is forward biased and the current flow through it.

$$L\frac{di_L}{dt} = -v_o \tag{2.5}$$

Equation (2.5) can be written as,

$$\frac{\Delta i_L}{\Delta t} = \frac{-v_o}{L} \tag{2.6}$$

Since the duration of time when the switch OFF is given as $\Delta t = (1 - D)T$, Equation (2.6) is given as,

$$\Delta i_{L(open)} = \frac{-v_o}{L}(1 - D)T \tag{2.7}$$

For steady state operation, the net change in inductor current must be zero over one period of time.

$$\Delta i_{L(closed)} + \Delta i_{L(open)} = 0 \tag{2.8}$$

Substituting Equation (2.4) and (2.7) into Equation (2.8) and simplifying it gives,

$$D = \frac{v_o}{v_{in}} \tag{2.9}$$

In other ways, since in steady-state operation waveform must repeat from one time period to the next, the integral of the inductor voltage V_L over one time period must be zero.

$$\int_0^T v_L\, dt = \int_0^{t_{on}} v_L\, dt + \int_{t_{on}}^T v_L\, dt = 0 \tag{2.10}$$

Where T is switching period (i.e. $T = t_{on} + t_{off}$) and integrating over switching period gives,

$$(v_{in} - v_o)t_{on} = v_o(T - t_{on}) \tag{2.11}$$

Simplifying Equation (2.11) gives,

$$D = \frac{v_o}{v_{in}} = \frac{t_{on}}{T} \tag{2.12}$$

Calculation for Inductor

Assuming no power losses in the converter, power absorbed by the load must be equal with power supplied by the source.

$$P_o = P_s \tag{2.13}$$

Equation (2.13) can be written as,

$$\frac{{v^2}_o}{R} = v_{in} I_s \tag{2.14}$$

Average source current is related to average inductor current as,

$$I_s = I_L D \tag{2.15}$$

Thus Equation 2.14 can be written as,

$$\frac{{v^2}_o}{R} = v_{in} I_L D \tag{2.16}$$

Since $v_o = v_{in} D$ Equation (2.16) can be written as,

$$I_L = \frac{v_o}{R} \tag{2.17}$$

This implies the average inductor current is equal to the load current.

The maximum and minimum inductor current is given as follows:

$$I_{Lmax} = I_L + \frac{\Delta i_L}{2} \tag{2.18}$$

$$I_{Lmin} = I_L - \frac{\Delta i_L}{2} \tag{2.19 }$$

Substituting for I_L and Δi_L from Equation (2.17) and (2.4) respectively into Equation (2.18) and (2.19) and simplifying gives,

$$I_{Lmax} = \frac{v_o}{R} + \frac{v_o}{2L}(1 - D)T \tag{2.20}$$

$$I_{Lmin} = \frac{v_o}{R} - \frac{v_o}{2L}(1 - D)T \tag{2.21}$$

For continuous current, the inductor current must remain positive. Therefore in order to determine the boundary between continuous conduction mode and discontinuous conduction mode minimum inductor current is set to zero.

$$I_{Lmin} = \frac{v_o}{R} - \frac{v_o}{2L}(1 - D)T = 0 \tag{2.22}$$

Rearranging and solving for the inductor from Equation (2.22) gives,

$$L_{min} = \frac{(1 - D)}{2f}R \tag{2.23}$$

Where f is the switching frequency ($f = \frac{1}{T}$).

This is the value of the inductor that determines the boundary between the CCM and DCM of operation. Thus for buck converter to operate in continuous conduction mode, the value of inductor used is greater than the minimum value of inductor.

The ripple inductor current is the difference between maximum and minimum value of the inductor current.

$$\Delta i_L = I_{L,max} - I_{L,min} \tag{2.24}$$

Substituting for $I_{L,max}$ and $I_{L,min}$ from Equation (2.20) and (2.21) respectively into Equation (2.24) and simplifying gives,

$$\Delta i_L = \frac{v_o}{L}(1 - D)T \tag{2.25}$$

Calculation for Capacitor

The output capacitor is assumed to be so large as to yield constant output voltage. But the ripple in the output voltage (ΔVo) with a practical value of capacitance can be calculated by considering the waveform shown in Figure 2.2 above for continuous conduction mode of operation. Assuming that the entire ripple component of the inductor current (Δi_L) flows through the capacitor and its average component flows through the load resistor, the maximum increase of the charge (ΔQ) which is stored in filter capacitor C is equal to shaded triangle area. Therefore by calculation the shaded triangle area shown by Figure 2.2 ΔQ is obtained for each cycle [11].

$$\Delta v_o = \frac{\Delta Q}{C} \tag{2.26}$$

$$\Delta Q = \frac{1}{2}\frac{T}{2}\frac{\Delta i_L}{2} \tag{2.27}$$

Substituting for ΔQ from Equation (2.27) into Equation (2.26) gives,

$$\Delta v_o = \frac{1}{C}\frac{1}{2}\frac{T}{2}\frac{\Delta i_L}{2} \tag{2.28}$$

Substituting for Δi_L from Equation (2.25) into Equation (2.28) and rearranging and solving for capacitor,

$$C_{min} = \frac{(1-D)v_o}{8\Delta v_o L f^2} \tag{2.29}$$

This is the minimum capacitance required. To limit the peak-to-peak value of the ripple voltage below a certain value and to minimize the voltage overshoot, the filter capacitance must be greater than the minimum capacitance.

2.4. Purpose of Different Components in the Buck Converter

Buck converter consists of the following different types of components. They are:

- Switch
- Inductor
- Capacitor
- Diode

A. Switch

In its crudest form a switch can be a toggle switch which switches between supply voltage and ground. Here transistors chosen for use in switching power supplies must have fast switching times and should be able to withstand the voltage spikes produced by the inductor. The most popular switching devices are SCR, GTO, IGBT and MOSFET.

Power MOSFETs are the key elements of high frequency power systems such as high-density power Supplies [12]. Therefore MOSFETs are used in designs operating at much higher frequencies but at lower voltages.

Operating Frequency

The operating frequency determines the performance of the switch. There is now a growing trend in research work and new power converter designs in increasing the switching frequencies. The higher is the switching frequency aims to reduce cost, smaller the physical size and component value. The reason for this is to reduce even further the overall size of the power converter.

B. Inductor

The function of the inductor is to limit the current slew rate (limit the current in rush) through the power switch when the circuit is ON. The current through the inductor cannot change suddenly. When the current through an inductor tends to fall, the inductor tends to maintain the current by acting as a source. The key advantage is when the inductor is used to drop voltage, it stores energy. Also the inductor controls the percent of the ripple and determines whether or not the converter is operating in the continuous conduction mode. The smaller inductor value enables a faster transient response; it also results in larger current ripple, which causes higher conduction losses in the switches, inductor, and parasitic resistances. Also the smaller inductor value requires a larger filter capacitor to decrease the output voltage ripple. Inductors used in switched supplies are sometimes wound on toroidal cores, often made of ferrite or powdered iron core with distributed air-gap to store energy.

C. Capacitor

Capacitor provides the filtering action by providing a path for the harmonic currents away from the load. Output capacitance (across the load) is required to minimize the voltage overshoot and ripple present at the output of a step-down converter. The capacitor is large enough so that its voltage does not have any noticeable change during the time the switch is off.

D. Diode

Since the current in the inductor cannot change suddenly, a path must exist for the inductor current when the switch is off (open). This path is provided by the freewheeling diode (catch diode). The purpose of this diode is not to rectify, but to direct current flow in the circuit and to ensure that there is always a path for the current to flow into the inductor. It is also necessary that this diode should be able to turn off relatively fast. Thus the diode enables the converter to convert stored energy in the inductor to the load. This is a reason why we have higher efficiency in a DC-DC Converters as compared to a linear regulator. When the switch closes, the current rises linearly. When the switch opens, the freewheeling diode causes a linear decrease in current.

2.5. Buck Converter Mode of Operation

There are usually two modes of operation for DC-DC buck converter based on the continuity of inductor current flow. These are:

- Discontinuous conduction mode (DCM) and
- Continuous conduction mode (CCM)

2.5.1. Discontinuous Conduction Mode (DCM)

This condition occurs when the inductor current has an interval of time staying at zero with no charge and discharge during a switching period. In the discontinuous conduction mode each switching cycle is divided into three parts that is DT, D˙T and D˙˙T (D+D˙+D˙˙=1). During the first and second mode that is in DT and D˙T the inductor current increase and decrease respectively, while during the third mode that is in D˙˙T the inductor current stay at zero with no charge and discharge.

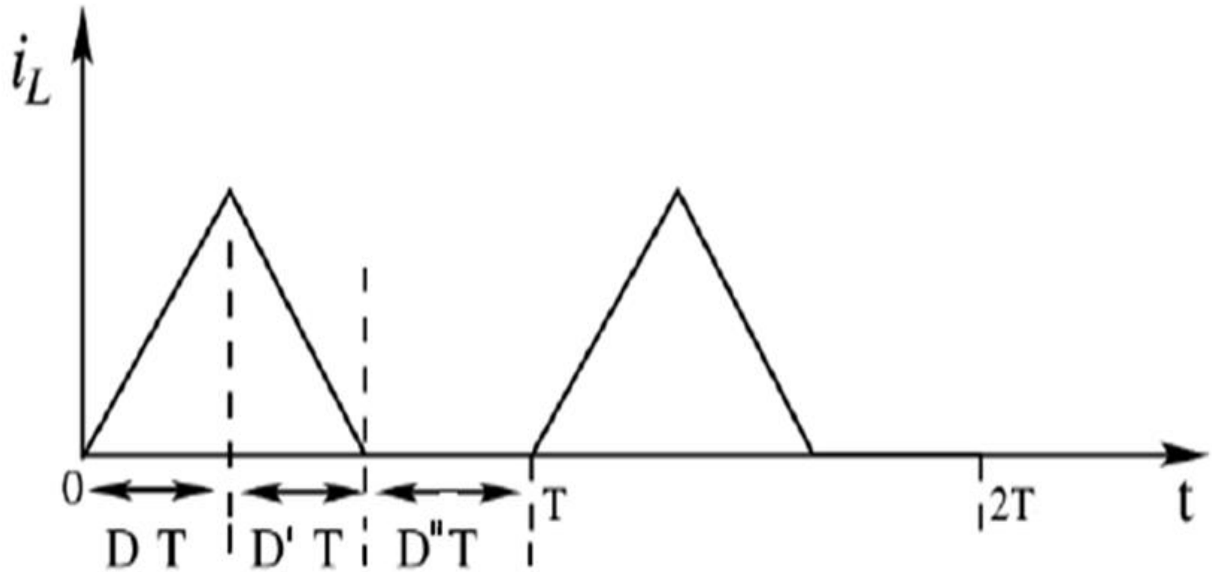

Figure 2.5: Waveform of inductor current for discontinuous conduction mode [2].

2.5.2. Continuous Conduction Mode (CCM)

This condition occurs when the inductor current flow is continuous of charge and discharge during a switching period. In CCM, each switching cycle consists of two parts that is DT and D˙T (D+ D˙=1). During the first and second mode that is in DT and D˙T the inductor current increase and decrease respectively.

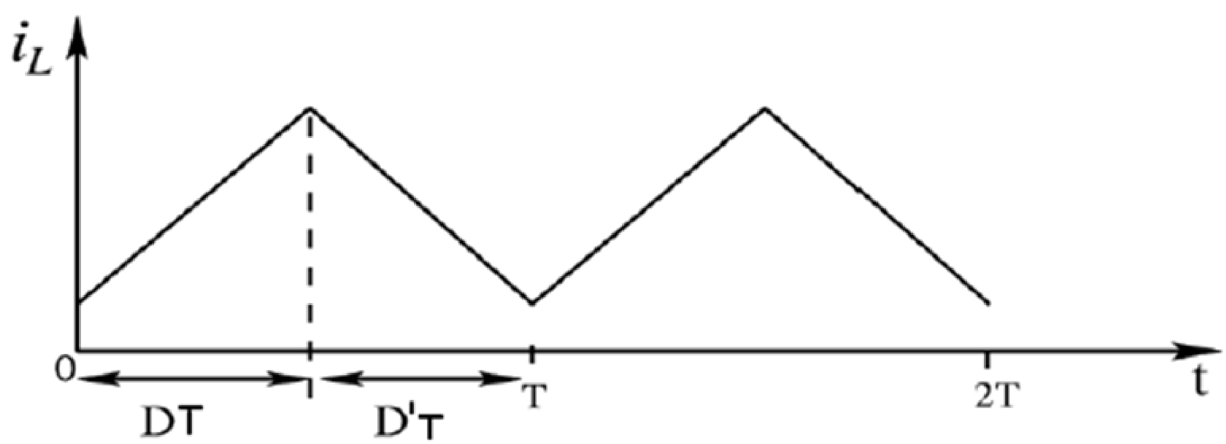

Figure 2.6: Waveform of inductor current for continuous conduction mode [2].

In continuous conduction mode, the buck converter has two states per switching cycle that are ON state and OFF state. The ON state is when the switch is closed and diode is in reverse biased mode. The OFF state is when the switch is open and diode is forward biased (conducting) mode. In this thesis only continuous conduction mode of buck converter is considered.

2.6. PID Controller

A proportional-integral-derivative controller (PID controller) is a generic control loop feedback mechanism widely used in industrial control systems. A PID controller attempts to correct the error between a measured process variable and a desired set point.

The PID controller calculation (algorithm) involves three separate parameters; the Proportional, the Integral and Derivative values. The Proportional value determines the reaction to the current error, the Integral determines the reaction based on the sum of recent errors and the Derivative determines the reaction to the rate at which the error has been changing. By "tuning" the three constants in the PID controller algorithm the PID can provide control action designed for specific process requirements. The response of the controller can be described in terms of the responsiveness of the controller to an error, the degree to which the controller overshoots the set-point and the degree of system oscillation [13].

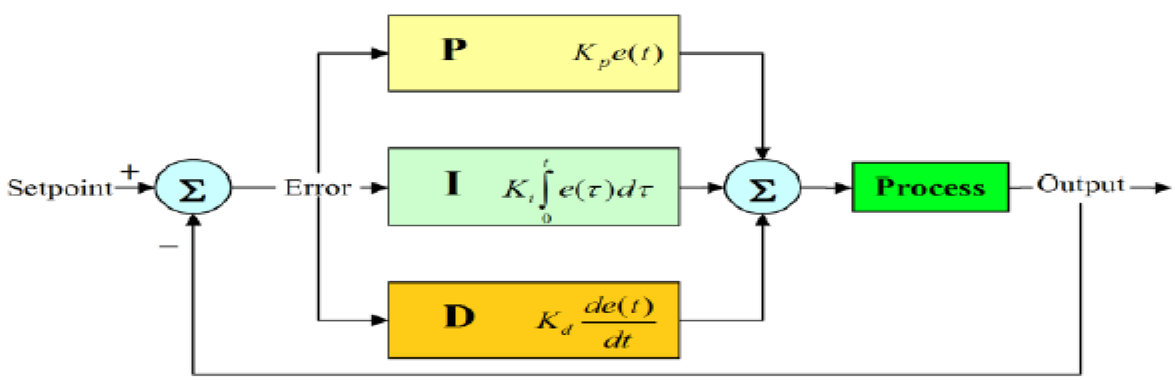

Figure 2.7: Block Diagram of PID Controller [14].

The output of PID controller u(t), is equal to the sum of three signals. The signal obtained by multiplying the error signal by a constant proportional gain K_p, plus the signal obtained by differentiating and multiplying the error signal by constant derivative gain K_d and the signal obtained by integrating and multiplying the error signal by constant internal gain K_i. The output of PID controller is given by $u(t)$, taking Laplace transform, and solving for transfer function, gives ideal PID transfer function given by U(s). It is given as,

$$u(t) = Kpe(t) + Kd\frac{de(t)}{dt} + Ki\int e(t)dt \tag{2.30}$$

Taking Laplace transform

$$G_c(s) = \left[Kp + \frac{Ki}{s} + Kds\right] \tag{2.31}$$

This is the transfer function of PID controller.

Conventional PID controllers have been extensively used in industry, due to their effectiveness for linear systems, ease to understand and simple to implement. Despite their effectiveness for linear systems, conventional PID controllers are not suitable for nonlinear systems. It does not reliable and satisfactorily in case of non-linear systems.

2.7. Sliding Mode Control (SMC)

Sliding mode controller (SMC) is versatile control technique that provides robustness to variable structure systems (VSS) under parameter variations and external disturbances. It is a nonlinear control strategy to stabilize the output(s) of a plant at a certain set point or around a certain trajectory. As compared to other nonlinear controllers, SMC is easier to implement and shows higher degree of design flexibility [6]. Two important steps in SM control is to design a sliding surface in state space and then prepare a control law to direct the system state trajectory starting from any arbitrary initial state to reach the sliding surface in finite time, and at the end it should arrive to a point where the system equilibrium state exists that is in the origin point of the phase plane [15]. Sliding mode controller (SMC) consists of two modes. One is the reaching mode in which the trajectory moves towards the sliding surface from any initial point. In reaching mode, the system response is sensitive to parameter uncertainties and disturbances. The other is the sliding mode in which, the state trajectory moves to origin along the switching surface and the states never leave the switching line [16].

As described above there are some advantages of first-order sliding mode control (1-SMC). In particular, those are the invariance property (immune to disturbances and parameter variations), the simplicity of implementation of the controller itself (no precise plant model is needed) and the order reduction when the system is on the sliding surface. But there are some problems when using the conventional sliding mode control. These are [17]:

Chattering: Defined as unwanted fast oscillations of the system trajectories near the sliding surface. It is dangerous high-frequency vibration of the controlled system. It needs to be considered in any first order sliding mode control (1-SMC) implementation. Because any 1-SMC control signal is fast switching nature, those fast switching will lead to chattering which may excite unmolded high frequency dynamics, which is not accounted in the design and the system performance degrades and may even lead to instability. Chattering is also caused when the control law is implemented in a sampled system, which is mostly the case. Because sampled control systems act as open-loop systems between two sampling instants, it is impossible for any sampled control algorithm to satisfy a constraint with infinite accuracy when acting on a plant with uncertain internal and/or external disturbances.

Accuracy: sampled first order sliding mode control (1-SMC) algorithms can keep the sliding constraint $s = 0$ no better than with an error proportional to the sampling time τ, i.e., the sliding error is $O(\tau)$.

Relative degree one limitation: Finite-time stabilization of the sliding variable at the origin, $s = 0$, is only guaranteed with a first order sliding mode control (1-SMC) algorithm if the algorithm can assign the sign of $\dot{s}$ directly. In other words, the input must appear explicitly in the first derivative of s, i.e. the system must have relative degree one with respect to s. If more levels of differentiation of s are required before the input appears, i.e., in case of higher relative degree, 1-SMC algorithms might not be able to stabilize the system at all.

Some methods were proposed to overcome the above difficulties. The main idea was to change the dynamics in small vicinity of the discontinuity surface in order to avoid real discontinuity and at the same time to preserve the main properties of the whole system. In particular, high-gain control with saturation approximates the sign-function and diminishes the chattering, while on-line estimation of the so-called equivalent control is used to reduce the discontinuous-control component. However, by using saturation function and equivalent control the accuracy and robustness of the sliding mode were partially lost. On the contrary, higher order sliding modes

(HOSM) generalize the basic sliding mode idea acting on the higher order time derivatives of the sliding surfaces instead of influencing the first derivative like it happens in standard sliding modes. Keeping the main advantages of the original approach, at the same time they totally remove the chattering effect and provide for even higher accuracy in realization [18]. The main features of HOSM are [19]:

- It can force the sliding variable and its (r-1) successive derivatives to zero.
- For this approach, there is no restriction on relative degrees.
- As the high frequency control switching is pushed in the higher derivative of the sliding variable, chattering is significantly reduced.
- No detailed mathematical model of the plant is required.
- As the integration of the signum function is utilized in synthesizing the control, it becomes continuous.

2.8. Sliding Order

The sliding order of HOSM characterizes the dynamic smoothness degree in the vicinity of the sliding mode. If the task is to provide for keeping a constraint given by equality of a smooth function s to zero, the sliding order is a number of continuous total derivatives (including the zero one) in the vicinity of the sliding mode. Hence, the r^{th} order sliding mode is determined by the equalities forming an r-dimensional condition on the state of the dynamic system [18].

$$s = \dot{s} = \cdots = s^{r-1} = 0 \tag{2.32}$$

Standard sliding mode is called 1-sliding mode. In 1-sliding mode $\dot{s}$ is discontinuous while in the r^{th} order sliding mode s^{r} is discontinuous, where r is the order of the sliding mode. In the following sections second order sliding mode control is discussed.

2.9. Second Order Sliding Mode Control (SOSMC)

Second order sliding mode controller is a special case of higher order sliding mode control which preserve the desirable properties, particularly invariance and order reduction but they achieve better accuracy and guarantee finite-time stabilization of relative degree two systems [20]. Based on Equation (2.32) above, second order sliding mode control is obtained when,

$$s = \dot{s} = 0 \tag{2.33}$$

The aim of second order sliding mode control is to steer s and $\dot{s}$ to zero in finite time. To design second order sliding mode, a number of algorithms are available. These algorithms differ with respect to the following aspects [17]:

- Some are formulated in continuous-time (which must be discretized) and others explicitly in discrete-time.
- Most 2-SMC algorithms are able to stabilize a sliding constraint s with relative degree r = 2. One notable exception is the super-twisting algorithm which can only stabilize r = 1 plants.
- The algorithms differ in the number of necessary parameters to tune.
- Most algorithms require knowledge of s and $\dot{s}$, but some require less real-time plant information (the super-twisting algorithm) and some require other information.

Now these algorithms are discussed one by one.

2.9.1. Twisting Algorithm

By considering local coordinates $y_1 = s$ and $y_2 = \dot{s}$, the SOSMC problem is the same as the finite time stabilization problem for uncertain second order systems having a relative degree 1.

$$\left.\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= \phi(t,x) + \gamma(t,x)\dot{u} \end{aligned}\right\} \tag{2.34}$$

Here $y_2(t)$ is immeasurable but the sign is possibly known, and $\phi(t,x)$ and $\gamma(t,x)$ are uncertain functions such that,

$$\Phi > 0, \qquad |\phi| \le \Phi, \qquad 0 < \Gamma_m \le \gamma \le \Gamma_M \tag{2.35}$$

The main feature of this algorithm is the twisting behavior around the origin as depicted in Figure 2.8. The trajectories converge to the origin after performing infinite number of rotations. The vibration magnitude and rotation time decrease in geometric progression. The control law is defined as,

$$\dot{u}(t) = \begin{cases} -u & \text{if } |u| > 1 \\ -v_m \text{sign}(y_1) & \text{if } y_1 y_2 \le 0;\ |u| \le 1 \\ -v_M \text{sign}(y_1) & \text{if } y_1 y_2 > 0;\ |u| \le 1 \end{cases} \tag{2.36}$$

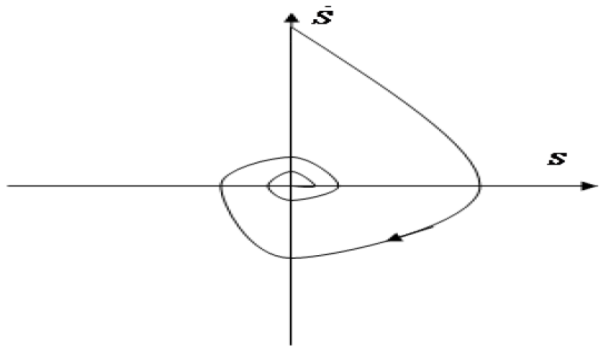

Figure 2.8: Phase trajectory of twisting algorithm [21].

The corresponding sufficient conditions for the finite time convergence are given below.

$$\left.\begin{aligned} v_M &> v_m \\ v_m &> \frac{4\Gamma_M}{s_o} \\ v_m &> \frac{\Phi}{\Gamma_M} \\ \Gamma_m v_M - \Phi &> \Gamma_M v_m + \Phi \end{aligned}\right\} \tag{2.37}$$

Where s_o, v_m and v_M are proper positive constants such that $V_M > V_m$ and V_M / V_m is sufficiently large. The above equations of controller can be used for relative degree 1 systems, for relative degree 2 systems, the following controller can be used.

$$u(t) = \begin{cases} -v_m \text{sign}(y_1) & \text{if } y_1 y_2 \leq 0 \\ -v_M \text{sign}(y_1) & \text{if } y_1 y_2 > 0 \end{cases} \tag{2.38}$$

2.9.2. Super-twisting Algorithm

Super-Twisting Algorithm has been employed for the systems having relative degree one for the purpose of chattering reduction. Since an n^{th}-order sliding-mode controller requires the information about s, $\dot{s}$, $\ddot{s}$ … $s^{(n-1)}$ in order to keep s = 0, therefore the knowledge about the values of higher-order derivatives of the sliding variable seemed to be a constraining requirement. However, this apparent restriction could be avoided by the use of this algorithm that does not require this extra piece of information.

The algorithm ensures that system trajectories twist around the origin in the phase portrait as shown in the Figure 2.9. Another advantage of this algorithm is that it is not sensitive to sampling time.

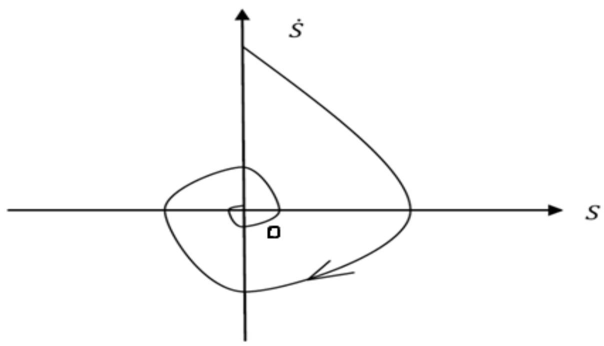

Figure 2.9: Super-twisting controller phase portrait [21].

The super twisting algorithm had the advantage over other algorithms in that it did not demand the time derivatives of sliding variable. The control law used in this algorithm was composed of two terms. The first term u_a was defined in term of discontinuous time derivative whereas the second term u_b was a continuous function of sliding variable.

$$u = u_a + u_b \tag{2.39}$$

$$\dot{u}_a = \begin{cases} -u & \text{when } |u| > 1 \\ -K\text{sign}(s) & \text{when } |u| \leq 1 \end{cases} \tag{2.40}$$

$$u_b = \begin{cases} -\lambda|s_o|^{\xi}\text{sign}(s) & \text{when } |s| > s_o \\ -\lambda|s|^{\xi}\text{sign}(s) & \text{when } |s| \leq s_o \end{cases} \tag{2.41}$$

The super twisting algorithm converged in finite time and the corresponding sufficient conditions were:

$$K > \frac{\Phi}{\Gamma_m}, \qquad \lambda^2 \geq \frac{4\Phi}{{\Gamma_m}^2}\frac{\Gamma_M(K+\Phi)}{\Gamma_m(K-\Phi)}, \qquad 0 < \xi \leq 0.5 \tag{2.42}$$

Where K, λ, s_o, Φ, Γ_m, Γ_M are some positive constants.

When controlled system was linearly dependent on u, the control law could be simplified as,

$$u = -\lambda|s|^{\xi}\text{sign}(s) + u_a \tag{2.43}$$

$$\dot{u}_a = -K\text{sign}(s) \tag{2.44}$$

When ξ=1 then this algorithm converges to the origin exponentially.

2.9.3. Sub-Optimal Algorithm

This algorithm represents sub-optimal feedback realization for a time-optimal control for a double integrator system. By considering relative degree 2, the auxiliary system can be defined as,

$$\left.\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= \phi(t,x) + \gamma(t,x)\dot{u} \end{aligned}\right\} \tag{2.45}$$

Using this algorithm, the system trajectories are restricted within limit parabolic arcs including origin. In this algorithm both twisting and leaping behaviors are possible as shown in Figure 2.10. After an initialization phase the algorithm is defined by the following control law:

$$u(t) = -\alpha(t)v_M\text{sign}\left(y_1(t) - \frac{1}{2}y_{1M}\right) \tag{2.46}$$

$$\alpha(t) = \begin{cases} \alpha^* & \text{if } \left[y_1(t) - \frac{1}{2}y_{1M}\right][y_{1M} - y_1(t)] > 0 \\ 1 & \text{if } \left[y_1(t) - \frac{1}{2}y_{1M}\right][y_{1M} - y_1(t)] \leq 0 \end{cases} \tag{2.47}$$

Where y_{1M} is the final singular value of the function $y_1(t)$.

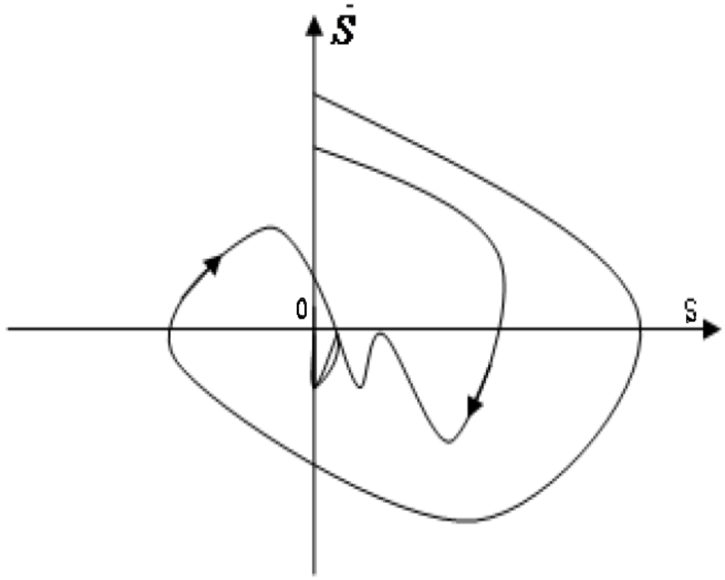

Figure 2.10: Phase trajectories of sub-optimal algorithm [21].

The corresponding sufficient conditions for the finite-time convergence are as follows:

$$\alpha^{*}\epsilon]0,1] \cap \left(0, \frac{3\Gamma_m}{\Gamma_M}\right) \tag{2.48}$$

$$v_M > max\left(\frac{\Phi}{\alpha^{*}\Gamma_m}, \frac{4\Phi}{3\Gamma_m - \alpha^{*}\Gamma_M}\right) \tag{2.49}$$

2.9.4. Drift Algorithm

In drift algorithm the system trajectories are moved to the 2nd order sliding mode while keeping s small. In other words trajectories are forced to move towards the origin along y_1-axis. The phase trajectories of this algorithm on 2-sliding manifold are characterized by loops having constant sign of sliding variable y_1 as shown in Figure 2.11.

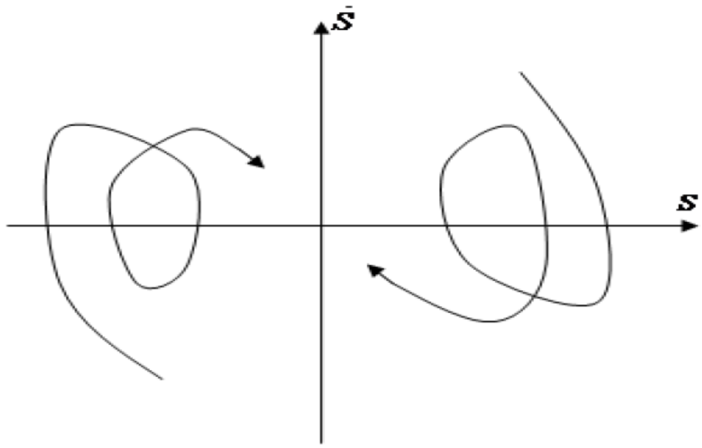

Figure 2.11: Phase trajectories of drift algorithm [21].

The control algorithm for the case of relative degree 1 is defined as follows:

$$\dot{u} = \begin{cases} -u & \text{if } |u| > 1 \\ -v_m \text{sign}(\Delta y_{1i}) & \text{if } y_1 \Delta y_{1i} \leq 0;\ |u| \leq 1 \\ -v_M \text{sign}(\Delta y_{1i}) & \text{if } y_1 \Delta y_{1i} > 0;\ |u| \leq 1 \end{cases} \quad (2.50)$$

Where V_m and V_M are proper positive constants such that $V_m < V_M$ and V_M / V_m is sufficiently large, also $\Delta y_{1i} = y_1(t_i) - y_1(t_i - \tau)$, $t \in [t_i, t_{1+i}[$. For relative degree 2, the control law is modified as shown below.

$$\dot{u} = \begin{cases} -v_m \text{sign}(\Delta y_{1i}) & \text{if } y_1 \Delta y_{1i} \leq 0 \\ -v_M \text{sign}(\Delta y_{1i}) & \text{if } y_1 \Delta y_{1i} > 0 \end{cases} \quad (2.51)$$

2.9.5. Control Algorithm with Prescribed Convergence Law

The second order sliding mode controller with prescribed convergence law is defined as

$$u = -\alpha \text{sign}\left(\dot{s} + \xi(s)\right), \qquad \alpha > 0 \quad (2.52)$$

Where $\xi(s)$ is a continuous function smooth everywhere except s=0. It is assumed that all solutions of the differential equation $\dot{s} + \xi(s) = 0$ converge to 0 in finite time. Choosing $\xi(s) = \beta |s|^{1/2} \text{sign}(s), \beta > 0$ in Equation (2.52) yields the controller,

$$u = -\alpha \text{sign}\left(\dot{s} + \beta |s|^{1/2} \text{sign}(s)\right) \tag{2.53}$$

The controller guarantees the establishment and maintenance of a 2-sliding mode $s \equiv 0$ for the sliding variable dynamics in finite time [22]. A sufficient condition for convergence is given as [23],

$$\alpha k_m - \Phi > \frac{\beta^2}{2} \tag{2.54}$$

It should be noticed that the algorithm needs not only just s to be known, but also its time derivative. This controller trajectory in the phase plane is shown in Figure 2.12.

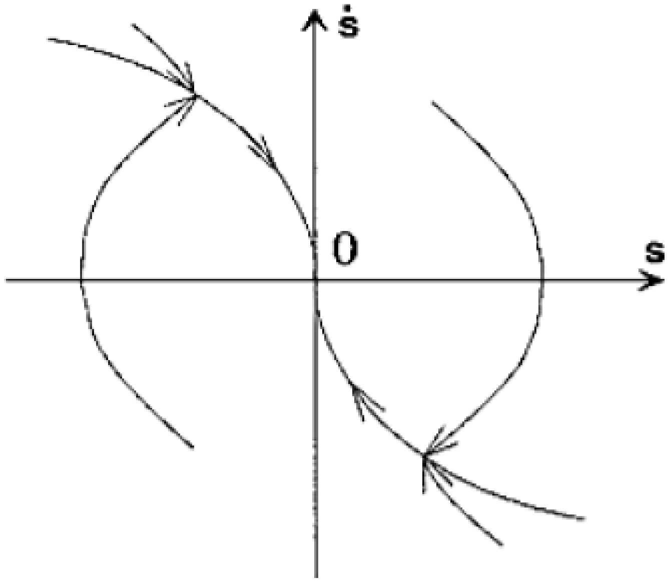

Figure 2.12: Trajectories of the controller with the prescribed convergence law in the phase plane [23].

2.9.6. Quasi-Continuous Control Algorithm

An important class of controllers comprises the recently proposed so-called quasi-continuous controllers, featuring control continuous everywhere except the 2-sliding manifold $s = \dot{s} = 0$ itself. This algorithm also needs not only just sliding surface to be known, but also its time derivative. The control law of the second order quasi-continuous sliding mode controller can be written as [22].

$$u = -\alpha \frac{\dot{s} + \beta |s|^{1/2} \text{sign}(s)}{|\dot{s}| + \beta |s|^{1/2}}, \qquad \alpha, \beta > 0 \tag{2.55}$$

The trajectory of Quasi-Continuous Control Algorithm in the phase plane is shown in Figure 2.13.

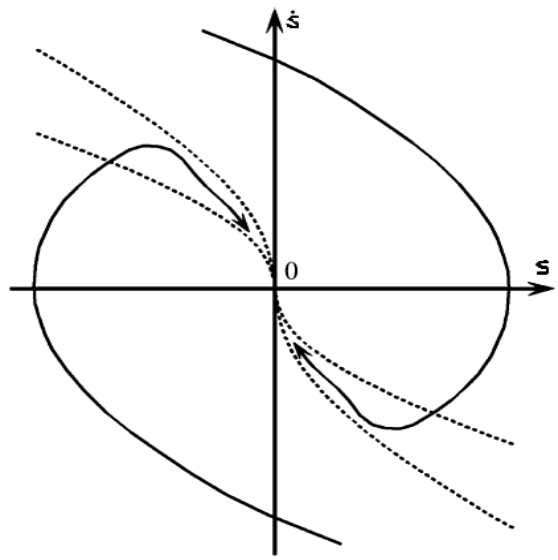

Figure 2.13: Trajectories of the quasi-continuous controller [22].

This control is continuous everywhere except the origin and it vanishes on the parabola $\dot{s} + \beta|s|^{1/2}\mathrm{sign}(s) = 0$. For sufficiently large α, there are numbers $\rho 1, \rho 2 : 0 < \rho 1 < \beta < \rho 2$ such that all the trajectories enter the region between the Curves $\dot{s} + \rho i|s|^{1/2}\mathrm{sign}(s) = 0, i = 1,2$ and cannot leave it (Figure 2.13).

CHAPTER THREE

METHODOLOGY

3.1. Introduction

In this chapter the mathematical model describing the DC-DC buck converter system is developed. Also linear controller, particularly PID controller is designed for buck converter to obtained desired output voltage. Since the converter is nonlinear, linear controller unable to produce best performance under input voltage and load variations. Therefore to overcome the drawback of linear controller, second order sliding mode controller which is nonlinear controller has been designed for buck converter in this chapter.

3.2. Mathematical Model of Buck Converter

The mathematical model of DC-DC buck converter in state space form is obtained by application of basic laws governing the operation of the system. The dynamics of this converter operating in continuous conduction mode can be easily obtained by applying Kirchhoff's voltage and current laws. The following assumption taken into consideration:

- The switch is closed for time DT and open for time (1-D)T, where D is duty ratio and T is switching period.
- The circuit operates at steady state.
- The inductor current is continuous (i.e. always positive).
- The capacitor is very large i.e. in order to limit the peak-to-peak value of the ripple voltage below a certain value and to minimize voltage overshoot; it must be greater than the minimum capacitance required for Continuous conduction mode.
- The inductor value must be greater than the minimum inductor to insure continuous conduction mode and to prevent higher conduction loss in switch, inductor and parasitic resistances.

A basic buck converter topology is shown in Figure 3.1.

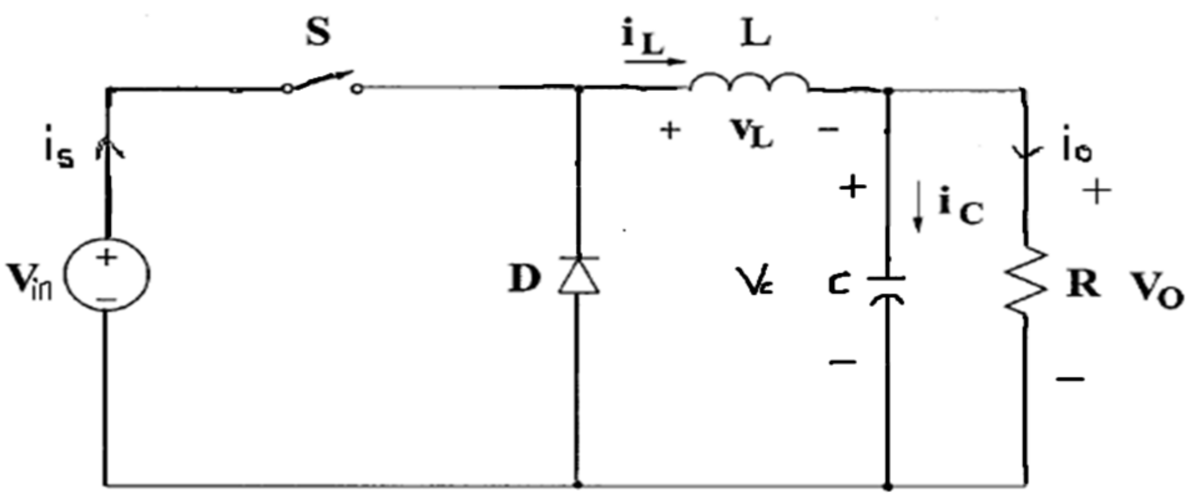

Figure 3.1: Buck converter

The equivalent circuit when the switch is closed and the diode is reverse biased is shown in Figure 3.2.

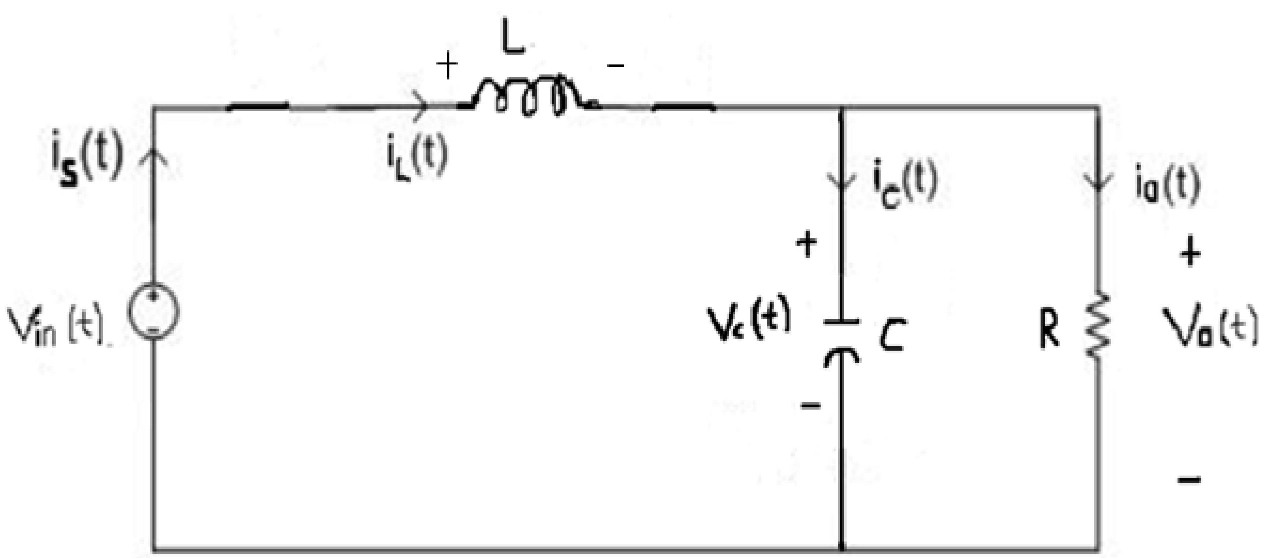

Figure 3.2: Buck converter when the switch is closed.

The differential equations related to state variables are:

$$\left.\begin{aligned} L\frac{di_L(t)}{dt} &= v_{in}(t) - v_o(t) \\ C\frac{dv_c(t)}{dt} &= i_L(t) - \frac{v_c(t)}{R} \end{aligned}\right\} \quad (3.1)$$

Where, $v_o(t) = i_o(t)R$

Applying Kirchhoff's voltage and current law to obtain state equation as:

$$\left.\begin{aligned} \frac{di_L(t)}{dt} &= \frac{v_{in}(t)}{L} - \frac{1}{L}v_c(t) \\ \frac{dv_c(t)}{dt} &= \frac{1}{C}i_L(t) - \frac{v_c(t)}{CR} \\ v_o(t) &= v_c(t) \\ i_s(t) &= i_L(t) \end{aligned}\right\} \quad (3.2)$$

Since $v_o(t) = v_c(t)$ the state equation matrices is given as,

$$\left.\begin{aligned} \begin{bmatrix} \frac{dv_o(t)}{dt} \\ \frac{di_L(t)}{dt} \end{bmatrix} &= \begin{bmatrix} \frac{-1}{RC} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_o(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} v_{in}(t) \\ \begin{bmatrix} v_0(t) \\ i_s(t) \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_c(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_{in}(t) \end{aligned}\right\} \quad (3.3)$$

This can be written as:

$$\left.\begin{aligned} \frac{dx(t)}{dt} &= A_1x(t) + B_1u(t) \\ y(t) &= C_1x(t) + D_1u(t) \end{aligned}\right\} \quad (3.4)$$

Where, $A_1 = \begin{bmatrix} \frac{-1}{RC} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}$, $C_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $D_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

The variables x(t), $u(t)$ and y(t) are the state variables consisting of output voltages and inductor currents, input voltages to the converter and output voltages and source currents respectively. Similarly the equivalent circuit when the switch is open and diode is forward biased (in conducting state) is as shown in Figure 3.3 below.

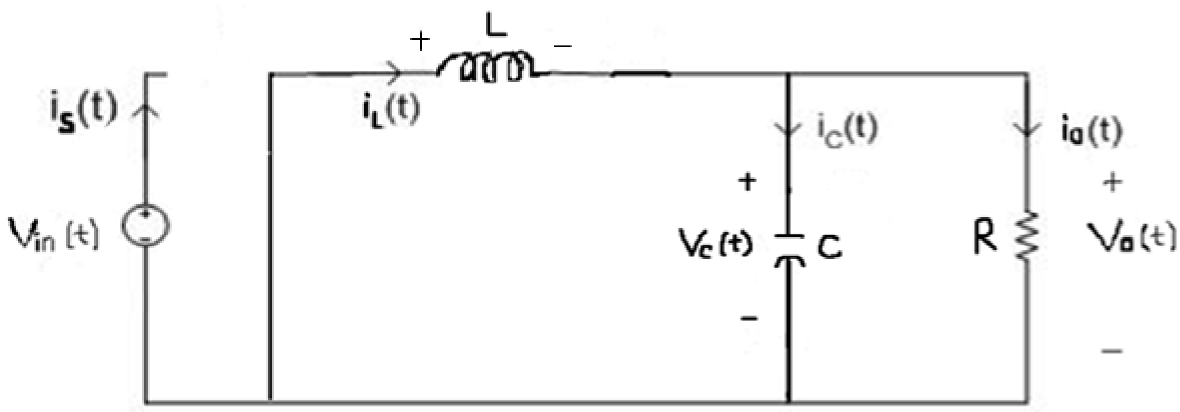

Figure 3.3: Buck converter when the switch is open.

State equation for above circuit is given as,

$$\left.\begin{aligned} L\frac{di_L(t)}{dt} &= -v_c(t) \\ C\frac{dv_c(t)}{dt} &= i_L(t) - \frac{v_c(t)}{R} \\ v_o(t) &= v_c(t) \\ i_s(t) &= 0 \end{aligned}\right\} \tag{3.5}$$

Substitute $v_o(t)$ instead of $v_c(t)$ the state equation matrices are given as,

$$\left.\begin{aligned} \left.\begin{bmatrix} \frac{dv_o(t)}{dt} \\ \frac{di_L(t)}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-1}{CR} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_o(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_{in}(t) \right\} \\ \left.\begin{bmatrix} v_0(t) \\ i_s(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_c(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_{in}(t) \right\} \end{aligned}\right\} \tag{3.6}$$

This can be written as:

$$\left.\begin{aligned} \frac{dx(t)}{dt} &= A_2x(t) + B_2u(t) \\ y(t) &= C_2x(t) + D_2u(t) \end{aligned}\right\} \tag{3.7}$$

Where, $A_2 = \begin{bmatrix} \frac{-1}{CR} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $C_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $D_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

3.3. State Space Average Model of Buck Converter

A state space average model is obtained by averaging the state equation derived above over switching cycle. Perform state-space averaging using the duty cycle as a weighting factor and combine state equations into a single averaged state equation. Then state-space averaged equation is given as follow:

$$\left.\begin{aligned}\frac{dx(t)}{dt} &= Ax(t) + Bu(t)\\ y(t) &= Cx(t) + Du(t)\end{aligned}\right\} \tag{3.8}$$

Where,

$A = dA_1 + (1-d)A_2$, $B = dB_1 + (1-d)B_2$, $C = dC_1 + (1-d)C_2$ and $D = dD_1 + (1-d)D_2$

$x(t) = \begin{bmatrix} v_o(t) \\ i_L(t) \end{bmatrix}$, $u(t) = v_{in}(t)$, $y(t) = \begin{bmatrix} v_o(t) \\ i_s(t) \end{bmatrix}$, and d = duty cycle

Therefore, substituting for the values of parameters into Equation (3.8) gives,

$$\left.\begin{aligned}&\begin{bmatrix} \frac{dv_o(t)}{dt} \\ \frac{di_L(t)}{dt} \end{bmatrix} = \left[d\begin{bmatrix} \frac{-1}{RC} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix} + (1-d)\begin{bmatrix} \frac{-1}{CR} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix}\right]\begin{bmatrix} v_o(t) \\ i_L(t) \end{bmatrix} + \left[d\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} + (1-d)\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right] v_{in}(t) \\ &\begin{bmatrix} v_o(t) \\ i_s(t) \end{bmatrix} = \left[d\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + (1-d)\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right]\begin{bmatrix} i_L(t) \\ v_c(t) \end{bmatrix}\end{aligned}\right\} \tag{3.9}$$

Simplifying the above matrices given in Equation (3.9), the state space average model of the converter is,

$$\left.\begin{aligned}&\begin{bmatrix} \frac{dv_o(t)}{dt} \\ \frac{di_L(t)}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-1}{RC} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix}\begin{bmatrix} v_o(t) \\ i_L(t) \end{bmatrix} + d\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} v_{in}(t) \\ &\begin{bmatrix} v_o(t) \\ i_s(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ d & 0 \end{bmatrix}\begin{bmatrix} i_L(t) \\ v_c(t) \end{bmatrix}\end{aligned}\right\} \tag{3.10}$$

3.4. Small Signal Model of Buck Converter

Since switching converters are nonlinear systems, it is desirable to construct ac small signal linearized models to design linear control system. This is accomplished by perturbing and linearizing the state space averaged model around precise operating point [24]. From the ac small signal model, the most important small signal transfer function can be derived.

Let introduce perturbation in state variables such that

$$i_L = I_L + \hat{i}_L, v_c = V_c + \hat{v}_c, v_{in} = V_{in} + \hat{v}_{in}, v_o = V_o + \hat{v}_o, d = D + \hat{d}, i_s = I_s + \hat{i}_s \tag{3.11}$$

Where the quantity $\hat{i}_L$, $\hat{v}_c$, $\hat{v}_{in}$, $\hat{v}_o$, $\hat{d}$ and $\hat{i}_s$ are small ac variations about the equilibrium solution. Substituting these state variables in the state space averaged model of the converter given in equation (3.10) gives,

$$\left.\begin{array}{l} \begin{bmatrix} \dfrac{d(V_o + \hat{v}_o)}{dt} \\ \dfrac{d(I_L + \hat{i}_L)}{dt} \end{bmatrix} = \begin{bmatrix} \dfrac{-1}{RC} & \dfrac{1}{C} \\ \dfrac{-1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_o + \hat{v}_o \\ I_L + \hat{i}_L \end{bmatrix} + \left(D + \hat{d}\right) \begin{bmatrix} 0 \\ \dfrac{1}{L} \end{bmatrix} (V_{in} + \hat{v}_{in}) \\ \\ \begin{bmatrix} V_o + \hat{v}_o \\ I_s + \hat{i}_s \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ D + \hat{d} & 0 \end{bmatrix} \begin{bmatrix} I_L + \hat{i}_L \\ V_c + \hat{v}_c \end{bmatrix} \end{array}\right\} \tag{3.12}$$

From Equation (3.12),

$$\left.\begin{array}{l} \dfrac{d(V_o + \hat{v}_o)}{dt} = -\dfrac{1}{CR}(V_o + \hat{v}_o) + \dfrac{1}{C}(I_L + \hat{i}_L) \\ \dfrac{d(I_L + \hat{i}_L)}{dt} = -\dfrac{1}{L}(V_o + \hat{v}_o) + \left(D + \hat{d}\right)\left(\dfrac{1}{L}\right)(V_{in} + \hat{v}_{in}) \\ V_o + \hat{v}_o = V_c + \hat{v}_c \\ I_s + \hat{i}_s = \left(D + \hat{d}\right)(I_L + \hat{i}_L) \end{array}\right\} \tag{3.13}$$

From Equation (3.13) equate ac and dc quantities and proceed with ac equations by neglecting second order ac quantities (eliminate the product of any ac terms because they are nonlinear, as they involve the multiplication of time-varying signals and also they are much smaller than the first-order ac terms) gives,

$$\left.\begin{aligned} C\frac{d\hat{v}_o}{dt} &= \hat{i}_L - \frac{1}{R}\hat{v}_o \\ L\frac{d\,\hat{i}_L}{dt} &= -\hat{v}_o + D\hat{v}_{in} + V_{in}\hat{d} \\ \hat{v}_o &= \hat{v}_c \\ \hat{i}_s &= D\hat{i}_L + I_L\hat{d} \end{aligned}\right\} \tag{3.14}$$

Take the Laplace transform of Equation (3.14)

$$\left.\begin{aligned} Cs\hat{v}_o(s) &= \hat{i}_L(s) - \frac{1}{R}\hat{v}_o(s) \\ Ls\hat{i}_L(s) &= -\hat{v}_o(s) + D\hat{v}_{in}(s) + V_{in}\hat{d}(s \\ \hat{v}_o(s) &= \hat{v}_c(s) \\ \hat{i}_s(s) &= D\hat{i}_L(s) + I_L\hat{d}(s) \end{aligned}\right\} \tag{3.15}$$

Arranging Equation (3.15), the small signal equivalent ac model of buck converter is obtained as shown in Figure 3.4.

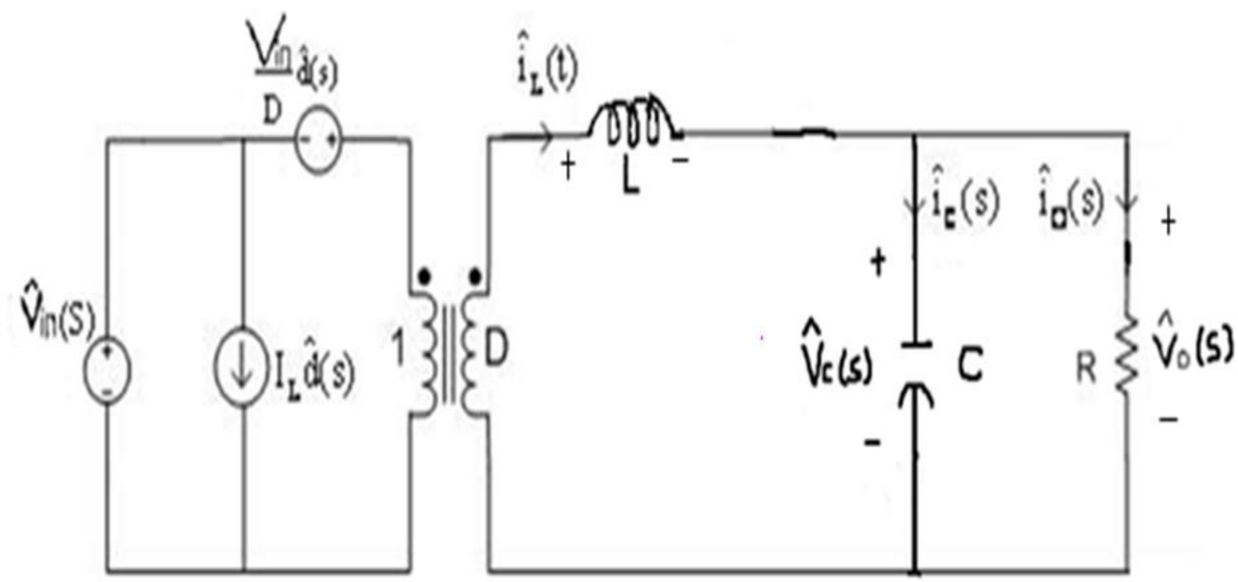

Figure 3.4: Small signal ac model of buck converter.

The most important converter transfer function is variation in output voltage with variation in input voltage. Therefore transfer function from the input voltage to the output voltage obtain by making $\hat{d}(s) = 0$. Since the transfer function from the input voltage to the output voltage expresses the effect of the input voltage changes on the output voltage when only the input voltage works, which implies that the perturbation of the duty cycle is equal to zero. To drive the transfer function of buck converter Equation (3.15) is given as,

$$Cs\hat{v}_o(s) = \hat{i}_L(s) - \frac{1}{R}\hat{v}_o(s) \tag{3.16}$$

$$Ls\hat{i}_L(s) = -\hat{v}_o(s) + D\hat{v}_{in}(s) \;\; \big| \hat{d}(s) = 0 \tag{3.17}$$

$$\hat{i}_s = D\hat{i}_L(s) \tag{3.18}$$

From Equation (3.16) solve for $\hat{i}_L(s)$ gives,

$$\hat{i}_L(s) = \left(Cs + \frac{1}{R}\right)\hat{v}_o(s) \tag{3.19}$$

Substitute for $\hat{i}_L(s)$ into Equation (3.17) from Equation (3.19) gives,

$$Ls\left(Cs + \frac{1}{R}\right)\hat{v}_o(s) = -\hat{v}_o(s) + D\hat{v}_{in}(s) \tag{3.20}$$

By rearranging and simplifying Equation (3.20) above, the transfer function of buck converter from input voltage to output voltage is given as,

$$\frac{\hat{v}_o(s)}{\hat{v}_{in}(s)} = D\left[\frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}\right] \tag{3.21}$$

3.5. Design of Proportional Integral Derivative (PID) Controller for Buck Converter

A PID controller is feedback loop controlling mechanism. It corrects the error between a measured process value and a desired set point by calculating and then a corrective action adjust the process as per the requirement. Its calculation involves three separate parameters; the Proportional, the Integral and Derivative values. The weighted sum of these three actions is used to adjust the process via a control element. By "tuning" the three constants in the PID controller, it can provide control action designed for specific requirements.

The overall structure of the control system is shown in the Figure 3.5 below.

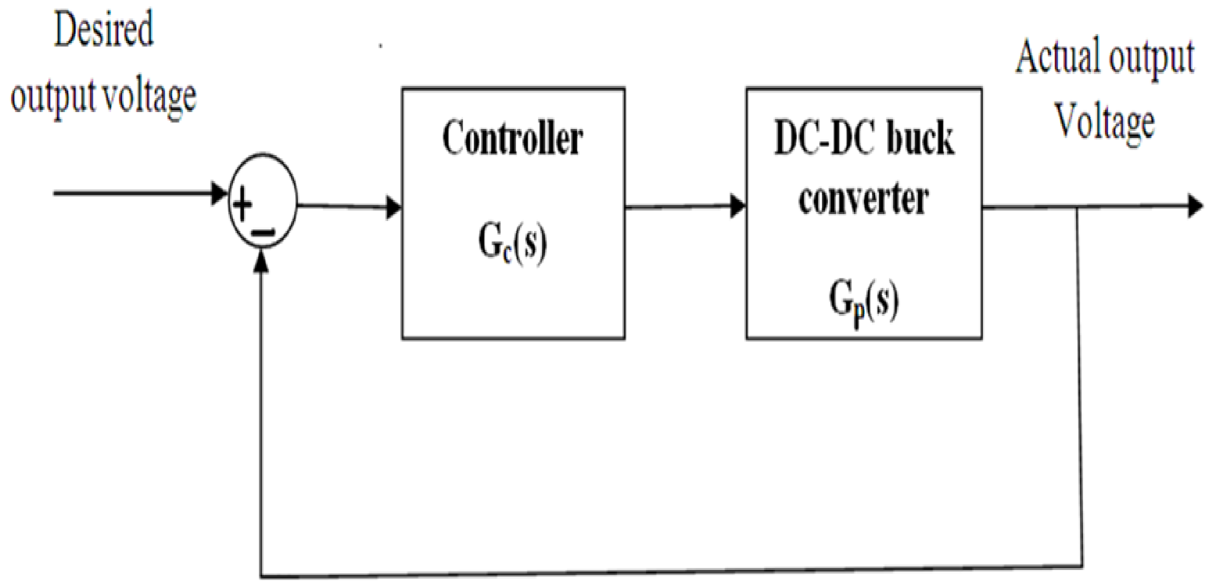

Figure 3.5: Over all structure of the system.

The open loop transfer function of buck converter as derived in Section 3.4 above is given as,

$$G_p(s) = \frac{\hat{v}_o(s)}{\hat{v}_{in}(s)} = D\left[\frac{\frac{1}{LC}}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}\right] \tag{3.22}$$

Here in order to design PID controller a guaranteed dominant pole placement method is used. Let consider a transfer function of integer order PID controller need to control second order plant (G_p) (with sluggish "*S*" shaped or oscillatory open loop dynamics).

$$G_c(s) = k_P + \frac{k_i}{s} + k_d s = \frac{k_d s^2 + k_p s + k_i}{s} \tag{3.23}$$

Where,

K_p: Proportional gain of the PID controller

K_i: Integral gain of the PID controller

K_d: Derivative gain of the PID controller

In general second order plant is characterized by the open loop transfer function,

$$G_p(s) = \frac{K}{s^2 + 2\xi^{ol} w_n{}^{ol} s + \left(w_n^{ol}\right)^2} \tag{3.24}$$

Where,

ξ^{ol} : Damping ratio of the second order (open loop) plant

$w_n{}^{ol}$: Natural frequency of the second order (open loop) plant

k: Gain of the second order (open loop) plant

Then, the closed loop transfer function becomes,

$$G_{cl}(s) = \frac{G_C(s)G_p(s)}{1 + G_C(s)G_p(s)} \tag{3.25}$$

Substituting for $G_C(s)$ and $G_p(s)$ from Equation (3.23) and (3.24) respectively into Equation (3.25) gives,

$$G_{cl}(s) = \frac{k(k_d s^2 + k_p s + k_i)}{s^3 + \left(2\xi^{ol} w_n{}^{ol} + kk_d\right)s^2 + \left(\left(w_n^{ol}\right)^2 + kk_p\right)s + kk_i} \tag{3.26}$$

From (3.24) it is clear that the open loop plant has two poles at $\left[-\xi^{ol} w_n{}^{ol} \pm j w_n{}^{ol}\sqrt{1-\left(\xi^{ol}\right)^2}\right]$ and from (3.23) is also evident that the PID controller has one pole at origin and also two zeros at $\left[-\frac{k_p}{2k_d} \pm j\sqrt{\frac{k_i}{k_d} - \frac{k_p{}^2}{4k_d{}^2}}\right]$. It is considered that both the plant poles and controller zeros are complex conjugates in the complex s-plane. From (3.26), it is seen that the closed loop system has two zeros and three poles in the complex s-plane and position of the closed loop zeros remains unchanged as in (3.23) while position of the closed loop poles changes depending on the PID controller gains.

For guaranteed pole placement with PID controllers, the closed loop system (3.26) should have one real pole which should be far away from the real part of the other two complex (conjugate) closed loop poles as discussed in Wang et al. [25]. It is also reported in [25] that the contribution of the real pole in the closed loop dynamics becomes insignificant if magnitude of the real closed loop pole be at least 3-5 (let us call this factor as m 'relative dominance') times greater than real part of the complex closed loop poles.

Now, if the desired closed loop performance of a second order system be given as the specifications on ξ^{cl} (Damping ratio of the closed loop system) and ${w_n}^{cl}$ (Natural frequency of the closed loop system) as in [26] one can easily replace the position of the real zero (α) by $(-m\xi^{cl}{w_n}^{cl})$, provided that $\alpha = -m\xi^{cl}{w_n}^{cl}$ is chosen to be large enough with respect to $(-\xi^{cl}{w_n}^{cl})$.

Therefore, With a proper choice of relative dominance (m), the third order closed loop system (3.26) will behave like an almost second order system having the user specified closed loop damping ratio ξ^{cl} (percentage of maximum overshoot) and closed loop natural frequency ${w_n}^{cl}$ (rise time). In this circumstance, the characteristic polynomial is written as:

$$\left(s + m\xi^{cl}{w_n}^{cl}\right)\left(s^2 + 2\xi^{cl}{w_n}^{cl}s + \left(w_n^{ol}\right)^2\right) = 0 \tag{3.27}$$

After multiplication Equation (3.27) is written as,

$$\Rightarrow \left[s^3 + (2+m)\xi^{cl}{w_n}^{cl}s^2 + \left(\left(w_n^{ol}\right)^2 + 2m\left(\xi^{cl}\right)^2\left(w_n^{ol}\right)^2\right)s + m\xi^{cl}\left(w_n^{ol}\right)^3\right] = 0 \tag{3.28}$$

Comparing the coefficients of Equation (3.28) with the denominator of Equation (3.26), the PID controller parameters are given as,

$$\left.\begin{aligned} k_p &= \frac{\left(w_n^{cl}\right)^2 + 2m\left(\xi^{cl}\right)^2\left(w_n^{cl}\right)^2 - \left(w_n^{ol}\right)^2}{k} \\ k_i &= \frac{m\xi^{cl}\left(w_n^{cl}\right)^3}{k} \\ k_d &= \frac{(2+m)\xi^{cl}{w_n}^{cl} - 2\xi^{ol}{w_n}^{ol}}{k} \end{aligned}\right\} \tag{3.29}$$

3.6. Parameters of Buck Converter

In order to test the performance of buck converter the selected parameter of the converter is given in the Table below.

Table 3.1: Parameters of buck converter system

No.	Description	parameter	Value	Unit
1	Input voltage	V_{in}	24	V
2	Output voltage required	V_{ref}	12	V
3	Capacitance	C	14.65	μF
4	Inductance	L	160	μH
5	Load resistance	R	8	Ω
6	Switching frequency	f	100	kHz
7	Duty cycle	D	0.5	-

Since the open loop transfer function of buck converter system is given above by Equation (3.22), so Equating it with Equation (3.24) the unknown open loop system parameters k, ξ^{ol} and ${w_n}^{ol}$ are obtained. Using the parameters given in table above,

$$k = \frac{D}{LC} = 2.5598 * 10^8$$

$${w_n}^{ol} = \sqrt{\frac{1}{LC}} = 2.2626 * 10^4$$

$$\xi^{ol} = \frac{1}{2RC{w_n}^{ol}} = 0.1885$$

Therefore, the open loop transfer function of the system becomes,

$$G_p(s) = \frac{2.56 * 10^8}{s^2 + 8532s + 5.12 * 10^8} \tag{3.30}$$

The unit step response of the open loop system without controller is shown in Figure 3.6 below.

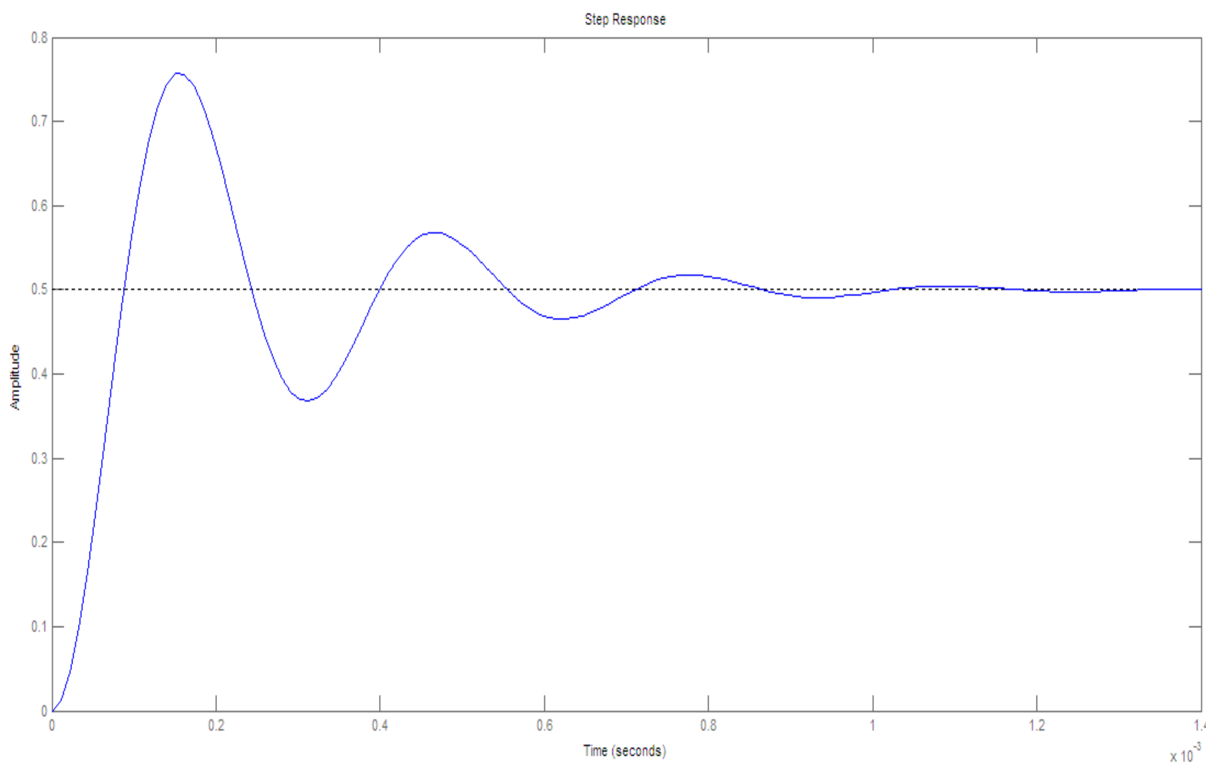

Figure 3.6: Unit step response of open loop system

Then selecting the desired parameters of closed loop system ξ^{cl} , $w_n{}^{cl}$ and using the relations in Equation (3.29), the gains of the PID controller are calculated which produces exact pole placement at desired damping and frequency, provided m is chosen iteratively by checking the accuracy of system response. Here the desired parameters of closed loop system chosen are $\xi^{cl} = 0.95, w_n{}^{cl} = 120 \text{ rad/sec}$ and selecting appropriate value of the relative dominance by trial and error by checking the accuracy of system response, the closed loop transfer function becomes,.

$$G_{cl}(s) = \frac{1.363 * 10^8 s^2 + 3.057 * 10^{10} s + 1.963 * 10^{12}}{s^3 + 1.363 * 10^8 s^2 + 3.109 * 10^{10} s + 1.963 * 10^{12}} \quad (3.31)$$

The unit step response of the closed loop system with PID controller is shown in Figure 3.7.

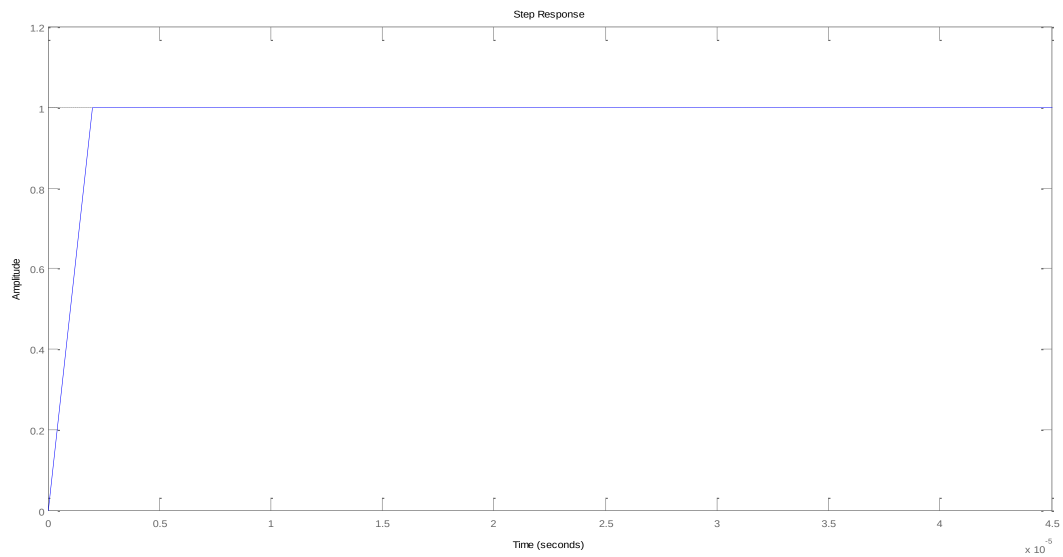

Figure 3.7: Unit step response of closed loop system

3.7. Design of Second Order Sliding Mode Controller for Buck Converter

The state space average of buck converter as derived in Section 3.3 is given as,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \frac{-1}{RC} & \frac{1}{C} \\ \frac{-1}{L} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{d}{L} \end{bmatrix} u \qquad (3.32)$$

Where, $[x_1 \quad x_2] = [v_o \quad i_L]$, $v_o =$ output voltage, $i_L =$ inductor current, d=duty cycle and let the control input of system $u = v_{in}$

In sliding mode control, the controller employs a sliding surface to decide its input states, u to the system. For the sliding mode voltage controller, the switching states, u which corresponds the turning on and off the power converters switch, is decided by the sliding line [27]. Therefore to design the control law, the sliding surface is designed first for the system.

3.7.1. Design of the sliding surface

The selected sliding surface for the DC-DC buck converter to achieve voltage regulation is given as,

$$s = x_1 - x_{ref} \tag{3.33}$$

Where, $x_1 -$ Represent the output voltage (v_o) and $x_{ref} -$ Represent the reference of output voltage/desired output voltage.

The derivative of the sliding surface gives,

$$\dot{s} = \dot{x}_1 - \dot{x}_{ref} \tag{3.34}$$

Since x_{ref} is constant, its derivative $\dot{x}_{ref} = 0$ and substituting for $\dot{x}_1$ from Equation (3.32) into Equation (3.34) gives,

$$\dot{s} = \frac{-1}{RC}x_1 + \frac{1}{C}x_2 \tag{3.35}$$

The second derivative of the sliding surface gives,

$$\ddot{s} = \frac{-1}{RC}\dot{x}_1 + \frac{1}{C}\dot{x}_2 \tag{3.36}$$

Substituting for $\dot{x}_1$ and $\dot{x}_2$ from Equation (3.32) into Equation (3.36) gives,

$$\ddot{s} = \underbrace{\left(\frac{1}{R^2C^2} - \frac{1}{LC}\right)x_1 - \frac{1}{RC^2}x_2}_{f} + \underbrace{\frac{d}{LC}}_{g}u \tag{3.37}$$

$$\ddot{s} = f + gu \tag{3.38}$$

Since the system relative degree is 2, it is more suitable to use second order sliding mode controller to improve the control accuracy.

3.7.2. Design of the control law

The second order sliding mode control algorithm which used here is given with prescribed convergence law and defined as,

$$u = -\alpha \mathrm{sign}\left(\dot{s} + \beta|s|^{1/2}\mathrm{sign}(s)\right) \tag{3.39}$$

Where, $\alpha > 0$ and $\beta > 0$

Buck converter consists of power switch semiconductor devices which are operated as electronic switch. Therefore the control signal for the power switch is 0 and 1. When the control signal $u = 1$, the power switch is on and when the control signal $u = 0$, the power switch is off. Then the control signal given above by Equation (3.39) is rewritten as,

$$u = \frac{1}{2}\left(1 - \mathrm{sign}\left(\dot{s} + \beta|s|^{1/2}\mathrm{sign}(s)\right)\right) \tag{3.40}$$

A sufficient condition for convergence is given as [28],

$$\alpha k_m - \gamma > \frac{\beta^2}{2} \tag{3.41}$$

Assume that,

$$0 < k_m \le g \le k_M \text{ and } |f| \le \gamma \tag{3.42}$$

Where, k_m, k_M and γ all are positive constant (i.e. $k_m, k_M, \gamma > 0$) and f and g are obtained from Equation (3.38). Since the value of $\alpha = 1$ in Equation (3.40), sufficient condition for convergence is given by Equation (3.41) is rewritten as,

$$k_m - \gamma > \frac{\beta^2}{2} \tag{3.43}$$

Therefore, in order to obtain faster response and constant stable output voltage in the case of input voltage and load variation, adjusting the design parameter β using MATLAB/simulink is required.

CHAPTER FOUR

SIMULATION RESULTS AND DISCUSSIONS

4.1. Open Loop Performance of Buck Converter

The open-loop performance of the buck converter is investigated using its MATLAB/Simulink model given in Fig. 4.1. The parameters used to carry out the simulation studies are provided in Table 4.1 below.

Table 4.1: Parameters of buck converter system

No.	Description	parameter	Value	Unit
1	Input voltage	Vin	24	V
2	Reference voltage	Vref	12	V
3	Capacitance	C	14.65	µF
4	Inductance	L	160	µH
5	Load resistance	R	8	Ω
6	Switching frequency	f	100	kHz
7	Duty cycle	D	0.5	-

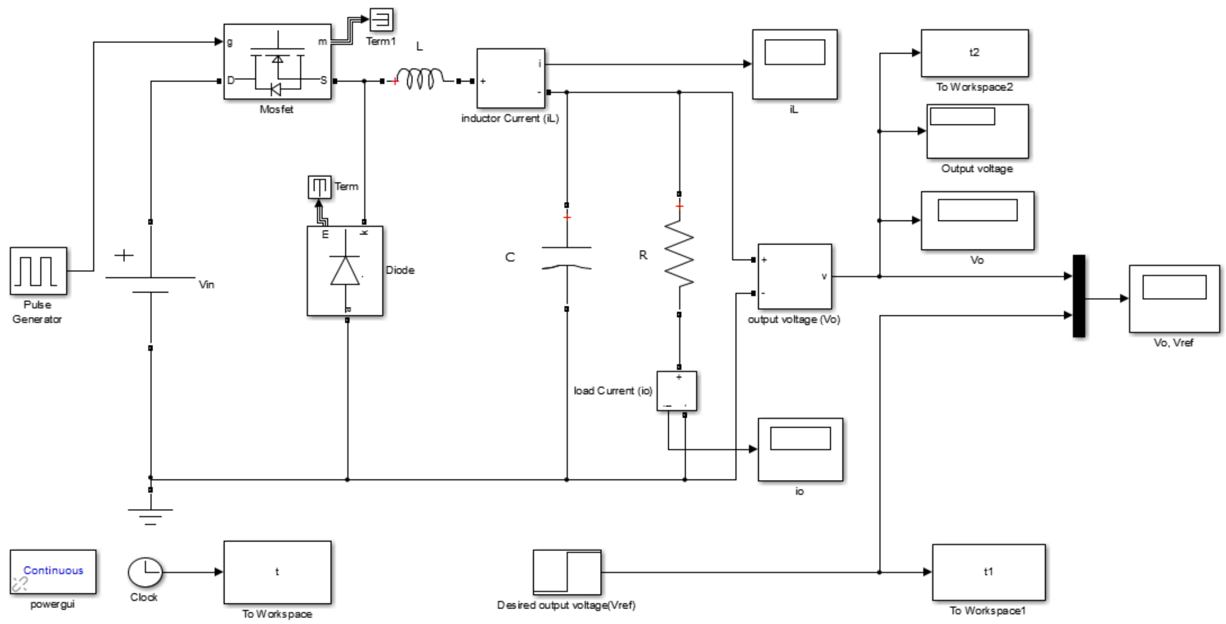

Figure 4.1: MATLAB/Simulink model of open loop buck converter.

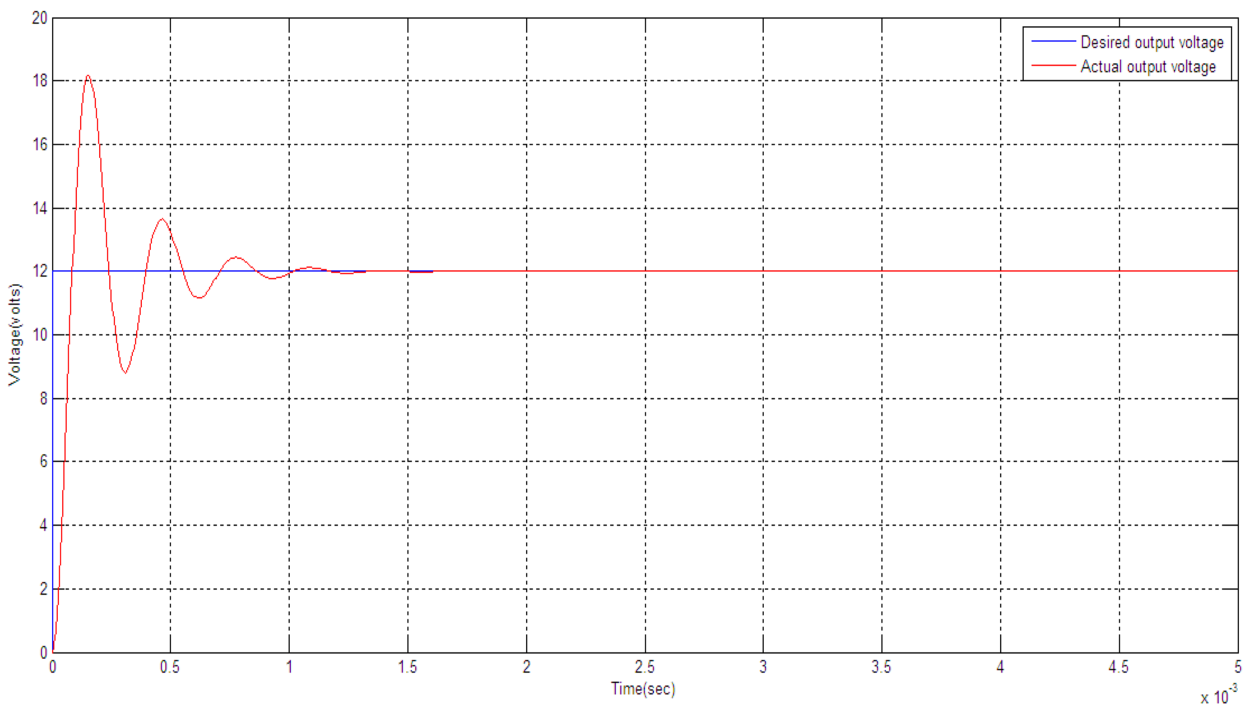

Figure 4.2: Output voltage of open loop buck converter.

The output voltage response obtained using the MATLAB/Simulink is shown in Fig. 4.1. It is observed that the overshoot of output voltage is 51.3% and rise time is 0.05865 milliseconds. The settling time is 0.82635 milliseconds. This large overshoot in output voltage is not required.

4.2. Disturbance of Input Voltage and Load Resistance in Open Loop Buck Converter

In order to test the open-loop performance of DC-DC buck converter under input voltage variation, the input voltage is increased from 24 to 34V and decreased to19V respectively. Also to test the effect load resistance variation, the load resistance is increased from 8 to 13Ω and decreased to 3Ω respectively. The simulation results are shown in Figure 4.3.

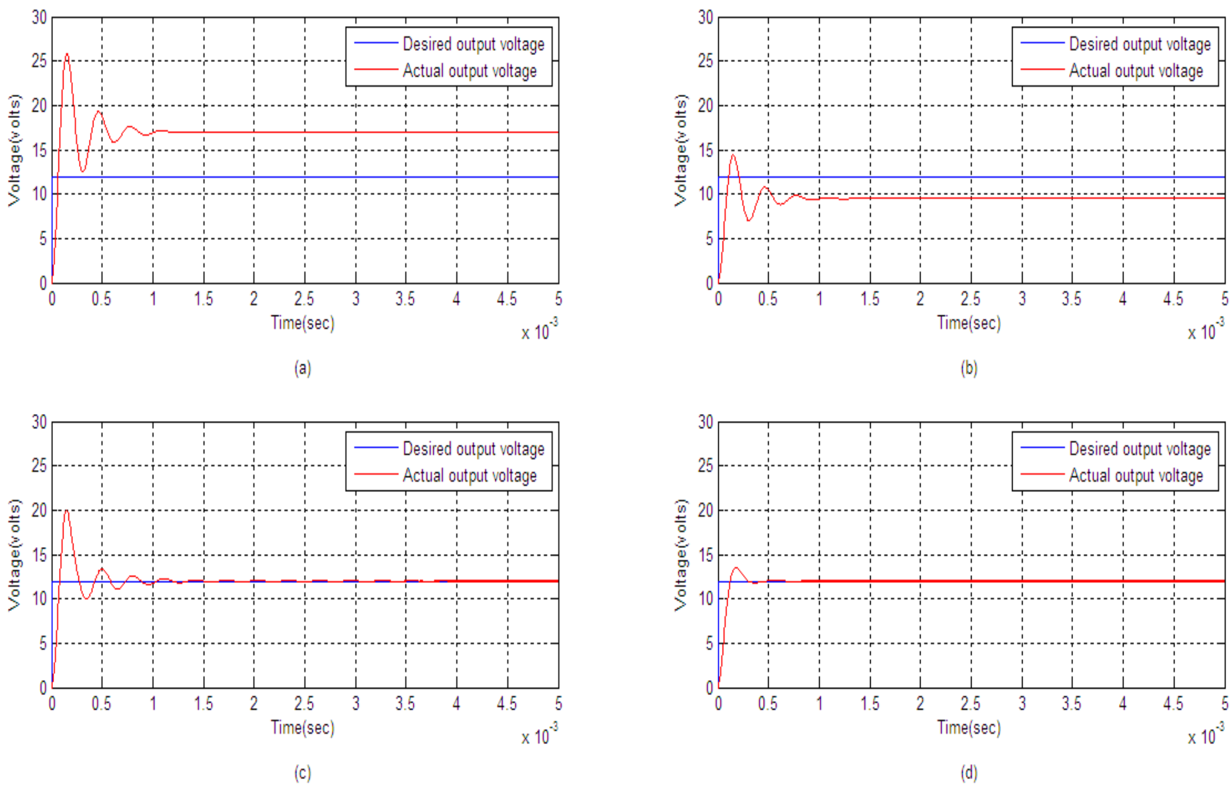

Figure 4.3: Simulation results under input voltage and load resistance variation. (a) Input voltage change from 24 to 34V; (b) For input voltage change from 24 to 19V; (c) For load resistance change from 8 to 13Ω; (d) For load resistance change from 8 to 3Ω.

As seen from simulation result above the output voltage of buck converter alone has high overshoot and oscillating. Also for input voltage variation the output voltage of the converter can't give the desired output voltage (Figure 4.3 (a) and (b)) and in addition if input voltage increases the overshoot is highly increased (Figure4.3 (a)). The decrease in load resistance does not introduce negative effect to achieve the desired output voltage (Figure 4.3 (d)) but the increase in load resistance increase the overshoot (Figure 4.3 (c)). Therefore to overcome the problem different controllers are used.

4.3. Performance of Buck Converter with PID Controller

To demonstrate the performance of DC-DC buck converter with PID controller using in MATLAB/Simulink, the parameters of the converter are same as given in Table 4.1 pp.45. The simulink model of buck converter with PID controller is given in Figure 4.4 below.

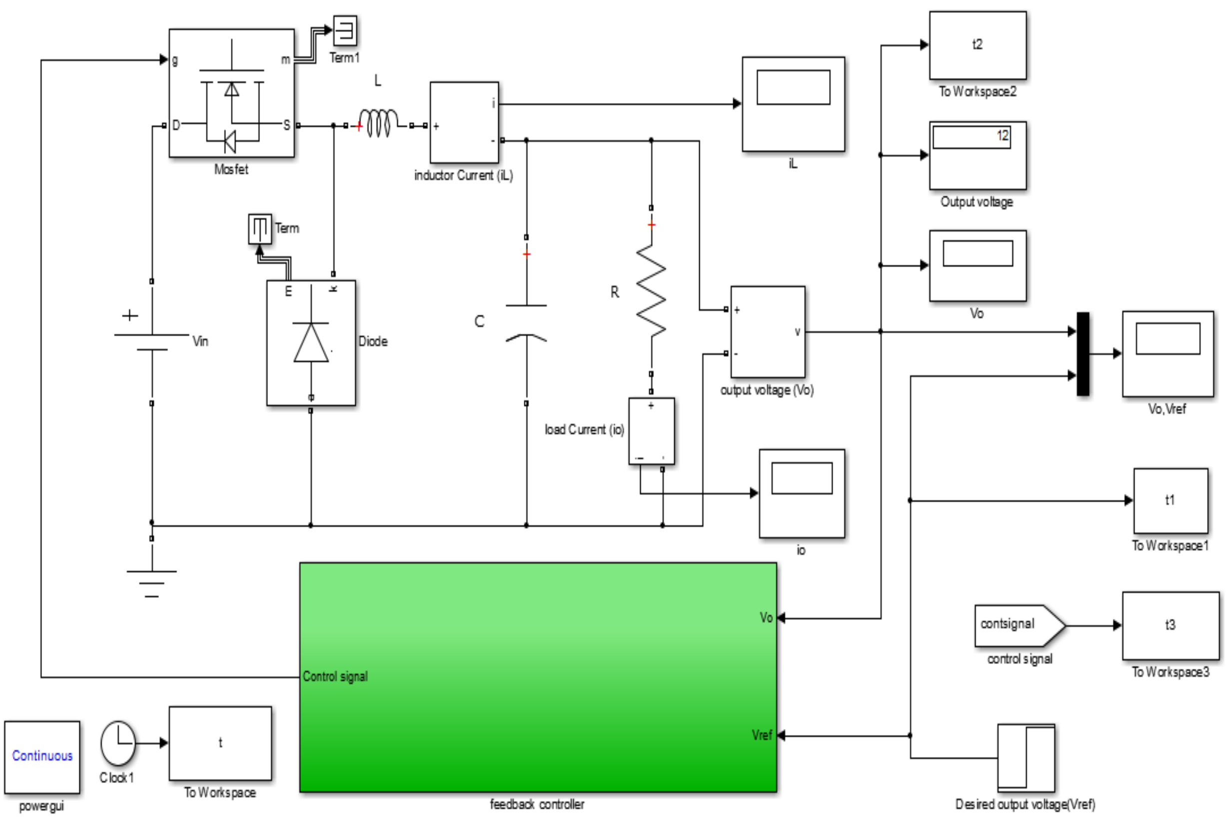

Figure 4.4: MATLAB/Simulink model of buck converter with PID controller.

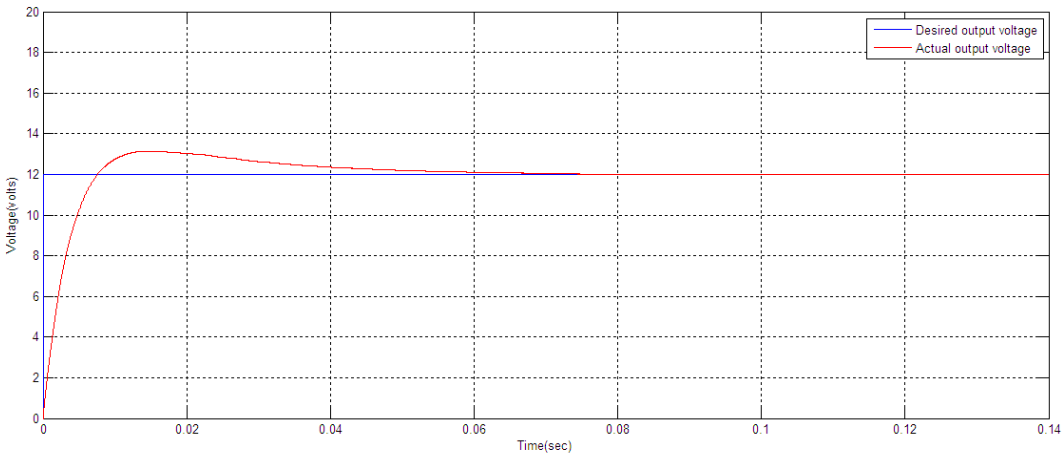

Figure 4.5: Output voltage of buck converter with PID controller.

From the result obtained in Figure 4.5 above; using PID controller for buck converter the overshoot is highly reduced. It reduced from 51.3% to 9.455%. But as it is seen clearly from the figure the rise and settling time are increased. The control signal is given below by Figure 4.6.

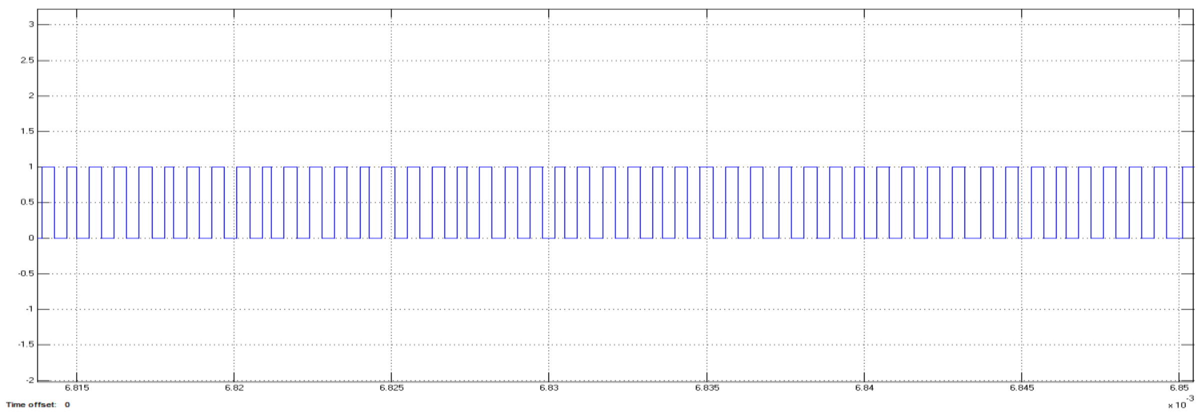

Figure 4.6: Control signal generated by PID controller.

4.4. Performance of Buck Converter with PID Controller Under Disturbance in Input Voltage and Load Resistance

In order to test the performance of DC-DC buck converter with PID controller under input voltage variation, the input voltage is increased from 24 to 34V and decreased to19V respectively. Also to test the effect load resistance variation, the load resistance is increased from 8 to 13Ω and decreased to 3Ω respectively. The simulation results are shown in Figure 4.7.

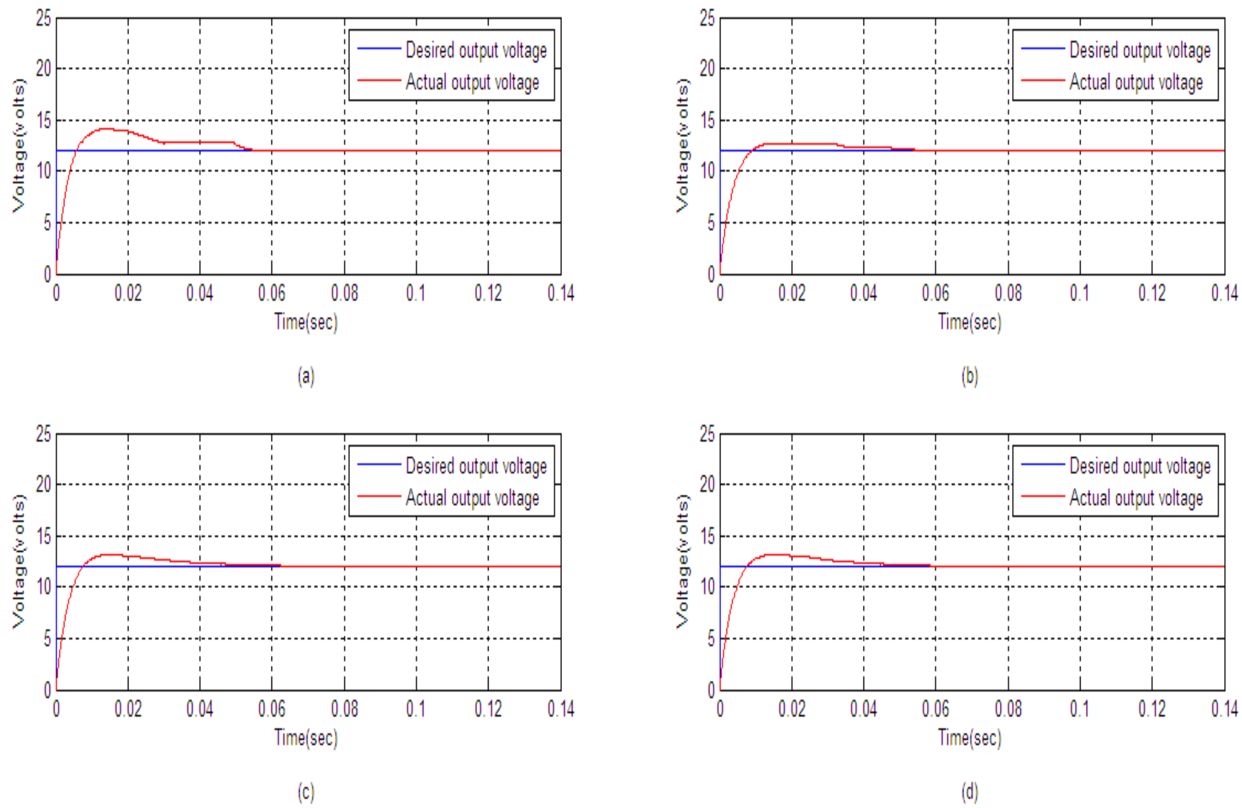

Figure 4.7: Simulation results of buck converter with PID controller under input voltage and load resistance variation. (a) Input voltage change from 24 to 34V; (b) For input voltage change from 24 to 19V; (c) For load resistance change from 8 to 13Ω; (d) For load resistance change from 8 to 3Ω.

The simulation result shows, using PID controller for buck converter the desired output voltage is obtained by reducing the overshoot to a very small value while the settling and rise time are increased (Figure 4.5). The output voltage of the converter does not affected by variation of load resistance to obtain the desired output voltage (Figure 4.7 (c) and (d)). Similarly the output voltage of the converter does not affected by decrease in the input voltage from operating point (Figure 4.7 (b)). But for the increase in the input voltage from operating point the converter does not give constant desired output voltage by using PID controller. The overshoot is increased and the rise and settling time become longer (Figure 4.7 (a)). This is because of the buck converter is nonlinear system. But the linear controllers were designed based on the small signal model of the converter after linearization. The controllers designed based on this technique were not able to respond effectively to changes in operating point. Because the small signal model of the converter change with the variations of the operating point. So that using linear controller the desired output voltage does not obtained under increase in the input voltage from operating point. Therefore to overcome the disadvantage of linear controller, nonlinear controllers are required.

4.5. Performance of Buck Converter with Second Order Sliding Mode Controller

To demonstrate the performance of DC-DC buck converter with second order sliding mode controller in MATLAB/Simulink, the parameters of the converter are same as given in Table 4.1 pp.45. The MATLAB/Simulink model of buck converter with second order sliding mode controller is given in Figure 4.8 below.

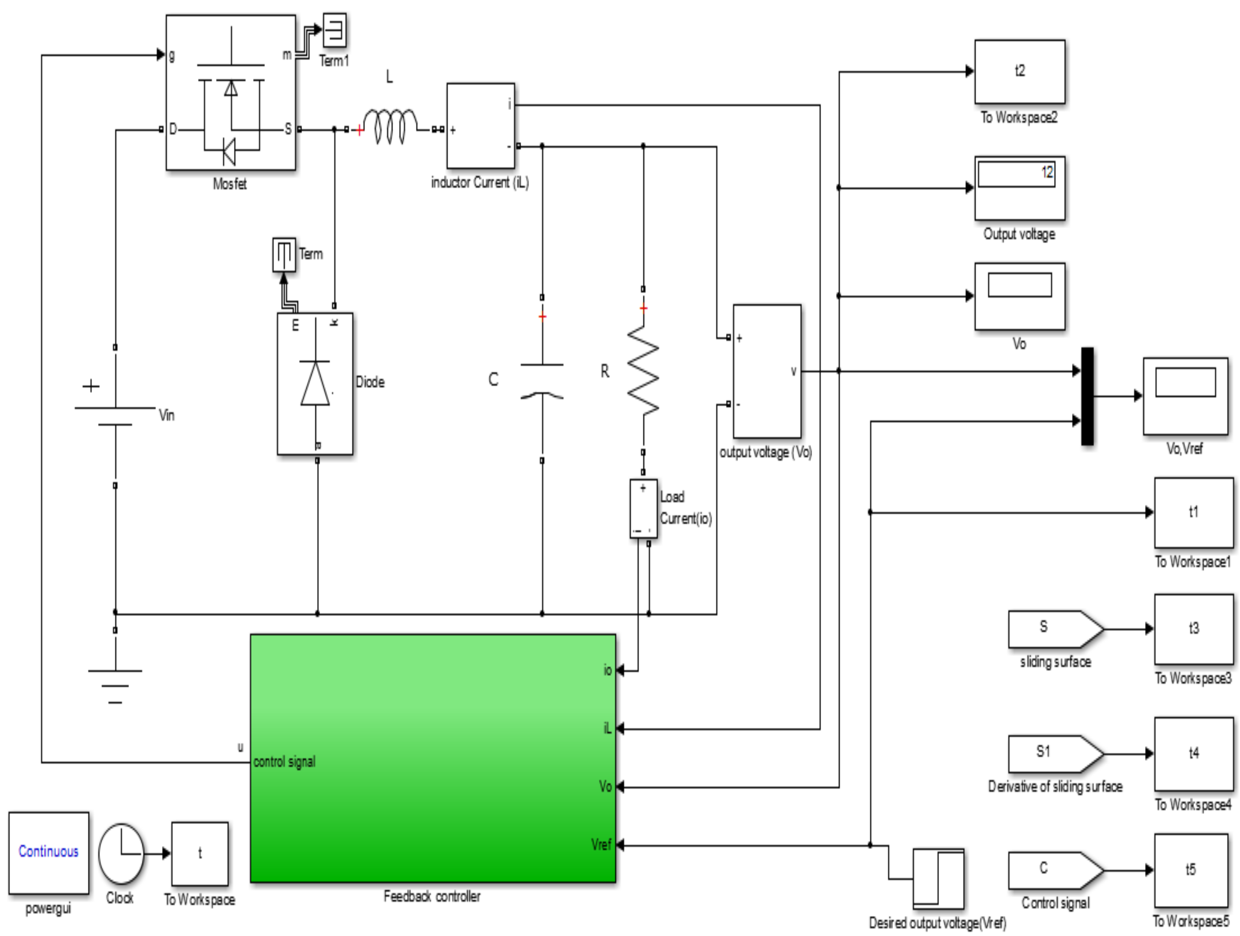

Figure 4.8: Simulink model of buck converter with second order sliding mode controller.

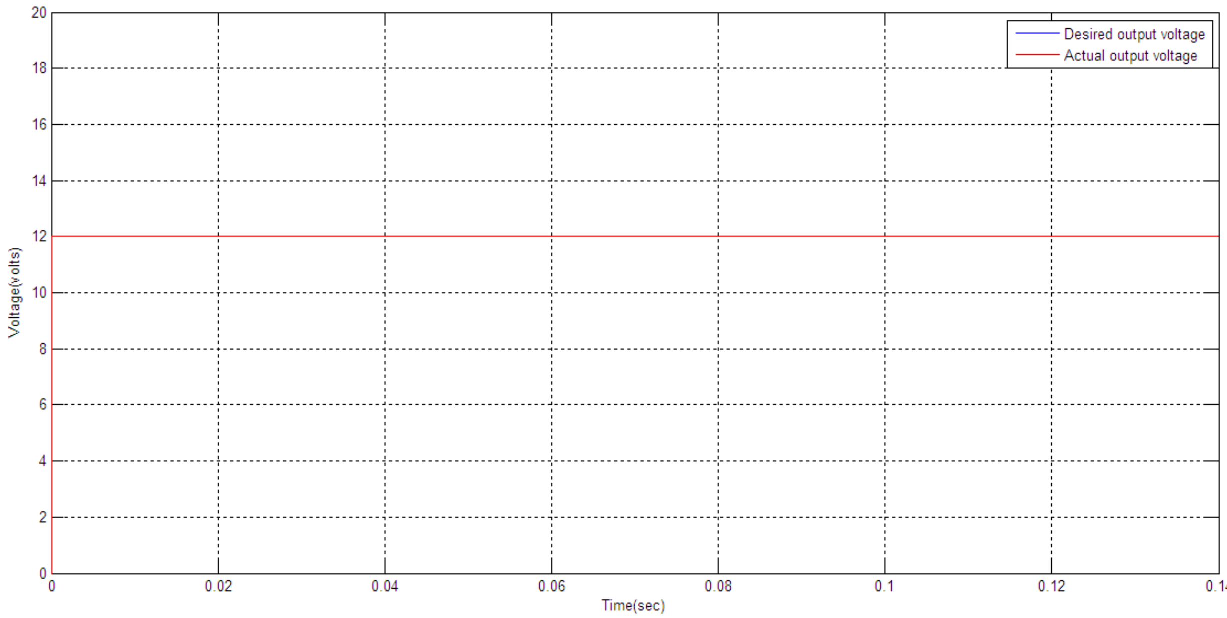

Figure 4.9: Output voltage of buck converter with second order sliding mode controller.

From the result obtained in Figure 4.9 above using second order sliding mode controller for buck converter the overshoot is totally removed. The desired output voltage is obtained at fast rise and settling time.

Figure 4.10, 4.11 and 4.12 below is the plot of control signal, sliding surface (s) and derivative of sliding surface ($\dot{s}$) respectively. The sliding surface is smooth and converge to zero as expected while the derivative of sliding surface ($\dot{s}$) converge to zero and has a little noisy.

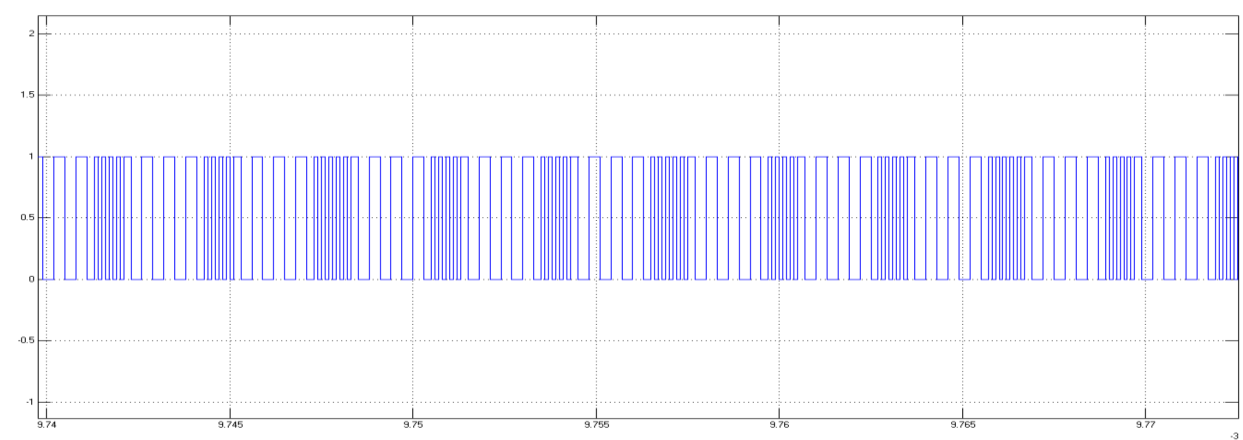

Figure 4.10: Control signal generated by SOSM controller.

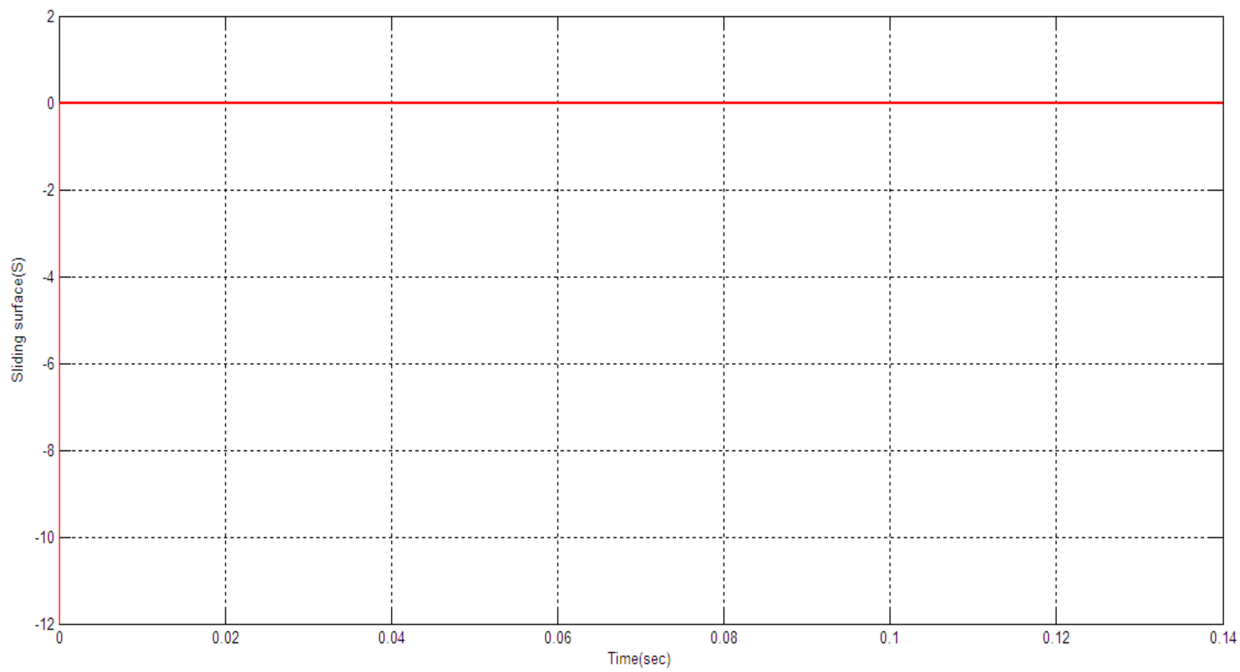

Figure 4.11: The sliding surface.

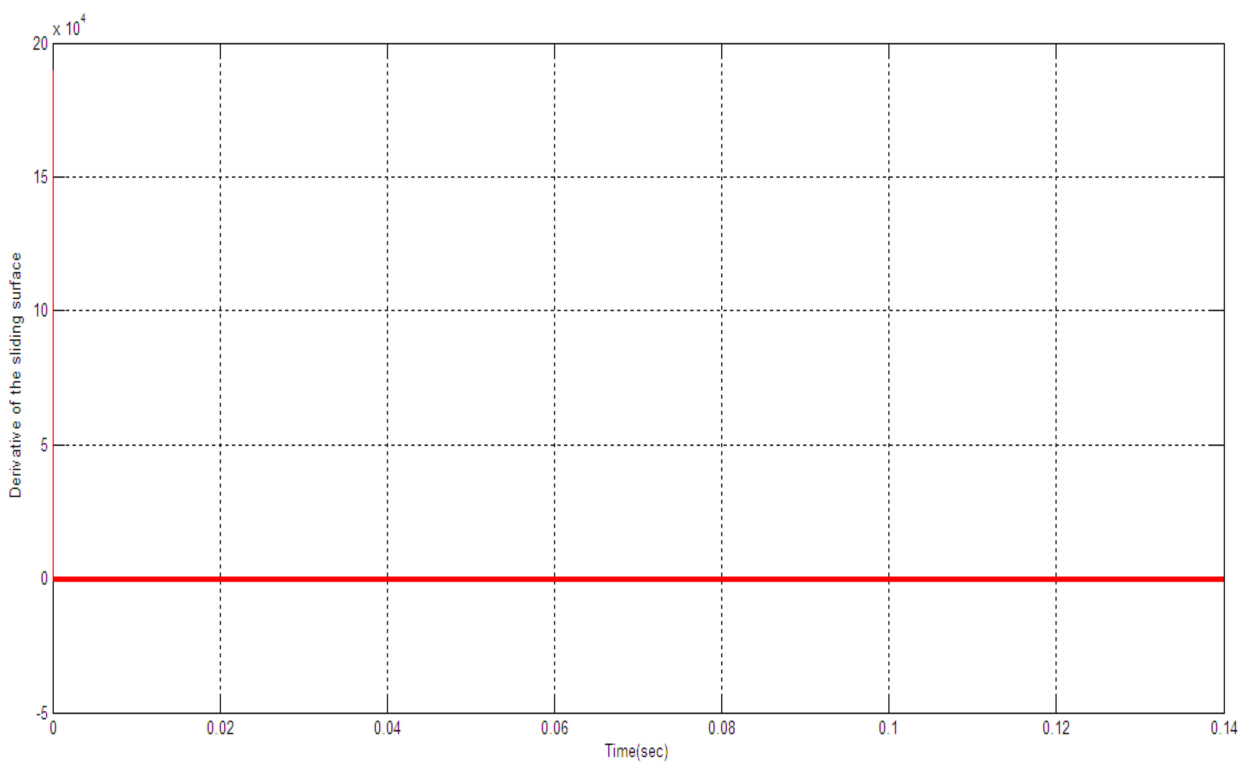

Figure 4.12: The derivative of the sliding surface.

4.6. Performance of Buck Converter with Second Order Sliding Mode Controller Under Disturbance in Input Voltage and Load Resistance

In order to test the performance of DC-DC buck converter with second order sliding mode controller under input voltage variation, the input voltage increased from 24 to 34V and decreased to 19V respectively. Also to test the effect of load resistance variation, the load resistance increased from 8 to 13Ω and decreased to 3Ω respectively. The simulation results are shown in Figure 4.13.

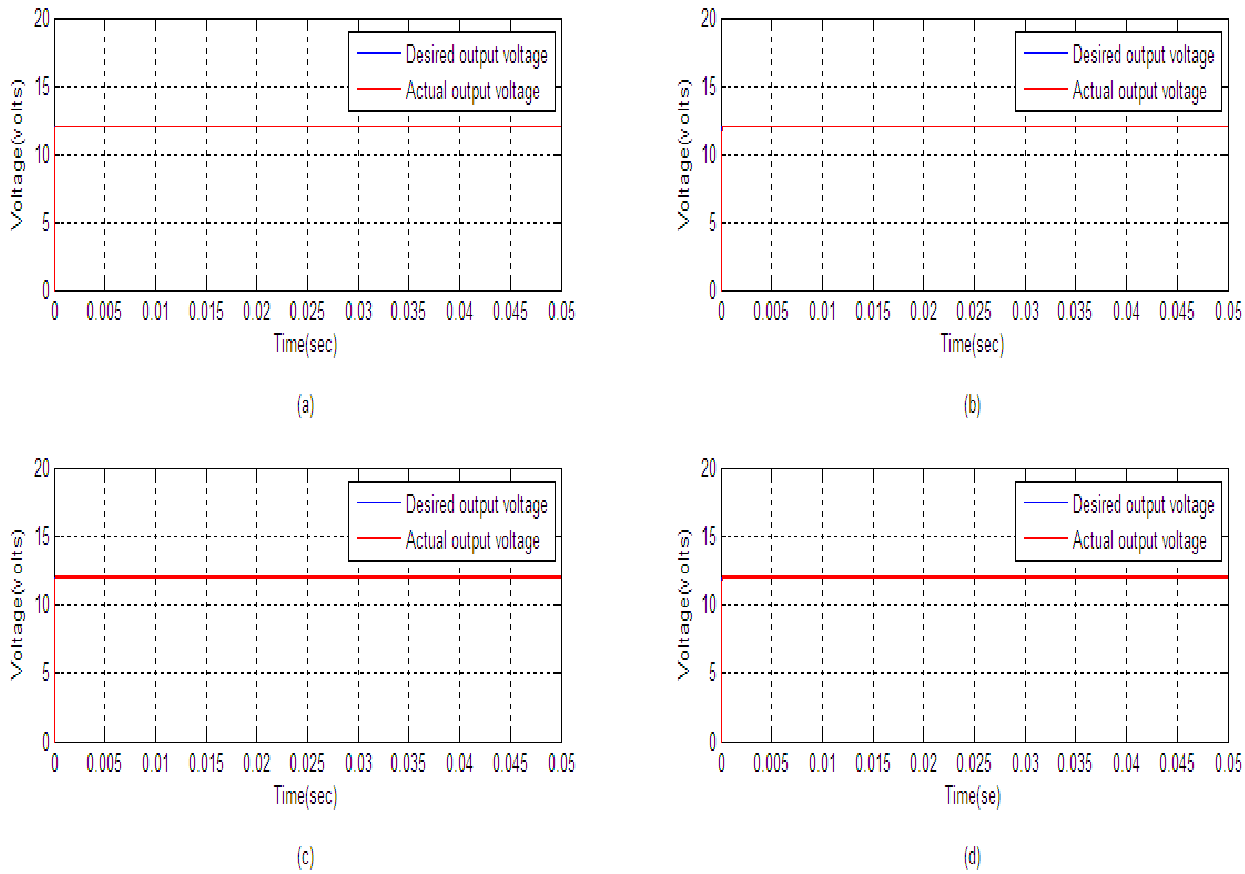

Figure 4.13: Simulation results of buck converter with second order sliding mode controller under input voltage and load resistance variation. (a) Input voltage change from 24 to 34V; (b) For input voltage change from 24 to 19V; (c) For load resistance change from 8 to 13Ω; (d) For load resistance change from 8 to 3Ω.

From the result obtained in Figure 4.13 above using second order sliding mode controller for DC-DC buck converter can overcome input voltage and load resistance variations. The actual output voltage overlaps the desired output voltage. This shows that the output voltage of the converter does not affected by input voltage and load resistance variations to give the desired output voltage. Even for large input voltage variation by using second order sliding mode controller for buck converter, the output voltage of the converter does not affected to give the desired output voltage as shown in Figure 4.14 below. It shows that for different input voltage the actual output voltage of the converter overlaps the desired output voltage which implies that for large input voltage variation the desired output voltage is obtained.

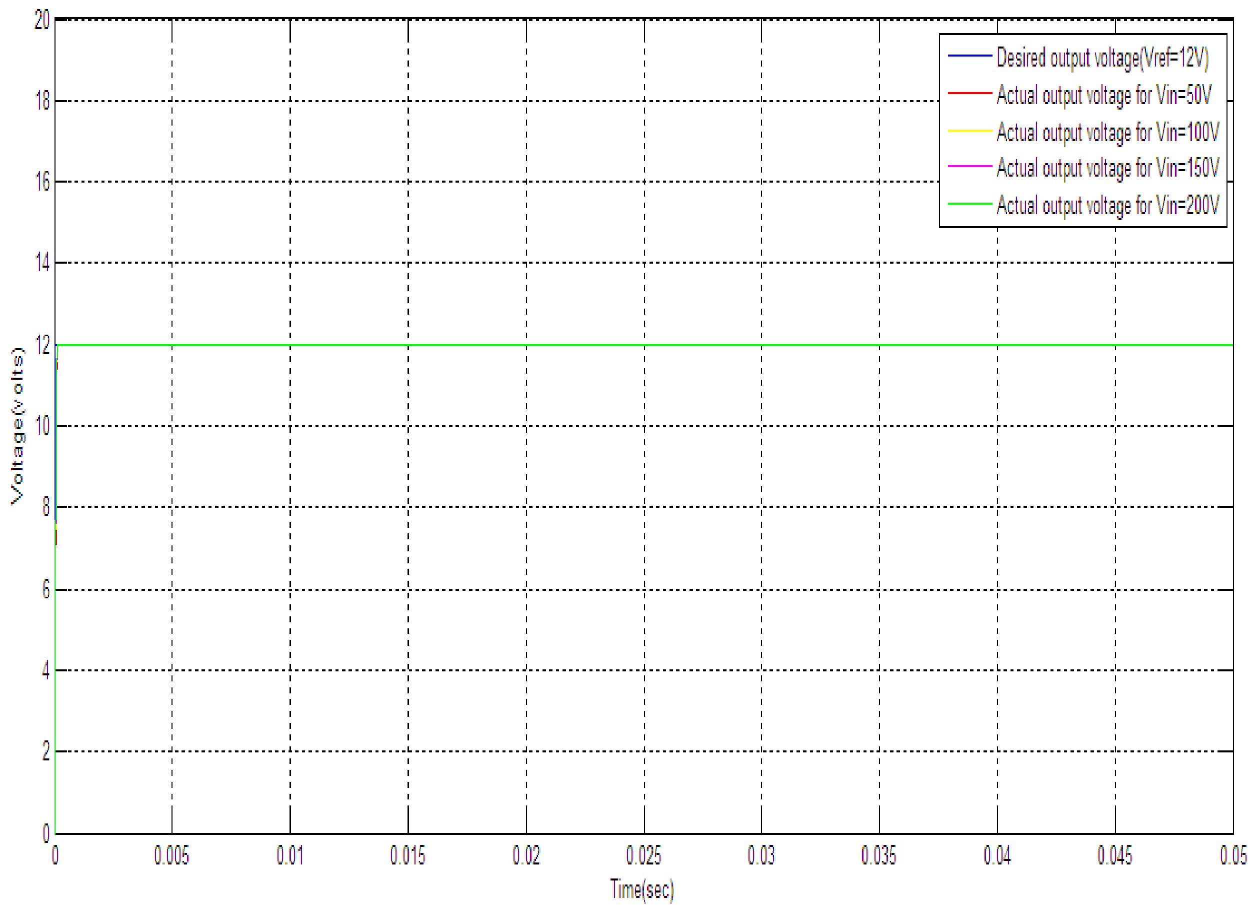

Figure 4.14: Output voltage of buck converter with second order sliding mode controller for different input voltage.

4.7. Performance Comparison of Second Order Sliding Mode Control and PID Control for Buck Converter

Figure 4.15 shows the response of SOSM control and PID control when the buck converter is required to give the desired output voltage of 12V. It is clear that second order sliding mode control gives the desired output voltage more rapidly than PID control. The rise and settling time is very small in the case of SOSM control while PID control has long rise and settling time. As seen in Figure 4.15 (a) using second order sliding mode controller the overshoot is totally removed while using PID controller it is reduced from 51.3% to 9.455%. Figure 4.15 (b) shows SOSM controller is insensitive to input voltage variation. When the input voltage is increased by 10V the actual output voltage is the same as the desired output voltage. But under increase in the input voltage by using PID controller the required output voltage is not obtained. As clearly seen from figure 4.15 (b) when the input voltage is increased by 10V the overshoot is increased by 8.045% beyond the overshoot exist at operating point. From simulation result it is possible to conclude that the performance of SOSM controller is better than PID controller. The transient performance and the output voltage deviation from desired value for input voltage variation is given in Table 4.2.

Table 4.2: Result of transient performance and the output voltage deviation.

controller	Rise time (msec)	Settling time (msec)	Overshoot (%)	Output voltage deviation for input voltage increased by 10V
PID	5.30142	46.5	9.455	-
SOSM	0.073384	0.11	-	-

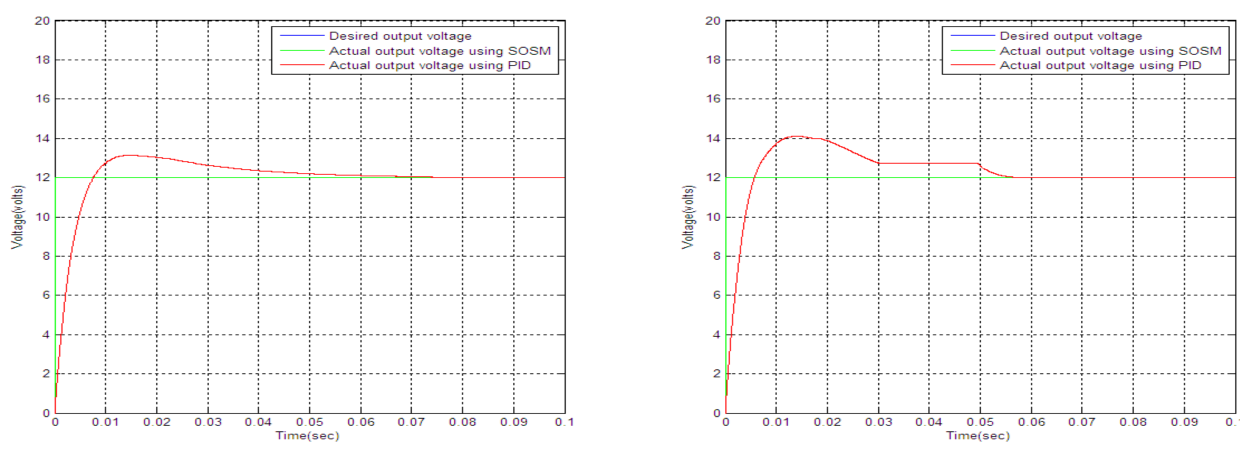

Figure 4.15: (a) Output voltage of buck converter using SOSMC and PID control; (b) Output voltage of buck converter using SOSMC and PID control when input voltage is changed from 24V to 34V.

CHAPTER FIVE

CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

5.1. Conclusions

This thesis presents two different control methods for voltage regulation of DC-DC buck converter. The first control technique is the conventional PID control, while the second control methodology is based on second order sliding mode (SOSM) technique. In particular, a second order sliding-mode controller (SOSMC) based on prescribed convergence algorithm is derived. The performance of DC-DC buck converter was tested using MATLAB/Simulink for both controllers (in transient region, under load variation, and under line variation). The settling and rise time is longer and the overshoot is reduced from 51.3% to 9.455% in the case of PID control. But in the case of second order sliding mode control the settling and rise time is very small (i.e. 0.11msec and 0.073384 msec respectively) and the overshoot is totally removed. The variation of load resistance had no effect in both cases of controller. For the increase in the input voltage PID controller is unable to produce the desired output voltage fastly, while second order sliding mode controller overcome the effect of increase in the input voltage and give the desired output voltage more rapidly. The SOSMC is showing a promising future in the application of DC-DC buck converter because it is a non linear control and can compensate the non linearity of the converter. Also complexities associated with nonlinear mathematical analysis are relatively low and selecting the control parameters and implementing it in MATLAB/Simulink is easier to design a converter with SOSMC than PID control.

5.2. Suggestions for Future Work

Although the theoretical of the voltage regulation of DC-DC buck converter using second order sliding mode control has been validated by computer simulations, it would be useful to investigate its practical implementation.

Another research issue is to test another second order sliding algorithm than the used prescribed convergence algorithm. A comparison of performance of SOSM controllers designed using different algorithm could lead to new insight.

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Appendix A: Parameter Specification of Buck Converter

The converter is designed for low power application and high switching frequency. So that converter parameters value used for MATLAB/simulink is given as follows:

- The switching frequency is 100 kHz.
- The input voltage is taken as 24V.
- The desired output voltage is 12V.
- The load resistance is taken as 8Ω.
- Assume the capacitor ripple voltage is 0.4% of the output voltage.

Based on these parameters it is possible to calculate the minimum inductor that determines the boundary between continuous conduction mode and discontinuous conduction mode and also minimum capacitor to limit ripple voltage below certain value and to minimize overshoot. Using the value given above the duty cycle is given as,

$$\mathrm{D} = \frac{\mathrm{V_o}}{\mathrm{V_{in}}} = \frac{12}{24} = \frac{1}{2} = 0.5$$

The minimum inductor is given as,

$$\mathrm{L_{min}} = \frac{(1-\mathrm{D})}{2\mathrm{f}}\mathrm{R} = \frac{(1-0.5)}{2*10^5} * 8 = 20\mu\mathrm{H}$$

Since the converter is designed for continuous conduction mode only, the value of inductor should be greater than the minimum value. Therefore to ensure continuous conduction mode of operation let the value inductor eight times the minimum inductor, so that $\mathrm{L} = 160\mu\mathrm{H}$.

The minimum capacitor required is given by,

$$\mathrm{C_{min}} = \frac{(1-\mathrm{D})}{8\Delta \mathrm{VLf}^2}\mathrm{V_o} = \frac{(1-0.5)}{8*0.048*160*10^{-6}(10^5)^2} * 12 = 9.7656\mu\mathrm{F}$$

To limit the peak-to-peak value of ripple voltage below a certain value and to minimize the voltage overshoot, the value of capacitor should be greater than the minimum value. For this purpose let the value of capacitor increased by 50% of minimum value of capacitor, so that $\mathrm{C} = 14.65\mu\mathrm{F}$.