



ADDIS ABABA UNIVERSITY
ADDIS ABABA INSTITUTE OF TECHNOLOGY
SCHOOL OF GRADUATE STUDIES
SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

**VEHICLES DEPARTURE HEADWAY MODELING AT SIGNALIZED
INTERSECTIONS: THE CASE OF ADDIS ABABA, ETHIOPIA**

By:

Habtamu Chalachew Goshu

A Thesis Submitted to the School of Graduate Studies in Partial Fulfillment of

the Requirements for the Degree of

Master of Science

In

Road and Transport Engineering

Research Advisor: **Bikila Teklu Wodajo (PhD)**

October, 2018

Addis Ababa, Ethiopia.

ADDIS ABABA UNIVERSITY
ADDIS ABABA INSTITUTE OF TECHNOLOGY
SCHOOL OF GRADUATE STUDIES
SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

VEHICLES DEPARTURE HEADWAY MODELING AT SIGNALIZED INTERSECTIONS:
THE CASE OF ADDIS ABABA, ETHIOPIA

By:

Habtamu Chalachew Goshu

A Thesis Submitted to the School of Graduate Studies in Partial Fulfillment of
the Requirements for the Degree of
Master of Science
In
Road and Transport Engineering

Approved by Board of Examiners:

Dr. Bikila Teklu

Advisor

Signature

Date

Internal Examiner

Signature

Date

External Examiner

Signature

Date

Chair Person

Signature

Date

UNDERTAKING

I, the undersigned, certify that this research work titled “***Vehicles Departure Headway Modeling at Signalized Intersections: The Case of Addis Ababa, Ethiopia***” is my original work performed under the supervision of my research advisor Dr. Bikila Teklu and has not been presented elsewhere for assessment and for a degree in any other university. All sources of materials used for this thesis have also been duly acknowledged.

Signature: _____

Undertaker’s Full Name: Habtamu Chalachew Goshu,

Place of Undertaking: Addis Ababa, Ethiopia.

Date: October, 2018

ABSTRACT

Addis Ababa, capital of Ethiopia, is one of the most congested cities in Ethiopia. Road network of the city has 33 registered signalized intersections. Signalized intersections affect the capacity and level of service of the intersecting roads through controlling the amount of traffic able to use them. So understanding traffic parameters at signalized intersection is essential. Departure headway is one of the microscopic traffic flow parameters at signalized intersections which is used in capacity estimation, level of service evaluation and signal timing. Adopting inaccurate departure headway values could lead to inefficient use of intersection facility. In this study a more accurate way of estimating departure headway is proposed by developing applicable and replicable models. Four signalized intersections were selected for this study. A unique way of data collection was used in the study. The data was collected at the selected locations using Phantom 4 drones which provide a high quality video (100 frame rate per second) and a preferable aerial view of the traffic movement. Data was extracted from the video with maximum error of 3 micro seconds.

Departure headway data for five vehicle types making left turning and through movement were fitted to a probability distribution. The 3-Parameter Weibull distribution fit departure headway data of left turning small bus, left turning large bus, left turning small truck & Large bus making through movement, with different shape, scale, and threshold parameters. The Largest Extreme Value distribution was found to fit departure headway data of Cars and Small bus making through movement with different location and scale parameters. The Log-Logistic distribution fit to departure headway data of large truck and small truck making through movement with different location and scale parameter. Cars making left turning movement follow a certain Gamma distribution and large trucks making left turning movement were found to follow lognormal distribution. Regression model was developed using multiple linear regression technique to predict departure headway. From the regression analysis vehicle type, Movement type, Lateral position of the vehicle, Approach Grade, Lane width, and Green time were identified as factors affecting departure headway and found to statistically significantly predict departure headway. The Developed models are applicable for signalized intersections in Addis Ababa. Interestingly, the developed regression model can also be used as a dynamic PCE value generator for various scenarios. PCE values can be computed by dividing estimated headway values of different vehicle classes to estimated headway values of a passenger car.

Key Words: Departure Headway, Model, Probability Distribution, and Signalized Intersections.

ACKNOWLEDGMENT

First I would like to thank God for being with me all the way from the beginning. I am highly indebted to my research advisor **Dr. Bikila Teklu** for his genuine guidance, valuable comments and help me achieve my goal in this research.

I have a heartfelt appreciation to my employer, **Etoile General Contractors Plc.**, for their unconditional support throughout the whole period of my graduate study.

It's my pleasure to express a profound gratitude to **Tewodros T/Wold, Lisanework Chalachew, Firehiwot Alebachew, Michael Amare, Gudeta Feysa, Samuel Birhanu, Sahele Aregani, and Maru Kassie** for helping me during the data collection and data extraction stage of this research.

Finally I would like to thank Ethiopian Roads Authority (ERA) for giving me this scholarship.

Dedication

To my mother, Aselefech Demssie Jimulu
(The most Independent, Brave, and Kind Women I know)

Table of Contents

UNDERTAKING.....	i
ABSTRACT.....	ii
ACKNOWLEDGEMENT.....	iii
List of Figures.....	viii
List of Tables.....	ix
CHAPTER ONE. INTRODUCTION.....	1
1.1. General Background	1
1.2. Statement of the problem.....	2
1.3. Research Questions	2
1.3.1. Research questions	2
1.4. Objectives of the study	2
1.4.1. General objectives	3
1.4.2. Specific objectives	3
1.5. Scope of the study and Limitations.....	3
1.6. Research Challenge	3
1.7. Organization of the study	4
CHAPTER TWO. LITERATURE REVIEW.....	5
2.1. Chapter Introduction.....	5
2.2. Traffic flow modeling	5
2.2.1. Macroscopic traffic flow modeling.....	5
2.2.2. Microscopic Traffic flow Modeling.....	6
2.2.3. Homogeneous and Heterogeneous Traffic flow Modeling	6
2.3. Departure Headway of Vehicles	7
2.3.1. Definition of Departure Headway	7
2.3.2. Importance of Departure Headway	7
2.4. Departure Headway Modeling.....	8
2.4.1. Average Departure Headway	8
2.4.2. Departure Headway Probability Distributions Models	10
2.4.2.1. Probability distribution Goodness of fit tests.....	12
2.4.3. Discharge Headway Regression Modeling	14
2.4.3.1. Factors Which Affect Departure Headway	14
2.4.3.2. Regression Parameter Estimation Methods.....	15
2.4.3.3. Regression Goodness of fit tests	16

2.4.4. Simulation/Computer based departure headway models	17
2.5. Study site selection criteria.....	17
CHAPTER THREE. RESEARCH METHODOLOGY	18
3.1. Description of the study area	18
3.2. Research Materials	19
3.2.1. Data Requirement.....	19
3.2.2. Vehicle Classification	20
3.2.3. Sample size	20
3.2.4. Sample Eligibility criteria	21
3.3. Research Methods.....	22
3.3.1. Data collection	22
3.3.1.1.Primary data collection	22
3.3.1.2.Secondary data collection	22
3.3.2. Data Extraction	22
3.3.3. Data Analysis	25
3.3.3.1.Probability distribution modeling	25
3.3.3.2.Departure headway regression modeling	26
CHAPTER FOUR. RESULTS AND DISCUSSION.....	29
4.1. Descriptive Statistics of Departure Headway Data.....	29
4.2. Departure Headway Distribution Modeling.....	34
4.3. Departure Headway Regression Modeling	43
4.3.1. Testing multiple linear regression assumptions	43
4.3.2. Regression Analysis.....	47
CHAPTER FIVE. CONCLUSIONS AND RECOMMENDATIONS	52
5.1. Conclusions.....	52
5.2. Recommendations.....	53
REFERENCES	57
APPENDICE-A	68
APPENDICE-B.....	63
APPENDICE-C.....	68

LIST OF FIGURES

Figure 1-1: Thesis Organization Flow Chart	4
Figure 3-1: Camera View at Lebu Mebrathail intersection	23
Figure 3-2: Camera View at Jemo Michael intersection.....	23
Figure 3-3: Camera View at Gerji Mebrathail intersection	24
Figure 3-4: Camera View at Saris Abo intersection	24
Figure 4-1: Box plot of Departure headway data.....	30
Figure 4-2: Individual plot of departure headway data.....	31
Figure 4-3: Scatter plot: Departure Headway vs. Lane Width.....	32
Figure 4-4: Scatter plot of Departure Headway vs. Approach Grade	32
Figure 4-5: Scatter plot of Departure Headway vs. Green time.....	33
Figure 4-6: Normal probability Plot of CAR (LT)	35
Figure 4-7: Histogram of CAR (LT) with Normal Curve.....	35
Figure 4-8: Probability Plot of CAR (LT) – Lognormal.....	36
Figure 4-9: Probability Plot of CAR (LT) – Gamma.....	37
Figure 4-10: Probability Plot of CAR (LT) – Log logistic	37
Figure 4-11: Fitted Distribution Curve for Car (TH)-Largest Extreme Value.....	39
Figure 4-12: Fitted distribution curves for each combinations of vehicle type and movement type	43
Figure 4-13: Scatter Plot of Residuals vs. Fitted Values	45
Figure 4-14: Scatter Plot of Residuals vs. Fitted Value (Transformed Data)	46
Figure 4-15: Histogram of Residuals with a Normal Curve	47

LIST OF TABLES

Table 3-1: Detailed Description of Study Area.....	18
Table 3-2: Vehicle Classes Used in the study	20
Table 3-3: ERA vehicle classification.....	20
Table 3-4: Sample Size of Departure Headway Data	21
Table 4-1: Descriptive Statistics of departure headway data	29
Table 4-2: Estimated Parameters and AD Test Result.....	38
Table 4-3: Summary of fitted Probability Distributions	39
Table 4-4: Correlation Matrix between Independent Variables.....	44
Table 4-5: Predictor Variables Variance Inflation Factor	44
Table 4-6: Step Wise Regression	49
Table 4-7: Final Regression Output	49

CHAPTER ONE. INTRODUCTION

1.1. General Background

Intersections play a significant role in the operation of transport network. Signalized intersections control the amount of traffic able to use the intersecting roads together with capacity and level of service of the road network. So understanding and characterizing the traffic parameters at signalized intersections is very important. Traffic parameters at signalized intersections include departure headway, saturation flow rate, and capacity.

Departure headway of vehicles is an important parameter at signalized intersections. It is the time gap between consecutive vehicles as vehicles cross stop line at a signalized intersection, after the light turns green (*Jin et al., 2009*). So understanding and characterizing the nature of vehicles departure headway at signalized intersections is crucial since it is used to estimate other different parameters such as saturation flow rate and start up lost time which are in turn to be used in signal timing and performance evaluation.

Due to the practical background, modeling of Discharge Headway of vehicles at signalized intersection attracts a permanent interest of researches. As is intelligible, the determination of distributions of discharge headways among vehicles may lead to a more accurate determination of intersection capacities, which finally results in an economic profit. Indeed, the importance of the topic is noticeable from a high frequency of recent scientific papers dealing with intersection analyses (*Krbalek and Sleis, 2014*). To use inaccurate departure headways may lead to wrong estimations of delays, etc. and thus results in economic losses. Due to its importance, lots of investigations had been carried out in the last forty years to study the statistics of departure headways concerning some external influence factors (*Jin et al., 2009*).

Several researches conducted in the area reveals that departure headway of vehicles in front of a signal differs from one place to another depending on the traffic characteristics of the area. In this study the researcher aims to fill the above mentioned gaps and minimize inaccuracies in capacity estimation and performance evaluation of signalized intersections through developing a more complete and flexible mathematical model and identifying the appropriate distribution model for vehicles departure headway at signalized intersections of Addis Ababa city.

1.2. Statement of the problem

Signalized Intersections play a very important role in the transportation network. Since they form a capacity bottlenecks in urban road network, the parameters at signalized intersections should be better understood and characterized. One of the major parameters at signalized intersection is departure headway which is the most important one because it is used to determine other parameters at signalized intersections such as saturation flow rate and capacity. So adopting inaccurate Departure headway values could lead to non-optimal signal operations, wrong capacity estimation and level of service evaluation. Due to the difference in driver's behavior and traffic flow characteristics from place to place departure headways developed for other cities should not be used for Addis Ababa.

Developing departure headway models which are applicable for Addis Ababa's signalized intersections is important in understanding and characterizing the stochastic nature of vehicles departure process. The models are also essential to quantify vehicles departure headway at Addis Ababa's signalized intersections to be used in capacity estimation, and level of service evaluation.

1.3. Research Questions

1.3.1. Research questions

- ✚ What are the factors which affects vehicles departure headway at signalized intersections of Addis Ababa city?
- ✚ What does the regression model reveal about the effects of each factors on vehicles departure headway at signalized intersections of Addis Ababa city?
- ✚ Which probability distribution are more suitable for the distribution of vehicles departure headway for different combinations of vehicle type and movement type?
- ✚ Does departure headway distributions vary among different combinations of vehicle type and movement type?

1.4. Objectives of the study

The overall objective of this research is to minimize inaccuracies in capacity estimation and performance evaluation of signalized intersections from a study of vehicular headways at some selected multi-lane signalized intersections of Addis Ababa city with heterogeneous traffic flow condition.

1.4.1. General objectives

The general objective of this research is to characterize and understand the nature of vehicles departure headway at signalized intersections and to develop applicable departure headway models for signalized intersections of Addis Ababa city.

1.4.2. Specific objectives

The following are the specific objectives of this research:-

- ✚ To determine factors which affect vehicles departure headway at signalized intersections of Addis Ababa city.
- ✚ To develop a mathematical model for vehicles departure headway at signalized intersections of Addis Ababa city using multiple linear regression.
- ✚ To fit probability distributions to vehicles departure headway for different combinations of vehicle type and movement type.
- ✚ To determine variability of departure headway distributions among different combinations of vehicle types and movement types.

1.5. Scope of the study and Limitations

The scope of the research covers vehicles saturation departure headway modeling (Regression and Distribution model) for signalized intersections from a study of some selected multi-lane signalized intersections in Addis Ababa city. In this study the effects of moisture, curvature and pavement distress on departure headway of vehicles is not considered.

1.6. Research Challenge

The following were some of the challenges faced during pre-data collection, data collection, and data extraction stage.

- ✚ During pre-data collection stage the major challenge was getting permission from authorities to use drones for data collection purposes.
- ✚ The other challenge encountered was capturing a continuous video data for two and half hour using drones. Fully charged drone battery is capable of powering the drone for only 18 minutes maximum making it challenging to capture uninterrupted data.
- ✚ Due to lack of software package for the extraction of headway data from recorded video the researcher had no choice but to extract the data manually.

1.7. Organization of the study

This study is organized in five chapters. Chapter one describes the general background of the study, statement of the problem, challenges encountered during the study, hypothesis and research question, the general and specific objectives of the study, and scope of the research. Chapter two deals with literature review including traffic flow modeling (macroscopic, microscopic, homogeneous, and heterogeneous), definition of departure headway, importance of departure headway, and departure headway modeling. Chapter three describes research methods and materials including description of the study area, data analysis methodology, sample size determination, study design, data collection methodology, and data extraction methodology. Chapter four deals with analysis and discussion, detailed data analysis and discussion of results is presented in this chapter. The last chapter of the study is conclusion and recommendation main findings of the study along with recommendation for future studies are presented in this chapter.

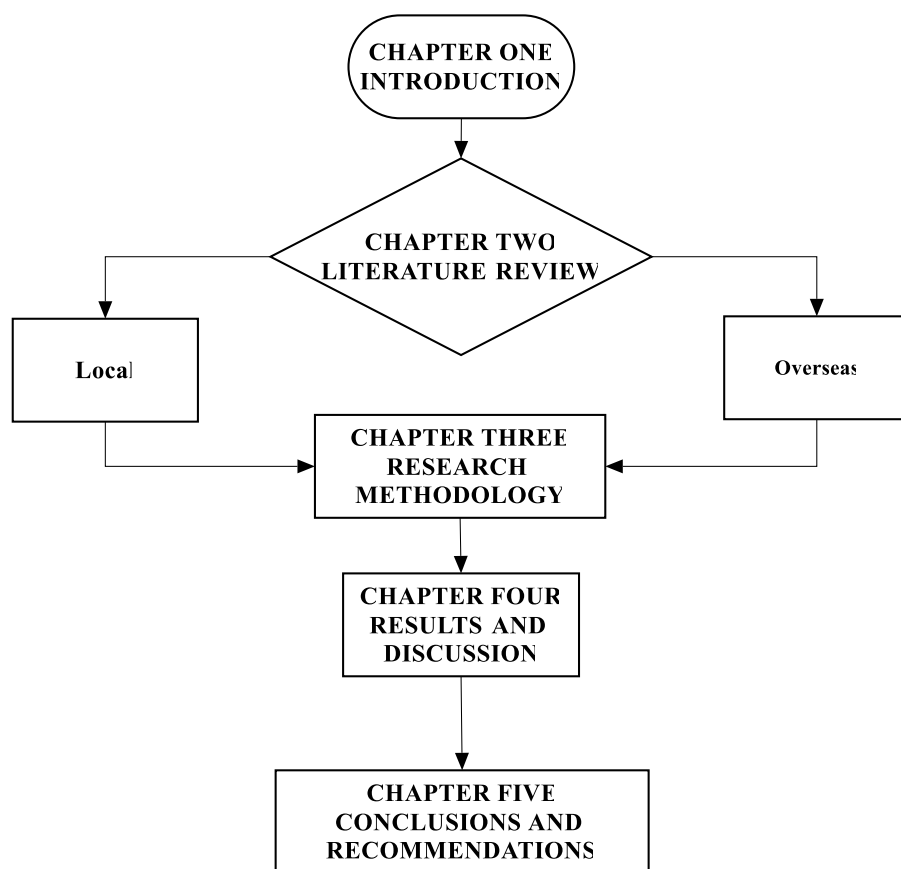


Figure 1-1: Thesis Organization Flow Chart

CHAPTER TWO. LITERATURE REVIEW

2.1. Chapter Introduction

Several literatures which are appropriate for this research are reviewed in this chapter. **Section 2.2** discusses Traffic flow modeling, Historical Background of traffic flow modeling, types of traffic flow modeling including Macroscopic, Macroscopic, Heterogeneous, and homogeneous traffic flow modeling. In **Section 2.3** the general definition of departure headway and importance of departure headway are discussed. **Section 2.4** presents different methods of departure headway modeling along with Goodness of fit tests of the models, Parameter estimation techniques, and factors affecting departure headway of vehicles. In the last section of this chapter, **section 2.5**, Study site selection criteria's are discussed.

2.2. Traffic flow modeling

Researchers have been studying traffic since the beginning of 20th century. Due to the increase in mobility demand and number of Automobiles, our roads began to face congestion problems. Several researches have been conducted to model this traffic phenomenon in between 1920 to 1950. And those models became a study ground for this new research field. The constant change in human behavior and driving characteristics makes traffic flow modeling a very difficult task though it is possible to model the overall traffic phenomena with a certain level of accuracy. Researchers nowadays use two types of traffic flow modeling, Microscopic traffic flow modeling and macroscopic traffic flow modeling (**Scott miller, 2011**).

2.2.1. Macroscopic traffic flow modeling

Macroscopic Traffic flow modeling deals with the general or average aspect of traffic flow. The first traffic flow model to be developed by researchers was macroscopic model in the 1950's and the model was based on average behavior rather than individual vehicles. The advantages of macroscopic traffic flow modeling are simplicity of its calculations and have to deal with lesser parameters than microscopic models. Although macroscopic modeling has the above advantages, it cannot incorporate specific details in the traffic stream that can be modeled with microscopic models (**Scott miller, 2011**).

➤ Macroscopic Traffic Stream Parameters

- ✓ Volume or rate of flow
- ✓ Speed
- ✓ Density/concentration

2.2.2. Microscopic Traffic flow Modeling

Microscopic traffic flow modeling deals with or describes behavior of individual vehicles in a stream (**Scott miller, 2011**). Microscopic traffic flow variables describe the characteristics of individual drivers intermingling with the adjacent vehicles (**S. Hoogendoorn and V. Knoop, 2013**).

➤ Microscopic traffic stream parameters

- ✓ Speed of individual vehicles
- ✓ Headway
- ✓ Spacing

2.2.3. Homogeneous and Heterogeneous Traffic flow Modeling

The principal traffic flow condition in different countries reveals the existence of mixed traffic where different types of vehicles which vary in physical and operational characteristics. The fact that these vehicles share the same space makes the traffic flow more complicated. As a result modeling heterogeneous traffic flow condition is a little more difficult than modeling homogeneous flow and Different literatures also agree to that.

The Operational characteristics of mixed traffic flow is significantly different from homogeneous traffic flow condition. Heterogeneous traffic is composed of different types of vehicles which vary in a lot of ways. There are motorized, non-motorized, slow moving, and fast moving vehicles which also varies in maneuverability, control, and static & dynamic characteristics. Since the traffic is not isolated by vehicle type and all vehicles use same space, vehicles with small size takes any available gap between large sized vehicles and move in disorganized manner (**Dey et al, 2013**). But Under homogeneous traffic flow condition there is limited interaction between vehicles, also less intrusion from other road users which makes it less difficult to determine departure headway or any other traffic parameter. However in most developing countries the prevailing traffic flow

condition indicates the existence of a heterogeneous or mixed traffic flow where different motorized and non-motorized vehicles use non isolated infrastructure (**Liu et al, 2011**). It has been noted that heterogeneous traffic flow characteristics is affected by geometric features of the roadway, and proportion and properties of type of vehicle in the traffic stream. Driver behavior in heterogeneous traffic flow condition is also significantly different from homogeneous traffic flow condition. Small cars following a truck trailer makes an unusual and unique decision to overpass than in homogeneous traffic (**Khan, S. and Maini, P., 1999**).

2.3. Departure Headway of Vehicles

2.3.1. Definition of Departure Headway

Different researchers have defined vehicles departure headway, also called entering headway or discharge headway, in front of a signal. Basically discharge headway is the time gap between consecutive vehicles departing from a queue after the signal light turns green.

“The vehicle discharge headway can be defined as the difference of passage time between the front or rear bumper of successive vehicles over the stop line. If the front bumper is used, the headway of the subject vehicle depends mainly on the characteristics of the preceding vehicle. For a larger preceding vehicle, a longer headway results for the subject vehicle. Thus, a passenger car following a bus has a longer headway than that of a bus following a passenger car. This condition appears to give misleading results because a passenger car should have a smaller headway than a bus. On the other hand, if the rear bumper is used, the measured headway depends essentially on the length and characteristics of the subject vehicle. Larger vehicles probably result in longer headways.” (**Hung et al, 2003**) Therefore, in this study, the vehicle discharge headway is defined as the difference of passage time between the rear bumpers of successive vehicles over the stop line.

2.3.2. Importance of Departure Headway

Discharge headway have been studied extensively for the last five decades due to its importance in traffic engineering. It is one of the major microscopic traffic stream parameter and is a very critical factor that affects a number of other variables at signalized intersections like safety, level of service, driver behavior, and intersection capacity (**Liu et al, 2011**). It is also a principal parameter in signal timing design especially coordinate strategy (**Tan et al, 2011**).

2.4. Departure Headway Modeling

Departure headway modeling has been an active research area due to its significance in signal timing and intelligent transport system researches (*Jin et al, 2009*).

2.4.1. Average Departure Headway

Average departure headway of vehicles tends to decline as we go towards the back of the queue until it gradually level out and that's where the saturation headway is attained. Different literatures suggested that saturation headway is attained at the fourth or fifth queue position.

According to (*Tan et al, 2011*), Average departure headway tends to decrease as we go backwards through the queue and usually gets constant at the fifth queue position. It was also found that as we move the reference line away from the stop line in the direction of the queue, the effect of queue position will eventually be insignificant. Different other studies also agrees to that. A study from collected data at near side legs of five selected signalized intersections in the city of Rasht reveals that the saturation discharge headway is achieved at the 4th or 5th queued vehicle. (*Bargegol et al, 2015*) studied average departure headways from a collected data at different signalized intersections in the city of Bhubaneswar, India for heterogeneous traffic flow condition. Main finding of the paper shows that the discharge headway of vehicles have gradual compression pattern.

In contrary to the above findings, (*Tang, K. and Nakamura, H., 2007*) claims that three completely different discharge patterns were observed in a study conducted based on seventeen through lanes from ten intersections. It was found that the average headway of the first queuing vehicle were significantly larger than the average headway of the second queuing vehicle at 16 lanes out of the observed 17 lanes. At the observed 8 lanes, the average headway tends to increase starting from the 13th queued vehicle. The third vehicle discharge pattern witnessed was a Constant average headway throughout the entire discharge period. The authors forwarded three potential causes for the observed different discharge pattern, Queue length per cycle, lane volume per cycle, and lane volume per hour.

Although there is a frequently observed common discharge pattern of vehicles from a queue at signalized intersections, the average values of departure headways at different locations could vary

significantly due to the difference in the fundamental traffic flow characteristics and driver behavior of the locations. As a result is not recommended to adopt headway values developed for of a specific city to another city.

A study of average discharge headway of the first queuing vehicle on the basis of data collected in Beijing reported the average discharge headway was found to be much longer than the one observed in other developed countries. Vehicle type and intersection complexity were the major factors which affects the first discharge headway, apart from that disturbances at the beginning of the green would dramatically increase the average headway of the first vehicle. Generally vehicle type and phase number were found to have a positive effect on first discharge headway, lane assignment and lateral position of vehicles also have a significant effect on first discharge headway (*Liu et al, 2011*).

Similarly (*Yin et al, 2011*) compared average departure headway of vehicles at each queue position in Beijing china and Atlanta USA and arrive at the same conclusion except for discharge headways of 3rd to 5th queued vehicles. The collected data at the two locations were found different in two features. The first difference is that the departure headway values of the first two queued vehicles determine the overall traffic delay in china, since there is a pedestrian interference in Chinese traffic flow. Second, departure headway for the 6th vehicle and above could be significantly different depending on weather all the queued vehicles are cleared or not. Unlike Atlanta the queued vehicles cannot be cleared off the intersection with the given green time in Beijing which makes the discharge headway of the 6th vehicle and above significantly different. It is apparent that the discharge headway of the first two queued vehicles in Beijing greatly depends on the startup condition, with or without interference. Entering headway from the 3rd to 5th vehicles at both locations were found to have a similar pattern. The study shows that intelligent traffic control systems used in Atlanta and other developed countries might not Represent Chinese scenario so they would require some calibration.

In a study conducted in Saudi Arabia, it was attempted to study the discharge headway of vehicles at signalized intersections and compare the results with previous research findings. Data was collected at eight selected intersections with two and three numbers of lane. Results of the study reveals that the observed departure headway values are higher than values from other researchers

conducted in other countries. Since the driver behavior and traffic flow condition differ as we go from place to place, it is not recommended to adopt headway values from other countries (***AL-Ghamdi, 1999***).

Average departure headway of vehicles vary not only between countries or towns but also between sites found in the same city. According to (***Gonzalez, C.L, 2006***), Observed average headway at three selected intersections in Monterrey, Mexico was found to be greater than the ones found in America. Also Variability of departure headway was observed between the three sites selected for the study. (***Li, H. and Prevedouros, P., 2002***) studied the effect of headway measurement methodologies on departure headway. Three field measurement methodologies for saturation headway were used. Method one was sampling characteristics of vehicles between from 4th to 12th queue position. The second method was the one recommended by Highway Capacity Manual which uses all queued vehicles as a sample. The third method take account of arrival of vehicles up to 140 ft. from the stop line. The calculated saturation headway using the above methodologies were found to be significantly different from each other.

2.4.2. Departure Headway Probability Distributions Models

Departure headway distribution is important for intelligent traffic control and traffic simulations. Headway distributions on various traffic scenarios could be a critical input for traffic engineers in roadway capacity estimation and evaluation of applied traffic management tactics. Appropriate headway distribution model is of a great importance in traffic flow forecasting. This indicates that discharge headway distribution is a fundamental research for intelligent transportation system (***Wu et al, 2010***).

Characteristics of Vehicular headways on a roadway and signalized intersection are significantly different. So some probability distributions which are used to fit vehicle headway data on a roadway are not suitable for modeling departure headway of vehicles in front of a signal. (***Wu et al, 2010***) suggests that vehicle arrival and departure patterns at roadways and intersections are different. Due to this, some distributions like M3 distribution and negative exponential distributions used to fit headways on the road way are not appropriate for headway data at signalized intersections.

Due to different traffic flow condition, driver behavior, and geometric characteristics, departure headway distributions vary from place to place. Most literatures conducted in the area reveals the variability of headway distributions from site to site.

(Wu et al, 2010) studied departure headway distributions for ordinary large cities from field data collected in Jilin. Data for free flow condition and peak hour was collected. Significant difference was found in headway distribution and statistical properties between extremely large cities and ordinary large cities. The departure headway for free flow morning and afternoon peak hour was found to be different. Log-Laplace distribution model was found to fit free flow morning and the log-logistic distribution model fits peak hour in the afternoon. **(Li, H. and Prevedouros, P., 2002)** put forward that for saturation departure headway of vehicles making a through movement lognormal probability distribution was found to fit the data and for vehicles making left turning movement the shifted lognormal probability distribution fit the data best. Similarly Variability of Distributions of saturation headway at signalized intersections were studied. Results shows that discharge headway distributions repeatedly vary and depart from the traditional theories. Statistical tests revealed that departure headway distributions significantly vary for different locations **(Ranasingle et al, 2017)**.

In contrary to the above results, in a study conducted to develop distribution model for departure headway of vehicles, Negative exponential distribution, Displaced(shift) negative exponential distribution, Erlang Distribution, Pearson type III distribution, Normal distribution, Log-Normal Distribution, Bunched exponential distribution, and double displaced negative exponential distribution were tested to fit the collected departure headway data. But none of them were found to fit the data. Nevertheless, except for the first queued vehicle, each position was found to follow a certain Log-Normal distribution with varying Mean and Variance. Discharge headway values between the first and second queued vehicles are significantly different from the others. The researchers came up with three reasons first, startup time of the first vehicle, second, the time lag of the first vehicle pass the stop line, third, the time lag before the front of the 2nd vehicle touches the stop line **(Jin et al, 2009)**. In contrary to that **(Hao, H. and Ma, W., 2017)** examined Discharge headway distributions from observed field data in Shanghai, China. Shifted Log-Normal distribution was found suitable to capture features of discharge headways at each position in a queue. The shifted Log-Normal distribution is appropriate to fit the departure headway data

between the fifth vehicle in a queue and the last queuing vehicle. Other conflicting result was found by **(Hung et al, 2003)**, in a study of departure headway based on data collected from 26 different sites. The sites were selected in such a way to represent different geometric characteristics. Probability distribution model was developed and Type I Extreme Value distribution was found to be a better fit of departure headway data for each queue position.

2.4.2.1. Probability distribution Goodness of fit tests

Goodness of fit test in distribution fitting is a way of determining whether a data comes from a specified distribution or not **(Anderson, T.W., 2011)**. There are two distribution goodness of fit tests which are frequently used to compare distributions, The Anderson-Darling, and The Kolmogorov-Smirnoff tests.

➤ The Anderson-Darling Test

The Anderson-Darling test was first introduced by T.W. Anderson and D.A. Darling in 1952. Its main purpose was to detect sample departure from normality and was commonly used in engineering fields **(Engmann. S. and Cousineau, D., 2011)**. The Anderson-Darling test is a refined form of the Kolmogorov- Smirnoff statics **(Vose, D., 2010)**.

The Anderson-Darling Test Statistics

$$A_n^2 = \int_{-\infty}^{\infty} |F_n(x) - F(x)|^2 \Psi(x) f(x) dx \dots\dots\dots \text{Eqn-2.1}$$

$$\Psi(x) = \frac{n}{F(x)\{1-F(x)\}} \dots\dots\dots \text{Eqn-2.2}$$

Where:-

n is total number of data points

F(x) is distribution function of the fitted distribution

f(x) is density function of the fitted distribution

F_n(x) is i/n

I is the cumulative rank of the data point.

➤ The Kolmogorov-Smirnoff Test

The Kolmogorov-Smirnoff test was first developed and introduced by Kolmogorov (1933, 1941) and Smirnoff (1939). It compares deviation or vertical distance of empirical distributions from proposed theoretical distributions (**Engmann. S. and Cousineau, D., 2011**). The only criteria of the K-S test is the maximum distance between the cumulative distribution function of the proposed distribution and the observed cumulative distribution of the data (**Vose, D., 2010**).

Kolmogorov-Smirnoff (K-S) test Statistic

$$D_n = \text{Max}[|F_n(x) - F(x)|] \dots\dots\dots \text{Eqn-2.3}$$

Where:-

D_n is known as K-S distance

n is total number of data points

$F(x)$ is Distribution function of fitted distribution

$F_n(x)$ is i/n

I is the cumulative rank of the data point.

According to (**Vose, D., 2010**), the Anderson-Darling test is more dominant over the Kolmogorov-Smirnoff test due to the following reasons.

- ✚ “ $\Psi(x)$ compensates for the increased variance of the vertical distances between distributions.”
- ✚ “ $f(x)$ weights the observed distance by the probability that a value will be generated at that x-value”
- ✚ “The vertical distances are integrated over all values of x to make maximum use of the observed data (the K-S statistic only looks at the maximum vertical distance).”

Generally speaking the Anderson-Darling test is a better goodness of fit measure than the Kolmogorov-Smirnoff test (**Vose, D., 2010**), the result of this study is also supported by (**Razali,**

N.M., and Wah, Y.B., 2011) who concluded that the Anderson-Darling test outperform the Kolmogorov-Smirnoff test.

2.4.3. Discharge Headway Regression Modeling

There are not sufficient researches using a regression approach to model vehicles departure headway. A study which used regression approach to model discharge headway of vehicles was conducted by *(Radhakrishnan, S. and Ramadurai, G., 2015)*. Data was collected at three selected intersections in Chennai, India at the morning and evening peak hour. Five vehicle classes were considered for the study and these are two wheelers, cars and three wheelers, light commercial vehicle, and heavy motor vehicle. In this study departure headway model was developed using multiple linear regression technique. The effect of different variables on departure headway of vehicles under Heterogeneous traffic flow condition was studied. From the results it was apparent that there was a significant difference between departure headway values for heterogeneous and homogeneous traffic flow condition. Among the candidate predictor variables Vehicle type, lateral position of vehicles, and green time were found to statistically affect departure headway of vehicles. According to *(Majeed et al, 2014)*, observed saturation headway at intersections of Alabama is different from the recommended value in highway capacity manual. Due to that the researcher developed an alternative way to estimate saturation headway using multiple linear regression technique. Volume of vehicle per approach, number of through lanes, and effective green time were found to statistically significantly affect saturation headway.

(Bogale, S., 2015) developed a predictive model for saturation flow rate at signalized intersections using multiple linear regression technique. Traffic data were collected at four signalized intersections in Addis Ababa city by using a video camera. The study suggest that lane number, approach grade, Green time, width of exit approach and % of heavy vehicle per phase are factors affecting saturation flow rate.

2.4.3.1. Factors Which Affect Departure Headway

Discharge headway of vehicles is affected by a lot of factors. Six factors affecting discharge headway of vehicles were studied in a research conducted in Lawrence, Kansas. Signal type and time of the day were found to have a slight effect on discharge headway and vehicles in the near

curb lane of an approach have lower discharge headway than vehicles in outside lane. Approaches with low speed posts have higher entering headway. The functional classification of roadway also have a positive influence on discharge headway. Longer queue length have lower entering headway. **(Lee, J. and Chen, R.L., 1986)** Similarly **(Tang, K. and Nakamura, H., 2007)** examined factors which affect the discharge headway of vehicles and found that queue length and lane volume per cycle affects discharge headway. The gradual reduction in saturation headway happens when the queue length exceeds 20pcu per cycle or the lane volume equals 30pcu per cycle, at small intersections, and curb side through lanes. However **(Li, H. and Prevedouros, P., 2002)** believes that Type of movement is more critical in determining saturation headway than the level of saturation. **(Liu et al, 2011)** Claims that Physical characteristics of intersections, vehicle characteristics, and traffic flow characteristics are the principal factors which affect the first departure headway of vehicles. Vehicle type and phase number were found to be directly proportional to first discharge headway. The increase in vehicle size and signal phase, result in increased first discharge headway. There is also a significant difference between the average first discharge headway at exclusive left turn lane, straight through lane, curb side straight lane, off side straight lane, and right turn lane. In other studies, departure headway of vehicles in front of a signal is found to be affected by Queue position, vehicle type, and reaction times of the subject and leading vehicle **(Tong, H.Y. and Hung, W.T, 2002)**.

2.4.3.2. Regression Parameter Estimation Methods

Two parameter estimators, Maximum Likelihood and Ordinary Least Squares which are equivalent for the normal theory regression model, are considered as the best linear unbiased estimators of the underlying population parameter **(Washington et al, 2010)**.

➤ Least Square Estimation

Least square estimation, also called Ordinary least squares, technique is one of frequently used methods for parameter estimation in regression analysis. It presents a method for estimating regression model parameters using the sample data. Least square estimation finds the least or minimum solution of the squared disturbances **(Washington et al, 2010)**.

➤ Maximum Likelihood Estimation

Maximum likelihood estimation is one of the popular and important regression parameter estimation technique. Maximum likelihood technique results in the maximum likelihood estimate (*Washington et al, 2010*).

2.4.3.3. Regression Goodness of fit tests

When performing a regression analysis it is critical to perform goodness of fit tests of the model parameters to determine model sufficiency. Linear regression significance tests determine whether or not there exist a linear relationship between the dependent and predictor variables (*Montgomery, D.C. and Runger, G.C., 2010*).

R² Statistics

R² statistics, Coefficient of Multiple Determination, is one of goodness of fit tests used in multiple linear regression. Its equation is written as:-

$$R^2 = 1 - \frac{SS_E}{SS_T} \dots\dots\dots \text{Eqn-2.4}$$

Where:-

SS_E is sum squared regression error

SS_T is sum squared total error

Since the R² value always increases as we add a predictor variable, we can't tell if the new independent variable actually affect the dependent variable or not (*Montgomery, D.C. and Runger, G.C., 2010*). This is where the Adjusted R² comes in handy. Adjusted R² Statics increases only if the newly added variable reduces the error mean square. For obvious reasons the Adjusted R² is preferred over the R² statistics.

Adjusted R² Statistics.

$$R^2_{Adj} = 1 - \frac{SS_E/(n-p)}{SS_T/(n-1)} \dots\dots\dots \text{Eqn-2.5}$$

Where;-

SS_E is sum squared regression error

SS_T is sum squared total error

n is sample size

p is number of predictors

2.4.4. Simulation/Computer based departure headway models

Some researchers tried to study discharge headways produced by different simulation software's. In a study conducted in Beijing, China Departure headways generated by three popular traffic simulation software's, VISSIM, PARAMICS, and, TransModeler, were examined and it was concluded that departure headways at different queue positions are independent with each other except for headways generated by VISSIM. Headways generated by VISSIM were found deterministically dependent (**Tan et al, 2013**). Due to variation in driver's behavior it is difficult to model the departure headway of individual queued vehicles. (**Tong, H.Y. and Hung, W.T, 2002**) used computer based neural network approach to model discharge headway of vehicles. Validation study result reveals that the model is capable of predicting departure headway data with 12% average absolute error.

2.5. Study site selection criteria

(**Hung et al, 2003**) suggests the following study site selection criteria.

- ✚ “Different geometrical characteristics;
- ✚ Significant proportion of heavy vehicles;
- ✚ Significant proportion of turning vehicles for mixed lanes;
- ✚ No parking; and
- ✚ Insignificant disturbances from bus stops”.

CHAPTER THREE. RESEARCH METHODOLOGY

3.1. Description of the study area

There are 33 signalized intersections currently functioning in Addis Ababa, Ethiopia. Study areas selected for this research are signalized intersections located in Addis Ababa, Ethiopia. Four signalized intersections were selected for this study. Namely, Lebu Mebrathail, Gerji Mebrathail, Jemo Michael, and Saris Abo Signalized intersections. Study site selection was based on the existence of heterogeneous traffic flow condition and the selected signalized intersections are found on heavy vehicle corridor of the city. The following table shows detailed description of the study area.

Table 3-1: Detailed Description of Study Area

Name of Intersection:- Gerji Mebrathail								
Item No	Approach Name	No of Lane	Approach Grade (%)	Green Period	Pavement Distress			
					No Distress	Low	Medium	Sever
1	Jackross-Gergi m/hail	3	-4.02	30	√	-	-	-
2	Megenagna-Gerji m/hail	3	4.16	25	√	-	-	-
3	Goro-gerji m/hail	3	-2.49	40	√	-	-	-
Name of Intersection:- Lebu Mebrathail								
Item No	Approach Name	No of Lane	Approach Grade (%)	Green Period	Pavement Distress			
					No Distress	Low	Medium	Sever
1	German-Lebu	3	2.48	40LT, 73TH	√	-	-	-
2	Kality-Lebu	4	-2.27	40LT, 73TH	√	-	-	-
3	Laphto-lebu	3	1.28	50	√	-	-	-
4	Sebeta-Lebu	3	-0.5	15	√	-	-	-
Name of Intersection:- Jemo Michael								
Item No	Approach Name	No of Lane	Approach Grade (%)	Green Period	Pavement Distress			
					No Distress	Low	Medium	Sever
1	Jemo-Michael	3	2.72	30	√	-	-	-
2	Ayertena-Michael	3	-1.26	40	√	-	-	-
3	Mexico-Michael	3	-0.81	46	√	-	-	-
4	German-Michael	3	1.26	40	√	-	-	-

Name of Intersection:- Saris Abo								
Item No	Approach Name	No of Lane	Approach Grade (%)	Green Period	Pavement Distress			
					No Distress	Low	Medium	Sever
1	Kality-Abo	4	1.56	45	√	-	-	-
2	Bole-Abo	4	2.24	45	√	-	-	-
3	Bulbula-Abo	3	4.98	40	√	-	-	-
4	Saris-Abo	2	-5.84	20	√	-	-	-

See **Appendix A** for detailed description of all intersections in Addis Ababa, Ethiopia.

3.2. Research Materials

According to different studies, departure headway of vehicles is affected by different factors. Intersection geometry, vehicle characteristics, signal characteristics, type of movement, and lateral position of vehicles are suggested as a major factors which affects departure headway. In this study, six variables are selected to capture the effect of the above stated factors. To capture the effect of intersection geometry, lane width and approach grade are considered. Type of vehicles is used to account for the effect of vehicle characteristics. The effect of signal characteristics is taken through green time. Through and left turning movements are considered for the effect of movement type. Middle lane, near curb lane, and near median lane are considered for the effect of lateral position of vehicles.

3.2.1. Data Requirement

Data required for the research are classified in to three categories and these are Traffic related data, Signal related data, and Geometric related data.

Traffic related data

- ✓ Vehicular Headway
- ✓ Vehicle type
- ✓ Movement type
- ✓ Lateral Position of Vehicles

Signal related data

- ✓ Green time

Intersection Geometry related data

- ✓ Lane width

✓ Approach Grade

3.2.2. Vehicle Classification

Five Type of vehicle classes are used for this particular study and these are Cars, small buses, large buses, small trucks, and large trucks. The following table demonstrates the five vehicle types used in this study.

Table 3-2: Vehicle Classes Used in the study

Item	Name	Description
1	Car	Class 1 and 2 in ERA Manual
2	Small Bus	Class 3 in ERA Manual
3	Large Bus	Class 4 in ERA Manual
4	Small Truck	Class 5 and 6 in ERA Manual
5	Large Truck	Class 7,8, and 9 in ERA Manual

Table 3-3: ERA vehicle classification

Class	Type	Axles	Description
1	Car	2	Passenger cars and taxies
2	Pick-up/4-wheel drive	2	Pick-up, minibus, Land Rovers, Land Cruisers
3	Small Bus	2	≤27 seats
4	Bus/Coach	2	>27 seats
5	Small truck	2	≤3.5 tones
6	Medium truck	2 or 3	3.5 - 7.5 tones
7	Large 2-axeled truck	2	>7.5 tones
8	3-axled truck	3	*
9	4-axled truck	4	*
10	5-axled truck	5	*
11	6-axled truck	6	*
12	2-axled truck trailer	2	*
13	3-axled truck trailer	3	*
14	Unknown vehicle types		

3.2.3. Sample size

Sample size determination depends on the selected significance level. The minimum required number of departure headway data for this study is determined as follows.

$$n = \left(\frac{Z_{\sigma/2} \sigma}{E} \right)^2 \dots \dots \dots \text{Eqn} - 3.1$$

Where: - E is margin of error, σ is standard deviation, and Z_C is the Z score

Selection of standard deviation value to be used in this formula should be based on previous studies. According to (**Dey et al, 2013**), standard deviation of Departure headway varies from 0.26 to 0.85 for 5th to 24th queue position so for this study σ of 0.5 is used. For a 95% confidence level, E is 0.05 and the minimum number of headway will be

$$n = \left(\frac{1.96 * 0.5}{0.05} \right)^2$$

$$n = 385 \text{ headways}$$

Although statistically calculated sample size is 385 headways, the minimum number of headway data required for fitting probability distributions is 50 and working with lesser data could lead to inaccurate result (**Maurya et al., 2015**). According to (**Vanvoorhis, C.W. and Morgan, B.L., 2007**), the general rule of thumb for determining sample size for regression analysis is $50+8*m$ where m is number of independent variables but the minimum number of sample should be 50. For this study $m=6$

$$N = 50 + 6*8 = 98$$

A total of 1637 departure headway data is collected for this study. The following table present sample size of departure headway data.

Table 3-4: Sample Size of Departure Headway Data

Item	Vehicle Type	Movement Type	Sample size
1	Small Car	Through	557
		Left Turn	391
2	Small Bus	Through	101
		Left Turn	72
3	Large Bus	Through	88
		Left Turn	60
4	Small Truck	Through	115
		Left Turn	77
5	Large Truck	Through	95
		Left Turn	81
Total			1637

3.2.4. Sample Eligibility criteria

The following samples were not eligible:-

- ✚ Vehicles in between Queue position 1 to 4 (to remove the effect of startup lost time)
- ✚ Vehicles involved in incidents (i.e. crash)
- ✚ Vehicles which are not already queued at the start of green phase.

3.3. Research Methods

3.3.1. Data collection

3.3.1.1. Primary data collection

In this study a unique way of data collection, in Ethiopia, was applied. Drones were used to record video data of traffic movement at the intersections. Phantom 4 drones which have 4k camera quality were positioned to provide a clear aerial view of the traffic movement. The drones were positioned at a distance greater than a hundred meters from the ground. Data collection was conducted at the specified four intersections (Lebu M/Hail, Jemo Michael, Gerji M/Hail, and Saris Abo) on working days, except for Monday and Friday because there exists an exaggerated traffic movement on these days, from 06:30 PM to 09:00 PM in order to capture the morning peak hour at the study locations. The exact dates for data collection was as follows:-

- ✚ At Gerji Mebrathail on Tuesday May, 29, 2018.
- ✚ At Saris Abo on Wednesday May, 30, 2018.
- ✚ At Lebu Mebrathail on Thursday, May, 31, 2018.
- ✚ At Jemo Michael on Tuesday, June, 5, 2018.

3.3.1.2. Secondary data collection

Geometric data (number of lane, Lane width, and Approach Grade) and signal related data (Green time) were collected at the selected study sites using the following specific methodology.

- ✚ *No of lane*: counting divided lane on each approach
- ✚ *Lane width*: measuring with roller meter
- ✚ *Approach grade*: Leveling device Was used to measure the grade at each approach
- ✚ *green time*: Recording at the intersections

3.3.2. Data Extraction

Departure headway data was extracted from the recorded video manually by using video playback technique. In order to reduce error the video was recorded with an aerial view of the traffic movement and a video media player, which supports a hundred frame rate per second was used. The maximum error in data extraction was three microseconds.



Figure 3-1: Camera View at Lebu Mebrathail intersection



Figure 3-2: Camera View at Jemo Michael intersection



Figure 3-3: Camera View at Gerji Mebrathail intersection



Figure 3-4: Camera View at Saris Abo intersection

3.3.3. Data Analysis

3.3.3.1. Probability distribution modeling

In this study the probability distribution model is developed for the collected headway data. Minitab 18 statistical analysis software is used as a major data analysis and manipulation tool. Probability distribution is fitted to departure headway data of each vehicle type and movement type. Before fitting any probability distribution the data is tested for outliers using Grubbs' test and are removed from the analysis. Then the data is tested for normality using Anderson-Darling normality test and for non-normal data other probability distributions are fitted. Probability-Probability (P-P) plots and Anderson-Darling test are considered to test the existing distribution models. The P-P plot compares an empirical cumulative distribution function of a variable with a specific theoretical cumulative distribution function. If the selected variable matches the test distribution, the points cluster around a straight line. The following are some common distributions that are usually used for fitting departure headway data. Normal, Lognormal, 3-Parameter Lognormal, Weibull, 3-Parameter Weibull, Smallest Extreme Value, Largest Extreme Value, Gamma, 3-Parameter Gamma, Logistic, Log-logistic, and 3-Parameter Log-logistic.

- ✚ Grubbs' test with the following Null and Alternate Hypothesis is used to determine outliers.

Ho: - All the data values come from the same normal population

H₁:- Smallest or largest data value is an outlier

- ✚ Anderson-Darling Test with the following Null and Alternate Hypothesis is used to test for normality.

Ho: - Data values do come from the specified probability distribution

H₁:- Data values do not come from the specified probability distribution.

➤ Parameter estimation

Parameter estimation for probability distribution is based on the maximum likelihood (ML) method. Maximum likelihood estimation begins with mathematical expression known as a likelihood function of the sample data. The likelihood of a set of data is the probability of obtaining that particular set of data given the chosen probability model. It provides a consistent approach to parameter estimation problems. This means that maximum likelihood estimates can be developed for a large variety of estimation situations. The method provide us with minimum variance unbiased estimators as the sample size increases.

➤ Goodness of fit test

For continuous distributions, the Anderson-Darling test or the Kolmogorov-Smirnov test is used to determine goodness of fit. The calculation consists of the following steps:

- ✚ Transform the data to a Uniform (0, 1) distribution
- ✚ Sort the transformed data to generate the Order Statistics
- ✚ Calculate the Anderson-Darling or Kolmogorov-Smirnov test statistic
- ✚ Compute the approximate p-value associated with the test statistic

The first two steps are common to both the Anderson-Darling and Kolmogorov-Smirnov tests. The original data are transformed to a Uniform (0, 1) distribution using the transformation. The recommended distribution is the one with the minimum value of the test statistic. However, if the p-value of the recommended distribution is less than 0.05, then the recommended distribution does not provide a close fit to the data. The Anderson-Darling goodness of fit test is used in this study.

3.3.3.2. Departure headway regression modeling

Multiple linear regression is used to model departure headway of vehicles. Minitab 18 statistical analysis software is used as a major data analysis tool. Individual departure headway of vehicles will be regressed against the below listed explanatory variables.

Dependent Variable

- ✚ Individual Vehicles Departure headway (Headway)

Predictor variables

- ✚ Green time (Green)
- ✚ Approach Grade (Grade)
- ✚ Lane width
- ✚ Vehicle Type (CAR, Small Bus, Large Bus, Small Truck, and Large Truck)
- ✚ Lateral position of vehicles (Near Median, Near Curb, & Middle Lane)
- ✚ Type of Movement (Left turn & through)

Vehicle type, Lateral position of vehicles, & Type of Movement will be dummy coded in the regression analysis with the following base categories.

Vehicle Type: - Base category is CAR.

Lateral position of vehicles: - Base category is Near Median.

Type of Movement: - Base category is through.

The general mathematical formulation of the regression model is written as follows:-

$$DH = \beta_0 + \beta_1 * AG + \beta_2 * GT + \beta_3 * LW + \beta_4 * ML + \beta_5 * NC + \beta_6 * LT + \beta_7 * SB + \beta_8 * ST + \beta_9 * LB + \beta_{10} * HT \dots \dots \dots \text{Eqn} - 3.2$$

Where:-

- ✓ DH is Departure Headway
- ✓ $\beta_0, \beta_1, \dots, \beta_{10}$, Are estimated coefficients from the regression analysis
- ✓ AG is approach Grade
- ✓ GT is Green Time
- ✓ LW is Lane Width
- ✓ ML is Middle Lane
- ✓ NC is Near Curb
- ✓ LT is Left Turn
- ✓ SB is Small Bus
- ✓ ST is Small Truck
- ✓ LB is Large Bus
- ✓ HT is Large Truck

➤ Multiple linear regression assumptions

The major part of a regression analysis is checking the assumptions. Before conducting any regression analysis the assumptions should be checked. The following multiple linear regression assumptions are checked for violation before performing the regression analysis.

- ✚ **Assumption 1:-** Linear relationship between dependent variable and each of independent variables: - There should be significant linear relationship between the dependent variable and each of the independent variables. Scatter plot of the each independent variables versus the dependent variable is used to check the assumption.
- ✚ **Assumption 2:-** Multicollinearity (the predictor variables should not be highly correlated with each other). Correlation matrix between each independent variable is used to check the assumption. Coefficients greater than 0.8 indicates that the independent variables are strongly correlated with each other.
- ✚ **Assumption 3:-** Homoscedasticity (residuals have a constant variance along the fitted value). Scatter plot of the residuals versus the fitted value and Breusch-Pagan / Cook-

Weisberg test is used to check the assumption. If the data show homoscedasticity, the data points will be fairly randomly distributed with a fairly even spread of residuals at all predicted values.

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity

Ho: Constant variance

✚ *Assumption 4:-* There should be no significant outlier, high leverage point or highly influential points, which represent observation in the data set. The data points in the data should provide the same amount of information in developing the predictive model. Unusual observations and data points with high residuals are identified and those which are considered as significant outliers, high leverage points, and highly influential points are removed from the analysis.

✚ *Assumption 5:-* Residuals are normally distributed. The residuals (Errors) should be approximately normally distributed. Histogram plot of the residuals with super imposed normal curve is used to check the assumption. If the residuals are normally distributed, histogram plot of the data will look like a bell shaped structure.

➤ Goodness of fit test

The developed model is tested with statistical goodness of fit tests in this case the adjusted R^2 test is conducted. The closer the Adjusted R^2 value to 1 the better the model. Adjusted R^2 test is preferable because $SS_E / - (n-p)$ is the error or residual mean square and $SS_T / - (n-1)$ is a constant, R^2_{adj} will only increase when a variable is added to the model which reduces the error mean square.

$$R^2_{Adj} = 1 - \frac{SS_E / (n - p)}{SS_T / (n - 1)} \dots \dots \dots \text{Eqn} - 3.3$$

Where;-

SS_E is sum squared regression error

SS_T is sum squared total error

n is sample size

p is number of predictors

CHAPTER FOUR. RESULTS AND DISCUSSION

4.1. Descriptive Statistics of Departure Headway Data

1637 departure headway data samples were extracted from the recorded video at the selected signalized intersections which will be used for both Probability Distribution modeling and multiple linear regression modeling. The tabular and graphical descriptive statistics of the collected departure headway data is presented below

Table 4-1: Descriptive Statistics of departure headway data

<i>Variable</i>	<i>Total</i>	<i>Mean</i>	<i>StDev</i>	<i>Minimum</i>	<i>Median</i>	<i>Maximum</i>
<i>CAR (Left Turn)</i>	391	2.0502	0.3462	1.1660	2.0330	3.1000
<i>CAR (Through)</i>	557	1.6472	0.3454	0.9660	1.6000	2.8000
<i>Small Bus (Left Turn)</i>	72	3.0505	0.4521	2.3340	2.9835	4.2330
<i>Small Bus (Through)</i>	101	2.5165	0.5465	1.4660	2.4000	4.1340
<i>Large Bus (Left Turn)</i>	60	4.2803	0.7414	3.1000	4.2165	6.1000
<i>Large Bus (Through)</i>	88	3.5794	0.7566	2.5000	3.3500	5.8670
<i>Small Truck (Left Turn)</i>	77	2.9003	0.5597	2.0000	2.9000	4.2000
<i>Small Truck (Through)</i>	115	2.2496	0.3620	1.2000	2.2000	3.4000
<i>Large Truck (Left Turn)</i>	81	3.8342	0.7487	2.4000	3.8330	6.0330
<i>Large Truck (Through)</i>	95	3.1126	0.6396	1.7000	3.0000	4.9000

As we can see from the above table the mean value of departure headway for left turning car is greater than cars making through movement. This pattern is also observed in all vehicle classes considered in this study. The above observed pattern implies that departure headway of vehicles is affected by type of movement. If we look into mean departure headways of each vehicle class, it indicates that as the size of the vehicle increases the mean departure headway also increases for a specific type of movement. What we have observed in **Table 4-1** can be seen clearly from the below box plot. **Figure 4-1** shows Box plot of departure headway data for each combination of vehicle type and movement type. In the box plot the yellow triangle indicates the mean departure headway value for each combination of vehicle type and movement type.

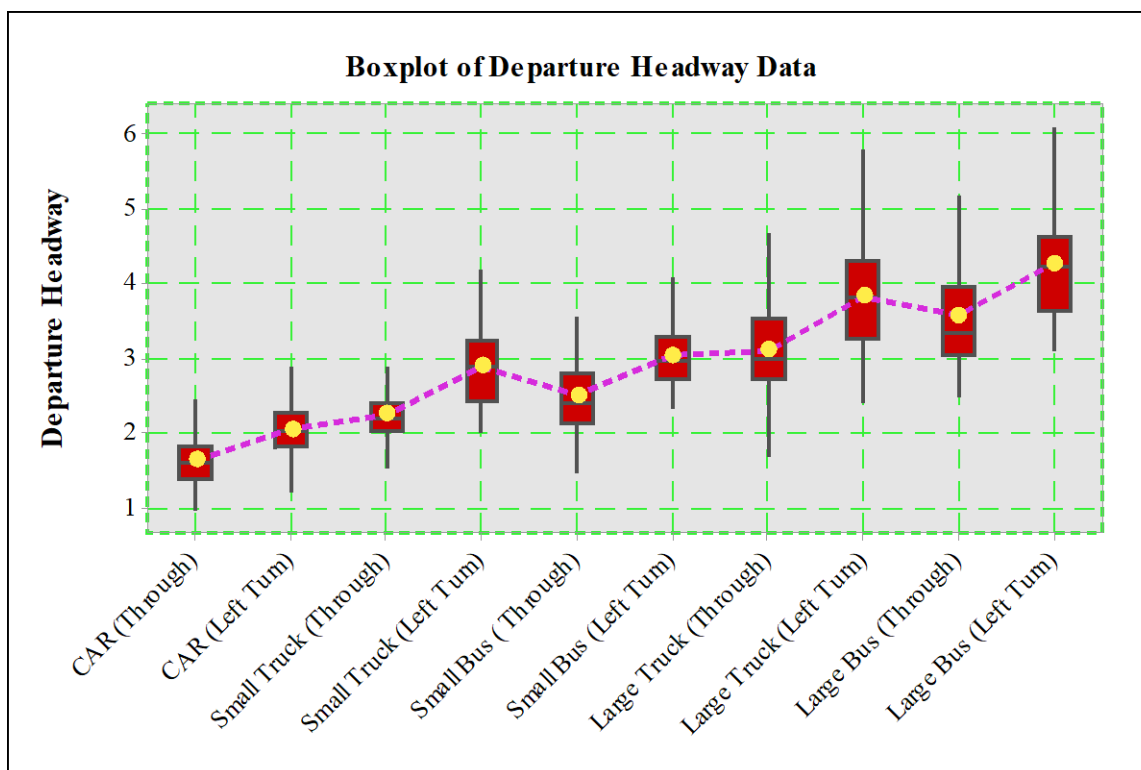


Figure 4-1: Box plot of Departure headway data

If we follow the dotted line connecting the mean headway values, the variability in mean departure headway values over the type of vehicle and type of movement is evident which also shows that departure headway increases as size of vehicle increases and left turning movements have a higher departure headway than through movement. Any potential non-normal distribution of data can also be identified from the box plots. If we look at the box plot of CAR (Through), CAR (left turn), and Small Truck (Through), the data seems to be fairly normally distributed but it has to be confirmed with Anderson-Darling Normality test result. But for Small bus (Left turn), Small bus (through), Small Truck (Left turn), Large Truck (Left turn), Large Bus (Left turn), Large Truck (Through), and Large Bus (Through) the data seems to be skewed to the right (Positively skewed) which implies that the data may not be normally distributed but as stated above it should be confirmed with normality test result. Some data values which are far away, usually 1.5 times the inter-quartile range, from the other data points are usually identified as outliers in box plots. In this case all data points are within 1.5 times the inter-quartile range

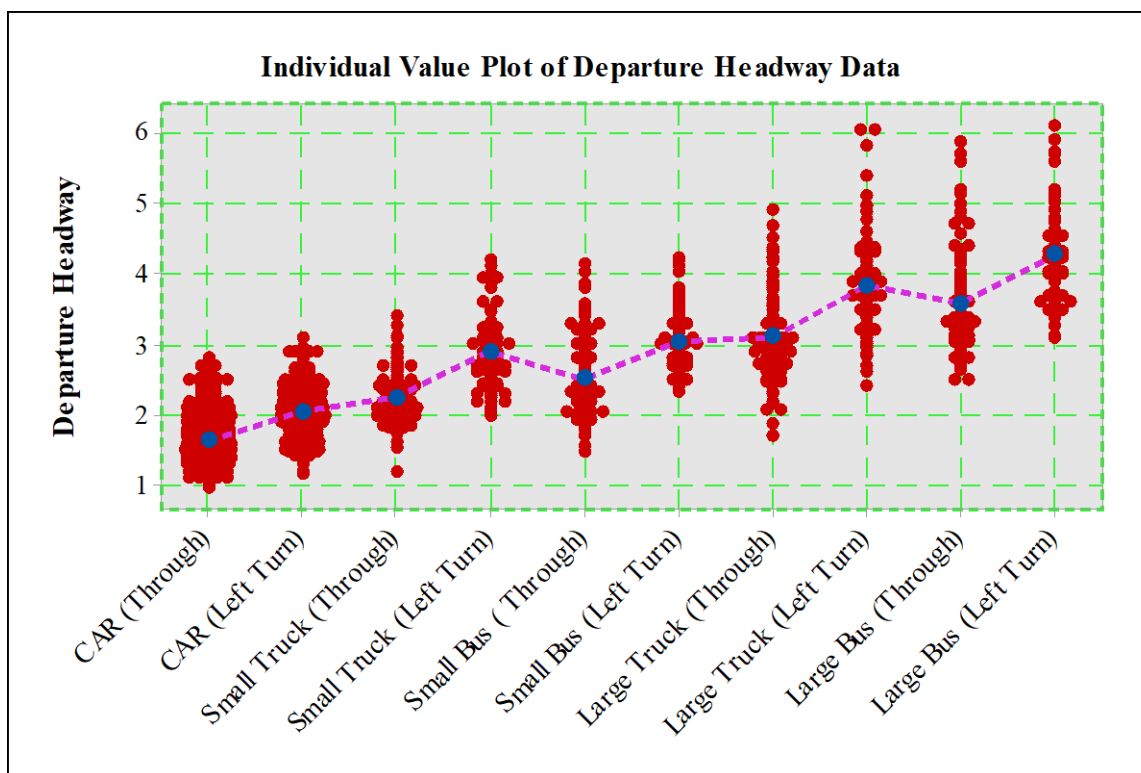


Figure 4-2: Individual plot of departure headway data

The individual plot of departure headway data provided above shows same pattern indicated in **Table 4-1** and **Figure 4-1**. The data points of CAR (Through), CAR (left turn), Small Bus (Left turn), and Small Truck (Through) are relatively clustered around the mean departure headway value than the others which implies that departure headway for the above stated vehicles and movement type is less variable than the others. Small Truck (Left turn) and Small Bus (Through) have a higher spread of data which indicates the existence of higher variability than variables stated earlier. And for Large Truck (Through), Large Truck (Left turn), Large Bus (Through), and Large Bus (Left Turn) the spread of the data is even higher which shows the existence of a higher variability. We can also look at the standard deviation values for each combination of vehicle type and movement type provided on **Table 4-1** to confirm the above observed pattern.

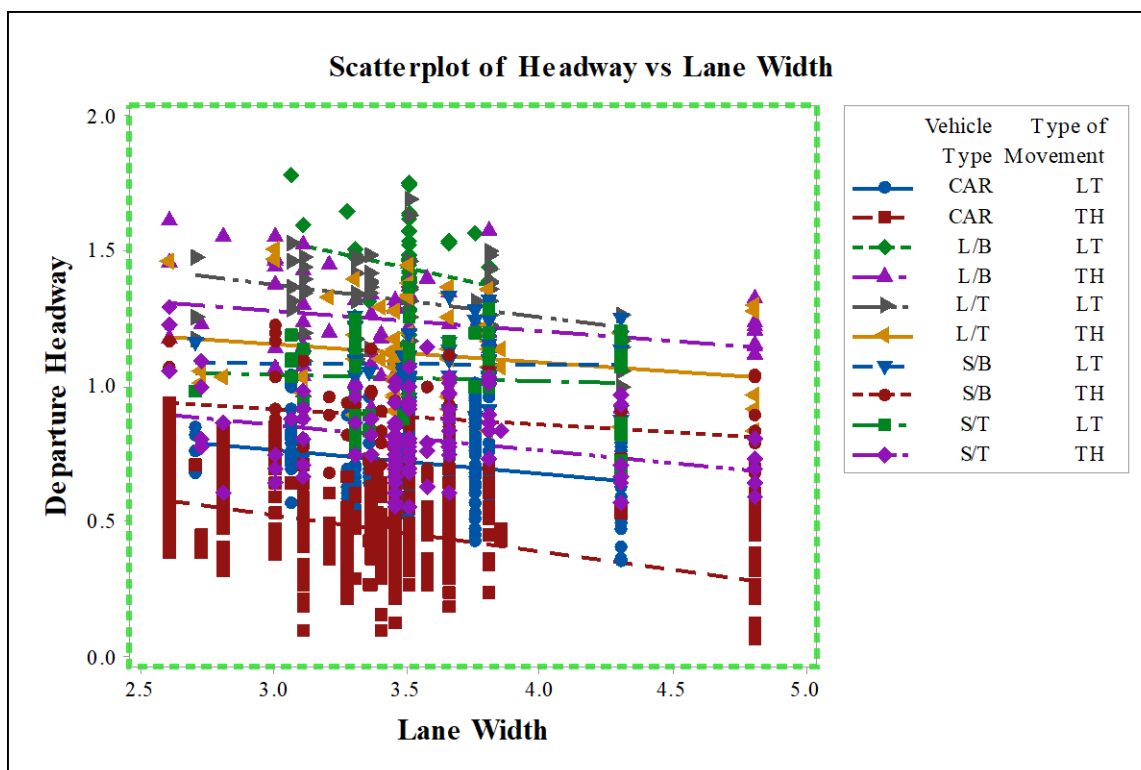


Figure 4-3: Scatter plot: Departure Headway vs. Lane Width

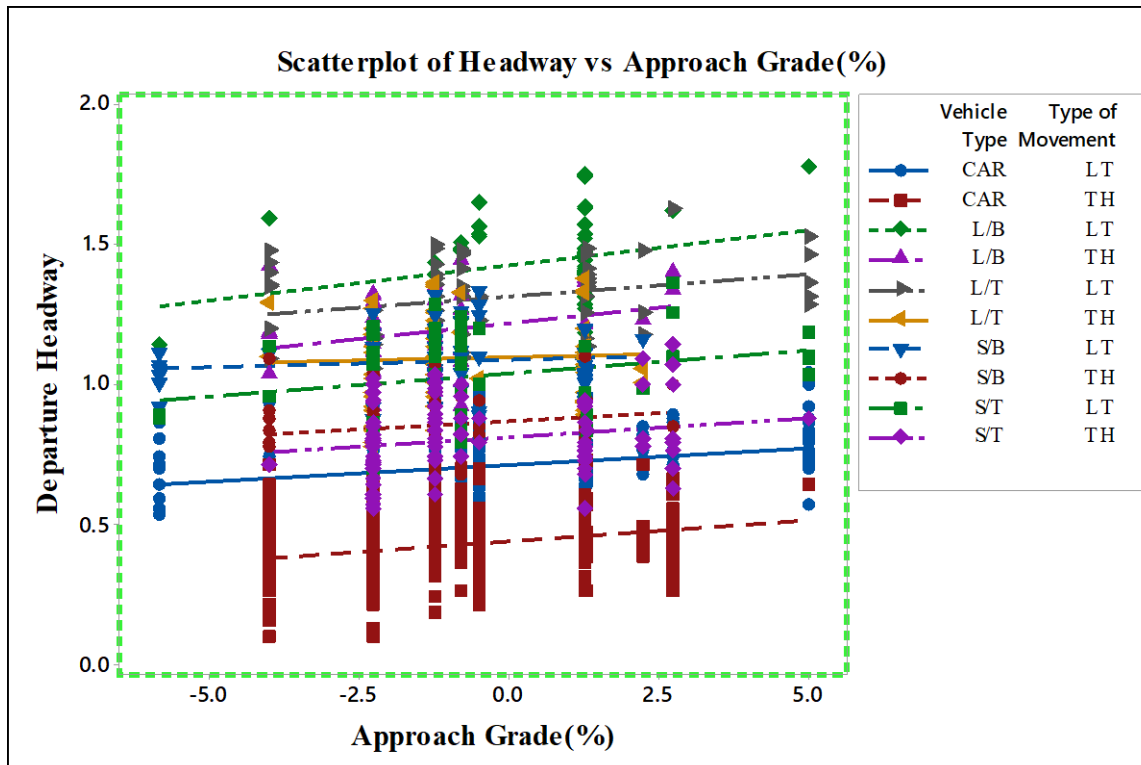


Figure 4-4: Scatter plot of Departure Headway vs. Approach Grade

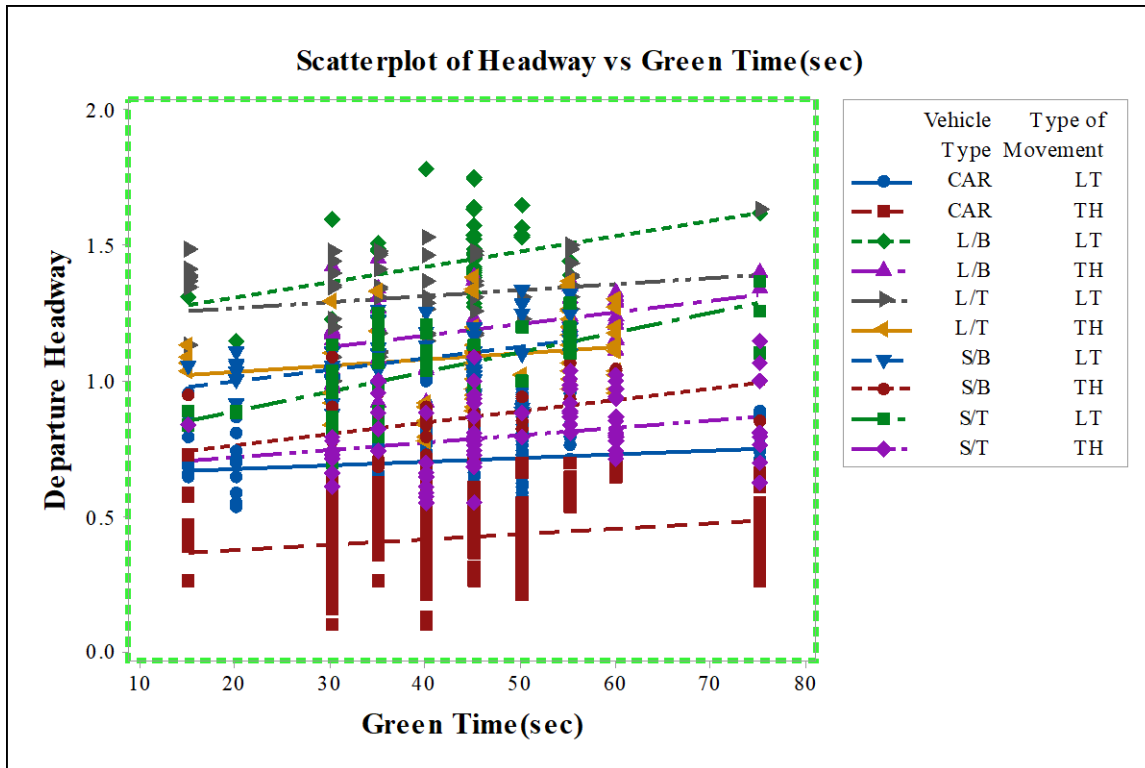


Figure 4-5: Scatter plot of Departure Headway vs. Green time.

Figure 4-3 shows the scatter plot of vehicles departure headway Vs. Lane Width for each combination of vehicle type and movement type. As can be seen in the figure, the relationship between Lane Width and departure headway of vehicles is linear for each combination of vehicle type and movement type. If we look in to the line of best fit in the figure, we can see that the general trend of relationship between the variables is negative. So as Lane Width increases departure headway of vehicles decreases.

Figure 4-4 provides scatter plot of Vehicles departure headway versus Approach Grade with line of best fit for each combination of vehicle type and movement type. The line of best fit indicates that there is a linear relationship between approach grade and departure headway. Generally as approach grade increases vehicles departure headway also increases.

As we can see from scatter plot of departure headway data versus Green Time for each combination of vehicle type and movement type in **Figure 4-5**, there is a linear relationship between departure headway and Green time. The general trend of relationship between the variables indicates the existence of positive linear relationship. So as Green Time increases departure headway also increases.

4.2. Departure Headway Distribution Modeling

In this study, the distribution model is developed for the collected headway data. Minitab 18 statistical analysis software is used as a major data analysis and manipulation tool. Since the traffic flow condition at the selected intersections is Heterogeneous, it is not reasonable to treat all vehicle types under the same category. So departure headway distributions will be fitted for each vehicle type and movement type. Before fitting any distribution to our data first we have to test it for normality. In order to try to fit any non-normal distributions to a data, the data should show a significant departure from normality. To determine this we should look at the probability plot of the data, histogram, and Anderson –Darling test results. The Null and Alternate hypothesis for Anderson-Darling test are as follows.

Anderson-Darling Test

Ho: - Data values do come from the specified probability distribution

H₁:- Data values do not come from the specified probability distribution.

Significance level: - 0.05

- Distribution fitting for departure Headway data of Small Cars making left turning movement

Probability plot of left turning small car shows that smaller and larger data values depart from the normality line (*see Figure 4-6*). The fit curve on the histogram is also a little bit skewed to the right implying a probable departure from normality, (*see Figure 4-7*), but we have to look at Anderson-Darling test result, provided on the probability plot, to be sure that the data significantly departs from normality. The p-value from Anderson-Darling test is **0.045**, less than significance level (**0.05**) which leads to the conclusion to accept the alternate hypothesis which states that the data does not come from a normal distribution.

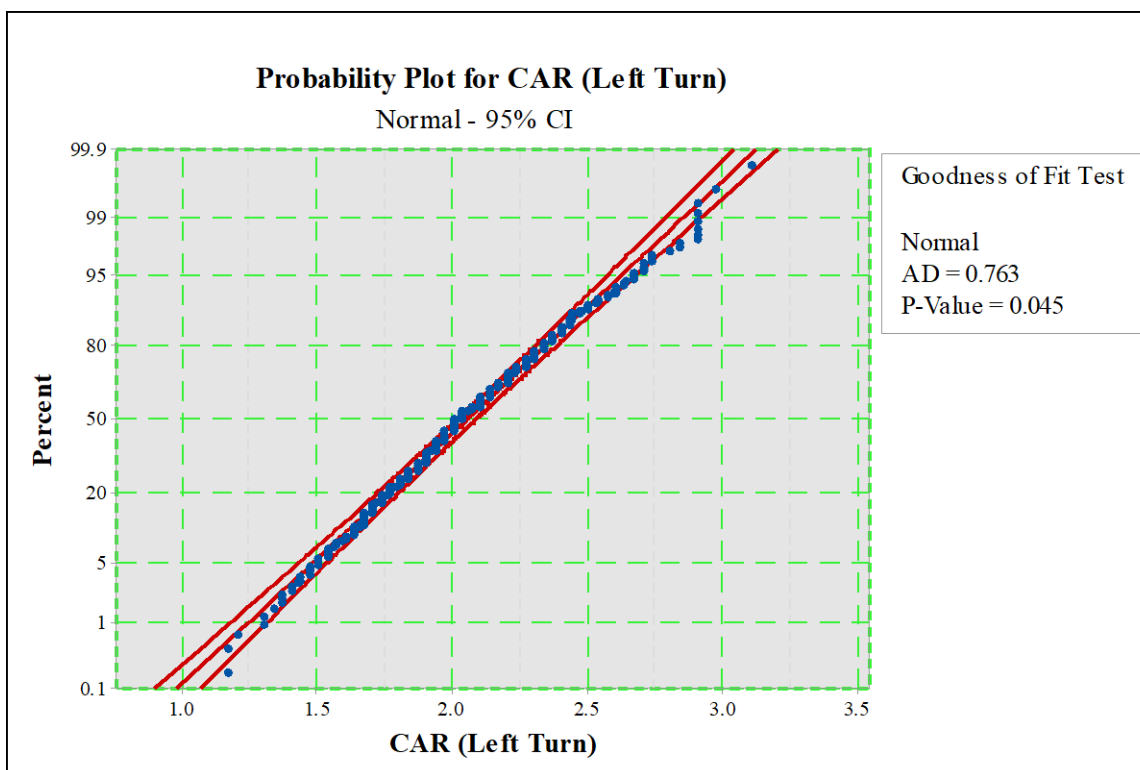


Figure 4-6: Normal probability Plot of CAR (LT)

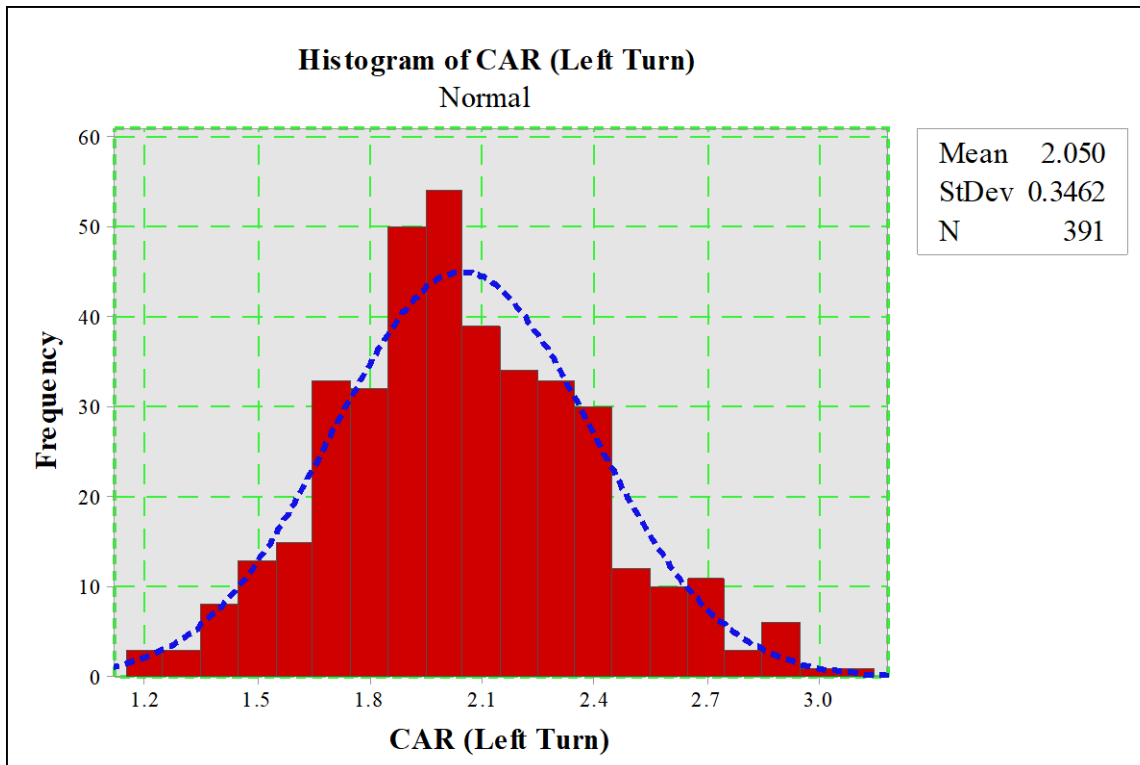


Figure 4-7: Histogram of CAR (LT) with Normal Curve

Since the data does not come from a normal distribution, we must find which probability distribution the data belongs to. Probability plot and Anderson-Darling test are used to determine which probability distribution fit the data best. All possible distributions were tested to fit departure headway data for left turning small cars. Lognormal, Gamma, and Log-logistic probability distributions were found to fit the data but we have to find out which probability distribution fit the data best. The smaller the Anderson-Darling test parameter (AD), the better the fit and the higher the p-value the better the fit. The AD value for the Gamma distribution is lesser and its p-value is significantly greater than the others. If a certain theoretical probability distribution function is a good fit of a data, the data points cluster in to the straight line in the probability plot. Gamma distribution probability plot has a less number of data points which lie of the 95% confidence interval limit and the points are also closer to the straight line than the other probability Distributions. So the Gamma probability distribution is a better fit for the headway data of small cars making a left turning movement.

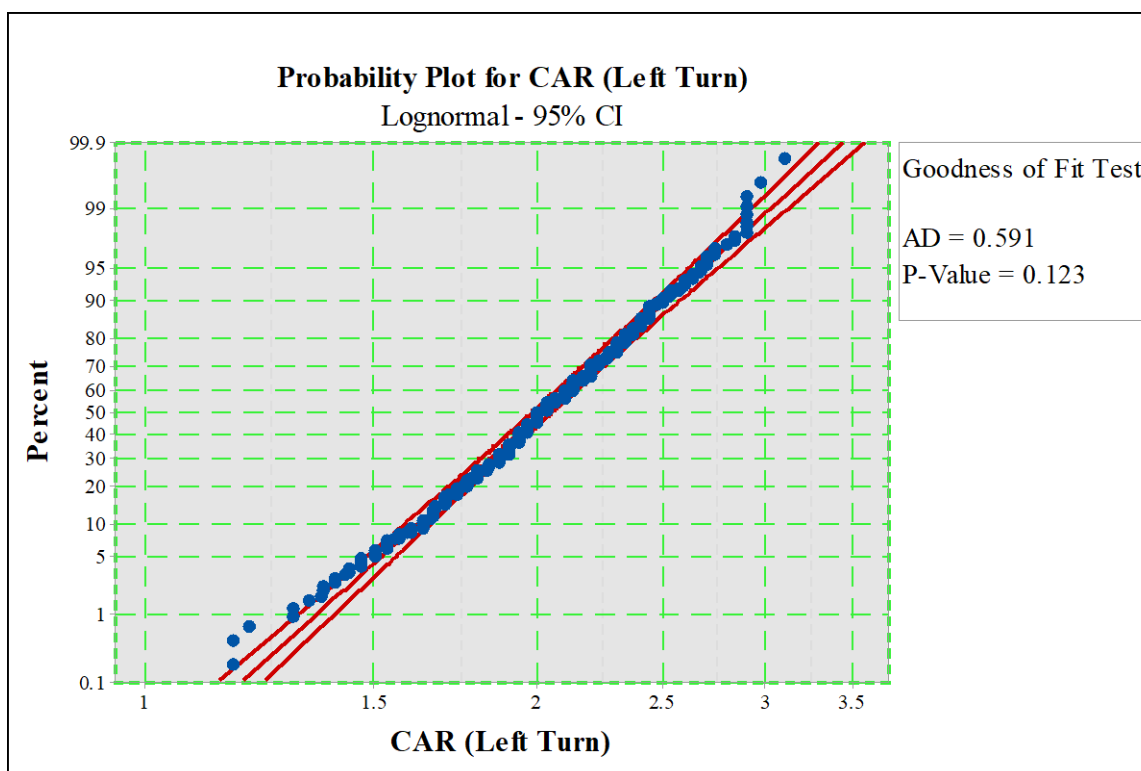


Figure 4-8: Probability Plot of CAR (LT) – Lognormal

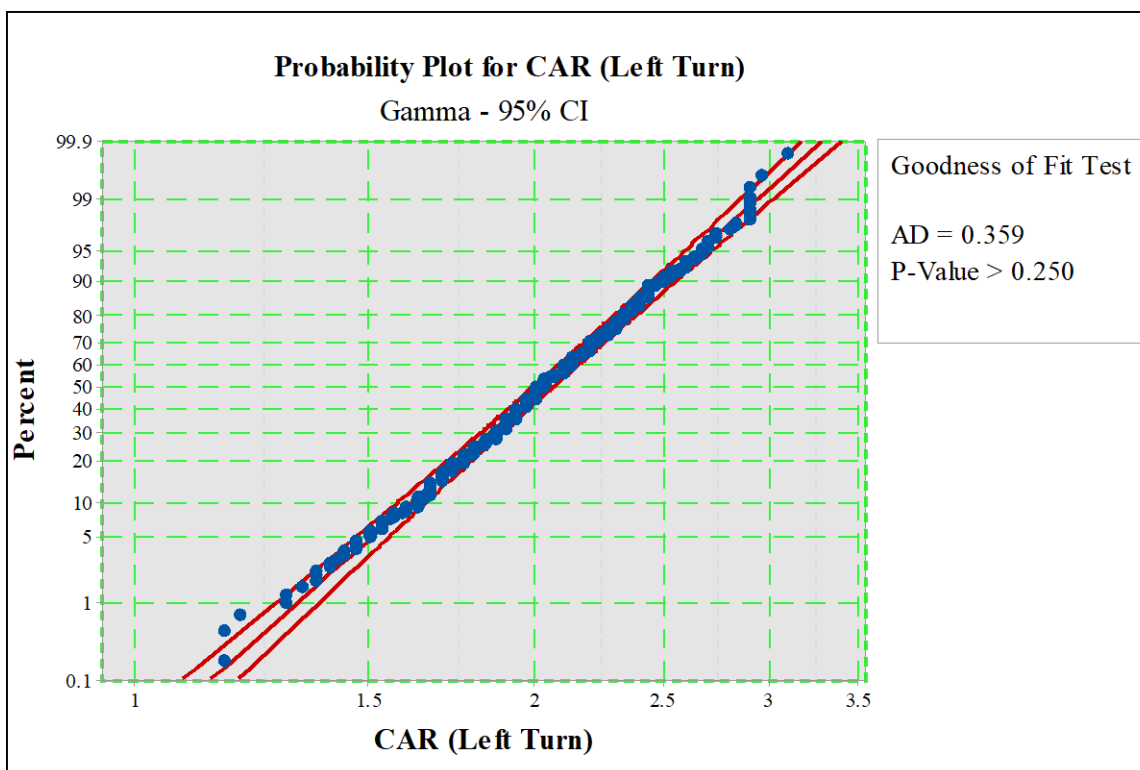


Figure 4-9: Probability Plot of CAR (LT) – Gamma

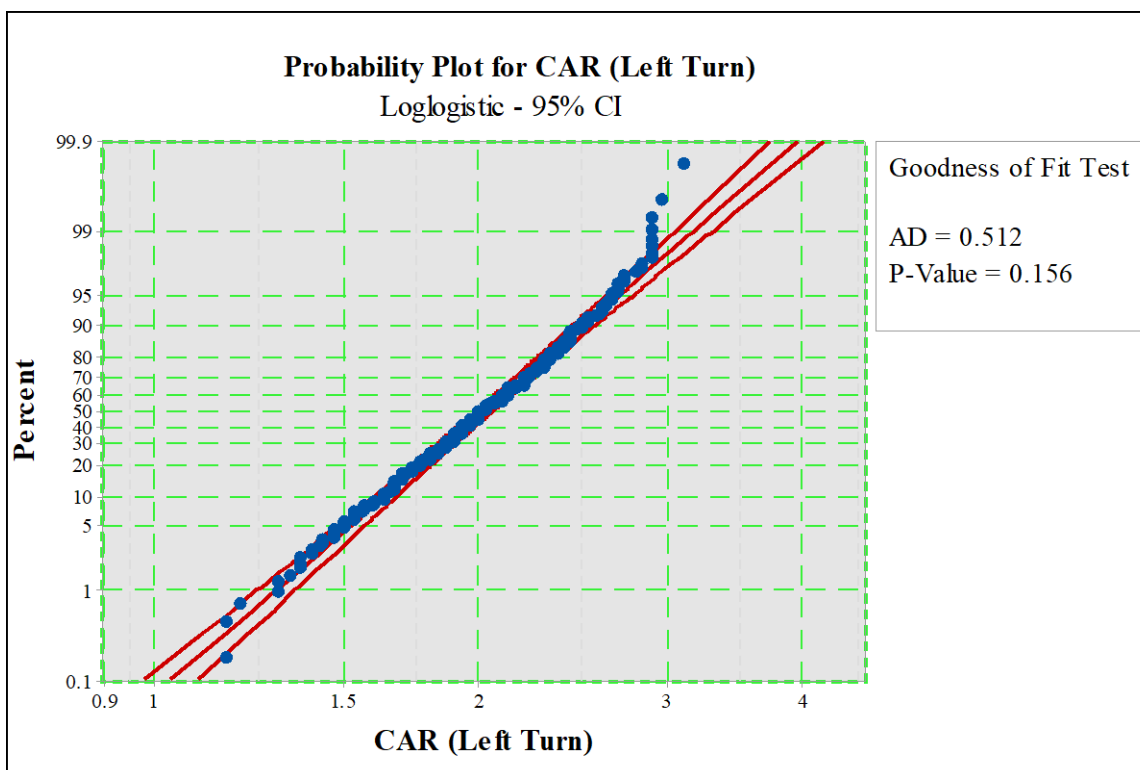


Figure 4-10: Probability Plot of CAR (LT) – Log logistic

The Gamma probability distribution function is generally written as follows.

$$f(x) = \frac{1}{\Gamma(x)b^x} x^{x-1} e^{-x/b}, x > 0 \dots \dots \dots \text{Eqn} - 4.1$$

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt \dots \dots \dots \text{Eqn} - 4.2$$

Where:-

F(x) is the General Gamma Distribution

k is shape parameter

b is shape parameter

$\Gamma(x)$ is the gamma function

Distribution parameters are estimated for the fitted probability distributions using maximum likelihood parameter estimation technique. The table below shows the estimated distribution parameter along with the Anderson Darling test result.

Table 4-2: Estimated Parameters and AD Test Result

Distribution	Estimated Parameters				Anderson-Darling Test Result		
	Location	Shape	Scale	Threshold	AD	P	LRT P
Lognormal	0.703		0.17167		0.591	0.1230	
Gamma		34.7324	0.05903		0.359	>0.2500	
Log-logistic		0.70728		0.09707	0.512	0.1560	

The table below demonstrates fitted probability distribution models for each combination of vehicle type and movement type along with estimated distribution parameters and Anderson-Darling test Results. For detailed analysis of fitting distributions for the nine combinations of vehicle type and movement type see **Appendix – C**.

Table 4-3: Summary of fitted Probability Distributions

Variables	Distribution	Estimated Parameters				Anderson-Darling Test Result		
		Location	Shape	Scale	Threshold	AD	P	LRT P
CAR(LT)	Gamma	-	34.7324	0.05903	-	0.359	>0.2500	-
CAR(TH)	Largest Extreme Value	1.48755	-	0.27828	-	0.503	0.220	-
S.B (LT)	3-Parameter Weibull	-	1.69611	0.83285	2.30669	0.262	>0.5000	0.0000
S.B(TH)	Largest Extreme Value	2.26773	-	0.43225	-	0.361	>0.250	-
L.B(LT)	3-Parameter Weibull	-	1.73792	1.39820	3.03328	0.245	>0.500	0.0000
L.B(TH)	3-Parameter Weibull	-	1.50983	1.22972	2.47	0.370	0.4460	0.0000
S.T(LT)	3-Parameter Weibull	-	1.74141	1.06827	1.94674	0.356	0.4740	0.0000
S.T(TH)	Log-Logistic	0.79547	-	0.08799	-	0.355	>0.2500	-
L.T(LT)	Lognormal	1.32605	-	0.18883	-	0.324	0.5200	-
L.T(TH)	Log-Logistic	1.11139	-	0.11407	-	0.542	0.1210	-

If we look at fitted distribution curve for departure headways of Cars making through movement, it is positively skewed so the median is less than the mean value which also means that most departure headway values of cars making through movement fall below the mean. In this case the mean value is pulled to the right because of the extended tail of the distribution curve. So in this kinds of situations the median is preferable in estimating departure headway value.

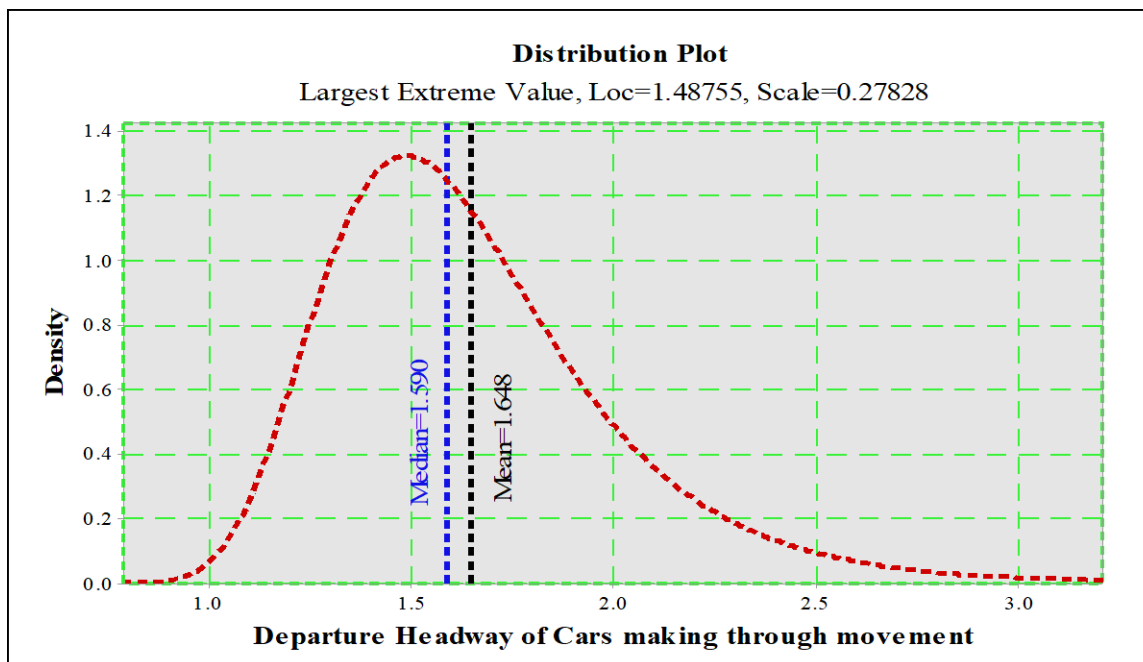
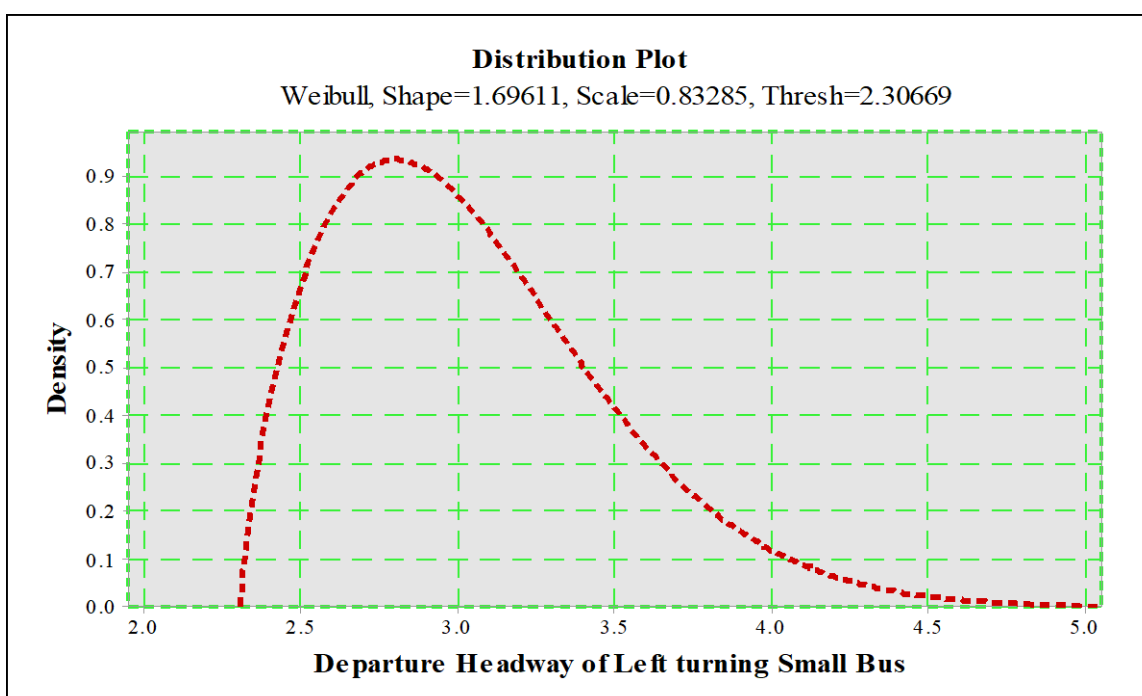
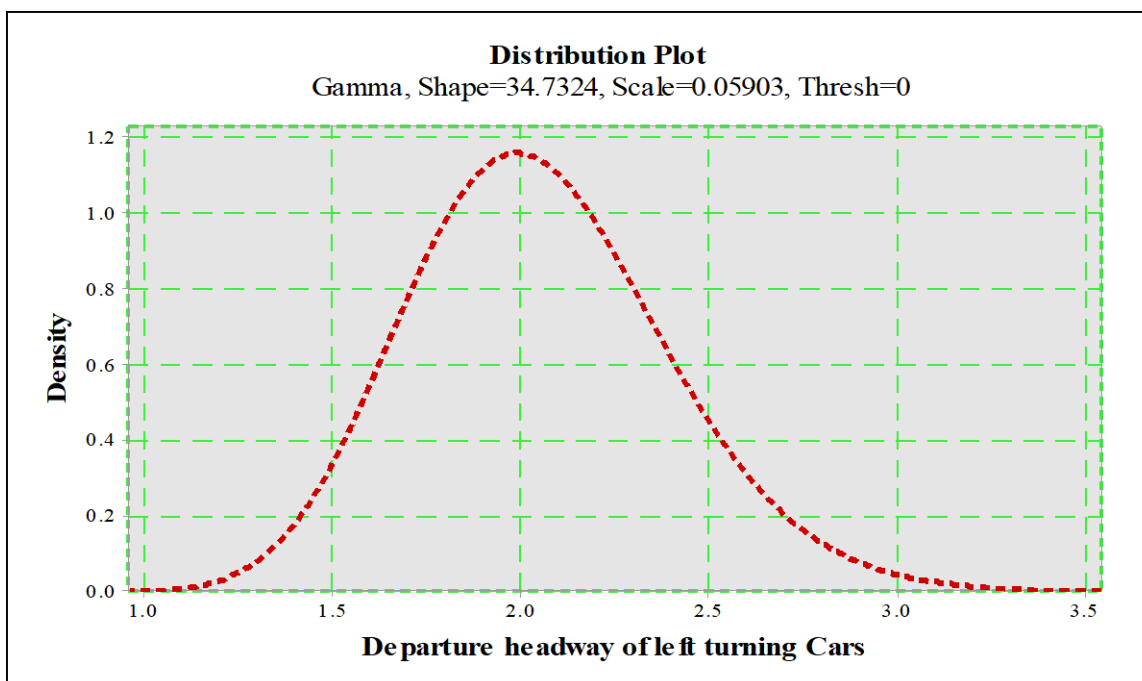
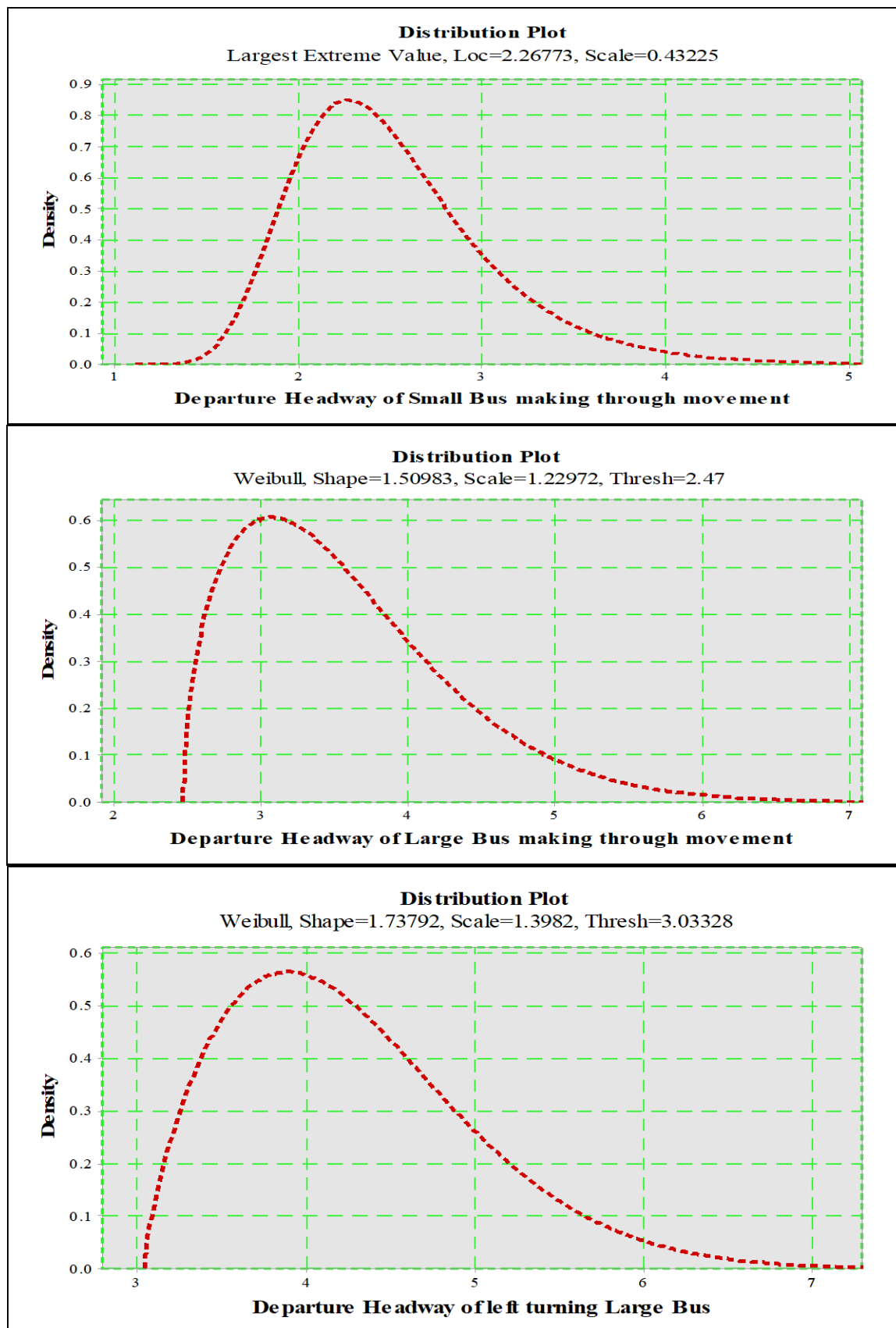
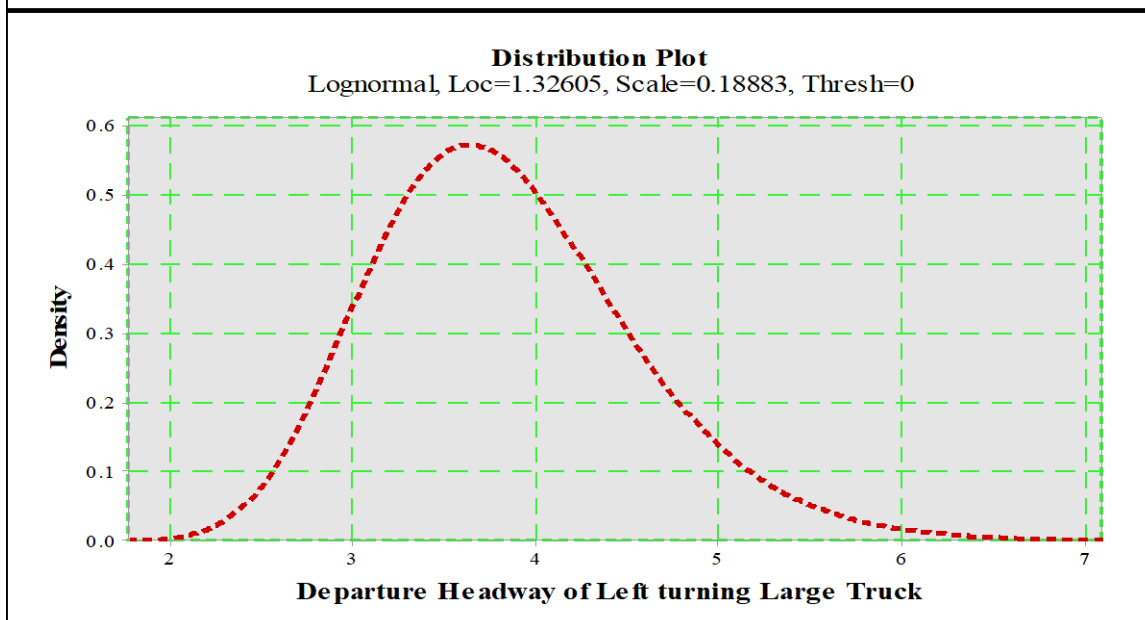
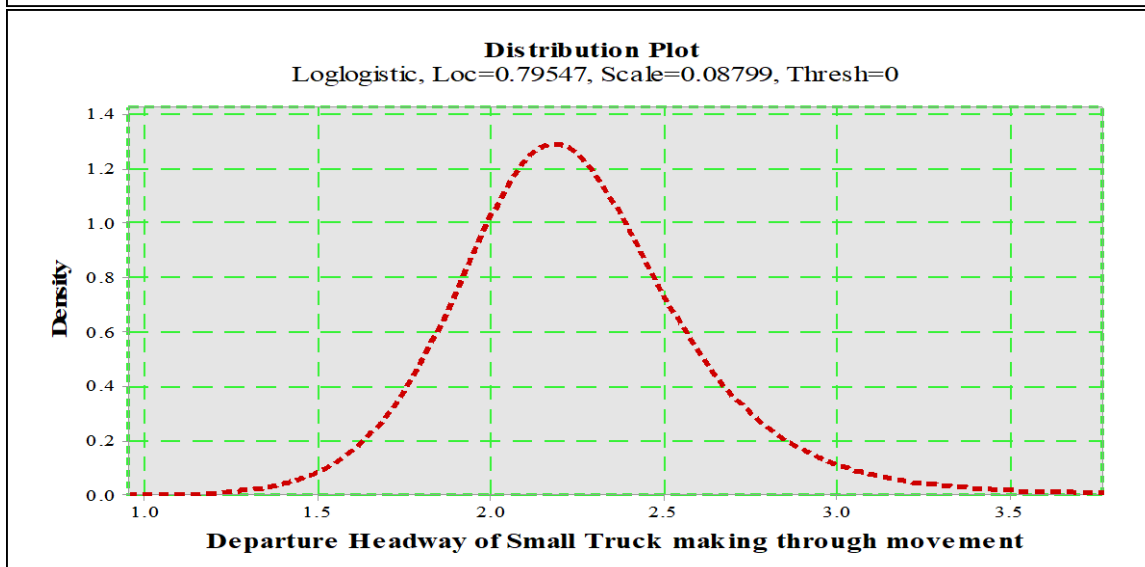
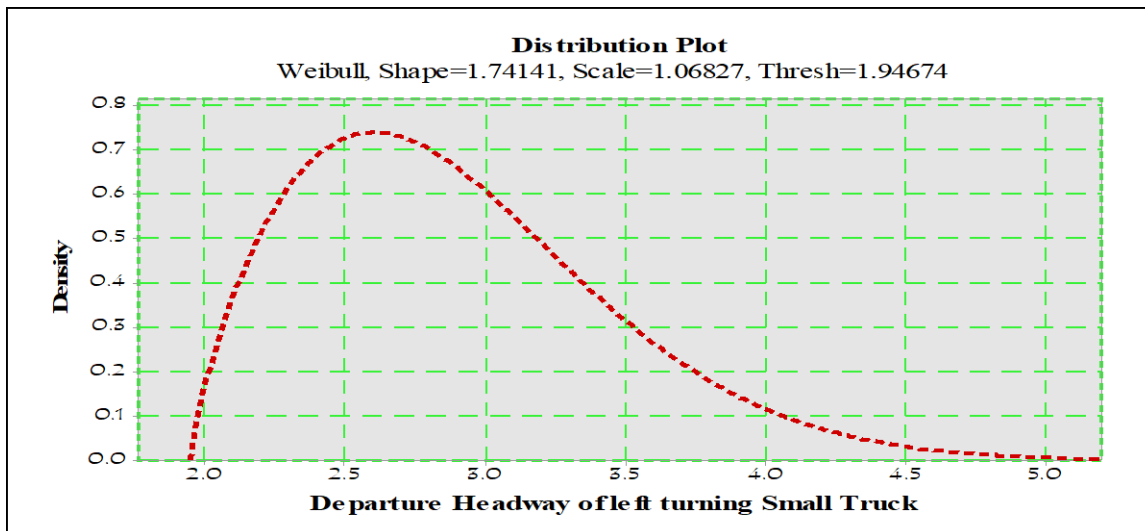


Figure 4-11: Fitted Distribution Curve for Car (TH)-Largest Extreme Value

Fitted probability distribution curves for the other nine combinations of vehicle type and movement type exhibit the same pattern observed in fitted distribution curve to departure headway data of cars making through movement. All the curves are positively skewed but the degree of the skewness differs in each of the distributions. The mean is always pulled to the right and the median is less than the mean. Fitted distribution curves are displayed in the following figure.







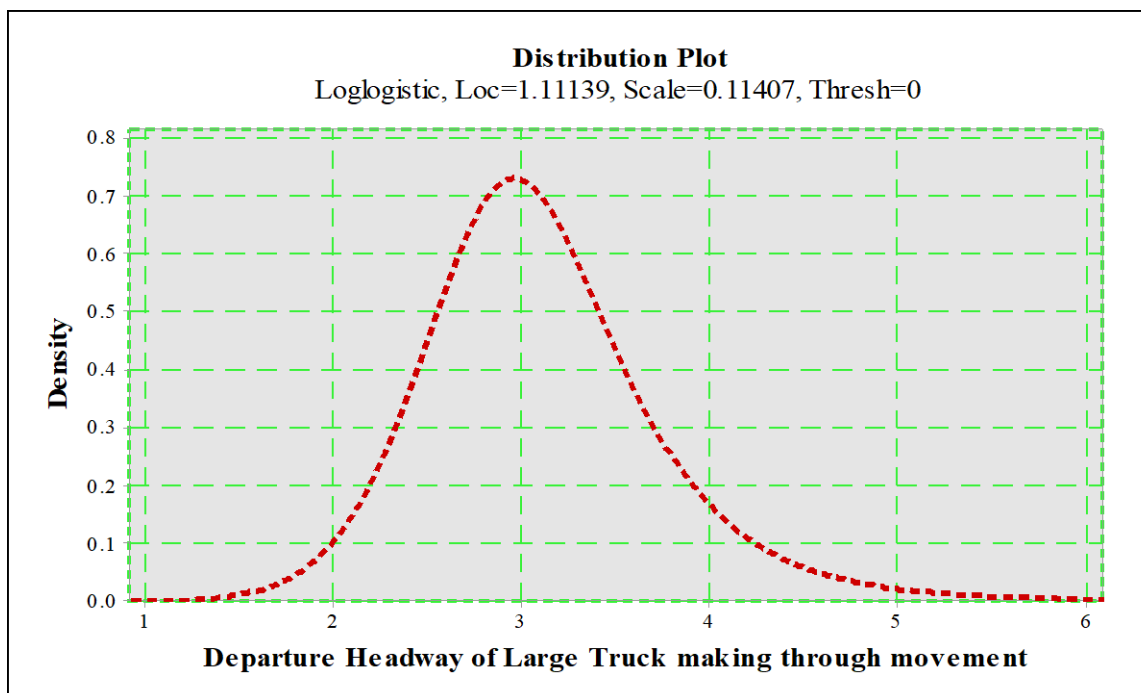


Figure 4-12: Fitted distribution curves for each combinations of vehicle type and movement type

4.3. Departure Headway Regression Modeling

Multiple linear regression technique is used to develop a mathematical model for vehicles departure headway. Minitab 18 statistical analysis software is used as a major data analysis tool. The total departure headway data extracted for the regression analysis is 1637. The general mathematical formulation of the regression model is written as **Eqn-3.2**.

4.3.1. Testing multiple linear regression assumptions

Multiple linear regressions have five major assumptions so before conducting any regression analysis these assumptions needs to be checked for violation. If one or more of the assumptions are violated, the output of the analysis will give a misleading result.

- *Assumption 1:-* Linear relationship between dependent variable and each of independent variables.

There should be linear relationship between the dependent variable and each of the independent variables. Scatter plot of the each independent variables versus the dependent variable shows that there is no significant evidence for the existence of nonlinear relationship. (See **Figure 4-3, 4-4, & 4-5**)

- *Assumption 2:-* Multicollinearity (the predictor variables should not be highly correlated with each other).

If the independent variables are highly correlated with each other, the standard error from the regression analysis tends to increase and give us a misleading result even if the adjusted R^2 value is significant. If there exists a correlation coefficient greater than 0.8 then we definitely have Multicollinearity. As we can see in the correlation matrix below the correlation coefficients for all the variables are much lesser than 0.8 (<0.8) so the independent variables are not strongly correlated with each other. **(See Table 4-4)**

The correlation matrix only give us an insight about whether any two variables are highly correlated or not but the problem could also be that one predictor variable is a linear function of a set of the other predictor variables. To identify problems at this level we can use variance inflation factor (VIF). Variance inflation factor basically tells us if there is inflation or an increase in standard error. If the VIF is greater than 10 then we have Multicollinearity. As we can see in the table below all the Variance inflation factors corresponding to each explanatory variables is much lesser than 10 (<10) so it can be concluded that there is no a Multicollinearity problem. **(See Table 4-5)**

Table 4-4: Correlation Matrix between Independent Variables

	Small truck	Small Bus	Large truck	Large bus	Near curb	Middle lane	Left turn	Lane Width	Green time	Approach Grade
Small truck	1									
Small bus	-0.13	1								
Large truck	-0.13	-0.12	1							
Large bus	-0.11	-0.1	-0.10	1						
Near curb	-0.07	0.04	-0.04	0.05	1					
Middle lane	0.03	-0.04	-0.04	-0.05	-0.33	1				
Left turn	-0.08	0.00	-0.02	0.01	-0.37	-0.37	1			
Lane Width	0.01	0.03	-0.02	-0.02	0.16	-0.20	0.08	1		
Green Time	0.1	-0.06	-0.01	0.03	-0.02	0.14	-0.08	0.12	1	
Approach Grade	0.02	-0.07	0.06	0.06	-0.15	-0.01	0.17	-0.24	0.39	1

Table 4-5: Predictor Variables Variance Inflation Factor

Variable	Approach Grade	Green time	Lane Width	Left turn	Middle Lane	Near curb	Small truck	Small bus	Large truck	large bus	Mean VIF
VIF	1.41	1.34	1.22	1.83	1.73	1.75	1.10	1.06	1.08	1.05	1.36

➤ *Assumption 3:- Homoscedasticity*

The data needs to show Homoscedasticity, which is where the variances along the line of best fit remain similar as we move along the line. If our data shows homoscedasticity, the residuals exhibit an overall fairly random display. the scatter plot of fitted value versus residuals shows that at lower values of y, the residuals are clustered around the line and then spread as we go to the higher values of y. what this basically means is that the model is doing a better job at predicting the lower values of y and is not efficient at predicting the higher values of y. so we can conclude that this is a violation of the constant variance assumption.

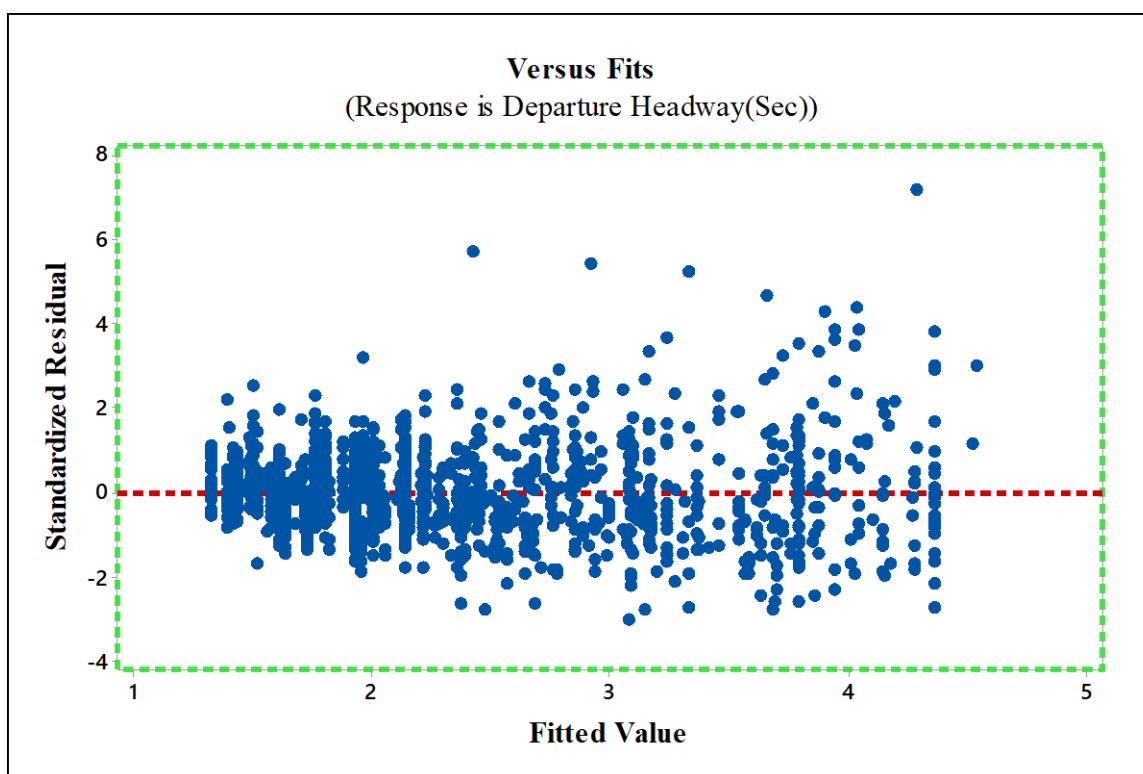


Figure 4-13: Scatter Plot of Residuals vs. Fitted Values

But in order to confirm our decision we should conduct Breusch-Pagan / Cook-Weisberg test for heteroskedasticity. According to the test result it is evident that the null hypothesis is rejected since the p-value is less than 0.05.

Breusch-Pagan / Cook-Weisberg test for heteroscedasticity

Ho: Constant variance

Variables: fitted values of departure headway

$$\chi^2(1) = 507.92$$

$$\text{Prob} > \chi^2 = 0.0000$$

Since the model shows heteroskedasticity, we cannot rely on the coefficients of the regression and significance test results so it is compulsory to remove heteroskedasticity from the model. The natural logarithmic transformation is used to remove heteroskedasticity from the model. The dependent variable, Departure headway, is transformed into $\ln(\text{Departure Headway})$. Scatter plot of fitted values versus residuals after transformation exhibit an overall fairly random display. The p-value from Breusch-Pagan / Cook-Weisberg test for heteroskedasticity is greater than the significance level. So the model now shows homoscedasticity.

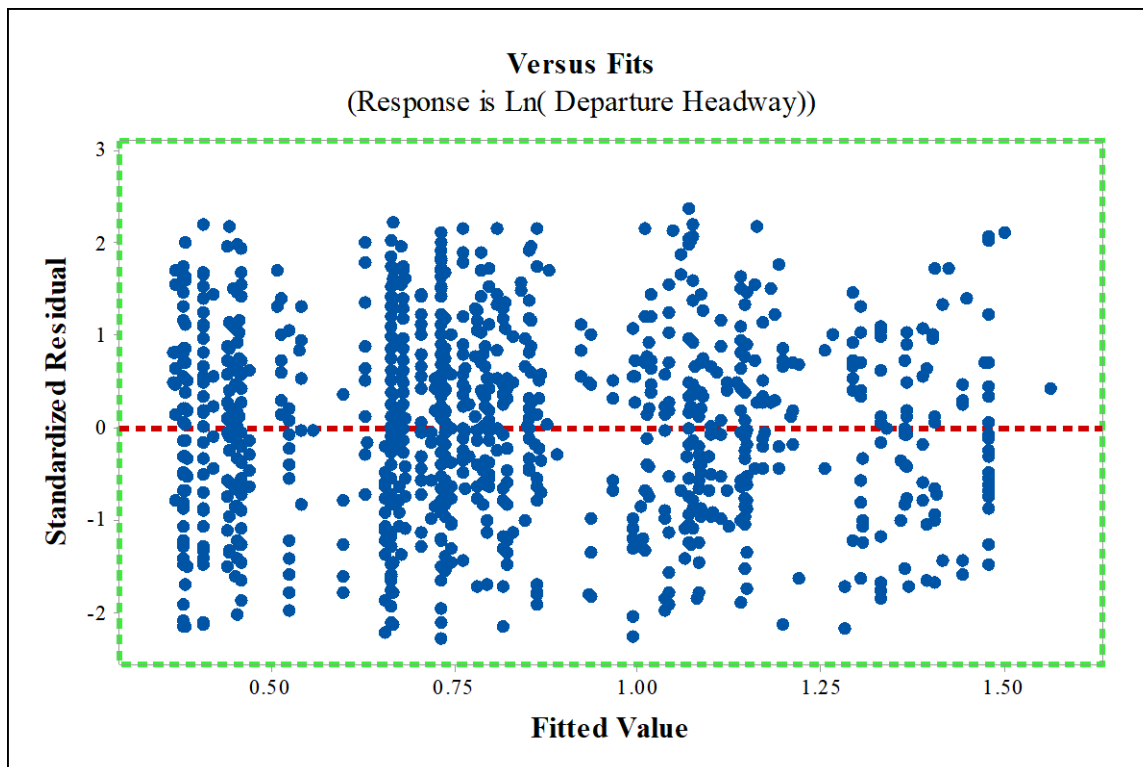


Figure 4-14: Scatter Plot of Residuals vs. Fitted Value (Transformed Data)

Breusch-Pagan / Cook-Weisberg test for heteroscedasticity

H₀: Constant variance

Variables: fitted values of departure headway

$$\chi^2(1) = 0.12$$

$$\text{Prob} > \chi^2 = 0.7273$$

- *Assumption 4:-* There should be no significant outlier, high leverage point or highly influential points, which represent observation in the data set.

We want all the data points in the data set to provide the same amount of information in developing the predictive model. Unusual observations and data points with high residuals are identified and those which are considered as significant outliers, high leverage points, and highly influential points are removed from the analysis.

- *Assumption 5:-* Residuals are normally distributed

The residuals (Errors) should be approximately normally distributed. Histogram plot of the residuals with super imposed normal curve have a bell shaped structure which shows that the residuals are normally distributed.

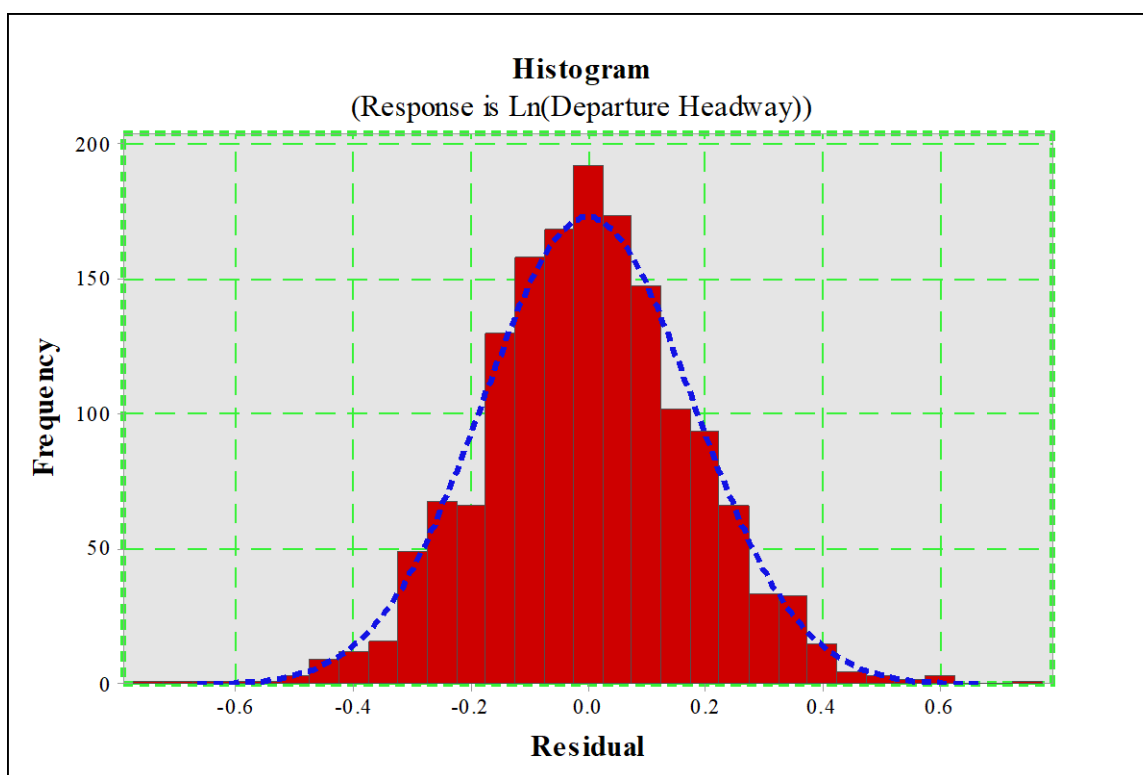


Figure 4-15: Histogram of Residuals with a Normal Curve

4.3.2. Regression Analysis

A stepwise multiple linear regression analysis was performed to model departure headway of vehicles at signalized intersections with the following explanatory variables. The explanatory

variables consists both continuous and categorical variables. Approach grade, green time, and lane width are continuous variables. Type of vehicle, type of movement, and lateral position of the vehicle are categorical variables. Vehicle type consists five categories which are presented in **Table 3-2**, Type of movement have two categories (through and left turn), lateral position of the vehicle have three categories (near curb, near median, and middle lane). The categorical predictor variables are encoded in the regression model as dummy variables which takes a value of 1 or 0.

Dependent Variable

- Ln(Departure headway)

Predictor variables

- Vehicle Type (CAR, Small Bus, Large Bus, Small Truck, & Large Truck)
- Type of Movement (Left turn & through)
- Lateral position of vehicles (Near Median, Near Curb, & Middle Lane)
- Approach Grade in % (Grade)
- Lane Width in meters (Width)
- Green time in seconds (Green)

Vehicle type, Lateral position of vehicles, & Type of Movement are dummy coded in the regression analysis with the following base categories.

Vehicle Type: - Base category is CAR.

Lateral position of vehicles: - Base category is Near Median (NM).

Type of Movement: - Base category is through.

Stepwise selection of terms is used for all candidate predictor variables. Only one variable is added at each step of the regression. As shown in the table below p-values for each added variables is less than the significance level which confirms that all predictor variables significantly predict departure headway of vehicles and there is an increment in the adjusted r squared value at each step of the regression.

Table 4-4: Step Wise Regression

	Step 1		Step 2		Step 3		Step 4		Step 5		Step 6	
	Coef	P	Coef	P	Coef	P	Coef	P	Coef	P	Coef	P
Constant	0.563		0.430		0.317		0.369		0.401		0.484	
Vehicle Type	0.751	0.0	0.754	0.0	0.749	0.0	0.744	0.0	0.741	0.0	0.740	0.00
Movement Type			0.265	0.0	0.270	0.0	0.225	0.0	0.220	0.0	0.223	0.00
Green Time					0.002	0.0	0.002	0.0	0.002	0.0	0.002	0.00
Lateral Position of Vehicles							-0.070	0.0	-0.068	0.0	-0.071	0.00
Approach Grade									0.008	0.0	0.006	0.00
Lane Width											-0.027	0.00
R ²	0.6623		0.8215		0.8319		0.8384		0.8407		0.8416	
R ² _{Adj}	0.6612		0.8207		0.8311		0.8373		0.8395		0.8403	

Coef is coefficient

P is p-value

The final regression analysis was run to predict Departure Headway from Lane width, Green time, Approach Grade, Lateral position of vehicles, type of vehicle, and Type of movement. The predictor variables were found to statistically significantly predict Departure headway, $p < 0.05$, and $R^2_{Adj} = 0.8403$. All predictor variables added statistically significantly to the prediction, $p < 0.05$.

Table 4-5: Final Regression Output

Departure Headway	Coef.	Std. Err.	t	P> t	[95% Conf.Interval]	
Approach Grade	0.0065159	0.0021108	3.09	0.0020	0.0023746	0.010657
Green Time	0.0025095	0.0003327	7.54	0.0000	0.0018568	0.003162
Lane width	-0.026953	0.0101117	-2.67	0.0080	-0.046791	-0.00711
Type of Movement						
Left Turn	0.2238567	0.010231	21.88	0.0000	0.2037843	0.243929
Lateral Position of Vehicles						
Middle Lane	-0.0716052	0.0102687	-6.97	0.0000	-0.091752	-0.05146
Near Curb	-0.0622018	0.0137845	-4.51	0.0000	-0.089246	-0.03516
Vehicle Type						
Small Truck	0.3382702	0.0119528	28.3	0.0000	0.3148198	0.361721
Small Bus	0.4089665	0.0129377	31.61	0.0000	0.3835838	0.434349
Large Truck	0.6281448	0.0126088	49.82	0.0000	0.6034073	0.652882
Large Bus	0.7409154	0.0145656	50.87	0.0000	0.7123388	0.769492
Constant	0.4846857	0.0359091	13.5	0.0000	0.4142349	0.555136

Since the dependent variable, departure headway, data is transformed in to a natural logarithm the prediction equation is written as follows.

$$\begin{aligned} \ln(DH) = & 0.4846857 + 0.0065159*AG + 0.0025095*GT - 0.026953*LW - 0.0716052*ML - \\ & 0.0622018*NC + 0.2238567*LT + 0.4089665*SB + 0.3382702*ST + 0.7409154*LB + \\ & 0.6281448*HT \dots\dots\dots \text{Eqn- 4.3} \end{aligned}$$

Where:-

- ✓ DH is Departure Headway in seconds
- ✓ AG is approach Grade in percent
- ✓ GT is Green Time in seconds
- ✓ LW is Lane Width in meters
- ✓ ML is Middle Lane (Dummy, takes a value of 1 or 0)
- ✓ NC is Near Curb (Dummy, takes a value of 1 or 0)
- ✓ LT is Left Turn (Dummy, takes a value of 1 or 0)
- ✓ SB is Small Bus (Dummy, takes a value of 1 or 0)
- ✓ ST is Small Truck (Dummy, takes a value of 1 or 0)
- ✓ LB is Large Bus (Dummy, takes a value of 1 or 0)
- ✓ HT is Large Truck (Dummy, takes a value of 1 or 0)

In the above equation the variables left turn, middle lane, near curb, small truck, small bus, large truck, and large bus are dummy variables and take a values of 1 or 0.

If we keep the effect of all other variable constant and examine the effect of each variable on departure headway, departure headway values for vehicle types in increasing order will be as follows :- CAR<Small Truck<Small Bus<Large Truck<Large Bus. The main reason for variability of departure headway among type of vehicles is vehicle size and Left turning vehicles also have higher departure headway values than through movements because left turning movements are more complex than through movements. Vehicles departing from the intersection in the middle lane were found to have smaller discharge headway than in the near curb and near median lane. The regression coefficient for Approach Grade is positive which indicate that vehicles on an upgrade approach would have a higher departure headway value. Similarly as Green time increases departure headway of vehicles increases which implies that during a smaller green time drivers would become frustrated thinking that they wouldn't clear the queue which results in speeding and they feel relaxed during higher green times because they become confident that they will clear the

queue with in the given green time. Lane width and departure headway of vehicles have an inverse relationship. When lane width increases, drivers get more space to speedup which results in smaller headway values and when lane width decreases drivers will not be comfortable to speedup. The developed regression model can be used in capacity estimation and level of service evaluation of signalized intersections. The model is also capable of generating passenger car equivalences for different circumstances. We can calculate PCE values by dividing estimated headway values of different vehicle classes by estimated headway values of passenger car. PCE values can be determined for different combinations of Approach grade, Lane width, Green time, Type of movement and lateral position of vehicles.

CHAPTER FIVE. CONCLUSIONS AND RECOMMENDATIONS

5.1. Conclusions

In this study the researcher aimed to understand the stochastic nature of vehicles departure process at signalized intersections and come up with a more accurate way of estimating vehicles departure headway at signalized intersections by developing probability distribution and multiple linear regression models.

- ✚ Departure headway distribution model was developed for five vehicle types making Left turning and through movement. Distribution parameters were estimated from the data for the selected probability distributions.
- ✚ The 3-Parameter Weibull distribution fit departure headway data of left turning small bus, Left turning large bus, Large bus making through movement, and left turning small truck with different shape, scale, and threshold parameters. The Largest Extreme Value probability distributions was found to fit departure headway data of Cars and Small bus making through movement with different location and scale parameter. The Log-Logistic probability distribution fit to departure headway data of large truck and small truck making through movement with different location and scale parameter. Cars making left turning movement follow a certain Gamma distribution and large trucks making left turning movement were found to follow lognormal distribution.
- ✚ Interestingly, as stated above, five different probability distributions were found to fit departure headway data of five vehicle types making left turning and through movement. It is evident from the result that discharge headway data for different vehicle type and movement type does not follow an identical probability distribution.
- ✚ All fitted distribution curves are positively skewed which implies that the mean value is pulled to the right due to extended tail of the curves and the median is less than the mean. So for estimating purposes it is better to use the median values of the distributions.
- ✚ Multiple linear regression model was developed to predict Individual Departure headway of vehicles from Vehicle type, Movement type, and Lateral position of the vehicle, Approach Grade, Lane width, and Green time. All variables were found to statistically significantly predict Departure headway at **95%** confidence level. The calculated R^2_{Adj} for the regression model is **0.8403**.

- ✚ Vehicle size is the main reason for departure headway variability among vehicle types. Vehicles making through movement have smaller departure headway than vehicles making left turning movement and it's mainly because left turning movement is more complex than through movement. Vehicles departing from intersection in the near median lane were found to have higher discharge headway than in the near curb and middle lane. Effect of Approach Grade on departure headway were found to be positive. Similarly as Green time increases departure headway of vehicles increases and it's mainly due to driver's behavior. Lane width have a negative effect on departure headway of vehicles.
- ✚ The developed regression model can also be used to generate passenger car equivalences for numerous conditions. PCE values can be derived by dividing estimated headway values for different vehicle classes to estimated headway values of passenger car.

5.2. Recommendations

Although it is a critical parameter in signal timing, capacity estimation, and level of service evaluation, departure headway of vehicles haven't been studied sufficiently in Ethiopia. So it is recommended to perform an extensive study, involving bigger number of intersections, on discharge headway of vehicles at signalized intersections of Addis Ababa and Ethiopia. It is also recommended to replicate this study to confirm the results. Since departure headway of vehicles vary from place to place due to prevailing traffic flow condition and driver's behavior, the researcher of this paper recommends the practical use of the developed models in this study only for intersections in Addis Ababa, Ethiopia. For future research in this area it is recommended to use a more complex and computer based Neural Network modelling technique. Future research areas related with this study are listed below.

- ✚ Effect of Signal type (Digital or Analog) on departure headway.
- ✚ Effect of pavement distress on departure headway.
- ✚ Effect of angle between the intersecting roads on departure headway.
- ✚ Effect of moisture on departure headway.
- ✚ Startup disturbances at the start of green phase.
- ✚ Effect of horizontal alignment of the interesting roads on departure headway.

REFERENCES

- Al-Ghamdi, A., 1999. Entering Headway for Through Movements at Urban Signalized Intersections. *Transportation Research Record: Journal of the Transportation Research Board*, 1678, 42–47.
- Anderson, T.W., 2011. Anderson-Darling tests of goodness of fit. In *International Encyclopedia of Statistical Science* (pp. 52-54). Springer, Berlin, Heidelberg.
- Bargegol, I., Amlashi A.T. and Gilani, V.N.M., 2015. Evaluation Average Headway at Nearside Legs of Signalized Intersections. *Jurnal UMP Social Sciences and Technology Management*, 2015 Vol 3, Issue 3.
- Bogale, S.S. , 2015, A Predictive Model for The Saturation Flow Rate at Signal Controlled Intersections In Addis Ababa, Addis Ababa Institute Of Technology- *School of Civil and Environmental Engineering Master Of Science Thesis, Addis Ababa, Ethiopia*.
- Dey, P.P., Nandal, S. and Kalyan, R., 2013. Queue discharge characteristics at signalized intersections under mixed traffic conditions. *European Transport-Transporti Europei*, 55, pp. 1-21.
- Engmann, S. and Cousineau, D., 2011. Comparing distributions: the two sample Anderson-Darling test as an alternative to the Kolmogorov-Smirnoff test. *Journal of Applied Quantitative Methods*, 6(3), pp. 1-17
- Gonzalez, C.L., 2006. Discharge headway at signalized intersections in Monterrey, Nuevo Leon, Mexico. *University of Texas, Doctor of Philosophy Thesis, Arlington, Texas*.
- Hao, H. and Ma, W., 2017. Revisiting distribution model of departure headways at signalized intersections. *Transportmetrica B: transport dynamics*, 5(1), pp. 1-14.
- Hoogendoorn, S. and Knoop, V., 2013 “Traffic flow theory and modeling. *The Transport System and Transport Policy: An Introduction*, pp.125-159.

- Hung, W.T., Tian, F. and Tong, H.Y., 2003. Discharge headway at signalized intersections in Hong Kong. *Journal of Advanced Transportation*, 37(1), pp. 105-117.
- Jin, X., Zhang, Y., Wang, F., Li, L., Yao, D., Su, Y. and Wei, Z., 2009, Departure headways at signalized intersections: A Log-normal distribution model approach. *Transportation research part C: emerging technologies*, 17(3), pp.318-327.
- Khan, S. and Maini, P., 1999. Modeling heterogeneous traffic flow. *Transportation Research Record: Journal of the Transportation Research Board*, (1678), pp.234-241.
- Krbalek, M. and Sleis, J., 2014. Vehicular headways on signalized intersections: theory, models, and reality. *Journal of physics A: mathematical and theoretical*, 48(1), p.012101.
- Lee, J. and Chen, R.L., 1986. Entering headway at signalized intersections in small metropolitan area. *Transportation Research Record*, 1091, pp.117-126.
- Li, H. and Prevedouros, P., 2002. Detailed observation of saturation headway and startup lost times. *Transportation Research Record: Journal of the Transportation Research Board*, (1802), pp.44-53.
- Liu, M.J., Mao, B.H., Chen, S.K., Gao, L.P., Sun, Q.X., 2011. An Exploratory Hazard-based Analysis of the First Discharge Headway. *Procedia - Social and Behavioral Sciences*, 16, 536–547.
- Majeed, A.A., Zephaniah, S.O., Mehta, G. and Jones, S., 2014. Field-Based saturation headway model for planning level applications. *International journal of Traffic and Transportation Engineering*, 3(5), pp.207-215.
- Maurya, A.K., Dey, S., and Das, S., 2015. Speed and Time Headway Distributions Under mixed Traffic Condition. *Journal of the Eastern Asia Society for Transportation Studies*, 11, pp.1774-1792.
- Miller, Scott, 2011, Traffic Modeling – December 5th, 2011 (retrieved from http://home2.fvcc.edu/~dhicketh/DiffEqns/Spring11projects/Scott_Miller/Project.pdf) (Pp. 1 – 3)

- Montgomery, D.C. and Runger, G.C., 2010. *Applied statistics and probability for engineers*: John Wiley and Sons.
- Radhakrishnan, S. and Ramadurai, G., 2015. Discharge headway model for heterogeneous traffic conditions. *Transportation research procedia*, 10, pp. 145-154.
- Ranasingle, D.U., Jayanthi, W., Bunker, J.M. and Bhaskar, A., 2017. Saturation headway variation at a signalized intersection approaches with downstream bus stop and bicycle lane. *Australasian Transport Research Forum 2017 Proceedings*, 27 – 29 November 2017, Auckland, New Zealand.
- Razali, N.M. and Wah, Y.B., 2011. Power of Shapiro-Wilk, Kolmogorov-Smirnov, Lilliefors, and Anderson-Darling tests. *Journal of Statistical modeling and analytics*, 2(1), pp.21-33.
- Tan, J., Li, L., Li, Z. and Zhang, Y., 2011, July. A comparison of departure headway at signalized intersection produced by VISSIM and TransModeler. In *Vehicular Electronics and Safety (ICVES), 2011 IEEE International Conference on* (pp. 291-296).
- Tan, J., Li, Li., Li, Z., and Zhang, Y., 2013. Distribution models for start-up lost time and effective departure flow rate. *Transportation Research Part A: Policy and Practice*, 51, pp. 1-11.
- Tang, K. and Nakamura, H., 2007, June. An analysis on saturation flow rate and its variability. In *Proceedings of the 11th World Conference on Transportation Research*, Berkeley, US.
- Tong, H.Y. and Hung, W.T., 2002. Neural network modeling of vehicle discharge headway at signalized intersection: model description and result. *Transportation Research Part A: Policy and practice*, 36(1), pp. 17-40.
- Vanvoorhis, C.W. and Morgan, B.L., 2007. Understanding power and rules of thumb for determining sample sizes. *Tutorials in Quantitative Methods for Psychology*, 3(3), pp. 43-50
- Vose, D., 2010. Fitting distributions to data. Retrieved March, 8, P.2016
- Washington, S.O., Karlaftis, M.G., and Mannering, F., 2010. *Statistical and Econometric Method for transportation data analysis*. Chapman and Hall/CRC

Wu, J., Hu, J. and Sun, J., 2010. November. Departure headway distribution modeling for ordinary large cities based on video information. *In Computer Sciences and Convergence Information Technology (ICCIT), 1010 5th International Conference on*, pp. 1040-1045. IEEE

Yin, S., Su, Y., Wang, C., Yao, O., Li, L., and Zhang, Y., 2011. *Comparison of vehicle departure headways in Beijing and Atlanta. Tsinghua Science and Technology*, 16(3), pp.332-336.

APPENDICE A

Detailed Description of Signalized Intersections in Addis Ababa

Table A-1: Detailed Description of Signalized Intersections in Addis Ababa (a)

Item No	Intersection Name	Parking	Local Bus Blockage	Channelized	No of Approaches							Working Condition
					Three			Four			≥5	
					T	Skewed	Y	Right Angle	Offset	Oblique		
1	Megenagna Round About	-	-	-	-	-	-	-	-	-	-	×
2	Lebu M/Hail	×	×	√						√		√
3	Bole Michael	×	×	×				√				√
4	Agona Cinema	-	-	-	-	-	-	-	-	-	-	×
5	Legehar	×	√	×				√				√
6	Buna ena Shay	×	√	×				√				√
7	Mexico Round About	-	-	-	-	-	-	-	-	-	-	×
8	Gerji M/Hail	×	×	√		√						√
9	Imperial	×	×	×				√				√
10	Coca Bridge	-	-	-	-	-	-	-	-	-	-	×
11	I8 Mazoria	-	-	-	-	-	-	-	-	-	-	×
12	Semen Hotel	×	×	×				√				√
13	4-Kilo Parliament	×	×	×				√				√
14	National Theater	√	×	×							√	√
15	Post Office	×	×	√				√				√
16	ETV	×	×	√						√		√
17	Sengatera	×	×	×				√				√
18	Harambe Hotel	×	×	×						√		√
19	Shola Gebeya	×	×	×	√							√
20	Goma Kuteba	×	×	×				√				√
21	Banko Diroma	×	×	√						√		√
22	Atlas	×	×	×						√		√
23	Meskel Square	√	√	×						√		√
24	St. Joseph	√	√	√						√		√
25	Lideta Tsebel	×	×	×			√					√
26	Jemo Michael	×	×	×						√		√
27	Kera to Gofa	×	×	×				√				√
28	Near Kera	×	×	×				√				√
29	St. Marry 5-Kilo	-	-	-	-	-	-	-	-	-	-	×
30	Kazanchis	-	-	-	-	-	-	-	-	-	-	×
31	Salitemihret	×	×	×				√				√
32	Saris Abo	×	×	×				√				√

Table A-2: Detailed Description of Signalized Intersections in Addis Ababa (b)

Item No	Approach Name	No of Lane	Approach Grade (%)	Approach Width(m)	Pavement Distress			
					No Distress	Low	Medium	Sever
Name of Intersection:-					BOLE MICHAEL			
1	Bulbula-Michael	1	-1.06	5	√			
2	Saris-Michael	3	3.81	11	√			
3	Bole-Michael	3	-1.71	11	√			
4	Rwanda-Michael	2	0.38	6	√			
Name of Intersection:-					ATLAS			
1	Medhanyalem-Atlas	3	-0.26	10	√			
2	Urael-Atlas	3	3.36	10	√			
3	Africa Avenue-Atlas	3	3.02	11	√			
4	Wuhalimat-Atlas	3	-5.41	10	√			
Name of Intersection:-					IMPERIAL			
1	Gerjy-Imperial	2	-3.03	7		√		
2	Bole-Imperial	4	3.28	12	√			
3	Megenagna-Imperial	3	-1.44	11	√			
4	Lemhotel-Imperial	2	3.7	6	√			
Name of Intersection:-					GERJI M/HAIL			
1	Jackross-Gergi m/hail	3	-4.02	9	√			
2	Megenagna-Gerji m/hail	3	4.16	10	√			
3	Goro-gerji m/hail	3	-2.49	9	√			
Name of Intersection:-					SALITEMIHRET			
1	Gurdsholla-Salitemihret	2	-2.39	6				√
2	Salitemihret-Salitemihret	3	-0.04	9		√		
3	Jackross-Salitemihret	3	-4.15	9	√			
4	Safari-Salitemihret	3	4.78	9	√			
Name of Intersection:-					SHOLLA			
1	Sholla Gebeya-Sholla	3	3.82	10	√			
2	British Embassy-Sholla	4	-3.11	14		√		
3	Megenagna-Sholla	4	3.01	14	√			
Name of Intersection: -					St. ESTIFANOS			
1	22-Estifanos	3	-1.23	10	√			
2	Bole-Estifanos	4	-2.31	14	√			
3	Mexico-Estifanos	6	0.99	20	√			
4	4-kilo - Estifanos	3	-1.41	11	√			
Name of Intersection:-					St. JOSEPH			
1	Saris-St.Joseph	3	-0.24	9	√			
2	Mexico-St.Joseph	3	-0.28	11	√			
3	Estifanos-St.Joseph	4	0.74	14	√			
4	Ambassador-St.Joseph	4	-0.17	14	√			

Vehicles Departure Headway Modeling at Signalized Intersections: The Case of Addis Ababa, Ethiopia

Item No	Approach Name	No of Lane	Approach Grade (%)	Approach Width(m)	Pavement Distress			
					No Distress	Low	Medium	Sever
Name of Intersection:-					LEGEHAR			
1	Mexico-Legehar	3	-1.01	9	√			
2	Leegehar bus station-Legehar	5	-3.22	17	√			
3	Stadium-Legehar	4	0.48	14	√			
4	Biherwi-Legehar	3	3.22	10	√			
Name of Intersection:-					HARAMBE/AMBASSADOR			
1	Piazza-Harambe	3	-1.84	9	√			
2	Filwiha-Harambe	3	-0.5	9	√			
3	Stadium-Harambe	3	1.02	9	√			
4	Biherawi-Harambe	3	-1.04	9	√			
Name of Intersection:-					BUNA ENA SHAY			
1	Shebele-kera	2	-1.47	8.406	√			
2	Federal police-Buna ena shay	2	1.61	7.061	√			
3	Kera-Buna ena shay	3	2.52	9.03		√		
4	Stadium-Buna ena shay	2	-0.67	7.125	√			
Name of Intersection:-					BANKODIROMA			
1	Mexico-Bankodiroma	3	-0.31	10.582	√			
2	Stadium-Bankodiroma	3	1.03	10.418	√			
3	Sengatera-Bankodiroma	2	-0.54	7	√			
Name of Intersection:-					SENGATERA			
1	Legehar-Sengatera	3	1.38	9.88	√			
2	Mexico-Sengatera		-0.95		√			
3	Biherawi-Sengatera		0.9					
4	Sengatera-legehar		-3.63					
Name of Intersection:-					SEMIEN HOTEL			
1	Giorgis-Semien hotel	3	1.92	10.035	√			
2	Merkato-Semien hotel	1	2.03	5		√		
3	Adisu gebeya-Semien hotel	3	-2.15	10	√			
4	Afnchober-Semien hotel	2	-0.38	7	√			
Name of Intersection:-					St. MERRY			
1	4Kilo-St Merry	2	3.53	7.035	√			
2	5Kilo-St.Merry	2	-3.59	6.296	√			
3	Piazza-St.Merry	2	-0.46	7.095		√		
4	Janmeda-St.Merry	2	1.03	7.035	√			
Name of Intersection:-					ETV			
1	Black lion-ETV	2	-5.46	6.948		√		
2	Ambassador-ETV	2	2.56	7.195	√			
3	Piazza-ETV	3	-2.65	10.06	√			
4	Biherawi-ETV	3	1.86	10.06	√			

Vehicles Departure Headway Modeling at Signalized Intersections: The Case of Addis Ababa, Ethiopia

Item No	Approach Name	No of Lane	Approach Grade (%)	Approach Width(m)	Pavement Distress			
					No Distress	Low	Medium	Sever
Name of Intersection:-					GOMAKUTEBA			
1	ETV-Gomakuteba	3	0.76	9.342	√			
2	Shebele-Gomakuteba	2	5.43	7.55			√	
3	St.Lideta-Goma kuteba	4	5.92	11.9	√			
4	Teklehaimanot-Gomakuteba	3	-4.68	9.351		√		
Name of Intersection:-					KERA			
1	Sarbet-Kera	2	4.96	6.86				√
2	Mexico-Kera	3	-2.02	10	√			
3	Saris-Kera	4	-3.9	12.312		√		
Name of Intersection:-					KERA TO GOFA			
1	Kera-Gofa	3	0.41	11.519			√	
2	Saris-Kera	4	2.23	12.296	√			
3	Gofa-Kera	3	-0.33	9	√			
4	Cherkos-kera	4	-0.91	12.021	√			
Name of Intersection:-					LEBU M/HAIL			
1	German-Lebu	3	2.48	13.535	√			
2	Kality-Lebu	4	-2.27	12.535	√			
3	Laphto-lebu	3	1.28	10	√			
4	Sebeta-Lebu	3	-0.5	10	√			
Name of Intersection:-					JEMO MICHAEL			
1	Jemo-Michael	3	2.72	10.7				√
2	Ayertena-Michael	4	-1.26	12.347		√		
3	Mexico-Michael	3	-0.81	10			√	
4	German-Michael	4	1.26	12.347			√	
Name of Intersection:-					SARIS ABO			
1	Bole-Abo	4	2.24	11.58	√			
2	Kality-Abo	4	1.56	11.94	√			
3	Bulbula-Abo	3	4.98	9.18	√			
4	Saris-Abo	2	-5.84	6.96	√			
Name of Intersection:-					POST OFFICE			
1	Biherawi-Post office	3	2.78	10.3	√			
2	Immigration-post office	2	-6.69	7.5	√			
3	Piazza-Post office	3	-5.35	10.3	√			
4	Ambassador-Post office	2	0.34	6.861	√			

APPENDICE B

Outlier Plot and Outlier Determination

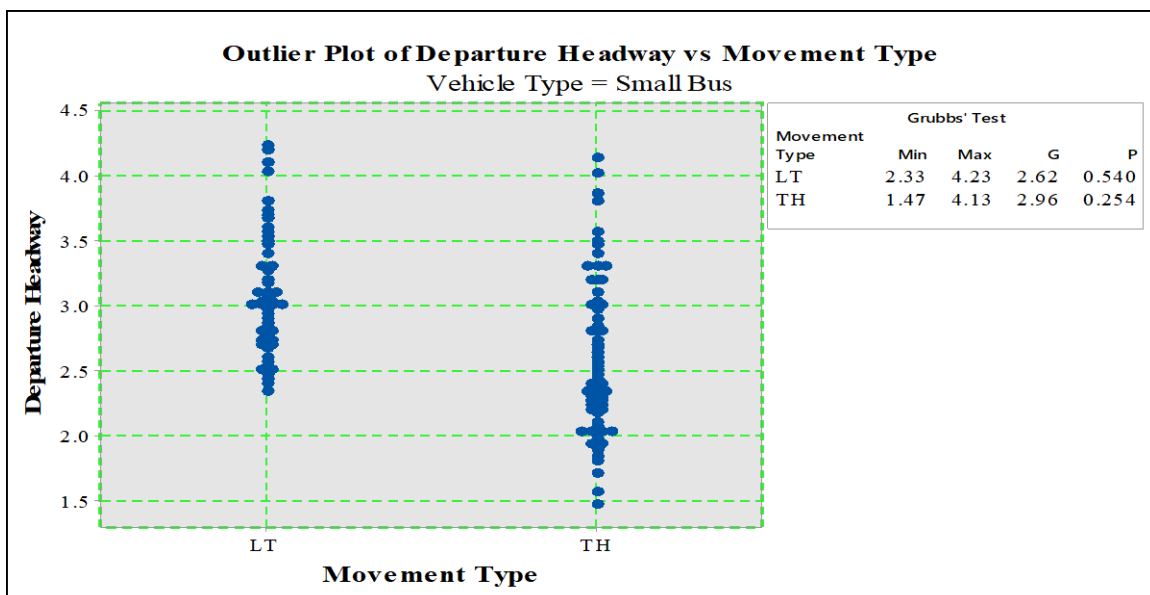
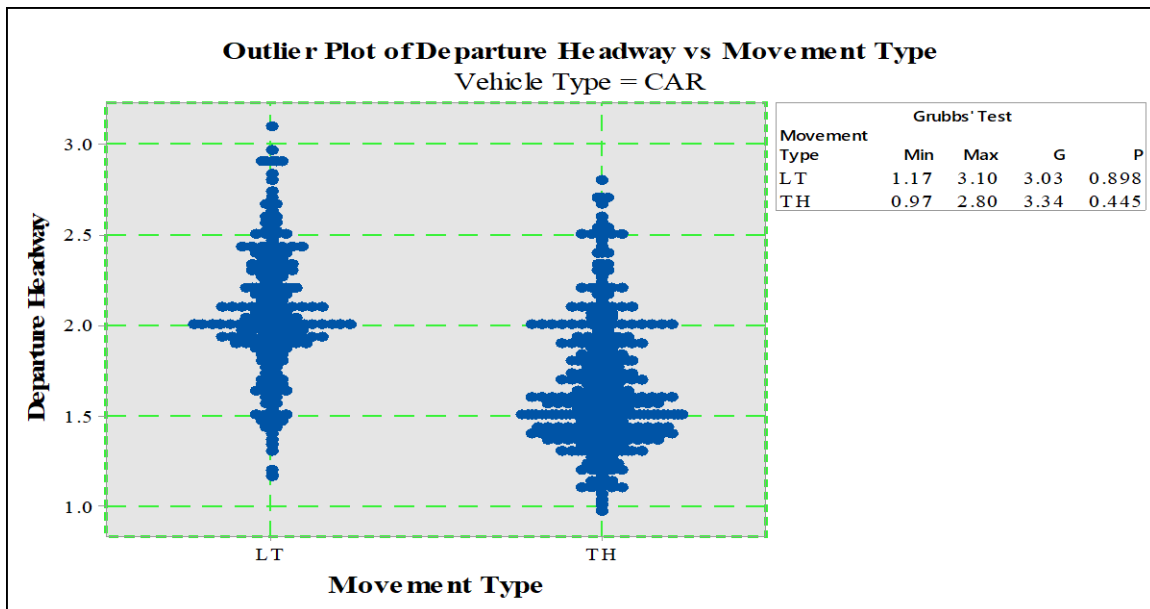
➤ **Determining outliers in the data set**

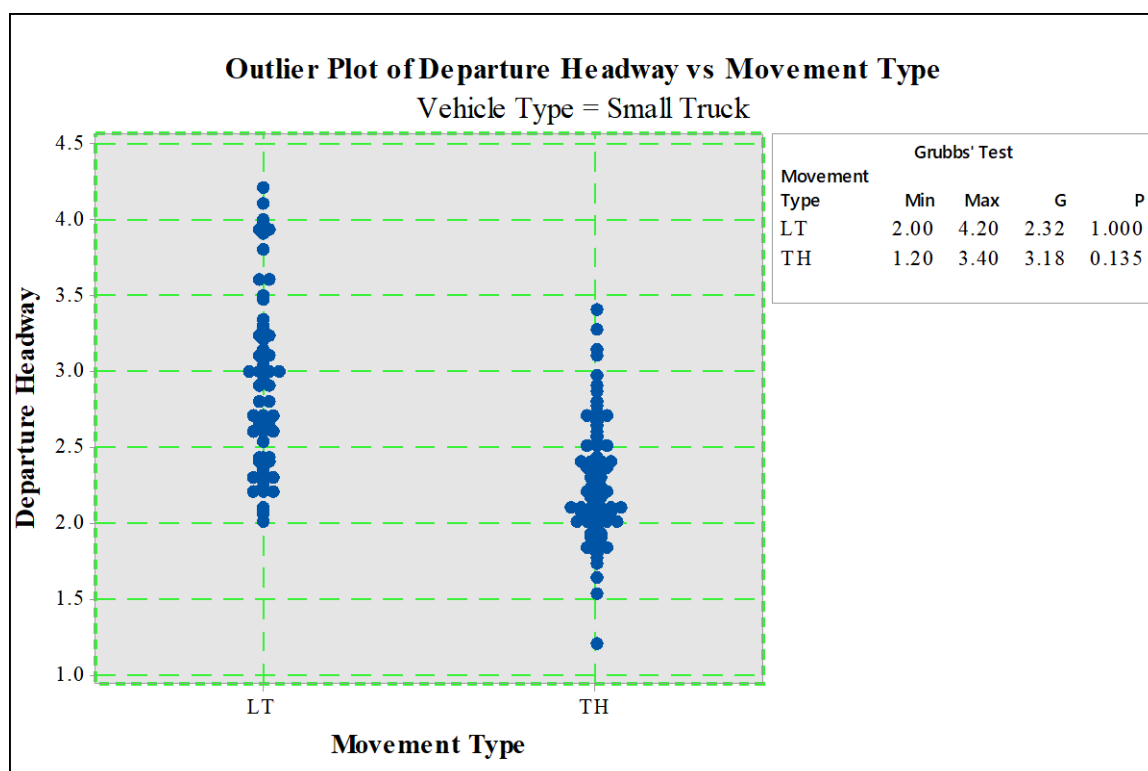
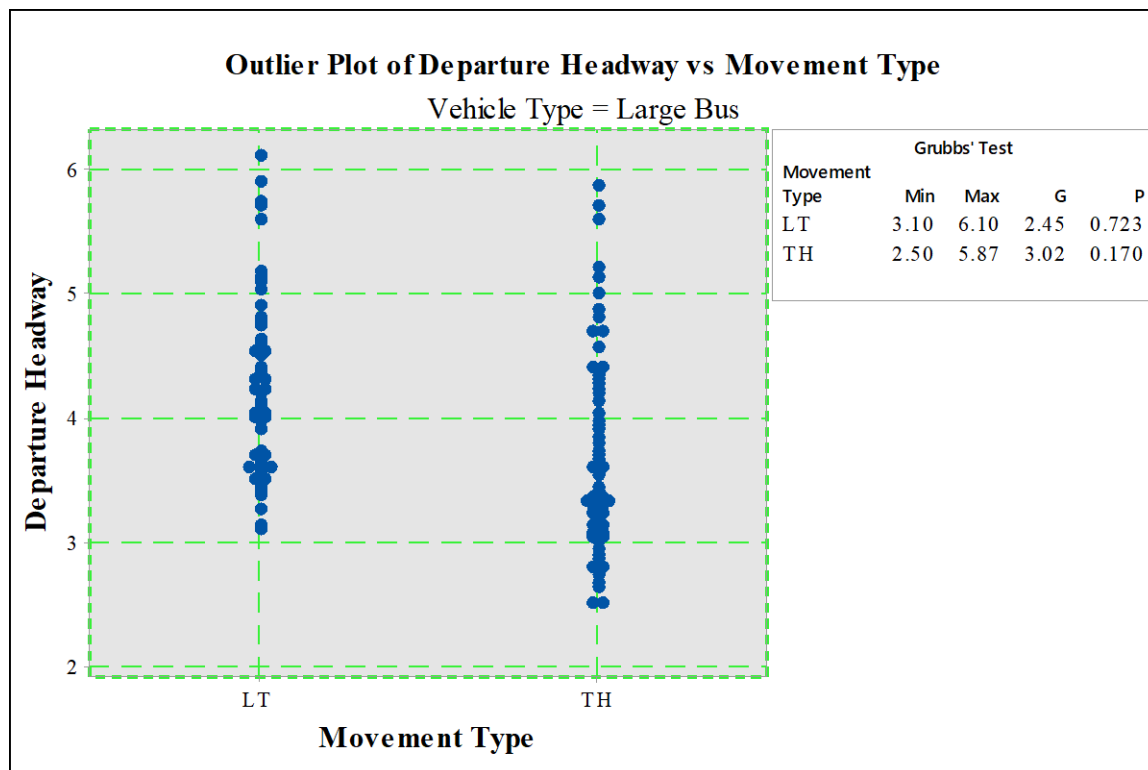
Before conducting the analysis the data set should be tested for outliers and influential points. This could be done with Grubbs' test with the following Null and Alternate Hypothesis.

H_0 : - All the data values come from the same normal population

H_1 : - Smallest or largest data value is an outlier

Significance Level: - 0.05





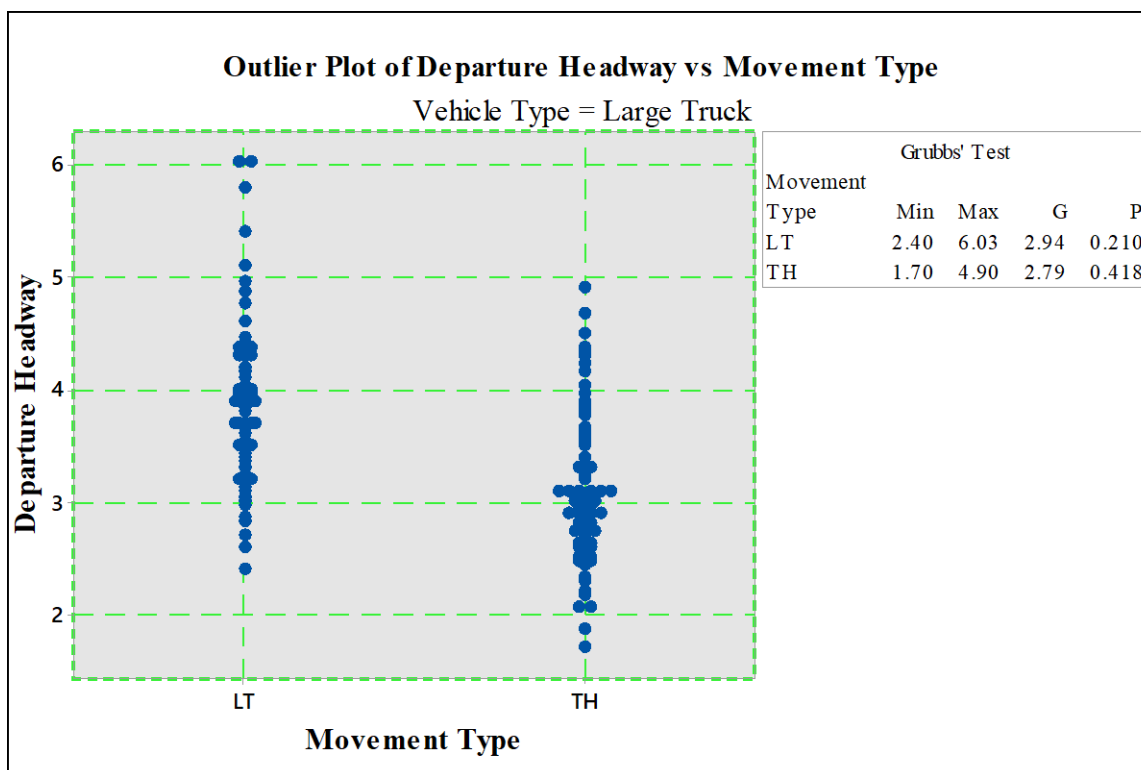


Figure B-1 Outlier plot for each vehicle type and movement type

Table B-1: Grubb's Test Result

Variable	N	Mean	StDev	Min	Max	G	P
CAR (Left Turn)	391	2.0502	0.3462	1.1660	3.1000	3.0300	0.8980
CAR (Through)	557	1.6472	0.3454	0.9660	2.8000	3.3400	0.4450
Small Bus (Left Turn)	72	3.0505	0.4521	2.3340	4.2330	2.6200	0.5400
Small Bus (Through)	101	2.5165	0.5465	1.4660	4.1340	2.9600	0.2540
Large Bus (Left Turn)	60	4.2803	0.7414	3.1000	6.1000	2.4500	0.7230
Large Bus (Through)	88	3.5794	0.7566	2.5000	5.8670	3.0200	0.1700
Small Truck (Left Turn)	77	2.9003	0.5597	2.0000	4.2000	2.3200	1.0000
Small Truck (Through)	115	2.2496	0.3620	1.2000	3.4000	3.1800	0.1350
Large Truck (Left Turn)	81	3.8342	0.7487	2.4000	6.0330	2.9400	0.2100
Large Truck (Through)	95	3.1126	0.6396	1.7000	4.9000	2.7900	0.4180

As we can see from the outlier plot of departure headway data for each vehicle type and movement type, there are no observations pointed out as an outlier. The p-value from Grubs test result in the above table for the all variables is greater than 0.05 (level of significance) which leads to the

conclusion to accept the Null hypothesis which is All the data values come from the same population.

APPENDICE C

Detailed analysis of distribution fitting for vehicles departure headway data

➤ Distribution fitting for departure Headway data of Small Cars making through movement

Histogram plot of departure headway data for small cars making through movement shows that the data is skewed to the right and the probability plot shows the data points are far from the straight line especially for lower and upper data values. The above results indicate that there is a probable departure from normality. The p-value from Anderson-Darling test result is less than **0.005** which is smaller than the significance level (**0.05**) and this confirms that the data does not come from normally distributed population.

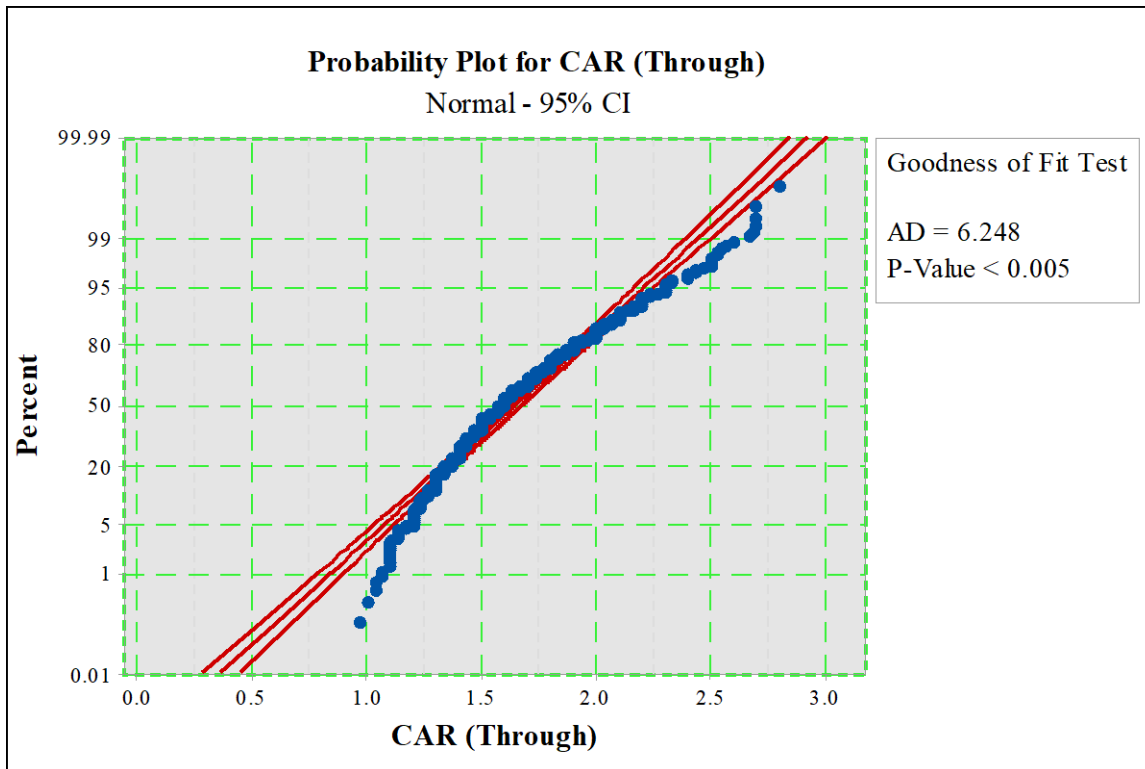


Figure C-1: Normal Probability Plot of CAR (TH)

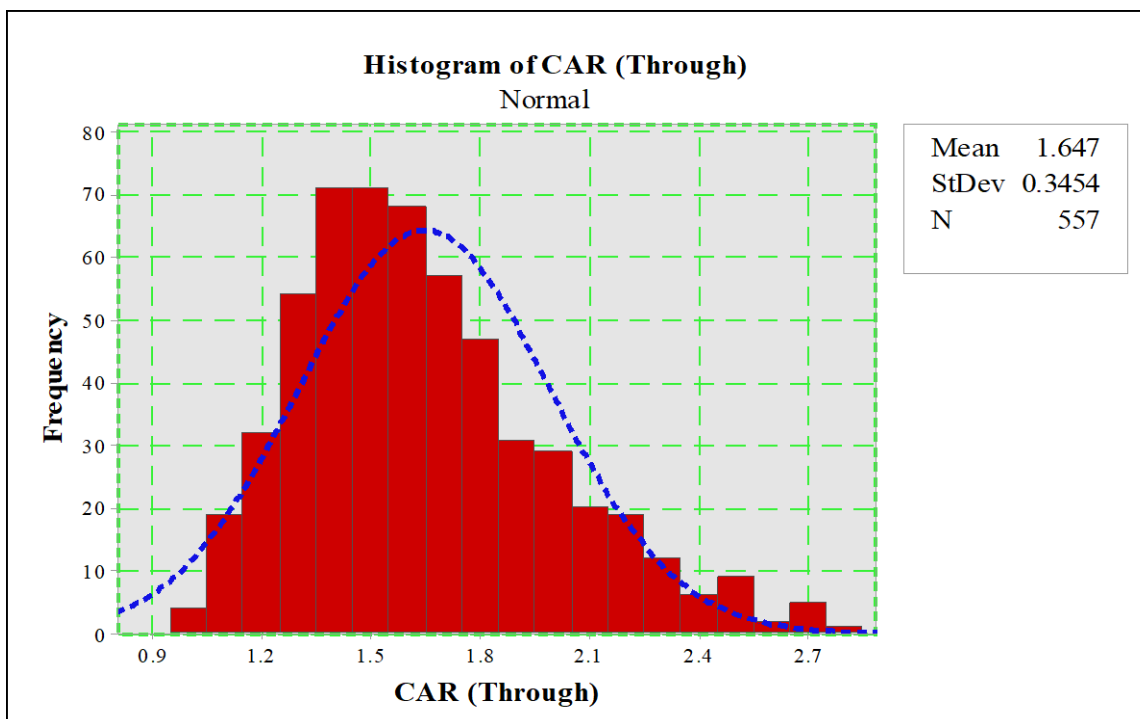


Figure C-2: Histogram of CAR (TH) With Normal Curve

Among all possible probability distributions tried to fit the data only the Largest Extreme Value was found to fit the data.

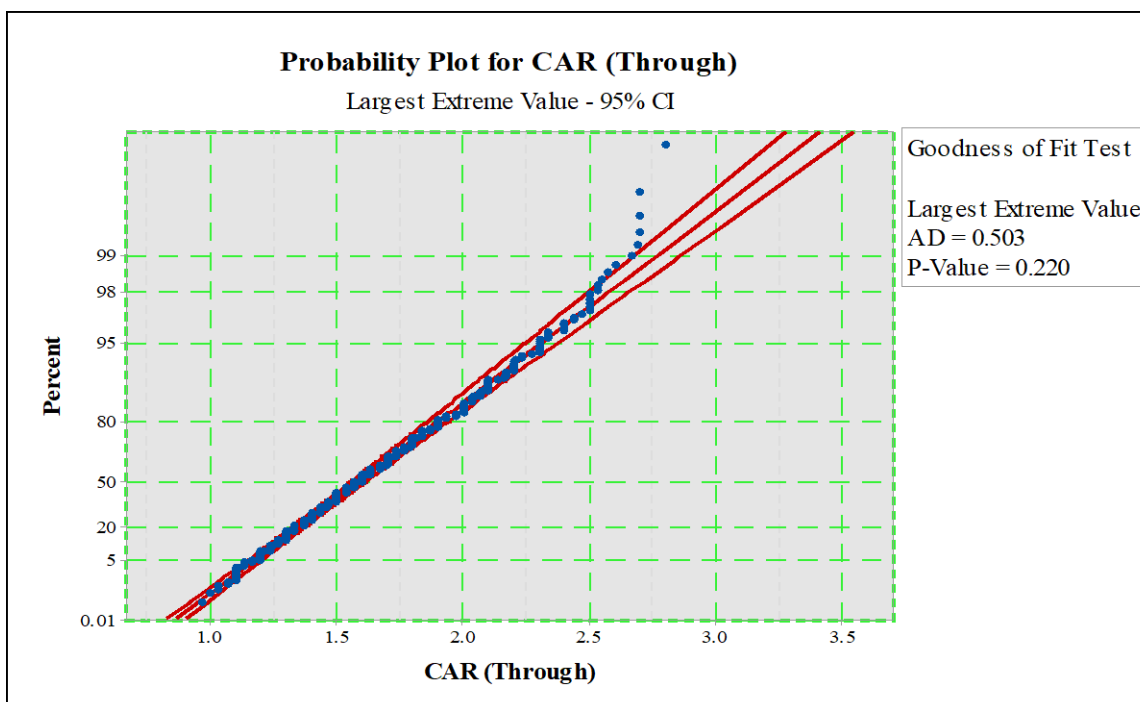


Figure C-3 : Largest Extreme Value Probability Plot for CAR (TH)

The p-value from the Anderson-Darling test is **0.220** which is greater than the significance level (**0.05**). The probability plot also confirms the test result, the data values are clustered near the straight line. The above results show that departure headway of small cars making a through movement follows Largest Extreme Value probability distribution.

The Largest Extreme Value probability distribution function is generally written as:-

$$f(x) = \frac{1}{\sigma} e^{\left[-\frac{1}{\sigma} (x - \mu) - e^{-\left(\frac{1}{\sigma} (x - \mu) \right)} \right]} \dots \dots \dots \text{Eqn - C1}$$

$$-\infty < \mu < \infty$$

$$\sigma > 0$$

Where:-

μ is Location Parameter

σ is scale Parameter

The table below shows estimated distribution parameter and Anderson-Darling test result

Table C-1: Estimated Parameter and AD Test Result for CAR (TH)

Distribution	Estimated Parameters				Anderson-Darling Test Result		
	Location	Shape	Scale	Threshold	AD	P	LRT P
Largest Extreme Value	1.48755		0.27828		0.503	0.220	

- Distribution fitting for departure Headway data of Small Buses making left turning movement

Normality test result, probability plot and histogram of departure headway data for left turning small bus indicate a significant departure from normality. The data points are not gathered around the straight line. The Anderson-Darling test result implies that the data does not follow a normal distribution. The p-value from the test is less than 0.005 which is less than the significance level (0.05) so we accept the Alternate hypothesis.

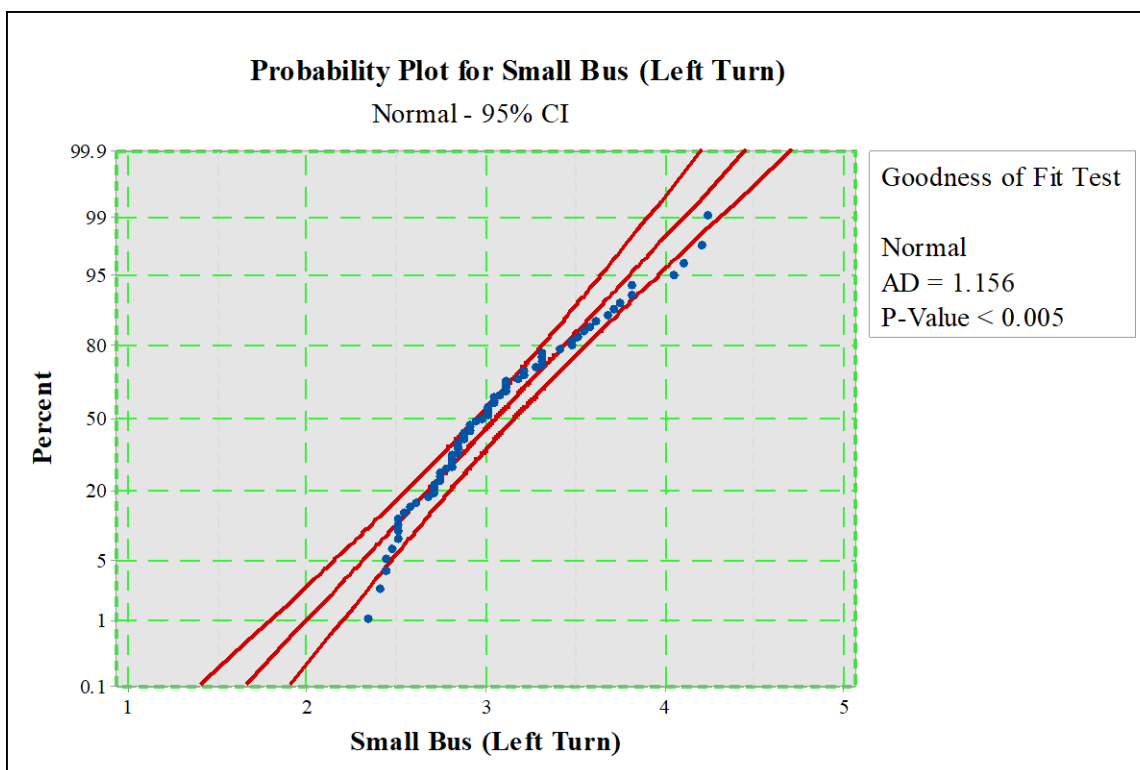


Figure C-4: Normal Probability Plot for Small Bus (LT)

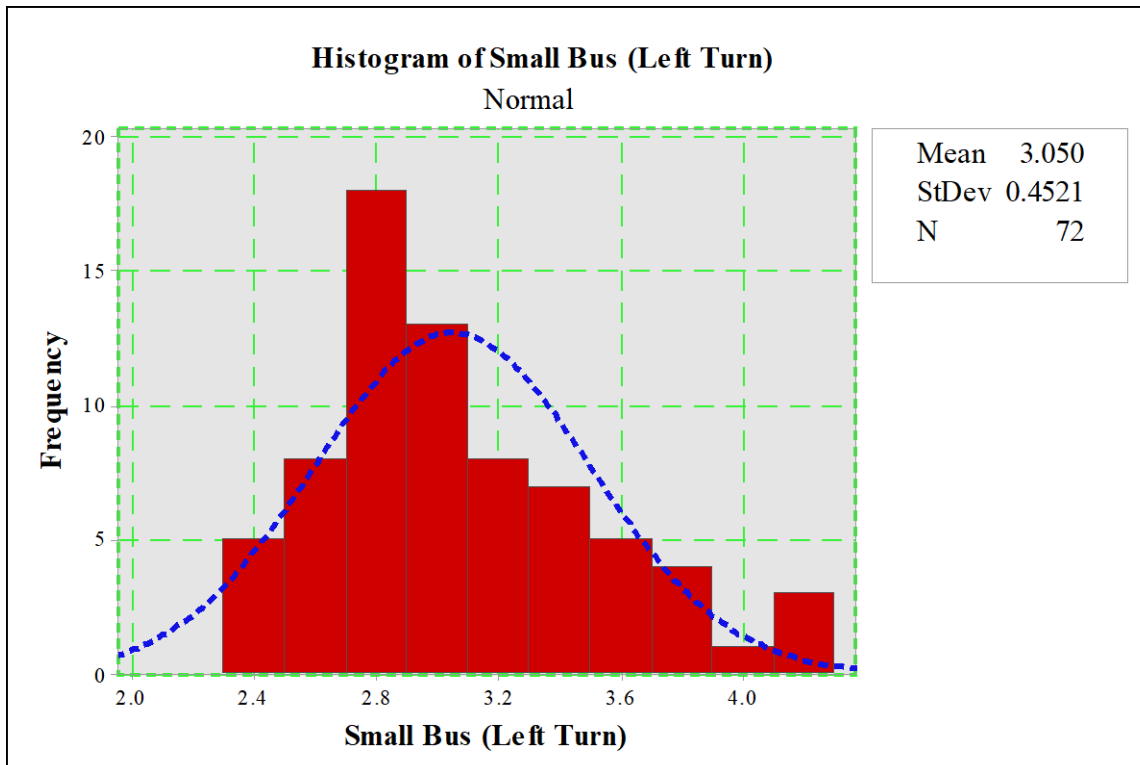


Figure C-5: Histogram of Small Bus with Normal Curve

Lognormal, 3-Parameter Weibull, Largest Extreme Value, and Log-Logistic probability distributions were found to fit the data. But 3-Parameter Weibull probability distribution have significantly lower AD value and larger p-value than the others which makes it a better fit for the data. The p-value from the test is greater than **0.500** which is greater than any significance level that can be set.

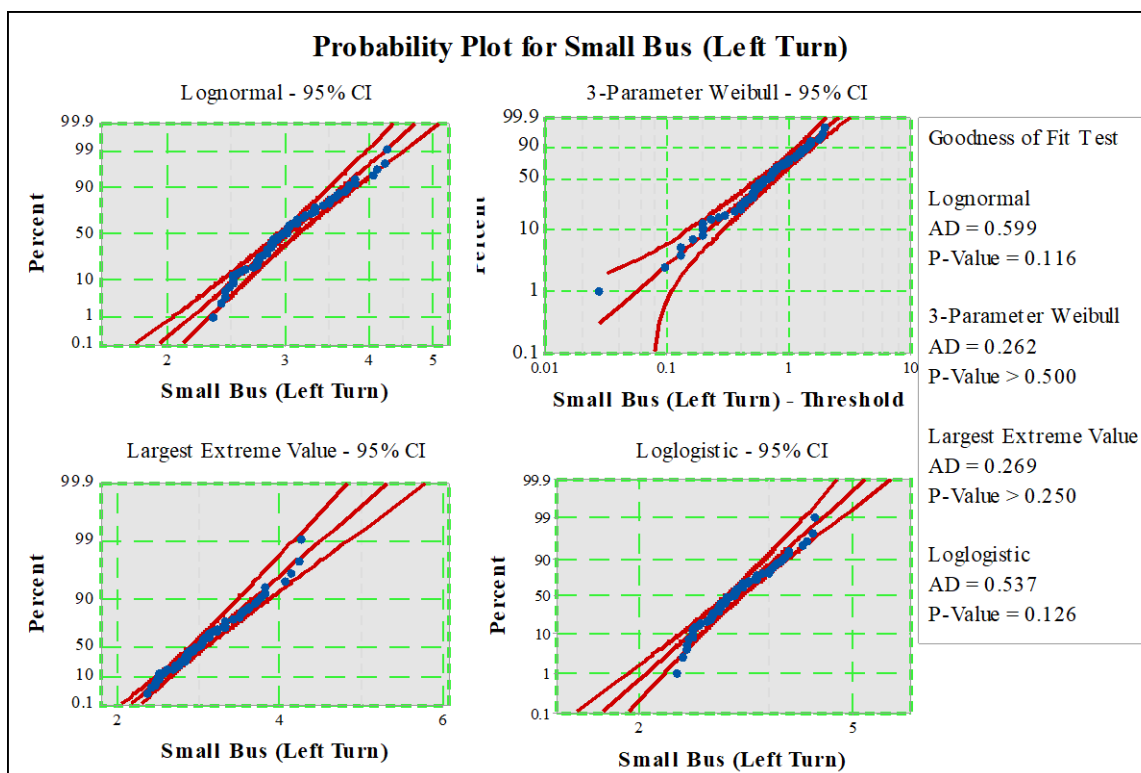


Figure C-6: Probability Plot of Small Bus (LT)

The 3-Parameter Weibull probability distribution function is generally written as:-

$$f(t) = \frac{\beta}{\eta} \left[\frac{t-\gamma}{\eta} \right]^{\beta-1} e^{-\left[\frac{t-\gamma}{\eta} \right]^{\beta}} \dots\dots\dots \text{Eqn-C2}$$

$$f(t) \geq 0, t \geq \gamma, \beta > 0, \eta > 0, \text{ and } -\infty < \gamma < +\infty$$

Where:-

η is scale parameter

β is shape parameter

γ is location parameter

The following table shows the estimated parameters along with the Anderson-Darling test result. The smaller LRT p-value implies that the addition of the third parameter actually improves the model.

Table C-2: Estimated Parameter and AD Test Result for Small Bus (LT)

Distribution	Estimated Parameters				Anderson-Darling Test Result		
	Location	Shape	Scale	Threshold	AD	P	LRT P
Lognormal	1.10497		0.14322		0.599	0.1160	
3-Parameter Weibull		1.69611	0.83285	2.30669	0.262	>0.5000	0.0000
Largest Extreme Value	2.84352		0.35450		0.269	>0.250	
Log-Logistic	1.09674		0.08192		0.537	0.126	

- Distribution fitting for departure Headway data of Small Buses making through movement

Histogram plot with a fit line signify that the data is skewed to the right and the p-value from normality test is **<0.005** which is significantly lesser than the significance level (**0.05**). The probability plot shows the data points are not close enough to the theoretical cumulative distribution function. The above points confirm that the data does not come from a normally distributed population.

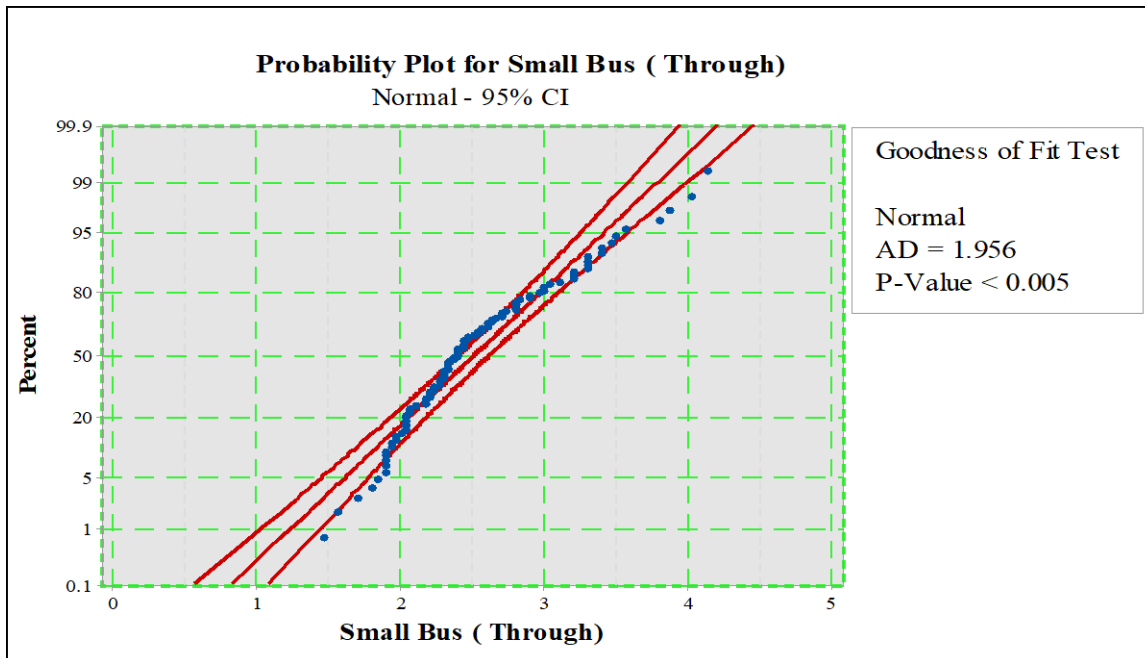


Figure C-7: Normal Probability Plot of Small Bus (TH)

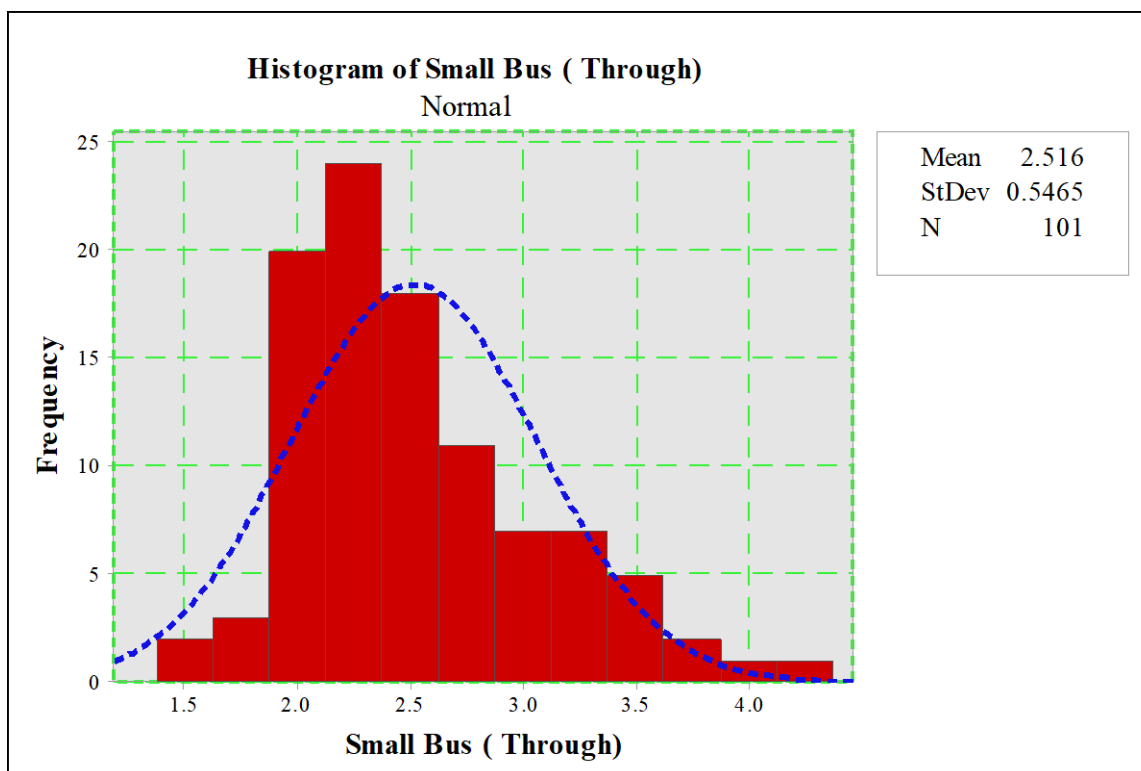


Figure C-8: Histogram of Small Bus (TH) With Normal Curve

Among the tested distributions Lognormal and Largest Extreme Value probability distributions significantly fit the data. The Largest Extreme Value probability distributions have smaller AD value and larger p-value than the other distributions which proves that it's a better fit for the data. The Largest Extreme Value probability distribution function is generally written as (*Eqn – C1*). The following table shows the estimated parameters along with the Anderson-Darling test result.

Table C-3: Estimated Parameters and AD Test Result for Small Bus (TH)

Distribution	Estimated Parameters				Anderson-Darling Test Result		
	Location	Shape	Scale	Threshold	AD	P	LRT P
Lognormal	0.90085		0.20866		0.747	0.050	
Largest Extreme Value	2.26773		0.43225		0.361	>0.250	

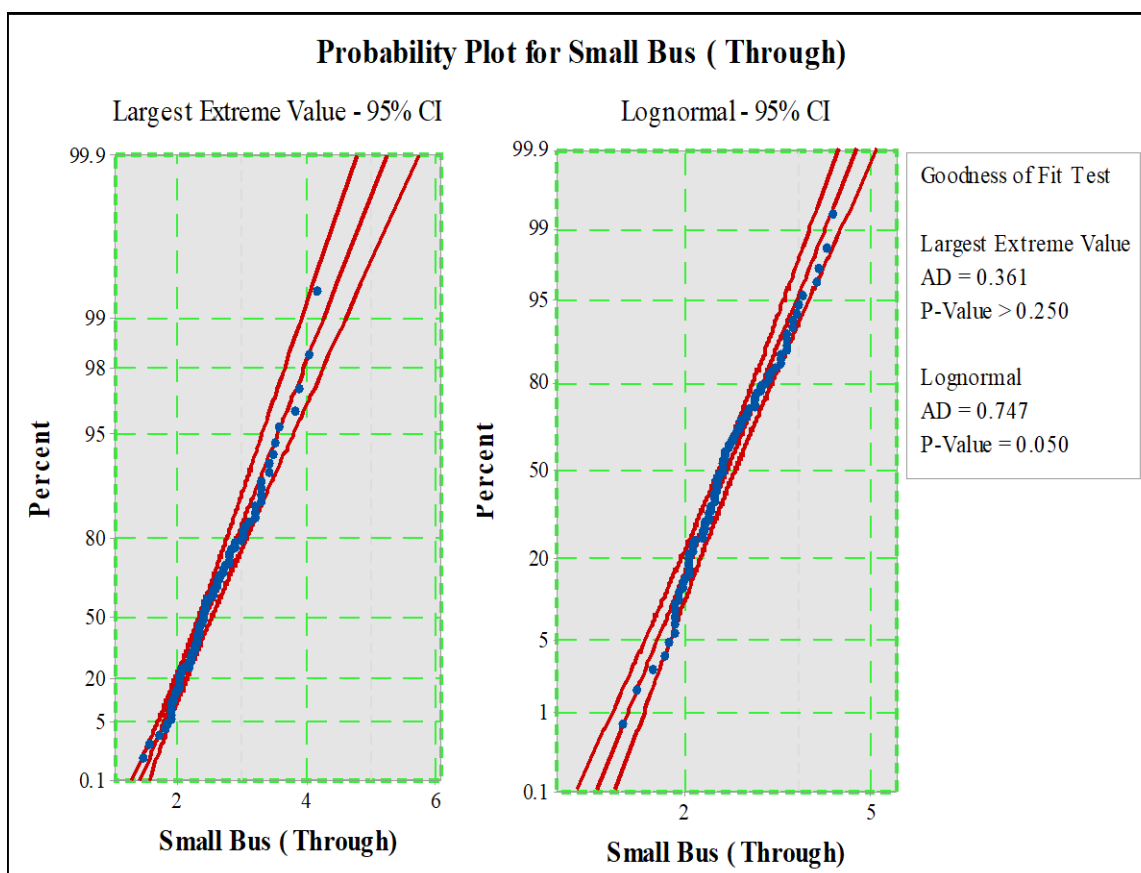


Figure C-9: Probability Plot for Small Bus (TH)

- Distribution fitting for departure Headway data of Large Buses making Left turning movement

Normal probability plot of the data shows that the data points do not cluster along the straight line which is a sign of departure from normality. The result from Anderson-Darling normality test also confirms that the data does not come from a normally distributed population. The p-value from the test is **0.028**, less than the significance level (**0.05**).

Lognormal, 3-Parameter Weibull, Largest Extreme Value, Gamma, Logistic, and Log-Logistic probability distributions significantly fits the data but 3-Parameter Weibull probability distribution have a smaller AD value and Higher p-value than the others so is a better fit for the data.

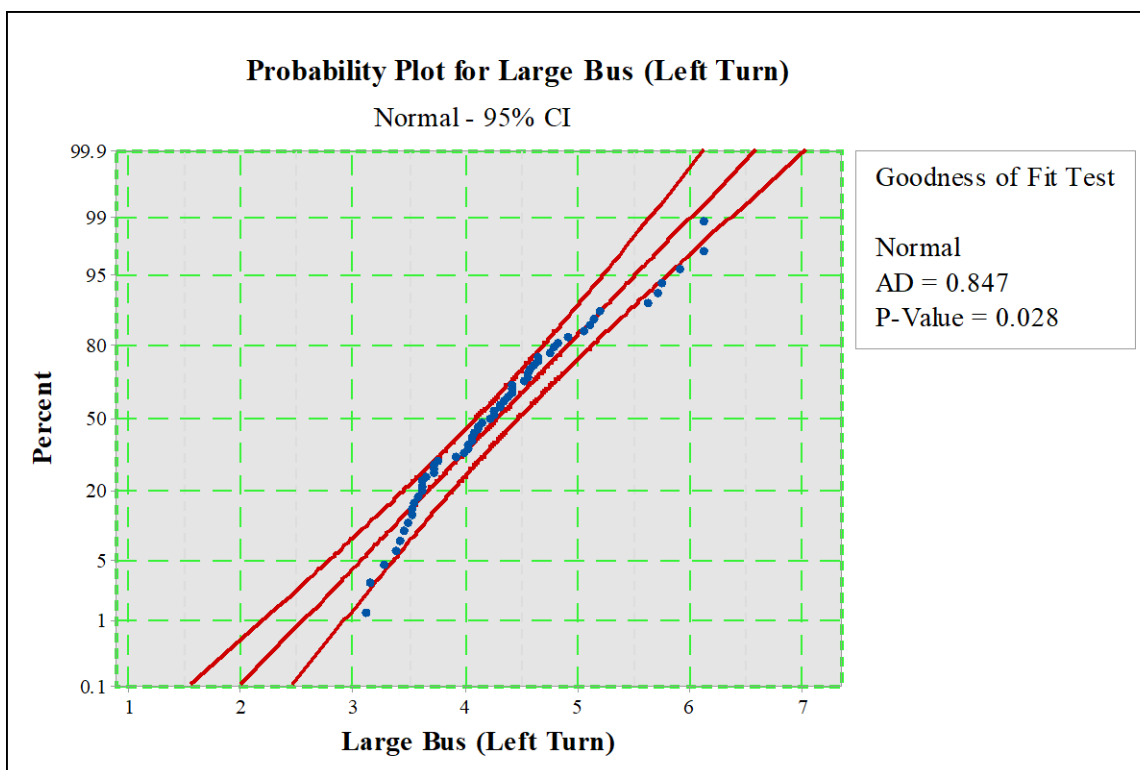


Figure C-10: Normal Probability Plot for Large Bus (LT)

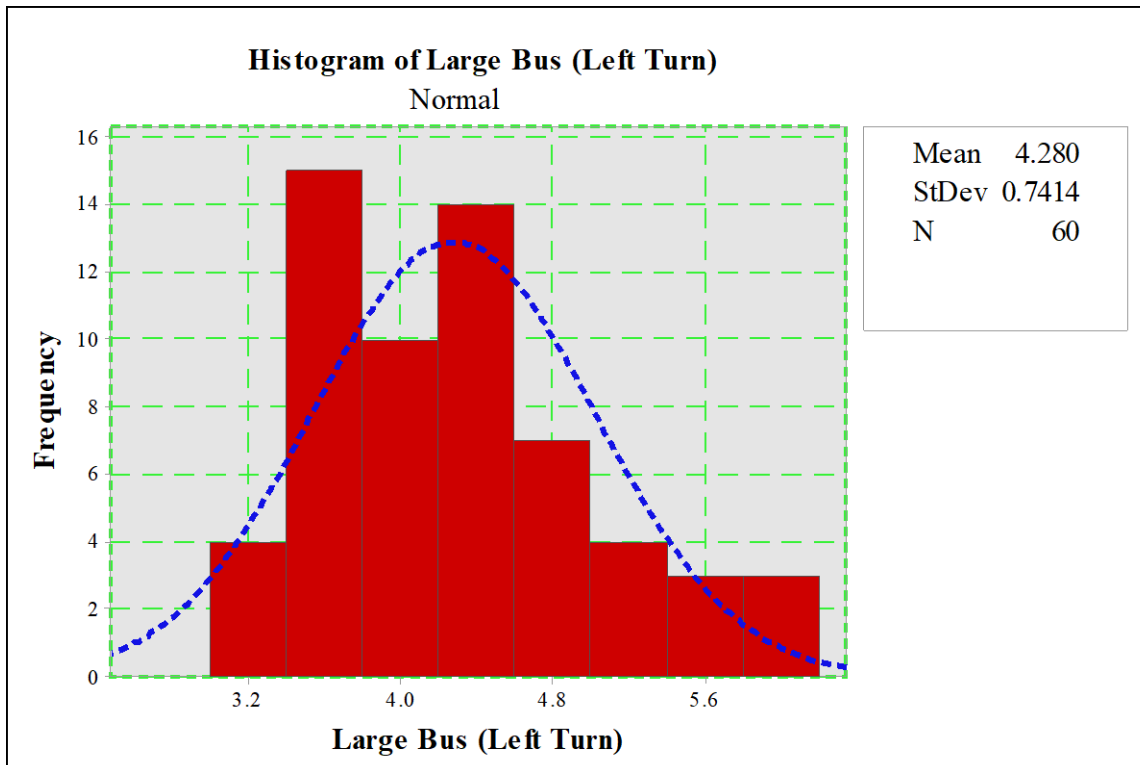


Figure C-11: Histogram of Large Bus (LT) With Normal Curve

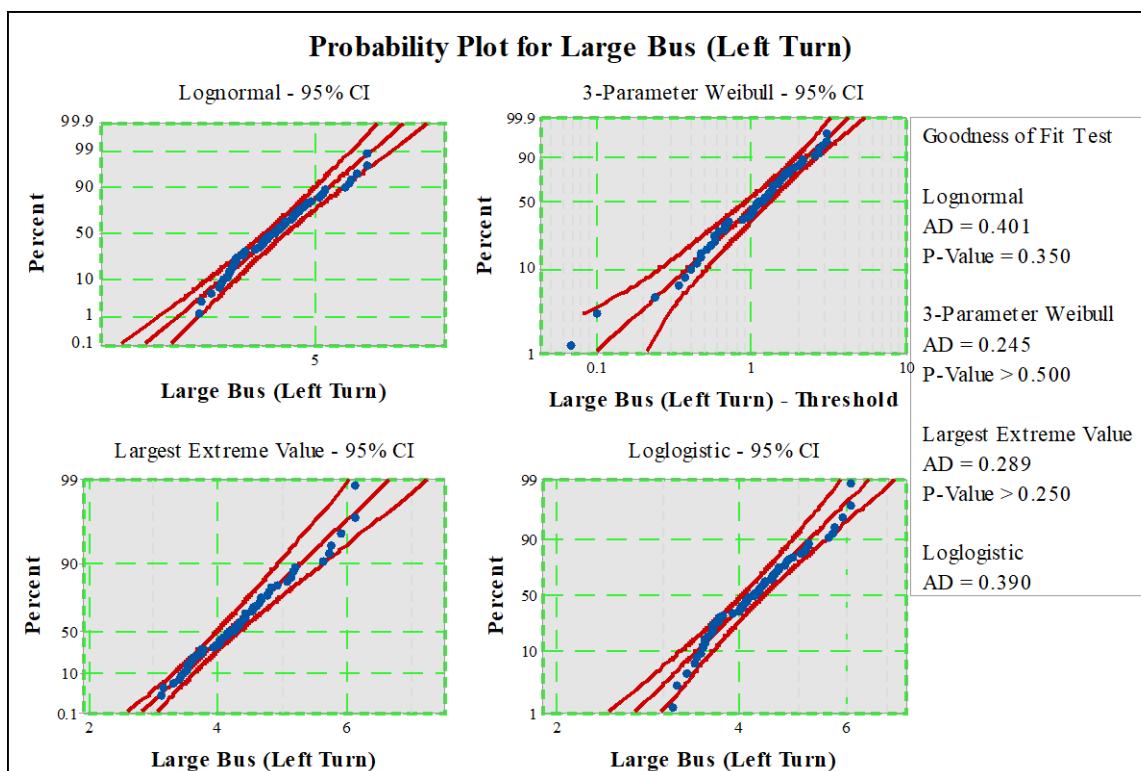


Figure C-12: Probability Plot of Large Bus (LT)

The 3-Parameter Weibull probability distribution function is generally written as (*Eqn-C2*). The following table shows the estimated parameters along with the Anderson-Darling test result. The smaller LRT p-value implies that the addition of the third parameter actually improves the model.

Table C-4: Estimated Parameter and AD Test Result for Large Bus (LT)

Distribution	Estimated Parameters				Anderson-Darling Test Result		
	Location	Shape	Scale	Threshold	AD	P	LRT P
Lognormal	1.43991		0.16783		0.401	0.350	
3-Parameter Weibull		1.73792	1.39820	3.03328	0.245	>0.500	0.0000
Largest Extreme Value	3.93914		0.58843		0.289	>0.250	
Log-Logistic	1.43343		0.09642		0.390	>0.250	

- Distribution fitting for departure Headway data of Large Buses making through movement

Histogram plot of departure headway data of large bus making through movement shows that the data is skewed to the right. The data points on the probability plot do not gather around the straight line. The p-value from the Anderson-Darling normality test is **<0.005** which is significantly less than the significance level. So the data do not follow a normal distribution.

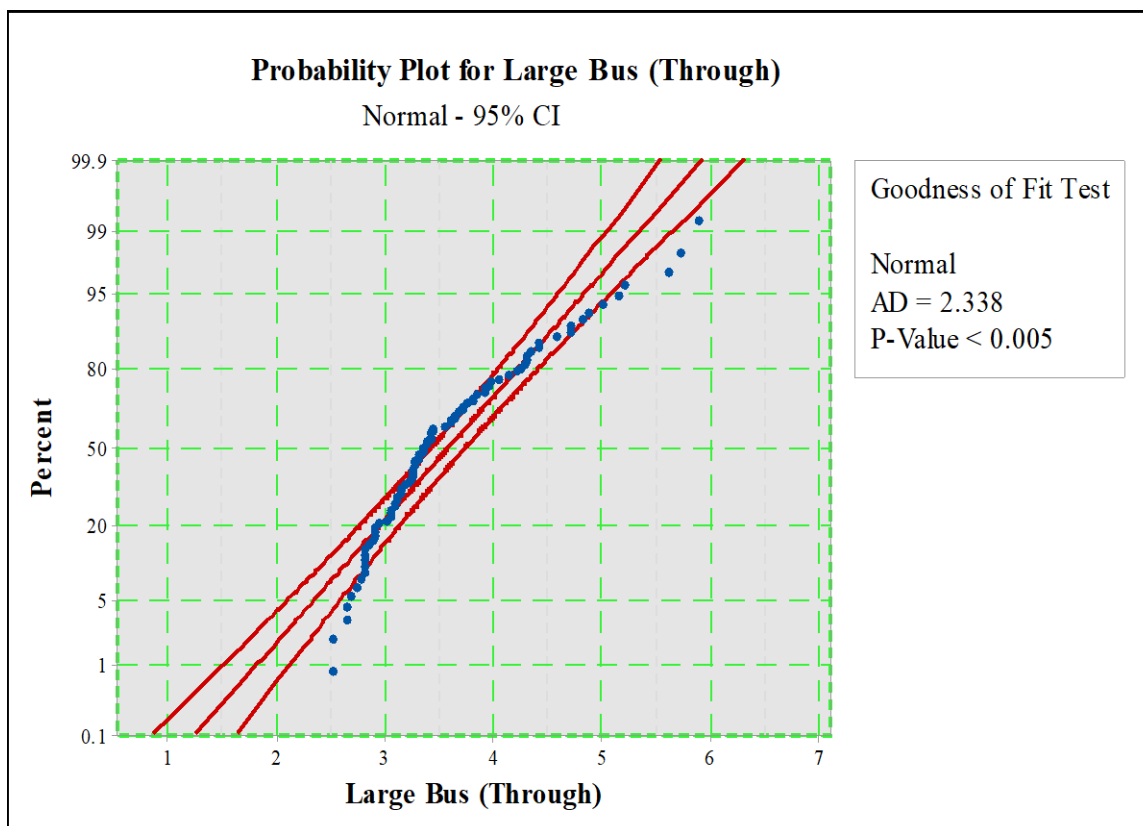


Figure C-13: Normal Probability Plot for Large Bus (TH)

Among the tested distributions 3-Parameter Weibull and Largest Extreme Value probability distributions are a good fit for the data. But we have to determine which one of the two is a better fit. 3-Parameter Weibull probability distribution has a smaller AD value and higher p-value than Largest Extreme Value probability distribution so it is a better fit for the data.

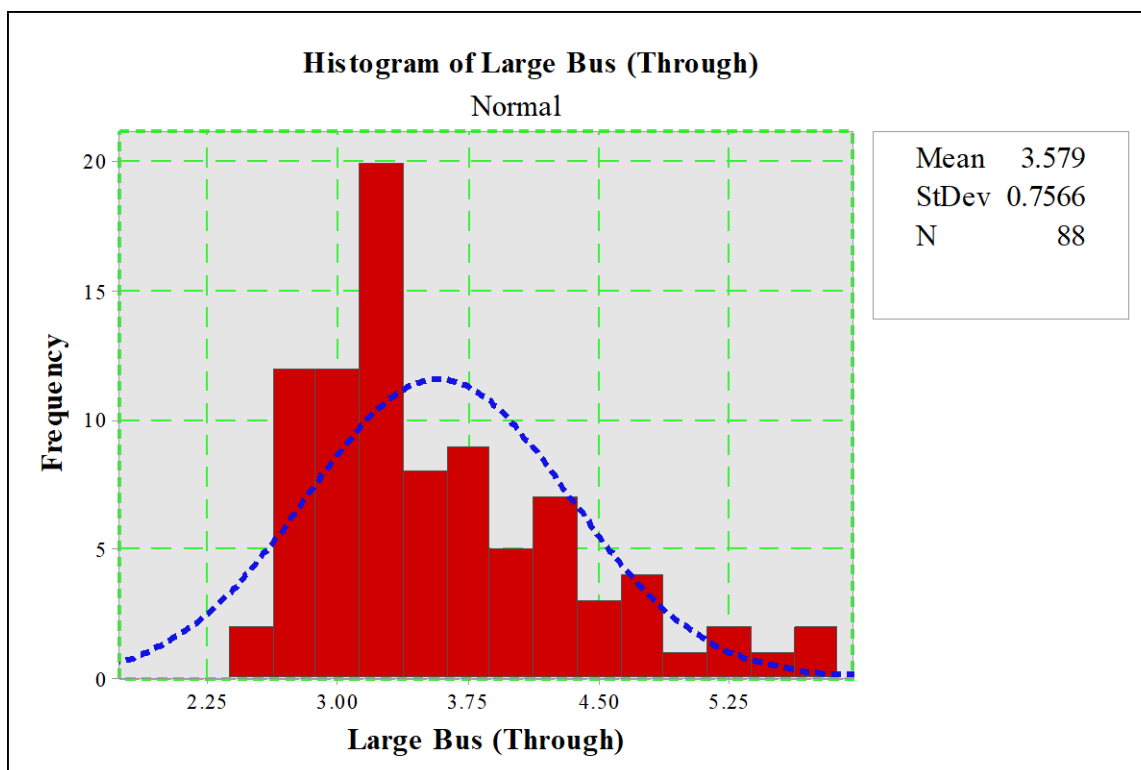


Figure C-14: Histogram of Large Bus (TH) With Normal Curve

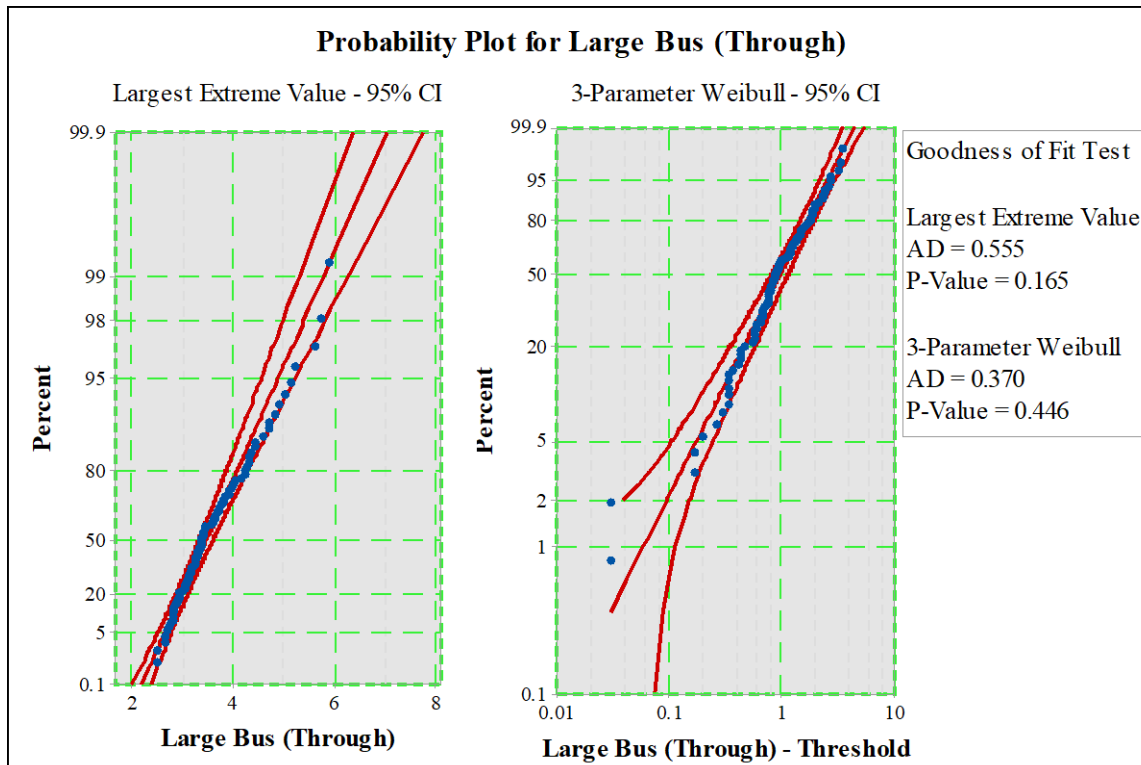


Figure C-15: Probability Plot of Large Bus (TH)

The 3-Parameter Weibull probability distribution function is generally written as (*Eqn-C2*). The following table shows the estimated parameters along with the Anderson-Darling test result. The smaller LRT p-value implies that the addition of the third parameter actually improves the model.

Table C-5: Estimated Parameters and AD Test Result for Large Bus (TH)

Distribution	Estimated Parameters				Anderson-Darling Test Result		
	Location	Shape	Scale	Threshold	AD	P	LRT P
Largest Extreme Value	3.24481		0.54894		0.555	0.1650	
3-Parameter Weibull		1.50983	1.22972	2.47	0.370	0.4460	0.0000

- Distribution fitting for departure Headway data of small trucks making left turning movement

Probability plot of departure headway data of left turning small truck demonstrate that the data points are not close enough to the theoretical cumulative normal distribution function. The p-value from the Anderson-Darling normality test is **0.029** which is less than the significance level (**0.05**). So we accept the alternate hypothesis and try to fit a non-normal distribution to the data.

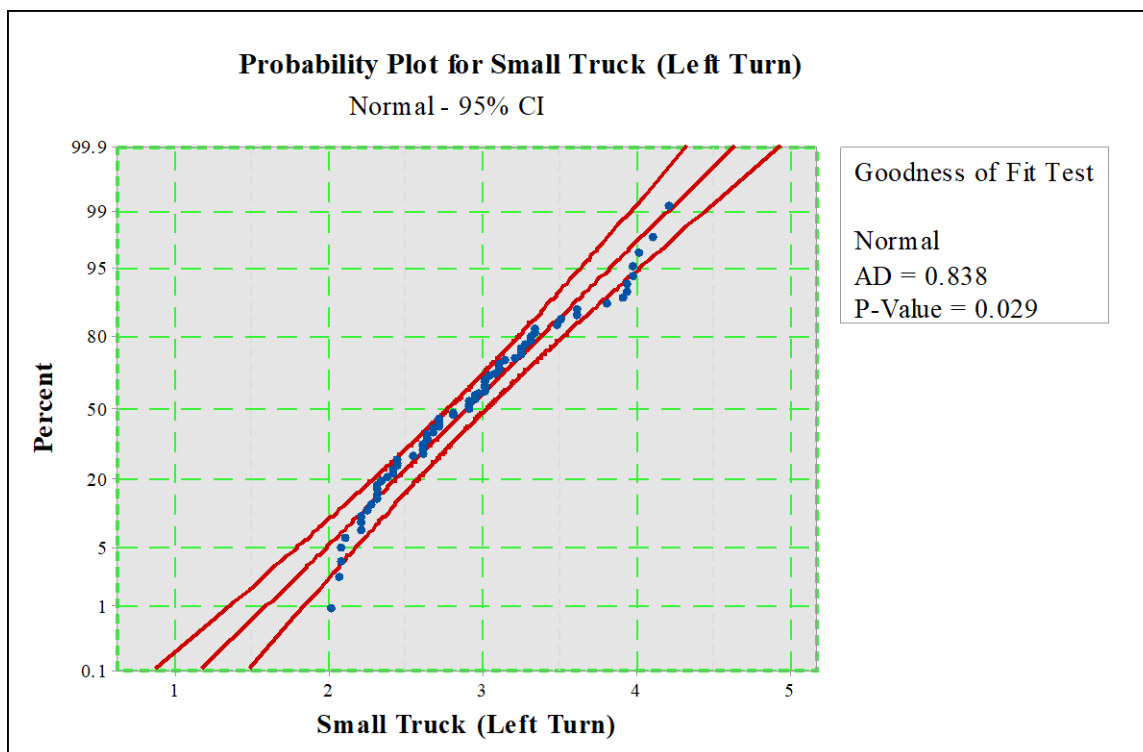


Figure C-16: Normal Probability Plot for Small Truck (LT)

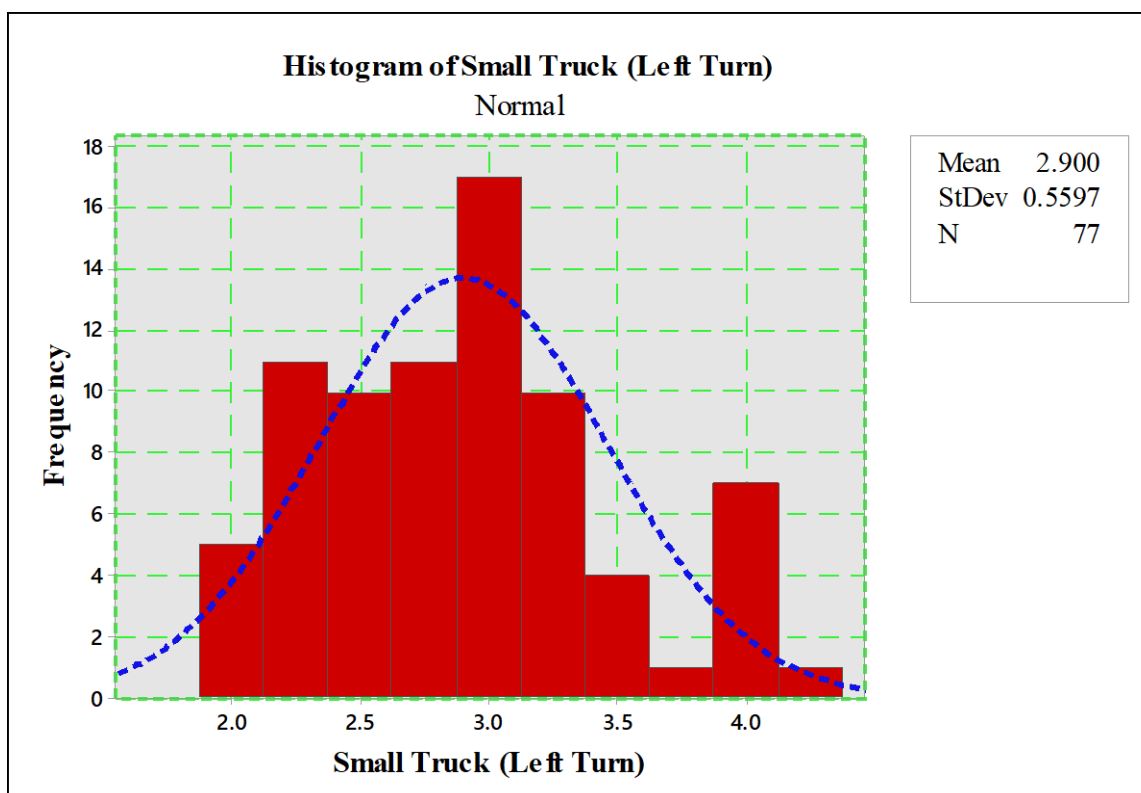


Figure C-17: Histogram of Small Truck (LT) with Normal Curve

Lognormal, 3-Parameter Weibull, Largest Extreme Value, Gamma, and Log-Logistic probability distributions are a good fit for the data. But the 3-Parameter Weibull probability distribution has a smaller AD value and a higher p-value than the others which makes it a better fit for the data.

The 3-Parameter Weibull probability distribution function is generally written as (*Eqn-C2*). **Table C-6** shows the estimated parameters along with the Anderson-Darling test result. The smaller LRT p-value implies that the addition of the third parameter actually improves the model.

Table C-6: Estimated Parameter and AD Test Result for Small Truck (LT)

Distribution	Estimated Parameters				Anderson-Darling Test Result		
	Location	Shape	Scale	Threshold	AD	P	LRT P
Lognormal	1.04683		0.19015		0.424	0.3110	
3-Parameter Weibull		1.74141	1.06827	1.94674	0.356	0.4740	0.0000
Largest Extreme Value	2.63659		0.46150		0.437	>0.250	
Gamma		27.98919	0.10362		0.523	0.2010	

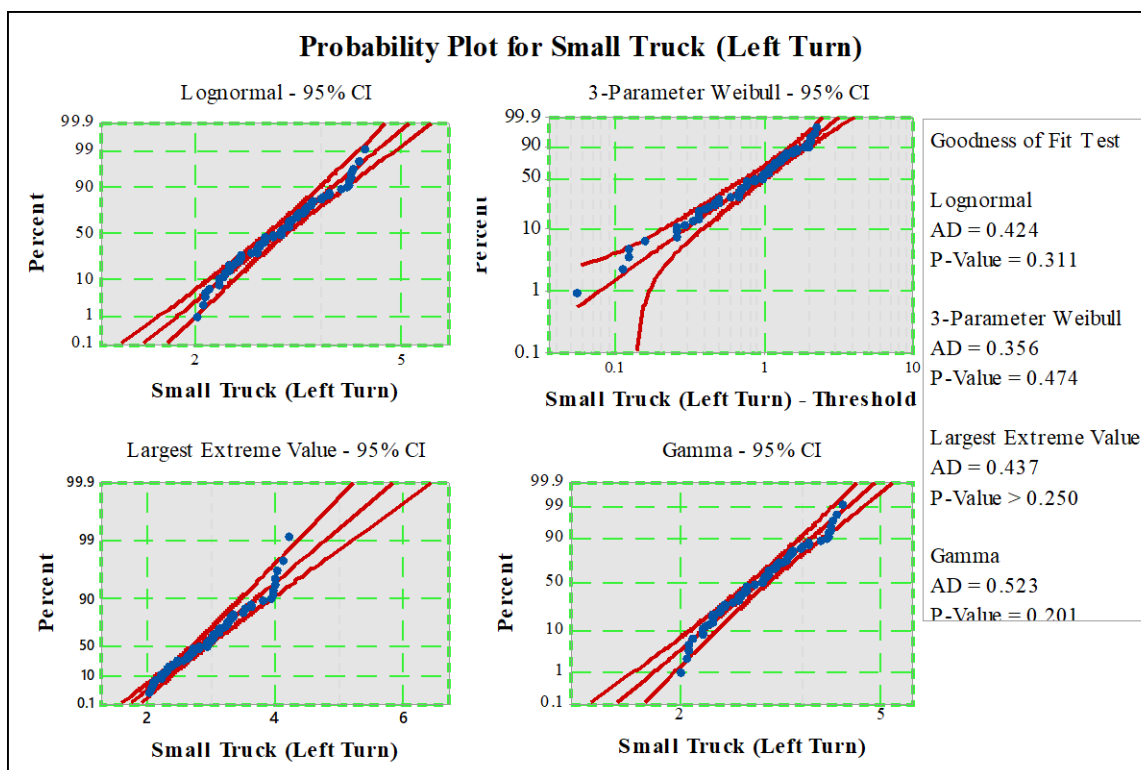


Figure C-18: Probability Pot for Small Truck (LT)

- Distribution fitting for departure Headway data of small truck making through movement

Normal Probability plot of departure headway data for small truck making through movement shows that the data points does not fall in the vicinity of the straight line. The p-value from Anderson-Darling normality test result is **0.005** that is less than the significance level (**0.05**). So we have enough evidence to say that the data do not come from a normally distributed population. Among the candidates Probability Distributions Lognormal, Gamma, and Log-Logistic probability distributions are a good fit for the data. But the Log-Logistic probability distribution has a smaller AD value and a higher p-value than the others. So the Log-Logistic probability distribution is a better fit for the data.

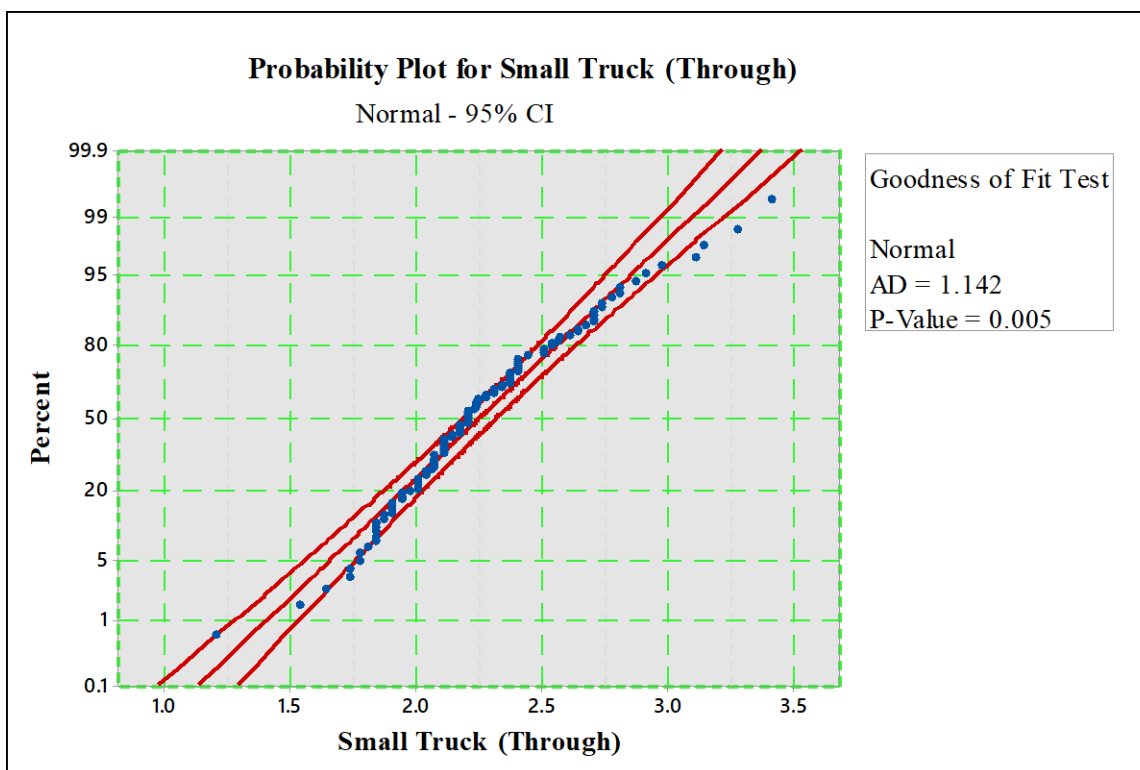


Figure C-19: Normal Probability Plot of Small Truck (TH)

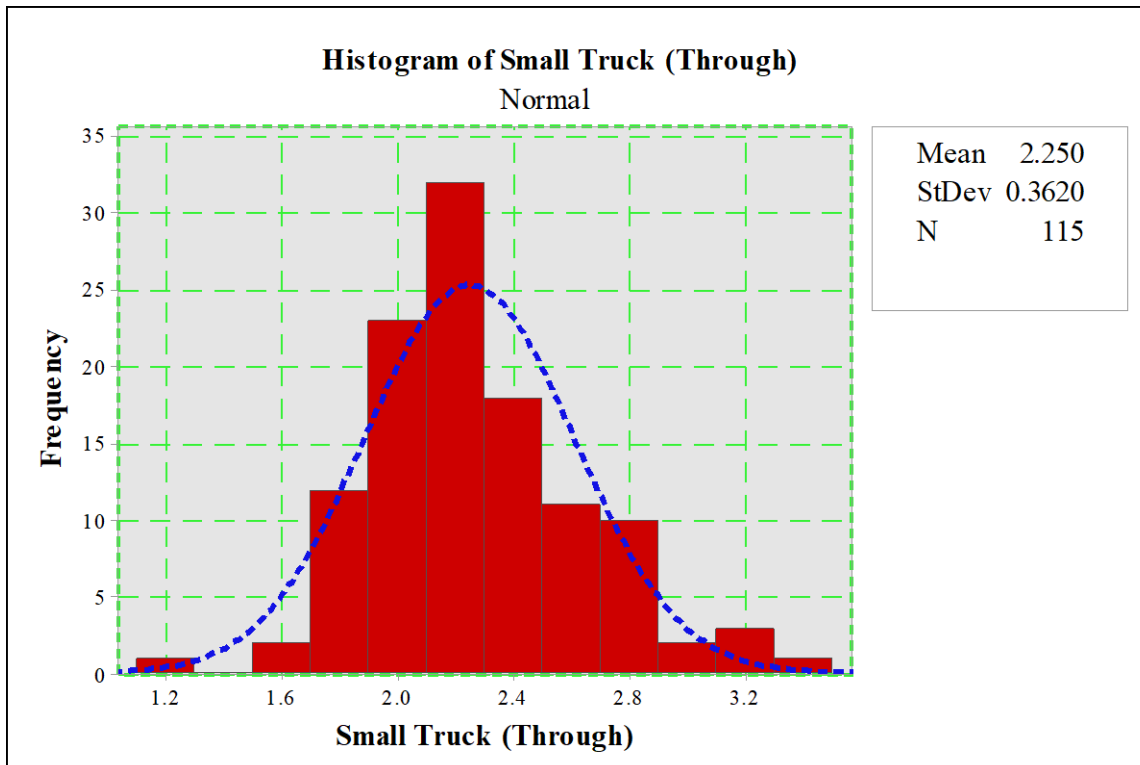


Figure C-20: Histogram of Small Truck (TH) with Normal Curve

The Log-Logistic probability distribution function is generally written as:-

$$f(x) = \frac{e^{\left[\frac{\ln x - \mu}{\sigma}\right]}}{\sigma \left[1 + e^{\left(\frac{\ln x - \mu}{\sigma}\right)}\right]^2} \dots \dots \dots \text{Eqn - C3}$$

Where:-

μ is location parameter

σ is scale parameter

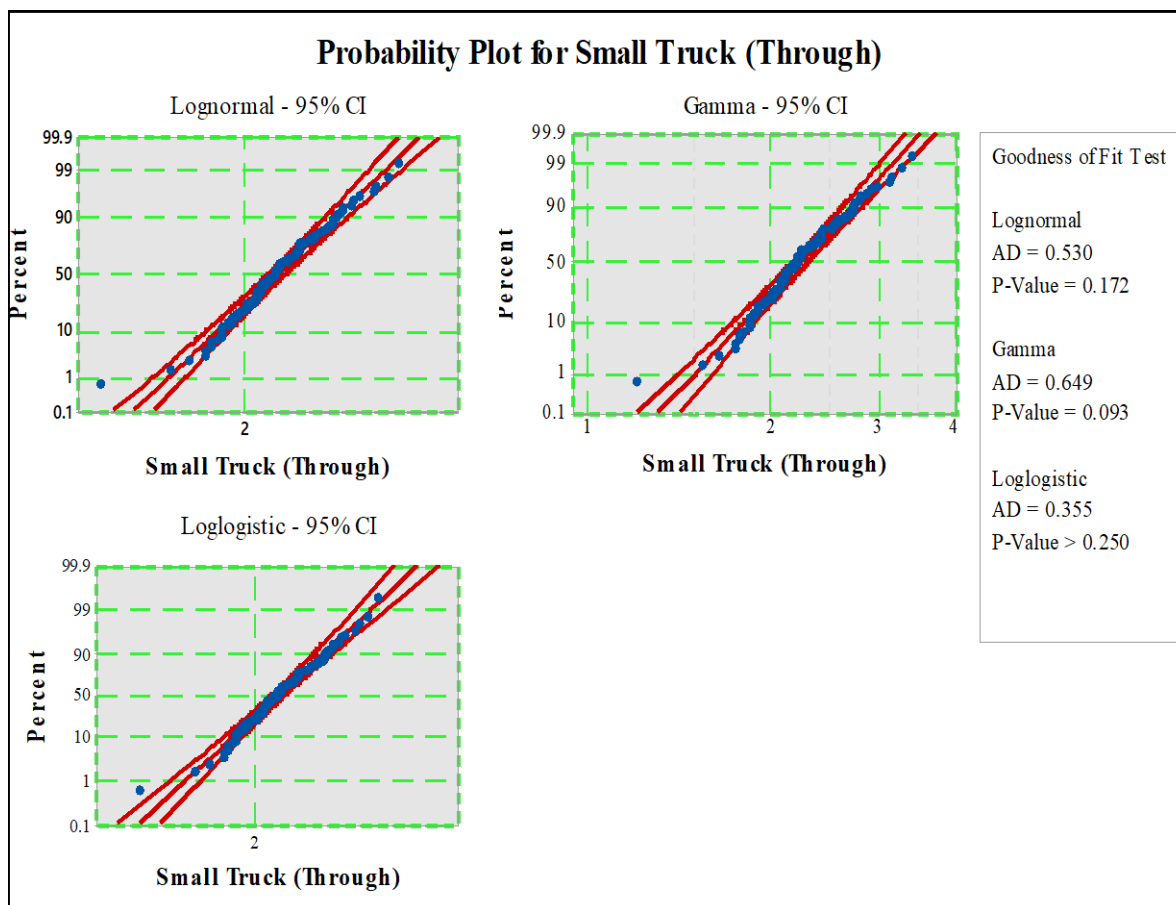


Figure C-21: Probability Plot for Small Truck (TH)

Estimated parameters and Anderson-Darling test result is presented in the table below.

Table C-7: Estimated Parameters and AD Test Result for Small Truck (TH)

Distribution	Estimated Parameters				Anderson-Darling Test Result		
	Location	Shape	Scale	Threshold	AD	P	LRT P
Lognormal	0.79805		0.16042		0.530	0.1720	
Gamma		39.5574	0.05687		0.649	0.0930	
Log-Logistic	0.79547		0.08799		0.355	>0.2500	

- Distribution fitting for departure Headway data of Large Truck making Left turning movement

Empirical cumulative distribution function of departure headway data for left turning large truck do not match with the theoretical cumulative distribution function as can be seen in the normal probability plot. The p-value from the normality test is also less than the significance level so we can say that the data does not follow a normal distribution.

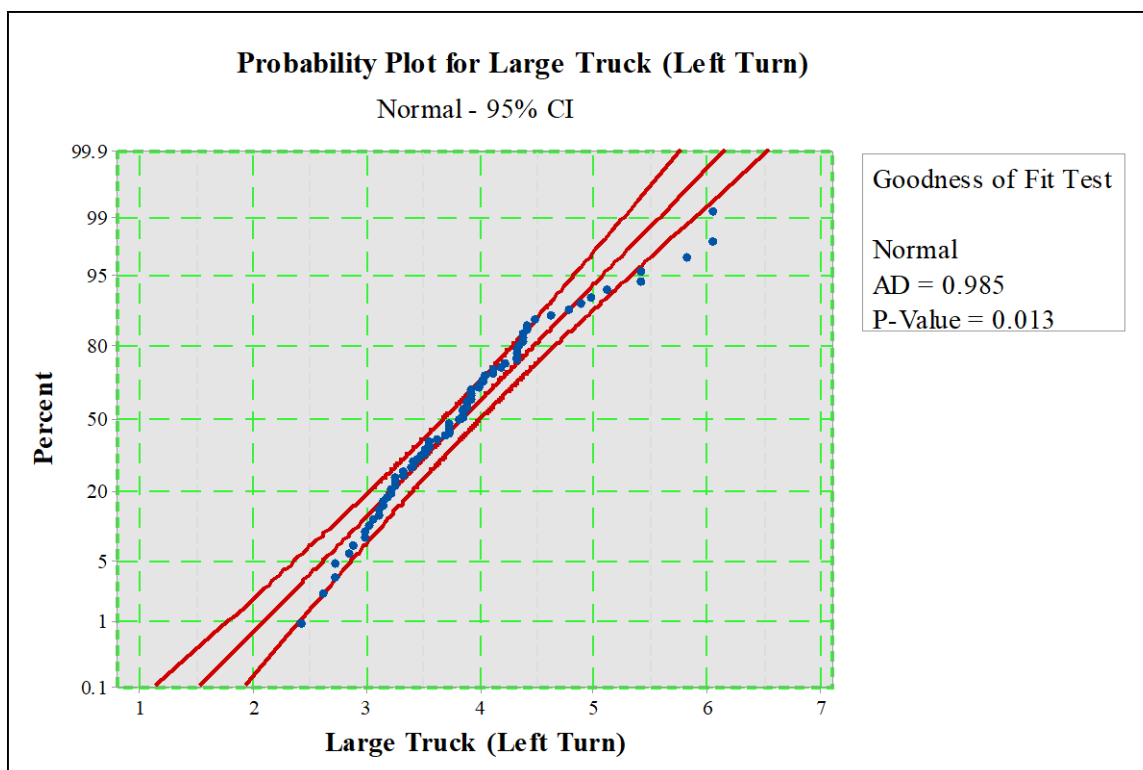


Figure C-22: Probability Plot of Large Truck (LT)

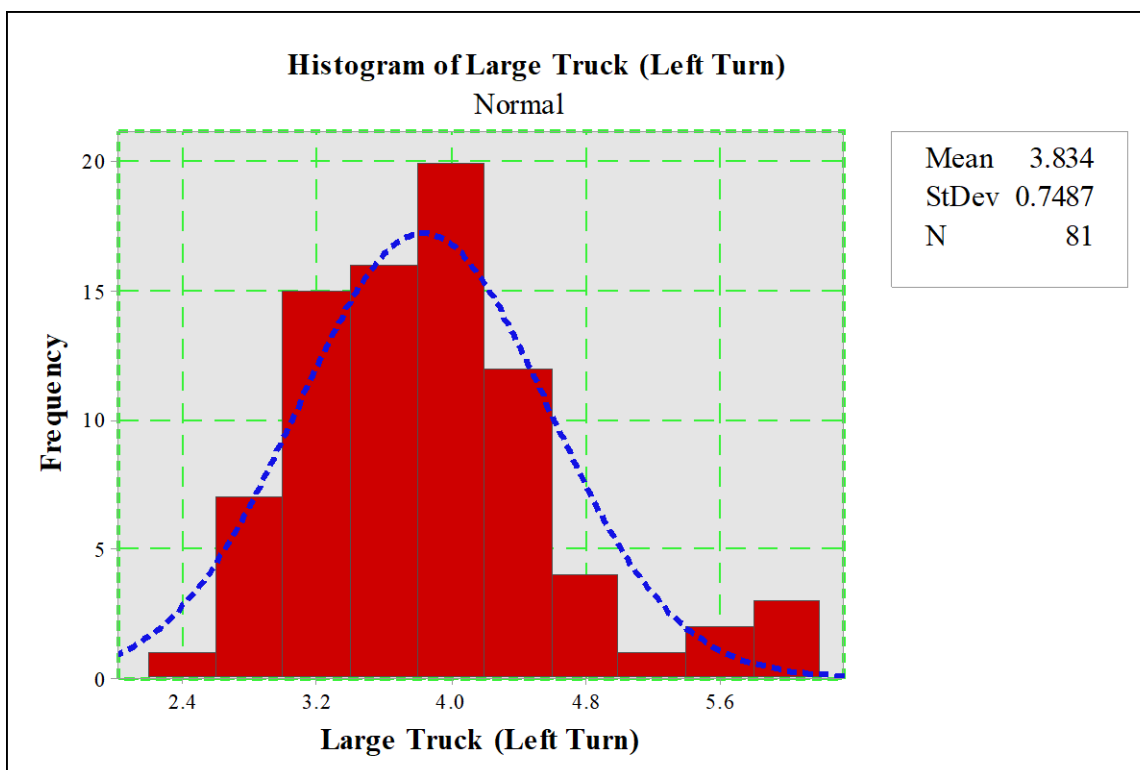


Figure C-23: Histogram of Large Truck (LT) with Normal Curve

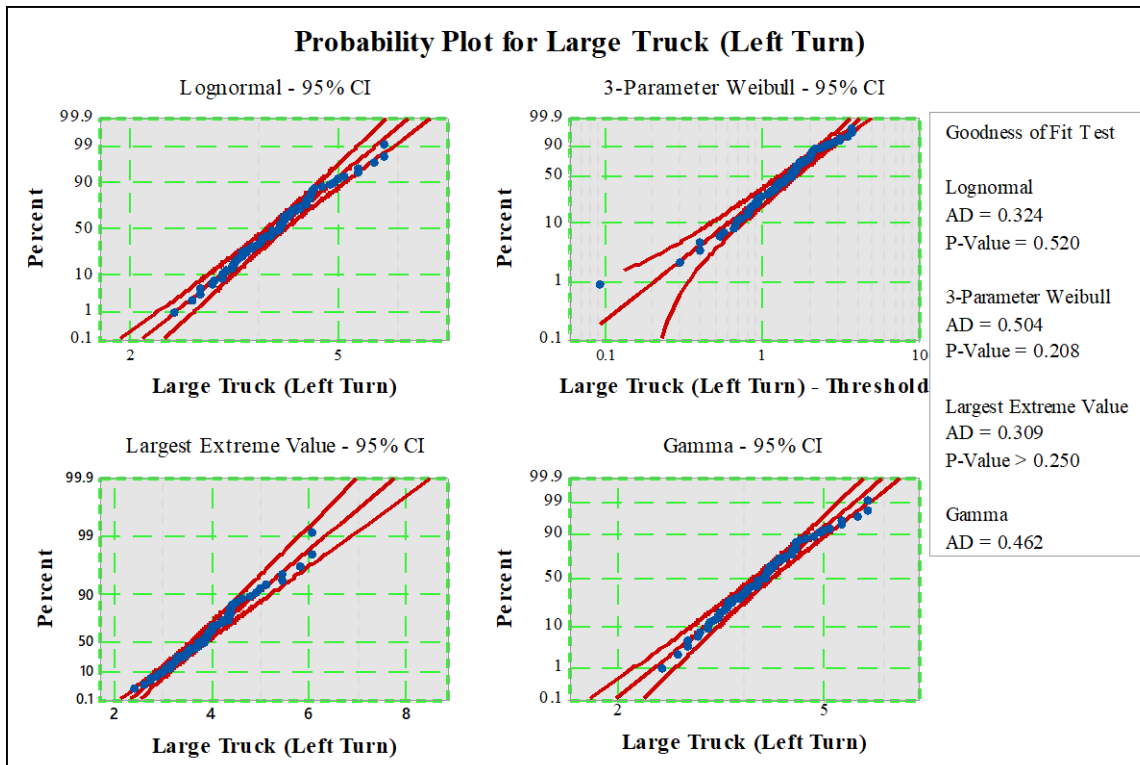


Figure C-24: Probability Plot of Large Truck (LT)

Among the tested distributions Lognormal, 3-Parameter Weibull, Largest Extreme Value, Gamma, Logistic, and Log-Logistic probability distributions are a good fit for departure headway data of left turning large truck. But the Lognormal probability distribution has a smaller AD value and higher p-value than the others, so it is a better fit for the data.

The Lognormal probability distribution function is generally written as:-

$$f(x) = \frac{1}{\sqrt{2\pi\sigma x}} e^{-\left[\frac{(\ln x - \mu)^2}{2\sigma^2}\right]} \dots \dots \dots \text{Eqn - C4}$$

Where:-

μ is location parameter

σ is scale parameter

Estimated distribution parameters and Anderson-Darling test result is presented in the table below.

Table C-8: Estimated Parameters and AD Test Result for Large Truck (LT)

Distribution	Estimated Parameters				Anderson-Darling Test Result		
	Location	Shape	Scale	Threshold	AD	P	LRT P
Lognormal	1.32605		0.18883		0.324	0.5200	
3-Parameter Weibull		2.14413	1.72032	2.30959	0.504	0.2080	0.0000
Largest Extreme Value	3.49099		0.61210		0.309	>0.250	
Gamma		28.10346	0.13643		0.462	>0.250	

- Distribution fitting for departure Headway data of Large Trucks making through movement

Probability plot of the data signify a substantial departure from normality. The empirical cumulative distribution does not much with the theoretical cumulative normal distribution. The p-value from Anderson-Darling normality test result is less than **0.005** which is less than the significance level (**0.05**). All the above points prove that the data do not follow normal distribution.

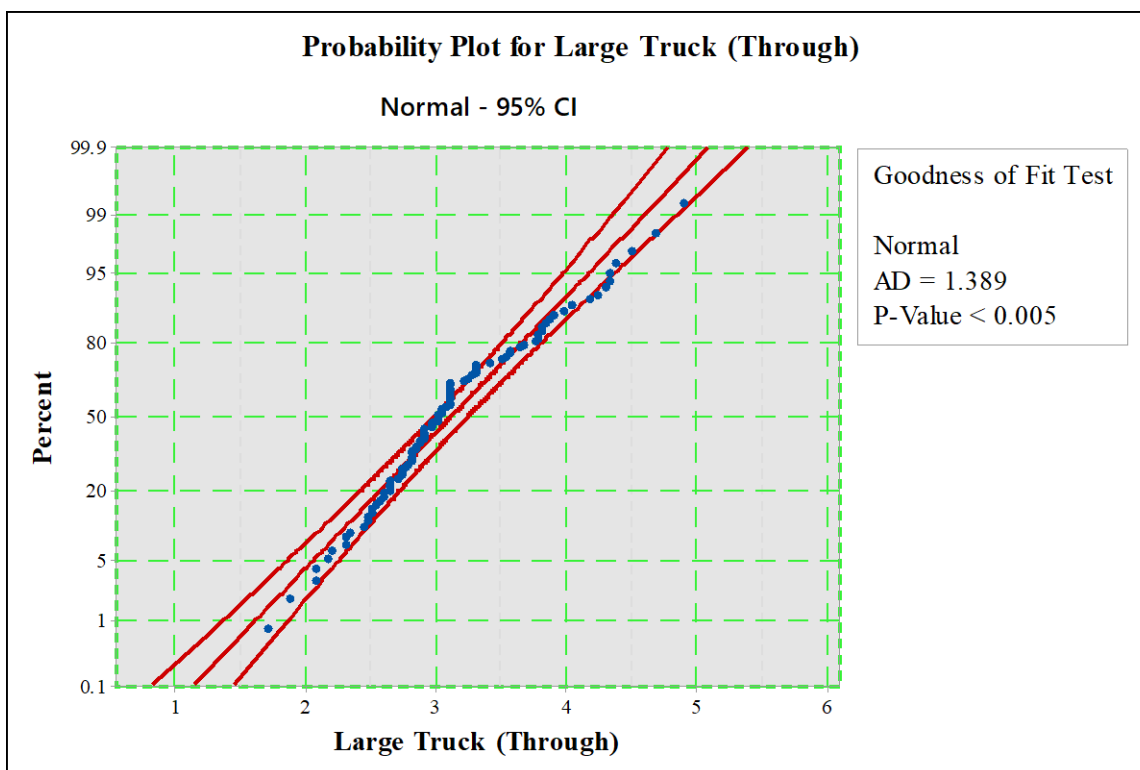


Figure C-25: Normal Probability Plot of Large Truck (LT)

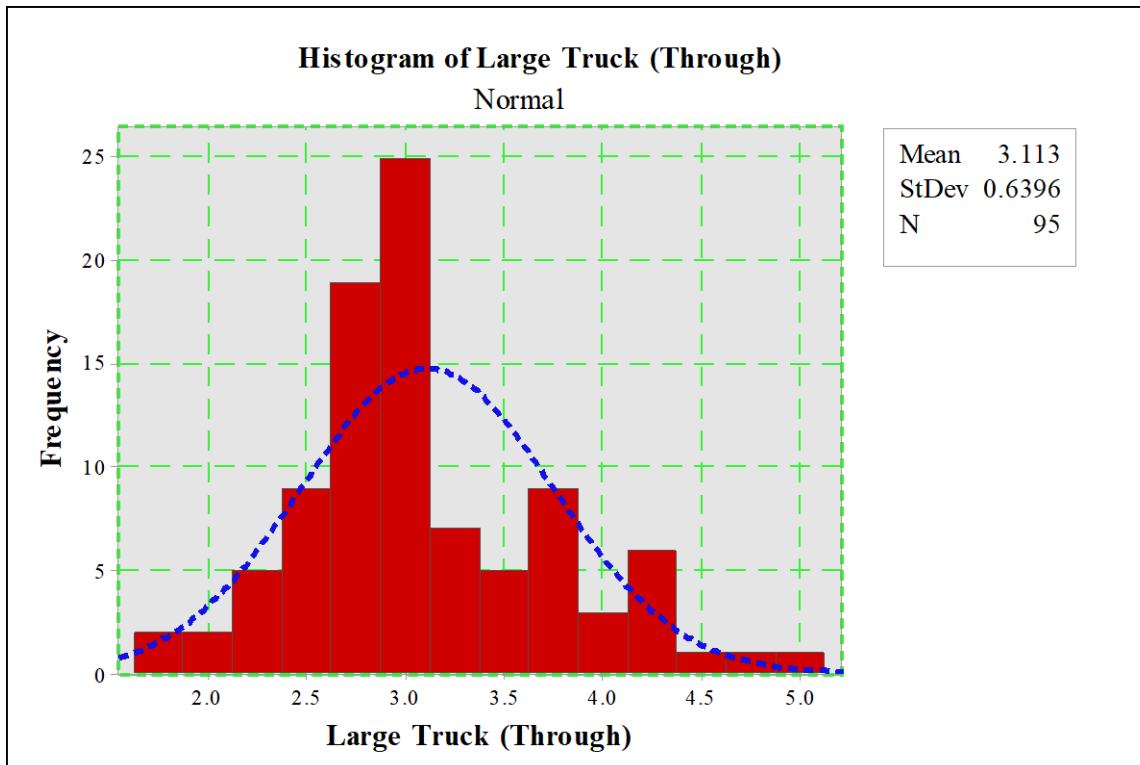


Figure C-26: Histogram of Large Truck (TH) with a Normal Curve

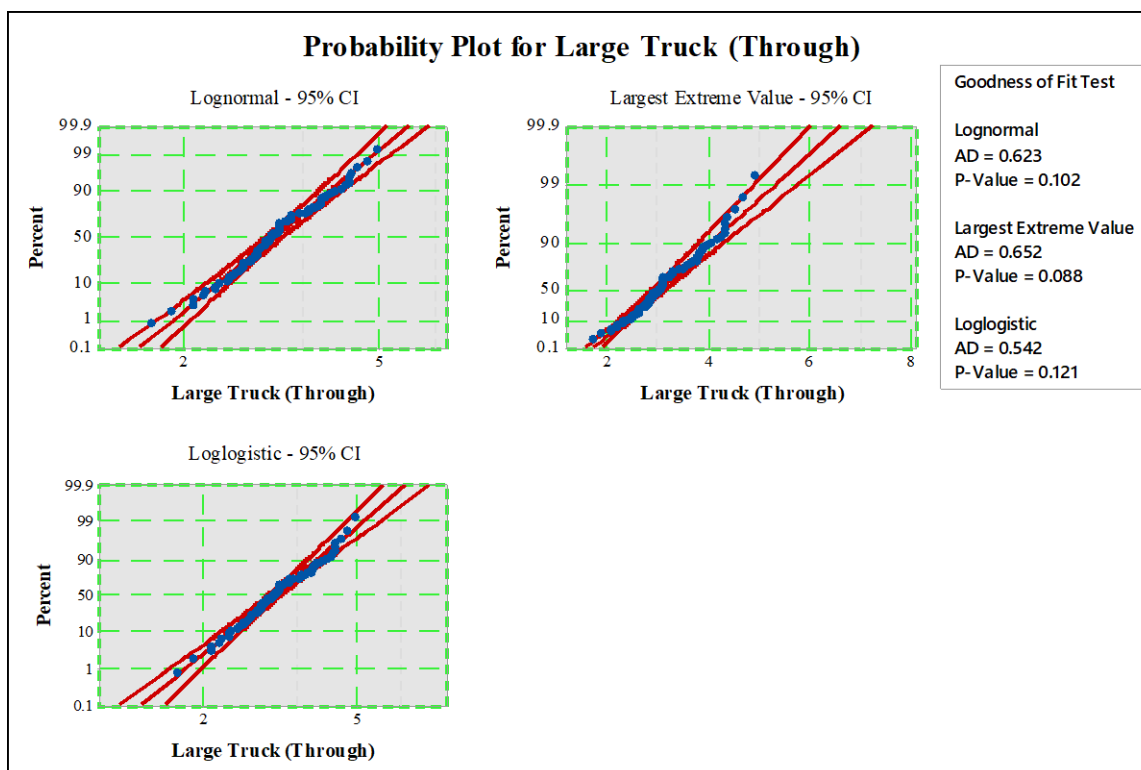


Figure C-27: Probability Distribution of Large Truck (TH)

Lognormal, Largest Extreme Values, and Log-Logistic probability distributions are good fits for the data. Log-Logistic probability distributions have smaller AD values and larger p-value than the others which implies that the data comes from a Log-Logistic probability distribution.

The Log-Logistic probability distribution function is generally written as (*Eqn – C3*). the estimated distribution parameters and Anderson-Darling test result are presented in the table below.

Table C-9: Estimated Parameters and AD Test Result for Large Truck (LT)

Distribution	Estimated Parameters				Anderson-Darling Test Result		
	Location	Shape	Scale	Threshold	AD	P	LRT P
Lognormal	1.11491		0.20392		0.623	0.1020	
Largest Extreme Value	2.81188		0.55478		0.652	0.0880	
Log-Logistic	1.11139		0.11407		0.542	0.1210	