



# SOME BASICS OF NUCLEAR REACTIONS

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# Abstract

In this paper I have seen some basics of nuclear reaction, Basically include in the introduction part starting from Rutherford experiment to the modern accelerating machine nuclear reactions. In the second chapter of this paper briefly describe about definition of nuclear reaction and also discussed about conservation laws that including Energetics conservation of linear momentum, other conservation laws and Isospin. Finally in the last chapter I have seen types of nuclear reaction i.e, compound, direct, resonance and heavy-ion nuclear reaction and also nuclear cross-section.

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# Chapter 1

## Introduction

Nuclear physics is the study of atomic nuclei from deuteron to Uranium there are almost 1700 species that occurs naturally on earth. In addition, large numbers of others are created in the laboratory and in interior of stars. The main force responsible for nuclear properties comes from strong interaction. However both weak and electromagnetic interactions also play important roles. For these reasons, nuclear physics serves as an important platform where basic properties of subatomic matter can be examined and fundamental laws of physics can be studied.

The beginning of nuclear physics may be traced to the discovery of radioactivity in 1896 by Becquerel. Almost by accident, he noticed that well-wrapped photographic plates were blackened when placed near certain minerals. To appreciate the significance of this discovery, it is useful to recall that the time was before the era of quantum mechanics. The only known fundamental interactions were gravity and electromagnetism in fact, just before the end of the nineteenth century, most of the observed physical phenomena were considered to be well understood in terms of what we now refer to as classical physics. Radioactivity was one of the few examples of unsolved problems. It was through the desire to understand these "exceptions" to other wise well-established set of physical laws that gave birth to modern physics.

Two years after Becquerel's discovery, Pierre and Marie Curie succeeded in separating a naturally occurring radioactive element, radium ( $Z=88$ ) from the ore (pitch-blende). soon

afterward, it was realized that the chemical property of an element where changed by such activities. when source was placed in a magnetic field, it was found that there were three different possible types of activity, as the trajectories of some of the "rays" emitted were deflected to one direction, some to the opposite direction, and some not affected at all. these were named  $\alpha, \beta$ , and  $\gamma$  -rays , as nothing more was known about them until much latter, subsequently, it was found that  $\alpha$  -rays consist of positively charged  ${}^4\text{He}$  nuclei,  $\beta$ -ray are made of electrons or positrons, and  $\gamma$  -rays are nothing but electromagnetic radiation that carries no net charge.

The existence of the nucleus as the small central part of an atom was first proposed by Rutherford in 1911, later in 1920 the radii of a few heavy nuclei were measured by Chadwick and were found to be of the order of  $10^{-14}\text{m}$ , much smaller than the order of  $10^{-10}\text{m}$  for atomic radii. The experiments involved scattering  $\alpha$ -particles, obtained from radioactive elements. Off such heavy elements as copper, silver, and gold, and measured cross section where found to be different from values expected of the Rutherford formula for coulomb scattering off point charges.

If energetic particles from a reactor or accelerator (or even from a radioactive source) are allowed to fall upon bulk matter. There is the possibility of a nuclear reaction taking place. The first such nuclear reactions were done in Rutherford's laboratory, using  $\alpha$ -particle from radioactive source. In some of these early experiments, the  $\alpha$ -particles merely rebounded elastically from the target nuclei, this phenomenon, known ever since as Rutherford scattering, gave us the first evidence for the existence of atomic nuclei. In other experiments, Rutherford was able to observe a change or transmutation of nuclear species. as in this reaction done in 1919;



The first particle accelerator capable of inducing nuclear reactions was built by Cockcroft and Walton, who in 1930 observed the reaction.



For the nucleons inside a nucleus, nuclear force is far stronger than that due to electromagnetic interaction as can be seen from the comparisons of the relative strengths, or coupling constant. This presented some difficulties in spontaneous  $\alpha$ -particle decay of some heavy nuclei in the early part of the twentieth century. If the interaction is strong, how can  $\alpha$ -decays have such long life times? For example nuclei such as  ${}^{238}\text{U}$  ( $T_{1/2} = 4.47 \times 10^9$  years) were created before the solar system was born and must have half-lives comparable to or longer than the age of the earth or else it can not be found as ores today. The solution of the puzzle is quantum-Mechanical tunneling, a direct evidence of the wave nature of the particles.

In light nuclei the threshold for  $\alpha$ -particle decay is comparable with that for nucleon emission. For this reason  $\alpha$ -particle decay is not energetically favored until about  $A > 150$ . Even for heavy nuclei the life times are long by strong interaction time scales. Furthermore, the energy of  $\alpha$ -particles emitted tends to be confined in narrow range of 5 to 9 MeV where as the half-lives vary by several orders of magnitude.

A nuclear reaction is a process that takes place when any projectile (a nucleon or combination of nucleons, or heavy ions) come in close contact with nucleon or combination of nucleons. It is a process in which a new system of new composition or a system of new energy or both are formed by bombarding a target nucleus by nuclear probe or with gamma ray. The study of nuclear reactions provides a knowledge of nuclear force and nuclear structure such as shell effect, nuclear density, and nuclear reaction mechanism and also useful in the construction of nuclear level schemes. In practice the beam of particles bombards the target. The high energy accelerating machines such as pelletron, cyclotron etc. The range of the projectile beams of nuclei have been obtained to energies varying from few MeV/nucleon to many GeV/nucleons. The projectile beams may be used for the bombardment with a variety of targets and enables us to study experimentally various

types of nuclear reaction in all elements of the periodic table.

The energy of alpha particles obtained from natural radio active nuclei distributes to all parts and discrete in nature and appreciably smaller than the coulomb barrier and they are not suitable to make nuclear reactions efficiently. But the alpha particle from the accelerator machines has continuous and higher energy and they are frequently used for study of the nuclear reaction mechanisms of different isotopes. The accelerator outgrow their structure. Now a days,physicists use particle accelerators to take particles up to speeds close to the speed of light and smash them in to other particles and study the out come. These accelerator led to the description of the structure of proton and neutron because physicists developed the theory of quark to explain all the particles that have been made in them. Modern accelerators play an effective complementary role in nuclear reactions, for they produce radioactive isotopes that have various applications in applied field of science as a residue of nuclear interaction between projectiles and target,

The emission of several particles is involving the higher energy nuclear reaction which was introduced by the study of models Analyzation of the experimental excitation function and the total energy spectra of the emitted particle had been applied by this model different degree of freedom shows the model which depend upon the incident energy of the projectile, if the emitted particle spectrum have the lower energy the formation of an intermediate equilibrium system asses to continue the reaction decaying independently of the formation mode. Particle or gamma ray emission is occurred to the decay of this excited compound nucleus and emission probability and available phase space governs. Nuclear reaction induced by heavy ion exhibit the characteristic both of compound nucleus and of the direct reaction (stripping and pick up mechanism), which depend up on time. The reaction mechanism showed the shape of the excitation function, namely the angle-integrated cross-section for a given reaction as a function of the projectile energy. This is very easily measured when the reaction produces, a radioactive residue by detecting the activity induced in the target. This gives the number of residues, which are produced

from which one deduces the total cross-section for the reaction concerned. In compound nucleus reaction the projectile is absorbed by the target nucleus and forming a compound nucleus which is subsequently decay.

some reaction that do not qualify neither as direct nor as compound nucleus these are called 'pre-equilibrium' or 'pre-compound reaction'. The experimental results of early 1950, showed that the nuclear reaction mechanism has also an intermediated character between one step direct reactions in which few degrees of freedom are involved and the compound nucleus reaction in which all the energy of the projectile is distributed among all nucleons of the compound nucleus in a completely statistical manner. All the intermediate processes are called pre-equilibrium reaction

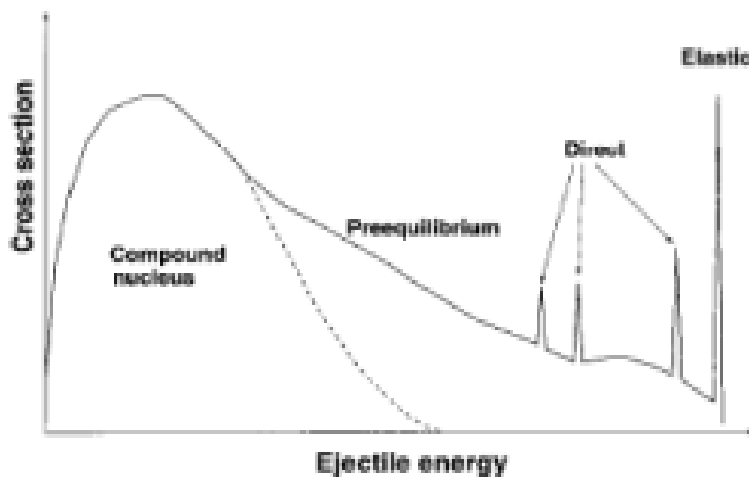


Figure 1.1: Neutron emission spectrum showing contributions of different reaction mechanisms in a nuclear reaction

In general in the low energy region up to about 10Mev mostly statistical processes dominate. The pre-equilibrium effects start manifesting themselves at energies around 20Mev, and at excitation energies above 50Mev their contribution starts becoming significant and also the time scale (nuclear time) means the time to cross a nucleus around 10 fermi radius by the nucleon of different energies. For the direct interaction is  $10^{-21}$ se, for pre-compound emission  $10^{-18}$ se and for compound nucleus evaporation  $10^{-15}$ se

# Chapter 2

## Nuclear Reaction and Conservation Laws

### 2.1 Nuclear Reactions

A large fraction of our knowledge on the properties of nuclei is derived from nuclear reactions. When an incoming particle is scattered off a target nucleus, the outcome depends on a combination of three factors: the reaction mechanism, interaction between the projectile and the target, and the internal structure of the nuclei involved. Different probes complement each other in what we can learn from an investigation. Furthermore, it is often possible to select the bombarding energy. Nuclear reaction a large subject by itself. We can give here only an overview of some of the important types of nuclear reaction will see in the next chapter.

### 2.2 Conservation Laws

For bombarding energies below 100 Mev, nuclear reactions usually produce tow products, i.e, they are of the type



Where a = bombarding particle

X = target(at rest in the lab.system)

b = light reaction product

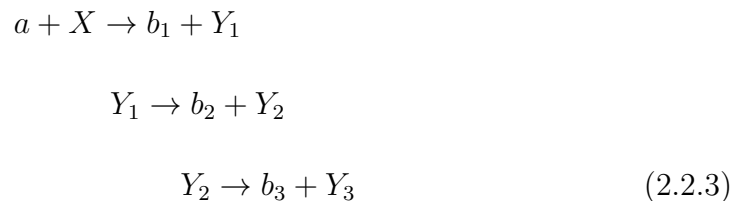
Y = heavy reaction product

To shorten the notation a reaction of type (equation 2.2.1)is designated by



Commonly, one reaction product is light and the other heavy because of the binding energies of nuclei involved In some cases b and Y have comparable masses (spallation reaction or fission),or are identical. If b is a gamma ray, We speak of a capture reaction in which Y is the compound nucleus.

In most cases in which more than two products appear, it is possible to describe the process as a rapid sequence of two-product reactions



The excitation energy of the compound nucleus is  $S + T_0$ ,where S is the separation energy of the incident particle from the ground state of the ground state of the compound nucleus, S is of the order of 8Mev for protons and neutrons.

If anti nucleons are counted negatively, the total number of nucleons in all known reactions is conserved.

Note that number of neutrons and protons is conserved. presently the number of known reactions is in the thousands.

### 2.2.1 Energetics Conservation of linear momentum

. Since the number of protons remains unchanged in a reaction, all masses can be Written as atomic masses if electron binding-energy differences of a few eV are ignored conservation of energy, therefore, gives for the reaction(2.2.1)

$$M_a C^2 + T_a + M_X C^2 = M_b C^2 + T_b + M_Y C^2 + T_Y \quad (2.2.4)$$

Where T represents the (lab.) kinetic energy of each particle. The masses of a and X are ground-state masses On the other hand, many reactions leave Y in excited states; in that case,  $M_Y$  represents the total mass energy of the state.

The Q value of the reaction is defined as the difference between the final and initial kinetic energies

$$Q = T_b + T_Y - T_a \quad (2.2.5)$$

$$Q = [M_a + M_X - (M_b + M_Y)]C^2 \quad (2.2.6)$$

If Q is positive the reaction is said to be exergic; if Q is negative, it is endoergic. A reaction cannot take place unless particle b and Y emerge with positive kinetic energy, that is,  $T_b + T_Y \geq 0$  Or

$$Q + T_a \geq 0 \quad (2.2.7)$$

Although this condition is necessary, it is not sufficient.

The Q value is an important quantity in a nuclear reaction. It can be determined from mass spectroscopy (Eq. 2.2.6) or by measuring kinetic energies (Eq 2.2.5). We can show, as a result of linear momentum conservation, only  $T_b$  and the angle  $\theta$  of b with respect to the direction of  $\phi$  need to be determined In the lab.system

$$\begin{aligned}
M_a v_a &= M_Y v_Y \cos \phi + M_b v_b \cos \theta \\
0 &= M_Y v_Y \sin \phi - M_b v_b \sin \theta
\end{aligned} \tag{2.2.8}$$

In order to eliminate  $\phi$ , substitute  $M_v = (2MT)^{1/2}$  for each particle and rewrite the equations

$$\begin{aligned}
(M_a T_a)^{1/2} - (M_b T_b)^{1/2} \cos \theta &= (M_Y T_Y)^{1/2} \cos \phi \\
(M_b T_b)^{1/2} \sin \theta - (M_Y T_Y)^{1/2} \sin \phi &= 0
\end{aligned} \tag{2.2.9}$$

Squaring both equations and adding

$$M_a T_a - 2(M_a T_a M_b T_b)^{1/2} \cos \theta + M_b T_b = M_Y T_Y \tag{2.2.10}$$

Eliminating  $T_Y$  With the help of (Eq 2.2.4)

$$Q = T_b \left(1 + \frac{M_b}{M_Y}\right) - T_a \left(1 - \frac{M_a}{M_Y}\right) - \frac{2}{M_Y} (M_a T_a M_b T_b)^{1/2} \cos \theta \tag{2.2.11}$$

This is called the Q equation special case of interest are those with  $\theta = 90^\circ$  and those with zero bombarding energy  $T_a$ . The latter reaction is possible only with neutrons, since the coulomb barrier prevents nuclear reactions with zero energy charged particles.

Part of the incident energy  $T_a$  is used up as kinetic energy of the center of mass and is not available for the nuclear reaction itself. Although we can study all the resulting effects by means of Eq.(2.2.11), more insight is gained if we consider the reaction in the c.m.system,

The Kinetic energy of the center of mass is

$$T_{c.m} = \frac{1}{2} (M_a + M_X) v_0^2 \tag{2.2.12}$$

Where  $v_0 = v_a / (M_a + M_X)$  is the speed of the center of mass. The kinetic energy  $T_0$  of the initial particles in the c.m.system can be calculated in two equivalent ways as

$$T_0 = T_a - T_{c.m.} \tag{2.2.13}$$

or

$$T_0 = \frac{1}{2}M_a V_a^2 + \frac{1}{2}M_X V_X^2 \quad (2.2.14)$$

Where V represents the speed of each particle in the c.m.system Equations(2.2.13) and (2.2.14) both yield

$$T_0 = \frac{M_X}{M_a + M_X} T_a \quad (2.2.15)$$

The energy available for the nuclear reaction is

$$Q + T_0 \quad (2.2.16)$$

Since the mass of the system changes from  $M_a + M_X$  to  $M_b + M_Y$ , the c.m.system is not identical for the initial and final products. As long as all velocities are non relativistic and the fractional mass difference is small, this effect can be ignored, it is more useful to define a center-of-momentum system, which does not change during the reaction. The speed of the center of mass is  $v_0 = v_a M_a / (M_a + M_X)$ . Also  $M_b V_b = M_Y V_Y$  Which is equal to the kinetic energy of the reaction products in the c.m system

$$Q + T_0 = \frac{1}{2}M_b V_b^2 + \frac{1}{2}M_Y V_Y^2 \quad (2.2.17)$$

This is easy to see, because if  $T_{c.m.}$  is added to both sides of Eq.(2.2.17), the result is identical to Eq(2.2.5).

A necessary and sufficient condition that the reaction proceed is that the right-hand side of Eq(2.2.17) be positive,i.e.,

$$Q + T_0 \geq 0 \quad (2.2.18)$$

This Would automatically satisfy Eq(2.2.7). Using Eq(2.2.15), the same condition is

$$T_a \geq \frac{-Q(M_a + M_X)}{M_X} \quad (2.2.19)$$

In the case of an endoergic reaction ( $Q < 0$ ), Eq(2.2.19) gives the threshold energy of the reaction. The threshold energy can also be derived by noting that at threshold, particles b and Y both move speed  $v_0$  in the lab.system

$$(T_b + T_Y)_{thresh} = \frac{1}{2}(M_b + M_Y)v_0^2 \quad (2.2.20)$$

After a short calculation, using  $M_b M_Y \approx M_a + M_X$ , Eq(2.2.19) is obtained.

We can return to the lab.system by adding the velocity  $v_0$  to the velocities. Numerous interesting situations can then be examined geometrically. For example endoergic reaction, particle b can appear with two different kinetic energies at the same lab.angle  $\theta$ , if  $T_a$  is only slightly above the threshold energy. This occurs because  $T_a$  determines only the c.m.speed  $V_b$  and note that  $M_b V_b = M_Y V_Y$  and not the direction of the velocity  $V_b$ . under suitable conditions

At certain lab.angle  $\theta$ , particle b appears with two different kinetic energies  $\frac{1}{2} M_b v_b^2$  and  $\frac{1}{2} M_b v_b'^2$ .

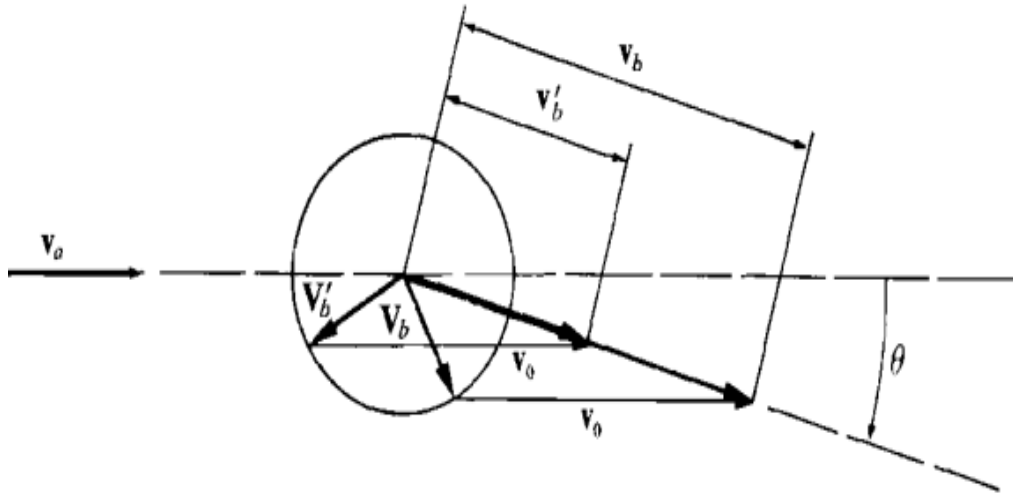


Figure 2.1: Vlocity diagram for particle b at an energy slightly above threshold of an endoergic reaction

two different velocities  $V_b$  and  $V_b'$  of the magnitude can produced particle b at a given lab.angle  $\theta$  with different speeds  $v_b$  and  $v_b'$

It is convenient to plot the c.m. mass-energy information for nuclear reaction on a diagram similar to fig.(2.2). The example illustrates an endoergic reaction( $Q < 0$ )

A famous closed-chamber alpha particles Where emitted by a thorium active deposit which produces principal alpha groups of 8.8Mev from  $Po^{212}$ (Th C') and 6.1Mev from  $Bi^{212}$  (Th C). From the path length of the interacting alpha particle, its energy at the

place of collision was calculated. The illustration applies to an endoergic reaction, that is,  $Q = (M_a + M_X)c^2 - (M_b + M_Y)c^2 < 0$ . The compound-system mass energy is also indicated.

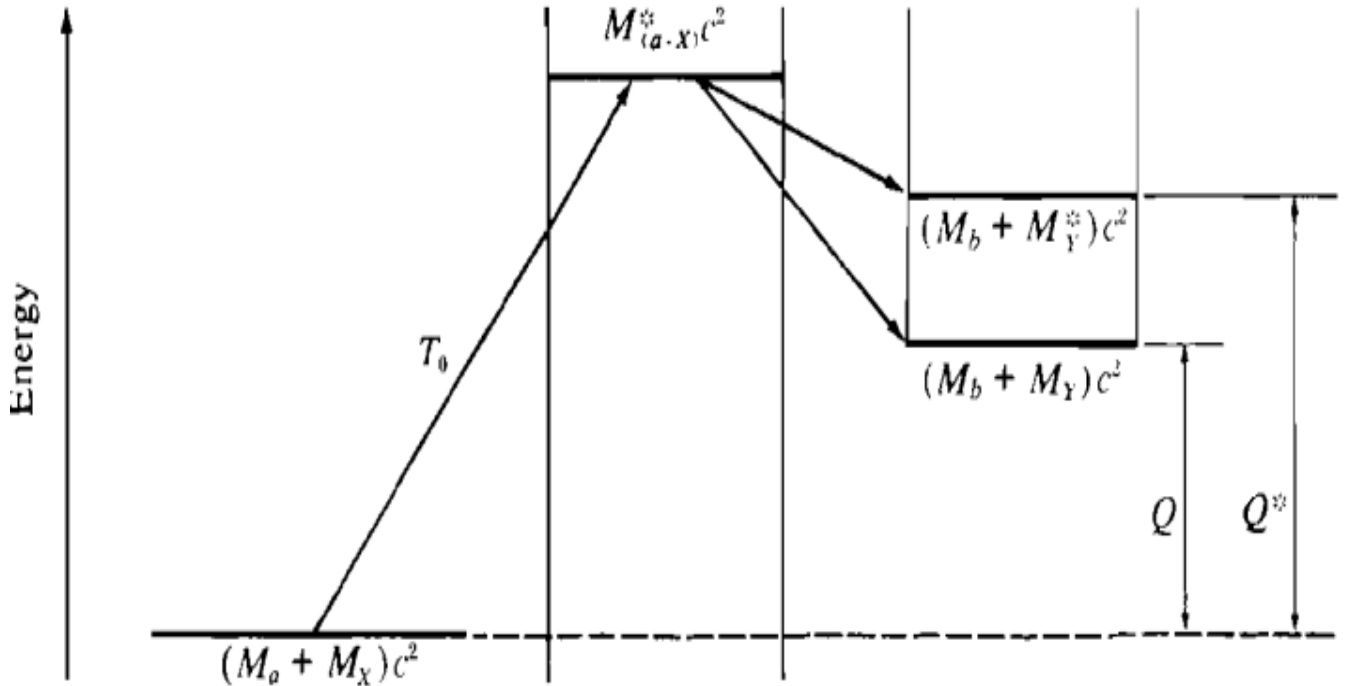


Figure 2.2: Center-of-mass energetics of a nuclear reaction

to be 3.9 Mev. The angle and range of the emitted proton allowed  $Q$  to be computed with the result  $Q = -1.2$  Mev. We can appreciate the small probability of a nuclear interaction by noting that in 400,000 alpha particle tracks only eight reactions of the type shown were found.

### 2.2.2 Other conservation laws.

Nuclear reactions are most conveniently discussed in the c.m.system. Conservation of angular momentum in the reaction  $X(a, b)Y$  then requires.

$$I_a + I_X + l_{a.X} = I_b + I_Y + l_{b.Y} \quad (2.2.21)$$

Where  $I$  is the total angular momentum of each nucleus (in units of  $\hbar$ ), and  $l$  is the orbital angular momentum of each pair of particles about the center of mass. parity conservation requires

$$\pi_a \pi_X (-1)^{l_{a \cdot x}} = \pi_b \pi_Y (-1)^{l_{b \cdot Y}} \quad (2.2.22)$$

Where  $\pi$  is the parity of each nuclear state involved in the reaction. These conservation laws impose restrictions on the reaction probability. But even if the conservation laws allow a reaction to proceed, the reaction rate sometimes may be so minute that its occurrence cannot be detected with available equipment.

### 2.2.3 Isospin

The interaction of a nucleon with its surroundings other nucleons, for instance most cases do not depend on whether the nucleon has spin components  $m = -1$  or  $m = -1/2$  relative to an arbitrarily choose axis is there is no need to distinguish in the formalism of nuclear physics between a spin-up nucleon and a spin-down nucleon. The multiplicity of spin orientations (two for a single nucleon) may enter into the equations, for example in the statics of the interaction, but the actual value of projection does not appear. The exception to this situation comes about when a magnetic field is applied; the magnetic interaction of a nucleon depends on its spin components relative to the direction of the external field.

The charge independence of nuclear forces means that in most instances we do not need to distinguish in the formalism between neutrons and protons. And this leads us to group them together as members of a common family, the nucleons. The formalism for nuclear interactions may depend on the multiplicity of nucleon states(two) but it is independent of whether the nucleons are protons or neutrons. The exception of courser, is the electromagnetic interaction, which can distinguish between protons and neutrons; with respect to the strong nuclear force alone. The symmetry between neutrons and protons remains valid.

This two-state degeneracy leads naturally to a formalism analogous to that of the magnetic interaction of a spin-1/2 particle. The neutron and proton are treated as two different states of a single particle. The nucleon is assigned a fictitious spin vector called the isospin. The two degenerate nuclear states of the nucleon in the absence of electromagnetic fields, like the two degenerate spin states of a nucleon in the absence of a magnetic field are then isospin-up which we arbitrarily assign to the proton and isospin-down the neutron, That is for a nucleon with isospin quantum number  $l = 1/2$ , a proton has  $m_l = 1/2$  and a neutron has  $m_l = -1/2$ . These projections are calculated with respect to an arbitrary axis called the 3 - axis in a coordinate system whose axes are labeled 1,2 and 3, in order to distinguish it from the laboratory axis of the x,y,z coordinate system. The isospin vector  $l$  of length  $l(l + 1)h$  and with 3-axis projection  $l_3 = m_l h$ .

For a system of several nucleons the isospin follows coupling rules identical with the rules of ordinary angular momentums of vectors. A two nucleon system for example can have total isospin vector,  $T_3$  is the sum of the 3-axis components of the individual nucleons and thus for any nucleus.

$$T_3 = \frac{1}{2}(Z - N) \quad (2.2.23)$$

expressed in units of  $h$  which will not be shown explicitly.

For a given nucleus,  $T_3$  is determined by the number of neutrons and protons. For any value of  $T_3$ , the total isospin quantum number  $T$  can take any value at least as great as  $|T_3|$ . Two related questions that immediately follow are; Can we assign the quantum number  $T$  to individual nuclear states? is such an assignment useful. for example, in predicting decay or reaction probability?

We consider as an example the two - nucleon system, which can have  $T$  of 0 or 1. There are thus four possible 3-axis components;  $T_3 = 1$  (two protons),  $T_3 = -1$  (two neutrons). and two combinations with  $T_3 = 0$  (one proton and one neutron), The first two states

must have  $T = 1$ , while the latter two can have  $T = 0$  and  $T = 1$ . If the nuclear reaction is perfectly charge independent (and if we "turn off" the electromagnetic interaction), then the three 3-axis projections of  $T = 1$  ( $+1, -1$ ) must have the same energy, while the single  $T = 0$  state may have a different energy. In fact, we know that the isospin triplet (Which is the  $I = 0$  singlet of ordinary spin) is unbound.

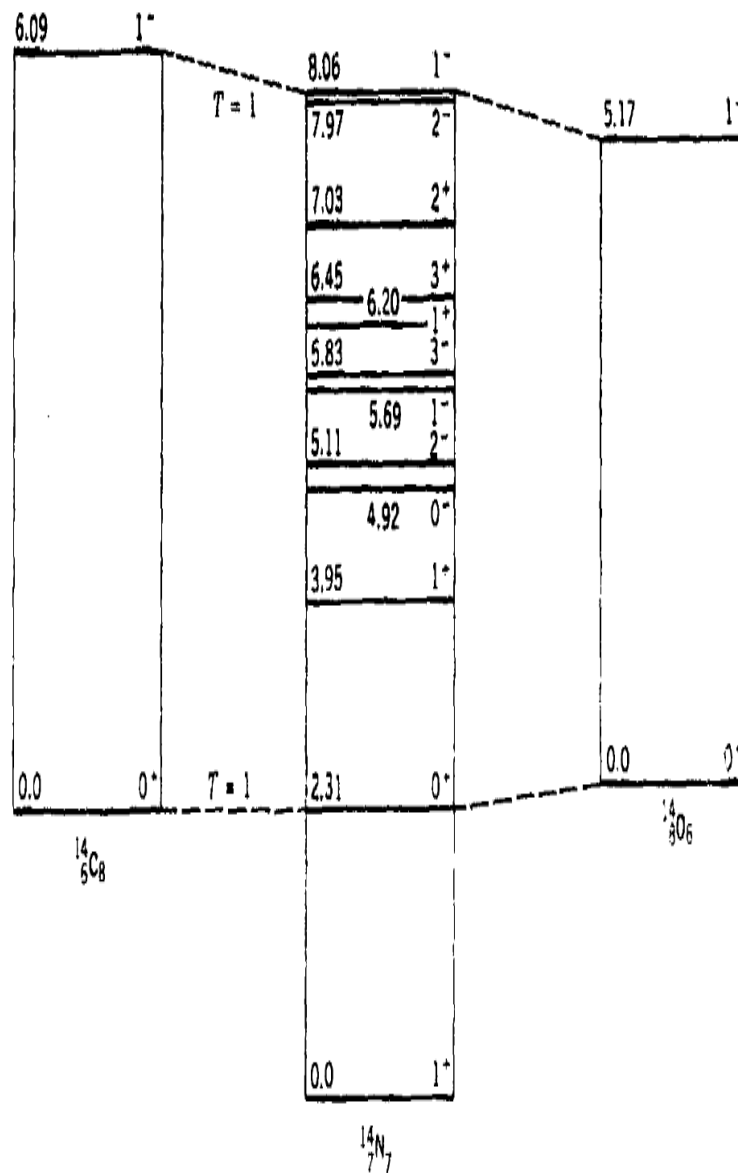


Figure 2.3: Lower energy level of  $A = 14$  isobars.

A clearer example of the isospin assignments can be found in the nuclei of the  $A = 14$

system. Fig shows the states of  $^{14}\text{C}$  ( $T_3 = -1$ ),  $^{14}\text{N}$  ( $T_3 = 0$ ) and  $^{14}\text{O}$  ( $T_3 = +1$ ). For  $^{14}\text{N}$ , we know that any integer  $T$  can give a 3-axis component of 0, and the possible values of  $T$  therefore range from 0 up to a maximum of  $A/2$  or 7. The tendency toward nuclear symmetry (reflected in the symmetry term of the semi empirical mass formula) implies that the lowest states will most likely have  $T = |T_3|$ , that is the smallest possible value of  $T$ . This will certainly apply to the ground state, but the excited states must be assigned on the basis of reaction or decay studies or symmetry arguments. In fig the energies have been adjusted so that the neutron-proton mass difference (an electromagnetic effect) and the coulomb energy of the nucleus have been removed. The energies of the  $0^+$  states in the three nuclei are nearly identical; these are the states of the  $T = 1$  triplet. similar agreement is obtained for the  $1^-$  triplet.

Such speculations regarding the  $T$  assignments can be verified through decay and reaction studies. For example, angular momentum coupling theory 1 leads to selection rule for  $E_1$  transitions;  $\Delta T$  must be 0 or  $\pm 1$ , except that transitions from  $T = 0$  to  $T = 0$  are forbidden and  $\Delta T = 0$  transitions are forbidden in nuclei with  $T_3 = 0$ . To test these rules, we examine the half-lives for the  $1^-$  to  $0^+ E_1$  transitions in  $^{14}\text{O}$ ,  $^{14}\text{C}$  and  $^{14}\text{N}$ . The measured half-lives of the analogous states are respectively,  $1.2 \times 10^{-4}$ ,  $< 7$ , and 27 fs. The  $^{14}\text{N}$  transition, which is a  $\Delta T = 0 E_1$  transition in a  $T_3 = 0$  nucleus, is forbidden by the isospin selection rule and is indeed strongly inhibited, as its longer half-life indicates. (The Weisskopf estimate for the half-life is about  $7 \times 10^{-3}$  fs.)

Consider also the decay of the  $1^-$ ,  $T = 0$  level at 5.69 MeV in  $^{14}\text{N}$ . the  $E_1$  decay to the  $1^+$ ,  $T = 0$  ground state should be inhibited by the selection rule, while the  $E_1$  decay to the  $0^+$ ,  $T = 1$  level at 2.31 MeV is permitted. The higher energy transition ought to be greater in intensity by about a factor of 5, owing to the  $E^3$  dependence of the  $E_1$  transition probability, yet the lower energy transition is observed to have about twice the intensity, The effect of the isospin selection rule is a reduction in the expected relative intensity of the 5.69-MeV  $E_1$  transition by about an order of magnitude.

Similar selection rules operate in  $\beta$  decay, The Fermi matrix element is forbidden unless  $\Delta T = 0$ . The non mirror decays are those with  $\Delta T = 1$ . and the Fermi contribution to the transition is reduced by several orders of magnitude by the violation of the isospin selection rule. The  $0^+$  to  $0^-$  decays, which on the basis of ordinary angular momentum alone should be pure Fermi decays of the super allowed category are inhibited by three orders of magnitude if  $\Delta T \neq 0$ ; the log ft values rise from about 3.5 for the  $\Delta T = 0$  decays permitted by the isospin selection rule to 7 or larger for the  $\Delta T \neq 0$  isospin forbidden decays.

Nuclear reactions also show effects of isospin, Because the nuclear force does not distinguish between proton and neutrons. The isospin must be absolutely conserved in all nuclear reactions. The 3-axis component is automatically conserved when the numbers of protons and neutrons remains constant, but it is also true that total isospin quantum number T remains invariant in reactions. Consider the reaction  $^{16}\text{O} + ^2\text{H} \rightarrow ^{14}\text{N} + ^4\text{He}$  leading to states in  $^{14}\text{N}$ , All four reacting particles have T = 0 ground states; thus T is conserved if the product particles remains in the ground states. Excitation of  $^4\text{He}$  is unlikely in low energy reactions, for its first excited states is above 20MeV, and thus it is expected that only T = 1 state should not be populated. Any small population observed for that state must arise from isospin impurities in the reacting particles. The cross-section to reach the 2.31MeV state is observed to be about 2 orders of magnitude smaller than the cross-section to reach the neighboring T = 0 states. showing the effectiveness of the isospin selection rule. In the similar reaction  $^{12}\text{C}(\alpha, d)^{14}\text{N}$  the cross section for the 2.31MeV state is 3 orders of magnitude smaller than the isospin-allowed cross sections, and in  $^{10}\text{B}(^6\text{Li}, d)^{14}\text{N}$  and  $^{12}\text{C}(^6\text{Li}, \alpha)^{14}\text{N}$  it is at least two orders of magnitude smaller. By way of contrast in  $^{10}\text{B}(^7\text{Li}, ^3\text{H})^{14}\text{N}$  the T = 1 level is populated with a strength comparable to that of the neighboring T = 0 level; the isospin selection rule does not inhibit the probability to reach the T = 1 level. (The initial nuclei have a total T of 1/2; the 1/2 isospin of  $^3\text{H}$  can couple to either T = 0 or T = 1 in  $^{14}\text{N}$  to give a resultant of

1/2)

The members of an isospin multiplet, as for example pairs of mirror nuclei or a set of the three states connected by the dashed lines in fig are called isobaric analog states, a term which was previously introduced in the discussion of  $\beta$  decay. The analog in neighboring nuclei have identical nucleon wave functions, except for the change in the number of protons and neutrons. In the  $^{14}\text{C}$  and  $^{14}\text{O}$  ground states, the nucleons are strongly coupled pairwise (With two coupled proton holes in  $^{14}\text{C}$  and two coupled neutron holes in  $^{14}\text{O}$ ), and the 2.31MeV analog state in  $^{14}\text{N}$  must have a similar wave function. with the odd proton hole and neutron hole paired.

Because analog states are obtained by exchanging a proton for a neutron. they tend to be strongly populated in  $\beta$  decay and in (P,n) or (n,P) reactions. In medium and heavy nuclei, placing a proton into a state formerly occupied by a neutron involves a large energy transfer, because with  $N > Z$  the newly placed neutron occupies a considerably higher shell-model state than the former proton. Analog states may appear in medium and heavy nuclei at energies of 10 MeV and above, and thus they generally do not contribute to low-energy reaction and decay studies.

## Chapter 3

# Types Of Nuclear Reaction And Reaction Cross Section

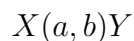
### 3.1 Types Of Nuclear Reaction

A typical nuclear reaction is written



Where  $a$  is the accelerated projectile,  $X$  is the target (usually stationary in the laboratory), and  $Y$  and  $b$  are reaction products. Usually,  $Y$  will be a heavy product that stops in the target and is not directly observed, while  $b$  is a light particle that can be detected and measured. Generally,  $a$  and  $b$  will be nucleons or light nuclei, but occasionally  $b$  will be a  $\gamma$  ray, in which case the reaction is called radiative capture. (If  $a$  is a  $\gamma$  ray, the reaction is called the nuclear photo effect.)

An alternative and compact way of indicating the same reaction is



which is convenient because it gives us a natural way to refer to a general class of reactions with common properties, for example  $(\alpha, n)$  or  $(n, \gamma)$  reactions.

We classify reactions in many ways, If the incident and outgoing particles are the same (and correspondingly  $X$  and  $Y$  are the same nucleus), it is a scattering process. Elastic if

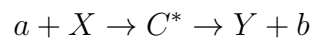
Y and b are in their ground state and inelastic if Y or b is in an excited state (from which it will generally decay quickly by  $\gamma$  emission). Sometimes a and b are the same particle, but the reaction causes yet another nucleon to be ejected separately (so that there are three particles in the final state); this is called a knockout reaction. In a transfer reaction, one or two nucleons are transferred between projectile and target, such as an incoming deuteron turning in to an outgoing proton or neutron. thereby adding one nucleon to the target X to form Y, Reactions can also be classified by the mechanism that governs the process. In direct reactions (of which transfer reactions are an important subgroup). Only very few nucleons take part in the reaction, with the remaining nucleons of the target serving as passive spectators Such reactions might insert or remove a single nucleon from a shell-model state and might therefore serve as ways to explore the shell structure of nuclei. Many excited states of Y can be reached in these reactions. The other extreme is the compound nucleus mechanism, in which the incoming and target and nuclei merge briefly for a complete sharing of energy before the outgoing nucleon is ejected, somewhat like evaporation of a molecule from a hot liquid. Between these two extremes are the resonance reactions, in which the incoming particle forms a "quasi bound" state before the outgoing particle is ejected.

### 3.1.1 Compound-Nucleus Reactions

Suppose an incident particle enters a target nucleus with an impact parameter small compared with the nuclear radius. It then will have a high probability of interacting with one of the nucleons of the target, possibly through a simple scattering. The recoiling struck nucleon and the incident particle (now with less energy) can each make successive collision with other nucleons, and after several such interactions. the incident energy is shared among many of the nucleons of the combined system of projectile + target. The average increase in energy of any single nucleon is not enough to free it form the nucleus, but as many more-or-less random collisions occur. There is a statistical distribution in

energies and a small probability for a single nucleon to gain a large enough share of the energy to escape, Much as molecules evaporate from a hot liquid.

Such reactions have a definite intermediate state, after the absorption of the incident particle but before the emission of the outgoing particle (or particles). This intermediate state is called the compound nucleus. Symbolically then the reaction  $a + X \rightarrow Y + b$  becomes



where  $C^*$  indicate the compound nucleus.

As might be assumed from seeing the reaction written in this form, we can consider a reaction that proceeds through the compound nucleus to be a two-step process: the formation and then the subsequent decay of the compound nucleus. A given compound nucleus may decay in a variety of different ways and essential to the compound nucleus model of nuclear reactions is the assumption that the relative probability for decay into any specific set of final products is independent of the means of formation of the compound nucleus. The decay probability depends only on the total energy given to the system; in effect, the compound nucleus "forget" the process of formation and decays governed primarily by statistical rules.

Let's consider a specific example.

The compound nucleus  ${}^{64}\text{Zn}^*$  can be formed through several reaction processes, including  $p + {}^{63}\text{Cu}$  and  $\alpha + {}^{60}\text{Ni}$ . It can also decay in a variety of ways. Including  ${}^{63}\text{Zn} + n$ ,  ${}^{62}\text{Zn} + 2n$ , and  ${}^{62}\text{Cu} + p + n$ . That is

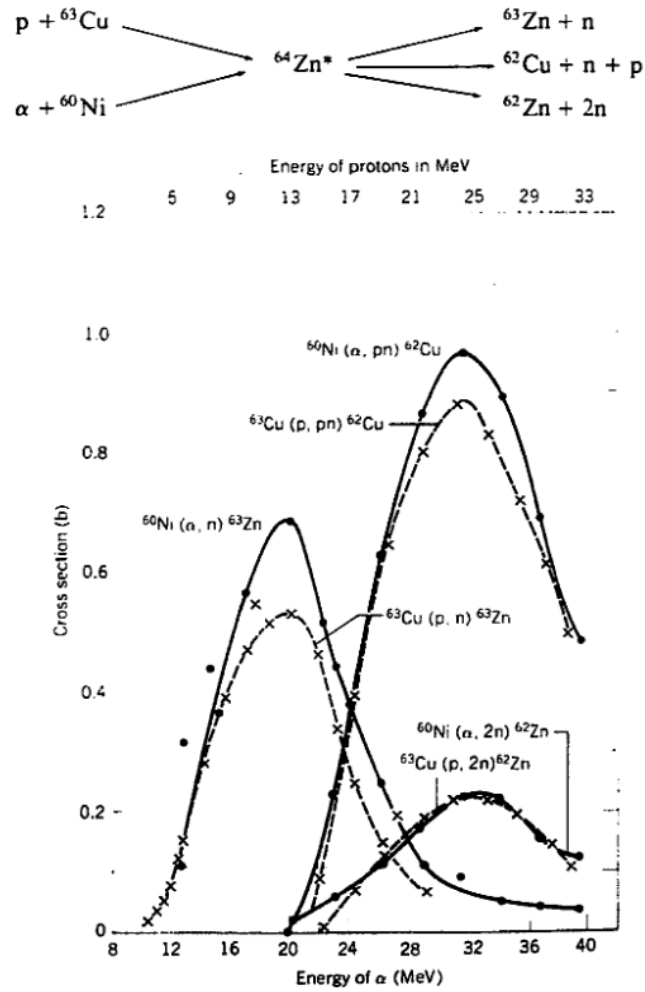


Figure 3.1: cross-section for different reactions leading to the compound nucleus Zn show very similar characteristics, consistent with the basic assumptions of the compound nucleus model, from S.N.Goshal, *phy, Rev.* 80, 939 (1950)

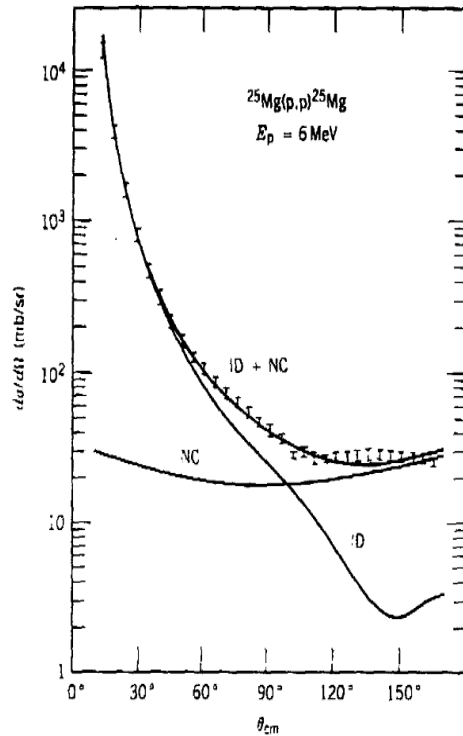


Figure 3.2: the curve marked NC shows the contribution from compound-nucleus formation to the cross-section of the reaction  $\text{Mg}(p,p)\text{Mg}$ , The curve marked ID shows the contribution from direct reaction. note that the direct part has a strong angular dependence, while the compound-nucleus part shows little angular dependence. from A Gallman et al.nucl.phys.88.654 (1966)

If this model were correct, we would expect for example that the relative cross section for  ${}^{63}\text{Cu}(p,n){}^{63}\text{Zn}$  and  ${}^{60}\text{Ni}(\alpha,n){}^{63}\text{Zn}$  would be the same at incident energies that give the same excitation energy to  ${}^{64}\text{Zn}^*$ .fig 3.1 shows the cross-sections for the three final state With the energy scales for the incident protons and  $\alpha$ 's shifted so that they correspond to a common excitation of the compound nucleus. The agreement between the three pairs of cross sections is remarkably good. Showing that indeed, the decay of  ${}^{64}\text{Zn}^*$  into any specific final state is nearly independent of how it was originally formed.

The compound-nucleus model works best for low incident energies (10-20MeV), where the incident projectile has a small chance of escaping from the nucleus with its identity and most of its energy intact. It also works best for medium-weight and heavy nuclei, where the nuclear interior is large enough to absorb the incident energy.

Another characteristic of compound nucleus reactions is the angular distribution of the products. Because of the random interactions among the nucleons, we expect the outgoing particle to be emitted with a nearly isotropic angular distribution (that is the same in all directions). This expectation is quite consistent with experiment, as shown in fig 3.2. In cases in which a heavy ion is the incident particle, large amounts of angular momentum can be transferred to the compound nucleus, and to extract that angular momentum the

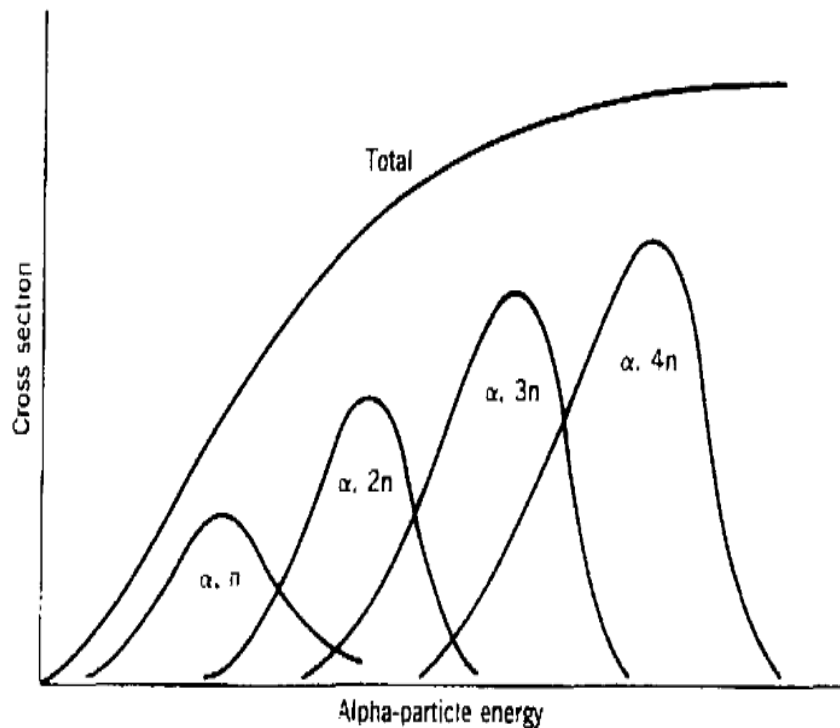


Figure 3.3: At higher incident energies, it is more likely that additional neutrons will "evaporate" from the compound nucleus.

emitted particles tend to be emitted at right angles to the angular momentum. And thus preferentially at  $0$  and  $180^\circ$  with light projectiles, this effect is negligible.

The "evaporation" analogy mentioned previously is really quite appropriate. In fact, the more energy we give to the compound nucleus, the more particles are likely to evaporate. For each final state, the cross section has the Gaussian-like shape shown in fig 3.1, fig 3.3 shows the cross sections for  $(\alpha, xn)$  reactions. Where  $x = 1, 2, 3, \dots$ . For each reaction, the

cross sections increases to a maximum and then decreases as the higher energy makes it more likely for an additional neutron to be emitted.

### 3.1.2 Direct Reactions

At the opposite extreme from compound-nucleus reactions are direct reactions, in which the incident particle interacts primarily at the surface of the target nucleus: such reactions are also called peripheral processes. As the energy of the incident particle is increased, its de Broglie wavelength decreases. Until it becomes more likely to interact with a nucleon-sized object than with a nucleus-sized object. A 1-MeV incident nucleon has a de Broglie wavelength of about 4 fm, and thus does not "see" individual nucleons; it is more likely to interact through a compound-nucleus reaction. A 20-MeV nucleon has a de Broglie wavelength of about 1fm and therefore may be able to participate in direct processes. Direct processes are most likely to involve one nucleon or very few valence nucleons near the surface of the target nucleus.

Of course, it may be possible to have direct and compound-nucleus processes both contribute to a given reaction. How can we distinguish their contributions or decide which may be more important? There are two principal differences that can be observed experimentally: (1)Direct processes occurs very rapidly, in a time of the order of  $10^{-22}$ s, while compound-nuclear processes typically take much longer, perhaps  $10^{-16}$  to  $10^{-18}$ s. This additional time is necessary for the distribution and reconstruction of the incident energy. There are ingenious experimental techniques for distinguishing between these two incredibly short intervals of time. (2)The angular distributions of the outgoing particles in direct reactions tend to be more sharply peaked than in the case of compound-nuclear reactions.

Inelastic scattering could proceed either through a direct process or a compound nucleus, largely depending on the energy of the incident particle. The deuteron stripping reaction (d,n), which is an example of a transfer reaction in which a single proton is transferred from projectile to target, may also go by either mechanism. Another deuteron

stripping reaction (d,p) may be more likely to go by direct process. For the "evaporation" of protons from the compound nucleus is inhibited by the Coulomb barrier. The ( $\alpha$ ,n) reaction is less likely to be a direct process for it would involve a single transfer of three nucleons into valence states of the target, a highly improbable process.

One particularly important application of a single-particle transfer reactions, especially (d,p) and (d,n), is the study of low-lying shell-model excited states. Several such states may be populated in a given reaction; We can choose a particular excited state from the energy of the outgoing nucleon. Once we have done so, we would like to determine just which shell-model state it is. For this we need the angular distribution of the emitted particles, which often give the spin and parity of the state that is populated in a particular reaction. Angular distributions therefore are of critical importance in studies of transfer reactions. (Pickup reactions, for example (p,d), in which the projectile takes a nucleon from the target, also give information on single-particle states.)

### 3.1.3 Resonance Reactions

The compound-nucleus model of nuclear reactions treats the unbound nuclear states as if they formed a structureless continuum. That is, there may be discrete nuclear states, but there are so many of them and they are so close together that they form a continuous spectrum. Each of these supposed discrete states is unstable against decay and therefore has a certain width; when the states are so numerous that their spacing is much less than the widths of the individual states the compound-nucleus continuum results.

The bound states studied by direct reactions are at the opposite end of the scale. Because they are stable against particle emission, their mean lives are much longer (for example, characteristic of  $\gamma$  decay) and their corresponding widths are much smaller. A state with a lifetime of 1ps, for instance, has a width of about  $10^3$ eV. Far smaller than the typical spacing of bound states. We are therefore justified in treating these as discrete states with definite wave functions.

Between these two extremes is the resonance region-discrete levels in the compound-nucleus region. These levels have a high probability of formation (large cross sections), and their width are very small because at low incident energy, where these resonances are most likely to occur. The quasi bound state that is formed usually has only two modes of decay available to it-re-ejecting the incident particle, as in elastic or inelastic scattering, or  $\gamma$  emission.

To obtain a qualitative understanding of the formation of resonances. We represent the nuclear potential seen by the captured particle as a square well. The oscillatory wave functions inside and outside the well must be matched smoothly, fig 3.4 shows several examples of how this might occur. Depending on the phase of the wave functions inside and outside the nucleus, In case (a), the incident particle has relatively little probability to penetrate the nucleus and form a quasi bound state; in case (c), there is a very the relative phase of the inner and outer wave functions; the location of the matching point and the relative amplitudes vary accordingly. Only for certain incident energies do we achieve the conditions shown in part(c) of fig 3.4 These are the energies of the resonances in the cross section.

In a single, isolated resonance of energy  $E_R$  and width  $\Gamma$ , the energy profile of the cross section in the vicinity of the resonance will have the character of the energy distribution of any decaying state of lifetime  $\tau = \frac{\hbar}{\Gamma}$ ;

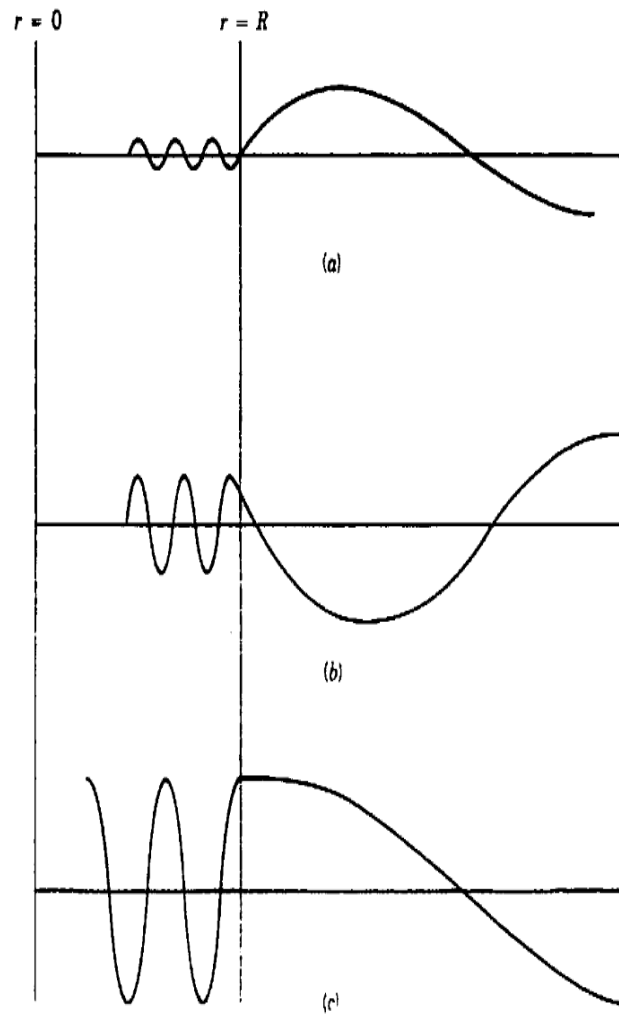


Figure 3.4: (a) Far from resonance, the exterior and interior wave functions match badly, and little penetration of the nucleus occurs. (b) As the match improves, there is a higher probability to penetrate. (c) At resonance the amplitudes match exactly, the incident particle penetrates easily, and the cross section rises to a maximum.

The resonance will occur where the total cross section has a maximum: assuming only one partial wave  $l$  is important for the resonant state, there will be a scattering resonance where  $\eta_l = -1$ , corresponding to phase shift  $\delta_l = \frac{\pi}{2}$ .

The shape of the resonance can be obtained by expanding the phase shift about the value  $\delta_l = \frac{\pi}{2}$ . Better convergence of the Taylor series expansion is obtained if we expand the cotangent of  $\delta_l$

$$\begin{aligned}
Cot\delta_l(E) &= Cot\delta_l(E_R) + (E - E_R) \left( \frac{\partial Cot\delta_l}{\partial E} \right)_{E=E_R} \\
&+ \frac{1}{2}(E - E_R)^2 \left( \frac{\partial^2 Cot\delta_l}{\partial E^2} \right)_{E=E_R} + \dots
\end{aligned} \tag{3.1.2}$$

in which

$$\left( \frac{\partial Cot\delta_l}{\partial E} \right)_{E=E_R} = - \left( \frac{\partial \delta_l}{\partial E} \right)_{E=E_R} \tag{3.1.3}$$

Defining the width  $\Gamma$  as

$$\Gamma = 2 \left( \frac{\partial \delta_l}{\partial E} \right)_{E=E_R}^{-\Gamma} \tag{3.1.4}$$

then it can be shown that the second-order term vanishes, and thus (neglecting higher-order terms)

$$Cot\delta_l = - \frac{(E - E_R)}{\Gamma/2} \tag{3.1.5}$$

Because  $\Gamma$  is the full width of the resonance, the cross section should fall to half of the central value at  $E - E_R = \pm\Gamma/2$ . From Equation 3.1.5 this occurs when  $Cot\delta_l = \pm 1$ , or  $\delta_l = \Pi/4, 3\Pi/4$  (compared with  $\delta_l = \Pi/2$  at the center of the resonance). The cross section depends on  $Sin^2\delta_l$  which does indeed fall to half the central value at  $\delta_l = \Pi/4$  and  $3\Pi/4$ , The width defined by Equation 3.1.4 is From Equation 3.1.5. We find

$$Sin\delta_l = \frac{\Gamma/2}{\left[ (E - E_R)^2 + \Gamma^2/4 \right]^{1/2}} \tag{3.1.6}$$

and the Scattering cross section becomes

$$\sigma_{sc} = \frac{\Pi}{K^2} (2l + 1) \frac{\Gamma^2}{(E - E_R)^2 + \Gamma^2/4} \tag{3.1.7}$$

This resultant be generalized in two ways. In the first place, we can account for the effect of reacting particles with spin. If  $s_a$  and  $s_X$  are the spins of the incident and target particles, and if  $I$  is the total angular momentum of the resonance.

$$I = s_a + s_X \div l \tag{3.1.8}$$

then the factor  $(2l + 1)$  in Equation 3.1.7 should be replaced by the more general statistical factor

$$g = \frac{2I + 1}{(2s_a + 1)(2s_X + 1)} \quad (3.1.9)$$

Note that  $g$  reduce to  $(2l + 1)$  for spineless particles.

The second change we must make is to allow for partial entrance and exit widths. If the resonance has many ways to decay, then the total width  $\Gamma$  is the sum of all the partial widths  $\Gamma_i$

$$\Gamma = \sum_i \Gamma_i \quad (3.1.10)$$

The  $\Gamma^2$  factor in the denominator of Equation 3.1.7 is related to the decay width of the resonance state and therefore to its lifetime:  $\Gamma = \hbar/\tau$ . The observation of only a single entrance or exit channel does not affect this factor, for the resonance always decays with the small lifetime  $\tau$ . In the analogous situation in radioactive decay, the activity decays with time according to the total decay constant, even though we might observe only a single branch with a very different partial decay constant. The  $\Gamma^2$  factor in the numerator, on the other hand, is directly related to the formation of the resonance and to its probability to decay in to a particular exit channel. In the case of elastic Scattering, for which equation 3.1.7 was derived, the entrance and exit channels are identical. That is, for the reaction  $a + X \rightarrow a + X$ , we should use the partial widths  $\Gamma_{aX}$  of the entrance and exit channels:

$$\sigma = \frac{\Pi}{K^2} g \frac{(\Gamma_{aX})^2}{(E - E_R)^2 + \Gamma^2/4} \quad (3.1.11)$$

Similarly for the reaction  $a + X \rightarrow b + Y$ . A different exit width must be used:

$$\sigma = \frac{\Pi}{K^2} g \frac{\Gamma_{aX} \Gamma_{bY}}{(E - E_R)^2 + \Gamma^2/4} \quad (3.1.12)$$

### 3.1.4 Heavy-Ion Reactions

From the point of view of nuclear reactions, a heavy ion is defined to be any projectile with  $A > 4$ . Accelerator devoted to the study of heavy-ion reactions can produce beams

of ions up to  $^{238}\text{U}$ . At typical energies of the order of 1-10 MeV per nucleon although much higher energies are also possible.

The variety of processes than can occur in heavy-ion reactions is indicated schematically in fig 3.5. At large impact parameters Coulomb effects dominate, and Rutherford scattering or Coulomb excitation may occur. When the nuclear densities of the target and projectile just begin to overlap. Nuclear reactions can occur and at small overlap ordinary elastic or inelastic scattering and few-nucleon transfer through direct reactions may occur,

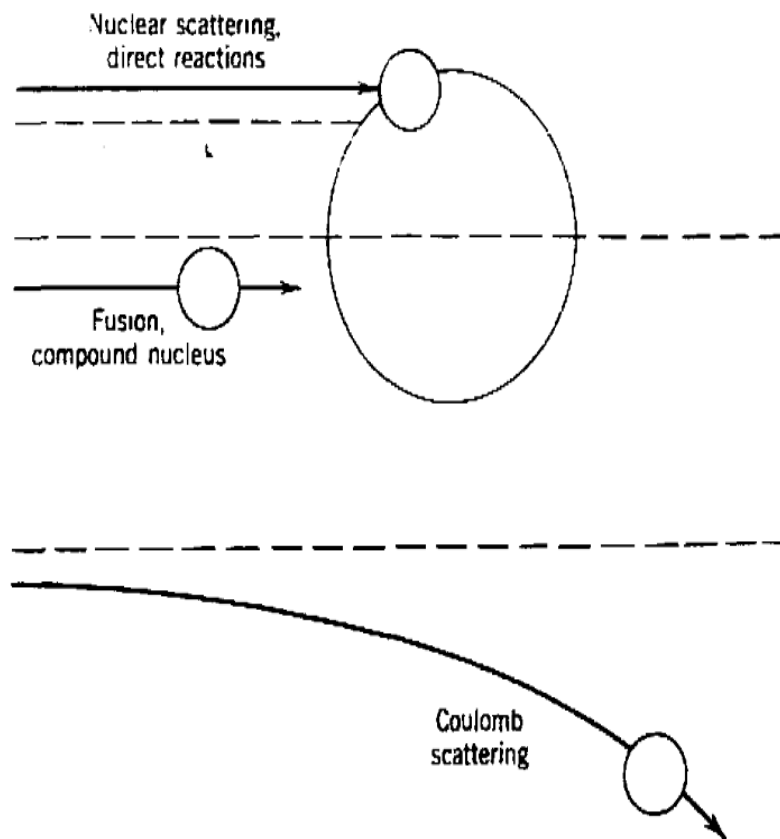


Figure 3.5: processes in heavy-ion scattering depend on the impact parameter, when energies are large enough to penetrate the Coulomb barrier.

At small impact parameters, new and unusual features emerge in these reactions representing complete fusion of the two nuclei can form as an intermediate state. However,

to overcome the repulsive Coulomb barrier. The incident ion must be quite energetic and thus the compound nucleus is formed with a considerable excitation energy. This compound nucleus may be an unusual state of nuclear matter that cannot be achieved in reactions with light nuclei. Because of large incident energy, the compound nucleus may achieve a density or a "temperature" (that is, a mean internal kinetic energy per nucleon) beyond what can be achieved in reactions with light ions. The analysis of these compound states and their decay modes thus represent a challenge for nuclear theory—can we extrapolate from an equation of state for "ordinary" nuclear matter to one for "extraordinary" nuclear matter?

## 3.2 Reaction Cross Section

In this section we give some more general definitions of various measurable quantities that are loosely grouped under the heading "cross section".

Roughly speaking the cross section is a measure of the relative probability for the reaction to occur. If we have a detector placed to record particle  $b$  emitted in a direction  $(\theta, \phi)$  with respect to the beam direction. The detector defines a small solid angle  $d\Omega$  at the target nucleus. Let the current of the incident particles be  $I_a$  particles per unit time and let the target show to the beam  $N$  target nuclei per unit area. If the outgoing particles appear at a rate  $R_b$ , then the reaction cross section is

$$\sigma = \frac{R_b}{I_a N} \quad (3.2.1)$$

Defined in this way,  $\sigma$  has the dimension of area per nucleus, but it may be very much larger or smaller than the geometrical area of the disc of the target nucleus seen by the incoming beam

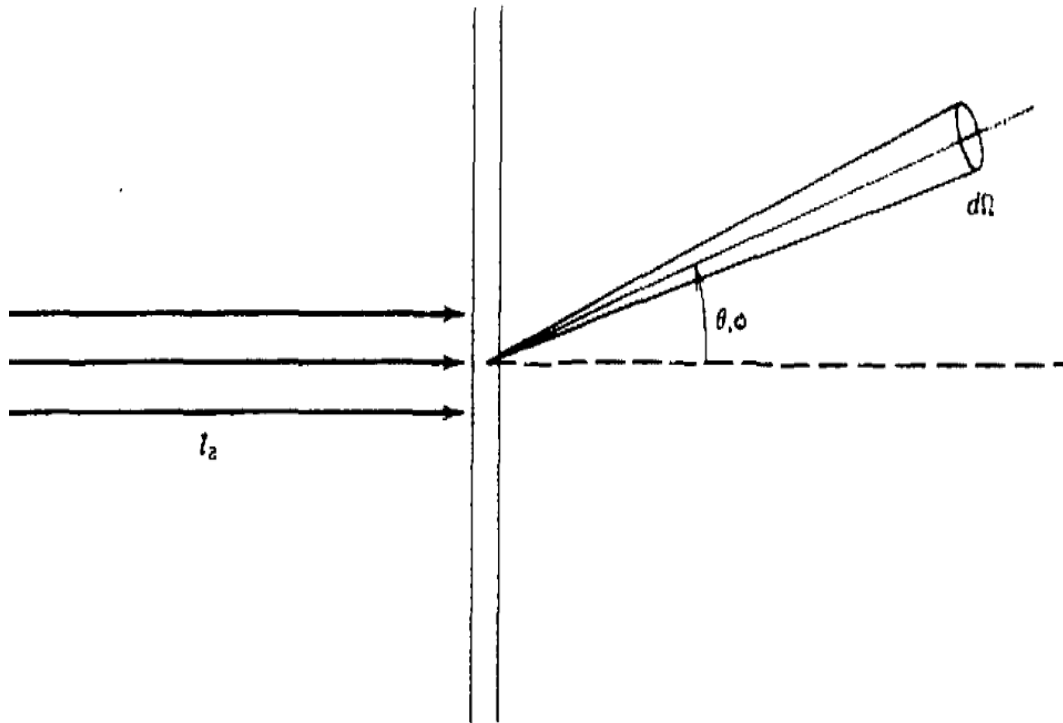


Figure 3.6: Reaction geometry showing incident beam, target and outgoing beam going into solid angle

For a typical nucleus of radius  $R = 6$  fm, the geometrical area  $\Pi R^2$  is about  $100 \text{ fm}^2 = 1$  b; for neutron capture by  $^{135}\text{Xe}$ , the cross section is about  $10^6$  b, while for other much more improbable reactions the cross section may be measured in millibars. You should think of  $\sigma$  as a quantity which has the dimension of an area, but which is proportional to the reaction probability.

Our detector occupies only a small solid angle  $d\Omega$  and therefore does not observe all of the outgoing particles; only a small fraction  $dR_b$  are actually counted, and therefore only a fraction of the cross section  $d\sigma$  will be deduced. Moreover, the outgoing particles will not in general be emitted uniformly in all directions. But will have an angular distribution that will depend on  $\theta$  and possibly also on  $\phi$ . We let this angular distribution function be arbitrarily represented by  $r(\theta, \phi)$ , then  $dR_b = r(\theta, \phi)d\Omega/4\pi$ . (The  $4\pi$  is introduced to

make  $d\Omega/4\pi$  a pure fraction.) Then

$$\frac{d\sigma}{d\Omega} = \frac{r(\theta, \phi)}{4\pi I_a N} \quad (3.2.2)$$

The quantity  $d\sigma/d\Omega$  is called the differential cross section and its measurement gives us important information on the angular distribution of the reaction products. In the literature, it is often called  $\sigma(\theta, \phi)$  or  $\sigma(\theta)$  or sometimes (unfortunately) just "cross section," (If you see a graph of "cross section" vs  $\theta$  you should know that what is intended is differential cross section.) Because solid angle is measured in steradians (the surface of a sphere subtends a solid angle of  $4\pi$  steradians at its center). Units of differential cross section are barns/steradian. The reaction cross section  $\sigma$  can be found by integrated  $d\sigma/d\Omega$  over all angles; with  $d\Omega = \sin\theta d\theta d\phi$  we have

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega = \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \frac{d\sigma}{d\Omega} \quad (3.2.3)$$

Notice that if  $d\sigma/d\Omega$  is constant (independent of angle), the integral gives  $\sigma = 4\pi(d\sigma/d\Omega)$ . This justifies the insertion of the constant  $4\pi$  into Equation (3.2.2) for now  $r(\theta, \phi)$  reduces to the constant  $R_b$  and Equation (3.2.2) agrees with Equation(3.2.1).

In many nuclear physics applications, we are not concerned simply with the probability to find particle b emitted at a certain angle: we also want to find it with a certain energy, corresponding to a particular energy of the residual nucleus Y. We therefore must modify the definition of cross section to give the probability to observe b in the angular range of  $d\Omega$  and in the energy range  $dE_b$ . This gives the so-called doubly differential cross section  $d^2\sigma/dE_b d\Omega$ . In the literature, this additional energy dependence is often not explicitly stated: usually the cross sections are plotted as  $d^2\sigma/dE_b d\Omega$ , although it may not be labeled as such. For discrete states, there may be only a single level within the energy range  $dE_b$ , and the

Cross section	Sec-	Symbol	Technique	Possible Application	Applica- tion
Total		$\sigma_1$	Attenuation of beam	of shielding	
Reaction		$\sigma$	Integrated over all angles and all energies of b (all excited states of Y)	Production of radioisotope Y in a nuclear reaction	
Differential (Angular)		$d\sigma/d\Omega$	Observe b at $(\theta, \phi)$ but integrate over all energies	Formation of beam of b particles in a certain direction (or recoil of Y in a certain direction)	
Differential (Energy)		$d\sigma/dE$	Don't observe b, but observe excitation of Y by subsequent $\gamma$ emission	study of decay of excited states of Y	
Doubly differential		$d^2\sigma/dE_b d\Omega$	Observe b at $(\theta, \phi)$ at a specific energy	Information on excited states of Y by angular distribution of b	

Table 3.1: Table of reaction cross section

distinction becomes unimportant, If on the other hand we do not observe the direction of particle b (by surrounding the target area  $4\pi$  solid angle of detectors, or by not observing b at all ), then we measure yet another differential cross section  $d\sigma/dE$  where now E may represent an excitation energy of Y. There is still another cross section that may be of interest, the total cross section  $\sigma_t$ . Here, for a specific incident particle a, we add the reaction cross sections  $\sigma$  for all possible different outgoing particles b, no matter what their direction or energy. Such a determination would tell us the probability for an incident particle to have any reaction at all with the target and thus be removed from the beam of incident particles, This can be deduced directly by measuring the loss in intensity of a collimated beam in passing through a certain thickness of the target material.

When we discuss a specific reaction then, the exact meaning of the term cross section will depend on exactly what we measure. Table summarizes these different measurements, how they might be put. For example, If wish to produce a radioactive isotope

as the residual nucleus  $Y$ , we have absolutely no interest in the direction of emission of particle  $b$ , nor in the excited states of  $Y$  that may be populated, for they will quickly decay by  $\gamma$  emission to the ground state of  $Y$ . The literature often does not discriminate carefully among these definitions, and often they are called merely "cross section." It is almost always obvious in context which cross section is meant, and therefore not strictly necessary to distinguish carefully among them.

### 3.2.1 Experimental Techniques

A typical nuclear reaction study requires a beam of particles, a target, and a detection system. Beams of charged particles are produced by a variety of different types of accelerators, and neutron beams are available from nuclear reactors and as secondary beams from charged-particle accelerators. To do precision spectroscopy of the outgoing particle  $b$  and the residual nucleus  $Y$ , the beam must satisfy several criteria:

- 1, It must be highly collimated and focused, so that we have a precise reference direction to determine  $\theta$  and  $\phi$  for angular distribution measurements.
- 2, It must have a sharply defined energy: otherwise, in trying to observe a specific excited state by finding  $Q_{ex}$  and  $E_{ex}$  from Equation, we might find that variation in  $T_a$  would give two or more different  $E_{ex}$  for the same  $T_b$ .
- 3, It must be of high intensity, so that we can gather the necessary statistics for precise experiments.
- 4, If we wish to do timing measurements (such as to measure the lifetimes of excited states of  $Y$ ). the beam must be sharply pulsed to provide a reference signal for the formation of the state. and the pulse must be separated in time by at least the time resolution of the measuring apparatus and preferably by a time of the order of the one we are trying to measure.
- 5, Under ideal circumstances, the accelerator beam should be easily selectable we should

be able to change the incident energy  $T_a$  or even the type of incident particle in a reasonable time. The stringent tuning requirements of modern large accelerator and the demands that high currents put on ion sources make this requirement hard to meet in practice. Accelerator beam time is often scheduled far in advance (6 months to a year is common). So that experiments with common beam requirements can be grouped together thus minimizing the beam tuning time.

6, The intensity of the incident beam should be nearly constant and easily measurable, for we must know it to determine the cross section. If we move a detector from one position to another, we must know if the change in the observed rate of detection of particle b comes from the angular dependence of the differential cross section or merely from a change in the incident beam intensity.

7, The beam may be polarized (that is, the spins of the incident particles all aligned in a certain direction) or unpolarized, according to the desire of the experimenters.

8, The beam must be transported to the target through a high-vacuum system so as to prevent beam degradation and production of unwanted products by collisions with air molecules.

Types of targets vary widely, according to the goals of the experiment. If we want to measure the yield of a reaction (that is,  $\sigma$  or  $\sigma_t$ ), perhaps through observation of the attenuation of the beam or the decay of radioisotope Y, then we may choose a thick, solid target. Such a target might degrade, scatter, or even stop the outgoing particles b, which does not bother us in this kind of measurement. On the other hand, if we wish to observe b unaffected by interactions in the targets, a very thin target is required. Thin metal foils are often used as targets but for nonmetals, including compounds such as oxides, the target material is often placed on a thin backing, which does not contribute to the reaction or affect the passage of particle b. For many applications, extremely rare (and often expensive) targets of separated isotopes are used. A high-intensity, highly focused

beam (typically a few mm in diameter) delivers considerable thermal power to the target (absorption of  $1 \mu A$  of 10 MeV protons delivers 10W), which is enough to burn up thin targets: therefore a way must be found to cool the target and extract the heat generated by the beam. As with the beam, it should be relatively easy to change targets so that valuable beam time is not wasted. For some applications, it may be desirable to polarize the spins of the target nuclei.

The detectors may consist of some or all of the following: particle detectors or detector telescopes to determine the energy and type of the outgoing particles, magnetic spectrometers for good energy resolution (sometimes necessary to identify close-lying excited states of Y), position-sensitive particle detectors (such as multi wire proportional counters) to do accurate angular distribution work  $\gamma$ -ray detectors to observe the de-excitation of the excited states of Y (Possibly in coincidence with particles b), polarimeters to measure the polarization of the particle b, and so on. Because beam time is a precious commodity at a modern accelerator facility, the emphasis is always on getting the largest amount of data in the shortest possible time. Therefore multi detector configurations are very common: many signals arrive simultaneously at the detectors and are stored by an on-line computer system for later "re-play" and analysis. (Keeping the beam and the detectors going during the experiment usually demands all the attention of the experimenters and leaves little time for data analysis)

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**Declaration**

This project is my work, has not been presented for a degree in any other University and that all the sources of material used for the project have been dully acknowledged.

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**Place and time of submission: Addis Ababa University, June 2011**

This project has been submitted for examination with my approval as University advisor.

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