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Sub-Structure Optimization for High Rise Building Using Brute Force Optimization

by

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the Degree of Master of Science in Structural Engineering

The undersigned have examined the thesis entitled “Sub Structure Optimization for High Rise Building using Brute Force Optimization” presented by Testi Yusuf, a candidate for the degree of Master of Science and hereby certify that it is worthy of acceptance.

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Testi Yusuf

ABSTRACT

This study is entitled as “Sub Structure Optimization for High Rise Building using Brute Force Optimization”, and includes the concept of two different streams i.e. Optimization and Design of Concrete Sub-Structure.

Previous researches such as “Optimum Design of Pile Foundation by Automatic Grouping Genetic Algorithms” by Xiaofeng Liu, Gengdong Cheng et al. optimized pile and pile cap separately, by first optimizing the piles and then designing a pile cap for the optimal pile design, however this does not consider the effect of piles in the optimization of the pile cap. This thesis employed *Brute Force Optimization* to optimize pile and pile cap simultaneously and take into account the effect of piles in the optimization of the pile cap.

For the optimization formulation of the foundation types, the desired variable chosen for pile cap are the depth, length and width of the pile cap, while for Pile foundation the desired variables chosen are the number of piles in the X, the number of piles in the Y, the diameter of the pile, the height of the pile and the depth of the pile cap. These variables will help minimize or optimize the objective function by reducing the cost of material and excavation. The foundations will be designed based on EC7.

A computer program using Java is developed to design an optimum foundation design for any particular case within the scope of this study. The optimized design is checked using hand calculation by simple example to determine its correctness and accuracy.

Keywords: Brute Force Optimization, Sub-Structure Optimization, Design of Concrete Sub-Structure, Java,

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1. INTRODUCTION

1.1. Background

Construction is a process of building something by human being for some purpose or another. Construction is very important for the development of a country, for example in the case of our country it has high contribution to the implementation of the GTP, the growth of the economy and is one of the major sources of employment thereby greatly contributing to the well-being of the people.

The importance of construction becomes evident when we notice that man was living in a cave before there was even a proper language. Because of the importance of shelter in the creation of towns, there was a boom in construction in ancient civilizations.

In recent years, because of lack of space in major towns, tall building are in demand, for this reason new and crucial technologies and techniques had to be tested and approved to make these buildings safe. Moreover, design codes and Design Procedures have also been developed to satisfy the safety of buildings.

It is to be noted that nowadays safety is not the only factor in construction. Cost consideration has become paramount in the construction business particularly in developing countries like Ethiopia. In Ethiopia, construction consumes most of the resources of the country and up to 40% of the countries investment. Saving cost is why optimization methods were developed and quickly became very urgent and important in day to day life.

In this regard optimizing foundations will help reduce construction cost, improve safety and save man hour that can in turn be used productively elsewhere.

It is interesting to note that high rise buildings and/or buildings on soil with low bearing capacity use Pile foundation. It is also a common practice that, the designer chooses the type of foundation by experience and designs the parameters considering only structural safety. This creates serious problem such as construction of uneconomical foundation, costly implementation and undue delay due to lack of money or exceeding budget.

This study addresses “Sub-Structure Optimization for High Rise Building using Brute Force Optimization”, and includes the concept of two different streams i.e. Structural Optimization and

Design of Concrete Structure. This thesis work makes use of Brute Force Optimization to optimize pile foundation by optimizing the piles and the pile cap simultaneously. Use of Brute Force Optimization made to help find optimal arrangements and dimensions of pile foundations based on the minimum cost.

To help visualize the optimization process let's start at isolated footing, as the load increase or as the soil loses bearing capacity the isolated footing will increase in height, width and depth. As the height and width increase, the neighboring isolated footings area of influence overlap making the isolated foundation eventually turn into a mat foundation. As the load increases even more or the bearing capacity of the soil decreases such as in the case of thick black cotton soil layer, the mat will not be able to hold the building, therefore the mat will have a small column type structures underneath it to increase its resistance to the applied load. Thus the substructure turns from mat foundation to pile foundation, where, the mat acts as pile cap.

1.2. Objectives and Scope of the Study

1.2.1. General Objectives

The general objective of this thesis is to obtain minimum cost design of Pile Foundation and select the appropriate dimensions using Brute Force Optimization for safe and economical design.

1.2.2. Specific Objectives

The specific objective of the thesis includes the following:

1. To help structural and geotechnical engineers to design efficient and safe foundations.
2. To reduce time of design and construction
3. To reduce the cost of foundation and increase the savings from this optimization to be used elsewhere and also reducing the implementation delay of most buildings.
4. To inspire others to optimize other structural components. For example one might optimize the type of slab to obtain the minimum cost.

1.2.3. Scope of the Study

Concrete Pile and Pile cap design is a very vast topic where the variable has an infinite amount of possibilities. This is why the scope is discussed to elaborate which parts of its design is included and exclude with their design.

In a perfectly optimized structure, the piles near a bigger load tend to have a bigger diameter, larger depth and smaller spacing between each other on the contrary piles near smaller load tend to have smaller diameter, smaller depth and bigger spacing with each other. The difference in applied loading of piles in one column is Moment and/or eccentricity. The moment that is applied on the foundation is due to lateral loads especially earthquake and wind loads. This load can act in any direction; therefore the design procedure is for the worst case scenario of the respective pile in every direction which dictates all the piles to have the same diameter, depth and spacing to resist the vertical and horizontal loads. Therefore:

- Pile Diameter under a pile cap are the same
- Pile spacing shall be assigned as a constant in each direction
- Pile depth under a pile cap is the same

The other reason this study does not consider varying pile diameter under a pile cap and the spacing between the piles as a design variable is because the number of design variables to be solved will be unknown; if one doesn't know the number of piles, It is not possible to know how many spacing's and how many different diameter of piles exist in the pile configurations. This means that one doesn't know how many unknowns exist when one start optimization.

Mat foundation is not considered in this thesis work because

- Mat foundations are often not used in high rise buildings,
- Mat foundation requires the precise location of all columns to establish planar area and then compute shear force and bending moment that is not ideal for creating a general case optimization and
- Similar design calculations are utilized as some of the procedures between mat foundation and pile cap are similar.

Edge column piles near property lines cannot be optimized in this study's program because the program considers the column to be at the center of the pile cap.

This study can be applied for all soil types because the software input is made to consider the reaction force per unit area of the soil. This will however require the user to calculate the soils reaction per unit area using the properties of the soil or field experiment data to eventually input to the software. This study is structural engineering thesis paper because no soil parameters are used in this paper except in the calculation of settlement. This is because this study's program uses only the applied load and the reaction to design safe and economical piles and pile caps.

2. LITERATURE REVIEW

2.1. Structural Optimization

Optimization is the act of obtaining the best result under the given circumstances. In design construction and maintenance of any engineering system, many technological and managerial decisions have to be taken at several stages. The ultimate goal of all such decisions is either to minimize the effort required or to maximize the desired benefit. Hence optimization can be defined as the process of finding the conditions that give the minimum or maximum value of a function, where the function represents the effort required or the desired benefit.

Optimization can include a wide range of problems with the aim of searching for certain optimality. Subsequently, there are many different ways of naming and classifying optimization problems, and typically the optimization techniques can also vary significantly from problem to problem. A unified approach is not possible, and the complexity of an optimization problem largely depends on the function forms of its objective functions and constraints.

It is possible to write most optimization problems in the generic form mathematically as indicated in equation 2.1 below.

$$\begin{aligned} &\text{Minimize} && f_i(x), && (i=1, 2, ..., M), && (2.1)\\ &\text{Subject to} && \phi_j(x) = 0 && (j=1, 2, ..., J),\\ & && \Psi_k(x) \leq 0, && (k=1, 2, ..., K), \end{aligned}$$

$$X = (x_1, x_2, ..., x_n)^T$$

Where: $f_i(x)$, $\phi_j(x)$ and $\Psi_k(x)$ are functions of the design vector

- X_i of X are called design or decision variables, and they can be real continuous, discrete or the mixed of these two.
- The functions $f_i(x)$ where i=1,2,..., M are called the objective functions (cost function/ energy function), in the case of M=1,there is only a single objective and in the case of M>1, there are multi objectives

- The space spanned by the decision variables is called the design space or search space R^n, while the space formed by the objective function values is called the solution space or response space.
- The equalities for ϕ_j and inequalities of Ψ_k are called constraints. It is worth pointing out that we can also write the inequalities in the other way ≥ 0 for Ψ_k, and we can also formulate the objectives as maximization problem.

According to Xin-SheYang [36], classification can be carried out in terms of the number of objectives, number of constraints, function forms, landscape of the objective functions, type of design variables, uncertainly in values, and computational effort.

2.1.1. Classification of Optimization

2.1.1.1. Optimization problems according to the number of objectives, there are two types:

- single objective M =1 and
- Multi objective M>1. Multi objective optimization is also referred to as multi criteria or even multi attributes optimization in literature. Most real world optimization problems are multi objective.

2.1.1.2. Optimization in terms of number of constraints J+K. If there is no constraint at all J=K=0, then it is called an unconstrained optimization problem. If K=0 and J≥1, it is called an equality constrained problem, while J=0 and K ≥1 becomes an inequality constrained problem.

2.1.1.3. Optimization classification based on function forms. The objective functions can be either linear or nonlinear. If the constraints ϕj and Ψk are all linear, then it becomes a linearly constrained problem. If both the constraints and the objective functions are all linear, it becomes a linear programming problem. However, if all fi, ϕj and Ψk are non-linear, then it is a nonlinear optimization problem.

2.1.1.4. Optimization classification based on landscape of the objective functions. If there is only a single valley or peak with a unique global optimum (the local optimum is the only optimum, which is also its global optimum), then the optimization problem is said to be uni-modal. In this case. A special and very important class of uni-modal optimization is the convex optimization, when the objective functions have certain convexity and the global optimal solution is guaranteed.

However, most objective functions have more than one mode, and such multimodal functions are much more difficult to solve.

2.1.1.5. Optimization classification based on the type of value of the design. If the values of all design variables are discrete, then the optimization is called discrete optimization or combinatorial optimization. If all these values are integers, then it becomes an integer programming problem. However, the design variables in many problems can be both discrete and continuous; such types are called the mixed type optimization.

2.1.1.6. Optimization classification based on Determinacy. When the values in both design variables and objective/constraints are exact, these are called deterministic. If there is any uncertainty in the design variables and the objective functions and/or constraints, then the optimization becomes a stochastic optimization problem, or a robust optimization problem. Stochastic problems must be redefined or reformulated so that they become meaningful when using standard optimization techniques. This often involves the averaging over some space and the objective should be evaluated in terms of mean and/or in combination with their related uncertainties.

2.1.2. Optimization Algorithms

For different types of optimization problems, it is necessary to use different optimization techniques because some algorithms such as the Newton-Raphson method are more suitable for certain types of optimization than others.

In general, optimization algorithms can be divided into two categories:

- Deterministic algorithms follow a rigorous procedure and its path and values of both design variables and the functions are repeatable. A good example of this method is simple x method. Deterministic algorithms can be further divide into Gradient based as it uses the function values and their derivatives, works extremely well for smooth uni-modal problems (Newton-Raphson algorithm) and Non-gradient based or gradient-free algorithms that do not use any derivative, but only the function values (Hooke-Jeeves pattern search, Nelder-Mead and downhill simplex)

- Stochastic algorithms always have some randomness. There are two general types: heuristic and metaheuristic, though their difference is small. Heuristic means ‘to find’ or ‘to discover by trial and error’. Quality solutions to a tough optimization problem can be found in a reasonable amount of time, but there is no guarantee that optimal solutions are reached. Meta-heuristic algorithms, here meta means 'beyond' or' higher level', and they generally perform better than simple heuristics. In addition, all meta-heuristic algorithms use certain trade off of randomization and local search
- Hybrid algorithm which is a mixture of deterministic and stochastic algorithms.

2.2. Optimization Techniques

Optimization is systematic tool to obtain the best result under a set of given circumstances. Optimum design is thus a search to obtain economic and light weight structures due to finite resources and energy limitations. For this, several optimization methods have been devised each with their own unique advantages and disadvantages.

Two distinct divisions of optimization methods

1) Numerical Methods: - programming methods based on numerical mathematics where a near optimal design is generated in an iterative manner.
2) Analytical method: - employ mathematical theory of calculus variation method suitable in studies of the fundamentals of single structural component.

Search methods can be classified as: calculus based, enumeration based, random search methods and natural algorithms

1) Calculus based approaches use derivatives to find local/global extrema for problem space
2) Enumerative methods try to find an optimal solution by searching through the problem space one point at a time
3) Random search methods greatly improve on the enumerative methods by selecting random points and testing for the optimality criteria.
4) Natural algorithms start out with an initial trial set of solutions and improve on those through successive iterations

2.3. Brute Force Optimization

The brute-force search is the simplest search method available. However, despite its requirements of good computing power, it remains a functional and useful tool. Moreover, its simplicity makes it a good method to study as an initiation to optimization.

A brute-force approach is any algorithm that tries possible solutions one after the other until it finds one that is acceptable or until a pre-set maximum number of attempts. A brute-force optimization algorithm would thus simply evaluate value after value for a given time, and return the value with the optimal result as its solution at the end.

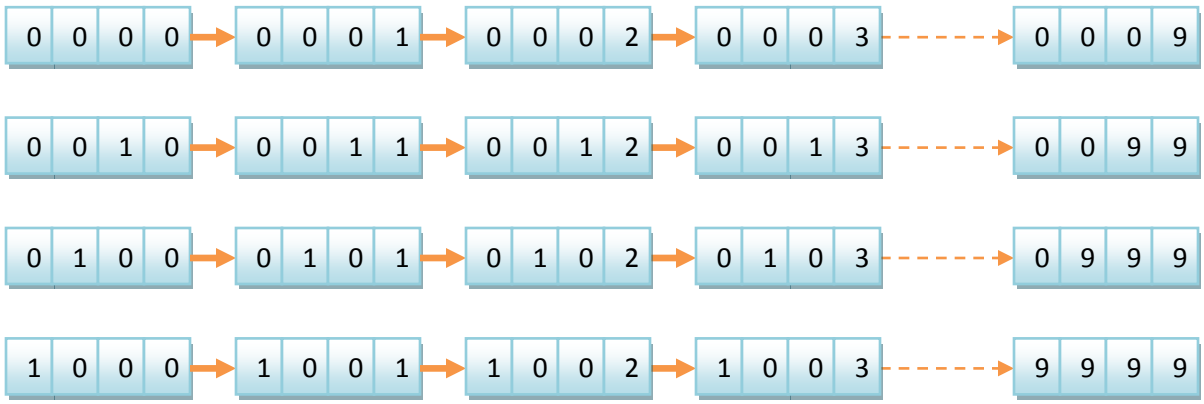
While such a brute-force approach may seem unsophisticated, it does have the advantage of being able to search any function, even one that has a complex and irregular shape, multiple local optima, and even discontinuities.

It is possible to take advantage of various kinds of partial knowledge one may have about the solution. Heuristics can also be used to make an early cutoff of parts of the search. In certain fields, such as language parsing, techniques such as chart parsing can exploit constraints in the problem to reduce an exponential complexity problem into a polynomial complexity problem. In many cases, such as in Constraint Satisfaction Problems, one can dramatically reduce the search space by means of Constraint propagation that is efficiently implemented in Constraint programming languages. The search space for problems can also be reduced by replacing the full problem with a simplified version.

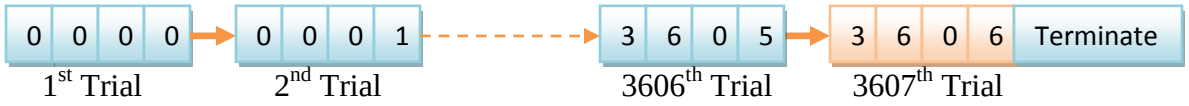
In the case of this study, for the purpose of making the optimized solution more applicable in the field, the possible solutions were heavily reduced. For instance, the length of a pile might be 3.28; this is impractical in the real world. The reduction of the possible solution has the added advantage of reducing the need for high computational power.

Brute Force is usually used in hacking pass codes. In some movies hacking pass codes is shown by some device that tries all possible combination. It is also used in real life and is the main

reason that PIN codes and other pass codes have only three attempts available. This hacking method tries every possible value until it satisfies the requirement set by the programmer; until it opens the locked object.



In this case, there are 10,000 possibilities for one code. For Instance, if the code required is 3606, then the solution will be reached at the 3607th trial. After reaching the correct code, the program terminates by opening the locked object.



In this study's case, there are seven variables that can attain certain values. Brute force will compute the cost of the sub-structure for those which satisfy the constraints by using each and every combination of those values. Then compares each cost of the sub-structure and select the one that has the minimum cost. The difference between the above example and the optimization of the sub-structure is that; the program won't terminate until it computes and compares all the costs generated by all the values of the variable.

Table 2-1The Pros and Cons of using Brute Force Method

Pros	Cons
Unsophisticated and Easily in Computation	**Needs a lot of Computing Power.**
Locates Optimum with ease even for: ➢ **Discontinuous Function** ➢ **Complex Functions** ➢ **Irregular Shaped Functions** ➢ **Functions that have Multiple Local Optima & Minima**	**The values used can be more accurate and obtain more optimal design. But for the ease of construction, limiting accuracy to one decimal number was found to be sufficient.**

2.4. Foundation Structure for High Rise Buildings

2.4.1. What is high rise building?

A building is an enclosed structure that has walls, floors, a roof, and usually windows. "A 'tall building' is a multi-story structure in which most occupants depend on elevators [lifts] to reach their destinations. The most prominent tall buildings are called 'high-rise buildings' in most countries and 'tower blocks' in Britain and some European countries. The terms do not have internationally agreed definitions. " However, a high-rise building can be defined as follows:

- The *New Shorter Oxford English Dictionary* defines a high-rise as "a building having many storeys".
- The *Emporis Standards Committee* defines a high-rise building as "a multi-story structure between 35–100 meters tall, or a building of unknown height from 12–39 floors"
- The *International Conference on Fire Safety in High-Rise Buildings* defined a high-rise as "any structure where the height can have a serious impact on evacuation"
- Some structural engineers define a high rise as any vertical construction for which wind is a more significant load factor than earthquake or weight.
- In the U.S., the National Fire Protection Association defines a high-rise as being higher than 75 feet (23 meters), or about 7 stories.
- According to the *building code of Hyderabad*, India, a high-rise building is one with four floors or more, or 15 to 18 meters or more in height.

In Our country, most building engineers, inspectors, architects and similar professionals define a high-rise as a building that is at least 35 m tall.

High Rise buildings are usually situated in city centers where the price of land is high. Constructing a High Rise building becomes justified if the price of land is so high that it makes economic sense to build upwards as to minimize the cost of the land per the total floor area of a building. Thus the construction of High Rise buildings is dictated by economics and results in High Rise building in a certain part of a large city unless a building code restricts the height of buildings.

High Rise buildings are rarely seen in small cities and they are characteristic of large cities, because of the critical importance of high land prices for the construction of High Rise building.

One problem with High Rise building is car parking. In cities where most people commute via public transport have fewer problems, but for cities which don't have public transport, a lot of parking spaces are needed. Multi-storey car parks are impractical to build very tall, so a lot of land area is needed.

The other problem is it requires a huge amount of investment. The amount of steel, concrete and glass needed to construct a single skyscraper is large, and these materials represent a great deal of money. High Rise buildings have considerable mass, which means that they must be built on a sturdier foundation than would be required for shorter, lighter buildings. Building materials must also be lifted to the top of a skyscraper during construction, requiring more energy and money than would be necessary at lower heights. Furthermore, a High Rise building consumes a lot of electricity because potable and non-potable water have to be pumped to the highest occupied floors, High Rise building are usually designed to be mechanically ventilated, elevators are generally used instead of stairs, and natural lighting cannot be utilized in rooms far from the windows and the windowless spaces such as elevators, bathrooms and stairwells.

Today in Addis Ababa and other major cities in Ethiopia, High Rise building are an increasingly common sight where land is expensive, as in the centers of big cities, because they provide such a high ratio of usable or rentable floor space per unit area of land.

The first high rise building which was more than 20 stories was Nani Building around Addis Ababa stadium. Public and Private Commercial banks are now involved in constructing high rise buildings of their own.

As the building becomes taller and taller, the load on the foundation will also increase, this makes it impossible for traditional foundation methods used for low rise buildings to provide the necessary reaction for these huge loads. High rise buildings other characterization is the presence of high wind loads, this is due to the large surface area they have due to their length, and they also may encounter high earthquake loads. Due to the above reasons, a foundation system that resists lateral loads is very important for the structural safety of the High rise building.

The above reasons are the major contributors for the selection of pile foundation for high-rise buildings.

2.4.2. Type of Foundation for Building Structure

Foundations may be classified based on where the load is carried by the ground, producing:

- Shallow foundations—termed bases, footings, spread footings, or mats. The depth is generally $D/B \leq 1$ but may be somewhat more.
- Deep foundations—piles, drilled piers, or drilled caissons. $L_p/B \geq 4^+$

2.4.3. Shallow and Deep foundation structures

2.4.3.1. Shallow Foundation

2.4.3.1.1. Spread or isolated footing

A spread (or isolated or pad) footing is provided to support an individual column. A spread footing might be circular, square or rectangle slab of uniform thickness. Sometimes, it is stepped or haunched to spread the load over a large area. The drawing below [18] shows an isolated foundation with column.

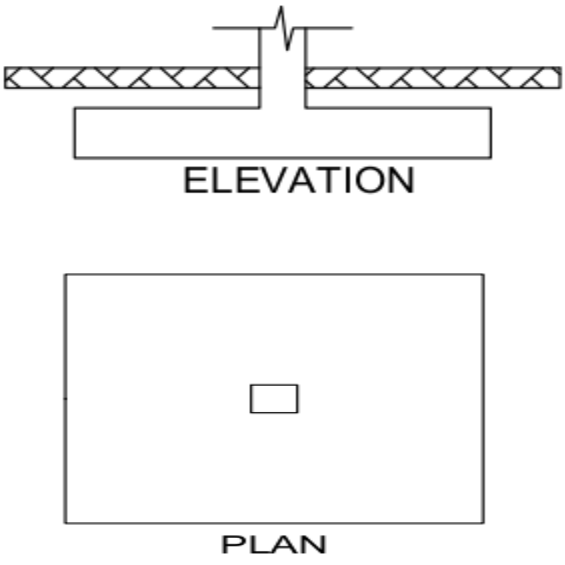


Figure 2-1 ***Plan and Elevation view of Isolated Footing***

2.4.3.1.2. Combined footing

A combined footing supports two columns. It is used when the two columns are so close to each other that their individual footings would overlap. It is also used when the property line is so close to one column that a spread footing would be eccentrically loaded when kept entirely within the property line. By combining it with that of an interior column, the load is evenly distributed.

2.4.3.1.3. Strap or cantilever footing

A strap (or cantilever footing consists of two isolated footings connected with a structural strap or a lever. The strap connects the two footings such that they behave as one unit. The strap simply acts as a connecting beam and does not take any soil reaction. The strap is designed as a rigid beam.

2.4.3.1.4. Mat foundations

A mat or raft foundation is a large slab supporting a number of columns and walls under the entire structure or a large part of the structure. A mat is required when the allowable soil pressure is low or where the column and walls are so close that the individual footing would overlap or nearly touch each other.

Mat foundations are useful in reducing the differential settlement on non-homogenous soils or where there is a large variation in the loads on individual columns. The drawing below [18] shows a mat foundation with columns and walls. In this paper however we will consider only columns with no walls.

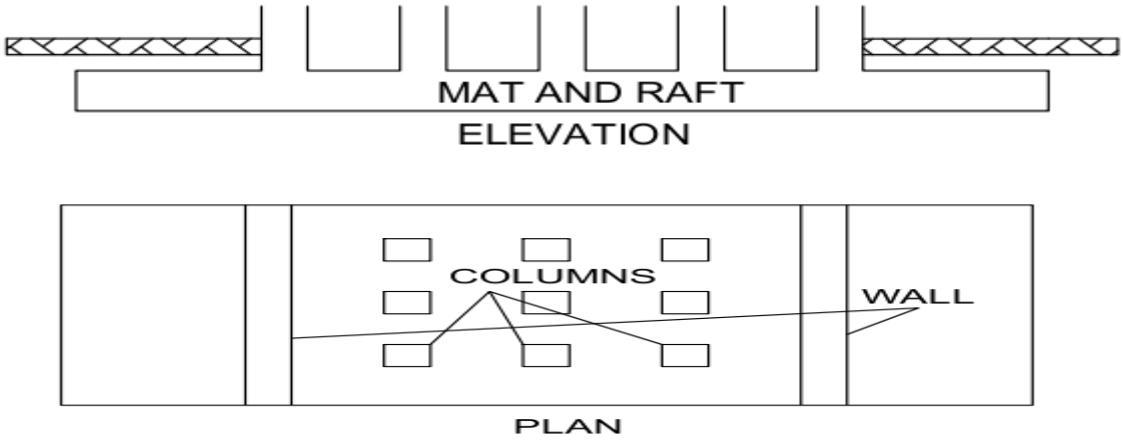
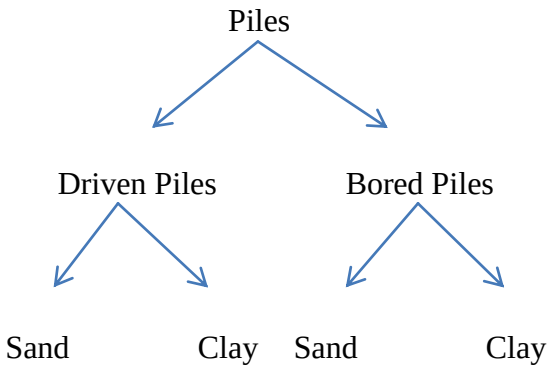


Figure 2-2 *Plan and Elevation view of Mat foundation*

2.4.3.2. Deep Foundation

Piles can be classified into two based on the method of their installation and the type of soil they are installed in. Briefly,



The following sections briefly illustrate design procedures for each type of pile mentioned above.

2.4.3.2.1. Driven Piles

Precast piles driven by force so as to develop additional resistance are classified as driven piles. They can be installed in both types of soils and need heavy equipment for installation. The design calculations and procedures for Driven Piles in Sandy Soil and Driven Piles in Saturated Clay differ. Therefore we must calculate both cases.

2.4.3.2.2. Bored Piles

Piles installed by excavating of the soil and then either filling it with in-situ or precast concrete are called Bored piles. The design calculations and procedures for Bored piles in Sandy Soil and Bored piles in Clay soil differ. Therefore we must calculate both cases.

2.4.3.2.3. Under reamed Piles for clay

Under reamed piles have one or more extruding bulbs at the lower half of their composition and this bulb serves as anchorage in addition to increasing the overall resistance of the pile.

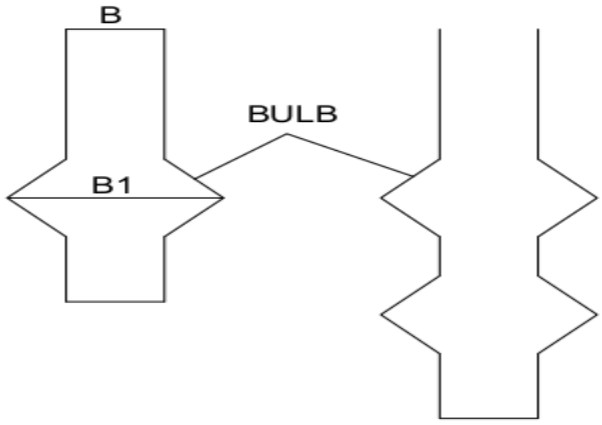


Figure 2-3 *Elevation view of undreamed piles*

2.4.3.2.4. Pile Group in Sand Gravel and Clay

Pile groups are groups of piles that have or assigned to a single property for the whole piles as one entity. Pile groups may possess different characteristics from the sum of single piles. Therefore, it is often necessary to check for group resistance and probability of group failure in piles by comparing the total factored applied load to the group resistance of the piles.

2.4.4. Foundation type selection

Table 2-2 Foundation Types and Typical Usage

Foundation types and typical usage		
Foundation type	**Use**	**Applicable soil conditions**
Shallow foundations (generally D/B ≤1)		
Spread footings, wall footings	Individual columns, walls	Any conditions where bearing capacity is adequate for applied load. May use on a single stratum; firm layer over soft layer or soft layer over firm layer. Check settlements from any source.
Combined footings	Two to four columns on footing and/or space is limited	Same as for spread footings above.
Mat foundations	Several rows of parallel columns; heavy column loads; use to reduce differential settlements	Soil bearing capacity is generally less than for spread footings, and over half the plan area would be covered by spread footings. Check settlements from any source.
Deep foundations (generally $L_P/B \geq 4^+$)		
Floating pile	In groups of 2^+ supporting a cap that interfaces with	Surface and near surface soils have low bearing capacity and competent soil is at

	column(s).	great depth. Sufficient skin resistance can be developed by soil-to-pile perimeter to carry anticipated loads.
Bearing pile	Same as for floating pile	Surface and near surface soils not relied on for skin resistance; Competent soil for point load is at a practical depth (8-2 0m).
Drilled piers or caissons	Same as for piles; use fewer; for large column loads.	Same as for piles. May be floating or point-bearing (or combination). Depends on depth to competent bearing stratum.
Retaining structures		
Retaining walls, bridge abutments	Permanent material retention	Any type of soil but a specified zone in back fill is usually of controlled fill.
Sheeting structures (sheet pile, wood sheeting, etc.)	Temporary or permanent for excavations, marine cofferdams for river work	Retain any soil or water. Backfill for water front and cofferdam systems is usually granular for greater drainage.

Source: Joseph E. Bowels, Foundation analysis and design,1997 page 8

Pile foundations are used in the following condition:

1) When the strata at or just below the ground surface is highly compressible and very weak to support the load transmitted by the structure.
2) When soft Stratum extends to great depth with no underlying stratum.
3) In case of expansive soils, such as black cotton soil, which swell or shrink as the water content changes, piles are used to transfer the load below the active zone.
4) Collapsible soils, such as loess, have a breakdown of structure accompanied by a sudden decrease in void ratio when there is an increase in water content. Piles are used to transfer the load beyond the zone of possible moisture changes in such soils and for topsoil stratum is susceptible to liquefaction.
5) Piles are required when the soil conditions are such that a wash out, erosion or scour of soil may occur from underneath a shallow foundation.
6) Foundation is subjected to large horizontal and vertical loads.
7) Piles are used for the foundations of some structures, such as transmission towers, off-shore platforms, which are subjected to uplift.
8) Pile foundations are used to resist horizontal forces in addition to support the vertical loads in earthling-retaining structures and tall structures that are subjected to horizontal forces due to wind and earthquake.

9) Foundation for highway signs and sound walls that are subjected to horizontal loads such as wind and earthquake.
10) When the plan of the structure is irregular relative to its outline and load distribution. It would cause non-uniform settlement if a shallow foundation is constructed. A pile foundation is required to reduce differential settlement.
11) Pile foundations are required for the transmission of structural loads through deep water to a firm stratum.

2.5. Design consideration of pile foundation

The ultimate bearing capacity of a pile, Qu, includes the resistance from the pile tip and the resistance from the exterior surface of the pile shaft. The bottom of a pile is also called the toe, tip, end, base, and point. The exterior surface of a pile is also called the skin, side, and shaft. The formulas below [18] are used to determine the ultimate bearing capacity of a pile.

$$Q_u = Q_s + Q_t \tag{2.2}$$

Where Q_u =ultimate bearing capacity of a pile,

Q_s =ultimate skin resistance of a pile,

Q_t =ultimate bearing capacity provided by the toe of a pile.

$$Q_s = f_s A_s \tag{2.3}$$

$$Q_t = q_t A_t \tag{2.4}$$

Where f_s =ultimate unit skin resistance,

A_s =skin area,

q_t =ultimate unit bearing capacity at the toe,

A_t =cross sectional area of pile tip.

Equation 2.2 assumes that both the pile toe and the pile shaft have moved sufficiently to simultaneously develop the maximum (ultimate) toe and skin resistances. Generally, however, the displacement needed to mobilize the skin resistance is smaller than that required to mobilize the toe resistance. The skin resistance generally includes adhesion and friction. Adhesion is because of the attraction between the pile and the surrounding soil; friction depends on the vertical effective stress and the external friction angle between the pile and the surrounding soil. The design load of a pile, also known as allowable bearing capacity, Q_{all}, is the ultimate bearing capacity divided by the factor of safety (FS) [18].

$$Q_{all} \leq \frac{Qu}{FS} \quad (2.5)$$

The FS typically ranges from 2 to 4, depending on the reliability of the analysis method, input design parameters, the effect and consistency of the proposed pile installation method, and the level of construction monitoring such as static load test and dynamic analysis. FS=2.0can be used if a static loading test is used to confirm the calculated Qu by a static analysis method. The typical value of FS is 3.0.

An alternative approach that uses partial factors of safety can also be employed for the calculation of the allowable bearing capacity Q_{all}. Such that:

$$Q_{all} \leq \frac{Qt}{\gamma t} + \frac{Qs}{\gamma s} \quad (2.6)$$

Where: γt andγs are partial factors of safety applied to the toe resistance Qt and skin resistance Qs, respectively.

2.5.1. α-method, for undrained cohesive soil

The α-method is a total stress method that uses un-drained soil shear strength parameters to calculate the static pile capacity in cohesive soils. It was originally proposed by Tomlinson (1971) and included in adhesion and friction. In un-drained saturated clay, it is assumed the internal friction angle of the soil (ϕ) and the external friction angle between the pile and the soil(δ) are zero, so the friction on the skin of the pile is assumed to be zero. The un-drained cohesion is expressed as cu. The unit skin resistance is determined by the adhesion, ca, which is the attraction between the soil and the pile:

The Common Procedure taken for Un-drained cohesive soil using the α-method (given γ_{sat}, C_u and ϕ=0). For ease of calculation, rather than using charts, it is appropriate to use the formula of Sladen (1992).Therefore the method of calculating Q_{all} based on Sladen's α-method has the following steps [24]:

Step 1: Compute α

$$\alpha = C_1 * \left(\frac{q_{eos}}{S_U}\right)^{0.45} \text{ where } S_u = C_u \quad (2.7)$$

Where: α is empirical adhesion factor, S_u is un-drained shear strength, q_{eos} is effective overburden stress, C_1 is 0.4 for bored piles and 0.5 for driven piles

Step 2: Compute F_s

$$F_s = \alpha * S_u \quad (2.8)$$

Step 3: Compute Q_s using equation (2.3)

$$Q_s = f_s * A_s$$

Step 4: Compute q_t where q_t is toe resistance

$$q_t = C_u * N_c \quad (2.9)$$

Step 5: Compute Q_t using equation (2.4)

$$Q_t = q_t * A_t$$

Step 6: Compute Q_u using equation (2.2)

$$Q_u = Q_s + Q_t$$

Step 7: Compute Q_{all} using equation (2.5)

$$Q_{all} \leq \frac{Qu}{FS}$$

Even though this is the right procedure, this paper only considers structure and optimization (i.e. no need to consider soil parameters). Therefore resistance to the load is computed by the given values of resistance of the soil per area $\frac{f_s}{FS}$ for skin resistance and $\frac{q_t}{FS}$ for toe resistance.

2.5.2. β-method, for drained cohesionless and cohesive soils

Theβ-method is based on effective stress and uses the drained shear strength of the soil. It was proposed by Burland (1973). It can be used to evaluate the long-term bearing capacity of a driven pile. However, cohesion is not considered in theβ-method. In this method, For drained cohesionless and cohesive soils using the β-method (given γ, ϕ and C1) has the following steps [24]:

Step 1: Compute β

$$\beta = K_s * \tan\delta \quad (2.10)$$

Where: β is Bjerrum-Burland beta coefficient

Ks =earth pressure coefficient,

δ =external friction angle between the pile and the soil.

Step 2: Compute F_s

$$f_s = \beta * \sigma_o' \quad (2.11)$$

Where: σ_o' is average vertical effective stress along the pile shaft

Step 3: Compute Q_s using equation (2.3)

$$Q_s = f_s * A_s$$

Step 4: Compute q_t

$$q_t = N_t * \sigma'_t \quad (2.12)$$

Where: N_t =bearing capacity factor at the toe,

σ'_t =effective overburden stress at the toe.

Step 5: Compute Q_t using equation (2.4)

$$Q_t = q_t * A_t$$

Step 6: Compute Q_u using equation (2.2)

$$Q_u = Q_s + Q_t$$

Step 7: Compute Q_{all} using equation (2.5)

$$Q_{all} \leq \frac{Qu}{FS}$$

Just like the α-method, resistance to the load is computed by the given values of resistance of the soil per area $\frac{f_s}{FS}$ for skin resistance and $\frac{q_t}{FS}$ for toe resistance.

2.5.3. Pile Cap Design

Pile cap is a reinforced concrete slab or block which interconnects a group of piles and acts as a medium to transmit the load from wall or column to the Piles. The Pile cap should normally be rigid so as to distribute the forces equally on the piles of a group. In general it is designed like a footing on soil but with the difference that instead of uniform reaction from the soil, the reactions in this case are concentrated either point loads or distributed.

2.5.3.1. Assumptions Involved in the Design of Pile Caps

1) Pile cap is perfectly rigid.
2) Pile heads are hinged to the pile cap and hence no bending moment is transmitted to piles from pile caps.
3) Since the piles are short and elastic columns, the deformations and stress distribution are planer.

2.5.3.2. Design of Pile Caps

In the design of pile cap, these must be satisfied:

1) The Maximum Permissible Shear Force ($V_{Rd,max}$) must not be exceeded at the face of the loaded area.
2) Punching shear at a critical distance of 2* Davg must be sufficient enough to resist shear force developed by the applied load.
3) Wide beam shear at a critical distance of A_V must be sufficient enough to resist shear force developed by the applied load.
4) The reinforcement provided must accommodate the bending moment developed by the applied load.

The design of pile cap is discussed in the next sections.

2.5.4. Settlement of pile groups

The settlement of a pile group includes the elastic compression of the piles, elastic (immediate) settlement of the soil at the toe of the piles, and consolidation settlement of the soils around and beneath the pile group. Pile groups that are supported in cohesion less soils are subjected only to immediate (elastic) settlement; consolidation settlement is negligible. Pile groups that are supported in cohesive soils are subjected to both immediate (elastic) settlement and consolidation settlement. The settlement of a pile group is much greater than the settlement of a single pile because the soil zone affected by the loading adjacent to and beneath the pile group is much larger.

2.5.4.1. Empirical equations for pile group settlement using field penetration data.

Meyerhof (1976) gave empirical equations for the settlement of pile groups using standard penetration test (SPT) and cone penetration test (CPT) data.

The settlement of a pile group in granular soils using SPT data is [24]:

$$s_g = \frac{\Delta q \sqrt{B/D}}{2N60D} (mm) \qquad (2.13)$$

Where Sg =settlement of a pile group (mm),

Δq =foundation pressure at the toe of the pile group, equal to the total foundation load divided by the group area at the toe,

B =width of the pile group; if the pile group's dimensions are width and length of a pile cap as shown in Figure 3-2, use the larger dimension as B,

D =individual pile diameter or width,

N60 =average SPT blow count in the zone from B to 2B below the pile toe.

The settlement of a pile group using CPT data is [24]:

$$S_g=\frac{k1B\Delta q}{2qc} \quad (2.14)$$

Where: $k1=1-\frac{L}{8B}\geq 0.5$

qc =average static cone tip resistance within the depth of B below the pile toe.

2.5.4.2. Consolidation settlement of a pile group in saturated cohesive soil

Terzaghi and Peck (1967) proposed that the consolidation settlement of a pile group in saturated clay could be evaluated using an equivalent footing situated at a depth of*L/3* above the pile toe. It is assumed that the soil in the top two-third of the pile length is not subjected to consolidation settlement. It is also assumed that the vertical stress increase from*L/3* above the pile toe follows the 2:1 method illustrated below [24].

$$S_{all} \geq S_g=C_c*\frac{H}{1+e_0}\log\frac{\sigma_0+\Delta\sigma}{\sigma_0} \quad (2.15)$$

Where: C_c is Compression Index

e_o is initial void ratio of layer i

H_i is thickness of layer i

$\Delta\sigma_i$ Is the increase in pressure at the middle of each layer.

σ_0 is the effective overburden pressure at the middle of each layer.

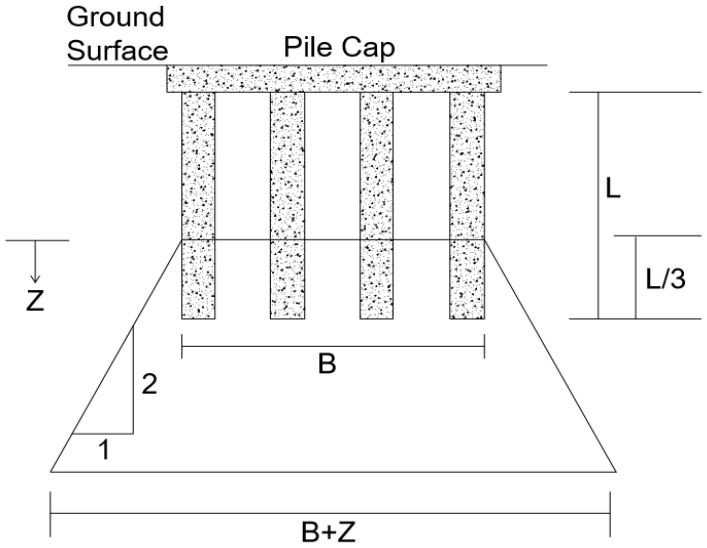


Figure 2-4 *2:1 method in the calculation of consolidation settlement of pile group*

Two important notes should be followed when calculating consolidation settlement.

1. The consolidation settlement is caused by vertical stress increase. When the vertical stress increase, $\Delta\sigma'z$, at a certain depth is less than one-tenth of the in-situ effective stress $\sigma'0$,it is assumed the consolidation settlement is negligible. Therefore, consolidation settlement should be considered for a depth that is either $\Delta\sigma'_z<0.1\sigma'_0$, or there is an underlying firm layer whose compressibility is negligible.
2. A thick soil layer should be divided into a number of thin layers; the consolidation settlement for each thin layer should be calculated using equation 2.15; and the individual settlements should be summed.

2.6. Java Programming Language

The great thing about computers is that they carry ideas efficiently. When you start dealing efficiently with ideas, all the overhead is gone. Instead of using paper and trees, you're using numbers and concepts. Without the overhead, you can do things much faster, and do things that are far more complex than ever before.

To tell a computer what to do, you have to speak a special language and write terse, unambiguous instructions in that language. A special language of this kind is called a computer programming language. A set of instructions written in such a language is called a program. When looked at as a big group, these instructions are called software or code.

A brief history of modern computer programming can be presented as follows [6]:

1. 1954–1957: FORTRAN is developed.

FORTRAN was the first modern computer programming language.

2. 1959: COBOL is created.

COBOL is all about business. The language's primary feature is the processing of one record after another, one customer after another or one employee after another. COBOL has become the most widely used language for business data processing.

3. 1972: Dennis Ritchie at AT&T Bell Labs develops the C programming language.

You can use C to solve the same problems that you can solve by using FORTRAN, Java, or any other modern programming language. The difference between one programming language and another isn't power. The difference is ease and appropriateness of use.

4. 1986: Bjarne Stroustrup (again at AT&T Bell Labs) develops C++.

Unlike its C language ancestor, the language C++ supports object-oriented programming. This represents a huge step forward.

5. May 23, 1995: Sun Microsystems releases its first official version of the Java programming language.

Java improves upon the concepts in C++. Java's "Write Once, Run Anywhere" philosophy makes the language ideal for distributing code across the Internet. Additionally, Java is a great general-purpose programming language. With Java, you can write windowed applications, build and explore databases, control handheld devices, and more.

6. 2002: Microsoft introduces a new language named C#.

Many of the C# language features come directly from features in Java.

7. May 2011: Java runs on more than 1.1 billion desktop computers.

Java runs on 3 billion mobile phones. Java technology provides interactive capabilities to all Blu-ray devices. Java is the most popular programming language in the TIOBE Programming Community.

Object-oriented languages

Java is object-oriented. Unlike languages, such as FORTRAN, which focus on giving the computer imperative "Do this/Do that" commands, object-oriented languages focus on data. Even though object-oriented programs still tell the computer what to do, they start by organizing the data, and the commands come later.

Object-oriented languages are better because they organize data in a way that helps people do all kinds of things with it. To modify the data, you can build on what you already have, rather than scrap everything you've done and start over each time you need to do something new.

Basic tools for creating Java programs

A computer programmer uses existing programs as tools to create new programs. The existing programs and new programs might perform very different kinds of tasks. For example, an existing program that looks for errors in your Java code. This error-finding program can find errors in any kind of Java code — customer-tracking code, weather-predicting code, gaming code, or the code for an app on your mobile phone. There are three major tools, they are [20]:

1. A compiler: A compiler takes the Java code that you write and turns that code into something that can run on your computer.
2. A Java virtual machine (JVM): A Java virtual machine runs your code (and other peoples' Java code) on your computer.
3. You need an integrated development environment (IDE): An integrated development environment helps you manage your Java code and provides convenient ways for you to write, compile, and run your code. In the olden days, writing and running a Java program involved opening several windows — a window for typing the program, another window for running the program, and maybe a third window to keep track of all the code that you've written. An integrated development environment seamlessly combines all this functionality into one well-organized application.

Categories of words in a Java program:

1. Keywords: A keyword is a word that has its own special meaning in the Java programming language, and that meaning doesn't change from one program to another. Examples of keywords in Java include if, for, and do.
2. Identifiers: An identifier is a name for something. The identifier's meaning can change from one program to another, but some identifiers' meanings tend to change more.

2.1. Identifiers created by the user: A Java programmer can create new names for classes and other things that he/she describes in his/hers programs. The first programmer may name something Optimum, and another programmer can name something else Optimum. That's okay because Java doesn't have a predetermined meaning for Optimum

2.2. Identifiers from the API: The JCP members have created names for many things and thrown almost 40,000 of these names into the Java API. The API comes with each version of Java, so these names are available to anyone who writes a Java program. Examples of such names are String, Integer, JWindow, JButton, JTextField, and File

A Java program that gives out "I love to optimize" can be written as shown next page (Figure 2-5) where, Class is a Java keyword and no matter who writes a Java program, class is always used the same way. On the other hand, Display is an identifier. I made up the word Display which is the name of this particular class.

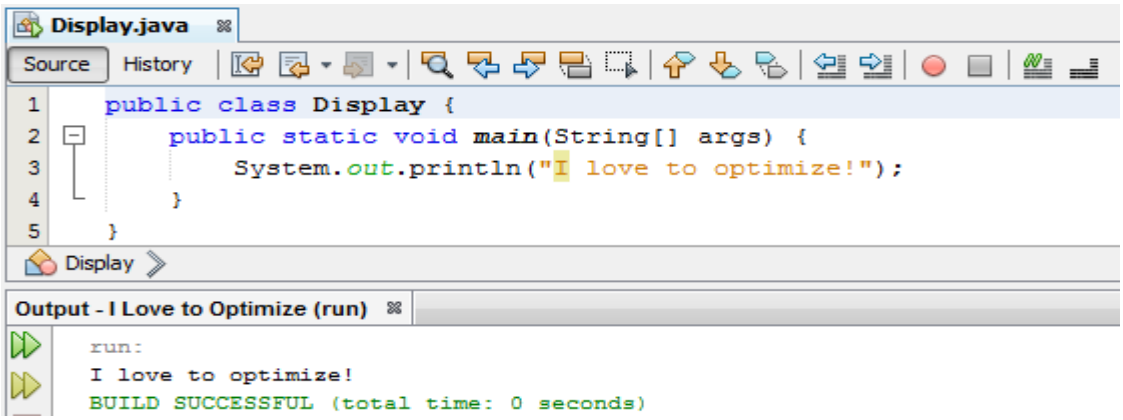


Figure 2-5 A Simple Example on Java

This research work has chosen Java for its support for writing graphic user interfaces (GUI), its speed, its ease of integration with other applications and customizability in using hardware.

2.7. Literature Review on Foundation Optimization

Optimization is used in a number of examples such as Optimal Design of Pre-Stressed Concrete Girder and Optimal Design of Gravity Dam. These share some of the objective with this thesis i.e. reduction of cost of Material and some of the constraints i.e. Deflection criteria and Sectional Capacity criteria. Although they share this, none of them are about foundation and therefore have different objective functions, Constraints and variables. This study takes into account the change of real life costs for excavation and formwork for the accurate calculation of the cost function/Objective function.

One of the two important literatures is from Xiaofeng Liu [36]

> "*This paper studies the optimum conceptual design of pile foundations at the initial design stage. A modular method is proposed, which divides the foundation into modules and each module is identified by its characteristics of pile length, diameter, number and layout. Modules with the same characteristics may be packed and represented by a design variable. A minimum-cost optimization model with multiple design constraints based on Chinese code and a cardinality constraint is built to achieve the concurrent optimization of pile size and layout. The model is solved by the improved automatic grouping genetic algorithms to obtain the design with optimal variables and optimal variable grouping. A practical example demonstrates the effectiveness of the proposed approach.*"

This study uses genetic optimization to optimize the piles only of a whole building without including pile caps. However, the number of piles can have the value of any integer for each column.

The second important literature is by Christos Letsios et al. [7] which include the following:

> "*In this work the problem of finding optimized designs of pile foundations (using finite element method) is examined and is performed in accordance to two design code recommendations, namely Eurocode 7 and DIN 4014. The proposed structural optimization procedure is implemented in two real-world cases both located in London, UK in order to assess the efficiency of the proposed design formulation."*
>
> *"Through the implementation of the proposed formulation in two different soil conditions, it was found that depending on the soil conditions DIN design procedure can become more conservative compared to the EC7 and vice versa. Furthermore, through the proposed formulation pile foundation designs are achieved which are more economic compared to the foundation solutions implemented in the original design. The benefit in terms of construction cost of the optimum designs varies from 7% to 33% compared to original solutions.*

The study above in its conclusion found a benefit in terms of construction cost savings ranging from 7% to 33%. The comparison of the above paper and the new thesis paper reveled that:

- The above literature optimizes the pile foundation using the finite element method while the thesis will optimize the foundation using Brute Force Optimization.
- The above literature optimizes only the piles and did not consider the pile cap, while the thesis considers the effect of piles in the optimization of pile cap.
- The above literature compares the effect of using different codes in the optimal design of foundation while the thesis only considers the Euro Code (Ethiopia uses Euro Norm).

In order to assess the performance of the designs obtained during the optimization process, the design procedures that will be considered in this work is the Euro code 7 [12] [13]. The standard is based on the following main design criteria: (i) axial bearing capacity, (ii) acceptable settlements and (iii) strength of pile as a structural element.

3. STRUCTURAL OPTIMAL DESIGN PROBLEM FORMULATION

3.1. General Structural Design Problem formulation

First it must be decided how the piles are placed when their number is increased and decreased. In this optimization, it starts with four piles in each corner (2x2), and then step by step piles will be included in the middle without affecting the symmetry.

(2x2) ≈ (Number of piles in the X-direction x Number of piles in the Y-direction) = 4 piles
First without changing the number of piles in X and increasing the number of piles in the Y by one gives pile configuration (2x2), (2x3), (2x4) then (2x5), then increase the number piles in X by one and repeat the same for piles in Y giving (3x2), (3x3), (3x4) and (3x5). It continues adding one until piles in x-direction reaches 5 piles.

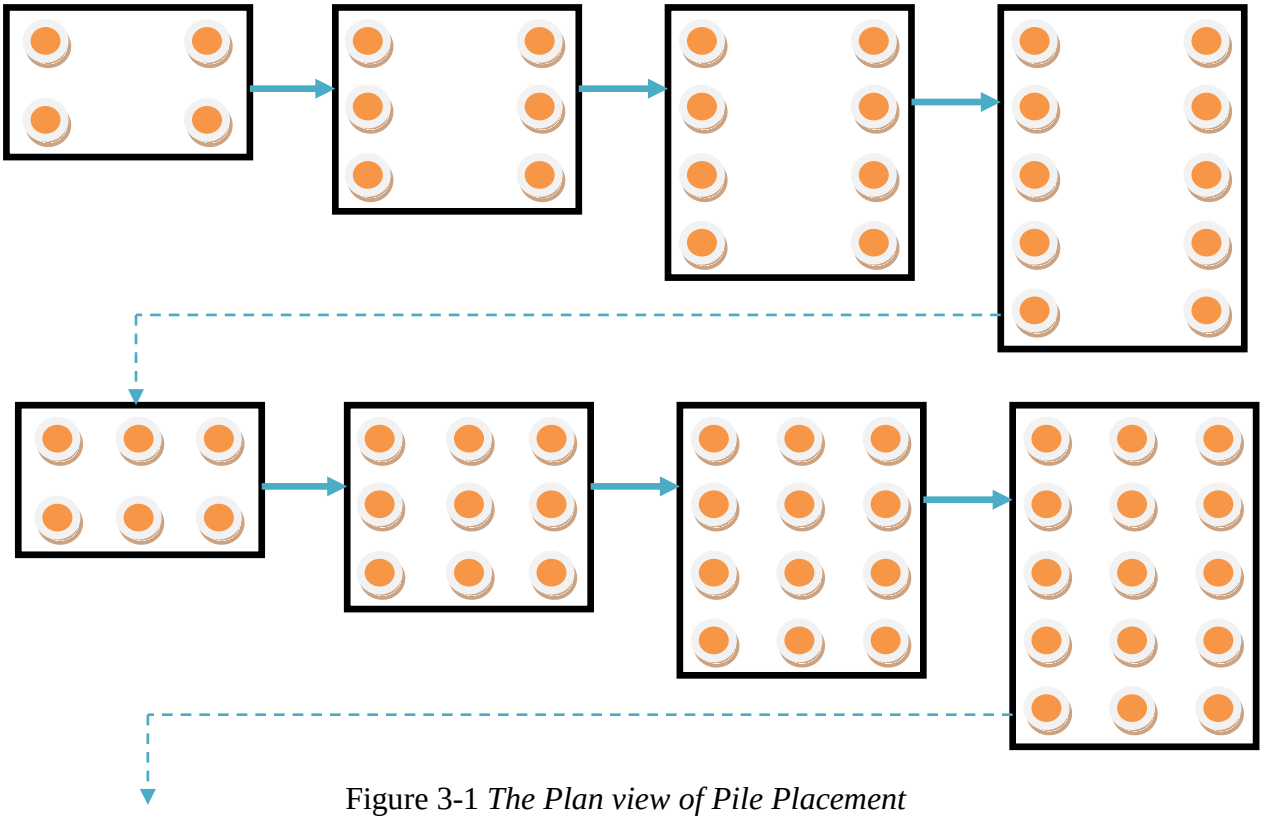


Figure 3-1 *The Plan view of Pile Placement*

Therefore the numbers of piles that are acceptable are 4(2x2), 6(2x3) & (3x2), 8(2x4) & (4x2), 9(3x3), 10(2x5)&(5x2), 12(3x4)&(4x3), 15(3x5)&(5x3), 16(4x4), 20(4x5)&(5x4) and 25(5x5). This will help keep the piles to be symmetrical and reduce the calculation in the pile cap. It is very important to know the Parameters that are given; this will dictate the design approach.

Here are the given parameters:

- Cost of Concrete per unit volume = C_c
- Cost of Steel(rebar) per unit volume relative to C_c =C_{SC}
- Column dimension in x and y direction = C_1 and C_2
- Concrete and Steel Properties = f_{ck} and f_{yk} and Diameter of Rebar Used = ϕ
- Average Cone Penetration Resistance =q_c
- The height of 3 soil types through which the pile passes = H1, H2, (H2+(Z3-H2))
- The factored frictional and toe reaction per unit area of the three soil types =R1, R11, R2, R22, R3, R33
- Total applied Factored Load = F_{app}

  A vertical load on a group of vertical piles with an axis of symmetry is considered to be distributed according to the following equation [25]:

$$P_n=\frac{N}{n} \pm \frac{N*e_{xx}}{I_{xx}}*y_n \pm \frac{N*e_{yy}}{I_{yy}}*x_n \qquad (3.1)$$

  Where: P_n is the axial load on an individual pile, N is the vertical load on the pile group, n is the number of piles, e_{xx} and e_{yy} are the eccentricity of the load N about centroidal axes XX and YY of the pile group, I_{xx} and I_{yy} are the second moment of area of the pile group about axes XX and YY, X_n and Y_n are the distance of the individual pile from axes YY and XX respectively.

  The F_{APP} used in the computer program is the highest P_n from all the piles multiplied by n

$$F_{app} = \text{Max } (Pn) *n = \text{Max } (\frac{N}{n} \pm \frac{N*e_{xx}}{I_{xx}}*y_n \pm \frac{N*e_{yy}}{I_{yy}}*x_n)*n \qquad (3.2)$$

- Mxx and Myy are moments in X and Y directions respectively, these moments are equivalent to the load applied multiplied with eccentricity. Mxx=N*e_{xx} and Myy=N*e_{yy}
- Cost of Drilling per unit meter relative to C_c = C_{DC1}, C_{DC2}, C_{DC3}, C_{DC4}, C_{DC5}

  Cost of drilling per unit meter is taken to change significantly in every 10 meter drilling and when the soil type changes. Therefore new C_{DC} values are introduced in that interval. For example if H1=5 meter, H2=25 meter and embedment depth (Z3) = 27 meter, then the C_{DC} used is shown in the table below.

Table 3-1 The Cost of Drilling per unit depth relative to C_c taken for different height

Height (m)	C_{DC} Considered	Reason
0-5	C_{DC1}	Soil Type 1 & First 10 meters
5-10	C_{DC2}	Soil Type 2 & First 10 meters
10-20	C_{DC3}	Soil Type 2 & Second 10 meters
20-25	C_{DC4}	Soil Type 2 & Third 10 meters
25-27	C_{DC5}	Soil Type 3 &Third 10 meters

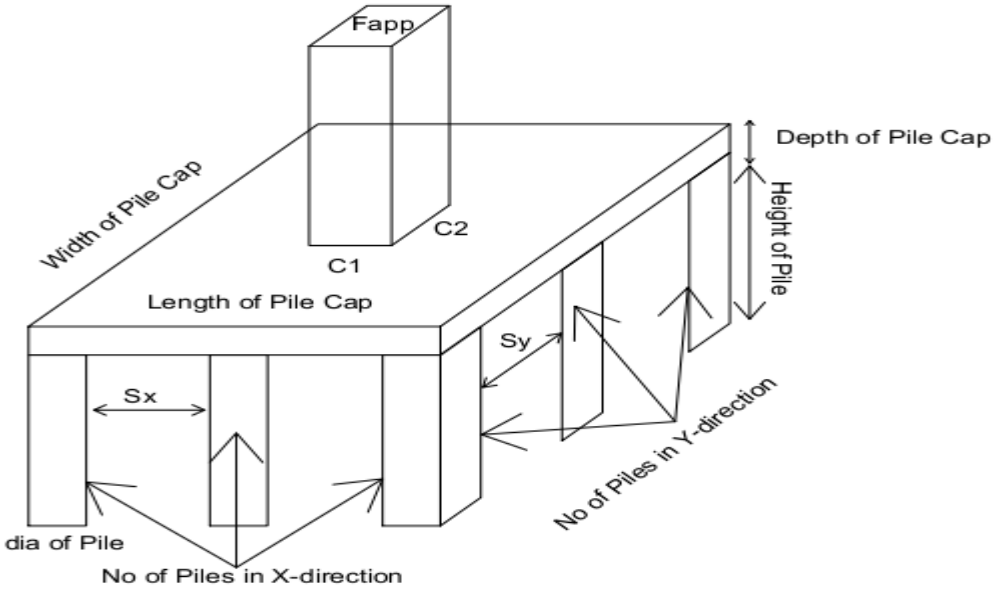


Figure 3-2 ***the Problem in Pictorial Form***

3.2. Design Variables, Objective Function and Constraints

3.2.1. Design Variable

- Number of piles in X-direction (Z1)
- Number of piles in Y-direction (Z2)
- Embedment depth (Z3)
- Diameter of the piles (Z4)
- Depth of pile cap (Z5)
- Length of pile cap (Z6)
- Width of pile cap (Z7)

3.2.2. Objective Function

1. The Total Cost of Concrete with placement is

$$\left(\left(\frac{\pi {Z_4}^2}{4}\right) * Z3 * Z2 * Z1\right) * C_c \qquad (3.3)$$

Where: $\left(\frac{\pi {Z_4}^2}{4}\right)$ = area of one pile

Z1*Z2 = Total Number of Piles

Z3 = Embedment depth

$\left[\left(\frac{\pi {Z_4}^2}{4}\right) * Z3 * Z1 * Z2\right]$ = Total volume of concrete without pile cap

2. The Total Cost of Drilling is given by

$$\{[Z3 * Z1 * Z2] * C_{dci}\} * C_c \quad (3.4)$$

It is important to note that C_{DC} varies per 10 meters and also vary when there is a change in soil type. Therefore the above equation changes to a summation of heights and C_{DC}'s.

$$[Z1 * Z2] * \sum H_i * C_{DCi} \quad (3.5)$$

Where: "i" is depth indicator

3. The Total Cost of Reinforcement for the piles is given by

$$\left\{\left(\left(\frac{\pi {Z_4}^2}{4}\right) * Z_3 * Z_2 * Z_1 * S_{R1}\right) * (C_{SC} - 1)\right\} * C_C \quad (3.6)$$

Where: S_{R1} =Steel Ratio of Pile (Because the load is mostly compression force use As_{min})

$(C_{SC} - 1)$ to reduce the volume of concrete where the rebar occupies the space.

4. The Total Cost of Concrete in a Pile cap is

$$\{Z_5 * Z_6 * Z_7\} * C_C \quad (3.7)$$

5. The Total Cost of Re-bar in a Pile cap due to bending moment and punching reinforcement is

$$\{(Z_5 * Z_6 * Z_7 * S_{R2}) * (C_{SC} - 1)\} * C_C \quad (3.8)$$

Where: S_{R2} =Steel Ratio of Pile cap (The Area of Re-bar must be computed and greater or equal to AS_{min})

6. The Total Cost of Pile Foundation with the pile cap will be

$$\psi_{(Z(x))} = \left[\left(\left(\frac{\pi {Z_4}^2}{4}\right) * Z3 * Z2 * Z1\right) + \{[Z3 * Z1 * Z2] * C_{dc}\} + \left\{\left(\left(\frac{\pi {Z_4}^2}{4}\right) * Z_3 * Z_2 * Z_1 * S_{R1}\right) * (C_{SC} - 1)\right\} + \{Z_5 * Z_6 * Z_7\} + \{(Z_5 * Z_6 * Z_7 * S_{R2}) * (C_{SC} - 1)\}\right] * C_C \quad (3.9)$$

Where: ψ is the total cost of Reinforced Concrete construction.

3.2.3. Constraint

3.2.3.1. General Constraints (Side Constraints)

General Constraints are constraints that exist to create a feasible region that is similar to real life. For instance, it is not possible to have the sum of pile diameters on one row greater than the width of the pile cap.

Therefore: $Z6 \geq ((Z4) + ((Z1-1)*3*Z4) + (0.6))$ (3.10)

$Z7 \geq ((Z4) + ((Z2-1)*3*Z4) + (0.6))$ (3.11)

Where: (Z4) is two times the radius of the pile (2 times Z4/2) as seen in Figure 3-6

(Z1-1) and (Z2-1) are the number of Spacing between the piles in X & Y direction.

(3*Z4) The minimum spacing must be 3 times the pile diameter [24]

(0.6) is two times the minimum spacing from end of pile cap to the edge pile (2 times 0.3)

Another constraint include, all piles in the pile cap must be in compression. This means P_n minimum must be greater than 0.

$$\text{Min } (P_n) = \text{Min } (\frac{N}{n} \pm \frac{N*e_{xx}}{I_{xx}}*y_n \pm \frac{N*e_{yy}}{I_{yy}}*x_n) \geq 0 \quad (3.12)$$

Other constraints due to practicality include:

- The Number of Piles in X and Y are restricted from2 to 5. This is because it is not practical to have more than 25 (5x5) piles for one column.
- The depth of the pile ranges from 5 to 30 in the interval of 0.5 meter. This is because depths of more than 30m are infeasible.
- The Diameters of the piles are restricted to the drills that have cutters with the diameter sizes of 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1 and 1.2.
- The depth of pile cap ranges like [0.4, 0.5, 0.6, …3], this is because pile cap depths of more than 3 meters are found to be excessive
- The length and width of pile caps are restricted to 12m (half of maximum spacing between two columns on the right and left side of the column). But because 12m spacing between columns is used rarely, 8m spacing was found to be sufficient in this program.

3.2.3.2. Allowable Load

From equation 2.5 and 2.2, $F_{applied} \leq Q_{ult} / F.S$ and $Q_{ult}=Q_s+Q_t$

Where $F_{applied}$ is the load applied, Q_{ult} is the ultimate bearing capacity of the pile, F.S. is the factor of safety (taken from 2-4 depending on foundation type), Q_s is skin/friction resistance and Q_t is toe/end resistance.

Here however the Q_s / (As*F.S) is changed to factored reaction of friction per surface area (R1, R2 and R3) and Q_t / (As*F.S) is changed to factored reaction of toe per cross-sectional area (R11, R22 and R33).

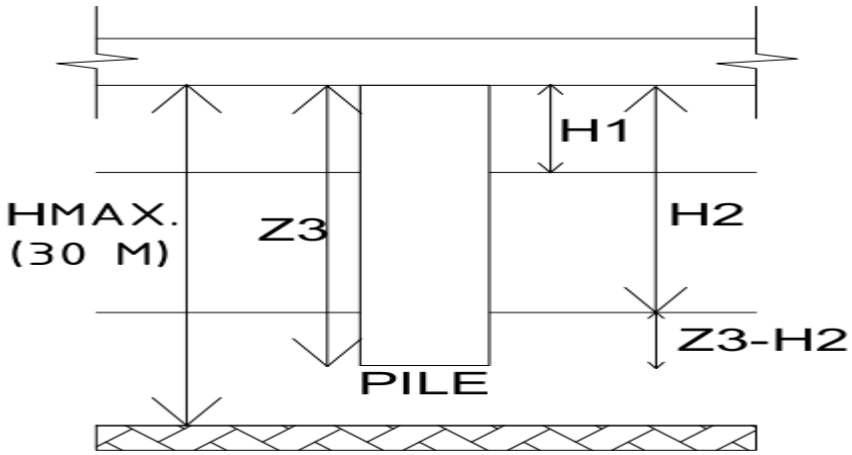


Figure 3-3 ***The Different Soil Types and their heights***

For Embedment Depth (Z_3) <H1

$$Fapp \leq ((3.1416*Z4*Z3*R1) + (0.7854*Z4^2*R11))*Z1*Z2 + (Z6*Z7-0.7854*Z4^2*Z1*Z2)*R11 \quad (3.13)$$

Where: (3.1416*Z4*Z3) is the surface area of the frictional resistance (π*D*H)

($0.7854*Z4^2$) is the area of toe resistance $\left(\frac{\pi D^2}{4}\right)$

($Z6*Z7-0.7854*Z4^2*Z1*Z2$) is the area of pile cap for toe resistance (Total pile cap area – area for total number of piles)

R1 is the frictional resistance of the first soil per unit area

R11 is the toe resistance of the first soil per unit area

For Embedment Depth (Z_3) H1≤ Z_3<H2

$$Fapp \leq ((3.1416*Z4*H1*R1) + (3.1416*Z4*(Z3-H1)*R2) + (0.7854*Z4^2*R22))*Z1*Z2 + (Z6*Z7-0.7854*Z4^2*Z1*Z2)*R11 \quad (3.14)$$

Where: R2 is the frictional resistance of the second soil per unit area

R22 is the toe resistance of the second soil per unit area

For Embedment Depth (Z_3) Z_3=H2

Fapp ≤ ((3.1416*Z4*H1*R1) + (3.1416*Z4*(H2-H1)*R2) +
(0.7854*Z42*R33))*Z1*Z2 + (Z6*Z7-0.7854*Z42*Z1*Z2)*R11 (3.15)

Where: R33 is the toe resistance of the third soil per unit area

For Embedment Depth (Z_3) Z_3>H2

Fapp ≤ ((3.1416*Z4*H1*R1) + (3.1416*Z4*(H2-H1)*R2) + (3.1416*Z4*(Z3-H2)*R3) +
(0.7854*Z42*R33))*Z1*Z2 + (Z6*Z7-0.7854*Z42*Z1*Z2)*R11 (3.16)

Where: R3 is the frictional resistance of the third soil per unit area

As for the reinforcement of the piles, as there are no lateral forces and moments, this means the pile is in compression, therefore minimum reinforcement As_{min} =0.01*Ac (Area of concrete) was taken to be sufficient for the piles.

3.2.3.3. Allowable Settlement

3.2.3.3.1. Group Settlement Using Static Cone Result (q_C)

From Equation (2.14):

$$S_{ALL} \geq S_G$$

$$S_G = \frac{K_1 * B_{MAX} * \Delta q}{2 * q_c}$$

Where: L is pile length (Z_3)

B_{MAX} is the width of the pile group; if the pile group's dimensions are Z_6 and Z_7, use the larger dimension as B_{MAX}

Δq is foundation pressure at the toe of the pile group, equal to the total foundation load F_{app} divided by the group area at the toe B6*B7

K_1=1- $\frac{L}{8*B_{Max}} \leq 0.5$, K_1= 1- $\frac{Z_3}{8*Z_6}$ or 1- $\frac{L}{8*Z_7}$ or 0.5 Choose the smallest

q_c is average static cone tip resistance within the depth of B_{MAX} below the pile toe.

3.2.3.3.2. Consolidated Settlement of a Pile Group in Saturated Cohesive soil (Clay)

The Settlement for one layer from Equation 2.15:

$$S_g = Cc * \frac{H_i}{1+e_{0i}} \log \frac{\sigma_0 + \Delta\sigma_i}{\sigma_0}$$

Where: C_c is Compression Index

e_o is initial void ratio of layer i

H_i is thickness of layer i

$\Delta\sigma_i$ Is the increase in pressure at the middle of each layer.

σ_0 is the effective overburden pressure at the middle of each layer.

The Settlement of the whole structure due to the many layers is given by Equation 3.14 [24]

$$S_{all} \geq \sum S_g \qquad (3.17)$$

For $H_1 > \frac{2}{3}Z_3$

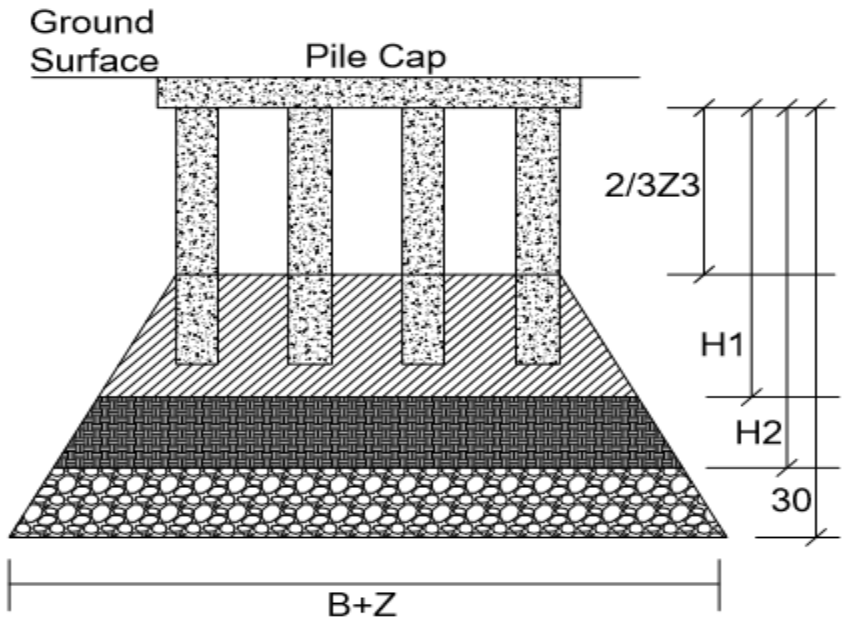


Figure 3-4 ***2:1 method using three different layer of consolidation settlement for H1*** $> \frac{2}{3}\mathbf{Z_3}$

Table 3-2 Consolidated Settlement parameters in cohesive soil for $\mathbf{H_1} > \frac{2}{3}\mathbf{Z_3}$

No of Layer (i)	H_i	σ_0	$\Delta\sigma_i = \frac{F_{app}}{Area}$, where Area is:
1	$H_1 - \frac{2}{3}*Z_3$	$\frac{2}{3}*Z_3*\gamma_1 + \left[\frac{H_1-\frac{2}{3}Z_3}{2}\right]*\gamma_1$	$\left\{Z_6 + \left[\frac{H_1-\frac{2}{3}Z_3}{2}\right]\right\}*\left\{Z_7 + \left[\frac{H_1-\frac{2}{3}Z_3}{2}\right]\right\}$
2	$H_2 - H_1$	$\frac{2}{3}*Z_3*\gamma_1 + [H_1 - \frac{2}{3}Z_3]*\gamma_1 + \left[\frac{H_2-H_1}{2}\right]*\gamma_2$	$\left\{Z_6 + H_1 - \frac{2}{3}Z_3 + \left[\frac{H_2-H_1}{2}\right]\right\} * \left\{Z_7 + H_1 - \frac{2}{3}Z_3 + \left[\frac{H_2-H_1}{2}\right]\right\}$
3	$30 - H_2$	$\frac{2}{3}*Z_3*\gamma_1 + [H_1 - \frac{2}{3}Z_3]*\gamma_1 + [H_2 - H_1]*\gamma_2 + \left[\frac{30-H_2}{2}\right]*\gamma_3$	$\left\{Z_6 + H_2 - \frac{2}{3}Z_3 + \left[\frac{30-H_2}{2}\right]\right\} * \left\{Z_7 + H_2 - \frac{2}{3}Z_3 + \left[\frac{30-H_2}{2}\right]\right\}$

Firstly, find $\Delta\sigma_i$ for each layer by dividing the applied force to the area given in the above table, then compute each three layers settlement by using equation (2.15) and finally add the individual settlements and compare it to the allowable settlement by using Equation (3.14).

For $H_1 < \frac{2}{3}Z_3 < H_2$

Table 3-3 Consolidated Settlement parameters in cohesive soil for H1 < $\frac{2}{3}$Z$_3$ < H2

No of Layer (i)	H_i	σ_0	$\Delta\sigma_i = \frac{F_{app}}{Area}$, where Area is:
1	$H_2 - \frac{2}{3}*Z_3$	$H_1*\gamma_1 + [\frac{2}{3}Z_3 - H_1]*\gamma_2 + \left[\frac{H_2 - \frac{2}{3}Z_3}{2}\right]*\gamma_2$	$\left\{Z_6 + \left[\frac{H_2 - \frac{2}{3}Z_3}{2}\right]\right\} * \left\{Z_7 + \left[\frac{H_2 - \frac{2}{3}Z_3}{2}\right]\right\}$
2	$30 - H_2$	$H_1*\gamma_1 + [\frac{2}{3}Z_3 - H_1]*\gamma_2 + \left[H_2 - \frac{2}{3}Z_3\right]*\gamma_2 + \left[\frac{30 - H_2}{2}\right]*\gamma_3$	$\left\{Z_6 + H_2 - \frac{2}{3}Z_3 + \left[\frac{30 - H_2}{2}\right]\right\} * \left\{Z_7 + H_2 - \frac{2}{3}Z_3 + \left[\frac{30 - H_2}{2}\right]\right\}$

Follow the same procedure as the above one with the only difference being, there are two soil layers that settle rather than three.

For $H_1 < H_2 < \frac{2}{3}Z_3 < 30$

Table 3-4 Consolidated Settlement parameters in cohesive soil for H1 < H2 < $\frac{2}{3}$Z$_3$<30

No of Layer (i)	H_i	σ_0	$\Delta\sigma_i = \frac{F_{app}}{Area}$, where Area is:
1	$30 - \frac{2}{3}*Z_3$	$H_1*\gamma_1 + H_2*\gamma_2 + [\frac{2}{3}Z_3 - H_2]*\gamma_3 + \left[\frac{30 - \frac{2}{3}Z_3}{2}\right]*\gamma_3$	$\left\{Z_6 + \left[\frac{30 - \frac{2}{3}Z_3}{2}\right]\right\} * \left\{Z_7 + \left[\frac{30 - \frac{2}{3}Z_3}{2}\right]\right\}$

Follow the same procedure as the above one with the only difference being, there is only one soil layers that settle rather than three and two in the previous cases.

3.2.3.4. Allowable Minimum pile spacing

Spacing in the x direction (S_x) and Spacing in the Y direction (S_y) must be greater or equal to 3 times the diameter of the piles ($3*Z_3$) to effectively utilize the soil capacity and avoid the sharing of influence area [24].

3.2.3.5. Pile Cap Design

3.2.3.5.1. Maximum permissible force at face of loaded area

Checks must be undertaken to ensure that the maximum permissible shear force ($V_{Rd,\ max}$) is not exceeded at the face of the loaded area based on Euro code 2 [10].

Load ≤ Maximum permissible shear force

$$V_{ED} \leq V_{Rd,\ max} \tag{3.18}$$

Where: V_{ED} is the existing shear force due to the applied load (F_{APP})

$V_{Rd,\ max}$ is the maximum permissible shear force at the face of column

The maximum permissible shear force according to Mosley [25] is given by:

$$V_{Rd,\ max} = 0.5*v_1*f_{cd}*u*d = 0.5*v_1*(f_{ck}/1.5)*u*d \tag{3.19}$$

Where: u is the perimeter of the loaded area and

v_1 is the strength reduction factor = $0.6*(1 - \frac{f_{ck}}{250})$ where f_{ck} is in MPa.

U = 2*C1+2*C2 (perimeter of pile cap just below column face)

D = H - Pile inside the section – cover - diameter or rebar used - $\frac{\text{diameter of rebar used}}{2}$

D_X = Z5 - 0.1 - 0.075 - D_{BAR} - $\frac{D_{BAR}}{2}$ and D_Y = Z5 - 0.1 - 0.075 - $\frac{D_{BAR}}{2}$

$D_{AVG} = \frac{D_X + D_Y}{2}$, as shown in Figure 3.5

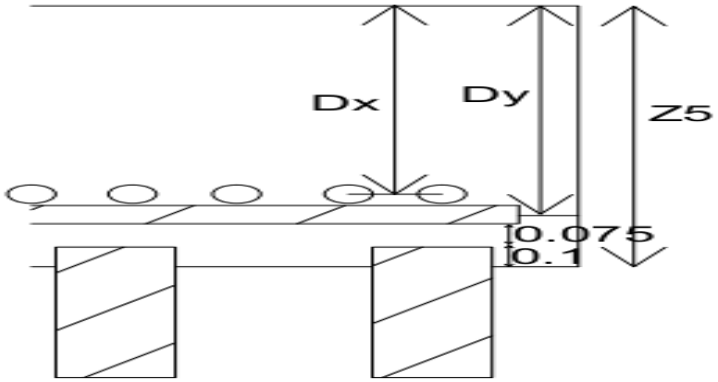


Figure 3-5 ***The Different depths in X and Y***

3.2.3.5.2. Punching Shear

If the spacing between piles is greater than three times the diameter of the piles, there is a need to calculate punching shear [25].

$$\text{Spacing (S)} \leq 3*D_{PILE} \tag{3.20}$$

$$(Z6 - 2*(0.3) - Z4)/ (Z1 - 1) \leq 3*Z4 \quad (3.21a)$$

$$(Z7 - 2*(0.3) - Z4)/ (Z2 - 1) \leq 3*Z4 \quad (3.21b)$$

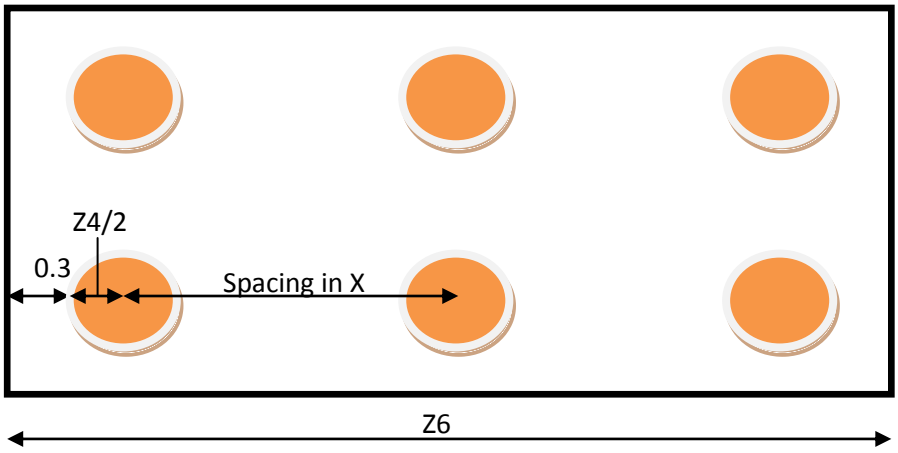


Figure 3-6 ***The Dimensions of the pile cap***

If the above requirement is not satisfied, the applied punching shear force must be less than the shear capacity at that particular section, in order to satisfy the shear requirement without shear reinforcement [10] [25].

$$\text{Load} \leq \text{Capacity}$$

$$V_{ED} \leq V_{RD} \quad (3.22)$$

Where: V_{ED} is the existing punching shear force due to the applied load (F_{APP})

V_{RD} is the resistance to punching shear at the critical section

$$V_{RD} = f_C*As \quad (3.23)$$

Where: $f_C = 0.12*K*(100*\rho*fck)^{1/3}$

$$K= 1+\sqrt{\frac{200}{D_{avg}}}$$

$A_S = U*D_{AVG}$

$U = 2*C1+2*C2+ 4*\pi*D_{AVG}$ (perimeter of shear critical region in the pile cap)

$4*\pi*D_{AVG} = 2*\pi*(2*D_{AVG})$ is the perimeter of the four quarter circles with the radius of $2*D_{AVG}$

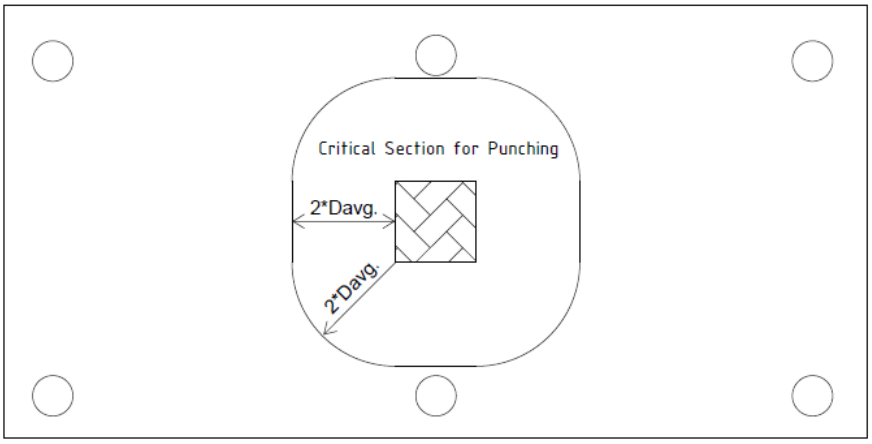


Figure 3-7 ***The Critical section of Punching Shear***

If the applied punching shear force is greater than the punching capacity of the concrete, then reinforcement must be provided [12] [25].

First find out the perimeter where reinforcement is not required

$$U_{out,\,ef} = \frac{V_{ED}}{f_c * d_{avg}} \tag{3.24}$$

Then find the distance X, in which the above perimeter is obtained.

$$U_{out,\,ef} = 2*(C_1 + C_2) + 2*\pi*d_{avg}*X \tag{3.25}$$

If the distance X is less than 3, then the reinforcement must be provided from at least $0.3*d_{avg}$ to $(X-1.5)*d_{avg}$. But if X is more than 3, then the reinforcement must be provided from at least $0.5*d_{avg}$ to $(X-1.5)*d_{avg}$.

Provided that the pile cap is greater than 200mm thick overall, the amount of reinforcement required is given by [12] [25]:

$$A_{sw} \geq \frac{v_{Rd,cs} - 0.75*f_c}{1.5*\left(\frac{f_{ywd,ef}}{S_r*U_1}\right)} \tag{3.26}$$

Where: A_{sw} is the total area of shear reinforcement in one perimeter (mm^2)

S_r is the radial spacing of perimeters of shear reinforcement

$f_{ywd,ef}$ Is the effective design strength of the shear reinforcement and is given by:

$f_{ywd,ef} = 250 + 0.25*d_{ang} \leq f_{ywd}$

$v_{Rd,cs}$ Is the punching shear resistance of the reinforced slab and is given by $\frac{V_{ED}}{U_1 d_{avg}}$

A check must also be made that the calculated reinforcement satisfies the minimum requirement that:

$$A_{sw,min} \geq \frac{0.053*\sqrt{f_{ck}}*(S_r S_t)}{f_{yk}} \quad (3.27)$$

Where: S_t is the spacing of links around the perimeter and $A_{sw,min}$ is the area of an individual link leg.

3.2.3.5.3. Wide Beam Shear

To satisfy the shear requirements, the applied shear force must be less than the sear capacity at that particular section [12] [25].

$$\text{Load} \leq \text{Capacity}$$

$$V_{ED} \leq V_{RD} \quad (3.28)$$

Where: V_{ED} is the existing Wide Beam shear due to the applied load (F_{APP})

V_{RD} is the resistance to Wide Beam shear at the critical location.

$$V_{EDX} = \sum \frac{\sum P_Y * a_V}{2*D_{AVG}} \quad (3.29a)$$

$$V_{EDY} = \sum \frac{\sum P_X * a_V}{2*D_{AVG}} \quad (3.29b)$$

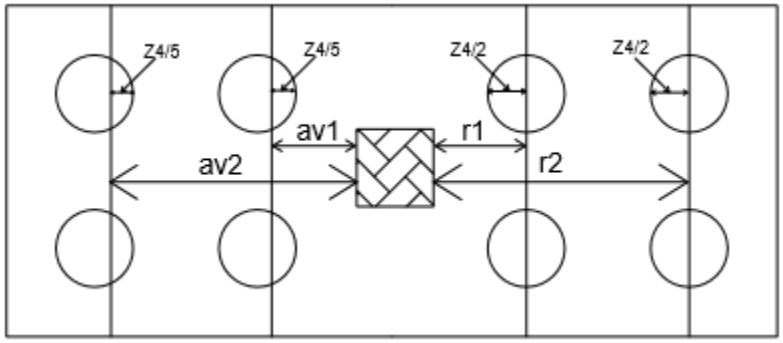


Figure 3-8*The Critical section of Av for Wide Beam and r for Bending*

Where: $\sum P_Y$ is the sum of all the resistance of piles which have the same centerline with respect to X. In the Case of the above drawing, there are two piles across the width.

$\sum P_X$ Is the sum of all the resistance of piles which have the same centerline with respect to Y. In the Case of the above drawing, there are four piles across the length.

A_V, is the distance from one fifth of pile diameter to the face of the column.

$$V_{RD} = f_C * As \quad (3.30)$$

Where: $f_C = [0.12*K*(100*\rho*fck)]^{1/3}$

$$K = 1+\sqrt{\frac{200}{D_{avg}}}$$

As = b*D_{AVG} and differs in the x and y directions: Asx = Z7*D_{AVG} and Asy = Z6*D_{AVG}

3.2.3.5.4. Bending

To satisfy the Bending Moment requirements, the applied Bending Moment must be less than the Bending Moment capacity at that particular section [12] [25].

$$MED \leq MRD \tag{3.31}$$

$$M_{EDX} = \sum(\sum P_Y * r) \tag{3.32a}$$

$$M_{EDY} = \sum(\sum P_X * r) \tag{3.32b}$$

Where: $\sum P_Y$ is the sum of all the resistance of piles which have the same centerline with respect to X. In the Case of the above drawing, there are two piles across the width.

$\sum P_X$ Is the sum of all the resistance of piles which have the same centerline with respect to Y. In the Case of the above drawing, there are four piles across the length.

r is the distance from one center of pile diameter to the face of the column.

$$M_{RD} = 0.87 * f_{yk} * Z * As \tag{3.33}$$

Where: $Z = D_{AVG} * (0.5 + \sqrt{0.25 - \frac{K}{1.134}})$

$K = \frac{M_{ED}}{BD^2 F_{CK}}$ has different values in the x ($\frac{M_{ED}}{Z7 * {D_{AVG}}^2 * F_{CK}}$) and Y ($\frac{M_{ED}}{Z6 * {D_{AVG}}^2 F_{CK}}$) directions.

Therefore, it is possible to find A_s, from Equation 3.29

$$A_s \geq \frac{M_{ED}}{0.87 * f_{YK} * Z}$$

3.3. Mathematical Presentation of Design Problem

Minimize from Equation 3.7:

$$\psi_{(Z(x))} = \left[\left(\left(\frac{\pi {Z_4}^2}{4}\right) * Z3 * Z2 * Z1\right) + \{[Z3 * Z1 * Z2] * C_{dc}\} + \left\{\left(\left(\frac{\pi {Z_4}^2}{4}\right) * Z_3 * Z_2 * Z_1 * S_{R1}\right) * (C_{SC} - 1)\right\} + \{Z_5 * Z_6 * Z_7\} + \{(Z_5 * Z_6 * Z_7 * S_{R2}) * (C_{SC} - 1)\}\right] * C_C$$

Subjected to: $F_{applied} \leq Q_{ult} / F.S$ Resistive force Requirement (Eq. 2.5)

$S_{ALL} \geq S_G$ Allowable Settlement Requirement (Eq. 2.14)

(S_x) and $(S_y) \geq 3*Z_3$	Spacing Requirement between piles (Eq. 3.8 & 3.9)
$V_{ED} \leq V_{RD}$	Maximum Permissible Shear force Requirement (Eq. 3.15)
$V_{ED} \leq V_{RD}$	Punching shear Requirement at 2*Davg (Eq. 3.18)
$V_{ED} \leq V_{RD}$	Wide Beam shear Requirement at Av (Eq. 3.24)
$M_{ED} \leq M_{RD}$	Reinforcement Requirement due to moment (Eq. 3.27)

3.4. General Optimality Criteria and Solution Scheme

Brute force optimization, takes a single value for each variable and then first checks if the constraints are sufficiently fulfilled. Then, for those values that meet the requirements of the constraint, it calculates the cost of the pile and pile cap. Finally, it compares and stores the values of the variables that give the least amount of cost. The possible values of the variables with the total number of iterations are presented in the table below.

Table 3-5 Possible Inputs for All Variables

Variables	Possible Values	Total Number of Values
Diameter of the piles (Z4)	[0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2]	10
Number of piles in X-direction (Z1)	[2, 3, 4, 5]	4
Number of piles in Y-direction (Z2)	[2, 3, 4, 5]	4
Embedment depth (Z3)	[5, 5.5, 6, 6.5, 7, …, 30]	51
Depth of pile cap (Z5)	[0.4, 0.5, 0.6, 0.7 …, 3]	27
Length of pile cap (Z6)	[2, 2.1, 2.2, 2.3, 2.4, …, 8]	61
Width of pile cap (Z7)	[2, 2.1, 2.2, 2.3, 2.4, …, 8]	61
THE TOTAL NUMBER OF POSSIBLE SOLUTIONS CHECKED		819,810,720

This Program takes 819,810,720 potential solutions, and checks weather each meets the requirements of the constraints. At this time more than half of the possible solutions are dropped. The next step is to compare the cost of the foundation. After this process the potential solutions are reduced to one final optimal value.

4. COMPUTER PROGRAM FOR PILE FOUNDATION

4.1. Algorithm of Design Solution with Flowchart and Description

4.1.1. Algorithms

4.1.1.1. Algorithm 1- General Constraints

Function analyzes

Number of piles in X-direction (Z1)
Number of piles in Y-direction (Z2)
Embedment depth (Z3)
Diameter of the piles (Z4)
Depth of pile cap (Z5)
Length of pile cap (Z6)
Width of pile cap (Z7)
Check Z6 < ((Z4) + ((Z1-1)*3*Z4) + (0.6))
Z7 < ((Z4) + ((Z2-1)*3*Z4) + (0.6))
End Check
Do Xn=(Z6/2)-0.3-(Z4/2)
Yn=(Z7/2)-0.3-(Z4/2)
Exx=Mxx/Load
Eyy=Myy/Load
Ixx=2*Z1*((Z7/2)-0.3-(Z4/2))2 For Z1<=3
Ixx=2*Z1*((Z7/2)-0.3-(Z4/2))2+(2*Z1*(Z7/6-0.1-Z4/6)2) For Z1<=4
Ixx=2*Z1*((Z7/2)-0.3-(Z4/2))2+(2*Z1*(Z7/4-0.15-Z4/4)2) For Z1<=5
Iyy=2*Z2*((Z6/2)-0.3-(Z4/2))2 For Z1<=3
Iyy=2*Z2*((Z6/2)-0.3-(Z4/2))2+(2*Z2*(Z6/6-0.1-Z4/6)2) For Z1<=4
Iyy=2*Z2*((Z6/2)-0.3-(Z4/2))2+(2*Z2*(Z6/4-0.15-Z4/4)2) For Z1<=5
Pn1= (Load/ (Z1*Z2)) + ((Load*Exx*Yn)/Ixx) + ((Load*Eyy*Xn)/Iyy)
Pn2= (Load/ (Z1*Z2)) + ((Load*Exx*Yn)/Ixx) - ((Load*Eyy*Xn)/Iyy)
Pn3= (Load/ (Z1*Z2)) - ((Load*Exx*Yn)/Ixx) + ((Load*Eyy*Xn)/Iyy)
Pn4= (Load/ (Z1*Z2)) - ((Load*Exx*Yn)/Ixx) - ((Load*Eyy*Xn)/Iyy)
Fapp = Max (Pn1, Pn2, Pn3, Pn4)*Z1*Z2
End Do
Check Min (Pn1, Pn2, Pn3, Pn4) ≥ 0
End Check

End Function

4.1.1.2. Algorithm 2 – Allowable Load

Function Analyze

A=3.1416*Z4*H1*R1
B=3.1416*Z4*(Z3-H1)*R2
C=3.1416*Z4*(H2-H1)*R2
D=3.1416*Z4*Z3*R1
E=3.1416*Z4*(Z3-H2)*R3
F = (Z6*Z7-(0.7854*$Z4^2$*Z1*Z2))*R11

Do FAPP > (D + (0.7854*$Z4^2$*R1))*Z1*Z2+F for Z3<H1
FAPP > (A+B + (0.7854*$Z4^2$*R2))*Z1*Z2 + F for Z3<H2
FAPP > (A+C + (0.7854*$Z4^2$*R3))*Z1*Z2 + F for Z3<=H2
FAPP> (A+C+E + (0.7854*$Z4^2$*R3))*Z1*Z2) +F for Z3<=30
End Do
End Function

4.1.1.3. Algorithm 3 – Group Settlement Requirement

Function Analyze
Unit Weight of each soil layer (γ): GM1, GM2 and GM3
Compression Index of each soil layer: CC1, CC2 and CC3
Void Ratio of each soil layer: EO1, EO2, and EO3
Check Soil whether it is Cohesive or Cohesion-less
End Check

For Cohesion-less Soil
Check Z6>=Z7 for BMAX
End Check
Q=FAPP / (Z6*Z7)
K2=1-(Z3/ (8*BMAX))
Check K2>=0.5
End Check
SG = (K2*BMAX*Q)/ (2*QC)
Check SG > 200mm
End Check

For Cohesive Soil
Z33 = (2*Z3)/3
Do PO1=Z33*GM1+ ((H1-Z33)*GM1)/2 for H1≥Z33
PO2=PO1+ (H1-Z33)*GM1/2+ (H2-H1)*GM2/2 for H1≥Z33
PO3=PO2+ ((H2-H1)*GM2)/2+ ((30-H2)*GM3)/2 for H1≥Z33
PO1=H1*GM1+ (Z33-H1)*GM2+ ((H2-Z33)*GM2)/2 for H2≥Z33
PO2=PO1+ ((H2-Z33)*GM2)/2+ ((30-H2)*GM3)/2 for H2≥Z33
PO1=H1*GM1+H2*GM2+ (Z33-H2)*GM3+ (30-Z33)*GM3/2 for 30≥Z33
CP1=FAPP/ ((Z6+ (H1-Z33)/2)*(Z7+ (H1-Z33)/2)) for H1≥Z33
CP2=FAPP/ ((Z6-Z33+H2/2+H1/2)*(Z7-Z33+H2/2+H1/2)) for H1≥Z33
CP3=FAPP/ ((Z6-Z33+15+H2/2)*(Z7-Z33+15+H2/2)) for H1≥Z33
CP1=FAPP/ ((Z6+ (H2-Z33)/2)*(Z7+ (H2-Z33)/2)) for H2≥Z33
CP2=FAPP/ ((Z6-Z33+15+H2/2)*(Z7-Z33+15+H2/2)) for H2≥Z33
CP1=FAPP/ ((Z6+ (30-Z33)/2)*(Z7+ (30-Z33)/2)) for 30≥Z33
HT1=H1-Z33 for H1≥Z33
HT2=H2-H1 for H1≥Z33
HT3=30-H2 for H1≥Z33
HT1=H2-Z33 for H2≥Z33
HT2=30-H2 for H2≥Z33
HT1=30-Z33 for 30≥Z33
ST1= ((HT1*CC1)/ (1+EO1))*LOG ((PO1+CP1)/PO1) for H1≥Z33

ST2= ((HT2*CC2)/ (1+EO2))*LOG ((PO2+CP2)/PO2) for H1≥Z33
ST3= ((HT3*CC3)/ (1+EO3))*LOG ((PO3+CP3)/PO3) for H1≥Z33
ST4= ST1+ST2+ST3 for H1≥Z33
ST1= ((HT1*CC1)/ (1+EO1))*LOG ((PO1+CP1)/PO1) for H2≥Z33
ST2= ((HT2*CC2)/ (1+EO2))*LOG ((PO2+CP2)/PO2) for H2≥Z33
ST4=ST1+ST2 for H2≥Z33
ST4= ((HT1*CC1)/ (1+EO1))*LOG ((PO1+CP1)/PO1) for 30≥Z33

End Do
Check ST4>0.2m
End Check
End Function

4.1.1.4. Algorithm 4- Bending Moment

Function Analyze
X1=0.5*(Z6-C1-0.6-Z4)
Do MED1= (FAPP/Z1)*X1 for Z1<=3
MED1= (FAPP/Z1)*((2*X1)+0.2-(Z6/3)) for Z1<=4
MED1= (FAPP/Z1)*((2*X1)+0.15-(Z6/4)) for the rest
End Do
X2=0.5*(Z7-C2-0.6-Z4)
Do MED2 = (FAPP/Z2)*X2 for Z2<=3
MED2 = (FAPP/Z2)*((2*X2)+0.2-(Z7/3)) for Z2<=4
MED2 = (FAPP/Z2)*((2*X2)+0.15-(Z7/4)) for the rest
End Do
Y1=MED1/ (Z7*Davg^2*FCK*1000)
Check (0.25-(Y1/1.134))<=0
End check
Y2=Davg*(0.5+(0.25-(Y1/1.134))**0.5)
AS2=MED1/ (0.87*FYK*1000*Y2)
Y3=MED2/ (Z6*Davg^2*FCK*1000)
Check (0.25-(Y3/1.134))<=0)<=0
End check
Y4=Davg*(0.5+(0.25-(Y3/1.134)) $^{0.5}$)
AS3=MED2/ (0.87*FYK*1000*Y4)
ASMIN1= (0.01*3.1416*Z4^2)/4
ASMIN2=0.0013*Z7*Davg
ASMIN3=0.0013*Z6*Davg
Check AS2<ASMIN2
AS3<ASMIN3
End Check
End Function

4.1.1.5. Algorithm 5- Maximum Permissible force at the column face

Function Analyze
Diameter of Re-bar used Dbar
Average Effective depth Davg

Davg = Z5-0.185-(Dbar/2)
H = 0.3-(0.0012*F_{CK})
F_{CD} = (F_{CK}*1000)/1.5
Check FAPP > (H*F_{CD}*2*(C1+C2)*Davg)
End Check
End Function

4.1.1.6. Algorithm 6- Punching shear

Function Analyze

SPX= (Z6-0.6-Z4)/ (Z1-1)

SPY= (Z7-0.6-Z4)/ (Z2-1)
Take the minimum of SPX and SPY as SP
Check (SP>=3*Z4)
End Check No need to calculate punching shear.
K1=1+ (200/ (DAVG*1000)) $^{0.5} \leq 2$
SRX=AS2/ (Z7*Davg)
SRY=AS3/ (Z6*Davg)
SRS = (SRX*SRY) $^{0.5}$
U1=2*(C1+C2+2*3.1416*davg)
VRDP=120*K1*((100*SRS*FCK) $^{(1/3)}$)*U1*Davg
UOUT=FAPP/ (120*K1*Davg*(100*SRS*FCK) $^{(1/3)}$)
XNCR= (UOUT-2*C1-2*C2)/ (2*3.1416*Davg)
VRDCS=FAPP/ (U1*Davg)
VRDC=120*K1*((100*SRS*FCK) $^{(1/3)}$)
FYWD= (250+0.25*Davg)*1000
SR=0.75*Davg
Do NO=1 for XNCR-1.5<0.4
NO=2 for XNCR-1.5<1.15
NO=3 for XNCR-1.5<1.9
NO=4 for XNCR-1.5<2.65
NO=5 for XNCR-1.5<3.4
NO=6 for XNCR-1.5<4.15
NO=7 for XNCR-1.5<4.9
NO=8 for XNCR-1.5<5.65
NO=9 for XNCR-1.5<6.4
NO=10 for the rest
ST=2*DAVG for XNCR-1.5>2
ST=1.5*DAVG for XNCR-1.5<2
End Do
Do AS4=0 for FAPP<=VRDP
AS4=NO*(VRDCS-0.75*VRDC)/((1.5*FYWD)/(SR*U1)) for AS4 ≥ AS4MIN
AS4MIN=NO*(0.053*(FCK) $^{0.5}$*SR*ST)/FYK for AS4MIN ≥ AS4
End Do punching shear reinforcement!
End Function

4.1.1.7. Algorithm 7- Wide Beam shear

Function Analyze

SR1 = (ASMIN1*4)/ (3.1416*Z4**2)

SR2=AS2/ (Z7*Davg) + AS3/ (Z6*DAVG) +AS4/ (2*(Z6 +Z7)*Davg)

AV1=0.5*(Z6-C1-0.6-1.6*Z4)

AV2=0.5*(Z7-C2-0.6-1.6*Z4)

G=FAPP/ (Z1*2*Davg)

H=FAPP/ (Z2*2*Davg)

Do VED1=G*AV1 for Z1<=3

VED1= (G*AV1) + (G*(AV1+0.2-Z6/3)) for Z1<=4

VED1= (G*AV1) + (G*(AV1+0.15-Z6/4)) for the rest

VED2=H*AV2 for Z2<=3

VED2= (H*AV2) + (H*(AV2+0.2-Z7/3)) for Z2<=4

VED2= (H*AV2) + (H*(AV2+0.15-Z7/4)) for the rest

End Do

K1=1+SQRT (200/ (Davg*1000))

VRD1=120*K1*((100*0.0013*FCK) $^{(1/3)}$)*Z7*Davg

VRD2=120*K1*((100*0.0013*FCK) $^{(1/3)}$)*Z6*Davg

Check VED1<VRD1

VED2< VRD2

End Check

End Function

4.1.1.8. Algorithm 8 – Optimizing total cost

Function Analyze

Re-bar and Drilling Cost Relative to Cost of concrete (CC) per unit volume and depth:

CSC & CDC The depth where the cost of drilling doesn't vary: F1, F2

Current and Best Total Cost: CTC, BTC; Where B stand for best: BZ1, BZ2, BZ3, BZ4...

I=Z1*Z2

J=0.7854*Z4^2*Z1*Z2*Z3

K=Z5*Z6*Z7

L=F1*CDC1+F2*CDC2+F3*CDC3

Do M=Z3*CDC1 for Z3< F1

M=F1*CDC1+ (Z3-F1)*CDC2 for Z3< (F2+F1)

M=F1*CDC1+F2*CDC2+ (Z3-F1-F2)*CDC3 for Z3< (F1+F2+F3)

M=L+ (Z3-F1-F2-F3)*CDC4 for Z3< (F1+F2+F3+F4)

M=L+F4*CDC4+ (Z3-F1-F2-F3-F4)*CDC5 for Z3< (F1+F2+F3+F4+F5)

End Do

CTC = ((I*M) +J+ (J*SR1*(CSC-1)) +K+ (K*SR2*(CSC-1)))*CC

BTC = Max (CTC), BZ1= Z1 of BTC, BZ2 = Z2 of BTC, BZ3 = Z3 of BTC …

End Function

Output: BTC, BZ1, BZ2, BZ3, BZ4, BZ5, BZ6, BZ7

4.1.2. Flowchart

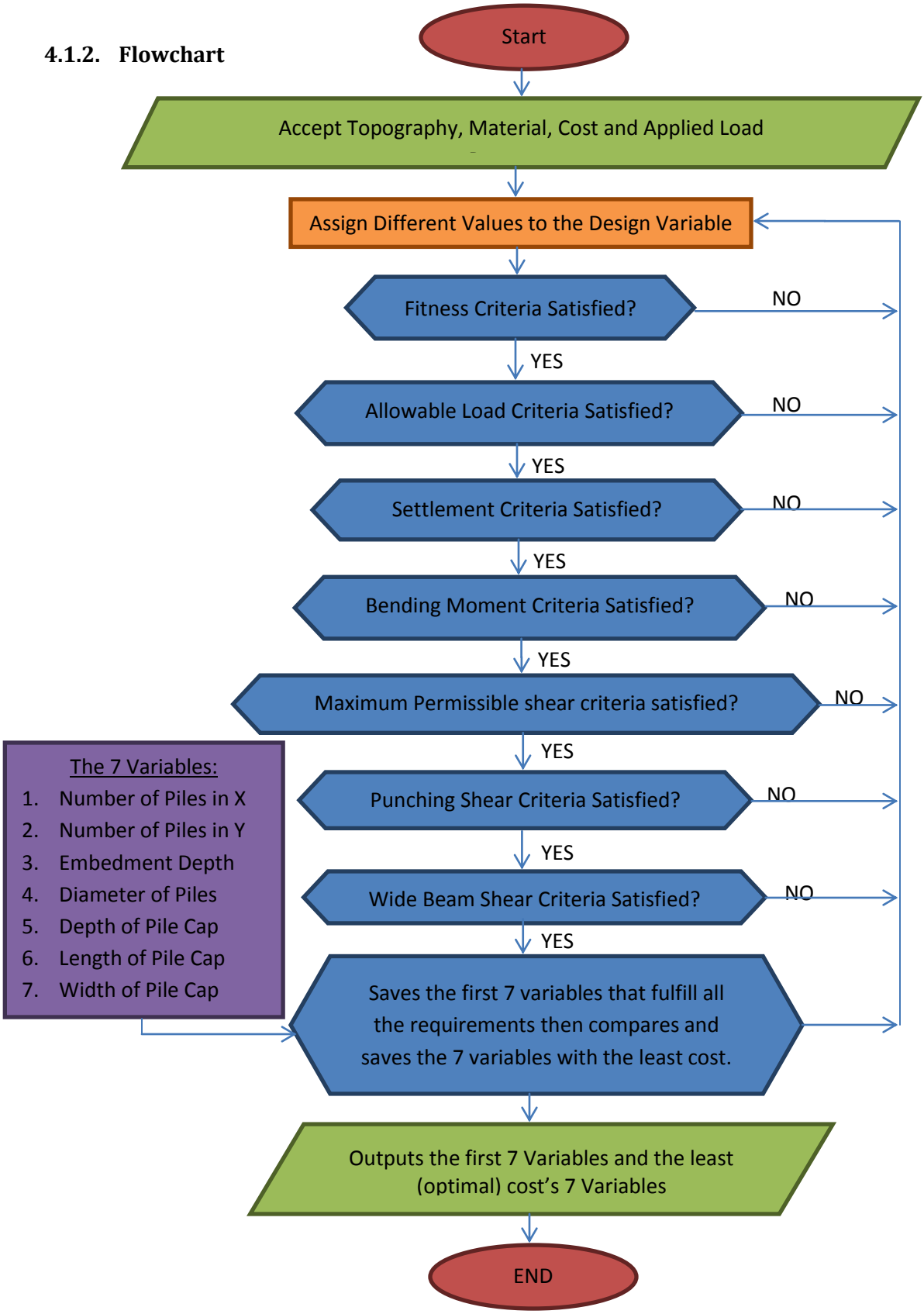


Figure 4-1 ***Flow Chart of the Program***

4.2. Computer Program Description

The Input mechanism is by keyboard and Mouse, and the user should carefully read the questions and answer with their appropriate units mentioned in the questions themselves. More information is given in pictures and when the user places the mouse pointer on the blank space.

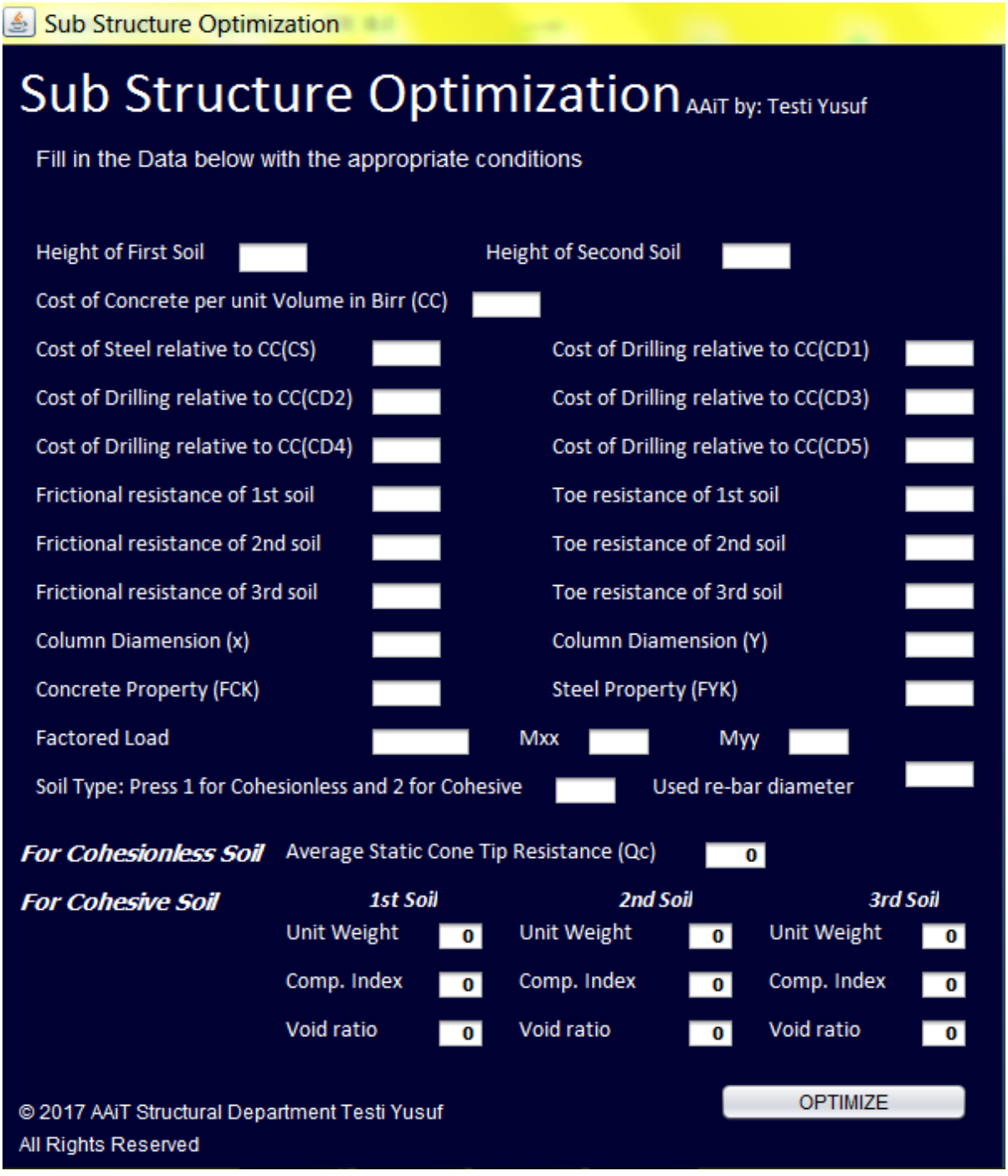


Figure 4-2 Java *Input Mechanism*

The output may take a few seconds, but all the optimal solutions, the non-optimal solutions and the time it took to optimize the Sub-Structure will be displayed together.

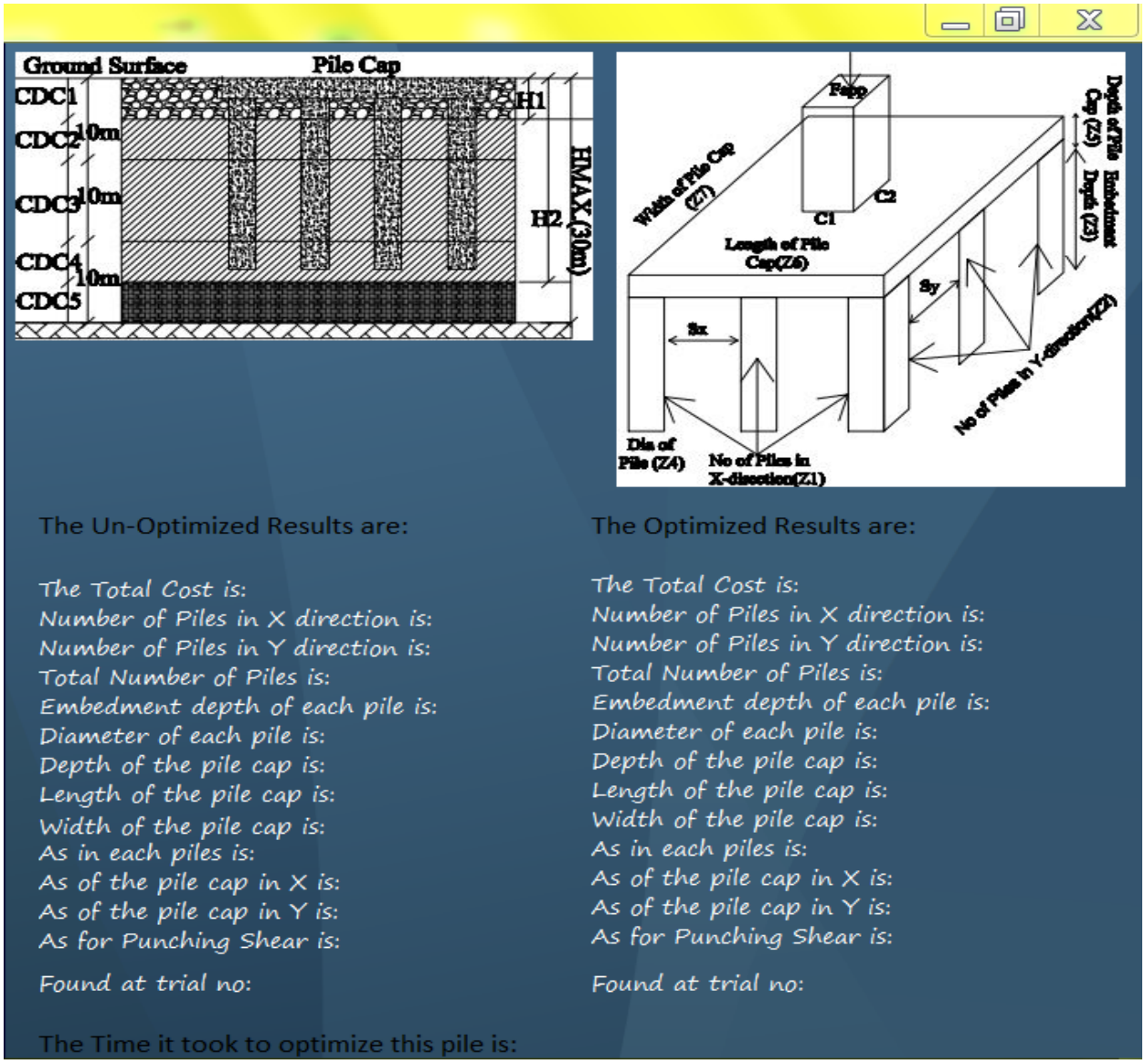


Figure 4-3 ***Java Output Mechanism***

Pressing the ENTER key will close the application.

4.3. Design Example

4.3.1. Input in the Example

A Design of a pile foundation was done using the computer program by utilizing the following inputs.

Table 4-1 the Values Entered into the Program

Property	Design Property Input	Property	Design Property Input
The height of the first soil type in (m)	5	The height of the second soil type in (m)	25
Cost of Concrete per unit volume (Cc) in (Birr)	5000	Cost of rebar per unit volume relative to $C_c = C_{SC}$	63
Cost of Drilling per unit depth relative to $Cc = C_{DC1}$	2.0	Cost of Drilling per unit depth relative to $C_c = C_{DC2}$	2.4
Cost of Drilling per unit depth relative to $C_c = C_{DC3}$	3.0	Cost of Drilling per unit depth relative to $C_c = C_{DC4}$	3.5
Cost of Drilling per unit depth relative to $C_c = C_{DC5}$	4.0	Total applied Factored Load (Load) (KN)	20,000
Moment in X Direction (Mxx) in (KNm)	0	Moment in Y Direction (Myy) in (KNm)	0
Select The Soil Type Cohesionless (1) / Cohesive(2)	1	Average Cone Penetration Resistance (q_c) in (KPa)	20
The factored frictional reaction per unit area of the first soil type (R1) in (KPa)	20	The factored toe reaction per unit area of the first soil types (R11) in (KPa)	100
The factored frictional reaction per unit area of the second soil type (R2) in (KPa)	30	The factored toe reaction per unit area of the second soil types (R22) in (KPa)	200
The factored frictional reaction per unit area of the third soil type (R3) in (KPa)	50	The factored toe reaction per unit area of the third soil type (R33) in (KPa)	500
Column dimension in the X (C_1) in (m)	0.9	Column dimension the Y (C_2) in (m)	0.9
Concrete Property (f_{ck}) in (MPa)	30	Steel Property (f_{yk}) in (MPa)	500
Diameter of Rebar Used (ϕ) in (m)	0.02		

4.3.2. **Output from the example**

Solution from the program is given in table 4.2

Table 4-2 the Values Outputted in the Program

Variable	Result
THE FIRST FEASIBLE RESULTS IS AS FOLLOWS	
The Total Cost in (Birr)	7847728.15
Number of piles in X-direction (Z1)	3
Number of piles in Y-direction (Z2)	5
Total Number of Piles (Z1*Z2)	15
Embedment depth (Z3) in (m)	30
Diameter of the piles (Z4) in (m)	0.3
Depth of pile cap (Z5) in (m)	2.4
Length of pile cap (Z6) in (m)	7.7
Width of pile cap (Z7) in (m)	8
Total Area of Steel in the Piles in (m2)	0.01060

Area of Steel in the pile cap in the x direction in (m2)	0.02293
Area of Steel in the pile cap in the Y direction in (m2)	0.02207
Area of Steel to resist Punching Shear in (m2)	0.02903
The Number of trial it took to reach this result	39,453,580
THE OPTIMIZED RESULTS IS AS FOLLOWS	
The Total Cost in (Birr)	3452333.61
Number of piles in X-direction (Z1)	2
Number of piles in Y-direction (Z2)	2
Total Number of Piles (Z1*Z2)	4
Embedment depth (Z3) in (m)	30
Diameter of the piles (Z4) in (m)	1.2
Depth of pile cap (Z5) in (m)	2.4
Length of pile cap (Z6) in (m)	6.3
Width of pile cap (Z7) in (m)	6.2
Total Area of Steel in the Piles in (m2)	0.04524
Area of Steel in the pile cap in the x direction in (m2)	0.01911
Area of Steel in the pile cap in the Y direction in (m2)	0.01856
Area of Steel to resist Punching Shear in (m2)	0.02818
The Number of trial it took to reach this result	715,417,010

4.3.3. Design Check from Hand Calculation

4.3.3.1. The first feasible Result

A. General Constraints

Spacing: $Z6 \geq ((Z4) + ((Z1-1)*3*Z4) + (0.6))$ $Z7 \geq ((Z4) + ((Z2-1)*3*Z4) + (0.6))$

$7.8 \geq ((0.3) + ((2)*3*0.3) + (0.6))$ $8 \geq ((0.3) + ((4)*3*0.3) + (0.6))$

$7.7 \geq 2.7m$ Spacing in X OK! $8m \geq 4.5m$ Spacing in Y OK!

Compression $Pn1= (Load/ (Z1*Z2)) + ((Load*Exx*Yn)/Ixx) + ((Load*Eyy*Xn)/Iyy) = 20000$

$Pn2= (Load/ (Z1*Z2)) + ((Load*Exx*Yn)/Ixx) - ((Load*Eyy*Xn)/Iyy) = 20000$

$Pn3= (Load/ (Z1*Z2)) - ((Load*Exx*Yn)/Ixx) + ((Load*Eyy*Xn)/Iyy) = 20000$

$Pn4= (Load/ (Z1*Z2)) - ((Load*Exx*Yn)/Ixx) - ((Load*Eyy*Xn)/Iyy) = 20000$

$Min (Pn1, Pn2, Pn3, Pn4) = 20000 \geq 0$ OK! All Piles are in Compression

B. Allowable Load

$$F_{applied} \leq Q_{ult} / F.S$$

$$Q_{ult}=Q_s+Q_t$$

For Embedment Depth (Z_3), Z3>H2

$Fapp \leq ((3.1416*Z4*H1*R1) + (3.1416*Z4*(H2-H1)*R2) + (3.1416*Z4*(Z3-H2)*R3) + (0.7854*Z4^2*R33))*Z1*Z2 + (Z6*Z7-0.7854*Z4^2*Z1*Z2)*R11$

$Fapp \leq ((3.1416*0.3*5*20) + (3.1416*0.3*(20)*30) + (3.1416*0.3*(5)*50) + (0.7854*0.3^2*500))*3*5 + (7.7*8-0.7854*0.3^2*3*5)*100 = 20,014.46KN$

$20,000KN \leq 20,014.46KN$ OK!

C. Allowable Settlement

Group Settlement Using Static Cone Result (q_C)

$$S_{ALL} \geq S_G$$

$$S_G = \frac{K_1 * B_{MAX} * q}{2 * q_c}$$

$$K_1 = 1 - \frac{L}{8 * B_{MAX}} = 1 - \frac{30}{8*8} = 0.53125 \geq 0.5$$

$$S_G = \frac{0.53125 * 8 * \frac{20000}{7.7*8}}{2*20}$$

200mm > 34.497mm OK!

D. Allowable Minimum pile spacing

Spacing in the x direction (S_x) and Spacing in the Y direction (S_y) must be greater or equal to 3 times the diameter of the piles ($3*Z_3$) to effectively utilize the soil capacity and avoid the sharing of influence area. This was satisfied by the general constraint.

E. Maximum Permissible Force at the face of loaded area

To satisfy the maximum shear requirements, the applied shear force must be less than the Maximum Permissible Force at the column face.

$$V_{ED} \leq V_{Rd,max}$$

$$V_{Rd,max} = 0.5 * v_1 * f_{cd} * u * D_{AVG} = 0.5 * v_1 * (f_{ck}/1.5) * u * D_{AVG}$$

$$v_1 = 0.6 * (1 - f_{ck}/250)$$

$$D_x = 2.4 - 0.075 - 0.1 - 0.01 - 0.02 = 2.195$$

$$D_y = 2.4 - 0.075 - 0.1 - 0.01 = 2.215$$

$$D_{AVG} = \frac{D_x + D_Y}{2} = 2.205, \; u = 4 * 0.9 = 3.6$$

$$V_{Rd,max} = 0.5 * 0.6 * (1 - f_{ck}/250) * (f_{ck}/1.5) * u * D_{AVG}$$

$$V_{Rd,max} = 0.5 * 0.6 * (1 - 30/250) * (30 * 1000/1.5) * 3.6 * 2.205$$

20,000KN ≤ 41,912.64KN OK!

F. Punching Shear

$$\text{Spacing (S)} \geq 3 * D_{PILE}$$

$(Z6 – 2*(0.3) – Z4)/ (Z1 - 1) \geq 3*Z4$ $\quad$ $(Z7 – 2*(0.3) - Z4)/ (Z2 - 1) \geq 3*Z4$

$(7.7 – 2*(0.3) – 0.3)/ (3 - 1) \geq 3*0.3$ $\quad$ $(8 – 2*(0.3) – 0.3)/ (5 - 1) \geq 3*0.3$

$3.4 \geq 0.9$ Punching Check required! $\quad$ $1.775 \geq 0.9$ Punching Check required!

$$V_{ED} \leq V_{RD}$$

$U1 = 2*C1+2*C2+ 4*\pi*D_{AVG} = 2*0.9+2*0.9+4*3.1416*2.205 = 31.309$

$A_S = U*D_{AVG} = 31.309*2.205 = 69.036$, $K= 1+\sqrt{\frac{200}{D_{avg}}} = 1+\sqrt{\frac{200}{2.205*1000}} = 1.301$

$SR_X=As2/ (Z7*D_{AVG}) = 0.02293/ (8*2.205) = 0.0013$,

$SR_Y=As3/ (Z6*D_{AVG}) = 0.02207/ (7.7*2.205) = 0.0013$,

$\rho = SRS =\sqrt{(SR_X * SR_Y)} = \sqrt{(0.0013 * 0.0013)}= 0.0013$

$f_C = 0.12*1000*K*(100*\rho*fck)^{1/3} = 0.12*1.301*(100*0.0013*30)^{1/3} = 245.774$

$V_{RD} = f_C*As = 245.775*69.036 = 16967.32$

$U_{out, ef} = \frac{V_{ED}}{f_c*d_{avg}} = \frac{20000}{245.775*2.205}= 36.905$

$U_{out, ef} = 2*(C_1 + C_2) + 2*\pi*d_{avg}*X$; $36.905= 3.6 +13.854*X$; $X = 2.404$

$v_{Rd,cs}= F_{APP}/ (U1*D_{AVG}) = 20000/ (31.309*2.205) = 289.703$

$f_{ywd}= (250+0.25*D_{AVG})*1000 = (250+0.25*2.205)*1000 = 250,551.25$

$S_r = 0.75* D_{AVG} = 0.75*2.205 = 1.65375$

$A_{sw} \geq \frac{v_{Rd,cs}-0.75*f_c}{1.5*\left(\frac{f_{ywd,ef}}{S_r*U_1}\right)} = \frac{289.703-0.75*245.774}{1.5*\frac{250551.25}{1.6538*31.301}} = 0.014517$

$S_t = 1.5* D_{AVG} = 1.5* 2.205 = 3.3075$

$A_{sw,min} \geq \frac{0.053*\sqrt{f_{ck}}*(S_r S_t)}{f_{yk}} = \frac{0.053*\sqrt{30}*1.65375*3.3075}{500} = 0.003176$

Therefore the total punching shear reinforcement area is: $No*A_{sw}=2*A_{sw}= 0.029034$

G. Wide Beam Shear

The applied shear force must be less than the sear capacity at that particular section.

$$V_{ED} \leq V_{RD}$$

$$V_{EDX} =\sum \frac{\sum P_Y*a_V}{2*D_{AVG}}$$

$$V_{EDY} =\sum \frac{\sum P_X*a_V}{2*D_{AVG}}$$

$Avx1 = 0.5*(Z6-C1-0.6-1.6*Z4) = 0.5*(7.7-0.9-0.6-1.6*0.3) = 2.86$

$$V_{EDX} = \frac{20000*2.86}{2*3*2.205} = 4323.51KN$$

$Avy1 = 0.5*(8-0.9-0.6-1.6*0.3) = 3.01$

$V_{EDY} = \frac{20000*3.01}{2*5*2.205} + (\frac{20000}{2*5*2.205}*(3.01+(0.3/4)+0.15-(8/4))) = 3850.34KN$

$$V_{RD} = f_C*As$$

$$V_{RD} = 0.12*(1+\sqrt{\frac{200}{D_{AVG}}})*(100*\rho*fck)*\ b*D_{AVG}$$

$$V_{RDX} = 245.77*8*2.205 = 4335.46KN$$

$$V_{RDY} = 245.77*7.7*2.205 = 4172.88KN$$

$$4323.51KN \leq 4335.46KN \qquad OK!$$

$$3850.34KN \leq 4172.88KN \qquad OK!$$

H. Bending

$$MED \leq MRD$$

$$M_{EDX} = \sum(\sum P_Y * r)$$

$$M_{EDY} = \sum(\sum P_X * r)$$

$$M_{EDX} = (\frac{20000}{3})*(3.85-0.45-0.3-0.15) = 19{,}666.6\ KNm$$

$$M_{EDY} = (\frac{20000}{5}) *(2*(4.0-0.45-0.3-0.15)+0.15-(8/4)+(0.3/4)) = 17{,}700.0\ KNm$$

$$M_{RD} = 0.87*f_{yk}*Z*As$$

$$K1 = \frac{19666.6}{8*30000*2.205^2} = 0.016854 \text{ and } K2 = \frac{17700.0}{7.7*30000*2.205^2} = 0.01576$$

$$Z1 = 2.205*(0.5+\sqrt{0.25-\frac{0.016854}{1.134}}) = 2.1717 \text{ and } Z2 = 2.205*(0.5+\sqrt{0.25-\frac{0.01576}{1.134}}) = 2.1739$$

$$As1 = \frac{19666.6}{0.87*500*1000*2.1717} = 0.0208 \geq (Asmin2)\text{: } 0.0013*Z7*D_{avg} = 0.02293$$

$$As2 = \frac{17700.0}{0.87*500*1000*2.1739} = 0.0187 \geq (Asmin3)\text{: } 0.0013*Z6*D_{avg} = 0.02207$$

$$MRD1 = 0.87*500000*2.1717*0.02293 = 21{,}663.88\ KNm$$

$$MRD2 = 0.87*500000*2.1739*0.02207 = 20{,}872.53\ KNm$$

$$19{,}666.7\ KNm \leq 21{,}663.88\ KNm \qquad OK!$$

$$17{,}700.0\ KNm \leq 20{,}872.53\ KNm \qquad OK!$$

4.3.3.2. The Optimized Result

A. General Constraints

Spacing: $Z6 \geq ((Z4) + ((Z1-1)*3*Z4) + (0.6))$ $Z7 \geq ((Z4) + ((Z2-1)*3*Z4) + (0.6))$

$6.3 \geq ((1.2) + ((1)*3*1.2) + (0.6))$ $6.2 \geq ((1.2) + ((1)*3*1.2) + (0.6))$

$6.3 \geq 5.4m$ Spacing in X OK! $6.2m \geq 5.4m$ Spacing in Y OK!

Compression $Pn1= (Load/ (Z1*Z2)) + ((Load*Exx*Yn)/Ixx) + ((Load*Eyy*Xn)/Iyy) = 20000$

$Pn2= (Load/ (Z1*Z2)) + ((Load*Exx*Yn)/Ixx) - ((Load*Eyy*Xn)/Iyy) = 20000$

$Pn3= (Load/ (Z1*Z2)) - ((Load*Exx*Yn)/Ixx) + ((Load*Eyy*Xn)/Iyy) = 20000$

$Pn4= (Load/ (Z1*Z2)) - ((Load*Exx*Yn)/Ixx) - ((Load*Eyy*Xn)/Iyy) = 20000$

$Min (Pn1, Pn2, Pn3, Pn4) = 20000 \geq 0$ OK! All Piles are in Compression

B. Allowable Load

$$F_{applied} \leq Q_{ult} / F.S$$

$$Q_{ult}=Q_s+Q_t$$

For Embedment Depth (Z_3), Z3>H2

Fapp ≤ ((3.1416*Z4*H1*R1) + (3.1416*Z4*(H2-H1)*R2) + (3.1416*Z4*(Z3-H2)*R3) + (0.7854*$Z4^2$*R33))*Z1*Z2 + (Z6*Z7-0.7854*Z42*Z1*Z2)*R11

Fapp ≤ ((3.1416*1.2*5*20) + (3.1416*1.2*20*30) + (3.1416*1.2*5*50) + (0.7854*1.2^2*500))*2*2 + (6.3*6.2-0.7854*1.2^2*2*2)*100 = 20,041.26KN

20,000KN ≤ 20,041.26KN OK!

C. Allowable Settlement

Group Settlement Using Static Cone Result (q_C)

$$S_{ALL} \geq S_G$$

$$S_G = \frac{K_1*B_{MAX}*q}{2*q_c}$$

$$K1=1-\frac{L}{8*B_{MAX}} = 1-\frac{30}{8*6.3} = 0.404 \geq 0.5$$

$$S_G = \frac{0.5*6.3*\frac{20000}{6.3*6.2}}{2*20}$$

200mm > 40.32mm OK!

D. Allowable Minimum pile spacing

Spacing in the x direction (S_x) and Spacing in the Y direction (S_y) must be greater or equal to 3 times the diameter of the piles ($3*Z_3$) to effectively utilize the soil capacity and avoid the sharing of influence area. This was satisfied by the general constraint.

E. Maximum Permissible Force at the face of loaded area

To satisfy the maximum shear requirements, the applied shear force must be less than the Maximum Permissible Force at the column face.

$$V_{ED} \leq V_{Rd,max}$$

$$V_{Rd,max} =0.5*v_1*f_{cd}*u* D_{AVG} = 0.5*v_1*(f_{ck}/1.5)*u* D_{AVG}$$

$$v_1=0.6*(1 - f_{ck}/250)$$

$$D_x = 2.4-0.075-0.1-0.01-0.02= 2.195$$

$$D_y = 2.4-0.075-0.1-0.01= 2.215$$

$$D_{AVG} = \frac{D_x+D_Y}{2} = 2.205, u=4*0.9 = 3.6$$

$$V_{Rd,max} =0.5*0.6*(1 - f_{ck}/250)*(f_{ck}/1.5)*u* D_{AVG}$$

$V_{Rd,max}$ =0.5*0.6*(1 - 30/250)*(30*1000/1.5)*3.6* 2.205

20,000KN ≤ 41,912.64KN OK!

F. Punching Shear

Spacing (S) ≥ 3*D_{PILE}

(Z6 – 2*(0.3) –Z4)/ (Z1 - 1) ≥ 3*Z4 (Z7 – 2*(0.3) - Z4)/ (Z2 - 1) ≥ 3*Z4

(6.3 – 2*(0.3) –1.2)/ (2 - 1) ≥ 3*1.2 (6.2 – 2*(0.3) –1.2)/ (2 - 1) ≥ 3*1.2

4.5 ≥ 3.6 Punching Check required! 4.4 ≥ 3.6 Punching Check required!

$V_{ED} \leq V_{RD}$

U1 = 2*C1+2*C2+ 4*π*D_{AVG} = 2*0.9+2*0.9+4*3.1416*2.205 = 31.309

A_S = U*D_{AVG} = 31.309*2.205 = 69.036, K= $1+\sqrt{\frac{200}{D_{avg}}} = 1+\sqrt{\frac{200}{2.205*1000}}$ = 1.301

SR_X=As2/ (Z7*D_{AVG}) = 0.019108/ (6.2*2.205) = 0.001398,

SR_Y=As3/ (Z6*D_{AVG}) = 0.018562/ (6.3*2.205) = 0.001336,

ρ = SRS =$\sqrt{(SR_X * SR_Y)}$ = $\sqrt{(0.001398 * 0.001336)}$= 0.001367

f_C = 0.12*1000*K*(100*ρ*fck)$^{1/3}$ = 0.12*1.301*(100*0.001367*30)$^{1/3}$ = 249.902

V_{RD} = f_C*As = 249.902*69.036 = 17252.24

$U_{out,\ ef} = \frac{V_{ED}}{f_c * d_{avg}} = \frac{20000}{249.902*2.205}$ = 36.295

$U_{out,\ ef}$ = 2*(C_1 + C_2) + 2*π*d_{avg}*X; 36.295= 3.6 +13.854*X; X = 2.36

$v_{Rd,cs}$= F_{APP}/ (U1*D_{AVG}) = 20000/ (31.309*2.205) = 289.703

f_{ywd}= (250+0.25*D_{AVG})*1000 = (250+0.25*2.205)*1000 = 250,551.25

S_r = 0.75* D_{AVG} = 0.75*2.205 = 1.65375

$A_{sw} \geq \frac{v_{Rd,cs} - 0.75 * f_c}{1.5 * \left(\frac{f_{ywd,ef}}{S_r * U_1}\right)} = \frac{289.703 - 0.75*249.902}{1.5 * \frac{250551.25}{1.65375*31.301}}$ = 0.014091

S_t = 1.5* D_{AVG} = 1.5* 2.205 = 3.3075

$A_{sw,min} \geq \frac{0.053*\sqrt{f_{ck}}*(S_r S_t)}{f_{yk}} = \frac{0.053*\sqrt{30}*1.65375*3.3075}{500}$ = 0.003176

Therefore the total punching shear reinforcement area is: No*A_{sw}=2*A_{sw}= 0.028181

G. Wide Beam Shear

The applied shear force must be less than the shear capacity at that particular section.

$V_{ED} \leq V_{RD}$

$V_{EDX} = \sum \frac{\sum P_Y * a_V}{2 * D_{AVG}}$

$$V_{EDY} = \sum \frac{\sum P_X * av}{2 * D_{AVG}}$$

Avx1 = 0.5*(Z6-C1-0.6-1.6*Z4) = 0.5*(6.3-0.9-0.6-1.6*1.2) = 1.44

$$V_{EDX} = \frac{20000 * 1.44}{2 * 2 * 2.205} = 3{,}265.31KN$$

Avy1 = 0.5*(6.2-0.9-0.6-1.6*1.2) = 1.39

$$V_{EDY} = \frac{20000 * 1.39}{2 * 2 * 2.205} = 3{,}151.93KN$$

$$V_{RD} = f_C * As$$

$$V_{RD} = 0.12*(1+\sqrt{\frac{200}{D_{AVG}}})*(100*\rho*fck)*\ b*D_{AVG}$$

V_{RDX} = 245.77*6.2*2.205 = 3,359.981KN

V_{RDY} = 245.77*6.3*2.205 = 3,414.175KN

3,265.31KN ≤ 3,359.981KN OK!

3,151.93KN ≤ 3,414.175KN OK!

H. Bending

MED ≤MRD

$$M_{EDX} = \sum(\sum P_Y * r)$$

$$M_{EDY} = \sum(\sum P_X * r)$$

$$M_{EDX} = (\frac{20000}{2})*(3.15-0.45-0.3-0.6) = 18{,}000.0\ KNm$$

$$M_{EDY} = (\frac{20000}{2})*(3.1-0.45-0.3-0.6) = 17{,}500.0\ KNm$$

$$M_{RD} = 0.87*f_{yk}*Z*As$$

$$K1 = \frac{18000.0}{6.2*30000*2.205^2} = 0.0199 \text{ and } K2 = \frac{17500.0}{6.3*30000*2.205^2} = 0.0190$$

$$Z1 = 2.205*(0.5+\sqrt{0.25 - \frac{0.0123}{1.134}}) = 2.1656 \text{ and } Z2 = 2.205*(0.5+\sqrt{0.25 - \frac{0.0118}{1.134}}) = 2.1673$$

$$As1 = \frac{18000}{0.87*500*1000*2.1656} = 0.0191 \geq (Asmin2)\text{: } 0.0013*Z7*D_{avg} = 0.01777$$

$$As2 = \frac{17500}{0.87*500*1000*2.1673} = 0.0186 \geq (Asmin3)\text{: } 0.0013*Z6*D_{avg} = 0.01806$$

MRD1 = 0.87*500000*2.1656*0.019108 = 18,000 KNm

MRD2 = 0.87*500000*2.1673*0.018563 = 17,500 KNm

18,000.0 KNm ≤ 18,000.0 KNm OK!

17,500.0 KNm ≤ 17,500.0 KNm OK!

4.4. Discussion of Results

The time it took for the program with an old computer (processor core i3) to give the first feasible and optimized design together is 37.0 seconds. The first feasible result is the first result obtained that fulfills all the constraints that are necessary to be a safe structure. However, this result is not optimized and does not take into account the cost of the materials. On the contrary, the Optimized result compares all the results that satisfy all the constraints and taking into account the cost of the material, chooses the most optimal one. The comparison table and the important points are discussed below.

Table 4-3 Comparison of the First Feasible and Optimal Results

Variable	The First Feasible Result	Optimized Result	Saving (Difference)	Saving (Percentage)
The Total Cost in (Birr)	7847728.154	3452333.61	4395394.544	56.01%
Number of piles in X-direction (Z1)	3	2	1	33.33%
Number of piles in Y-direction (Z2)	5	2	3	60.00%
Total Number of Piles (Z1*Z2)	15	4	11	73.33%
Embedment depth (Z3) in (m)	30	30	0	0.00%
Diameter of the piles (Z4) in (m)	0.3	1.2	-0.9	-300.00%
Depth of pile cap (Z5) in (m)	2.4	2.4	0	0.00%
Length of pile cap (Z6) in (m)	7.7	6.3	1.4	18.18%
Width of pile cap (Z7) in (m)	8	6.2	1.8	22.50%
Total Area of Steel in the Piles in (m^2)	0.0106029	0.04523904	-0.03463614	-326.67%
Area of Steel in the pile cap in the x direction in (m^2)	0.022932	0.019107609	0.003824391	16.68%
Area of Steel in the pile cap in the Y direction in (m^2)	0.02207205	0.018561989	0.003510061	15.90%
Area of Steel to resist Punching Shear in (m^2)	0.029034031	0.028181126	0.000852904	2.94%

1. The pile foundation will follow the soil with a larger resisting force. If the first soil layer has a good resisting force, the piles will only move to the first soil type. And if the load is further increased, the diameter of the pile and the number of the piles will increase to

accommodate the additional load. For instance in the above example, the third soil layer has a good resisting force therefore the piles moved into the third soil type.

2. Most of the time the governing factor or constraint is the resistance force of the soil. This was deduced because the result of the applied load and the resistance force of the soil are very close. This can be clearly seen in chapter 4.3.3.1.B and 4.3.3.2.B. The second important governing factor is the bending moment capacity. The depth of the pile cap must provide sufficient depth for bending moment and fulfill the shear requirements with shear reinforcement. This can be clearly seen in chapter 4.3.3.1.H and 4.3.3.2.H.

3. The saving of the total cost resembles the saving of drilling cost (decreasing the number of piles to be drilled). As drilling is the most expensive task, this dictates the overall cost of the foundation. This can clearly be seen on the saving (percentage) column in the above table where the total number of piles is the largest positive result (73.33%).

4. The second expensive task in pile foundation is the reinforcement. However, the increase in area of steel in each pile is -1500%, this area of steel must be multiplied with the number of piles to obtain the total area of steel in the piles. The area of steel in each pile in the first feasible result must be multiplied by 15, while the area of steel in each pile in the optimized result must only be multiplied by 4. This reduces the loss of the total area of steel in the piles significantly (-326.67%). There is also a substantial saving of area of steel in the pile cap.

5. The number of trial it took to find the first feasible result is very high (39,453,580). This is mainly because the applied load is very high (20,000KN) for the program and the resistive force of the soil is very weak at the top. This made some portions of the 0.4 pile diameter inadequate to resist the applied load. Other examples with smaller loads (medium & low rise buildings) have given out the first feasible result when the trial is in its thousands.

6. Even if everything is same in the x and y direction, there is a good possibility of using different number of piles and length of pile cap with respect to x and y direction. If a (2 by 2) pile configuration does not satisfy the requirement, the optimization moves on to a (2 by 3). However in real life, if a designer is tasked to design a foundation with similar properties, then he/she will have a non-optimal design with (3 by 3) pile configuration.

5. CONCLUSION AND RECOMMENDATION

5.1. Conclusion

The foundation cost, of real-world structural systems, can vary from 5% to 20% of the construction cost of the superstructure while the number of piles required might exceed several hundreds or even thousands. This is the main reason why the general objective of this thesis is to obtain minimum cost design of Pile Foundation and select the appropriate dimensions using Brute Force Optimization for safe and economical design. To determine the optimum, a number of background information is used including the observation of some practical performances, relevant literature review, data collection and analysis, modeling and computer software programming to find out the optimum solution to be applicable in Ethiopia.

Although, the proposed framework is used for the design of building structures, it can also be applied with proper modification and implementing the requirements and specifications imposed for other type of structures (such as bridges, dams, power plant stations, offshore platforms and museums).

The First Feasible and Optimal solution were obtained in the 39,453,580th and 715,417,010th iteration respectively out of the total 819,810,720 iterations. Brute force is the best optimization method for this problem because the values are not continuous which might affect gradient based optimization techniques and the number of values the variable can take is relatively small.

The amount of saving cannot be pin pointed accurately due to the fact that designs vary from designer to designer. However, in this study, the saving is significant, depending on the costing the saving ranges from 35 to 60 percent from the first feasible sub-structure. Even though this study's example result is on the higher side, in other literatures as seen in the literature review up to 33% worth of savings were found in the optimization of pile foundations using other methods.

In conclusion, it was found to be of paramount importance to optimize any structural member, to fully utilize the capacity of the structure while reducing a significant amount of cost. Here in this study, an optimization program using brute force optimization was developed to optimize a pile foundation with its pile cap.

5.2. Recommendation

These are the recommendations of this study:

1. The “number of piles” variable used in this study can be more optimized to incorporate non-regular pile configurations. For instance, total number of piles that have 5 and 7 piles
2. It is recommend that the use of this program or other similar versions developed to optimize any structural member carefully, and it will be very beneficial and indeed ensure the precision of the solution to double check the results with hand calculations (as done in this study) or other computer programs related to foundation.
3. Other structural components must be optimized, to reduce wastage in construction business. A lot of rebar, formwork, and PVC pipes are left unused or sold in pennies on the dollar because of wastage. If the whole construction works was optimized, a lot of money could have been saved.
4. Optimization course is offered to structural area of the school; however students in geotechnical engineering and other fields in AAiT do not have that advantage. Education in optimization will greatly help develop cost cautious engineers.

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