STUDY OF STRUCTURE AND LOCALIZATION OF VORTICES IN HIGH-TEMPERATURE SUPERCONDUCTORS

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Addis Ababa University

In partial Fulfilment of the Requirements for the
Degree of Master of Science in Physics

By

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The undersigned hereby certify that they have read and recommended to the Faculty of Science School of Graduate Studies for acceptance a thesis entitled "**study of structure and localization of vortices in High-temperature superconductors**" by **Ermias Libnedengel** in partial fulfillment of the requirements for the degree of **Master of Science in Physics**.

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To my family

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Abstract

In the present paper we study stable lattice configuration for a vortex in HTSC using the Ginzburg-Landau(GL) theory for a superconductor with d-wave paring. We calculate the minimum G-L free energy interms of evaluating the Abrikosov parameter (β_A) for the ratio of unit-cell dimensions(R). For $R = 1$ square lattice $\beta_A = 1.08643$ and $R = \sqrt{3}$ triangular lattice,$\beta_A = 1.25881$. That shows square lattice is relatively stable, Which is in agrement with recent theoretical and experimental work by kogan et al(1997),Maki(1991) by working on Ginzburg-Landau treatments,combined with Fermi-surface anisotropy[24]. Also we have studied localization of vortices theoretically using strong pining columnar defect[20]. During our work we calculate pinning potential $U_r(R)$ and critical current density j_c and show the effect of temperature and thermal fluctuation. Finally using scaling theory we see the temperature dependence of resistivity and have shown the vortex motion is prohibited around Bose-glass temperature.

Introduction

Superconductivity was discovered in 1911 by H.Kamerlingh Onnes in Leinden. In 1933 two German physicist, Meissner and Ochenfeld found that a superconductor expel magnetic field[1]. This is a phenomena in which a superconductor in its normal state is put in a magnetic field and the temperature is lowered below the critical temperature (where the material becomes superconductor)the magnetic field is expelled. The way in which this process takes place divides the superconducting materials in two types, Type-I and Type-II as shown in fig.1 Pure type-I superconductor excludes a magnetic field until a thermodynamical critical field H_c at which superconductivity is destroyed suddenly (complete Messiner state and perfect diamagnetism). In a type-II material Messiner state is incomplete and field will be excluded until a lower critical field H_{c1}. At fields above the upper critical field H_{c2} the magnetic field penetrates completely and the material is in its normal state. In the intermediate region between H_{c1} and H_{c2} there is a mixed state at which the magnetic field penetrates into the superconductor in the form of quantized flux lines or vortices. It is a new state of matter where both superconductivity and magnetic field coexist. It is usually called vortex state.

The behavior of vortices under various temperature and magnetic field has been interest both from the practical and scientific point of view. Vortex interaction for

pure system create stable lattice configuration, which means a vortex line can be in static equilibrium at any given position only if the superfluid velocity from all other source is zero there[1]. The main objective of condensed matter physicist and material scientists has been to raise the transition temperature to make superconducting materials technologically useful. In this endeavor handling the motion and determining stable lattice configuration of vortex lines becomes crucial because raising the temperature results in the motion of vortices and thermal depinning which destroys zero resistivity property of a superconductor. Recently scientists have found that vortices can be pinned using different pinning mechanisms. Such as:pinning by oxygen defect or oxygen vacancies(Thinkam,1988)[7], planar defects such as twin boundaries, grain boundaries[20], dislocation[22] and (Civeal et al and Nelson et al) found that columnar defects are strong pinning mechanism to enhance critical current[3,8].

In the present paper we study the structure and behavior of vortices around H_{c2} using Ginzburg-Landau(GL) free energy in a d-wave superconductor and localization of vortices using vortex pinning by columnar defects. A number of experimental investigations on a vortex lattice have been made using different techniques such as magnetic decoration [13],neutron scattering [14], electron microscopy [15], and electron holography [16]. Oblique lattice structure is observed by a small-angle neutron scattering study of the vortex lattice in $YBa_2CU_3O_7$ in magnetic field region of $0.5T \leq H \leq 5T$ applied parallel to the c-axis [14]. At low fields of 35G and 65G ,(Dolan et al) reported the regular triangular lattice by the Bitter-pattern observation of the vortices, where the slight distortion of the vortex lattice is explained by the anisotropy of the coherence lengths between the a-axis and b-axis directions. It is suggested that this deformation from the regular triangular lattice in the high field region of a $d_{x^2-y^2}$ wave

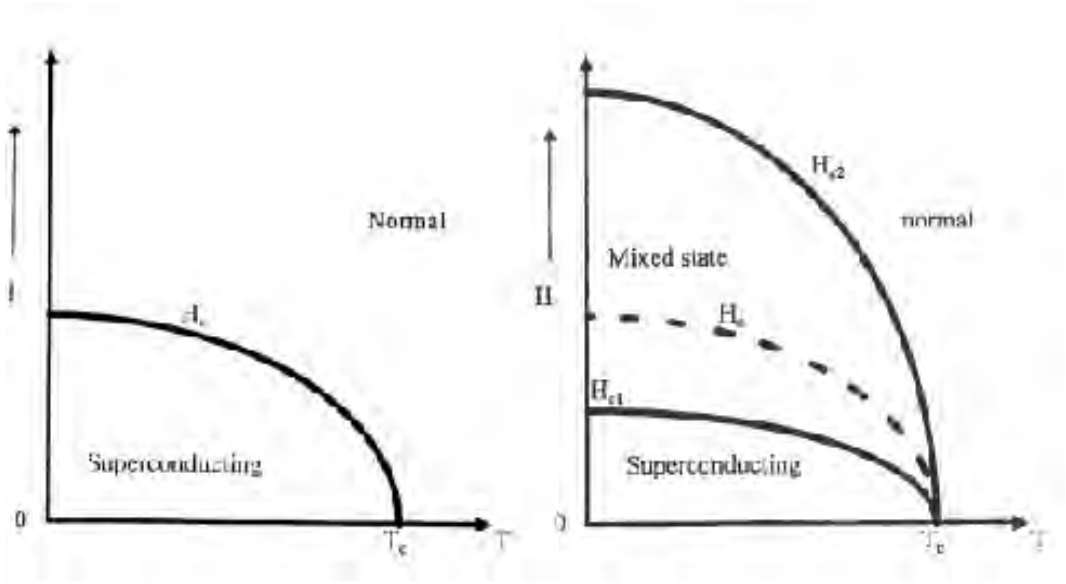


Figure 1: H-T phase diagrams of Type I(left) and Type II superconductors(right)

superconductor is due to the effect of the four fold symmetric vortex core structure. There have been theoretical investigations of structure of vortex lattice following the early work of Abrikosov such as (W.H.Kleiner et al) who calculated the Abrikosov parameter(β_A) for the minimum free energy solution of Ginzburg-Landau equation and obtain a result of $\beta_a = 1.1596$ for $R = \sqrt{3}$ which is for triangular lattice. And $\beta_A = 1.1803$ for $R = 1$ square lattice.

In summary the structure of vortex lattice have been described many years ago by Abrikosov and few years latter by W.H.Kleiner et al. Also recently by Berlinsky et al and Masanori et al. for the two -component Ginzburg landau theory for s-and d-wave superconductivity[4,21]. In this paper we try to find the minimum energy for a vortex and there by obtain the stable lattice configuration for a vortex lattice.

A general theory describing vortex pinning by many point defects was established

already long before the discovery of High-T_c superconductors. In the theory of weak collective pinning a single vortex is pinned by many pinning centers, that are randomly distributed and have a random pinning force [7]. In the second section of this paper we try to show localization of a vortex lines and pinning by strong correlated linear defect:columnar defect and temperature dependence of the critical current and pinning potential.

Chapter 1

Literature Review and background information

Historically vortex lines first discussed for the He^4 problem by Onsagar and Feynman. The generalization to the superconductivity is due to Abrikosov(1956). Vortex lines are very analogous to the vortex lines found in superfluid He^4. The important difference is that helium atoms are not charged marking e= 0.

In a vortex state of type-II superconductors the applied magnetic field can penetrate the sample in a tube like configuration. These tubes of magnetic flux referred to as superconducting vortices. This penetration takes place in the form of quantized magnetic flux lines crossing the sample each one containing a quantum of flux $\Phi_o = 2.07 \times 10^{-7} G.cm^2$[23]. A schematic representation of the penetration of these flux lines (vortex)into the sample is shown in fig.1.1. These vortices are formed by a core of unbinded charge carriers surrounded by superconducting currents as shown in the fig 1.2a. With increasing field the density of flux lines (which form a triangular lattice [2]) increase until the vortex cores overlap when the upper critical field H_{c2} is reaches,beyond this we recover the normal metallic state. The appearance of normal region in the superconduction region lowers the Gibbs free energy so that it may be

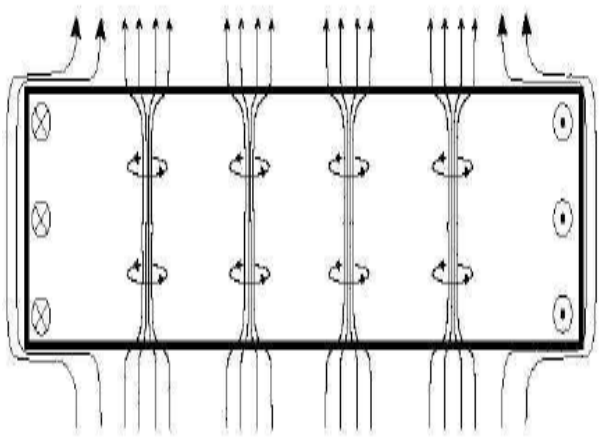


Figure 1.1: schematic representation of the magnetic flux inside a type II superconductor in the mixed state. Flux lines penetrate into the sample in quantized form,as vortices[17].

energetically favorable for the material to split up into a large number of small normal filaments to produce a maximum boundary area relative to the volume of the normal material.

High-temperature superconductors (HTSC) are extremely type-II materials they have short coherence lengths and long penetrations depths. Which results in a large values of the Ginzburg-Landau parameter κ. A direct consequence of a large κ is that the mixed state extends over a large part of the H-T phase diagram.$YBa_2CU_3O_{7-\delta}$ has been one the most investigated HTSC superconductors due to their potential applications. The influence of microstructure has been widely studied in $YBa_2CU_3O_{7-\delta}$ single crystals where artificial and controlled pinning centers have been successfully generated by irradiation.

The structure of an individual vortex depends on the coherence length and penetration depth. The vortex has a normal core whose radius is the coherence length.

The normal core is surrounded by supercurrents that produce magnetic field, thus in the middle of the normal core, the magnetic field has its highest value and it decays exponentially with a characteristic length λ fig.1.2[10]. Vortices have dynamic nature in the presence of magnetic field. There motion is strongly enhanced in HTSC due to the large thermal energies present. In the presence of externally applied field vortices start to move under the action of Lornez force ($f_L = J_s \times \Phi_o/c$). This motion of vortices generates voltage, and the resultant heat is likely to destroy the superconducting state. Thus, in order to enable the possible application of these materials, the motion of vortices in HTSC has to be understood and controlled. Since the motion of the cores leads to dissipation(Tsuyoshi Matsuda etal)[19], it has been found that the interaction between vortices and microstructural defects with in the superconducting material strongly enhance superconducting properties, which acts as vortex pinning centers[17].

1.1 vortex state lattice configuration

In 1957, the Russian physicist Alexi A.Abrikosov predicted the existence of type-II superconductors by working on the solution of Ginzburg Landau equations for $K = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}}$ where λ_L is the London penetration depth and ξ is the coherence length[2]. In particular he worked the case where the applied magnetic field H is only around the vicinity of H_{c2}. The solution shows the existence of periodic microscopic magnetic field distribution , transverse to the applied magnetic field. More precisely his efforts predicted a periodic square array of thin filaments of magnetic fields as vortex lines and the array of filaments as a vortex lattice,And he further study on the Gibbs free energy and obtain β_A which is called Abrikosov parameter,that determines

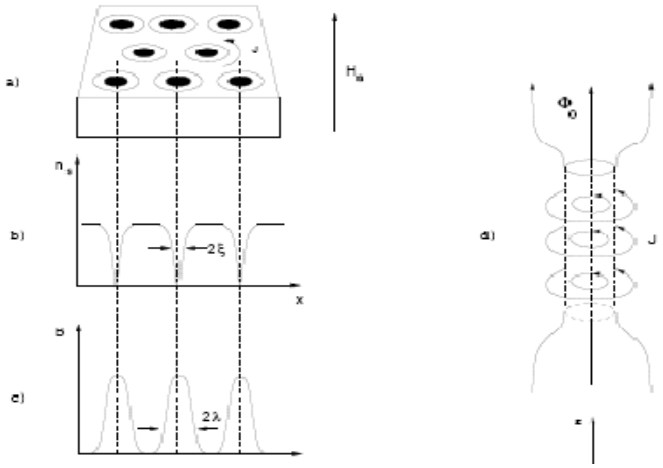


Figure 1.2: The mixed state in an applied magnetic field of strength greater than Hc1. a) A lattice of cores with encircling currents. b) The variation with position of concentration of superelectrons. c) The variation of the flux density. d) A schematic of a vortex[10].

the relative favorability of various possible solution[1]. If $\beta_A = 1$ for the usual condensation energy. If $\beta_A > 1$ and the more β_A increases,the less favorable is the energy. Numerical calculation shows that for the square lattice of Abrikosov,$\beta_A = 1.18$ for triangular lattice confirmed by latter work to be $\beta_A = 1.16$ this clearly shows that numerical calculation error led Abrikosov to conclude that square array was more stable configuration.Kleiner et al [9] who corrects this error and showed that triangular array had in fact the most favorable value of β_A of all possible periodic structure of flux lattice. A vortex line can be in static equilibrium at any given position only if the superfluid velocity from all other sources is zero there[1]. This will happen if each vortex is surrounded by a symmetrical array, as in the square or triangular arrays. However , it turns out that the square array has only unstable equilibrium,because of small displacements that tend to grow. But the triangular array is stable [A.L.Fettre.et al 1966], because it is shown to have lowest energy.

1.2 Vortex State Phase Transition and localization

The phase diagram of vortices in the HTSC in general described in the appearance of a vortex liquid phase occupies a substantial portion of the phase diagram below the $H_{c2}(T)$ line fig.1.3. It is very interesting that vortex lattice can melt not only as a consequence of increasing the temperature but also as a consequence of decreasing the magnetic field near $H_{c1}(T)$[7]. With decreasing field ,the distance between the vortices increases and eventually grows beyond the London penetration depth ,λ. In this region the vortex-vortex interaction is exponentially small, and consequently the

shear modulus $c_{66} = exp(-a_0/x)$ rapidly leading to a melting of the flux -line lattice. The Abrikosov vortex lattice is melted over a substantial part of the phase diagram. The thermodynamic phase transition is shifted to the melting line $B_m(T)$. With the upper critical -field lineH_{c2} marking only a cross over line where the modulus of the order parameter increases rapidly. The melting line $B_m(T)$ separates the vortex - liquid phases above $B_m(T)$. There are two symmetries break while going from the high-temperature, normal metallic phase to the Abrikosov vortex lattice. These two symmetries are the transition invariance,which is broken in the lattice phase, and the Guage symmetry producing longitudinal superconductivity [7].

Depending on the field strength,temperature, sample thickness and pin density a number of other interesting reigns are possible. Mott-insulator is a Phase at low temperature when fluxon density exactly matches the density of columnar pins (N.R.Nelson)[3]. Fisher et al(1989)[18] also Present qualitative arguments giving strong evidence for localization of the Bose particles and propose a phase diagram involving a superfluid phase, a Mott-insulatior (number of sites=number of particles) and what has been called a Bose- glass phase, separating the first two phases. The Mott-insulator is buried deep with in the Bose-glass phase, where relaxation times are very long. The transition from Bose-glass to the entangled flux-liquid of "superfluid " phase, which identify with the experimentally observed "Irreversibility line ". vortex glass is a distinct thermodynamic phase ,with zero linear resistivity, dominated by point disorder. By Bose-glass we mean an alternative thermodynamic phase, again with zero linear resistivity, but now dominated by correlated disorder, in the form of lines or planes which extend across an appreciable fraction of the sample thickness. In

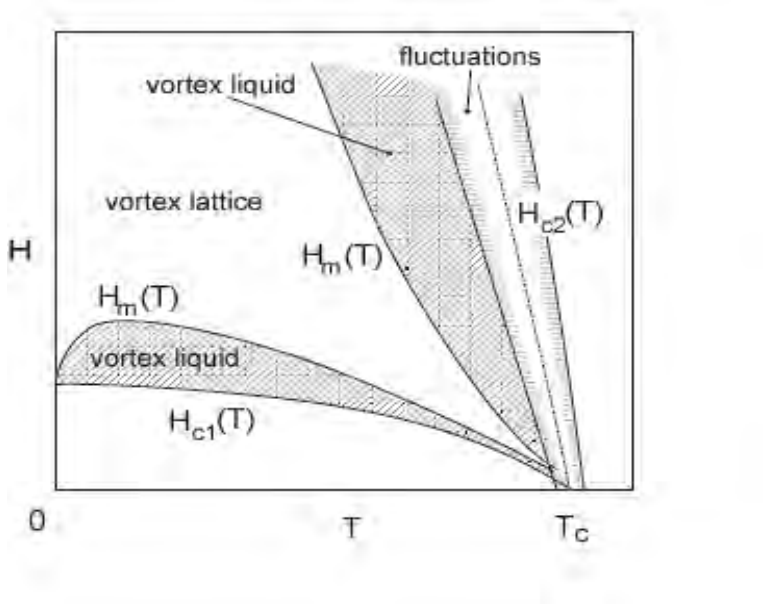


Figure 1.3: phenomenological phase diagram for the high-temperature superconductors including the effects of thermal fluctuation and of quenched disorder(pinning). For the vortex lattice with its finite shear modulus the disorder is relevant, and the lattice turns into a vortex glass, which is a true superconductor with $\rho(j \to 0) \to 0$ for the close to the melting line $B_m(T)$ the (finite) barriers against vortex motion are still large note the importance of thermal fluctuations is strongly suppressed in the diluted vortex-liquid phase close to $H_{c1}(T)$[7].

contrast to point disorder which promotes flux-line wandering and entanglement, correlated disorder inhibits wandering and promotes localization[3]. These are disorder-induced "glasses" phases which have dramatically different response to tipping the external magnetic field away from the axis. The corresponding three phases are expected to occur in the vortex system considered here is characterized by structural (i.e off diagonal) disorder and thus some kind of localization of the vortices to the tracks is expected to occur, leading to a Bose-glass phase at low enough temperatures.

This localization phenomenon gives rise to the occurrence of infinite barriers against vortex system, and hence the term "Bose-glass" seems to be appropriate for the description of a vortex system localized on the columnar defects. At higher temperature we expect a delocalization of the vortex systems from the tracks, resulting in a vortex-liquid phase analogous to the superfluid phase in Bose system.

The Moot- insulator, finally, correspond to a vortex density in which every columnar defect is occupied by a single vortex line. As a result the vortex density is pinned over some field regime $B = B_\phi$, leading a Messiner Ochsenfield -like effect at a fine magnetic field. The uncorrelated disorder in the vortex-glass phase induces line wandering, where as the correlated disorder due to columnar defects or due to twin boundaries promotes localization of the vortex lines. In general random pinning turns the vortex lattice phase into a vortex glass phase. At low fields where elastic interaction dominates, vortex lattice is only weakly destroyed, resulting in quasilattice phase in which a relative well-defines lattice structure is observed. This phase was recently termed Bragg-glass [12]in which there are no dislocation in the lattice. At higher fields, where the pinning energy dominates, the vortex is dramatically destroyed resulting in an entangled solid phase where elementary cells of vortex lattice

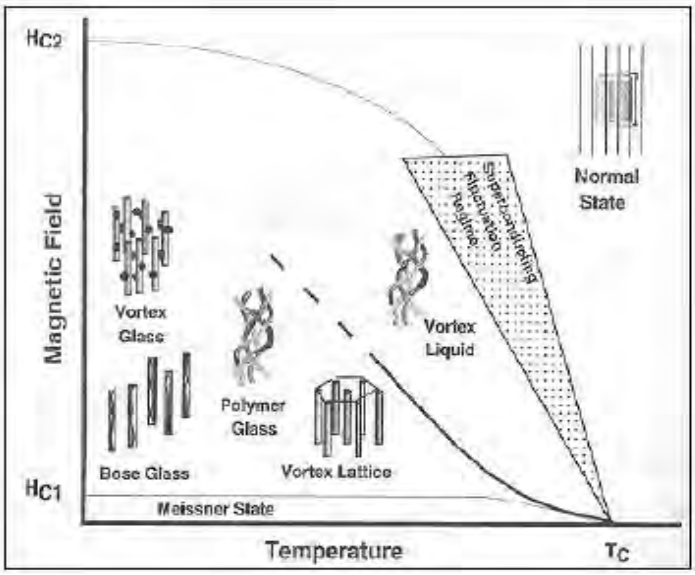


Figure 1.4: schematic representation of a magnetic phase diagram,showing the different phases that may appear depending on the energy balance[17].

are twisted and dislocation proliferates.

To summarize the behavior of vortex matter in HTSC arises from the four energies:

I)vortex interaction energy :favors a perfect vortex lattice.

II)Thermal energy: vortex depinning is enhanced and therefore a vortex liquid appears.

III) vortex pinning energy: distorts the vortex lattice into a glass of vortices.

iv) coupling energy determines the flexibility of the flux line. The melting temperature depends dramatically on the flexibility of the line. In materials with weak coupling and thus highly flexible vortices, the melting line resides at much lower temperatures than in martials with strong coupling.

Generally there are two major transitions in the phase diagram that can be a first

order transition (melting)or a second order transition, depending on the density of defects present in the samples. This transition separates a solid vortex state from a liquid vortex state appear in phase diagram fig.1.4

solid vortex state : The vortex thermal energy is lower than the energy barrier given by the vortex-vortex interaction and vortex-pinning interaction. A finite force is required in order to promote vortex motion.

liquid vortex state: The vortex thermal energy is large enough to overcome the back ground potential. Vortex lines easily moves when external forces are applies (even in the limit of a vanishingly small force). However, just above the solid-liquid transition,the pining energy is still able to compute against the thermal energy and modify vortex motion.

1.3 vortex Pinning mechanisms

Even though triangular lattice is found to be stable with reasonably lowest energy, vortices will feel a force transverse to any transport current so that the vortices will move unless pinned by inhomogeneities, against the Lorentz force exerted by the current. Some of the major pinning centers are illustrated in fig1.5. In perfectly clean and ideal material, the vortex liquid freezes into a regular lattice on decreasing the temperature. However, real materials always have chemical and structural imperfections. When such materials are cooled to below the melting temperature, the vortices are pinned by material imperfections, since it is energetically favorable[10]. This pinning distorts the regular lattice pattern that would otherwise form in ideal and pure materials. There are two competing effects: the repulsive interaction between the vortices favors lattice structure, but randomly located material defects tend to

pin the lines at random positions, distorting the lattice configuration. The motion of flux lines say with a velocity $\mathbf{V}$, essentially induces an electric field of magnitude $E = B \times V/c$, Which is parallel to j. This acts like a resistive voltage, and power is dissipated[1]. Any static disorder affecting the superconducting order parameter will contribute to a finite pinning force density F_{pin}. As a result the technological application of the type-II superconductors is reestablished ,However increasing the current density j above $J_c = cF_{pin}/B$[assume $j \perp B$] leads to the depinning of the vortices and hence to the reappearance of dissipation[7].

The current density j_c is the critical deppining current density, which is always less than the depairing current density $j_0 = cH_c/3\pi$ that determines by thermodynamical field $H_c, j_c < j_o$. If there is no net pinning for instance if pinning centers were distributed randomly and so weak as to leave the vortex lattice rigid. In this case force on the whole lattice at different centers would add up to zero. Thus the whole lattice would translate freely if it were not deformed either elastically (with out tearing of plastically(split into domains)[19]. The ratio j_c/j_o is a measure of the strength of the pinning force density f_{pin}. This also characterizes the strength of disorder. Thermal fluctuations affect the phenomenology of type-II superconductors and the dynamical behavior of the vortex system in different ways. The first one is the motion of vortex lines due to thermally activated jumps over the pinning barriers even at current density $j < j_c$(Anderson 1962)[7]correlated disorder affects the configurations and dynamics of vortex assemblies at both high and low temperatures. One significant source of correlated disorder is surely twin boundaries potentially even more important for application are columnar pins, produced by bombardment of crystals with energetic heavy ions of tin, iodine of lead [3]. In YBCO for example Civeal et al.[8]

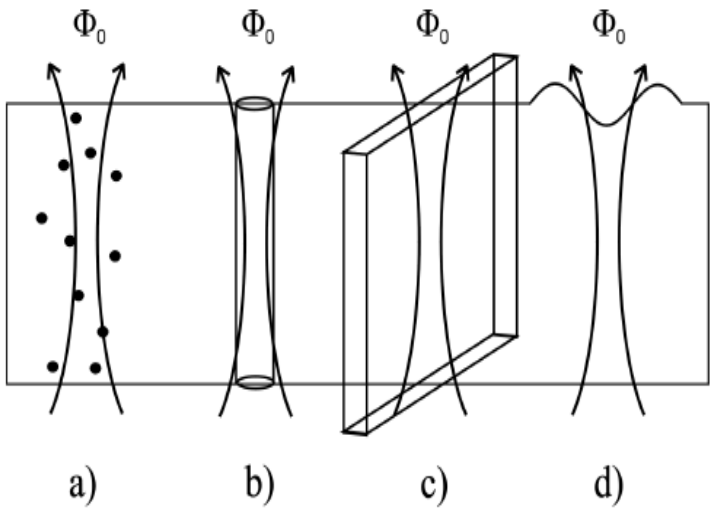


Figure 1.5: schematic illustration of various pinning centers .The vortex is indicated by the magnetic field lines and is pinned by (a)many point defects (b)a line defect (c)a planar defect and (d)surface roughness,which reduces locally the thickness of the film[20].

reported greatly enhanced pinning due to long aligned columns of damaged materials.

Generally we have two kinds of pinning centers:

point-like pinning centers when the defect has dimensions of the order of coherence length. Consequently, the region where the defect is present will have suppressed superconducting properties and will act as a point-like pinning center which promote a vortex -glass[17].

Linear-like pinning centers When one of the dimensions of the defect (parallel to the vortex) is larger than the coherence length while the other two are of the order of ξ_{ab}. The vortex will, therefore, be pinned over a large length (see figure) forming a Bose-glass[17].

1.4 Disorders /Defects

The dynamics of the vortex system in type-II superconductors is strongly dependent on the kind of disorder. Disorder affects the properties of the vortex state, it destroys the triangular vortex lattice and thus changes the H-T phase diagram of the vortex matter. Material defects or disorders are obtained to be centers for the occurrence of dissipation free current flow. There are different kinds of disorders. Among them quenched and thermal disorders are the major ones[7]. There main difference is the first one is static where as the latter is dynamical. Thermal disorder has influence both for the phase diagram of the vortex system and for its dynamical properties. Concerning quenched disorders one can distinguish between, randomly distributed defects, such as structural point defects, oxygen vacancies in high temperature superconductors, grain boundaries and defect with a correlated spatial distribution, for example,parallel pinning planes or artificially formed defects, caused by heavy ion radiation. Uncorrelated "point"disorder in the form of say oxygen vacancies can be used as pinning mechanism however evidently quite weak at the elevated temperature of the melting transition expected in pure systems. Another kind of disorder is introduced into the vortex system by thermal fluctuations or thermal disorder which influence both the phase diagram of the vortex system and its dynamical property[6].

Vortices can be pinned by secondary phase inclusions or spatial inhomogeneities of the crystal lattice, such a disorder as,for example, cavities impurities, secondary phases inclusions (grain boundaries, dislocations and twin planes). They are also called planar defects, are effective in pinning the vortices perpendicular to the plane of the defect only, whereas they form channels for easy flux flow along the plane. This result in a highly anisotropic pinning forces and critical current density[11]. All of

these defects usually suppress the order parameter Ψ in their vicinity. This means they present energetically favorable locations for a vortex[6]. Concerning statistical mechanics of the vortex lattice, we have a melting transition transforming the vortex-solid into a vortex-liquid phase, which is due to large thermal fluctuations. Thermal disorder also opposes quenched disorder via a smoothing of the disorder potential and there by reduces the critical current density j_c in the system. Thermal depinning is not an abrupt transition but rather a continuous cross-over from a pinned to unpinned situation.

1.5 Columnar defects

Pinning in a type-II superconductor can be optimized by introducing defects that trap the individual vortex lines all along their linear dimensions. While at the same time destroying a small fraction of superconducting material itself. In vortex state of high temperature superconductor, columnar defects produced by heavy ion bombardment of crystals with energetic heavy ions of tin, iodine, or lead are strong pinning centers of flux lines and dramatically enhance the critical current[12]. A defect structure reaching this goal can be obtained by introducing columnar defects into the material with cylinders of non-superconducting material of diameter ξ, the vortex core size. The resulting pinning properties will be highly anisotropic, with optimal pinning obtained for a configuration in which the magnetic field is aligned with the linear defect structure.

Since thermal softening of the linear pinning potential is much more gradual than for the point like pins, one can also expect less of a decrease in the critical current

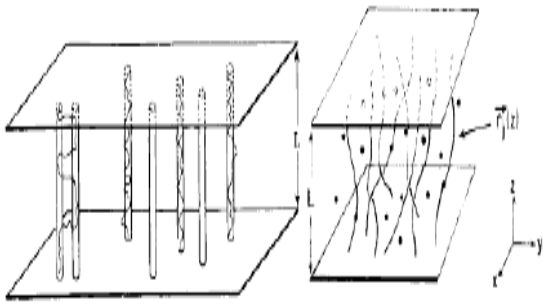


Figure 1.6: Influence of linear like(left)and point like defects (right)
on vortex position [17]

density with increasing temperature [8]. The point defects created by light-ion irradiation are not the optimum to pin the vortex array. A random distribution of point defects will pin only small fraction of the total length of each vortex and the net pinning energy per unit volume will be small and promote vortex wondering. The point defects created by light -ion irradiation are not the optimum to the vortex array. A dilute random distribution of point defects will be small. In this situation, as described by the collective pinning theory[7,8], the localization of the flux line, and the critical current will be significantly smaller than expected from a single sum of elemental pinning forces. Obviously,defects that confine a longer section of vortex core should provide better pinning.

The optimum pinning sites should consist of columns of non-Superconducting material that completely traverse the sample in the direction of the applied field. To

provide the maximum pinning force, the diameter of these columns should be of the order of ξ. Mkrtchy and Schmidt(1972)[3] who determined the pinning force acting on a vortex due to the presence of a cylindrical cavity (This analysis provides an upper estimate for the pinning energy /force/ critical current density). At low temperature we find a "Bose-glass" phase with flux lines localized on columnar pins, separated by a sharp phase transition from an entangled liquid of delocalized lines depending on the field strength temperature.

Chapter 2

Ginzburg-Landau Theory of a superconductor with d-wave pairing

The free energy in terms of 2-components of order parameter $\Psi_s(r)$ and $\Psi_d(r)$ for mixed s and d-wave paring[21,25]is given by:

$$f = \alpha_s|\Psi_s|^2 + \alpha_d|\Psi_d|^2 + \beta_1|\Psi_s|^4 + \beta_2|\Psi_d|^4 + \beta_3|\Psi_s|^2|\Psi_d|^2 + \beta_4((\Psi_s^*)^2\Psi_d^2 + \Psi_s^2{\Psi_d}^{*2})$$
$$+\gamma_s|\mathbf{D}\Psi_s|^2 + \gamma_d|\mathbf{D}\Psi_d|^2 + \gamma_v[(\mathbf{D}_y\Psi_s)^*(\mathbf{D}_y\Psi_d) - (\mathbf{D}_x\Psi_s)^*(\mathbf{D}_x\Psi_d) + c.c] + \frac{h^2}{8\Pi} \quad (2.0.1)$$

here we symbolize $\mathbf{D} = -i\nabla - \frac{e^*A}{\hbar c}$ and take $\alpha_s = \alpha'(T - T_s)$,$\alpha_d = \alpha'(T - T_d)$ with $T_s < T_d$.$\beta_1, \beta_2, \beta_3, \beta_4, \gamma_s, \gamma_d$ are all positive $\gamma_i = \frac{\hbar^2}{2m_i^*}$for $i = s, d, v$. To study for the vortex lattice the first necessary step is to write the above free energy equation by

varying with Ψ_d^* and Ψ_s^*.

$$\begin{aligned}\frac{\partial f}{\partial \Psi_s^*} &= \alpha_s \Psi_s + 2\beta_1 |\psi_s|^2 \psi_S + \beta_3 \psi_S |\psi_d|^2 + 2\beta_4 (\psi_S^* \psi_d^2) \\ &+ \gamma_v \mathbf{D}_Y^* (\mathbf{D}_Y \psi_d) - \mathbf{D}_x^* (\mathbf{D}_x \psi_d) + \gamma_s |\mathbf{D}|^2 \end{aligned} \tag{2.0.2}$$

$$= (\alpha_s + \gamma_s |\mathbf{D}|^2)\Psi_s + \gamma_v (\mathbf{D}_y^2 - \mathbf{D}_x^2)\psi_d + 2\beta_1 |\psi_S|^2 \psi_s + 2\beta_4 (\psi_S^* \psi_d^2) \tag{2.0.3}$$

$$+ \beta_3 \psi_S |\psi_d|^2 + 2\beta_4 |\psi_d|^2 \psi_S^* = 0$$

similarly for ψ_d^*

$$= (\alpha_d + \gamma_d |\mathbf{D}|^2)\Psi_d + \gamma_v (\mathbf{D}_y^2 - \mathbf{D}_x^2)\psi_s + 2\beta_2 |\psi_d|^2 \psi_d + 2\beta_4 (\psi_S^2 \psi_d^*) \tag{2.0.4}$$

$$+ \beta_3 \psi_S |\psi_d|^2 + 2\beta_4 |\psi_s|^2 \psi_d^* = 0$$

in a similar fashion one can obtain the current density in the x-y plane by varying with magnetic field potential($\mathbf{A}$):

$$\begin{aligned}\mathbf{j} &= \frac{e^*\hbar}{2m_d^*}[\Psi_d^*(\mathbf{D}\Psi_d) + (\mathbf{D}\Psi_d)^*\Psi_d] + \frac{e^*\hbar}{2m_s^*}[\Psi_s^*(\mathbf{D}\Psi_s) + (\mathbf{D}\Psi_s)^*\Psi_s] \\ &- \hat{x}\frac{e^*\hbar}{2m_v^*}[\Psi_s^*(\mathbf{D}_x\Psi_d) + (\mathbf{D}_x\Psi_s)^*\Psi_d + c.c] \\ &+ \hat{y}\frac{e^*\hbar}{2m_v^*}[\Psi_s^*(\mathbf{D}_y\Psi_d) + (\mathbf{D}_y\Psi_s)^*\Psi_d + c.c]\end{aligned} \tag{2.0.5}$$

2.1 vortex lattice solution

The variational solution for general case in the form of ground state wave function of harmonic oscillator

$$\phi_n^{\pm}(x) = \sqrt{\frac{\sigma_{\pm}}{\ell\sqrt{\Pi}}} exp{-\sigma_{\pm}^2 (x - x_n)^2 / 2\ell^2} \tag{2.1.1}$$

To construct a periodic vortex lattice, consider a liner superposition of the the basis functions of the form

$$\Psi_{\pm}(r) = \sum_n c_n e^{inqy} \phi_n^{\pm}(x) \tag{2.1.2}$$

To create y-direction periodicity of the function we constrained the values of k to $k_n = nq$ $n = 0, \pm 1, \pm 2, \pm 3$ $\psi_s$andψ_d order parameter can be written as:

$$\Psi_d = [\Psi_+(r) + \Psi_-(r)]/2, \Psi_s = [\Psi_+(r) - \Psi_-(r)]/2 \tag{2.1.3}$$

For periodicity in x the constants c_n satisfy the condition $c_{n+N} = c_n$ for some integer n taking $N = 2$ we have $c_{2n} = c_o$and$c_{2n+1} = c_1$ $L_x = \frac{2\ell^2}{q}$, $L_y = \frac{2\pi}{q}$ are periodicity in x and y rectangular $L_x \times L_y$ unit cell contains $HL_x \times L_y = 2(\frac{hc}{e^*} = 2\Phi_o)$ where Φ_o is a flux quantum.The parameter q controls the shape of the unit cell $R = \frac{L_x}{L_y} = (\frac{\ell^2}{\pi})q$ where R=1 corresponds to the square ,$R = \sqrt{3}$corresponds to the triangular and intermediate values $1 < R < \sqrt{3}$ implies the oblique vortex lattice. The solution that we have constructed for GL equations has three independent variable $c_0, c_1$and R. These materials determine the structure of vortex lattice near H_{c2}. The solution of GL free energy has degeneracy in energy. It is the forth order term that lifts the degeneracy and determine the equilibrium lattice. To find this minimum we have take into account the fourth order term in the free energy as well as the effect of screening which were so far ignored. The complete average free energy density can be written as

$$< f >=< f_2 > + < f_4 > + \frac{< h^2 >}{8\pi} \tag{2.1.4}$$

where f_2 and $f_4$represents for quadratic and quartic invariants respectively, $h = H + h_S$ is the local magnetic field. Let us now account the effect of screening on the gradient terms in $< f_2 >$.

$$\mathbf{D}_o + \mathbf{D}_s = (-i\nabla - \frac{e^* A_o}{c\hbar}) - \frac{e^* A_s}{c\hbar} \tag{2.1.5}$$

where $\mathbf{H} = \nabla \times \mathbf{A}_o$,$\varepsilon$ is corresponds to the uniform applied field,and $h_s = \nabla \times \mathbf{A}_s$ is screening field induced by the supercurrent j_S in the sample. A typical term will be of the form

$$< |\mathbf{D}\Psi_d|^2 >=< |\mathbf{D}_o\Psi_d\mathbf{D}_s\Psi_d|^2 > + < |\mathbf{D}_o\Psi_d|^2 > + < \mathbf{D}_s[\Psi_d^*\mathbf{D}_o\Psi_d + c.c] > + \frac{< h^2 >}{8\pi} \tag{2.1.6}$$

similarly expanding all the remaining gradient terms and neglecting terms higher than four.

$$< f_2 >=< f_2^o > + \frac{\hbar}{e^*} < \mathbf{D}_s.\mathbf{j}_s > \tag{2.1.7}$$

wheref_2^o is part of f_2 containing $\mathbf{D}_o$ piece of Gauage invariant gradient.which is given in equation 2.1.5 for $\mathbf{D} = \mathbf{D}_o$ from property of the variational solution we have $< f_2^o >= E < |\Psi_s|^2 + |\Psi_d|^2 >$ using $\mathbf{D}_s = \frac{e^*\mathbf{A}_s}{c\hbar}$ we can write

$$< f >= E < |\Psi_s|^2 + |\Psi_d|^2 > -\frac{1}{c} < \mathbf{A}_s.\mathbf{j}_s > + < f_4 > + \frac{< (H + h_s)^2 >}{8\pi} \tag{2.1.8}$$

The second term on RHS can be simplified by expressing j_s through the Maxwell equation ($curl\mathbf{h}_s = 4\frac{\pi}{c}\mathbf{j}_s$) Integrating by parts and neglecting the surface term one obtains $\frac{1}{c} < \mathbf{A}_s \cdot \mathbf{j}_s >= \frac{<h_s^2>}{4\pi}$ The last term on the RHS can be written as by recalling the definition $B = H + < h_s >$ of the magnetic induction yields:

$$\begin{aligned} \frac{< h^2 >}{8\pi} = \frac{< (\mathbf{H} + \mathbf{h}_s)^2 >}{8\pi} &= \frac{< H^2 >}{8\pi} + \frac{H}{4\pi} < h_s > + \frac{< h_s^2 >}{8\pi} \\ &= \frac{< H^2 >}{8\pi} + \frac{H}{4\pi}(B - H) + \frac{< h_s^2 >}{8\pi} \\ &= \frac{< -H^2 >}{8\pi} + \frac{\mathbf{H}.\mathbf{B}}{4\pi} + \frac{< h_s^2 >}{8\pi} \end{aligned} \tag{2.1.9}$$

Now the average free energy becomes

$$< f >= E < |\Psi_s|^2 + |\Psi_d|^2 > -\frac{< h_s^2 >}{8\pi} - \frac{< H^2 >}{8\pi} + \frac{\mathbf{H}.\mathbf{B}}{4\pi} \tag{2.1.10}$$

The relation between Gibbs free energy to $< f >$ is given by $< g >=< f > -\frac{B.H}{4\pi}$ then The average free energy converts to:

$$< g >= E < |\Psi_s|^2 + |\Psi_d|^2 > -\frac{< h_s^2 >}{8\pi} - \frac{< H^2 >}{8\pi} \tag{2.1.11}$$

having the relation $\frac{\partial < g >}{\partial H} = -\frac{B}{4\pi}$ and differentiating both sides with H after rearranging terms we get:

$$< h_s > \;\; = \;\; \frac{B-H}{4\pi} = -\frac{\partial E}{\partial H} < |\Psi_s|^2 + |\Psi_d|^2 > \tag{2.1.12}$$

In the vicinity of H_{c2} we have $E \approx -\frac{\partial E}{\partial H}(H - H_{c2})$ and $< f_2^o >= E < |\Psi_s|^2 + |\Psi_d|^2 >$

$$\begin{aligned} < f_2^o > \;\; &= \;\; \frac{\partial E}{\partial H}(H - H_{c2}) < |\Psi_s|^2 + |\Psi_d|^2 > \\ &= \;\; (H - H_{c2})\frac{< h_s >}{4\pi} \end{aligned} \tag{2.1.13}$$

The expression for Gibbs free energy which shows the dependence on H is given by

$$< g > - < g >_n = \frac{1}{4\pi}(H_{c2} - H) < h_s > + < f_4 > -\frac{1}{8\pi} < h_s^2 > \tag{2.1.14}$$

where $< g >_n = \frac{-H^2}{8\pi}$ (The normal state contribution to the Gibbs free energy).Consider now a simple scaling transformation$(\psi_s, \psi_d) \rightarrow (\lambda\psi_s, \lambda\psi_d)$ whereλ is a real number under such a transformation $< h_s > \rightarrow \lambda^2 < h_s >, < f_4 > \rightarrow \lambda^4 < f_4 >$and $< h_s^2 > \rightarrow \lambda^4 < h_s^2 >$ with this scaling the Gibbs free energy become

$$< g > - < g >_n = \frac{1}{4\pi}(H_{c2} - H)\lambda^2 < h_s > +\lambda^4 < f_4 > -\frac{1}{8\pi}\lambda^4 < h_s^2 > \tag{2.1.15}$$

minimizing with respect to λ^2 and equating forλ^2 we get the following expression

$$\lambda^2 = \frac{-(H_{c2} - H) < h_s >}{8\pi < f_4 > - < h^2 >} \tag{2.1.16}$$

Inserting λ^2 back to the free energy expression after simplification and taking the leading term we get the following expression for Gibbs free energy

$$< g > - < g >_n = -\frac{1}{8\pi}[\frac{(H_{c2} - H)^2 < h_s >^2}{8\pi < f_4 > - < h_s^2 >}] \tag{2.1.17}$$

which is independent of the Abrikosov ratio β_A and Ginzburg-Landau parameter k and which are given by $\beta_A = \frac{<h_s^2>}{<h_s>^2}$,$k^2 = \frac{4\pi<f4>}{<h_s^2>}$ Then the above equation converted to the following Abrikosov free energy expression form

$$< g > - < g >_n = -\frac{1}{8\pi}\frac{(H_{C2} - H)^2}{(2k^2 - 1)\beta_A} \tag{2.1.18}$$

which is the Gibbis free energy for the Abrikosov lattice in the familiar form.The minimum Gibbs free enegrgy corresponds to the minimum $\beta_A.2k^2 - 1$ simply serve for type-II behavior i.e occur only for $k > \frac{1}{\sqrt{2}}$.

2.2 Magnetic field distribution

Our aim is to determine the Abrikosov parameter β_A for the triangular case $R = \sqrt{3}$ and square $R = 1$.To find β_A it is necessary to evaluate the spatial averages $< h_s^2 >$ and $< h_s >$. These two averages can be computed in since they are directly related to the vortex lattice solutions $\Psi_{\pm}(r)$. which is simple linear superposition of the gaussian wave functions $\phi_n^{\pm}$ given by

$$\phi_k^{\pm}(x) = \sqrt{\frac{\sigma_{\pm}}{\ell\sqrt{\pi}}} exp - \sigma_{\pm}^2(x - x_k)^2/2\ell^2 \tag{2.2.1}$$

$$\Psi_{\pm}(r) = \sum_n c_n e^{inqy}\phi_n^{\pm}(x) \tag{2.2.2}$$

and $k_n = nq$,$n = 0, \pm 1, \pm 2...$ Let us now calculate the spatial distribution of the screening field.

The Maxwell equation for screening field A_s can be written as $\nabla^2 A_s = -4\frac{\pi}{c} j_s$ where Landau gauge satisfies $\nabla . A_s = 0$. Let us convert the equation to fourier space

$$\mathbf{j}_s(r) = \sum_k expi\mathbf{k}.\mathbf{r}\ \mathbf{j}_s(k) \quad \mathbf{A}_s(r) = \sum_k expi\mathbf{k}.\mathbf{r}\ \mathbf{A}_s(k) \tag{2.2.3}$$

the summation goes over the reciprocal lattice vectors $k = (k_x, k_y) = (\frac{2\pi k_1}{L_x}, \frac{2\pi k_2}{L_y}) and (k_1, k_2) = 0, \pm 1, \pm 2....$ the above Maxwel equation becomes

$$\mathbf{h}_s(k) = \frac{4\pi i \mathbf{k} \times \mathbf{j}_s(k)}{ck^2} \qquad where \ \ k \neq 0 \tag{2.2.4}$$

in order to evaluate this it is helpful to write the supercurrent using wave functions $\Psi_\pm$ instead of $\Psi_s \ and \ \Psi_d$:

$$\begin{aligned} j &= \frac{e^*\hbar}{2m_d^*}[\Psi_d^*(\mathbf{D}\Psi_d) + (\mathbf{D}\Psi_d)^*\Psi_d] + \frac{e^*\hbar}{2m_s^*}[\Psi_s^*(\mathbf{D}\Psi_s) + (\mathbf{D}\Psi_s)^*\Psi_s] \\ &- \hat{x}\frac{e^*\hbar}{2m_v^*}[\Psi_s^*(\mathbf{D}_x\Psi_d) + (\mathbf{D}_x\Psi_s)^*\Psi_d + c.c] \\ &+ \hat{y}\frac{e^*\hbar}{2m_v^*}[\Psi_s^*(\mathbf{D}_y\Psi_d) + (\mathbf{D}_y\Psi_s)^*\Psi_d + c.c] \end{aligned} \tag{2.2.5}$$

Now substituting $\Psi_s \ \ and \ \ \Psi_d$ for the expression $\mathbf{j}$ And setting $\epsilon_v = \frac{\gamma_v}{\gamma_d}$,$\gamma_s = \gamma_d$ and $\mathbf{D} = \mathbf{D}_x\hat{x} + \mathbf{D}_y\hat{y}$ one can write the total current in fourier form

$$\mathbf{j}_s(k) = \frac{e^*\hbar}{4m}\sum_{\alpha=\pm}[\hat{x}(1-\alpha\epsilon_v) < \psi_\alpha^*\mathbf{D}_x\Psi_\alpha >_k + \hat{y}(1+\alpha\epsilon_v) < \Psi_\alpha^*\mathbf{D}_y\Psi_\alpha >_k + < c.c >_{-k}] \tag{2.2.6}$$

$\Psi\alpha$ is a linear combination of gaussian.We take a short hand representation of

$$< ... >_k = \frac{1}{L_x L_y}\int_0^{L_x} dx \int_0^{L_x} dy ... e^{-ik.r} \tag{2.2.7}$$

and after integration we obtain the following result

$$< \psi_\alpha^*\mathbf{D}_x\psi_\alpha >_k = \frac{\sigma_\alpha}{2}(\frac{k_x}{\sigma_\alpha} - ik_y\sigma_\alpha) < |\psi_\alpha|^2 >_k \tag{2.2.8}$$

$$< \psi_\alpha^* \mathbf{D}_y \psi_\alpha >_k = \frac{i}{2\sigma_\alpha}(\frac{k_x}{\sigma_\alpha} - ik_y\sigma_\alpha) < |\psi_\alpha|^2 >_k \tag{2.2.9}$$

For the fourier component$< |\psi_\alpha|^2 >_k$ we have

$$< |\psi_\alpha|^2 >_k = \frac{ak}{L_x} exp - \frac{\ell^2}{4}[(\frac{k_x}{\sigma_\alpha})^2 + (k_y\sigma_\alpha)^2] \tag{2.2.10}$$

$$a_k = e^{i\frac{\pi}{2}k_1k_2}[c_o c_{k_2}^* + (-1)^{k_1} c_1 c_{k_2+1}^*] \tag{2.2.11}$$

substituting these terms one obtains

$$\mathbf{j}_s(k) = \frac{e^*\hbar}{4m}\sum_{\alpha=\pm}[\hat{x}(1-\alpha\epsilon_v)[-ik_y\sigma_\alpha] + \hat{y}(1+\alpha\epsilon_v)[\frac{ik_x}{\sigma_\alpha^2}]] < |\psi_\alpha|^2 >_k \tag{2.2.12}$$

substituting $\mathbf{j}(k)$ for the modified Maxwell equation one obtains the real space field distribution of the form

$$\begin{aligned}\mathbf{h}_s(r) = -\hat{z}\pi\frac{e^*\hbar}{mc}[R_o\sum_k e^{ik.r} < |\psi_+|^2 + |\psi_-|^2 >_k \\ + R_1\sum_{k\neq 0} e^{ik.r}(\frac{k_x^2 - k_y^2}{k_x^2 + k_y^2}) < |\psi_+|^2 - |\psi_+|^2 >_k]\end{aligned} \tag{2.2.13}$$

where we have defined the numerical factors

$$R_0 = [(\sigma_-^2 + \sigma_+^2) + \epsilon_v(\sigma_-^2 - \sigma_+^2)]/2 \tag{2.2.14}$$

$$R_1 = [(\sigma_-^2 - \sigma_+^2) + \epsilon_v(\sigma_-^2 + \sigma_+^2)]/2 \tag{2.2.15}$$

The fourier components $\mathbf{h}_s(k)$ can be deduced from the above equation as

$$\mathbf{h}_s(k) = -\pi\frac{e^*\hbar}{mc}[R_o < |\psi_+|^2 + |\psi_-|^2 >_k \; + R_1 f_k |\psi_+|^2 - |\psi_+|^2 >_k]$$

where the quantity

$$\mathrm{f}_k = \begin{cases} \frac{k_x^2 - k_y^2}{k_x^2 + k_y^2}, & \text{if k}\neq 0; \\ 0 & \text{if k=0.} \end{cases} \tag{2.2.16}$$

Let us now calculate $< h_s^2 >$ using the fourier identity:

$$< h_s^2 >= \sum_k \mathbf{h}_s(k)\mathbf{h}_s(-k) \tag{2.2.17}$$

having the relation $< \psi_\alpha^* \psi_\alpha >_k = a_k \Theta_{\alpha\alpha}(k) = \frac{a_k}{L_x} exp - \frac{\ell^2}{4}[(\frac{k_x}{\sigma_\alpha})^2 + (k_y \sigma_\alpha)^2]$ where $\alpha = \pm$ one can find

$$\begin{aligned} < h_s^2 >= \frac{\pi^2 e^* \hbar^2}{m^2 c^2} \sum_k [R_o < |\psi_+|^2 + |\psi_-|^2 > k + R_1 f_k |\psi_+|^2 - |\psi_+|^2 >_k] \\ * [R_o < |\psi_+|^2 + |\psi_-|^2 > -k + R_1 f_{-k} < |\psi_+|^2 - |\psi_+|^2 >_{-k}] \end{aligned} \tag{2.2.18}$$

$$< \mathbf{h}_s^2 >= \frac{\pi^2 e^* \hbar^2}{m^2 c^2} \sum_k a_k a_{-k} [(R_o + R_1 f_k)\Theta_{++} + (R_o - R_1 f_k)\Theta_{--}]^2 \tag{2.2.19}$$

Now we can evaluate β_A following the work of Kleiner, Roth, and Autler,[9]it is practical to carry out the minimization with respect to the constants c_o and c_1 analytically.This reduces the relevant parameter space to a single parameter, the geometric ratio $R = L_x/L_y$.Recalling that by assumption $c_0 = 2k$ and $c_1 = 2k + 1$.Theminimization with respect to c_0 and c_1; carried out and obtains that a condition for the minimum is $c_1 = \pm i c_o$. So we can arbitrarily choose $c_1 = 1$ and $c_o = i$ by this choice we have an identity

$$a_k a_k = [(-1)^{k_1} + (-1)^{k_2}]^2$$

which sets a domain that is both k_1 and k_2 should be either even or odd otherwise β_A will be zero. Using these in mind we can easily calculate the average magnetic field which is found to be:

$$\begin{aligned} < h_s >= h_s(k=0) &= -\pi z_o a_o (\frac{e^* \hbar}{mc})[\Theta_{++}(0) + \Theta_{--}(0)] \\ &= -2\pi \frac{R_o}{L_x} (\frac{e^* \hbar}{mc})(|c_0|^2 + |c_1|^2) \end{aligned} \tag{2.2.20}$$

Therefore $< h_s >^2$ is given by:

$$< \mathbf{h}_s >^2 = \frac{4\pi^2 R_o^2}{L_x^2}(\frac{e^*\hbar}{mc})^2(|c_0|^2 + |c_1|^2)^2 \tag{2.2.21}$$

Finally the expression for β_A is given by

$$\beta_A = \frac{< h_s^2 >}{< h_s >^2} = \frac{L_x^2}{4R_0^2}\sum_k{}'[(R_0 + R_1 f_k)\Theta_{++} + (R_o - R_1 f_k)\Theta_{--}]^2 \tag{2.2.22}$$

for $k = 0$ β_A becomes one. Prime sign in the summation refers to the domain . Thus in the limit of pure d-wave state where $\epsilon_v \to 0$ and $\sigma_\pm \to 1$ we have $R_0 \to 1$,$R_1 \to 0$ and $|\psi_s(r)| \to 0$(no s-wave sate) The Abrikosov parameter β_A reduces to the following expression:

$$\beta_A = \frac{L_x^2}{4R_0^2}\sum_k{}'[\Theta_{++} + \Theta_{--}]^2 \tag{2.2.23}$$

using the relation $< |\psi_\alpha|^2 >= a_k\theta_{\alpha\alpha}(k)$,$L_x = \frac{2\ell^2}{q}$, $L_y = \frac{2\Pi}{q}$ and $R = \frac{L_x}{L_y} = (\frac{\ell^2}{\Pi})q$ we find :

$$\beta_A = \sum_k{}'[exp - \frac{\pi R}{4}(\frac{k_1^2}{R^2} + k_2^2)]^2 \tag{2.2.24}$$

for square lattice of $R = 1$

$$\beta_A = 1.08643$$

for triangular lattice $R = \sqrt{3}$

$$\beta_A = 1.25881$$

β_A is smaller for R=1 and is relatively large for triangular ($R = \sqrt{3}$) which shows square lattice is energetically favorable.

Chapter 3

Localization of vortices by correlated disorder:columnar defects

Using the frame work of strong linear correlated disorder we study the localization of vortices by columnar defects[20]. Which is described by Bose-glass phase. Nelson et al [3] presented there work on localization of vortices following the work of Mkrtchyan et al. However there work presented a theory valid only when the correlated disorder is much larger or much smaller than the vortex core. The theory is in adequate to explain localization by columnar defects ,which have defect radius nearly equal to vortex core radius. Using the method mentioned we study localization by columnar defect. In this paper we have mapping of 2D bosons to the concept of vortex state superconductivity. In this view the pinning energy U_r correspond to the potential minimum for the boson U_o and elastic energy is related to the inertia of the vortex or equivalently to the mass m of the bosons[7]. The width of the trapping potential a in the boson system is transferred to the extension of the pinning potential $b_o \equiv max(r_d, \sqrt{2}\xi)$($r_d$ is linear defect radius and $\sqrt{2}\xi$ the vortex core radius). The boson vortex mapping concerns both the free energy of both systems and the elementary excitation from one defect

(potential minimum) to another[20]. As there is no essential difference between natural linear defects and columnar tracks,as resulting from heavy-ion irradiation. We will use the same theory to describe pinning by columnar defects. An essential parameter which describe the ratio $y = \frac{r_d}{\sqrt{2}\xi}$ between the radius of a columnar defect and the vortex core. In this study both quantity are comparable in size. Irradiation of single crystals or films with heavy ions creates columnar tracks,where the crystal structure is heavily distorted ,there by causing a suppression of the superconducting order parameter[8]. In this section we describe the essential features of Bose glass theory (Nelson et al and Blater et al) to study vortex localization using columnar defect,which are defects that have equivalently the same radius as the vortex core.

3.1 pinning potential and critical current

Before calculating pinning potential and critical current for a vortex line in the presence of columnar defects. We like to start by stating the pinning potential in the limit of very large $y >> 1$,and very small ,defects ,$y << 1$,the pinning potential per unit length yields .[3,7] where y is given by the ratio$\frac{r_d}{\sqrt{2}\xi}$

$$U_r(R) = \frac{\varepsilon_o}{2} ln(1 - \frac{r_d^2}{R^2}) \qquad \text{for} \qquad y << 1 \tag{3.1.1}$$

$$U_r(R) = -\frac{\varepsilon_o}{2} \frac{r_d^2}{R^2 + 2\xi^2} \qquad \text{for} \qquad y >> 1 \tag{3.1.2}$$

Where R is the distance of vortex from the center of pinning site. Since the pinning potential diverges in equation(3.1.1) at $R = r_d$ a cutoff of the pinning potential is used for small R inside the defects.

$$U_r(R < r_d) = -\varepsilon_o ln(\frac{r_d}{\sqrt{2}\xi}) \tag{3.1.3}$$

The critical current j_c is evaluated by equating the Lorentz force per unit length$f_L = j_c\Phi_o$ and the maximum pinning force $-\frac{dU_r}{dR}|_{max}$ for large defects the pinning force on a vortex at $R = r_d + \sqrt{2}\xi$ is used to determine j_c.Giving $f_{pin} = \frac{U_r}{\sqrt{2}}$[7]. And the critical current is given by

$$j_c = \frac{3\sqrt{3}}{4\sqrt{2}} j_o \simeq j_o \qquad for \qquad y >> 1 \tag{3.1.4}$$

$$j_c = \frac{27\sqrt{2}}{64}(\frac{r_d}{2\xi(T)})^2 j_o \qquad for \qquad y << 1 \tag{3.1.5}$$

3.2 The pinning potential U_r around the vortex core $(y \simeq 1)$

As we have already mentioned the derivation of Bose glass theory for the pinning potential. It does not work when the vortex radius is nearly equal to that of the defect radius. Both equations of the Bose glass theory diverge strongly in the limit $y \to 1$. Even the propose pinning potential $U_r(R)$ vanishes for $y << 1$ which is physically meaning less. In order to make a quantitative comparison between theory and experiment,a more general derivation for the pinning potential is thus required. There is always a crossover in behavior from $Y > 1$to$Y < 1$,namely at $T = T_{r\xi}$.There fore we need to have general expression valid at $Y \approx 1$. In order to find the pinning potential for all defect sizes. We need to evaluate how much the order parameter is reduced at the position of defect. For both vortex and defect are comparable in size ,the superconducting order parameter $|\psi(R)|^2$ varies considerably inside the defect. We can find the reduction of order parameter $(1 - |\psi(R)|^2)$ over the defect. For a

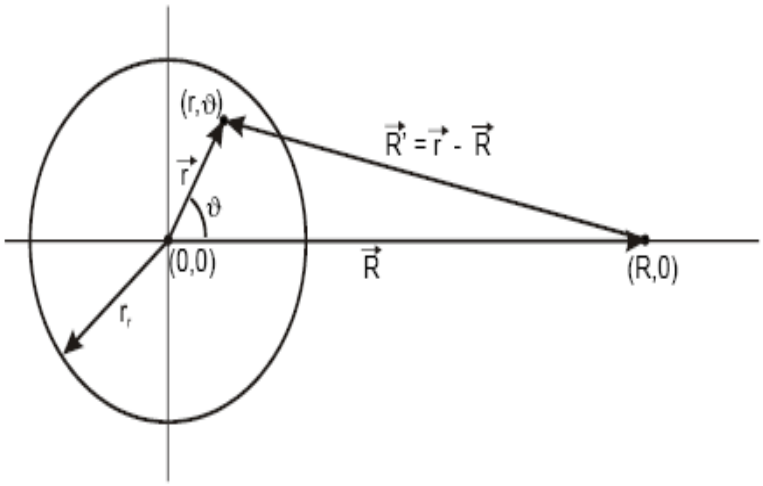


Figure 3.1: Geometrical situation for calculating the vortex pinning potential $U_r(R)$.The linear defect with radius r_d is at the origin of the coordinate system.The center of the vortex is at position (R,0).To obtain the pinning potential,we integrate the reduction of the order parameter at (r,θ) over the full defect,i.e.from r=0$\rightarrow$ r_d and $\theta = 0 \rightarrow 2\pi$

vortex having its center at a position R from the center of the defect, situated in the origin of the coordinate system, as shown in the figure 3.1.

The pinning potential is given by:

$$U_r = -\frac{1}{2}\mu_o H_o^2 \int_0^{r_d} \int_0^{2\pi} [1 - |\psi(R')|^2] r dr d\theta \tag{3.2.1}$$

we integrate over θ from 0 to 2π and defect radius 0 to r_d with $R' = |\vec{r} - \vec{R}| = r^2 + R^2 - 2rRcos\theta$,R is distance from the center of the vortex to the center of defect,and $R\prime$ is the distance from the vortex to the point (r, θ) and $\psi(|\vec{r} - \vec{R}|)$ the size of the superconducting order parameter at a point (r, θ).taking vortex core $\psi(R') =$

$f(R')exp(i\varphi)$,with $f(R') = \frac{R'}{(R'^2+2\xi^2)^{\frac{1}{2}}}$ one can get:

$$U_r(R) = -\frac{\varepsilon_o}{2\pi}\int_0^{r_d}\int_0^{2\pi}\frac{rdrd\theta}{r^2+R^2-2rRcos\theta+2\xi^2} \tag{3.2.2}$$

integration over θ yields:

$$U_r(R) = -\varepsilon_o\int_0^{r_d}\frac{rdr}{\sqrt{(r^2+R^2-2rR+2\xi^2)(r^2+R^2-2rR+2\xi^2)}} \tag{3.2.3}$$

we can simplify and write as:

$$U_r(R) = -\varepsilon_o\int_0^{r_d}\frac{rdr}{\sqrt{(r^2-R^2+2\xi^2)^2+8R^2\xi^2}} \tag{3.2.4}$$

after integration one can get:

$$U_r(R) = -\frac{\varepsilon_o}{2}[\mathbf{arcsinh}(\frac{r_d^2-R^2+2\xi^2}{2\sqrt{2}\xi R}) - \mathbf{arcsinh}(\frac{2\xi^2-R^2}{2\sqrt{2}\xi R})] \tag{3.2.5}$$

The depth of pinning potential can be found by putting $R = 0$ in the expression U_r in equation 3.2.2 and is given by:

$$U_r(0) = -\frac{\varepsilon_0}{2}ln[1+\frac{r_d^2}{2\xi^2}] \tag{3.2.6}$$

It have been proposed that the maximum pinning force is ,when $R = r_d$. physically this seems reasonable since the vortex is exactly on the edge of being dragged into the defect(pinning)and flying off from the defect(depinning). Therefore using this reason one can find the maximum pinning force which can be used to calculate critical current by derivation of pinning potential equation 3.2.5 with respect to R and evaluate for $R = r_d$ the result is:

$$f_{pin} \approx -\frac{dU_r(R)}{dR}|_{R=r_d} = \frac{\varepsilon_o}{2r_d}(\frac{\xi^2+r_d^2}{\xi\sqrt{2r_d^2+\xi^2}} - 1) \tag{3.2.7}$$

before we calculate the critical density ($j_c(T)$) we have to consider the influence of temperature on the critical current density. There are two sources of temperature

effect on $j_c(T)$,the first source is due to the Ginzburg landau parameters $\lambda(T)$ and $\xi(T)$. The second effect is directly connected to the vortex movements:it is the effect of thermal fluctuations.We consider the temperature dependence of the Ginzburg-Landau parameter $\lambda(T)$ and $\xi(T)$ as it directly affect the critical current.The critical current can be found by equating,the maximum pinning force equation 3.2.7 with that of Lornez force($f_L = j_c\Phi_o$) which is given by :

$$\begin{aligned} j_c &\approx \frac{\varepsilon_o}{2r_d\Phi_o\xi}\left(\frac{\xi^2+r_d^2}{\sqrt{2r_d^2+\xi^2}}-\xi\right) \\ &= 3\frac{\sqrt{3}}{8}\left(\frac{\xi^2+r_d^2}{r_d\sqrt{2r_d^2+\xi^2}}-\frac{\xi}{r_d}\right)j_o \end{aligned} \tag{3.2.8}$$

where depairing current(j_o)is given by :

$$j_o = \frac{4c\varepsilon_o}{3\sqrt{3}\xi\Phi_o}$$

Intrinsic temperature dependence of critical current is given in terms of temperature dependence of penetration depth $\lambda(T)$ and coherence length $\xi(T)$ which is given by $\lambda(T) = \frac{\lambda(0)}{\sqrt{1-t^4}}$, $\xi(T) = \xi(0)\sqrt{\frac{1+t^2}{1-t^2}}$ and $j_o = \frac{1}{\lambda^2(T)\xi(T)}$ where t is $\frac{T}{T_c}$

using these intrinsic temperature dependence the critical current for defect radius ,$r_d = 3\xi(0)$ becomes:

$$j_c = \frac{\sqrt{3}}{8}\left[\frac{9+\frac{1+t^2}{1-t^2}}{\sqrt{18+\frac{1+t^2}{1-t^2}}}-\sqrt{\frac{1+t^2}{1-t^2}}\right](1-t^4)\sqrt{\frac{1-t^2}{1+t^2}} \tag{3.2.9}$$

We investigate the influence of thermal fluctuations on pinning potential by the expansion of equation 3.2.5 in Taylor serious which is similar to the calculation by Blatter etal[7]. Except we are using here Taylor expansion given by :

$$U_r = U_{R=0} + \frac{\varepsilon_o}{2}\frac{r_d^2}{(r_d^2+2\xi^2)^2}R^2 + \frac{\varepsilon_o}{2}\frac{r_d^2(r_d^2-4\xi^2)}{2(r_d^2+2\xi^2)^4}R^4 + O(R^6) \tag{3.2.10}$$

To lowest order approximation we can write the expansion as:

$$\begin{aligned} U_r(R) &\approx U_r(R=0) + \frac{\varepsilon_o}{2}\frac{r_d^2}{(r_d^2+2\xi^2)^2}R^2 \\ &= U_r(0) + \frac{1}{2}kR^2 \end{aligned} \tag{3.2.11}$$

where k is given by $k = \frac{\varepsilon_o r_d^2}{(r_d^2+2\xi^2)^2}$

The first term of the above equation gives the depth of the potential and the second term is a harmonic oscillator potential. The binding energy of a particle trapped in this harmonic oscillator potential in two dimensions is given by[7]:

$$E_B = U_o + \hbar\omega \tag{3.2.12}$$

with $\omega = \sqrt{\frac{K}{m}}$

To translate the above equation for a vortex picture according to the usual mapping $\hbar \rightarrow T, m \rightarrow \varepsilon_l, U_o \rightarrow \varepsilon_r, T_{dp}^r = b_o\sqrt{\varepsilon_l\varepsilon_r}$,and the pinning potential at low temperature is given by equation $U_r(0) = -\frac{\varepsilon_0}{2}ln[1+\frac{r_d^2}{2\xi^2}]$ and equation 3.2.12 become

$$U_r = U_r(T=0)(1-\eta\frac{T}{T_{dp}^r}), \qquad with \qquad \eta = \frac{\sqrt{2}y^2}{\sqrt{ln(1+y^2)(1+y^2)}} \tag{3.2.13}$$

Both intrinsic and thermal fluctuation influence on pinning potential is given by

$$U_r'(R,T) \approx U_r'(R=0,T) + \frac{\varepsilon_o}{2}\frac{r_d^2}{(r_d^2+2\xi^2)^2}R^2 \tag{3.2.14}$$

which can be written as

$$U_r(R,T) \sim U_r'(R,T)(1-\eta\frac{K_BT}{T_{dp}^r}) \tag{3.2.15}$$

which shows linear temperature dependence of pinning potential.

3.3 localization

Among the different phase transition of vortex state superconductor one of particular interest is the transition at T_{BG} from the Bose glass to the entangled flux-liquid or "superfluid" phase, which is identified with the experimentally observed "Irreversibility line". Although, an exact theory of this transition is not available, its critical behavior can be parameterized in terms of a scaling theory with just two undetermined critical exponents as in the vortex glass phenomenology[3]. The root-mean squared transverse distance to the field direction is expected to diverge as $T \to T_{BG}$ from below

$$\ell_{\perp}(T) \sim \frac{1}{(T - T_{BG})^{x}} \tag{3.3.1}$$

where x is the critical exponent

The time scale τ for relaxation of a fluctuation with this size is assumed to diverge with a new critical exponent η.

$$\tau \sim \ell_{\perp}^{\eta}$$

Now

$$\tau \sim [\frac{1}{(T - T_{BG})^{x}}]^{\eta} \sim \frac{1}{(T - T_{BG})^{x\eta}}$$

using free vortex /particle concept we write conductivity as

$$\sigma = \frac{ne^2\tau}{m}$$

m here scales as $\tilde{\epsilon}_1$ (energy per unit length square)This implies

$$\sigma \sim \frac{\tau}{\ell_{\perp}^2} = (T - T_{BG})^{2x} \frac{1}{(T - T_{BG})^{x\eta}}$$

where ne^2 is constant and we don't explicitly require here.

$$\sigma \sim \frac{1}{(T - T_{BG})^{x\eta - 2x}} \sim \frac{1}{(T - T_{BG})^{x(\eta - 2)}}$$

Resistivity is given by $\frac{1}{\sigma}$

$$R \sim (T - T_{BG})^{x(\eta-2)}$$

resistivity when there is no pinning that is $T_{BG} \to 0$ given by:

$$R_o \sim T^{x(\eta-2)}$$

The ratio of the two resistivity becomes

$$\frac{R}{R_o} \sim [1 - \frac{T_{BG}}{T}]^{x(\eta-2)}$$

Where T_{BG} is Bose-glass temperature.

Chapter 4

Results and discussion

We have studied the structure of Abrikosov flux lattice using the frame work of the two component Ginzburg-Landau free energy for s and d paring.we drive the Gibbs-free energy for Abrikosov vortex lattice by minimizing the free energy and obtain a result independent of particular normalization ψ_s and ψ_d given by:

$$< g > - < g >_n = -\frac{1}{8\pi}\frac{(H_{C2} - H)^2}{(2k^2 - 1)\beta_A}$$

which is similar to the Abrikosov first derived.As we can see from the equation the minimum of β_A determines minimum of Gibbs- free energy.For pure d-wave superconductor in the case($\epsilon_v = 0$)that means the mixing condition has been omitted ,Since we are dealing with high temperature superconductors which are predominantly d-wave.We calculate Abrikosov parameter (β_A) by evaluating spatial averages$< h_s^2 >$ and $< h_s >$obtain a result for Abrikosov square ($R = 1$)and triangular lattice($R = \sqrt{3}$) $\beta_A = 1.08643$ and $\beta_A = 1.2588889$ respectively.which shows square lattice is more energetically favorable having relatively low energy.

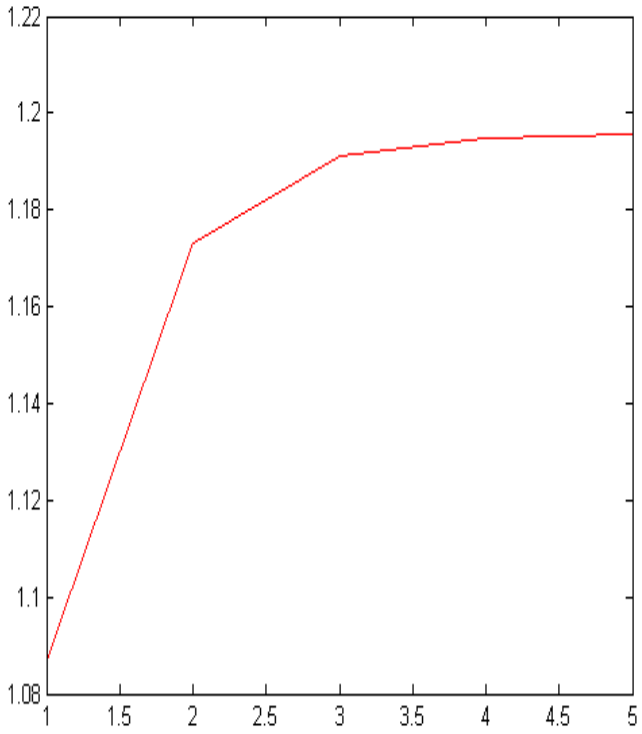


Figure 4.1: Graph of β_A(vertical line) vs unit cell ratio(R)(horizontal line)

We also study localization of vortices,using scaling theory.What we try to show is localization of vortices by columnar defects in terms of investigating how resistivity decrease around Bose- glass phase.The phase diagram of vortex states in the plane of density of columnar defects Vs temperature is obtained by large -scale MonteCaralo simulations by Y.Nonomura and X.HU [12].They found that coexistence of two phases between T=0.368$\frac{J}{k_B}$ and 0.374 $\frac{J}{k_B}$ .They are also evaluate the BG transition temperature T_{BG},by measuring the tilt modulus c_{44} ,and found out that flux lines in glass phases look frozen but those in liquid phase are already moving around in their time scale at T=0.368$\frac{J}{k_B}$.Using the scaling theory we try to find a relation of Bose-Glass temperature with resistivity. Recent simulations of a simplified model show that the value of x is equal to 1,and $\eta = 6.5$[3]. Using these values we plot resistivity vs temperature as shown in the fig.4.1.

From the Graph we can see that when Bose-Glass temperature becomes zero resistivity becomes maximum ,which is non-localized state that,there is no enough pinning to prevent vortex motion which results in dissipation.Where as temperature goes to the Bose-glass temperature (T_{BG})resistivity decreases that shows the localization of vortices.

we study pinning by columnar tracks,formed by irradiating single crystals with heavy ions. The irradiation damage forms columnar tracks of typically $30A^o - 50A^o$[20]. In our work we try to calculate pinning potential and critical current density for HTSC. For defect radius approximately equal to the radius vortex core. Considering intrinsic temperature dependence through the parameters $\lambda(T)$ and $\xi(T)$[1] and thermal fluctuation including theoretical assumption made by Bose-theory for large ($Y \gg 1$) and small defect($Y \ll 1$)[3]. We also calculate pinning potential and critical

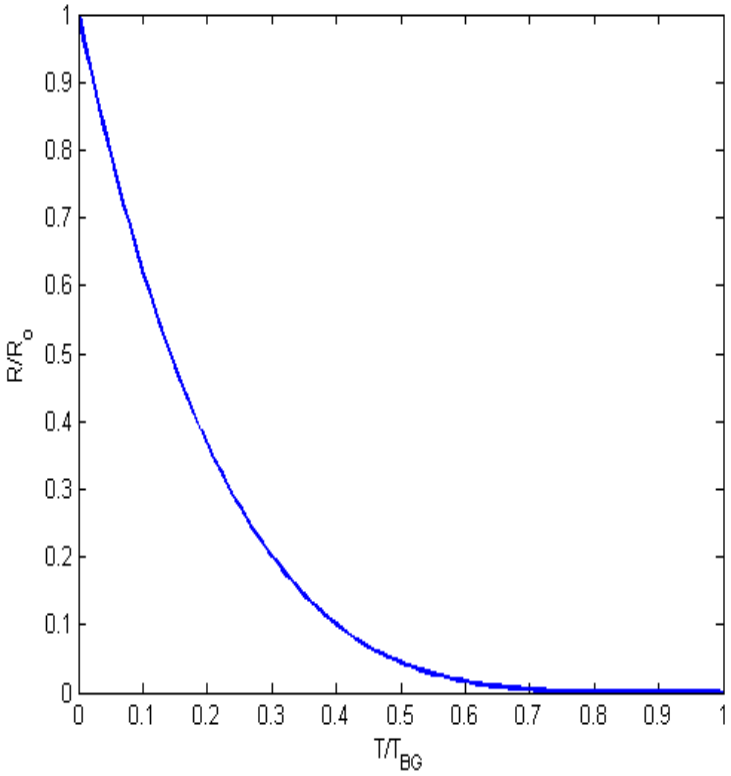


Figure 4.2: Graph of resistivity vs temperature

current for typical radius of columnar tracks $35A^o$ for YBa_2CUO_7 irradiated with 580 Mev [8]. Theoretically we take defect radius $r_d \simeq 3\xi(0)$ where $\xi(0) = 12A^o$ coherence length for YBa_2CUO_7(YBCO) at T=0[7].At low magnetic field and for $H \parallel c$ we use the approximation $3\xi(0) \approx 35A^o$ and plot both the effect of intrinsic temperature dependence and thermal fluctuation on critical current density for compound YBCO. And from the graph we can see that the critical current for heavy ion irradiated YBCO with columnar radius(r_d)equal to $35A^o$ is found between the theoretical limits for large and small defect and it decreases when the temperature approaches the critical temperature,T_c for YBCO. We also plot for thermal fluctuation dependence of critical current density on temperature which shows that thermal fluctuations have strong influence on the pinning potential and there by decrease the critical current density as shown in the figure 4.2 and 4.3 The results are in broad agreement with experiments.

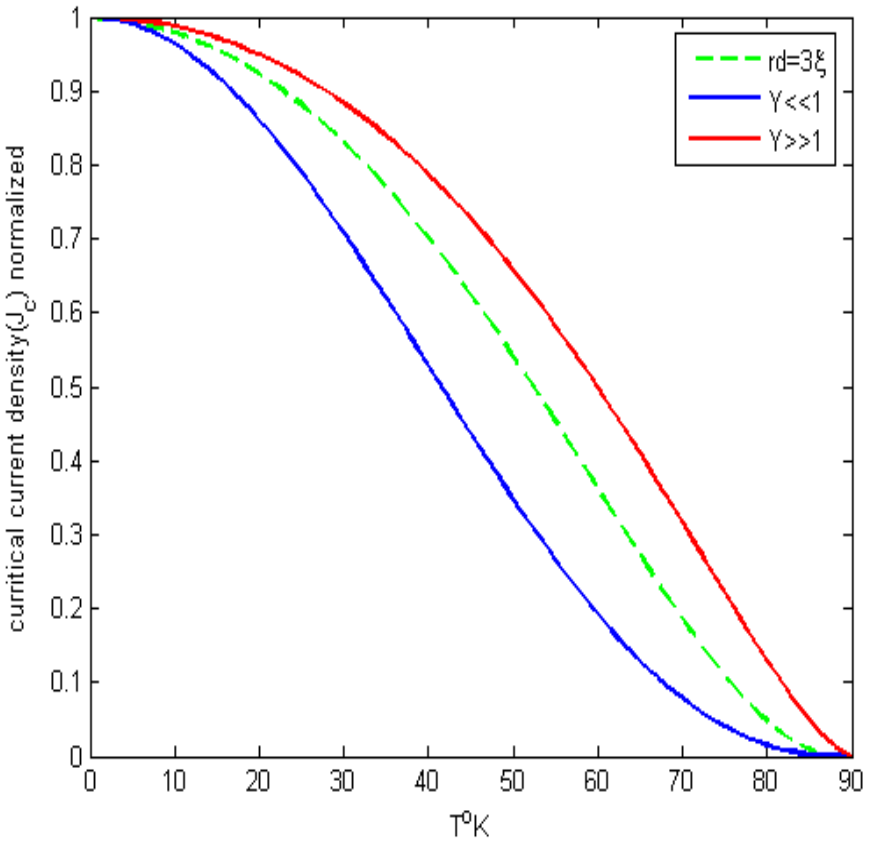


Figure 4.3: Graph of critical current density vs temperature.For small($y \ll 1$)(blue) and large($y \gg 1$) defect[red] and for defect radius is equal to $35A^o$(heavy ion irradiated YBCO)[dashed green line]

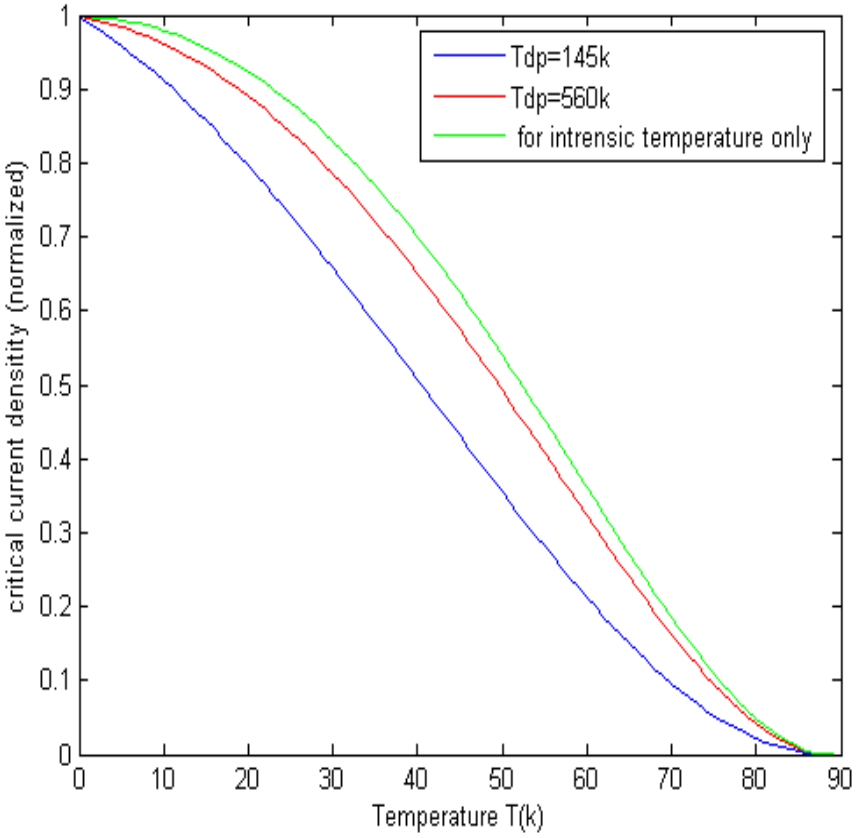


Figure 4.4: Graph of critical current density vs temperature. Which shows thermal fluctuation effect,that are indicated in the lower curves for $T^r_{dp} = 145K$(blue) $T^r_{dp} = 560K$(red) where T^r_{dp} the deppnining energy.Together with for intrinsic temperature dependence

Chapter 5

conclusion

In the present work we study structure and localization of a vortex lines. From our work we can conclude that square lattice configuration still have lowest free energy. Which is in agrement with recent theoretical work by kogan et al(1997),Maki(1991) by working on Ginzburg-Landau treatments ,combined with Fermi-surface anisotropy. Riseman et al(1998),kealey et al,(2000) observed a square flux lattice by small-angle neutron scattering ,A square lattices was also deduced from studies of muon spin rotation(Aegerter et al.(1998),Luke et al(2000))[24].

Using the method of linear defect mechanism we drive critical current and pinning for heavy ion irradiated YBCO,and conclude that critical current density is strongly affected by thermal fluctuation which will be aggravated in HTSC due to the presence of relatively high thermal energy. Finally using scaling theory we see temperature dependence of resistivity and has shown that vortex motion is prohibited around Bose-glass temperature. From this we can conclude that vortex lines are localized at this temperature.

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Declaration

I hereby declare that this thesis is my original work and has not been presented for a degree in any other university. All sources of material used for the thesis have been duly acknowledged.

Name: Ermias Libnedengel

Signature: .....

This thesis has been submitted for the examination with my approval as university advisor.

Name: Prof. P. Singh

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