



**GENERATION OF SQUEEZED LIGHT BY
OPTICALLY PUMPED TWO-LEVEL ATOM IN A
CAVITY CONTAINING PARAMETRIC AMPLIFIER
AND COUPLED TO SQUEEZED VACUUM
RESERVOIR**

By

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Abstract

In this thesis, we have analyzed the quantum properties of light generated by coherently driven two level atom in a cavity containing a parametric amplifier and coupled to a squeezed vacuum reservoir. The equations of evolution for the expectation values of cavity mode and atomic operators are determined with the aid of the system's master equation. Applying the large time approximation to the cavity mode quantum Langevin equation, we obtain the steady-state solutions for the expectation values of cavity mode and atomic operators. Utilizing these solutions, we calculate the mean photon number, the power spectrum, second-order correlation function, quadrature variance and squeezing of the cavity mode.

Our results indicate that the two-level atom has larger probability of being found in the lower level than being found in the upper level and the photons of the fluorescent light produced by the two-level atom are found to be anti-bunched. Moreover, the cavity mode is observed to be squeezed, with squeezing being occurring in the plus quadrature.

In addition, the squeezing increases as the amplitude of driving coherent light increases; with a maximum squeezing of 60% for ($\epsilon = 0.1$) and 50% in the absence of parametric amplifier. And the degree of squeezing enhances with the increase of stimulated emission decay constant and reaches a maximum value of 70% below the vacuum state level.

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Chapter 1

Introduction

Quantum optics deals with the study of the quantum properties of radiation generated by various optical systems and its interaction with matter. The interaction of radiation with the atom involves absorption and emission of radiation by the atom depending on the initial energy level of the atom. The quantum properties of light are determined by the quantum states of light such as the squeezed state, the coherent state, chaotic state and the number state[1]. When the uncertainty in one of the quadrature is below the vacuum state level at the expense of enhanced fluctuations in the other quadrature, with the product of the uncertainties in the two quadratures satisfying the uncertainty relation, the light is said to be in a squeezed state[1, 2].

Two level lasers are the source of coherent or chaotic light that is emitted by two level atom in a cavity coupled to a reservoir via a port mirror [1, 2, 3]. The effect of the squeezed light on the properties of the fluorescent light emitted by the two-level atom has attracted a great deal of interest[1-4]. The squeezing properties of the fluorescent light emitted by a two-level atom has attracted a great deal of interest. It has been recently reported that a two-level atom interacting with coherently driven cavity mode generates squeezed light with maximum squeezing of 50% below the vacuum state level[5]. In a parametric amplifier, a pump photon interacts with the nonlinear crystal inside a cavity and is down converted in to two correlated photons[1-9]. In a degenerate Subharmonic generator a pump photon of frequency ω is down converted into highly correlated signal and idler photons which have the same frequency ω_a . Moreover, during non-degenerate Subharmonic generator a pump photon of frequency ω is down converted into highly correlated signal and idler photons which have different frequencies ω_a and ω_b . In this paper, we analyze the quantum properties of the cavity light produced by a two-level atom driven by coherent light and in a cavity containing parametric amplifier

and coupled to squeezed vacuum reservoir.

The equations of evolution for the expectation values of cavity mode and atomic operators are determined with the aid of the system's master equation. Applying the large time approximation to the cavity mode quantum Langevin equation, we obtain the steady-state solutions for the expectation values of cavity mode and atomic operators. Utilizing these solutions, we calculate the mean photon number, the power spectrum, second-order correlation function, quadrature variance and squeezing of the cavity mode.

Chapter 2

Operator Dynamics

We consider a two level atom driven by a coherent light and in a cavity containing degenerate subharmonic generating system and coupled to a squeezed vacuum reservoir. We represent the upper and lower levels of the atom by $|a\rangle$ and $|b\rangle$, respectively. The upper and lower levels of the atom are coupled by driving coherent light.

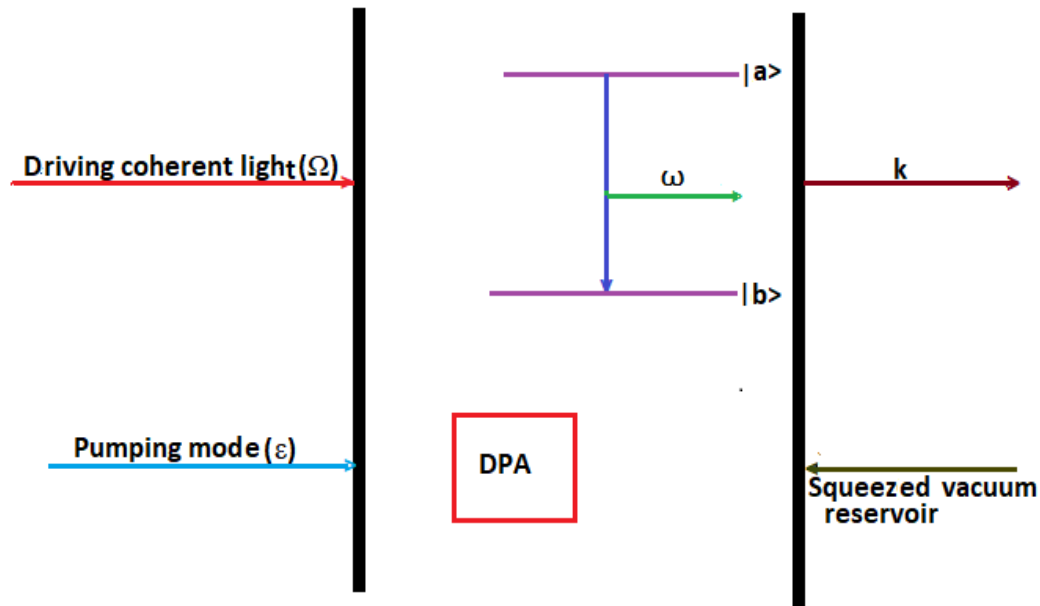


Figure 2.1: Schematic representation of a two-level atom in a cavity containing a parametric amplifier, a driving coherent light and a squeezed vacuum reservoir.

2.1 Equation of evolution for the expectation values of cavity mode and atomic operators

The Hamiltonian describing the coupling of the top and bottom levels of the atom by the coherent light can be expressed as [4-9]

$$\hat{H}_1 = i\lambda(\hat{\sigma}_a^\dagger \hat{b} - \hat{b}^\dagger \hat{\sigma}_a), \quad (2.1)$$

where \hat{b} is annihilation operator for the deriving coherent light, λ is coupling constant between the two level atom and the coherent light, $\hat{\sigma}_a = |b\rangle\langle a|$ is lowering atomic operator. Treating the deriving coherent light classically, we can replace \hat{b} by a c-number μ_1 and re-write the above Hamiltonian as

$$\hat{H}_1 = i\frac{\Omega}{2}(\hat{\sigma}_a^\dagger - \hat{\sigma}_a), \quad (2.2)$$

where $\Omega = 2\mu_1\lambda$ is a real constant proportional to the amplitude of the deriving coherent light. Moreover, the interaction of a two level atom with a cavity mode is described by the Hamiltonian [10]

$$\hat{H}_2 = ig(\hat{\sigma}_a^\dagger \hat{a} - \hat{a}^\dagger \hat{\sigma}_a), \quad (2.3)$$

where \hat{a} is annihilation operator for the cavity mode and g is the atom-cavity mode coupling constant. The process of parametric interaction with the pump mode is described by

$$\hat{H}_3 = i\frac{\beta}{2}(\hat{a}^2 \hat{c}^\dagger - \hat{c} \hat{a}^{\dagger 2}), \quad (2.4)$$

where β is the coupling constant, \hat{c} is annihilation operator for the pump mode and \hat{a} is annihilation operator for the cavity mode. For the pump mode treated classically, \hat{c} can be replaced by c-number μ_2 and the Hamiltonian in Eq. (2.4) can be put as

$$\hat{H}_3 = i\frac{\varepsilon}{2}(\hat{a}^2 - \hat{a}^{\dagger 2}), \quad (2.5)$$

where $\varepsilon = \beta\mu_2$ is a positive and real constant proportional to the amplitude of the pump mode. Now the total Hamiltonian operator for the parametric interaction and the interaction of a two-level atom with the coupling coherent light and cavity mode can be expressed as

$$\hat{H} = ig(\hat{\sigma}_a^\dagger \hat{a} - \hat{a}^\dagger \hat{\sigma}_a) + i\frac{\Omega}{2}(\hat{\sigma}_a^\dagger - \hat{\sigma}_a) + i\frac{\varepsilon}{2}(\hat{a}^2 - \hat{a}^{\dagger 2}). \quad (2.6)$$

The master equation for the light produced by the quantum optical system under consideration can be written as [1]

$$\begin{aligned} \frac{d\hat{\rho}}{dt} = & -i[\hat{H}, \hat{\rho}] + \frac{\kappa}{2}(N+1)(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}) + \frac{\kappa}{2}N(2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger) \\ & - \frac{\kappa}{2}M(2\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger - \hat{a}^{\dagger 2}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger 2}) - \frac{\kappa}{2}M(2\hat{a}\hat{\rho}\hat{a} - \hat{a}^2\hat{\rho} - \hat{\rho}\hat{a}^2), \end{aligned} \quad (2.7)$$

where κ is the cavity damping constant and $\hat{\rho}$ is the density operator. The effects of the reservoir are incorporated via the parameters N and M as

$$\begin{aligned} N &= \sinh^2 r \\ &= \frac{e^{2r} + e^{-2r} - 2}{4}, \end{aligned} \quad (2.8)$$

and

$$\begin{aligned} M &= \sinh r \cosh r \\ &= \frac{e^{2r} - e^{-2r}}{4}, \end{aligned} \quad (2.9)$$

with r being the squeeze parameter. The time evolution of the expectation value of an operator in the Schrödinger picture can be written as [11]

$$\frac{d\langle \hat{A}(t) \rangle}{dt} = Tr\left(\frac{d\hat{\rho}(t)}{dt} \hat{A}\right). \quad (2.10)$$

Now taking into account Eq. (2.7) and Eq. (2.10), the time evolution of $\langle \hat{a}(t) \rangle$ can be written as [1]

$$\begin{aligned} \frac{d}{dt}\langle \hat{a}(t) \rangle &= Tr\left(\frac{d\hat{\rho}}{dt} \hat{a}\right) \\ &= -iTr([\hat{H}, \hat{\rho}]\hat{a}) \\ &+ \frac{\kappa}{2}(N+1)Tr(2\hat{a}\hat{\rho}\hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^\dagger\hat{a}^2) \\ &+ \frac{\kappa}{2}NTr(2\hat{a}^\dagger\hat{\rho}\hat{a}^2 - \hat{a}\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{\rho}\hat{a}\hat{a}^\dagger\hat{a}) \\ &- \frac{\kappa}{2}MTr(2\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger\hat{a} - \hat{a}^{\dagger 2}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^{\dagger 2}\hat{a}) \\ &- \frac{\kappa}{2}MTr(2\hat{a}\hat{\rho}\hat{a}^2 - \hat{a}^2\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^3) \\ &= -iT_1 + \frac{\kappa}{2}(N+1)T_2 + \frac{\kappa}{2}NT_3 - \frac{\kappa}{2}MT_4 - \frac{\kappa}{2}MT_5, \end{aligned} \quad (2.11)$$

where

$$T_1 = Tr([\hat{H}, \hat{\rho}]\hat{a}), \quad (2.12)$$

$$T_2 = Tr(2\hat{a}\hat{\rho}\hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^\dagger\hat{a}^2), \quad (2.13)$$

$$T_3 = Tr(2\hat{a}^\dagger\hat{\rho}\hat{a}^2 - \hat{a}\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{\rho}\hat{a}\hat{a}^\dagger\hat{a}), \quad (2.14)$$

$$T_4 = Tr(2\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger\hat{a} - \hat{a}^{\dagger 2}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^{\dagger 2}\hat{a}), \quad (2.15)$$

$$T_5 = Tr(2\hat{a}\hat{\rho}\hat{a}^2 - \hat{a}^2\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^3). \quad (2.16)$$

Now by applying cyclic property of trace operation, we can calculate the traces in Eqs. (2.12)-(2.16) as

$$\begin{aligned} T_1 &= Tr([\hat{H}, \hat{\rho}]\hat{a}) \\ &= Tr((\hat{H}\hat{\rho} - \hat{\rho}\hat{H})\hat{a}) \\ &= Tr(\hat{H}\hat{\rho}\hat{a} - \hat{\rho}\hat{H}\hat{a}) \\ &= Tr(\hat{\rho}\hat{a}\hat{H} - \hat{\rho}\hat{H}\hat{a}) \\ &= Tr\hat{\rho}(\hat{a}\hat{H} - \hat{H}\hat{a}) \\ &= \langle [\hat{a}, \hat{H}] \rangle, \end{aligned} \quad (2.17)$$

$$\begin{aligned} T_2 &= Tr(2\hat{a}\hat{\rho}\hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^\dagger\hat{a}^2) \\ &= Tr(2\hat{\rho}\hat{a}^\dagger\hat{a}^2 - \hat{\rho}\hat{a}^\dagger\hat{a}^2 - \hat{\rho}\hat{a}\hat{a}^\dagger\hat{a}) \\ &= Tr(\hat{\rho}\hat{a}^\dagger\hat{a}^2 - \hat{\rho}\hat{a}\hat{a}^\dagger\hat{a}) \\ &= Tr\hat{\rho}(\hat{a}^\dagger\hat{a}^2 - \hat{a}\hat{a}^\dagger\hat{a}) \\ &= Tr\hat{\rho}([\hat{a}^\dagger\hat{a}, \hat{a}]) \\ &= Tr\hat{\rho}(\hat{a}^\dagger[\hat{a}, \hat{a}] + [\hat{a}^\dagger, \hat{a}]\hat{a}) \\ &= Tr(-\hat{\rho}\hat{a}) \\ &= -\langle \hat{a} \rangle, \end{aligned} \quad (2.18)$$

$$\begin{aligned}
T_3 &= Tr(2\hat{a}^\dagger \hat{\rho} \hat{a} \hat{a} - \hat{a} \hat{a}^\dagger \hat{\rho} \hat{a} - \hat{\rho} \hat{a} \hat{a}^\dagger \hat{a}) \\
&= Tr(2\hat{\rho} \hat{a}^2 \hat{a}^\dagger - \hat{\rho} \hat{a}^2 \hat{a}^\dagger - \hat{\rho} \hat{a} \hat{a}^\dagger \hat{a}) \\
&= Tr(\hat{\rho} \hat{a}^2 \hat{a}^\dagger - \hat{\rho} \hat{a} \hat{a}^\dagger \hat{a}) \\
&= Tr \hat{\rho} (\hat{a} \hat{a} \hat{a}^\dagger - \hat{a} \hat{a}^\dagger \hat{a}) \\
&= Tr \hat{\rho} ([\hat{a}, \hat{a} \hat{a}^\dagger]) \\
&= Tr \hat{\rho} (\hat{a} [\hat{a}, \hat{a}^\dagger] + [\hat{a}, \hat{a}] \hat{a}^\dagger) \\
&= Tr(\hat{\rho} \hat{a}) \\
&= \langle \hat{a} \rangle,
\end{aligned} \tag{2.19}$$

$$\begin{aligned}
T_4 &= Tr(2\hat{a}^\dagger \hat{\rho} \hat{a}^\dagger \hat{a} - \hat{a}^{\dagger 2} \hat{\rho} \hat{a} - \hat{\rho} \hat{a}^{\dagger 2} \hat{a}) \\
&= Tr(2\hat{\rho} \hat{a}^\dagger \hat{a} \hat{a}^\dagger - \hat{\rho} \hat{a} \hat{a}^\dagger \hat{a}^\dagger - \hat{\rho} \hat{a}^\dagger \hat{a}^\dagger \hat{a}) \\
&= Tr(2\hat{\rho} \hat{a}^\dagger \hat{a} \hat{a} \hat{a}^\dagger - \hat{\rho} (\hat{a}^\dagger \hat{a} + 1) \hat{a}^\dagger - \hat{\rho} \hat{a}^\dagger (\hat{a} \hat{a}^\dagger - 1)) \\
&= Tr(2\hat{\rho} \hat{a}^\dagger \hat{a} \hat{a}^\dagger - 2\hat{\rho} \hat{a}^\dagger \hat{a} \hat{a}^\dagger + \hat{\rho} \hat{a}^\dagger - \hat{\rho} \hat{a}^\dagger) \\
&= 0,
\end{aligned} \tag{2.20}$$

$$\begin{aligned}
T_5 &= Tr(2\hat{a} \hat{\rho} \hat{a} \hat{a} - \hat{a} \hat{a} \hat{\rho} \hat{a} - \hat{\rho} \hat{a} \hat{a} \hat{a}) \\
&= Tr(2\hat{\rho} \hat{a} \hat{a} \hat{a} - \hat{\rho} \hat{a} \hat{a} \hat{a} - \hat{\rho} \hat{a} \hat{a} \hat{a}) \\
&= Tr(2\hat{\rho} \hat{a} \hat{a} \hat{a} - 2\hat{\rho} \hat{a} \hat{a} \hat{a}) \\
&= 0.
\end{aligned} \tag{2.21}$$

Substituting Eqs. (2.17)-(2.21) into Eq. (2.11), we obtain

$$\frac{d}{dt} \langle \hat{a} \rangle = -i \langle [\hat{a}, \hat{H}] \rangle - \frac{\kappa}{2} \langle \hat{a}(t) \rangle. \tag{2.22}$$

Following the same procedure used to arrive at Eq. (2.22), we easily obtain

$$\frac{d}{dt} \langle \hat{a}^2 \rangle = -i \langle [\hat{a}^2, \hat{H}] \rangle - \kappa \langle \hat{a}^2 \rangle + \kappa M, \tag{2.23}$$

$$\frac{d}{dt} \langle \hat{a}^\dagger \hat{a} \rangle = -i \langle [\hat{a}^\dagger \hat{a}, \hat{H}] \rangle - \kappa \langle \hat{a}^\dagger \hat{a} \rangle + \kappa N, \tag{2.24}$$

$$\frac{d}{dt} \langle \hat{a} \hat{a}^\dagger \rangle = -\langle [\hat{a} \hat{a}^\dagger, \hat{H}] \rangle - \kappa \langle \hat{a} \hat{a}^\dagger \rangle + \kappa(N + 1). \tag{2.25}$$

Moreover, taking into account Eq. (2.7) and Eq. (2.10), the time evolution of $\langle \hat{\sigma}_a(t) \rangle$ can be written as

$$\begin{aligned}
\frac{d}{dt} \langle \hat{\sigma}_a(t) \rangle &= Tr\left(\frac{d\hat{\rho}(t)}{dt} \hat{\sigma}_a\right) \\
&= -iTr([\hat{H}, \hat{\rho}] \hat{\sigma}_a) \\
&+ \frac{\kappa}{2}(N+1)Tr(2\hat{a}\hat{\rho}\hat{a}^\dagger \hat{\sigma}_a - \hat{a}^\dagger \hat{a} \hat{\rho} \hat{\sigma}_a - \hat{\rho} \hat{a}^\dagger \hat{a} \hat{\sigma}_a) \\
&+ \frac{\kappa}{2}NTr(2\hat{a}^\dagger \hat{\rho} \hat{a} \hat{\sigma}_a - \hat{a} \hat{a}^\dagger \hat{\rho} \hat{\sigma}_a - \hat{\rho} \hat{a} \hat{a}^\dagger \hat{\sigma}_a) \\
&- \frac{\kappa}{2}MTr(2\hat{a}^\dagger \hat{\rho} \hat{a}^\dagger \hat{\sigma}_a - \hat{a}^{\dagger 2} \hat{\rho} \hat{\sigma}_a - \hat{\rho} \hat{a}^{\dagger 2} \hat{\sigma}_a) \\
&- \frac{\kappa}{2}MTr(2\hat{a} \hat{\rho} \hat{a} \hat{\sigma}_a - \hat{a}^2 \hat{\rho} \hat{\sigma}_a - \hat{\rho} \hat{a}^2 \hat{\sigma}_a) \\
&= -iP_1 + \frac{\kappa}{2}(N+1)P_2 + \frac{\kappa}{2}NP_3 - \frac{\kappa}{2}MP_4 - \frac{\kappa}{2}MP_5, \tag{2.26}
\end{aligned}$$

where

$$P_1 = Tr([\hat{H}, \hat{\rho}] \hat{\sigma}_a), \tag{2.27}$$

$$P_2 = Tr(2\hat{a}\hat{\rho}\hat{a}^\dagger \hat{\sigma}_a - \hat{a}^\dagger \hat{a} \hat{\rho} \hat{\sigma}_a - \hat{\rho} \hat{a}^\dagger \hat{a} \hat{\sigma}_a), \tag{2.28}$$

$$P_3 = Tr(2\hat{a}^\dagger \hat{\rho} \hat{a} \hat{\sigma}_a - \hat{a} \hat{a}^\dagger \hat{\rho} \hat{\sigma}_a - \hat{\rho} \hat{a} \hat{a}^\dagger \hat{\sigma}_a), \tag{2.29}$$

$$P_4 = Tr(2\hat{a}^\dagger \hat{\rho} \hat{a}^\dagger \hat{\sigma}_a - \hat{a}^{\dagger 2} \hat{\rho} \hat{\sigma}_a - \hat{\rho} \hat{a}^{\dagger 2} \hat{\sigma}_a), \tag{2.30}$$

$$P_5 = Tr(2\hat{a} \hat{\rho} \hat{a} \hat{\sigma}_a - \hat{a}^2 \hat{\rho} \hat{\sigma}_a - \hat{\rho} \hat{a}^2 \hat{\sigma}_a). \tag{2.31}$$

We wish to evaluate the traces of Eqs. (2.27)-(2.31) as follows:

$$\begin{aligned}
P_1 &= Tr([\hat{H}, \hat{\rho}] \hat{\sigma}_a) \\
&= Tr((\hat{H}\hat{\rho} - \hat{\rho}\hat{H}) \hat{\sigma}_a) \\
&= Tr(\hat{H}\hat{\rho}\hat{\sigma}_a - \hat{\rho}\hat{H}\hat{\sigma}_a) \\
&= Tr(\hat{\rho}\hat{\sigma}_a\hat{H} - \hat{\rho}\hat{H}\hat{\sigma}_a) \\
&= Tr\hat{\rho}(\hat{\sigma}_a\hat{H} - \hat{H}\hat{\sigma}_a) \\
&= \langle [\hat{\sigma}_a, \hat{H}] \rangle, \tag{2.32}
\end{aligned}$$

$$\begin{aligned}
P_2 &= Tr(2\hat{a}\hat{\rho}\hat{a}^\dagger\hat{\sigma}_a - \hat{a}^\dagger\hat{a}\hat{\rho}\hat{\sigma}_a - \hat{\rho}\hat{a}^\dagger\hat{a}\hat{\sigma}_a) \\
&= Tr(2\hat{\rho}\hat{a}^\dagger\hat{\sigma}_a\hat{a} - \hat{\rho}\hat{\sigma}_a\hat{a}^\dagger\hat{a} - \hat{\rho}\hat{a}^\dagger\hat{a}\hat{\sigma}_a) \\
&= Tr(2\hat{\rho}\hat{a}^\dagger\hat{a}\hat{\sigma}_a - \hat{\rho}\hat{\sigma}_a\hat{a}^\dagger\hat{a} - \hat{\rho}\hat{\sigma}_a\hat{a}^\dagger\hat{a}) \\
&= 2Tr(\hat{\rho}[\hat{a}^\dagger\hat{a}, \hat{\sigma}_a]) \\
&= 0,
\end{aligned} \tag{2.33}$$

$$\begin{aligned}
P_3 &= Tr(2\hat{a}^\dagger\hat{\rho}\hat{a}\hat{\sigma}_a - \hat{a}\hat{a}^\dagger\hat{\rho}\hat{\sigma}_a - \hat{\rho}\hat{a}\hat{a}^\dagger\hat{\sigma}_a) \\
&= Tr(2\hat{\rho}\hat{a}\hat{\sigma}_a\hat{a}^\dagger - \hat{\rho}\hat{\sigma}_a\hat{a}\hat{a}^\dagger - \hat{\rho}\hat{a}\hat{a}^\dagger\hat{\sigma}_a) \\
&= Tr(2\hat{\rho}\hat{a}\hat{a}^\dagger\hat{\sigma}_a - \hat{\rho}\hat{\sigma}_a\hat{a}\hat{a}^\dagger - \hat{\rho}\hat{\sigma}_a\hat{a}\hat{a}^\dagger) \\
&= Tr(2\hat{\rho}\hat{a}\hat{a}^\dagger\hat{\sigma}_a - 2\hat{\rho}\hat{\sigma}_a\hat{a}\hat{a}^\dagger) \\
&= 2Tr(\hat{\rho}[\hat{a}\hat{a}^\dagger, \hat{\sigma}_a]) \\
&= 0.
\end{aligned} \tag{2.34}$$

Following a similar procedure, we arrive at

$$P_4 = 0, \tag{2.35}$$

and

$$P_5 = 0. \tag{2.36}$$

Inserting Eqs. (2.32)-(2.36) into Eq. (2.26), we easily get

$$\frac{d}{dt}\langle\hat{\sigma}_a\rangle = -i\langle[\hat{\sigma}_a, \hat{H}]\rangle. \tag{2.37}$$

Following the same procedure used to arrive at Eq. (2.37), one can verify that

$$\frac{d}{dt}\langle\hat{\eta}_a\rangle = -i\langle[\hat{\eta}_a, \hat{H}]\rangle, \tag{2.38}$$

$$\frac{d}{dt}\langle\hat{\eta}_b(t)\rangle = -i\langle[\hat{\eta}_b, \hat{H}]\rangle. \tag{2.39}$$

We now seek to evaluate the commutators in Eqs. (2.22), (2.23), (2.24), (2.25), (2.37), (2.38) and (2.39) using the Hamiltonian specified by Eq. (2.6) along with the commutation property

$$\begin{aligned}
[\hat{A}\hat{B}, \hat{C}\hat{D}] &= \hat{A}[\hat{B}, \hat{C}\hat{D}] + [\hat{A}, \hat{C}\hat{D}]\hat{B} \\
&= \hat{A}\hat{C}[\hat{B}, \hat{D}] + \hat{A}[\hat{B}, \hat{C}]\hat{D} + \hat{C}[\hat{A}, \hat{D}]\hat{B} + [\hat{A}, \hat{C}]\hat{D}\hat{B}.
\end{aligned}$$

We thus note that

$$\begin{aligned}
\langle [\hat{a}, \hat{H}] \rangle &= \langle [\hat{a}, ig(\hat{\sigma}_a^\dagger \hat{a} - \hat{a}^\dagger \hat{\sigma}_a) + i\frac{\Omega}{2}(\hat{\sigma}_a^\dagger - \hat{\sigma}_a) + i\frac{\varepsilon}{2}(\hat{a}^2 - \hat{a}^{\dagger 2})] \rangle \\
&= ig\langle (\hat{a}\hat{\sigma}_a^\dagger \hat{a} - \hat{a}\hat{a}^\dagger \hat{\sigma}_a - \hat{\sigma}_a^\dagger \hat{a}\hat{a} + \hat{a}^\dagger \hat{\sigma}_a \hat{a}) \rangle \\
&+ i\frac{\Omega}{2}\langle (\hat{a}\hat{\sigma}_a^\dagger - \hat{a}\hat{\sigma}_a - \hat{\sigma}_a^\dagger \hat{a} + \hat{\sigma}_a \hat{a}) \rangle \\
&+ i\frac{\varepsilon}{2}\langle (\hat{a}^3 - \hat{a}\hat{a}^{\dagger 2} - \hat{a}^3 + \hat{a}^{\dagger 2}\hat{a}) \rangle \\
&= ig\langle (\hat{\sigma}_a^\dagger [\hat{a}, \hat{a}] + \hat{\sigma}_a [\hat{a}^\dagger, \hat{a}]) \rangle + i\frac{\Omega}{2}\langle ([\hat{a}, \hat{\sigma}_a^\dagger] + [\hat{\sigma}_a, \hat{a}]) \rangle + i\frac{\varepsilon}{2}\langle [\hat{a}^{\dagger 2}, \hat{a}] \rangle. \tag{2.40}
\end{aligned}$$

Applying the commutation relations

$$[\hat{a}^\dagger, \hat{a}] = -1, \tag{2.41}$$

$$[\hat{a}^{\dagger 2}, \hat{a}] = -2\hat{a}^\dagger, \tag{2.42}$$

$$[\hat{a}, \hat{\sigma}_a^\dagger] = 0, \tag{2.43}$$

$$[\hat{\sigma}_a, \hat{a}] = 0, \tag{2.44}$$

we can put Eq. (2.40) as

$$\langle [\hat{a}, \hat{H}] \rangle = -ig\langle \hat{\sigma}_a \rangle - i\varepsilon\langle \hat{a}^\dagger \rangle. \tag{2.45}$$

Now in view of Eq. (2.45), the time evolution described by Eq. (2.22) becomes

$$\frac{d}{dt}\langle \hat{a}(t) \rangle = -g\langle \hat{\sigma}_a \rangle - \varepsilon\langle \hat{a}^\dagger \rangle - \frac{\kappa}{2}\langle \hat{a} \rangle. \tag{2.46}$$

Following the same procedure used to arrive at Eq. (2.46), one can re-write Eqs. (2.23), (2.24), (2.25), (2.37), (2.38) and Eq. (2.39) as

$$\frac{d}{dt}\langle \hat{a}^2 \rangle = -g(\langle \hat{a}\hat{\sigma}_a \rangle + \langle \hat{\sigma}_a \hat{a} \rangle) - \varepsilon(\langle \hat{a}\hat{a}^\dagger \rangle + \langle \hat{a}^\dagger \hat{a} \rangle) - k\langle \hat{a}^2 \rangle + \kappa M, \tag{2.47}$$

$$\frac{d}{dt}\langle \hat{a}^\dagger \hat{a} \rangle = -g(\langle \hat{\sigma}_a^\dagger \hat{a} \rangle + \langle \hat{a}^\dagger \hat{\sigma}_a \rangle) - \varepsilon(\langle \hat{a}^2 \rangle + \langle \hat{a}^{\dagger 2} \rangle) - \kappa\langle \hat{a}^\dagger \hat{a} \rangle + \kappa N, \tag{2.48}$$

$$\frac{d}{dt}\langle \hat{a}\hat{a}^\dagger \rangle = -g(\langle \hat{a}\hat{\sigma}_a^\dagger \rangle + \langle \hat{\sigma}_a \hat{a}^\dagger \rangle) - \varepsilon(\langle \hat{a}^2 \rangle + \langle \hat{a}^{\dagger 2} \rangle) - k\langle \hat{a}\hat{a}^\dagger \rangle + \kappa(N + 1), \tag{2.49}$$

$$\frac{d}{dt}\langle\hat{\sigma}_a\rangle = g(\langle\hat{\eta}_b\hat{a}\rangle - \langle\hat{\eta}_a\hat{a}\rangle) + \frac{\Omega}{2}(\langle\hat{\eta}_b\rangle - \langle\hat{\eta}_a\rangle), \quad (2.50)$$

$$\frac{d}{dt}\langle\hat{\eta}_a\rangle = g(\langle\hat{\sigma}_a^\dagger\hat{a}\rangle + \langle\hat{a}^\dagger\hat{\sigma}_a\rangle) + \frac{\Omega}{2}(\langle\hat{\sigma}_a^\dagger\rangle + \langle\hat{\sigma}_a\rangle), \quad (2.51)$$

$$\frac{d}{dt}\langle\hat{\eta}_b\rangle = -g(\langle\hat{\sigma}_a^\dagger\hat{a}\rangle + \langle\hat{a}^\dagger\hat{\sigma}_a\rangle) - \frac{\Omega}{2}(\langle\hat{\sigma}_a^\dagger\rangle + \langle\hat{\sigma}_a\rangle). \quad (2.52)$$

Taking Eq. (2.46) in to consideration, we can write the quantum Langevin equation for the cavity mode operator as

$$\frac{d}{dt}\hat{a}(t) = -g\hat{\sigma}_a(t) - \frac{\kappa}{2}\hat{a}(t) - \varepsilon\hat{a}^\dagger(t) + \hat{f}(t), \quad (2.53)$$

where $\hat{f}(t)$ is a cavity mode noise operator whose correlation properties can be determined. Then the expectation value of Eq. (2.53) is equal to Eq. (2.46), if and only if

$$\langle\hat{f}(t)\rangle = 0. \quad (2.54)$$

We observe that Eqs. (2.47), (2.48), (2.49), (2.50), (2.51) and Eq. (2.52) are nonlinear coupled differential equations and hence we can not obtain the exact solutions of these equations. Thus we seek to overcome this problem by using the large-time approximation scheme to Eq. (2.53) and write its solution as [12]

$$\hat{a}(t) = -\frac{2g}{\kappa}\hat{\sigma}_a(t) - \frac{2\varepsilon}{\kappa}\hat{a}^\dagger(t) + \frac{2}{\kappa}\hat{f}(t), \quad (2.55)$$

and its adjoint is

$$\hat{a}^\dagger(t) = -\frac{2g}{\kappa}\hat{\sigma}_a^\dagger(t) - \frac{2\varepsilon}{\kappa}\hat{a}(t) + \frac{2}{\kappa}\hat{f}^\dagger(t). \quad (2.56)$$

Substituting Eq. (2.56) into Eq. (2.55), we have

$$\begin{aligned} \hat{a}(t) &= -\frac{2\varepsilon}{k}\left[-\frac{2\varepsilon}{k}\hat{a}(t) - \frac{2g}{\kappa}\hat{\sigma}_a^\dagger(t) + \frac{2}{\kappa}\hat{f}^\dagger(t)\right] - \frac{2g}{\kappa}\hat{\sigma}_a(t) + \frac{2}{\kappa}\hat{f}(t) \\ &= \frac{4\varepsilon^2}{k^2}\hat{a}(t) + \frac{4g\varepsilon}{k^2}\hat{\sigma}_a^\dagger(t) - \frac{4\varepsilon}{k^2}\hat{f}^\dagger(t) - \frac{2g}{k}\hat{\sigma}_a(t) + \frac{2}{k}\hat{f}(t) \\ &= \frac{4g\varepsilon}{k^2 - 4\varepsilon^2}\hat{\sigma}_a^\dagger(t) - \frac{2kg}{k^2 - 4\varepsilon^2}\hat{\sigma}_a(t) - \frac{4\varepsilon}{k^2 - 4\varepsilon^2}\hat{f}^\dagger(t) + \frac{2k}{k^2 - 4\varepsilon^2}\hat{f}(t), \end{aligned} \quad (2.57)$$

and its adjoint is

$$\hat{a}^\dagger(t) = \frac{4g\varepsilon}{k^2 - 4\varepsilon^2}\hat{\sigma}_a(t) - \frac{2kg}{k^2 - 4\varepsilon^2}\hat{\sigma}_a^\dagger(t) - \frac{4\varepsilon}{k^2 - 4\varepsilon^2}\hat{f}(t) + \frac{2k}{k^2 - 4\varepsilon^2}\hat{f}^\dagger(t). \quad (2.58)$$

Then by inserting Eq. (2.57) and Eq. (2.58) into Eqs. (2.46), (2.47), (2.48), (2.49), (2.50), (2.51) and (2.52), we can obtain the decoupled differential equations. Now multiplying Eq. (2.57) from the right by $\hat{\sigma}_a(t)$ and taking the expectation value of the product, we obtain

$$\begin{aligned}
\langle \hat{a}(t)\hat{\sigma}_a(t) \rangle &= \langle [\frac{4g\varepsilon}{k^2 - 4\varepsilon^2}\hat{\sigma}_a^\dagger(t) - \frac{2kg}{k^2 - 4\varepsilon^2}\hat{\sigma}_a(t) - \frac{4\varepsilon}{k^2 - 4\varepsilon^2}\hat{f}^\dagger(t) + \frac{2k}{k^2 - 4\varepsilon^2}\hat{f}(t)]\hat{\sigma}_a(t) \rangle \\
&= \frac{4g\varepsilon}{k^2 - 4\varepsilon^2}\langle \hat{\sigma}_a^\dagger(t)\hat{\sigma}_a(t) \rangle - \frac{2kg}{k^2 - 4\varepsilon^2}\langle \hat{\sigma}_a(t)\hat{\sigma}_a(t) \rangle - \frac{4\varepsilon}{k^2 - 4\varepsilon^2}\langle \hat{f}^\dagger(t)\hat{\sigma}_a(t) \rangle \\
&\quad + \frac{2k}{k^2 - 4\varepsilon^2}\langle \hat{f}(t)\hat{\sigma}_a(t) \rangle \\
&= \frac{4g\varepsilon}{k^2 - 4\varepsilon^2}\langle \hat{\eta}_a(t) \rangle - \frac{4\varepsilon}{k^2 - 4\varepsilon^2}\langle \hat{f}^\dagger(t)\hat{\sigma}_a(t) \rangle + \frac{2k}{k^2 - 4\varepsilon^2}\langle \hat{f}(t)\hat{\sigma}_a(t) \rangle, \tag{2.59}
\end{aligned}$$

where

$$\langle \hat{\sigma}_a\hat{\sigma}_a \rangle = \langle |b\rangle\langle a|b\rangle\langle a| \rangle = 0. \tag{2.60}$$

and

$$\langle \hat{\sigma}_a^\dagger\hat{\sigma}_a \rangle = \langle |a\rangle\langle b|b\rangle\langle a| \rangle = \langle |a\rangle\langle a| \rangle = \langle \hat{\eta}_a \rangle. \tag{2.61}$$

Similarly, multiplying Eq. (2.57) from the left by $\hat{\sigma}_a(t)$ and taking the expectation value of the product and using Eq. (2.60), we arrive at

$$\langle \hat{\sigma}_a(t)\hat{a}(t) \rangle = \frac{4g\varepsilon}{k^2 - 4\varepsilon^2}\langle \hat{\eta}_b(t) \rangle - \frac{4\varepsilon}{k^2 - 4\varepsilon^2}\langle \hat{\sigma}_a(t)\hat{f}^\dagger(t) \rangle + \frac{2k}{k^2 - 4\varepsilon^2}\langle \hat{\sigma}_a(t)\hat{f}(t) \rangle, \tag{2.62}$$

where

$$\langle \hat{\sigma}_a\hat{\sigma}_a^\dagger \rangle = \langle |b\rangle\langle a|a\rangle\langle b| \rangle = \langle |b\rangle\langle b| \rangle = \langle \hat{\eta}_b \rangle. \tag{2.63}$$

Adding Eq. (2.59) and Eq. (2.62), we get

$$\begin{aligned}
\langle \hat{a}(t)\hat{\sigma}_a(t) \rangle + \langle \hat{\sigma}_a(t)\hat{a}(t) \rangle &= \frac{4g\varepsilon}{k^2 - 4\varepsilon^2}(\langle \hat{\eta}_a(t) \rangle + \langle \hat{\eta}_b(t) \rangle) - \frac{4\varepsilon}{k^2 - 4\varepsilon^2}(\langle \hat{f}^\dagger(t)\hat{\sigma}_a(t) \rangle + \langle \hat{\sigma}_a(t)\hat{f}^\dagger(t) \rangle) \\
&\quad + \frac{2k}{k^2 - 4\varepsilon^2}(\langle \hat{f}(t)\hat{\sigma}_a(t) \rangle + \langle \hat{\sigma}_a(t)\hat{f}(t) \rangle). \tag{2.64}
\end{aligned}$$

Assuming that the cavity mode noise and atomic operators are not correlated [4], and taking Eq. (2.54) into account, we obtain

$$\langle \hat{\sigma}_a(t)\hat{f}(t) \rangle = \langle \hat{\sigma}_a(t) \rangle\langle \hat{f}(t) \rangle = 0, \tag{2.65}$$

$$\langle \hat{f}(t)\hat{\sigma}_a(t) \rangle = \langle \hat{f}(t) \rangle\langle \hat{\sigma}_a(t) \rangle = 0, \tag{2.66}$$

$$\langle \hat{\sigma}_a(t)\hat{f}^\dagger(t) \rangle = \langle \hat{\sigma}_a(t) \rangle\langle \hat{f}^\dagger(t) \rangle = 0, \tag{2.67}$$

$$\langle \hat{f}^\dagger(t) \hat{\sigma}_a(t) \rangle = \langle \hat{f}^\dagger(t) \rangle \langle \hat{\sigma}_a(t) \rangle = 0. \quad (2.68)$$

Therefore in view of Eqs. (2.65)-(2.68), we can reduce Eq. (2.64) to

$$\langle \hat{a}(t) \hat{\sigma}_a(t) \rangle + \langle \hat{\sigma}_a(t) \hat{a}(t) \rangle = \frac{4g\varepsilon}{k^2 - 4\varepsilon^2} (\langle \hat{\eta}_a(t) \rangle + \langle \hat{\eta}_b(t) \rangle). \quad (2.69)$$

Substituting Eq. (2.69) into Eq. (2.47), we get

$$\frac{d}{dt} \langle \hat{a}^2 \rangle = \frac{-\eta\varepsilon}{\kappa} (\langle \hat{\eta}_a \rangle + \langle \hat{\eta}_b \rangle) - \varepsilon (\langle \hat{a} \hat{a}^\dagger \rangle + \langle \hat{a}^\dagger \hat{a} \rangle) - k \langle \hat{a}^2 \rangle + kM, \quad (2.70)$$

where

$$\eta = \frac{4g^2}{\kappa(1 - \frac{4\varepsilon^2}{\kappa^2})}. \quad (2.71)$$

Following the same procedure used to arrive at Eq. (2.70), one can express

Eqs. (2.48), (2.49), (2.50), (2.51) and Eq. (2.52) as

$$\frac{d}{dt} \langle \hat{a}^\dagger \hat{a} \rangle = \eta \langle \hat{\eta}_a \rangle - \varepsilon (\langle \hat{a}^2 \rangle + \langle \hat{a}^{\dagger 2} \rangle) - \kappa \langle \hat{a}^\dagger \hat{a} \rangle + \kappa N, \quad (2.72)$$

$$\frac{d}{dt} \langle \hat{a} \hat{a}^\dagger \rangle = \eta \langle \hat{\eta}_b \rangle - \varepsilon (\langle \hat{a}^2 \rangle + \langle \hat{a}^{\dagger 2} \rangle) - k \langle \hat{a} \hat{a}^\dagger \rangle + \kappa(N + 1), \quad (2.73)$$

$$\frac{d}{dt} \langle \hat{\sigma}_a \rangle = -\frac{\eta}{2} \langle \hat{\sigma}_a \rangle - \frac{\eta\varepsilon}{\kappa} \langle \hat{\sigma}_a^\dagger \rangle + \frac{\Omega}{2} (\langle \hat{\eta}_b \rangle - \langle \hat{\eta}_a \rangle), \quad (2.74)$$

$$\frac{d}{dt} \langle \hat{\eta}_a \rangle = -\eta \langle \hat{\eta}_a \rangle + \frac{\Omega}{2} (\langle \hat{\sigma}_a^\dagger \rangle + \langle \hat{\sigma}_a \rangle), \quad (2.75)$$

$$\frac{d}{dt} \langle \hat{\eta}_b \rangle = \eta \langle \hat{\eta}_a \rangle - \frac{\Omega}{2} (\langle \hat{\sigma}_a^\dagger \rangle + \langle \hat{\sigma}_a \rangle). \quad (2.76)$$

Now taking into account the completeness relation

$$\hat{\eta}_a + \hat{\eta}_b = \hat{I}, \quad (2.77)$$

we notice that

$$\langle \hat{\eta}_a \rangle + \langle \hat{\eta}_b \rangle = 1, \quad (2.78)$$

where $\langle \hat{\eta}_a \rangle$ is probability of finding a two level atom in the upper level and $\langle \hat{\eta}_b \rangle$ is probability of finding it in the lower level.

2.2 Correlation property

In this section, we seek to determine the correlation properties of the cavity mode noise operator $\hat{f}(t)$. Now applying the relation

$$\frac{d}{dt}\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle = \left\langle\frac{d\hat{a}^\dagger(t)}{dt}\hat{a}(t)\right\rangle + \langle\hat{a}^\dagger(t)\frac{d\hat{a}(t)}{dt}\rangle, \quad (2.79)$$

along with Eq.(2.53) and its conjugate, it follows that

$$\begin{aligned} \frac{d}{dt}\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle &= -g\langle\hat{\sigma}_a^\dagger(t)\hat{a}(t)\rangle - \frac{k}{2}\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle - \varepsilon\langle\hat{a}^2(t)\rangle + \langle\hat{f}^\dagger(t)\hat{a}(t)\rangle \\ &\quad - g\langle\hat{a}^\dagger(t)\hat{\sigma}_a(t)\rangle - \frac{k}{2}\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle - \varepsilon\langle\hat{a}^{\dagger 2}(t)\rangle + \langle\hat{a}^\dagger(t)\hat{f}(t)\rangle \\ &= -g(\langle\hat{a}^\dagger(t)\hat{\sigma}_a(t)\rangle + \langle\hat{\sigma}_a^\dagger(t)\hat{a}(t)\rangle) - k\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle \\ &\quad - \varepsilon(\langle\hat{a}^2(t)\rangle + \langle\hat{a}^{\dagger 2}(t)\rangle) + \langle\hat{f}^\dagger(t)\hat{a}(t)\rangle + \langle\hat{a}^\dagger(t)\hat{f}(t)\rangle. \end{aligned} \quad (2.80)$$

Then in view of Eq. (2.48) and Eq. (2.80), we get

$$\langle\hat{f}^\dagger(t)\hat{a}(t)\rangle + \langle\hat{a}^\dagger(t)\hat{f}(t)\rangle = \kappa N. \quad (2.81)$$

The solution of Eq. (2.53) can be written as

$$\hat{a}(t) = \hat{a}(0)e^{-\frac{kt}{2}} + \int_0^t e^{-\frac{k(t-t')}{2}}(-g\hat{\sigma}_a(t') - \varepsilon\hat{a}^\dagger(t') + \hat{f}(t'))dt', \quad (2.82)$$

and its conjugate is

$$\hat{a}^\dagger(t) = \hat{a}^\dagger(0)e^{-\frac{kt}{2}} + \int_0^t e^{-\frac{k(t-t')}{2}}(-g\hat{\sigma}_a^\dagger(t') - \varepsilon\hat{a}(t') + \hat{f}^\dagger(t'))dt'. \quad (2.83)$$

Then multiplying Eq. (2.82) from the left by $\hat{f}^\dagger(t)$ and taking the expectation value of the product, we get

$$\langle\hat{f}^\dagger(t)\hat{a}(t)\rangle = \langle\hat{f}^\dagger(t)\hat{a}(0)\rangle e^{-\frac{kt}{2}} + \int_0^t e^{-\frac{k(t-t')}{2}}(-g\langle\hat{f}^\dagger(t)\hat{\sigma}_a(t')\rangle - \varepsilon\langle\hat{f}^\dagger(t)\hat{a}^\dagger(t')\rangle + \langle\hat{f}^\dagger(t)\hat{f}(t')\rangle)dt'. \quad (2.84)$$

Again multiplying Eq. (2.83) from the right by $\hat{f}(t)$ and taking the expectation value of the product, we get

$$\langle\hat{a}^\dagger(t)\hat{f}(t)\rangle = \langle\hat{a}^\dagger(0)\hat{f}(t)\rangle e^{-\frac{\kappa t}{2}} + \int_0^t e^{-\frac{\kappa(t-t')}{2}}(-g\langle\hat{\sigma}_a^\dagger(t')\hat{f}(t)\rangle - \varepsilon\langle\hat{a}(t')\hat{f}(t)\rangle + \langle\hat{f}^\dagger(t')\hat{f}(t)\rangle)dt'. \quad (2.85)$$

Assuming that a noise operator at a time "t" has no effect on a system operator at earlier times, one can re-write Eq. (2.84) and Eq. (2.85) respectively as follows:

$$\langle \hat{f}^\dagger(t) \hat{a}(t) \rangle = \int_0^t e^{-\frac{\kappa(t-t')}{2}} \langle \hat{f}^\dagger(t) \hat{f}(t') \rangle dt', \quad (2.86)$$

$$\langle \hat{a}^\dagger(t) \hat{f}(t) \rangle = \int_0^t e^{-\frac{\kappa(t-t')}{2}} \langle \hat{f}^\dagger(t') \hat{f}(t) \rangle dt'. \quad (2.87)$$

Now combining Eqs. (2.86) and (2.87) with Eq. (2.81), we find that

$$\int_0^t e^{-\frac{\kappa(t-t')}{2}} [\langle \hat{f}^\dagger(t) \hat{f}(t') \rangle + \langle \hat{f}^\dagger(t') \hat{f}(t) \rangle] dt' = \kappa N. \quad (2.88)$$

Assuming that

$$\langle \hat{f}^\dagger(t) \hat{f}(t') \rangle = \langle \hat{f}^\dagger(t') \hat{f}(t) \rangle, \quad (2.89)$$

we obtain

$$\int_0^t e^{-\frac{\kappa(t-t')}{2}} \langle \hat{f}^\dagger(t) \hat{f}(t') \rangle dt' = \frac{\kappa N}{2}, \quad (2.90)$$

which on the basis of the relation

$$\int_0^t e^{-\frac{\kappa(t-t')}{2}} \kappa N \delta(t-t') dt' = \frac{\kappa N}{2}, \quad (2.91)$$

we arrive at

$$\langle \hat{f}^\dagger(t) \hat{f}(t') \rangle = \kappa N \delta(t-t'). \quad (2.92)$$

In a similar procedure, it can be verified that

$$\langle \hat{f}(t) \hat{f}^\dagger(t') \rangle = \kappa(N+1) \delta(t-t'), \quad (2.93)$$

$$\langle \hat{f}(t) \hat{f}(t') \rangle = \kappa M \delta(t-t'), \quad (2.94)$$

$$\langle \hat{f}^\dagger(t) \hat{f}^\dagger(t') \rangle = \kappa M \delta(t-t'). \quad (2.95)$$

2.3 Steady-state solutions

Here we need to obtain the steady-state solutions for the equations of evolution for the expectation values of cavity mode and atomic operators. The steady-state solution of Eq. (2.74), can be written as

$$\langle \hat{\sigma}_a \rangle = -\frac{2\varepsilon}{\kappa} \langle \hat{\sigma}_a^\dagger \rangle + \frac{\Omega}{\eta} (\langle \hat{\eta}_b \rangle - \langle \hat{\eta}_a \rangle), \quad (2.96)$$

and its adjoint is

$$\langle \hat{\sigma}_a^\dagger \rangle = -\frac{2\varepsilon}{\kappa} \langle \hat{\sigma}_a \rangle + \frac{\Omega}{\eta} (\langle \hat{\eta}_b \rangle - \langle \hat{\eta}_a \rangle). \quad (2.97)$$

Substituting Eq. (2.97) into Eq. (2.96), we obtain

$$\begin{aligned} \langle \hat{\sigma}_a \rangle &= -\frac{2\varepsilon}{\kappa} \left[-\frac{2\varepsilon}{\kappa} \langle \hat{\sigma}_a \rangle + \frac{\Omega}{\eta} (\langle \hat{\eta}_b \rangle - \langle \hat{\eta}_a \rangle) \right] + \frac{\Omega}{\eta} (\langle \hat{\eta}_b \rangle - \langle \hat{\eta}_a \rangle) \\ &= \frac{4\varepsilon^2}{\kappa^2} \langle \hat{\sigma}_a \rangle - \frac{2\varepsilon\Omega}{\kappa\eta} (\langle \hat{\eta}_b \rangle - \langle \hat{\eta}_a \rangle) + \frac{\Omega}{\eta} (\langle \hat{\eta}_b \rangle - \langle \hat{\eta}_a \rangle) \\ &= \frac{\Omega(1 - \frac{2\varepsilon}{\kappa})}{\eta(1 - \frac{4\varepsilon^2}{\kappa^2})} (\langle \hat{\eta}_b \rangle - \langle \hat{\eta}_a \rangle) \\ &= \frac{\Omega}{\eta(1 + \frac{2\varepsilon}{\kappa})} (\langle \hat{\eta}_b \rangle - \langle \hat{\eta}_a \rangle). \end{aligned} \quad (2.98)$$

Again substituting Eq. (2.96) into Eq. (2.97) and following the same procedure to arrive at Eq. (2.98), we obtain

$$\langle \hat{\sigma}_a^\dagger \rangle = \frac{\Omega}{\eta(1 + \frac{2\varepsilon}{\kappa})} (\langle \hat{\eta}_b \rangle - \langle \hat{\eta}_a \rangle). \quad (2.99)$$

In view of Eq. (2.98) and Eq. (2.99), we note that

$$\langle \hat{\sigma}_a \rangle = \langle \hat{\sigma}_a^\dagger \rangle. \quad (2.100)$$

In addition, the steady-state solution of Eq. (2.75) can be written as

$$\langle \hat{\eta}_a \rangle = \frac{\Omega}{2\eta} (\langle \hat{\sigma}_a \rangle + \langle \hat{\sigma}_a^\dagger \rangle), \quad (2.101)$$

so that taking into account Eqs. (2.98) and Eq. (2.100), we can express Eq. (2.101) as

$$\langle \hat{\eta}_a \rangle = \frac{\Omega^2}{\Omega^2 + \eta^2(1 + \frac{2\varepsilon}{\kappa})} \langle \hat{\eta}_b \rangle, \quad (2.102)$$

which with the aid of Eq. (2.78) becomes

$$\langle \hat{\eta}_a \rangle = \frac{\Omega^2}{2\Omega^2 + \eta^2(1 + \frac{2\varepsilon}{\kappa})}. \quad (2.103)$$

On the base of Eq. (2.78) and Eq. (2.103), we obtain

$$\langle \hat{\eta}_b \rangle = \frac{\Omega^2 + \eta^2(1 + \frac{2\varepsilon}{\kappa})}{2\Omega^2 + \eta^2(1 + \frac{2\varepsilon}{\kappa})}. \quad (2.104)$$

In view of Eq. (2.103) and Eq. (2.104), we observe that the probability of finding two level atom in the lower level is greater than the probability of finding it in the upper level.

Substituting Eq. (2.103) and Eq. (2.104) into Eq. (2.98), we get

$$\langle \hat{\sigma}_a \rangle = \frac{\Omega\eta}{(2\Omega^2 + \eta^2(1 + \frac{2\varepsilon}{\kappa}))}. \quad (2.105)$$

The steady-state solution of Eq. (2.46) can be written as

$$\langle \hat{a} \rangle = -\frac{2g}{\kappa} \langle \hat{\sigma}_a \rangle - \frac{2\varepsilon}{\kappa} \langle \hat{a}^\dagger \rangle, \quad (2.106)$$

from which follows

$$\langle \hat{a}^\dagger \rangle = -\frac{2g}{\kappa} \langle \hat{\sigma}_a^\dagger \rangle - \frac{2\varepsilon}{\kappa} \langle \hat{a} \rangle. \quad (2.107)$$

Now substituting Eq. (2.107) into Eq. (2.106) and taking into account Eqs. (2.100) and (2.105), we obtain

$$\begin{aligned} \langle \hat{a} \rangle &= -\frac{2g}{\kappa} \langle \hat{\sigma}_a \rangle - \frac{2\varepsilon}{\kappa} \left[-\frac{2g}{\kappa} \langle \hat{\sigma}_a \rangle - \frac{2\varepsilon}{\kappa} \langle \hat{a} \rangle \right] \\ &= -\frac{2g}{\kappa} \langle \hat{\sigma}_a \rangle + \frac{4g\varepsilon}{\kappa^2} \langle \hat{\sigma}_a \rangle + \frac{4\varepsilon^2}{\kappa^2} \langle \hat{a} \rangle \\ &= \frac{4\varepsilon^2}{\kappa^2} \langle \hat{a} \rangle - \frac{2g}{\kappa} \left(1 - \frac{2\varepsilon}{\kappa} \right) \langle \hat{\sigma}_a \rangle \\ &= \frac{-\frac{2g}{\kappa} \left(1 - \frac{2\varepsilon}{\kappa} \right)}{\left(1 - \frac{2\varepsilon}{\kappa} \right) \left(1 + \frac{2\varepsilon}{\kappa} \right)} \langle \hat{\sigma}_a \rangle \\ &= \frac{-2g}{\kappa \left(1 + \frac{2\varepsilon}{\kappa} \right)} \langle \hat{\sigma}_a \rangle \\ &= \frac{-2g\Omega\eta}{\kappa \left(1 + \frac{2\varepsilon}{\kappa} \right) (2\Omega^2 + \eta^2 \left(1 + \frac{2\varepsilon}{\kappa} \right))}. \end{aligned} \quad (2.108)$$

Similarly substituting Eq. (2.106) into Eq. (2.107) and in view of Eq. (2.100) and Eq. (2.105), we get

$$\langle \hat{a}^\dagger \rangle = \frac{-2g\Omega\eta}{\kappa \left(1 + \frac{2\varepsilon}{\kappa} \right) (2\Omega^2 + \eta^2 \left(1 + \frac{2\varepsilon}{\kappa} \right))}. \quad (2.109)$$

On account of Eq. (2.108) and Eq. (2.109), we see that

$$\langle \hat{a} \rangle = \langle \hat{a}^\dagger \rangle. \quad (2.110)$$

Again the steady state solution of Eqs. (2.70), (2.72) and (2.73) can be, respectively, written as

$$\langle \hat{a}^2 \rangle = \frac{-\eta\varepsilon}{\kappa^2} (\langle \hat{\eta}_a \rangle + \langle \hat{\eta}_b \rangle) - \frac{\varepsilon}{\kappa} (\langle \hat{a}\hat{a}^\dagger \rangle + \langle \hat{a}^\dagger\hat{a} \rangle) + M, \quad (2.111)$$

$$\langle \hat{a}^\dagger\hat{a} \rangle = \frac{\eta}{\kappa} \langle \hat{\eta}_a \rangle - \frac{\varepsilon}{\kappa} (\langle \hat{a}^2 \rangle + \langle \hat{a}^{\dagger 2} \rangle) + N, \quad (2.112)$$

$$\langle \hat{a}\hat{a}^\dagger \rangle = \frac{\eta}{\kappa} \langle \hat{\eta}_b \rangle - \frac{\varepsilon}{\kappa} (\langle \hat{a}^2 \rangle + \langle \hat{a}^{\dagger 2} \rangle) + N + 1. \quad (2.113)$$

The conjugate of Eq. (2.111) is expressible as

$$\langle \hat{a}^{\dagger 2} \rangle = \frac{-\eta\varepsilon}{\kappa^2} (\langle \hat{\eta}_a \rangle + \langle \hat{\eta}_b \rangle) - \frac{\varepsilon}{\kappa} (\langle \hat{a}\hat{a}^\dagger \rangle + \langle \hat{a}^\dagger\hat{a} \rangle) + M. \quad (2.114)$$

In view of Eq. (2.78), we can write Eqs. (2.111) and (2.114) as

$$\langle \hat{a}^2 \rangle = \frac{-\eta\varepsilon}{\kappa^2} - \frac{\varepsilon}{\kappa} (\langle \hat{a}\hat{a}^\dagger \rangle + \langle \hat{a}^\dagger\hat{a} \rangle) + M, \quad (2.115)$$

$$\langle \hat{a}^{\dagger 2} \rangle = \frac{-\eta\varepsilon}{\kappa^2} - \frac{\varepsilon}{\kappa} (\langle \hat{a}\hat{a}^\dagger \rangle + \langle \hat{a}^\dagger\hat{a} \rangle) + M, \quad (2.116)$$

from which we conclude that

$$\langle \hat{a}^2 \rangle = \langle \hat{a}^{\dagger 2} \rangle. \quad (2.117)$$

Moreover, in view of Eqs. (2.78), (2.116), and (2.117), the sum of Eqs. (2.112) and (2.113) can be written as

$$\langle \hat{a}^\dagger\hat{a} \rangle + \langle \hat{a}\hat{a}^\dagger \rangle = \frac{\eta}{\kappa} - \frac{4\varepsilon}{\kappa} \langle \hat{a}^2 \rangle + 2N + 1. \quad (2.118)$$

Substituting Eq. (2.118) into Eq. (2.115), we get

$$\langle \hat{a}^2 \rangle = \frac{-2\eta\varepsilon - \varepsilon\kappa(2N + 1) + M\kappa^2}{\kappa^2 - 4\varepsilon^2}. \quad (2.119)$$

In view of Eq. (2.117), we have

$$\langle \hat{a}^{\dagger 2} \rangle = \frac{-2\eta\varepsilon - \varepsilon\kappa(2N + 1) + M\kappa^2}{\kappa^2 - 4\varepsilon^2}. \quad (2.120)$$

Adding Eq. (2.119) and Eq. (2.120), we get

$$\langle \hat{a}^2 \rangle + \langle \hat{a}^{\dagger 2} \rangle = \frac{-4\eta\varepsilon - 2\varepsilon\kappa(2N + 1) + 2M\kappa^2}{\kappa^2 - 4\varepsilon^2}. \quad (2.121)$$

Substituting Eq. (2.104) and Eq. (2.121) into Eq. (2.113), one can obtain

$$\begin{aligned} \langle \hat{a}\hat{a}^\dagger \rangle &= \frac{\eta}{\kappa} \left(\frac{\Omega^2 + \eta^2(1 + 2\frac{\varepsilon}{\kappa})}{2\Omega^2 + \eta^2(1 + 2\frac{\varepsilon}{\kappa})} \right) - \frac{\varepsilon}{\kappa} \left[\frac{-4\eta\varepsilon - 2\varepsilon\kappa(2N + 1) + 2M\kappa^2}{\kappa^2 - 4\varepsilon^2} \right] + (N + 1) \\ &= \frac{\eta[\Omega^2 + \eta^2(1 + 2\frac{\varepsilon}{\kappa})]}{\kappa[2\Omega^2 + \eta^2(1 + 2\frac{\varepsilon}{\kappa})]} + \frac{\frac{\eta}{\kappa} \frac{4\varepsilon^2}{\kappa^2} + \frac{2\varepsilon^2}{\kappa^2} - 2M\frac{\varepsilon}{\kappa} + N}{1 - \frac{4\varepsilon^2}{\kappa^2}} + 1. \end{aligned} \quad (2.122)$$

Now Eqs. (2.103), (2.104), (2.105), (2.108), (2.109), (2.119), (2.120) and (2.122) represent the steady-state solutions for the equations of evolution for the expectation values of cavity mode and atomic operators.

Chapter 3

Photon Statistics

In this section, we seek to study the statistical properties of the light produced by a two level atom driven by a coherent light and available in a cavity containing degenerate subharmonic generator and coupled to squeezed vacuum reservoir. To this end we calculate the mean photon number, power spectrum and second order correlation function for the cavity mode.

3.1 Mean photon number

The mean photon number of the cavity light can be expressed as

$$\begin{aligned}\bar{n} &= Tr(\hat{\rho}\hat{a}^\dagger\hat{a}) \\ &= \langle\hat{a}^\dagger\hat{a}\rangle.\end{aligned}\tag{3.1}$$

Now combining Eqs. (2.103), (2.112), and (2.121), the steady-state mean photon number of the cavity mode can be expressed as

$$\begin{aligned}\bar{n} &= \frac{\eta}{\kappa}\left(\frac{\Omega^2}{2\Omega^2 + \eta^2(1 + \frac{2\varepsilon}{\kappa})}\right) - \frac{\varepsilon}{\kappa}\left[\frac{-4\eta\varepsilon - 2\kappa\varepsilon(2N + 1) + 2M\kappa^2}{\kappa^2 - 4\varepsilon^2}\right] + N \\ &= \frac{\eta}{\kappa}\left(\frac{\Omega^2}{2\Omega^2 + \eta^2(1 + \frac{2\varepsilon}{\kappa})}\right) + \frac{4\eta\frac{\varepsilon^2}{\kappa} + 4N\varepsilon^2 + 2\varepsilon^2 - 2M\kappa\varepsilon + N\kappa^2 - 4N\varepsilon^2}{\kappa^2 - 4\varepsilon^2} \\ &= \frac{\eta}{\kappa}\left(\frac{\Omega^2}{2\Omega^2 + \eta^2(1 + \frac{2\varepsilon}{\kappa})}\right) + \frac{4\eta\frac{\varepsilon^2}{\kappa} + 2\varepsilon^2 - 2M\kappa\varepsilon + N\kappa^2}{\kappa^2 - 4\varepsilon^2} \\ &= \frac{\eta\Omega^2}{\kappa[2\Omega^2 + \eta^2(1 + \frac{2\varepsilon}{\kappa})]} + \frac{\frac{\eta}{\kappa}\frac{4\varepsilon^2}{\kappa^2} + \frac{2\varepsilon^2}{\kappa^2} - 2M\frac{\varepsilon}{\kappa} + N}{1 - \frac{4\varepsilon^2}{\kappa^2}},\end{aligned}\tag{3.2}$$

where

$$\eta = \frac{\gamma_c}{1 - \frac{4\varepsilon^2}{\kappa^2}},\tag{3.3}$$

in which $\gamma_c = \frac{4g^2}{\kappa}$ is the stimulated emission decay constant.

Furthermore in view of Eqs. (2.8), (2.9) and (3.3), we can re-write Eq. (3.2) as

$$\bar{n} = \frac{\gamma_c \Omega^2 (1 - \frac{4\varepsilon^2}{\kappa^2})}{\kappa [2\Omega^2 (1 - \frac{4\varepsilon^2}{\kappa^2})^2 + \gamma_c^2 (1 + \frac{2\varepsilon}{\kappa})]} + \frac{\frac{4\gamma_c \varepsilon^2}{\kappa(\kappa^2 - 4\varepsilon^2)} + \frac{2\varepsilon^2}{\kappa^2} - \frac{2\varepsilon}{\kappa} [\frac{e^{2r} - e^{-2r}}{4}] + \frac{e^{2r} + e^{-2r} - 2}{4}}{1 - \frac{4\varepsilon^2}{\kappa^2}}. \quad (3.4)$$

This is the steady-state mean photon number of the cavity light. For the case in which the parametric amplifier is removed from the cavity ($\varepsilon = 0$), Eq. (3.4) has the form

$$\bar{n} = \frac{\gamma_c \Omega^2}{\kappa(2\Omega^2 + \gamma_c^2)} + \frac{e^{2r} + e^{-2r} - 2}{4}. \quad (3.5)$$

The first term represents the mean photon number due to the interaction of the two-level atom with the cavity mode and the driving coherent light and the second term represents the mean photon number due to the squeezed vacuum reservoir.

For the case in which the driving coherent light is absent ($\Omega = 0$) and the cavity is coupled to vacuum reservoir ($r = 0$), the mean photon number specified by Eq. (3.4) reduces to

$$\bar{n} = \frac{4\kappa\gamma_c\varepsilon^2}{(\kappa^2 - 4\varepsilon^2)^2} + \frac{2\varepsilon^2}{\kappa^2 - 4\varepsilon^2}. \quad (3.6)$$

This is the mean photon number produced by a two-level atom in a cavity containing parametric amplifier and coupled to a vacuum reservoir.

Moreover, neglecting the interaction of the two-level atom with cavity mode ($\gamma_c = 0$) and considering a cavity coupled to ordinary vacuum ($r = 0$), the cavity mean photon number in Eq. (3.4) reduced to

$$\bar{n} = \frac{2\varepsilon^2}{\kappa^2 - 4\varepsilon^2}. \quad (3.7)$$

This represents the mean photon number of the degenerate parametric amplifier in a cavity coupled to vacuum reservoir. Again in the absence of both parametric amplifier ($\varepsilon = 0$) and driving coherent light ($\Omega = 0$), we obtain the mean photon number due to a cavity coupled to squeezed vacuum reservoir:

$$\bar{n} = \frac{e^{2r} + e^{-2r} - 2}{4}. \quad (3.8)$$

For the case in which there is no parametric amplifier ($\varepsilon = 0$) and the cavity is coupled to vacuum reservoir ($r = 0$), the mean photon number becomes

$$\bar{n} = \frac{\gamma_c}{\kappa} \frac{\Omega^2}{(2\Omega^2 + \gamma_c^2)}. \quad (3.9)$$

This represents the mean photon number due to the interaction of the two-level atom with the cavity mode and the driving coherent light.

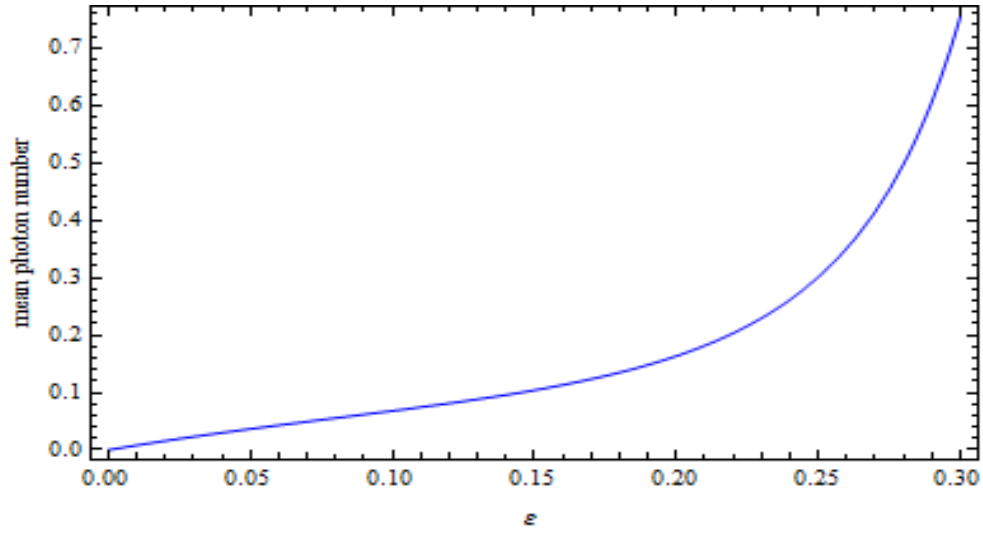


Figure 3.1: The plot of cavity mode mean photon number [Eq. (3.4)] versus ε for $\gamma_c = 0.7$, $\kappa = 0.8$, $\Omega = 0.6$ and $r = 0.7$.

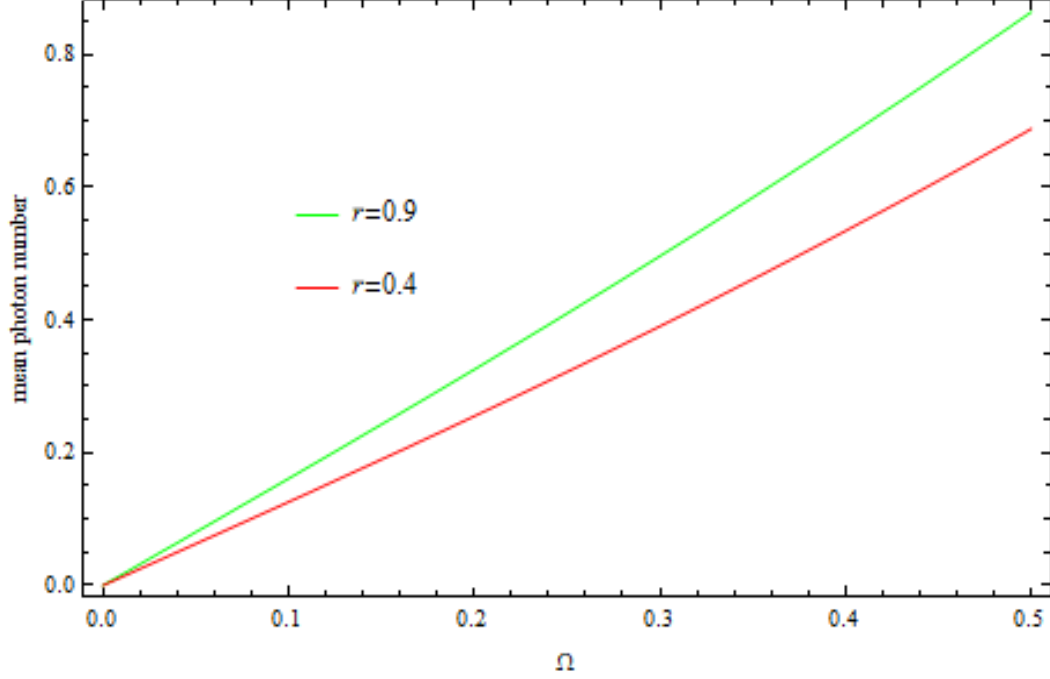


Figure 3.2: The plots of cavity mode mean photon number [Eq. (3.4)] versus Ω for $\gamma_c = 0.7$, $\kappa = 0.8$, $\varepsilon = 0.27$ and different values of r .

The plots in Figure 3.1 and Figure 3.2 indicate that the squeeze parameter, the amplitudes of the driving coherent light and the parametric pump mode have the effect of increase the mean photon number of the cavity mode.

3.2 Power spectrum of cavity mode

In this section, we need to determine the power spectrum and local mean photon number of a cavity light produced by our quantum optical system under consideration. The power spectrum of the cavity light with the central frequency ω_o can be expressed as [1]

$$P(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \langle \hat{a}^\dagger(t) \hat{a}(t + \tau) \rangle_{ss} e^{i(\omega - \omega_o)\tau} d\tau. \quad (3.10)$$

Integrating both side with respect to ω , we have

$$\begin{aligned} \int_{-\infty}^{\infty} P(\omega) d\omega &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \langle \hat{a}^\dagger(t) \hat{a}(t + \tau) \rangle_{ss} e^{i(\omega - \omega_o)\tau} d\tau d\omega \\ &= \int_{-\infty}^{\infty} \langle \hat{a}^\dagger(t) \hat{a}(t + \tau) \rangle_{ss} e^{-i\omega_o\tau} d\tau \times \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega\tau} d\omega. \end{aligned} \quad (3.11)$$

In view of the relation

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega\tau} d\omega = \delta(\tau), \quad (3.12)$$

we can put Eq. (3.11) as

$$\int_{-\infty}^{\infty} P(\omega) d\omega = \int_{-\infty}^{\infty} \langle \hat{a}^\dagger(t) \hat{a}(t + \tau) \rangle_{ss} e^{-i\omega_o \tau} \delta(\tau) d\tau. \quad (3.13)$$

Moreover in view of the relation

$$\int_{-\infty}^{\infty} A(\tau) \delta(\tau) d\tau = A(\tau) /_{\tau=0}, \quad (3.14)$$

we get

$$\int_{-\infty}^{\infty} P(\omega) d\omega = \langle \hat{a}^\dagger(t) \hat{a}(t) \rangle_{ss}. \quad (3.15)$$

On the basis of Eqs. (3.1) and (3.15) one can conclude that $P(\omega) d\omega$ represents the steady-state mean photon number of light mode from the frequency interval ω to $\omega + d\omega$ [2, 3]. For convenient, Eq. (3.10) can be written as

$$P(\omega) = \frac{1}{2\pi} \int_{-\infty}^0 \langle \hat{a}^\dagger(t) \hat{a}(t + \tau) \rangle_{ss} e^{i(\omega - \omega_o)\tau} d\tau + \frac{1}{2\pi} \int_0^{\infty} \langle \hat{a}^\dagger(t) \hat{a}(t + \tau) \rangle_{ss} e^{i(\omega - \omega_o)\tau} d\tau. \quad (3.16)$$

Now replacing t by $t + \tau$ and τ by $-\tau$ in the first term of the above equation, we have

$$P(\omega) = \frac{1}{2\pi} \int_0^{\infty} \langle \hat{a}^\dagger(t + \tau) \hat{a}(t) \rangle_{ss} e^{-i(\omega - \omega_o)\tau} d\tau + \frac{1}{2\pi} \int_0^{\infty} \langle \hat{a}^\dagger(t) \hat{a}(t + \tau) \rangle_{ss} e^{i(\omega - \omega_o)\tau} d\tau. \quad (3.17)$$

The first term is the complex conjugate of the second term. Thus the power spectrum can be written as

$$P(\omega) = \frac{1}{\pi} \text{Re} \int_0^{\infty} \langle \hat{a}^\dagger(t) \hat{a}(t + \tau) \rangle_{ss} e^{i(\omega - \omega_o)\tau} d\tau, \quad (3.18)$$

where "Re" represents the real part of the integration. In view of Eqs. (2.74), (2.78), (2.100) and (2.101), one can obtain

$$\begin{aligned} \frac{d}{dt} \langle \hat{\sigma}_a(t) \rangle &= \frac{-\eta\varepsilon}{\kappa} \langle \hat{\sigma}_a(t) \rangle - \frac{\eta}{2} \langle \hat{\sigma}_a(t) \rangle + \frac{\Omega}{2} (1 - \frac{\Omega}{\eta} (2 \langle \hat{\sigma}_a(t) \rangle)) \\ &= -v \langle \hat{\sigma}_a(t) \rangle + \frac{\Omega}{2}, \end{aligned} \quad (3.19)$$

where

$$v = \eta \left(\frac{1}{2} + \frac{\varepsilon}{\kappa} + \frac{\Omega^2}{\eta^2} \right). \quad (3.20)$$

A formal solution of Eq. (3.19) can be written as

$$\begin{aligned} \langle \hat{\sigma}_a(t + \tau') \rangle &= \langle \hat{\sigma}_a(t) \rangle e^{-v\tau'} + \frac{\Omega}{2} e^{-v\tau'} \int_0^{\tau'} e^{v\tau''} d\tau'' \\ &= \langle \hat{\sigma}_a(t) \rangle e^{-v\tau'} + \frac{\Omega}{2v} e^{-v\tau'} (e^{v\tau'} - 1) \\ &= \langle \hat{\sigma}_a(t) \rangle e^{-v\tau'} + \frac{\Omega}{2v} (1 - e^{-v\tau'}). \end{aligned} \quad (3.21)$$

Applying the quantum regression theorem to Eq. (3.21), we get

$$\langle \hat{a}^\dagger(t) \hat{\sigma}_a(t + \tau') \rangle = \langle \hat{a}^\dagger(t) \hat{\sigma}_a(t) \rangle e^{-v\tau'} + \frac{\Omega}{2v} \langle \hat{a}^\dagger(t) \rangle (1 - e^{-v\tau'}). \quad (3.22)$$

In view of Eqs. (2.46) and (2.110), we obtain

$$\begin{aligned} \frac{d}{dt} \langle \hat{a}(t) \rangle &= -\frac{\kappa}{2} \langle \hat{a}(t) \rangle - g \langle \hat{\sigma}_a(t) \rangle - \varepsilon \langle \hat{a}(t) \rangle \\ &= -\kappa \left(\frac{1}{2} + \frac{\varepsilon}{\kappa} \right) \langle \hat{a}(t) \rangle - g \langle \hat{\sigma}_a(t) \rangle \\ &= -\alpha \langle \hat{a}(t) \rangle - g \langle \hat{\sigma}_a(t) \rangle, \end{aligned} \quad (3.23)$$

where

$$\alpha = \kappa \left(\frac{1}{2} + \frac{\varepsilon}{\kappa} \right). \quad (3.24)$$

A formal solution of Eq. (3.23) can be written as

$$\langle \hat{a}(t + \tau) \rangle_{ss} = \langle \hat{a}(t) \rangle_{ss} e^{-\alpha\tau} - g e^{-\alpha\tau} \int_0^\tau e^{\alpha\tau'} \langle \hat{\sigma}_a(t + \tau') \rangle_{ss} d\tau'. \quad (3.25)$$

Applying the quantum regression theorem to Eq. (3.25), we get

$$\langle \hat{a}^\dagger(t) \hat{a}(t + \tau) \rangle_{ss} = \langle \hat{a}^\dagger(t) \hat{a}(t) \rangle_{ss} e^{-\alpha\tau} - g e^{-\alpha\tau} \int_0^\tau e^{\alpha\tau'} \langle \hat{a}^\dagger(t) \hat{\sigma}_a(t + \tau') \rangle_{ss} d\tau'. \quad (3.26)$$

Inserting Eq. (3.22) into Eq. (3.26) and carrying out the integration, we get

$$\begin{aligned} \langle \hat{a}^\dagger(t) \hat{a}(t + \tau) \rangle_{ss} &= \langle \hat{a}^\dagger(t) \hat{a}(t) \rangle_{ss} e^{-\alpha\tau} - g e^{-\alpha\tau} \int_0^\tau [\langle \hat{a}^\dagger(t) \hat{\sigma}_a(t) \rangle e^{-(v-\alpha)\tau'} \\ &\quad + \frac{\Omega}{2v} \langle \hat{a}^\dagger(t) \rangle (e^{\alpha\tau'} - e^{-(v-\alpha)\tau'})] d\tau' \\ &= \langle \hat{a}^\dagger(t) \hat{a}(t) \rangle_{ss} e^{-\alpha\tau} + \frac{g\Omega}{2v(\alpha - v)} \langle \hat{a}^\dagger(t) \rangle (e^{-v\tau} - e^{-\alpha\tau}) \\ &\quad + \frac{g}{(\alpha - v)} \langle \hat{a}^\dagger(t) \hat{\sigma}_a(t) \rangle (e^{-\alpha\tau} - e^{-v\tau}) + \frac{g\Omega}{2\alpha v} \langle \hat{a}^\dagger(t) \rangle (e^{-\alpha\tau} - 1) \end{aligned} \quad (3.27)$$

Upon multiplying Eq. (2.58) from the right by $\hat{\sigma}_a(t)$ and taking expectation values and using the assumption that the atomic and cavity mode noise operators are not correlated and in view of Eq. (2.103), we obtain

$$\langle \hat{a}^\dagger(t) \hat{\sigma}_a(t) \rangle = \frac{g\Omega^2}{4\kappa \left(\frac{1}{2} + \frac{\varepsilon}{\kappa} \right) \left(\frac{\varepsilon}{\kappa} - \frac{1}{2} \right) (\Omega^2 + \eta^2 \left(\frac{1}{2} + \frac{\varepsilon}{\kappa} \right))}. \quad (3.28)$$

Moreover, on account of Eqs. (2.109), (3.1), (3.3) and (3.28), we can re-write Eq. (3.27)

as

$$\langle \hat{a}^\dagger(t)\hat{a}(t+\tau) \rangle_{ss} = \bar{n}e^{-\alpha\tau} + \beta(e^{-v\tau} - e^{-\alpha\tau}) + \theta(e^{-\alpha\tau} - e^{-v\tau}) + \phi(e^{-\alpha\tau} - 1), \quad (3.29)$$

where

$$\beta = \frac{-\gamma_c^2 \Omega^2}{64v(\alpha - v)\left(\frac{1}{4} - \frac{\varepsilon^2}{\kappa^2}\right)\left(\frac{1}{2} + \frac{\varepsilon}{\kappa}\right)\left(\Omega^2 + \frac{\gamma_c^2}{16\left(\frac{1}{4} - \frac{\varepsilon^2}{\kappa^2}\right)^2}\left(\frac{1}{2} + \frac{\varepsilon}{\kappa}\right)\right)}, \quad (3.30)$$

$$\theta = \frac{-\gamma_c \Omega^2}{16(\alpha - v)\left(\frac{1}{2} - \frac{\varepsilon}{\kappa}\right)\left(\frac{1}{2} + \frac{\varepsilon}{\kappa}\right)\left(\Omega^2 + \frac{\gamma_c^2}{16\left(\frac{1}{4} - \frac{\varepsilon^2}{\kappa^2}\right)^2}\left(\frac{1}{2} + \frac{\varepsilon}{\kappa}\right)\right)}, \quad (3.31)$$

$$\phi = \frac{-\gamma_c^2 \Omega^2}{64\alpha v\left(\frac{1}{4} - \frac{\varepsilon^2}{\kappa^2}\right)\left(\frac{1}{2} + \frac{\varepsilon}{\kappa}\right)\left(\Omega^2 + \frac{\gamma_c^2}{16\left(\frac{1}{4} - \frac{\varepsilon^2}{\kappa^2}\right)^2}\left(\frac{1}{2} + \frac{\varepsilon}{\kappa}\right)\right)}. \quad (3.32)$$

Substituting Eq. (3.29) into Eq. (3.18), we obtain

$$\begin{aligned} P(\omega) &= \frac{\bar{n}}{\pi} \text{Re} \int_0^\infty e^{-(\alpha - i(\omega - \omega_o))\tau} d\tau \\ &+ \frac{\beta}{\pi} \text{Re} \int_0^\infty [e^{-(v - i(\omega - \omega_o))\tau} - e^{-(\alpha - i(\omega - \omega_o))\tau}] d\tau \\ &+ \frac{\theta}{\pi} \text{Re} \int_0^\infty [e^{-(\alpha - i(\omega - \omega_o))\tau} - e^{-(v - i(\omega - \omega_o))\tau}] d\tau \\ &+ \frac{\phi}{\pi} \text{Re} \int_{-0}^\infty [e^{-(\alpha - i(\omega - \omega_o))\tau} - e^{i(\omega - \omega_o)\tau}] d\tau. \end{aligned} \quad (3.33)$$

Carrying out the integration

$$\begin{aligned} \text{Re} \int_0^\infty e^{-(v - i(\omega - \omega_o))\tau} d\tau &= \text{Re} \frac{e^{-(v - i(\omega - \omega_o))\tau} \Big|_0^\infty}{-(v - i(\omega - \omega_o))} \\ &= \text{Re} \frac{-1}{-(v - i(\omega - \omega_o))} \\ &= \text{Re} \frac{v + i(\omega - \omega_o)}{v^2 + (\omega - \omega_o)^2} \\ &= \frac{v}{v^2 + (\omega - \omega_o)^2}. \end{aligned} \quad (3.34)$$

Similarly, we obtain

$$\text{Re} \int_0^\infty e^{-(\alpha - i(\omega - \omega_o))\tau} d\tau = \frac{\alpha}{\alpha^2 + (\omega - \omega_o)^2}. \quad (3.35)$$

Moreover, applying Dirac delta function, we have

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i(\omega - \omega_o)\tau} d\tau = \delta(\omega - \omega_o). \quad (3.36)$$

In view of Eqs. (3.34), (3.35) and (3.36), the power spectrum described by Eq. (3.33) can be expressed as

$$\begin{aligned} P(\omega) &= \frac{\bar{n}}{\pi} \left[\frac{\alpha}{\alpha^2 + (\omega - \omega_o)^2} \right] \\ &+ \frac{\beta}{\pi} \left[\frac{v}{v^2 + (\omega - \omega_o)^2} - \frac{\alpha}{\alpha^2 + (\omega - \omega_o)^2} \right] \\ &+ \frac{\theta}{\pi} \left[\frac{\alpha}{\alpha^2 + (\omega - \omega_o)^2} - \frac{v}{v^2 + (\omega - \omega_o)^2} \right] \\ &+ \frac{\phi}{\pi} \left[\frac{\alpha}{\alpha^2 + (\omega - \omega_o)^2} \right] \\ &- \phi \delta(\omega - \omega_o). \end{aligned} \quad (3.37)$$

And with the aid of Eq. (3.3), one can write Eq. (3.20) as

$$v = \frac{\gamma_c^2 \kappa + 2\gamma_c + \Omega^2 (1 - \frac{4\epsilon^2}{\kappa^2})^2}{2\gamma_c \kappa (1 - \frac{4\epsilon^2}{\kappa^2})}. \quad (3.38)$$

Now on account of Eq. (3.15), the mean photon number in the frequency interval $-z$ to z can be written as

$$\bar{n}_{\pm z} = \int_{-z}^z P(\omega') d\omega', \quad (3.39)$$

where $\omega' = \omega - \omega_o$. Substituting Eq. (3.37) in to (3.39), we have

$$\begin{aligned} \bar{n}_{\pm z} &= \frac{\bar{n}}{\pi} \alpha \int_{-z}^z \frac{d\omega'}{\alpha^2 + \omega'^2} + \frac{\beta}{\pi} \left[v \int_{-z}^z \frac{d\omega'}{v^2 + \omega'^2} - \alpha \int_{-z}^z \frac{d\omega'}{\alpha^2 + \omega'^2} \right] \\ &+ \frac{\theta}{\pi} \left[\alpha \int_{-z}^z \frac{d\omega'}{\alpha^2 + \omega'^2} - v \int_{-z}^z \frac{d\omega'}{v^2 + \omega'^2} \right] + \frac{\phi}{\pi} \alpha \int_{-z}^z \frac{d\omega'}{\alpha^2 + \omega'^2} - \phi \int_{-z}^z \delta(\omega') d\omega'. \end{aligned} \quad (3.40)$$

Carrying out the integration, by using the relation

$$\int_{-z}^z \frac{dy}{x^2 + y^2} = \frac{2}{x} \tan^{-1}\left(\frac{z}{x}\right), \quad (3.41)$$

and

$$\int_{-z}^z \delta(\omega') d\omega' = 1, \quad (3.42)$$

the local mean photon number is expressible as

$$\begin{aligned} \bar{n}_{\pm z} &= \frac{2\bar{n}}{\pi} \tan^{-1}\left(\frac{z}{\alpha}\right) + \frac{2\beta}{\pi} \left[\tan^{-1}\left(\frac{z}{v}\right) - \tan^{-1}\left(\frac{z}{\alpha}\right) \right] \\ &+ \frac{2\theta}{\pi} \left[\tan^{-1}\left(\frac{z}{\alpha}\right) - \tan^{-1}\left(\frac{z}{v}\right) \right] + \frac{2\phi}{\pi} \tan^{-1}\left(\frac{z}{\alpha}\right) - \phi \end{aligned} \quad (3.43)$$

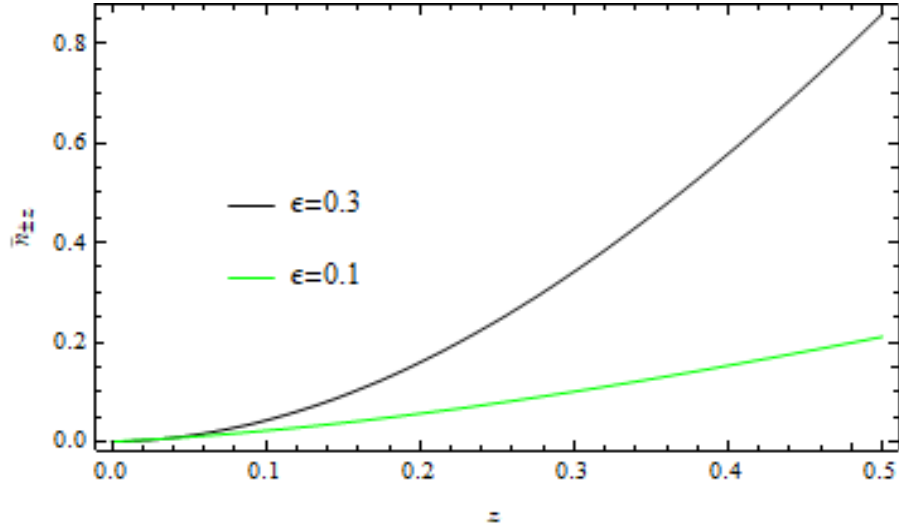


Figure 3.3: The plots of local mean photon number [Eq. (3.43)] versus z , for $\gamma_c = 0.7$, $\kappa = 0.8$, $\Omega = 0.5$, $r = 0.2$ and different values of ϵ .

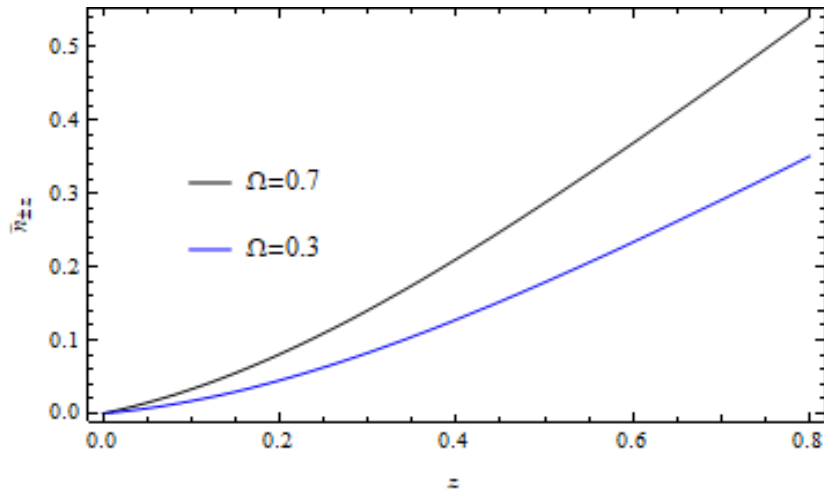


Figure 3.4: The plots of local mean photon number [Eq. (3.43)] versus z , for $\gamma_c = 0.7$, $\kappa = 0.8$, $\epsilon = 0.2$, $r = 0.3$ and different values of Ω .

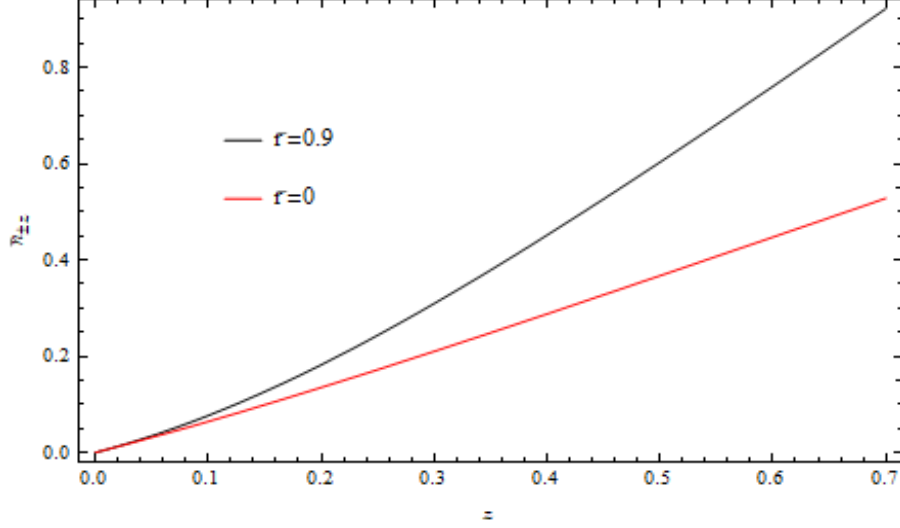


Figure 3.5: The plots of local mean photon number [Eq. (3.43)] versus z , for $\gamma_c = 0.7$, $\kappa = 0.8$, $\Omega = 0.5$, $\varepsilon = 0.27$ and different values of r .

We observe from the plots in figures 3.3, 3.4 and 3.5 that the cavity mode local mean photon number increases significantly with the increase of frequency interval, the squeeze parameter, the amplitudes of the driving coherent light and parametric pump mode. Moreover, in view of Figures 3.2 and 3.5, we notice that the local mean photon number approaches to the global mean photon number as the frequency interval increases for a squeezed parameter(r)=0.9.

3.3 Second order correlation function

The normalized second order correlation function for the light emitted by a two level atom in a cavity can be expressed, in terms of atomic operators as[12-16]

$$g^{(2)}(\tau) = \frac{\langle \hat{\sigma}_a^\dagger(t) \hat{\sigma}_a^\dagger(t+\tau) \hat{\sigma}_a(t+\tau) \hat{\sigma}_a(t) \rangle}{\langle \hat{\sigma}_a^\dagger(t) \hat{\sigma}_a(t) \rangle^2}. \quad (3.44)$$

In view of Eqs. (2.98) and (2.100), we can express Eq. (2.75) as

$$\begin{aligned} \frac{d}{dt} \langle \hat{\eta}_a(t) \rangle &= -\eta \langle \hat{\eta}_a(t) \rangle + \Omega \langle \hat{\sigma}_a(t) \rangle \\ &= -\eta \langle \hat{\eta}_a(t) \rangle + \frac{\Omega^2}{\eta(1 + \frac{2\varepsilon}{\kappa})} (\langle \hat{\eta}_b(t) \rangle - \langle \hat{\eta}_a(t) \rangle). \end{aligned} \quad (3.45)$$

Again on the base of Eq. (2.78), we arrive at

$$\frac{d}{dt} \langle \hat{\eta}_a(t) \rangle = -\lambda \langle \hat{\eta}_a(t) \rangle + \frac{\Omega^2}{\eta(1 + \frac{2\varepsilon}{\kappa})}, \quad (3.46)$$

where

$$\lambda = \frac{(2\Omega^2 + \eta^2(1 + \frac{2\varepsilon}{\kappa}))}{\eta(1 + \frac{2\varepsilon}{\kappa})}. \quad (3.47)$$

Moreover, with the base of Eq. (3.3), we can rewrite Eq. (3.47) as

$$\lambda = \frac{2\Omega^2(1 - \frac{4\varepsilon^2}{\kappa^2})^2 + \gamma_c^2(1 + \frac{2\varepsilon}{\kappa})}{\gamma_c(1 + \frac{2\varepsilon}{\kappa})(1 - \frac{4\varepsilon^2}{\kappa^2})}. \quad (3.48)$$

A formal solution of Eq. (3.46) can be expressed as

$$\begin{aligned} \langle \hat{\eta}_a(t + \tau) \rangle &= \langle \hat{\eta}_a(t) \rangle e^{-\lambda\tau} + \frac{\Omega^2}{\eta(1 + \frac{2\varepsilon}{\kappa})} \int_0^\tau e^{-\lambda(\tau-\tau')} d\tau' \\ &= \langle \hat{\eta}_a(t) \rangle e^{-\lambda\tau} + \frac{\Omega^2}{\lambda\eta(1 + \frac{2\varepsilon}{\kappa})} (1 - e^{-\lambda\tau}), \end{aligned} \quad (3.49)$$

so that using the relation

$$\langle \hat{\sigma}_a^\dagger(t) \hat{\sigma}_a(t) \rangle = \langle \hat{\eta}_a(t) \rangle, \quad (3.50)$$

Eq. (3.49) can be expressed as

$$\langle \hat{\sigma}_a^\dagger(t + \tau) \hat{\sigma}_a(t + \tau) \rangle = \langle \hat{\sigma}_a^\dagger(t) \hat{\sigma}_a(t) \rangle e^{-\lambda\tau} + \frac{\Omega^2}{\lambda\eta(1 + \frac{2\varepsilon}{\kappa})} (1 - e^{-\lambda\tau}). \quad (3.51)$$

Applying the quantum regression theorem to Eq. (3.51), we obtain

$$\begin{aligned} \langle \hat{\sigma}_a^\dagger(t) \hat{\sigma}_a^\dagger(t + \tau) \hat{\sigma}_a(t + \tau) \hat{\sigma}_a(t) \rangle &= \langle \hat{\sigma}_a^{\dagger 2}(t) \hat{\sigma}_a^2(t) \rangle e^{-\lambda\tau} + \frac{\Omega^2 \langle \hat{\sigma}_a^\dagger(t) \hat{\sigma}_a(t) \rangle}{\lambda\eta(1 + \frac{2\varepsilon}{\kappa})} (1 - e^{-\lambda\tau}) \\ &= \frac{\Omega^2 \langle \hat{\sigma}_a^\dagger(t) \hat{\sigma}_a(t) \rangle}{\lambda\eta(1 + \frac{2\varepsilon}{\kappa})} (1 - e^{-\lambda\tau}). \end{aligned} \quad (3.52)$$

Substituting Eq. (3.52) into Eq. (3.44) and using Eq. (3.50), we obtain

$$g^{(2)}(\tau) = \frac{\Omega^2}{\lambda\eta \langle \hat{\eta}_a(t) \rangle (1 + \frac{2\varepsilon}{\kappa})} (1 - e^{-\lambda\tau}). \quad (3.53)$$

With the aid of Eqs. (2.103) and (3.47), one can express Eq. (3.53) as

$$\begin{aligned} g^{(2)}(\tau) &= \frac{\Omega^2}{(\frac{\Omega^2}{2\Omega^2 + \eta^2(1 + \frac{2\varepsilon}{\kappa})}) (\frac{(2\Omega^2 + \eta^2(1 + \frac{2\varepsilon}{\kappa}))}{\eta(1 + \frac{2\varepsilon}{\kappa})}) \eta(1 + \frac{2\varepsilon}{\kappa})} (1 - e^{-\lambda\tau}) \\ &= 1 - e^{-\lambda\tau}. \end{aligned} \quad (3.54)$$

This is the steady state second order correlation function and it is similar to the result described in references[3] and [12].

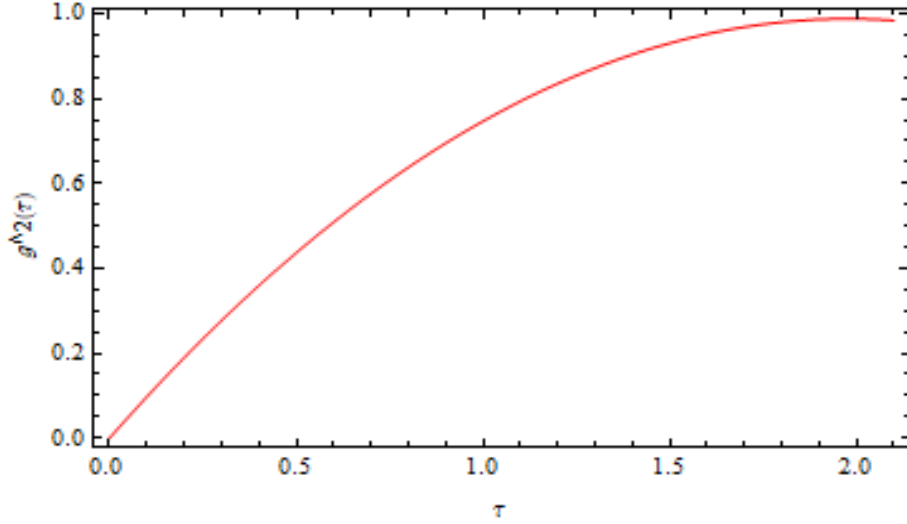


Figure 3.6: The plot of second order correlation function $g^{(2)}(\tau)$ in [Eqs.(3.48) and (3.54)] versus τ , for $\Omega = 0.7, \gamma_c = 0.7, \kappa = 0.8$ and $\varepsilon = 0.2$.

In view of Figure 3.6, we observe that $g^{(2)}(0) = 0$ and $g^{(2)}(\tau) > 0$ for $\tau > 0$. This implies $g^{(2)}(\tau) > g^{(2)}(0)$, and the fluorescent light thus exhibits the phenomenon of photon anti-bunching. Because a two level atom can not emit two or more photons simultaneously. After each emission the atom returns to the lower level and absorb a photon before another emission takes place and the photons have a tendency to arrive at a detector separately rather than in pair.

Chapter 4

Quadrature Squeezing

4.1 Quadrature variance for the cavity mode

Here, we seek to analyze the quadrature variance and the quadrature squeezing of the light emitted by the system under consideration. The squeezing properties of a single-mode light can be described by plus and minus quadrature operators[11-14]

$$\hat{a}_+ = \hat{a}^\dagger + \hat{a}, \quad (4.1)$$

and

$$\hat{a}_- = i(\hat{a}^\dagger - \hat{a}). \quad (4.2)$$

The minus quadrature variance for a light beam in terms of the minus quadrature operator can be expressed as

$$(\Delta a_-)^2 = \langle \hat{a}_-^2 \rangle - \langle \hat{a}_- \rangle^2, \quad (4.3)$$

which on the base of Eq. (4.2) becomes

$$(\Delta a_-)^2 = -\langle \hat{a}^{\dagger 2} \rangle + \langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{a} \hat{a}^\dagger \rangle - \langle \hat{a}^2 \rangle + \langle \hat{a}^\dagger \rangle^2 + \langle \hat{a} \rangle^2 - 2\langle \hat{a}^\dagger \rangle \langle \hat{a} \rangle. \quad (4.4)$$

Applying the same procedure, the plus quadrature variance is expressed as

$$(\Delta a_+)^2 = \langle \hat{a}^{\dagger 2} \rangle + \langle \hat{a}^2 \rangle + \langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{a} \hat{a}^\dagger \rangle - \langle \hat{a}^\dagger \rangle^2 - \langle \hat{a} \rangle^2 - 2\langle \hat{a}^\dagger \rangle \langle \hat{a} \rangle. \quad (4.5)$$

In view of Eqs. (2.110) and (2.117), we can re-write Eqs. (4.4) and (4.5) as

$$(\Delta a_-)^2 = -2\langle \hat{a}^2 \rangle + \langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{a} \hat{a}^\dagger \rangle, \quad (4.6)$$

and

$$(\Delta a_+)^2 = 2\langle \hat{a}^2 \rangle + \langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{a} \hat{a}^\dagger \rangle - 4\langle \hat{a} \rangle^2. \quad (4.7)$$

Moreover, in view of Eqs. (2.108), (2.110) and (3.3), we have

$$\langle \hat{a}^\dagger \rangle^2 = \langle \hat{a} \rangle^2 = \frac{\gamma_c^3 \Omega^2}{16\kappa(1 - \frac{4\varepsilon^2}{\kappa^2})^2(\frac{1}{2} + \frac{\varepsilon}{\kappa})^2[\Omega^2 + (\frac{\gamma_c}{1 - \frac{4\varepsilon^2}{\kappa^2}})^2(\frac{1}{2} + \frac{\varepsilon}{\kappa})]^2} \quad (4.8)$$

Substituting Eqs. (2.119), (2.122) and (3.2) into Eq. (4.6) and applying Eqs. (2.8), (2.9) and (3.3), we obtain

$$(\Delta a_-)^2 = \Phi + \Psi, \quad (4.9)$$

and again substituting Eqs. (2.119), (2.122), (3.2), and (4.8) into Eq. (4.7) and taking into account Eqs. (2.8), (2.9) and (3.3), we get

$$(\Delta a_+)^2 = \Phi - \Psi - \Gamma, \quad (4.10)$$

where

$$\Phi = \frac{\frac{\gamma_c(\kappa^2 + 4\varepsilon^2)}{\kappa(\kappa^2 - 4\varepsilon^2)} - \frac{\varepsilon}{\kappa}[e^{2r} - e^{-2r}] + \frac{e^{2r} + e^{-2r} - 2}{2} + 1}{1 - \frac{4\varepsilon^2}{\kappa^2}}, \quad (4.11)$$

$$\Psi = \frac{\frac{4\varepsilon\gamma_c}{(\kappa^2 - 4\varepsilon^2)} + \frac{2\varepsilon}{\kappa}(\frac{e^{2r} + e^{-2r} - 2}{2} + 1) - (\frac{e^{2r} - e^{-2r}}{2})}{1 - \frac{4\varepsilon^2}{\kappa^2}}, \quad (4.12)$$

$$\Gamma = \frac{\gamma_c^3 \Omega^2}{16\kappa(1 - \frac{4\varepsilon^2}{\kappa^2})^2(\frac{1}{2} + \frac{\varepsilon}{\kappa})^2[\Omega^2 + (\frac{\gamma_c}{1 - \frac{4\varepsilon^2}{\kappa^2}})^2(\frac{1}{2} + \frac{\varepsilon}{\kappa})]^2}. \quad (4.13)$$

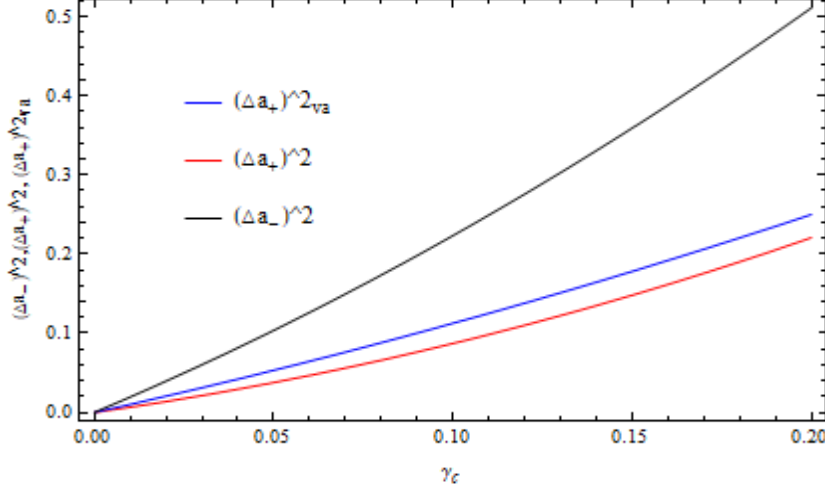


Figure 4.1: The plots of minus and plus quadrature variances in [Eqs. (4.9) and (4.10)] versus γ_c , for ($\varepsilon = 0.3$, $\Omega = 0.7$, $\kappa = 0.8$, $r = 0.5$) and plus quadrature variance in vacuum level.

The plots show that the plus quadrature variance is less than the plus quadrature variance in vacuum level. This shows that the cavity light is in a squeezed state and the squeezing occurs in the plus quadrature.

4.2 Quadrature squeezing

Here, we seek to study the squeezing properties of the light generated by two level atom. The quadrature squeezing of the cavity light, relative to the vacuum level, can be expressed as [15- 18]

$$S = \frac{(\Delta a_+)_{va}^2 - (\Delta a_+)^2}{(\Delta a_+)_{va}^2}, \quad (4.14)$$

where S is quadrature squeezing of the cavity light and $(\Delta a_+)_{va}^2$ is the quadrature variance of the cavity light in vacuum state. The quadrature variance of the cavity light in vacuum state can be obtained by putting Ω , ε and $r = 0$ in Eq. (4.10). We therefore obtain

$$(\Delta a_+)_{va}^2 = \frac{\gamma_c}{\kappa} + 1. \quad (4.15)$$

Now substituting Eqs. (4.10) and (4.15) into (4.14), we obtain

$$S = 1 - (\theta + \xi + \lambda), \quad (4.16)$$

where

$$\theta = \frac{\frac{\gamma_c(\kappa^2+4\epsilon^2)}{\kappa(\kappa^2-4\epsilon^2)} - \frac{\epsilon}{\kappa}[e^{2r} - e^{-2r}] + (\frac{e^{2r}+e^{-2r}-2}{2}) + 1}{(\frac{\gamma_c}{\kappa} + 1)(1 - \frac{4\epsilon^2}{\kappa^2})}, \quad (4.17)$$

$$\xi = \frac{-\frac{4\epsilon\gamma_c}{(\kappa^2-4\epsilon^2)} - \frac{2\epsilon}{\kappa}(\frac{e^{2r}+e^{-2r}-2}{2} + 1) + \frac{e^{2r}-e^{-2r}}{2}}{(\frac{\gamma_c}{\kappa} + 1)(1 - \frac{4\epsilon^2}{\kappa^2})}, \quad (4.18)$$

$$\lambda = \frac{-\gamma_c^3\Omega^2}{16\kappa(\frac{\gamma_c}{\kappa} + 1)(1 - \frac{4\epsilon^2}{\kappa^2})^2(\frac{1}{2} + \frac{\epsilon}{\kappa})^2[\Omega^2 + (\frac{\gamma_c}{1-\frac{4\epsilon^2}{\kappa^2}})^2(\frac{1}{2} + \frac{\epsilon}{\kappa})^2]}. \quad (4.19)$$

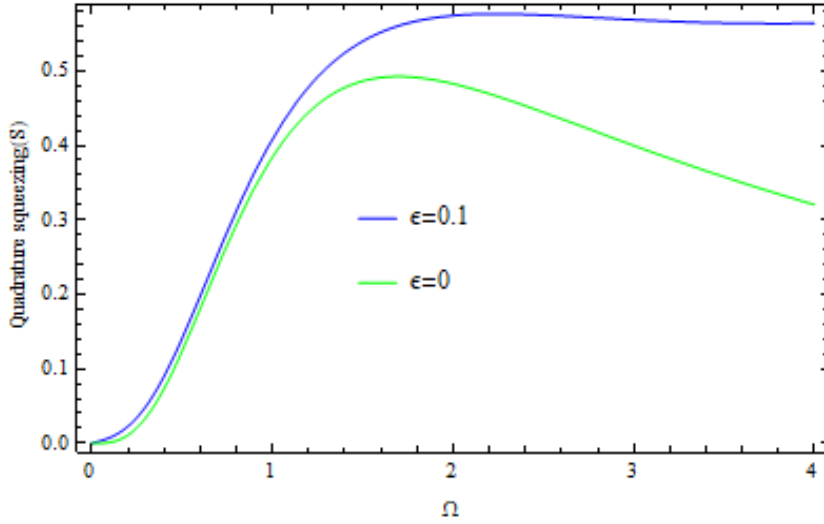


Figure 4.2: The plots of quadrature squeezing [Eq. (4.16)] versus Ω , for $(\gamma_c = 0.7, \kappa = 0.8, r = 0.2)$ and different values of ϵ .

The plot shows that the squeezing increases as the amplitude of driving coherent light increases; with a maximum squeezing of 60% for $(\epsilon = 0.1)$ and 50% in the absence of parametric amplifier. We thus observe that the presence of the parametric amplifier enhances the degree of squeezing and this result is similar with the results given in references[17] and [18].

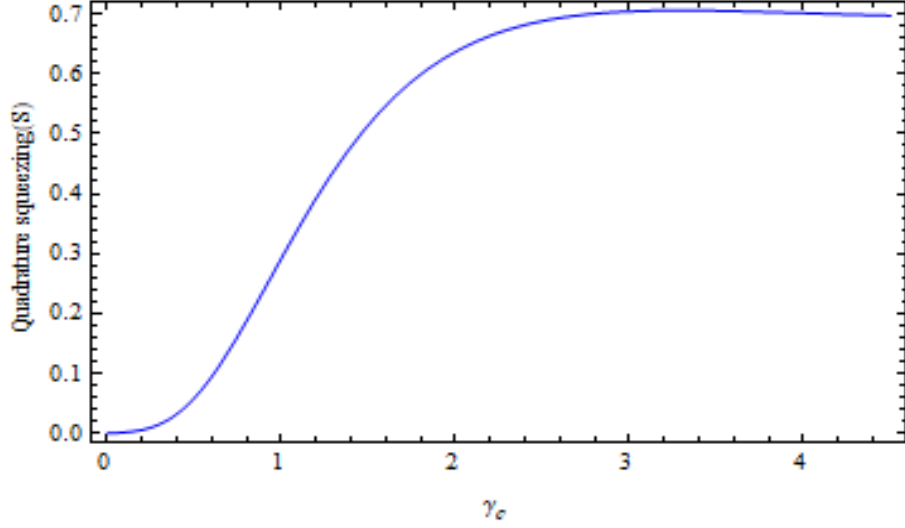


Figure 4.3: The plot of quadrature squeezing [Eq. (4.16)] versus γ_c for $\Omega = 0.9$, $\kappa = 0.8$, $\epsilon = 0.3$ and $r = 0.2$.

The plot in Figure 4.3 shows that the degree of squeezing enhances with the increase of stimulated emission decay constant and reaches a maximum value of 70% below the vacuum state level and this result is similar with the result given in reference[17].

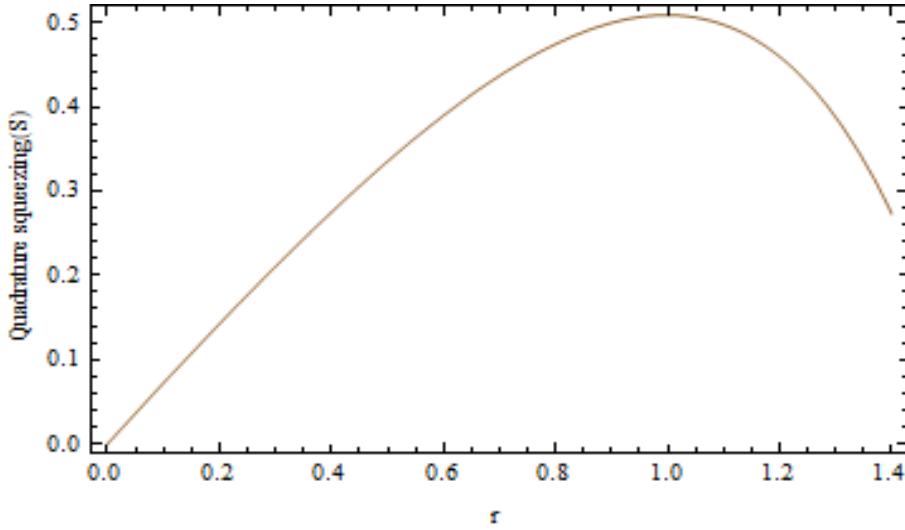


Figure 4.4: The plot of quadrature squeezing [Eq. (4.16)] versus r for $\Omega = 0.8$, $\kappa = 0.8$, $\epsilon = 0.3$ and $\gamma_c = 0.7$.

The plot in Figure 4.4 shows that the degree of squeezing enhances with the increase of squeezed parameter (r) and reaches a maximum value of 50% below the vacuum state level.

Chapter 5

Conclusion

We have studied the quantum properties of a light produced by coherently driven two-level atom available in a cavity containing parametric amplifier and coupled to squeezed vacuum reservoir. Applying the Master equation, the time evolution of the expectation values of the cavity and atomic operators have been determined. Applying the large -time approximation scheme, we obtained the steady-state solutions of the cavity mode and atomic operators.

The mean photon number, the power spectrum of the cavity mode, second order correlation function and quadrature squeezing are also calculated employing the steady-state solutions. Our results indicated that the probability of finding two level atom in the lower level is greater than the probability of finding it in the upper level. As we easily observed from Figures, 3.1 and 3.2, the presence of parametric amplifier, driving coherent light and squeezed vacuum reservoir enhance the global mean photon number significantly.

Again in view of Figures, 3.3, 3.4 and 3.5, we note that the increase of the amplitude (Ω) of the atomic level coupling coherent light, the amplitude (ϵ) of the light pumping the parametric amplifier, and squeeze parameter (r) enhance the local mean photon number of the cavity light. We have observed that the photons of the fluorescent light are anti-bunched. Moreover, our results have shown that the light generated by the system is squeezed and the squeezing occurs in the plus quadrature. In addition, the squeezing increases as the amplitude of driving coherent light increases; with a maximum squeezing of 60% for ($\epsilon = 0.1$) and 50% in the absence of parametric amplifier. And the degree of squeezing enhances with the increasing of stimulated emission decay constant and reaches a maximum value of 70% below the vacuum state level.

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DECLARATION

▷I, hereby declare that this thesis is my original work and has not been presented for a degree in any other un

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