DYNAMICS OF A COHERENTLY DRIVEN NONDEGENERATE THREE-LEVEL ATOM

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Abstract

In this thesis we study the statistical and squeezing properties of the light emitted by a nondegenerate three-level atom driven by coherent light and in a cavity coupled to a vacuum reservior via a single-port mirror. We carry out our analysis by putting the noise operators associated with the vacuum reservior in normal order and applying the large-time approximation.

It is found that the mean of the two cavity modes are equal. In addition, the photon number statistics is Poissonian for $\eta < 0.13$ and sub-Poissonian for $\eta \geq 0.13$. Our result shows that the photon numbers of modes a and b are uncorrelated. We have also calculated the quadrature variance for the separate modes and for the superposition of the two modes. It turns out that mode a is in a chaotic state for arbitrary values of γ_c and Ω but mode b is in chaotic state for $\gamma_c << \Omega$ and in a coherent state for $\gamma_c >> \Omega$ and the superposed cavity modes are in a squeezed state. Furthermore, we have determined the quadrature squeezing of the superposed cavity modes. The maximum quadrature squeezing is found to be 43% below the coherent-state level.

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Chapter 1

Introduction

The interaction of a three-level atom with a cavity mode has attracted a great deal of interest in recent years [1-7]. It is belived that atomic coherence is found to be responsible for various important quantum features of the emitted photons. In general, the atomic coherence can be induced in a three-level atom by preparing the atom initially in a coherent superposition of the top and bottom levels or by coupling these levels by coherent light after it is injected into the cavity [2, 4, 5, 6, 8, 9]. The superposition or the coupling of the top and bottom levels is responsible for the interesting nonclassical features of the emitted photons.

In a three-level atom the top, intermediate, and the bottom levels are denoted by $|a\rangle$, $|b\rangle$, and $|c\rangle$ in which the transitions between levels $|a\rangle$ and $|b\rangle$ and $|b\rangle$ and $|c\rangle$ are assumed to be dipole allowed, with direct transition between levels $|a\rangle$ and $|c\rangle$ to be dipole forbidden. When the atom makes a transition from the top to the intermediate level and then from the intermediate to the bottom level, two photons are emitted. If the two photons have different frequencies, then the three-level atom is called a nondegenerate three-level atom otherwise it is called degenerate.

Some authors have studied the statistical and the squeezing properties of the light produced by a three-level atom in which the crucial role is played by the superposition of the top and bottom levels [2, 3, 4, 5, 10, 11,12, 13]. It is found that the cavity modes exhibit squeezing under certain conditions. On the other hand, a three-level atom in which the top and bottom levels coupled by a coherent light has been studied by different authors [3, 5, 7, 14]. They have predicted that such a system can generate squeezed light over a large-range of the amplitude of the coherent light. The squeezing in this case is due to the coupling of the top and bottom levels.

In this thesis, we consider a nondegenerate three-level atom, driven by coherent light and inside a cavity coupled to a vacuum reservior via a single-port mirror. We carry out our calculation by putting the noise operators associated with the vacuum reservior in normal order. With the aid of the cavity mode operators obtained by solving the quantum Langevin equations using the large-time approximation, we calculate the probability for the atom to be in the top, intermediate, or bottom level.

In addition, applying the steady-state solutions of the quantum Langevin equations for the cavity mode operators, we calculate the mean of the photon number sum and difference, the variance of the photon number sum and difference and the photon number correlation. Moreover, we determine the quadrature variance for the separate modes and for the superposition of the two modes. We then calculate the quadrature squeezing for the superposed cavity modes.

Chapter 2

OPERATOR DYNAMICS

We consider here a nondegenerate three-level atom available in a closed cavity. We denote the top, intermediate, and bottom levels by $|a\rangle$, $|b\rangle$, $|c\rangle$, respectively. When the atom makes a transition from $|a\rangle$ to $|b\rangle$ and from $|b\rangle$ to $|c\rangle$, with different frequencies, ω_{ab} and ω_{bc} the quantum optical system is said to be a nondegenerate three-level atom. The direct transition between $|a\rangle$ and $|c\rangle$ is dipole forbidden.

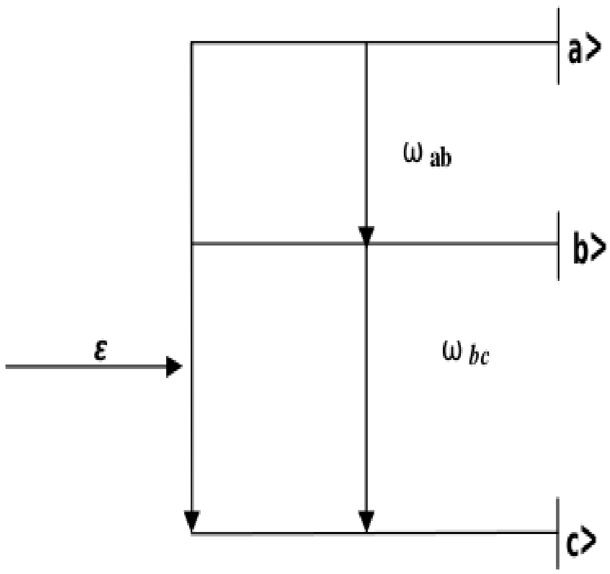

Figure 2.1: Coherently driven nondegenerate three-level atom.

The interaction of a nondegenerate three-level atom with two cavity modes can be described by the Hamilitonian

$$\hat{H}' = ig(\hat{\sigma}_a^\dagger \hat{a} - \hat{a}^\dagger \hat{\sigma}_a + \hat{\sigma}_b^\dagger \hat{b} - \hat{b}^\dagger \hat{\sigma}_b), \tag{2.0.1}$$

where g is the coupling constant between the atom and the cavity modes, $\hat{a}$, $\hat{b}$ are the annihilation operators for the cavity modes. On the other hand, the coupling of the top and the bottom levels by the coherent light can be described by the Hamilitonian

$$\hat{H}'' = \frac{i\Omega}{2}(\hat{\sigma}_c^\dagger - \hat{\sigma}_c), \tag{2.0.2}$$

where $\Omega = 2g\varepsilon$ is the Rabi frequency, with ε being the amplitude of the coherent light. Hence, on the basis of Eqs. (2.0.1) and (2.0.2) the interaction of a coherently deriven three-level atom with the cavity modes can be described by the Hamilitonian

$$\hat{H} = ig(\hat{\sigma}_a^\dagger\hat{a} - \hat{a}^\dagger\hat{\sigma}_a + \hat{\sigma}_b^\dagger\hat{b} - \hat{b}^\dagger\hat{\sigma}_b) + \frac{i\Omega}{2}(\hat{\sigma}_c^\dagger - \hat{\sigma}_c), \tag{2.0.3}$$

where

$$\hat{\sigma}_a = |b\rangle\langle a|, \tag{2.0.4}$$

$$\hat{\sigma}_b = |c\rangle\langle b|, \tag{2.0.5}$$

$$\hat{\sigma}_c = |c\rangle\langle a|, \tag{2.0.6}$$

are lowering atomic operators. We assume that the cavity modes are coupled to a vacuum reservoir via single-port mirror. In addition, we carry out our calculation by putting the noise operators associated with the vacuum reservoir in normal order. Thus the noise operators will not have any effect on the cavity mode operators. Therefore, we can drop the noise operator and write the quantum Langevin equation for the operators $\hat{a}$ and $\hat{b}$ as

$$\frac{d}{dt}\hat{a} = -\frac{\kappa}{2}\hat{a} - i[\hat{a}, \hat{H}] \tag{2.0.7}$$

and

$$\frac{d}{dt}\hat{b} = -\frac{\kappa}{2}\hat{b} - i[\hat{b}, \hat{H}], \tag{2.0.8}$$

where κ is the cavity damping constant. Then with the aid of Eq. (2.0.3), we easily find

$$\frac{d}{dt}\hat{a} = -\frac{\kappa}{2}\hat{a} - g\hat{\sigma}_a \tag{2.0.9}$$

and

$$\frac{d}{dt}\hat{b} = -\frac{\kappa}{2}\hat{b} - g\hat{\sigma}_b. \quad (2.0.10)$$

Furthermore, applying the Heisenberg equation

$$\frac{d}{dt}\langle\hat{A}\rangle = -i\langle[\hat{A},\hat{H}]\rangle \quad (2.0.11)$$

along with Eq.(2.0.3), one easily obtains

$$\frac{d}{dt}\langle\hat{\sigma}_a\rangle = \langle[\hat{\sigma}_a, g(\hat{\sigma}_a^\dagger\hat{a} - \hat{a}^\dagger\hat{\sigma}_a + \hat{\sigma}_b^\dagger\hat{b} - \hat{b}^\dagger\hat{\sigma}_b) + \frac{\Omega}{2}(\hat{\sigma}_c^\dagger - \hat{\sigma}_c)]\rangle, \quad (2.0.12)$$

$$\frac{d}{dt}\langle\hat{\sigma}_b\rangle = \langle[\hat{\sigma}_b, g(\hat{\sigma}_a^\dagger\hat{a} - \hat{a}^\dagger\hat{\sigma}_a + \hat{\sigma}_b^\dagger\hat{b} - \hat{b}^\dagger\hat{\sigma}_b) + \frac{\Omega}{2}(\hat{\sigma}_c^\dagger - \hat{\sigma}_c)]\rangle, \quad (2.0.13)$$

$$\frac{d}{dt}\langle\hat{\sigma}_c\rangle = \langle[\hat{\sigma}_c, g(\hat{\sigma}_a^\dagger\hat{a} - \hat{a}^\dagger\hat{\sigma}_a + \hat{\sigma}_b^\dagger\hat{b} - \hat{b}^\dagger\hat{\sigma}_b) + \frac{\Omega}{2}(\hat{\sigma}_c^\dagger - \hat{\sigma}_c)]\rangle. \quad (2.0.14)$$

We then see that

$$\frac{d}{dt}\langle\hat{\sigma}_a\rangle = g\langle\hat{\eta}_b\hat{a}\rangle - g\langle\hat{\eta}_a\hat{a}\rangle + g\langle\hat{b}^\dagger\hat{\sigma}_c\rangle + \frac{\Omega}{2}\langle\hat{\sigma}_b^\dagger\rangle, \quad (2.0.15)$$

$$\frac{d}{dt}\langle\hat{\sigma}_b\rangle = g\langle\hat{\eta}_c\hat{b}\rangle - g\langle\hat{\eta}_b\hat{b}\rangle - g\langle\hat{a}^\dagger\hat{\sigma}_c\rangle - \frac{\Omega}{2}\langle\hat{\sigma}_a^\dagger\rangle, \quad (2.0.16)$$

$$\frac{d}{dt}\langle\hat{\sigma}_c\rangle = g\langle\hat{\sigma}_b\hat{a}\rangle - g\langle\hat{\sigma}_a\hat{b}\rangle + \frac{\Omega}{2}\langle\hat{\eta}_c\rangle - \frac{\Omega}{2}\langle\hat{\eta}_a\rangle, \quad (2.0.17)$$

where

$$\hat{\eta}_a = |a\rangle\langle a|, \quad (2.0.18)$$

$$\hat{\eta}_b = |b\rangle\langle b|, \quad (2.0.19)$$

$$\hat{\eta}_c = |c\rangle\langle c|. \quad (2.0.20)$$

Next we seek to calculate the probability for the three-level atom to be in the top, intermediate, and bottom level by using Heisenberg equation

$$\frac{d}{dt}\langle|a\rangle\langle a|\rangle = -i\langle[|a\rangle\langle a|, \hat{H}]\rangle. \tag{2.0.21}$$

Since $\langle|a\rangle\langle a|\rangle = \rho_{aa}$, this can be rewritten as

$$\frac{d}{dt}\rho_{aa} = -i\langle[|a\rangle\langle a|, \hat{H}]\rangle, \tag{2.0.22}$$

so that with the aid of Eq. (2.0.3), we have

$$\frac{d}{dt}\rho_{aa} = g\langle\hat{\sigma}_a^\dagger\hat{a}\rangle + g\langle\hat{a}^\dagger\hat{\sigma}_a\rangle + \frac{\Omega}{2}(\rho_{ac} + \rho_{ca}), \tag{2.0.23}$$

$$\frac{d}{dt}\rho_{bb} = -g\langle\hat{\sigma}_a^\dagger\hat{a}\rangle - g\langle\hat{a}^\dagger\hat{\sigma}_a\rangle + g\langle\hat{\sigma}_b^\dagger\hat{b}\rangle + g\langle\hat{b}^\dagger\hat{\sigma}_b\rangle, \tag{2.0.24}$$

$$\frac{d}{dt}\rho_{cc} = -g\langle\hat{\sigma}_b^\dagger\hat{b}\rangle - g\langle\hat{b}^\dagger\hat{\sigma}_b\rangle - \frac{\Omega}{2}(\rho_{ac} + \rho_{ca}), \tag{2.0.25}$$

$$\frac{d}{dt}\rho_{ac} = g\langle\hat{a}^\dagger\hat{\sigma}_b^\dagger\rangle - g\langle\hat{b}^\dagger\hat{\sigma}_a^\dagger\rangle + \frac{\Omega}{2}(\rho_{cc} - \rho_{aa}). \tag{2.0.26}$$

in which

$$\rho_{bb} = \langle b|\rho|b\rangle, \tag{2.0.27}$$

$$\rho_{cc} = \langle c|\rho|c\rangle, \tag{2.0.28}$$

$$\rho_{ac} = \langle a|\rho|c\rangle, \tag{2.0.29}$$

with ρ_{bb} and ρ_{cc} being the probability for the atom to be the intermediate and bottom levels, respectively.

We see that Eqs. (2.0.15-2.0.17) and (2.0.23- 2.0.26) are nonlinear and coupled differential equations and hence it is not possible to obtain exact time-dependent solution of these equations. We intend to overcome this problem by applying the large-time approximation [15]. Thus applying the large-tiime approximation scheme, we obtain from Eqs. (2.0.9) and (2.0.10) the approximately valid relations

$$\hat{a}(t) = -\frac{2g}{\kappa}\hat{\sigma}_a(t) \tag{2.0.30}$$

and

$$\hat{b}(t) = -\frac{2g}{\kappa}\hat{\sigma}_b(t). \tag{2.0.31}$$

Evidently, these turn out to be exact relations at steady state ($t \to \infty$). Now introducing Eqs. (2.0.30), and (2.0.31) into Eqs. (2.0.15), (2.0.16), (2.0.17), (2.0.23), (2.0.24), (2.0.25), and (2.0.26), we get

$$\frac{d}{dt}\langle\hat{\sigma}_a\rangle = -\gamma_c\langle\hat{\sigma}_a\rangle + \frac{\Omega}{2}\langle\hat{\sigma}_b^\dagger\rangle, \tag{2.0.32}$$

$$\frac{d}{dt}\langle\hat{\sigma}_b\rangle = -\frac{\gamma_c}{2}\langle\hat{\sigma}_b\rangle - \frac{\Omega}{2}\langle\hat{\sigma}_a^\dagger\rangle, \tag{2.0.33}$$

$$\frac{d}{dt}\langle\hat{\sigma}_c\rangle = -\frac{\gamma_c}{2}\langle\hat{\sigma}_c\rangle + \frac{\Omega}{2}(\langle\hat{\eta}_c\rangle - \langle\hat{\eta}_a\rangle), \tag{2.0.34}$$

$$\frac{d}{dt}\rho_{aa} = -\gamma_c\rho_{aa} + \frac{\Omega}{2}(\rho_{ac} + \rho_{ca}), \tag{2.0.35}$$

$$\frac{d}{dt}\rho_{bb} = -\gamma_c\rho_{bb} + \gamma_c\rho_{aa}, \tag{2.0.36}$$

$$\frac{d}{dt}\rho_{cc} = \gamma_c\rho_{bb} - \frac{\Omega}{2}(\rho_{ac} + \rho_{ca}), \tag{2.0.37}$$

$$\frac{d}{dt}\rho_{ac} = -\frac{\gamma_c}{2}\rho_{ac} + \frac{\Omega}{2}(\rho_{cc} - \rho_{aa}), \tag{2.0.38}$$

where γ_c defined by

$$\gamma_c = \frac{4g^2}{\kappa} \tag{2.0.39}$$

is the stimulated emission decay constant. Based on the definition of this decay constant, we infer that an atom in the top level and inside a closed cavity emits photons due to its interaction with the cavity modes. We certainly identify this process to be stimulated photon emission.

We easily find the steady-state solutions of Eqs. (2.0.32-2.0.38) to be

$$\langle\hat{\sigma}_a\rangle = \frac{\Omega}{2\gamma_c}\langle\hat{\sigma}_b^\dagger\rangle, \tag{2.0.40}$$

$$\langle\hat{\sigma}_b\rangle = -\frac{\Omega}{\gamma_c}\langle\hat{\sigma}_a^\dagger\rangle, \tag{2.0.41}$$

$$\rho_{aa} = \frac{\Omega}{2\gamma_c}(\rho_{ac} + \rho_{ca}), \tag{2.0.42}$$

$$\rho_{bb} = \rho_{aa}, \tag{2.0.43}$$

$$\rho_{ac} = \frac{\Omega}{\gamma_c}(\rho_{cc} - \rho_{aa}), \tag{2.0.44}$$

With the aid of the identity

$$\rho_{aa} + \rho_{bb} + \rho_{cc} = 1 \tag{2.0.45}$$

along with Eq. (2.0.44), we obtain

$$\rho_{ac} = \frac{\Omega}{\gamma_c} - \frac{3\Omega^2}{2\gamma_c^2}(\rho_{ac} + \rho_{ca}). \tag{2.0.46}$$

Since ρ_{ac} is real, we see that $\rho_{ca} = \rho_{ac}$. In view of this, Eq. (2.0.46) can be put in the form

$$\rho_{ac} = \frac{\gamma_c\Omega}{\gamma_c^2 + 3\Omega^2}. \tag{2.0.47}$$

Now on substitute Eq. (2.0.47), into Eq. (2.0.42), there follows

$$\rho_{aa} = \frac{\Omega^2}{\gamma_c^2 + 3\Omega^2}, \tag{2.0.48}$$

$$\rho_{bb} = \frac{\Omega^2}{\gamma_c^2 + 3\Omega^2}, \tag{2.0.49}$$

$$\rho_{cc} = \frac{\gamma_c^2 + \Omega^2}{\gamma_c^2 + 3\Omega^2}. \tag{2.0.50}$$

Expressions (2.0.48), (2.0.49) and (2.0.50) represent the probabilities for the atom to be in the top, intermediate, and bottom levels. Moreover, from Eq. (2.0.43), we see that at steady state $\rho_{aa} = \rho_{bb}$, which shows that the probability for the atom to be in the top level $|a\rangle$ is equal to that in the intermediate level $|b\rangle$. Then introducing Eqs. (2.0.41) and (2.0.40), into Eqs. (2.0.32) and (2.0.33) respectively. we get

$$\frac{d}{dt}\langle\hat{\sigma_a}\rangle = -\left(\gamma_c + \frac{\Omega^2}{2\gamma_c}\right)\langle\hat{\sigma_a}\rangle \tag{2.0.51}$$

and

$$\frac{d}{dt}\langle\hat{\sigma_b}\rangle = -\left(\frac{\gamma_c}{2} + \frac{\Omega^2}{4\gamma_c}\right)\langle\hat{\sigma_b}\rangle. \tag{2.0.52}$$

At steady state, we have

$$\langle\hat{\sigma_a}\rangle = 0 \tag{2.0.53}$$

and

$$\langle\hat{\sigma_b}\rangle = 0. \tag{2.0.54}$$

From the above results, we note that the atomic operators $\hat{\sigma}_a$ and $\hat{\sigma}_b$ are Gaussian variables with zero mean.

Chapter 3

PHOTON STATISTICS

The statistical properties of the light emitted by a nondegenerate three-level atom driven by coherent light are described by the mean and the variance of the photon number sum and difference as well as the photon number correlation.

3.1 The mean of the photon number sum and difference

We defined the photon number sum and difference by

$$\hat{n}_{\pm} = \hat{n}_a \pm \hat{n}_b. \tag{3.1.1}$$

where

$$\hat{n}_a = \hat{a}^{\dagger}\hat{a} \tag{3.1.2}$$

and

$$\hat{n}_b = \hat{b}^{\dagger}\hat{b}, \tag{3.1.3}$$

are the photon number operators for cavity modes a and b, respectively. The mean of the photon number sum and difference for the cavity modes can be expressed as

$$\bar{n}_{\pm} = \bar{n}_a \pm \bar{n}_b. \tag{3.1.4}$$

We first find the mean photon number for cavity modes a and b separately. Now using Eqs. (3.1.2) and (3.1.3), one can write

$$\bar{n}_a = \langle \hat{a}^\dagger \hat{a} \rangle \tag{3.1.5}$$

and

$$\bar{n}_b = \langle \hat{b}^\dagger \hat{b} \rangle. \tag{3.1.6}$$

Using the steady state solutions of Eqs. (2.0.9) and (2.0.10), we have

$$\hat{a} = -\frac{2g}{\kappa}\hat{\sigma_a} \tag{3.1.7}$$

and

$$\hat{b} = -\frac{2g}{\kappa}\hat{\sigma_b}. \tag{3.1.8}$$

We then see that

$$\bar{n}_a = \frac{\gamma_c}{\kappa}\rho_{aa} \tag{3.1.9}$$

and

$$\bar{n}_b = \frac{\gamma_c}{\kappa}\rho_{bb}, \tag{3.1.10}$$

with ρ_{aa} and ρ_{bb} given by Eqs. (2.0.48) and (2.0.49). On account of Eq. (2.0.43), we observe that

$$\bar{n}_a = \bar{n}_b. \tag{3.1.11}$$

Therefore, using Eqs. (3.1.9) and (3.1.10), the mean of the photon number sum and difference at steady state can be written as

$$\bar{n}_\pm = \frac{\gamma_c}{\kappa}(\rho_{aa} \pm \rho_{bb}). \tag{3.1.12}$$

Moreover, applying Eq. (2.0.43) the mean of the photon number sum for cavity modes a and b can be expressed as

$$\bar{n}_+ = \frac{2\gamma_c}{\kappa}\rho_{aa}. \tag{3.1.13}$$

or

$$\bar{n}_+ = \frac{2\gamma_c}{\kappa}\rho_{bb}. \tag{3.1.14}$$

Using Eq. (2.0.48), this can be rewritten as

$$\bar{n}_+ = \frac{2\gamma_c}{\kappa}\left(\frac{\Omega^2}{\gamma_c^2 + 3\Omega^2}\right). \tag{3.1.15}$$

Finally dividing both the numerator and denominator by γ_c^2, we easily obtain

$$\bar{n}_+ = \frac{2\gamma_c}{\kappa}\left(\frac{\eta^2}{1+3\eta^2}\right), \tag{3.1.16}$$

where

$$\eta = \frac{\Omega}{\gamma_c}. \tag{3.1.17}$$

On the other hand, the mean of the photon number difference can also be put in the form

$$\bar{n}_- = 0. \tag{3.1.18}$$

This shows that the mean photon numbers of the two cavity modes are equal.

3.2 The variance of the photon number sum and difference

The variance of the photon number sum and difference can be expressed as

$$(\Delta n_\pm)^2 = \langle(\hat{n}_a \pm \hat{n}_b)^2\rangle - \langle\hat{n}_a \pm \hat{n}_b\rangle^2. \tag{3.2.1}$$

This can be rewritten as

$$(\Delta n_\pm)^2 = (\Delta n_a)^2 + (\Delta n_b)^2 \pm 2(\langle\hat{n}_a\hat{n}_b\rangle - \langle\hat{n}_a\rangle\langle\hat{n}_b\rangle). \tag{3.2.2}$$

On the other hand, the photon number variance for cavity mode a can be written as

$$(\Delta n_a)^2 = \langle\hat{a}^{\dagger 2}\hat{a}^2\rangle + \langle\hat{a}^\dagger\hat{a}\rangle - \langle\hat{a}^\dagger\hat{a}\rangle^2. \tag{3.2.3}$$

In addition, employing Eq. (3.1.7), we find

$$\langle\hat{a}^{\dagger 2}\hat{a}^2\rangle = \left(\frac{\gamma_c}{\kappa}\right)^2\langle\hat{\sigma}_a^{\dagger 2}\hat{\sigma}_a^2\rangle, \tag{3.2.4}$$

so that in view of the fact that

$$\hat{\sigma}_a^\dagger\hat{\sigma}_a^\dagger = 0 \tag{3.2.5}$$

and

$$\hat{\sigma}_a\hat{\sigma}_a = 0, \tag{3.2.6}$$

there follows

$$\langle \hat{a}^{\dagger 2}\hat{a}^2 \rangle = 0. \tag{3.2.7}$$

Therefore, applying Eqs. (3.1.9) and (3.2.7), the photon-number variance for cavity mode a can be put in the form

$$(\Delta n_a)^2 = (1 - \bar{n}_a)\bar{n}_a. \tag{3.2.8}$$

Following a similar procedure, the photon-number variance for cavity mode b can also be written as

$$(\Delta n_b)^2 = (1 - \bar{n}_b)\bar{n}_b. \tag{3.2.9}$$

Using Eqs. (3.1.2) and (3.1.3), we have

$$\langle \hat{n}_a\hat{n}_b \rangle = \langle \hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b} \rangle \tag{3.2.10}$$

and using the fact that $\hat{a}$ and $\hat{b}$ are Gaussian variables with zero mean, we readily get

$$\langle \hat{n}_a\hat{n}_b \rangle = \langle \hat{a}^\dagger\hat{a} \rangle\langle \hat{b}^\dagger\hat{b} \rangle + \langle \hat{a}^\dagger\hat{b}^\dagger \rangle\langle \hat{a}\hat{b} \rangle + \langle \hat{a}^\dagger\hat{b} \rangle\langle \hat{a}\hat{b}^\dagger \rangle. \tag{3.2.11}$$

Moreover, with the help of Eqs. (3.1.5), (3.1.6) and taking into account Eqs. (3.1.7) and (3.1.8), one obtains

$$\langle \hat{n}_a\hat{n}_b \rangle = \bar{n}_a\bar{n}_b + \left(\frac{\gamma_c}{\kappa}\right)^2 \langle \hat{\sigma}_a^\dagger\hat{\sigma}_b^\dagger \rangle\langle \hat{\sigma}_a\hat{\sigma}_b \rangle + \left(\frac{\gamma_c}{\kappa}\right)^2 \langle \hat{\sigma}_a^\dagger\hat{\sigma}_b \rangle\langle \hat{\sigma}_a\hat{\sigma}_b^\dagger \rangle. \tag{3.2.12}$$

Employing the identity

$$\hat{\sigma}_a^\dagger\hat{\sigma}_b^\dagger = |a\rangle\langle c|, \tag{3.2.13}$$

$$\hat{\sigma}_a\hat{\sigma}_b = 0, \tag{3.2.14}$$

$$\hat{\sigma}_a^\dagger\hat{\sigma}_b = 0, \tag{3.2.15}$$

and

$$\hat{\sigma}_a\hat{\sigma}_b^\dagger = 0, \tag{3.2.16}$$

we have

$$\langle\hat{n}_a\hat{n}_b\rangle = \bar{n}_a\bar{n}_b. \tag{3.2.17}$$

In addition, one can also write

$$\langle\hat{n}_a\rangle\langle\hat{n}_b\rangle = \bar{n}_a\bar{n}_b. \tag{3.2.18}$$

Hence substitution of Eqs. (3.2.8), (3.2.9), (3.2.17), and (3.2.18) into Eq. (3.2.2), yields

$$(\Delta n_\pm)^2 = \bar{n}_a - \bar{n}_a^2 + \bar{n}_b - \bar{n}_b^2 \pm 2\bar{n}_a\bar{n}_b \mp 2\bar{n}_a\bar{n}_b, \tag{3.2.19}$$

so that on account of (3.1.11), the variance of the photon number sum and difference can be put in the form

$$(\Delta n_\pm)^2 = 2[\bar{n}_a - \bar{n}_a^2 \pm \bar{n}_a^2 \mp \bar{n}_a^2] \tag{3.2.20}$$

or this can also be put in the form

$$(\Delta n_\pm)^2 = 2[\bar{n}_b - \bar{n}_b^2 \pm \bar{n}_b^2 \mp \bar{n}_b^2]. \tag{3.2.21}$$

Therefore, the variance of the photon number sum can be expressed as

$$(\Delta n_+)^2 = (1 - \bar{n}_a)2\bar{n}_a \tag{3.2.22}$$

or

$$(\Delta n_+)^2 = (1 - \bar{n}_b)2\bar{n}_b, \tag{3.2.23}$$

and the variance of the photon number difference can also be written as

$$(\Delta n_-)^2 = (1 - \bar{n}_a)2\bar{n}_a. \tag{3.2.24}$$

Taking into account (3.1.9), one can express Eqs. (3.2.23) and (3.2.24) in the form

$$(\Delta n_+)^2 = (1 - \frac{\gamma_c}{\kappa}\rho_{aa})2\frac{\gamma_c}{\kappa}\rho_{aa} \tag{3.2.25}$$

and

$$(\Delta n_-)^2 = (1 - \frac{\gamma_c}{\kappa}\rho_{aa})2\frac{\gamma_c}{\kappa}\rho_{aa}. \tag{3.2.26}$$

Moreover, with the help of (2.0.48) , one easily obtains

$$(\Delta n_+)^2 = \left(1 - \frac{\gamma_c}{\kappa}\left(\frac{\Omega^2}{\gamma_c^2 + 3\Omega^2}\right)\right)\frac{2\gamma_c}{\kappa}\left(\frac{\Omega^2}{\gamma_c^2 + 3\Omega^2}\right) \tag{3.2.27}$$

and

$$(\Delta n_-)^2 = \left(1 - \frac{\gamma_c}{\kappa}\left(\frac{\Omega^2}{\gamma_c^2 + 3\Omega^2}\right)\right)\frac{2\gamma_c}{\kappa}\left(\frac{\Omega^2}{\gamma_c^2 + 3\Omega^2}\right) \tag{3.2.28}$$

Dividing both the numerator and the denominator by γ_c^2, we readily get

$$(\Delta n_+)^2 = \left(1 - \frac{\gamma_c}{\kappa}\left(\frac{\eta^2}{1 + 3\eta^2}\right)\right)\frac{2\gamma_c}{\kappa}\left(\frac{\eta^2}{1 + 3\eta^2}\right) \tag{3.2.29}$$

and

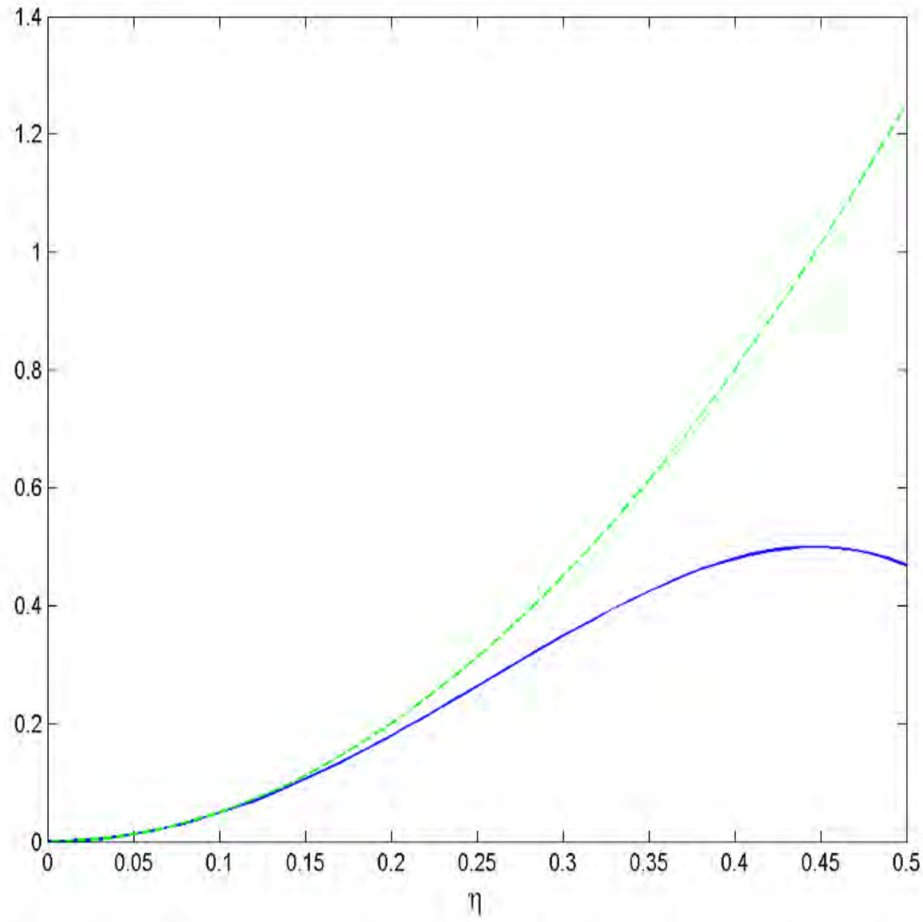

Figure 3.1: Plots of the mean of the photon number sum $\bar{n}_+$ [Eq. (3.1.16) dashed curve] and the variance of the photon number sum $(\Delta n_+)^2$ [Eq. (3.2.29) solid curve] versus η for $\gamma_c = 0.5$ and $\kappa = 0.8$.

$$(\Delta n_-)^2 = \left(1 - \frac{\gamma_c}{\kappa}\left(\frac{\eta^2}{1 + 3\eta^2}\right)\right)\frac{2\gamma_c}{\kappa}\left(\frac{\eta^2}{1 + 3\eta^2}\right), \tag{3.2.30}$$

with η is given by Eq. (3.1.17). We clearly see from Fig. 3.1 that the photon statistics of the light emitted by the coherently driven three-level atom is Poissonian for $\eta < 0.13$ and sub-Poissonian for $\eta \geq 0.13$.

3.3 Photon number correlation

In the previous sections, we have considered the mean and the variance of the photon number sum and difference. In this section we study the correlation property of the photon numbers for a nondegenerate three-level atom driven by coherent light. To this end, we introduce the photon number correlation defined by

$$g_{ab}^{(2)}(0) = \frac{\langle \hat{n}_a \hat{n}_b \rangle}{\bar{n}_a \bar{n}_b}. \tag{3.3.1}$$

where $\hat{n}_a$ and $\hat{n}_b$ are the photon number operators for cavity modes a and b, respectively. If $g_{ab}^{(2)}(0) = 1$, the photon numbers of the cavity modes are uncorrelated. If on the other hand $g_{ab}^{(2)}(0) \neq 1$, the photon numbers of the cavity modes are correlated. With the help of (3.1.2) and (3.1.3), one can express Eq. (3.3.1) in the form

$$g_{ab}^{(2)}(0) = \frac{\langle \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b} \rangle}{\bar{n}_a \bar{n}_b}. \tag{3.3.2}$$

Recalling that $\hat{a}$ and $\hat{b}$ are Gaussian variables with zero mean, we readily find

$$g_{ab}^{(2)}(0) = \frac{\langle \hat{a}^\dagger \hat{a} \rangle \langle \hat{b}^\dagger \hat{b} \rangle + \langle \hat{a}^\dagger \hat{b}^\dagger \rangle \langle \hat{a} \hat{b} \rangle + \langle \hat{a}^\dagger \hat{b} \rangle \langle \hat{a} \hat{b}^\dagger \rangle}{\bar{n}_a \bar{n}_b} \tag{3.3.3}$$

Now on account of Eqs. (3.1.5) and (3.1.6), we easily get

$$g_{ab}^{(2)}(0) = 1 + \frac{\langle \hat{a}^\dagger \hat{b}^\dagger \rangle \langle \hat{a} \hat{b} \rangle + \langle \hat{a}^\dagger \hat{b} \rangle \langle \hat{a} \hat{b}^\dagger \rangle}{\bar{n}_a \bar{n}_b}. \tag{3.3.4}$$

Moreover, using Eqs. (3.1.7) and (3.1.8), we have

$$g_{ab}^{(2)}(0) = 1 + \frac{(\frac{\gamma_c}{\kappa})^2 \bigg(\langle \hat{\sigma}_a^\dagger \hat{\sigma}_b^\dagger \rangle \langle \hat{\sigma}_a \hat{\sigma}_b \rangle + \langle \hat{\sigma}_a^\dagger \hat{\sigma}_b \rangle \langle \hat{\sigma}_a \hat{\sigma}_b^\dagger \rangle \bigg)}{\bar{n}_a \bar{n}_b}, \tag{3.3.5}$$

and taking into account Eqs. (3.2.13), (3.2.14), (3.2.15) and (3.2.16), we arrive at

$$g_{ab}^{(2)}(0) = 1. \tag{3.3.6}$$

This result shows that the photon numbers for cavity modes a and b are uncorrelated.

Chapter 4

QUADRATURE VARIANCE

In this chapter we proceed to determine single-mode quadrature variance and two-mode quadrature variance for the cavity modes produced by a nondegenerate three-level atom driven by coherent light.

4.1 Single-mode quadrature variance

The squeezing properties of a cavity mode a are described by two quadrature operators defined by

$$\hat{a}_+ = \hat{a} + \hat{a}^\dagger \tag{4.1.1}$$

and

$$\hat{a}_- = i(\hat{a}^\dagger - \hat{a}). \tag{4.1.2}$$

The operators $\hat{a}_+$ and $\hat{a}_-$ represents physical quantities called the plus and minus quadratures. Using Eqs. (4.1.1) and (4.1.2), one can write

$$[\hat{a}_-, \hat{a}_+] = i\langle[\hat{a}^\dagger - \hat{a}, \hat{a} + \hat{a}^\dagger]\rangle. \tag{4.1.3}$$

with the aid of the identity

$$[\hat{A} + \hat{B}, \hat{C} + \hat{D}] = [\hat{A}, \hat{C}] + [\hat{A}, \hat{D}] + [\hat{B}, \hat{C}] + [\hat{B}, \hat{D}], \tag{4.1.4}$$

We have

$$[\hat{a}_-, \hat{a}_+] = i\bigg(\langle[\hat{a}^\dagger, \hat{a}]\rangle + \langle[\hat{a}^\dagger, \hat{a}^\dagger]\rangle - \langle[\hat{a}, \hat{a}]\rangle - \langle[\hat{a}, \hat{a}^\dagger]\rangle\bigg). \tag{4.1.5}$$

So that in view of Eq. (3.1.7), one can express this commutation relation in the form

$$[\hat{a}_-, \hat{a}_+] = i\frac{\gamma_c}{\kappa}\bigg(\langle[\hat{\sigma}_a^\dagger, \hat{\sigma}_a]\rangle + \langle[\hat{\sigma}_a^\dagger, \hat{\sigma}_a^\dagger]\rangle - \langle[\hat{\sigma}_a, \hat{\sigma}_a]\rangle - \langle[\hat{\sigma}_a, \hat{\sigma}_a^\dagger]\rangle\bigg). \tag{4.1.6}$$

Using the commutation relations

$$[\hat{\sigma}_a, \hat{\sigma}_a^\dagger] = |b\rangle\langle b| - |a\rangle\langle a| \tag{4.1.7}$$

and

$$[\hat{\sigma}_a, \hat{\sigma}_a] = 0, \tag{4.1.8}$$

along with Eq. (4.1.6), we obtain

$$[\hat{a}_-, \hat{a}_+] = 2i\frac{\gamma_c}{\kappa}(\rho_{aa} - \rho_{bb}), \tag{4.1.9}$$

and taking into account (2.0.43), one easily gets

$$[\hat{a}_-, \hat{a}_+] = 0. \tag{4.1.10}$$

Furthermore, we recall that if

$$[\hat{A}, \hat{B}] = i\hat{c}, \tag{4.1.11}$$

then

$$\Delta\hat{A}\Delta\hat{B} \geq \frac{1}{2}|\langle\hat{c}\rangle|. \tag{4.1.12}$$

Thus on account of (4.1.10) and (4.1.12), the uncertainty relation for cavity mode a can be expressed as

$$\Delta a_+ \Delta a_- \geq 0. \tag{4.1.13}$$

The quadrature variance for mode a is defined by

$$(\Delta a_\pm)^2 = \langle\hat{a}_\pm^2\rangle - \langle\hat{a}_\pm\rangle^2. \tag{4.1.14}$$

In view of (4.1.1) and (4.1.2), the quadrature variance can be put in the form

$$(\Delta a_\pm)^2 = \pm(\langle\hat{a}^{\dagger 2}\rangle + \langle\hat{a}^2\rangle \pm \langle\hat{a}^\dagger\hat{a}\rangle \pm \langle\hat{a}\hat{a}^\dagger\rangle) \mp (\langle\hat{a}^\dagger\rangle^2 + \langle\hat{a}\rangle^2 \pm 2\langle\hat{a}^\dagger\rangle\langle\hat{a}\rangle). \tag{4.1.15}$$

We now proceed to determine the various expectation values involved in Eq. (4.1.15). Then using Eq. (3.1.7), we find

$$\langle\hat{a}\rangle = -\frac{2g}{\kappa}\langle\hat{\sigma_a}\rangle. \tag{4.1.16}$$

Using the steady state solution of (2.0.51), we have

$$\langle\hat{\sigma}_a\rangle = 0, \tag{4.1.17}$$

and hence

$$\langle\hat{a}\rangle = 0. \tag{4.1.18}$$

We observe on the basis of Eqs. (2.0.9) and (4.1.18) that $\hat{a}$ is a Gaussian variable with zero mean. In addition, one can also write

$$\langle\hat{a}^\dagger\rangle\langle\hat{a}\rangle = \langle\hat{a}\rangle\langle\hat{a}^\dagger\rangle = 0. \tag{4.1.19}$$

With the help of (3.1.7), we also find

$$\langle\hat{a}\hat{a}^\dagger\rangle = \frac{\gamma_c}{\kappa}\langle\hat{\sigma}_a\hat{\sigma}_a^\dagger\rangle. \tag{4.1.20}$$

Hence on account of the fact that

$$\hat{\sigma}_a\hat{\sigma}_a^\dagger = |b\rangle\langle b|, \tag{4.1.21}$$

we easily get

$$\langle\hat{a}\hat{a}^\dagger\rangle = \frac{\gamma_c}{\kappa}\rho_{bb}. \tag{4.1.22}$$

Using Eq. (3.1.7), one can also write

$$\langle\hat{a}^2\rangle = \frac{\gamma_c}{\kappa}\langle\hat{\sigma}_a\hat{\sigma}_a\rangle. \tag{4.1.23}$$

Now with the help of the identity

$$\hat{\sigma}_a\hat{\sigma}_a = 0, \tag{4.1.24}$$

we have

$$\langle\hat{a}^2\rangle = 0. \tag{4.1.25}$$

Applying Eqs. (4.1.18), (4.1.19), (4.1.22), and (4.1.25), we arrive at

$$(\Delta a_{\pm})^2 = \bar{n}_a + \frac{\gamma_c}{\kappa}\rho_{bb}, \tag{4.1.26}$$

so that in view of (3.1.10), the quadrature variance can be rewritten as

$$(\Delta a_{\pm})^2 = \bar{n}_a + \bar{n}_b. \tag{4.1.27}$$

Finally, with the help of Eq. (3.1.11), the quadrature variance can be put in the form

$$(\Delta a_+)^2 = (\Delta a_-)^2 = 2\bar{n}_a, \tag{4.1.28}$$

where $\bar{n}_a$ is given by Eq. (3.1.9). We then see that cavity mode a is in a chaotic state.

The squeezing properties of cavity mode b are described by two quadrature operators defined by

$$\hat{b}_+ = \hat{b} + \hat{b}^\dagger \tag{4.1.29}$$

and

$$\hat{b}_- = i(\hat{b}^\dagger - \hat{b}). \tag{4.1.30}$$

These operators are Hermitian and satisfy the commutation relation

$$[\hat{b}_-, \hat{b}_+] = 2i\frac{\gamma_c}{\kappa}(\rho_{bb} - \rho_{cc}). \tag{4.1.31}$$

It then follows that

$$\Delta b_+ \Delta b_- \geq \frac{\gamma_c}{\kappa}|\rho_{bb} - \rho_{cc}|. \tag{4.1.32}$$

The quadrature variance for mode b is defined by

$$(\Delta b_{\pm})^2 = \langle \hat{b}_{\pm}^2 \rangle - \langle \hat{b}_{\pm} \rangle^2. \tag{4.1.33}$$

With the help of (4.1.29) and (4.1.30), The quadrature variance can be put in the form

$$(\Delta b_{\pm})^2 = \pm(\langle \hat{b}^{\dagger 2} \rangle + \langle \hat{b}^2 \rangle \pm \langle \hat{b}^\dagger \hat{b} \rangle \pm \langle \hat{b}\hat{b}^\dagger \rangle) \mp (\langle \hat{b}^\dagger \rangle^2 + \langle \hat{b} \rangle^2 \pm 2\langle \hat{b}^\dagger \rangle \langle \hat{b} \rangle). \tag{4.1.34}$$

Then, we find the various expectation values involved in Eq. (4.1.34). Using Eq. (3.1.8), we have

$$\langle\hat{b}\rangle = -\frac{2g}{\kappa}\langle\hat{\sigma}_b\rangle, \tag{4.1.35}$$

Now introducing Eq. (2.0.54) into Eq. (4.1.35), we get

$$\langle\hat{b}\rangle = 0. \tag{4.1.36}$$

On the basis of Eqs. (2.0.10) and (4.1.36), we also note that $\hat{b}$ is a Gaussian variable with zero mean. Then we see that

$$\langle\hat{b}^\dagger\rangle\langle\hat{b}\rangle = 0. \tag{4.1.37}$$

Applying Eq. (3.1.8), one obtains

$$\langle\hat{b}\hat{b}^\dagger\rangle = \frac{\gamma_c}{\kappa}\langle\hat{\sigma}_b\hat{\sigma}_b^\dagger\rangle. \tag{4.1.38}$$

With the aid of the identity

$$\hat{\sigma}_b\hat{\sigma}_b^\dagger = |c\rangle\langle c|, \tag{4.1.39}$$

we have

$$\langle\hat{b}\hat{b}^\dagger\rangle = \frac{\gamma_c}{\kappa}\rho_{cc}. \tag{4.1.40}$$

On account of (3.1.8), one can also find

$$\langle\hat{b}^2\rangle = 0. \tag{4.1.41}$$

Thus in view of Eqs. (4.1.36), (4.1.37) (4.1.40), and (4.1.41), the quadrature variance can be written as

$$(\Delta b_+)^2 = (\Delta b_-)^2 = \bar{n}_b + \frac{\gamma_c}{\kappa}\rho_{cc}, \tag{4.1.42}$$

in which

$$\rho_{cc} = \frac{\gamma_c^2 + \Omega^2}{\gamma_c^2 + 3\Omega^2}, \tag{4.1.43}$$

and we note that for $\gamma_c << \Omega$, this result can be put as

$$\rho_{cc} = \frac{1}{3}. \tag{4.1.44}$$

On account of this, Eq. (4.1.42) takes the form

$$(\Delta b_+)^2 = (\Delta b_-)^2 = 2\bar{n}_b, \tag{4.1.45}$$

where

$$\bar{n}_b = \frac{\gamma_c}{3\kappa}. \tag{4.1.46}$$

We note that cavity mode b is in a chaotic state for $\gamma_c << \Omega$. In addition, for $\gamma_c >> \Omega$, we have

$$(\Delta b_+)^2 = (\Delta b_-)^2 = \frac{\gamma_c}{\kappa}, \tag{4.1.47}$$

and

$$\Delta b_+ \Delta b_- \geq \frac{\gamma_c}{\kappa} \tag{4.1.48}$$

Hence on the basis of this result, we conclude that cavity mode b is in a coherent state .

4.2 Two-mode quadrature variance

In the previous section, we have considered the squeezing properties of modes a and b. We now extend our analysis to the superposed cavity modes. The squeezing properties of the superposed cavity modes is described by the quadrature operators defined by

$$\hat{c}_+ = \hat{a}_+ + \hat{b}_+ \tag{4.2.1}$$

and

$$\hat{c}_- = \hat{a}_- + \hat{b}_-, \tag{4.2.2}$$

where $\hat{a}_\pm$ and $\hat{b}_\pm$ are defined by (4.1.1), (4.1.2), (4.1.29) and (4.1.30). The operators $\hat{c}_+$ and $\hat{c}_-$ are Hermitian. Using Eqs. (4.2.1) and (4.2.2), one can write

$$[\hat{c}_-, \hat{c}_+] = \langle[\hat{a}_- + \hat{b}_-, \hat{a}_+ + \hat{b}_+]\rangle. \tag{4.2.3}$$

In view of (4.1.4), this can be rewritten as

$$[\hat{c}_-, \hat{c}_+] = \langle[\hat{a}_-, \hat{a}_+]\rangle + \langle[\hat{a}_-, \hat{b}_+]\rangle + \langle[\hat{b}_-, \hat{a}_+]\rangle + \langle[\hat{b}_-, \hat{b}_+]\rangle, \tag{4.2.4}$$

thus with the aid of (4.1.10) and (4.1.31), we arrive at

$$[\hat{c}_-, \hat{c}_+] = 2i\frac{\gamma_c}{\kappa}(\rho_{bb} - \rho_{cc}). \tag{4.2.5}$$

It then follows that

$$\Delta c_+ \Delta c_- \geq \frac{\gamma_c}{\kappa}|\rho_{bb} - \rho_{cc}|. \tag{4.2.6}$$

Now with the help of (2.0.49) and (4.1.43), we get

$$\Delta c_+ \Delta c_- \geq \frac{\gamma_c}{\kappa}\left|\frac{\gamma_c^2}{\gamma_c^2 + 3\Omega^2}\right|. \tag{4.2.7}$$

We note that for $\gamma_c >> \Omega$, Eq. (4.2.7) reduces to

$$\Delta c_+ \Delta c_- \geq \frac{\gamma_c}{\kappa} \tag{4.2.8}$$

The variance of the quadrature operators is defined by

$$(\Delta c_\pm)^2 = \langle \hat{c}_\pm^2 \rangle - \langle \hat{c}_\pm \rangle^2. \tag{4.2.9}$$

Now with the help of Eqs. (4.2.1) and (4.2.2), one can write Eq. (4.2.9) as

$$(\Delta c_\pm)^2 = \langle \hat{a}_\pm^2 \rangle + \langle \hat{b}_\pm^2 \rangle + \langle \hat{a}_\pm \hat{b}_\pm \rangle + \langle \hat{a}_\pm \hat{b}_\pm \rangle - \langle \hat{a}_\pm \rangle^2 - \langle \hat{b}_\pm \rangle^2 - 2\langle \hat{b}_\pm \rangle \langle \hat{a}_\pm \rangle. \tag{4.2.10}$$

Then, we determine the various expectation values involved in Eq. (4.2.10). Using Eqs. (4.1.1), (4.1.2), (4.1.29) and (4.1.30), we find

$$\langle \hat{b}_\pm \hat{a}_\pm \rangle = \pm\langle \hat{b}^\dagger \hat{a}^\dagger \rangle + \langle \hat{b}\hat{a}^\dagger \rangle + \langle \hat{b}^\dagger \hat{a} \rangle \pm \langle \hat{b}\hat{a} \rangle, \tag{4.2.11}$$

So that in view of (3.1.7) and (3.1.8), this can be rewritten as

$$\langle \hat{b}_\pm \hat{a}_\pm \rangle = \pm\frac{\gamma_c}{\kappa}\langle \hat{\sigma}_b^\dagger \hat{\sigma}_a^\dagger \rangle + \frac{\gamma_c}{\kappa}\langle \hat{\sigma}_b \hat{\sigma}_a^\dagger \rangle + \frac{\gamma_c}{\kappa}\langle \hat{\sigma}_b^\dagger \hat{\sigma}_a \rangle \pm \frac{\gamma_c}{\kappa}\langle \hat{\sigma}_b \hat{\sigma}_a \rangle \tag{4.2.12}$$

and on account of the fact that

$$\hat{\sigma}_b^\dagger \hat{\sigma}_a^\dagger = 0, \tag{4.2.13}$$

$$\hat{\sigma}_b \hat{\sigma}_a^\dagger = 0, \tag{4.2.14}$$

$$\hat{\sigma}_b^\dagger \hat{\sigma}_a = 0, \tag{4.2.15}$$

$$\hat{\sigma}_b\hat{\sigma}_a = |c\rangle\langle a|, \tag{4.2.16}$$

along with Eq. (4.2.12), we have

$$\langle \hat{b}_\pm \hat{a}_\pm \rangle = \pm \frac{\gamma_c}{\kappa}\rho_{ac}, \tag{4.2.17}$$

in which

$$\rho_{ac} = \frac{\gamma_c \Omega}{\gamma_c^2 + 3\Omega^2}. \tag{4.2.18}$$

Similarlly

$$\langle \hat{a}_\pm \hat{b}_\pm \rangle = \pm \frac{\gamma_c}{\kappa}\rho_{ca}. \tag{4.2.19}$$

So that on account of the fact that $\rho_{ac} = \rho_{ca}$. This can be rewritten as

$$\langle \hat{a}_\pm \hat{b}_\pm \rangle = \pm \frac{\gamma_c}{\kappa}\rho_{ac}. \tag{4.2.20}$$

Now with the aid of Eqs. (4.1.1) and (4.1.2), we verify that

$$\langle \hat{a}_\pm \rangle^2 = \pm \langle \hat{a}^\dagger \rangle^2 \pm \langle \hat{a} \rangle^2 + 2\langle \hat{a}^\dagger \rangle \langle \hat{a} \rangle. \tag{4.2.21}$$

Thus on substituting Eqs. (4.1.18) and (4.1.19) into Eq. (4.2.21), there follows

$$\langle \hat{a}_\pm \rangle^2 = 0. \tag{4.2.22}$$

On the other hand,with the aid of (4.1.29) and (4.1.30), we get

$$\langle \hat{b}_\pm \rangle^2 = \pm \langle \hat{b}^\dagger \rangle^2 \pm \langle \hat{b} \rangle^2 + 2\langle \hat{b}^\dagger \rangle \langle \hat{b} \rangle, \tag{4.2.23}$$

hence on account of (4.1.36) and (4.1.37), we have

$$\langle \hat{b}_\pm \rangle^2 = 0. \tag{4.2.24}$$

Then, we can also see that

$$\langle \hat{a}_\pm \rangle \langle \hat{b}_\pm \rangle = \langle \hat{b}_\pm \rangle \langle \hat{a}_\pm \rangle = 0. \tag{4.2.25}$$

Using Eqs.(4.1.1) and (4.1.2), we find

$$\langle \hat{a}_\pm^2 \rangle = \pm \langle \hat{a}^{\dagger 2} \rangle \pm \langle \hat{a}^2 \rangle + \langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{a}\hat{a}^\dagger \rangle. \tag{4.2.26}$$

We recall that

$$\langle \hat{a}^2 \rangle = 0, \tag{4.2.27}$$

$$\langle \hat{a}\hat{a}^\dagger \rangle = \frac{\gamma_c}{\kappa}\rho_{bb}, \tag{4.2.28}$$

and

$$\langle \hat{a}^\dagger \hat{a} \rangle = \frac{\gamma_c}{\kappa}\rho_{aa}. \tag{4.2.29}$$

So that in view of this resut Eq. (4.2.26), takes the form

$$\langle \hat{a}_\pm^2 \rangle = \frac{\gamma_c}{\kappa}\rho_{aa} + \frac{\gamma_c}{\kappa}\rho_{bb}. \tag{4.2.30}$$

Applying Eqs. (3.1.9) and (3.1.10), one get

$$\langle \hat{a}_\pm^2 \rangle = \bar{n}_a + \bar{n}_b. \tag{4.2.31}$$

Moreover, on account of (3.1.11), one obtains

$$\langle \hat{a}_\pm^2 \rangle = 2\bar{n}_a. \tag{4.2.32}$$

On the other hand, with the aid of (4.1.29) and (4.1.30), we arrive at

$$\langle \hat{b}_\pm^2 \rangle = \pm\langle \hat{b}^{\dagger 2} \rangle \pm \langle \hat{b}^2 \rangle + \langle \hat{b}^\dagger \hat{b} \rangle + \langle \hat{b}\hat{b}^\dagger \rangle. \tag{4.2.33}$$

Hence in view of (3.1.10), (4.1.36), and (4.1.40), Eq. (4.2.33) takes the form

$$\langle \hat{b}_\pm^2 \rangle = \bar{n}_b + \frac{\gamma_c}{\kappa}\rho_{cc} \tag{4.2.34}$$

and for $\gamma_c << \Omega$, this result can be rewritten as

$$\langle \hat{b}_\pm^2 \rangle = 2\bar{n}_b. \tag{4.2.35}$$

with $\bar{n}_b$ is given by Eq. (4.1.46). Finally, applying Eqs. (4.2.17), (4.2.20), (4.2.22), (4.2.24), (4.2.25), (4.2.32) and (4.2.34) into Eq. (4.2.10), the quadrature variance for the superposed cavity modes can be expressed as

$$(\Delta c_\pm)^2 = 2\bar{n}_a + \bar{n}_b + \frac{\gamma_c}{\kappa}\rho_{cc} \pm 2\frac{\gamma_c}{\kappa}\rho_{ac}. \tag{4.2.36}$$

So that employing Eq. (3.1.11) into (4.2.36), we readily obtain the quadrature variance for the superposed cavity modes

$$(\Delta c_{\pm})^2 = 3\bar{n}_a + \frac{\gamma_c}{\kappa}\rho_{cc} \pm 2\frac{\gamma_c}{\kappa}\rho_{ac} \tag{4.2.37}$$

and taking into account of (3.1.9), we get

$$(\Delta c_{\pm})^2 = 3\frac{\gamma_c}{\kappa}\rho_{aa} + \frac{\gamma_c}{\kappa}\rho_{cc} \pm 2\frac{\gamma_c}{\kappa}\rho_{ac}. \tag{4.2.38}$$

Therefore, the quadrature variance of the superposed cavity modes can be put in the form

$$(\Delta c_{+})^2 = 3\frac{\gamma_c}{\kappa}\rho_{aa} + \frac{\gamma_c}{\kappa}\rho_{cc} + 2\frac{\gamma_c}{\kappa}\rho_{ac} \tag{4.2.39}$$

and

$$(\Delta c_{-})^2 = 3\frac{\gamma_c}{\kappa}\rho_{aa} + \frac{\gamma_c}{\kappa}\rho_{cc} - 2\frac{\gamma_c}{\kappa}\rho_{ac}. \tag{4.2.40}$$

Now substitution of Eqs. (2.0.48), (4.1.43) and (4.2.18) into Eqs. (4.2.39) and (4.2.40), yields

$$(\Delta c_{+})^2 = \frac{\gamma_c}{\kappa}\left(\frac{4\Omega^2 + \gamma_c^2 + 2\gamma_c\Omega}{\gamma_c^2 + 3\Omega^2}\right) \tag{4.2.41}$$

and

$$(\Delta c_{-})^2 = \frac{\gamma_c}{\kappa}\left(\frac{4\Omega^2 + \gamma_c^2 - 2\gamma_c\Omega}{\gamma_c^2 + 3\Omega^2}\right). \tag{4.2.42}$$

Then dividing both the numerator and denominator by γ_c^2, one easily obtains

$$(\Delta c_{+})^2 = \frac{\gamma_c}{\kappa}\left(\frac{1 + 4\eta^2 + 2\eta}{1 + 3\eta^2}\right) \tag{4.2.43}$$

and

$$(\Delta c_{-})^2 = \frac{\gamma_c}{\kappa}\left(\frac{1 + 4\eta^2 - 2\eta}{1 + 3\eta^2}\right), \tag{4.2.44}$$

where

$$\eta = \frac{\Omega}{\gamma_c}. \tag{4.2.45}$$

For $\eta << 1$, Eqs. (4.2.43) and (4.2.44) reduce to

$$(\Delta c_{+})^2 = (\Delta c_{-})^2 = \frac{\gamma_c}{\kappa}. \tag{4.2.46}$$

On the basis of this result and Eq. (4.2.8), the superposed cavity modes are in a coherent state. The two cavity modes are said to be in squeezed state if either $\Delta c_+ < \frac{\gamma_c}{\kappa}$ or $\Delta c_- < \frac{\gamma_c}{\kappa}$ such that the uncertainty relation $\Delta c_+ \Delta c_- \geq \frac{\gamma_c}{\kappa}$ is not violated. Fig 4.1 clearly indicates that the superposed cavity modes are in

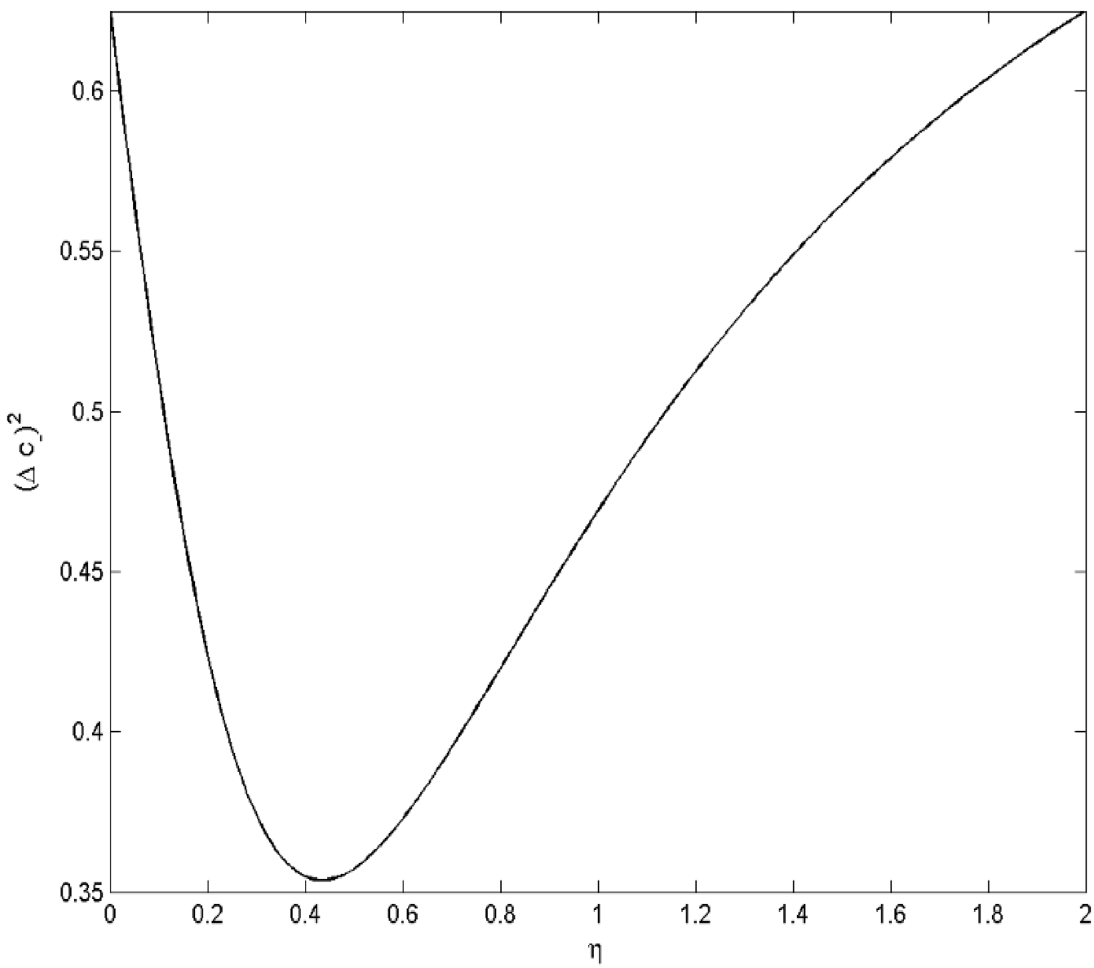

Figure 4.1: Plot of the quadrature variance$(\Delta c_-)^2$ [Eq. (4.2.44)], at steady state versus η for $\kappa = 0.8$ and $\gamma_c = 0.5$

squeezed state for all values of η between 0 and 2 and the squeezing occurs in the minus quadrature.

We next proceed to obtain the quadrature squeezing of the superposed cavity modes relative to the quadrature variance of the superposed coherent light.

We define the quadrature squeezing of the superposed modes by

$$s = \frac{\frac{\gamma_c}{\kappa} - (\Delta c_-)^2}{\frac{\gamma_c}{\kappa}}. \tag{4.2.47}$$

Now employing Eq. (4.2.44), one can express Eq. (4.2.47) in the form

$$s = 1 - \left(\frac{1 + 4\eta^2 - 2\eta}{1 + 3\eta^2}\right). \tag{4.2.48}$$

We clearly see from Fig. 4.2 that the maximum quadrature squeezing of the

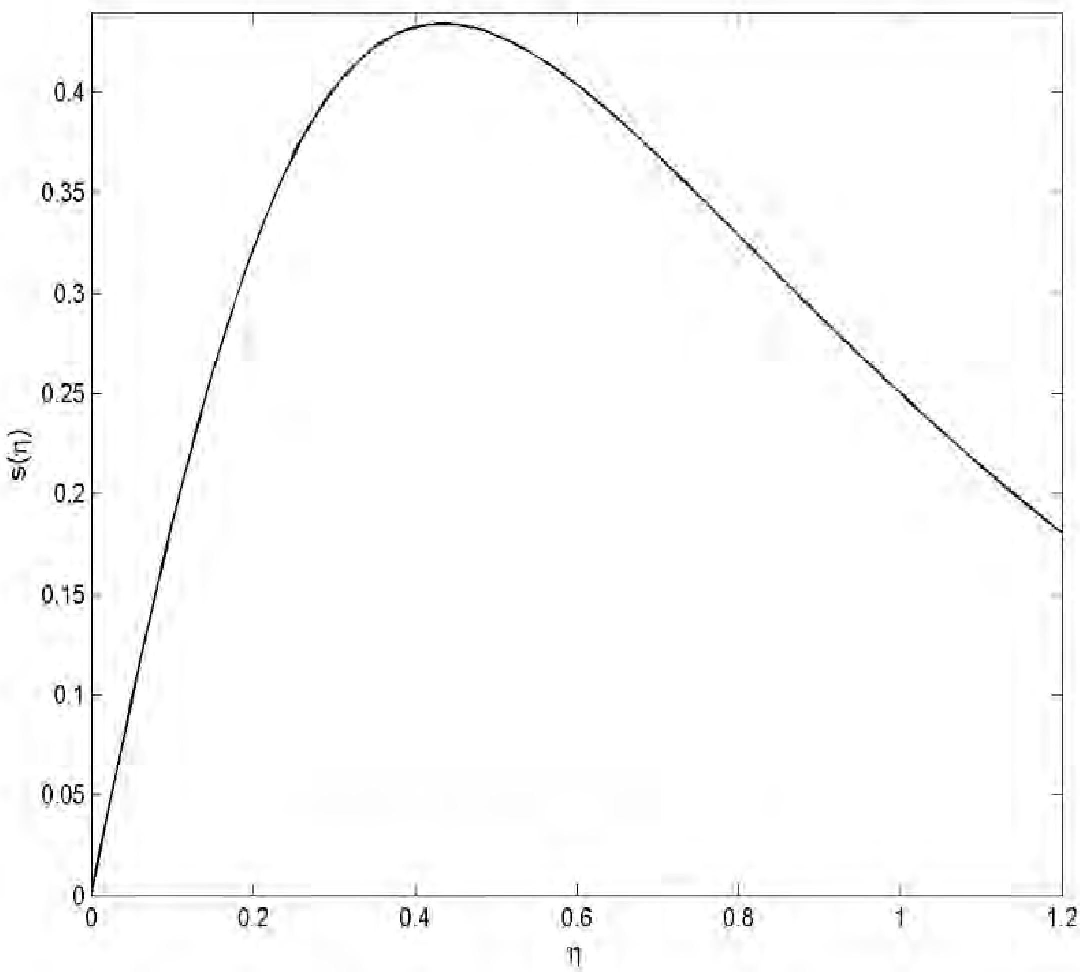

Figure 4.2: Plot of the quadrature squeezing $s(\eta)$ versus η .

superposed cavity modes is 43% below the coherent-state level and this occurs at $\eta = 0.42$. Hence we observe that the degree of squeezing for the superposed cavity modes increases in the interval between 0 and 0.42 and it also decreases in the interval between 0.42 and 1.2.

Chapter 5

CONCLUSION

In this thesis we have studied the statistical and the squeezing properties of the light emitted by a nondegenerate three-level atom driven by coherent light and in a cavity coupled to a vacuum reservior via a single-port mirror.

Employing the solutions of the quantum Langevin equations, we have determined the probability for the atom to be in the top, intermediate, or bottom level. We have carried out our analysis by putting the noise operators associated with the vacuum reservior in normal order. Our result shows that at steady state the atom has equal probability to be in the top or intermediate level.

On the other hand, applying the steady- state solutions of the quantum Langevin equations for the cavity mode operators, we have calculated the mean and the variance of the photon number sum and difference. From the result obtained, we have seen that the mean of the two cavity modes are equal. The photon number statistics is Poissonian for $\eta < 0.13$ and sub-Poissonian for $\eta \geq 0.13$. Furthermore, applying the same solutions, we have determined the photon number correlation. This shows that the photon numbers of the cavity modes a and b are uncorrelated. In addition, we have calculated the quadrature variance for the separate modes and for the superposition of the two modes. From the result we have found, we note that cavity mode a is in a chaotic state for arbitrary values of γ_c and Ω but mode b is in chaotic state for $\gamma_c << \Omega$ and in a coherent state for $\gamma_c >> \Omega$ and we have also observed that the superposed cavity modes are in a squeezed state for all values of η between 0 and 2.

Finally, we have determined the quadrature squeezing of the superposed cavity modes. The superposed cavity modes are in a squeezed state, with the maximum quadrature squeezing being 43% below the coherent-state level. This occurs at $\eta = 0.42$. We also seen that the degree of squeezing of the superposed cavity modes increases in the interval between 0 and 0.42 and decreases in the interval between 0.42 and 1.2.

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Declaration

I, the undersigned, declare that this thesis is my original work and it has not been presented before for a degree in any other University. Moreover, I declare that all the sources of material used for the thesis have been dully acknowledged.

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This thesis has been submitted for examination with my approval as University advisor.

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