An investigation of Attenuation and Dispersion in propagation mode characteristics of optical fiber material leading to non-linearity

By: Belay Getachew
Advisor: Dr. S.K.Ghoshal

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By: BELAY GETACHEW

MATERIALS SCIENCE PROGRAM

FACULITY OF SCIENCE

Approved by the Examining board

Prof. A.V. Gholap ______________________________

EXAMINER

Dr. S.K Ghoshal ______________________________

ADVISOR

Dr. Mesfin Redi ______________________________

EXAMINER

Dr. Teketel Yohannes ______________________________

CHAIRPERSON

DECLARETION:

I, the undersigned, declare that this thesis work is my original work, has not been presented for a degree in this or any universities and all sources of materials used for the thesis work have been fully acknowledged.

Name: Belay Getachew

Place: Addis Ababa

Date of submission: 2009-07-15 Signature…………

This thesis has been submitted for examination with my approval as a university advisor.

Advisor's Name Dr. S.K. Ghoshal

Signature……….....................

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Abstract

Study of non-linearity in optical fiber material has attracted tremendous attention over last two decades. Despite of many experimental and theoretical efforts to find the origin of non-linearity and to quantify the loss due to non-linearity still a debatable issue. The aim of this research work is to find a mechanism and to quantify to non-linearity that causes attenuation and dispersion in optical fiber. We specifically calculated solution to non-linear envelope equation (non-linear Schrödinger equation) from Maxwell theory choosing appropriate envelope function. Our calculations are in agreement with other observation as far as loss and non-linearity concerned. We found the **soliton** solution that is a very interesting result that can be further looked at in detail to observe the non-linear effect in fiber material. It is found that the non-linearity balances dispersion and a stable pulse is formed which does not alter its shape during propagation.

This simple calculation is to understand the mechanism of the origin of non-linearity how to further control it by tuning some of the fiber material.

II. OBJECTIVES

General objective

- Understand the propagation of pulses in optical fiber and how one can able to choose a physical parameter for the reduction of unwanted effects and comes to final result and conclusion using different equations and solutions of electromagnetic waves with major influence of non-linearity.

Specific objectives

- Investigation propagation of electromagnetic pulses through an optical fiber.
- Derive the wave equation for a general electromagnetic wave traveling along the fiber.
- Consider the dispersive character of the optical fiber material in connection with the existence of light pulses as a solution of to the wave equation.
- Introduce attenuation and non-linearity leading non-linear Schrödinger equation for the envelope of the electromagnetic pulses.
- Investigate properties of light pulses using envelope equation.
- Derive the width and intensity of pulse equation as a function of the traveled distance (for linearized envelope and for the full non linear equation)
- Find the equation that keeps the shape of the pulse during propagation.

Methodology

- Choosing an optical fiber material, which is made from silicon glass (low-loss fiber). This optical fiber is non-conducting and non-magnetic and for our application we also assume no free charges along the fiber.
- Investigating propagation of light pulses through an optical fiber in connection with the transmission of signals and problems that arise when dealing with attenuation and dispersion. This will be done by:-

- Using Maxwell equations to derive electromagnetic wave equation.
- Taking 1-D wave equation with refractive index depending on frequency and intensity.
- Considering solutions that describe the light pulses and deriving envelope equation for a light pulse – non-linear second order partial differential equation.
- Examining solutions to the lineralized envelope equations and investigating the influence of attenuation and dispersion on pulse propagation.
- Calculating a special solution to the non-linear envelope equation in the absence of attenuation – non-linear Schrödinger equation.
- Using MATLAB for drawing pictures.

Acknowledgement

I am very grateful to Dr. S.K.Ghoshal, my advisor, who inspired me to develop how to write the thesis, get resources and develop my confidence to engage my self in a more advanced research. I greatly appreciate the material science program director Dr. Teketel Yohannes who provided time and materials to defend first and second progress report because it helped me to write this thesis in a better way. Special thanks go to Prof. A.V.Gholap and Dr. Mesfin Redi for their useful and harmonious suggestions while I was presenting second progress report for improving final report.

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To my daughter

MARANATA

Reason for …

Chapter one

Introduction

1.1 Introduction on background

Light beam because of its frequency is capable of carrying far more information in comparison to frequencies of radio waves and micro waves [1]. It is necessary to have a guiding medium through which light waves could be propagated. The best guiding medium with minimum loss is an optical fiber which is made from silica.

In the construction of new telephone lines and the replacement of old lines the telephone companies use silica fibers for optical communication. Information is transformed into light pulses by solid state lasers; the pulses are then sent along optical fibers and detected by receivers for conversion into electrical digital signals or even analog signals.

In transferring information in the form of light signal in optical fiber researchers have successfully overcome numerous obstacles along this path, many of which when first discovered looked as through they would impede further increases in capacity and transmission distance. The evolution of optical fiber transmission systems: An early system using LEDs over multimode fiber. A system using MLM lasers over single mode fiber in 1.3μm band to overcome intermodal dispersion in multimode fiber, and a later system using SLM laser to over come chromatic dispersion limits and a current generation WDM system using multiple wave length at 1.55μm and optical amplifier instead of regenerator [2]

Attenuation and dispersion are two unwanted effects in propagation of mode in optical fiber in the linear case. They are a cause for power loss and limitation of band width, respectively. Different mechanisms are taking to minimize these undesired effects. Some mechanisms have bean tried as we have seen above and researchers are also finding different mechanisms.

Non-linearity is also one cause for attenuation and dispersion in optical fiber. In fact, the origin of non linearity and to quantify the loss due to non-linearity stills a debatable issue. But it has its own contribution in increasing the capacity and transmission distance. This research work will find a mechanism and to quantify non-linearity that causes these unwanted effects and to get fast, effective and error free information flux.

This paper has five chapters. In Chapter one – started with introduction and optical fiber, light propagation and modes in optical fiber are introduced. Optical fiber is a circular wave guide; components of electromagnetic wave are explained mathematically. Chapter two - explains theory of attenuation and dispersion as a review literature. Chapter three - the dispersive character of the fiber material is considered in connection with the existence of light pulses as a solution to the wave equation-this leads to the non-linear Schrödinger equation for the envelope of electromagnetic pulses. The width and the intensity of a pulse, as a function of the traveled distance, first for the linearized envelope equation are derived and a special solution to the non-linear envelope equation is calculated, in the absence of damping. Chapter four – result and discussion (results are shown and depending on the result important things are discussed). Chapter five – the last chapter contains conclusion.

1.2. An optical fiber

An optical fiber consists of a cylindrical core surrounded by a cladding. The cross-section of an optical fiber is shown in fig1. Both the core and the cladding are made primarily of silica (SiO_2), which has a refractive index of approximately 1.45. The refractive index of a material is the ratio of the speed of light in a vacuum to the speed of light in the material. During the manufacturing of the fiber certain impurities (or dopants) are introduced in the core and/or the cladding so that the refractive index is slightly higher in the core than in the cladding. Materials such as germanium and phosphorus increase the refractive index of silica and used as dopants for the core, where as materials such as boron and fluorine that decreases the refractive index of silica are used as dopants for the cladding. The resulting higher refractive index of core enables the light to be guided by the core, and thus propagates through the fiber [2].

We consider a typical commercial fiber which consists of a central core with the refractive index n_c surrounded by a cladding layer whose refractive index n_{cl} is slightly lower than that of the core. The final layer is the so called buffer (jacket) which has a refractive index lower than n_{cl} [5].

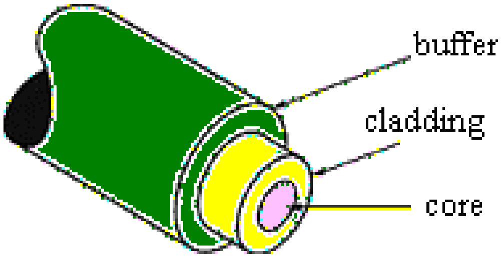

Fig1. Schematic illustration of the cross section of an optical fiber.

The material choice for low –loss optical fiber is silica glass which is non magnetic and non-conducting. For our application we also assume no free charges along the fiber.

1.2.1 Types of fiber

There are two types of fiber. These are multi-mode fiber and single-mode fiber.

1.2.1.1 Multi mode fiber

Fiber with a large core diameter (greater than 10 μm) is called multi mode fiber. This may be analyzed by geometric optics. In step index (due to the step discontinuity in the index of refraction at the core cladding interface) multi mode fiber, rays of light are guided along the fiber core by total internal reflection. Since it has a high numerical aperture many modes allowed propagating. A key parameter that describes the mode structure is the V- parameter (normalized frequency). The V- parameter is important because it determines the number of electromagnetic modes in the fiber. For any value of V larger than 2.405, more than one mode exists. An estimate of the total number of modes N supported by any given large value of V (V>2.405) [4].

$$N \approx \frac{V^2}{2} \quad \text{(for V>2.4)} \qquad \text{where, } V = \frac{2\pi a}{\lambda}\sqrt{n_1^2 - n_2^2} \tag{1.1}$$

This will be seen in the next section.

In graded index (the index of refraction is maximum at the center –with value n_1 and tapes off to a minimum value-n_2 at the edge of the core) the confinement process is a consequence of the solution of the electromagnetic propagation problem in a dielectric medium with such a refractive index profile. These waves follow a sinusoidal path.

It used in application that when a band width (< 1GHz) over a relatively short distance (<3km) such as local area network, campus network, home network and car network is required [5].

The major benefits of multimode fiber are

- It is relatively easy to work with.
- Because of its larger core size, light is easily coupled to and from it.
- It can be used with both lasers and LEDs as sources.
- Coupling losses are less than those of the single mode fiber.

1.2.1.2 Single mode fiber

Fiber with small core diameter (5 μm-10 μm) called single mode fiber. Fiber with a core diameter less about ten times the wavelength of the propagating light can not be modeled using geometric optics. Instead, it must be analyzed as an electromagnetic structure, by solution of Maxwell's equation as reduced to the electromagnetic wave equation (it will be explained later). As an optical waveguide, the fiber supports one or more confined transverse mode by which light can propagate along the fiber. Fiber supporting only one mode is called single mode fiber. The waveguide analysis shows that the light energy in the fiber is not completely confined in the core. Instead, especially in single mode fibers, a significant fraction of the energy in the bound mode travels in the cladding as an evanescent wave.

The most common type of a single mode fiber has a core diameter 8 μm-10 μm and is designed for use in the near – infrared. The mode structure depends on the wavelength of the light used, so that this fiber actually supports a small number of additional modes at visible wave length.

Single mode fiber is used in high band width, long distance application such as long distance telephone trunk lines, TV head – ends, and high speed local and wide area network (LAN and WAN) back bones. The major draw back of single mode fiber is that it is relatively difficult to work with (i.e. splicing and termination) because of its small core size. Also, single mode fiber is typically used only with laser sources because of the high coupling losses – associated with LEDs.

1.3 Light propagation in optical fiber

An optical fiber is a cylindrical structure made up of pure silica. As light is launched into the fiber, the light is confined to the fiber by virtue of the phenomenon of Total Internal Reflection (TIR). To understand TIR, we need to look at the transmission of light through materials having different refractive indices [6]. The refractive index determines the speed of the wave inside the material. The speed of light **v** inside any material is given by the following expression,

$$v = \frac{c}{n} \tag{1.2}$$

where c is the velocity of light in free space ($3\text{x}10^8$ m/s) and n is the refractive index. We can now explain the phenomenon of total internal reflection using Figure 2. In Figure 2a, a ray of light is going from a medium of lower index (rarer) to one of a higher index (denser). As the ray enters the second medium it moves towards the normal (the dotted line). In Figure 2b, light goes from a higher index to a lower index and the light moves away from the normal. As the angle of incidence θ_i is increased and becomes equal to θ_c, the light in the second medium grazes the interface (Figure 2c). We now have total internal reflection if the angle of incidence goes above θ_c, known as the critical angle. All the light now stays in the medium of higher index.

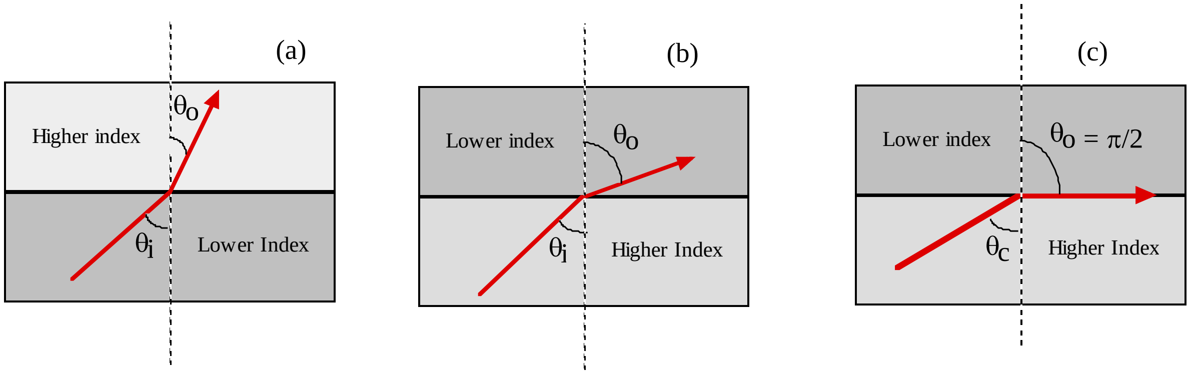

Figure 2 Reflection at an interface (a) Lower index to higher index (b) Higher index to lower index(c) Higher index to lower index at critical angle θ_c.

Condition of TIR allows light to be guided. To understand this, consider a simple experiment as shown in Figure 3. A beaker full of water has a glass tube on the right through which water can flow out freely. A beam of laser light is allowed to enter from the left as shown. If the water is now allowed to flow out, the water will appear red as it flows freely out of the tube. The higher index water (1.33) surrounded by lower index air (1.0) creates TIR conditions, and the light that was present in the beaker is now provided guiding conditions as it comes out because of water being surrounded by air.

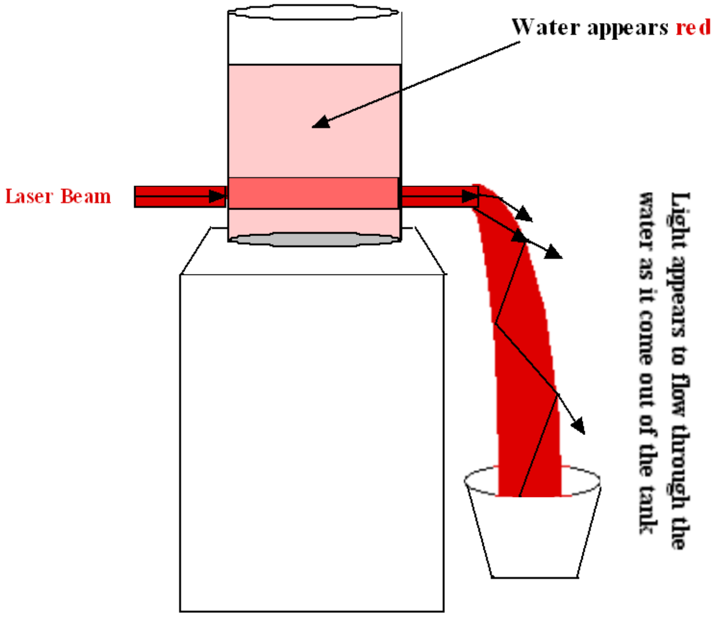

Figure 3 Concept of Total Internal reflection

The conditions of total internal reflection in optical fibers are created by having the fiber with an inner core of high index silica glass surrounded by silica glass of slightly lower index. A number of ways exist for the creation of the required index difference between the core and cladding. Thus there are a number of index profiles that are used in the fabrication of fibers. Some of these are shown in Figure 4.

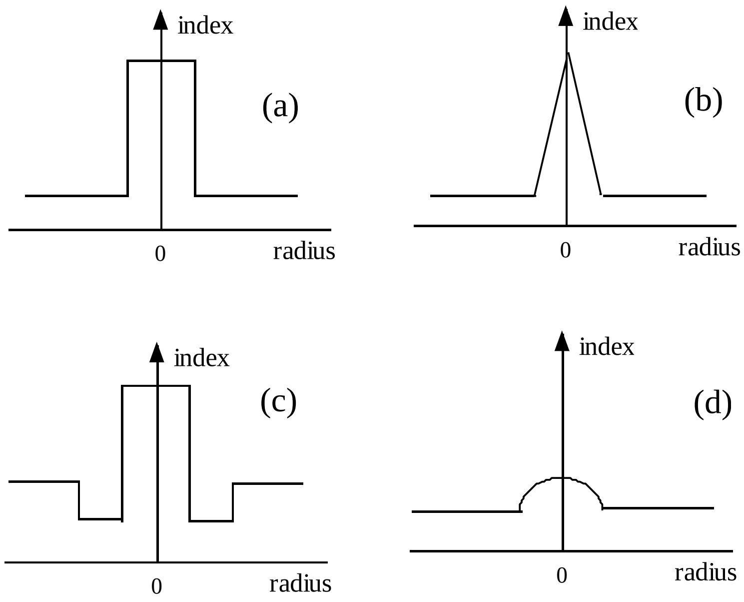

Figure 4 Index profiles
(a) Step Index
(b) Triangular
(c) W-type
(d) Graded index

We will look at the case of the step index fiber. The structure is shown in Figure 4a. The index of the core is n_c and index of the cladding is n_{cl}. Note that air has refractive index of unity, water has a refractive of 1.33, and ordinary glass has a refractive index of around 1.45. A cut out of the fiber is shown in Figure 5.

Thus, if we have an inner cylinder (core) made up of a higher index, surrounded by material (cladding) of lower index, total internal reflection conditions can be met. Indeed, the material, Silica, is used for the core and cladding. By doping the core with a small percentage of Germanium, the index of the core region can be increased by about 10^{-3} to 10^{-4} above the cladding index (~1.456). We now have three physical parameters that characterize the fiber, the radius of the core (a), the index of the core (n_c), and the index of the cladding (n_{cl}). We will normally assume that the cladding is infinitely thick.

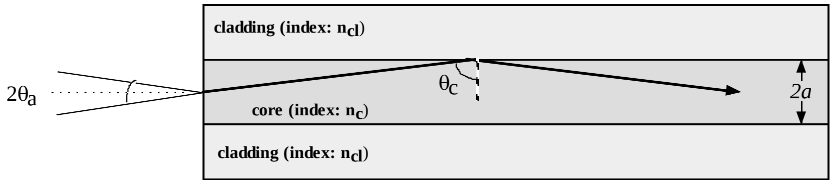

Figure 5 The critical angle at the interface of the core/cladding is θ_c. This determines the maximum angle (θ_a) for accepting light into the fiber. Light beyond the cone of $2\theta_a$ will be lost.

The acceptance angle θ_a is determined by the indices of the core and cladding and is given by

$$\theta_a = \sqrt{n_c^2 - n_{cl}^2}\ rad\ . \tag{1.3}$$

The acceptance angle expressed in radians is also known as the Numerical Aperture (NA) of the fiber. Higher values of numerical apertures point to better light gathering capacity of the fibers. Note that NA does not depend on the radius of the fiber.

1.4 Modes in optical fiber

An optical mode refers to a particular solution to the equation governing the propagation of light inside the fiber, subject to the boundary conditions existing from the physical properties of the fiber such as the core diameter, index of the core, index of the cladding and the operating wavelength. The mode has the property that its spatial distribution does not change with length or distance. The only effect of propagation is a change in the instantaneous phase or a change in amplitude induced due to losses. No change in shape of the spatial distribution (i.e., in a direction at right angle to the propagation) takes place. A fiber can support many modes. We can also fabricate a fiber that only supports a single mode as we have seen single mode fiber. If we take a multimode fiber and reduce the radius of the fiber, the number of modes supported in the fiber goes down, and, it is possible to reach a point when only a single mode can be supported. Before we look at the simple equations governing the number of modes supported by the fiber, let us try to understand the concept of modes using a simple analogy of a ‘highly flexible vertical rod’ in a long box. Let us look at Figure 6. If the box is small in height (box *aa*), then the rod that can be fitted in the box can only take one possible shape (mode 1). If we increase the height now to *bb*, it can fit mode 1 and another shape (mode 2). For the box of height *cc*, the rod can also take another shape (mode 3). Thus, the transverse dimensions determine how many different ‘shapes’ can be present in the box. These shapes can be identified as the modes in an optical waveguide.

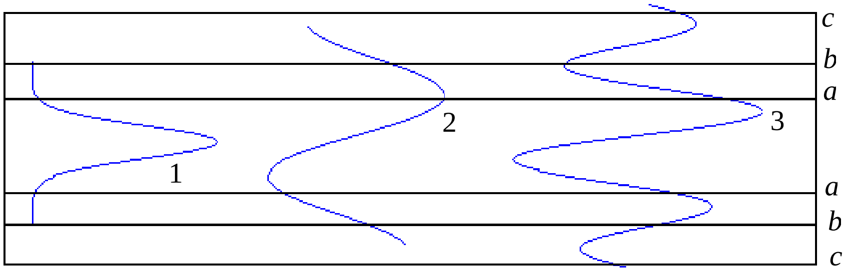

Figure 6 Concept of modes

The number of modes and their shapes will be determined by the cross sectional geometry of the fiber.

The parameter that determines whether a fiber is single mode or multimode is referred to as the V parameter of the fiber given by

$$V = \frac{2\pi}{\lambda} a \sqrt{n_c^2 - n_{cl}^2} \,. \tag{1.4}$$

Making an approximation that the indices are very close,

$$V = \frac{2\pi}{\lambda} a\, n_c \sqrt{2\Delta} \quad , \tag{1.5}$$

where Δ is the index difference defined as

$$\Delta = \frac{n_c^2 - n_{cl}^2}{2n_c} . \tag{1.6}$$

If the V value is less than 2.405, the fiber is considered to be a single mode fiber. The number of modes, N, supported in a step index fiber is given by

$$N = \frac{V^2}{2} . \tag{1.7}$$

It might appear that for V=2.405, N~2. This is not a mistake. It simply reflects the fact that the single mode contains two polarizations, which are indistinguishable from each other because of circular symmetry of the optical fiber. In other words, a single mode fiber contains two modes that are together and cannot be separated. One way to separate them is to use an elliptical fiber. An elliptical fiber cross section is shown Figure 6 which has the capability of separating the two modes of a single mode fiber. These fibers are also known as polarization preserving single mode fibers. Such fibers are used in coherent fiber communications and in fiber sensors.

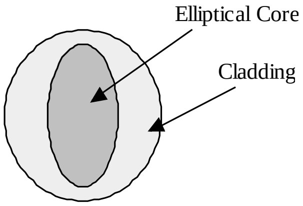

Figure 7A typical elliptical fiber cross section. It allows the LP01 modes to become two separate modes, LP_{01}^{x} and LP_{01}^{y}.

A few modes of an optical fiber are shown in Figure 8. As the mode order increases, the patterns become more and more complex. The lowest order mode has almost a Gaussian field pattern as shown in Figure 9.

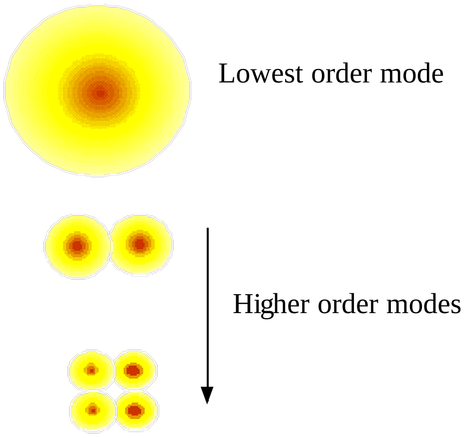

Figure 8 Modes in an optical fiber

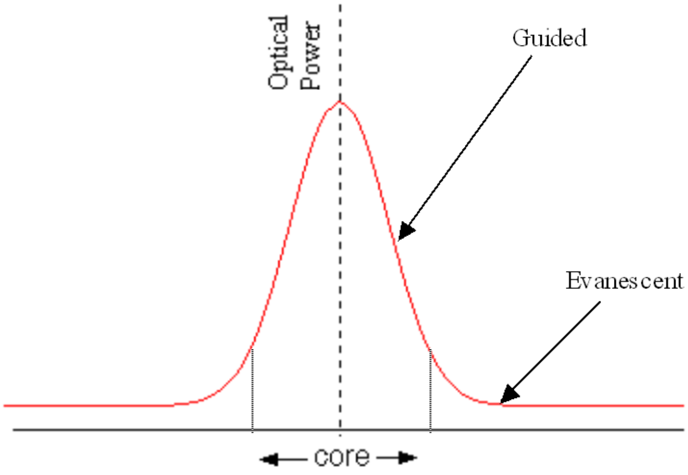

Figure 9 Intensity (power) profile of the fundamental mode

The pattern of the lowest order shows a ‘guided’ part of the mode inside the core and an ‘evanescent’ part in the cladding which has an exponential decay in the radial direction. The lowest order mode intensity profile can be approximated to a Gaussian pattern. As the V –value increases, the fundamental mode becomes ‘tighter and tighter’ and all most all the power in the lowest order power stays within the core. In the case of a single mode fiber of a step index fiber (V<2.405), about 85% of the power travels through the core and the rest travels through the cladding.

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1.5 Circular Wave Guide

This kind of surface is important because it includes both circular and the coaxial line, which is in the form of two concentric cylinders. The wave propagates in the angular space between the concentric regions [7].

We have to solve

$$(\nabla^2 + k_c^2)\varphi = 0 \tag{1.8}$$

Using cylindrical coordinate (r, θ, z)

Let ‘a’ be the radius of the circular cross section

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} \tag{1.9}$$

We have

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}\right]\varphi + k_c^2\varphi = 0$$

Subjected to the boundary condition

$$\varphi \Big|_{r=0} = 0 \,\&\, \frac{\partial\varphi}{\partial n}\Big|_{r=a} = 0 \quad \text{for all values of } \theta$$

1. For TE-mode $E_z(r,\theta) = 0$ every where; $H_z(r,\theta)$ can be determined by solving the wave equation, $H_z(r,\theta)$

2. For TM-mode $H_z(r,\theta)$ = 0 every where; $E_z(r,\theta)$ can be determined by solving the wave equation, $E_z(r,\theta)$

Free space Maxwell’s equations:

$$\vec{\nabla} x \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{and} \qquad \vec{\nabla} x \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

$$\begin{vmatrix} \hat{n}_r & \hat{n}_\theta & \hat{n}z \\ \frac{\partial}{\partial r} & \frac{1}{r}\frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ \varphi_r & \varphi_\theta & \varphi_z \end{vmatrix} \qquad \text{where } \varphi \equiv \begin{Bmatrix} \vec{E} \\ \vec{B} \end{Bmatrix}$$

Now we can write

$$\hat{n}_r\left[\frac{1}{r}\frac{\partial E_z}{\partial \theta} - \frac{\partial E_\theta}{\partial z}\right] + \hat{n}_\theta\left[\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r}\right] + \hat{n}_z\frac{1}{r}\left[\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial E_r}{\partial \theta}\right] = -\left[\frac{\partial B_r}{\partial t}\hat{n}_r + \frac{\partial B_\theta}{\partial r}\hat{n}_\theta + \frac{\partial B_z}{\partial t}\hat{n}_z\right]$$

and

(1.13)

$$\hat{n}_r\left[\frac{1}{r}\frac{\partial B_z}{\partial \theta} - \frac{\partial B_\theta}{\partial z}\right] + \hat{n}_\theta\left[\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right] + \hat{n}_z\frac{1}{r}\left[\frac{\partial}{\partial r}(rB_\theta) - \frac{\partial B_r}{\partial \theta}\right] = \frac{1}{c^2}\left[\frac{\partial E_r}{\partial t}\hat{n}_r + \frac{\partial E_\theta}{\partial r}\hat{n}_\theta + \frac{\partial E_z}{\partial t}\hat{n}_z\right]$$

Note: 3rd term

$$\begin{aligned} &\frac{\partial E_\theta}{\partial r} - \frac{1}{r}\frac{\partial E_r}{\partial \theta} \\ &= \frac{1}{r}\frac{\partial}{\partial r}(rE_\theta) - \frac{1}{r}\frac{\partial E_r}{\partial \theta} \\ &= \frac{1}{r}r\frac{\partial E_\theta}{\partial r} + \frac{E_\theta}{r} - \frac{1}{r}\frac{\partial E_r}{\partial \theta} \\ &E_\theta = 0 \ \ everywhere \end{aligned}$$

Collecting the coefficients of $\hat{n}_r, \hat{n}_\theta$ & $\hat{n}_z$ from both sides

$$\left.\begin{aligned} &\frac{1}{r}\frac{\partial E_z}{\partial \theta} - \frac{\partial E_\theta}{\partial z} = -\frac{\partial B_r}{\partial t} \\ &\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -\frac{\partial B_\theta}{\partial t} \\ &\frac{1}{r}\left[\frac{\partial}{\partial r}(rE_\theta) - \frac{\partial E_r}{\partial \theta}\right] = -\frac{\partial B_z}{\partial t} \end{aligned}\right\} \qquad (1.10)$$

$$\left.\begin{aligned}
&\frac{1}{r}\frac{\partial B_z}{\partial \theta}-\frac{\partial B_\theta}{\partial z}=-\frac{\partial E_r}{\partial t}\\
&\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial r}=-\frac{\partial E_\theta}{\partial t}\\
&\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rB_\theta\right)-\frac{\partial B_r}{\partial \theta}\right]=-\frac{\partial E_z}{\partial t}
\end{aligned}\right\} \tag{1.11}$$

Here

$$\boldsymbol{E}(r,\theta,z,t)=\boldsymbol{E}(r,\theta)\ e^{i(kz-\omega t)}$$

$$\boldsymbol{B}(r,\theta,z,t)=\boldsymbol{B}(r,\theta)\ e^{i(kz-\omega t)}$$

$$\frac{\partial}{\partial z}\rightarrow ik \qquad \text{and} \qquad \frac{\partial}{\partial t}\rightarrow -i\omega$$

Substituting in 1.10 & 1.11we get

$$\frac{1}{r}\frac{\partial E_z}{\partial \theta}-ik_zE_\theta=i\omega B_r \tag{1.12a}$$

$$ikE_r-\frac{\partial E_z}{\partial r}=i\omega B_\theta \tag{1.12b}$$

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rE_\theta\right)-\frac{\partial E_r}{\partial \theta}\right]=i\omega B_z \tag{1.12c}$$

$$\frac{1}{r}\frac{\partial B_z}{\partial \theta}-ik_zB_\theta=-\frac{i\omega}{c^2}B_r \tag{1.12d}$$

$$ikB_r-\frac{\partial B_z}{\partial r}=-\frac{i\omega}{c^2}E_\theta \tag{1.12e}$$

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(rB_\theta\right)-\frac{\partial B_r}{\partial \theta}\right]=-\frac{i\omega}{c^2}i\omega E_z \tag{1.12f}$$

Eliminate B_r from (1.12 a& e) we get

$$E_\theta = -\frac{i\omega}{\left(\frac{\omega^2}{c^2} - k^2\right)}\left[\frac{\partial B_z}{\partial r} - \frac{1}{r}\frac{k}{\omega}\frac{\partial E_z}{\partial \theta}\right] \tag{1.13}$$

From (1.12b & 1.12d) eliminate B_θ then

$$E_r = \frac{i\omega}{\left(\frac{\omega^2}{c^2} - k^2\right)}\left[\frac{k}{\omega}\frac{\partial E_z}{\partial r} + \frac{1}{r}\frac{\partial B_z}{\partial \theta}\right] \tag{1.14}$$

From (1.12a&1.12e) eliminate E_r then

$$B_\theta = \frac{i}{\left(\frac{\omega^2}{c^2} - k^2\right)}\left[\frac{\omega}{c^2}\frac{\partial E_z}{\partial r} + \frac{k}{r}\frac{\partial B_z}{\partial \theta}\right] \tag{1.15}$$

Eliminate E_θ from (1.12b&1.12d) we have

$$B_r = \frac{i}{\left(\frac{\omega^2}{c^2} - k^2\right)}\left[k\frac{\partial B_z}{\partial r} - \frac{\omega}{c^2 r}\frac{\partial E_z}{\partial \theta}\right] \tag{1.16}$$

These equations shows that all the transverse components of the field E_r, E_θ, B_r & B_θ can be obtained from B_z and E_z. So we solve (1. 9) for E_z and B_z.

Case I. TM mode B_z = 0

Substituting B_z = 0 in eq (1.13-1.16) we have

$$E_r = \frac{ik}{\left(\frac{\omega^2}{c^2} - k^2\right)}\frac{\partial E_z}{\partial r} \tag{1.17a}$$

$$E_\theta = \frac{ik}{\left(\frac{\omega^2}{c^2} - k^2\right)} \frac{\partial E_z}{\partial \theta} \tag{1.17b}$$

$$B_r = \frac{-i\omega}{\left(\frac{\omega^2}{c^2} - k^2\right) c^2 r} \frac{\partial E_z}{\partial \theta} \tag{1.17c}$$

$$B_\theta = \frac{i\omega}{\left(\frac{\omega^2}{c^2} - k^2\right) c^2} \frac{\partial E_z}{\partial r} \tag{1.17d}$$

It is clear that all transverse components of the TM mode are expressible in terms of E_z .

Solve the wave equation for E_z

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right] E_z + k_c^2 E_z = 0 \tag{1.18}$$

Boundary condition is $E_z(r,\theta) = 0$ at $r = a$ for all θ

We can solve it using the method of separation of variables

$$\boldsymbol{E_z(r,\theta) = R(r)\Theta(\theta)} \tag{1.19}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}(R\Theta)\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta^2}(R\Theta) + k^2 R\Theta = 0$$

$$\frac{1}{rR}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) + \frac{1}{r^2\Theta}\frac{\partial^2\Theta}{\partial \theta^2} + k_c^2 = 0 \tag{1.20}$$

Multiplying eq (1.20) by r^2, we get

$$\frac{r}{R}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) + k_c^2 r^2 = -\frac{1}{\Theta}\frac{\partial^2\Theta}{\partial \theta^2} \tag{1.21}$$

LHS function of r and RHS function of Θ only

$$-\frac{1}{\Theta}\frac{\partial^2\Theta}{\partial\theta^2} = n^2 \tag{1.22}$$

$$\frac{r}{R}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) + k_c^2 r^2 = n^2 \tag{1.23}$$

$$\frac{\partial^2\Theta}{\partial\theta^2} + n^2\Theta = 0 \tag{1.24}$$

Solution

$\Theta = A_n \cos(n\theta) + B_n \sin(n\theta)$

A_n and B_n determine the orientation of the field in the guide. For a circular guide and for any particular n one can orient θ axes in such a way so that either A_n or B_n equal to zero. Let us choose B_n = 0

$$\Theta = A_n \cos n\theta \tag{1.25}$$

For R

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial R}{\partial r}\right) + \left(k_c^2 - \frac{n^2}{r^2}\right)R = 0$$

$$\frac{1}{r}\frac{\partial}{\partial(k_c r)}\left[r\frac{\partial R}{\partial(k_c r)}\right] + \left[1 - \frac{n^2}{(k_c r)^2}\right]R = 0 \tag{1.26}$$

This is the Bessel's equation variable $(k_c r)$. Two independent solutions are Bessel's function $J_n(k_c r)$ and Neumann's function $N_n(k_c r)$. They are having the following properties.

I. At x = 0 $\quad J_n(x) \propto x^n$ and $N_n(x) \to \infty$

II. For large x $\quad J_n(x)$ and $N_n(x)$ goes to

$$J_n(x) \to \sqrt{\frac{2}{\pi x}}\cos\left(x - \frac{2n+1}{4}\pi\right)$$

(1.27)

$$N_n(x) \to \sqrt{\frac{2}{\pi x}}\sin\left(x - \frac{2n+1}{4}\pi\right)$$

In a circular guide at r = 0 field must be finite so we use only Bessel's function $J_n(k_c r)$

Therefore $R = J_n(k_c r)$ (1.28)

Hence $E_z(r,\theta) = R(r)\Theta(\theta)$

$$= J_n(k_c r)\,A_n \cos n\theta$$

$$= A_n\, J_n(k_c r)\cos n\theta \tag{1.29}$$

Finally, we find

$$E_z(r,\theta,z,t) = A_n\, J_n(k_c r)\cos n\theta\; e^{i(kz-\omega t)} \tag{1.30}$$

The boundary condition for TM mode is

$$E_z\big|_{r=a} = 0 \Rightarrow J_n(k_c a) = 0 \tag{1.31}$$

This equation has infinite number of roots, but for the propagation to be possible, $k = \sqrt{k_o^2 - k_c^2} > 0$ real & $k_c^2 << k_o^2 or k_o^2 << \frac{\omega^2}{c^2}$ otherwise high frequencies will be required. Therefore, only first few roots of (1.31) will be the practical interest.

$J_n(x_{nm}) = 0$ first few roots are $x_{01}, x_{11}, x_{21}, x_{10}, x_{22}...$, $TM_{01}, TM_{02}, TM_{11}...$ and so on TM_{nm} in general. No root for x_{00} i.e. $(k_c a)_{00} \Rightarrow TM_{00}$ Waves cannot exist.

$[x_{01} = 2.405, x_{11} = 3.832, x_{10} = 5.520...]$

Physical meaning of n & m:-

- n represents the number of cycles of variation of E_z found as θ varies around the complete cylinder. $(n \to \theta)$
- m represents the number of zeros of electric field along a radial path from the center to the inside surface of the guide wall. $(m \to r)$
- If we take x_{nm} the m^{th} value of x for which $J_n(x) = 0$ then cut off value is

$$k_c a = x_{nm} \Rightarrow k_c = \frac{x_{nm}}{a} \tag{1.32}$$

[Note n comes through the algebra. The subscript m refers to the roots in their order of magnitude.]

Cutoff wavelength $\lambda_c = \frac{2\pi}{k_c} = \frac{2\pi a}{x_{nm}}$ (1.33)

Phase velocity

$$\upsilon_p = \frac{\omega}{k} = \frac{\omega}{\sqrt{k_o^2 - k_c^2}} = \frac{\omega}{k_o\sqrt{1-\frac{k_c^2}{k_o^2}}}$$

$$= \frac{c}{\sqrt{1-\frac{k_c^2}{k_o^2}}} = \frac{c}{\sqrt{1-\frac{\omega_c^2}{\omega^2}}} > c \qquad (1.34)$$

Group velocity

$$\upsilon_g = \frac{\partial\omega}{\partial k} = \frac{\partial}{\partial k}\left(ck_o\right) = \frac{\partial}{\partial k}\left[c\left(k^2 + k_c^2\right)^{\frac{1}{2}}\right]$$

$$= \frac{ck}{\sqrt{k^2 + k_c^2}} = \frac{ck}{k_o} = \frac{c^2 k}{\omega} \qquad (1.35)$$

For a given k_c υ_g varies $0 \rightarrow c$

k_c varies $0 \rightarrow \infty$

We have from

$$E_z(r,\theta,z,t) = A_n\, J_n(k_c r)\cos n\theta\; e^{i(kz-\omega t)}$$

$$k_c^2 = k_o^2 - k^2 = \frac{\omega^2}{c^2} - k^2$$

From (1.17) transverse field components are

$$E_r = \frac{ik}{k_c^2}\, A_n\, J_n'(k_c r)\cos n\theta\; e^{i(kz-\omega t)}$$

$$E_\theta = \frac{-ikn}{k_c^2 r} A_n J_n(k_c r) \sin n\theta \; e^{i(kz-\omega t)}$$

$$B_r = \frac{i\omega n}{k_c^2 c^2 r} A_n J_n(k_c r) \sin n\theta \; e^{i(kz-\omega t)}$$

$$B_\theta = \frac{i\omega}{k_c^2 c^2} A_n J_n^{'}(k_c r) \cos n\theta \; e^{i(kz-\omega t)}$$

where $J_n^{'} = \frac{\partial}{\partial r}(J_n)$

Case II. TE mode $E_z = 0 \qquad E_z(r,\theta)$

We have from (1.13-1.16) putting $E_z = 0$

$$E_r = \frac{i\omega}{\left(\frac{\omega^2}{c^2} - k^2\right)} \frac{\partial B_z}{\partial \theta} \quad ; \qquad E_\theta = \frac{-i\omega}{\left(\frac{\omega^2}{c^2} - k^2\right)} \frac{\partial B_z}{\partial r}$$

(1.36)

$$B_r = \frac{ik}{\left(\frac{\omega^2}{c^2} - k^2\right)} \frac{\partial B_z}{\partial r} \quad ; \qquad B_\theta = \frac{ik}{\left(\frac{\omega^2}{c^2} - k^2\right)} \frac{\partial B_z}{\partial \theta}$$

All transverse components are expressible interms of B_z

From equation (1.8) we get the solution for B_z

$$(\nabla_\perp^2 + k_c^2)\varphi = 0 \tag{1.37}$$

Boundary condition for TE mode is $\frac{\partial B_z}{\partial n} = \frac{\partial B_z(r,\theta)}{\partial r} = 0$ at r = a for all θ and

$E_z(r,\theta) = 0$ every where

$B_z(r,\theta)$ is the solution of (1.37) subjected to boundary condition.

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right] B_z(r,\theta) + k_c^2 B_z(r,\theta) = 0$$

Let $B_z(r,\theta) = R(r)\Theta(\theta) = R\Theta$

$$\Rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\Theta\frac{\partial R}{\partial r}\right)+\frac{1}{r^2}R\frac{\partial^2\Theta}{\partial\theta^2}+k^2R\Theta=0$$

Divided by $R\Theta/r^2$

$$\Rightarrow \frac{r}{R}\left[\frac{\partial R}{\partial r}+r\frac{\partial^2 R}{\partial r^2}\right]+k_c^2r^2=-\frac{1}{\Theta}\frac{\partial^2\Theta}{\partial\theta^2} \tag{1.38}$$

L.H.S function of r and R.H.S function of θ only

Hence
$$-\frac{1}{\Theta}\frac{\partial^2\Theta}{\partial\theta^2}=n^2\Rightarrow\frac{\partial^2\Theta}{\partial\theta^2}+n^2\Theta=0 \tag{1.39}$$

and
$$\frac{r}{R}\left[\frac{\partial R}{\partial r}+r\frac{\partial^2 R}{\partial r^2}\right]+k_c^2r^2=n^2$$

$$\Rightarrow r^2\frac{\partial^2 R}{\partial r^2}+r\frac{\partial R}{\partial r}+\left(k_c^2r^2-n^2\right)R=0$$

$$\Rightarrow \frac{\partial^2 R}{\partial r^2}+\frac{1}{r}\frac{\partial R}{\partial r}+\left(k_c^2-\frac{k^2}{r^2}\right)R=0 \tag{1.40}$$

The solution of (1.39) is

$$\Theta=A_n\cos(n\theta)+B_n\sin(n\theta) \tag{1.41}$$

A_n and B_n are constants

From (1.40)

$$\frac{\partial^2 R}{k_c^2\partial r^2}+\frac{1}{k_c^2r}\frac{\partial R}{\partial r}+\left(1-\frac{n^2}{k_c^2r^2}\right)R=0$$

$$\Rightarrow \frac{\partial^2 R}{\partial(k_cr)^2}+\frac{1}{k_cr}\frac{\partial R}{\partial(rk_c)}+\left(1-\frac{n^2}{(rk_c)^2}\right)R=0 \tag{1.42}$$

We can write it

$$\frac{\partial^2 y}{\partial x^2}+\frac{1}{x}\frac{\partial y}{\partial x}+\left(1-\frac{n^2}{x^2}\right)y=0$$

Solution of this equation is in the form of Bessel's function $J_n(x)$ and Neumann function $N_n(x)$. These functions are

$$J_n(x) = \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{2n+1}{4}\pi\right)$$

$$N_n(x) = \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{2n+1}{4}\pi\right)$$

At x = 0 $J_n(x) \propto x^n$ and $N_n(x) \to \infty$

For circular wave guide field must be finite at r = 0 so we throw $N_n(x)$ and we use only $J_n(x)$. Hence (1.42) gives

$$R = J_n(rk_c)$$

The over all solution is

$$B_z(r,\theta) = R(r)\Theta(\theta) = J_n(k_c r)\, A_n \cos n\theta + B_n \sin n\theta \qquad (1.43)$$

Applying boundary condition

$$\frac{\partial B_z(r,\theta)}{\partial r} = \frac{\partial J_n(rk_c)}{\partial r}\Theta = 0 \quad \text{at r = a for all } \theta$$

$$\Rightarrow J_n'(ak_c) = 0 \qquad (1.44)$$

$$\left(J_n'(ak_c) = \frac{\partial J(rk_c)}{\partial r} = \frac{n}{k_c r} J_n(k_c r) - J_{n+1}(k_c r)\right)$$

For a given ‘n’ equation (1.44) has infinite number of solution for a k_c. If m^{th} solution is S_{nm} then $k = \frac{S_{nm}}{a}$ and equation (1.43) yields

$$B_z(r,\theta) = J_n(rS_{nm})[\, A_n \cos n\theta + B_n \sin n\theta\,] \qquad (1.45)$$

[First few roots of $J_n'(S_{nm}) = 0$ are $S_{01} = 3.832, S_{11} = 1.842, S_{02} = 7.016$ & so on.]

Cut off wavelength $\lambda_c = \frac{2\pi a}{S_{nm}}$

Guide wavelength $\lambda_g = \lambda_o \sqrt{\frac{1}{(1-\lambda_o/\lambda_c)^2}}$

Impedance offered $= \sqrt{\frac{\mu}{\varepsilon}} \frac{\lambda_g}{\lambda_o}$

$$= \sqrt{\frac{\mu}{\varepsilon}} \frac{1}{\sqrt{\left[1-\left(\frac{\lambda_o S_{nm}}{2\pi a}\right)^2\right]}}$$

The Z component of phase velocity

$$\upsilon_p^z = \frac{1}{\sqrt{\varepsilon\mu}} \frac{1}{\sqrt{1-\left(\lambda_o S_{nm}/2\pi a\right)^2}} = \frac{\upsilon_o}{\sqrt{1-\frac{\omega_c^2}{\omega^2}}} \quad ; \quad \upsilon_o = \frac{1}{\sqrt{\varepsilon\mu}}$$

The Z component of group velocity

$$\upsilon_g^2 = \frac{1}{\sqrt{\varepsilon\mu}} \sqrt{1-\left(\frac{\lambda_o S_{nm}}{2\pi a}\right)}^2$$

Look for E_r, E_θ, B_r & B_θ: from (1.13-1.16) by making $E_z = 0$

$$E_r = \frac{i\omega}{\left(\frac{\omega^2}{c^2}-k^2\right)} \frac{\partial B_z}{\partial \theta} \quad ; \quad E_\theta = -\frac{i\omega}{\left(\frac{\omega^2}{c^2}-k^2\right)} \frac{\partial B_z}{\partial r}$$

(1.48)

$$B_r = \frac{ik}{\left(\frac{\omega^2}{c^2}-k^2\right)} \frac{\partial B_z}{\partial r} \quad ; \quad B_r = \frac{ik}{\left(\frac{\omega^2}{c^2}-k^2\right)} \frac{\partial B_z}{\partial r}$$

E_r, E_θ, B_r & B_θ are expressed interms of B_z. So solution of B_z is

$$B_z(r,\theta) = A_n J_n(k_c r)\cos n\theta \tag{1.49}$$

The boundary condition is $\frac{\partial B_z}{\partial r} = 0$ at r = 0 demand $B_n = 0 \Rightarrow$

$$\left.\frac{\partial J_n(k_c r)}{\partial r}\right|_{r=a} = 0 \Rightarrow J_n^{'}(k_c a) = 0$$

$$J_n^{'}(S_{nm}) = 0$$

m$^{\text{th}}$ value of S for which $J_n^{'}(x)=0$ then cut off is fixed by:

$$k_c a = S_{nm} \quad ; \quad \lambda_c = \frac{2\pi}{k_c} = \frac{2\pi a}{S_{nm}}$$

So we can calculate the cut off wavelength of various modes. The expression for phase and group velocity will remain the same as those obtained for TM mode except we replace x_{nm} by S_{nm}.

The field components for TE mode may be expressed as:

$$E_z = 0 \quad ; \quad B_z = A_n J_n(k_c r)\cos n\theta \; e^{i(kz-\omega t)}$$

$$E_r = -\frac{i\omega n}{k_c^2 r} A_n J_n(k_c r)\sin n\theta \; e^{i(kz-\omega t)}$$

$$E_\theta = -\frac{i\omega n}{k_c^2} A_n J_n^{'}(k_c r)\cos n\theta \; e^{i(kz-\omega t)}$$

$$B_r = \frac{ik}{k_c^2} A_n J_n^{'}(k_c r)\cos n\theta \; e^{i(kz-\omega t)}$$

$$B_\theta = -\frac{ikn}{k_c^2 r} A_n J_n^{'}(k_c r)\sin n\theta \; e^{i(kz-\omega t)}$$

1.6 Background of the problem

A critical area to control in either single mode or multimode optical fibers is attenuation. Attenuation is the decrease in magnitude of the optical signal power as it is transmitted between two points. The attenuation loss overcomes the optical signal power as it propagates until the signal is lost in the noise at the receiver. The lower the attenuation is, the better optical transmission qualities the fiber has.

An additional parameter of interest is dispersion. Dispersion is the source of bandwidth limitations in an optical fiber. Dispersion relates to the spread or increase in width of a light pulse as it propagates along the length of the fiber. In single mode fibers, total dispersion is primarily affected by the material dispersion and waveguide dispersion. Material dispersion is caused by the differential delay of various wavelengths of light in the waveguide material. Waveguide dispersion is attributed to light pulses traveling in both the core and the cladding of the fiber. Both of these dispersion sources are generally influenced by the interdependence of the refractive index of the core and cladding material on wavelength. However, both types of dispersion are viewed separately and can have a cumulative effect on the transmission properties of the optical fiber.

[5]Single mode fibers of the prior art are generally operated at wavelengths ranging from about 1310 nm to 1550 nm. To minimize the total dispersion of a single mode fiber, it is desirable to operate at a wavelength longer than about 1270 nm. At a wavelength of approximately 1310 nm, the small positive material dispersion cancels out the small negative wavelength dispersion causing a net dispersion of zero for high silica content optical fibers. The upper range of the operating range, around 1550 nm, is preferred, however, to keep the attenuation loss of the optical fiber low for many reasons. Optical fibers exhibit a hydroxyl peak at around 1380 nm due to the introduction of water which appear as hydroxyl (-OH) groups during the manufacturing process. The hydroxyl peak is a significant source of attenuation for an optical fiber.

Further, the theoretical minimum loss for glass fiber, 0.16 dB/km, occurs at a wavelength of 1550 nm.

One solution, to minimize total dispersion and attenuation loss in an optical fiber, is to shift the dispersion to wavelengths around 1550 nm. Optical fibers which are designed for operation at wavelengths around 1550 nm are called dispersion shifted fibers. In dispersion shifted fibers, the zero of total dispersion lies between 1330 nm to 1620 nm rather than at 1300 nm. Conventional methods to shift the zero dispersion above 1300 nm include lowering the V value for the fiber, altering the refractive index profile of the fiber, and/or doping of the core of the fiber with germanium oxide (Ge02) or other additives.

Increasing the operating wavelength to around 1550 nm to minimize total dispersion and attenuation loss is challenging. These two objectives, unfortunately, compete against one other. Waveguide dispersion is very sensitive to changes in the optical fiber parameters. As mentioned earlier, lowering the V value to about 1.5 may cause more of the light pulse to propagate through the cladding than through the central core region. This would have a negative impact on waveguide dispersion. In addition, waveguide dispersion is also influenced by the diameter of the central core region, the refractive index profile and the fractional change delta of the refractive indices of the central core region (n_1) and the cladding (n_2) of the optical fiber. A decrease in the core region diameter would require a doubling of the A to keep the V value constant. Material dispersion, on the other hand, is principally effected by doping of the central core region.

The lower limit of the range may be preferred to minimize total dispersion, the water or hydroxyl peak at 1380 nm is a major source of attenuation loss for a fiber. Thus, the upper limit of this range is generally preferred to keep the attenuation of the optical fiber low. Even fibers that have lowered, or eliminated, the hydroxyl peak at 1380 nm experience losses due to Rayleigh scattering. In addition, as the wavelength of the fiber is increased, more of the optical signal is propagated in the cladding area rather than in the central core region.

This creates a major source of attenuation loss for single mode optical fibers, known as bending loss, when the optical fiber is cabled or bent. Eventually, the fundamental propagating mode LPo (linear polarization) becomes lossy.

Chapter Two

Theory of Attenuation and Dispersion

The capacity of transmission is not unlimited. In order to increase the bit-rate we have to look at properties of the medium such as attenuation and dispersion.

2.1 Attenuation

The attenuation of any fiber is the degradation in the power of the input signal due to the fiber properties. Given input power P_{in} over a fiber of length L and output power P_{out}, the mean attenuation constant α of the fiber, in units 'dB/km', is defined as [2].

$$\alpha = \frac{10}{L}\log\frac{P_{in}}{P_{out}} \tag{2.1}$$

The decibel unit (dB) is used to represent power ratios. However, it is sometimes convenient to represent absolute power levels on a logarithmic scale. The most commonly used unit is dBm, which corresponds to power referenced to 10mW.

$$P_{dBm} = 10\log\frac{P}{10x10^{-3}} \tag{2.2}$$

One of the main causes of attenuation is absorption of energy (or photons). Absorption is caused by atomic defects which result when the fiber is exposed to radiation: extrinsic absorption by impurity atoms, and intrinsic absorption by constituent atoms of the material. The dominant mechanism is extrinsic absorption, primarily by metallic ions (Iron, Cobalt, etc) and OH^{-} ions [5].

In early optical fibers, the transmission distance was primarily limited by absorption by OH^{-} ions. These ions were introduced in the material from the presence of water or water vapor during the manufacture process. Attenuation caused by this ion is greatest at 1400nm, 1250nm, 950nm and 750nm, leaving “windows” for transmission between these wavelengths. The advent of the vapor phase axial deposition (VAD) manufacture method led to tremendous reduction in the OH^- concentration n fibers. Peaks on Fig 10 are due to OH^- [5].

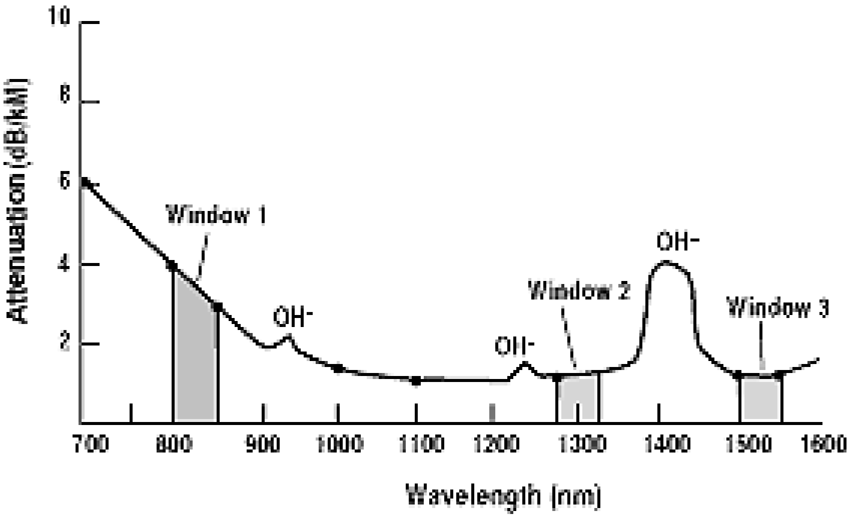

Fig 10 attenuation versus wavelength (showing peaks due to OH^-)

Losses in modern fibers are caused by ultraviolet absorption, infrared losses and scattering losses. The scattering losses, modeled by Rayleigh scattering are caused by the interaction of the light wave with the constituent molecules which are on the order of the light wavelength. Rayleigh scattering loss is $\sim\frac{1}{\lambda^4}$, so it can be reduced by increasing the wavelength. On the other hand, infrared absorption loss tends to increase with λ, and usually worst above 1.5 μm. The point where this loss starts to increase to unacceptably large levels can be pushed out by doping the SiO_2 halides. In general, the combined effect of such losses is minimum at 1.3 μm. Fig 11 shows losses due to all these effects [5].

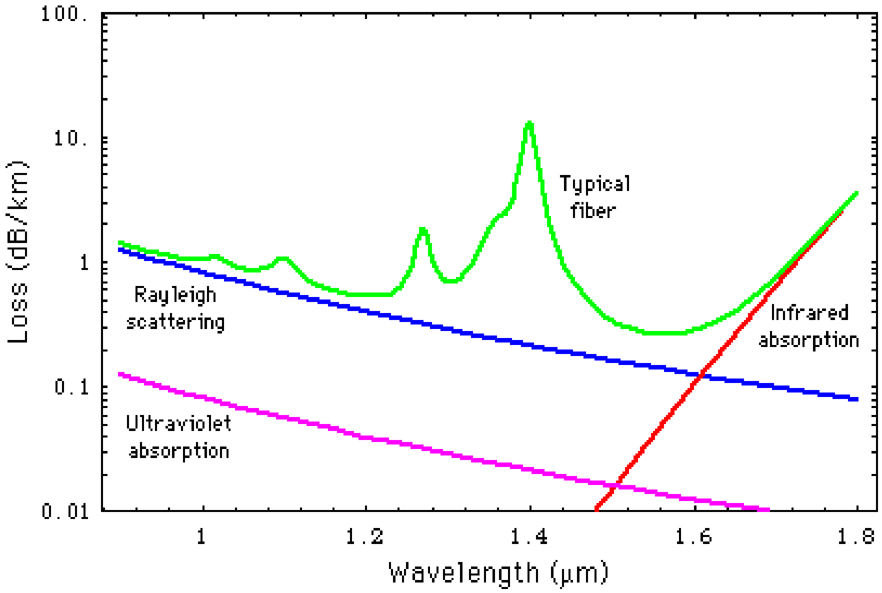

Fig 11 Attenuation valley b/n Rayleigh scattering and absorption in silica fiber.

There are also losses caused by macrobends and microbends: Large bends of the cable fibers are macrobends and microbend is a tiny crinkle or imperfection in the surface of the fiber, on the dimensions of several wavelengths, and causes a perturbation in the field. Thus, a microbend leads to coupling to higher order modes, which do not have the desired transmission, characteristics, and also causes power loss. Macro and microbending can occur while the fiber is being manufactured, especially during the spooling process. Spooling a fiber to minimize macrobends and microbends is not trivial when we consider that very long continues fibers, of lengths 1km or more, are manufactured. The reason for manufacturing such long fiber is that splicing or coupling fiber segments together can introduce significant losses. The basic reason for loss when splicing fiber is the faces of the two segments are not properly aligned, so not all the output power of one segment is inserted to the other. Losses in modern fibers can be kept down to as little as 0.01dB/km.

2.2 Dispersion in optical fiber

One of the major limitations of fiber communication systems is the pulse broadening introduced by dispersion. Dispersion is present in both single mode and multimode fibers. If a fiber is multimode, it supports a number of modes. Each of these modes will travel down the fiber at different speeds, resulting in a significant difference in times of arrival of the modes at the output end. Consider now the launch of a single pulse of light as shown in Figure 12. As soon as the pulse enters the fiber, replicas of the pulse will be created corresponding to each of the modes. Since these replicas will arrive at the receiver at different times, the resultant pulse will be broader (dark line envelope of the replicas at the output). The broadening of the pulse is the result of differential time delays and is said to come from the multimode dispersion in the fiber [3].

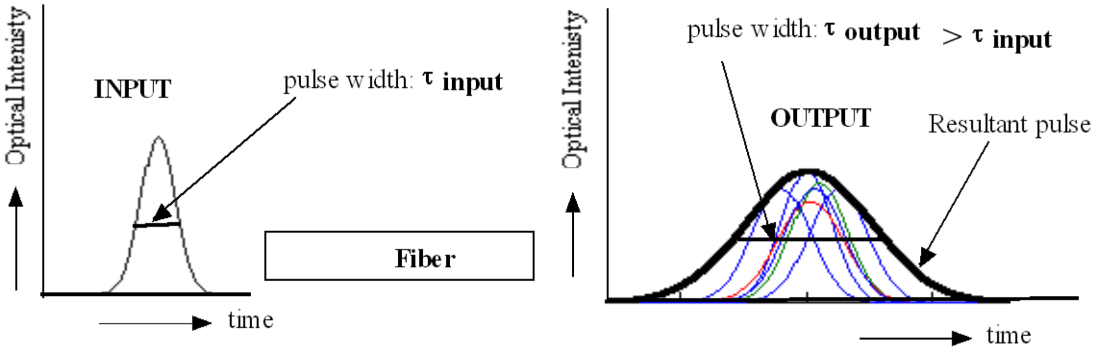

Figure 12 Pulse broadening in fibers is shown. The output pulse is an envelope of all the replicas and is thus, broader than the input pulse.

The dispersion in multimode fibers limits the data rate of transmission. As the transmission distance increases, two adjoining pulses transmitted through the fiber broaden and it becomes difficult to separate the pulses. For a multimode fiber, the upper limit of the distance-bit-rate product *BL* is given by

$$BL < \frac{8c}{n_c \Delta^2} \text{ bits/s.Km.} \tag{2.3}$$

Dispersion also is present in single mode fibers. The primary cause of dispersion in single mode fiber is the dispersive nature of the refractive index of glass. A material is considered to be dispersive if the speed depends on the wavelength. The refractive index of glass indeed is a function of wavelength, i.e., $n \equiv n(\lambda)$. Since all the optical sources have a finite spectral width, the dispersive nature of the index in conjunction with the finite spectral width of the source leads to dispersion in single mode fibers. We can go back to Figure and visualize that each of the replicas of the pulses now correspond to the individual wavelengths that constitute the spectrum of the source, the output pulse will certainly be broadened. The pulse broadening in single mode fibers will be much smaller than the corresponding value in multimode fibers.

There are two components to the dispersion in single mode fibers. One is the material dispersion, determined purely by the material properties. The other one, waveguide dispersion is dependent on the waveguide parameters such as the V-value, the index profile and the core diameter. This also means that while material dispersion remains a constant for a given material, the waveguide dispersion can be tailored by the choice of an appropriate index profile and diameter. The material and waveguide dispersion are interdependent. The total dispersion is the sum of these two and is shown in Figure along with the material and waveguide dispersion. The point at which the dispersion value is zero gives the operating wavelength for minimum dispersion. If the waveguide dispersion is not tailored, the fiber operated around this wavelength is referred to as a *zero* dispersion fiber (typically around 1310 nm). If the waveguide dispersion is tailored, the zero dispersion point can be moved and fibers operated at such a wavelength is identified as dispersion shifted fiber (around 1550 nm). By having negligible dispersion around a band wavelengths we can have a dispersion flattened fiber (around 1550 nm). The dispersion characteristics of a typical single mode fiber are shown in Figure 13 and Figure 14. The unit of dispersion (D) is ps/nm.Km. Figure 13 shows the two components of the dispersion in single mode fibers. Since they are interdependent, the total dispersion in single mode fibers is given by

$$\delta\tau_{total} = \delta\tau_{mat} + \delta\tau_{waveguide}. \tag{2.4}$$

If we have a multimode fiber, the total dispersion in a multimode fiber sill be given by

$$\Delta\tau_{total} = \sqrt{\delta\tau_{total}^{2} + \delta\tau_{mod\,al}^{2}}, \tag{2.5}$$

where the first term is the contribution from the material and waveguide dispersion while the second term is arising from the multimode.

Figure 13compares the dispersion characteristics of the three single mode fiber types. The dispersion flattened fiber has very low dispersion over a broad range of wavelengths. These fibers will find applications in wavelength division multiplex fiber optic systems.

The ‘nm’ (nanometer) refers to the spectral width of the source and the ‘Km’ refers to the transmission distance in kilometers. The pulse broadening is given by

$$\Delta\tau = |D| L \sigma_{\lambda} \text{ ps}, \tag{2.6}$$

where D is given in Figure 13 [5].

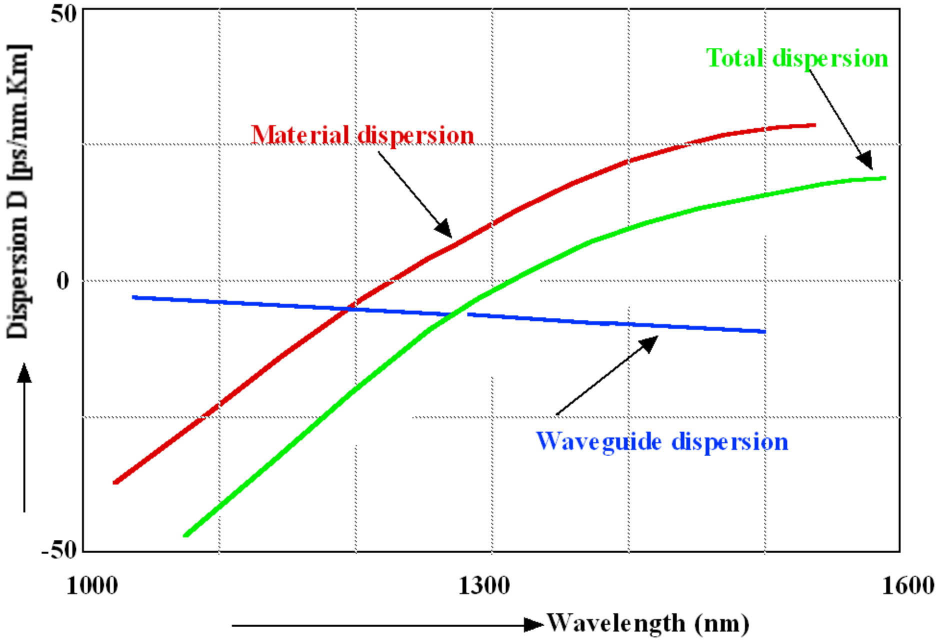

Figure 13 The material and waveguide dispersion are shown. The total dispersion is the sum of the two [5].

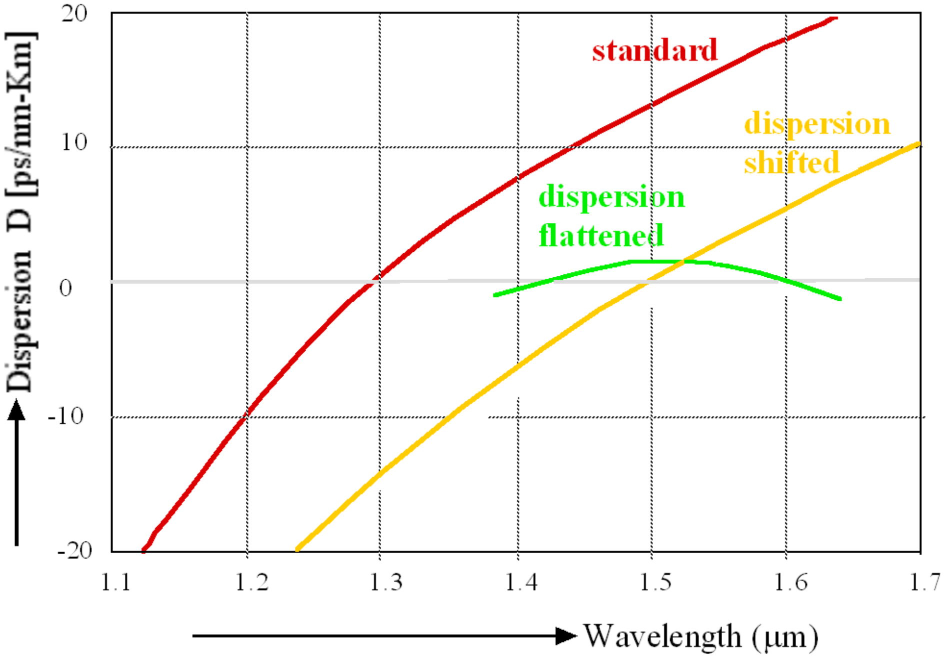

Figure 14 Dispersion in single mode fibers [5].

Dispersion limits the ability to transmit data at high rates over long distances. As the data rate-distance product increases, we will be dealing with a weaker signal as the transmission distance increases. We also will be facing increased dispersion leading overlapping of the adjoining bits resulting in intersymbol interference as shown in Figure 15. The ISI will increase the likelihood of errors in the detection of 0's and 1's. The weaker signal also increases the likelihood of increased errors because the signal-to-noise ratio may fall below the threshold value required for acceptable performance. This means that the optical signal needs to be detected, pulses reshaped, and retransmitted. This is accomplished by using a repeater. Repeater is a receiver/transmitter combination. Before we look at a repeater

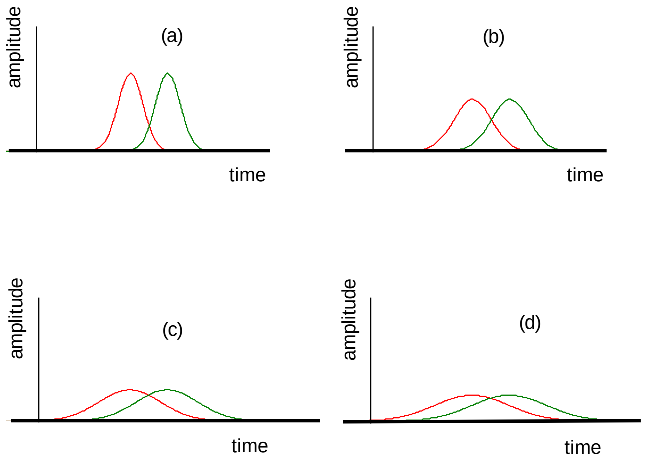

Figure 15 Two pulses are injected into the fiber. Pulses broaden as they travel down the fiber

(a) Closest to the input end

(b) Away from the input end

(c) Further away

(d) Farthest from the input end.

Figure 15 shows the problems arising out of dispersion. As the pulses travel down the fiber, they broaden and start overlapping. It becomes difficult to separate the two adjoining pulses, increasing the error at the receiver. Reducing the data-rate-distance product will help alleviate the problem. However, this reduces our ability to transmit data at higher rates over long distances. We will see that we use a soliton based communication system to overcome the limitations imposed by dispersion.

Chapter Three

Formulation of the problem

3.1 Maxwell Equations

When we analyze the propagation of electromagnetic waves we have to consider the Maxwell equations given by [3].

$$\nabla.\vec{D} = \rho \tag{3.1a}$$

$$\nabla.\vec{B} = 0 \tag{3.1b}$$

$$\nabla x\vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{3.1c}$$

$$\nabla x\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{3.1d}$$

Where $\vec{E}$ is the electric field vector and $\vec{H}$ the magnetic field vector. $\vec{B}$ and $\vec{D}$ are the corresponding magnetic and electric flux densities. The current density $\vec{J}$ and the charge density ρ represent the sources of the electro-magnetic field. Optical fibers are made of silica glass. In such a medium we have no free charges, so we have $\vec{J} = 0$ and $\rho = 0$. The relations between the magnetic and electric field and the flux densities are given by

$$\vec{B} = \mu_o \vec{H} + \vec{M} \tag{3.2}$$

$$\vec{D} = \varepsilon_o \vec{E} + \vec{P} = \varepsilon_o (1 + \vec{\chi})\overline{E} \tag{3.3}$$

Where, μ_o is permeability in vacuum, ε_o is the vacuum permittivity. $\vec{P}$ and $\vec{M}$ are the electric and magnetic polarization respectively, arising from the electric and magnetic field. The relation between $\vec{P}$ and $\vec{E}$ is given by $\vec{P} = \varepsilon_o \chi \vec{E}$, where χ is the electric susceptibility. The optical fibers is non magnetic medium, so $\vec{M} = 0$.

We use the Maxwell equations to derive a wave equation for the light propagation in optical fiber.

By taking the curl of eq. (3.1c) and making use of eq. (3.1d), we obtain

$$\nabla x(\nabla x\vec{E}) = -\frac{\partial(\nabla x\vec{B})}{\partial t} = -\mu_o \frac{\partial(\nabla x\overrightarrow{H})}{\partial t} \quad (3.4)$$

$$= \mu_o \frac{\partial}{\partial t}(\vec{J} + \frac{\partial\vec{D}}{\partial t}) \quad (3.5)$$

Equation (3.5) can now be made a function of only $\vec{E}$ by incorporating eq. (3.2 and 3.3) as follows:

$$\nabla x(\nabla x\vec{E}) = -\frac{\partial}{\partial t}[\varepsilon_o(1+\chi)\frac{\partial}{\partial t}\vec{E}] \quad (3.6)$$

However, using the vector operation, we find that

$$\nabla x(\nabla x\vec{E}) = \nabla(\nabla.\vec{E}) - (\nabla.\nabla)\vec{E}$$

$$= \nabla(\nabla.\vec{E}) - \nabla^2\vec{E}$$

WE notice that $\nabla.\vec{D} = \rho = 0$ and therefore $\nabla.\vec{E} = 0$.

Thus eq. (3.6) reduces to

$$\nabla^2\vec{E} = \mu_o\varepsilon_o(1+\chi)\frac{\partial^2\vec{E}}{\partial t^2} \quad (3.7)$$

By using the relations $c^2 = \frac{1}{\varepsilon_o\mu_o}$, $n = \sqrt{1+\chi}$ (>1) (3.8)

Where c is the speed of light and n is the refractive index of the medium, and substituting eq. (3.8) into eq. (3.7), we get

$$\nabla^2 \overrightarrow{E} = \frac{n^2}{c^2}\frac{\partial^2 \overrightarrow{E}}{\partial t^2} \tag{3.9}$$

This is differential equation.

We define the Z-axis to be along the fiber and assume that the light propagation of the incoming pulse is only along the fiber.

We assume a constant polarization and choose the x-axis along the $\overrightarrow{E}$ vector, i.e.

$$\overrightarrow{E} = E_x(z).\overrightarrow{e_x}$$

Hence we obtain the one – dimensional wave equation

$$\frac{\partial^2 \overrightarrow{E}(z,t)}{\partial Z^2} - \frac{n^2}{c^2}\frac{\partial^2 \overrightarrow{E}(z,t)}{\partial t^2} = 0 \tag{3.10}$$

Any solution of this partial differential equation has the form

$$\overrightarrow{E}(z,t) = f_1(z+\frac{n}{c}t) + f_2(z-\frac{n}{c}t)$$

Where $f_1, f_2 \, \varepsilon \, c^2(\Re)$ of a particular interest is the solution

$$\overrightarrow{E}(z,t) = Ae^{i(\omega_o t - k_o z)} \tag{3.11}$$

This equation is damped harmonic oscillatory solution.

Where, $k_o = \pm \frac{n}{c}\omega$

$\overrightarrow{E}(z,t)$ is the electric field as a function of z and t.

$e^{i\omega_o}$ is high frequency carrier wave

A is slowly varying envelope function

3.2 The Envelope Equation

We consider the wave equation (3.10). In practice, however, one finds that the refractive index, n, depends on the frequency ω of the considered mode $Ae^{i(\omega t - kz)}$. In this case we say that the system is dispersive. This will be justified in the next section. Further more, for light with high intensity; $I \sim |E|^2$ the refractive index n also depends on the intensity. We assume that this is the case and, therefore, write

$$n = n(\omega, |E|^2)$$

and

$$E_{zz} - \frac{n^2(\omega, |E|^2)}{c^2} E_{tt} = 0\,. \tag{3.12}$$

In the following we shall seek pulse-like solution of (3.12), of the form

$$E(z,t) = A(\varepsilon t, \varepsilon z)e^{i(\omega_o t - k_o z)} \tag{3.13}$$

where A is slowly varying function $(\varepsilon << 1)$. In order to do so, we need to derive a partial differential equation for A. For the wave number we have

$$k = k(\omega, |E|^2). \tag{3.14}$$

As we are searching for the envelope of a pulse, we should look at waves with frequencies located near the high carrier-wave frequency:

$$\omega = \omega_o + \upsilon \tag{3.15}$$

$$k = k_o + \kappa \tag{3.16}$$

$$|\upsilon|, |\kappa| << 1 \tag{3.17}$$

where ω_o is the frequency of the carrier-wave and υ the frequencies in the signal due to the much more slowly varying envelope.

Making a Taylor expansion of (3.14), we find:

$$k = k(\omega, |E|^2). = k(\omega_o + \upsilon, |E|^2)$$

$$= k(\omega, |E|^2) + (\frac{\partial k}{\partial \omega}\upsilon + \frac{\partial k}{\partial |E|^2}|E|^2) + \frac{1}{2}[\frac{\partial^2 k}{\partial \omega^2}\upsilon^2 + \ldots]$$

$$= k(\omega, |E|^2) + \frac{\partial k}{\partial \omega}\upsilon + \frac{1}{2}\frac{\partial^2 k}{\partial \omega^2}\upsilon^2 + \frac{\partial k}{\partial |E|^2}|E|^2 + \ldots$$

$$\kappa = k_o^{'}\upsilon + \frac{1}{2}k_o^{''}\upsilon^2 + \alpha|E|^2, \tag{3.18}$$

Where,

$$k_o^{'} = \frac{\partial k(\omega_{o,},0)}{\partial \omega} \tag{3.19}$$

$$k_o^{''} = \frac{\partial^2 k(\omega_o.0)}{\partial \omega^2} \tag{3.20}$$

$$\alpha = \frac{\partial k(\omega_o,0)}{\partial |E|^2} \tag{3.21}$$

are constants. $k_o^{'}$ is the inverse group-velocity of the pulse and $k_o^{''}$ is called the dispersion. They are dependent only on the carrier-frequency and the medium, so the function $k_o^{''}(\omega_o)$ is a characteristic of the fiber.

Consider the forward and backward Fourier transforms $E(z,t)$ in both coordinates:

$$F(k,\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} E(z,t) e^{-i(kz-\omega t)} dz dt \tag{3.22a}$$

$$E(z,t) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(k,\omega) e^{i(kz-\omega t)} dk d\omega \tag{3.22b}$$

Shifting the coordinate system according to

$$F(k,\omega) = F(k_o + \kappa, \omega_o + \upsilon) =: \tilde{F}(\kappa,\upsilon)$$

Eq. (3.22b) becomes:

$$E(z,t) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{F}(\kappa,\upsilon) e^{i(kz-\upsilon t)} d\kappa d\upsilon . e^{i(k_o z - \omega_o t)}$$

If we now define $q(z,t)$ as Fourier transform of $\tilde{F}(\kappa,\upsilon)$, i.e.

$$q(z,t) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\tilde{F}(\kappa,\upsilon)e^{i(kz-\upsilon t)}d\kappa d\upsilon, \tag{3.23}$$

Then we have

$$E(z,t) = q(z,t)\, e^{i(k_o z-\omega_o t)}, \tag{3.24}$$

this gives us the decomposition of the solution in the high frequency carrier wave and the envelope-function $q(z,t)$. We note that

$$q_t = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(-i\upsilon)\tilde{F}(\kappa,\upsilon)e^{i(\kappa z-\upsilon t)}d\kappa d\upsilon \tag{3.25a}$$

$$q_{tt} = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(-\upsilon^2\right)\tilde{F}(\kappa,\upsilon)e^{i(\kappa z-\upsilon t)}d\kappa d\upsilon \tag{3.25b}$$

$$q_z = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(i\kappa)\tilde{F}(\kappa,\upsilon)e^{i(\kappa z-\upsilon t)}d\kappa d\upsilon \tag{3.25c}$$

Using eq. (3.25a-c) we can have the following relations:

$$q_t \sim -i\upsilon$$

$$q_{tt} \sim -\upsilon^2 \text{ and} \tag{3.25d}$$

$$q_z \sim i\kappa$$

From Eq. (3.25d) and the expansion in (3.18) we obtain

$$q_z = -k_o^{'} q_t - \frac{1}{2} i k_o^{''} q_{tt} + i\alpha|q|^2 q. \tag{3.26}$$

Here we have used Eq. (3.25a-c), $|E|^2 = |q|^2$, and that these are independent of κ and υ. Eq.(3.26) is the desired envelope-equation.

It is the equation for the envelope $q(z,t)$ that, together with the carrier-wave, results in a pulse-type solution for the non-linear dispersive wave-propagation described by Eq. (3.12).

Note that if $k_o^{'}$ were a linear function of ω, that is $k_o^{''} = 0$, then $q(z,t) = F(t - k_o^{'}z)$, when F is an arbitrary function that satisfies eq. (3.26). Then $q(z,t) = A(t - k_o^{'}z)$ for all z and t, and arbitrary pulse shapes propagate without change in shape (and at a velocity $\frac{1}{k_o^{'}}$). In other words, if the group velocity is independent of ω, no broadening of the pulse occurs. Thus $k_o^{''}$ is key parameter governing group velocity or dispersion. It is termed the group velocity dispersion parameter.

In equation (3.26), the term $\frac{1}{2} i k_o^{''} q_{tt}$ incorporates the effect of dispersion (chromatic) and the term $i\alpha|q|^2 q$ incorporates the intensity dependent phase shift.

Since this equation incorporates the effect of dispersion also, the combined effect of dispersion and non-linearity on pulse propagation can be analyzed using this equation as a starting point. These effects are qualitatively different from that of dispersion and non linearity (especially due to self phase modulation) acting alone. This equation will be investigated linearly and non-linearly.

To Eq. (3.26) we can add a term representing damping of the signal, i.e. a loss of energy. We do not derive this term from assumptions on the constitutive equation concerning wave propagation in the fiber material, but just add it to the equation with an unknown parameter γ, determining the magnitude of the damping. We then obtain the following equation:

$$q_z + k_o^{'} q_t + \frac{1}{2} i k_o^{''} q_{tt} - i\alpha|q|^2 q + \frac{\gamma}{2} q = 0 \tag{3.27}$$

This equation holds group velocity, dispersion, attenuation and intensity dependent of phase shift. This equation will be examined in the sections to follow

3.3 Investigation of the envelope- equation

3.3.1 Linearizing and scaling

To make things easier, we first linearize the envelope equation. Here we assume that the third-order term $|q|^2 q$ is small compared to the first-order terms so we can neglect it. The associated linearized envelope equation reads

$$\frac{\partial q}{\partial Z} + k_o^{'} \frac{\partial q}{\partial t} + \frac{i}{2} k_o^{''} \frac{\partial^2 q}{\partial t^2} + \frac{\gamma}{2} q = 0 \tag{3.28}$$

We choose the initial conditions to be a Gaussian curve of the form

$$q(0,t) = \exp\left(-\left(\frac{t}{w_o} \right)^2 \right)$$

which typically represents a bit-like signal sent in the fibers. ω_o is a parameter which is a measure of the initial width of the pulse. We want to look at the dimensionless form of equation (3.28) so we introduce the following scaling of the variable:

$$u = k_q q, \quad \xi = k_\xi (Z - \upsilon_g t), \quad z = k_z Z \ , \ \delta = k_{w_o} w_o$$

With

$$k_q = \gamma \ , \ k_\xi = k_o^{'} \sqrt{-\frac{\gamma}{k_o^{''}}} \ , \ \upsilon_g = \frac{1}{k_o^{'}} \ , \ k_Z = \gamma \ , \ k_{\omega_o} = \sqrt{-\frac{\gamma}{k_o^{''}}}$$

We will see later that ξ is the coordinate traveling along with the pulse. After some calculation we obtain the scaled envelope equation with initial conditions

$$i \frac{\partial u}{\partial z} + \frac{1}{2} \frac{\partial^2 u}{\partial \xi^2} + i \frac{u}{2} = 0 \ , \tag{3.29a}$$

$$u(0,\xi) = \gamma \exp\left(-\left(\frac{\xi}{\delta} \right)^2 \right). \qquad (3.29b)$$

3.3.2 Solving the scaled envelope equation

Equation (3.29) is solved by means of the Fourier transform

$$\hat{u}(z,k) = \int_{-\infty}^{\infty} u(z,\xi) e^{ik\xi} d\xi. \qquad (3.30)$$

If we take the Fourier transform of the scaled envelope equation with respect to the ξ variable i.e.

We have

$$i\frac{\partial u}{\partial z} + \frac{1}{2}\frac{\partial^2 u}{\partial \xi^2} i\frac{u}{2} = 0$$

using

$$F\left(f^{n}(t)\right) = i\omega F\left[f^{n-1}(t)\right] = (i\omega)^2 F\left(f^{n-2}(t)\right) = \ldots = (i\omega)^n F(\omega)$$

we can write

$$\Rightarrow \frac{1}{2}u''(z) = \frac{1}{2}(ik)^2 \hat{u}(z,k) = -\frac{k^2}{2}\hat{u}(z,k)$$

then we obtain

$$i\frac{\partial}{\partial z}\hat{u}(z,k) - \frac{k^2}{2}\hat{u}(z,k) + \frac{i}{2}\hat{u}(z,k) = 0$$

This equation is a first-order ordinary differential equation. It can easily be solved,

$$i\hat{u}' - \hat{u}\left(\frac{k^2}{2} - \frac{i}{2}\right) = 0$$

$$i\hat{u}' = \hat{u}\left(\frac{k^2 - i}{2i}\right)$$

$$\frac{d\hat{u}}{dz} = -\hat{u}\left(\frac{ik^2 + 1}{2}\right)$$

$$\int \frac{d\hat{u}}{\hat{u}} = -\left(\frac{ik^2 + 1}{2}\right)\int dz$$

$$\ln \hat{u} = -\left(\frac{ik^2 + 1}{2} z\right)$$

$$\Rightarrow \hat{u}(z,k) = \exp\left(-\frac{ik^2+1}{2} z\right) \text{ or}$$

$$\Rightarrow \hat{u}(z,k) = \exp\left(-\frac{ik\ \ +1}{2} z\right)\hat{f}(k).$$

Here $\hat{f}(k)$ denotes the Fourier transform of our initial condition. Since we have chosen a Gaussian function as initial condition, the Fourier transform is known and we obtain a formula for $\hat{u}$. The inverse Fourier transform can be calculated analytically and we obtain the following solution

$$u(z,\xi) = \gamma \frac{1}{\sqrt{1 + \frac{2iz}{\delta^2}}} \exp\left(\frac{-\xi^2}{\delta^2 + 2iz} - \frac{z}{2}\right)$$

We are only interested in the modulus, which is

$$|u(z,\xi)| = \gamma \frac{1}{\sqrt[4]{1 + \frac{4z^2}{\delta^4}}} \exp\left(\frac{-\xi^2}{\delta^2 + 4z^2\delta^{-2}} - \frac{z}{2}\right) \tag{3.31}$$

Going back to the unscaled variables we find

$$|q(Z,t)| = \frac{1}{\sqrt[4]{1+\frac{4z^2 k_o^{''2}}{w_o^4}}} \exp\left(\frac{-\left(k_o^{'} Z - t\right)^2}{w_o^2 + 4Z^2 w_o^{-2} k_o^{''2}} - \frac{\gamma}{2} Z \right) \tag{3.32}$$

The solution (3.31) gives an idea where the interesting effects come in.
This result will be discussed in chapter four (result and discussion)

3.4 Exact Solution of the Nonlinear Envelope-Equation

One again we consider the differential equation for the envelope (without adding attenuation term), but now with the nonlinear term

$$q_z + k_o^{'} q_t + \frac{1}{2} i k_o^{''} q_{tt} - i\alpha |q|^2 q = 0 \tag{3.33}$$

We scale the variable according to

$$u = \alpha^{\frac{1}{3}} q, \quad \xi = \frac{k_o^{'2}}{k_o^{''}}, \quad z = \frac{k_o^{'2}}{k_0^{''}} \left(Z - \frac{t}{k_o^{'}} \right).$$

And get dimensionless differential equation for the envelope

$$iu_z + \frac{1}{2} u_{\xi\xi} + |u|^2 u = 0 \tag{3.34}$$

This equation is nonlinear Schrödinger equation. We introduce the solution ansatz

$$u(z,\xi) = \rho(\xi) e^{i\sigma(z,\xi)} \tag{3.35}$$

$$u_z = \rho e^{i\sigma} i \sigma_z \tag{3.36a}$$

$$u_\xi = \rho_\xi e^{i\sigma} + \rho e^{i\sigma} i \sigma_\xi \tag{3.36b}$$

$$u_{\xi\xi} = \rho_{\xi\xi} e^{i\sigma} + 2i\rho_{\xi}\sigma_{\xi} e^{i\sigma} - \rho\sigma_{\xi}^{2} e^{i\sigma} + i\rho\sigma_{\xi\xi} e^{i\sigma} \tag{3.36c}$$

$$|u|^{2} u = \rho^{3} e^{i\sigma} \tag{3.36d}$$

Substituting eq. 3.36a, c & d in eq. 3.34 the differential equation has the form

$$i[\rho i \sigma_{z} e^{i\sigma}] + \frac{1}{2}\rho_{\xi\xi} e^{i\sigma} + i\rho_{\xi}\sigma_{\xi} e^{i\sigma} - \frac{1}{2}\rho\sigma_{z}^{2} e^{i\sigma} + \frac{1}{2} i\rho\sigma_{\xi\xi} e^{i\sigma} + \rho^{3} e^{i\sigma} = 0$$

Multiplying both sides by 2, and we get

$$e^{i\sigma}\left[-2\rho\sigma_{z} + \rho_{\xi\xi} + 2i\rho_{\xi}\sigma_{\xi} - \rho\sigma_{\xi}^{2} + i\rho\sigma_{\xi\xi} + 2\rho^{3}\right] = 0 . \tag{3.37}$$

From this we derive the relations for the imaginary and real parts

$$-2\rho\sigma_{z} + \rho_{\xi\xi} - \rho\sigma_{\xi}^{2} + 2\rho^{3} = 0 \tag{3.38}$$

$$2\rho_{\xi}\sigma_{\xi} + \rho\sigma_{\xi\xi} = 0 \tag{3.39}$$

Transforming and integrating the imaginary part eq. (3.39), we obtain

$$2\rho_{\xi}\sigma_{\xi} = -\rho\sigma_{\xi\xi}$$

$$\frac{2\rho_{\xi}}{\rho(\xi)} = -\frac{\sigma_{\xi\xi}}{\sigma_{\xi}}$$

$$\int 2\frac{\rho_{\xi}(\xi)}{\rho(\xi)} d\xi = -\int \frac{\sigma_{\xi\xi}(z,\xi)}{\sigma_{\xi}(z,\xi)} d\xi$$

$$\ln \rho(\xi)^{2} = -\ln \sigma_{\xi}(z,\xi) + C(z)$$

$$\ln \rho(\xi)^{2} + \ln \sigma_{\xi}(z,\xi) = C(z)$$

$$\ln \rho(\xi)^{2} \sigma_{\xi} = C(z)$$

$$\sigma_{\xi}(z,\xi) = \frac{\tilde{C}(z)}{\rho(\xi)^{2}} . \tag{3.40}$$

For $\tilde{C}(z) = 0, \sigma$ is only a function of z. Using this fact we find for the real part

$$\rho_{\xi\xi} + 2\rho^{3} - 2\rho\sigma_{z} = 0$$

and accordingly

$$\sigma_z(z) = \frac{\rho_{\xi\xi} + 2\rho^3}{2\rho}$$

While the left-hand side is only a function of z the right hand size is only a function of ξ. Consequently, σ_z is constant and $\sigma(z) = kz$, where k is integration constant. Thus

$$\rho_{\xi\xi} + 2\rho^3 - 2\rho k = 0$$

One solution of this differential equation is

$$\rho(\xi) = \frac{\sqrt{2k}}{\cosh\left(\sqrt{2k}\xi\right)}, \quad k = \frac{\sigma(z)}{z} \tag{3.41}$$

Hence we get a specific solution to the differential equation of the scaled envelope equation eq. (3.34)

$$u(z.\xi) = \frac{\sqrt{2k}}{\cosh\left(\sqrt{2k}\xi\right)} e^{ik\xi}, \qquad k \in R \tag{3.42}$$

and the absolute value squared

$$|u(z,\xi)|^2 = \frac{2k}{\cosh^2\left(\sqrt{2k}\xi\right)}, \qquad k \in R \tag{3.43}$$

This equation will be seen in the next chapter.

Chapter four

Result and discussion

4.1 For linear case

We start from eq. (3.32) result of unscaled variables i.e.

$$|q(Z,t)| = \frac{1}{\sqrt[4]{1+\frac{4z^2 k_o^{''2}}{w_o^4}}} \exp\left(\frac{-\left(k_o^{'}Z - t\right)^2}{w_o^2 + 4Z^2 w_o^{-2} k_o^{''2}} - \frac{\gamma}{2} Z \right)$$

One can split the sum in exponential into attenuation and dispersion term. The only influence of the attenuation parameter γ is to decrease the amplitude by the factor $\exp\left(-\frac{\gamma}{2}Z\right)$. To investigate the dispersion we define the width w of the signal implicitly by writing the modulus in the form

$$|q| = A(Z)\exp\left(-\left(\frac{k_o^{'}Z - t}{w}\right)^2\right)$$

where $A(Z)$ is a factor that only depends on Z. We get for the width w

$$w(Z, w_o) = w_o \sqrt{1 + \frac{4Z^2 k_o^{''2}}{w_o^4}} \tag{4.1}$$

The function w is a measure of the signal's width at position Z along the fiber. Starting at Z = 0 the signal has a characteristics time width w_o. When it moves through the fiber, w increases with the distance Z and the light-pulse is widened. We can see that this dispersive effect depends on the coordinate Z along the fiber. The longer the signal travels, the wider it is spread. Also, we can see from (4.1) that if the dispersion-coefficient of the medium at the carrier frequency, $k_o''(w)$, is zero, the light pulse will not be spread out. The initial width w_o also influences the dispersion. Shorter initial signals are stretched out faster. This is undesired effect, because the information through the fiber is sent digitally, with a pulse representing a bit. So, if the width of the pulse increases it can overlay others and information can be lost. One objective for optical fiber communication is to increase the bit-rates, by making the pulse narrower, but the dispersive effect sets limits to this.

4.2 Numerical values

For the results in the next section, we used the following values:

$$k_o' = \frac{1.14}{c} \qquad \left[\frac{s}{m}\right] \qquad c = 3\times 10^8 \qquad \left[\frac{m}{s}\right]$$

$$k_o'' = -0.0053\left(1-\frac{\lambda_D}{\lambda}\right)\frac{\lambda}{2\pi c^2} \qquad \left[\frac{s^2}{m}\right] \qquad \lambda_D = 1.312\times 10^{-6} \qquad [m]$$

Here λ denotes the wave length of the carrier wave and the values in the square brackets are the physical units of the parameter. Note that the dispersion parameter k_o'' depends on λ. This relation is found empirically. It is interesting to note that at the wavelength λ_D, k_o'' vanishes, which means there is no dispersion.

The damping parameter γ also depends on λ. We can determine it by use of a graph of γ versus λ [8]**.** Of course these parameter values differ for different fibers. They depend on the radius, the structure and the material. However, our values are valid for typically used fibers.

First we plot the width of a pulse with initial width $w_o = 1000\ ps,$ as a function of the traveled distance. The wavelength of the carrier wave is $\lambda = 1.55 \mu m.$ The plot shows no visible effect of dispersion. The widening of the pulse, relative to w_o is negligible. However, assuming the same carrier wave-frequency, consider a pulse with an initial width of $w_o = 10\ ps$ (in fig $16 = 1 \times 10^{-11} s$), then the widening is very large, as can be seen from figure 16. We find that dispersion has a major effect 100 km. If one wants to use this width of the initial pulse, one has to use a wave-length corresponding to no dispersion, $\lambda = 1.312 \mu m$.

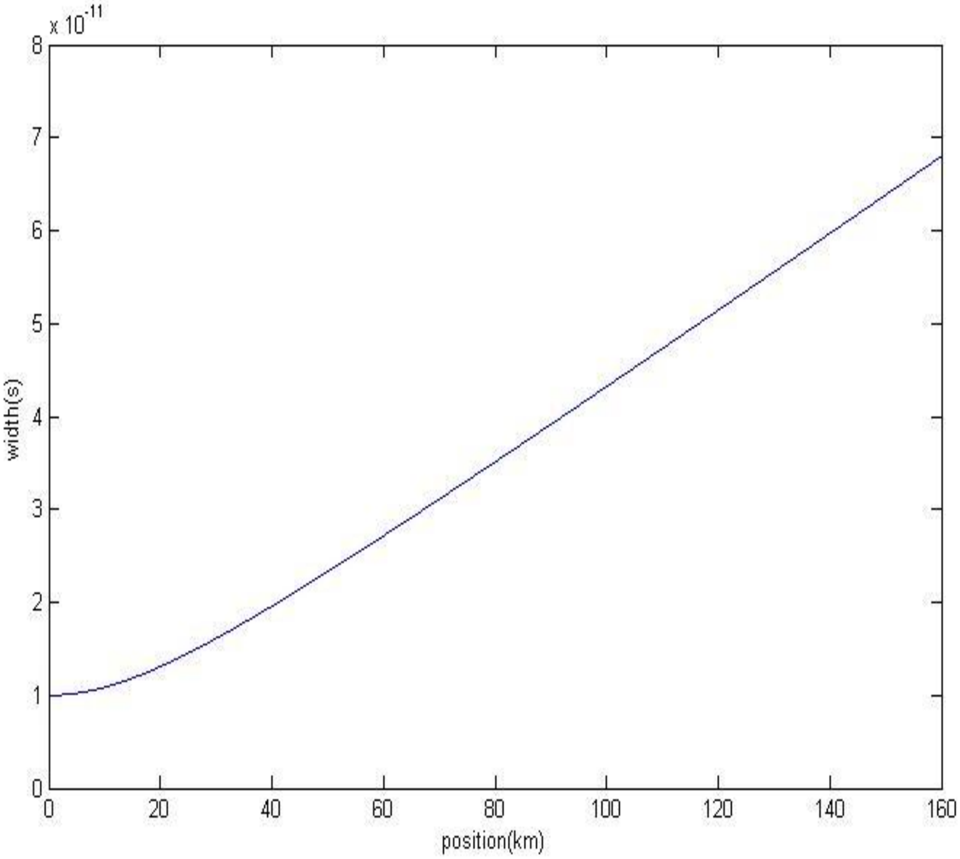

Figure 16 pulse width versus distance (position)

This figure shows how the pulse width all changes with the distance the pulse has traveled, when the initial pulse width is 10ps (1x10^{-11}s). The drawing is shown for a wavelength of the carrier wave of $\lambda = 1.55\mu m$.

Then we plot the intensity of a pulse as a function of the traveled distance, at the carrier wave length of $\lambda = 1.55\mu m$. We find that, due to attenuation, the intensity falls under 10% of the initial intensity after about 50km. In this case the signal has to be generated every 50km. But, as we just saw, at $\lambda = 1.55\mu m$ the effect of dispersion is unacceptably high when using very short pulses of about $10ps$. In order to use these short pulses, we would like to switch to the carrier wave-length of $\lambda = \lambda_D = 1.312\,\mu m$ to get dispersion. However, if we plot the intensity of a pulse versus the traveled distance at $\lambda = \lambda_D$, as we did in figure 17, then we see that the pulses have to be regenerated every 25 km (if we want to stay above 10% of the initial intensity, which is twice as often. So the effect of attenuation is much worse in this case.

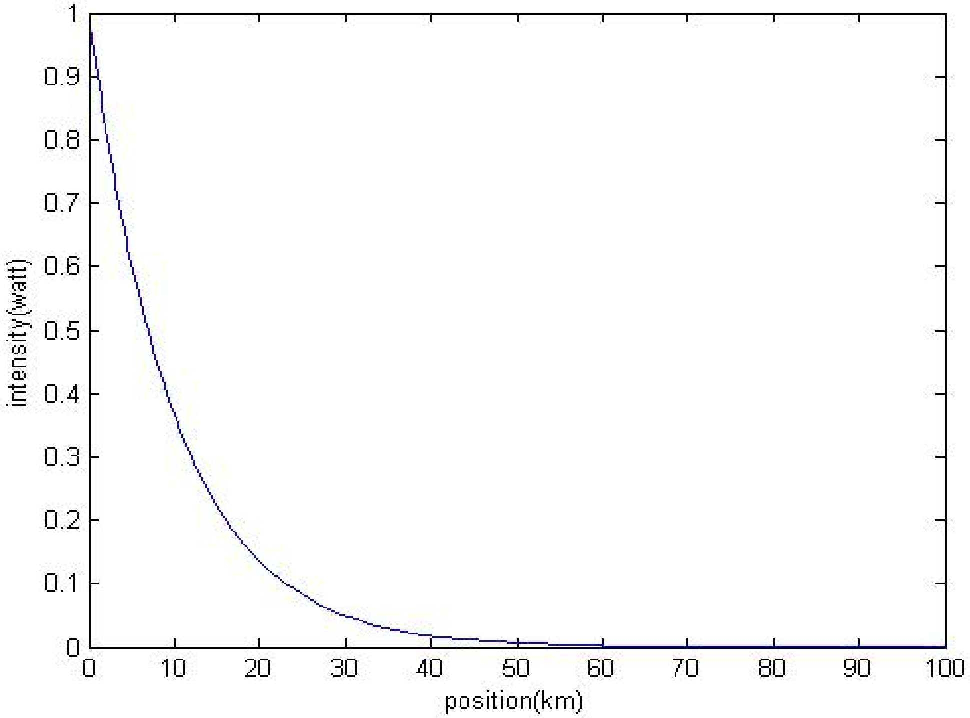

Figure 17 Intensity versus distance (position)

This figure shows how the intensity of a pulse decreases due to damping, while the pulse is traveling along the fiber. The wavelength of the carrier wave is $\lambda = 1.312\mu m$.

We have to conclude that there is no obvious best choice of the carrier wave-length in the presence of attenuation and dispersion, if we wash to have short pulses (i.e. high bit rate). We will see in the next section that when the non-linear term is not neglected, the dispersion is balanced by the non-linearity, i.e. pulses will not get wider at any carrier wave-length.

Using equation (3.31) it is possible to have the following graph (for linear case) which is scaled variables.

$$|u(z,\xi)| = \gamma \frac{1}{\sqrt[4]{1+\frac{4z^2}{\delta^4}}} \exp\left(\frac{-\xi^2}{\delta^2 + 4z^2\delta^{-2}} - \frac{z}{2} \right)$$

By writing modulus of the scaled variable

$|u(z=0,\xi)| = \gamma \exp\left(\frac{-\xi^2}{\delta^2} \right)$, we get:-

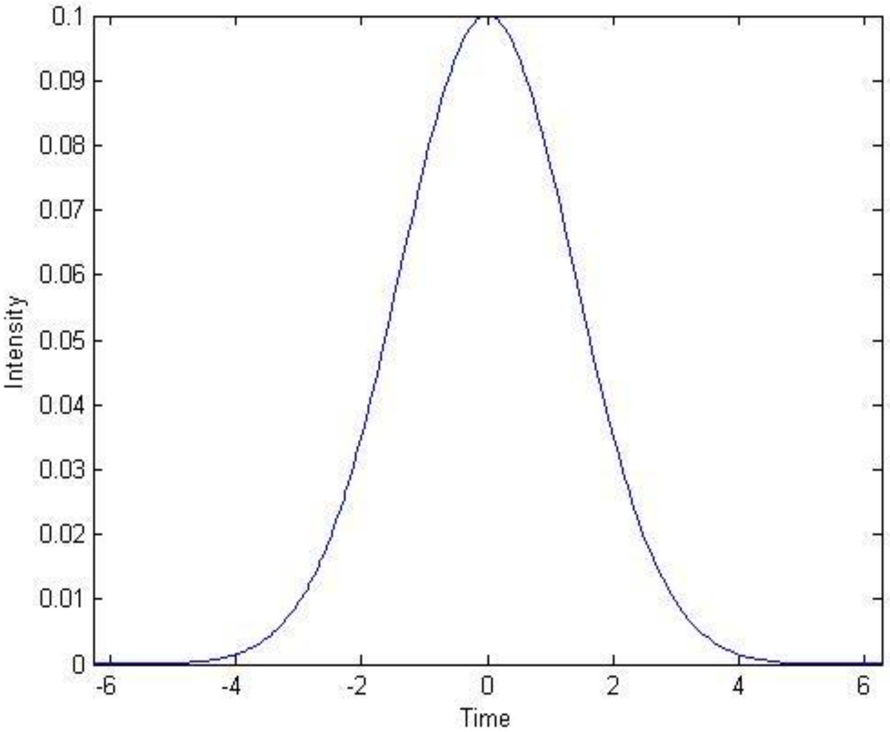

Figure 18 shape for the pulse in linear case (fundamental mode)

4.3 For non-linear case

From eq. (3.43) we have

$$|u(z,\xi)|^2 = \frac{2k}{\cosh^2\left(\sqrt{2k}\xi\right)}, \qquad k \in R \qquad \text{(Exact solution of non-linear envelope equation)}$$

This equation can be written as $|u(z,\xi)|^2 = \dfrac{1}{\cosh^2(\xi)}$ by taking k = 0.5 using this graph can be drawn like:-

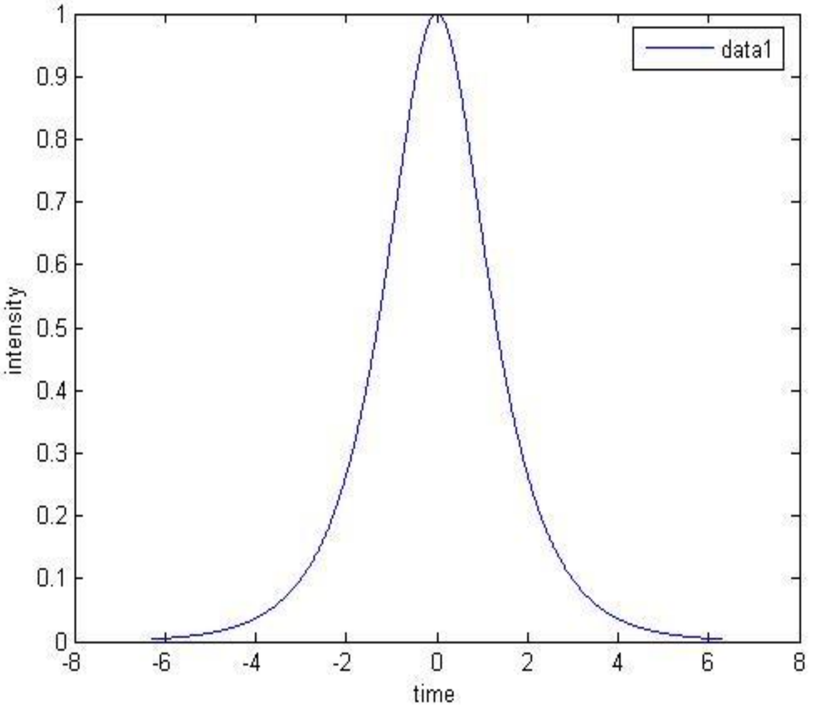

Figure 19 shape of soliton [$\left|u(z,\xi)\right|^2 = \frac{1}{\cosh^2(\xi)}$ (k = 0.5)]

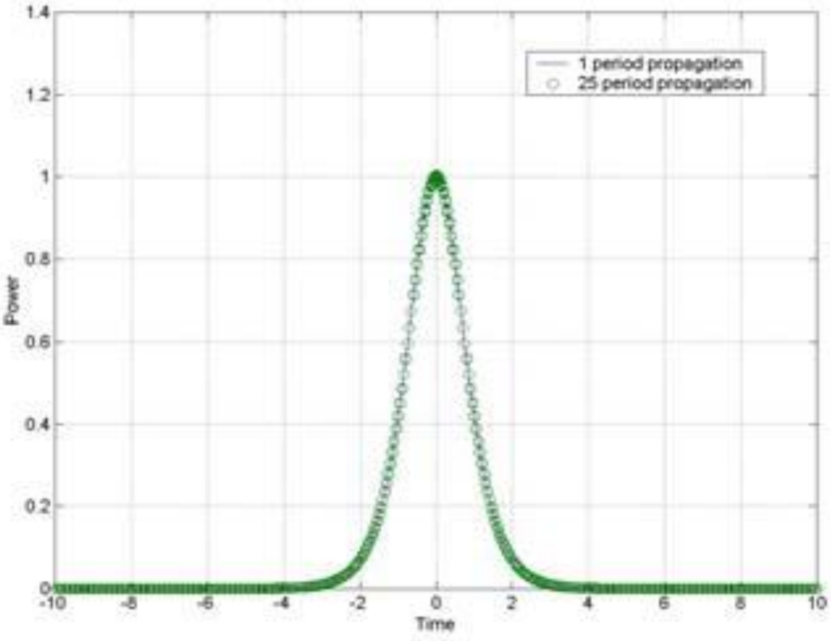

Figure 20 Propagation of signal with taking into consideration the Dispersion and Nonlinear soliton. (Experimentally obtained) [13].

Transmission system operating at $\lambda = 1.55\,\mu$ m with $\alpha_s = 0.22$ dB/km (≈0.0506 /km)

Fig.20 shows experimentally obtained shape of soliton. Comparing the two figures (fig. 19 and fig 20), possible to see an agreement in between them.

Other experimentally obtained solitons are

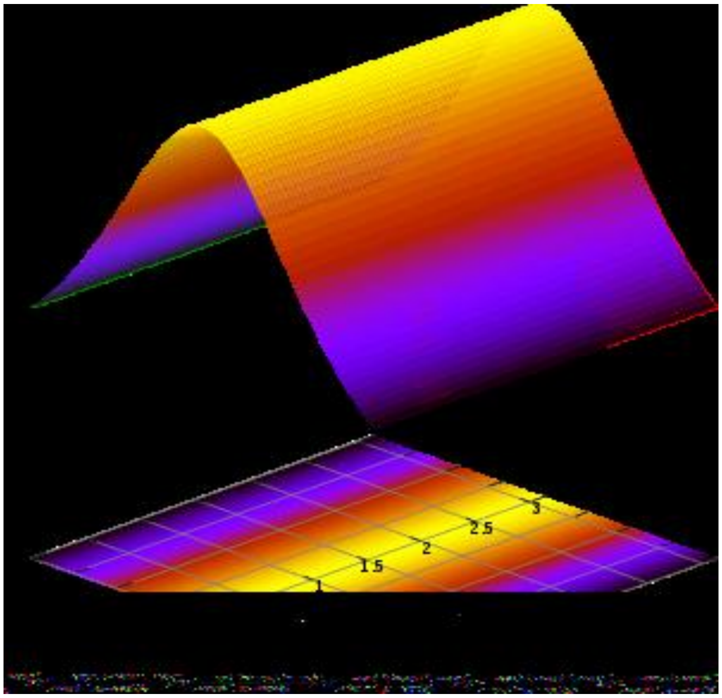

Figure 21 Soliton's shape while propagating, it does not change its shape [5]

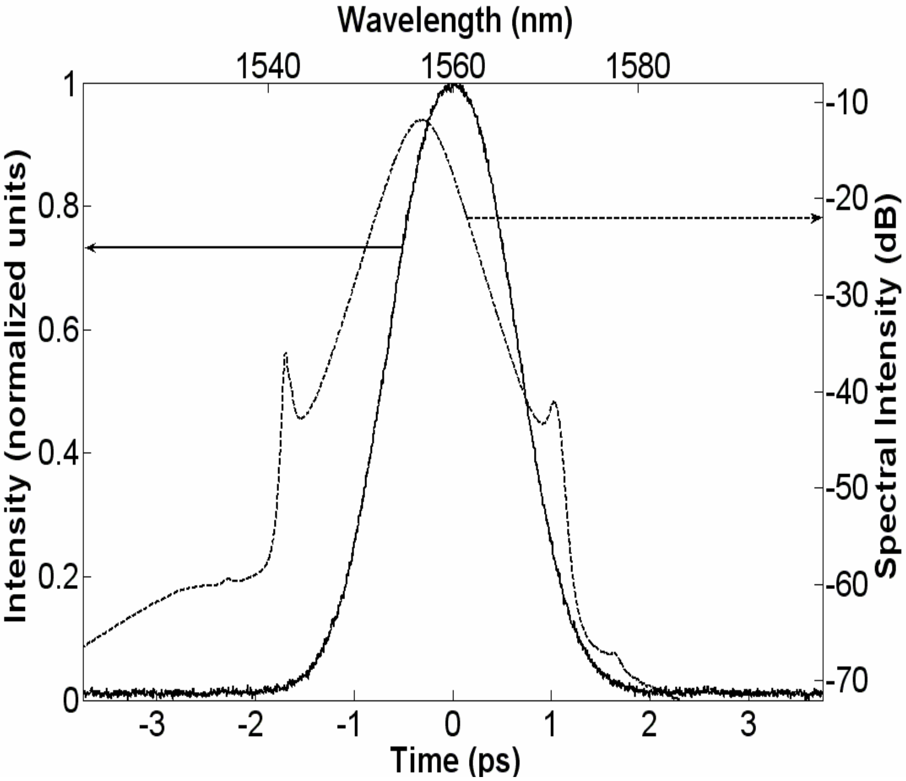

Figure 22. A typical vector soliton experimentally obtained optical spectrum and the corresponding autocorrelation trace; [13]

See Figure 19 for the shape of this function. In physical coordinate, we have

$$|q(Z,t)|^2 = \frac{2k}{\cosh^2\left(\sqrt{2k}\,\frac{k_o^{'2}}{k_o^{''}}\left(Z-\frac{t}{k_o^{'}}\right)\right)}. \tag{4. 2}$$

This solution keeps its shape during propagation, showing that the nonlinearity balances the dispersion. Therefore, we could choose the optimal wavelength for minimal loss. The above solitary wave solution is termed ‘soliton’.

Soliton is a narrow pulse with high peak power and a special shape as you see from fig. 19. Most pulses under go broadening (spreading in time) due to the group velocity dispersion when propagating through optical fiber. However, soliton pulse takes advantage of non-linear effects in silica, especially self modulation to overcome the pulse spreading of group velocity dispersion. Thus pulse can propagate for long distance with no change in shape [11, 14].

Self phase modulation arises because the refractive index of the fiber has intensity dependent component. This non-linear refractive index causes an induced phase shift that is proportional to the intensity of the pulse. Thus different parts of the pulse under go a different phase shift, which gives rise to chirping of the pulses. Pulse chirping intern enhances the pulse broadening effects of chromatic dispersion. This chirping effect is proportional to the transmitted single power so that self phase modulation effects are more pronounced in systems using high transmitted powers. The self phase modulation induced chirp affects the pulse broadening effects of chromatic dispersion and thus important to consider for high-bit rate systems that already have significant chromatic dispersion limitation.

Chapter Five

Conclusion

Summing up, we can say that for the linear case, the result stays unsatisfactory for a real situation. There are two major effects occurring, attenuation and dispersion. These two effects oppose against the goal to have a fast effective and error-free information flux. The model shows that one physical parameter – the wavelength of the carrier wave – can be chosen to reduce the undesired effects. However, the linear case sets limits to the effectiveness of the communication system because attenuation and dispersion can not be optimized simultaneously – low dispersion does not necessarily mean low attenuation. The way out of this dilemma is to increase the amplitude of the signal into the range, where non-linearity has major influence. In this case the non-linearity balances the dispersion and we can choose a carrier wave with a wavelength corresponding to minimal loss, without penalty. This is soliton.

An optical soliton communication system can be in the limelight as a system that will break the bottleneck in the speedup of transmission by the dispersion characteristic and the nonlinear optical effect.

The optical soliton communication system is a system that positively utilizes the dispersion characteristic and nonlinear optical effect of optical fibers which are major factors to the degradation of characteristics of the conventional transmission systems and that transmits optical short pulses intact by mutually balancing optical pulse width expansion owing to the dispersion in the optical fibers and pulse width compression based on the nonlinear optical effect. In case of using, as repeaters, optical amplifiers which compensate for a loss in optical fibers, it is possible to realize soliton communication with practically no waveform variations of optical pulses like ideal soliton pulses, by setting an average power between repeaters and an average dispersion of optical fibers to soliton conditions.

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Declaration