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Constructions of Partial Diallel Cross through Partially Balanced Incomplete Block Designs with block size two

A thesis submitted to the Department of Statistics as partial fulfillment of the requirements for the Degree of Master of Science in Statistics

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Table of Contents

Acronyms

ANVOA	Analysis Of Variance
BIBD	Balanced Incomplete Block Design
CRD	Completely Randomized Design
CDC	Complete Diallel Cross
C	Cyclic
F_1	First Generation (Crosses of the form $\boldsymbol{i} \times \boldsymbol{j}$)
GCA	General Combining Ability
GD	Group Divisible
IBD	Incomplete Block Design
LS	Latin Square
M	Miscellaneous
Max	Maximum
Min	Minimum
PBIBD($\boldsymbol{m}$)	Partially Balanced Incomplete Block Design with $\boldsymbol{m}$ associate
PDC	Partial Diallel Cross
PG	Partial Geometry
RGD	Regular Group Design
SCA	Specific Combining Ability
SGD	Singular Group Divisible
SRGD	Semi-Regular Group Divisible

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Abstract

Various forms of the diallel crosses play an important role in evaluating the breeding potential of genetic material in plant and animal breeding. In this paper we give the simple method of construction of partial diallel cross design through partially balanced incomplete block design as auxiliary design with the method of analysis and also see the importance of partial diallel cross design through partially balanced incomplete block design when large number of inbred line exist in plant and animal breeding experiment. We compare the efficiencies of our proposed designs with other existing designs for partial diallel cross in the literature and found that several our designs have higher A- and D-efficiency in comparison to the existing designs.

Key words: A-efficiency, D-efficiency, MS-optimality, Ortogonality, Partial Diallel Dross, Complete Diallel Cross, General Combining Ability, Specific Combining Ability

CHAPTER ONE

1. Introduction

1.1. Background of the Study

The subject of statistics deals with random phenomena and uncertainty. Its foundation is based on mathematics and probability, and that part of the subject has acquired the name of "mathematical statistics". As an applied science, however, it has grown into a body of knowledge which deals with measuring uncertainty surrounding the data generating in a given discipline like Biology, Agriculture, and Engineering etc. In fact, it is at the interface of statistical methods and the discipline of application that statistics really exists in the form of a science of the meaning and use of data. This makes statistical science an inter- and cross-disciplinary research activity.

The foundation of statistical approach to experimentation was laid by R.A.Fisher in the early1930s. The subject evolved in agriculture but is now applicable to almost all sciences, to genetics, to engineering and to other areas/fields.

Research in genetics is based almost exclusively on experimental investigation. The interpretation of and conclusions drawn from these experiment is often based on statistical analyses of some sort. This implies in turn that the design of such experiments must be based, implicitly or explicitly, on statistical considerations in order to obtain data that are relevant to experimenter's questions in the sense that if analyzed appropriately they will provide answers to these questions.

One of the main aims of genetic investigation involving crops is to develop improved crop varieties using genetic architecture. When there are a numbers of strains of crop, one type of investigation consisting of crossing between such strains to create new varieties. Through these types of investigation the combining capabilities of the strains can be studied. Normally highly inbred parental lines are considered for such investigation.

If there are p such lines or strains, and each line is crossed with each other, then there will be p^2 crosses in all. All these crosses constitute what is called complete diallel crosses. For assessing the yield of these all crosses are tried in an experiment where these crosses are taken as treatments. The usual designs adopted for such experiments are completely randomized designs (CRD) or randomized block design (RBD). The purpose of analyzing the data collected from such trials is to estimate, using appropriate model, the effects or yielding abilities of the original p lines. When there are large numbers of crosses incomplete block designs can be adopted to keep the homogeneity of experimental material within a block. Again, when p is large, the number of crosses becomes large and sometimes unmanageable. It is obviously impossible to cover and summarize all aspects of design for genetical experiments. A selection therefore has to be made based on personal preference and on the theme of study. A natural choice then is to discuss experiments in quantitative genetics.

1.2. Diallel Crosses

The term **diallel** is a Greek word and implies all possible crosses among a collection of male and female animals. Hyman (1954a, 1954b) defined diallel cross as the set of all possible crosses between several genotypes, which may be individuals, clones, homozygous lines etc. Diallel cross is the most balanced and systematic experiment to examine continuous variation. The genetic information related to parental population is available in early generation. Thus, it is useful for breeding strategy without loss of much time. Many improvements with respect to generalization of diallel crosses widens its scope and utility.

Diallel crosses are used mainly:

- To estimate the genetic components of variation of a quantitative character, and
- To estimate the combining abilities of different inbred lines involved in the crosses.

The concept of combining ability is becoming increasingly important in plant and animal breeding. It is the measure of gene action and helps in the evaluation of inbreds in terms of their genetic value and in the selection of parents for hybridization.

Superior cross combinations can be identified by this technique. It is specially useful in connection with 'testing' procedures, in which it is desired to study and compare the performance of lines or hybrid combination.

There are two types of combining abilities: general combining ability(GCA) and specific combining ability(SCA). The terms general and specific combining ability where originally defined by Sprague and Tatum (1942). They defined the terms as follows "the term 'general combining ability' is used to designate the average performance of a line in hybrid combination and the term 'specific combining ability' is used to designate those cases in which certain combinations do relatively better or worse than would be expected on the basis of average performance of a line involved". General combining ability is analogous to the main effect of a factorial experiment. It is estimated from half-sib families. Specific combining ability refers to a pair of inbred lines involved in a cross. It indicates cases in which certain combinations do relatively better or worse than would be expected on the basis of GCA effects of the two lines involved in it.

It is the derivation of a particular cross from the experimentation on the basis of average GCA effects of the two lines involved. It is analogous to an interaction effect of factorial experiment.

A diallel is a mating scheme used by plant breeders and geneticists to test general and specific combining ability as well as genetic effects behind quantitative traits Hallauer and Miranda Filho (1988).

In a full diallel, all parents are crossed to make hybrids in all possible combinations. Variations include partial diallel with and without parents. Full diallel require twice as many crosses and entries in experiments, but allows for testing for maternal effects. If maternal effects are assumed to be negligible, then a partial diallel without reciprocals is effective.

Common analysis methods utilize general linear models to identify heterotic groups Griffing (1956), estimate general combining ability (GCA) and specific combining ability(SCA) Gardner and Eberhart (1966), interactions with testing environments, and estimates of additive, dominant, and epistatic genetic effects Hayman (1954) and Sprague and Tatum (1942).

Other common mating designs used to study quantitative genetic effects include regression, generation Hayman (1958), triple test cross Kearsey and Jinks (1968), and the mating schemes proposed by Comstock and Robinson (1948) called by plant breeders NC I nested design, NC II factorial design, and NC III test cross mating designs.

By diallel crossing system is meant a system in which a set of inbred lines is chosen and crosses among this lines made. This procedure gives rise to a maximum of p^2 combinations. Data from such combinations can be most conveniently set out in a pxp table in which y_{ii} represents the mean value for the i^{th} inbred and y_{ij} the mean value for the F_1 resulting from crossing the i^{th} and j^{th} inbreeds and y_{ji} represents its reciprocal.

Thus the p^2 combinations are divided into three groups.

1. The p parental lines themselves.

2. One set of $p(p-1)/2$ F1's hybrid.

3. The set of $p^{C}{}_{2}= p(p-1)/2$ F1's reciprocal hybrid.

Diallel crossing techniques may vary depending up on whether or not the parental inbreeds or the reciprocal F_1's are included or both. With this as a basis for classification, there are four possible experimental methods defined by Griffing (1956).

- Parents, one set of F_1's and reciprocal F_1's are included (all p^2 combinations).

- Parents and one set of F_1's are included but not the reciprocal hybrids ($p(p-1)/2$ combinations), i.e., include p parents and $p^{C}{}_{2}$ F_1's hybrids.

- One set of F_1's and reciprocal F_1's are included but not the parents i.e., including $p^{C}{}_{2}$ F_1's hybrids and $p^{C}{}_{2}$ F_1's reciprocals.

- One set of F_1's but neither parents nor reciprocal F_1's is included ($p(p-1)/2$ combinations)), i.e., $p^{C}{}_{2}$ F_1's only.

With regard to sampling assumptions it is necessary to distinguish between:

- The situation in which the parental lines or the experimental material as a whole are assumed to be a random sample from some population about which inferences are to be made, and

- The situation in which the lines are deliberately chosen and cannot be regarded as a random sample from any population. This second assumption can be expressed somewhat differently by stating that the experimental material constitutes the entire population about which valid inferences can be made.

These two different assumptions give rise to different estimation problems and different tests of hypotheses regarding combining ability effects Griffing (1956).

Various forms of diallel mating designs (for a description see,e.g.,Hinkelmann, 1975) are commonly used in plant and animal breeding to investigate the genetic properties and potentials of inbred lines or individuals (for applications in plant breeding see,e.g.,Hallauer and Miranda,1988). These mating designs consist of all possible crosses or a subset of all crosses among a set of inbred lines or individuals. This, however, is only one component of the actual experiment since the offspring of these crosses have to be reared(grown) by using what we shall refer to as environment design (see,e.g., Hinkelmann, 1975). In other words, the offspring have to be grown in the "field" in order to obtain observations, which can then be analyzed.

Most commonly, diallel cross experiments have been evaluated using completely randomized designs (CRD) or randomized complete block designs (RCBD), with suitable number of replicates, as environmental design. In most cases, however, the number of crosses is too large, leading to an overall inefficient experiment. It is for this reason that the use of incomplete block designs as environmental design has been advocated by Braaten (1965), Aggarwal (1974), Ceranka and Kielezewska (1986), Ceranka and Mejza (1988), Aggarwal and Das (1990), and Givecha and Ghosh (1994).

We shall be concerned here with incomplete block designs for the partial diallel cross.Among the four types of diallel crosses discussed by Griffing, method 4 is the most commonly used diallel in plant breeding. Specifically, we have p lines, denoted by $i = 1, 2, ..., p$. We then consider crosses of the form $i \times j$ with $i < j$. With all possible $p(p-1)/2$ crosses this is sometimes also referred to as the modified or half diallel. We shall refer to it as a complete diallel (CDC). If a sample of all possible crosses (say $n_c = ps/2$) is used then the mating design is referred to as a partial diallel cross (PDC) (e.g., Kempthorne and Curnow, 1961). Here $s < (p-1)$ refers to the number of lines in which each line is crossed with. For this type of diallel one assumes that there are no differences between reciprocal crosses, i.e., no differences between crosses $i \times j$ and $j \times i$. This has received much attention nowadays because GCA's are important while selecting inbred lines for hybrid.

1.3. Statement of the Problem

Diallel crosses as mating designs are used to study the genetic properties of inbred lines in plant and animal breeding experiments. Most of the theory of diallel cross designs are based on standard linear assumption.

Most commonly, diallel cross experiments have been evaluated using completely randomized designs or randomized complete block designs as the environment design. In most cases, however, the number of crosses is too large, leading to an overall inefficient experiment. In order to overcome this problem we give a simple method of sampling PDC through PBIB design having block size k=2 along with their A-optimal and D-optimal efficiencies.

1.4. Objectives

1.4.1. General Objective

- To obtain large number of PDC plans by linking them with some of PBIB designs in the tables of Clatworthy (1973) and to give the method of analysis corresponding to the method of analysis of incomplete block designs having C-matrix.

1.4.2. Specific Objectives

- To obtain PDC plans which will allow estimation of **general combining ability** (GCA's) such that there are only two types of variances for estimates of contrasts like $g_i - g_j$ using two associate PBIB designs.
- To develop a general method of analysis of all such plans.
- To compare the efficiency (optimality) of partial diallel cross (PDC) plans through PBIBD in relation to randomized block design (RBD).

1.5. Limitations of the Study

The study has the following limitations.

- The mating designs for diallel crosses are not available for every value of inbred line p.

- In partial diallel cross experiment, it is impossible to estimate the specific combing ability(SCA) of each line since all possible crosses of a line are not included in the plan.

CHAPTER TWO

2. Literature Review

2.1. Mating and Environmental Designs

Incomplete block designs have become highly developed as statistical tool for the planning, conducting, analysis, and interpretation of scientific experimentation conducted in laboratories and test installations where a high degree of experimental control is desirable or even necessary to cope with the experimental variability that invariably arises throughout the conduct of the experiment. Historically use of the designs began with balanced incomplete block (BIB) designs introduced by Yates (1936a) and developed by Fisher and Yates (1938), and Bose and Nair (1939) and extensively tabled in Fisher and Yates (1963) and Cochran and Cox (1957) along with the lattice designs of Yates (1936b, 1937) and Cochran and Cox (1957) all of which were developed for use in agricultural and biological experiments. Further refinements in statistically based experimental arrangements of the first type were developed by Youden (1937, 1951) and the second by Harshbarger (1946, 1947) and Nair (1951). Bose and Nair(1939) developed a very general class of incomplete block designs, which they called partially balanced incomplete block designs with m associate classes (abbreviated PBIB (m) designs). These designs include the balanced incomplete block (BIB) designs and the square and rectangular lattice designs as special cases. Generally, they have the advantage of an extremely large selection of experimental arrangements from which investigators may choose the ones best suited to their needs. Beside the advantage of control of experimental variation through blocking technique, they offer (1) the opportunity to select plans of smaller size (few repetitions of the treatments, varieties, test conditions etc), (2) statistical analysis of the experimental data by relatively simple computational procedures, and (3) simplicity of presentation and interpretation of the results from comparative experiments in terms of clear cut statements about their precision based on modern statistical developments.

In the United States and England during World War II, a strong interest was developed in industry and in government to adapt the developments in probability and small sample theory to control the quality of production processes in industry. The interest was extended after the war to the introduction of statistical procedures to the experimental problems of the physical and engineering sciences. This effort has been particularly significant in the vast expensive investigative activity associated with atomic power and aerospace developments. Today there is intense interest in current developments in probability and statistics for possible application to research and development in all branches of sciences, engineering, health and medical research, business, and in fact, in any area where decisions need be made on the basis of incomplete information. Thus the incomplete block designs have potential for application to many fields of investigation.

By 1952 R.C. Bose and a group of his advanced degree students at the University North Calorina (including S.S. Shrikhande, W.S. Connor, T. Shimamoto, and W. H. Clatworthy) has sufficiently extended and developed the partially balanced incomplete block designs with two associate classes (PBIB(2) designs) that several hundreds were now available. With the encouragement from Gertrude Cox, Director of the Institute of Statistics, University of North Calorina, and with support of the Agricultural Experiment Station, North Calorina State College, some 375 of these PBIB(2) designs were classified, catalogued and put into convenient form for use by experimenters. A monograph was published by Bose, Clatworthy and Shrikhande (1954) very soon after Bose and Shimamoto (1952) had developed the concept of association schemes, on which the classification of PBIB (2) designs, is based. Since 1952 the concept of association schemes has been the subject of intensive research by statisticians interested in combinatorial problems of the design of experiments. The intimate relations between association schemes and certain types of graphs have attracted many mathematicians interested in graph theory to join in their investigation. The results improved insight into the construction problems of PBIB(2) designs accompanying the improved knowledge of association schemes, leading to the construction of literally hundreds of new designs of PBIB(2) type since the publication by Bose, Clatworthy, and Shrikhande (1954).

The vast development of new designs of PBIB (2) type are the extremely widespread frequent referencing of Clatworthy (1973) by scholars all over the world, and the very high potential for use of these combinatorial arrangements in scientific experimentation.

A major objective of plant and animal breeding programmes is to improve the genetic potential of plants and animals. The breeding experiment comprise two types of design namely, mating and environmental design. Mating design is a procedure of producing the progenies. This, however, is only one component of the actual experiment since the offspring of these crosses have to be reared subjecting these progenies to environmental conditions in a systematic manner by using an environmental design.

Diallel cross planes are one of the commonly used mating designs. With limited facilities available for testing, a diallel cross may only be possible for relatively small number of inbred lines. However, if only a small number of lines are included, the estimate of the variance of the general combining abilities(GCA) in the whole population of potentially available inbred lines is subject to a large sampling error and also many potentially high yielding inbred lines may be left out completely untested. It is, therefore, necessary to have a large number of inbred lines but raise only a sample of all possible crosses among them. Such a diallel cross is known as a partial diallel cross(PDC). Gilbert(1958), Hinkelmann and Stern(1960), Kepthorne and Curnow(1961) have discussed the advantages of performing only a sample of all possible crosses among a larger number of parents.

Through PDC a plant breeder is not only able to estimate the GCA of a large number of parental lines but can also select among crosses from a wide range of parents. GCA of each line may be estimated with relatively low precision but larger genetic gains may result from the more intense selection that can be applied to them. For enabling the plant breeders to make use of PDC, in a given situation, is the most efficient in sense that it gives the least average variance of the difference between GCA effects of a pair of lines.

There are different ways of approaching the problem of sampling a diallel cross. The first approach involves developing a PDC based on circulant structure (Kempthorne and Curnow,1961). In this method, the line i is crossed with lines $(k+i)$, $(k+i+1)$, …, $(k+i+s-1)$ where $k=(v+1-s)/2$ and s is the number of times a line is involved in crosses.

All the numbers above are to be reduced to modulo v and for k to be a whole number (clearly, v and s can not both be odd). Arya (1983) suggested a modified circulant plan for obtaining PDC. The analysis of PDC data based on circulant plan can be worked out through the software package SPAR1(1991) developed by IASRI.

In the second one, the association scheme of a partially balanced incomplete block (PBIB) design is used in constructing the PDC. PBIB designs are an important class of block designs that can be used in plant and animal breeding trials to investigate the genetic properties and potentials of inbred lines or individuals.

In the third approach for sampling the diallel cross, the average variance over all the comparisons is minimized as shown in Mathur and Narain (1976),Venkatesan (1985) and Singh and Hinkelmann (1995). Furthermore, without going for mating design separately, one can go for a combined mating- environmental design and the sample is obtained by making all possible crosses within blocks of a proper environmental design (PBIB design). In this paper, we prefer to adopt the second method due to its simplicity, availability of a good number of PBIB designs that provides better choice, and relatively good efficiency of the selected plan. Gilbert (1958) pointed out the analogy between PDCs and incomplete block designs with blocks of size two. Since a complete diallel cross corresponds to a balanced incomplete design Kempthorne and Curnow(1961), it is expected that partially balanced incomplete block designs will be related to certain types of PDCs. Kempthorne and Curnow(1961), Curnow(1963), Fyfe and Gilbert(1963) and Hinkelmann and Kempthorne (1963), Ghosh and Divecha (1997) gave some PDCs derived from PBIB with two associate classes. The total number of crosses involved in a PDC using a two associate –class association scheme is likely to be large resulting in difficulty to handle all of them effectively. Fyfe and Gilbert(1963) further introduced PDCs derived from a PBIB design with three associate classes. If for a given v (number of lines) there is a design with higher associate classes, the number of crosses is likely to be small and there is more flexibility in the choice of associate classes Das and Sivaram(1968).Das and Sivaram (1968) obtained plans for partial diallel crosses using PBIB designs with any block size, any values of λ and any number of associate classes. Some work on this aspect has been done by Arya and Narain (1977), Narain *et al.* (1974) Narain and Arya (1981), Agrawal (1985), Kaushik and Puri (1989), Narain (1993) and Kaushik (1999).

2.2. The Use of Mating Designs in Environmental Designs

A Diallel is a mating scheme used by plant breeders and geneticists to test for general and specific combining ability as well as genetic effects behind quantitative traits Hallauer and Filho (1988). In a full diallel, all parents are crossed to make hybrids in all possible combinations. Variations include partial diallel with and without parents. Full diallel require twice as many crosses and entries in experiments, but allows for testing maternal effects. If maternal effects are assumed to be negligible, then a partial diallel without reciprocals is effective. Common analysis methods utilize general linear models to identify heterotic groups (Griffing, 1956), estimate general combining ability (GCA), estimate specific combining ability (SCA) Gardner and Eberhart (1966), interactions with testing environments, years, and estimates of additive, dominant, and epistatic genetic effects discussed by Hayman (1958), and Sprague and Tatum (1942).

Other common mating designs used to study quantitative genetic effects include regression, generation (Hayman, 1958), triple test cross (Kearsey and Jinks, 1968), and the mating schemes proposed by Comstock and Robinson (1948) called by plant breeders North Carolina(NC) I nested design, NC II factorial design, and NC III testcross mating designs.

Plant breeders frequently study the genetic properties of inbred lines using complete diallel crosses (CDC) or partial diallel crosses (PDC) as mating designs. Most common diallel cross experiment (CDCs or PDCs) have been evaluated using a completely randomized design (CRD) or a randomized complete block design (RCBD) with suitable number of replications as environmental design (e. g., Kempthorne and Curnow, 1961; Curnow, 1963; Hinkelmann and Kempthorne, 1963; Arya, 1983). Due to the limitation of homogeneous experimental units in a block to accommodate all the chosen crosses, the estimates of genetic parameters would not be precise enough if a complete block design was adopted for the large number of crosses. As the number of inbred lines increases, the number of crosses increases rapidly. For example, with $p = 5$ lines there are only 10 crosses. While for $p = 10$ the number of crosses is 45 and when $p = 15$ it becomes 105. Laying out the design, as a randomized complete block design, even with a moderately large number of lines will result in large blocks and consequently large intrablock variances due to loss of homogeneity. It results in high coefficient of variation (CV) and hence reduced precision on the comparisons of interest.

In order to overcome this problem, one may use incomplete block designs like balanced incomplete block (BIB) designs and partially balanced incomplete block (PBIB) designs with two-associate classes, cyclic designs, nested designs etc by treating the crosses as treatments for one-way elimination of heterogeneity settings. Under such circumstances, the use of suitable incomplete block designs is inevitable and is suggested by Braaten (1965),Aggrawal (1974), Das and Giri (1986), Ceranka and Mejza (1988), Agarwal and Das (1990),and Divecha and Ghosh (1994). Hinkelmann (1975) stresses the importance of embedding mating designs in environment designs. Gupta and Kageyama (1994), Dey and Midha (1996),Mukerjee (1997), Das, Dey and Dean (1998), Ghosh and Divecha (1997), Parsad, Gupta, and Srivastava (1999), Sharma (2000) and Sharma (2005) addressed the problem of finding optimal designs by using nested incomplete block designs (NBIB), triangular PBIB designs, nested balanced block (NBB) designs, GD PBIB designs, PBIB designs and circular designs respectively. Singh and Hinkelmann (1995) consider the use of partial diallel crosses in incomplete block designs, and give methods for constructing mating-environment designs and evaluate their efficiencies.

These designs have interesting optimality properties while making inferences on a complete set of orthonormalized treatment contrasts. However, in diallel cross experiments the main interest of the experimenter is making comparisons among general combining ability of lines and, therefore, using these designs as mating designs may result in poor precision for the comparisons among lines. Further, the analysis of a diallel cross experiments in incomplete blocks depends on the incidence of lines in blocks, rather than the incidence of experiments or crosses in blocks. Another approach advocated in the literature is to start with an incomplete block design, write all the pairs of treatments within a block, and identify these pairs of treatments as crosses by treating treatments of the original incomplete block designs as lines and use the resulting design as a design for diallel crosses. Das and Giri (1986) used this approach for complete diallel crosses (CDC) experiments by using BIB designs. Later designs have been obtained for partial diallel crosses (PDC) experiments using the same technique with PBIB designs. Sharma (1998) obtained designs for partial diallel crosses through circular designs.

In this paper, we consider PDC experiment in incomplete block design due to three advantages claimed by Kemptherne and Curnow(1961):

i. The variance of general combining ability among the population of which the parents are a sample can be estimated more accurately,

ii. Selection can be made among crosses from a wider range of parents, and

iii. The general combining abilities of a larger number of parents can be estimated. Each parent will be assessed with a relatively low precision but larger genetic gains may result from the more intense selection that can be applied to the parents.

CHAPTER THREE

3. Methods

3.1. Concepts of Partially Balanced Incomplete Block Designs

Though Balanced Incomplete Block Designs have many optimal properties, they do not fit well to many experimental situations as these designs require a large number of replications. Moreover, these designs are not available for all numbers of treatments and block sizes. To overcome these difficulties a class of binary, equireplicate and proper designs that are called Partially Balanced Incomplete Block (PBIB) designs were introduced. In these designs the variance of every estimated elementary contrast among treatment effects is not the same.

The definition of PBIB designs is based on the association scheme. Therefore first we give the concept of association scheme.

3.1.1. Association Scheme

Given v treatment symbols $1,2,\ldots,v$, a relation satisfying the following conditions is called an m -class association scheme($m \geq 2$).

i. Any two treatment symbols are either $1^{st}, 2^{nd}, \ldots, m^{th}$ associate; the relation of association being symmetric, i.e., if the symbol α is the i^{th} associate of β, then β is the i^{th} associate of α.

ii. Each symbol has n_i i^{th} associates, the number n_i being independent of α.

iii. If any two symbols α and β are i^{th} associates, then the number of the symbols that are j^{th} associates of α and k^{th} associates of β is $p^i{}_{jk}$ and is independent of the pair of i^{th} associates of α and β.

The numbers v , n_i and p^{i}_{jk} ($i, j, k = 1,2,...,m$) are called the parameters of the m^{th} association scheme. These parameters are not all independent but connected as the following parametric relations:

$$\sum_{i=1}^{m} n_i = v - 1; \tag{3.1}$$

$$\sum_{k=1}^{m} p^{i}{}_{jk} = n_j - \delta_{ij} \tag{3.2}$$

Where δ_{ij} is the Kronecker's delta i.e., $\delta_{ij} = 1$ if $i = j$ and is zero otherwise.

And

$$n_i p^{i}{}_{jk} = n_j p^{i}{}_{kj} \tag{3.3}$$

The relation $p^{i}{}_{jk} = p^{i}{}_{kj}$ is always holds because of symmetry, that is, from the fact that α is an i^{th} associate of β, then β is an i^{th} associate of α.

3.1.2. Definition of Partially Balanced Incomplete Block Designs

Given an association scheme with m classes $(m \geq 2)$ we have a PBIB design with m associate classes based on the association scheme, if the v treatment symbols can be arranged into b blocks, such that

i. Every symbol occurs at most once in a block.

ii. Every symbol occurs in exactly r blocks.

iii. If two symbols are i^{th} associates, then they occur together in λ_i blocks, the number λ_i being independent of the particular pair of i^{th} associates α and β.

The integers λ_i , b, r, k, are called the parameters of the PBIB design. Note that the definition of PBIB design is based on the existence of an association scheme. Consequently, if some specified values of v, n_i, $p^i{}_{jk}$, there are no association scheme with some m, there is no PBIB design with m-association classes based on the scheme. The following relations connect the parameters of PBIB design as also the parent association scheme:

$$vr = bk \tag{3.4}$$

$$\sum_{i=o}^{m} n_i \lambda_i = rk \tag{3.5}$$

where $\lambda_0 = r$ and $n_o = 1$

Let **N** be the incidence matrix of PBIBD with m-associate classes. Then from the above definition, it follows that the diagonal elements of **NN'** are all equal to r. Further, in each row there are precisely n_1 positions filled with λ_1, n_2 position filled with $\lambda_2, \ldots, n_m$ positions filled with λ_m, apart from the single entry r. Thus

$$\mathbf{NN'1} = \left(r + \sum_{i=1}^{m} n_i \lambda_i \right) \mathbf{1} \tag{3.6}$$

where, **1** is $vx1$ vector of ones.

Again,

$$\mathbf{NN'1} = \mathbf{N}(\mathbf{N'1}) = \mathrm{k}(\mathbf{N1}) = \mathrm{rk}\mathbf{1} \tag{3.7}$$

From(3.6) and (3.7), we have

$$rk = r + \sum_{i=1}^{m} n_i \lambda_i \tag{3.8}$$

or

$$\sum_{i=1}^{m} n_i \lambda_i = r(k-1) \tag{3.9}$$

3.1.3. Two- Associate Classes of Partially Balanced Incomplete Block Designs

Among the association schemes and PBIB designs, the two-associate class schemes and the two-associate classes of PBIB designs have received most attention. In this section, we study two class association schemes and the two-associate classes of PBIB designs.

3.2. Partially Balanced Incomplete Block Designs with Two Associate Classes

Following Bose, Clatworthy, and Shrikhande (1954), an incomplete block design is said to be partially balanced incomplete block design with two associate classes if it satisfies the following requirement:

i. The experimental material is divided into b blocks of k units each, different treatment being applied to the unit in the block.
ii. There are $v\,(>k)$ treatments each of which occurs in r blocks.
iii. There can be established a relation of association between any two treatments satisfying the following requirements:
    a. Two treatments are either first associates or second associates.
    b. Each treatment has exactly n_i i^{th} associates (i = 1, 2).
    c. Given any two treatments which are i^{th} associates, the number of treatments common to the j^{th} associate of the first and the k^{th} associate of the second is $p^i{}_{jk}$ and is independent of the pair of treatments we start with.

    Also $p^i{}_{jk} = p^i{}_{kj}\,(i, j, k = 1,2)$.

iv. Two treatments which are i^{th} associates occur together in exactly λ_i blocks (i =1 ,2).

For a proper partially balanced incomplete block design $\lambda_1 \neq \lambda_2$ .If $\lambda_1 = \lambda_2$ the design becomes a balanced incomplete block design.

The numbers v, r, k, b , n_1, n_2, λ_1 and λ_2 are called the parameters of the first kind, where as the numbers $p^i{}_{jk}$ $(i, j, k$ =1,2) are called the parameters of the second kind.

It is to be noted that the association relations between the treatments of partially balanced incomplete block design are governed solely by the requirements of (iii) on the above and do not depend up on how the treatments are distributed in blocks. The association scheme depends only on the parameters n_1, n_2 and $p^i{}_{jk}$

$$\begin{aligned} vr &= bk \\ n_1 + n_2 &= v - 1 \\ n_1\lambda_1 + n_2\lambda_2 &= r(k-1) \end{aligned} \tag{3.10}$$

$$\begin{aligned} p^1{}_{11} + p^1{}_{12} &= n_1 - 1 \\ p^1{}_{21} + p^1{}_{22} &= n_2 \\ p^2{}_{11} + p^2{}_{12} &= n_1 \\ p^2{}_{21} + p^2{}_{22} &= n_2 - 1 \end{aligned} \tag{3.11}$$

$$\begin{aligned} n_1 p^1{}_{12} &= n_2 p^2{}_{11} \\ n_1 p^1{}_{22} &= n_2 p^2{}_{12} \end{aligned} \tag{3.12}$$

We can exhibit the parameters $p^i{}_{jk}$ of the second kind as the element of the two symmetric matrices

$$\mathbf{P_1} = \begin{bmatrix} p^1{}_{11} & p^1{}_{12} \\ p^1{}_{21} & p^1{}_{22} \end{bmatrix} \qquad \mathbf{P_2} = \begin{bmatrix} p^2{}_{11} & p^2{}_{12} \\ p^2{}_{21} & p^2{}_{22} \end{bmatrix} \tag{3.13}$$

Since the above matrices are symmetric, it is unnecessary to write down the elements $p^1{}_{21}$ and $p^2{}_{21}$ explicitly. These values will accordingly be omitted in the rest of the monograph. The computed quantities $\mathbf{P_1}$ and $\mathbf{P_2}$ arise in the analysis of these designs.

3.3. The Classification of PBIB Designs

From practical point of view , two associate classes of PBIB designs are important and exhaustive list of these designs is available in Clatworthy (1973) and Clatworthy (1956).

Here we briefly define some of the well known two-associate class association schemes and give their parameters. Bose and Shimamoto(1952) have classified the two associate classes of PBIB designs into many categories, namely, Group Divisible(GD), consisting of three subtypes: singular(S), semi-regular (SR) and regular (R), Triangular (T), Latin square type (LSi), cyclic (C), partial geometry (PG) and miscellaneous (M).

3.3.1. Group Divisible (GD) PBIB (2) Designs

The largest, simplest and perhaps most important class of PBIB (2) design. A GD designs is a PBIB (2) designs for which the treatments may be divided into m groups of n distinct treatments each such that belong to the same group are the first associates and two treatments that belongs to different groups are second associates. The GD designs association scheme is an mxn array of the treatment symbols (1,2,...,$mn = v$, say).

For GD designs,it is clear that

$$v = mn\,,\; n_1 = n-1\,, n_2 = n(m-1) \tag{3.14}$$

From the general condition $n_1\lambda_1 + n_2\lambda_2 = r(k-1)$, we have for GD designs

$$(n-1)\lambda_1 + n(m-1)\lambda_2 = r(k-1) \tag{3.15}$$

From the definition of the parameters $p^i{}_{jk}$ (i, j, k =1,2) and consideration of the association scheme it is clear that

$$\mathbf{P_1}=(p^i{}_{jk})=\begin{bmatrix} n-2 & 0 \\ 0 & n(m-1) \end{bmatrix},\; \mathbf{P_2}=(p^i{}_{jk})=\begin{bmatrix} 0 & n-1 \\ n-1 & n(m-1) \end{bmatrix} \tag{3.16}$$

For any GD design $\lambda_2 > 0$ is necessary condition, since if $\lambda_2 = 0$ then the treatments that belong to groups that never occur together in a block and the design breaks up into m disconnected set of blocks of a degenerate form of the design (called a disconnected design) that does not permit estimation of comparison of treatments belong to different groups. When zero-valued λ in GD design is λ_2 , it is always be put into the proper form with λ_1=0 by renaming the associate classes.

Combinatorial Properties of GD Designs

Bose and Connor (1952) have made a careful study of the combinatorial properties of GD designs, and classified into three subtypes depending on values assumed by the quantities

$$Q = r - \lambda_1 \text{ and } rk - v\lambda_2 = p \tag{3.17}$$

They considered the *vxb* incidence matrix $\mathbf{N} = (n_{ij})$ of a GD design. The determinant of the treatment structure matrix, **NN**' is

$$\det(\mathbf{NN}') = rk(rk - v\lambda_2)^{m-1}(r - \lambda_1)^{m(n-1)} \tag{3.18}$$

They classified GD designs into three types from a consideration of the expression for det(**NN'**) as follows

1. Singular (S) if $r - \lambda_1 = 0$
2. Non-singular
    a. Semi-regular (SR) if $r - \lambda_1 > 0$ and $rk - v\lambda_2 = 0$
    b. Regular (R) if $r - \lambda_1 > 0$ and $rk - v\lambda_2 > 0$

For GD designs, the parameters are v, r, $k, m, n, \lambda_1, \lambda_2$ and p^i_{jk} (i, j, k = 1,2). The number of experimental units in the design is $n = vr = bk$ and is the size of the design.

We list below several of the more important results from the Bose and Connor (1952) combinatorial properties of GD designs:

1. The necessary and sufficient condition for PBIB (2) designs to be GD is that $p^1{}_{12} = 0$ or $p^2{}_{12} = 0$. Note that to be consistent with the definition of the association scheme, we need to interchange the associate classes when $p^2{}_{12} = 0$. This involves an interchange (1) the values of λ_1 and λ_2 (2) the values of n_1 and n_2, and (3) reordering of the quantities $p^i{}_{jk}$ in the matrices $\mathbf{P_1}$ and $\mathbf{P_2}$. Once this has been done so that $p^1{}_{12}$ assumes the value 0, then the expression $rk - v\lambda_2$ may be correctly evaluated.
2. For any group divisible designs, it is necessary that $r - \lambda_1 \geq 0$ and $rk - v\lambda_2 \geq 0$.
3. For a singular group divisible design (SGD) $b \geq m$.
4. For a SGD design (since $r = \lambda_1$), det(**NN'**) = 0.
5. For a semi-regular group divisible (SRGD) design k is divisible by m. If $k = cm$, then every block must contain c treatments from each group.
6. For a semi-regular group divisible (SRGD) design $b \geq v - m + 1$.
7. A necessary condition for the existence of a symmetrical ($v = b$) regular GD design is that $(r^2 - v\lambda_2)^{m-1}(r - \lambda_1)^{m(m-1)} = p^{(m-1)}Q^{m(n-1)}$ is perfect square.
8. If a symmetrical ($v = b$) regular GD design exists, then:

    a. If m is even, then $p = r^2 - v\lambda_2$ must be a perfect square and further if $m = 4t + 2$ and n is even, the Hilbert norm residue symbols $(Q\ ,(1))_p = -1$ for all primes p.

    b. If m is odd and n is even, $Q = r - \lambda_1$ is perfect square and $((-1)^{\alpha} n\lambda_2, p)_p = +1$ for all primes p, where $\alpha = \frac{1}{2}m(m-1)$.

    c. If m and n are both odd, $((-1)^{\alpha} n\lambda_2, p)_p, ((-1)^{\beta} n, Q) = +1$ for all primes p, where

    $\alpha = \frac{1}{2}m(m-1)$ and $\beta = \frac{1}{2}n(n-1)$.

3.3.2. Triangular PBIB (2) Designs

A partially balanced incomplete block designs with two associate classes is said to be triangular if the number of treatments is $v = \frac{1}{2}p(p-1)$ and the associate scheme is an array of p rows and p columns with the following properties.

1. The positions in the principal diagonals (running from upper left-hand to lower right-hand corner) are left blank.
2. The $\frac{1}{2}p(p-1)$ positions above the principal diagonal are filled by the numbers 1, 2…, $\frac{1}{2}p(p-1)$ corresponding to the treatments.
3. The $\frac{1}{2}p(p-1)$ positions below the principal diagonal are filled by the numbers which is similar to the above so that the array is symmetrical about the principal diagonal.
4. For any treatment i the first associates are exactly those treatments which lie in the same rows and in the same columns as i .

The following relations hold:

$$v = \frac{1}{2}p(p-1), \quad n_1 = 2(p-2), \quad n_2 = \frac{1}{2}9(p-2)(p-3) \tag{3.19}$$

$$\mathbf{P_1} = \begin{bmatrix} p-2 & p-3 \\ p-3 & \frac{1}{2}(p-3)(p-4) \end{bmatrix}, \quad \mathbf{P_2} = \begin{bmatrix} 4 & 2p-8 \\ 2p-8 & \frac{1}{2}(p-4)(p-5) \end{bmatrix} \tag{3.20}$$

where $p \le 5$. (The case p = 4 has $p^2{}_{12} = 0$ so that it is also the group divisible type and is classified in this way. We note further that for p = 2 or 3, n_2 = 0 , so that there is only one associate class and the design, if it exists, degenerates to the PBIB (1) design.)

3.3.3. Latin Square Types (LS) PBIB (2) Designs

Consider a set of $v = p^2$ treatments and association scheme which consist of an *pxp* array of the distinct symbols 1,2, ,..., p^2 on which a set of $i - 2$ mutually orthogonal Latin square (MOLS) are superimposed. Let LS_1 first associate of a treatment α consist of the $p-1$other treatments lying in the same row of an array with α , the set of $p-1$ other treatments in the same column of an array withα, and then $p-1$ other treatments corresponding to the same letters as α in each of the Latin squares. All other treatments are second associates of α. Such an association scheme is called a Latin square types (LS_i) PBIB (2) design association scheme.

A PBIB (2) designs based on a Latin square type (LS_i) association scheme is called a Latin square type (LS_i) PBIB (2) design.

For these association schemes, it is clearly that

$$v = n^2, \quad n_1 = i(n-1), \quad n_2 = (n-1)(n-i+1) \tag{3.21}$$

$$\mathbf{P_1}=\begin{bmatrix} i^2-3i+n & (i-1)(n-i+1) \\ (i-1)(n-i+1) & (n-i)(n-i+1) \end{bmatrix} \qquad \mathbf{P_2}=\begin{bmatrix} i(i-1) & i(n-i) \\ i(n-i) & (n-i)^2+i-2 \end{bmatrix} \tag{3.22}$$

From these relations and the definition, it is clearly that certain restrictions on the parameters *i* and *n* are necessary. First of all, for a given *n* the definition depends upon the existence of a set of MOLS so that a positive integral values of *n* for which a maximum number of squares exist $0 \le i-2 \le n-1$ or $2 \le i \le n+1$.

We note that when $i = n+1$, n_2 becomes 0, and any design based on such association scheme would necessary degenerate to a PBIB (2) design. If $i = n$, then $p^1_{12} = 0$ the design, if it exits, must be of GD type. To avoid these probabilities we shall require $i \le n-1$.

3.3.4. Cyclic (C) PBIB (2) Designs

Let the treatment be designated by the integers 1,2,..., v . Let first associates of treatment i be given by $i+d_1,\ i+d_2,\ldots,\ i+d_{n_1}$ (mod$v$)

Provided the *d's* satisfying the following conditions:

1. The *d's* are different and $1 \le d_j \le v$, j= 1,2,..., n_1;

2. Among the $n_1(n_1-1)$ differences $d_j - d_{j'}$ (j , j'' =1,2,..., n_1 :j $j \neq j'$)

Reduced mod v each of the numbers $d_1,\ d_2,\ldots,\ d_{n_1}$ occurs at α times whereas each of the numbers $e_1, e_2,\ldots, e_{n_2}$ occurs at β times where $\alpha \neq \beta$ and where $d_1,\ d_2,\ldots,\ d_{n_1},\ e_1, e_2,\ldots, e_{n_2}$ are the numbers 1,2,..., v, such an association scheme is said to be a cyclic design, or a PBIB (2) design based on the cyclic association scheme.

To reduce an integer $x \le v$ and positive, which is an unconventional procedure for retention of treatment symbols 1,2,..., v .

A PBIB (2) design for which the relationships among the v treatments are given by the cyclic association scheme defined above is itself is known as a cyclic types of partially balanced design.

For a cyclic PBIB (2) design it is clearly necessary that

$$n_1\alpha + n_2\beta = n_1(n_1 - 1) \tag{3.23}$$

The parameters of the second kind are determined by α , β ($\alpha \neq \beta$), n_1 and n_2 so that

$$\mathbf{P_1}=\begin{bmatrix} \alpha & n_1-\alpha-1 \\ & n_2-n_1+\alpha+1 \end{bmatrix} \qquad \mathbf{P_2}=\begin{bmatrix} \beta & n_1-\beta \\ & n_2-n_1+\beta+1 \end{bmatrix} \tag{3.24}$$

3.3.5. Partial Geometries (PG) PBIB (2) Designs

A partial geometry (r,k,t) has been defined by Bose (1963) a system of undefined points and lines and an undefined relation called incidence satisfying the following four axioms:

i. Any two points are incident with not more than one line.
ii. Each point is incident with r lines.
iii. Each line is incident with k points.
iv. If the point P is not incident with the line l, then there pass through P exactly t lines ($t \geq 1$) intersecting l.

From this beginning Bose showed that the number v points and the number b lines in the partial geometry as

$$v = \frac{1}{t}k\left[(r-1)(k-1)+t\right] \tag{3.25}$$

$$b = \frac{1}{t}r\left[r(r-1)(k-1)+t\right] \tag{3.26}$$

We also proved that a necessary condition for the existence of partial geometry (r,k,t) is that the number

$$\alpha = \frac{1}{t}rk\left[(r-1)(k-1)(k+r-t-1)\right] \tag{3.27}$$

is integral.

Another of Bose's basic results is the following: Given a partial geometry (r,k,t) there exists a dual partial geometry (r,k,t) obtained by interchanging the role of points and lines in the given geometry.

The partial geometry (r,k,t) has an association scheme wherein two points are first associates if they are incident with a line of the geometry and they are second associates if they are not incident with a line.

It then follows from Bose (1963) that the association scheme of a partial geometry (r,k,t) has

Parameters

$$n_1 = r(k-1),\ \ n_2 = \frac{1}{t}(r-1)(k-1)(k-t) \tag{3.28}$$

$$p^1{}_{11} = (t-1)(r-1)+k-2\ ,\ \ p^2{}_{11} = rt \tag{3.29}$$

$$1 \le t \le r\ ,\ 1 \le t \le k \tag{3.30}$$

Then it becomes evident that if a partial geometry (r,k,t) exits, it is a PBIB (2) designs with parameters r , k , $\lambda_1 = 1$, $\lambda_2 = 0$ and having other parameters. Thus, the corresponding PBIB (2) design would have the parameters

$$\begin{aligned} v &= \frac{1}{t}k[(r-1)(k-1)+t], r = r, \lambda_1 = 1 \\ b &= \frac{1}{t}r[(r-1)(k-1)+t], k = k, \lambda_2 = 0 \\ n_1 &= r(k-1), n_2 = \frac{1}{t}(r-1)(k-1)(k-t) \end{aligned} \tag{3.31}$$

$$\mathbf{P_1} = \begin{bmatrix} (t-1)(r-1)+k-2 & (r-1)(k-t) \\ & \frac{1}{t}(r-1)(k-1)+(k-t-1) \end{bmatrix} \tag{3.32}$$

$$\mathbf{P_2} = \begin{bmatrix} rt & r(k-t-1) \\ & \frac{1}{t}[(r-1)(k-1)(k-2t)+t(rt-k)] \end{bmatrix}$$

Where $1 \le t \le r, 1 \le t \le k$

3.3.6. Miscellaneous (M) PBIB (2) Designs

Those simple design of Bose, Clatworthy and Shrikhande (1954) for which $\lambda_1 \geq 2$ and $\lambda_2 = 0$ were constructed in all cases by replicating entire plans of our partial geometries.

Since they cannot be classified as partial geometries (for which $\lambda_1 = 1$), a new category called miscellaneous was formed to include these and other new designs for which the other classifications are not appropriate. Most of the miscellaneous (M) PBIB (2) designs are characterized by one zero λ but there are 8 instances of designs that have both λ's different from zero. Note that when a PBIB design with two associate classes is said to be simple if either (i) $\lambda_1 \neq 0$, $\lambda_2 = 0$ or (ii) $\lambda_1 = 0$, $\lambda_2 \neq 0$. It may happen that a design of the simple category may belong to some other category as well.

CHAPTER FOUR

4.Ortogonality and Optimality of the Designs

4.1 Blocked and Unblocked Models

We consider diallel cross experiments involving p inbred lines, giving rise to a total of n_c = $p(p-1)/2$ possible distinct crosses. Let r_{di} denote the number of times the i^{th} cross appears in a design d, (i = 1, 2, . . . , $p(p-1)/2$) and, similarly, let s_{dj} denote the total number of times that the j^{th} line occurs among the crosses in the design d, (j = 1, 2, . . . , p). Further, define $\mathbf{r_d}$ and $\mathbf{s_d}$ to be $\mathbf{r_d}$ = (r_{d1}, . . . , r_{dn_c})′, $\mathbf{s_d}$ = (s_{d1}, . . . , s_{dp})′, and let n denote the number of crosses (observations) in the design d. Then $\mathbf{1}'_{n_c}\mathbf{r_d}=\frac{1}{2}\mathbf{1_p}'\mathbf{s_d}$ where ′ denotes transpose of a matrix and $\mathbf{1_t}$ denotes a t-component column vector of all ones. We use the following model for an unblocked (completely randomized) diallel cross experiment:

Model M1 : $\mathbf{Y}=\mu\mathbf{1_n}+\mathbf{\Delta_1 g}+\boldsymbol{\varepsilon}$

and the following model for a blocked diallel cross experiment:

Model M2: $\mathbf{Y}=\mu\mathbf{1_n}+\mathbf{\Delta_1 g}+\mathbf{\Delta_2}\boldsymbol{\beta}+\boldsymbol{\varepsilon}$

where $\mathbf{Y}$ is the $nx1$ vector of observed responses, μ is a general mean effect, $\mathbf{g}$ and $\boldsymbol{\beta}$ are vectors of p general combining ability effects and b block effects respectively, $\mathbf{\Delta_1}$ and $\mathbf{\Delta_2}$ are the corresponding design matrices, that is, the $(h, l)^{th}$ element of $\mathbf{\Delta_1}$(respectively, of $\mathbf{\Delta_2}$) is 1 if the h^{th} observation pertains to the l^{th} line (respectively, to the l^{th} block), and is zero otherwise; $\boldsymbol{\varepsilon}$ is the vector of random error components, these components being distributed with mean zero and constant variance σ^2.

As is usual for the analysis of PDC experiments, it is assumed that the genetic effect of the cross (i, j) is represented sufficiently well by the general combining ability of the two parental lines (see Singh and Hinkelmann (1995) for a detailed comment on such a model).

Let $D_o(p,n)$ denote the class of all completely randomized designs with p lines and n crosses. For a design, $d_o \in D_o(p,n)$, under model M1, it can be shown that the information matrix of the reduced normal equations for estimating linear functions of general combining ability effects **g** is

$$C_{d_o} = G_{d_o} - \frac{1}{n}\mathbf{s_{do}}'\mathbf{s_{do}} \tag{4.1}$$

where $G_{d_o} = (g_{d_o ii'})$, $g_{d_o ii} = s_{d_o i}$ and for $i \neq i'$, $g_{d_o ii'}$ is the number of times the cross (i,i') appears in d_o. Also, $\sum_{i\le}\sum_{i'} g_{d_o ii'} = n$.

Similarly, let $D(p,b,k)$ denote the class of all block designs with p lines, and b blocks each with k crosses. For a block design $d \in D(p,b,k)$, under model M2, the information matrix for **g** is given by

$$C_d = G_d - \frac{1}{k}\mathbf{N_d}\mathbf{N_d}' \tag{4.2}$$

where $\mathbf{N_d} = (n_{dij})$, n_{dij} is the number of times that line i occurs in block j of d and $G_d = G_{d_o}$ is defined below (4.1). For such a block design, $n = bk$ and $\mathbf{N_d 1_b = s_d = s_{do}}$.

A design d will be called connected if and only if the rank of its information matrix is $p-1$. Equivalently, d is connected if and only if all elementary comparisons among the general combining ability effects are estimable. A connected design $d^*_o \in D_o(p,n)$ is said to be MS-optimal if

$$\max_{d_o \in D_o(p,n)} tr(C_{d_o}) = tr(C_{d^*_o}) \quad \text{and} \quad \min_{d_o \in D^*_o(p,n)} tr(C^2_{d_o}) = tr(C^2_{d^*_o}),$$

where $D^*_o(p,n)$ is the sub-class of all designs $d_o \in D_o(p,n)$ for which $tr(C_{d_o})$ is maximum.

Let $z_{d_o 1} \le z_{d_o 2} \le ... \le z_{d_o p-1}$ be the non-zero eigen values of C_{d_o} .Then, design $d^*{}_o$ is said to be A-optimal if

$$\min_{d_o \in D_o(p,n)} tr(C_{d_o}{}^{-}) = tr(C_{d_o^*}{}^{-})$$

and is said to be D-optimal if

$$\max_{d_o \in D_o(p,n)} \prod_{i=1}^{p-1} z_{d_o i} = \prod_{i=1}^{p-1} z_{d^*{}_o i} .$$

Similarly, MS-, A-and D-optimality are defined for connected block design $d \in D(p,b,k)$.

4.2. MS-Optimality of PDC Designs

In Theorem 4.2.1, below, we characterize MS-optimal designs in the class of completely randomized designs $D_o(p,n)$, with $s = 2n/p$ an integer. We need the following well known lemma which is easy to prove.

Lemma 4.2.1. For given positive integers α and β, the minimum of $\sum_{i=1}^{\alpha} m_i^{\,2}$ subject to $\sum_{i=1}^{\alpha} m_i = \beta$, where the m_i 's are non-negative integers, is obtained when $\beta - \alpha[\beta/\alpha]$ of the m_i 's are equal to $[\beta/\alpha]+1$ and $\alpha - \beta + \alpha[\beta/\alpha]$ are equal to $[\beta/\alpha]$, where [z] denotes the largest integer not exceeding z. The corresponding minimum of $\sum_{i=1}^{\alpha} m_i^{\,2}$ is $\beta(2[\beta/\alpha]+1) - \alpha[\beta/\alpha]([\beta/\alpha]+1)$.

Theorem 4.2.1. A design d^*_o with p lines is MS-optimal in $D_o(p,n)$ if and only if

(i) Every line occurs $s = 2n/p$ times in d^*_o, and

(ii) The number of times $g_{d^*_{o}ii'}$ that cross (i,i') occurs in d^*_o satisfies

$$\left|g_{d^*_{o}ii'} - s/(p-1)\right| < 1 \text{ for } i \neq i',\ i,\ i' = 1, \ldots, p.$$

From the above Theorem, PDC designs in which every line appears the same number $s = 2n/p$ of times and in which each cross appears either $\lambda = [s/(p-1)]$ or $\lambda + 1$ times are MS-optimal. A common way to construct a PDC design is to take a conventional binary incomplete block design with p treatments each occurring s times, n distinct blocks of size 2 and treatment concurrences λ and $\lambda + 1$ (called the auxiliary design by Singh and Hinkelmann, 1995) and to form crosses between the two treatments in each block. Any such PDC satisfies the conditions of Theorem 4.2.1 and is MS-optimal. Among others, this includes the M-designs of Singh and Hinkelmann (1995), the first series of PDCs of Mukerjee(1997), and the PDCs formed from the basic plans listed by Gupta, Das and Kageyama (1995).

We consider now the class $D(p,b,k)$ of block designs with $n = bk$ crosses among the p lines, divided into b blocks of size k crosses and cross concurrences $\lambda = 0$ or 1. Ignoring the division into blocks, the $n = bk$ crosses involved in a design $d \in D(p,b,k)$ forms a PDC completely randomized design $d_o \in D_o(p,bk)$.

Thus to every block design d in $D(p,b,k)$, there corresponds a completely randomized design d_o in $D_o(p,bk)$.

Following Gupta, Das and Kageyama (1995), we define a block design $d \in D(p,b,k)$ to be an orthogonal block design if the i^{th} line occurs in every block s_i/b times for $i = 1, \ldots p$ where s_i is the replication of the i^{th} line in the design, that is

$$\mathbf{N_d} = b^{-1}\mathbf{s_d}\mathbf{1_b}',$$

where $\mathbf{N_d}$ is the line-block incidence matrix of the design d.

From (4.1) and (4.2) and the fact that $\mathbf{N_d 1_b = s_d}$, it follows that

$$C_d = G_d - \frac{1}{k}\mathbf{N_d N_d'} = C_{d_o} - \frac{1}{k}\mathbf{N_d}(\mathbf{I_b} - \frac{1}{b}\mathbf{1_b 1_b'})\,\mathbf{N_d'} \quad (4.3)$$

Thus, $C_d \le C_{d_o}$,where for a pair of non-negative definite matrices A and B, A $\le$ B implies that B − A is non-negative definite. Equality is achieved if and only if $\mathbf{N_d} = b^{-1}\mathbf{s_d 1_b'}$, which is the condition for an orthogonal block design.

Now, consider a non-increasing optimality criterion ϕ. (An optimality criterion ϕ is non-increasing if ϕ(B) $\le \phi$(A) whenever B − A is non-negative definite). If the unblocked PDC design $d^*{}_o \in D_o(p,bk)$ corresponding to an orthogonal block design $d^* \in D(p,b,k)$ is ϕ-optimal, then d^* is also ϕ-optimal since $\phi(C_{d^*})$ = $\phi(C_{d^*{}_o}) \le \phi(C_{d_o}) \le \phi(C_d)$ for any $d \in D(p,b,k)$ and corresponding $d_o \in D_o(p,n)$. The MS-, A- and D-criteria are included in the ϕ-criterion. Thus, in particular, we have the following theorem.

Theorem4.2.2. An orthogonal block design $d^* \in D(p,b,k)$ is MS-optimal in $D(p,b,k)$ if the corresponding design $d^*{}_o \in D_o(p,bk)$ satisfies the conditions of Theorem 4.2.1.

4. 3. A- and D- Efficiency

In this section, we show that the Series A and B PDC orthogonal block designs constructed in Section 5.1, are not only MS-optimal, but also have high efficiencies with respect to the A- and D-optimality criteria. We also show that these efficiencies compare extremely well with the best Series C designs obtained from a number of different sources.

Let $z_{d1} \le z_{d2} \le ... \le z_{dp-1}$ be the non-zero eigenvalues of C_d for a connected design $d \in (p,b,k)$ .For any such design, the A-value is defined as $\phi_A(d) = tr(C^-{}_d) = \sum z_{di}{}^{-1}$ and the D-value as $\phi_D(d) = \prod z_{di}{}^{-1}$.

Let $d_A(d_D)$ be the A-optimal (D-optimal) design in $D(p,b,k)$, then the A- and D- efficiencies of design d are defined as

$$e_A(d) = \phi_A(d_A)/\phi_A(d)$$

and

$$e_D(d) = \{\phi_D(d_D)/\phi_D(d)\}^{1/(p-1)} \quad .$$

We now give the following result on lower-bounds for A- and D- efficiencies in $d \in (p,b,k)$.

Lemma 4.3.1. The A- and D-efficiency lower-bounds $e'_A(d)$ and $e'_D(d)$ for design $d \in (p,b,k)$ are given by

$$e'_A(d) = \frac{(p-1)^2}{s(p-2)\phi_A(d)} \tag{4.4}$$

and

$$e'_D(d) = \frac{(p-1)}{s(p-2)\{\phi_D(d)\}^{1/(p-1)}} \tag{4.5}$$

where $s = 2n/p$.

It is also that the lower bounds (4.4) and (4.5) also hold for designs in $d_o \in (p,n)$. We note that (4.4) is equivalent to the average efficiency factor for a PDC design relative to a complete diallel cross as calculated by Singh and Hinkelmann (1995) with 2r replaced by s in their formula (17). we can see the efficiency of Series A and B PDC orthogonal block designs under table I and II at the end of chapter five.

CHAPTER FIVE

5. A Class of MS-optimal Designs

5.1. Construction method of Mating-Environment Design

The genetic experiment depends on or is specified by both the mating and the environment design. We shall refer to the combination of these designs as the mating-environment design, or as the M-E design for short, and to its component designs as the M-design (for mating design) and E-design (for environment design), respectively. Both component designs and hence the combined design will be related to a certain incomplete block design as described in what follows.

Orthogonally blocked MS-optimal PDC designs for p lines with each cross occurring $\lambda = 0$ or1 times can be constructed from resolvable or 2-resolvable auxiliary incomplete block designs with p treatments each occurring s times, n blocks of size 2 and treatment concurrences $\lambda = 0$ or 1. The PDC design is obtained by forming a cross from the pair of treatments in each of the n blocks. Each resolvable (or 2-resolvable) set of blocks in the conventional design partitions the crosses of the PDC into orthogonal blocks. We call such designs Series C designs.

Even in the absence of blocks, each cross in partial diallel cross(PDC) design can be formally identified with a 'block' of size two and these design itself can be identified with a binary block design.

So we consider a partially balanced incomplete block design (PBIBD) with parameters $v = p$, b , r , k =2, $n_{1,} n_{2,} \lambda_i$ =0 or 1,where i =1,2. We, now present methods of constructing mating-environment designs for PDC.

We have two methods:

Method 1

Unblocked case p *even(odd)*:- Consider a partially balanced incomplete block design with parameters $(v = p, b, r, k = 2, \lambda_i = 0 or 1, n_1, n_2, p^i{}_{jk}; i = j = k = 1,2)$, we will refer it as the auxiliary design or A-design.Take all possible distinct pair of treatments in each block of the partially balanced incomplete block design.

Thus we get one pair per block. Consider treatments of a partially balanced incomplete block design as lines and the pair in block as cross. The resulting design is a partial diallel cross design with b crosses from v lines, we call it as mating-environment(M-E) design . Each cross is repeated either λ_1 or λ_2 times depending upon the value of λ's.We denote the resulting design by a class of $D(p,n)$, where n is number of experimental units in the design.

Example 5.1:- Let us consider the construction of a partial diallel crosses design involving 8 lines.We take a Regular group design 29 with parameter $v = 8, r = 6, k = 2, b = 24, m = 4, n = 2, \lambda_1 = 0, \lambda_2 = 1$ (Clatworthy,1973) as auxiliary design. The plan of the RGD29 and partial diallel cross (M-E design), with 24 crosses out of 28 crosses, are shown below.

Plans of RGD29 and resulting partial diallel crosses

RGD29 plan(Auxiliary design)

B_1: 1 2	B_5: 1 3	B_9: 1 4	B_{13}: 1 6	B_{17}: 1 7	B_{21}: 1 8
B_2: 3 5	B_6: 2 7	B_{10}: 2 3	B_{14}: 2 4	B_{18}: 2 8	B_{22}: 2 5
B_3: 4 7	B_7: 4 6	B_{11}: 5 6	B_{15}: 3 8	B_{19}: 3 6	B_{23}: 3 4
B_4: 6 8	B_8: 5 8	B_{12}: 7 8	B_{16}: 5 7	B_{20}: 4 5	B_{24}: 6 7

Partial Diallel Cross Plan(M-E design)

1 x 2	1 x 3	1 x 4	1 x 6	1 x 7	1 x 8
3 x 5	2 x 7	2 x 3	2 x 4	2 x 8	2 x 5
4 x 7	4 x 6	5 x 6	3 x 8	3 x 6	3 x 4
6 x 8	5 x 8	7 x 8	5 x 7	4 x 5	6 x 7

Example 5.2:- Now let us consider the construction of a partial diallel crosses involving 9 lines. We take a RGD34 with parameters $v = 9, r = 6, k = 2, b = 27$ and $m = 3, n = 3, \lambda_1 = 0, \lambda_2 = 1$ (Clatworthy,1973).The plan of RGD34 and partial diallel crosses, with 27 crosses out of 36 crosses, are shown below.

Plans of RGD34 and resulting partial diallel crosses

RGD34 plan(Auxiliary design)			Partial Diallel Cross Plan(M-E design)		
B_1: 1 2	B_{10}: 1 9	B_{19}: 1 6	1 x 2	1 x 9	1 x 6
B_2: 3 1	B_{11}: 5 1	B_{20}: 4 2	3 x 1	5 x 1	4 x2
B_3: 2 3	B_{12}: 2 6	B_{21}: 3 4	2 x 3	2 x 6	3 x 4
B_4: 4 9	B_{13}: 9 2	B_{22}: 5 3	4 x 9	9 x 2	5 x 3
B_5: 6 4	B_{14}: 3 8	B_{23}: 7 9	6 x 4	3 x 8	7 x 9
B_6: 9 5	B_{15}: 6 7	B_{24}: 2 7	9 x 5	6 x 7	2 x 7
B_7: 5 7	B_{16}: 7 3	B_{25}: 8 1	5 x 7	7 x 3	8 x 1
B_8: 8 6	B_{17}: 8 4	B_{26}: 6 5	8 x 6	8 x 4	6 x 5
B_9: 7 8	B_{18}: 4 5	B_{27}: 9 8	7 x 8	4 x 5	9 x 8

Method2

Here we have two cases:

(i)*Blocked case(p odd):* For blocked case for p odd, we divide the b blocks of PBIB design (Auxiliary design) by v in such a way that each b/v blocks contains every line equal number of times.Thus,we get block design for PDC design with parameters $v = p, b' = b/v, k' = v, r' = 1$. we denote these designs by a class $D(p, b', k')$.

We call such designs Series B designs.

Example 5.3: In Example 5.2 we presented a partial diallel cross plan(M-E design) with auxiliary design for odd line. This partial diallel cross plan for odd line v=9 can be converted in to block design by forming b/v=3 blocks in such away that each line occurs equal number of times in each block. The resulting partial diallel cross design is shown below.

Resulting partial diallel crosses design

Partial Diallel Cross Plan(M-E design)

B_1	B_2	B_3
1 x 2	1 x 9	1 x 6
3 x 1	5 x 1	4 x 2
2 x 3	2 x 6	3 x 4
4 x 9	9 x 2	5 x 3
6 x 4	3 x 8	7 x 9
9 x 5	6 x 7	2 x 7
5 x 7	7 x 3	8 x 1
8 x 6	8 x 4	6 x 5
7 x 8	4 x 5	9 x 8

This design is MS-optimal because it follows the conditions given in theorem 4.2.1.

(ii) ***Blocked case**(p even):*For blocked case for p even, we divided the b blocks of PBIB design in r blocks such that each block contains each line equal number of times. Thus we get PDC design in which each cross occurs only once with parameters $v = p, b' = r, k' = b/r, r' = 1$. we denote these designs by a class $D(p, b', k')$. We call such designs Series A designs.

Example5.4: In Example 5.1 we presented a partial diallel cross plan(M-E design) with auxiliary design for even line. This partial diallel cross plan for even line v=8 can be converted in to block design by forming $r = 6$ blocks in such away that each line occurs equal number of times in each block. The resulting block design for partial diallel cross plan is shown below.

Resulting block design for partial diallel crosses plan

Partial Diallel Cross Plan(M-E design)

B_1	B_2	B_3	B_4	B_5	B_6
1 x 2	1 x 3	1 x 4	1 x 6	1 x 7	1 x 8
3 x 5	2 x 7	2 x 3	2 x 4	2 x 8	2 x 5
4 x 7	4 x 6	5 x 6	3 x 8	3 x 6	3 x 4
6 x 8	5 x 8	7 x 8	5 x 7	4 x 5	6 x 7

This design is MS-optimal because it follows the conditions given in theorem 4.2.1.

Example5.5:In Example5.4, we presented an MS-optimal design d in $D(8,6,4)$ .The A-value and D-value for this design are $\phi_{Ap}(d)$ =1.4168 and $\phi_{Dp}(d)$ = 1.207x10^{-5}.The lower bounds(4.4) and (4.5) on the A- and D-efficiencies are $e'_{Ap}(d)$ =0.9607 and $e'_{Dp}(d)$ =0.9805.Similarly in Example5.3: For $D(9,2,9)$, $\phi_{Ap}(d)$ =2.8278 and $\phi_{Dp}(d)$ = 1.0036x10^{-4}.The lower bounds(4.4) and (4.5) on the A- and D-efficiencies are $e'_{Ap}(d)$ =0.8083 and $e'_{Dp}(d)$ =0.9031.And also the information matrix(C_d) and eigenvalues of the above designs, respectively, are

$$C_d = \begin{bmatrix} 4.5000 & -0.5000 & -0.5000 & -0.5000 & -1.5000 & -0.5000 & -0.5000 & -0.5000 \\ -0.5000 & 4.5000 & -0.5000 & -0.5000 & -0.5000 & -1.5000 & -0.5000 & -0.5000 \\ -0.5000 & -0.5000 & 4.5000 & -0.5000 & -0.5000 & -0.5000 & -1.5000 & -0.5000 \\ -0.5000 & -0.5000 & -0.5000 & 4.5000 & -0.5000 & -0.5000 & -0.5000 & -1.5000 \\ -1.5000 & -0.5000 & -0.5000 & -0.5000 & 4.5000 & -0.5000 & -0.5000 & -0.5000 \\ -0.5000 & -1.5000 & -0.5000 & -0.5000 & -0.5000 & 4.5000 & -0.5000 & -0.5000 \\ -0.5000 & -0.5000 & -1.5000 & -0.5000 & -0.5000 & -0.5000 & 4.5000 & -0.5000 \\ -0.5000 & -0.5000 & -0.5000 & -1.5000 & -0.5000 & -0.5000 & -0.5000 & 4.5000 \end{bmatrix}$$

and its eigenvalues are -0.0000,4.0000,4.0000,4.0000,6.0000,6.0000,6.0000,6.0000.

And

$$C_d = \begin{bmatrix} 3.5556 & 0.5556 & 0.5556 & -0.4444 & 0.5556 & -0.4444 & -0.4444 & -0.4444 & 0.5556 \\ 0.5556 & 3.5556 & 0.5556 & -0.4444 & -0.4444 & 0.5556 & -0.4444 & -0.4444 & 0.5556 \\ 0.5556 & 0.5556 & 3.5556 & -0.4444 & -0.4444 & -0.4444 & 0.5556 & 0.5556 & -0.4444 \\ -0.4444 & -0.4444 & -0.4444 & 3.5556 & 0.5556 & 0.5556 & -0.4444 & 0.5556 & 0.5556 \\ 0.5556 & -0.4444 & -0.4444 & 0.5556 & 3.5556 & -0.4444 & 0.5556 & -0.4444 & 0.5556 \\ -0.4444 & 0.5556 & -0.4444 & 0.5556 & -0.4444 & 3.5556 & 0.5556 & 0.5556 & -0.4444 \\ -0.4444 & -0.4444 & 0.5556 & -0.4444 & 0.5556 & 0.5556 & 3.5556 & 0.5556 & -0.4444 \\ -0.4444 & -0.4444 & 0.5556 & 0.5556 & -0.4444 & 0.5556 & 0.5556 & 3.5556 & -0.4444 \\ 0.5556 & 0.5556 & -0.4444 & 0.5556 & 0.5556 & -0.4444 & -0.4444 & -0.4444 & 3.5556 \end{bmatrix}$$

and its eigenvalues -0.0000,1.4038,2.0000,2.4679,2.8174,3.6527,4.5157,5.2631,5.8794.

5.2. Method of Analysis

We will consider here the analysis method for $D(p,b,k)$ design. The reduced normal equations for GCA under model M2 are given by

$$C_d \hat{g} = Q_g \tag{5.1}$$

C_d can be obtained by using equation (4.2) and Q_g is the adjusted line total vector which can be obtained by the following equation

$$Q_g = \mathbf{T_g} - \frac{1}{k}\mathbf{NB} \tag{5.2}$$

where $\mathbf{T_g} = \mathbf{\Delta'_1 Y}$ and $\mathbf{B} = \mathbf{\Delta'_2 Y}$

Since the rank(C_d) $\leq p-1$,a solution for equation (5.1) is given by

$$\hat{g} = C_d^{-} Q_g \tag{5.3}$$

Where C_d^{-} is a generalized inverse of C_d such that $C_d C_d^{-} C_d = C_d$. The estimator for any contrast $c'g$, among the GCA is then given by

$$c'\hat{g} = c' C_d^{-} Q_g \tag{5.4}$$

with variance

$$\text{var}(c'\hat{g}) = c'C_d^{-}c\sigma^2 \tag{5.6}$$

where c is a $px1$ vector.

Finally, the analysis of variance associated with Model M2 is as given below

ANOVA for Model M2

Source of variation	df	SS	E(ms)
Blocks	$b-1$	$\frac{1}{k}\sum_{j=1}^{b} B_j^2 - \frac{G^2}{bk}$	
GCA	$p-1$	$\hat{g}'Q_g$	$\sigma^2 + \hat{g}'C_d\hat{g}/p-1$
Error	By subtraction	By subtraction	
Total	$bk-1$	$\sum_i \sum_j y_{ij}^2 - \frac{G^2}{bk}$	

B_j - Total for j^{th} block; G - Grand total

In Table I and II below, we give A- and D-efficiency of the proposed designs along with A- and D-efficiency of the designs constructed by Das, Dean and Gupta(1998).

Table I. Series A orthogonal block designs with block size b/r together with their A- and D- efficiencies.

S.№	Reference	p	n	Proposed Design		DDG Design	
				$e'_{Ap}(d)$	$e'_{Dp}(d)$	$e'_{A}(d)$	$e'_{D}(d)$
1	SR1	4	4	2.25	1.1906	-	-
*2	SR6	6	9	1.5627	1.0035	0.7143	0.8532
3	SR6	6	6	1.1719	0.9699	-	-
4	R18	6	12	0.8929	0.9473	0.8929	0.9473
5	R18	6	9	0.7143	0.8532	-	-
6	R18	6	6	0.5859	0.7786	-	-
*7	SR9	8	16	1.3611	0.9571	0.8229	0.9112
*8	SR9	8	12	1.2099	0.9481	0.6405	0.8242
9	SR9	8	8	1.8148	1.0567	-	-
10	R29	8	24	0.9607	0.9805	0.9608	0.9806
*11	R29	8	20	0.9029	0.9519	0.9026	0.9520
*12	R29	8	16	0.8229	0.9113	0.8229	0.9112
13	R29	8	12	0.6405	0.8242	0.6405	0.8242
14	R29	8	8	0.8167	0.8668	-	-
15	R36	10	40	0.9797	0.9899	0.9798	0.9900
*16	R36	10	35	0.9593	0.9798	0.9530	0.9767
17	R36	10	30	0.9157	0.9587	0.9184	0.9593
18	R36	10	25	0.8707	0.9355	0.8782	0.9373
19	R36	10	20	0.7526	0.8880	0.7826	0.8915
20	R36	10	15	0.4875	0.7735	0.5571	0.7920
21	R36	10	10	0.6329	0.8044	-	-
22	SR11	10	25	0.3164	0.7867	0.8782	0.9373
*23	SR11	10	20	1.1866	0.9372	0.7826	0.8915
*24	SR11	10	15	1.0313	0.9137	0.5571	0.7920
25	SR11	10	10	0.6328	0.8044	-	-
*26	T1	10	30	0.9264	0.9617	0.9184	0.9593
27	T1	10	25	0.8663	0.9336	0.8782	0.9373
*28	T1	10	20	0.7999	0.8992	0.7826	0.8915
*29	T1	10	15	0.6284	0.8201	0.5571	0.7920
30	T1	10	10	0.6328	0.8044	-	-
31	R38	12	48	0.9308	0.9698	0.9537	0.9769
32	R38	12	42	0.9054	0.9571	0.9304	0.9650
33	R38	12	36	0.8647	0.9382	0.8975	0.9484

Table I. Series A orthogonal block designs with block size b/r together with their A- and D- efficiencies, continued.

S.№	Reference	p	n	Proposed Design		DDG Design	
				$e'_{Ap}(d)$	$e'_{Dp}(d)$	$e'_{A}(d)$	$e'_{D}(d)$
34	R38	12	24	0.6238	0.8426	0.7590	0.8803
35	R38	12	12	0.5185	0.7618	-	-
36	R39	12	54	0.9679	0.9849	0.9725	0.9863
37	R39	12	48	0.9488	0.9753	0.9537	0.9769
38	R39	12	42	0.9252	0.9633	0.9304	0.9650
39	R39	12	36	0.8955	0.9479	0.8975	0.9484
40	R39	12	30	0.8434	0.9223	0.8595	0.9271
41	R39	12	24	0.7108	0.8670	0.7590	0.8803
42	R39	12	12	1.7286	1.0328	-	-
*43	SR13	12	36	1.2098	0.9345	0.8975	0.9484
*44	SR13	12	30	1.1615	0.9345	0.8595	0.9271
*45	SR13	12	24	1.0814	0.9229	0.7590	0.8803
*46	SR13	12	18	0.8997	0.8851	0.5698	0.7941
47	SR13	12	12	0.5185	0.7618	-	-
48	R40	12	60	0.9878	0.9939	0.9878	0.9939
49	R40	12	54	0.9718	0.9861	0.9725	0.9863
50	R40	12	48	0.9308	0.9608	0.9537	0.9769
51	R40	12	42	0.9054	0.9517	0.9304	0.9650
52	R40	12	36	0.8203	0.9246	0.8975	0.9484
53	R40	12	30	0.7642	0.8990	0.8595	0.9271
*54	R40	12	24	1.0676	0.9203	0.7590	0.8803
*55	R40	12	18	0.9490	0.8955	0.5698	0.7941
56	R40	12	12	2.0167	1.0328	-	-
57	SR14	14	49	0.5156	0.8736	0.9156	0.9573
58	SR14	14	42	0.2633	0.8020	0.8824	0.9406
*59	SR14	14	35	1.0922	0.9265	0.8393	0.9182
*60	SR14	14	28	0.9995	0.9099	0.7467	0.8748
*61	SR14	14	21	0.8019	0.8628	0.6142	0.8029
62	SR14	14	14	0.4402	0.7307	-	-
*63	SR15	16	64	1.3393	0.7067	0.9282	0.9637
64	SR15	16	56	0.2315	0.8114	0.9054	0.9521
*65	SR15	16	48	1.0918	0.9291	0.8730	0.9359
*66	SR15	16	40	1.0286	0.9191	0.8282	0.9128
*67	SR15	16	32	1.2857	1.0875	0.7387	0.8708
*68	SR15	16	24	1.1479	0.9246	0.5904	0.7959
69	SR15	16	16	0.7839	0.8504	-	-

Table I. Series A orthogonal block designs with block size b/r together with their A- and D- efficiencies, continued.

S.№	Reference	p	n	Proposed Design		DDG Design	
				$e'_{Ap}(d)$	$e'_{Dp}(d)$	$e'_{A}(d)$	$e'_{D}(d)$
*70	LS3	16	48	0.8929	0.6933	0.8730	0.9359
71	LS3	16	24	0.3568	0.6376	0.5904	0.7959
72	LS3	16	16	0.6221	0.8184	-	-
73	LS4	16	72	0.9441	0.9725	0.9453	0.9793
74	LS4	16	56	0.8648	0.9416	0.9054	0.9521
*75	LS4	16	48	1.2418	0.9235	0.8730	0.9359
*76	LS4	16	40	1.0285	0.9189	0.8282	0.9128
*77	LS4	16	32	0.9643	0.9047	0.7387	0.8708
*78	LS4	16	24	1.1479	0.9246	0.5904	0.7959
79	LS4	16	16	1.6917	1.0230	-	-
80	M1	16	40	0.7810	0.8962	0.8282	0.9128
81	M1	16	24	0.4153	0.7475	0.5904	0.7959
82	M1	16	16	0.7840	0.8504	-	-
*83	M2	16	48	0.8929	0.9423	0.8730	0.9359
*84	M2	16	40	0.8419	0.9181	0.8282	0.9128
*85	M2	16	32	0.9754	0.9072	0.7387	0.8708
*86	M2	16	24	0.8221	0.8685	0.5904	0.7959
87	M2	16	16	0.7840	0.8504	-	-
88	SR16	18	81	1.1289	0.9336	-	-
89	SR16	18	72	1.1132	0.9339	-	-
90	SR16	18	63	1.0877	0.9309	-	-
91	SR16	18	54	1.0522	1.0607	-	-
92	SR16	18	45	0.9888	0.9799	-	-
93	SR16	18	36	0.8705	0.9194	-	-
94	SR16	18	27	0.6618	0.8192	-	-
95	SR16	18	18	0.3386	0.6283	-	-
96	SR17	20	100	1.1142	0.9351	-	-
97	SR17	20	90	1.1004	0.9348	-	-
98	SR17	20	80	1.0831	0.9338	-	-
99	SR17	20	70	1.0577	0.9307	-	-
100	SR17	20	60	1.0272	0.9249	-	-
101	SR17	20	50	0.9752	0.9141	-	-
102	SR17	20	40	1.1418	0.9208	-	-
103	SR17	20	30	0.9960	0.8990	-	-
104	SR17	20	20	0.6183	0.7967	-	-

Table II. Series B orthogonal block designs with block size v together with their A- and D- efficiencies.

S.№	Reference	p	n	Proposed Design		DDG Design	
				$e'_{Ap}(d)$	$e'_{Dp}(d)$	$e'_{A}(d)$	$e'_{D}(d)$
1	R34	9	18	0.8083	0.9031	0.8472	0.9168
2	R41	15	45	0.8748	0.9372	0.9080	0.9500
3	R41	15	30	0.6906	0.8927	0.8869	0.9292
4	LS5	25	100	0.9222	0.9577	-	-
5	LS5	25	75	0.8717	0.9330	-	-
6	LS5	25	50	0.6198	0.8389	-	-

We denote that $e'_{Ap}(d)$ and $e'_{Dp}(d)$ are the lower bounds of the proposed designs and $e'_{A}(d)$ and $e'_{D}(d)$ are the lower bounds of Das, Dean and Gupta(1998).

'*' - Denotes the higher efficiency of the proposed design in comparison to Das, Dean and Gupta(1998).

DDG - Das, Dean and Gupta

In Table III below, we give A- and D-efficiency of the proposed designs along with A- and D-efficiency of the designs constructed by Parsad, Gupta and Gupta (2005).

Table III. Series A and B orthogonal block designs with block size b/r and v respectively together with their A- and D- efficiencies.

S.№	Reference	p	n	Proposed Design		PGG Design	
				$e'_{Ap}(d)$	$e'_{Dp}(d)$	$e'_{A}(d)$	$e'_{D}(d)$
1	R18	6	12	0.8929	0.9473	0.8929	0.9473
*2	SR9	8	16	1.3611	0.9571	0.9423	0.9661
3	R29	8	16	0.8229	0.9113	0.9423	0.9661
4	R36	10	20	0.7526	0.8880	0.9265	0.9618
5	R36	10	15	0.4875	0.7735	0.7168	0.9196
6	R39	12	30	0.8434	0.9223	0.9308	0.9698
*7	SR13	12	30	1.1615	0.9345	0.9308	0.9698
8	R40	12	30	0.7642	0.8990	0.9308	0.9698
*9	SR15	16	32	1.2857	1.0875	0.8929	0.9473
*10	SR15	16	24	1.1479	0.9246	0.9259	0.9572
*11	LS4	16	32	0.9643	0.9047	0.8929	0.9473
*12	LS4	16	24	1.1479	0.9246	0.9259	0.9572
13	M1	16	24	0.4153	0.7475	0.9259	0.9572
*18	M2	16	32	0.9754	0.9072	0.8929	0.9473
19	M2	16	24	0.8221	0.8685	0.9259	0.9572
20	R34	9	18	0.8083	0.9031	0.8448	0.9185
21	R41	15	30	0.6906	0.8927	0.8206	0.9311

We denote that $e'_{Ap}(d)$ and $e'_{Dp}(d)$ are the lower bounds of the proposed designs and $e'_{A}(d)$ and $e'_{D}(d)$ are the lower bounds of Parsad, Gupta and Gupta(2005).

'*' - Denotes the higher efficiency of the proposed design in comparison to Parsad, Gupta and Gupta (2005).

PGD – Parsad, Gupta and Gupta

As it is seen from the above tables (Table I, II and III) some of the proposed designs have higher efficiency than the design proposed by Das, Dean and Gupta(1998) and Parsad, Gupta and Gupta(2005).

CHAPTER SIX

6. Discussion and Conclusion

6.1. Discussion

Diallel cross plans are one of the commonly used mating designs. With limited facilities available for testing, a diallel cross may only be possible for relatively small number of inbred lines. However, if only a small number of lines are included, the estimate of the variance of the general combining abilities (GCA) in the whole population of potentially available inbred lines is subject to a large sampling error and also many potentially high yielding inbred lines may be left out completely untested. It is, therefore, necessary to have a large number of inbred lines but pick only a sample of all possible crosses among them. Such a diallel cross is known as a partial diallel cross (PDC).

The appropriateness of the various kinds of diallel crossing methods depends on the experimental material and the objectives of the experiment. When information on general combining ability for a set of lines is only described in connection with a plant or animal breeding problem and if it can be assumed that there will be no genotypic reciprocal effects a PDC design is appropriate.

6.2. Conclusion

The complete diallel cross, which is composed of all possible single crosses among a group of inbred lines, is a common investigatory tool for plant and animal breeders. Partial diallel crosses (PDC),which represent a subset of the complete diallel cross, have attracted the attention of several workers during last three decades.

Most commonly, diallel cross experiments have been evaluated using completely randomized designs(CRD) or randomized complete block designs (RCBD), with suitable number of replicates, as environmental design. In most cases, however, the number of crosses is too large, leading to an overall inefficient experiment. It is for this reason that the use of incomplete block designs as environmental design has been advocated.

This study shows a simple method of construction of partial diallel cross design through partially balanced incomplete block design as auxiliary design with the method of analysis and also give comparisons of the efficiencies of the proposed designs with other existing designs for partial diallel cross in the literature.

The MS-optimal M-E designs proposed in Section 5.1 through PBIBD(2) can be obtained very easily,once the breeder has determined which M-design to use based on the available techniques as described in Chapter five. The simplicity of the proposed design carries over to the analysis.

It is quite obvious from Table I,II,III that some of the proposed designs have higher A-and D-efficiency than the existing designs given by Das, Dean and Gupta(1998) and Parsad, Gupta and Gupta(2005) and found that several our designs have higher A- and D-efficiency in comparison to the existing designs.

The study also shows the importance of partial diallel cross design through partially balanced incomplete block design when large number of inbred lines exist in plant and animal breeding experiment.

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