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DEPARTMENT OF CIVIL ENGINEERING

A COMPUTER PROGRAM FOR THE ANALYSIS AND DESIGN OF REINFORCED CONCRETE DOMES

WENDWESEN FEKEDE

2017

A COMPUTER PROGRAM FOR THE ANALYSIS AND DESIGN OF REINFORCED CONCRETE DOMES

BY: WENDWESEN FEKEDE

A THESIS SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES OF ADDIS ABABA UNIVERSITY IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE IN CIVIL ENGINEERING

THESIS ADVISOR

DR. ASNAKE ADAMU

ADDIS ABABA UNIVERSITY

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APPROVED BY BOARD OF EXAMINERS

DR. ASNAKE ADAMU	________________	**JUNE 2017**
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Declaration

I, the undersigned, declare that this thesis is my work and all sources of materials used for this thesis have been duly acknowledged.

Name: Wendwesen Fekede

Signature:

Place: Addis Ababa University

Addis Ababa Institute of Technology

Date of Submission: June 2017

This Thesis is dedicated to my father.

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Thanks to God for each and every success in my life and satisfactory accomplishment of this thesis. He has endowed me courage and strength as well as precious health throughout my school time and the entire life as well.

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LIST OF SYMBOLS

fck = Characteristic compressive strength of concrete

fcu = Cube compressive strength of concrete

fcd = Design compressive strength of concrete

ν = Poisson's ratio of concrete

Ec = Modulus of elasticity of concrete

fyk = Characteristic yield strength of steel

fyd = Design yield strength of steel

Es = Modulus of elasticity of steel

eyd = Design yield strain of steel

DL = Dead uniformly distributed load on the dome

LL = Live uniformly distributed load on the dome

SuDL = Super dead uniformly distributed load on the dome

q = Total uniformly distributed load on the dome

f = Rise of the spherical dome

D1 = Span of the spherical dome

a = Radius of the spherical dome

α = Half the central angle of the spherical dome

Φ = Meridional angle

t = Thickness of the spherical dome

h = Depth of the ring beam

b = Width of the ring beam

e = Eccentricity b/n center of beam and center line of dome

r = Radius of projected horizontal circle of the dome

D10D, D20D, D11D, D12D, D22D = Influence coefficients of dome

D10R, D20R, D11R, D12R, D22R = Influence coefficients of ring beam

D10, D20, D11, D12, D22 = Total influence coefficients

H = Uniformly distributed radial force on ring beam

$M\boldsymbol{\alpha}$ = Twisting couple on ring beam

$N\phi$ = Meridional force

$N\theta$ = Hoop force

$M\phi$ = Meridional bending moment

$M\theta$ = Hoop bending moment

w = Normal displacement

v = Meridional displacement

ε_{ϕ} = Meridional strain

ε_{θ} = Hoop strain

qcr = Critical buckling load

ABSTRACT

This thesis addresses a computer program for the analysis and design of shells of revolution, specifically reinforced concrete domes with edge rings. The study examines the method used to analyze and design such shells and integrates these methods in a programming language to create a simple and easily understandable tool for analysis and design of shells of revolution. A comparison of the outputs of the program developed with the results of finite element method is also made. The results associated with this thesis work minimizes the time required and the errors involved in the course of manual calculations for analysis and design of reinforced concrete shells of revolution. In addition, it makes the analysis and design process of shell structures simple so that designers will be initiated to design and recommend such kinds of structures for Clients.

1.0. INTRODUCTION

1.1. Background

Shell structures are one of the most prominent structures that exist in nature. Shell structures are used to encase from the delicate eggs of birds and human skull to roofs of huge arenas, from simple tankers to the most sophisticated space shuttles, they can be used for wide range of application from foundation to frames of building structures.

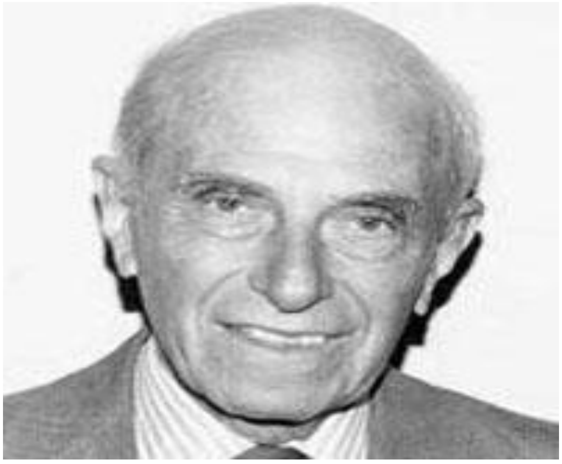
Shells that exist in nature are abundant, example of such shell include, eggs, sea shells, turtles shell, skulls, nuts and bird nests built by instinct. The abundant existence of shells in nature indicates how much naturally efficient shells can be under certain conditions. The most efficient shell that exist in nature is egg shells, whose strength to weight ratio is so high especially when you consider that thickness of hen egg is only 0.4mm, it amazes how nature is perfect by itself. The average load to break the eggs when standing up is 53lb (24.04kg) (source: info.admet.com/videos/bid/54676/Egg-Compression-Strength-Test).

Shell structures have been utilized by mankind for several millennia. Although nobody can pinpoint exactly when the construction of shell structures is, they were occasionally built by different civilizations all around the world. Nobody for sure can tell how ancient civilizations were able to construct such structures, in fact it is not possible to deduce weather those shells were designed and constructed instinctively or on basis of scientific understanding of the behavior.

One of the famous dome shaped shell structure in the world is the Pantheon in Rome, which is the oldest known concrete shell which was completed about 125 AD and it is still standing. It has a massive concrete dome 43m in diameter, with an oculus at its center. A monolithic structure, it appears to have been sculpted in place by applying thin layers on top of each other in decreasing diameter. Massively thick at the bottom and thinning (with aerated volcanic pumice as part of the concrete mix) at the top, the pantheon is a remarkable feat of engineering.

Figure 1.1. Pantheon aerial view and interior.

Despite the construction of shell structures since the beginning of civilizations, the design theories have been predominantly instinctive until the 19th century. It was in the early 1920s that intensive researches were carried out to develop the theories and practices used in the design of shell structures. The invention of cement and fabrication of formworks has facilitated the development of this design theories.



During the last century many scientists have contributed their fair share for the development and improved their applicability. Few of the famous engineers include Pier Luigi Nervi (1891-1979), Eduardo Torroja (1891-1961), Felix Candela (1910-1997), the South brothers Randy, Barry, and David B. South, Dyckerhoff, Widmann, Anton Tedesko (1903-1994).

Figure 1.2. Anton Tedesko German born Civil Engineer

During the late 1950s and early 1960s scientists were trying to invent a numerical technique for finding approximate solutions which is known as finite element method. Although the method originated from the need to solve complex elasticity and structural analysis in aeronautical engineering, it has been widely used in solving civil engineering structures, including shells. As a result of this invention, the last three decades have witnessed the advancement of shell analysis. In 1961, A. Adini and R.W. Clough first applied a hybrid finite element method for stress analysis of shell structures, this has attracted several researchers to develop different techniques for this purpose. The invention of computers have also accelerated the development effective techniques of analysis. Ever since the first invention of computers, the overall processing power of computers has doubled every two years. The introduction of high speed digital computers have opened another frontier in the field in search of a simplification in theoretical formulations as well as making a better approximations in the numerical computations. With the advent of high speed computers, various surfaces, buckling of shells, nonlinear behavior and many other aspects of shell research which were otherwise intractable, could now be analyzed in detail with the aid of these computers. The objectives for such refinements in both theories and solutions are to know the exact behavior and characteristics of shells in detail and also to know the relative importance of various assumptions and approximations considering safety and stability in relation to economy for such shell structures.

The invention of computers and finite element method have facilitated the development of computer aided design and analysis software in the 1960s and 1970s. UC Berkeley developed finite element program SAP IV. Other software programs include SAFE, FEMTA, sas-civil, ETABS, AxisVM . . .

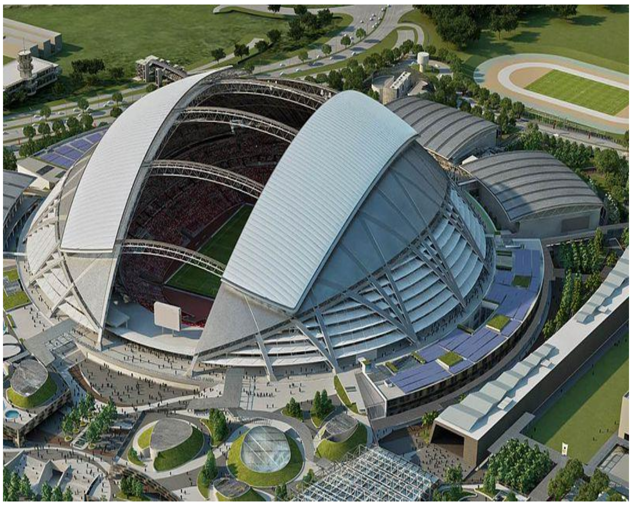
1.1.1. Renowned shell structures in the world

Even though the construction of shell structures begun before thousands of years, their construction has been limited to developed nations. This is as a result of the complexity of the analysis of shell structures and the building technology and the precision required in the construction of this structures. Therefore, most of the early shell structures were constructed in Europe and USA. The world first double curvature lattice thin shell structure was constructed in Russia. Most of the shell structures listed below are from Japan, Russia, USA and western countries.

Table 1.1.List of famous shell structures.

Name of the structure	Location	Type	Span	Rise	Thickness
Nagoya Dome	Japan	Dome	187.2m	66.9m	
Kresge Auditorium	USA	1/8th sphere	30.5m	15.2m	10.8cm
Shukhv Rotunda	Russia	lattice	68m	16m	
Europe 1 Transmitter Building	Germany		82mX63m		
Denver Coliseum	USA	cylinder			9-13cm

Tokyo Dome	Japan	Dome	100m	56.18m	
Singapore National Stadium (largest dome)	Singapore	Dome	310m		
Kingdome (largest reinforced concrete dome)	USA	Dome	201.1m	33.5m	12.5cm



A. Singapore National Stadium B. Nagoya Dome, Japan

C. Kingdome, USA D. Tokyo Dome

Figure 1.3. Famous Dome Structures of the World.

1.1.2. Shell structures in Ethiopia

Ethiopia is one of the ancient countries in the world, it has a rich history constructing beautiful palaces, churches and mosques. Ethiopia is home to many cultures and many of this cultures have used the principles of shells to construct their houses. The traditional house, Gojo of Sidama and Gurage people, temporary house of Afar and Somali people, Lalibela rock churches, and ancient mosques of Harar are some of the historic and cultural shell practices. The knowledge of construction of traditional houses has been transferred from generation to generation and the practices still exist specially in the rural parts of Ethiopia.

A. Somali girl constructing

B. Nomad House, Somali region

C. Traditional Gojo Bet, Arbaminch

Figure 1.4. Traditional shell structures

Modern construction of shell structures in Ethiopia is believed to have begun in early 1950s and 1960s, during the regime of Emperor Hailesilassie II. During this period construction of shell structures boomed but it has declined ever since, and now it is almost not in existence. Some of the shells constructed in Ethiopia include Bole airport old terminal, Axum hotel hall, and St. Marry Tsion church found in Axum town.

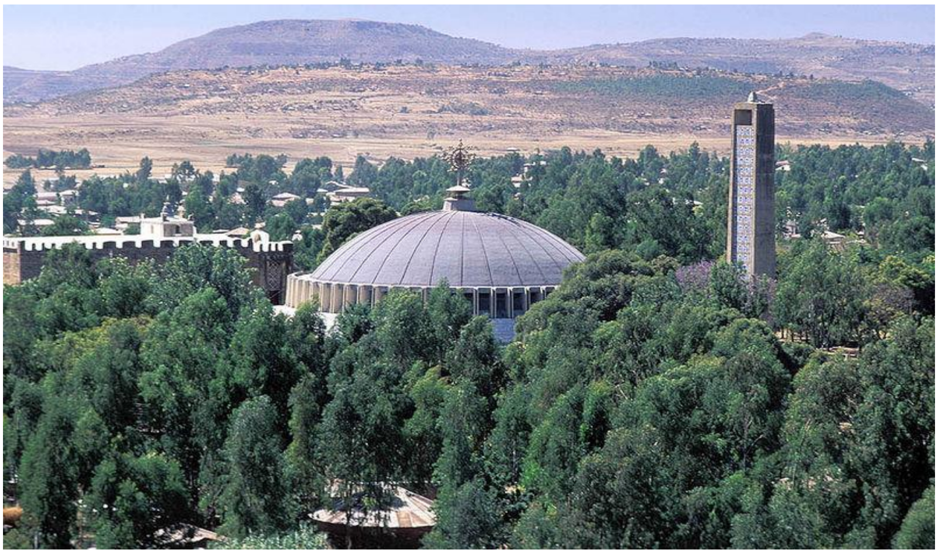


Figure 1.5. Saint Marry Tsion church, Axum.

1.2. Objective

The main objective of this thesis is to prepare a computer design program for the analysis and design of reinforced concrete domes following the EBCS code and force method of analysis so that we can significantly reduce the time and resources required to analyze and design shells of revolution.

In due course, the following benefits are obtained

- Avoid the tedious manual calculation of analysis and design;
- Eliminate the possible errors that could be made through the process of manual calculation;
- Prepare an easily understandable analysis and design tool;
- Appreciate the enormous advantages of shell structures by providing a tool for easily analyzing and designing them so that designers will be initiated to design and recommend such kinds of structures for clients.

1.3. Content of the Thesis

This thesis consists of five chapters. The first chapter deals with the general background of construction of shell structures in the world and the objectives of the thesis, and outcome of the study.

The second chapter is devoted to discuss the literature survey carried on parameters, types and classification of shells.

The third chapter focused on design consideration of Reinforced Concrete Domes. Moreover, it addresses the design specification to be considered during analysis and design of domes as per the standards stated in the Design Manuals and general standards. The core and specific input of the study is presented under this chapter, indicating all the necessary steps and calculations used in developing the program using “Microsoft visual basic 2010”.

The flow charts and algorithm for analysis and design of Reinforced Concrete Domes, the analysis of Reinforced Concrete Domes using Finite Element Methods, demonstrating the application of the developed program and illustration using practical examples and summary of the outputs for the example using both the developed program and Finite Element Methods are also presented in chapter four.

The last chapter of this thesis is made to contain the conclusions drawn from the outputs based on the developed program and the recommendations on the basis of the findings.

1.4. Applications and Limitations

1.4.1. Applications

- The computer program developed will be applicable for Analysis and Design of Reinforced Concrete Domes of any span length.
- The program shall benefit Engineering Consultants and shell constructors (contractors) in the checking and design review works and simplifies to look matters of optimal solution. Moreover, the school may use this for future research in line with up grading the program to handle many other aspect of shell design.

1.4.2. Limitations

- The computer program developed will be applicable only for Analysis and Design of Spherical Reinforced Concrete Domes, it does not analyze domes made of other materials and domes of other shapes.
- The computer program is developed only for domes subjected to symmetric and uniformly distributed dead and live loads, therefore, it is not applicable for domes subjected to other kinds of loads.
- The computer program is developed only for domes without sky light and to calculate membrane forces for domes with sky light, therefore, it is not applicable to calculate bending field forces of domes with sky light.

2.0. BASICS OF SHELL PARAMETERS

2.1. General

Shells are spatially curved surface structures which support own weight and externally applied loads. Shells are found in a variety of natural structures such as eggs, plants, leaves, skeletal bones, and geological forms. Shell structures have also been built by man since the most ancient times. Many shell domes built of masonry and stone in ancient times are still in existence in some parts of the world.

Shell structures can be efficiently and economically used in various fields of engineering and architecture. A great variety of shell roofs, water retaining shells, tall silos (up to 60 meters high) and other containment structures, structurally efficient doubly curved high arch dams (up to 300 meters high), the containment shells of nuclear power plants, requiring high degree of safety, tall chimneys and also huge cooling towers (as high as 200 meters) have been built of steel or reinforced concrete shells.

The behavior of shell structures is different from that of so-called "framed structures". This feature originates mainly from the geometrical features of shells which make the internal force system in shells differ from those in other types of structural forms. The internal force distribution in shells is, in general, three dimensional, i.e., spatial. Moreover, shell structures carry the applied forces mostly by the so-called membrane forces, whereas other structural forms carry the applied loads by bending mechanisms.

The salient features of shells, as compared with other structural forms such as beams, frames, and plates can be outlined as follows: [M. Farshad, 1992].

- Efficiency of load carrying behavior
- High degree of reserved strength and structural integrity
- High strength to weight ratio
- Very small thickness ratio to other dimensions (span, radius of curvature)
- Very high stiffness
- Containment of space

2.1.1. Thin Shells

The surface passing through the mid thickness of the shell at each point is, by definition, called the middle-surface of the shell. If the thickness of the shell is very small compared with the radii of curvature of the shell mid-surface, then the shell is considered as a thin shell. The thickness to radii ratio, or sometimes the thickness to span ratio, of about 1/200, occurring in reinforced concrete shells, puts the actual shells well in the range of being "thin shell" structures. For metallic and composite shells, this ratio is in practice much smaller, of the order of 1/300. [S. Timoshenko, and S. Woinowsky-Krieger, 1987].

2.2. Classification of Shells

Shells are classified on the basis of the following characteristics: [M. Farshad, 1992].

- Sign of Gaussian curvature as synclastic surfaces, anticlastic surfaces and surfaces with zero Gaussian curvature at a point.
- Development as developable and non-developable surfaces.
- Method of generation as surfaces of revolution, surfaces of translation and ruled surfaces.

2.2.1. Surfaces of Revolution

Surfaces of revolution are those generated by the rotation of a plane curve, including a straight line, called the meridian, about an axis known as the axis of the shell. During such rotation, points on the meridian draw parallel circles in planes normal to the axis of the shell. A shell of revolution is called spherical, conical, elliptical, cycloidal, etc., depending upon its meridian being part of a circle, straight line, ellipse, cycloid, etc. Examples of surfaces of revolution are shown in Figure 2.1.

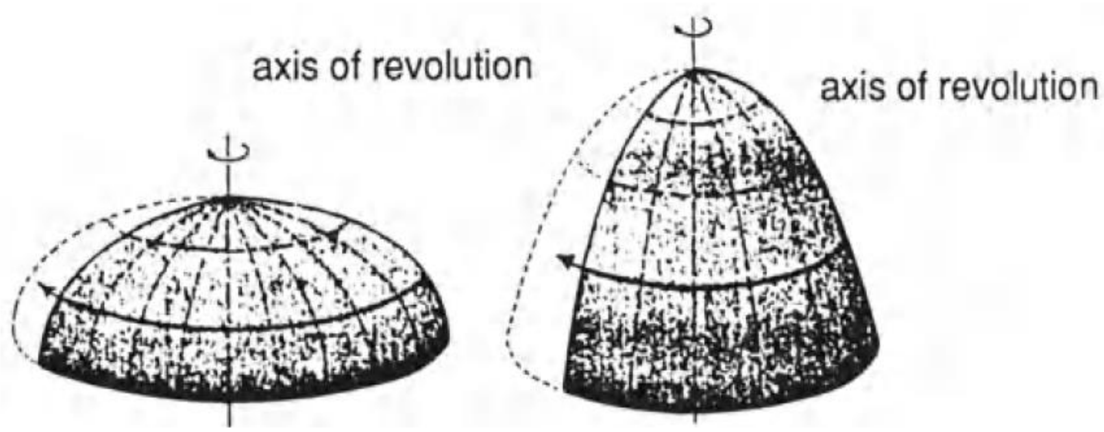


Figure 2.1. Surfaces of revolution

2.2.2. Surfaces of translation

Surfaces of translation are defined as the surfaces generated by sliding a plane curve along another plane curve, while keeping the orientation of the sliding curve constant. The second curve on which the original plane curve slides, is called the generator of the surface. In the special case in which the generator is a straight line, the resulting translational surface is called a cylindrical surface. Examples of translational surfaces are shown in Figure 2.2.

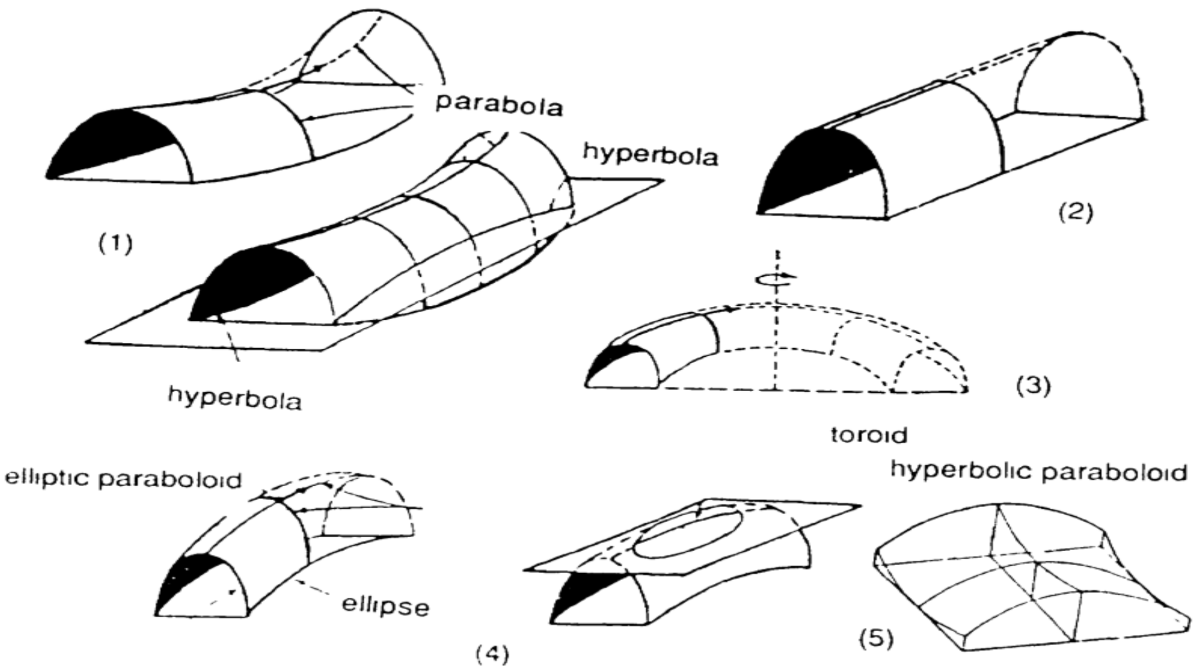


Figure 2.2. Surfaces of translation

2.2.3. Ruled surfaces

Ruled surfaces are obtained by sliding a straight line, two ends of which remain on two generating curves, in such a fashion that it remains parallel to a prescribed direction or plane. The generating straight line is not necessarily at right angles to the planes containing the director curves. Some examples of ruled surfaces are shown in Figure 2.3.

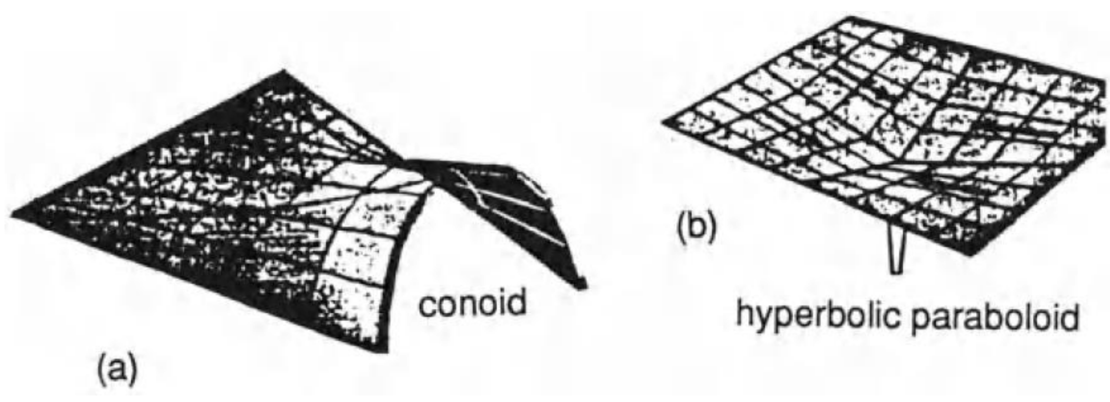


Figure 2.3. Ruled surfaces.

3.0. ANALYSIS AND DESIGN OF REINFORCED CONCRETE DOMES

3.1. General

3.1.1. Internal Force System in a Shell

Shell structures support applied external forces efficiently by virtue of their geometrical forms. Shells, having their spatial curvature, are much stronger and stiffer than other structural forms. For this reason shells are sometimes referred to as form resistant structures.

The general state of stress in a shell structure subjected to applied external loading, temperature changes, support settlements, and deformation constraints consists of membrane normal and shear stresses lying in the shell surface, as well as the transverse shear stresses. In thin shells, the component of stress normal to the shell surface, compared with other components of the internal stresses, is very small and is neglected in the classical shell theories. [S. Timoshenko, and S. Woinowsky-Krieger, 1987].

The internal forces at each point of the shell may be placed in one of two groups of force fields:

- Membrane forces, and
- Bending forces.

The membrane forces, as the name implies, are the resultant internal forces which lie "inside" the mid-surface of the shell. The membrane force field causes the stretching or contraction of the shell, as a membrane, without producing any bending and / or local curvature changes. The membrane force field consists of two membrane normal resultant forces and a membrane shear force.

The second group of internal forces are called the bending forces, since they cause bending and twisting of the shell cross-sections. The bending force field consists of bending moments, twisting couples, and transverse shear forces.

The internal forces at any point of a shell, consist of ten component internal force resultants (Nx, Ny, Nxy, Nyx, Mx, My, Mxy, Myx, Qx, Qy). These components, can be separated into two groups, entitled membrane and bending internal force field, as follows:

- Membrane field: Nx, Ny, Nxy, Nyx
- Bending field: Mx, My, Mxy, Myx, Qx, Qy

In this terminology, Mx and My stand for bending moments while Mxy and Myx represent the twisting couples. Qx and Qy represent the out-of-plane shear forces.

For a material body in spatial equilibrium there are six governing equilibrium equations. Since there are more than six force resultants, we conclude that a shell is, in general, an internally statically indeterminate structure.

The internal force redundancy, although it is an indication of additional load carrying mechanism, is not always required for shell equilibrium. Let us imagine a shell subjected to applied loading in which only the membrane force field has been produced and the bending field is absent. By writing the moment equation of equilibrium about the normal to the shell element (z axis) we can conclude that Nxy = Nyx. Therefore, the membrane force field will consist of the forces Nx, Ny, and Nxy = Nyx.

In a shell in which only the membrane field exists, three of the six equilibrium moment equations (Mx = 0, My = 0, Mz = 0) are identically satisfied. We are then left with three remaining force equilibrium equations and three internal membrane forces to be determined. Since the number of equilibrium equations and the number of unknown forces are equal, the membrane shell is statically determinate and its internal force system can be determined by the use of the equilibrium equations alone, without the need of any auxiliary relations.

The membrane force field is, of course, associated with the membrane normal and shear forces, which are assumed to be uniformly distributed through the thickness of the shell. A shell in which only the membrane force field exists is said to have a membrane behavior. The resultant theory is called the membrane theory of shells.

A shell will have a pure membrane behavior provided certain boundary requirements, loading conditions, and geometrical configurations are satisfied. In order that a membrane theory be totally applicable, the forces and the displacements at the shell boundaries must be force-compatible and deformation- compatible with the true membrane behavior of the shell.

There may be some conditions in which the pure membrane action of a given shell could be disturbed and thus the premises of a membrane theory would be violated. The most prominent of these conditions are the following: [M. Farshad, 1992].

- Deformation constraints and some boundary conditions which are incompatible with the requirements of a pure membrane field.

- Application of concentrated forces, and change in the shell geometry and / or sudden change of curvature.

Laboratory and field experiments, as well as elaborate theoretical calculations, show that the bending field produced in any one of the above-mentioned situations would mostly remain confined to the region in which the membrane conditions are violated.

The bending forces, being confined to a small region, leave the rest of the shell virtually free of bending actions. Therefore, in most cases, the major part of a shell structure behaves as a true membrane. This very interesting and unique character of shells is the result of the inherent curvature in the spatial shell form. It is this salient feature of shells that is responsible for the most profound and efficient structural performance of shells observed in nature, as well as in the shells designed and constructed in engineering practice.

To summarize, shell structures carry the applied external forces mostly by the mechanism of membrane action. In some regions of the shell a bending force field may develop to satisfy specific equilibrium or deformation requirements. The range of influence of the bending field is local and is confined to the vicinity of loading and geometrical discontinuities and / or the deformation incompatibilities. The rest of the shell is virtually free from bending actions and can be analyzed and designed as a membrane. Depending on the nature of the applied forces, this membrane shell may be in tension or compression or partly both. The extent of the domain of influence of bending depends on the particular shell geometry and its edge and loading conditions. [M. Farshad, 1992].

3.2. Shell Theories

Any shell theory is founded on three set of relations. These relations are the equilibrium equations, kinematical relations, and constitutive relations. To be complete, these three sets of field equations must be accompanied by the appropriate boundary conditions of the particular shell problem.

3.2.1. Assumptions of Classical Shell Theories

The classical theories of shells are based on the following assumptions: [M. Farshad, 1992].

- The shell is assumed to be thin, i.e., its thickness is small compared with its representative minimum radius of curvature, or lateral dimensions.
- Plane sections originally normal to the shell mid-surface remain plane and perpendicular to the deformed mid-surface. This is equivalent to ignoring the shear deformations.

- The stress component normal to the shell mid-surface is very small compared with other stress components, and can be neglected.
- The displacements and strains are so small that their higher powers can be neglected.

3.2.2. Force Method of Shell Analysis

To perform a shell structural analysis, one of the two general well known methods of structural analysis may be used. These are:

- The force method (or compatibility method), and
- The displacement method (or the stiffness method).

Many Finite Element shell computer programs are based on the stiffness method. However, for manual calculations, the compatibility method offers certain advantages over the stiffness method.

The analysis of a given shell, according to the force method includes the following stages: [M. Farshad, 1992].

- First, a membrane analysis of the given shell subjected to an applied distributed external loading is carried out. In this stage, the boundary conditions are assumed to be compatible with the requirements of the membrane action of the shell. The internal forces as well as the edge displacements and rotations are to be determined from this membrane analysis. At this stage, the shell is statically determinate.
- The unknown corrective redundant bending forces are applied to the shell from which the distributed loading is now removed. The shell is analyzed with the help of an appropriate bending theory. The internal forces as well as the edge displacements and rotations are obtained. This stage of analysis yields the corrections, i.e., the redundant forces, due to the bending field which exist at the shell boundaries. The internal forces and edge deformations of the shell are obviously expressed in terms of unknown bending forces.
- The results of analyses performed in the above two stages are combined to satisfy the compatibility requirements at the shell boundaries. The compatibility requirements, expressed in terms of known membrane displacements and unknown edge forces, yield a set of simultaneous algebraic equations from which the redundant edge forces can be determined.
- Having performed the membrane analysis and having obtained the corrective boundary bending effects, one can now superimpose these two fields to determine the complete force and deformation field in the shell. This completes the force method analysis of the shell.

3.2.3. Shells of Revolution

3.2.3.1. Geometrical Description

The middle surface of a shell of revolution with non-zero positive Gaussian quadrature can be described rather like the earth. Thus through any point we may take two sections. These sections create two plane curves with two local principal radii of curvature, r_1 and r_2. One of these sections is called the meridional curve while the projection of another section on plane perpendicular to the axis of revolution creates the parallel circles on the shell surface. At any point, the radius of curvature of the meridian is called r_1, and the radius of parallel circ1e, r, is projected value of another principal radius of curvature which has been denoted by r_2. (See Figure 3.1.)

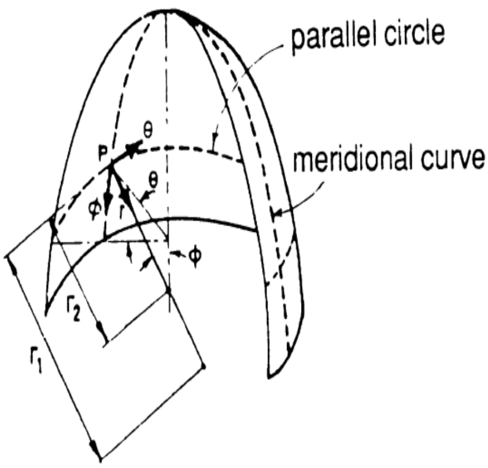


Figure 3.1. A partial perspective view of a surface of revolution

Parallel circles form the perimeter of the base of a cone the apex of which is the center of curvature for r_2. Due to rotational symmetry, the center of curvature of r_2 always lies on the axis of revolution. However, the center of curvature of r_1 does not have to lie on this axis.

Denote the angle between the normal to the surface at P with the axis of revolution by ϕ. We also denote the horizontal angular position of P, from some arbitrary origin, by the angle θ. The direction of the axis of revolution is assumed to coincide with the z axis.

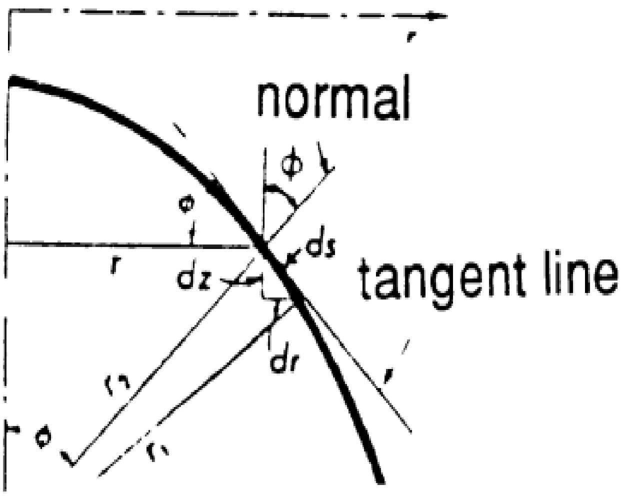


Figure 3.2. A meridional section of rotational shell.

Referring to Figure 3.2, the following relations exist among the shell geometrical parameters. [M. Farshad, 1992].

$$r = r_2 Sin\phi \qquad (3.1)$$

$$ds = r_1 d\phi \qquad (3.2)$$

$$dr = ds Cos\phi \qquad (3.3)$$

$$dz = ds Sin\phi \qquad (3.4)$$

$$\frac{dr}{d\phi} = r_1 Cos\phi \qquad (3.5)$$

$$\frac{dz}{d\phi} = r_1 Sin\phi \qquad (3.6)$$

Combining the above relations, we obtain the following inter-relation between the surface parameters r_1, r_2, and ϕ.

$$\frac{1}{r}\frac{dr}{d\phi} = \frac{r_1}{r_2} Cot\phi \tag{3.7}$$

3.2.3.2. Governing Membrane Equations

To derive the membrane equilibrium equations for shells of revolution, we consider the free body diagram of an element of the shell, Figure 3.3. The element shown in figure 3.3 is taken out from the shell by two pairs of infinitesimally adjacent sections. The first pair of sections are meridians while the second pair contains the normals at the corner points. Since these two intersections are principal sections, they are mutually orthogonal to each other.

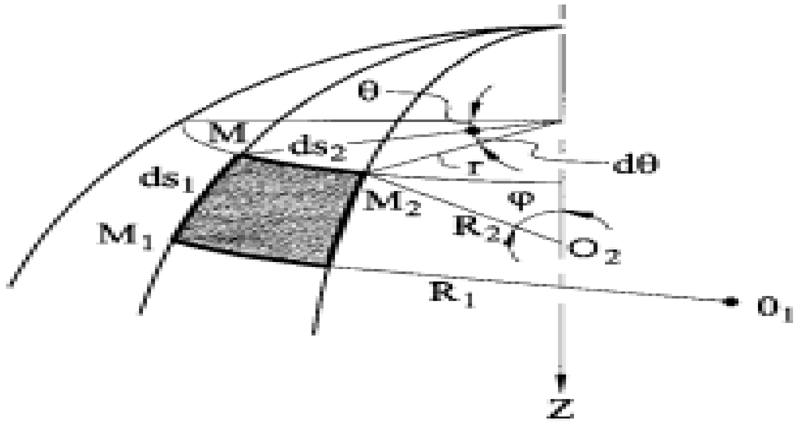


Figure 3.3. An infinitesimal element of a rotational surface.

The free body diagram of Figure 3.4 shows the internal membrane forces, Nϕ, Nθ, Nϕθ. and their differential variations, Nϕ designates the meridional force, Nθ the hoop force, and Nϕθ the membrane shear force; the quantities Pr, Pϕ and Pθ represent the intensity of external distributed applied loading, in the r, ϕ and θ directions, respectively. [M. Farshad, 1992].

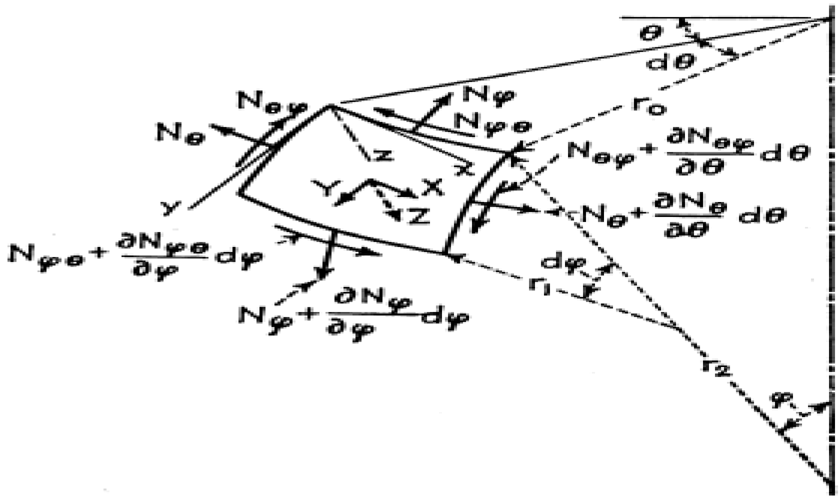


Figure 3.4. Free body diagram of a rotational shell element.

We write the equations of equilibrium in the ϕ, θ, and r directions. Because of the double curvature, the membrane forces have projections in all three directions and thus contribute to all three equilibrium equations. Figure 3.5 shows the contributions of $N\phi$ and $N\theta$ in various directions.

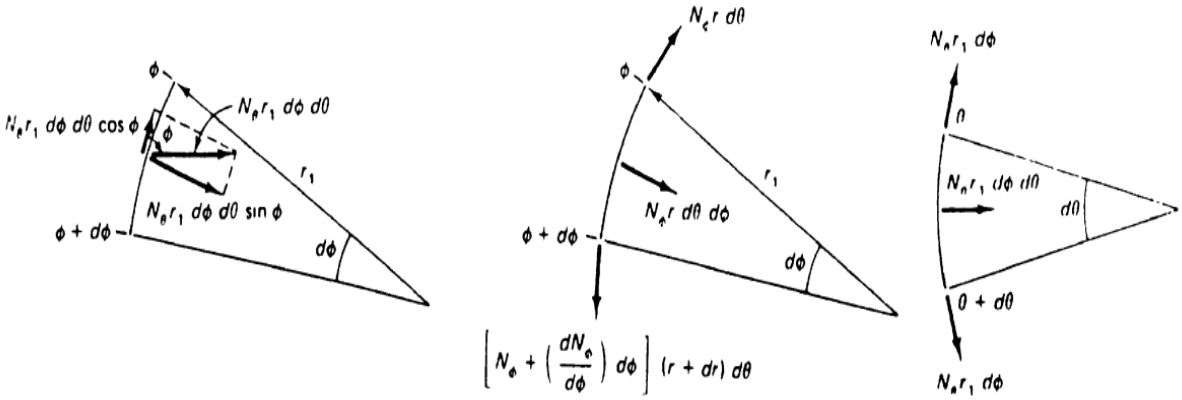


Figure 3.5. Meridional and hoop sections through the shell of revolution.

The equilibrium equation in the hoop direction is:

$$\frac{\partial N_{\theta\phi}}{\partial\theta} r_1 d\theta d\phi + \frac{\partial}{\partial\phi}\left(rN_{\phi}\right)d\theta d\phi - N_{\theta} r_1 d\theta d\phi Cos\phi + P_{\phi} r r_1 d\theta d\phi = 0 \qquad (3.8)$$

If we divide both side of this equation by $(d\theta d\phi)$ we obtain

$$\frac{\partial N_{\theta\phi}}{\partial\theta} r_1 + \frac{\partial}{\partial\phi}\left(rN_{\phi}\right) - N_{\theta} r_1 Cos\phi + P_{\phi} r r_1 = 0 \qquad (3.9)$$

We derive the equilibrium equation in the ϕ direction in a similar fashion.

$$\frac{\partial N_{\theta}}{\partial\theta} r_1 + \frac{\partial}{\partial\phi}\left(rN_{\phi\theta}\right) + N_{\theta\phi} r_1 Cos\phi + P_{\theta} r r_1 = 0 \qquad (3.10)$$

The third equilibrium equation is obtained by projecting all the forces in the direction normal to the shell, i.e., in the r direction. By doing so, we obtain:

$$N_{\theta} r_1 Sin\phi + N_{\phi} r - P_r r r_1 = 0$$

Which, upon division by (rr_1) yields:

$$\frac{N_{\phi}}{r_1} + \frac{N_{\theta}}{r_2} = P_r \qquad (3.11)$$

Equations (3.9), (3.10), and (3.11) constitute the governing equilibrium equations of the membrane theory for shells of revolution. These relations yield $N\phi$, $N\theta$ and $N\phi\theta$. i.e., the membrane force field in the shell. [M. Farshad, 1992].

Note that the meridional and hoop forces $N\phi$, $N\theta$ appear in all three equations. This indicates that a doubly curved shell is a complex and efficient structure; all three forces $N\phi$, $N\theta$ and $N\phi\theta$

contribute to carrying the load in any direction. The spatial interaction of internal forces, manifested in their presence in all equilibrium equations, is indicative of an efficient and profound behavior of doubly curved shells. This spatial collaboration is very rare in framed structures.

3.2.3.3. Rotational Shells with Axisymmetric Loading

In a number of important loading cases, such as the dead weight and internal fluid pressure loading, geometrically complete shells of revolution have axisymmetric behavior.

Axisymmetric behavior is independent of the variable θ. The loading, internal forces and deformations can vary in the ϕ direction.

The membrane behavior of axially symmetric shell subjected to axisymmetric loading is axisymmetric, so that the membrane shear force, $N_{\phi\theta}$, is identically zero; and the directions of principal stresses coincide with the meridional and hoop directions. [M. Farshad, 1992].

The governing equations of axisymmetrically loaded shells of revolution can be easily obtained from the equations (3.9) to (3.11) by setting all derivatives with respect to θ equal to zero:

3.2.4. Spherical Domes

3.2.4.1. Membrane Forces

Consider a constant thickness spherical dome of radius a acted upon by its own dead weight of intensity q. We analyze the shell by the method of sections.

The membrane forces for spherical domes with axisymmetric loading are: [M. Farshad, 1992].

$$N_{\phi} = -\frac{aq(1 - Cos\phi)}{Sin^2\phi} = -\frac{aq}{1 + Cos\phi} \tag{3.12a}$$

$$N_{\theta} = aq\left(\frac{1}{1 + Cos\phi} - Cos\phi\right) \tag{3.12b}$$

Several interesting observations can be made concerning this solution. First, the expression (3.12a) always yields negative values for N_ϕ throughout the shell. Hence, the meridional force in a dome under its own weight is always compressive. Secondly, the hoop force, N_θ is compressive at the top, but changes sign somewhere along the meridian and becomes tensile in the lower part of the shell. N_θ is zero when:

$$\frac{1}{1 + Cos\phi} - Cos\phi = 0 \tag{3.13}$$

The root of this transcendental equation is ϕ = 51° 50’.

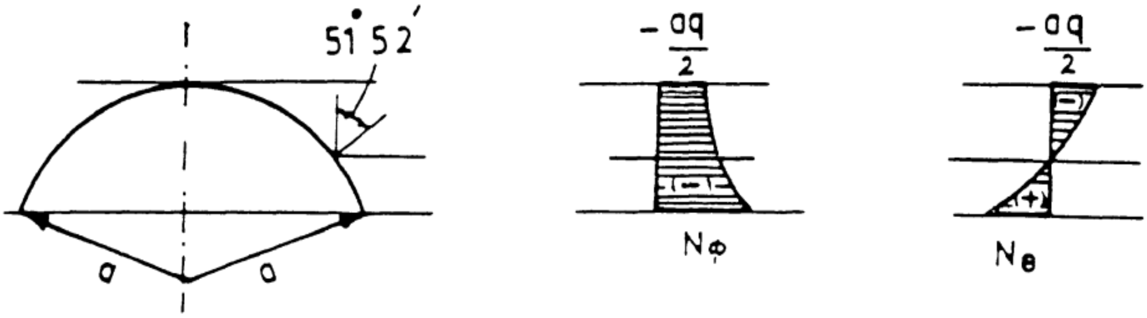


Figure 3.6. A spherical dome under its own weight.

3.2.4.2. Domes with skylight

In some occasions, the top sector of the domes is removed for some purpose. For example, domes can be provided with an open top for natural lighting; these roofs are called domes with skylight. In such cases, the shells are usually provided with a stiffening ring at the top, as well as one at the base, Figure 3.10.

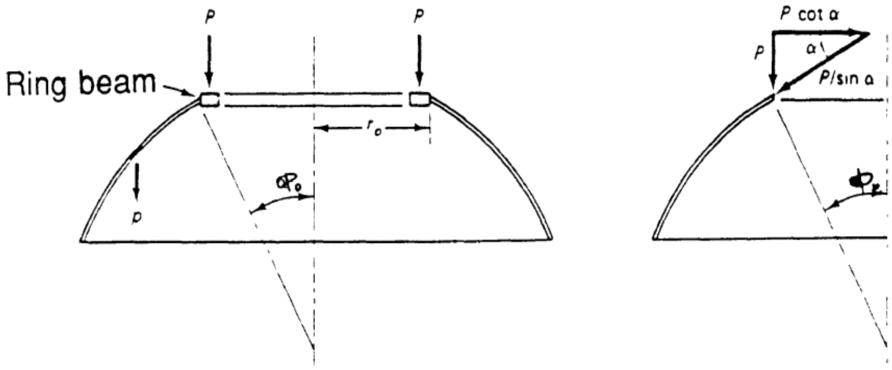


Figure 3.7. A spherical dome with a skylight and a ring at the top.

The ring at the top region acts in compression and is meant to reduce the internal forces in the shell body. The weight of this ring is applied to the shell as a uniformly distributed line loading.

Assume that the weight per unit width of the top stiffening ring of the spherical dome of Figure (3.10) is equal to P. Then, for a dome with skylight we have: [M. Farshad, 1992].

$$N_{\phi} = -aq\frac{Cos\phi_o - Cos\phi}{Sin^2\phi} - P\frac{Sin\phi_o}{Sin^2\phi} \tag{3.14a}$$

$$N_{\theta} = aq\left(\frac{Cos\phi_o - Cos\phi}{Sin^2\phi} - Cos\phi\right) + P\frac{Sin\phi_o}{Sin^2\phi} \tag{3.14b}$$

3.2.5. Displacements of Axisymmetric Shells

The displacement vector in a rotational shell of double curvature generally has meridional, hoop, and normal components. If the applied loading is symmetrical, then the hoop component of the displacement vector is zero. In these truly axisymmetric problems there are only the displacement components along the meridional and normal to the shell to be determined.

Consider an infinitesimal element, AB, taken from the meridional section of the shell. This element is deformed into A'B', as shown in the Figure 3.11. The positive meridional displacement, v, is taken in the direction of increasing ϕ, the positive normal displacement, w, is taken inwards.

The change of length of element AB is composed of two parts: one part arises from the meridional differential displacement, (dv/dϕ) x dϕ; the other from the normal displacement, (w) x dϕ. [M. Farshad, 1992].

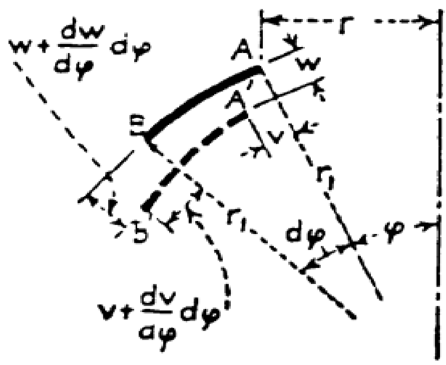


Figure 3.8. A meridional element of the shell and its symmetrically deformed configuration.

The meridional strain is obtained by dividing the above change of length to the undeformed length of the element (r_1 dϕ). So, the expression for meridional strain is:

$$\varepsilon_\phi = \frac{1}{r_1}\frac{dv}{d\phi} - \frac{w}{r_1} \tag{3.15}$$

As we see, the meridional component of strain is also affected by the normal displacement, w, as well as the meridional displacement, v.

The meridional strain is equal to the change of diameter divided by the Original diameter, i.e., (w/r).

To determine the hoop strain, we consider a hoop element of the shell. Figure 3.12 shows the change of radius, Δr, of the parallel circle passing through this element. Referring to this figure we find the following expression:

$$\Delta r = vCos\phi - wSin\phi \quad (+\leftarrow) \tag{3.16}$$

The arrow indicates the assumed positive direction.

Since the circumferential length change is proportional to the change in the radius, so the hoop strain is:

$$\varepsilon_\theta = \frac{1}{r}(vCos\phi - wSin\phi)$$

And since r = r_2 sin ϕ we may write

$$\varepsilon_\theta = \frac{v}{r_2}Cot\phi - \frac{w}{r_2} \tag{3.17}$$

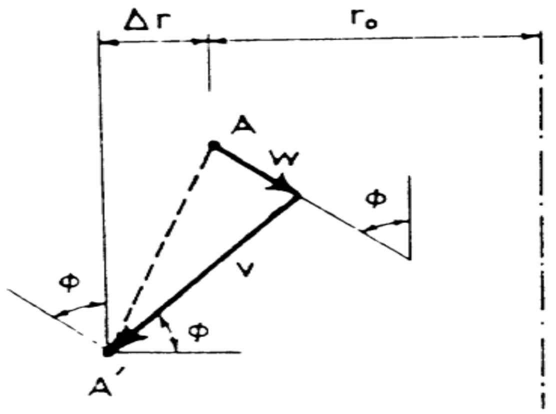


Figure 3.9. Shell displacement components leading to the change of radius of a typical parallel circle.

Expressions (3.15) and (3.17) constitute the strain-displacement relations of a rotational shell undergoing axisymmetric deformation. If we eliminate the normal displacement function, w, between these relations, we obtain the following differential equation for the meridional displacement component v. [M. Farshad, 1992].

$$\frac{dv}{d\phi} - vCot\phi = r_1\varepsilon_\phi - r_2\varepsilon_\theta \tag{3.18}$$

Having obtained the kinematic relations and equilibrium equations we now write down the third group of governing relations, i.e., the constitutive relations. If the shell is linearly elastic and isotropic, the two dimensional elastic constitutive relations, for a local state of plane stress, are: [M. Farshad, 1992].

$$\varepsilon_\phi = \frac{1}{Et}(N_\phi - \nu N_\theta) \tag{3.19a}$$

$$\varepsilon_\theta = \frac{1}{Et}(N_\theta - \nu N_\phi) \tag{3.19b}$$

3.2.5.1. Membrane Deformation of Spherical Domes

Consider the spherical dome of Figure (3.9) subjected to its own weight. The membrane forces in this dome were obtained earlier and are given in relations (3.14). The membrane deformation field in this shell (i.e. the meridional displacement function, v, and the normal component of displacement, w.), are: [M. Farshad, 1992].

$$v = \frac{a^2 q(1+\nu)}{Et} Sin\phi \left[\ln(1+Cos\phi) - \frac{1}{1+Cos\phi}\right] + C\,Sin\phi \qquad (3.20)$$

At ϕ=**α**, we have v=0, so that

$$C = \frac{a^2 q(1+\nu)}{Et}\left[\frac{1}{1+Cos\alpha} - \ln(1+Cos\alpha)\right]$$

With v (ϕ) determined, we can find w (ϕ) from either of the two relations (3.15) or (3.17); the latter gives:

$$w = -a\varepsilon_\theta + vCot\phi = -\frac{a}{Et}(N_\theta - \nu N_\phi) + vCot\phi \qquad (3.21)$$

In particular we may find the horizontal displacement at the base of the dome ΔH (at ϕ=**α**).

$$\Delta H = r\varepsilon_\theta = \frac{a^2 q}{Et}\left(\frac{1+\nu}{1+Cos\alpha} - Cos\alpha\right)Sin\alpha \qquad (3.22)$$

3.2.6. Qualitative Description of Dome Behavior

The membrane field of internal forces in domes consists of a meridional force, a hoop force, and a membrane shear force, Figure 3.13. For axisymmetric loading of domes, the membrane shear is zero throughout and the internal force field consists of meridional and hoop forces only, Figure 3.13. [M. Farshad, 1992].

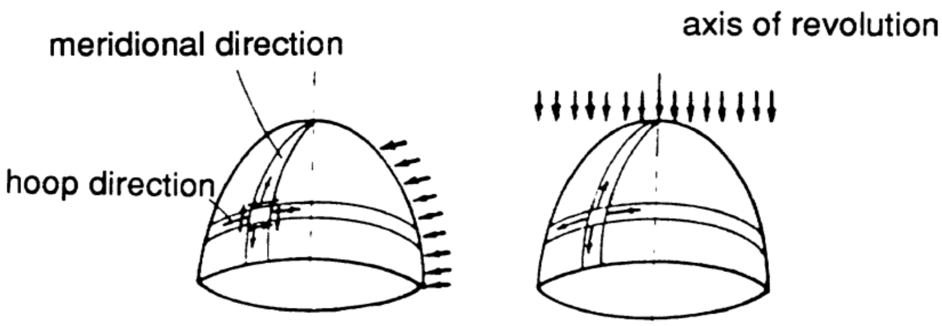


Figure 3.10. State of internal membrane force field in domes.

For axisymmetric loading of domes, the stress trajectories, i.e., the directions of principal normal stresses, will coincide with meridional and hoop curves; the shear stress is identically zero along these directions, Figure 3.14a.

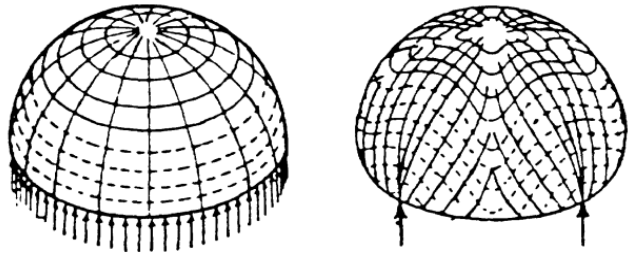


Figure 3.11. Compressive principal stresses (solid lines) and tensile principal stresses (dashed lines) in hemispherical domes under vertical loading; (a) distributed vertical support, (b) four point supports

Figure 3.14a shows the stress trajectories for a symmetrically loaded continuously supported spherical dome. As we have seen before, from our membrane analysis of domes, the meridional force is compressive throughout the shell, while the hoop force has a change of sign from compression to tension. In this figure, the compression field of principal stresses are plotted by solid curves while the tension stress trajectories are sketched by dashed lines.

Figure 3.14b shows the stress trajectories for a spherical dome with four concentrated supports under vertical symmetric loading. This figure reveals the flow of forces towards the supports and the resulting stress concentration near the point supports. A bending field will develop at these supports to compensate for the shortcomings of the membrane behavior.

The structural behavior of domes can be conceived as the interaction of two mechanisms:

- Arch action of the shell along the meridional direction;
- Ring action of the shell in the hoop direction.

The interaction of these two mechanisms gives rise to an efficient spatial behavior of the doubly curved shell.

Sometimes domes are provided with edge supporting and/or stiffening rings. For example, when there is only a vertical support (such as a supporting wall) the horizontal thrust must be absorbed by a ring, Figure 3.15. To stiffen a dome the designer may place a stiffening ring at the intersection of the dome with other structural elements.

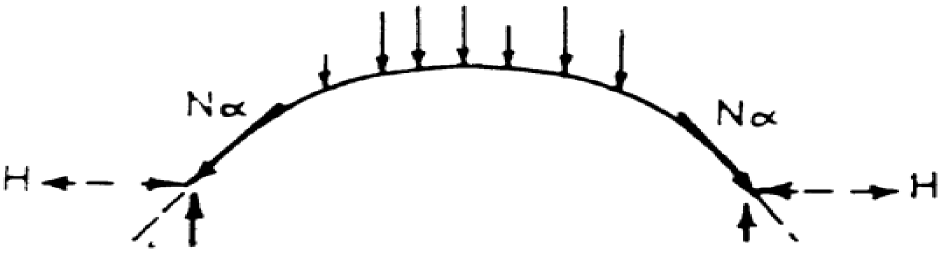


Figure 3.12. Vertical and horizontal edge forces in a dome.

Figure 3.16a shows a dome roof with an edge ring. Figure 3.16b shows a liquid storage tank with a cylindrical wall, a dome roof and a stiffening ring at the intersection of the two shell types.

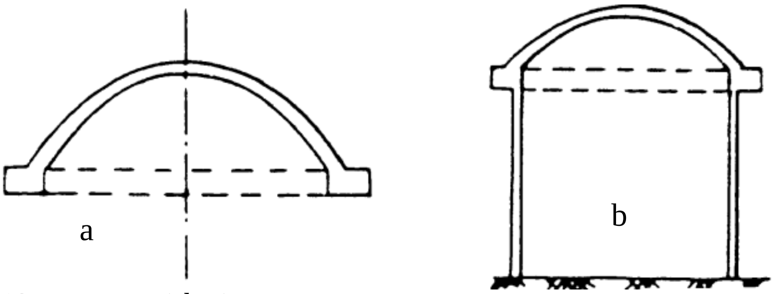


Figure 3.13. Domes with rings.

The overall membrane behavior of domes with or without rings is graphically represented in Figures 3.17. All these domes have distributed supports and are subjected to axisymmetric vertical loading. [M. Farshad, 1992].

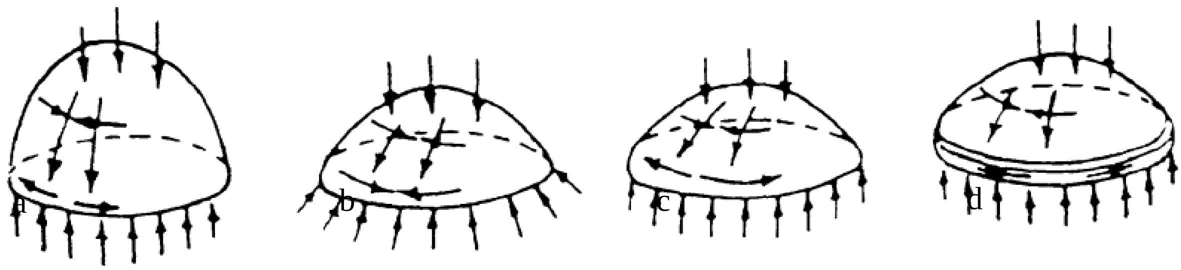


Figure 3.14. Membrane behavior of axisymmetrically loaded domes.

The arrows in Figure (3.17a) show the sign of membrane field in a high rise dome with no ring and only a vertical support. The tangent to the meridional curve at the lower edge is assumed to be vertical. The meridional force remains compressive, but the hoop stress changes sign. In this shell, the hoop tension is taken by the shell itself and the edge meridional force is carried by the vertical support.

The membrane field (meridian and hoop stresses) in the low rise shell of Figure (3.17b) is totally compressive. At the edges of this shell, the inclined meridional force is carried through the support, which is assumed to sustain vertical as well as lateral thrusting forces. The equilibrium requirements of membrane behavior are satisfied for this shell.

The support of low rise shell of Figure (3.17c) can only carry vertical forces. Therefore, the horizontal thrust developed by meridional compression must be carried through the shell itself mainly by the mechanism of hoop action. Some tension will be induced in the lower parallel circles, as demonstrated in Figure (3.17c). This is obviously contrary to the predictions of membrane theory; there must be some bending field in the lower part of this shell to satisfy the equilibrium requirements.

The low rise shell of Figure (3.17d) has a supporting ring at the edge together with a vertical support; the horizontal thrust is totally carried by the ring.

3.2.7. Bending Analysis of Axisymmetric Shells

3.2.7.1. Governing Equations for Axisymmetric Shells

3.2.7.1.1. Equilibrium Equations

Consider a shell of revolution subjected to axisymmetric loading. Figure (3.18) shows the free-body diagram of an element of this shell. Figure (3.18a) shows the membrane forces and the applied distributed loading while the complementary Figure (3.18b) demonstrates the bending force field developed in this shell element. Due to axial symmetry of geometrical and loading conditions, all variables involved are independent of the hoop parameter, q.

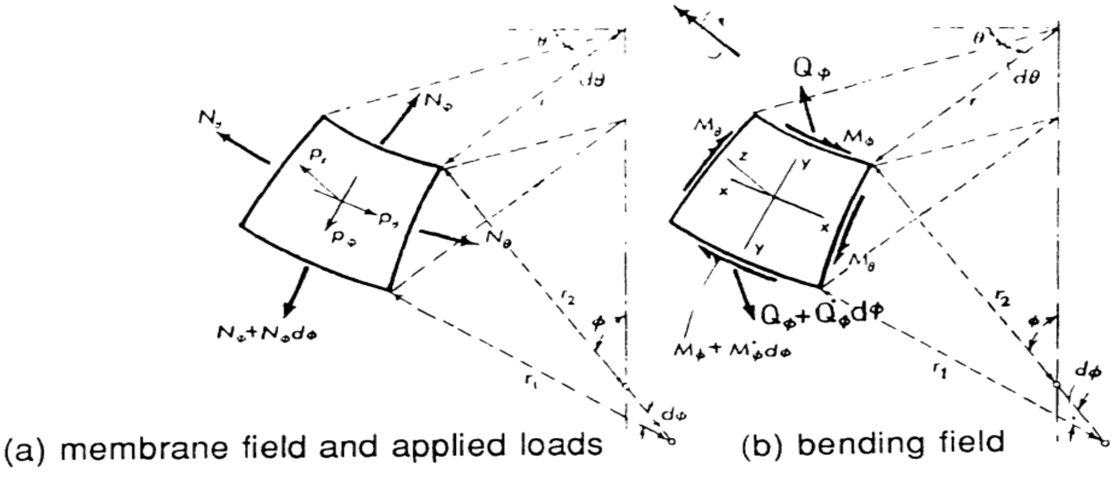


Figure 3.15. Free body diagram of a rotational shell with axisymmetric loading.

The equations of equilibrium consist of three force equations and three moment relations. Due to axisymmetric conditions, and assuming the applied load in θ direction, P_θ, to be zero, the force equation of equilibrium along the θ direction is satisfied identically as are the moment equations of equilibrium about the r and ϕ directions. We are left with three equations of equilibrium which we will now write down.

Because the shell has double curvature, the internal forces have projections in all directions. For example, when writing down the equilibrium of forces in the ϕ direction we should take into consideration the contribution of the shear force, Qϕ, as well as the membrane forces, Nϕ and Nθ. Figure (3.19) shows the projections of membrane and the bending shear force in the ϕ and r directions. [M. Farshad, 1992].

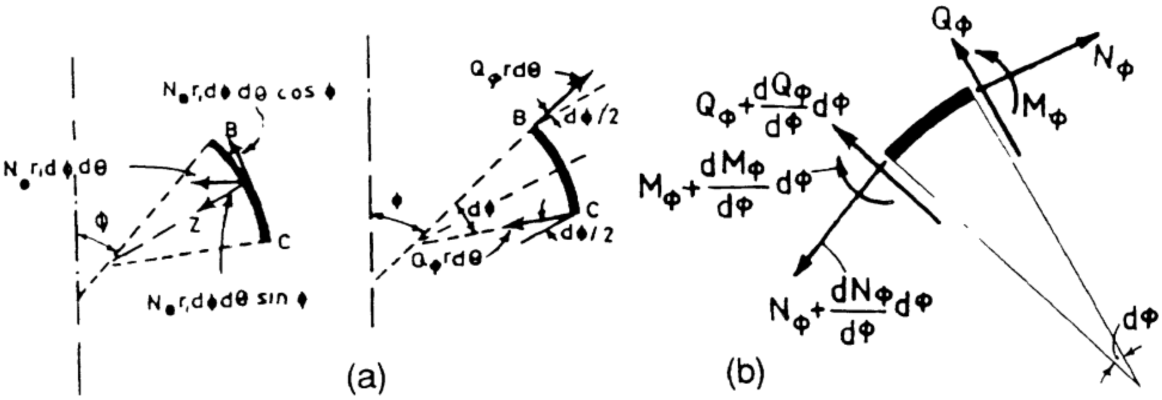


Figure 3.16. Meridional section through a shell element showing the internal forces and their projections, (a) membrane forces, (b) bending shear force.

The force equations of equilibrium are, with the help of Figure (3.19), written as follows:

$$\frac{d}{d\phi}\left(N_{\phi}r\right) - N_{\Theta}r_1 Cos\phi - rQ_{\phi} + rr_1P_{\phi} = 0 \qquad (3.23)$$

$$N_{\phi}r - N_{\Theta}r_1 Sin\phi - \frac{d(Q_{\phi}r)}{d\phi} - rr_1P_r = 0 \qquad (3.24)$$

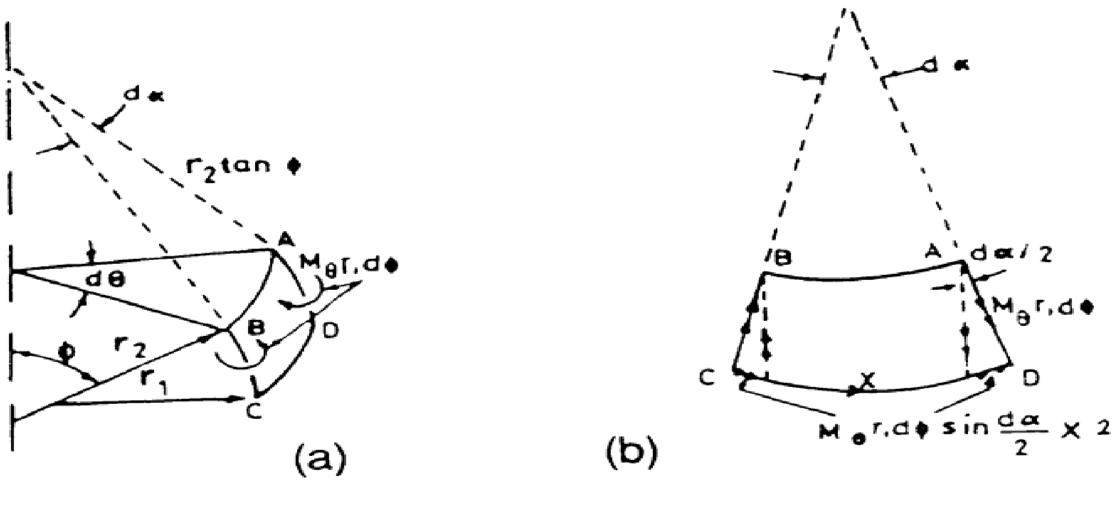


Figure 3.17. A rotationally symmetric shell element with geometrical parameters and hoop bending moment.

To derive the moment equation of equilibrium about the θ axis, we make use of Figure (3.20). This figure shows the spatial configuration of shell element and the projections of bending as well as twisting couples. Some useful relations among the geometrical parameters represented in this figure are:

$$r = r_2 Sin\phi$$

$$AB = rd\theta = r_2 Sin\phi d\theta$$

$$AB = r_2 tan\phi d\alpha$$

$$d\alpha = Cos\phi d\theta$$

The moment equation of equilibrium for the shell element about the θ axis is:

$$\left(M_\phi + \frac{dM_\phi}{d\phi} d\phi\right)\left(r + \frac{dr}{d\phi} d\phi\right) d\theta - M_\phi r d\theta - M_\theta r_1 Cos\theta d\phi d\theta - Q_\phi r_2 Sin\phi r_1 d\phi d\theta = 0$$

Which can be simplified to:

$$\frac{d}{d\phi}(M_\phi r) - M_\theta r_1 Cos\phi - Q_\phi r_1 r = 0 \qquad (3.25)$$

Equations (3.23), (3.24), and (3.25) constitute three relations among six unknown force quantities $N\phi$, $N\theta$, $M\phi$, $M\theta$ and $Q\phi$. This means that the shell is statically indeterminate and three more relations are needed to find the internal forces. These additional relations are provided by the kinematic and constitutive equations.

3.2.7.2. Influence Coefficients for Axisymmetric Shells

As a useful by-product of this analysis, we now obtain the flexibility influence coefficients of axisymmetric shells, i.e., the displacements due to unit edge forces.

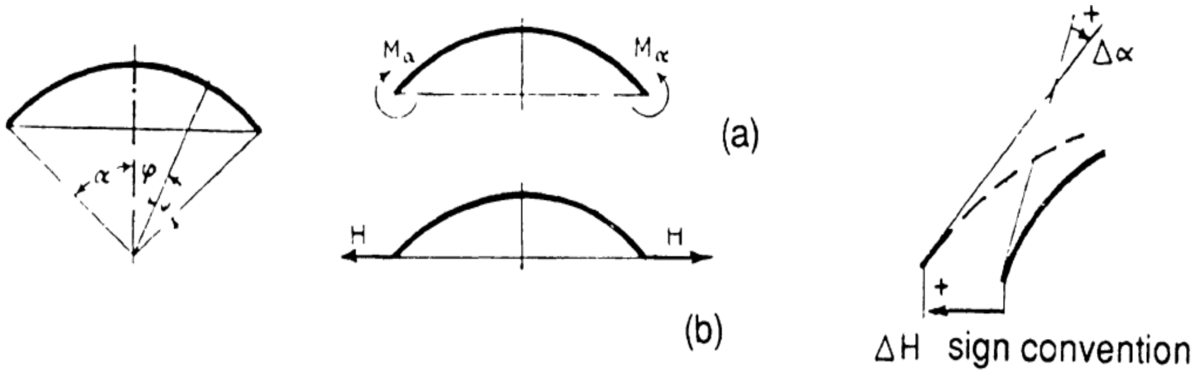


Figure 3.18. Axisymmetric shell under separate application of edge forces, (a) shear force, (b) bending moment

Consider Figure (3.26b) in which the shell is acted upon by a uniformly distributed edge moment M**α**. The boundary conditions are:

$$(M_\phi)_{\phi=\alpha} = M_\alpha \tag{3.26a}$$

$$(N_\phi)_{\phi=\alpha} = 0 \tag{3.26b}$$

We can write down the expressions for internal edge forces and edge displacements due to the edge moment M**α**. These values are tabulated in the third column of table (3-1). Specifically, for M**α** = 1.0 we shall get the bending moment flexibility influence coefficients.

As another basic solution, we consider the shell of Figure (3.26c); the edge conditions for this shell are:

$$(M_\phi)_{\phi=\alpha} = 0 \tag{3.27a}$$

$$(N_\phi)_{\phi=\alpha} = -H\ Cos\alpha \tag{3.27b}$$

We can write the expressions for internal edge forces and edge displacements due to a distributed edge shear force, H. These results are tabulated in the second column of table (3-1). Again, for H = 1.0, these expressions give the flexibility influence coefficients of the shell due to a unit edge shear force. [M. Farshad, 1992].

Table 3.1. Flexibility influence coefficients for axisymmetric shells.

		
N_ϕ	$-\sqrt{2}\ \mathrm{Cot}\ (\alpha-\psi)\mathrm{Sin}\ \alpha\ e^{-\lambda\psi}\ \mathrm{Sin}\left(\lambda\psi-\frac{\pi}{4}\right)\mathrm{H}$	$-\frac{2\lambda}{a}\mathrm{Cot}\ (\alpha-\psi)\ e^{-\lambda\psi}\mathrm{Sin}(\lambda\psi)\mathrm{M}_\alpha$
N_θ	$-2\lambda\ \mathrm{Sin}\ \alpha\ e^{-\lambda\psi}\ \mathrm{Sin}\left(\lambda\psi-\frac{\pi}{2}\right)\mathrm{H}$	$-\frac{2\sqrt{2}}{a}\lambda^2 e^{-\lambda\psi}\mathrm{Sin}\left(\lambda\psi-\frac{\pi}{4}\right)\mathrm{M}_\alpha$
M_ϕ	$\frac{a}{\lambda}\mathrm{Sin}\ \alpha\ e^{-\lambda\psi}\ \mathrm{Sin}\lambda\psi\ \mathrm{H}$	$\sqrt{2}e^{-\lambda\psi}\mathrm{Sin}\left(\lambda\psi+\frac{\pi}{4}\right)\mathrm{M}_\alpha$
ΔH	$\frac{2a\lambda Sin^2\alpha}{Et}H$	$\frac{2\lambda^2 Sin\alpha}{Et}\mathrm{M}_\alpha$
$\Delta\alpha$	$\frac{2\lambda^2 Sin\alpha}{Et}\mathrm{H}$	$\frac{4\lambda^3\mathrm{M}_\alpha}{Eat}$

3.3. Reinforced Concrete Domes

3.3.1. General Features of Domes

Reinforced concrete domes are used to cover large spans of stadiums, memorial buildings, meeting halls, and other large assembly halls. They are also used to cover the roofs of liquid retaining structures, silos, as well as the roofs of containment shells of nuclear power plants.

A dome is often provided with an edge ring at its lower edge and / or with a ring somewhere along its parallel circles.

Edge rings stiffen the shell and / or provide lateral support for the shell structure. The lateral support action of the rings is specially needed in cases where there are only vertical supports and thus the lateral thrusts are to be absorbed by the structure itself. For combined shells, the stiffening ring between two shells acts as a strengthening member which absorbs part of the bending field created by the curvature change from one shell to the other.

Edge beams in a shell structure create some bending field in the vicinity of the ring. This is due to the difference in stiffness between the shell and the ring and the ensuing violation of the membrane assumptions. [M. Farshad, 1992].

From the structural analysis point of view, a force field composed of shear force and bending moment as well as membrane forces would exist between the shell and its edge beam. The magnitudes of bending effects would be such that the deformation compatibility requirements are satisfied. These forces of dome-ring interaction are shown in Figure 3.27.

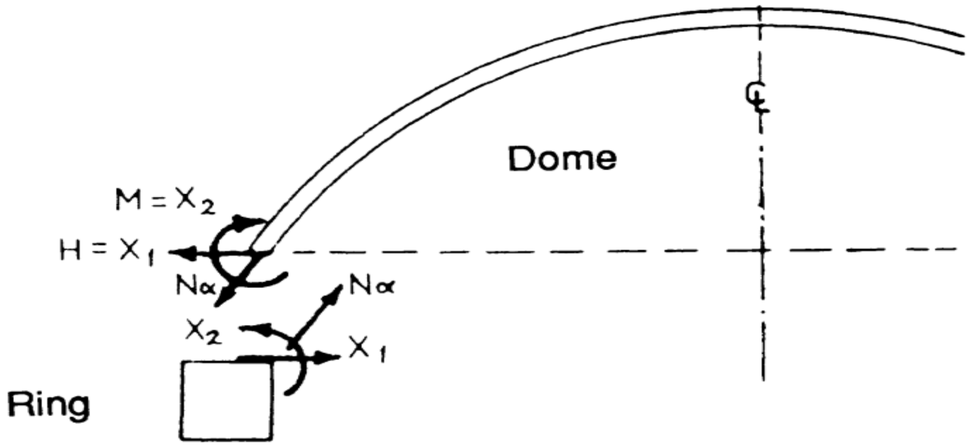


Figure 3.19. Interaction between an axisymmetric shell and its edge ring.

3.3.2. Force Method of "Dome-Ring" Analysis

3.3.2.1. General Methodology

The general force method of structural analysis can be used in the analysis of domes with rings and also domes with cylindrical walls. The ring in a dome acts as a tie capable of absorbing the horizontal forces; it is a deformable body integral with the shell and must be analyzed along with the shell.

This analysis can be applied to a shell with a ring. Figure 3.28a shows a dome with a ring. Figure 3.28b depicts the same dome without the ring, acting as a (statically determinate) membrane shell. The membrane deformations in the dome and the deformations in the ring, due to membrane forces, are also defined alongside this figure. Figure 3.28c demonstrates the edge forces and corresponding flexibility influence coefficients related to the dome. Finally, Figure 3.28d shows the ring together with related influence coefficients. In all these figures, $D^{D}ij$ and $D^{R}ij$ refer to influence coefficients related to dome and ring, respectively. The corresponding membrane deformations are denoted by $D^{D}io$ and $D^{R}io$.

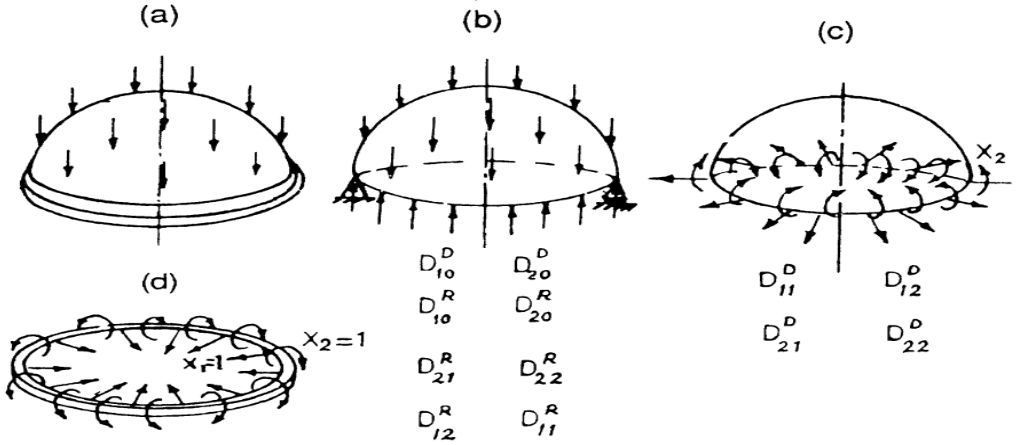


Figure 3.20. Ingredients of force method of "dome-ring" analysis.

Figure 3.29 shows the details of the decomposition of the dome and its related deformation parameters. Figure 3.30 demonstrates the decomposition scheme of ring analysis as well as the deformation parameters to be calculated in the course of the ring analysis.

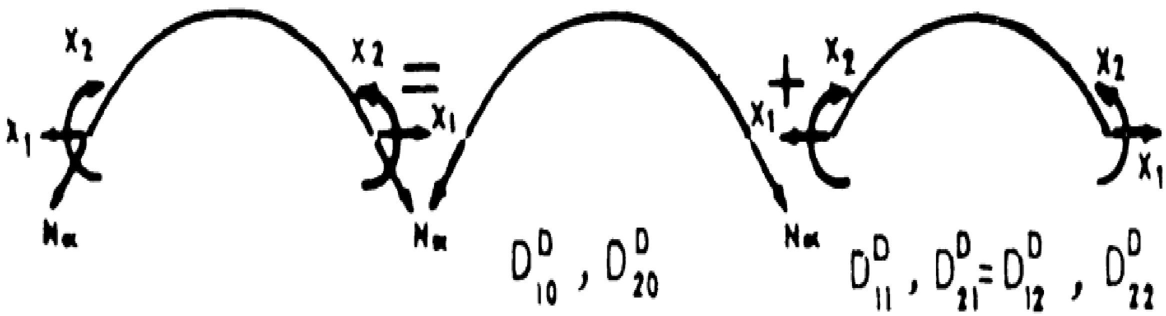


Figure 3.21. Decomposition of internal forces in the dome into membrane and bending fields.

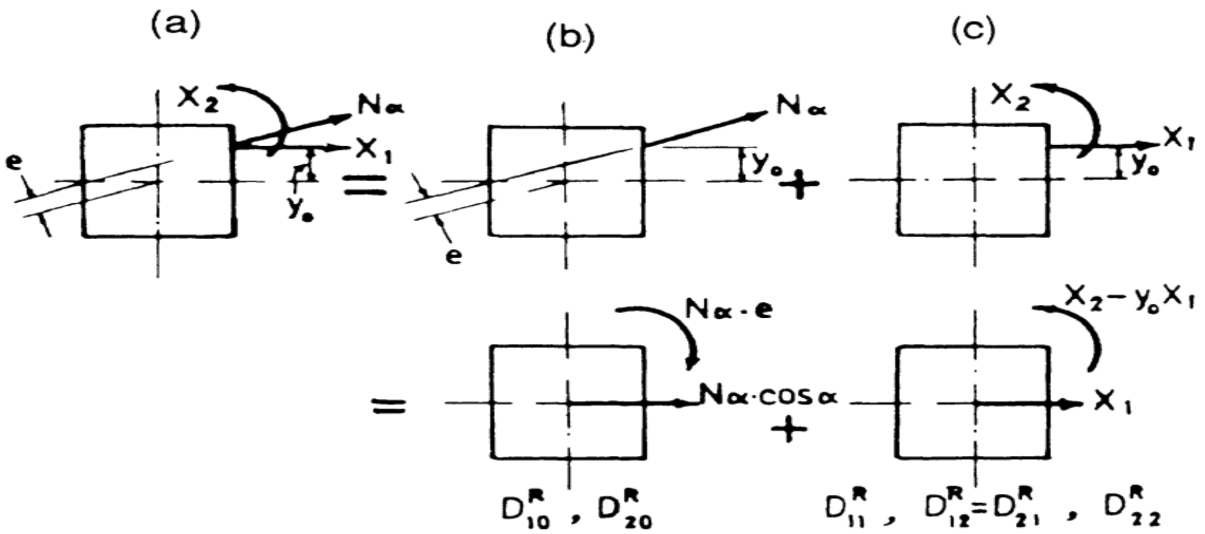


Figure 3.22. Decomposition of internal forces in the ring and their related deformations.

3.3.3. Analysis Procedures

Based on the foregoing discussion, we may now state the stages of any “dome-ring" analysis problem as follows: [M. Farshad, 1992].

- Analysis of the ring under hoop force, unit radial force, and unit torsional couple.
- Membrane analysis of dome for distributed forces as well as bending analysis of dome for unit value of edge effects.
- Matching of the dome and ring deformations by imposition of compatibility relations. Determination of unknown "dome-ring" interaction forces from these relations.
- Superposition of membrane and bending effects to find the total force and deformation in the dome-ring structure.

3.3.3.1. Analysis of the Ring

Consider a linearly elastic circular ring of internal radius r and rectangular section b x h. The ring is subjected to a uniformly distributed radial force, H, and a uniformly distributed twisting couple, M**α**. Figure 3.31 shows the free-body diagrams of this ring segment.

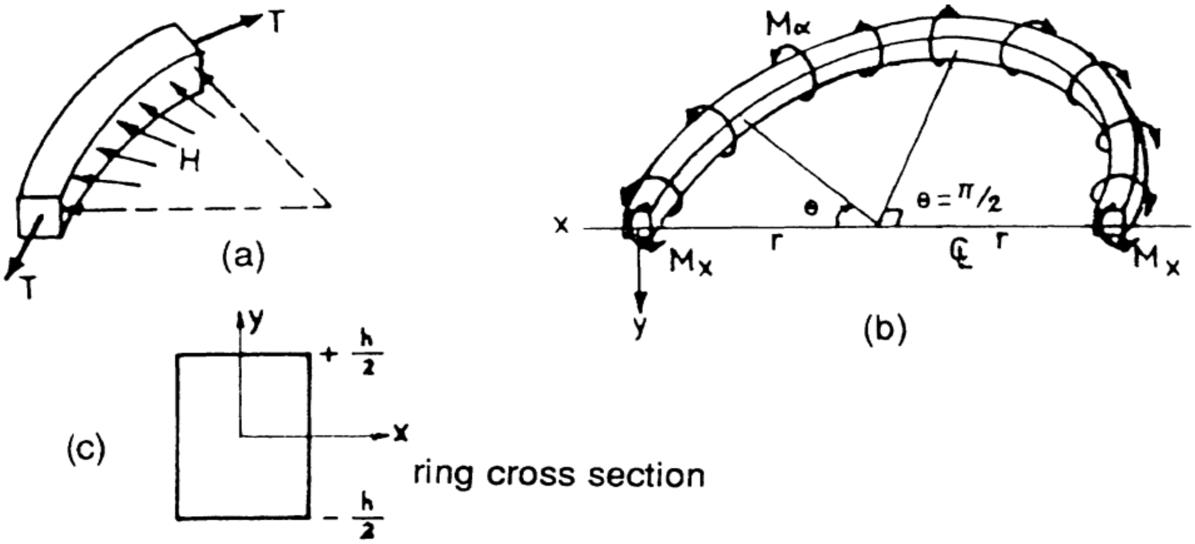


Figure 3.23. Free-body diagrams of a ring segment under radial force and twisting couple.

Figure 3.31a shows that the hoop force and the hoop stress are:

$$T = H.r \qquad , \qquad \sigma_\theta = \frac{T}{A_R} \tag{3.28}$$

Now consider the free body diagram of half ring shown in Figure 3.31b. The equation of moment equilibrium about the x-axis gives:

$$M_x = M_\alpha.r[Sin\theta]_0^{\pi/2} = M_\alpha.r \tag{3.29}$$

Figure 3.32a shows the deformation of a section of this ring under the action of twisting couple M**α**.

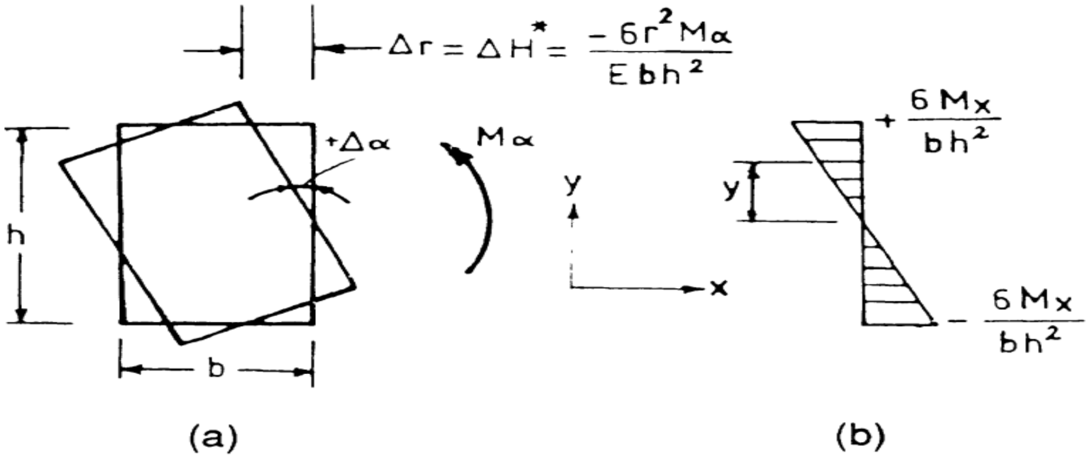


Figure 3.24. Torsional-bending deformation of the ring.

Referring to Figure 3.32a, and using the classical formula for bending of beams, the relations for the radius change and cross-sectional rotation of the ring under the uniformly distributed radial force H and twisting couple M**α** are: [M. Farshad, 1992].

$$\Delta H = \frac{-r^2}{Ebh} H \quad , \quad \Delta r = \frac{-12r^2 Y}{Ebh^3} M_\alpha \quad , \quad \Delta \alpha = \frac{12r^2}{Ebh^3} M_\alpha \qquad (3.30)$$

3.3.3.2. Analysis of Domes under Edge and Distributed Forces

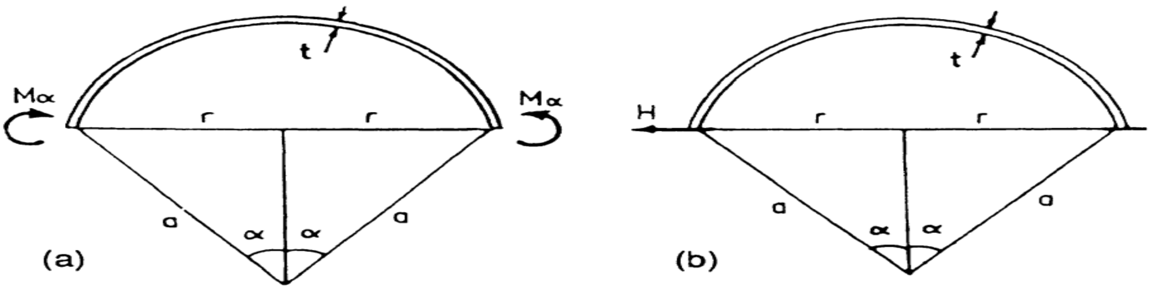


Figure 3.25. Dome subjected to uniformly distributed edge forces.

Figure 3.33 shows the dome under edge forces.

The related influence coefficients are: [M. Farshad, 1992].

$$D_{11}^D = \frac{+2a\lambda Sin^2\alpha}{Et} \qquad D_{12}^D = \frac{+2\lambda^2 Sin\alpha}{Et}$$

$$D_{21}^D = \frac{+2\lambda^2 Sin\alpha}{Et} \qquad D_{22}^D = \frac{4\lambda^3}{Eat}$$

$$\lambda^4 = 3(1-\nu^2)(\frac{a}{t})^2$$

3.3.3.3. "Dome-Ring" Interaction

Figure 3.34a shows part of a "dome-ring" structure resting on a vertical support. Figure 3.34b shows the forces of interaction between the dome and the ring.

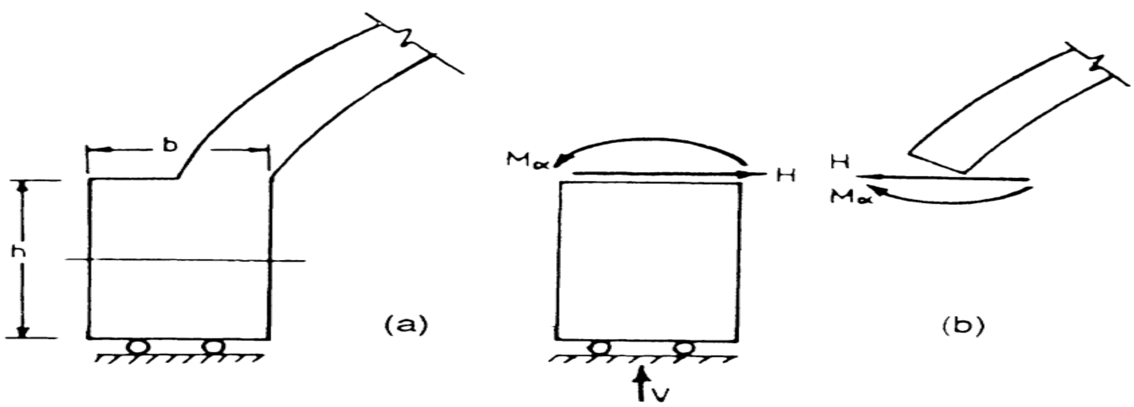


Figure 3.26. Bending forces of "dome-ring" interaction.

Figure 3.35a shows the membrane field of "dome-ring" interaction. The deformations caused by these sort of interaction and the adopted sign convention are shown in Figures 3.35b and 3.35c, respective1y.

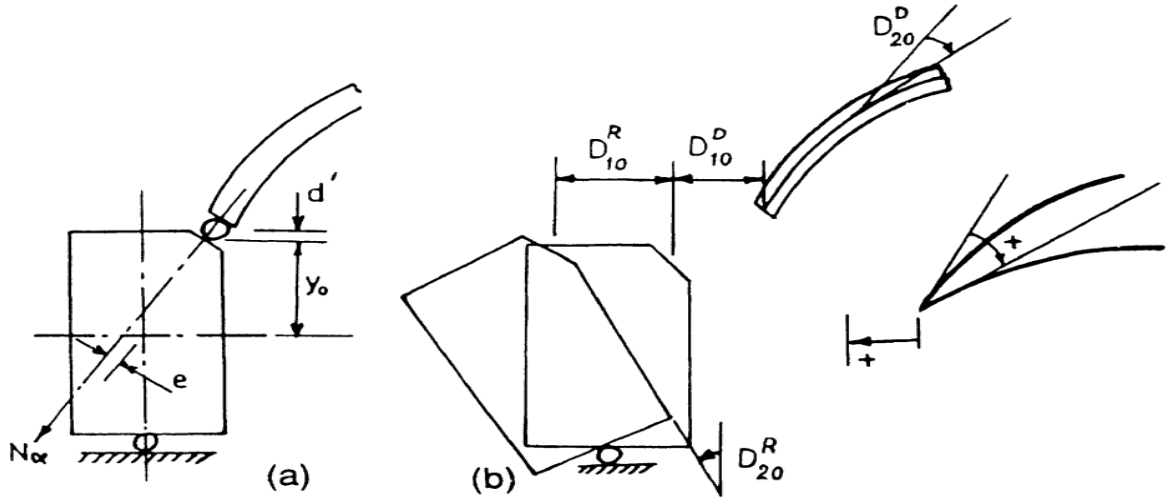


Figure 3.27. Membrane "dome-ring" interaction, (a) membrane meridional force, (b) membrane ring and dome deformations.

In considering the “dome-ring" interaction, we use the spherical dome approximation. In what follows, we assume that the spherical dome has a radius a, and a half central angle of **α**.

Figure 3.35a shows that the components of meridional force at the base of the shell are:

$$H_O = N_\alpha Cos\alpha \qquad V_O = N_\alpha Sin\alpha \tag{3.31}$$

The vertical component is absorbed by the vertical support while the horizontal component is taken by the ring. The radial displacement of the ring due to this horizontal component is,

$$\Delta_{OH} = \frac{r^2}{EA_R} H_O = \frac{r^2}{EA_R} N_\alpha Cos\alpha \tag{3.32}$$

In the general case, the meridional force acts on the ring section with an eccentricity. Thus, assuming an eccentricity of e, we find that the torsional couple, induced by the membrane force, acting on the ring is $M_{o\alpha} = N_\alpha$ e. The radial displacement of the ring due to this couple, derived in the previous section, is:

$$\frac{r^2 Y}{EI_R} M_{o\alpha} = \frac{r^2 Y}{EI_R} N_\alpha e \tag{3.33}$$

We seek the radial displacement of the ring at the "dome-ring" junction. At this point, we have

$$Y_O = \frac{h}{2} - d'$$

In which

$$d' = \frac{t}{2} Cos\alpha$$

Since d' is usually very small, we may use the approximation Y_0 = h / 2. Therefore, the total radial displacement of the ring is: [M. Farshad, 1992].

$$\Delta_H^R = D_{10}^R = (\frac{r^2}{EA_{R_\alpha}} Cos\alpha + \frac{r^2 Y_O e}{EI_R}) N_\alpha \tag{3.34}$$

$$\Delta_\alpha^R = D_{20}^R = -\frac{r^2 e}{EI_R} N_\alpha \circlearrowleft \tag{3.35}$$

For a ring with rectangular cross section, the above relations become:

$$D_{10}^R = (Cos\alpha + \frac{12 Y_O e}{h^2}) \frac{r^2 N_\alpha}{Ebh} \tag{3.36}$$

$$D_{20}^R = -\frac{12 r^2 e}{Ebh^3} N_\alpha \circlearrowleft \tag{3.37}$$

For example, if a spherical dome is acted upon by uniform dead weight of intensity q. then as we know:

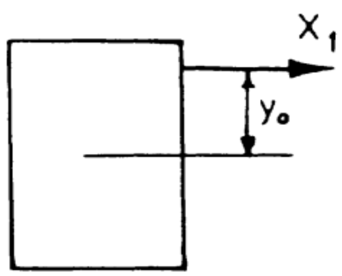
$$N_\alpha = \frac{-qa}{1 + Cos\alpha}$$

Then we would have:

$$D_{10}^D = +\frac{a^2 q}{Et} \left(\frac{1+\nu}{1 + Cos\alpha} - Cos\alpha \right) Sin\alpha \tag{3.38a}$$

$$D_{20}^D = +\frac{aq}{Et} (2+\nu) Sin\alpha \qquad \circlearrowright \tag{3.38b}$$

As another step in "dome-ring" interaction analysis, we subject the dome and the ring to the edge forces H = X_1 and M**α** = X_2, separately. Figure 3.36 shows that the ring deformation due to force X1, applied at Y_0, is: [M. Farshad, 1992].



$$\Delta_1 H = \frac{r^2}{EA_R} X_1$$

$$\Delta_2 H = \frac{r^2 y_O^2}{EI_R} X_1 \qquad (e = Y_O)$$

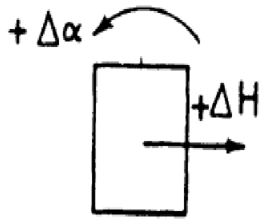
$$\Delta_\alpha = -\frac{r^2 Y_O}{EI_R} X_1$$

Figure 3.28. Edge force applied to the ring-beam.

The ring deformation due to a torsional couple X2 is: [M. Farshad, 1992].

$$\Delta H = -\frac{r^2 Y_O}{EI_R} X_2 \qquad\qquad \Delta_\alpha = \frac{r^2}{EI_R} X_2$$

Therefore the ring influence coefficients, i.e., the ring deformation for unit radial force and unit twisting couple, observing the sign convention of Figure 3.37, are:



$$D_{11}^R = (1 + \frac{12Y_O^2}{h^2})\frac{r^2}{Ebh} \tag{3.39}$$

$$D_{12}^R = -\frac{12r^2 Y_O}{Ebh^3} = D_{21}^R \tag{3.40}$$

$$D_{22}^R = \frac{12r^2}{Ebh^3} \tag{3.41}$$

Figure 3.29. Sign conventions for ring deformation.

At this stage, we are prepared to combine the influence coefficients of the dome and the ring to determine the influence coefficients for the "dome-ring" system. The system influence coefficients are:

$$D_{11} = D_{11}^D + D_{11}^R$$

$$D_{12} = D_{12}^D + D_{12}^R = D_{21} \tag{3.42}$$

$$D_{22} = D_{22}^D + D_{22}^R$$

This completes the "dome-ring" interaction analysis.

3.3.3.4. Summary of "Dome-Ring" Analysis Relations

In the following relations, we adopt the sign conventions shown in Figure 3.38.

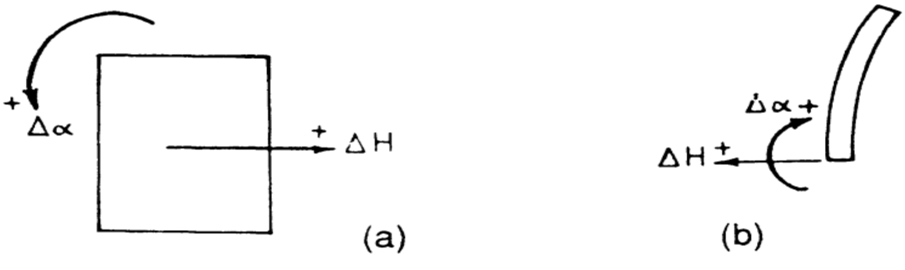


Figure 3.30. Sign conventions, (a) for the ring, (b) for the dome.

Membrane deformation field

- **The dome**

$$\Delta_{OH}^D = D_{10}^D = \frac{r_2 Sin\phi}{Et}(N_\theta - \nu N_\phi) \tag{3.43}$$

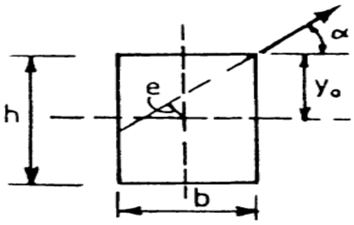
$$\Delta_{O\alpha}^{D} = D_{20}^{D} = \frac{1}{Et}[N_{\phi}(r_1 + r_2) - N_{\theta}(r_2 + \nu r_1)] \quad (3.44)$$

For spherical rings with radius $r_1 = r_2 = a$,

$$\Delta_{OH}^{D} = D_{10}^{D} = \frac{a^2 q}{Et}\left(\frac{1+\nu}{1+Cos\phi} - Cos\phi\right)Sin\phi \quad (3.45)$$

$$\Delta_{O\alpha}^{D} = D_{20}^{D} = -\frac{aq}{Et}(2+\nu)Sin\phi \quad (3.46)$$

- **The ring, Figure 3.39:**



$$D_{10}^{R} = \left(Cos\alpha + \frac{12Y_O e}{h^2}\right)\frac{r^2 N_\alpha}{Ebh} \quad (3.47)$$

$$D_{20}^{R} = -\frac{12r^2 e}{Ebh^3}N_\alpha \quad (3.48)$$

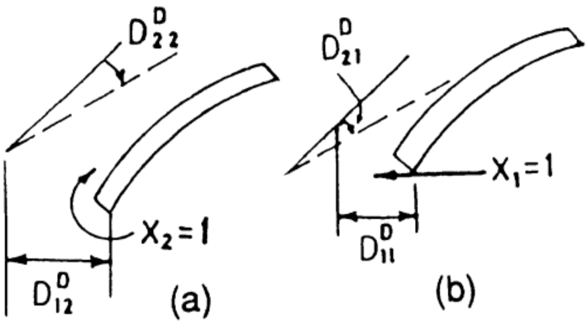
Figure 3.31. Eccentrically applied membrane force to the ring

For a spherical dome acted upon by uniform dead weight of intensity q:

$$N_\alpha = \frac{-qa}{1+Cos\alpha} \quad (3.49)$$

Bending field - The influence coefficients

- **The dome, Figure 3.40:**



$$D_{11}^{D} = \frac{+2a\lambda Sin^2\alpha}{Et} \quad (3.50)$$

$$D_{21}^{D} = D_{12}^{D} = \frac{+2\lambda^2 Sin\alpha}{Et} \quad (3.51)$$

$$D_{22}^{D} = \frac{4\lambda^3}{Eat} \quad (3.52)$$

$$\lambda^4 = 3(1-\nu^2)\left(\frac{a}{t}\right)^2$$

Figure 3.32. Positive sign convention far the influence coefficients of the dome.

- **The ring, Figure 3.41:**

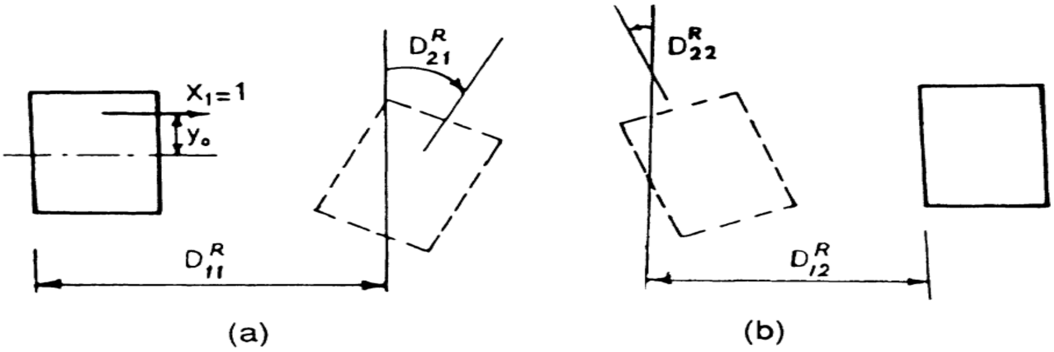


Figure 3.33. Positive sign convention for the ring influence coefficients.

$$D^R_{11} = (1 + \frac{12Y_O^2}{h^2})\frac{r^2}{Ebh} \tag{3.53}$$

$$D^R_{12} = -\frac{12r^2Y_O}{Ebh^3} = D^R_{21} \tag{3.54}$$

$$D^R_{22} = \frac{12r^2}{Ebh^3} \tag{3.55}$$

3.3.3.5. Application of the Force Method

Having obtained all necessary influence coefficients and membrane deformations, we are now prepared to apply the final relations of the force method to the "dome-ring" system. We must satisfy the compatibility relations which express the continuity of radial displacement and rotation at the "dome-ring" junction. These are:

$$(D^D_{10} + D^D_{11}X_1 + D^D_{12}X_2) = -(D^R_{10} + D^R_{11}X_1 + D^R_{12}X_2) \tag{3.56a}$$

$$(D^D_{20} + D^D_{21}X_1 + D^D_{22}X_2) = -(D^R_{20} + D^R_{21}X_1 + D^R_{22}X_2) \tag{3.56b}$$

Using the parameters defined in relations (3.99) we write the compatibility relations as:

$$D_{11}X_1 + D_{12}X_2 + D_{10} = 0 \qquad D_{10} = D^D_{10} + D^R_{10} \tag{3.57a}$$

$$D_{12}X_1 + D_{22}X_2 + D_{20} = 0 \qquad D_{20} = D^D_{20} + D^R_{20} \tag{3.57b}$$

By solving these linear simultaneous algebraic equations, we determine the two unknown redundant forces X1 and X2; they are:

$$X_1 = H = -\frac{D_{22}D_{10} - D_{12}D_{20}}{D_{22}D_{11} - D^2_{12}} \tag{3.58a}$$

$$X_2 = M_\alpha = -\frac{D_{11}D_{20} - D_{12}D_{10}}{D_{22}D_{11} - D_{12}^2} \tag{3.58b}$$

The final step in the "dome-ring" problem is to combine the bending field induced by these forces with the membrane field.

3.3.4. Buckling of Concrete Domes

Domes are doubly curved, synclastic, and non-developable surfaces. Therefore, they are generally very strong and highly stable. The critical stability loads of concrete domes are usually much higher than those of concrete shells with single curvature. Nevertheless, thin concrete domes with large spans are susceptible to buckling; indeed the buckling considerations are one of the main design criteria of such shells.

The theoretical buckling load for a doubly curved elastic shell having the values of principal curvature 1/R1 and 1/R2, under the dead load, is: [D.P. Billington, 1982].

$$q_{cr} = \frac{2Et^2}{\sqrt{3(1-\nu^2)}}\frac{1}{R_1}\frac{1}{R_2} \tag{3.59}$$

In this relation, q_{cr} is the intensity of buckling dead load, E and ν are the Young's modulus and Poisson's ratio, respectively.

For a spherical shell, in which R1 = R2 = a, relation (3.116) yields:

$$q_{cr} = \frac{2E}{\sqrt{3(1-\nu^2)}}(\frac{t}{a})^2 = \alpha E(\frac{t}{a})^2 \quad , \quad \alpha = \frac{2}{\sqrt{3(1-\nu^2)}} \tag{3.60}$$

Experimental investigations yield buckling loads much less than the theoretical ones given here. This is due to imperfection sensitivity of shells which dramatically affects the stability behavior of shells. On this basis, some building codes recommend values of a reduction parameter for design purposes which lies in the region 0.05 and 0.1.

Based on these considerations, a more practical formula for the buckling strength of shells with double curvature would be

$$q_{cr} = 0.05E\frac{t^2}{R_1R_2}$$

Which, for a spherical shell of radius a, yields

$$q_{cr} = 0.05E(\frac{t}{a})^2$$

3.4. Design of Reinforced Concrete Domes

3.4.1. Preliminary Design Dimensions of Spherical Domes

When spherical shells under normal loading conditions are considered, experience suggests that they have some relationships between span and height and also between span and thickness that make them more economical than other domes.

A meaningful value for evaluating the efficiency of thin shells is the relationship between the thickness and the span of the structure. If t is the thickness of the shell in a certain unit and L the span in the same unit, then: t = K X L where K is a coefficient of proportionality. The smaller K is, the more efficient the shell is, because it uses less material to perform an equivalent task. [M. Melaragno, 1991].

Considering the egg as a thin-shelled structure produced by nature, then for an average egg the relationship above becomes:

$$t = \frac{1}{230} L$$

This value should be kept in mind when designing man-made thin-shelled structures. The recommended values for various spans for dome height and thickness are shown in Table 3.2. [M. Melaragno, 1991].

Table 3.2. Recommended dimensions for dome and ring-beam with span of the dome.

Span (m)	Minimum Height (m)	Maximum Height (m)	Thickness (cm)	Depth of Ring-Beam (cm)	Width of Ring-Beam (cm)
30.48	3.35	4.11	10.8	30.5	25.4
60.96	8.53	10.36	9.5	45.7	30.5
91.44	15.09	17.83	10.1	50.8	40.6
121.92	23.16	25.6	15.2	61	45.7

3.4.2. Reinforcement for Spherical Domes

Shell reinforcement shall be provided: [ACI 318M-05, 2005].

- To resist tensile stresses from internal membrane forces,
- To resist tension from bending and twisting moments,
- To limit shrinkage and temperature crack width and spacing, and

- As special reinforcement at shell boundaries, load attachments, and shell openings.

Area of Shell Reinforcement

The area of shell tension reinforcement shall be limited so that the reinforcement will yield before either crushing of concrete in compression or shell buckling can take place.

For a design tension force, Nsd, the area of reinforcement required is calculated using:

$$A_S = \frac{N_{sd} b}{f_{yd}} \tag{3.61}$$

Where: b is the design width, (b =1 m), and fyd is the design yield strength of reinforcement.

For a design bending moment, Msd, the area of reinforcement required is calculated using:

$$A_S = \frac{M_{sd}}{Z f_{yd}} \tag{3.62}$$

Where: Z is the moment arm between tensile and compressive force.

The area of shell reinforcement at any section as measured in two orthogonal directions shall not be less than the slab shrinkage or temperature reinforcement required.

Area of shrinkage and temperature reinforcement area to gross concrete area shall not be less than 0.0014. [ACI 318M-05, 2005].

The minimum concrete cover for reinforcement shall not be less than 20 mm. In corrosive environments or other severe exposure conditions, amount of concrete protection shall be suitably increased, or other protection shall be provided. [ACI 318M-05, 2005].

Spacing of Shell Reinforcement

Shell reinforcement in any direction shall not be spaced farther apart than 450 mm nor farther apart than five times the shell thickness. [ACI 318M-05, 2005].

The minimum clear spacing between parallel bars in a layer shall be d_b (i.e. diameter of bar), but not less than 25 mm. [ACI 318M-05, 2005].

4.0. COMPUTER PROGRAM FOR THE ANALYSIS AND DESIGN OF REINFORCED CONCRETE DOMES

4.1. Developing the Computer Program

4.1.1. General

A programming language is a formal computer language or constructed language designed to communicate instructions to a machine, particularly a computer. Programming languages can be used to create programs to control the behavior of a machine or to express algorithms.

4.1.2. Visual Basics

Microsoft Visual Basic is a computer programming environment used to create graphical applications for the Microsoft Windows family of operating systems. It uses a computer language of the same name. Like every computer language, Visual Basic is used to give instructions to a computer. The instructions can be written from a text editor such as Notepad. Another way is to use a programming environment that is equipped with many tools that make it easy to work on projects, to create the necessary files, and to distribute a completed application. [Evangelos Petroutsos, 2010].

Microsoft Visual Basic is not just a production environment. It also includes a fully functional language that can stand on its own. The language started as Basic, the QuickBasic or QBasic. It first went through various changes. In 2002, Microsoft created a new serious language that can understand and use the .NET Framework. Microsoft Visual Basic is used to create graphical applications, also referred to as Graphical User Interface (GUI) applications, web-based applications, and other types of applications. In order to effectively create these applications, you must be familiar with the language used in this programming environment. [Michael Halvorson, 2010].

4.1.3. Flow Charts

The simplified form of the computer program developed to analyze and design spherical reinforced concrete domes is shown in Figure 4.1 and Figure 4.2. Figure 4.1 shows the flow chart for the calculation of geometric parameters using equations developed based on recommended dimensions, Table 3.2 and input span of the dome to be designed. Figure 4.2 shows the calculation of internal actions (i.e. meridional forces, hoop forces and meridional moments) and the calculation of reinforcement based on the equations given in chapter three of this paper and input loads and material properties.

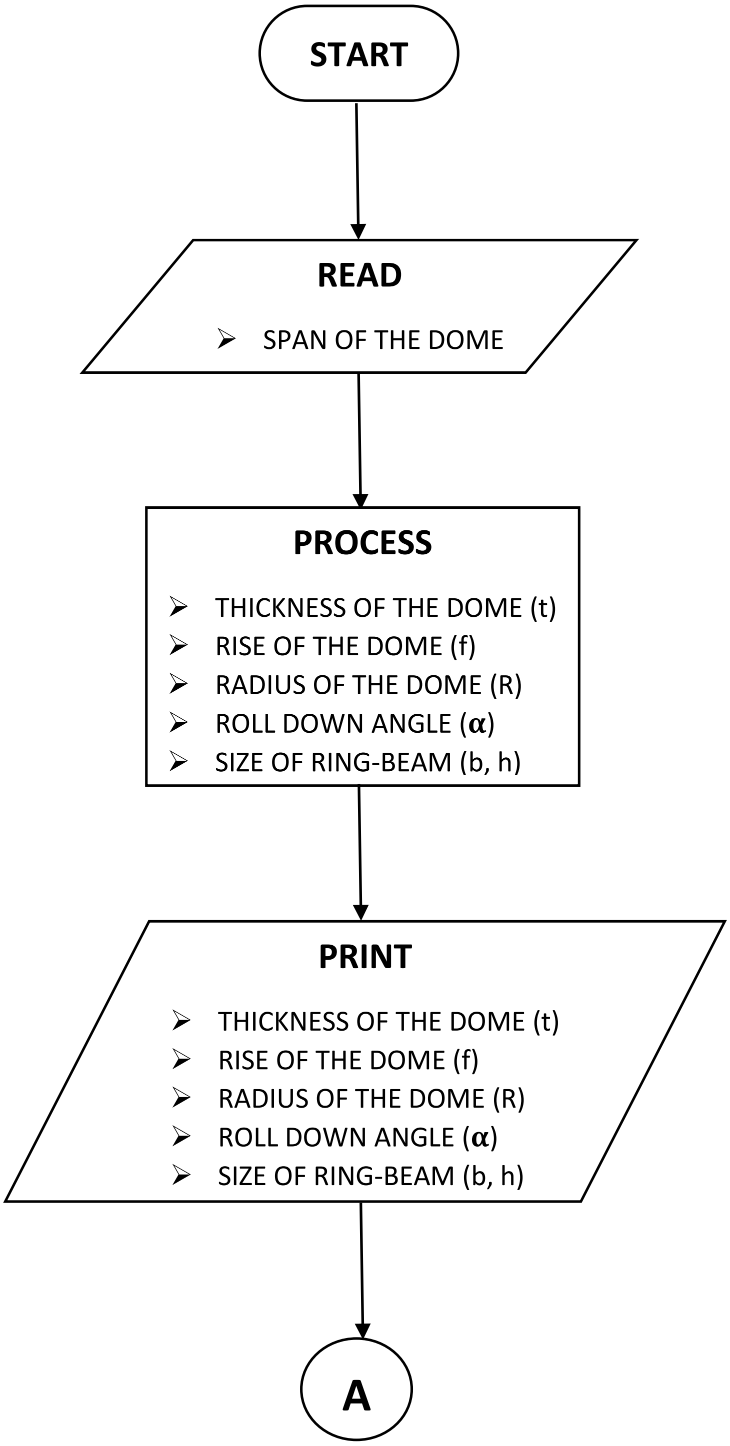


Figure 4.1. Flow Chart for the Calculation of Geometric parameters.

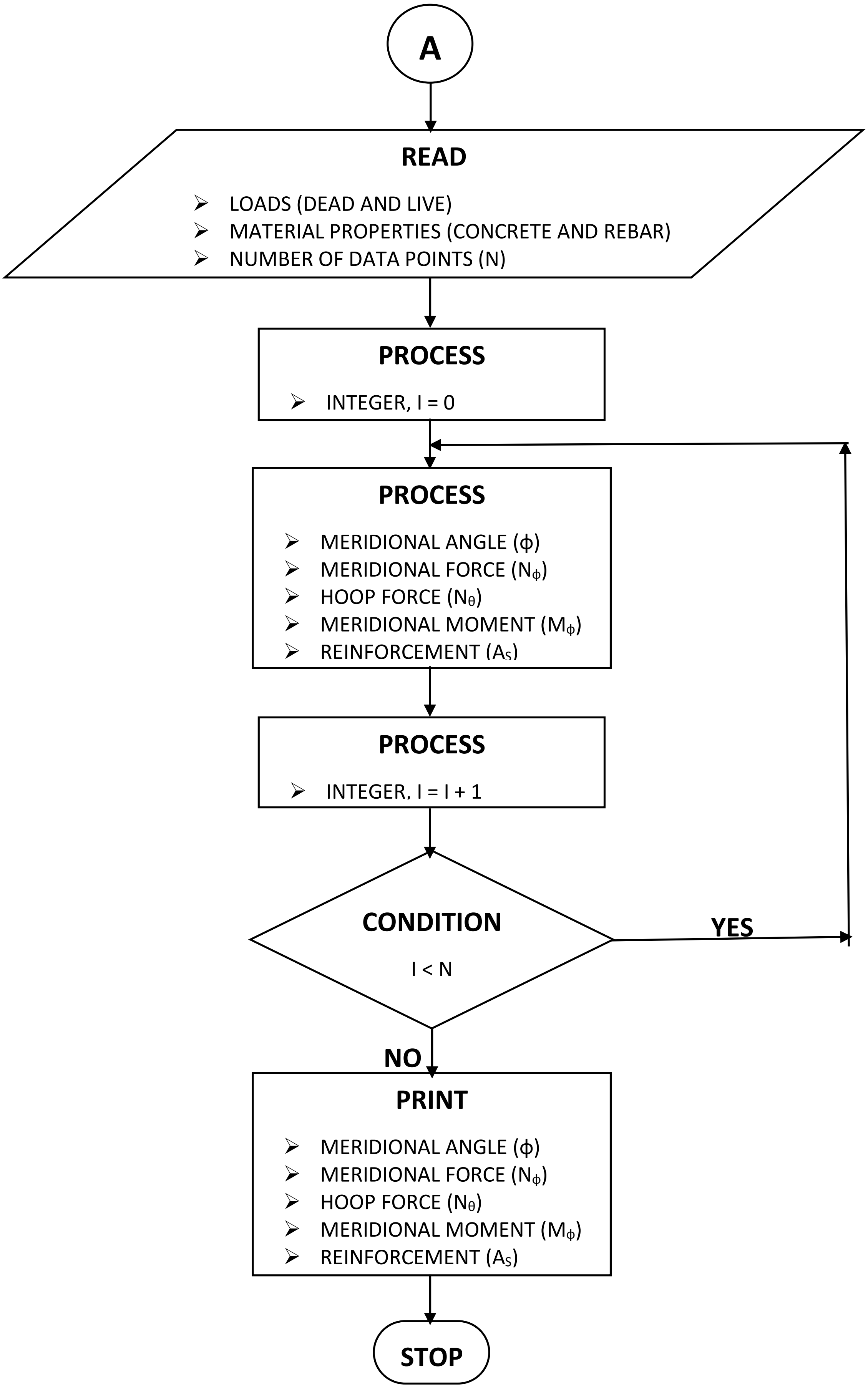


Figure 4.2. Flow Chart for Analysis and Design.

4.2. How to Use the Computer Program

4.2.1. General

The computer program is written in such a way that anybody who knows about spherical dome parameters and material properties of concrete and reinforcement bar can easily use it to analyze spherical reinforced concrete domes. This is because, windows form application and dialog boxes have been used to simplify the import of parameters and export of outputs.

To use this computer program the necessary steps are the followings:

4.2.2. Procedures

4.2.2.1. Creating a New Model

To create a new model click on the file menu strip then the new model sub menu under file menu strip which will lead you to a new dialog box to create the model.

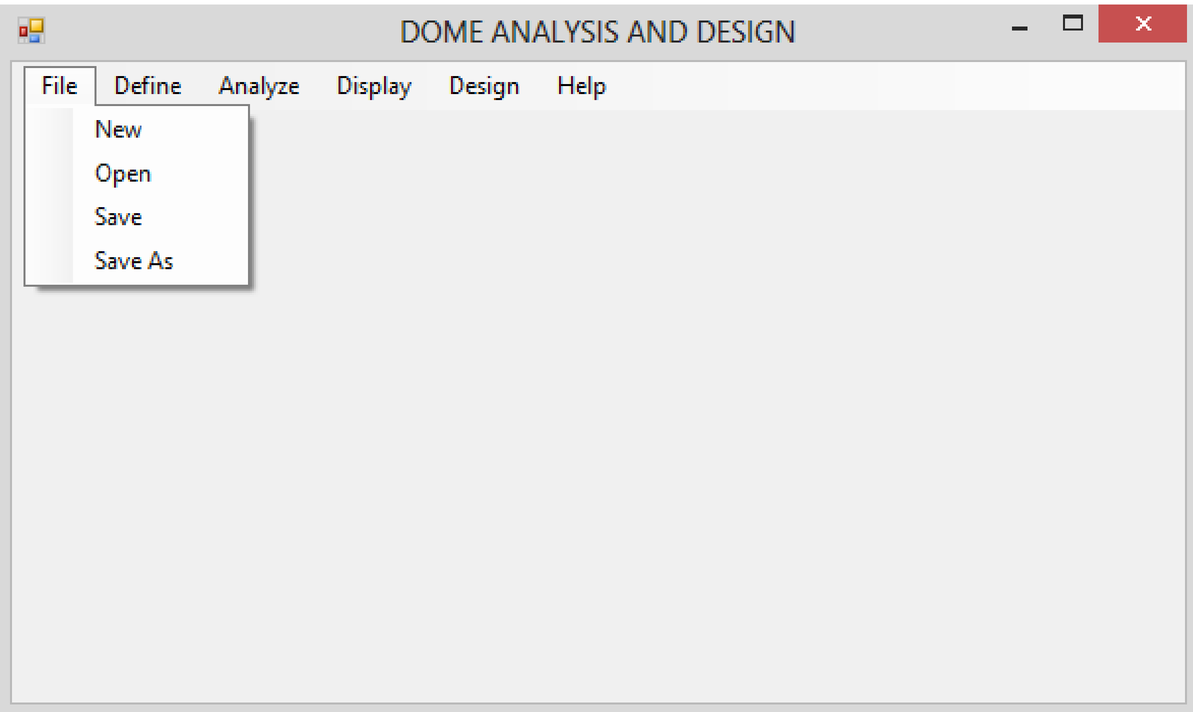


Figure 4.3. Interface for Dome Analysis and Design and File Menu

In this dialog box select the dome type from dome type combo box. As you select your dome type a group box with different parameters will appear on the existing dialog box.

Fill the text boxes with the appropriate values for the respective dome dimensions in meter. If the labels of the dome dimensions are not clear click on the parametric definition button on the right to see the dome parameters with a picture. After understanding click ok to go back to the dialog for filling the values for your dome parameters. Fill all the text boxes then click ok to finish creating your model.

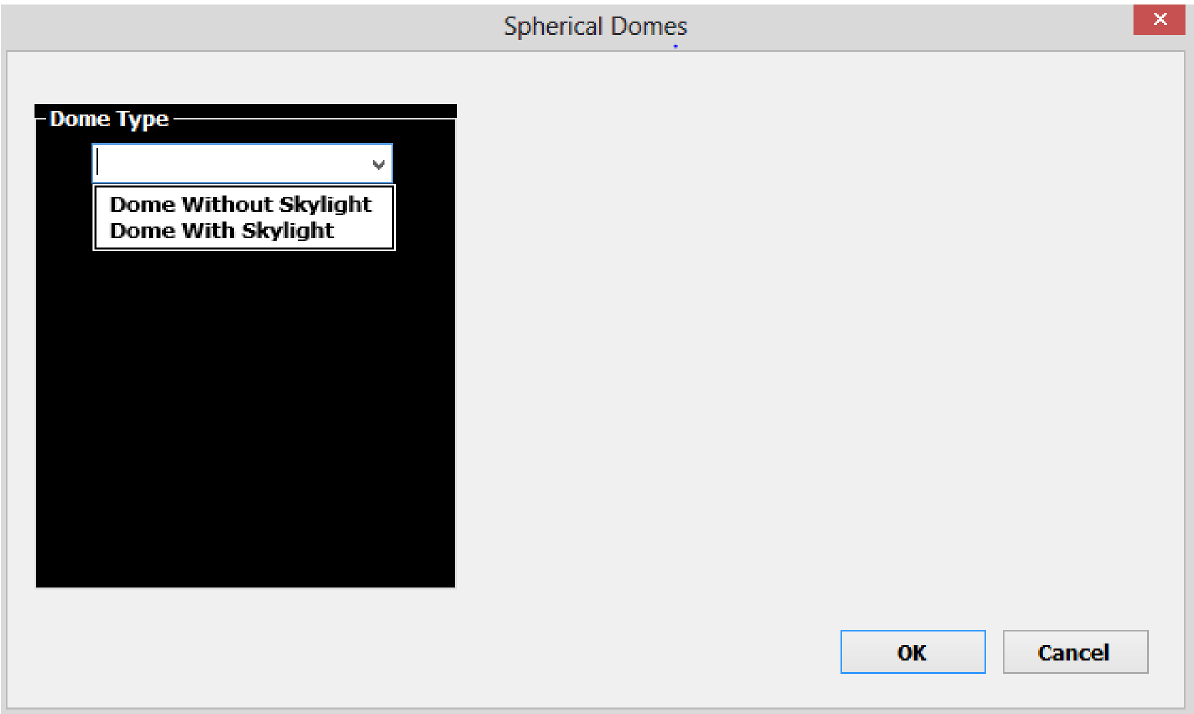


Figure 4.4. New Model Dialog Box

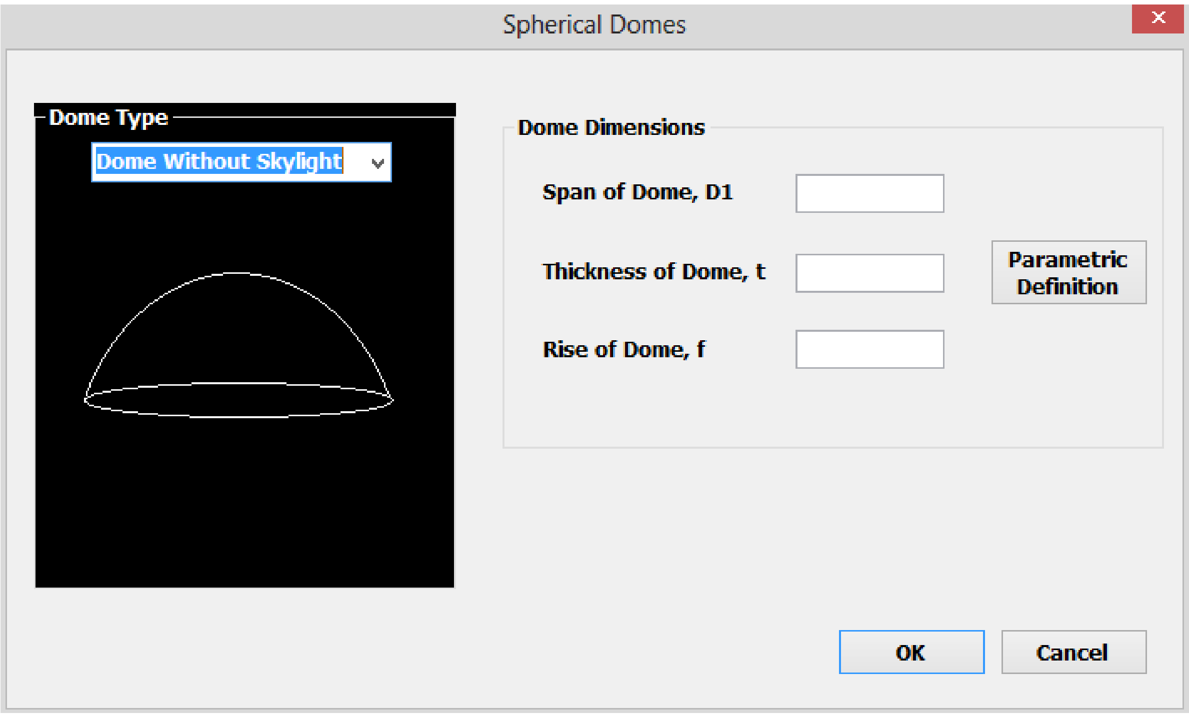


Figure 4.5. Dimensions for Dome and Parametric Definition Button

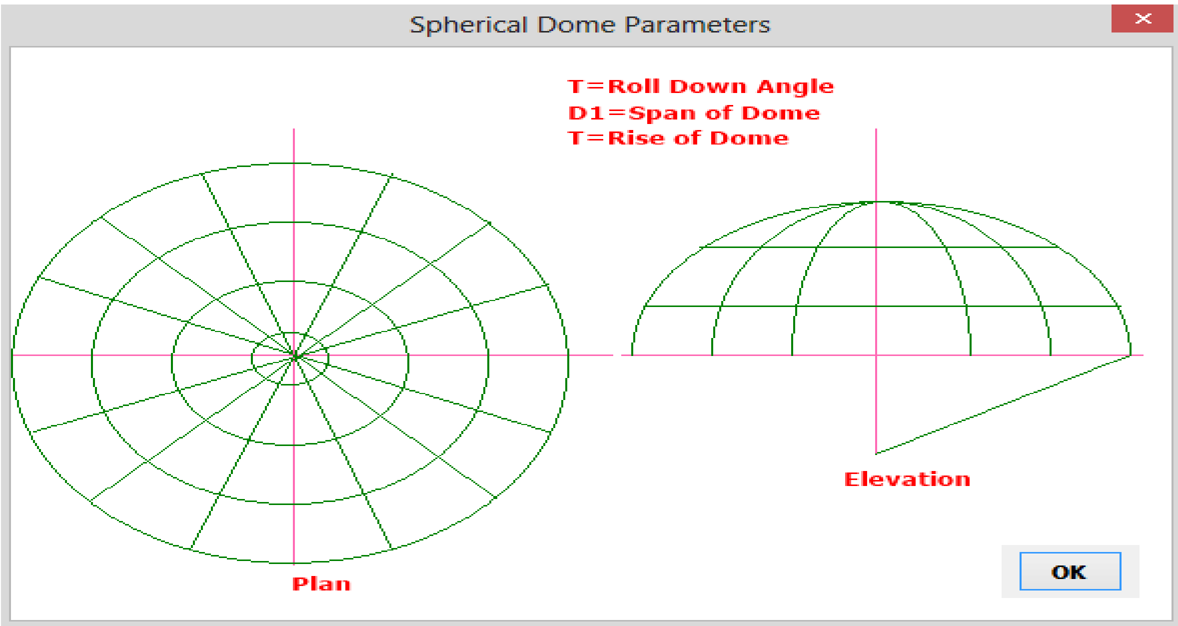


Figure 4.6. Parameters of Dome

4.2.2.2. Defining Materials

4.2.2.2.1. Defining Concrete Material

To define a concrete material click on the define menu strip then the material sub menu under which click on the concrete material sub menu which will lead you to define material dialog box to define the material for concrete.

Click on add new material button on define material dialog box which will lead you to material property data dialog box. Write down the name of the concrete material in the material name text box, select the material type and the class of work from the respective combo boxes. At last select the concrete grade from its combo box and click ok to add the material. Then click ok to the define materials dialog box if you finish adding the concrete material. Otherwise click on add new material button and repeat the steps before to add another concrete material.

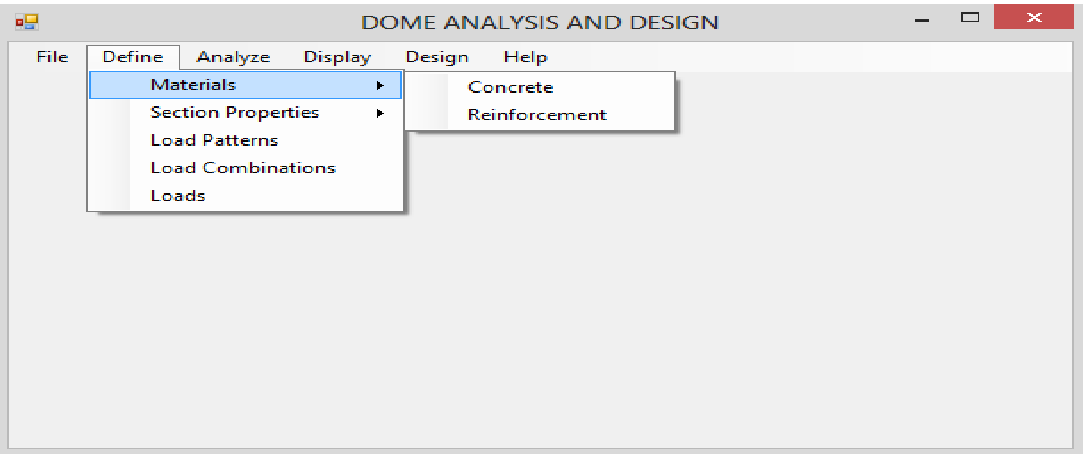


Figure 4.7. Define Menu

4.2.2.2.2. Defining Rebar Material

To define a rebar material click on the define menu strip then the material sub menu under which click on the steel material sub menu which will lead you to define material dialog box to define the material for rebar.

Click on add new material button on define material dialog box which will lead you to material property data dialog box. Write down the name of the steel material in the material name text box, select the material type and the class of work from the respective combo boxes. At last select the steel grade from its combo box and click ok to add the material. Then click ok to the define materials dialog box if you finish adding the rebar material. Otherwise click on add new material button and repeat the steps before to add another rebar material.

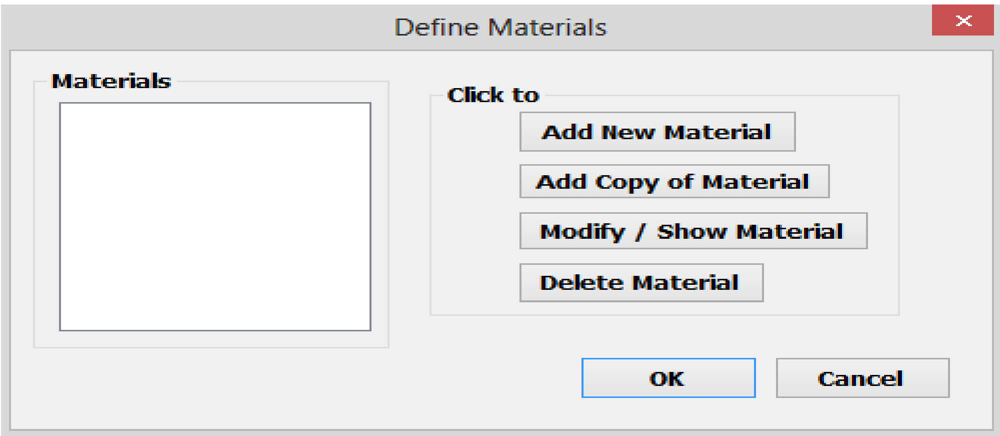


Figure 4.8. Material Definition Dialog Box

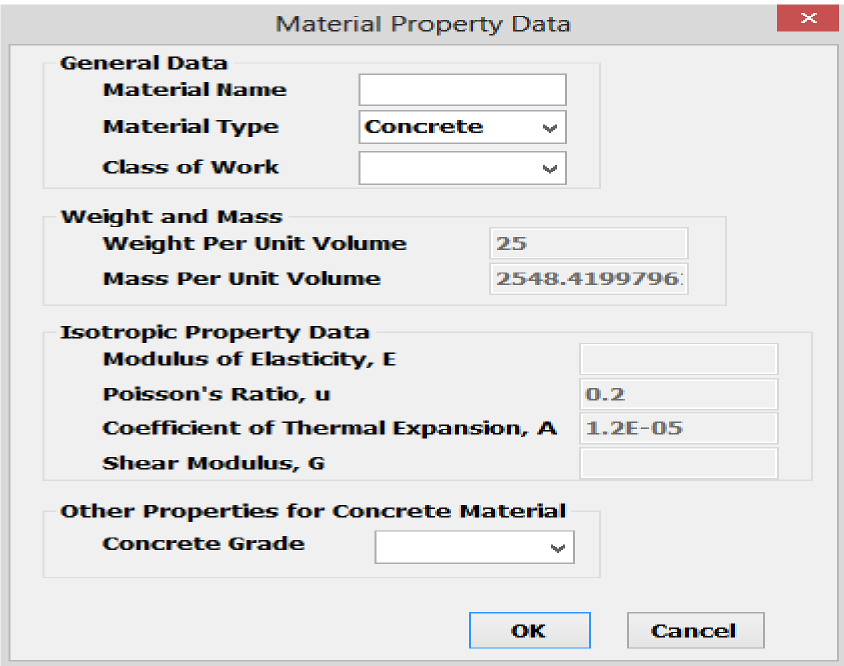


Figure 4.9. Concrete Material Property Data

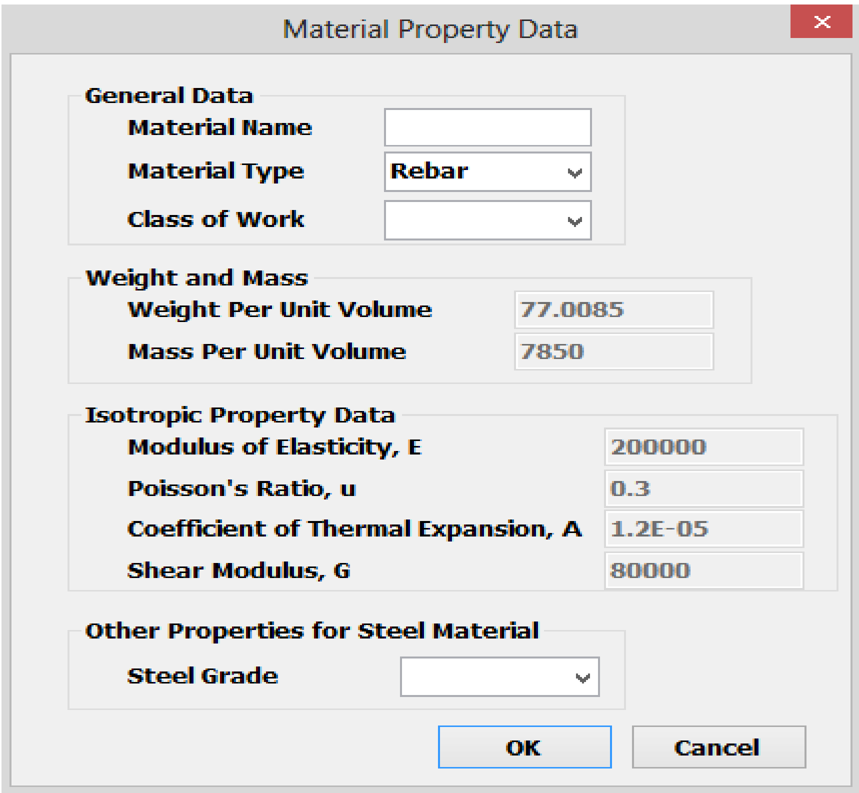


Figure 4.10. Rebar Material Property Data

4.2.2.3. Defining Section Properties

To define a section property click on the define menu strip then the section property sub menu under which click on the frame section sub menu which will lead you to frame properties dialog box to define the frame section.

Click on add new property button on frame properties dialog box which will lead you to add frame section property dialog box. Select the property type from its combo box and select the shape of the concrete section by clicking on the shape of your selection, in this case I have assumed a rectangular ring beam. This will lead you to the rectangular section dialog box where you can fill the name of the section on its text box, select the material from its combo box, fill the depth and the width of the ring beam in their own text boxes in meter. Finally click on concrete reinforcement button to select the rebar material and to give the bar size for both longitudinal and confinement bars in the reinforcement data dialog box. Then click ok to go back to the rectangular section dialog box. Then click ok to the rectangular section dialog box and click close to the add frame section property dialog box. If you finish adding the frame section, click ok to the frame properties dialog box. Otherwise click on add new property button and repeat the steps before to add another frame section.

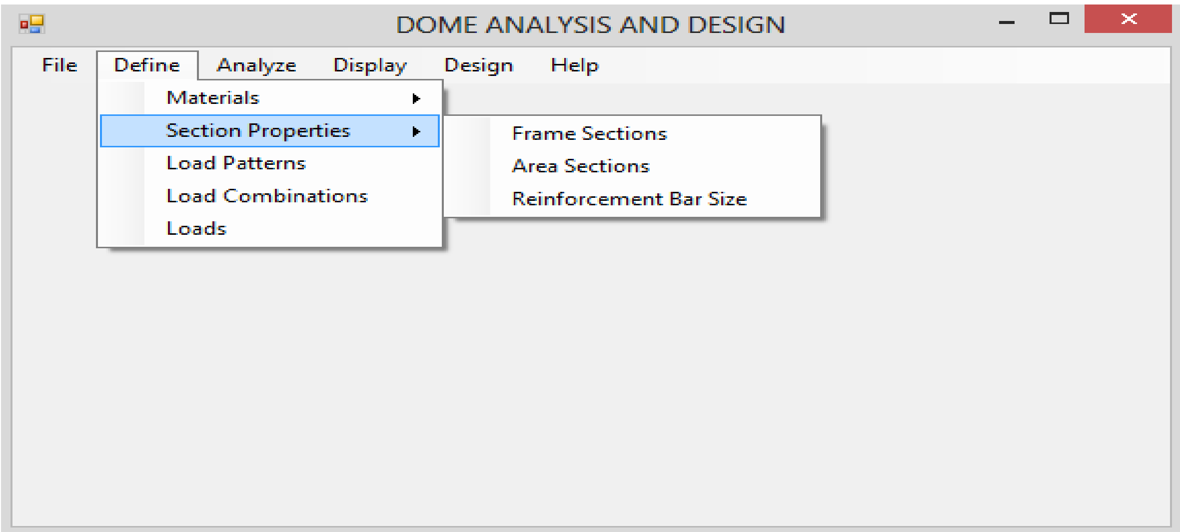


Figure 4.11. Section Property Definition Menu

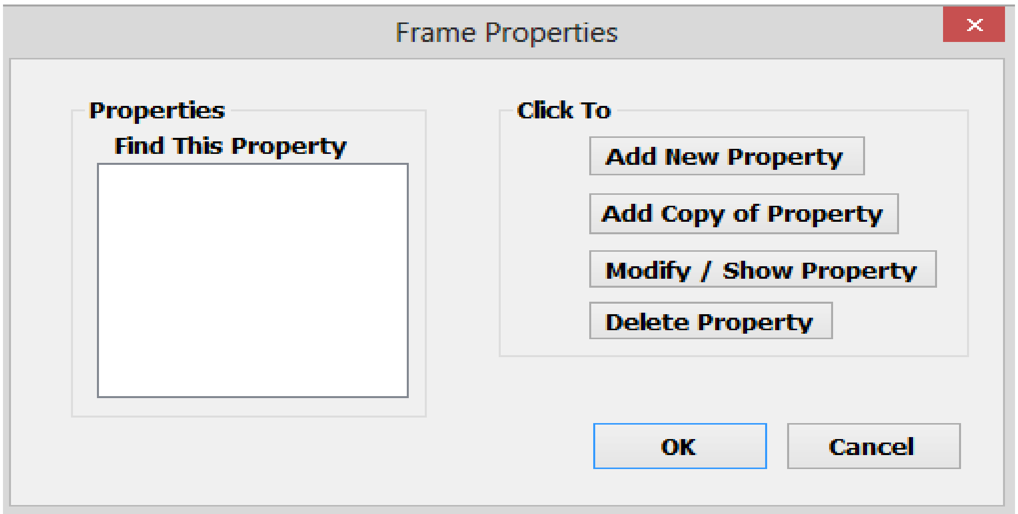


Figure 4.12. Frame Property Dialog Box

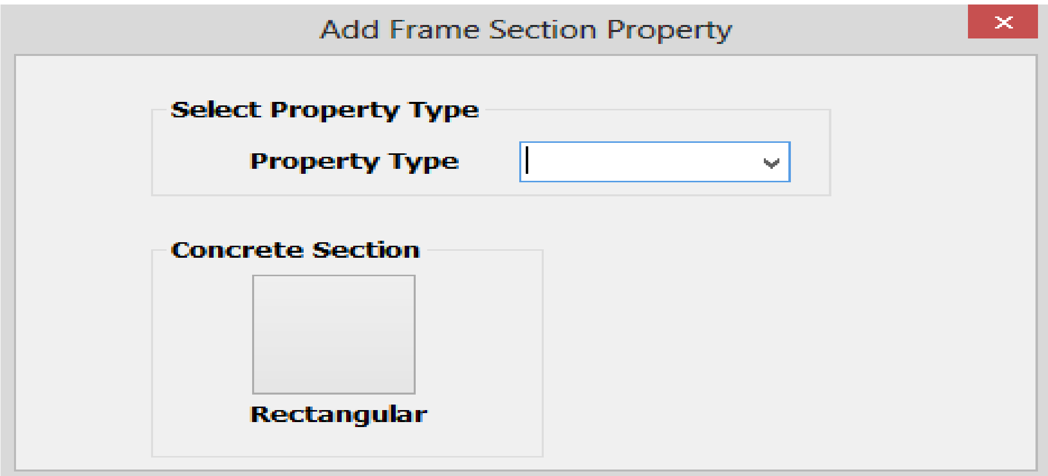


Figure 4.13. Frame Section Property Type and Shape

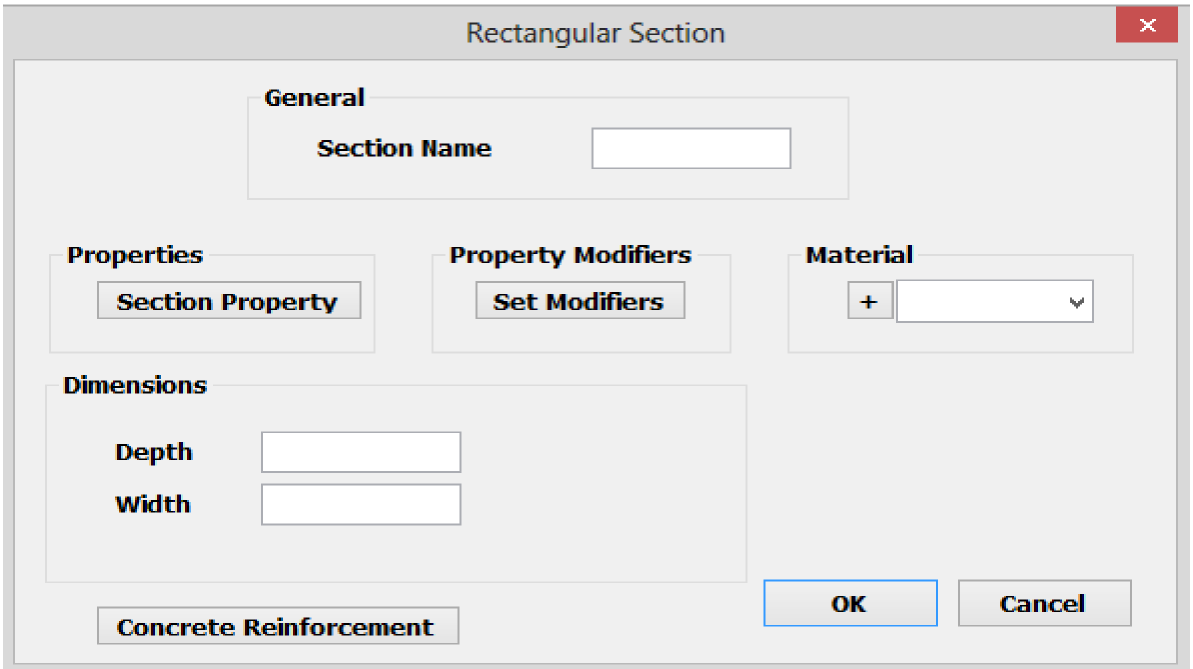


Figure 4.14. Rectangular Frame Section

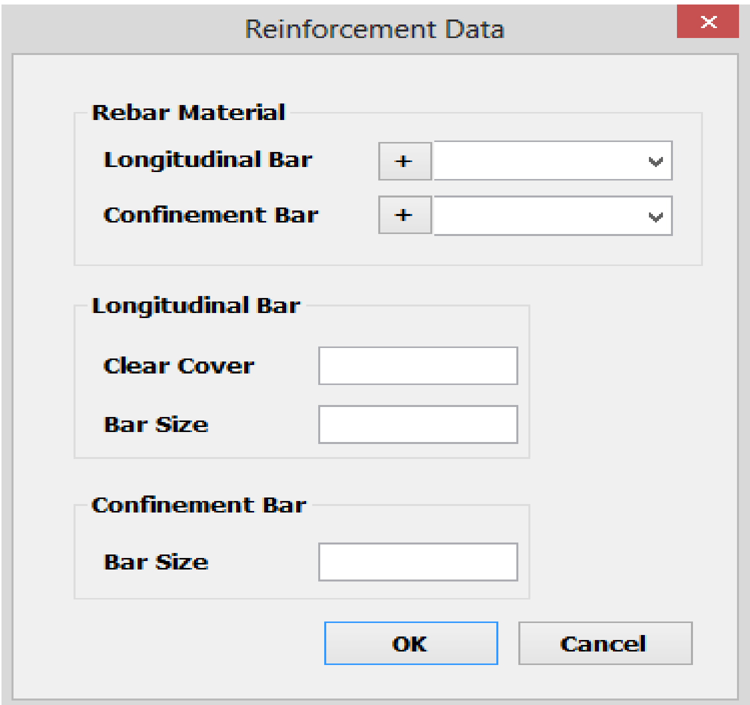


Figure 4.15. Reinforcement Data for Frame Section

4.2.2.4. Assigning Load

To assign a load click on the define menu strip then the loads sub menu which will lead you to load assignment dialog box to assign loads.

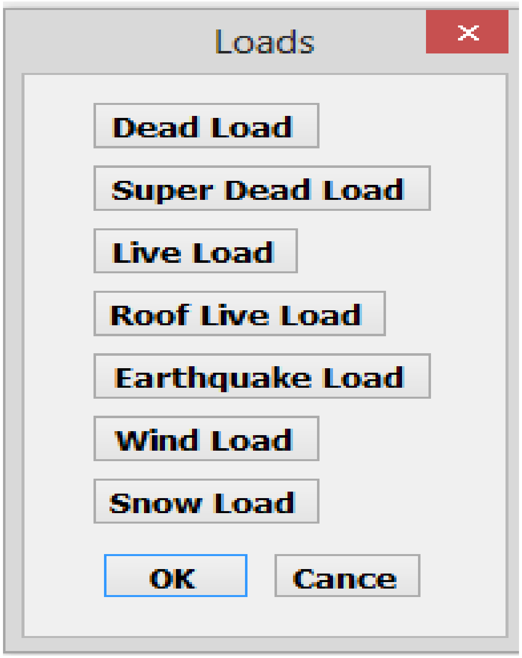


Figure 4.16. Load Assignment Dialog Box.

Click on each of the load buttons on load assignment dialog box to import the distributed load on the spherical dome.

4.2.2.5. Displaying Results

To display results click on the display menu strip then the show tables sub menu which will lead you to tables dialog box to display tables.

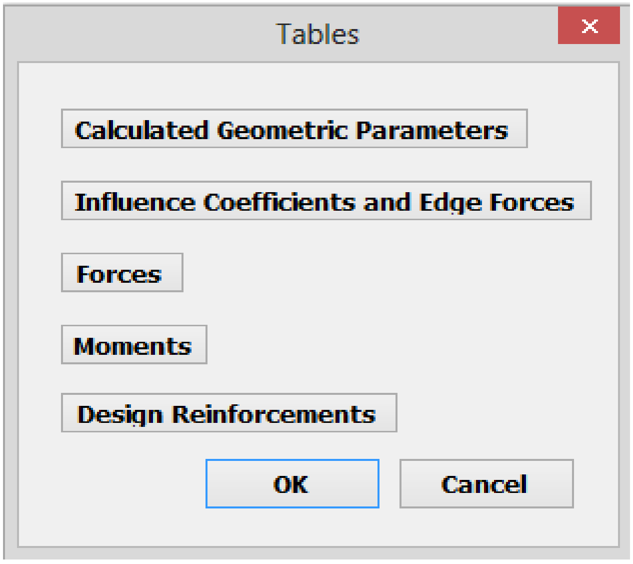


Figure 4.17. Tables Dialog Box.

Click on each of the output buttons on tables dialog box to display tabular outputs.

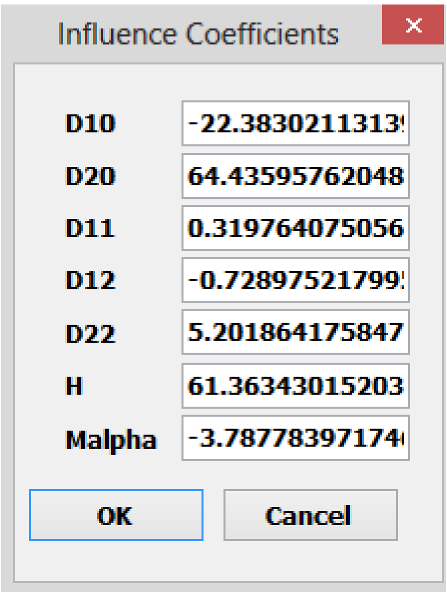
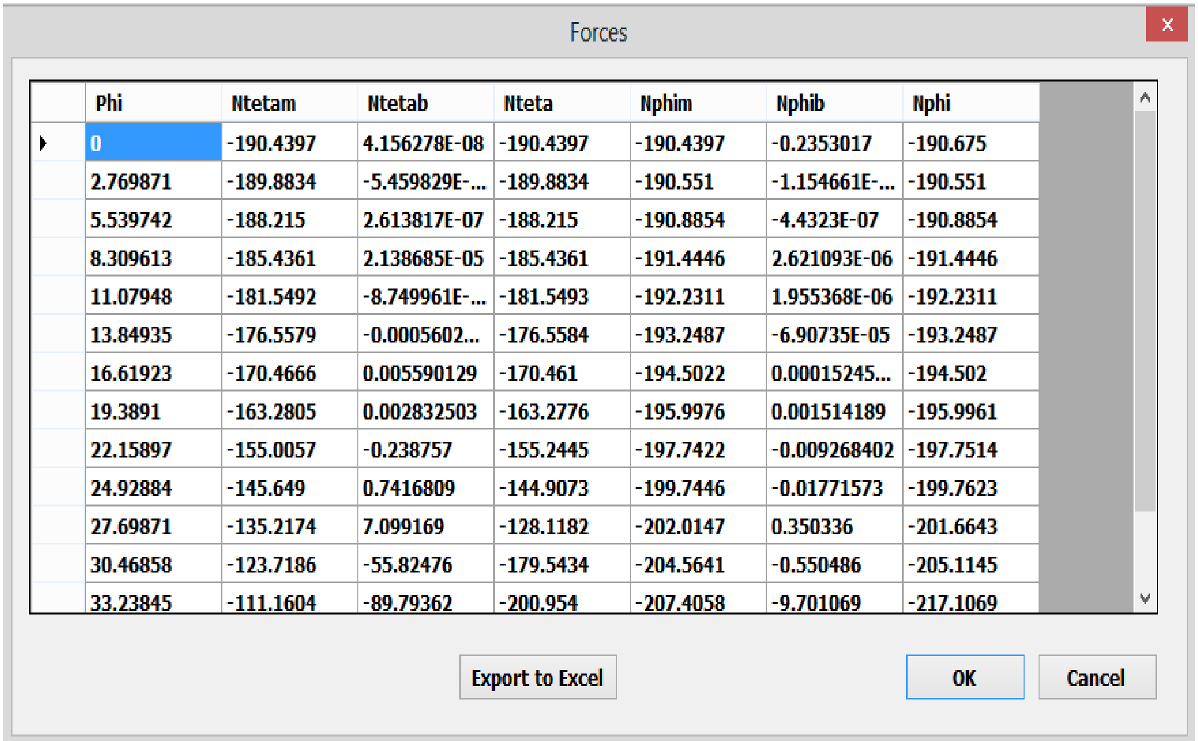


Figure 4.18. Outputs for Influence Coefficients.



Phi	Ntetam	Ntetab	Nteta	Nphim	Nphib	Nphi
0	-190.4397	4.156278E-08	-190.4397	-190.4397	-0.2353017	-190.675
2.769871	-189.8834	-5.459829E-...	-189.8834	-190.551	-1.154661E-...	-190.551
5.539742	-188.215	2.613817E-07	-188.215	-190.8854	-4.4323E-07	-190.8854
8.309613	-185.4361	2.138685E-05	-185.4361	-191.4446	2.621093E-06	-191.4446
11.07948	-181.5492	-8.749961E-...	-181.5493	-192.2311	1.955368E-06	-192.2311
13.84935	-176.5579	-0.0005602...	-176.5584	-193.2487	-6.90735E-05	-193.2487
16.61923	-170.4666	0.005590129	-170.461	-194.5022	0.00015245...	-194.502
19.3891	-163.2805	0.002832503	-163.2776	-195.9976	0.001514189	-195.9961
22.15897	-155.0057	-0.238757	-155.2445	-197.7422	-0.009268402	-197.7514
24.92884	-145.649	0.7416809	-144.9073	-199.7446	-0.01771573	-199.7623
27.69871	-135.2174	7.099169	-128.1182	-202.0147	0.350336	-201.6643
30.46858	-123.7186	-55.82476	-179.5434	-204.5641	-0.550486	-205.1145
33.23845	-111.1604	-89.79362	-200.954	-207.4058	-9.701069	-217.1069

Figure 4.19. Outputs for Hoop and Meridional Forces.

As you can see from figure 4.17 it is possible to export the outputs to Microsoft Excel by clicking on export to excel button below the displayed tabular outputs.

4.3. Defining Design Example Problem

This example problem is basically about the analysis and design of a spherical dome structure for the purpose of auditorium. The proposed site of the project is located in Addis Ababa, around Entoto Comprehensive High School. The major objective of this work is to replace the existing roof of the auditorium of the Ethiopian National Center for Culture. See Figures 4.1 and 4.2 below. This is needed because the existing structure is reaching its design period and its capacity is small as compared to the prevailing demand for different occasions. For this the basic architectural dimensions based on the demand and recommended preliminary dimensions of the existing auditorium is considered, then analysis of the structure subjected to the appropriate loadings as recommended is done.

Figure 4.20. The existing Ethiopian nation and nationalities culture center

4.3.1. Load Path

It is necessary to have clear understanding of how load is transferred from the dome to the foundation, to model the structure accurately. The typical load transfer path is shown below.

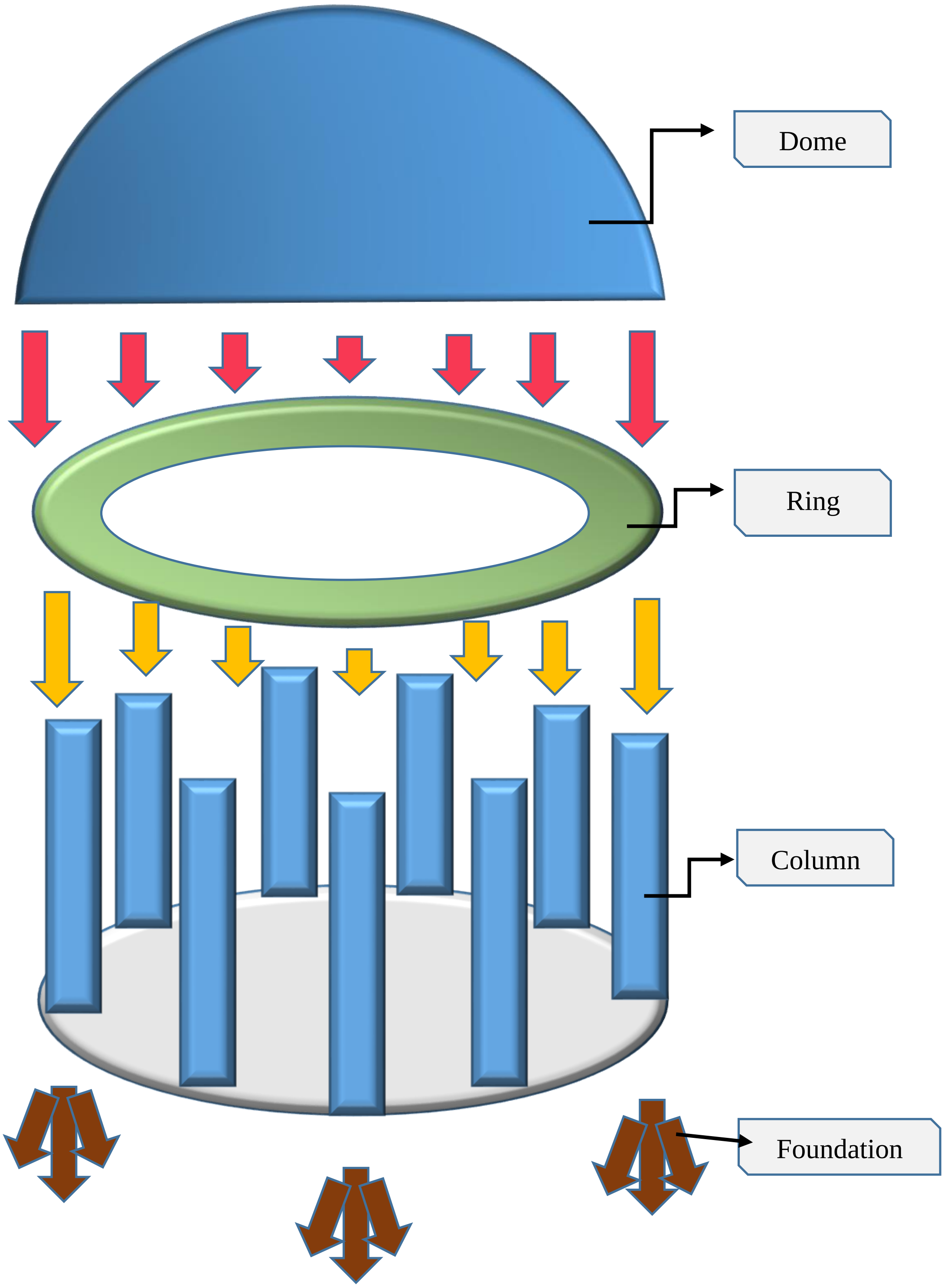


Figure 4.21. Structural Components and Load Transfer

4.3.2. Preliminary Dimensions and Material Properties

4.3.2.1. Preliminary Dimensions

- Number of Seat = 3000
- Area per Seat = 0.7 m^2
- 30% Additional Area for Stage, Circulation and Others.
- Span of the Shell (d = 2r = 80m)
- Height of the Shell (h)

Table 4.1. Recommended Dome Heights for Various Spans: Spherical Dome Shells

Span (m)	Minimum Height (m)	Maximum Height (m)
30.48	3.35	4.11
60.96	8.53	10.36
91.44	15.09	17.83
121.92	23.16	25.6

From the recommended dome heights h=13.8m

- Radius of Sphere (R)

$$R^2 = (R-h)^2 + r^2 = (R-13.8)^2 + 40^2 \Rightarrow R = 64.871m$$

$$\tan\alpha = \frac{r}{R-h} = \frac{40}{64.871-13.8} \Rightarrow \alpha = 38.069^0$$

Where: R= Radius of Sphere

r = Radius of Ring Beam

α = Roll-Down Angle (Angle from Axis of Revolution up to Ring-Beam)

- From recommended dimensions, Table 3.2, the thickness of the Shell (ts) =10cm

4.3.2.2. Material Property

- Concrete – C-30, f_{CK} = 0.8*30= 24 MPa
- Steel – S-400, f_{YK} = 400 MPa

4.3.3. Load Calculation

Permanent Load

- Self-Weight

$$G_S = t_S * \gamma_C = 0.1 * 25 = 2.5\ kN/m^2$$

- Plastering

$$G_P = t_P * \gamma_P = 0.02 * 23 = 0.46\ kN/m^2$$

Transient Load

- Roof Live Load

$$L_L = 1.0 \text{ kPa}$$

Load Combination

$$q = 1.35\ D_L + 1.5\ L_L$$

$$D_L = G_S + G_P + G_{WP} = 2.5 + 0.46 = 2.96\ \text{kN/m}^2$$

$$q = 1.35 * 2.96 + 1.5 * 1.0 = 5.496\ \text{kN/m}^2$$

4.3.4. Example Solving Using the Developed Program

4.3.4.1. Analysis of the Dome

The numerical calculations based on the force method of shell analysis are carried out as follows.

From recommended dimensions, Table 3.2, take dimension of ring beam to be 40cm by 50cm

The numerical values of parameters y_o, e, d' and $\boldsymbol{\lambda}$ are

$$d' = \frac{t}{2} * \text{Cos}\ \alpha = 5 * \text{Cos}\ 38.069 = 3.93635\text{cm}$$

$$Y_O = \frac{h}{2} - d' = 25 - 3.93635 = 21.063649\text{cm}$$

$$b' = \frac{b}{2} - \frac{t}{2} * \text{Sin}\ \alpha = 20 - 5 * \text{Sin}\ 38.069 = 16.9169589\text{cm}$$

$$e = Y_O * \text{Cos}\ \alpha - b' * \text{Sin}\ \alpha = 21.063649 * \text{Cos}\ 38.069 - 16.9169589 * \text{Sin}\ 38.069$$

$$e = 6.1516468\text{cm}$$

$$r = a * \text{Sin}\ \alpha - \frac{t}{2} * \text{Sin}\ \alpha = 4000 - 5 * \text{Sin}\ 38.069 = 3996.916958\text{cm}$$

$$\lambda^4 = 3 * (1 - \nu^2)(\frac{a}{t})^2 = 3 * (1 - 0.2^2)(\frac{64.871}{0.1})^2 = 1234291.837 \qquad \lambda = 33.17978$$

Now we calculate the numerical values for the flexibility influence coefficients and the membrane deformations. By direct substitution of assumed data in the appropriate formulas we obtain:

$$D_{11}^{D} = \frac{1}{E} * \frac{2a\lambda \text{Sin}^2\alpha}{t} = \frac{1}{E} * \frac{2 * 64.871 * 33.17978\ \ * \text{Sin}^2\ 38.069}{0.1} = \frac{1}{E}16367.138$$

$$D_{12}^{D} = \frac{1}{E} * \frac{2\lambda^2 \text{Sin}\ \alpha}{t} = \frac{1}{E} * \frac{2 * 33.17978\ ^2 * \text{Sin}\ 38.069}{0.1} = \frac{1}{E} 13576.4502 = D_{21}^{D}$$

$$D_{22}^{D} = \frac{1}{E} * \frac{4\lambda^3}{at} = \frac{1}{E} * \frac{4 * 33.17978\ ^3}{64.871 * 0.1} = \frac{1}{E} 22523.1797$$

$$D_{11}^{R} = \frac{1}{E} * \frac{\left(1 + \frac{12Y_O^2}{h^2}\right) r^2}{bh} = \frac{1}{E} * \frac{\left(1 + \frac{12 * 21.063649^2}{50^2}\right) * 39.96916958^2}{0.4 * 0.5} = \frac{1}{E} 24998.6284$$

$$D_{12}^{R} = -\frac{1}{E} * \frac{12r^2 Y_O}{bh^3} = -\frac{1}{E} * \frac{12 * 39.96916958^2 * 21.063649}{0.4 * 0.5^3} = -\frac{1}{E} 80759.7758$$

$$D_{22}^{R} = \frac{1}{E} * \frac{12r^2}{bh^3} = \frac{1}{E} * \frac{12 * 39.96916958^2}{0.4 * 0.5^3} = \frac{1}{E} 383408.284$$

Deformation of the dome and the ring due to distributed loading:

$$D_{10}^{D} = \frac{a^2 q}{Et} * \left(\frac{(1 + \nu)}{1 + \text{Cos}\ \alpha} - \text{Cos}\ \alpha\right) \text{Sin}\ \alpha$$

$$= \frac{64.871^2 q}{E * 0.1} * \left(\frac{(1 + 0.2)}{1 + \text{Cos}\ 38.069} - \text{Cos}\ 38.069\right) \text{Sin}\ 38.069$$

$$= -\frac{1}{E} 5909.9484\ q$$

$$D_{20}^{D} = \frac{aq}{Et} * (2 + \nu) \text{Sin}\ \alpha = \frac{64.871q}{E * 0.1} * (2 + 0.2) \text{Sin}\ 38.069 = \frac{1}{E} 800\ q$$

$$D_{10}^{R} = \left(\text{Cos}\ \alpha + \frac{12Y_O e}{h^2}\right) \frac{r^2}{Ebh} \left(\frac{-aq}{1 + \text{Cos}\ \alpha}\right)$$

$$= \left(\text{Cos}\ 38.069 + \frac{12 * 0.21063649 * 0.06125}{0.5^2}\right) * \frac{39.96916958^2}{E * 0.4 * 0.5}$$

$$* \left(\frac{-64.871q}{1 + \text{Cos}\ 38.069}\right) = -\frac{1}{E} 408567.995\ q$$

$$D_{20}^{R} = \frac{-12r^2 e}{Ebh^3} \left(\frac{-aq}{1 + \text{Cos}\ \alpha}\right) = \frac{-12 * 39.96916958^2 * 0.06125}{E * 0.4 * 0.5^3} \left(\frac{-64.871q}{1 + \text{Cos}\ 38.069}\right)$$

$$= \frac{1}{E} 856078.0713\ q$$

Influence coefficients of the dome ring system:

$$ED_{11} = 16367.138 + 24998.6284 = 41365.77$$

$$ED_{12} = 13576.4502 - 80759.7758 = -67183.3$$

$ED_{22} = 22523.1797 + 383408.284 = 405931.5$

$ED_{10} = -5909.9484\ q - 408567.995\ q = -414478q$

$ED_{20} = 800\ q + 856078.0713\ q = 856878.1q$

If we substitute these values in the parametric solution of the compatibility relations we find

$$H = -\left(\frac{D_{22}D_{10} - D_{12}D_{20}}{D_{22}D_{11} - {D_{12}}^2}\right) = 9.014q$$

$$M_\alpha = -\left(\frac{D_{11}D_{20} - D_{12}D_{10}}{D_{22}D_{11} - {D_{12}}^2}\right) = -0.618q$$

So, for q=5.496kN/m2

$H = 9.014q = 9.014 * 5.496 = 49.014153kN/m$

$M_\alpha = -0.618q = -0.618 * 5.496 = -3.490509kNm/m$

Having obtained the edge forces, we can now use the following expressions to determine the bending field in the dome. The appropriate expressions are:

$$M_\phi = -\frac{a}{\lambda}\text{Sin}\ \alpha\ e^{-\lambda\psi}\ \text{Sin}\lambda\psi\ H + \sqrt{2}e^{-\lambda\psi}\text{Sin}\left(\lambda\psi + \frac{\pi}{4}\right)M_\alpha$$

$$N_\theta = -2\lambda\ \text{Sin}\ \alpha\ e^{-\lambda\psi}\ \text{Sin}\left(\lambda\psi - \frac{\pi}{2}\right)H - \frac{2\sqrt{2}}{a}\lambda^2 e^{-\lambda\psi}\text{Sin}\left(\lambda\psi - \frac{\pi}{4}\right)M_\alpha$$

$$N_\phi = -\sqrt{2}\ \text{Cot}\ (\alpha - \psi)\text{Sin}\ \alpha\ e^{-\lambda\psi}\ \text{Sin}\left(\lambda\psi - \frac{\pi}{4}\right)H - \frac{2\lambda}{a}\text{Cot}\ (\alpha - \psi)\ e^{-\lambda\psi}\text{Sin}(\lambda\psi)M_\alpha$$

Here $\psi = \boldsymbol{\alpha} - \phi$. If we substitute the numerical values in the above relations, we obtain the following expressions for our problem.

$$M_\phi = -59.2065e^{-\lambda\psi}\ \text{Sin}\lambda\psi - 4.768e^{-\lambda\psi}\text{Sin}\left(\lambda\psi + \frac{\pi}{4}\right)$$

$$N_\theta = -2009.535\ e^{-\lambda\psi}\ \text{Sin}\left(\lambda\psi - \frac{\pi}{2}\right) + 161.8558e^{-\lambda\psi}\text{Sin}\left(\lambda\psi - \frac{\pi}{4}\right)$$

$$N_\phi = -42.8259\text{Cot}\ (\alpha - \psi)\ e^{-\lambda\psi}\ \text{Sin}\left(\lambda\psi - \frac{\pi}{4}\right) + 3.4493\text{Cot}\ (\alpha - \psi)\ e^{-\lambda\psi}\text{Sin}(\lambda\psi)$$

To determine the complete internal force field, we must add to these bending forces the internal membrane forces. The calculations related to the determination of bending and membrane fields are summarized in the tables shown below.

The values for the membrane field are calculated using the following expressions:

$$N_\theta = aq\left(\frac{1}{1 + \cos\phi} - \cos\phi\right)$$

$$N_\phi = -aq\left(\frac{1}{1 + \cos\phi}\right)$$

Table 4.2. Tabulated Values for Analysis.

ϕ Deg.	ψ Deg.	$\sin\lambda\psi$	$\sin\left(\lambda\psi + \frac{\pi}{4}\right)$	$\sin\left(\lambda\psi - \frac{\pi}{2}\right)$	$\sin\left(\lambda\psi - \frac{\pi}{4}\right)$	$\cot(\alpha - \psi)$	$e^{-\lambda\psi}$
0.000	38.069	-0.054	-0.745	0.999	0.668		2.67E-10
2.928	35.140	0.998	0.755	-0.071	0.655	19.54868852	1.45E-09
5.857	32.212	-0.194	0.556	-0.981	-0.831	9.748767099	7.92E-09
8.785	29.284	-0.949	-0.894	0.315	-0.448	6.470719299	4.32E-08
11.713	26.355	0.431	-0.333	0.902	0.943	4.823095013	2.35E-07
14.642	23.427	0.842	0.977	-0.540	0.213	3.827589474	1.28E-06
17.570	20.499	-0.641	0.090	-0.768	-0.996	3.158088487	6.99E-06
20.499	17.570	-0.682	-0.999	0.732	0.035	2.674817661	3.81E-05
23.427	14.642	0.811	0.160	0.585	0.987	2.307879634	2.08E-04
26.355	11.713	0.479	0.960	-0.878	-0.281	2.018433077	1.13E-03
29.284	8.785	-0.930	-0.399	-0.366	-0.917	1.783164218	6.17E-03
32.212	5.857	-0.247	-0.860	0.969	0.510	1.587228371	3.37E-02
35.140	2.928	0.992	0.613	0.125	0.790	1.420720624	1.83E-01
38.069	0.000	0.000	0.707	-1.000	-0.707	1.276775362	1.00E+00

Table 4.3. Analysis Results (Bending and Membrane Fields) of the Dome from Computer Program.

ϕ	Bending Field			Membrane Field		
Deg.	M_ϕ(kNm/m)	N_θ (kN/m)	N_ϕ(kN/m)	M_ϕ(kNm/m)	N_θ(kN/m)	N_ϕ(kN/m)
0.000	1.84E-09	-5.04E-07	64.40024	0	-178.266	-178.266
2.928	-9.11E-08	3.65E-07	-6.95E-07	0	-177.684	-178.382
5.857	6.92E-08	1.45E-05	2.69E-06	0	-175.938	-178.732
8.785	2.61E-06	-3.05E-05	4.40E-06	0	-173.031	-179.317
11.713	-5.61E-06	-0.00039	-4.40E-05	0	-168.965	-180.141
14.642	-7.00E-05	0.001436	-3.00E-05	0	-163.745	-181.208
17.570	0.000262	0.009598	0.00089	0	-157.375	-182.523
20.499	0.001723	-0.0557	-0.0004	0	-149.862	-184.094
23.427	-0.01012	-0.20947	-0.01884	0	-141.213	-185.929
26.355	-0.03745	1.939933	0.031417	0	-131.434	-188.038
29.284	0.351609	3.587165	0.394896	0	-120.536	-190.434
32.212	0.634926	-62.5196	-1.21193	0	-108.525	-193.129
35.140	-11.3108	-21.6017	-7.8743	0	-95.4114	-196.139
38.069	-3.49051	1887.082	38.58738	0	-81.2027	-199.484

Table 4.4. Analysis Results (Total Field) of the Dome from Computer Program.

ϕ		Total Field		
Deg.	Rad	M_ϕ(kNm/m)	N_θ(kN/m)	N_ϕ(kN/m)
0.000	0.000	1.84E-09	-178.266	-113.865
2.928	0.051	-9.11E-08	-177.684	-178.382
5.857	0.102	6.92E-08	-175.938	-178.732
8.785	0.153	2.61E-06	-173.031	-179.317
11.713	0.204	-5.61E-06	-168.966	-180.141
14.642	0.256	-7.00E-05	-163.743	-181.208
17.570	0.307	0.000262	-157.365	-182.522
20.499	0.358	0.001723	-149.918	-184.094
23.427	0.409	-0.01012	-141.422	-185.948
26.355	0.460	-0.03745	-129.495	-188.007
29.284	0.511	0.351609	-116.949	-190.039
32.212	0.562	0.634926	-171.045	-194.341
35.140	0.613	-11.3108	-117.013	-204.014
38.069	0.664	-3.49051	1805.879	-160.896

4.3.5. Example Solving Using FEM

Using the same dimensions and material types, used for the spherical dome analyzed using the developed program, for the analysis of the spherical dome supported by ring beam using SAP 2000 (i.e. finite element analysis method) I have found out the following tabulated analysis results.

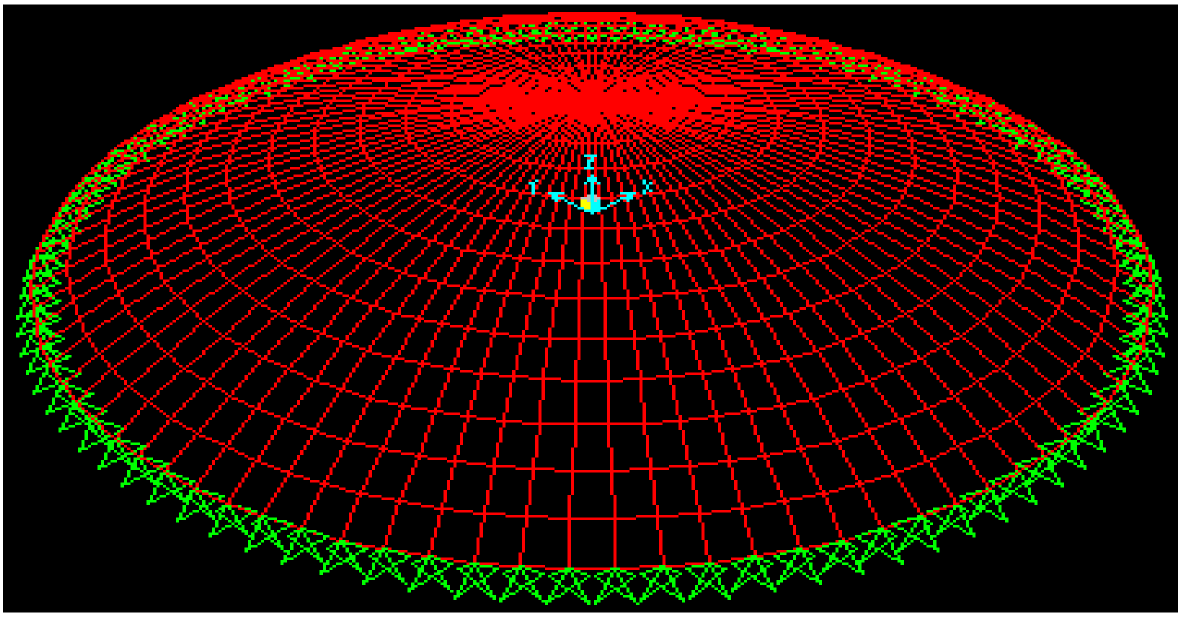


Figure 4.22. SAP 2000 Model.

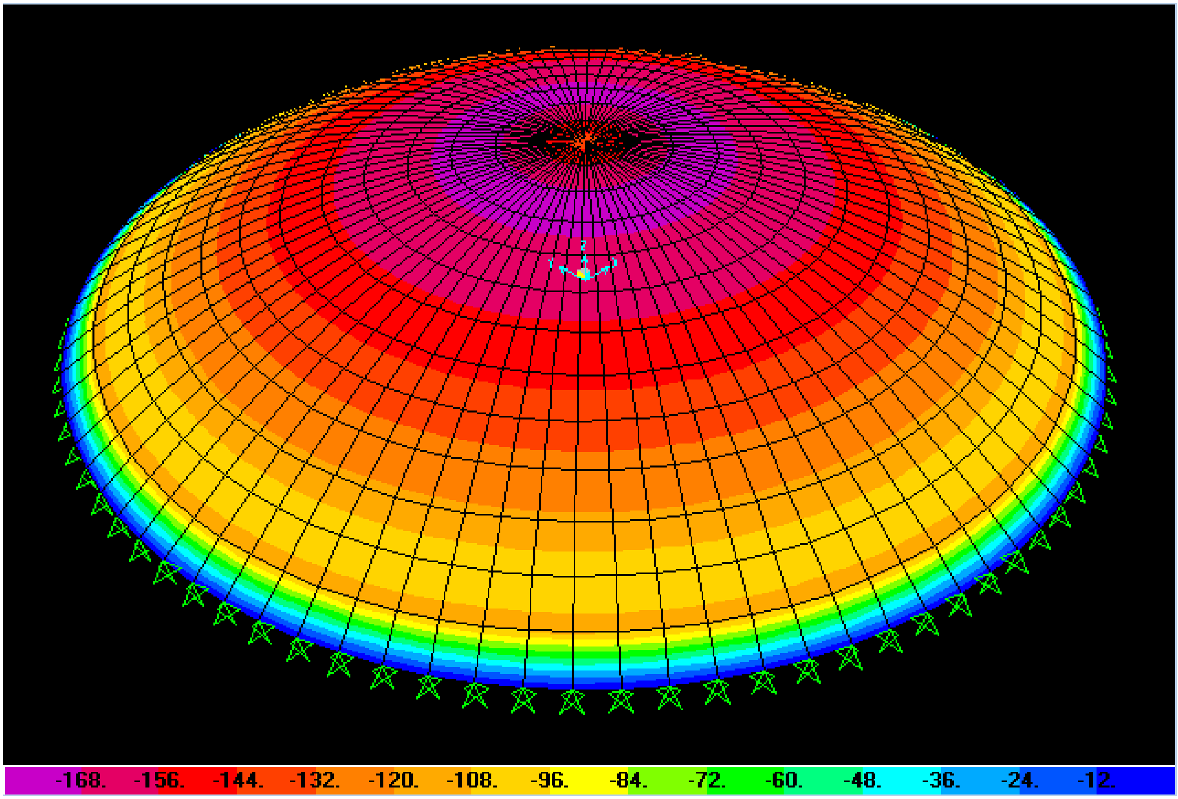


Figure 4.23. Hoop Force Diagram.

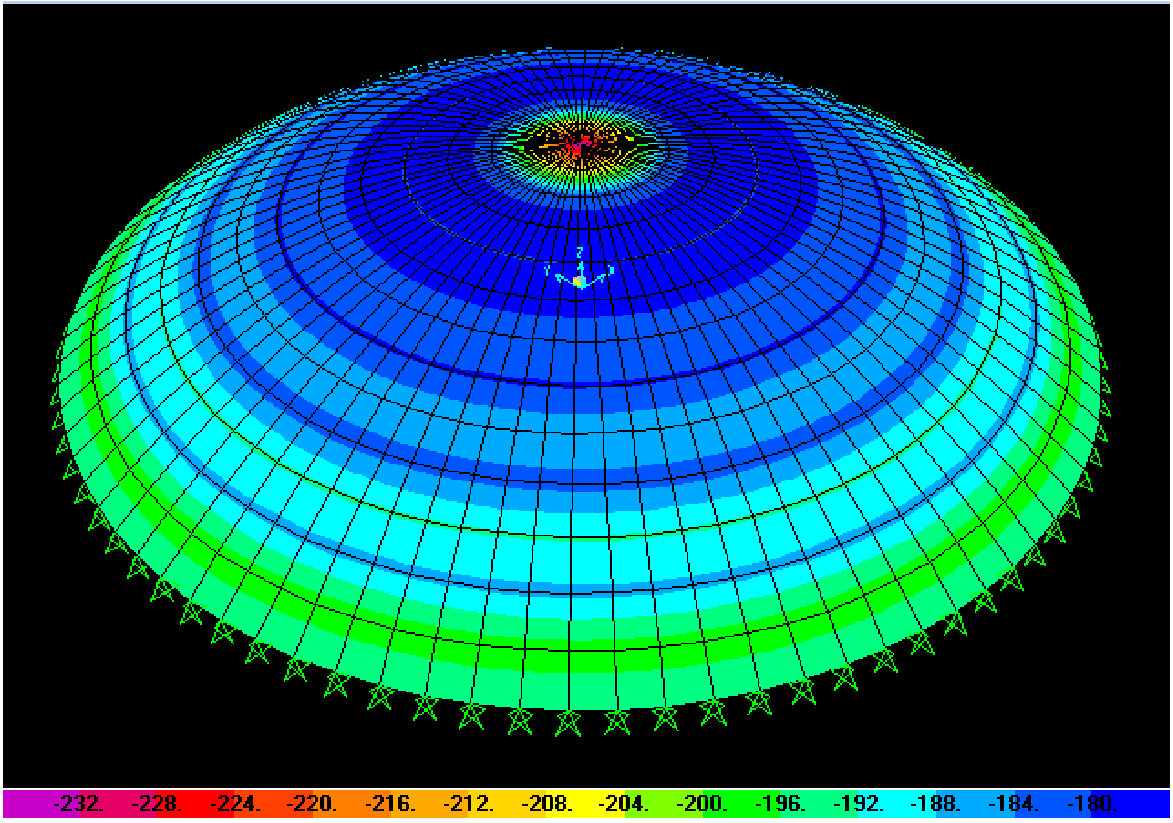


Figure 4.24. Meridional Force Diagram.

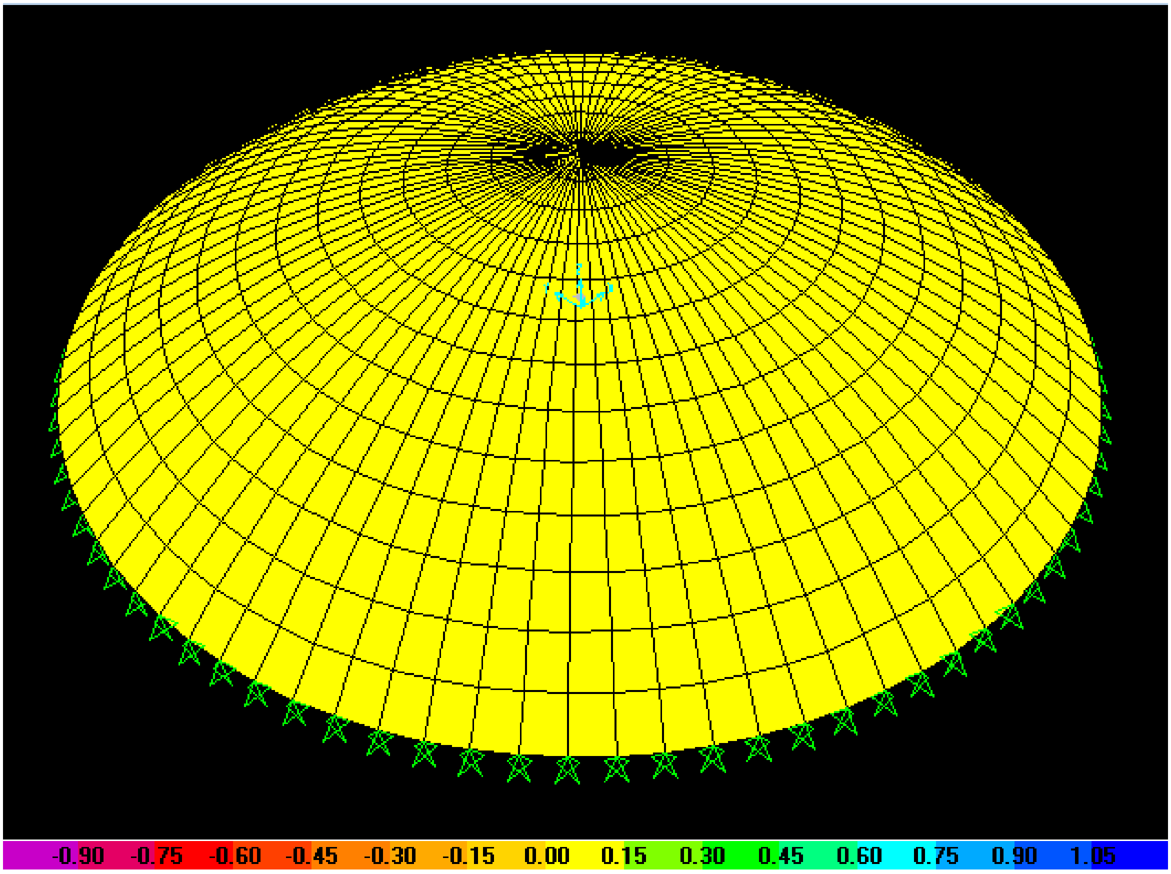


Figure 4.25. Meridional Bending Moment Diagram.

Table 4.5. Analysis Results (Membrane Field) of the Dome from SAP 2000.

ϕ		**Membrane Field**		
Deg.	**Rad**	**M_{ϕ}(kNm/m)**	**N_{θ}(kN/m)**	**N_{ϕ}(kN/m)**
0.000	0.000	0	-144.34	-238.27
2.928	0.051	0	-157.76	-211.49
5.857	0.102	0	-171.86	-183.29
8.785	0.153	0	-171.20	-181.13
11.713	0.204	0	-167.95	-181.21
14.642	0.256	0	-163.02	-181.80
17.570	0.307	0	-157.07	-183.14
20.499	0.358	0	-149.46	-184.31
23.427	0.409	0	-141.46	-186.56
26.355	0.460	0	-130.98	-187.96
29.284	0.511	0	-121.37	-191.28
32.212	0.562	0	-107.38	-192.58
35.140	0.613	0	-96.88	-197.36
38.069	0.664	0	-74.99	-195.30

4.4. Comparison of Result

Using the analysis results of the analytical methods based on the developed program and software analysis based on finite element method given in the previous tables I have plotted the following graphs to compare membrane components of meridional forces, hoop forces and bending moments

Table 4.6. Comparison of Hoop Force (N_θ) Analysis Results.

ϕ		Hoop Force N_θ(kN/m)		
Deg.	**Rad**	***Program***	**SAP 2000**	**Difference**
0.000	0.000	-178.266	-144.34	33.926
2.928	0.051	-177.684	-157.76	19.924
5.857	0.102	-175.938	-171.86	4.078
8.785	0.153	-173.031	-171.20	1.831
11.713	0.204	-168.965	-167.95	1.015
14.642	0.256	-163.745	-163.02	0.725
17.570	0.307	-157.375	-157.07	0.305
20.499	0.358	-149.862	-149.46	0.402
23.427	0.409	-141.213	-141.46	-0.247
26.355	0.460	-131.434	-130.98	0.454
29.284	0.511	-120.536	-121.37	-0.834
32.212	0.562	-108.525	-107.38	1.145
35.140	0.613	-95.4114	-96.88	-1.4686
38.069	0.664	-81.2027	-74.99	6.2127

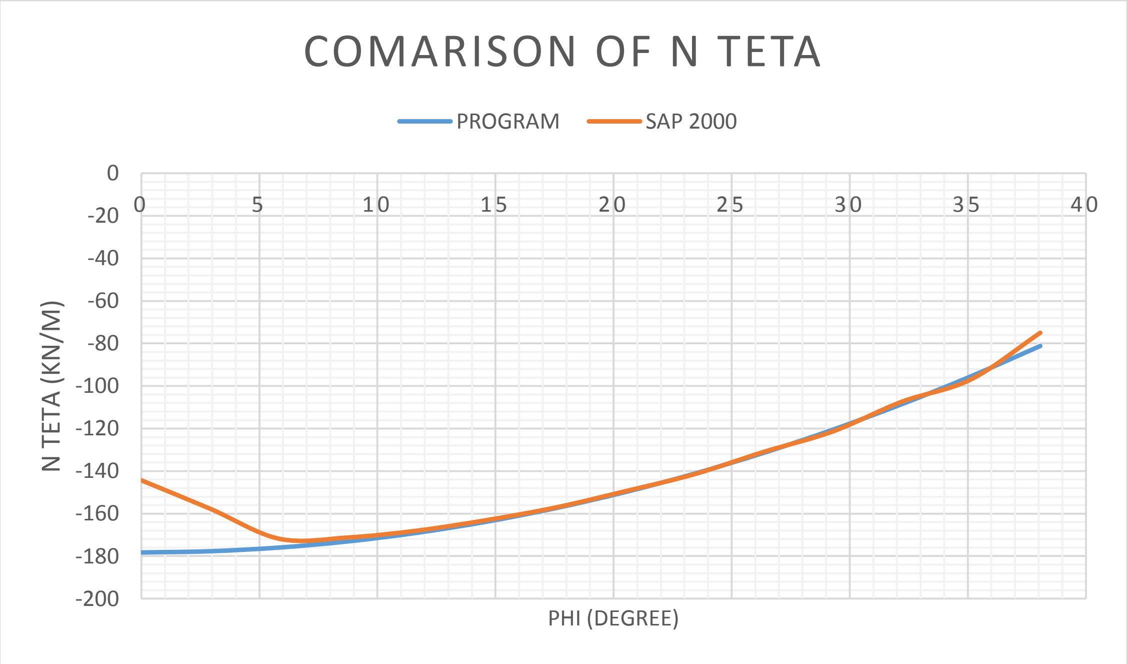


Figure 4.26. Comparison of Hoop Force (N_θ).

Table 4.7. Comparison of Meridional Force ($\boldsymbol{N_\phi}$) Analysis Results.

ϕ		Hoop Force N_ϕ(kN/m)		
Deg.	**Rad**	***Program***	**SAP 2000**	**Difference**
0.000	0.000	-178.266	-238.27	60.004
2.928	0.051	-178.382	-211.49	33.108
5.857	0.102	-178.732	-183.29	4.558
8.785	0.153	-179.317	-181.13	1.813
11.713	0.204	-180.141	-181.21	1.069
14.642	0.256	-181.208	-181.80	0.592
17.570	0.307	-182.523	-183.14	0.617
20.499	0.358	-184.094	-184.31	0.216
23.427	0.409	-185.929	-186.56	0.631
26.355	0.460	-188.038	-187.96	-0.078
29.284	0.511	-190.434	-191.28	0.846
32.212	0.562	-193.129	-192.58	-0.549
35.140	0.613	-196.139	-197.36	1.221
38.069	0.664	-199.484	-195.30	-4.184

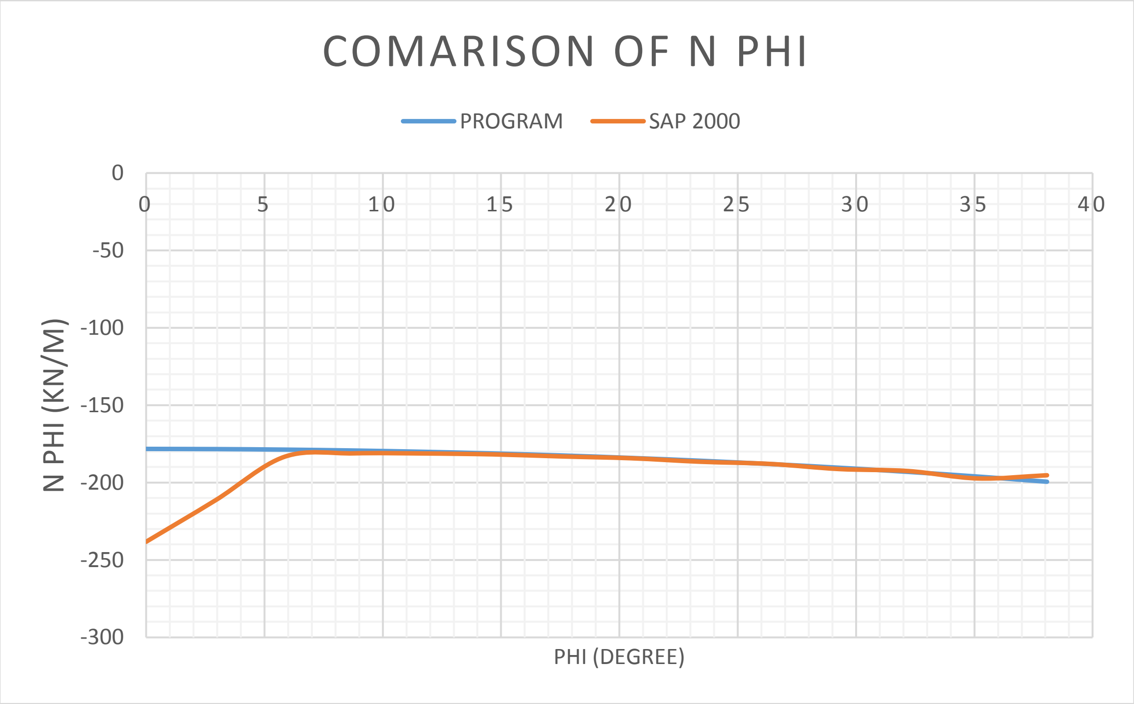


Figure 4.27. Comparison of Meridional Force ($\boldsymbol{N_\phi}$).

Observing the comparison graphs for the stresses, we can conclude that the software and analytical method have some divergence at the bottom and top of the shell.

5.0. CONCLUSION AND RECOMMENDATION

5.1. Conclusions

This study tries to address the problem of tedious manual calculations involved to calculate the internal actions of shells of revolution, specifically spherical reinforced concrete domes, using force method of analysis. The developed program eliminates this problem for symmetrically loaded spherical domes by providing a user interface with different form and dialog boxes to give input to the program and to get outputs from it.

It is observed that analysis results based on finite element methods and force method are very close to each other except for the edges and apex. Therefore, we can conclude that:

1. The developed program can be used as a simple analysis tool to calculate internal actions in spherical domes and to cross check analysis results from other methods.
2. By adjusting the limitations, it is possible to develop and use analysis software based on force method of analysis and design software based on our code.

5.2. Recommendations

From the study that has been carried out, the followings are the recommendations drawn from the results:

1. The school can update this program for domes with other shapes and subjected to unsymmetrical loadings and even upgrade it to other types of shell structures.

With appropriate adjustments

2. As the developed program is easy to apply, the school can use the program to do examples in related courses.
3. Consulting firms can make use of the program to cross check the analysis results of spherical reinforced concrete domes subjected to symmetric loadings which are found from finite element methods.

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APPENDEX

Computer Program

```
Option Explicit On
Imports System.Windows.Forms
Imports Excel = Microsoft.Office.Interop.Excel
#Region "PUBLIC VARIABLES"
    Dim fck 'characteristic compressive strength of concrete
    Dim fcu 'cube compressive strength of concrete
    Dim fcd 'design compressive strength of concrete
    Dim poissonsratio 'poissonsratio of concrete
    Dim fyk 'characteristic yield strength of steel
    Dim fyd 'design yield strength of steel
    Dim Es 'modulus of elasticity of steel
    Dim Econcrete 'modulus of elasticity of concrete
    Dim eyd 'design yield strain of steel
    Dim DL 'dead uniformly distributed load on the dome
    Dim LL 'live uniformly distributed load on the dome
    Dim SuDL 'super dead uniformly distributed load on the dome
    Dim q 'total uniformly distributed load on the dome
    Dim f 'rise of the spherical dome in meter
    Dim D1 'span of the spherical dome in meter
    Dim a 'radius of the spherical dome in meter
    Dim alpha 'half the central angle of the spherical dome in radian
    Dim t 'thickness of the spherical dome in meter
    Dim h 'depth of the ring beam in meter
    Dim b 'width of the ring beam in meter
    Dim ec 'eccentricity b/n center of beam and center line of dome in meter
    Dim r 'radius of projected horizontal circle of the dome in meter
    Dim d2, b2, yo
#End Region
```

```
#Region "CALCULATED GEOMETRIC PARAMETERS"
        D1 = Dialog1.TextBox1.Text
        t = Dialog1.TextBox2.Text
        f = Dialog1.TextBox3.Text
        h = Dialog6.TextBox2.Text
        b = Dialog6.TextBox3.Text
        a = (f / 2) + (D1 * D1 / (8 * f))
        alpha = Math.Asin(D1 / (2 * a))
        d2 = (t / 2) * Math.Cos(alpha)
        yo = (h / 2) - d2
        b2 = (b / 2) - ((t / 2) * Math.Sin(alpha))
        ec = (yo * Math.Cos(alpha)) - (b2 * Math.Sin(alpha))
        r = (a * Math.Sin(alpha)) - ((t / 2) * Math.Sin(alpha))
#End Region
#Region "INFLUENCE COEFFICIENTS"
        poissonsratio = Dialog3.TextBox5.Text
        DL = Dialog1.TextBox2.Text * 25 'DL=Dialog19.TextBox4.Text
        LL = Dialog21.TextBox1.Text
        SuDL = Dialog20.TextBox7.Text
        q = 1.3 * (DL + SuDL) + 1.6 * LL 'the total uniformly distributed load on the dome
        Econcrete = 9.5 * ((fck + 8) ^ (1 / 3)) * 10 ^ 6
        Dim lamda, D10D, D20D, D10R, D20R, D11D, D12D, D22D, D11R, D12R, D22R, D10,
D20, D11, D12, D22, H1, Malpha
        lamda = (3 * (1 - (poissonsratio * poissonsratio)) * (a / t) * (a / t)) ^ (1 / 4)
        D11D = (1 / (t * Econcrete)) * (2 * a * lamda * Math.Sin(alpha) * Math.Sin(alpha))
        D12D = (1 / (t * Econcrete)) * (2 * lamda * lamda * Math.Sin(alpha))
        D22D = (1 / (t * a * Econcrete)) * (4 * lamda * lamda * lamda)
        D11R = (1 / (b * h * Econcrete)) * (r * r) * (1 + ((12 * yo * yo) / (h * h)))
        D12R = - (1 / (b * h^3 * Econcrete)) * (r * r) * (12 * yo)
        D22R = (1 / (b * h^3 * Econcrete)) * (r * r) * 12
        D10D = ((a^2 * q) / (t * Econcrete)) * (((1 + poissonsratio) / (1 + (Math.Cos(alpha)))) -
(Math.Cos(alpha))) * (Math.Sin(alpha))
        D20D = ((a * q) / (t * Econcrete)) * ((2 + poissonsratio) * (Math.Sin(alpha)))
```

```
        D10R = ((Math.Cos(alpha)) + ((12 * yo * ec) / (h^2))) * ((r * r) / (b * h * Econcrete)) * ((-
a * q) / (1 + (Math.Cos(alpha))))
        D20R = - (1 / (b * h^3 * Econcrete)) * (r^2* 12 * ec) * ((-a * q) / (1 + (Math.Cos(alpha))))
        D10 = D10D + D10R
        D20 = D20D + D20R
        D11 = D11D + D11R
        D12 = D12D + D12R
        D22 = D22D + D22R
        H1 = - (((D22 * D10) - (D12 * D20)) / ((D22 * D11) - (D12 * D12)))
        Malpha = - (((D11 * D20) - (D12 * D10)) / ((D22 * D11) - (D12 * D12)))
#End Region
#Region "FORCE"
    Private Function CreateDataSet1() As DataSet
        Dim dataset As DataSet = New DataSet()
        Dim Forces As DataTable = CreateForceTable()
        dataset.Tables.Add(Forces)
        Return dataset
    End Function
    Private Function CreateForceTable() As DataTable
        Dim Forces As DataTable
        Forces = New DataTable("Force")
        AddNewColumn1 (Forces, "System.String", "Phi")
        AddNewColumn1 (Forces, "System.String", "Ntetam")
        AddNewColumn1 (Forces, "System.String", "Ntetab")
        AddNewColumn1 (Forces, "System.String", "Nteta")
        AddNewColumn1 (Forces, "System.String", "Nphim")
        AddNewColumn1 (Forces, "System.String", "Nphib")
        AddNewColumn1 (Forces, "System.String", "Nphi")
        bucklingload = (2 * Econcrete * t * 10 ^ 3 / Math.Sqrt(3 * (1 - poissonsratio *
poissonsratio))) * (t / a) ^ 2
         MsgBox("THE CRITICAL BUCKKLING LOAD =" & vbTab & bucklingload & vbTab
& "kN/m")
        Dim i As Integer
        Dim N As Integer
```

```
        N = InputBox("enter the number of data points required:")
        Dim phi(N) As Single, phid(N) As Single, psi(N) As Single
        Dim Ntetam(N) As Single, Ntetab(N) As Single, Nteta(N) As Single, Nphim(N) As Single, Nphib(N) As Single, Nphi(N) As Single
        For i = 0 To N
            phi(i) = i * alpha / N
            phid(i) = phi(i) * 180 / Math.PI
            psi(i) = alpha - phi(i)
            Ntetab(i) = ((-2 * lamda) * (Math.Sin(alpha)) * (Math.Sin((lamda * psi(i)) - (90 * Math.PI / 180))) * H1 * (Math.Exp(-lamda * psi(i)))) - (((2 * Math.Sqrt(2)) / a) * (lamda * lamda) * (Math.Sin((lamda * psi(i)) - (45 * Math.PI / 180))) * (Malpha) * (Math.Exp(-lamda * psi(i))))
            Nphib(i) = ((-Math.Sqrt(2)) * ((Math.Cos(alpha - psi(i))) / (Math.Sin(alpha - psi(i)))) * (Math.Sin(alpha)) * (Math.Sin((lamda * psi(i)) - (45 * Math.PI / 180))) * H1 * (Math.Exp(-lamda * psi(i)))) - (((2 * lamda) / a) * ((Math.Cos(alpha - psi(i))) / (Math.Sin(alpha - psi(i)))) * (Math.Sin(lamda * psi(i))) * (Malpha) * (Math.Exp(-lamda * psi(i))))
            Ntetam(i) = ((a * q) * ((1 / (1 + (Math.Cos(phi(i))))) - (Math.Cos(phi(i)))))
            Nphim(i) = -((a * q) * ((1 / (1 + (Math.Cos(phi(i)))))))
            Nteta(i) = Ntetab(i) + Ntetam(i)
            Nphi(i) = Nphib(i) + Nphim(i)
            AddNewRow1(Forces, phid(i), Ntetam(i), Ntetab(i), Nteta(i), Nphim(i), Nphib(i), Nphi(i))
        Next i
        Return Forces
        If bucklingload < Math.Max(Math.Abs(Nteta(i)), Math.Abs(Nphi(i))) Then
            MsgBox("CRITICAL BUCKLING LOAD IS EXCEEDED")
        Else : MsgBox("CRITICAL BUCKLING LOAD IS NOT EXCEEDED")
        End If
    End Function
    Private Sub AddNewColumn1(ByRef table As DataTable, _
    ByVal columnType As String, ByVal columnName As String)
        Dim column As DataColumn = _
         table.Columns.Add(columnName, Type.GetType(columnType))
    End Sub
```

```
    Friend Sub AddNewRow1(ByRef table As DataTable, ByRef phid As Single, _
        ByRef Ntetam As Single, ByRef Ntetab As Single, ByRef Nteta As Single, _
ByRef Nphim As Single, ByRef Nphib As Single, ByRef Nphi As Single)
        Dim newrow As DataRow = table.NewRow()
        newrow("Phi") = phid
        newrow("Ntetam") = Ntetam
        newrow("Ntetab") = Ntetab
        newrow("Nteta") = Nteta
        newrow("Nphim") = Nphim
        newrow("Nphib") = Nphib
        newrow("Nphi") = Nphi
        table.Rows.Add(newrow)
    End Sub
    Private Sub Button3_Click(ByVal sender As System.Object, ByVal e As System.EventArgs)
Handles Button3.Click
        Dim ds As New DataSet
        ds = CreateDataSet1()
        Dialog11.DataGridView1.DataSource = ds.Tables("Force")
    End Sub
#End Region
#Region "MOMENT"
    Private Function CreateDataSet() As DataSet
        Dim dataset As DataSet = New DataSet()
        Dim Moments As DataTable = CreateMomentTable()
        dataset.Tables.Add(Moments)
        Return dataset
    End Function
    Private Function CreateMomentTable() As DataTable
        Dim Moments As DataTable
        Moments = New DataTable("Moment")
        AddNewColumn(Moments, "System.String", "Phi")
        AddNewColumn(Moments, "System.String", "Mphim")
        AddNewColumn(Moments, "System.String", "Mphib")
        AddNewColumn(Moments, "System.String", "Mphi")
```

```
        Dim i As Integer
        Dim N As Integer
        N = InputBox("enter the number of data points required:")
        Dim phi(N) As Single, phid(N) As Single, Mphim(N) As Single, Mphib(N) As Single, Mphi(N) As Single, psi(N) As Single
        For i = 0 To N
            phi(i) = i * alpha / N
            phid(i) = phi(i) * 180 / Math.PI
            psi(i) = alpha - phi(i)
            Mphim(i) = 0
            Mphib(i) = (-(a / lamda) * (Math.Sin(alpha)) * (Math.Sin(lamda * psi(i))) * H1 * (Math.Exp(-lamda * psi(i)))) + ((Math.Sqrt(2)) * (Math.Sin((lamda * psi(i)) + (45 * Math.PI / 180))) * (Math.Exp(-lamda * psi(i))) * (Malpha))
            Mphi(i) = Mphib(i) + Mphim(i)
            AddNewRow(Moments, phid(i), Mphim(i), Mphib(i), Mphi(i))
        Next i
        Return Moments
    End Function
    Private Sub AddNewColumn(ByRef table As DataTable, _
    ByVal columnType As String, ByVal columnName As String)
        Dim column As DataColumn = _
         table.Columns.Add(columnName, Type.GetType(columnType))
    End Sub
    Friend Sub AddNewRow(ByRef table As DataTable, ByRef phid As Single, _
    ByRef Mphim As Single, ByRef Mphib As Single, ByRef Mphi As Single)
        Dim newrow As DataRow = table.NewRow()
        newrow("Phi") = phid
        newrow("Mphim") = Mphim
        newrow("Mphib") = Mphib
        newrow("Mphi") = Mphi
        table.Rows.Add(newrow)
    End Sub
    Private Sub Button4_Click(ByVal sender As System.Object, ByVal e As System.EventArgs) Handles Button4.Click
```

```
        Dialog12.Visible = True
        Dim ds As New DataSet
        ds = CreateDataSet()
        Dialog12.DataGridView1.DataSource = ds.Tables("Moment")
    End Sub
#End Region
#Region "DESIGN REINFORCEMENT"
    Private Function CreateDataSet2() As DataSet
        Dim dataset As DataSet = New DataSet()
        Dim Reinforcements As DataTable = CreateReinforcementTable()
        dataset.Tables.Add(Reinforcements)
        Return dataset
    End Function
    Private Function CreateReinforcementTable() As DataTable
        Dim Reinforcements As DataTable
        Reinforcements = New DataTable("Reinforcement")
        AddNewColumn2 (Reinforcements, "System.String", "Phi")
        AddNewColumn2 (Reinforcements, "System.String", "Nteta")
        AddNewColumn2 (Reinforcements, "System.String", "ASNteta")
        AddNewColumn2 (Reinforcements, "System.String", "NbNteta")
        AddNewColumn2 (Reinforcements, "System.String", "SNteta")
        AddNewColumn2 (Reinforcements, "System.String", "Nphi")
        AddNewColumn2 (Reinforcements, "System.String", "ASNphi")
        AddNewColumn2 (Reinforcements, "System.String", "NbNphi")
        AddNewColumn2 (Reinforcements, "System.String", "SNphi")
        AddNewColumn2 (Reinforcements, "System.String", "Mphi")
        AddNewColumn2 (Reinforcements, "System.String", "ASMphi")
        AddNewColumn2 (Reinforcements, "System.String", "NbMphi")
        AddNewColumn2 (Reinforcements, "System.String", "SMphi")
        If Dialog3.ComboBox2.SelectedItem = "C-12/16" Then
            fcu = 16, fck = 12
        ElseIf Dialog3.ComboBox2.SelectedItem = "C-16/20" Then
            fcu = 20, fck = 16
        ElseIf Dialog3.ComboBox2.SelectedItem = "C-20/25" Then
```

```
    fcu = 25, fck = 20
ElseIf Dialog3.ComboBox2.SelectedItem = "C-25/30" Then
    fcu = 30, fck = 25
ElseIf Dialog3.ComboBox2.SelectedItem = "C-30/37" Then
    fcu = 37, fck = 30
ElseIf Dialog3.ComboBox2.SelectedItem = "C-35/45" Then
    fcu = 45, fck = 35
ElseIf Dialog3.ComboBox2.SelectedItem = "C-40/50" Then
    fcu = 50, fck = 40
ElseIf Dialog3.ComboBox2.SelectedItem = "C-45/55" Then
    fcu = 55, fck = 45
ElseIf Dialog3.ComboBox2.SelectedItem = "C-50/60" Then
    fcu = 60, fck = 50
ElseIf Dialog3.ComboBox2.SelectedItem = "C-55/67" Then
    fcu = 67, fck = 55
ElseIf Dialog3.ComboBox2.SelectedItem = "C-60/75" Then
    fcu = 75, fck = 60
ElseIf Dialog3.ComboBox2.SelectedItem = "C-70/85" Then
    fcu = 85, fck = 70
ElseIf Dialog3.ComboBox2.SelectedItem = "C-80/95" Then
    fcu = 95, fck = 80
ElseIf Dialog3.ComboBox2.SelectedItem = "C-90/105" Then
    fcu = 105, fck = 90
End If
Dim yc As Single 'material safety factor for concrete
If Dialog3.ComboBox3.SelectedItem = "I" Then
    yc = 1.5
ElseIf Dialog3.ComboBox3.SelectedItem = "II" Then
    yc = 1.65
End If
fcd = 0.85 * fck / yc 'calculating design compressive strength of concrete
If Dialog14.ComboBox4.SelectedItem = "S-300" Then
    fyk = 300
ElseIf Dialog14.ComboBox4.SelectedItem = "S-400" Then
```

```
        fyk = 400
    End If
    Dim ys As Single 'material safety factor for reinforcement bar
    If Dialog14.ComboBox3.SelectedItem = "I" Then
        ys = 1.15
    ElseIf Dialog14.ComboBox3.SelectedItem = "II" Then
        yc = 1.2
    End If
    fyd = fyk / ys 'calculating the design yield strength of steel
    Es = Dialog14.TextBox4.Text
    eyd = fyd / Es 'calculating the design yield strain of steel
    Dim i As Integer, N As Integer
    N = InputBox("enter the number of data points required:")
    Dim phi(N) As Single, phid(N) As Single, Mphim(N) As Single, Mphib(N) As Single,
Mphi(N) As Single, psi(N) As Single
    Dim Ntetam(N) As Single, Ntetab(N) As Single, Nteta(N) As Single, Nphim(N) As
Single, Nphib(N) As Single, Nphi(N) As Single
    Dim roughmin1, ASmin, bs, ast, db, Smax1, Smax, d, concretecover, X(N), Z(N) As
Single
    Dim ASNphi(N), NbNphi(N), SNphi1(N), SNphi(N), ASNteta(N), NbNteta(N),
SNteta1(N), SNteta(N), ASMphi(N) As Single
    Dim NbMphi(N), SMphi1(N), SMphi(N)
    db = 8, concretecover =15, bs = 1000, roughmin1 = 0.0018
    ast = Math.PI * db * db / 4
    ASmin = roughmin1 * bs * t * 1000
    Smax1 = bs * ast / ASmin
    If 5 * t * 1000 > 450 Then
        If Smax1 > 450 Then
            Smax = 450
        Else : Smax = Smax1
        End If
    ElseIf 5 * t * 1000 < 450 Then
        If Smax1 > 5 * t * 1000 Then
            Smax = 5 * t * 1000
```

```
        Else : Smax = Smax1
        End If
    End If
    d = (t * 1000) - concretecover - (db / 2)
    For i = 0 To N
        phi(i) = i * alpha / N
        phid(i) = phi(i) * 180 / Math.PI
        psi(i) = alpha - phi(i)
        Ntetab(i) = ((-2 * lamda) * (Math.Sin(alpha)) * (Math.Sin((lamda * psi(i)) - (90 * Math.PI / 180))) * H1 * (Math.Exp(-lamda * psi(i)))) - (((2 * Math.Sqrt(2)) / a) * (lamda * lamda) * (Math.Sin((lamda * psi(i)) - (45 * Math.PI / 180))) * (Malpha) * (Math.Exp(-lamda * psi(i))))
        Nphib(i) = ((-Math.Sqrt(2)) * ((Math.Cos(alpha - psi(i))) / (Math.Sin(alpha - psi(i)))) * (Math.Sin(alpha)) * (Math.Sin((lamda * psi(i)) - (45 * Math.PI / 180))) * H1 * (Math.Exp(-lamda * psi(i)))) - (((2 * lamda) / a) * ((Math.Cos(alpha - psi(i))) / (Math.Sin(alpha - psi(i)))) * (Math.Sin(lamda * psi(i))) * (Malpha) * (Math.Exp(-lamda * psi(i))))
        Ntetam(i) = ((a * q) * ((1 / (1 + (Math.Cos(phi(i))))) - (Math.Cos(phi(i)))))
        Nphim(i) = -((a * q) * ((1 / (1 + (Math.Cos(phi(i)))))))
        Nteta(i) = Ntetab(i) + Ntetam(i)
        Nphi(i) = Nphib(i) + Nphim(i)
        Mphim(i) = 0
        Mphib(i) = (-(a / lamda) * (Math.Sin(alpha)) * (Math.Sin(lamda * psi(i))) * H1 * (Math.Exp(-lamda * psi(i)))) + ((Math.Sqrt(2)) * (Math.Sin((lamda * psi(i)) + (45 * Math.PI / 180))) * (Math.Exp(-lamda * psi(i))) * (Malpha))
        Mphi(i) = Mphib(i) + Mphim(i)
        Dim steelstrain(N), X2(N) As Single
        X(i) = 1.25 * (d - (Math.Sqrt((d * d) - ((2 * Math.Abs(Mphi(i)) * 1000000) / (bs * fcd)))))
        steelstrain(i) = 0.0035 * ((d / X(i)) - 1)
        If steelstrain(i) <= 0.01 Then
            If steelstrain(i) >= eyd Then
                Z(i) = d - 0.4 * X(i)
            ElseIf steelstrain(i) < eyd Then
                MsgBox("REINFORCEMENT BAR DOES NOT YIELD")
```

```
    End If
ElseIf steelstrain(i) > 0.01 Then
    X2(i) = d * (0.0035 / 0.0135)
    Z(i) = d - 0.4 * X2(i)
End If
ASMphi(i) = Math.Abs(Mphi(i)) * 1000000 / (Z(i) * fyd)
NbMphi(i) = ASMphi(i) / ast
SMphi1(i) = bs / NbMphi(i)
If NbMphi(i) > 0.1 Then
    If SMphi1(i) > Smax Then
        SMphi(i) = Smax
    Else
        SMphi(i) = SMphi1(i)
    End If
Else
    NbMphi(i) = "NO REINFORCEMENT FOR BENDING"
    SMphi(i) = "INFINITY SPACING"
End If
If Nphi(i) > 0 Then
    ASNphi(i) = Nphi(i) * bs / fyd
    NbNphi(i) = ASNphi(i) / ast
    SNphi1(i) = bs / NbNphi(i)
    If SNphi1(i) > Smax Then
        SNphi(i) = Smax
    Else
        SNphi(i) = SNphi1(i)
    End If
Else
    SNphi(i) = Smax
    NbNphi(i) = bs / Smax
    ASNphi(i) = NbNphi(i) * ast
End If
If Nteta(i) > 0 Then
    ASNteta(i) = Nteta(i) * bs / fyd
```

```
NbNteta(i) = ASNteta(i) / ast
SNteta1(i) = bs / NbNteta(i)
If SNteta1(i) > Smax Then
   SNteta(i) = Smax
Else
   SNteta(i) = SNteta1(i)
End If
Else
   SNteta(i) = Smax
   NbNteta(i) = bs / Smax
   ASNteta(i) = NbNteta(i) * ast
End If
AddNewRow2(Reinforcements, phid(i), Nteta(i), ASNteta(i), NbNteta(i), SNteta(i), _
        Nphi(i), ASNphi(i), NbNphi(i), SNphi(i), _
        Mphi(i), ASMphi(i), NbMphi(i), SMphi(i))
Next i
Return Reinforcements
End Function
Private Sub AddNewColumn2(ByRef table As DataTable, _
ByVal columnType As String, ByVal columnName As String)
   Dim column As DataColumn = _
    table.Columns.Add(columnName, Type.GetType(columnType))
End Sub
Friend Sub AddNewRow2(ByRef table As DataTable, ByRef phid As Single, _
   ByRef Nteta As Single, ByRef ASNteta As Single, ByRef NbNteta As Single, ByRef
SNteta As Single, _
ByRef Nphi As Single, ByRef ASNphi As Single, ByRef NbNphi As Single, ByRef SNphi As
Single, _
ByRef Mphi As Single, ByRef ASMphi As Single, ByRef NbMphi As String, ByRef SMphi
As String)
   Dim newrow As DataRow = table.NewRow()
   newrow("Phi") = phid
   newrow("Nteta") = Nteta
   newrow("ASNteta") = ASNteta
```

```
        newrow("NbNteta") = NbNteta
        newrow("SNteta") = SNteta
        newrow("Nphi") = Nphi
        newrow("ASNphi") = ASNphi
        newrow("NbNphi") = NbNphi
        newrow("SNphi") = SNphi
        newrow("Mphi") = Mphi
        newrow("ASMphi") = ASMphi
        newrow("NbMphi") = NbMphi
        newrow("SMphi") = SMphi
        table.Rows.Add(newrow)
    End Sub
    Private Sub Button5_Click(ByVal sender As System.Object, ByVal e As System.EventArgs)
Handles Button5.Click
        Dialog13.Visible = True
        Dim ds As New DataSet
        ds = CreateDataSet2()
        Dialog13.DataGridView1.DataSource = ds.Tables("Reinforcement")
    End Sub
#End Region
```