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Determination of Parameters for Datum Transformation between WGS 84 and ADINDAN-Ethiopia

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Advisor: Tulu Besha Bedada (PhD)

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School of Civil and Environmental Engineering
Addis Ababa Institute of Technology (AAiT)
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Declaration

I, the undersigned, declare that this thesis work is my original work carried out under the supervision of *Tulu Besha Bedada (PhD)*. It has not been presented for a degree in any other universities and all sources of materials used for the thesis work have been properly acknowledged.

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Abstract

Global Positioning System (GPS) becomes preferable technology in Ethiopia as a result of its numerous advantages over classical methods of surveying. However, it provides geocentric coordinates of a point defined based on a global datum (WGS84) which differs significantly compared to those coordinates realized with respect to a local datum. Ethiopia uses ADINDAN as the recognized local datum and the Geospatial Information Institute (GII) is the only responsible institution to determine the transformation parameters between the national (modified Clarke1880) and global (WGS84) datums. However, the current official transformation parameters in use by the GII are slightly different from the parameters determined by previous researches and adopted in widely used geospatial software packages. In addition to this, the rotation and scale changes are totally ignored from the transformation parameters; while they are inclusive for the region of Sudan. The main objective of this study is to determine datum transformation parameters between WGS84 and modified Clarke1880 reference ellipsoid for the region of Ethiopia using the conventional and conformal transformation models. Orthometric height to ellipsoidal height conversion approaches are also while computing the transformation parameters. Nine co-located Ground Control Points (GCPs) that are defined with reference to the two reference elliposids were obatined from the GII and this study contributed to the realization of one new co-located GCP. Results of the study revealed that, the parameters estimated with orthometric height method and iterative solutions gives better results for all models and the later approach were great. The Molodensky-Badekas seven parameters model with iteration solution is the most stable and suitable transformation model for Ethiopia. The transformation parameters from WGS84 to Clarke1880 with inherent uncertainties are 169.674 m± 0.132 m, 14.801 m ± 0.132 m, -204.841 m ± 0.132 m translation, -0.216 ± 0.114, 0.046 ± 0.127, 2.815 ± 0.173 rotations in the x-, y-, and z- components respectively and 0.288 ± 0.503 scale change. The centroid points used are 4990956.0195 m, 3807764.401 m and 1105492.297 m in x, y, and z axes respectively.

Key words: Transformation Models, Parameters, Orthometric height, WGS-84, ADINDAN-Ethiopia

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Dedicated to my beloved family

Aba, Ema, Jemi, Fewzi, Nu'am, Se'ida, Semira, Meryem

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List of Acronyms

BM	Bench Mark
CORS	Continuously Operating Reference Station
D	Dimension (i.e. 1D, 2D, 3D)
DGPS	Differential Global Positioning System
DMA	Defense Mapping Agency
EGM	Earth Gravitational Model
EPSG	European Petroleum Survey Group
GCP	Ground Control Point
GII	Geospatial Information Institute
GIS	Geographic Information System
GPS	Global Positioning System
ICSM	Intergovernmental Committee of Surveying and Mapping
IHO	International Hydrographic Organization
KTH	Kungl Tekniska Högskolan
LGO	Leica Geo Office
EMGI	Ethiopian Mapping and Geography Institute
MSL	Mean Sea Level
NGA	National Geospatial-Inelegance Agency
NIMA	National Imagery and Mapping Agency
QGIS	Quantum Geographic Information System
RMSE	Root Mean Square Error
UCAA	University College of Addis Ababa
UTM	Universal Transverse Mercator
WGS 84	World Geodetic System 1984

CHAPTER ONE

1. INTRODUCTION

1.1 Background

Knowing the position of the point on earth enables the production of geospatially-referenced data sets that are needed to address economic, social and environmental issues. Coordinates are the most fundamental information in geospatial databases that used to relate the desired data to a specific locality. Coordinates shall be defined consistently and help us to determine the shape of the Earth precisely.

The shape of the real earth is very complex, consisting of continental land masses, open oceans and seas. The oceans cover about 75 percent of the earth's surface, thus could serve as a reference to approximate the shape of the earth globally (Fan, 2007). Globally, the shape of the earth best fits to the Mean Sea Level (MSL) or the geoid (an equipotential surface of the earth). The geoid coincides with the MSL on the oceans and extends below the continents. Historically, the geoid has served as the height reference surface for geodetic levelling. With respect to the MSL, the earth's natural surface reaches its highest point at Mount Everest, about 8,848 m above the MSL and the deepest ocean bottom (in the Pacific) has depth of 11,000 m below the MSL. It is very difficult to formulate a simple mathematical model that can capture a very extreme topographic irregularities that consists of the highest and lowest elevations with respect to the MSL (Hofmann and Moritz, 2006; Fan, 2007).

However, at a global scale the shape of the earth can be mathematically modeled as sphere and ellipsoid that approximate the geoid and simple to perform computations on them. Sphere is used when low accuracy is required with R= 6371 km and ellipsoid of revolution is used when high accuracy positioning is required. Ellipsoid is established by revolving a meridian ellipse around its minor axis that is normally assumed to coincide with the mean axis of earth rotation on which coordinates may be defined, and computations are made. An ellipsoid of revolution is uniquely defined by its semi-major axis (a) and flattening (f); and the semi-minor axis (b), first (e') and second eccentricity (e'^2) also can be computed from them. The reference system can be global or local (regional) based on how the attached ellipsoid is oriented to fit all over the globe or local areas in a least square sense respectively (Fan, 2007).

The World Geodetic System (specifically the WGS 84) is a global 3D earth centered reference system developed by U.S. Defense Mapping Agency (NIMA or currently named as National Geospatial-Intelligence Agency (NGA)) based on Doppler stations and then refined many times to bring it close to ITRF system and by this time it is very close in decimetre accuracy level. It is an official reference system for Global Positioning System (GPS) with semi-major axis (a) = 6378137.000 m and inverse flattening (1/f) = 298.257223563 m, which is a right-handed and orthogonal system whose center coincides with the mass center of the earth. The axes are oriented as the z-axis directed to the mean rotation axis of the earth, x-axis toward the intersection of Greenwich meridian and equator from the origin and y-axis obeys the right hand rule with respect to z and x-axis (see *Figure 2.1*). Due to its numerous advantages over classical methods of surveying, GPS becomes preferable technology in Ethiopia through supporting it by geospatial legal frameworks and practices. However, GPS gives geocentric coordinates of a point determined based on a global datum (WGS84) which differs significantly (up to several hundreds of meters) compared to geodetic coordinates observed with a tie to a local datum. Local datums are primarily needed to satisfy surveying and mapping requirements for specific regions of the earth (El-Rabbany, 2002; MUDC, 2015; Hofmann and Moritz, 2006).

Ethiopia uses ADINDAN (the Modified Clarke1880) as the recognized local datum, defined with the semi-major axis (a) = 6378249.145 m and inverse flattening (1/f) = 293.465 m. It is a non-geocentric datum, which originated in Southern Egypt near Al Nasser city at 22°10'07.110"N astronomic latitude (Φ_o) and 31°29'21.608"E longitude (Λ_o). Currently, this local datum is used by around six African countries, namely Burkina Faso, Senegal, Mali, Cameron and Sudan with slightly different parameters for each country referenced to GPS datum of WGS-84 (IHO, 2008; MUDC, 2015).

In many previous studies and literatures (Gedamu, 2009; Ahmed and Mergia, 2009), the datum used in Ethiopia is called the Adindan datum. However, few scholars claim Adindan is not the name of the datum instead it is the name of the origin near Lake Al Nasser. It also state, it should be called as the Blue Nile Datum (Clifford, 2003). In this study, the most well-known and popular name ADINDAN-Ethiopia or Adindan is used interchangeably as the name of the local datum of Ethiopia.

1.2 Problem Statement

In the present GPS era, transforming coordinates from WGS84 to the ADINDAN-Ethiopia and/or vice versa become common and frequent tasks. It is widely used in geodesy, surveying, photogrammetry and related disciplines. With the increasing exchange of geospatial information, positional information needs to be available in terms of both local and global datums. Hence, there is a definite need for accurate and reliable transformation parameters between datums.

The Geospatial Information Institute (the former Ethiopian Mapping Agency (EMA) or Ethiopian Geospatial Information Agency) is the responsible government institution for the determination, updating and maintaining of the transformation parameters between the national and global datums on top of evaluating the quality of geospatial information. Normally, there should be only one set of parameters in a country to achieve consistency and avoid confusion. However, the current official transformation parameters are slightly different from the parameters used in different projects in Ethiopia and various international institutions and widely used geospatial software in worldwide. Basically, the transformation parameters shall contain the translations (creating common origin for the two systems), rotations (making the reference axes of the two systems parallel) and scaling (creating equal dimensions in the two systems) (Ghilani, 2010). The Geospatial Information Institute (GII) uses the translation parameters only, which are 162 m, 12 m, -206 m in X, Y and Z directions to transform in to local system (MUDC, 2015). Inconsistently, the international institutions and geospatial software packages like ESRI's ArcGIS and EPSG (and QGIS) uses 165 m, 11 m, -206 m and 166 m, 15 m, -204 m respectively. Similarly, the local irrigation projects like Megech and Koga used 162 m, 12 m, -206 m and 163 m, 12 m, -206 m respectively (Dubois, 2011; ESRI, 2012). In addition to this, the rotations and scale change are totally ignored from the transformation parameters. However, the study conducted in Sudan, the neighbor country for Ethiopia indicates the availability of rotations and scale factor whatever its magnitude small (AbdElrahim and Nagi, 2013).

Coordinates of common points in both systems are the main dataset for the computation of transformation parameters. The number, distribution and coordinate quality of these points may influence the accuracy of the transformation and the parameters itself (Elena et.al, 2015). Recently, the transformation parameters also determined using coordinates of five zero order ground control points (GCP's) located at centre, north, south, east and west of the country (Dubois, 2017). These points are not on the historical astro-geodetic network except the western Asossa station. The local coordinates computed through applying the previously known transformation parameters on the GPS coordinates with LGO software then the parameters determined again using these data (Dubois, 2017). Due to this, the residual from such data is error free, which is unacceptable approach and far from the reality in using the classical astro-geodetic technique (i.e. local coordinates (3D) = triangulation (2D) + geometric leveling (1D)). However, the NGA published the Ethiopian transformation parameters as ΔX = 165 m ± 3 m, ΔY = 11 m ± 3 m and ΔZ = -206 m ± 3 m using eight (8) number of collocated stations. The mean solution for Ethiopia and Sudan as ΔX = 166 m ± 5 m, ΔY = 15 m ± 5 m and ΔZ = -204 m ± 5 m using twenty-two (22) stations (IHO, 2008).

On the other hand, identifying the most suitable methodology mainly the transformation models to the specific region and context is a challenging stage in determination of the transformation parameters (Matej et.al, 2015). Because, the accuracy of transformation depends on accuracy of the transformation parameters and the convergence of the transformation models. Previous studies failed to indicate the types of transformation models used for the region of Ethiopia to compute the transformation parameters. Usually, most models use Cartesian coordinates that are of problematic for the local datums due to absence of the ellipsoidal heights.

Most European (Sweden, Australia, Croatia, Netherland, Russia…) (Constantin-Octavian, 2006; Featherstone, 2011; Fan and Chymyrov, 2015; Matej et.al, 2015) and few African (Ghana, Liberia, Sudan…) (AbdElrahim and Nagi, 2013; Solomon, 2013; Ziggah et.al, 2013) countries have conducted numerous empirical studies on datum transformation models and the parameters. However, no published papers exist on methodological aspects of datum transformation performed across the region of Ethiopia, except unpublished MSc thesis conducted in 2009 at KTH-Sweden (Ahmed and Mergia, 2009). It evaluates the accuracy of few models using an iterative solution method with eight collocated GCPs and suggested for Ethiopia to use Molodensky-Badekas with four parameters: i) three translation parameters in X, Y and Z directions as 169.337 m ± 0.155 m, 14.573 m ± 0.155 m and -204.877 m ± 0.155 m respectively, and ii) rotation only in the X-axis accounting for 0.404 " ± 0.127 " (Ahmed and Mergia, 2009). In this previous study, the scale change and rotations in Y and Z-axes are ignored as well as the results were not evaluated against plane coordinates.

1.3 Objectives

The main objective of this thesis was to determine parameters for datum transformation between WGS 84 and modified Clarke1880 reference ellipsoid (sometimes called ADINDAN-Ethiopia) using selected conventional and conformal transformation models. The specific objectives were the following:

- to identify and evaluate the most suitable transformation models;
- to assess the effect of the local orthometric height to ellipsoidal height conversion methods on the transformation parameters; and
- to compare and analyze the newly determined parameters with the existing ones;

1.4 Thesis Outline

This thesis is divided in to five parts: Chapter one offers a general background, statement of the problem and states the objectives of the study. Chapter two presents fundamental concepts associated with determination of parameters for datum transformation (i.e. datum, coordinate systems, map projections and transformation models). It also gives historical overview about the Ethiopian geodetic networks. Chapter three presents the methods s employed for the determination of transformation parameters and evaluation of the transformation models using statistical methods. While chapter four discusses the results of the study such as accuracy of the modes, and effect of using different height system on the transformation parameters. Finally, conclusions and recommendations are presented in the last chapter.

CHAPTER TWO

2. LITERATURE REVIEW

2.1 Coordinate Systems: Cartesian and Geodetic Coordinates

Geodetic datum is a set of constants specifying the coordinate system for a collection of points on the Earth surface. The datum can be global like WGS-84 and local like ADINDAN-Ethiopia. Coordinate systems defined by its origin (3 components), its orientation (3 components, usually the direction cosines of one axis and one component of another axes, and definition of handedness) and its scale. There are many different coordinate systems, based on a variety of geodetic datums, units, projections, and reference systems in use today. Coordinates can be plane or curvilinear, two-dimensional or three-dimensional, or even one-dimensional (Phang and Halim, 2007; Elena et.al, 2015).

In geodesy and surveying, various coordinate systems have been used to define the position of a point on or near the surface of the earth. One of the most commonly used is the rectangular coordinate system which is global, geocentric and terrestrial coordinate system where: O is the mass centre of the earth (geocentre); 'z' goes from O toward the mean position of the North pole; 'x' is along the intersection of the equator and the Greenwich meridian; 'y' follows so that O — xyz forms a right-handed system (Fan, 2007).

Similarly, the position of a ground point P can be defined by its geodetic coordinates (ϕ, λ, h): *Geodetic latitude* ϕ is the angle between the equatorial plane and ellipsoidal normal through P; *Geodetic longitude* λ is the angle between meridian plane at Greenwich and meridian plane at P; *Ellipsoidal height* h is the height of P above the reference ellipsoid along the ellipsoidal normal (Singh, 2002; Fan, 2007).

The figure below (Figure 2.1) give us 3D graphical representation and relationship of these two coordinate systems in detail. Their relationship helps us to conduct forward and inverse coordinate transformations between them. This is one of the fundamental steps in conducting datum transformation and determining the transformation parameters.

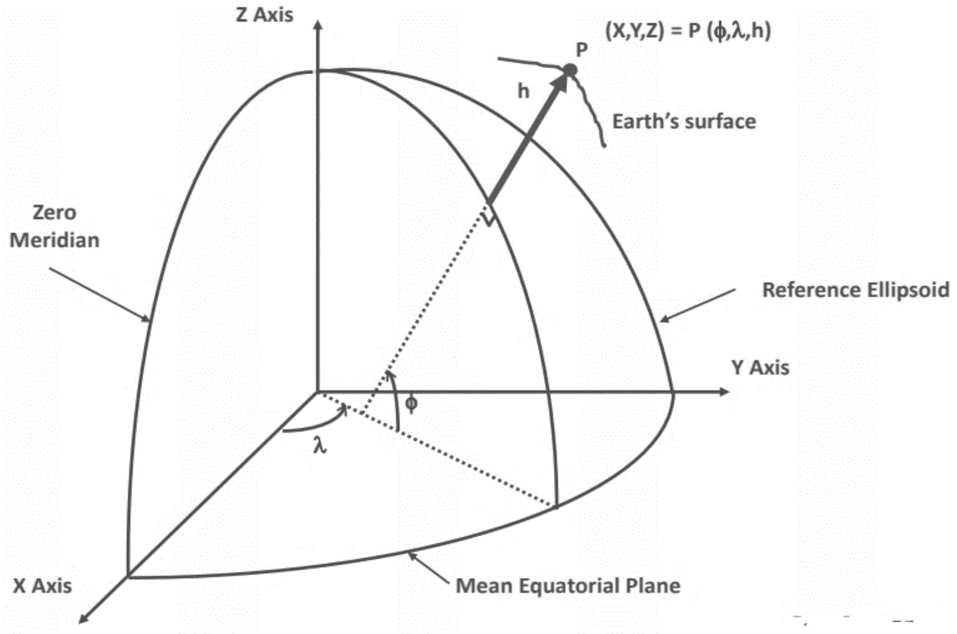

Figure 2.1: Cartesian Coordinates (x, y, z) and Geodetic Coordinates (ϕ, λ, h)

The process of mathematically converting data from one coordinate system in to another is referred to as *coordinate transformation* (Elena et.al, 2015). The relationship between the rectangular Cartesian coordinates (x, y, z) and geodetic coordinates (ϕ, λ, h) can be expressed as:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (N+h)\cos\phi\cos\lambda \\ (N+h)\cos\phi\sin\lambda \\ [N(1-e^2)+h]\sin\phi \end{bmatrix} \quad \text{..… [2.1]}$$

Where 'e' denotes the first eccentricity of the reference ellipsoid and 'N' is the radius of curvature in the prime vertical at latitude ϕ. They can be defined as:

$$N = \frac{a}{\sqrt{1-e^2\sin^2\phi}} \quad \text{..… [2.2]}$$

$$e^2 = 2f - f^2 \quad \text{..… [2.3]}$$

If point ***P*** is exactly on the surface of the reference ellipsoid, i.e. $h = 0$, the transformation from (x, y, z) back to ϕ, λ becomes very simple:

$$\tan\phi = \frac{1}{1-e^2}\frac{z}{\sqrt{x^2+y^2}} \qquad \tan\lambda = \frac{y}{x} \qquad \text{..... [2.4]}$$

If P is not on the reference ellipsoid, i.e. h ≠ 0, λ can still be obtained from the simple relation

$$\tan\lambda = \frac{y}{x} \qquad \text{..... [2.5]}$$

However, the reverse computation of ϕ and h becomes more complicated when h ≠ 0. There exist a number of methods to calculate (ϕ, h) from (x,y,z) whatever they have their own drawback depending on the context. The widely used ones are the following two approaches:

Closed Approximate Formulas (HMK, 1996 in (Fan, 2007))

$$\tan\phi = \frac{z + \frac{a.e^2}{\sqrt{1-e^2}}\sin^3\theta}{p - a.e^2.\cos^3\theta} \qquad h = \frac{p}{\cos\phi} - N \qquad \text{..... [2.6]}$$

Where p, θ, N are calculated as

$$p = \sqrt{x^2+y^2} \qquad \tan\theta = \frac{z}{p\sqrt{1-e^2}} \qquad N = \frac{a}{\sqrt{1-e^2\sin^2\phi}} \qquad \text{..... [2.7]}$$

Iterative Formulas (Heiskanen Och Moritz, 1967 in (Fan, 2007))

$$\tan\phi_n = \frac{z}{p(1-e^2\,\frac{N_{n-1}}{N_{n-1}+h_{n-1}})} \qquad (n = 1,2,3,\ldots) \qquad \text{..... [2.8]}$$

Where N_n is expressed as

$$N_n = \frac{a}{\sqrt{1-e^2\sin^2\phi_n}} \qquad (n = 1,2,3,\ldots) \qquad \text{..... [2.9]}$$

$$h_n = \frac{p}{\cos\phi_n} - N_n \qquad (n = 1,2,3,\ldots) \qquad \text{..... [2.10]}$$

The iteration starts with initial value for h: $h_o = 0$, and ends when changes in the two successive iterations are small enough:

$$\left.\begin{array}{ll} |\emptyset_n - \emptyset_{n-1}| < 10^{-8} & degree \\ |h_n - h_{n-1}| < 10^{-3} & metre \end{array}\right\} \qquad \text{..... [2.11]}$$

2.2 Map Projections

A map projection is a systematic and mathematical transformation of three-dimensional coordinates representing a location on a sphere or an ellipsoid into a two-dimensional location on a plane surface. Any maps cannot be created without map projections (Sahl, 2014). Hundreds of map projections already published and infinite number, which are theoretically possible. However, these projection can be classified on the basis of property (equal-area, conformal, equidistant, azimuthal, and so forth), type of construction (cylindrical, conical, azimuthal, and so forth), or both (DMA, 1989).

The U.S. Army adopted the Universal Transverse Mercator (UTM) in 1947 for accurate rectangular coordinates on large-scale military maps. The UTM is an ellipsoidal Transverse Mercator to which specific parameters, such as central meridians, have been applied. This projection divides the world into 60 longitude zones, where each zone is 6º wide. These zones are valid between 84ºN and 80ºS. For the remaining part of the respective poles, the Universal Polar Stereographic (UPS) projection is used instead. (Snyder, 1987; DMA, 1989). The zone number can be calculated based on the following formula.

$$Z = 1 + INT(\frac{\lambda + 180}{w}) \qquad \text{..… [2.12]}$$

Where Z is the zone number in UTM, INT means the whole integer, rounded down, λ is the longitude in degrees, 180 is the amount of degrees on each hemisphere, and w is the width of a zone in degrees.

The position is given by values of Easting and Northing along with the zone number in meters. The UTM system utilizes false values of Easting and Northing to avoid negative values. The false Easting (FE) is set as 500 000 m west of the zones central meridian. This means that Easting values are given as meters east from a line which is located 500 000 m west of the zones central meridian. Northings are given as meters north of Equator for positions in the Northern Hemisphere. For the Southern Hemisphere, a false Northing (FN) of 10 000 000 m is applied. This means that the negative values of Northings south of the equator are subtracted from the FN (Snyder, 1987; DMA 1989). Ethiopia uses this projection system.

Converting From Geodetic Coordinates to Northing and Easting (UTM)

The Northing and Easting values can be Computed using the Thomas-UTM series with accurate precision (Snyder 1987; DMA 1989):

$$E = FE + k_o v[A + (1 - t + c)\frac{A^3}{6} + (5 - 18T + T^2 + 72C - 58e'^2)\frac{A^5}{120}] \qquad \ldots\ldots [2.13]$$

$$N = FN + k_o[M - M_O + v\, tan\emptyset\, (\frac{A^2}{2} + (5 - T + 9C + 4C^2)\frac{A^4}{24} + (61 - 58T + T^2 + 600C - 330e'^2)\frac{A^6}{720}] \qquad \ldots\ldots [2.14]$$

where all angles are expressed in radians, A, T, C, M, and M_o are auxillary parameters given below, FE and FN are the false Easting and false Northing respectively, e' is the second eccentricity, *v* is the prime vertical radius of curvature using radians, and k_0 is the scale factor of the central meridian.

The auxillary parameters are given as:

$$T = tan^2\emptyset \qquad\qquad C = \frac{e^2cos^2\emptyset}{1-e^2}$$

$$A = (\lambda - \lambda_O)cos\emptyset$$

$$M = a(\left(1 - \frac{e^2}{4} - \frac{3e^4}{64} - \frac{5e^6}{256} - \cdots\right)\emptyset - \left(\frac{3e^2}{8} + \frac{3e^4}{32} + \frac{45e^6}{1024} + \cdots\right)\sin(2\emptyset) + \left(\frac{15e^4}{256} + \frac{45e^6}{1024} + \cdots\right)\sin(4\emptyset) - \left(\frac{35e^6}{3072} + \cdots\right)\sin(6\emptyset) + \cdots$$

$$M_o = a(\left(1 - \frac{e^2}{4} - \frac{3e^4}{64} - \frac{5e^6}{256} - \cdots\right)\emptyset_o - \left(\frac{3e^2}{8} + \frac{3e^4}{32} + \frac{45e^6}{1024} + \cdots\right)\sin(2\emptyset_o) + \left(\frac{15e^4}{256} + \frac{45e^6}{1024} + \cdots\right)\sin(4\emptyset_o) - \left(\frac{35e^6}{3072} + \cdots\right)\sin(6\emptyset_o) + \cdots$$

where the angles are still expressed in radians, e is the first eccentricity (Equation 2.3), $\emptyset_o$ and λ_o is the latitude and longitude of natural origin respectively:

$$\emptyset_o = 0$$

$$\lambda_o = (3 + W(Z-1) - 180)$$

Converting From Northing and Easting (UTM) to Geodetic Coordinates

The latitude and longitude values can be calculated using the Thomas-UTM series (Snyder 1987; DMA 1989):

$$\emptyset = \emptyset_1 - \frac{v_1 tan\emptyset_1}{p_1}\left[\frac{D^2}{2} - (5 + 3T_1 + 10C_1 - 4C_1^2 - 9e'^2)\frac{D^4}{24} + (61 + 90T_1 + 298C_1 + 45T_1^2 + 25e'^2 - 3C_1^2)\frac{D^4}{24}\right] \quad \ldots\ldots [2.15]$$

$$\lambda = \lambda_o + \frac{1}{cos\emptyset_1}\left[D - (1 + 2T_1 + C_1)\frac{D^3}{6} + (5 - 2C_1 + 25T_1 - 3C_1^2 + 8e'^2 + 24T_1^2)\frac{D^5}{120}\right]\ldots [2.16]$$

Where - and are latitude and longitude respectively, in radians, $\emptyset_1$, D, T_1, C_1, v_1 and p_1 are auxillary parameters given below along with μ_1, e_1, M_1, and M_o:

$$T_1 = tan^2\emptyset_1 \qquad C_1 = e'^2cos^2\emptyset_1 \quad v_1 = \frac{a}{\sqrt{1-e^2sin^2\emptyset_2}} \qquad p_1 = \frac{a(1-e^2)}{\sqrt{(1-e^2sin^2\emptyset_2)^3}}$$

$$\emptyset_1 = \mu_1 + \left(e_1\frac{3}{2} + e_1^3\frac{3}{2} + \cdots\right)\sin(2\mu_1) + \left(e_1^2\frac{21}{16} - e_1^4\frac{55}{32} + \cdots\right)\sin(4\mu_1) + e_1^3\frac{151}{96} + \cdots)\sin(6\mu_1) + e_1^4\frac{1097}{512} + \cdots)\sin(4\mu_1) + \cdots$$

$$e_1 = \frac{1-\sqrt{1-e^2}}{1-\sqrt{1+e^2}}$$

$$\mu_1 = \frac{M_1}{a(1 - e^2\frac{1}{4} - e^4\frac{3}{64} - e^6\frac{5}{256} - \cdots}$$

$$\lambda_o = 3 + 6(Z-1) - 180)$$

$$\emptyset_o = 0$$

where angles are still expressed in radians, e and e' are the first and second eccentricity respectively, a is the major-axis of the ellipsoid, M_0 is given in the above Equation, FN and FE stands for false Northing and false Easting respectively, k_o is the scale factor of the central meridian, and Z indicates the zone number.

2.3 Transformation Models

Transformation models are mathematical relationships that employed in accomplishing coordinate transformation from global to local system and/or vice versa using common coordinates in both reference systems (Constantin-Octavian, 2006). The transformation can be three-dimensional (3-D), two-dimensional (2-D) or one-dimensional (1-D), depending on the given requirement and datasets. However, this study and review focused more on the 3D models, which is more advantageous than others especially in this GPS era.

There are several transformation models that can be categorized in to *conventional techniques*, which considered as a mathematical model that addresses the problem with geometrical explanations that are under consideration or the *numerical techniques* that address the problem by taking the mathematical model and its properties into consideration to obtain the best result that satisfies the accuracy that is required. The well-known transformation models like Bursa-Wolf, Molodensky-Badekas, and Block shift models and others are considered as conventional and the conformal ones. The numerical models include multiple regression equation (MRE), least squares collocations (LSC), artificial neural networks (ANN) and minimum curvature surface (MCS) (El-Shambaky et.al, 2018). This review focused more on the conventional and conformal transformation models specifically the Block shift (Geocentric), Bursa-Wolf, Molodensky-Badekas, Standard and Abridged Molodenskey that are more tested, used and recommended by several countries, software packages, DMA and ICSM. Many countries in the world tried to conduct empirical test of this and others models for their specific country with specific context (number of data, distribution, quality and others). In Ethiopia, most of them are tested for the first time by this thesis with better number of collocated points and approaches. The data constraints, their merits and priority of testing the models leads the study to focus more on these conventional and conformal models.

The advantages of conformal (conventional) transformation models discussed by (Burford, 1985), (Harvey, 1986), (DMA, 1991) and others. These models become more popular due to the small number of parameters involved, the simplicity of the model, which is more easily implemented into software and the fact that it is adequate for relating two coordinate systems in the case when they are homogenous (no local distortion in scale or orientation).

2.3.1 Block Shift (Geocentric) Model

The block shift also knows as the geocentric model (or the molodensky three parameters model), which simply applies a three-dimensional origin shift of the geometrical centre of the reference ellipsoids, with little regard for any scale changes or rotations. Therefore, it is coarse, but also extremely simple to implement (Featherstone, 2011). The Cartesian co-ordinates from the initial datum are simply added to the origin shift (translation) and then converted to curvilinear coordinates on the new datum. This model expressed as

$$X_t = T + X_s \qquad \text{...... [2.17]}$$

where X_t and X_s are two column vector sets of collocated 3D coordinates in two different systems assuming as source (X_s) and target (X_t), $T = (\Delta X, \Delta Y, \Delta Z)^T$ denotes the three translation parameters.

For this thesis, the model expressed as

$$X_{\text{AD-Ethiopia}} = T + X_{\text{WGS 84}}$$

These target (in ADINDAN-Ethiopia), translations (Δ) and source (in WGS 84) vectors can be expanded as

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{AD-E} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS\ 84}$$

As three equations (one point data) are sufficient to solve the above equation, but the least square solution can be obtained for data points more than one. Thus, the equation for the least square solution becomes:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{AD-E} - \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS\ 84}$$

where

$$\mathrm{L}=\begin{bmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \end{bmatrix}_{\mathrm{t}} - \begin{bmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \end{bmatrix}_{\mathrm{s}}, \qquad \mathrm{A}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \mathrm{DX}=\begin{bmatrix} \Delta \mathrm{X} \\ \Delta \mathrm{Y} \\ \Delta \mathrm{Z} \end{bmatrix} \quad \text{and} \quad \varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix}$$

The equation to be solved by least square adjustment is:

For one point: $\mathrm{L}_{3\mathrm{x}1} - \varepsilon_{3\mathrm{x}1} = \mathrm{A}_{3\mathrm{x}3}\mathrm{DX}_{3\mathrm{x}1}$

For more than one points:

$$\begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ \vdots \\ . \\ L_n \end{bmatrix} - \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \vdots \\ . \\ \varepsilon_n \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ . \\ A_n \end{bmatrix} - \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$$

where n is the number of points

For this and others models, the least square estimates of the transformation parameters, the variance covariance matrix, the unit weight standard errors and the residuals can be thus computed by equations (3.3) to (3.5) using coefficient (A) and observation (L) matrices.

2.3.2 Bursa–Wolf Model

The Bursa–Wolf (Bursa, 1962; Wolf, 1963) also known as the Helmert model is one of the well-known seven-parameter conformal (similarity) three-dimensional model for transforming three-dimensional Cartesian co-ordinates between datums, which especially suited to satellite datums on a global scale (Krakiwsky and Thomson, 1974). This comprises an origin shift from the geocentre in three-dimensional space, rotation of the vector positions and a scale change (*ds*). This also can be expressed as

$$X_t = T + S \cdot R \cdot X_s \qquad \text{......} \ [2.18]$$

where X_t and X_s are two column vector sets of collocated 3D coordinates in two different systems assuming as source (X_s) and target (X_t), $T = (\Delta X, \Delta Y, \Delta Z)^T$ denotes three translation parameters, S refers the scale parameter (1+*ds*) often expressed as part per million (ppm), and the 3 × 3 rotation matrix R contains three rotation parameters. Obviously, to determine the seven parameters, the number of collocated coordinates X_t, and X_s should be greater than or equal to three (Fan, 2007). However, one problem with the Bursa-Wolf model is that the adjusted parameters are highly correlated when a network of points used to determine the parameters covers only a small portion of the earth (DMA, 1991; Deakin, 2006).

Similarly, this also can be expressed as:

$$X_{\text{AD-Ethiopia}} = T + S \cdot R \cdot X_{\text{WGS 84}}$$

$$\begin{bmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \end{bmatrix}_{\mathrm{AD-E}} = \begin{bmatrix} \Delta\mathrm{X} \\ \Delta\mathrm{Y} \\ \Delta\mathrm{Z} \end{bmatrix} + (1+\delta\mathrm{s})\mathrm{R}(\mathrm{r_x r_y r_z}) \begin{bmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \end{bmatrix}_{\mathrm{WGS\ 84}}$$

The equation to be solved by least square adjustment

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{AD-E} - \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS\ 84} - \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} = + \begin{bmatrix} 1 & 0 & 0 & X_W & 0 & -Z_W & Y_W \\ 0 & 1 & 0 & Y_W & Z_W & 0 & -X_W \\ 0 & 0 & 1 & Z_W & -Y_W & 0 & \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \delta s \\ r_x \\ r_y \\ r_z \end{bmatrix}$$

Where

$$\mathrm{L}=\begin{bmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \end{bmatrix}_{\mathrm{t}} - \begin{bmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \end{bmatrix}_{\mathrm{s}}, \qquad \mathrm{A}=\begin{bmatrix} 1 & 0 & 0 & X_W & 0 & -Z_W & Y_W \\ 0 & 1 & 0 & Y_W & Z_W & 0 & -X_W \\ 0 & 0 & 1 & Z_W & -Y_W & X_W & 0 \end{bmatrix}, \qquad \mathrm{DX}=\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \delta s \\ r_x \\ r_y \\ r_z \end{bmatrix} \quad \text{and} \quad \varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix}$$

For a number of points: $\mathrm{L_{3nx1}} - \varepsilon_{\mathrm{3nx1}} = \mathrm{A_{3nx7}DX_{7x1}}$

2.3.3 Molodensky-Badekas Model

The Molodensky-Badekas model (Molodensky *et al.*, 1962; Badekas, 1969) is also a seven-parameter conformal transformation of three-dimensional Cartesian co-ordinates between datums, but is more suited to the transformation between terrestrial and satellite datums (Krakiwsky and Thomson, 1974). Similarly, this model also comprises an origin shift from the geocentre in three-dimensional space, rotation of the vector position and a scale change. However, additional three parameters of the centroid of the network is required and this leads the number of transformation parameters to become from seven to ten.

$$X_t = T + X_m + S \cdot R \cdot (X_s - X_m) \qquad \text{...... [2.19]}$$

where X_m defines the Cartesian centroid (the mean) coordinates of the points in the source geodetic datum (in metres). In this case, the rotations and the scale are only applied on the vector ΔXsm between any terrain point (P_i) and the centroid point (P_o) (El-Shambaky et.al, 2018). Unlike to Bursa-Wolf model, this model removes the extraordinary correlation between parameters by relating the parameters to the barycenter (centroid) of the network or any convenient point within the network.

In principle, the Bursa-Wolf and Molodensky-Badekas models should give us the same results when the same data are used to determine the respective sets of transformation parameters. The adjusted coordinates, baseline lengths, scale factor, rotation angles, their variance-covariance matrices and the a posteriori variance factor computed by this model are the same as those from the corresponding Bursa-Wolf solution. However, the translations are different and their precisions are generally an order of magnitude smaller (Harvey, 1986; Deakin, 2006). The difference is due to the different scaling and rotating of the centroid of the network. It should be noted that when working with global network of points, the Molodensky-Badekas model has centroid coordinates equal the centre of the ellipsoid ($X_m = Y_m = Z_m = 0$) and therefore reduces to the Bursa-Wolf model.

The single 3x3 rotation matrix is simplified from three separate rotation matrices by assuming that each axial rotation is differentially small, typically less than five arc seconds for most geodetic networks(Featherstone, 2011), thus permitting binomial series expansions of the sine and cosine terms for radian measure. Thus, the rotations for both (Bursa-Wolf and Molodenskey-Badekas) models is an orthogonal matrix that is composed of three successive rotations that can be expressed as:

$$\underset{3\times3}{\mathbf{R}} = \mathbf{R}(\alpha_1,\alpha_2,\alpha_3) = \mathbf{R}_3(\alpha_3)\cdot\mathbf{R}_2(\alpha_2)\cdot\mathbf{R}_1(\alpha_1)$$

$$= \begin{bmatrix} \cos\alpha_3 & \sin\alpha_3 & 0 \\ -\sin\alpha_3 & \cos\alpha_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos\alpha_2 & 0 & -\sin\alpha_2 \\ 0 & 1 & 0 \\ \sin\alpha_2 & 0 & \cos\alpha_2 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_1 & \sin\alpha_1 \\ 0 & -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix}$$

where α_1, α_2, α_3 denote the three rotation angles around the *x*, *y* and *z* axes respectively.

Similarly, this also can be expressed as:

$$X_{\text{AD-Ethiopia}} = T + X_m + S \cdot R \cdot (X_{\text{WGS 84}} - X_m)$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\mathrm{AD-E}} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + (1+\delta s)R\left(r_x r_y r_z\right)\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{\mathrm{WGS\ 84}} - \begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix}$$

The equation to be solved by least square adjustment

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{AD-E} - \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS\,84} - \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & X_W - X_c & 0 & -(Z_W - Z_c) & Y_W - Y_c \\ 0 & 1 & 0 & Y_W - Y_c & Z_W - Z_c & 0 & -(X_W - X_c) \\ 0 & 0 & 1 & Z_W - Z_c & -(Y_W - Y_c) & X_W - X_c & 0 \end{bmatrix} \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \delta s \\ r_x \\ r_y \\ r_z \end{bmatrix}$$

Where

$$L = \begin{bmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \end{bmatrix}_{\mathrm{t}} - \begin{bmatrix} \mathrm{X} \\ \mathrm{Y} \\ \mathrm{Z} \end{bmatrix}_{\mathrm{s}}, \; A = \begin{bmatrix} 1 & 0 & 0 & X_W - X_c & 0 & -(Z_W - Z_c) & Y_W - Y_c \\ 0 & 1 & 0 & Y_W - Y_c & Z_W - Z_c & 0 & -(X_W - X_c) \\ 0 & 0 & 1 & Z_W - Z_c & -(Y_W - Y_c) & X_W - X_c & 0 \end{bmatrix}, \; DX = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \delta s \\ r_x \\ r_y \\ r_z \end{bmatrix} \;\&\; \varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix}$$

The centroids can be computed as $X_c = \frac{1}{n}\sum_i^n X_i$, $Y_c = \frac{1}{n}\sum_i^n Y_i$ and $Z_c = \frac{1}{n}\sum_i^n Z_i$

2.3.4 Standard Molodensky Model

The Standard Molodensky transformation model is a five-parameter transformation model linking the translations δX, δY, δZ between the X, Y, Z Cartesian axes, and changes in the ellipsoid parameters δa and δf with changes in the curvilinear coordinates $\delta\emptyset$, δλ, δh (Deakin, 2004).

$$\delta\emptyset = \frac{1}{M+h}\left\{-\delta X sin\emptyset cos\lambda - \delta Y sin\emptyset sin\lambda + -\delta Z cos\emptyset + \frac{Ne^2 sin\emptyset cos\emptyset}{a}\delta a + sin\emptyset cos\emptyset \left[\frac{M}{1-f}N(1-f)\right]\delta f\right\} \quad \ldots\ldots [2.20]$$

$$\delta\lambda = \frac{1}{(N+h)cos\emptyset}(-\delta X sin\lambda + \delta Y cos\lambda) \quad \ldots\ldots [2.21]$$

$$\delta h = \delta X cos\emptyset cos\lambda + \delta Y cos\emptyset sin\lambda + \delta Z sin\emptyset - \frac{a}{N}\delta a + N(1-f)sin^2\emptyset\delta f \quad \ldots\ldots [2.22]$$

Where

$\emptyset\ and\ \lambda$ denote latitude and longitude respectively.

$\Delta\emptyset$, $\Delta\lambda$ and Δh is changes in latitude, longitude and ellipsoidal heights of a common point between the two datums respectively.

a and b represents the semi major axis and semi minor axis respectively.

$e = \frac{\sqrt{a^2+b^2}}{a}$ and $f = \frac{a-b}{a}$ denotes first eccentricity and flattening respectively

Δa and Δf change in semi major axis and in flattening between the two datums.

N is the radius of curvature of prime vertical of origin datum described in Equation [2.2],

M is the radius of curvature of meridian, which can be mathematically defined as

$$M = \frac{a(1-e^2)}{\sqrt{1-e^2 sin^2\emptyset}} \qquad \text{...... [2.23]}$$

For this thesis, the Standard Molodensky transformation model expressed as

$$\delta\emptyset(M+h) = \left\{ -\Delta X sin\emptyset cos\lambda - \Delta Y sin\emptyset sin\lambda + \Delta Z cos\emptyset + \frac{Ne^2 sin\emptyset cos\emptyset}{a}\delta a \right.$$
$$\left. + sin\emptyset cos\emptyset \left[\frac{M}{1-f}N(1-f)\right]\delta f \right\}$$

$$\delta\lambda(N+h)cos\emptyset = (-\Delta X sin\lambda + \Delta Y cos\lambda)$$

$$\delta h = \Delta X cos\emptyset cos\lambda + \Delta Y cos\emptyset sin\lambda + \Delta Z sin\emptyset - \frac{a}{N}\delta a + N(1-f)sin^2\emptyset\delta f$$

The equation to be solved by least square adjustment, arranged as

$$L = \begin{bmatrix} \delta\emptyset(M+h) - \dfrac{e^2 sin\emptyset_W cos\emptyset_W}{(1-e^2 sin^2\emptyset_W)^{\frac{1}{2}}}\delta a + sin\emptyset_W cos\emptyset_W(M\dfrac{a}{b} + N\dfrac{b}{a})\delta f \\ \delta\lambda(N+h)cos\emptyset \\ \delta h - -(1-e^2 sin^2\emptyset_W)^{\frac{1}{2}}\delta a + \dfrac{a(1-f)}{(1-\varepsilon^2 sin^2\emptyset_W)^{\frac{1}{2}}} + sin^2\emptyset_W\delta f \end{bmatrix} - \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix}$$

$$A = \begin{bmatrix} -sin\emptyset_W cos\lambda_W & -sin\emptyset_W sin\lambda_W & cos\emptyset_W \\ -sin\lambda_W & cos\lambda_W & 0 \\ cos\emptyset_W cos\lambda_W & cos\emptyset_W sin\lambda_W & sin\emptyset_W \end{bmatrix} \quad \text{and} \quad DX = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$$

2.3.5 Abridged Molodensky Model

The Abridged Molodensky model is a modified version of standard molodensky transformation model obtained by certain simplifying assumptions. This transformation model equations do not certain the ellipsoidal heights *h* of points to be transformed (Deakin, 2004).

$$\delta\emptyset = \frac{1}{M}\{-\delta X sin\emptyset cos\lambda - \delta Y sin\emptyset sin\lambda + \delta Z cos\emptyset + (f\delta a + a\delta f)sin2\emptyset\} \qquad \text{...... [2.24]}$$

$$\delta\lambda = \frac{1}{Ncos\emptyset}(-\delta X sin\lambda + \delta Y cos\lambda) \qquad \text{...... [2.25]}$$

$$\delta h = \delta X cos\emptyset cos\lambda + \delta Y cos\emptyset sin\lambda + \delta Z sin\emptyset - \delta a + (f\delta a + a\delta f)sin^2\emptyset \qquad \text{...... [2.26]}$$

This model used to compute the ellipsoidal height of the local system using δh. Because the height from local system is orthometric height H (from MSL) through geometrical leveling.

The choice of the most appropriate transformation model may influenced by the factors as whether the model is to be applied to a small area, or over a large region; whether one (or both) networks have significant distortions; whether the networks are three-dimensional (3-D) in nature, 2-D or even 1-D and the accuracy required (Ziggah et.al., 2016).

The Abridged Molodensky transformation model expressed as

$$\delta\emptyset = \frac{1}{M}\{-\Delta X sin\emptyset cos\lambda - \Delta Y sin\emptyset sin\lambda + \delta Z cos\emptyset + (f\delta a + a\delta f)sin2\emptyset\}$$

$$\delta\lambda = \frac{1}{Ncos\emptyset}(-\Delta X sin\lambda + \Delta Y cos\lambda)$$

$$\delta h = \Delta X cos\emptyset cos\lambda + \Delta Y cos\emptyset sin\lambda + \Delta Z sin\emptyset - \delta a + (f\delta a + a\delta f)sin^2\emptyset$$

Similarly, these equation to be solved by least square adjustment, arranged as

$$L = \begin{bmatrix} \delta\emptyset M - (f\delta a + a\delta f)sin2\emptyset \\ \delta\lambda N cos\emptyset_W \\ \delta h + \delta a - (f\delta a + a\delta f)sin^2\emptyset \end{bmatrix} - \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix},$$

$$A = \begin{bmatrix} -sin\emptyset_W cos\lambda_W & -sin\emptyset_W sin\lambda_W & cos\emptyset_W \\ -sin\lambda_W & cos\lambda_W & 0 \\ cos\emptyset_W cos\lambda_W & cos\emptyset_W sin\lambda_W & sin\emptyset_W \end{bmatrix} \quad \text{and} \quad DX = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}$$

2.4 The Ethiopian Geodetic Networks

The Geospatial Information Institute (GII) or by its former name as the Ethiopian Mapping and Geography Institute (EMGI) and later as the Ethiopian Mapping Agency (EMA) and the Ethiopian Geospatial Information Agency (EGIA) is one of the oldest geospatial organization in Ethiopia with different structure and numerous responsibilities. This organization was established in 1954 under the reign of Emperor Haile Selassie I as a department in the Imperial Ethiopian Ministry of Education to produce graphic materials and geography textbooks for educational purposes. During its early stages, the GII benefited from States assistance, both in terms of financial assistance from the *Point Four Programs* and a comprehensive base mapping operation of the country undertaken by the United States Army. It later become as was a department under the Ministry of Finance and

Cooperation with the responsibility of cartographic mapping and remote sensing activities in the country (EMGI, 1961; Blackwell, 1962).

The Ethiopian Geodetic Control Project, in operation from 1957 to 1961, completed basic horizontal and vertical control networks throughout approximately 120,000 square miles of west-central Ethiopia, the area comprising the Ethiopian watershed of the Blue Nile River. The survey establishing the networks were conducted with first-order methods and procedures by a combined organization of Ethiopia and United States personnel, financed by a joint Imperial Ethiopia Government and United State Government cooperative service [Green Book, 1961]. In this project, 905 permanent benchmarks of the level net and 370 triangulation stations are established (*see figure 2.2*). Twenty-three triangulation stations were established as bench marks, and these stations furnished the control for the vertical-angle adjustment. The elevations so obtained for the triangulation stations were then used to compute sea-level corrections for the horizontal directions. The triangulation network includes first, second, and third-order stations, 11 Laplace azimuths, 9 invar-taped bases, and 78 Tellurometer measurements. The triangulation was computed on the Clarke spheroid of 1880 and is based on the Adindan, or 30th meridian, datum. The two stations held fixed in geographic position for this adjustment were obtained from the Survey Department Sudan, E. Africa, in October of 1958. The vertical control network is referred to mean sea level at Alexandria, Egypt, and is made up of over 1976 miles of spirit leveling. The completed level lines consist of a line from the Sudanese connection eastward to the eastern edge of the triangulation and a large loop southward through Addis Ababa which connects back into the first line. The final elevations obtained as the result of the least-squares adjustment are probably good within ±3 meters, except those for stations above 13º latitude. The elevations of these stations may be good only to ±10 meters (EMGI, 1961; Blackwell, 1962). Refer the below figure (figure 2.2) which shows the historical astrogeodetic triangulation and level networks that conducted in the year of 1957 to 1961.

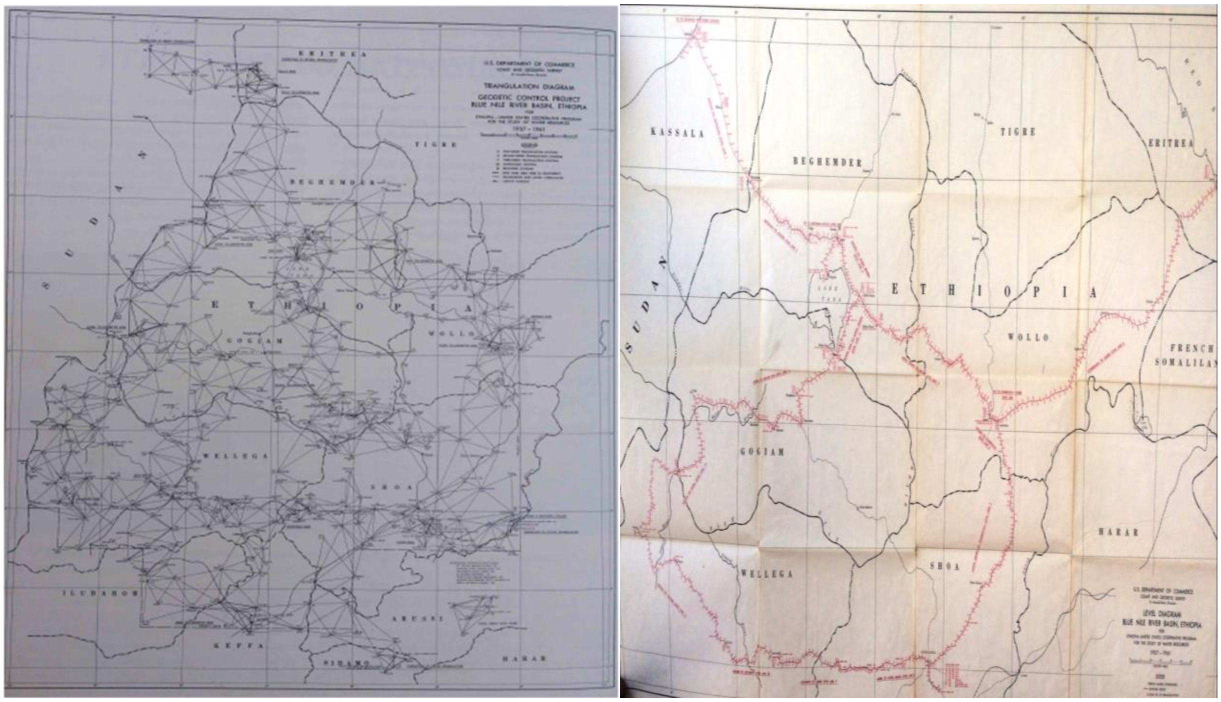

Figure 2.2: The Blue Nile Geodetic Triangulation (left) and level (right) Networks

Source: (EMGI, 1961)

In Ethiopia, the geodetic control networks (or points) categorized as *zero, first, second, third and fourth* orders depending on the duration of measurement and quality of such points. Everyone uses these points as a benchmark for any geospatial tasks in the country. According to the Geodetic survey team leader and experts, currently in Ethiopia about 5 CORS, 30 zero order, 150 first order, 1300 second order, 5000 third order and above 5000 fourth order geodetic infrastructures found.

CHAPTER THREE

3. METHODOLOGY

3.1 Study Area

Ethiopia is a country located in the horn of Africa, bordered with Eritrea in the north, Sudan and South Sudan in the west, Somalia and Kenya in the south and Somalia and Djibouti in the east. It geographically defined as between the latitudes 3°N and 15°N and the longitudes between 33°E and 48°E. In the last few decades, a number of projects, organizations and individuals who uses geospatial information has become increased. For instance, the current urban legal and rural cadastre projects can be taken as one of the core projects in the country with huge budget and resources. In this project, GPS is the preferable instrument rather than using the classical techniques that give a positional data with the global reference system (the WGS 84). However, these datas should be transformed to local datum (ADINDAN-Ethiopia) for any practical applications.

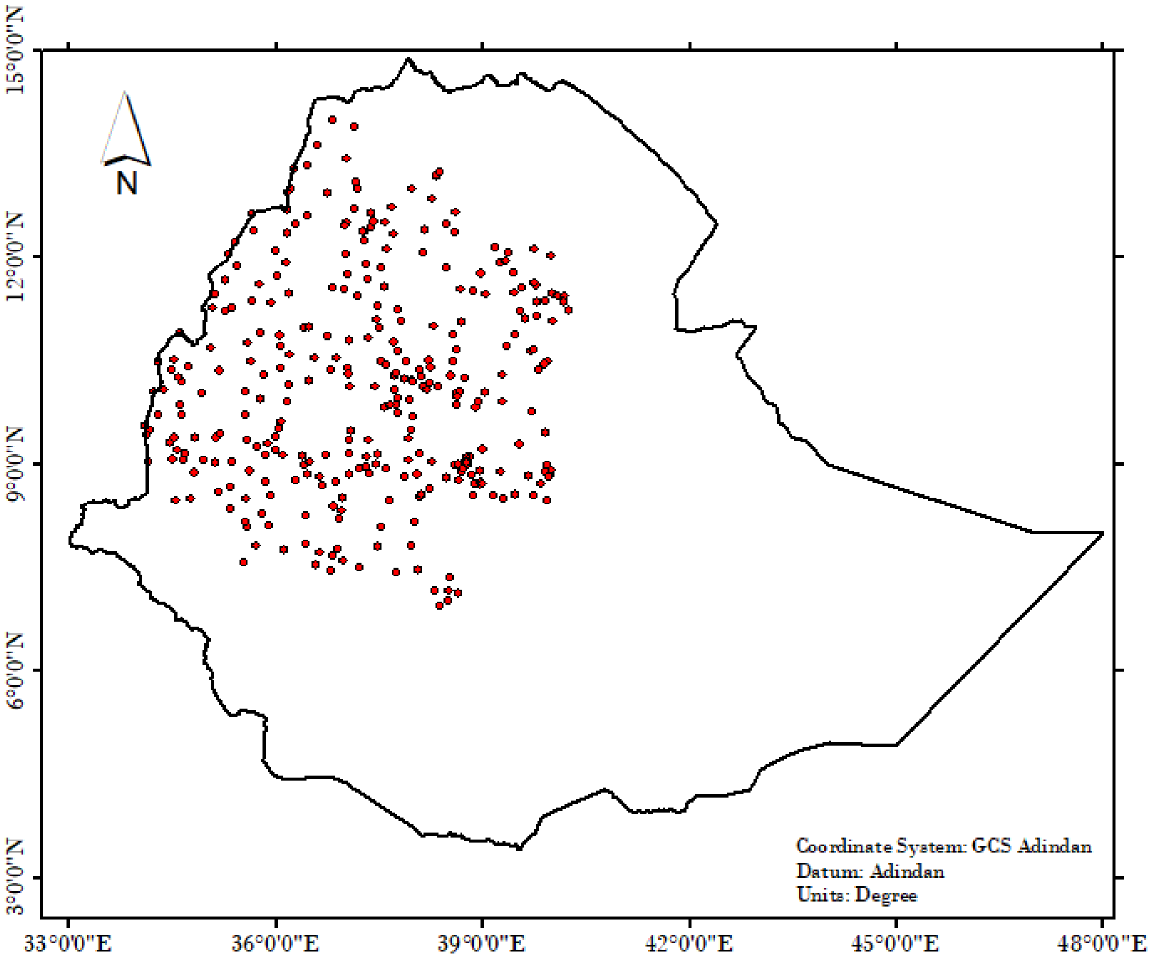

Figure 3.1: Location map (with Astro geodetic points) of the study area

3.2 Datasets and Sources

Collocated coordinates of identical points needed to compute the transformation parameters between any two datums. This study intends to use coordinates of nine (9) collocated ground control points in the WGS 84 and ADINDAN-Ethiopia datums, which obtained from Geospatial Information Institute (GII). In addition, the researcher had established one additional DGPS GCP (Bench Mark 1 Observatory UCAA) on the historical astrogeodetic triangulation networks, which found in Addis Ababa city to increase the number of collocated points and the quality of this work. There was no collocated points in the city before that used for parameter computation. Static GPS surveying with precise ephemeris in post-processing stage had been conducted. Besides, EGM08 used to compute the geoidal heights for estimating the local heights. The shape files (i.e. Ethiopia) used for visualization and making the location map of the study area.

Table 3.1: The collocated GCPs in WGS 84 and ADINDAN-Ethiopia (h and H in meter)

Station Name	ADINDAN-Ethiopia			WGS 84			
	Latitude	Longitude	H	Latitude	Longitude	h	Remark
Gondar Astro	12.522	37.418	2137.529	12.522	37.419	2136.321	48 hrs.
Tie	11.808	38.239	3113.883	11.809	38.240	3112.919	48 hrs.
Gore	8.154	35.550	2024.864	8.155	35.550	2018.633	48 hrs.
Asosa	10.049	34.551	1665.675	10.050	34.552	1661.317	48 hrs.
Tulu Amara	9.102	37.321	3049.974	9.103	37.321	3045.124	24 hrs.
Elin	9.859	38.621	3323.447	9.860	38.622	3318.881	24 hrs.
Wolonkomi	9.031	38.264	2503.431	9.032	38.265	2497.400	24 hrs.
Tabor	7.037	38.467	1821.203	7.038	38.468	1812.325	24 hrs.
Metema	12.951	36.174	820.010	12.952	36.167	816.544	48 hrs.
BM1 Obs. UCAA	9.033	38.764	2442.775	9.034	38.7649	2436.689	5 hrs.

Source: GII in [Ahmed A.W. and Mergia W.A. 2009]; Green Book, 1961 and GPS Survey, 2019

The datasets specifically the local dataset crosschecked with the Green book to detect mistakes. Then, corrections made on the naming of the stations and height values. These data used for parameter determination and evaluation, validation of the models and parameters in the country whatever the data more on the central-western part. The following figure used to visualize the location and distribution of such collocated points.

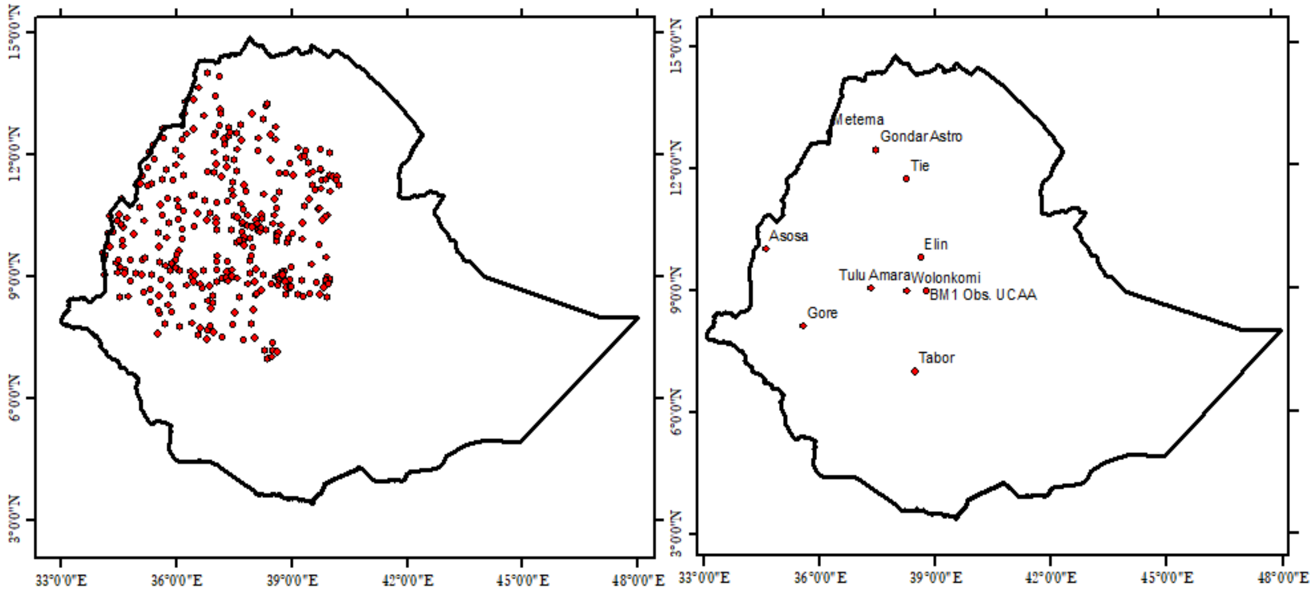

Figure 3.2: Astro-geodetic control points (left) and collocated data sets used to derive the transformation parameters and evaluation of the models (right)

3.3 Materials and Software

The materials used to collect all necessary data, to process and analyze in this thesis are:

For data collection;

- Leica Viva AS10 Differential GPS for static observation
- Google Earth and HGPS for identifying location of selected astrogeodetic points

For data processing, analysis and presenting the results;

- Computer
- Leica Geo Office (LGO) software and OPUS online used for post processing,
- MATLAB program used to develop codes (scripts) to handle the coordinates, the transformation models and used to plot the point residual errors,
- Arc GIS 10.6 for processing, analysis and visualization,
- Selected Microsoft Office (MS Word, MS Excel and MS PowerPoint) for analysis, presenting and reporting the findings.

3.4 Methods and Procedures

In this study, desk review, ground survey and computation approaches for determination of the parameters for datum transformation between WGS 84 and ADINDAN-Ethiopia using the selected transformation models with least square technique had employed as the main methodology. After collecting all necessary data in both systems, the following two basic tasks and steps used to achieve the objective of this thesis.

3.4.1 Computation of Transformation Parameters and Accuracy Assessment

Step 1: Transforming Geodetic Coordinates to Cartesian Coordinates system

First, the curvilinear geodetic coordinates (ϕ, λ, h) of the common points in both the WGS84 and ADINDAN-Ethiopia systems will be converted into Cartesian coordinates (X, Y, Z) using the above formula [2.1]. However, the height of the local system is Orthometric height (H) not ellipsoidal height (h). Therefore, in this study the Orthometric height (H) converted to ellipsoidal height (h) using orthometric height method (OHM), abridged molodensky, EGM08 and iteration. In addition, assuming both heights as zero is also the other approach used in this study to compute the Cartesian coordinates for the local system.

Step 2: Computing the Transformation Parameters

Then, the parameters for the datum transformation between WGS 84 and ADINDAN-Ethiopia has been computed using different local height assumptions and selected transformation models via least square adjustment. Here, the results are the transformation parameters between the two system.

Step 3: Accuracy Assessment

The accuracy of the transformation parameters were assessed in two ways: direct and plane surface analysis. The direct way is using the least square method (i.e. residual (error), standard deviation and standard errors). This shows the overall quality of the parameters.

In plane surface analysis, the distortion of the transformation models had assessed using the grid based transformation analysis techniques (point error residual analysis). For this task, the following steps are used:

Step 3.1: Transformation of the dataset coordinates using computed transformation parameters (using the step 2 results) from WGS 84 to ADINDAN-Ethiopia:

$$(X,Y,Z)_{\text{WGS-84}} \rightarrow \text{transformation} \rightarrow (X,Y,Z)_{\text{WGS-84} \rightarrow \text{ADINDAN-Ethiopia}}$$

Step 3.2: Conversion of the dataset transformed Cartesian coordinates to plane coordinates in the projection using the Equation [2.13 and 2.14]:

$$(X,Y,Z)_{\text{WGS 84}\rightarrow\text{ADINDAN-Ethiopia}} \rightarrow (\phi,\lambda)_{\text{WGS 84}\rightarrow\text{ADINDAN-Ethiopia}} \rightarrow (\text{E},\text{N})_{\text{WGS 84}\rightarrow\text{ADINDAN-Ethiopia}}$$

Step 3.3: Conversion of the local dataset transformed Cartesian coordinates to plane coordinates in the projection using the Equation [2.13 and 2.14]:

$$(\phi, \lambda)_{\text{ADINDAN-Ethiopia}} \rightarrow (\text{E}, \text{N})_{\text{ADINDAN-Ethiopia}}$$

Step 3.4: Then, the plane difference (residuals shift) between the two datasets will be computed to assess the systematic error (distortion) for E and N components:

$$\text{resE} = \text{E}_{\text{ADINDAN-Ethiopia}} - \text{E}_{\text{WGS 84} \rightarrow \text{ADINDAN-Ethiopia}}$$

$$\text{resN} = \text{N}_{\text{ADINDAN-Ethiopia}} - \text{N}_{\text{WGS 84} \rightarrow \text{ADINDAN-Ethiopia}}$$

Using the plane difference (residuals) for easting and northing components (resE and resN), the positional (horizontal) residual can be computed using as

$$\text{res}_{\text{pos}} = \sqrt{\text{resE}^2 + \text{resN}^2} \qquad \ldots\ldots [3.1]$$

These resE, resN and resPos used to measure the quality of transformation model. Statistical measures (minimum, maximum, mean, standard deviation and root mean square error) were calculated only for ‘resPos’ as it gives positional difference (residual) and unites both Easting and Northing residual components. The standard deviation (SD) and root mean square errors (RMSE) were calculated using the equations:

$$SD = \sqrt{\frac{\sum_{i=1}^{n}\left(res_{pos}^{i} - \mu\right)^2}{n-1}} \quad \text{and} \quad RMSE = \sqrt{\frac{\sum_{i=1}^{n}\left(res_{pos}^{i}\right)^2}{n-1}} \qquad \ldots\ldots [3.2]$$

where µ is the mean of res_{pos} : $\mu = \frac{1}{n}\sum_{i=1}^{n} res_{pos}^{i}$, n is the number of collocated points.

3.4.2 Orthometric to Ellipsoidal Height Conversion Methods

In estimating the parameters for datum transformation, it is obvious to use the transformation models that are only accept the Cartesian coordinates. As described in the Equation [2.1], the Cartesian coordinates can be computed only from the latitude, longitude and ellipsoidal heights of a given system. Nevertheless, the local systems does not have ellipsoidal heights (h) rather than the orthometric heghts (H) which is the challenge in estimating the transformation parameters. The following methods and assumptions were used in this thesis to get Cartesian coordinates of the local system i.e. ADINDAN-Ethiopia (Ahmed A.W. and Mergia W.A. 2009; Ziggah, Y.Y. et.al, 2016).

Abridged Molodensky Model

As described in section 2.3.5, this model used to estimate the height difference between the global and local datum ellipsoids. In this study, the ellipsoidal height of ADINDAN-Ethiopia computed as

$$h_{ADINDAN-Ethiopia} = h_{WGS\,84} - \Delta h \qquad \ldots\ldots [3.3]$$

where $h_{WGS\,84}$ is the GPS ellipsoidal height based on WGS 84 computed using the formula [2.20], Δh is the ellipsoidal height correction factor to transform $h_{WGS\,84}$ to $h_{ADINDAN-Ethiopia}$. Here, $h_{ADINDAN-Ethiopia}$ is the ellipsoidal height to be determined for the ADINDAN-Ethiopia Clarke 1880 ellipsoid.

Orthometric Height Method

The Orthometric Height Method (OHM) employs a direct conversion of the local latitudes, longitudes and Orthometric Height (*H*) to its associated Cartesian coordinates using the standard forward equation. The above equation [2.1] had modified as

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (N + H)\cos\phi\cos\lambda \\ (N + H)\cos\phi\sin\lambda \\ [N(1 - e^2) + H]\sin\phi \end{bmatrix} \qquad \ldots\ldots [3.4]$$

Here, the only change in this formula is using orthometric heights of the local system instead of using the ellipsoidal height for the local system.

Earth Gravitational Model (EGM08)

The Earth Gravitational Model (EGM2008) released by the US National Geospatial Intelligence Agency that used for determination of the earth's gravity field. It is a spherical harmonic model complete to degree and order of 2159, with additional spherical harmonic coefficients extending up to degree of 2190 and order 2159. This offers an unprecedented level of spatial resolution and accuracy for the recovery of gravity and potential fields over the whole globe. Now, the accuracy of the EGM08 derived geoid model is at the decimetre level: ±15 cm with 5 arcminute (~ 9 km) resolution at global scale. In general, its accuracy varies geographically depending on the quality and completeness of the terrestrial, airborne and marine gravity data included in the model development (Bedada, 2010).

In this study, the EGM08 used to compute the geoidal undulation of the collocated points. The geometric relation between the geoid, ellipsoid and the topography was then applied to estimate the ellipsoidal height for the local geodetic network.

$$h=H+N \qquad \text{...... [3.5]}$$

where h is the ellipsoidal height to be estimated; H is the orthometric height within the local geodetic network and N is the geoidal undulation value determined based on the EGM 2008.

Iteration and Ignoring Heights Method

The iteration method also can be used to determine the nearest local ellipsoidal heights. The steps used in the iteration is as follows:

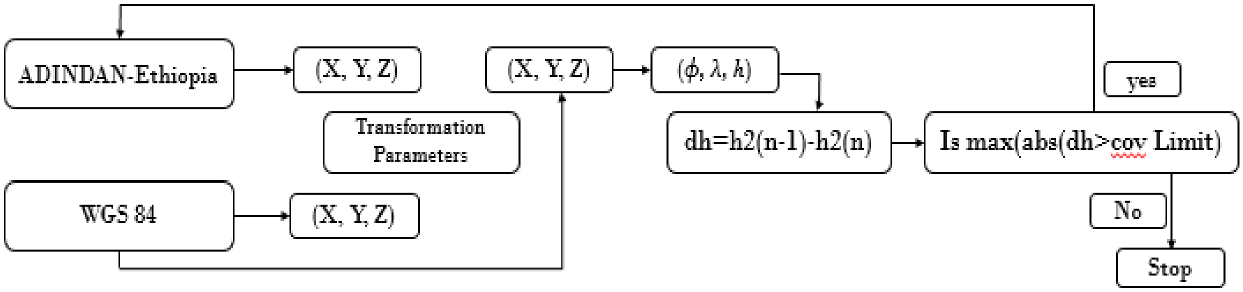

Figure 3.3: Iteration method computation steps

Besides, the heights of the both system can be totally ignored or taking as null (assuming on the reference ellipsoid) to compute the Cartesian coordinates of both systems.

3.5 Least Square Estimation of the Transformation Parameters and Residuals

Least square (LS) method, also known as least square approximation, in statistics, a method for estimating the true value of some quantity based on the consideration of errors in observations or measurements. A wide range of least squares adjustment techniques are known and utilized in geodetic sciences for coordinate transformations between geocentric and non-geocentric datums. In this thesis, the least square method used to estimate the transformation parameters and residuals in the *Block Shift (Geocentric), Bursa-Wolf, Molodensky-Badekas, Standard and Abridged Molodensky models*. The matrix of unknown parameters can be estimated (assuming a unit weight ($C^{-1} = 1$) due to absence of the errors data for the local and global systems) as

$$DX = (A^T C^{-1} A)^{-1} A^T C^{-1} A \qquad \ldots\ldots [3.6]$$

where DX is vector of unknown parameters, A is the design matrix and L is the observation matrix.

The variance covariance matrix of the transformation parameters can be estimated as:

$$C_{xx} = \sigma_o{}^2 (A^T C^{-1} A)^{-1} \qquad \ldots\ldots [3.7]$$

And, the standard error of the parameters can then be obtained from the variance covariance matrix as:

$$\sigma = \sqrt{C_{xx}} \qquad \ldots\ldots [3.8]$$

Where $\sigma_o{}^2 = \frac{\varepsilon^T C^{-1} \varepsilon}{3n-m}$, $\varepsilon = A\,DX - L$ are the posterior standard error and the residuals respectively. n is the number of observation and m is the number of unknowns.

In the previous section (section 2.3), the selected transformation models for this thesis tried to be reviewed from theorotical and mathematical point of view and prepared for least square adjustement to estimate the parameters, variance covarince, standard errors and others. Specifically, the observation, the unknowns and the coefficient matrices prepared to be ready for handling with coding in the MATLAB program. These tasks minimize the complexity and leads to be time effective.

CHAPTER FOUR

4. RESULTS AND DISCUSSIONS

4.1 Parameters Estimation between WGS 84 and ADINDAN-Ethiopia

The parameters for datum transformation between WGS84 and ADINDAN-Ethiopia were estimated with ten collocated datasets and selected transformation models. The models are mathematically defined in a Cartesian coordinates system and hence the models require only ellipsoidal heights as an input for the vertical components. However, the local system does not have the ellipsoidal heights and to solve challenges related to the transformation of leveled height/orthometric heights/ to ellipsoidal heights, the following five methods and assumptions were used in the computation of the transformation parameters (Ahmed and Mergia, 2009; Ziggah, et.al, 2016). These approaches are:

- the Orthometric height method (OHM),
- Ignoring heights of both systems,
- the EGM08 height,
- the Abridged Height and
- the Iteration method (dh=< 0.001m)

On the basis of these assumptions, the transformation models applied on the collocated datasets to check and detect extreme values (outliers) using the error vector in *xyz* system. The residuals of these models were almost similar, and the Molodensky-Badekas (seven parameters) and Standard Molodensky (three parameters) are selected as sample models for visualization (see, Fig. 4.1).

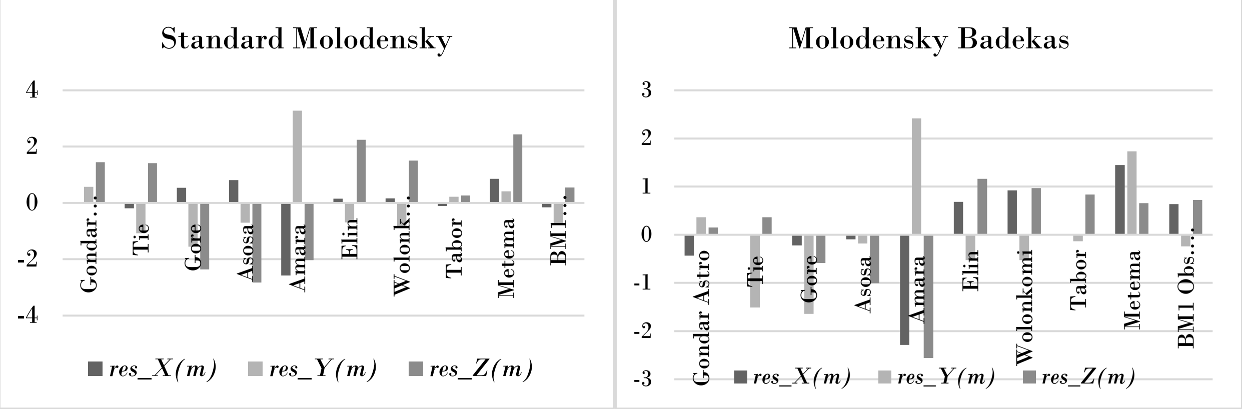

Figure 4.1: The residual histogram of xyz for all ten collocated GCPs

Figure 4.1 illustrates measurements from the station named Amara differ significantly from others; it has large residuals in all the x, y and z directions. In addition to improve the accuracy of the parameter estimation, we recalculated the residual errors by excluding the ‘Amara’ station. This will helps us numerically to assess its effect on the quality of the parameters. This assessment was applied by using all models and specifically a case study was conducted using the Standard Molodensky and Molodensky Badekas models. The variability of the residual errors in response to observation datasets are depicted in Table 4.1.

Table 4.1: the unit weight standard errors with ten, nine and eight collocated GCPs

Model	10 GCPs	9 GCPs	8 GCPs
Standard Molodensky	1.862 m	1.597 m	1.636 m
Molodensky-Badekas	1.286 m	0.888 m	0.925 m

The inclusion of Amara station and the newly established collocated data has resulted in low accuracy of parameter estimation when using both models (Standard Molodensky and Molodensky Badekas). When observations from these two stations are excluded, the transformation parameters are estimated with a better accuracy; 1.636m and 0.925 m accuracy for Standard Molodensky and Molodensky Badekas, respectively. Also, the parameters are estimated at good accuracy with the incorporation of measurements acquired from the newly established station. Therefore, the table also give us similar information about the collocated station ‘Amara’ that leads to be counted as an outlier. Since the residuals from the ‘Amara’ station an outlier, our study only used nine collocated GCPs obtained from GII and observations taken from the newly established station.

Data from the newly established station has a positive influence on the quality of the parameter estimation. It has improved the quality of the data by 0.39 m and 0.37 m, respectively for Standard Molodensky and Molodensky Badekas models. Many previous studies have used large number of collocated datasets (up to 5000 in Croatia) with good spatial distribution to determine transformation parameter between WGS84 and local reference ellipsoids. The minimum number of collocated GCPs used in Ghana, Liberia and Sudan, were below 20 collocated points. The density of data coverage was also very poor and hence reducing the accuracy of the transformation parameters.

Consequently, the following sub sections present the numerical results of parameter estimation and model evaluation with different heights. The results were also tested on plane surface and the quality of the plane point residuals were also evaluated.

4.1.1 Numerical Results and Discussions

Based on the nine collocated dataset, the estimated parameters, standard deviations and unit weight standard errors of the *Block Shift (Geocentric), Bursa-Wolf (Helmert), Molodensky-Badekas, Standard and Abridged Molodensky* models with different heights and height assumptions are presented below (Tables 4.2 – 4.6).

Table 4.2: Parameters using Block Shift (Geocentric) model

Parameters	h=H	h=H=0	$h=h_{EGM08}$	$h=h_{Abrdg}$	$h=h_{iterated}$
ΔX (m)	169.668 ± 0.532	166.007 ± 0.796	165.030 ± 0.840	162.368 ± 1.207	168.645 ± 0.163
ΔY (m)	14.789 ± 0.532	12.026 ± 0.796	11.233 ± 0.840	9.201 ± 1.207	13.947 ± 0.163
ΔZ (m)	-204.794 ± 0.532	-205.496 ± 0.796	-205.747 ± 0.840	-206.279 ± 1.207	-205.061 ± 0.163
σo (m)	1.597	2.387	2.519	3.620	0.489
n	--	--	--	--	700

Table 4.3: Parameters using Bursa Wolf (Helmert) model

Parameters	h=H	h=H=0	$h=h_{EGM08}$	$h=h_{Abrdg}$	$h=h_{iterated}$
ΔX (m)	110.193 ± 8.371	99.201 ± 5.043	102.950 ± 7.458	88.041 ± 10.024	-108.477 ± 3.469
ΔY (m)	78.477 ± 9.178	74.273 ± 5.528	66.883 ± 8.176	70.089 ± 10.990	279.263 ± 3.803
ΔZ (m)	-165.693 ± 8.037	74.273 ± 5.528	-92.031 ± 7.159	-35.438 ± 9.623	134.657 ± 3.330
ds (ppm)	0.273 ± 0.995	-0.488 ± 0.599	-0.684 ± 0.886	-1.224 ± 1.191	-0.048 ± 0.412
rX (")	0.653 ± 0.224	1.929 ± 0.135	2.131 ± 0.200	3.199 ± 0.269	5.280 ± 0.093
rY (")	-1.131 ± 0.250	-2.852 ± 0.150	-3.042 ± 0.223	-4.564 ± 0.299	-9.996 ± 0.104
rZ (")	2.820 ± 0.341	2.924 ± 0.205	2.664 ± 0.304	3.032 ± 0.408	12.119 ± 0.141
σo (m)	0.786	0.473	0.700	0.941	0.326
n	--	--	--	--	2891

Table 4.4: Parameters using Molodensky Badekas model

Parameters	h=H	h=H=0	h=h_{EGM08}	h=h_{Abrdg}	h=$h_{iterated}$
ΔX (m)	169.668 ± 0.296	166.007 ± 0.486	165.030 ± 0.540	162.368 ± 0.806	169.674 ± 0.132
ΔY (m)	14.789 ± 0.296	12.026 ± 0.486	11.233 ± 0.540	9.201 ± 0.806	14.801 ± 0.132
ΔZ (m)	-204.794 ± 0.296	-205.496 ± 0.486	-205.747 ± 0.540	-206.279 ± 0.806	-204.841 ± 0.132
ds (ppm)	0.595 ± 1.129	0.271 ± 1.855	0.133 ± 2.059	-0.014 ± 3.072	0.288 ± 0.503
rX (")	0.283 ± 0.256	1.019 ± 0.420	1.163 ± 0.466	1.755 ± 0.695	-0.216 ± 0.114
rY (")	0.872 ± 0.286	2.076 ± 0.469	2.215 ± 0.521	3.284 ± 0.778	0.046 ± 0.127
rZ (")	2.769 ± 0.388	2.750 ± 0.638	2.479 ± 0.708	2.741 ± 1.056	2.815 ± 0.173
Xm (m)	4990956.020	4989240.373	4990956.020	4990956.020	4990956.020
Ym (m)	3807764.401	3806441.447	3807764.401	3807764.401	3807764.401
Zm (m)	1105492.298	1105111.789	1105492.298	1105492.298	1105492.298
σo (m)	0.888	1.459	1.619	2.417	0.395
n	--	--	--	--	45

Table 4.5: Parameters using Standard Molodensky model

Parameters	h=H	h=H=0	h=h_{EGM08}	h=h_{Abrdg}	h=$h_{iterated}$
ΔX (m)	169.670 ± 0.532	166.008 ± 0.796	165.032 ± 0.840	162.369 ± 1.207	168.620 ± 0.163
ΔY (m)	14.793 ± 0.532	12.030 ± 0.796	11.237 ± 0.840	9.205 ± 1.207	13.931 ± 0.163
ΔZ (m)	-204.804 ± 0.532	-205.506 ± 0.796	-205.757 ± 0.840	-206.289 ± 1.207	-205.076 ± 0.163
Δa (m)	112.145	112.145	112.145	112.145	112.145
Δf (m)	5.475E-05	5.475E-05	5.475E-05	5.475E-05	5.475E-05
σo (m)	1.597	2.387	2.519	3.620	0.489
n	--	--	--	--	710

Table 4.6: Parameters using Abridged Molodensky model

Parameters	h=H	h=H=0	h=h_{EGM08}	h=h_{Abrdg}	h=$h_{iterated}$
ΔX (m)	169.671 ± 0.531	166.034 ± 0.796	165.033 ± 0.840	162.370 ± 1.207	169.079 ± 0.161
ΔY (m)	14.833 ± 0.531	12.050 ± 0.796	11.278 ± 0.840	9.245 ± 1.207	14.321 ± 0.161
ΔZ (m)	-205.143 ± 0.531	-205.882 ± 0.796	-206.095 ± 0.840	-206.627 ± 1.207	-205.313 ± 0.161
Δa (m)	112.145	112.145	112.145	112.145	112.145
Δf (m)	5.475E-05	5.475E-05	5.475E-05	5.475E-05	5.475E-05

σo (m)	1.594	2.387	2.519	3.620	0.484
n	--	--	--	--	9565

n is the number of iteration

Based on the above tabular results (Tables 4.2 to 4.6), the estimated transformation parameters differ significantly while using different ellipsoidal height conversion methods for each models. Small difference also exist between the tested models; except for the Bursa Wolf model. For all tested models, the parameters estimated by using the Orthometric height method (OHM) and iteration solution gives better results and the later approach were great. The Standard and Abridged Molodensky models differ from the Block Shift model only in two parameters that explicitly defines the ellipsoidal change (Δa and Δf). This means, Clarke 1880 ellipsoid used for the ADINDAN-Ethiopia and WGS84 for the GPS system. The Abridged Molodensky also differ from Standard Molodensky model by considering heights that the later models includes the ellipsoidal heights. The estimated parameters for each models with the same heights have almost the same value. The difference is within the standard error of each parameters. Nevertheless, the Bursa-Wolf gives different parameters from all the other models with better unit weight standard error and coarse standard deviations. This might exist due to a seriously affected by the correlation in the parameters (i.e. translation, rotations and scale change). However, these models were implemented in Australia to conduct transformation between AGD 84 and WGS 84 (Featherstone, 2011).

Again, except for the Bursa-Wolf model, the estimated parameters using EGM heights for all models is more close to the parameters published by NGA and implemented in ArcGIS software [IHO, 2008, ESRI, 2012]. However, these parameters has less quality relative to others height determination methods. The parameter estimation using Abridged Molodensky height has resulted less accuracy across the study area. In contrary, good results were obtained in a case study carried out in Ghana (Ziggah et.al, 2016).

In theory, the Bursa-Wolf and Molodensky-Badekas models should give us the same results when the same data are used. Nevertheless, the estimated transformation parameters in this study using the Molodensky-Badekas model shows significant disparity from the Bursa-Wolf model in the translation parameters and slight difference in the rotations and scale change components. It is the expected difference due to the Molodensky-Badekas model uses the three centroid in the computation and no for the Bursa Wolf model. These removes the correlation between the parameters by relating the parameters to the barycenter (centroid) of the network or any convenient point within the network.

Similar to other studies (Ahmed and Mergia, 2009; Matej et.al, 2015; Fan and Chymyrov, 2015), the parameters estimated by the Molodensky Badekas seven parameters model with iteration solution has greater reliability and stability due to good standard deviations (0.132 m for the translations and below 0.2 arc second for the axial rotations) and unit weight standard errors (0.395 m). Relatively bigger rotational angles are observed. Our results are superior compared to previous studies performed in the region of Ethiopia. However, others models also can give us comparable results below 0.163 m and 0.484 m standard deviations and unit weight standard errors respectively, except for Bursa Wolf.

4.1.2 Plane Residuals Analysis

The other best approach to assess the reliability of the estimated transformation parameters is looking at the point residuals or point error vectors on the plane surface (Fan and Chymyrov, 2015). In practice, GPS data are transformed to a local coordinate system and defined in a planimetric coordinates (E, N) using Universal Transverse Mercator (UTM) map projection for our study area and then deviations of the GPS data derived planer coordinates from an independent local coordinates were calculated to give point residual errors. The residual magnitude and directions from all selected models implied by height information are used to assess uncertainty of parameter estimation models.

Table 4.7: Statistics of positional residuals using Block Shift model

	h=H	**h=H=0**	**$h=h_{EGM08}$**	**$h=h_{Abrdg}$**	**$h=h_{iterated}$**
Min (m)	0.241	0.326	0.373	0.443	0.205
Max (m)	1.107	1.198	1.230	1.343	1.147
Mean (m)	0.748	0.753	0.771	0.813	0.734
St.dev.(m)	0.314	0.304	0.307	0.337	0.333
RMSE (m)	0.853	0.855	0.873	0.926	0.847

Table 4.8: Statistics of positional residuals using Burse-Wolf model

	h=H	h=H=0	h=h_{EGM08}	h=h_{Abrdg}	h=$h_{iterated}$
Min (m)	0.028	0.064	0.075	0.089	0.131
Max (m)	1.147	1.068	1.065	0.989	0.591
Mean (m)	0.503	0.485	0.487	0.475	0.416
St.dev.(m)	0.368	0.341	0.337	0.307	0.176
RMSE (m)	0.649	0.617	0.617	0.590	0.475

Table 4.9: Statistics of positional residuals using Molodensky-Badekas model

	h=H	h=H=0	h=h_{EGM08}	h=h_{Abrdg}	h=$h_{iterated}$
Min (m)	0.081	0.106	0.481	0.574	0.057
Max (m)	1.21	1.156	1.583	1.631	1.222
Mean (m)	0.517	0.639	0.817	0.984	0.519
St.dev.(m)	0.314	0.425	0.411	0.369	0.385
RMSE (m)	0.753	0.839	0.893	0.953	0.672

Table 4.10: Statistics of positional residuals using Standard Molodensky models

	h=H	h=H=0	h=h_{EGM08}	h=h_{Abrdg}	h=$h_{iterated}$
Min (m)	0.237	0.322	0.368	0.452	0.203
Max (m)	1.112	1.191	1.223	1.335	1.143
Mean (m)	0.747	0.752	0.770	0.812	0.734
St.dev.(m)	0.315	0.303	0.306	0.335	0.332
RMSE (m)	0.853	0.853	0.872	0.924	0.847

Table 4.11: Statistics of positional residuals using Abridged Molodensky models

	h=H	h=H=0	h=h_{EGM08}	h=h_{Abrdg}	h=$h_{iterated}$
Min (m)	0.300	0.337	0.368	0.424	0.303
Max (m)	1.266	1.178	1.159	1.132	1.304
Mean (m)	0.805	0.802	0.810	0.839	0.809
St.dev.(m)	0.339	0.293	0.277	0.266	0.347

RMSE (m)	0.919	0.900	0.903	0.928	0.926

The standard deviation and the root mean square error (RMSE) of the point error residuals were used as the main evaluation criterion to evaluate the accuracy of the models and the estimated transformation parameters. The statistics of positional residuals presented in the above tables (4.7 – 4.11) indicate that the parameter estimation with the iteration solution for all models gives good results compared to methods that use height conversion approaches. Here, the Bursa Wolf model has better results compared to all other models, but the direct solutions as described in the previous section (*section 4.1.1*) were not stable and good especially in the standard deviations.

The Molodensky-Badekas model with iteration solution gives better RMSE (i.e. 0.672 m) which also were good in the direct solutions. Our result shows that values of the standard deviation and of the root mean square errors of the positional differences decreases as the number of transformation parameters increases. This means, the reliability of the models and the estimated parameters will become valid as the number of the parameters used in the transformation computation increases. In this study, the Block Shift and the Standard Molodensky models gave almost an identical result, while they are not in a good agreement with Abridged Molodensky. This might be due to the Abridged Molodensky model did not considered the ellipsoidal heights in estimating the parameters. The Block Shift also work as the mean shift of the Cartesian coordinate in each axes.

On the other hand, the figure of point residual direction may tell us the characteristics of the residuals in the plane surface. However, due to few number of collocated points used in this study, it became very challenging to visualize and identify information about the residuals. Figures 4.2-4.6 show the point residuals of the collocated points with different heights.

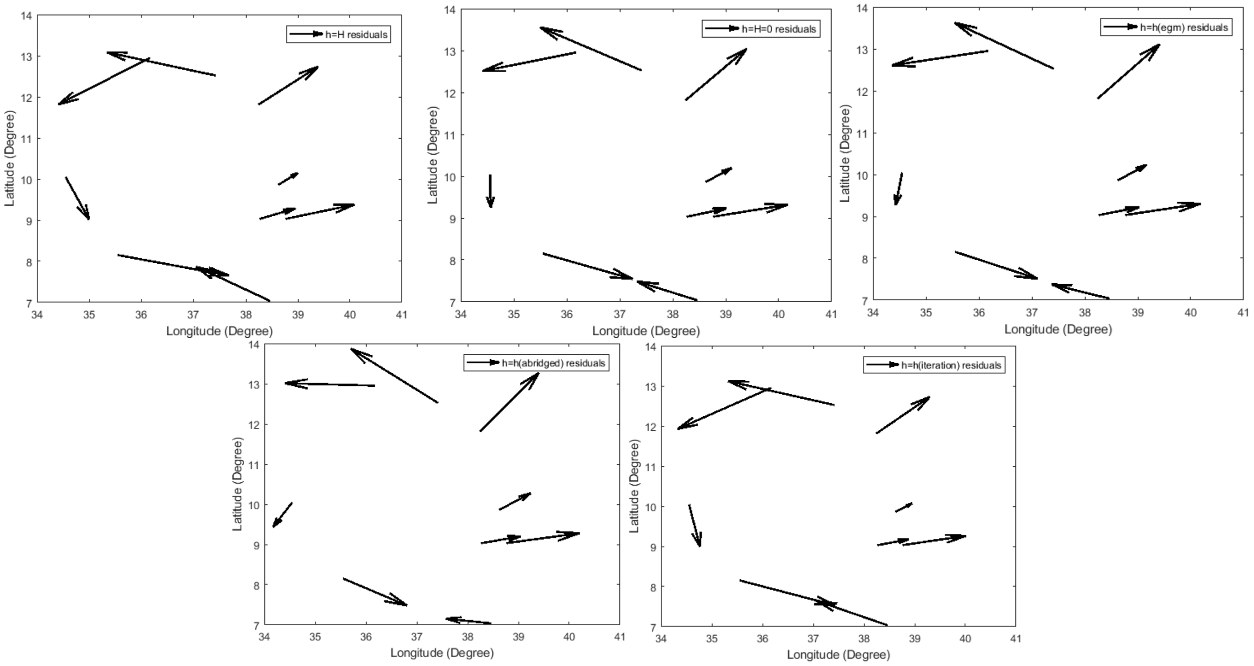

Figure 4.2: Point residuals of Block Shift model with different heights

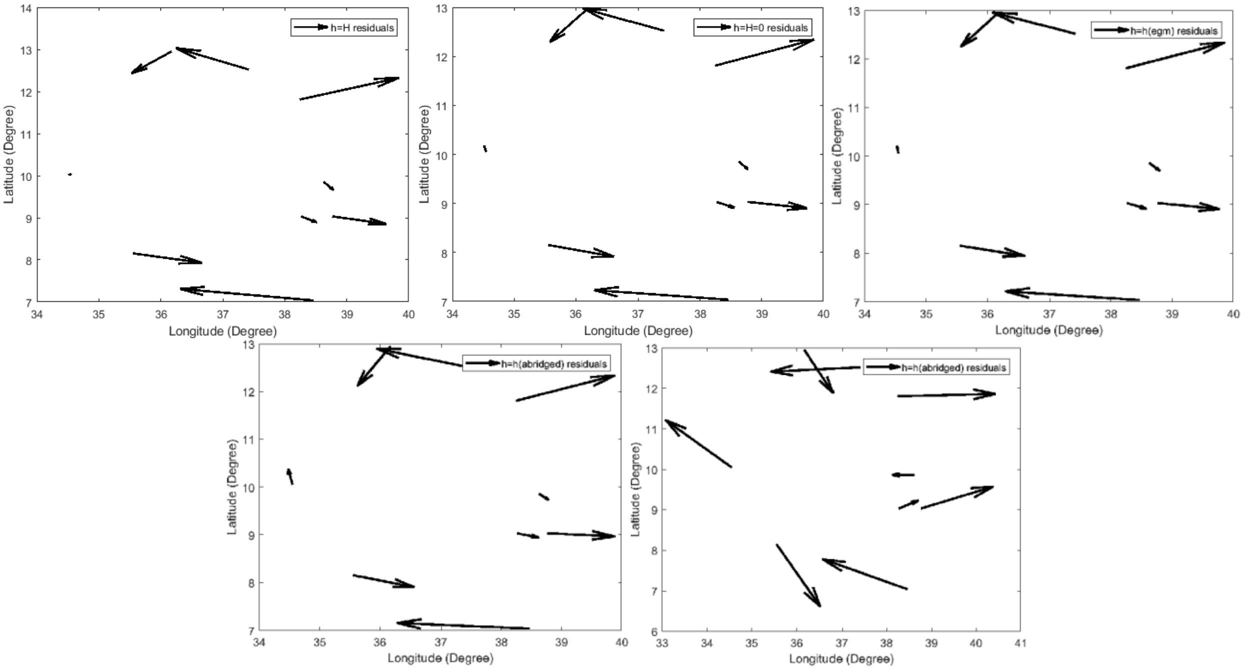

Figure 4.3: Point residuals of Bursa Wolf model with different heights

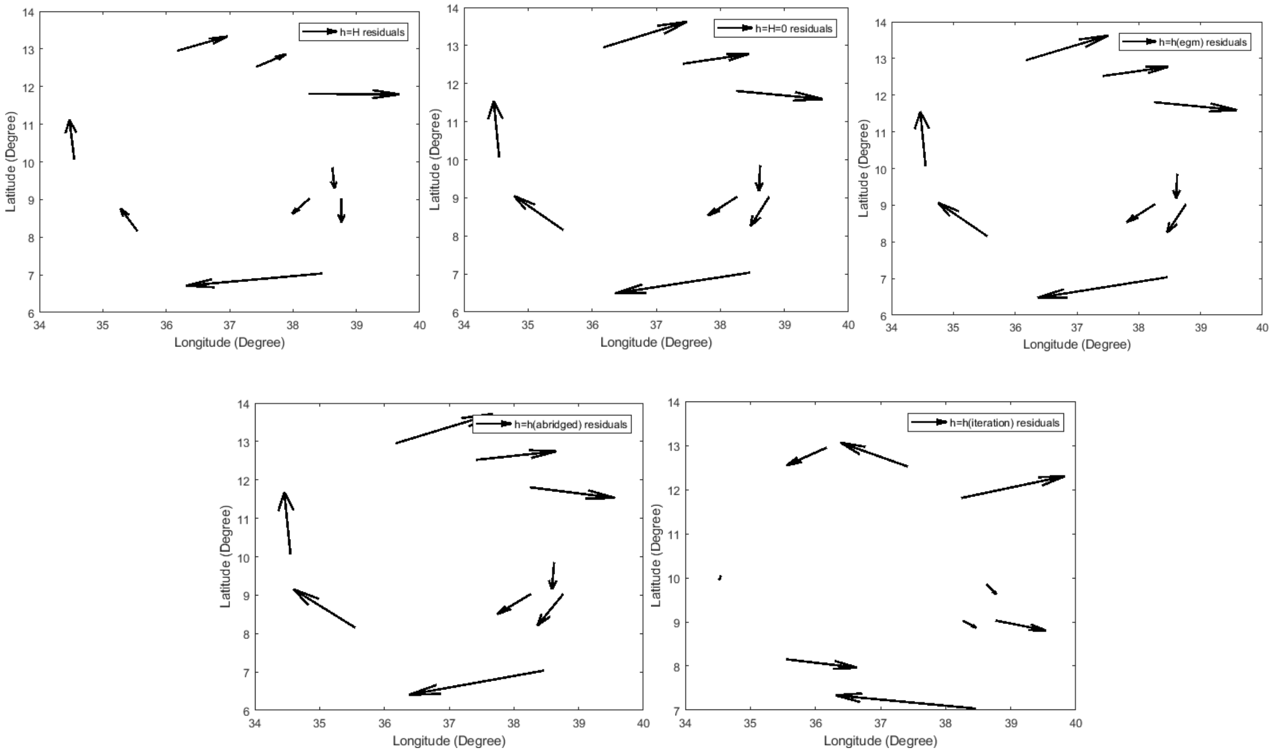

Figure 4.4: Point residuals of Molodensky-Badekas model with different heights

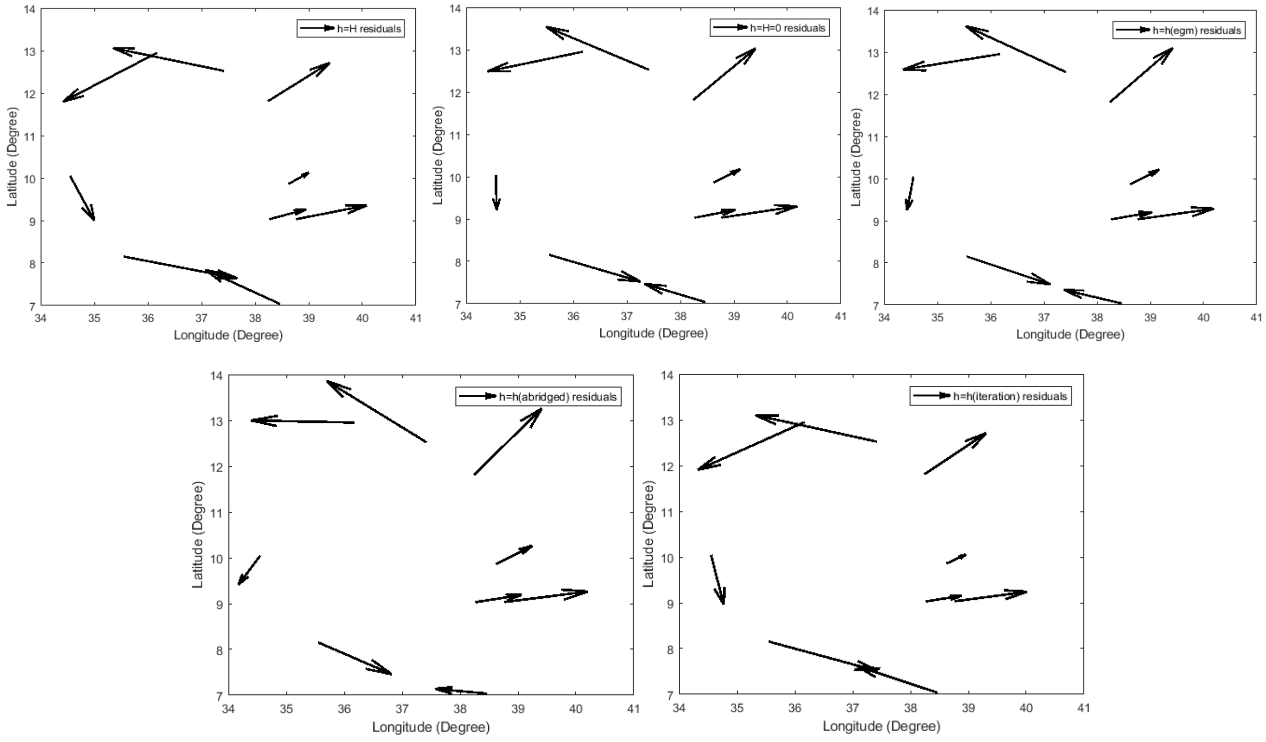

Figure 4.5: Point residuals of Standard Molodensky models with different heights

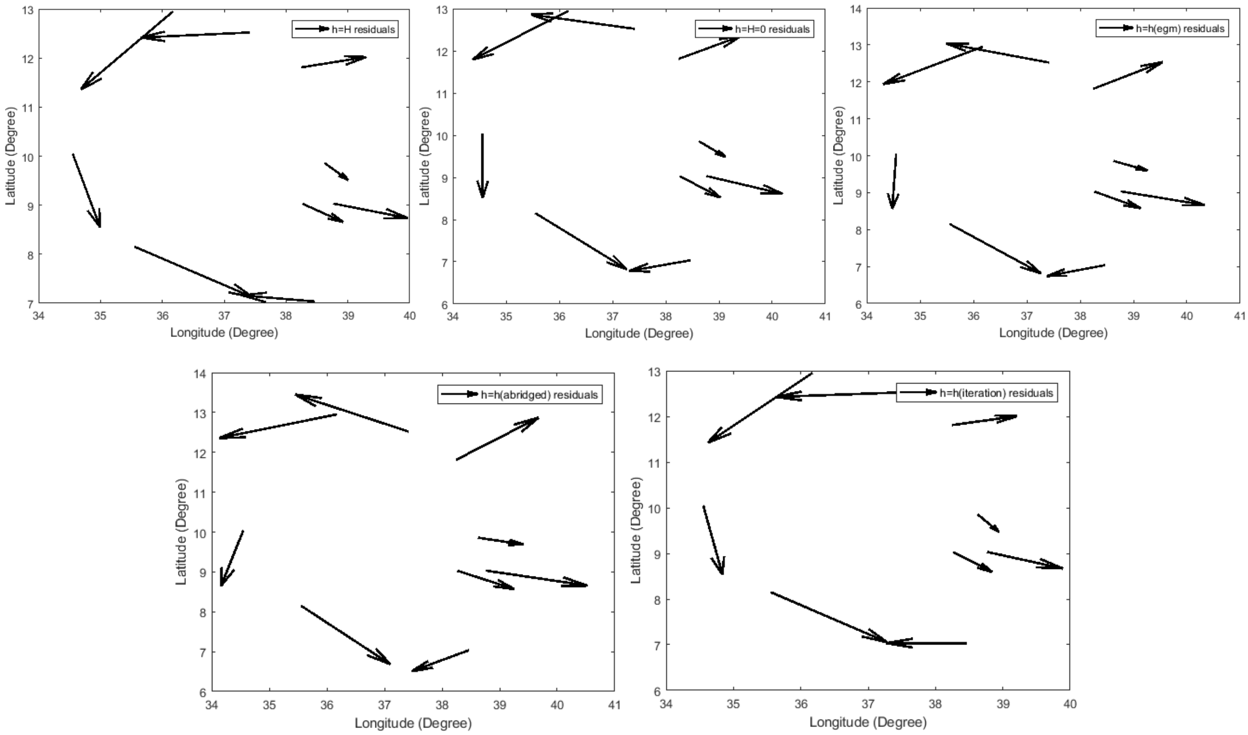

Figure 4.6: Point residuals of Abridged Molodensky models with different heights

As the above figures presents, the positional errors from the orthometric height method (OHM) and iteration solution have significantly compressed towards the center while errors from the other approaches more expanded outside the network. However, similar trends are also observed in all height estimation methods. The point residuals are small in the mid latitude for the collocated data set and big shift was observed in the northern and southern parts.

4.2 Comparative Analysis

Basically, there are three sets of transformation parameters that were defined for a region of Ethiopia: Parameters used by the National Mapping Agency/Geospatial Information Institute/, NGA for Ethiopia and NGA-Mean solution. The Federal Democratic Republic of Ethiopia Geospatial Information Institute uses the following ΔX = 162 m, ΔY = 12 m and ΔZ = -206 m as an official transformation parameters in order to transform GPS data to ADINDAN-Ethiopia system. Unfortunately, uncertainties of the transformation parameters and description of transformation models in use not well recognized and communicated to the end users. These parameters have been implementing extensively in the present urban cadastre project through incorporating it in their legal frameworks and standards as the mandatory procedure. Other local projects and individuals also use the same parameters in the geospatial data conversions. On the other hand, there is a growing serious concerns from the users side about the accuracy of the transformation parameters and the reliability of the models used. Also, many practitioners are reporting that the quality of their data are mostly affected by the uncertainty related to the transformation parameters.

In addition, NGA (IHO, 2008) published two sets of transformation parameters for Ethiopia ($\Delta X = 166$ m, $\Delta Y = 11$ m and $\Delta Z = -206$ m) and mean solution for Ethiopia and Sudan ($\Delta X = 166$ m, $\Delta Y = 15$ m and $\Delta Z = -204$ m). Again, due to the absence of meta-description about the parameters especially the overall quality, it is difficult to compare the official and NGA's parameters with the models tested and estimated parameters in this study. However, the NGA parameters have ±3 m and ±5 m standard deviation for each translation parameters with NGA-Ethiopia and NGA-mean solution for Ethiopia and Sudan, respectively. The parameters estimated using selected models has achieved better quality from these NGA parameters, except for the Bursa-Wolf model.

Besides, comparative analysis of the models performance is made in a local coordinates. Table 4.12 shows positional residual errors of the models.

Table 4.12: Statistics of positional residuals using official and NGA parameters

	Min (m)	Max (m)	Mean (m)	St. dev.(m)	RMSE (m)
Official	1.386	3.289	2.430	0.7469	2.703
NGA-Ethiopia	0.145	1.297	0.765	0.409	0.915
NGA-Mean (EPSG)	1.665	3.519	2.731	0.668	2.995
Molodensky-Badekas	0.057	1.222	0.519	0.385	0.672

In this study, the Molodensky-Badekas model is the most accurate, stable and suitable transformation models with the orthometric heights and iterative approaches. To compare the existing official parameters of Ethiopia and other with the selected transformation models, the steps listed in section 3.4.1 were applied.

Table 4.12 indicates that, the Official and NGA mean solution transformation parameters have poor quality. However, the NGA parameters defined for the region of Ethiopia is good in both the standard deviation and the RMSE error. This means, it is better to use the NGA-Ethiopia transformation parameters than the nationally adopted parameters. All the estimated parameters using the selected models are more accurate than the official parameters. The Molodensky-

Badekas model that tested its goodness in this study gives the possibility to achieve high accuracy than existing parameters. Using these official and well-known parameters may have an effect on achieving high accuracy requirements of the current cadastral projects, specifically for boundary survey and mapping. The point residuals on plane surface are presented in Figure 4.7 below.

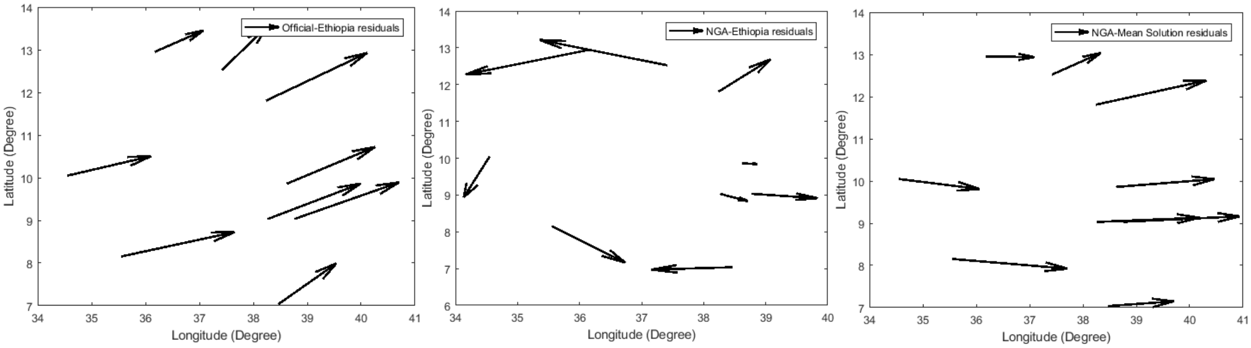

Figure 4.7: The point residuals of the Official Ethiopian (left), NGA-Ethiopia (middle) and NGA-mean solution parameters (left)

The point error residuals (Fig. 4.7) also shows that the point error from the official and NGA mean solution parameters has a clearly systematic trend toward the east. The points systematically shifted to east. However, the errors in the NGA Ethiopia has no uniform shift in magnitude and directions, and point errors are observed to smallest in mid latitude for the given dataset. This may remove the positional errors through compensating the positive and negative components thereby giving better results.

CHAPTER FIVE

5. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

In this study, five transformation models that are empirically tested and widely used in the world and implemented in most geospatial institutions and software packages were empirically studied for Ethiopia. Nine collocated GCPs were used to estimate the transformation parameters between WGS 84 and ADINDAN-Ethiopia using five methods and assumption of converting orthometric to ellipsoidal heights. In order to assess the reliability of the estimated transformation parameters, the point residuals were computed on the plane surface in addition to estimating the residuals, variance-covariance, standard deviation and unit weight standard errors. Based on the forgoing numerical results, discussions and analysis, the following conclusions are forwarded.

- The collocated datasets that used to determine parameters for datum transformation between WGS 84 and ADINDAN-Ethiopia are more on the west central part due to the Blue Nile basin historical astro-geodetic points.
- The parameters estimated with Orthometric height method (OHM) and iteration solution have better accuracy than the ones computed by using other methods. The iteration is more closer to the real values. However, it needs much time and high performance computer when the number of collocated points increase.
- The number and quality of collocated points have a vital influence on the quality of the transformation.
- The numerical results and statistics show that, the Molodensky-Badekas seven parameters model with iteration solution is the most stable and suitable model for Ethiopia to achieve high accuracy transformations. The model uses the centroid of the network that removes the correlation with the rotations and the scale. The other models also give comparable accuracy and may be used for practical works.
- Relative to the suggested stable and suitable model, the official transformation parameters of Ethiopia and the NGA-mean solution have poor quality. However, the NGA derived parameters are good for the region of Ethiopia and may satisfy the mapping requirements.

5.2 Recommendations

Before giving the recommendations, it is better to shortlist the contribution of this MSc thesis to the study area within the given dataset, methods and overall setups. Therefore, the contributions of this thesis are as follows:

- This study tried to increase the number of collocated GCPs that used to determine the transformation parameters between WGS 84 and ADINDAN-Ethiopia. There was no any other collocated points in Addis, the capital city of Ethiopia. This established point has positive influence on the quality of the estimated parameters.
- With the constraint of collocated dataset, the study attempted to test five conventional and conformal transformation models, statistically evaluated and then suggested the most suitable transformation model for Ethiopia.
- The orthometric height to ellipsoidal height conversion approaches and assumption evaluated for the study area within the given data.
- Comparative analysis and evaluation conducted between the suggested model, the existing official and other transformation parameters.

In general, this study tried to show the gaps, challenges and approaches, which could be used as a reference for future extensive and empirical studies. Based on the findings, the following recommendations are given.

- The geospatial society shall use customize transformation parameters rather than using the default one to achieve high quality positioning and mapping.
- The researchers and government also shall conduct detail empirical studies on this issue and provide the findings for the geospatial societies, institutions and software developers to avoid the inconsistency and confusion.
- Future works shall be focused more on densifying the number and distribution of the collocated GCPs for the determination of transformation parameters.

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