ROLES OF MODELS ON STUDENTS' CONCEPTUAL UNDERSTANDING OF GEOMETRYIN ADDIS ZEMENPREPARATORY SCHOOL

By

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This is to certify the thesis prepared by Meaza Zemene entitled: Roles of models on students' conceptual Understanding of geometry in addis zemen preparatory school and submitted in Partial Fulfillment of the Requirements for the Degree of Master of Education in Mathematics complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

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ABSTRACT

The main objective of the study was to investigate the role of models on students' conceptual understanding of geometry in Addis Zemen higher education preparatory secondary school. Specifically, the study examined students' understanding on their ability to identify the effect and contribution of models on the students' geometric concept understanding, providing different type of instructional material and teaching methods to the students in order to understand geometric concepts simply. The study employed quasi-experimental design. The data were collected from 92 students and grouped in to two one treated group the other the control group. The data were collected using test in the actual classroom situation, and analyzed using a t-test and ANOVA test were used for data analysis. Then the result indicated that the use of models in teaching geometry has significance in geometry classes. Hence, the study implied that teachers/curriculum expert's taught to aware of the students, concept understanding to diagnose and involve models approach in the teaching and learning process of geometry. Finally, the study suggested that the concepts of geometry are abstract in nature and it requires more visualization tools to the students' geometrical concept understanding and for spatial thinking .

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Table of content

Content page

CHAPTER THREE: METHODS OF THE STUDY

CHAPTER FOUR: DATA ANALYSIS AND INTERPRITATION

CHAPTER FIVE: SUMMARY, CONCLUSION AND RECOMMENDATION

LIST OF FIGURES

LIST OF TABLES

LIST OF ACRONOMYS

ETP Education and training policy

MOE Ministry of education

NCTM National council of teachers of mathematics

TGE Transitional government of Ethiopia

CHAPTER ONE

INTRODUCTION

1.1 Back ground of the study

Problem solving plays a very important role in mathematics education. The study of mathematics at this level that is grade 11 and 12 contribute to learners' growth in to good, balanced and educated individuals (TGE, 1994). To do this, the learners' should know the effect and contribution of models on the students' geometric concept understanding that are useful in their further studies, working life, hobbies and all rounded personal development. Consequently this aims will fulfilled by integrated mathematical learning with their daily life, paying attention to the practical application of mathematical concepts, methods and procedures in different circumstances. That is why in 1994, the Ethiopian government issued education and training policy that can address the countries development needs.

The Policy emphasizes on the development of problem solving capacity and culture in the content of education, curriculum, structure and approach focusing on the acquisition of scientific knowledge (TGE, 1994). One of the objectives of Ethiopian education is to develop the physical and mental potential of problem solving of individuals by expanding education throughout the country. The focus of mathematics education is problem solving. For these essential needs, the school curriculum emphasizes on the completion of these objectives and to make citizens who are problem solvers and to give solution for every problem that they may face in their day to day activities (TGE, 1994). Conceptual knowledge can be defined as knowledge of mathematical structure (concept and its elements) and give them with symbols; and benefiting from its utilities; the knowledge of procedural techniques of mathematics and give them with symbols; formatting the connections and relation among methods, symbols and concepts.

- Learning becomes joyful and students enjoy learning.
- Students can remember topics they learn with more ease and learning become permanent.

- New concepts are learned more easily, students can be self- learners and they need less help.

According to TGE (1994), the main challenges of Ethiopian education to create equal access of education to all citizens of the country. School facilities are also some of the major problems of our education system (MOE, 2006). As it is stated above, one of the main problem to attain the intended behavior of students is the mode of presentation of some topics using appropriate teaching method based on the understanding level of students. Other forms such as curricular change were also part of the interventions in this regard.

Geometry is one of the main content of mathematics in secondary cycle educational system of Ethiopia and the students always respond to geometry: we could never solve geometrical problems; many of them do not understand it at grade 11 even if they have a prerequisite knowledge about the concept of Euclidian geometryHoffer (`1981). These responses may be based on the nature of the geometric contents that use the abstraction approach of geometrical structures. According to Hoffer (`1981) most learners consider geometry to be difficult. That is if we ask every students, which part of mathematics (algebra or geometry) is difficult to learn, their answer is geometry. This is may be due to for some students at secondary school of first, and second cycle (grade 9 -10& 11-12), learning geometry is not easy and many of them fail to develop an adequate understanding of geometry concepts, geometry reasoning and geometry problem solving skills (Battista, 1999). From these points of view, without a good foundation in geometry at the secondary level this problem may worsen at secondary level of second cycle where students may face difficulty in mastering more complex geometry concepts.

Referring to the Van Hiel model for the learning of geometry, which suggests that learners advance through level of thought in geometry, is generally considered to be a fairly useful model to describe learners’ behavior in geometry (Battista, 2007). For meaningful learning, the students should learn geometry using appropriate teaching approach to master more abstract geometric concepts in their higher levels.

According to Bell (1978) it is difficult to teach theorems by providing traditional teaching method since it cannot supported by models. In relation to this when the researcher observes and compares the students’ performance of doing conic section particularly circle, parabola, ellipse and hyperbola at Grade 11, they score and perform much less than algebra and trigonometry. From these point of view, the researcher tried to understand that the learner have faced many difficulties in term of calculating and drawing of coordinate geometry mainly the conic sections.

Therefore, the purpose of this study was to investigate the role of models on the teaching learning process and its effect on the students’ performance of geometry and to fulfill the gap which exists among other researchers’ research findings. Almost all Ethiopian teachers do not use additional physical models in their classroom to assist their students to understand the abstract concepts of geometry that are crucial for problem solving with regard to mathematics or other areas of specialization.

As I have mentioned above from my teaching experience of teaching geometry at different level for 8 years the factor that led to this study are:

- Poor teaching of geometry methods as the nature of geometry contents that they use the abstract approach of geometric structures.
- The outlook of learners regarding geometry lesson and less attention of them to give more time to geometry part when it compared to other part of mathematics.
- Students low achievement particularly in coordinate geometry part.
- Lack of spatial geometrical knowledge and considering the importance of geometry for their future activity and life.

Therefore, this study investigates whether teaching geometric concepts using models would improve disadvantaged learners and performance in geometry. The investigator intended to study how students develop their geometric concept understanding which is crucial for solving geometrical problems when teachers use geometrical models to teach geometrical concepts and carry out exploration and investigation tasks.

1.2 Statement of the problem

All students, whatever their socio-economic, ethno cultural, or linguistic background, must have opportunities to learn and to grow, cognitively and socially. When students can make personal connections to their learning, and when they feel secured in their learning environment, their true capacity will be realized in their achievement. To create effective conditions for learning, teachers should avoid all forms of bias and stereotyping in resources and learning activities, which can quickly alienate students and limit their learning. Teachers should be aware of the need to provide a variety of experiences and to encourage multiple perspectives, so that the diversity of the class can be recognized and all students feel respected and valued. Learning activities and resources for teaching mathematics should be inclusive, providing examples and illustrations and using approaches that recognize the range of experiences of students with diverse backgrounds, knowledge, skills, interests, and learning styles.

Concepts are formed by students through a reconstruction of reality, not through an imitation of it. It is perplexing that relatively fewer programs incorporate experimental components while so many others concentration merely on completing. Most of the time mathematics teachers considering that meaningful learning for geometry come only through traditional learning and most of the time they do not use concrete means of teaching method in their class room. This situation is poor teaching of geometry where by learners are encouraged to learn geometry by memorizing theorems and are expected to reproduce in test or examination. It is important in teaching and learning geometry and acknowledging that appropriate use of visualization can facilitate the integration of representations in classroom instruction. This research investigates the role of models which is a tool for visualization in a whole class setting and useful for students' understanding of several important concepts in geometry. It also investigated the effect that using model has on students' understanding in learning geometry.

1.3 Objectives of the study

The main objective of this study was to examine the role of models on students' learning and the conceptual understanding of geometry.

The specific objectives of this study are to:

i. Identify the effect and contribution of models on the students' geometric concept understanding.

ii. To examine whether there is a significant between different groups of students those who learn using models and conventional method

1.4 Main Research Questions

This research will answer the following basic questions.

1. Does using models contribute to the development of students' geometry concept understanding?
2. Is there significance difference between different groups of students those who learn using models and conventional method?

1.4 Significance of the study

This study has a vital role for every teacher to improve the method for assisting students in teaching geometry. The effective attainment of school programs greatly depends on a delicate adjustment of many factors. This is so because when there is a gap between the intended out come and the actual achievement of students; attempts are made to resolve those problems. In addition to this the following key points can be raise as significance of the study:

- Apply models in teaching learning of geometric concepts.
- Use this knowledge of geometry for problem solving activities.
- Providing opportunities for students to work both independently and interdependently with others.

- Provide information for other researchers for those who are interested in doing this kind of research.
- Use different learning and assessment activities that make students to work with various learning style (e.g. Auditory, visual) to participate meaningfully.
- It helps students to give more time for mathematics by making the teaching learning process interactive.

1.6 Delimitation of the study

The study was conducted in Addis Zemen preparatory school, South Gondar zone which is one of the districts in Amhara National Regional state. It was only employed to compare the performance of students between the targeted groups. The study focused on the role of models on the students' geometric concept understanding and their ability to apply geometric concepts to solve geometric problems. This study specifically focused on the effect of models on other geometric competencies and other elements of spatial ability rather on visualization.

1.7 Limitation of the study

During conducting this study, some students are not voluntary to participate in the class room and to follow the necessary steps. The researcher also face a problem of selecting appropriate teaching method which is applicable for the treatment group and to identify who will participate in the treatment group.

1.8 Definition of concepts and basic terms

1. Geometry: The area of mathematics relating to the study of space and relationship between points, lines, curves and surfaces.

2. Conceptual understanding: Comprehension of mathematical concepts, operations and relation in test.

3. Model: a model is broadly defined as a simplified representation of a limited part of reality with related elements to reproduce the characteristics of the geometrical phenomena, system, or process that help to understand better the geometrical concepts. It helps to represent and view already existing information or to visualize abstract concepts of geometry.

CHAPTER TWO

REVIEW RELATED LITRATURE

2.1 Introduction

This chapter established the need for studying students' conceptions of geometry in a physical models environment. This chapter also examines some of the trends, research, and theories that inform the nature of research findings in the area. In doing so, it steps back to include work from the field of geometry research. Broadening the scope of the discussion frames my work in a historical context and acknowledges the continued importance of established theories. The literature review sorted by the three themes of this research-geometry, model, and problem solving. And the aim of this review part is to introduce some of the special features of geometry, and its teaching and learning.

2.2 Characteristics of Effective Teaching of Mathematics

Mathematics, it is widely understood, plays a key role in shaping how individuals deal with the various spheres of private, social, and civil life. Yet today, as in the past, many students struggle with mathematics and become disaffected as they continually confront obstacles to engagement. In order to break this pattern it is imperative, therefore, that we understand what effective mathematics teaching looks like. Many have looked research to seek evidence about what kinds of pedagogical practices contribute to desirable student outcomes (Anthony &Walshaw, 2007). Characteristics of Effective Teaching of Mathematics: Collectively, these reviews are closely aligned with recent mathematics initiatives within western education systems that shift teaching and learning away from a traditional emphasis on learning rules for manipulating symbols. National Council of Teachers of Mathematics (2000)focuses on developing communities of practice in which students are actively engaged with mathematics. Effective pedagogy within such communities is at the heart of this paper. We ask: What does research tell us about the characteristics of effective pedagogy in the west? From our investigations that have helped us answer that question, we have developed a set of principles that underpin the kinds of pedagogical approaches found to develop mathematical capability and disposition within an effective learning community.

In that theoretical framework, individual and collective knowledge emerge and evolve within the dynamics of the spaces people share and within which they participate.

In this paper our focus will be on the classroom as a community of practice. Our starting point is in the understanding that teachers who foster positive student outcomes do so through their beliefs in the rights of all students to have access to mathematics education in a broad sense understanding of the big ideas of curriculum and an appreciation of their value and application in everyday life. Additionally, Anthony and Walshaw (2001), claim that effective mathematics pedagogy:

1) Acknowledges that all students, irrespective of age, can develop positive mathematical identities and become powerful mathematical learners.

2) Is based on interpersonal respect and sensitivity and is responsive to the multiplicity of cultural heritages, thinking processes, and realities found in everyday classrooms.

3) Is focused on optimizing a range of desirable academic outcomes that include conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning.

4) Is committed to enhancing a range of social outcomes within the mathematics classroom that will contribute to the holistic development of students for productive citizenship.

Effective teachers facilitate learning by truly caring about their students' engagement (Nodding, 1995). They work at developing interrelationships that create spaces for students to develop their mathematical and cultural identities. They have high yet realistic expectations about enhancing students' capacity to think, reason, communicate, reflect upon and critique their own practice, and they provide students opportunities to ask why the class is doing certain things and with what effect (Watson, 2002). The relationships that develop in the classroom become a resource for developing students' mathematical competencies and identities. Students want to learn in a togetherness environment (Boaler, 2008; Ingram, 2008). Teachers can make everyone feel included by respecting and valuing the mathematics and the cultures that students bring to the classrooms. Ensuring that all students feel safe allows every student to get involved. However, it is important that the caring relationships that are developed do not encourage students to become overly dependent on their teachers. Effective teachers promote classroom relationships that allow students to think for them to ask questions, and to take intellectual risks (Angier &Povey, 1999).

Effective teachers draw on a range of representations and tools to support learners' mathematical development. Tools to support and extend mathematical reasoning and sense-making come in many forms including the number system itself, algebraic symbolism, graphs, diagrams, models, equations, notations, images, analogies, metaphors, stories, textbooks, and technology. Organize their mathematical reasoning and support their sense-making (Blanton & Kaput, 2005).Tools are helpful in communicating ideas that are otherwise difficult to talk about or write about. Teachers and students can use representations, such as stories, pictures, symbols, concrete objects, and virtual manipulative, to assist in communicating their thinking to others. As well as ready-made tools, effective teachers acknowledge the value of students generating and using their own representations, but it an invented notation, or a graphical, pictorial, tabular, or geometric representation. For example, young children frequently create their own pictorial representations to tell stories before using the more formal graphical tools that are fundamental to the statistics curriculum (Chick, Pfannkuch, & Watson, 2005) Theorems and definitions arise from real situations. This suggests that the study of space around the object, the position of the object, moments of objects and the space around objects improves students' spatial imagination. Many students approach geometry with hesitation because they lack the foundation to be successful in the course as it is designed. Successfully introducing geometry to students should be done over the course of years. The study of geometry is secondary school has been restricted to address specifically about conic section (circle, hyperbola, parabola and ellipse), triangles, quadrilaterals, prism, pyramid and the properties of each.

2.3 Overview of Geometry

1. What is geometry?

There is no fixed definition of geometry. There are different views on what constitute geometry. According to Wylie (1964) geometry is composed of Euclidian and non-Euclidean geometries. Historical background on coordinate geometry Coordinate system, or Cartesian coordinate system is sometimes known, is a rectangular system used to uniquely determine a point in two or three dimensional space by its distance from the origin of the coordinate system. It gained its name from a French mathematician and philosopher René Descartes (1596-1650), most famously known for his work on merging algebra and geometry into algebraic geometry. He developed ideas about this system in his book – Discourse on Method (published in 1637), to which an appendix the Geometry (La Geometric) was added trying to mathematically show the application of his philosophy. To most people geometry consists of theorems, proofs and diagrams (Giles, 1982). This suggests that Coordinate geometry has been reduced to a geometry that does not address itself to a real physical world. The meaning of geometry is not a straightforward as one may think (Hansen, 1993). Originally, the Greeks called it 'earth measure', the science of earth measuring. Coxeter (1969) described geometry as: 'the most elementary of the science that enable man, by purely intellectual processes, to make predictions about physical world based on observation. The power of geometry, in the sense of accuracy and utility of these deductions, is impressive, and has been a powerful motivation for the study of logic in geometry.' This suggests that, geometry can also be seen as the mathematics of logic and reasoning.

Freudenthal (1973) defines geometry as a science of physical space. In other words, the study of real physical space around the learner and the concrete object constitute geometry. Freudethal further argues that geometry does not start from formulating definitions and theorems, but it starts from organizing spatial experiences that lead to the formulation of definitions and theorems. In other words, geometry is about real life situations. Theorems and definitions arise from real situations. This suggests that the study of space around the object, the position of the object, moments of objects and the space around objects improves students' spatial imagination. Many students approach geometry with hesitation because they lack the foundation to be successful in the course as it is designed. Successfully introducing geometry to students should be done over the course of years. The study of geometry is secondary school has been restricted to address specifically about conic section (circle, hyperbola, parabola and ellipse), triangles, quadrilaterals, prism, pyramid and the properties of each.

2. The historical development of geometry

Different researchers and sources of knowledge have written about the historical overview of the development of geometry in different ways with no difference in content rather with different structures and organizations. But the emphasis is given for the history of geometry written by Carlin (1983) and the brief history and its influence on other fields of mathematics is shown in the following literature exposition. The early Egyptians had practical approach to geometry. They used geometry to help solve life's problems. The geometry of early Egypt was influenced by their need for practicality to tackle problem associated with construction and economics. The reason that the Egyptians did not develop a more formal looks in to geometric concepts an unclear. The Greeks used the concept of geometry that they learned from the Egyptian to develop geometry in to the deductive science.

3. Levels of Geometric thought

Pierre Van Hiele and Dina van Hiele-Geld of explored the development of geometric ideas in children and adults (Van de Walle& Folk, 2005). In their work, they proposed a five-level hierarchical model of geometric thinking that describes how individuals think and what types of geometric ideas they think about at each level of development. Advancement from one level to the next is more dependent on the amount of a student's experience with geometric thought than on the student's age and level of maturation.

- **Level 0:** Visualization. Students recognize and identify two-dimensional shapes and three-dimensional figures by their appearance as a whole. Students do not describe properties (defining characteristics) of shapes and figures. Level 0 represents the geometric thinking of many students in the early primary grades.

• **Level 1:** Analysis. Students recognize the properties of two-dimensional shapes and three-dimensional figures. They understand that all shapes or figures within a class share common properties (e.g., all rectangles have four sides, with opposite sides parallel and congruent). Level 1 represents the geometric thinking of many students in the later primary grades and the junior grades.

• **Level 2:** Informal Deduction. Students use informal, logical reasoning to deduce properties of two-dimensional shapes and three-dimensional figures (e.g., if one pair of opposite sides of a quadrilateral is parallel and congruent, then the other sides must be parallel and congruent). Students at this level can use this informal, logical reasoning to describe, explore, and test arguments about geometric forms and their properties. Level 2 represents the geometric thinking required in mathematics programs at the intermediate and secondary levels.

• **Level 3**: Deduction. As students continue to explore the relationships between and among the properties of geometric forms, they use deductive reasoning to make conclusions about abstract geometric principles. Level 3 represents the geometric thinking required in secondary and postsecondary mathematics courses.

• **Level 4:** Rig ours. Students compare different geometric theories and hypotheses. Level 4 represents the geometric thinking required in advanced mathematics course. The levels described by the van Hieles are sequential, and success at one level depends on the development of geometric thinking at the preceding level. Typically, students at the primary level demonstrate characteristics of level 0 and are moving toward level 1 of the van Hieles' levels of geometric thought. Students entering the junior grades are most likely functioning in the visualization and analysis levels (0 and 1) of geometric thought. The goal of the junior program is to provide instructional activities that will encourage children to develop the thinking and reasoning skills needed to move toward level 2 of the hierarchy, informal deduction. However, it involves developing explanations that are meaningful and making sense to others (Clements & Battista, 1990).

2.4 Geometry and associated concepts

1. What is Concept?

A concept is defined as knowledge structure of common characteristics of different substance and events captured by human brain. A circle, ellipse, hyperbola might be example of concepts. Concepts can also be defined as perspective characteristics of events and subjects derived from life experience in the real world. Therefore, characteristics of concepts are continuously investigated and concepts are redefined by time. For different people, the perspective characteristics of subject and events may not be the same. In order to fully understand, one need to know the meaning of words that are related with concepts. Educational program need to be prepared based consecutive and uninterrupted concepts. Teaching all the characteristics of concepts should be equally distributed throughout the educational period. Some information in mind can be built by learning of concept resulting in distinguishing some categories of stimulants. The learning of concept, especially in primary and secondary school, will help to learn the new things that are being used in life (Ulgen, 1996).

2. Cognitive development of geometric concepts

Before focusing on the growth of symbolic thinking, we briefly consider the very different cognitive development in geometry. This is rooted perception of objects in the world, initially recognized as whole gestalts. Some are specific individual perceptions, such as a child’s mother, or the family pet, but more often they are perceived as prototypes that apply to a wide range of percept. For instance, dog, cat, birds’ are prototype for various kind of living creatures. Some creatures are evidently birds (birds that can fly), whereas others, such as an ostrich, are also classified as birds even though they fail to fly. It is interesting to note that these classifications do not begin from the bottom up, or from the top down, but in terms of centrally typical level of recognition. For instance, children usually recognize dog before the most specific types of dog, or more general notion such as mammal, animal. Likewise in mathematics, the recognition of concepts such as circle, ellipse, hyperbola, parabola, square, rectangle, parallelogram, quadrilateral take time to organize into a conceptual hierarchy which is done neither bottom up top down.

This development involves various cognitive reconstructions. For instance, on the early stages, square and rectangle are initially considered by young children as disjoint concepts (a square is not a rectangle, because a square has four equal sides whilst a rectangle has only opposite sides equal). Disjoint categories of geometric shapes must be reconstructed to give hierarchies of shapes. Further reconstruction is necessary to see a shape not as a physical object, but as a mental object with perfect properties, and then to imagine geometry not just in terms of two and three dimensional Euclidean or non-Euclidean geometry, but as a variety of different geometry. Such a cognitive development and its succession of cognitive stages have been documented in the work of Van hiele (1986).

2.5 The nature of secondary school mathematics education

The term geometry comes originally from Greek, meaning literally, to measure the earth. It is an ancient branch of mathematics, but its modern meaning depend on context. Geometry meaning “measuring the earth” is the branch of mathematics that has to do with spatial relationships. In other words, geometry is a type of mathematics used to measure things that are impossible to measure with devices. However, geometry is more than measuring the size of objects.

For Ethiopian secondary school students, geometry has two flavors: synthetic and analytic. Synthetic geometry uses deductive proof to study the properties of points, lines, angles, triangles, circles, and other plane and solid figures, roughly following the plan laid out by the Greek textbook writer Euclid around 300 B.C. analytical geometry follows the pioneering work of the French mathematicians Rene Descartes (1596-1650) and Pierre Fermat (1601-1665) to impose a coordinate grid on the plane, making it possible to study geometric objects (example, lines, parabolas, and circle) by means of algebra (example, linear equations and quadratic equations) and vice versa. If we ask someone who had taken geometry in secondary school what it is that he/she remembers the answer would most likely be “proof” and if we ask him/ her what is that he or she liked the least, the answer would probably be “proofs”. The reason that secondary school geometry almost always spends a lot of time with proofs is that the first great geometry textbook, "the elements", was written exclusively with proofs. This text book is based on elementary geometry.

1 Teaching geometry in secondary schools

Geometry is one of the fundamental methods which people use to understand and to explain the physical environment by measuring length, surface area and volume. For this reason, enhancing geometric thinking is very important for high level mathematical thinking, and it should be developed with spatial interaction and manipulation in daily life (Clements &Battista, 1992). However, in the Ethiopian traditional class room, geometry learning is usually conducted only through the description of text, 2-dimensional graphs and mathematical formulas on blackboard or paper with some demonstrations with the help of plasma television. In some important topics, such as measuring the area and volume of 2 or 3 dimensional objects, traditional teaching methods often focus too heavily on the application of mathematical formulas, and lack opportunities for students to manipulate the object under study. Consequently, many students can memorize the formula and even appear to succeed in their work without fully understanding the physical meaning of the mathematics formula or geometry concepts (Tan, 1994). He also suggested that the development of understanding of concepts such as the measurement of area and volume should come from the experience of covering and stacking manipulations, so that, when formula mathematical concepts and formula are introduced or applied, students would actually understand the formula and their meaning.

2 Problem of teaching and learning of geometry

A number of factors have been proposed to explain what makes geometry learning difficult. First, the geometry language, which involves a specific terminology, is unique and needs particular attention and understanding before it can be used meaningfully. Misuse of geometry terminology can lead to misconception of geometry knowledge (Bishop, 1986). Next, geometry requires visualizing ability but many students cannot visualize two and three dimensional objects in a two dimensional perspective. Visualizing cross section of solid is very difficult for students lacking ample prior concrete experience with solid objects (Ben-Chaim et al., 1989). Due to their limited geometric experiences, students may not have had enough opportunities to develop and exercise their spatial thinking skills for effective geometry learning. For example, students consider a circle and a sphere as the same in a two dimensional view.

Another problem is that traditional approaches of geometry instruction do not seem to help students achieve the intended learning outcomes in the curriculum. By using just textbooks and chalk boards, class room geometry experience of students in public examination as it is seen in the actual classroom situations. There is an argent need to change the traditional mode of geometry instruction to one that is more rewarding for both teachers and students. Specifically, learners must be given opportunities to personally investigator and discover geometry to enable understanding of the subject in depth and also in relation to other fields of mathematics. However, students could be discouraged by certain hindrance in learning geometry. Among the problems are geometry language, visualizing space and instruction in geometry. These problems are discussed in greater detail below.

1. **Geometry language:** a pertinent problem with many geometry students is their weakness in the language of geometry (Bishop, 1986). The vocabulary in geometry is specific and carries meaning, descriptions and even properties. Knowing a geometric name like triangles and squares may not imply the student understands their exact meanings or their properties involving angle sums, perimeter or area. Norain-Idris (1999) observed that some students were unable to explain simple term like perimeter and triangle. Words like area, isosceles, scalene, and equilateral gave rise to much confusion among students. Evidently, geometry language, especially in the comprehension of geometry terms, plays a very important role in learning and understanding of geometric concepts (khoo&clements, 2001). In a study on the van heile levels of thinking in geometry among high school students, Fuys et al.(1988) found that the ability to advance in a kevel of thinking may be related to students' deficiencies in Language, both in knowledge of geometry vocabulary and ability to use precisely and consistently. In a study by Noraini- Idris (1999), students were involved in geometry activity requiring them to communicate geometry with each other and the teacher.

2. **Visualization:** another problem of geometry learning involves the ability to visualize, many concepts in geometry require students to visually perceive the objects and identify their properties by comparing them with their previous experiences involving similar objects. These geometrical concepts also require visual interpretations as many geometry problems are presented. Thus students who are unable to extract geometric information about solid object drawn on paper will face difficulty in interpreting questions involving solid geometry (Lappan et al., 1984). Research has shown that there exists a strong relationship between spatial abilities and geometry achievement (Noraini-Idris, 1998). According to Hershkowitz (1989) visualization is a necessary tool in geometry concept formation. Some mathematics educators recommend more visual activities in the class room to help students understand geometric concepts (Chong, 2001). It would therefore seem helpful for students if geometry lesson could be carried out with hands-on activities. By being able to "touch-see-and-do" and interacting with the object of their learning, students can learn geometry in a more imaginative and successful way (Bishop, 1983). This view is supported by Howard Gardner sighted by Campbell et al.(1996), in the theory of multiple intelligences. Gardner suggested that some learners are kinesthetically inclined, meaning that they learn best when actively involved with the object of their learning. For this type of learner, the visual, hands-on and process oriented approach might be most beneficial.

3. **Geometry instruction:** in the past, geometry lesson were pictured as students copying diagrams and properties of figures and shapes from blackboards and doing repetitive exercises to calculate angles, lengths, and areas of geometric figures Noraini-Idris (1998). This approach posed problems to both teachers and students, and both groups began to dread geometry. Teachers become frustrated because their poor conceptual understanding led to poor geometry achievement. Even in many geometry classrooms today, teachers introduce students to facts about coordinate geometry and the drill them with concepts in deductive reasoning. Students are seldom given opportunity to discover and conceptualize geometry on their own. Geometry instruction must therefore be evaluated and improved for more effective geometric understanding Noraini-Idris (1998). In Ethiopian schools, geometry is usually taught

using mainly the textbooks, the chalkboard, plasma TV and occasionally the compass and protractor.

3 Instructional methods in geometry

Instructional methods have been described as presentation forms. They are instructional procedures that are identified to enable learners to achieve the desired outcomes. They help to internalize the content that has to be learned. According to Seopo-sengwe (2001), the following are some of the method applicable to learners of all ages in teaching/learning mathematics with emphasis in geometry: presentation form, drill and practice, tutorial, gaming, simulation, discovery and problem solving.

A. **Presentation method:** in presentation method of instruction the source may be an educator, a text book, and audio tape, a film and so forth. The source tells, dramatizes or disseminates information to learners. There is no interaction or immediate response expected form the learner. It is one way communication controlled by the source. Reading a book, listening to a tape recorder, viewing a film are good examples of presentation method (Mbambisa et al,.1990). Objectives that are sometimes raised against presentation method which is sometimes known as narrative method focuses mainly on the following: that it causes the learner to listen passively, that more talking is done by the educator and he/she dominates the learning activities and it encourages memorization rather than understanding (Kruger et al., 1983).

B. **Demonstration method:** in the demonstration of instruction, the learner views a real life-like exemplar of the skill or procedure to be learned. Demonstration may be recorded and played by means of media such as video or film. This could be made more effective the learner practice what he sees under the supervision of the facilitator (Krugerr et al., 1983). Demonstration ensures that the learner is not passive observer. All they need in an educator who can give them immediate feedback while observing their performance (Rowntree, 1990) according to Van der stoep in (kruger et al., 1983)the educator can better explain certain concept by means of demonstration than by any other method and the demonstration varies from one situation to another. He further states that demonstration does not necessarily consist of mere performance of exemplary act for the learner. It is aimed at the formation of a mental image.

C. **Drill and practice method:** in drill and practice method, first of all the learner has to receive instruction on the concept to be learned, the principle or the procedure that has to be practiced (Kruger et al., 1983). Drill and practice exercise are designed to increase fluency of a new skill or to refresh an existing one (Mbambise et al., 1990). Drill and practice is commonly used in intensifying mathematical concepts. Certain media format and delivered system lend themselves particularly well to learners drill and practice exercise. In the case of young learners this method can be effective when the learner works on puzzles, building blocks, logs, and lotto. They do not free from repeating, which is in actual fact practice of the process.

D. **Tutorial method:** in tutoring, the method entails a one-to- one approach it is often used to teach basic skills such as reading and mathematics. The educator poses a question or problem and requests a learner to respond. Practice is provided until the learner demonstrates include educator-too- learner, learner- to- learner, computer-to-learner and print-to- learner (Heinich et al., 1990).

E. **Gaming method:** gaming can provide the learner attractive and stimulating effective frameworks for learning activities (Rowntree, 1974). Some instructional games add motivation to topics that attract young learners' interests (Heinich et al., 1990) such as visual perception, number concepts, sequencing, etc. the instructional game is an activity structured with set rules for play. It is aimed at reading set objectives. This method is suitable for young learners. Simple activities such as sorting and classification can be though through this method. Gaming provides a playful environment in which the learner follows rules as they strive to attain a challenging goal (Gerlach and Ely, 1971). For example, in an activity of additive classification, the learners' ability is explored by placing objects in categories based on the likeness or difference of the object.

F. **Problem solving:** problem solving means engaging in the task for which the solution method is not known in advance. In order to find a solution, students must draw on their knowledge, and through this process, they will often develop new mathematical understandings. Solving problem is not only a goal of learning mathematics but also a major means of doing so. Students should have frequent opportunities to formulate, grapple with, and solve complex problem that require a significant amount of effort and then be encourage reflecting on their learning (NCTM, 2000). This suggested that, by learning problem solving in mathematics, students should acquire way of thinking, habits of persistence and curiosity, and confidence in uniform situation that will serve them well outside the mathematics classroom.

4 Mathematical problem solving and ability

Problem solving has a special importance in the study of mathematics. A primary goal of mathematics teaching and learning is to develop the ability to solve a wide variety of complex mathematics problem. Reitman (1987) traced the rule of problem solving in school mathematics and illustrated a rich history of the topic. To many mathematically literate people, mathematics is synonymous with solving problems- doing word problems, creating patterns, interpreting figures, developing geometric construction, proving theorems, etc. on the other hand, persons not enthralled with mathematics may describe any mathematics activity as problem solving. Clearly, the aim of education in mathematics is to help to develop thinking abilities and to help in developing problem solving skills. Mathematics is bodybuilding for our mind. Now just as we don’t walk in to a gymnasium and start throwing all the weights on to a single bar, both would we sit down and expect to solve the most difficult problems. Our ability to solve problem must be developed, and one of the main ways to develop our problem solving ability is to do mathematics starting with simple problems and working our way up to the more complicated problems.

Problem solving as a process

Garofola and Lester (1985) have suggested that students are largely unaware of the processes involved in problem solving and that addressing this issue within the problem solving instruction may be important. We will discuss various areas of research pertaining to the process of problem solving.

1. **Domain specific knowledge**

Kantowski (1981) found that those students with a good knowledge base were most able to use the heuristics in geometry instruction. It is found that novices attended to surface features of problem whereas experts categorized problem on the basis of the fundamental principles involved. Silver (1985) found that successful problem solvers were more likely to categorize math problem on the basis of their underlying similarities in mathematical structure. These views suggest that, to become a good problem solver in mathematics one must develop a base of mathematics knowledge. How effective one is in organizing that knowledge also contributes to successful problem solving.

2. Algorithms

An algorithm is a procedure, applicable to a particular type of exercise, which, if followed correctly, is guaranteed to give the answer to the exercise. Algorithms are important in mathematics and our instruction must develop them but the process of carrying out an algorithm, even a complicated one, is not problem solving. The process of creating an algorithm, however, and generalizing it to a specific set application can be problem solving. Thus problem solving can be incorporated into the curriculum by having students create their own algorithms. Research involving these approaches is currently more prevalent at secondary level within the context of constructivist theories.

3. Heuristics

Heuristics are kind of information, available to students in making decisions during problem solving, that are aid to the generation of a solution, plausible in nature rather than perspective, seldom providing infallible guidance, and variable in results. Somewhat synonymous terms are strategies, techniques, and rules-of-thumb. For example, admonitions to "simplify an algebraic expression by removing parentheses", to "make a table'' to "restate the problem in our words", or to "draw a figure to suggest the line of argument for a proof" are heuristic in nature. Out of context, they have no particular value, but incorporated in to situations of doing mathematics they can be quite powerful. Theories of mathematics problem solving have placed a major focus on the role of heuristics. Surely it seems that providing explicit instruction on the development and use of heuristic should enhance problem solving performance; yet it is not that simple. Schoenfield (1986) have pointed out the limitation of such a simplistic analysis.

5 Geometry problem solving strategies

The skill of problem solving could and should be thought- it is not something that you are born with (George Polya, 1948}. There are a number of books written on the subject of mathematical problem solving. One of the best, and most famous, it is how to solve it by George Polya. He identifies four principles that form the basis for any serious attempt at problem solving:

1. **Understand the problem**: before beginning to solve any problem understanding what it is that trying to solve. There are two parts, what are given/s and what are trying to show. Clearly identify these parts:
   - What are the givens?
   - What are asked to find out or show?
   - It is reasonable that there is connection between the two?
   - Can we draw a picture or diagram to help to understand the problem?
   - Can we restate the problem in our own words?
   - Can we work out some numerical examples that would help make the problem clearer?

2. **Device a plan:** once we understand the problem that we are trying to solve we need to find a way to connect what we are given to what we are trying to show, in other words, we need a plan. Mathematicians are not very original and often use the same ideas over and over, so look for similar problems, i.e., problem with the same conclusion or the same given information. Trying solving a simpler version of the problem, or break he problem in to smaller parts. Work through an example. Is there other information that would help in solving the problem? Can we get that information from what you have? Are we using all of the given information?

   A partial list of problem solving strategies includes:

Guess and check	solve simpler information
Make an organized list	Experimental

Draw a picture or diagram — Act it out

Make a table — Use deduction

Use a variable — Change your point of view

3. **Carry out the plan:** once you have plan, carry it out. Check it step. Can we see clearly that the step is correct?
   - Carrying out the plan is usually easier than devising the plan
   - Be patient-most problem are not solved quickly nor on the first attempt
   - If the plan doesn't work immediately, be persistent
   - Do not let yourself get discouraged
   - If one strategy is not working, try a different one
4. **Look back (reflect):** with the problem finished, look at the solution.
   - Is there a way to check the answer?
   - Is the answer reasonable?
   - Can we see the solution at a glance?
   - Can we give a different proof?
   - Does the answer make sense?
   - Did the answer the entire question?
   - What did we learn by doing this?
   - Could we have done this problem another way-may be even an easier way?

We should review this process several times. When we feel like we have run in to a wall on a problem come back and start working through the questions. Often times are just a matter of understanding the problem that prevents its solution.

2.6 The place and role of models in teaching mathematics

Models

The term model has a different meaning in model theory. According to the famous Oxford Dictionary, a model is defined as: a three-dimensional representation of a person or thing, typically on smaller scale, a figure made in clay or wax which is then reproduced in a more durable material, something used as example or a simplified mathematical description of a system or process, used to assist calculations and predictions.

Another dictionary, such as the Fishery Glossary provides the definitions of a model and modeling as; a model is simplified representation of a limited part of reality with related elements and Modeling is the construction of physical, conceptual or mathematical simulations of the real world. A model is a three dimensional representation of real thing. It can be the same size of larger, or even smaller than the real thing. Learners should be informed of the actual size of the model in order to avoid misconceptions (Mbambisa et, 1990). From Wikipedia, the free encyclopedia, a model is a branch of mathematical logic. An artifact which is used to illustrate mathematical idea is also called a; mathematical model. A mathematical model uses mathematical language to describe a system the process of developing mathematical model is termed mathematical modeling.

Eykhoff(1974) cited Seopo-Sengwe(2001) defined a mathematical model as "a representation of the essential aspects of an existing system (or a system to be constructed) which presents knowledge of that system in usable form". The above definitions indicate that, a model can come in many shapes, sizes, and styles. It is important to emphasize that a model is not the real world but merely a human construct to help us better understand real world systems. A model is not the real world but merely a human construct to help us better understand real world systems. A model can represent a living or a non-living thing. Physical mathematical models include reproductions of plane and solid geometric figures made of cardboard, wood, plastic or other substances; or other substances; models of conic sections, curves in space, or three-dimensional surfaces of various kinds made of wire, plaster, or thread strung from frames; and models of surfaces of higher order that make it possible to visualize abstract mathematical concepts. Some of the most common three dimensional models or teaching/learning aids are relief maps, globes, dioramas and others. Since the learners are normally encouraged to handle and manipulate models especially those models which can assembled and disassembled to provide interior views, it is a good idea to store them out of sight when not being use for instruction to avoid learner attention being drifted away from other learning activities. The learners derive interest in solving problems in which they actively and intimately involved (Gerlach and Ely, 1971).

The concept of the model and the modeling in the mathematics, is focusing for problem solving, must be an instance of the concept defined above. That is, a model represents a part of reality which was specified by a modeling view and described by modeling facilities in order to the purposes for recognition, understanding, and manipulation of the target as shown in figure1.

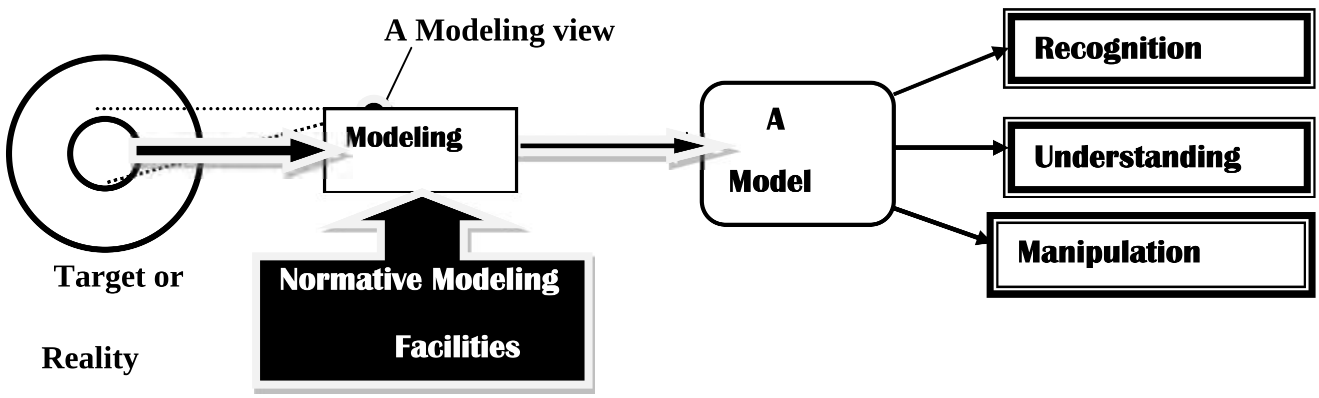

Figure 2: Modeling

2.7 Model Types

1. Conceptual models

Conceptual Models are qualitative models that help highlight important connections in real world system and process. They are used as a first step in the development of more complex models.

2. Teaching with Interactive Demonstrations

Interactive demonstrations are physical models of systems that can be easily observed and manipulated and which have characteristics similar to key features of more complex systems in the real world. These models can help bridge the gap between conceptual models and models of more complex real world systems.

3. Mathematical and statistical Models

Involves solving relevant equation(s) of a system or characterizing a system based up on its statically parameters such as mean, mode, variance or regression coefficients.

Mathematical models include Analytical models and Numerical Models, Statistical models are useful in helping identify patterns and underlying relationships between data sets

5. **Teaching with Visualizations**

By this we mean anything that can help one visualize how a system works. A visualization model can be direct link between data and some graphic or image output or can be linked in series with some other type of model so to convert its output into a visually useful format. Examples include 1-, 2-, and 3- dimensional graphics packages, map overlays, animations, image analysis.

2.8 Models as Instructional Media

Media are carriers of information from a source to a receiver Clark (1994) such carriers are known as instructional media when they are used to convey messages intended to change behavior. Educators have their disposal a very large variety of media that can be used to achieve their lessons objectives. According to Seopo-Sengwe (2001) Media is divided into three categories namely visual, auditive media and audio-visual aids. Visual media are carriers of images, auditive media are carriers of sound and audio-visual media are carriers of sound and images. These categories are further divided into the following: Under visuals we have projecting and no-projecting media. Under projecting media are overhead projectors, slides, filmstrips, silent motion pictures, opaque materials etc. Under non-projecting media are models, fieldstrips, illustrations, charts and diagrams, flannel boards, maps, posters, photographs etc. Under auditive media are compact discs, tapes, radio, telephone etc. projected audio-visual media are sound films, television, printed material with recorded sound etc. Media can only be effective if it is accomplished by relevant methods have been described of instruction (Kozma, 1994). Using relevant teaching method students can visualize the abstract concepts of geometry through models which emphasized under study which carries images of the abstract concepts of geometry.

The Role of Models in Learning Geometry

To visualize or visualization is a way of figuratively represent something, and this is propose for using visualization tools in Models environment to represent and view already existing information that is, geometry models, drawings etc.

There is a special importance of geometrical structures called as semi-concrete on teaching mathematics. An important component of forming concrete or at least semi-concrete of our mental representation of a concept is an external or physical reference (Konyaliogly et al., 2003 By any means of these, human reason sets up a relation between physical or external world and the abstract concepts(Konyalioglu,2003). It can be considered concepts such as geometric structures and mathematical-physical models for meaningful teaching mathematics. Also, mathematical concepts are abstract that one needs highly cognitive achievements to assimilate them (Baki, 1998). Specifically, there are many reasons that substantiate the use of visualization for learning and teaching of mathematics at all levels of schooling, from elementary to university passing through the middle and high school levels. The activity of ‘seeing’ differently is not a self-evident, innate process, but something created and learned (Whitely, 2000) As cognitive is not self-evident, innate process, but something created and learned (Whiteley, 2000) As cognitive science suggests, we see; we create what we see; visual reasoning or ‘seeing to think’ is learned, it can also be taught and it is important to teach it (Whiteley, 2004). Thus teachers who have learned and became skillful in the use of visualization and ‘seeing to think’ would be able to reinforce mathematical concepts and improve the learning process in the classroom.

Brain imaging, neuroscience, and anecdotal evidence confirm that visual and diagrammatic reasoning is congenitally distinct from verbal reasoning (Butterworgh, 1999). Also, studies of cognition suggest that visuals are widely used, in a variety of ways, by mathematics users and mathematicians (Brown, 1993). Moreover, whitely (2004) stated that “I work with future and in-service teachers of mathematics: elementary, secondary and post-secondary. They are surprised to learn than modern abstract and applied mathematics can be intensively visual, combining a very high level of reasoning with a solid grouping in the sense”.

The children's early ideas and actions may be more similar in geometry than in arithmetic. In geometry, physical shapes and children's concepts of shapes may be nearly identical, particularly in early stages of learning. For example, children initially identify shapes by their appearance (e.g., it's circle because it looked like a Frisbee) (van Hiele, 1986). In contrast, on addition problems, children ideas could be about parts and wholes while their actions could involve moving tile pieces around. Because ideas and actions are more similar in geometry, co-evolution and distributing work to the environment may operate more easily. Children may develop their ideas and actions faster if these are more similar, on the other hand, this very similarity may impede development, For example, in fraction problem solving, children transferred better when they learned about adding fractions with Models that were less like fractions (tile pieces) than with models that were more like fractions (pie pieces) (Martin & Schwartz, 2005). Children may not have had to constrict fraction structures mentally with the pie pieces because they could leave that distributed to the environment. Consequently, they learned less. From these views we conclude that models are more useful for geometry learning than algebra.

2.9 Research findings about models in teaching and learning

Geometry is one of fundamental methods which people use to understand and to explain the physical environment by measuring length, surface area and volume. For this reason, enhancing geometric thinking is very important for high-level mathematical thinking, and it should be developed with spatial interaction and manipulation in daily life (Clements & Battista, 1992). However, in traditional classrooms of Ethiopia, geometry learning is usually conducted only through the description of text, 2-dimentional graphs, plasma TV and mathematical formulas on blackboard or paper. In some important topics, such as measuring the area and volume of 2 or3-dimentional objects, traditional teaching methods often focus too heavily on the application of mathematical formulas, and lack opportunities for students to manipulate the objects under study. Consequently, many students can memorize the formulas and even appear to succeed in their course work without fully understanding the physical meaning of the mathematics formulas or geometry concepts (Tan, 1994).

Tan (1994)suggested that the development of understanding of concepts such as the measurement of area and volume should come from the experience of covering and stacking manipulations, so that when formal mathematical concepts and formulas are introduced or applied, students would actually understand the formulas and their meaning. This implies that the construction of the brainstorming (imagery concept) and writing symbolic solutions (abstract representation) (Battista & Clements, 1996).

Therefore, to provide an environment to facilitate such deep, rich learning, researchers employed simulations to create a multi representative construction model, offering learners more flexible ways to organize their thinking with manipulation (like coordinating, restructuring and comparing operations) and symbolic terms, such as text, graphics and speech. By providing students with an easy way to move and forth from the concrete to the symbolic, the tool facilitates student's thinking in geometry problem solving as per the pedagogical theory which states: students should construct their geometry concept from multiple representations like mapping the concrete items to abstract ideas through physical or mental manipulation. Mathematical concepts are abstract concepts. These concepts cannot be understood exactly without presenting some concrete things. If these concepts stay only in our minds, not touched and felt, they become unimportant. That it must be considered important to explain mathematical concepts (graphic, drawings, and tables etc.) with their visual presentations (Chiappini et al., 2001).

Generally, it is stated that abstract concepts are difficult to articulate, and this may be the reason why the students have difficulty in understanding it, however, this problem can be eliminated or at least reduced by making the concepts concrete or giving concrete means. That is, the more concrete but less abstract subjects can learned more easily is fact admitted by everybody (Baki, 2002). Moreover, the concretization of mathematics has been frequently used recently used recently as an essential idea in literature for solving problems and constituting the mathematical concepts in the minds of students meaningfully (Hitt, 2001).

From these views, we understand that the abstraction in mathematics affects the students negatively from the point of sensational learning, and as result of this, they get the opinion that they cannot learn mathematical concepts, so they try to learn mathematics by memorization. The use of concretization or concrete materials method in learning and teaching of abstract concepts affect students in a positive way both emotionally and mentally. As it is, concretization of concepts in mathematics makes it possible to give meaning to abstract mathematical concepts and to better understanding of the mathematical relations (Soylu, 2001). The mathematical concepts are generally abstract and require a high level of mental activity. It can be said that though with the help of geometrical models to help students in showing how a logical theory can be formed through a physical or external model. Because drawing pictures about abstract concepts causes to an interpretation in mind. Students may learn the concepts permanently and properly by Classifying putting into sequence, and outlining the concepts they learned concrete or formally. A number of studies suggest that spatial skills can be developed through instruction (Battista et al, 1982). Appropriate materials should be used in order to improve spatial skills, there exist different types of manipulative used in geometry classes. Physical manipulative are often used in classrooms to help students understand abstract concepts. For instance, when students learn about the volume and surface area of 3-dimensional shapes, teachers often use physical manipulative. This teaching method enables students to visualize the volume and surface area of each manipulative.

CHAPTER THREE

METHODS OF THE STUDY

This chapter is to address those issues related to the research design, research method and the total samples of the study with the selection mechanisms. Additionally, instruments and procedures of data collection were also addressed. Finally, the systems of pilot testing the instruments and data analysis techniques were discussed in this chapter.

3.1 Design of the Study

In this study quasi-experimental research design was used as the main type of research for the investigation. To achieve the research objective quantitative data collection and analytical techniques were employed. Therefore, the overall configuration of the study was made by quantitative research. The main intent of this study is to find out the role of models on students' geometry learning and to investigate the geometric understanding levels of the students when they use concrete means in doing geometry.

The research design was quasi-experimental will treatment and comparison groups. Two tests(pre-test and post-test) for intervention and comparison group were developed and given By the quantitative aspect, numerical data (student test) collected through using two tests. This is because it is difficult to control the interaction of students during the process of treatment. Students can communicate not only in the school but also in their home.

3.2 **Sampling techniques**

The study was conducted in South Gondar, especially in Addis Zemen preparatory school. The target population of the study was grade 11 students. The total population was 648 (which were 380 male and 268 females in grade 11).Out of these grade 11th students' (92 students) were selected as a sample population and organized in two sections. Since the research is quasi experiment the sampling techniques that the researcher used were convenience sampling. In addition to this in order to measure the validity and reliability of the study the researcher selected 40 students from Woreta preparatory school.

3.3 Participants

The subjects of this study were 2 sections of grades 11 in Addis Zemen preparatory school. Section A consisted of 22 female and 24 male students, section B also consisted of 19 female and 27 male students, totally female 41 and male 51. The total numbers of samples were a total of 92 students from two sections.

The background and achievement levels of students were compared according to baseline data collected during the semester of the school that was preceded the study to control the variable which the achievement of the students might affect the result of the study. One section of comparison students who are being taught by different teacher. The teacher is selected randomly out of nine teachers who were teaching the same school teaching sections 11 A and 11th B were taught by the researcher. The research was conducted with students who have similar knowledge in order to study a level of mathematics that had not been the subject of extensive prior research.

3.4 Experimental Treatment

The study had one phase using the investigation of quasi-experiment by taking two tests for the groups of students as treatment and comparison group. The quasi-experimental design of the study conducted for the two groups of students to receive different types of classroom instruction for a period of four weeks in a month. During the data were collected to determine the effectiveness of the treatment that gives the treatment group, the comparison group was thought by traditional instruction using black board and text book for five periods per a week. These tools are standard equipment's in most high school mathematics classrooms, and they are common tools which are used by the teachers in the class room.

The intervention group is known as the visualization group and was taught geometric concepts by using models or physical manipulative such as slides, pictures, maps.

In the chosen two sections, Section A (n=46) was identified as the comparison group and received traditional teaching method was taught by the selected teacher who has taught high school mathematics for at least five years. Similarly, section B (n=46) was considered as an intervention group by the researcher. Similar instruction was applied on the treatment group with the same material which contains blackboard, text book, some reference book and problems with more exercises and more visualization to the treatment groups. Most of the problems and activities were taken from the text book.

The pacing and sequencing of the content was determined by the researcher who was responsible for the creation of all classroom assessments with a careful planning and detailed activities. The same lessons were presented for the two groups. However, the experimental lessons were contained the same content to be presented in a more visual manner whenever possible, taking advantage of models or manipulative.

3.5 Data gathering instruments

Based on the nature of the study geometrical achievement test were used for the selected students. In the process of the quasi-experiments pre-test and post-test from the concepts of geometry were prepared. The items of these tests were based on the exercises and activities of the text book lesson which are taught in grade 11 provide to the students in the form of choice and work out. To measure the validity and reliability of the data, the researcher used Woreta preparatory school students a total of 40 students in grade 11 students. Form the data gathering, the reliability of the data to be 0 .75 crombach alpha, which is suggesting good internal consistency for the scale with this data. Moreover, the tests were evaluated before administration by colleagues to assure their quality to measure students' geometric concept understanding and problem solving. The test contained both procedural and conceptual problem type to evaluate three levels of students (low-medium-high achievers) and the three Bloom's domains of objectives (Cognitive-affective-psychomotor domains) to validate the test.

3.6 Data collection Procedures

During the study the intended data were collected using interview before the pre-test was held in order to know the intention of students towards the content. Classroom tests were administered to both classes. In the beginning of the session, students took the individual pre-test before they had without any treatment for two groups, in which they needed to answer questions related to geometric concepts. At the end of the treatment, the intervention and comparison groups were given post-test which was similar to the pre-test. The questions in the tests were chosen to account all difficulty levels on the geometric concepts, which can be solved directly by applying the geometric definitions and concepts and most of them requires the students' visual perception.

The scoring of data was done based on test of the students. The results of students who had taken all tests and students with the same achievement by comparing their preceded test were scored for data analysis and interpretation. In Addis Zemen preparatory school the result of 46 (22 female and 24 male in section A) students were scored out of 46 participates in the comparison section of the selected teacher, and the researchers' sections the results of 45 (19 female and 27 male in section B) students were scored out of 45 participates in the intervention section. The scoring was done after the some sample problems were scored by the one assistant teachers and the investigator. It was allowed to their correct, partially correct, incorrect answers and no response of the students' geometrical problem solving and geometrical concept understanding level. As clearly underlined in Gagatisis& Demetriadou (1998), in this research a modified measuring model for students scoring was used for the student's problem-solving in much smaller scale based on their responses as shown in table 1indicated below. It was measured by other teachers

Table 1: Expert grader item scoring rubric of students' responses

Score	Responses	Solution Stage
1	Correct Answer	Complete and correct answer with indication of correct understanding of concept. Answer accurate and presented in form specified or almost complete, nearly correct except for minor computational error.
0.5	Partially-Correct Answer	Partially correct with indication of some misunderstanding of concept, may also include computational errors or partially correct with significant lack of understanding evident, likely to include computational errors as well.
0	Incorrect Answer	Some work shown but little evidence that student understands concept or is making progress toward the desired result
0	No Response	Leaving the section blank or offered only a minimal effort at solving the problem

3.6 Data Analysis

The data gathered through tests was analyzed quantitatively. The numerical data which was collected through tests was analyzed using comparison test such as t-test.

This team of section independently scored every item using an item scoring rubric. The scores were then compared and discussed in a face-to face meeting until a consensus was reached for each item completed by each student in both classes. Averages of the scores on each item by group were compiled so that, a direct class-to-class comparison could be made. This served as in assessing differences between the intervention and comparison groups.

This focused on classroom episodes in which the intervention group used visualization tool to produce visual images, partially which was involving model manipulation, and the corresponding presentations used in the comparison group. The results were presented by percentages, frequencies and t-test. At the end the results was interpreted and discussed.

CHAPTER FOUR

DATA ANALYSIS AND INTERPRETATION

The aim of the data collection was to determine the students' understanding of the geometric concepts when they were taught using physical geometric models to solve geometrical problems. The focus was on the concept of conic section and in particular on the concepts of circle, ellipse, hyperbola and parabola.

The chapter presents the data that were gathered in the empirical investigation as analyzed by t-test to report significance at the 0.05 level. To facilitate location of the results, t-value and level of significance were given in tabular form. Discussion of the results was based on not only values and level of significance of the comparison shown on the tables. Implication of both the results and observation were considered. The profile of the respondents with regard to sex, age and previous mathematics knowledge test equivalence are presented first. This chapter deals with the analysis and interpretation of the data obtained from students through questionnaires. The data has been tabulated and discussed as follows.

4.1 Profile and background knowledge of the respondents

Table 2 respondents’’ profile

		Addis Zemen preparatory school			
Category		**11-A**		**11-B**	
		No	%	No	%
SEX	Male	24	52.17	27	58.69
	Female	22	47.82	19	41.3
	Total	46	100	46	100
Age	17	10	21.7	12	26.1
	18	15	32.6	12	26.1
	19	9	19.5	8	17.4
	20	10	21.7	9	19.6
	21	1	2.2	1	2.2
	22	0	0	3	6.5
	23	1	2.2	1	2.2
	Total	46	100	46	100
	Mean age	18.92		18.4	
	Mode age	19		18	

As it is shown in table 2 above, the student respondents were from grade 11 in Addis Zemen Preparatory school, the sex distribution is almost similar with the number of male and female students (the sum of male and female students in 2sections is 55.43% and 44.56% respectively) and in section 11-A the number of female students (47.82%) were less than male students (52.17%), section B the number of female students (41.3%) were less than male students (58.69%). Which implies the number of female Students in all classes is less than male students which does not make any significance difference on the study.

The students' age distribution ranges from 17-23, In Addis Zemen preparatory school, the mean ages for section 11-A and B are 18.92 and 18.4 and the modal ages are 19 and 18 respectively. The respondents in the school were at the same age which guaranteed the result of the quasi-experimental should not be affected by age level of the students (see appendix-B).

Table: 3 comparison of the previous knowledge of the respondents' on mathematics

Variable	*N*	*Mean*	*SD*	*T*	*Df*	*P*
Comparison	46	64.18	6.76	-1.780	90	0.092
intervention	46	64.28	6.84			

As indicated that table 3, the results indicated that there was no significant difference in the mean score between the intervention and comparison group in their previous geometric knowledge. The result of the t-test calculated with a value of (t=-1.780, df=90, p=0.092). It showed no significance difference in mean scores between the comparison and in the intervention group of students in their previous geometric understanding.

4.2 Geometrical knowledge test of the intervention and comparison groups

A per-test was given for the intervention and comparison groups of the student in the school to see their back ground geometry knowledge and the applicability of the test at the start of the study. After the end of different treatment both groups were given post-test which contains conceptual geometric problems.

Comparison of profile of understanding of the intervention group pre-intervention and post intervention

Table 4: ***Comparison by Prior understanding Between Comparison and*** **intervention** ***groups***

Variable	*n*	*Mean*	*SD*	*MD*	*T*	*Df*	*P*
Comparison	46	3.0435	1.0046	-.0978	-.483	90	0.63
intervention	46	3.1413	.9349				

Table 4 presents the results of an independent t-test to compare the prior understanding of Geometry between the comparison and intervention groups. From Leven's test equality of variance was assumed. From this table one can observe that there is no statistically significant difference between the comparison and intervention groups (t = -.483, df = 90, p = .63). This justifies that these two groups are comparable for the purpose of investigating the impact of the intervention of instruction by using modeling. It will also help assess the role of using models for instructional purposes and for students understanding.

Table5: ***Comparison of understanding by post intervention Between Comparison and*** **intervention** ***groups***

Variable	*n*	*Mean*	*SD*	*MD*	*T*	*df*	*P*
Comparison	46	4.7500	.68109	1.72826	13.514	85.37	0.000
intervention	46	3.0217	.53703				

Table 5 presents the results of an independent t-test to compare the understanding of Geometry of the students after the intervention between the comparison and intervention groups. From Leven's test equality of variance was not assumed. From this table one can observe that there is statistically significant difference between the comparison and intervention groups (t = 13.514, df = 90.9, p = .000). This justifies that these two groups had different level of understanding after the intervention of instruction by using models. This justifies that the use of models brought about positive impact on the students understanding.

Pair wise comparison was also conducted to see the effect of using models during instruction. To do this pair wise comparison for both the comparison and intervention groups was conducted. The reason why each groups were compared pair wise was to see whether there is no significant difference on the comparison groups and there is significant difference on the intervention groups. It will also help to check the mean gain among the intervention groups.

Table 6Paired Samples Correlations

	N	Correlation	Sig.
Aggregate Pre for Comparison & Aggregate Post for Comparison	46	.414	.004
Aggregate Pre for intervention &Aggregate Post for intervention	45	.304	.043

The aggregate comparison groups and the intervention groups were assessed for correlation and r = .414,p = .004 for comparison group and r = .304, p = .043 for intervention groups reveals that the understanding level at pre intervention and post intervention were significantly correlated.

Table 7: ***Pair wise Comparison of understanding Between Comparison and*** **intervention** ***groups***

Variable		*N*	*Mean*	*SD*	*MD*	*T*	*df*	*P*
Comparison	Pre	46	3.0435	1.0046	.3235	-12.157	45	0.000
	Post		4.7500	.6811				
intervention	Pre	45	3.1444	.9455	..4024	.876	44	.386
	Post		3.0222	.5431				

As can be observed from table 7, the results of the study show that there was statistically significant difference comparison group between pre &post-intervention (t=-12.157,df=45,p=0.000) and the intervention group is statistically no significant difference between pre & post-intervention (t=.876,df=44,p=0.386).This justifies that these two groups had different level of understanding after the intervention of instruction. This justifies that the use of models did not bring about positive change in the understanding of the students.

4.3Findings and Discussion

The data analysis has resulted in the researcher making the following findings:

1. The t-test result

Statistical results of the geometrical achievement test administered before the study and after the study are provided in table 2, 3, 4, 5, 6, To make best comparison all type of students that is low, medium and higher achiever students' were participated in the study. The researcher selected those students' carefully by giving more time and attention. In order to know their prior knowledge of geometry both the intervention and comparison groups were administered in the pre-test. As indicated in table 3 and 4, students prior knowledge measured through the first semester result and pre-test did not differ significantly ($p>0.05$). In this respect, it can be suggested that the prior knowledge level of the intervention group and the comparison group was significantly equal at the beginning of the intervention. From table 5 one can simply observe that there is statistically significant difference between the comparison and intervention groups during an independent t-test to compare the understanding of Geometry of the students after the intervention between the comparison and intervention groups. This justifies that these two groups had different level of understanding after the intervention of instruction. This justifies that the use of models did not bring about positive impact on the students understanding. From the paired sample correlated table both the intervention and comparison groups on the understanding level at the pre intervention and post intervention were significantly correlated. Table 6 indicated that, based on Pair wise Comparison of understanding Between Comparison and intervention groups suggested that the comparison group show a change in the understanding of geometry during the post test on the other hand the intervention group has no changes. Several reasons were seen during the process no change in the intervention group. One is students did not give more attention on visualization process rather than they focus on the teacher centered process and they considered the visualization teaching learning process as time killing. Most of the time students need more exercise from the teacher. Whereas on the comparison group they show that change due to more time of teaching and learning process and the teacher act as the master of the subject and use more time than they exercise in the class room. In general, a statistically significance difference was found between the post-test score of the intervention group and the comparison group in the setting and school with regarding to the conceptual learning ($p<0.05$). That is, the mean scorers of the students who were exposed to models instruction were significantly lower than that of the comparison group a probability value of 0.05 in conceptual geometric problem solving. As careful analysis of table 6 indicates that the students in the comparison group who were exposed to the traditional instruction had higher scores in the post- test in comparison to their scores in the pre-test, either their mean score in the post-test increase.

In general, the t-test result shows that the students who were exposed to the comparison group students are, on average, more successful than intervention group of students at the 0.05 significance level in the experiment in geometry learning achievement

The result supports the studies conducted by Chappell and Killpatrick (1995), Sherard (2002), Baki (2002), Hitt (2001), Soylu (2001) and Jones (2004). They generally stated that abstract concepts are difficult to articulate, and this may be the reason why the students have difficult in understanding it: however, this problem can be eliminated or at least reduced by making the concepts concrete or giving concrete means and use more time on focusing the concept. Although these concepts were given under almost equal circumstances to two groups, students' performance in solving these problems are found to be different. An explanation to this could be that though these two groups received the same concept of geometry. They were exposed to different experiences which resulted in forming different concept image.

2. Findings with regard to models' contribution

By comparing the students written solutions the following are the contributions of models that enhance the students' geometric competency in particular in geometrical problem solving:

- ❖ To visualize/draw the correct graph of the question that is helpful in labeling the concept and to identify the center and foci of the given question.
- ❖ To identify similar concepts like circle and ellipse.

- ❖ To clarify the relationship and the difference that exists between related and different geometrical concepts.
- ❖ To make learning enjoyable and facilitates students to participate in the classroom.
- ❖ To make students long memory about geometric concepts like parabola, ellipse and hyperbola.
- ❖ To state easily the geometric concepts and help them to relate the concept with the pre-requisite knowledge what they have learned in grade 10.
- ❖ It helps students to examine the similarity and difference between geometric concepts.

CHAPTER FIVE

SUMMARY, CONCLUSION AND RECOMMENDATIONS

This chapter deals with the summary and conclusion drawn from the findings of the study. It also presents possible recommendations forwarded by the researcher based on the conclusions.

5.1 Summary

The main objective of the study was to examine role of models on students' conceptual understanding of geometry in Addis zemen preparatory school. More specifically, the study attempted to answer the following question:

1. Does using models contribute to the development of students' geometry concept understanding?
2. Is there significance difference between different groups of those who learn using models and conventional method?

In order to meet the objectives of the study, data were gathered through the tests investigation of the students' results and convenience selected sampling population. Through review of literature was also made to arrive at the research gaps. The students' response were analyzed and interpreted using the statistical t-test to compare their average test scores of the intervention and comparison groups of students using a 0.05 significance level. Moreover the result shows that the students who were exposed to the comparison group students on average more successful than intervention group of students in the experiment in geometry learning achievement. Then the use of models did not bring about positive change in the understanding of students.

5.2 conclusions

From the result of the study, the research drew the following major conclusion. Model is suitable usage of semi-concrete structure pointed out as geometric system in teaching of the abstract concept in mathematics. Graph, diagram, pictures and geometrical shape or models are tool for visualization of the abstract concept in mathematics. By using models which is a toll of visualization approach many mathematical concepts can become concrete and clear for students to understand. Individual establishes a strong connection between an internal construct and something to which access is gained through the sense. Such a connection can be made in to directions. An act of visualization may consist of any mental construction of object or processes that an individual associates with objects or event perceived by an external source. Alternatively, it may consist of the construction, on some external medium such as paper of objects or events. Consequently, the act of visualization is translation from external to mental. Models can be alternative method and powerful resource for students doing mathematics, a resource that can upon the way to different ways of thinking about different perspective. Models have been described as the creation of a mental image of a given concept.

Findings of the study suggested that students in the comparison group who were exposed to the traditional instruction were slightly more successful than the intervention group students who were exposed to the physical models instruction in a significance level $p = 0.05$. In this result the use of models did not bring about positive change in the understanding of students.

5.3 Recommendations

In the light of findings mentioned in chapter 4 sections 4.3 the use of models brought negative impact on the students understanding. It does not create significant change between the groups. During the process the teacher start the instruction with materials which leads students to explore and discover the necessary structure, give opportunity for the students to pull together what they have learned, planning tasks, directing students attention to the subject matter , introducing terminology and engaging students in discussion using different terms and encouraging explanations and problem solving approaches

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Appendix A

Addis Ababa University

College of Education and behavioral studies

Department of Science and Mathematics Education

PRE-TEST AND POST-TEST

I. PRE-TEST

Name________________________grede&section_________No________

Instruction: answer the following question.

1. The planets revolve around the sun. the shape of the paths is
   A. Circle B. Hyperbolic C. Ellipse D. Parabolas
2. Satellite receivers are made in __________shape.
   A. Circle B. Ellipse C. parabola D. Hyperbolic
3. The section formed by the intersection of a cone and a plane are called_________.
   A. Spheres B. Squares C. Conic sections D. Rectangles
4. A conic section are include
   A. Circle B. Parabola C. Ellipse D. All
5. If the line intersects the circle at two points then the line is ________
   A. Tangent line B. Secant line C. A&B D. none
6. Smoke vents in nuclear reactors are made in __________ shape.
   A. Circle B. Hyperbolic C. Ellipse D. Parabolas
7. Find the center and radius of the circle with equation $(x-5)^2 + (y+3)^2 = 9$

II. Post- test First Phase

Instruction: answer the following question and show the necessary steps as needed to illustrate the method you used.

1. Find the equation of the tangent line to the circle with equation $(x\text{-}3)^2 + (y\text{-}4)^2 = 20$ at
   P (1, 0).
2. Write the equation of the circle with end point of its diameter given to be A(3,-7)& B(4,3).
3. A hyperbola for which a=b is called equatorial if and only if asymptote are perpendicular to each other (True/False).
4. Draw the graph of the ellipse given by the equation $\frac{(x-1)2}{16}+\frac{(y+3)2}{4}=1$.
5. Find the equation of the ellipse with foci at (-1,0) & (1,0) and the length of the major axis 6 unit.
6. Find the vertex, foci and directory of the following parabola $(x\text{-}2)^2 = 8(y+1)$.
7. Identify the center, foci and the length of major axis and minor axis of an ellipse whose equation is $\frac{(x-2)2}{9}+\frac{(y-2)2}{1}=1$

Appendix-B

The students first semester score with regard to sex and age in 2006 E.C. of Addis Zemen higher education preparatory secondary school.

Addis Zemen higher education preparatory secondary school						
Comparison and Experimental class						
	Co (11 A)			Ex (11 B)		
No	Sex	Age	Score (100%)	Sex	Age	Score (100%)
1	M	17	57	M	19	55
2	F	19	70	F	21	61.5
3	M	18	56	M	17	67.5
4	M	18	54	M	19	54.5
5	M	19	65	M	17	73
6	F	17	62	M	19	61.5
7	M	19	57.5	M	17	70.5
8	M	20	61.5	M	22	72
9	F	18	56.5	F	17	68.5
10	M	18	65.5	M	20	63
11	F	17	66.5	F	17	72
12	M	20	65.5	M	20	68
13	M	18	54.5	M	17	55
14	F	19	62.5	F	20	61
15	F	17	65.5	F	19	72
16	M	19	81.5	M	17	75
17	F	18	57.5	M	20	64
18	M	17	64.5	M	18	56
19	M	20	73.5	M	17	71
20	M	17	56.5	M	22	77
21	F	20	78.5	F	18	55
22	F	17	63.5	F	17	59.5
23	M	19	79.5	M	19	75
24	M	20	58.5	M	18	65
25	M	18	60.5	M	17	79
26	M	20	60	M	18	73.5
27	F	18	62.5	F	20	68.5
28	F	20	59.05	F	17	67
29	M	17	57.55	M	18	70.5
30	F	18	70.5	F	19	57
31	M	17	68.5	M	18	63
32	M	19	64.5	M	22	62
33	F	20	67	F	18	74
34	F	18	57.5	F	17	70
35	F	19	68.5	M	23	73
36	F	17	64.5	F	18	66.5
37	M	18	67	M	20	60.5
38	F	18	57.5	F	19	61.5
39	M	20	68.5	M	18	60
40	M	20	62.5	M	20	72
41	F	18	67	F	18	57
42	F	21	66	F	20	66
43	F	18	70	F	18	74.5

44	F	19	70.5	F	19	73
45	M	23	57	M	20	62.5
46	F	18	63	F	18	62.5

Appendix-C

Pre-test and post-test results with regard to total geometry achievement				
	Pre-test		**post-test**	
No	Co (11A)	EX (11B)	Co (11A)	EX (11B)
1.	3	2.5	5	3
2.	2.5	3	4.5	3
3	3	2	4	2.5
4	4.5	3.5	6	3
5	3	3	4	3
6	2.5	2.5	3	2
7	3.5	5	5	4
8	3	3	3	2.5
9	3	3	3	2.5
10	2.5	2.5	3.5	2
11	3.5	3	3.5	2.5
12	2	4	5	3
13	4	5	5	5
14	2.5	3	4	3
15	3	3	4	2.5
16	5	2.5	6	3
17	4	2	4	3
18	2	3.5	4.5	3
19	3.5	2.5	5	3
20	3	3	5	3
21	5	2	5	2.5
22	6	5.5	6	4
23	5	2	5	3
24	4	3.5	4	3.5
25	3	4.5	3.5	3
26	3.5	2	6	3
27	2	3.5	4	2.5
28	2.5	3	3	3
29	2	3.5	4	2
30	2	2	5	4
31	2	4.5	5	2
32	2.5	2	5	3
33	2	3.5	4	3
34	5	3	5.5	2
35	2	3	6	2.5
36	2.5	5	5	5
37	3	3	4	3
38	2	3	2	3
39	4	2.5	3	4.5
40	3	3	5	2.5
41	2	2.5	3	4
42	2	2.5	5	3
43	4.5	3	5	3
44	3	5	6	3
45	2	4	4	5
46	2	2	4	3

Declaration

I, undersigned, hereby declare that thesis is my original work done under the guidance Kasa.M (PhD)

Name MeazaZemen Signature_________________

Date_________________

This thesis has been submitted for the examiners with my approval as a university advisor

Kasa.M (PhD) Signature _____________

Date_________________

Feb 2015

Addis Ababa University