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MODELING THE VOLATILITY OF ETHIOPIAN BIRR

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This is to certify that the thesis prepared by Ashenafi Alemu, entitled: **Modeling the Volatility of Ethiopian Birr** and submitted in partial fulfillment of the requirements for the Degree of Master of Science in Statistics complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

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Abstract

Modeling the Volatility of Ethiopian Birr

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Volatility models cover a wide range of topics in econometrics and Statistics. They are broadly divided in to two categories: ARCH type and SV models based on the determination of variance at time (t-1) having all information at time t. The objective of this study is modeling the volatility of exchange rate of ETB deterministically and stochastically using ARCH type and SV models, respectively. ARCH type volatility models are considered as deterministic whereas SV models are stochastic. The data series used for the study is obtained from the NBE from February 2, 2001 to March 15, 2013 and consists of about 3092 daily observations. The major currencies selected for the study based on the availability of data documentation were: EURO, GBP and USD FOREX rate per ETB. A variety of time series models such as ARCH (1), ARCH (2), sGARCH (1, 1); EGARCH (1, 1), APARCH (1, 1) and basic SV model were estimated. The models used for the analysis were selected based on their performance reflected in the literature of econometrics and Statistics. Among the major currencies selected for the study; a EURO FOREX rate failed to pass the ARCH effect test and model estimation was done only for GBP and USD FOREX rate. The results of the study indicate that sGARCH (1, 1) outperforms other ARCH type time series models for modeling the volatility of exchange rate of ETB per major currencies (GBP and USD). Furthermore, SV model was modeled alone without any comparison for both currencies return. SV model fulfilled all the basic assumptions and can be a candidate for fitting the volatility of GBP and USD return series per ETB.

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List of Acronyms

ACF	Autocorrelation functions
sADF	Augmented Dickey-Fuller
AIC	Akaike Information Criterion
APARCH	Asymmetric power ARCH model
AR	Auto regression
ARCH	Autoregressive Conditional Heteroscedasticity
ARMA	Autoregressive Moving Average
BIC	Bayesian Information Criterion
DIC	Deviance Information Criteria
EURO	Official currency of euro zone/ European Union
EGARCH	Exponential GARCH
ETB	Ethiopian Birr
FOREX	Foreign Exchange
GARCH	Generalized ARCH
GED	Generalized Exponential distribution
GMM	Generalized Method of Moments
LM	Lagrange Multiplier
LR	Likelihood Ratio
MA	Moving Average
MCMC	Markov Chain Monet Carlo
NBE	National Bank of Ethiopia
N-SV	Nonlinear SV
PACF	Partial Autocorrelation Functions
PP	Phillips-Perron
QML	Quasi Maximum likelihood
sGARCH	Standard GARCH
SIC	Schwarz Information Criterion
SMM	Simulated Method of Moments
SV-M	SV in mean
SV	Stochastic Volatility
TGARCH	Threshold GARCH
USD	United States Dollar

Chapter 1: Introduction

1.1. Background of the Study

Time series models have been widely used in many disciplines in the science. Many econometricians and statisticians devote themselves to develop new models and improve the existing ones. Since 1980s, there has been growing interest in time series models with changing variance over time which is shown by most of the financial data. Such time series models with hetroscedastic errors are specifically useful for modeling high frequency data like stock returns and exchange rates.

A volatility model is a specification of dynamics of the volatility process. There are different ways for modeling changes in volatility over time. A commonly used model is the autoregressive conditionally hetroscedastic (ARCH) model introduced by Engle (1982) in which the conditional variance is a function of the squared past values of the series including time t-1. Consequently, the volatility is observable at time t-1. This model has been extended in different directions. The most popular of them is generalized autoregressive conditionally hetroscedastic (GARCH) model which was proposed by Bollerslev after four years of introduction of ARCH model and it lets conditional variance depend on the squared past observations and previous variances. In GARCH models the volatility is also known at time t-1. However, the volatility may be treated as an unobserved variable and this yields another class of models which consider the variance of the process as stochastic and model the logarithm of volatility as a linear stochastic process such as auto regression. Models of this kind are called stochastic variance or stochastic volatility (SV) models. The interest in SV models has been very strong in the last few decades. These models are important alternatives to the famous ARCH-type models. They have similar properties but they are different with respect to the observability of variance, σ_t^2, at time t-1, which means, the distinction between the two models relies on whether the volatility is observable or not.

Volatility of exchange rate can be defined as the variation of price at which two different countries currencies are traded. Volatility models are important to the policy makers, since they use to observe the effect of economic factors on foreign exchange rate as well as to formulate the policies related to the money supply in the economy and the policies associated with the government expenditures and incomes.

Taylor (2005) mentions that foreign exchange rate volatility inputs are helpful in certain financial decisions associated with portfolio optimization, hedging, risk management, pricing of financial derivatives. Kemal (2005) states that foreign exchange rate volatility influences the long-term decision unfavorably by thrilling the volume of worldwide marketing and decisions to allocate resources for investment, and government's sales and procurement policies.

Corporate policy formulators also employ exchange rate variation models as instruments for constructing portfolio, risk management as well as an input for derivate assets pricing. International transactions oriented countries call attention to more emphasis on the foreign exchange rate variation in formulating various economic policies since their economic growth is affected by the foreign exchange dealings significantly (Kamal et al., 2012).

Exchange rates are quoted as foreign currency per unit of domestic currency or domestic currency per unit of foreign currency. From the view point of extent of government control on exchange rates, foreign exchange rates (FOREX) system may be either fixed system or freely floating system .That is, fixed exchange rate is treated as stable one or permitted to be changed merely in a slight range and the floating exchange rate may drift, up and down, according to certain market trends. Floating FOREX rates are expected to be more volatile as they are free to fluctuate. The volatility in FOREX rates result in an increase of exchange rate risk and adversely affects the international trade and investment decisions (Madura, 2006). In case of Ethiopia; FOREX rate follows the floating system (Yohannes, 2007) where the market forces the system solely.

Most of the studies done on the financial time series data like exchange rate; where volatility clustering is an important issue do not consider both stochastic and deterministic volatility models together for modeling their volatility. They focuses either on deterministic part (for example, Ahmed, 2012, Kamal, 2012) or stochastic (for example, Yusuf; 2009). But in this study, both classes of volatility models were used. The exchange rates of ETB against major currencies were considered and suitable deterministic and stochastic volatility models are constructed.

1.2. Statement of the Problem

Financial markets are generally highly competitive. Information flows rapidly and different volumes of instruments can change hands more or less continuously with minimal friction. The variance of returns on an asset tends to change over time, rendering the assumption of a homoscedastic error term in an econometric model invalid. Since exchange rate data is a financial data; it fulfils all the properties of the financial time series. That is, volatility is inherent in exchange rate. Hence, the foreign exchange (FOREX) rate volatility is an important factor involved in the decision making of investors and policy makers. This study is attempted in Ethiopia to capture the volatility of ETB per major currencies. Since volatility model helps the investors and other financial application as an instrument, this thesis was designed to fill the gap. That is, there was no volatility model developed yet for exchange rate of ETB to the knowledge of the researcher. In addition to this, the thesis is designed to fill the gap of using volatility as deterministic and stochastic. Most of the researchers; to the knowledge of the researcher does not consider volatility as deterministic or stochastic. They simply model the ARCH or GARCH model as volatility model.

1.3. Objectives of the Study

1.3.1. General Objective

The main purpose of this study is modeling and quantifying the volatility of ETB per major currencies FOREX rate.

1.3.2. Specific Objectives

The specific objectives of this thesis are:

- ✓ Modeling the Volatility of FOREX of ETB per major currencies deterministically by using autoregressive conditional hetroscedastic (ARCH) type models.
  - ➢ Select an ARCH type model for each currency that best fits its volatility
- ✓ Modeling the Volatility FOREX of ETB per major currencies as stochastic by using basic stochastic volatility (SV) model.

1.4. Scope of the Study

This study focuses on the ARCH type and basic stochastic volatility models to model the FOREX rate volatility of Ethiopia Birr. Since the FOREX rate volatility is the main concern, the study uses the daily data. The data were obtained from NBE for the period between January 02/ 2001 and March 15/2013.

The choice of the major currencies such as EURO, GBP and USD per ETB was based on the relative proportion in the Bank's foreign exchange investment portfolio, and currency composition of imports as well as the record of the data series. The foreign exchange reserves portfolio was held mainly in EURO, GBP and US dollars. These three major currencies have equal documentation of the data series and preferred for the study.

1.5. Significance of the Study

Modeling FOREX rate has many practical applications in econometrics, statistics and finance with wide discussion in the literature. The basic ARCH/GARCH models are frequently applied and quoted to describe the volatility in financial markets. Volatility is the most important variable in the pricing of derivative securities. Primary source for measuring the volatility of an exchange rate, distribution of exchange rate data, has important implications for several financial models and is characterized by mild and volatile periods. Exchange rate volatility is important for developing asset and pricing models, constructing optimal portfolios, and understanding how the exchange rate markets function. This study tries to use financial time series data analysis using deterministic and non-deterministic models to see the volatility of FOREX of EURO per ETB, GBP per ETB and USD per ETB.

The result of this study could help.

- ✓ In understanding FOREX rate volatility; deterministically and non-deterministically.
- ✓ To formulate the policies related to the money supply in the economy.
- ✓ To provide information on the situation of FOREX volatility.
- ✓ As a basis to other researchers for further study.

Chapter 2: Literature Review

Analyzing financial time series data with volatility models has become very common since 1980's and huge literatures have been established. One of the most important tools that characterize the changing of the variance is the ARCH model. Engle (1982) proposes to model time-varying conditional variance with the ARCH process that use past disturbances to model the variance of the series. Early empirical evidence shows that high ARCH order has to be selected in order to catch the dynamic of the conditional variance. The GARCH model of Bollerslev (1986) is an answer to this issue. Several excellent surveys on ARCH/GARCH models are available in Bollerslev, Chou and Kroner (1992), Bollerslev, Engle and Nelson (1994) and Bera and Higgins (1993).

The maximum likelihood based inference procedures for the ARCH class of models under normality assumption are discussed in Engle (1982) and Pantula (1985). Generalized Method of Moments (GMM) estimation of ARCH type models are discussed in Mark (1988), Bodurtha and Mark (1991), Glosten, Jagannathan, and Runkle (1991) and Simon (1989). In addition to these, the Bayesian inference procedures within the ARCH type of models are developed by Geweke (1988) who uses Monte Carlo methods to determine the exact posterior distributions. As an alternative estimation technique, Gallant, Rossi and Tauchen (1990) use a semi parametric approach while Robinson (1987), Pagan and Ullah (1999), Whistler (1988) use a nonparametric method.

The search for model specification and selection is always guided by empirical stylized facts. Stylized facts about volatility have been well documented in the ARCH literature, for instance in Bollerslev, Engle and Nelson (1994). Since the early sixties, it was observed by Mandelbrot (1963) and Fama (1965) and among others that asset returns have leptokurtic distribution with thick tails. As a result numerous papers have proposed to model the returns from fat-tailed distributions. In addition to thick tails, the volatility clustering is also common. ARCH models introduced by Engle (1982) and the numerous extensions as well as SV models are built to capture this volatility clustering. Leverage effect is another fact about the financial time series. Leverage effect suggests that stock price movements are negatively correlated with volatility.

The thick tails property of financial time series data often is not fully captured by GARCH models. This has naturally led to the use of non normal distributions to better model this excess kurtosis. Bollerslev (1987), Baillie and Bollerslev (1989) use Student–t distribution while Nelson (1991) and Kaiser (1996) suggest the Generalized Error Distribution (GED). Other propositions such as: the normal-lognormal (Hsieh, 1989) and the Bernoulli-normal (Vlaar and Palm, 1993). Moreover, to better capture the skewness, Liu and Brorsen (1995) applied an asymmetric stable density. A promising distribution that models both the skewness and kurtosis is the skewed Student-t of Fernandez and Steel (1998), extended to the GARCH framework by Lambert and Laurent (2000).

A comparison of 330 different ARCH-type models in terms of their ability to describe the conditional variance is given in Hansen and Lune (2003). The main findings are that there is no evidence that a GARCH (1, 1) model is outperformed by other models.

Over the last few decades, there has been a tendency to employ the ARCH type models to analyze the volatilities of financial data while ignoring the specification and estimation of the conditional mean. Most recently, Li, Ling and McAleer (2002) define the ARMA-GARCH model which can be reduced to ARMA-ARCH, AR-ARCH, MA-ARCH by simply imposing some restrictions to the process.

Hsieh (1989) used 10 years (1974 – 1983) of daily closing-bid prices, consisting of 2,510 observations, for five countries such as British pound sterling (GBP), French franc (FF), Italian lira (IL), Norwegian Krone (NOK) and Spanish peseta (SP) in comparison of US dollar to estimate the ARCH and GARCH models along with the other modified/altered types of ARCH and GARCH. The findings of Hsieh (1989) proved that the two under study models were capable of removing all heteroscedasticity in price changes. It was also concluded that the standardized residuals from all the ARCH and GARCH models using the standard normal density were highly leptokurtic, and the standard GARCH (1,1) and EGACH (1,1) were found to be more efficient for removing conditional heteroscedasticity from daily exchange rate movements. The EGARCH proved to fit the data better than GARCH.

Mundaca (1991) modeled the Norwegian Krone (NOK)/US Dollar exchange rate through ARCH and GARCH models, the results of which supported that three out of four analyzed series fitted better through GARCH(1, 1) than the ARCH model.

Application of GARCH model by Chong et al. (2002), to capture FOREX rate volatility in the data of Malaysian Ringgit/Pound Sterling, for the period 1990-1997, resulted in their suggestion to possibly reject the hypothesis of constant variance model, arguing that the GARCH models were better ones than native random walk models

Hussein and Jalil (2007) applied the parametric and non-parametric techniques on daily exchange rate of Pakistan Rupee / US Dollar exchange rate and tried to measure the success of intervention in foreign exchange market in Pakistan, which was done either in shape of alteration in the exchange rate level or smoothing the exchange rate fluctuations. The GARCH results, as reported by Hussein and Jalil (2007) proved that intervention was successfully altered, in both direction of exchange rate and smoothed the fluctuations in exchange rate while the event study confirmed that the intervention was successful for level and volatility of the exchange rate.

A weekly Thai Baht (THB) per US Dollar exchange rate, for the period 1999-2005, was analyzed by Jithitikulchai (2005) to study the application of parametric and non-parametric volatility models in which ARCH and ARCH-M were found more realistic, both theoretically and empirically, because of their low volatility around zero mean whereas, the asymmetric coefficients of EGARCH and APARCH showed insignificant results. The APARCH model was declared the best model of modeling the exchange rate volatility in out-of-sample case by the researcher who suggested that non-parametric models could be the best for the conditional volatility prediction, while in the case of high frequency data; it is more preferable than any other volatility model.

Andersen et al. (1999) stated that,

...exchange rate returns are well-known to be unconditionally symmetric but highly leptokurtic. Standardized daily or weekly returns from ARCH type models also appear symmetric but leptokurtic; that is, the distributions are not only unconditionally, but also conditionally leptokurtic, although less than unconditionally. . .Accordingly, Milhoj (1987), Bollerslev (1987), Hsieh (1989) and Baillie and Bollerslev (1989) assert that while the simple symmetric linear GARCH (1, 1) model might provide a good description for most exchange rate series under free float, the assumption of conditional normality does not capture all the excess kurtosis observed in daily or weekly data.

In order to deal with this problem, Bollerslev (1987) successfully presents the leptokurtic property of daily data for foreign exchange rates and finds that the GARCH (1, 1) model with Student-t distribution fits quite well though not fully removes leptokurtic property. In the same way, Baillie and Bollerslev (1989) compare two leptokurtic distributions, Student-t and power exponential distributions, in order to produce a more adequate representation of data. They claim that the Student-t distribution compares favorably to the power exponential and captures the excess kurtosis for most of the rates. The Student-t distribution is also estimated by Hsieh (1989), together with the generalized error distribution, a normal-Poisson, and a normal-lognormal mixture distribution.

Zakaria (2012) examined the daily exchange rates series of nineteen Arab countries. The currencies considered are the United Arab Emirates dirham (AED), Bahraini Dinar (BHD), Djiboutian franc (DJF), Algerian Dinar (DZD), Egyptian Pound (EGP), Iraqi Dinar (IQD), Jordanian Dinar (JOD), Kuwaiti Dinar (KWD), Lebanese Pound (LBP), Libyan Dinar (LYD), Moroccan Dirham (MAD), Mauritanian ouguiya (MRO), Omani Rial (OMR), Qatari riyal (QAR), Saudi Arabian Riyal (SAR), Somali Shilling (SOS), Syrian Pound (SYP), Tunisian Dinar (TND), and Yemeni rial (YER), all against the US dollar. He employed two univariate specifications of the GARCH model, including both symmetric and asymmetric models that capture most common stylized facts about exchange rate returns such as volatility clustering and leverage effect. The empirical results show that the conditional variance (volatility) is an explosive process for the ten of nineteen currencies, while it is quite persistent for seven currencies which are required to have a mean reverting variance process. Finally, he concluded that the exchange rates volatility can be adequately modeled by the class of GARCH models.

Ahmed (2012) estimated the volatility of the exchange rate of Sudanese pound (SDG) using EGARCH (1, 1) and found that leverage effect term is negative and statistically different from zero, indicating the existence of the leverage effect (negative correlation between past returns and future volatility).

Chipili (2007) studied the sources of volatility in the real and nominal Zambian kwacha exchange rates with respect to the currencies of the major trading partners using GARCH models. The results reveal that exchange rates are characterized by different conditional volatility dynamics based on the three GARCH models, i.e., GARCH (1, 1), APARCH (1, 1) and EGARCH (1, 1) and found that EGARCH(1,1) best fit the kwacha exchange rate.

Maana, Mwita, and Odhiambo, (2010) estimated the Kenyan Shilling (Ksh) foreign exchange market data rate per USD, Euro, Sterling pound, and Japanese Yen for the period 1993 – 2006 using GARCH(1, 1). Their analysis showed that the exchange rates are leptokurtic and slightly positively skewed.

Another type of volatility process is stochastic volatility model. Due to the fact that, in SV models the mean and the variance are driven by separate stochastic process SV models are much harder to estimate than the GARCH models. Evaluating the likelihood function of ARCH type models is a relatively easy task. In contrast, for SV model, it is not easy to obtain explicit expression for the likelihood function. The lack of estimation procedures for SV models made them for a long time an unattractive class of models in comparison to ARCH type models. In the past two decades, however, several estimation methods have been developed with the increasing performance of the programming languages and computers (Kim, Shepherd and Chib, 1998).

The early attempts to estimate SV models used a GMM procedure due to Melino and Turnbull (1990). GMM considers the basic SV model with normal innovation processes. Another estimation method is called quasi-maximum likelihood estimation developed by Harvey, Ruiz and Shephard (1994). Harvey, Ruiz and Shephard (1994) have employed Kalman filtering to estimate the parameters by maximizing the quasi likelihood function.

Comparison of GMM and QML can be found in Ruiz (1994), Harvey and Shephard (1995). The general conclusion is QML gives estimates with smaller mean square error. The GMM and QML methods do not involve simulations. However, increasing computer power has made simulation-based estimation techniques increasingly popular. The simulated method of moments (SMM) or simulation based GMM approach proposed by Duffie and Singleton (1993) was a first attempt in simulation based estimation methods.

The strategy of SMM is to simulate data from the model for a particular value of the parameters and match moments from the simulated data with sample moments as substitutes.

Another simulation based approach to inference in the SV model is based on Markov Chain Monte Carlo methods, namely the Metropolis-Hastings algorithm (Jacquier, Polson and Rossi, 1994) and Gibbs sampling algorithm (Kim, Shephard and Chib, 1998). These methods have had a widespread influence on theory and practice of Bayesian inference.

The SV in mean (SV-M) model was developed by Koopman and Uspensky (2000) to incorporate the unobserved volatility as an explanatory variable in the mean equation. The estimation is based on importance sampling techniques.

Chib, Nardari and Shephard (2001) developed an MCMC procedure to analyze the SV model defined by heavy-tailed Student-t distribution with unknown degrees of freedom. They consider the SV_t model with Student-t observation errors and also the SV_t plus jump model which contains a jump component in the mean equation to allow for large, transient movements.

Yu, Yang and Zhang (2002) propose a new class of SV models, namely, nonlinear SV (N-SV) models. They include the lognormal SV model as a special case, which adds great flexibility on the functional form. The estimation procedure is again MCMC. Jacquier, Polson and Rossi (2002) extend their earlier work to analyze the SV model. They replace the Gaussian innovation by a fat-tailed distribution and they consider the leverage effect.

Hol and Koopman (2002) consider the exact maximum likelihood method based on the Monte Carlo simulation technique such as importance sampling and they state that more accurate estimates of the likelihood function are obtained when the number of simulations is increased.

The distribution of exchange rate news is fat-tailed as is widely established in the literature. Like ARCH models, stochastic volatility can explain part of the fat-tailedness. But given the evidence for ARCH models one would expect that time-varying volatility does not fully account for the tail behavior (Baillie and Bollerslev, 1989; Engle and Bollerslev, 1986).

Ronald and Peter (1998) have empirically studied the performance of the first-order stochastic volatility model using a dataset of weekly exchange rates of bilateral exchange rates among the major currencies (US dollar, British pound sterling, Japanese yen, and German mark) from 3 January 1973 to 9 February 1994. The model has been estimated for different specifications of the distribution of the standardized exchange rate innovations. They found that an estimate of the persistence of the volatility process depends crucially on the stochastic specification of the model. And finally concluded that volatility estimates based on a univariate time-series model have large standard errors that they are hardly informative for the purpose of option pricing or currency hedging.

Craine, Lochstoer, and Syrtveit (2000) modeled the Norwegian-British exchange rate volatility. The goal is to find a parsimoniously parameterized model that captures the essential features in the high frequency financial returns data which display potential jumps, volatility clustering, skewness, and excess kurtosis. The main results of their paper are: (1) Capturing these features requires a specification that allows both jumps and stochastic volatility. (2) A specification that only allows for jumps badly misrepresents the data. And (3) reasonably accurate estimates of the parameters of the jump distribution require a very large sample. They used a simulation-based technique to estimate a stochastic volatility (SV) model.

Jun Yu and Meyer (2006) compared fully likelihood-based estimation with multivariate stochastic volatility (MSV) models of Australian dollar (AUD) and New Zealand dollar (NZD) against USD exchange rates. They illustrated the ideas by fitting, bivariate time series data of weekly exchange rates, multivariate SV models, including the specifications with Granger causality in volatility, time-varying correlations, heavy-tailed error distributions, additive factor structure, and multiplicative factor structure. Their result suggested that the best specifications are those that allow for time-varying correlation coefficients.

Kulikova and Taylor (2013) studied ZAR/USD volatility and suggested that, daily South African Rand (ZAR) exchange rates are highly volatile. The QML estimators perform effectively and the SV models fit the market data well. Using the range return instead of the absolute return as the volatility proxy produces QML estimates that are both less biased and less variable. Thus, the log range return QML estimator is superior in performance to the log absolute return QML estimator.

Chapter 3: Methodology

3.1. Source of data

The data used in this study is secondary; which was obtained from National Bank of Ethiopia (NBE) department of Foreign Exchange Monitoring Statistics and Reserve Management (FEMSRM). The Birr, denoted by ETB, is the unit of currency used in Ethiopia. The first currency of Ethiopia was introduced in 1894 and was known as the "Menelik taler". Officially, Abyssinian Birr and Talar were circulated as Ethiopian currencies until the mid 1930. Early 1931 the bank of Ethiopia was formed which now a day's called the National Bank of Ethiopia. In 1934, the occupying Italian forces made the Italian lira the legal currency in Ethiopia until their expulsion in 1942. The Ethiopian currency; the Birr was finally introduced in 1945 and the official exchange rate of Ethiopian currency with the US dollar was created (with the official exchange rate of 2.48 birr per US dollar) on July 23, 1945. Hereafter in 1976 the Birr was officially renamed as ETB and revalued to 2.30 USD. From May 1993 up to the unification of Transitional Government of Ethiopia (TGE) the official and the auction exchange rates on 25 July 1995, the exchange rate was partly determined by government decree (applicable to the official rate) and partly by quasi-market forces (applicable to the auction rate) as represented by auctions (Derrese, 2008).

Since the date of unification, the exchange rate of the birr against the major currencies and the resultant cross-rates has been determined only through the auction system. From the date of unification up to the present day, we have a quasi-market determined exchange rate which is a freely floating exchange rate (Derrese, 2008).

The foreign exchange (FOREX) rate data of ETB per currencies is a daily organized data. These daily observations of the exchange rate data were gathered for the major currencies such as: EURO, GBP and USD from the NBE and its volatility were modeled. The observations of the data are about 3092. The data were obtained from NBE for the period between January 02/2001 and March 11/2013. On January 02/2001 the ETB was at a rate of 7.87, 12.47 and 8.41 per EURO GBP and USD respectively. On March 11/2013, ETB was at a rate of 23.717, 28.118 and 18.33 per EURO, GBP and USD respectively. The observations are daily records which vary from day to day and exclude weekends and holidays. Since the exchange rate of ETB varies daily; this study considered the daily observations since the main intention of the thesis is dealt with volatility modeling that is, modeling the variations of the observations floating freely at different time.

The study considered only the FOREX rate of ETB per three major currencies such as EURO, GBP and USD based on the data record of the NBE. Each of them had equal number of observations whereas others have no enough organized data.

3.2. Concepts and Definitions

A time series is a set of random variables $\{Y_t\}$. The random variables sequentially ordered in time are called a stochastic process. The realization of $\{Y_t\}$ is denoted as $\{y_t\}$. A time series can be continuous or discrete.

The stochastic process y_t can be defined in terms of its moments,

$$E(y_t) = \mu_t \,, \quad var(y_t) = \sigma_t^2 \quad \text{and } cov(y_t, y_{t-s}) = \gamma_{t,t-s}$$

Which are functions of time, t. If the unknown parameters; μ_t , $\sigma_t^2$and $\gamma_{t,t-s}$change with time, an essential restriction on the stochastic process is needed to avoid an estimation problem. The restriction is called stationary, which reduces the number of parameters to be estimated and leads to stable processes over time. A time series having a constant mean and variance is mean and variance stationary respectively. A time series is said to be weakly stationary if the mean and variance of the process are constant and the covariance of it depends only on lag s but not on time t (Kuan, 2003).

For a weak stationary process, the autocorrelation between y_t and y_{t-s} is defined as

$$\rho_s = \frac{\gamma_s}{\gamma_0},$$

Where γ_0 *is* the variance of y_t. Since γ_0 and γs are time-independent, the autocorrelation coefficients ρ_s are also time-independent. The autocorrelation between y_t and y_{t-1} can be different from the autocorrelation between y_t and y_{t-2}, however the autocorrelation between y_t and y_{t-1} must be identical to that between y_{t-s} and y_{t-s-1} (Enders, 1995).

The plot of γs, the auto covariance at lag s, against s depicts auto covariance function. Similarly, the plot of ρ_s against s displays the autocorrelation function denoted as ACF.

The other function related to the correlations between $\{y_t\}$ is called partial autocorrelation function, denoted by PACF. Different than the autocorrelation, the partial autocorrelation is simply the correlation between y_t and y_{t-s} after the effects of $y_{t-1} \ldots y_{t-s+1}$ are excluded.

A stronger form of week stationarity is called strong stationarity which is defined in terms of the distribution function of the random variable. A time series is strictly stationary if the joint distribution of the series of observations $\{Y_{t1}, Y_{t2} \ldots ..Y_{tn}\}$ is the same as that for $\{Y_{t1+s}, Y_{t2+s} \ldots Y_{tn+s}\}$; for all t and s (Türker, 1999). The strict stationarity imposes no restriction on moments. If a strict stationary series has a finite second order moment, it must be weakly stationary.

Since the stationarity defined in terms of the distribution functions is difficult to verify in practice, strict stationarity is not preferable. In this study, the term stationary is used whenever the criteria for weak-stationary are satisfied.

3.2.1. Simple Linear Processes

3.2.1.1. White Noise Process

A white-noise process contains a sequence of uncorrelated zero mean variables with constant variance σ^2.The financial time series will follow white noise patterns very rarely, but this process is the key for the formulation of more complex models.

3.2.1.2. Autoregressive Processes

The process y_t is said to be an autoregressive (AR) process if it can be expressed as,

$$\psi(B)y_t = \psi_0 + \varepsilon_t$$

where ψ_0 is a real number, ε_t *is* a white noise process with mean zero and variance σ^2 and ψ (B) is polynomial in terms of back-shift operator B.

The back-shift operator applied to a time series y_t is defined as $By_t = y_{t-1}$. Similarly, B^2y_t = B $(By_t) = y_{t-2}$, B^3y_t = B $(B^2y_t) = y_{t-3}$, and so on. The back-shift operator is also called lag operator which is denoted by L.

When the order of the polynomial is p, i.e. $\psi(B) = 1 - \psi_1 B - \psi_2 B^2 - \ldots - \psi_p B^p$, the process y_t is referred to as an AR process of order p, AR (p), which can be written as

$$y_t = \psi_0 + \psi_1 y_{t-1} + \psi_2 y_{t-2} + \ldots + \psi_p y_{t-p} + \varepsilon_t.$$

As it is stated in Enders (1995), an AR (1) process with $\psi(B) = 1 - \psi_1 B$ can be written as

$y_t = \psi_0 + \psi_1 y_{t-1} + \varepsilon_t$.

Assuming the process is started at period zero so that y_0 is the known initial condition, the solution of this equation by forward or backward iteration is,

$$y_t = \psi_0 \sum_{i=0}^{t-1} \psi_1^i + \psi_1^t y_0 + \sum_{i=0}^{t-1} \psi_1^i \varepsilon_{t-1} \tag{3.1}$$

Taking the expected value of (3.1)

$$E(y_t) = \psi_0 \sum_{i=0}^{t-1} \psi_1^i + \psi_1^t y_0 \tag{3.2}$$

Updating (3.2), by s periods yields.

$$E(y_{t+s}) = \psi_0 \sum_{i=0}^{t+s-1} \psi_1^i + \psi_1^{t+s} y_0$$

For $|\psi_1| < 1$ and if the limiting value of y_t is considered in equation (3.1) it can be shown that, the expression $(\psi_1^t)y_0$ converges to zero as t becomes infinitely large and the sum $\psi_0[1 + \psi_1 + (\psi_2)^2 + (\psi_3)^3 + \ldots]$ converges to $\psi_0/(1-\psi_1)$. Thus, as t approaches to infinity and if $|\psi_1| < 1$,

$$\text{Lim}y_t = \frac{\psi_0}{1-\psi_1} + \sum_{i=0}^{\infty} \psi_1^i \varepsilon_{t-i} \tag{3.3}$$

The expected value of equation (3.3) is $\psi_0/(1-\psi_1)$, which is finite and time independent.

The variance of y_t is calculated from equation (3.1) as:

$\text{Var}(y_t) = \sigma^2/(1-(\psi_1)^2)$

For the stationarity condition, i.e., for $|\psi_1| < 1$ is satisfied.

Finally, it is demonstrated by Kuan (2003) that the limiting values of all autocovariances are finite and time independent:

$\text{Cov}(y_t, y_{t-s}) = \sigma^2 \psi_1^s / [1-(\psi_1)^2]$

In summary, for an AR(1) process to be stationary, the coefficient of the lagged dependent variable must be less than one in absolute value and t must be sufficiently large. Solution by the iterative methods is not possible in higher-order systems. In these cases, the theory of difference equations is used to get the solution and the stability conditions of the system. For an AR (p) process defined as,

$\Psi(B)\, y_t = \psi_0 + \varepsilon_t$

For a stationary AR (p) process, the autocorrelation function is non-zero at all lags and should converge to zero geometrically. On the other hand, the partial autocorrelation function of an AR (p) process should die out for all lags greater than p.

3.2.1.3. Moving Average Processes

The process is said to be moving average (MA) process if it can be expressed as,

$$y_t = \pi_0 + \Pi(B)\, \varepsilon_t$$

Where π_0 is a real number, ε_t is a white noise process with mean zero and variance $\sigma^2$and $\Pi(B)$ is polynomial in terms of back-shift operator B. When the order of the polynomial is q, i.e. $\Pi(B) = 1 + \Pi_1 B + \Pi_2 B^2 + \ldots + \Pi_q B^q$, the process y_t is referred to as a MA process of order q, MA (q):

$$y_t = \Pi_0 + \Pi_1 \varepsilon_{t-1} + \Pi_2 \varepsilon_{t-2} + \ldots + \Pi_q \varepsilon_{t-q} .$$

In this case,

$$E(y_t) = \Pi_0$$

$$Var(y_t) = \gamma_0 = \sigma^2 (1 + \Pi_1^2 + \ldots + \Pi_q^2)$$

$$\text{cov}(y_t, y_{t-s}) = \gamma_s = \sigma^2 \sum_{i=0}^{q-s} \pi_i \pi_{i+s}$$

For s=0, 1, 2,. . . , q.

Since the mean, variance and covariance functions are all time-independent; the MA process is always stationary regardless of its coefficients. The autocorrelation function is obtained by dividing the γ_s by γ_0 so for the MA (q) process; the ACF has cut off property for the lags greater than q. On the other hand, the PACF of any MA (q) process should approach to zero.

Following the work of Enders (1995), a MA (q) process in the form of,

$$y_t = \Pi(B)\, \varepsilon_t$$

The residual can be calculated as:

$$\varepsilon_t = [\Pi(B)]^{-1} y_t$$

Provided that the $[\Pi(B)]^{-1}$ converges (which is satisfied when the roots of $\Pi(B)$ lie outside the unit circle). This condition is called the invert ability condition and implies that a MA (q) can be written as an AR (∞) process uniquely (kuan, 2003).

3.2.1.4. Autoregressive Moving Average Processes

Combining an AR (p) process and MA (q) process yields an Autoregressive Moving Average (ARMA) process. An ARMA process of order (p, q) is denoted by ARMA (p, q) and illustrated as:

$$\Psi(B)\, y_t = c + \Pi(B)\, \varepsilon_t$$

Where π_0 is a real number, ε_t is a white noise process with mean zero and variance σ^2,

$\Psi(B) = 1 - \Psi_1 B - \Psi_2 B^2 - \ldots - \Psi_p B^p$,

$\Pi(B) = 1 + \pi_1 B + \pi_2 B^2 + \ldots + \pi_q B^q$

For a stationary and invertible ARMA (p, q) process, neither ACF nor PACF has cut off points; they both decay to zero gradually.

3.3. Criteria for Model Selection

After estimating the ARMA models, the most appropriate model for the data set should be chosen. At this point, some model selection methods are considered. One of them is called the Box-Jenkins methodology (Kuan, 2003).

The standard Box-Jenkins approach contains the following four steps:

1. Transform the original time-series to a weakly stationary process.
2. Identify a preliminary ARMA (p, q) model for the transformed series.
3. Estimate the unknown parameters in this preliminary model.
4. Apply the diagnostic checks and re-estimate the model if the Preliminary model is found inappropriate.
5. Repeat these steps until a suitable model is found.

In practice, financial time series are usually no stationary and most of them include a trend component. If a series includes a trend component, it should be removed by taking the first difference. However, if it is a deterministic trend, the differencing is not appropriate; in that case a simple trend variable **t** may be included in the model. Seasonal patterns are other common reasons for non stationarity and they can be eliminated by taking the seasonal difference or by using seasonal dummies.

After obtaining a stationary process, the second step of Box-Jenkins methodology is to estimate a preliminary ARMA model. In order to do this, the properties of ACF and PACF functions are used.

In the third step, the unknown parameters of the preliminary ARMA (p, q) model should be estimated. Finally, diagnostic checks of the residuals are conducted. If the estimated model is correct, the residuals should behave like a white noise process.

Alternatively, the structure of the ARMA process can be determined by using model selection criteria. The most famous ones are the Akaike Information Criterion (AIC) and Schwartz Information Criterion (SIC or BIC):

$AIC = -2 \log likelihood + 2n,$

$BIC = -2 \log likelihood + n\frac{\log(T)}{T},$

Where **T** is the number of usable observations, and **n** is the number of parameters to be estimated. In practice, several ARMA models are estimated, and the one with the smallest AIC or SIC or BIC is selected as the best model (Enders, 1995).

3.4. Unit Root Tests

In order to make inferences on time series, the observations must be stationary. However, most of the financial time series do not satisfy the requirements of stationarity so that they have to be converted to stationary processes before modeling. Many test statistics have been developed to check whether the series contains unit roots or not. The most popular of them are the Augmented Dickey Fuller test (ADF) and Phillips Perron (PP) (Zivot and Wang, 2006).

3.4.1. The Augmented Dickey-Fuller (ADF) unit root tests

Dickey and Fuller (1979) introduced Dickey – Fuller (DF) test statistic to test whether the series contains unit root or not. They assume that the underlying process is a simple AR (1) model.

As explained in Türker (1999), in the simplest form of the test, the model is given as,

$$y_t = a_1 y_{t-1} + \varepsilon_t,$$

Where ε_t is a white noise process with zero mean and variance σ^2.

To obtain the test statistic, subtract y_{t-1} from both sides,

$$\Delta y_t = \gamma y_{t-1} + \varepsilon_t,$$

Where $\Delta = y_t - y_{t-1}$ and $\gamma = a_1 - 1$

So that testing the hypothesis that $a_1 = 1$ is equivalent to testing $\gamma = 0$. Dickey and Fuller (1979) consider three different equations that can be used to test:

$$\Delta y_t = \gamma y_{t-1} + \varepsilon_t \tag{3.4}$$

$$\Delta y_t = \alpha + \gamma\, y_{t-1} + \varepsilon_t, \tag{3.5}$$

$$\Delta y_t = \alpha + \gamma y_{t-1} + \beta_t + \varepsilon_t. \tag{3.6}$$

The first equation (3.4) written above is a pure random walk model, the second equation (3.5) adds an intercept (or drift term), and the last one (3.6) includes both a drift and linear time trend so that it is possible to test whether the trend that series exhibits is deterministic or stochastic (Enders, 1995).

In all of the above equations, H0: $\gamma = 0$ against H1: $\gamma < 0$ is tested and evaluated using a conventional t-ratio as:

$t = \frac{\hat{\gamma}}{se(\hat{\gamma})}$,

and is distributed as non-standard distribution. Mathematically:

$$t \sim \frac{\int w dw}{\int w^2 dw}$$

Where w is the Brownian motion or winner process.

In conducting the DF test as in (3.4), (3.5), or (3.6), it was assumed that the error term ε_t was uncorrelated. But in case the $\varepsilon_{t\,s}$' are correlated, it may lead to wrong conclusions if the data generating process is autoregressive of higher order. Dickey and Fuller have developed a test, known as the Augmented Dickey–Fuller (ADF) test. This test is conducted by "augmenting" the preceding three equations by adding the lagged values of the dependent variable ΔYt. To be specific, suppose we use (3.6). The ADF test here consists of estimating the following regression:

$$\Delta y_t = \alpha + \beta_t + \gamma y_{t-1} + \sum_{i=1}^{m} \theta_i \Delta y_{t-i} + u_t \tag{3.7}$$

Where $\mathbf{u_{t1}}$ is the pure white noise term and $\Delta \mathbf{y_{t-1}} = (y_{t-1} - y_{t-2})$, $\Delta \mathbf{y_{t-2}} = (y_{t-2} - y_{t-3})$, etc and m is lag length in the model; Δ is the difference operator. The number of lagged difference terms to include is often determined empirically, the idea being to include enough terms so that the error term in (3.7) is serially uncorrelated. In ADF we still test whether $\gamma = 0$ and the ADF test follows the same asymptotic distribution as the DF statistic, so the same critical values can be used obtained from the DF table.

A useful tool of thumb for determining lag length (P_{max}) suggested by Schwert (1989) is:

$$p_{max} = \left[12\left(\frac{T}{100}\right)^{1/4}\right]$$

This choice allows the lag length (p_{max}) to grow with the sample so that the ADF test regression are valid if the errors follow an ARMA process with unknown order.

3.4.2. Phillips Perron (PP) Unit Root test

An important assumption of the DF test is that the error terms ε_t is independently and identically distributed. The ADF test adjusts the DF test to take care of possible serial correlation in the error terms by adding the lagged difference terms of the regressand. Phillips and Perron (1988) developed a generalization of the DF procedure that allows for a fairly mild assumption concerning the distribution of the errors. The PP test is based on the statistic:

$$z_t = \frac{\widehat{\sigma_\varepsilon^2}}{\widehat{\lambda^2}} t_\gamma - \frac{1}{2}\left(\frac{\widehat{\lambda^2} - \widehat{\sigma_\varepsilon^2}}{\widehat{\lambda^2}}\right)\left(\frac{T.Se(\hat{\gamma})}{\widehat{\sigma_\varepsilon^2}}\right)$$

where T-is the number of observations; $\widehat{\sigma_\varepsilon^2}$ and $\widehat{\lambda^2}$ is consistent estimates of the residual variance and the heteroscedasticity corrected variance respectively; $\hat{\gamma}$ is the estimate of γ and t_γ is the t-ratio of γ; $Se(\hat{\gamma})$ is the coefficient standard error. The function z_t follows a student t distribution.

3.5. VOLATILITY MODELS

Modeling the volatility of a stochastic process has received much more attention in recent years. Volatility is the amount of price movement of a stock, bond or the market in general during a specific period. If the price moves up and down rapidly over short time periods, it has high volatility; if the price almost never changes, it has low volatility (Kuan, 2003).

There are so many methods which have been developed for modeling the mean value of the variable of interest, one of them is the Box-Jenkins approach. However, the random component of the series may also show changes in variability. As Campbell, Lo and MacKinlay stated in 1997, "It is both logically inconsistent and statistically inefficient to use volatility measures that are based on the assumption of constant volatility over some period when the resulting series moves through time". In some cases, the assumption of constant variance is not satisfied and this is called as the heteroscedasticity problem.

More efficient estimators and better forecast values can be obtained if the heteroscedasticity is handled properly. Because of this, the model which is used in estimating and forecasting the time series should satisfy the constant variance assumption. In most of the financial time series, volatility clustering is usual in the sense that large changes are followed by large changes, and small changes are followed by small changes. Moreover, volatility asymmetry is also quite common. Therefore, volatility models that accommodate all of the above features are needed to be constructed (Kuan, 2003).

The volatility models can be divided into two main classes: deterministic and stochastic volatility models. In deterministic volatility models, the conditional variance is a deterministic function of past observations. These are called as Autoregressive Conditionally Hetroscedastic (ARCH) type models. In stochastic case, on the other hand, the variance equation has its own innovation component which makes the process stochastic rather than deterministic (Pederzoli, 2003).

3.5.1. Deterministic Volatility Models

3.5.1.1. Autoregressive Conditionally Hetroscedastic Models

Engle (1982) introduced the autoregressive conditional hetroscedastic (ARCH) model, which was a first attempt in econometrics to model the volatility. The aim is to simultaneously model the conditional mean and conditional variance of the time series. To model the conditional mean and the conditional variance, Engle used the principle: "In order to model the conditional mean of y_t given y_{t-1}, y_{t-2}, y_{t-3}... write y_t as a conditional mean plus white noise. To allow the non-constant conditional variance in the model, multiply the white noise term by the conditional standard deviation."

To illustrate the principle, consider a time series $\{y_t\}$ such that:

$$y_t = \mu_t + \sigma_t \varepsilon_t$$

$$\mu_t = a + \beta_1 x_{1,t} + \beta_2 x_{2,t} \ldots + \beta_k x_{k,t}$$

Where, $\boldsymbol{\mu_t}$ denotes the conditional mean which is a function of explanatory variables $\mathbf{x_{i,t}}$ that may contain both lagged exogenous and dependent variable; y_t represents the dependent variable over time period. The disturbance term ε_t, is identically and independently distributed with zero mean and unit variance (or constant variance, $\sigma^2 =$

1). Usually, the assumption of normality for ε_t is added and σ_t^2 is the conditional variance of the process.

The ARCH (1) process is in the form:

$$y_t = \mu_t + \sigma_t \varepsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 (y_{t-1} - \mu_{t-1})^2, \alpha_0 > 0, \alpha_1 \geq 0.$$

If the mean part of the process is taken as zero, that is if $\mu_t = 0$, then, the ARCH (1) process can be written as,

$$y_t = \sigma_t \varepsilon_t \quad (3.8)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 \quad (3.9)$$

Where, $\alpha_0 > 0$ and $\alpha_1 \geq 0$ are unknown parameters; and

$$E(y_t/\Omega^{t-1}) = 0 \text{ and } E(y_t^2/\Omega^{t-1}) = \sigma_t^2 E(\varepsilon_t^2)$$

Ω^{t-1} is the information set which contains all the available information at time t.

The argument in Triantafyllopoulos (2003) considered, $y_t^2 = \sigma_t^2 + (y_t^2 - \sigma_t^2)$

By using equation (3.9),

$$y_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + v_t$$

Where, $v_t = \sigma_t^2 (\varepsilon_t^2 - 1)$

The process y_t^2 as defined above follows a non-normal AR (1) model with the innovations

$$v_t = \sigma_t^2 (\varepsilon_t^2 - 1).$$

By the law of iterated expectation, $E(y_t) = E[E(y_t/\Omega^{t-1})]$, and $var(y_t) = E(y_t^2) = \alpha_0 + \alpha_1 var(y_{t-1})2$. If $\alpha_1 < 1$, the process is stationary and $var(y_t) = \alpha_0/(1-\alpha_1)$. Assuming that y_t are conditionally normally distributed, $E(y_t^4/\Omega^{t-1}) = 3E(\sigma_t^4)$, assuming that $E(y_t^4)$ is constant for all t so that,

$$E(y_t^4) = [3\alpha_0^2 (1 + \alpha_1)] / [(1-\alpha_1)(1-3\alpha_1^2)]$$

This implies that $0 \leq \alpha_1^2 \leq 1/3$. The kurtosis coefficient of y_t is then,

$$M_4 / [var(y_t)]^2 = 3(1 - \alpha_1^2)/(1-3\alpha_1^2) > 3.$$

Since the kurtosis of a normal distribution is 3 so that its excess kurtosis can be calculated as

$M_4 / [var(y_t)]^2 - 3]$. According to this result, it can be noted that the unconditional distribution of y_t is leptokurtic. That means, even y_t are conditionally normally distributed, the resulting ARCH (1) process cannot be normal (Kuan, 2003).

An ARCH (1) process is easily generalized to an ARCH (q) process such that,

$$y_t = \sigma_t \varepsilon_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i y_{t-i}^2,$$

Where, $\alpha_0 > 0$, $\alpha_i \geq 0$ (i= 1, 2 . . . q). For stability of the process $\alpha_1 + \alpha_2 + \ldots + \alpha_q$ should be less than one (Li, Ling, McAleer, 2002). Similar to ARCH (1) model, ARCH (q) model can be represented by an AR representation with order q.

There are some problems with ARCH (q) models. The required value of q might be very large and the non-negativity constraints on coefficients might be violated. Because of these reasons, Generalized Autoregressive Conditionally Hetroscedastic (GARCH) models are introduced.

3.5.1.2. Generalized Autoregressive Conditionally Hetroscedastic Models

Generalized Autoregressive Conditionally Hetroscedastic (GARCH) models are first introduced by Bollerslev in 1986.

The standard GARCH (1, 1) process is specified as:

$$y_t = \sigma_t \varepsilon_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \alpha 0 > 0, \alpha 1, \beta 1 \geq 0. \quad (3.7)$$

The conditional variance equation of GARCH (1, 1) model contains a constant term, news about volatility from the previous period measured as the lag of previous term squared residual y_{t-1}^2 (the ARCH term), and last period's forecast variance σ_{t-1}^2 (the GARCH term).The unconditional mean and variance of GARCH (1, 1) process can be obtained by using law of iterative expectations for a weak stationary process such that,

$$E(y_t) = E[E(y_t/\Omega^{t-1})] = 0.$$

$$Var(y_t) = E(y_t^2) = \alpha_0 / [(1-\alpha_1)(1-\beta_1)]$$

Thus, $\alpha_1 + \beta_1$ must be less than one for the stationarity purpose. As in the ARCH process, in GARCH (1, 1) model the marginal distribution of y_t is leptokurtic even if the conditional distribution is normal (Kuan, 2003).

As illustrated in Enders (1995), the more general GARCH (p, q) model is,

$$y_t = \sigma_t \varepsilon_t$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i y_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

$$= \alpha_0 + \alpha(B)y_{t-1}^2 + \beta(B)\sigma_{t-1}^2 \qquad (3.8)$$

Here, it is quite obvious to observe the similar structure of Autoregressive Moving Average (ARMA (p, q)) of GARCH (p, q) processes: a GARCH (p, q) has a polynomial β(B) of order "p" ; the autoregressive term, and a polynomial α (B) of order "q" which is the moving average term (Kuan, 2003).

3.5.1.3. Exponential GARCH Models

In GARCH models, due to the presence of y_t^2 in the variance equation, the positive and negative values of the lagged innovations have the same effect on the conditional variance. However, volatility responds to positive and negative shocks differently, so in the case of volatility asymmetry; GARCH models are not good choices (Kuan, 2003). For this reason, exponential GARCH (EGARCH) models were introduced by Nelson in 1991. A simple EGARCH (1, 1) model is,

$y_t = \sigma_t \varepsilon_t$, with conditional variance

$$\sigma_t^2 = \exp\left[\alpha + \beta \ln\left(\sigma_{t-1}^2\right) + \theta[(\frac{y_{t-1}}{\sigma_{t-1}}) + \gamma\left|\frac{y_{t-1}}{\sigma_{t-1}}\right|]\right]$$

Here a positive y_{t-1} contributes $\frac{\theta(1+\gamma)}{\sigma_{t-1}}$. Thus, θ signifies the leverage effect of y_{t-1}. Again we expect θ to be negative in real applications.

In EGARCH process positive and negative shocks of the same magnitude do not have the same effect on volatility due to the exponential function; a larger innovation has a larger effect on σ_t^2. These are the basic differences between GARCH and EGARCH models. EGARCH (1, 1) process can be extended to EGARCH (p, q) process such that,

$y_t = \sigma_t \varepsilon_t$

$$\sigma_t^2 = \exp\left[\alpha_0 + \sum_{i=1}^{q} \beta_i \ln\left(\sigma_{t-i}^2\right) + \sum_{j=1}^{p} \theta_j (\frac{y_{t-j}}{\sqrt{h_{t-j}}}) + \sum_{j=1}^{p} \gamma_j \left|\frac{y_{t-j}}{\sqrt{h_{t-j}}}\right|\right]$$

3.5.1.4. APARCH Models

It has often been observed that a large negative return increases volatility more than does a positive return of the same size. This phenomenon is called the leverage effect and makes sense, since we can expect investors to become more nervous after a large negative return than after a positive return of the same magnitude. The GARCH models cannot model the leverage effect, because σ_t is a function only of past values of y^2_t, so information about the sign of y_t is lost. To model the leverage effect, the APARCH (asymmetric power ARCH) models replace the square function with a more flexible class of nonnegative functions that includes asymmetric functions. The APARCH (p, q) model for the conditional standard deviation is

$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i \left(|y_{t-i}| - \gamma_i y_{t-i}\right)^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta$$

Where $\delta>0$, $-1<\gamma_j<1$, j = 1, 2. . . P. The parameters $\alpha_1, \alpha_2, . . . , \alpha_p$ and $\beta_1, \beta_2, . . ., \beta_q$ satisfy the same constraints as GARCH model. Note that $\delta = 2$ and $\gamma_1 = \gamma_2 = \cdots = \gamma_p = 0$ gives a standard GARCH model. A positive value of γ_j indicates a leverage effect, because for a positive γ, as x increases, the function $|x| - \gamma x$ increases more quickly when x is negative than when x is positive. This cause the conditional variance to increase more for negative than positive values of y_{t-i}

3.5. 1.5.Estimation of ARCH/GARCH Models

3.5.1.5.1. Estimation of ARCH Models

Based on the assumption of the normality made on the ε_t the method of maximum likelihood estimation is adopted. Let $y_1, y_2, . . . , y_t$; be a realization from an ARCH (1) process, then the likelihood of the data can be written as a product of the conditionals as:

$$f(y_1, y_2, \ldots, y_t/\boldsymbol{\theta}) = f(y_t/y_{t-1}) f(y_{t-1}/y_{t-2}) \ldots f(y_2/y_1) f(y_1/\boldsymbol{\theta})$$

Where $\boldsymbol{\theta} = (\alpha_0, \alpha_1)'$. It is more practical to set condition on y_1 since the form f (y_1/θ) is difficult to obtain. Usually y_1 is assumed to be known and equal to its observed value. This allows us to use the conditional likelihood given by

$$f(y_1, y_2, \ldots, y_t/\boldsymbol{\theta}; \mathbf{y_1}) = f(y_t/y_{t-1}) f(y_{t-1}/y_{t-2}) \ldots f(y_2/y_1) f(y_1/\boldsymbol{\theta}; y_1)$$

Since $y_t/\Omega^t \sim N(0, \sigma_t^2)$ it follows that

$$f(y_t/\Omega^t) = \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp\left\{-\frac{y_t^2}{2\sigma_t^2}\right\}$$

Where $\boldsymbol{\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2}$. The conditional log-likelihood is expressed as:

$$\ell = \ln f(y_2, \ldots, y_t, \theta)$$

$$= -\tfrac{1}{2}\textstyle\sum_{i=2}^{t} \ln(2\pi\sigma_i^2) - \tfrac{1}{2}\sum_{i=2}^{t} \frac{y_i^2}{\sigma_i^2} \qquad (3.9)$$

The maximum likelihood estimates are obtained by maximizing this function with respect to α_0, α_1, (Tsay, 2002). Note that the function is non linear in these parameters and thus its maximization must be done using appropriate non linear optimization routine. Let a process $[y_t]_{t=1}^{T}$ be a series generated by an ARCH (1) process, where T is the sample size. Conditioning on the initial observation, the joint density function can be written as:

$$f(y) = \prod_{t=2}^{T} f(y_t/\Omega^t)$$

To find the conditional maximum likelihood estimates of α_0 and α_1, first one needs the derivatives of the conditional log-likelihood (equation (3.9)) with respect to α_0 and α_1 respectively. The derivative results in two systems of equations with two unknowns. Then the systems of equations can be solved for $\widehat{\alpha_0}$ and $\widehat{\alpha_1}$ which are the maximum likelihood estimates of α_0 and α_1 respectively.

3.5.1.5.2. Estimation of GARCH (1, 1) Model

Estimation of the parameters of the GARCH (1, 1) model is performed in the same approach as in the ARCH (1) model. However, since the conditional variance of the GARCH (1, 1) model depends also on the past conditional variance, an initial value of the past conditional variance is needed. Suppose as before that we have a sample of log-returns $y_1 \ldots y_n$ and we wish to find estimates $\widehat{\alpha_0}, \widehat{\alpha_1}$ and $\widehat{\beta_1}$ that maximize the log-likelihood function.

The likelihood function is:

$$L(\alpha_0, \alpha_1, \beta_1; y^n) = p(y_1, \ldots, y_n/\alpha_0, \alpha_1, \beta_1),$$
$$= \prod_{t=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{y_t^2}{2\sigma_t^2}\right)$$

Whereas from (3.7), $\sigma^2_t = \alpha_0 + \alpha_1 y_{t-1}^2 + \beta_1 \sigma_t^2$ the log-likelihood function of α_0, α_1, β_1 is:

$$\ell(\alpha_0, \alpha_1, \beta_1; y^n) = \log L(\alpha_0, \alpha_1, \beta_1; y^n)$$
$$= -\frac{n}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{n} \log\sigma_t^2 - \frac{1}{2}\sum_{t=1}^{n} \frac{y_t^2}{\sigma_t^2} \quad (3.10)$$

To find the maximum likelihood estimates of α_0, α_1, β_1, first we can obtain the partial derivatives of equation (3.10) with respect to α_0, α_1, β_1 respectively and equate each to zero which results in a system of three equations with three unknowns. Then the systems of equations can be solved for $\widehat{\alpha_0}, \widehat{\alpha_1}$ and $\widehat{\beta_1}$ which are the maximum likelihood estimates of α_0, α_1, and β_1 respectively.

3.5.1.6. Test of ARCH Effect and Model Diagnosis

For ease in notation, let $a_t = y_t - \mu_t$ be the residuals of the mean equation. The squared series a^2_t is then used to check for conditional heteroscedasticity, which is also known as the ARCH effects. Two tests are available. The first test is to apply the Ljung–Box statistics Q (m) to the $\{a^2_t\}$ series. The second test is running a Lagrange multiplier test, that is a squared residuals of the regression are regressed on their q lags such that, $\varepsilon_t^2 = \gamma_0 + \gamma_1\varepsilon_{t-1}^2 + \gamma_2\, \varepsilon_{t-2}^2 + \ldots + \gamma_q\varepsilon_{t-q}^2 + v_t$ and the R^2 of the regression equation multiplied by the number of usable observations, T. The test statistic TR^2 is distributed as chi-square with degree of freedom q which is the number of restriction on the null hypothesis, Ho: $\gamma_1 = \gamma_2 = \ldots = \gamma_q = 0$. If the test value is greater than the critical value the conditional variance has to be modeled, otherwise there is no need for ARCH family models (Engle, 1982).

For a properly specified ARCH model, the standardized residuals

$$\hat{a}_t = \frac{a_t}{\sigma_t}$$

form a sequence of iid random variables. Therefore, one can check the adequacy of a fitted ARCH model by examining the series $\{\hat{a}_t\}$. In particular, the Ljung–Box statistics of $\hat{a}_t$ can be used to check the adequacy of the mean equation and that of $\hat{a}_t^2$can be used to test the validity of the volatility equation assumptions. The skewness, kurtosis, and quantile-to-quantile plot (i.e., QQ-plot) of $\{\hat{a}_t\}$ may also be used to check the validity of the distribution assumption.

3.5.2. Stochastic Volatility

The stochastic volatility (SV) model is an important alternative to the ARCH type models and has attracted much attention recently. In ARCH / GARCH models, the volatility is considered as deterministic however, in SV models it is modeled as stochastic. That means SV considers the shocks affecting volatility in contrast to ARCH type models.
As illustrated in Kuan (2003), a simple SV process is,

$$y_t = \sigma_t \varepsilon_t \qquad (3.11)$$

$\ln(\sigma^2_t) = \alpha_0 + \alpha_1 \ln(\sigma_{t-1}^2) + v_t$

If $|\alpha_1| < 1$; $\ln(\sigma^2_t)$ of stationarity.

The volatility equation has innovation term v_t which is independent of ε_t. The inclusion of new innovations makes the model more flexible but estimation of the process becomes much more difficult.

If the assumption of normality is added, that is if $\varepsilon_t \sim N(0, 1)$ and $v_t \sim N(0, \sigma_v^2)$, then,

$$E[\ln(\sigma^2_t)] = \alpha_0 + \alpha_1 E[\ln(\sigma_{t-1}^2)] + E[v_t]$$

Since $|\alpha_1| < 1$, for stability and v_t has zero mean the expectation and variance becomes,

$$E[\ln(\sigma^2_t)] = \alpha_0 / (1 - \alpha_1)$$

$$Var[\ln(\sigma^2_t)] = \sigma_v^2 / (1 - \alpha_1^2)$$

That means, $\ln(\sigma^2_t)$ is distributed as normal with mean $\alpha_0/(1-\alpha_1)$ and variance $\sigma_v^2/(1-\alpha_1^2)$.

If $\ln(\sigma^2_t)$ is distributed as normal then σ^2_t is distributed as log-normal and the lognormal distribution can be specified in terms of the parameters of normal distribution. It is shown that,

$\ln(\sigma^2_t) \sim N(\alpha_0/(1-\alpha_1), \sigma_v^2/(1-\alpha_1^2))$

Then,

$$\sigma_t^2 \sim \log-normal\left\{\exp\left[\frac{\alpha_0}{1-\alpha_1} + \frac{\sigma_v^2}{2(1-\alpha_1^2)}\right], \exp\left[\frac{2\alpha_0}{(1-\alpha_1)} + \frac{\sigma_v^2}{(1-\alpha_1^2)}\right]\exp\left[\frac{\sigma_v^2}{1-\alpha_1^2} - 1\right]\right\}$$

Knowing that $E(y_t) = 0$ and using the above information the higher order moments of y_t can be calculated:

$$E(y^2) = E(\sigma_t^2)E(\varepsilon_t^2) = \exp\left[\frac{\alpha_0}{(1-\alpha_1)} + \frac{\sigma_V^2}{2(1-\alpha_1^2)}\right],$$

$$E(y^4) = E(\sigma_t^4)E(\varepsilon_t^4) = 3\exp\left[\frac{2\alpha_0}{(1-\alpha_1)} + \frac{2\sigma_V^2}{(1-\alpha_1^2)}\right],$$

When the kurtosis of y_t, M_4, is calculated,

$$M_4 = \frac{E(y_t^4)}{[E(y_t^2)]^2} = 3\exp\left[\frac{\sigma_V^2}{(1-\alpha_1^2)}\right] > 3,$$

Thus, y_t is also leptokurtic.

An alternative and more commonly used representation of SV models are given in Kim, Shephard and Chib (1998) such that,

$$y_t = \beta \exp(h_t/2)\, \varepsilon_t$$
$$h_t = \mu + \varphi(h_{t-1} - \mu) + \sigma_\eta \eta_t \quad (3.12)$$

Where the log-volatility is denoted by h_t such that, $h_t = \ln(\sigma^2_t)$ and $h_1 \sim N(\mu, v_t^2)$; where $v_t^2 = \frac{\sigma^2}{1-\phi^2}$. The log-volatility follows a stationary process if $|\varphi| < 1$; ε_t and η_t are uncorrelated standard normal white noise shocks and σ_η is the volatility of the log-volatility. The parameter β or exp ($\alpha/2$) is constant scaling factor.

3.5.2.1. Stochastic Volatility Estimation

Unlike the ARCH/GARCH models, a SV model include error terms in both mean and variance equations. The likelihood function is difficult to evaluate and several methods have been developed to solve this estimation problem. Such methods include generalized method of moments (GMM), quasi-maximum likelihood (QML) estimation, and Monte Carlo Markov chain (MCMC) methods. In a Monte Carlo study, Andersen, Chung and Sqrensen (1999) compared the performances of various procedures and the MCMC method is found to be the most efficient tool in making inferences about SV models. MCMC is a Bayesian approach while others are based on an ad hoc approach. Therefore, in this study, MCMC approach is used to estimate the parameters of the basic SV model. Since MCMC is a Bayesian approach the basic ideas in Bayesian analysis will be described below.

3.5.2.1.1. Bayesian Theory

As explained in Koop (2003), Bayesian econometrics is based on a few simple rules of probability. For two random variables A and B, it is known that,

$$p(B/A) = \frac{p(A/B)p(B)}{p(A)}$$

Similarly,

$$p(\theta/y) = \frac{p(y/\theta)p(\theta)}{p(y)}$$

where y is the data set and θ contains the unknown parameters. Bayesians treats the θ as a random variable and p (θ |y) is the fundamental of interest. It gives all the information about the parameters after observing the data. Ignoring p(y),

$$P(\theta | y) \propto p(y | \theta)\, p(\theta)$$

The term P (θ |y) is referred to as the posterior density, p (y | θ) is the likelihood function and P (θ) is the prior density.

If the mean of the posterior density, called posterior mean, is wanted to be estimated,

$$E(\theta/y) = \int \theta P(\theta/y)d\theta$$

If g (θ) is of interest rather than θ, then

$$E(g(\theta)/y) = \int g(\theta)P(\theta/y)d\theta$$

In general, the above integral cannot be evaluated analytically. Usually a numerical method is needed and in Bayesian econometrics this method is called as posterior simulation. The simplest posterior simulator is referred as Monte Carlo integration.

The Monte Carlo integration has the following steps:

Step1: Take a random draw θ^s, from the posterior of θ.

Step2: Calculate g (θ^s), where g (.) is a function of interest, keep the result.

Step3: Repeat step 1 and 2 S times.

Step4: Take the average of the S draws of g (θ^1)... g (θ^s). The average value converges to E [g (θ) |y] as S goes to infinity.

These steps give an estimate of E [g (θ) |y] for any function g (.).

In many cases, it is not possible to take random draws from p (θ|y) because of the functional forms. However, dividing the parameter space θ into various blocks such that θ = ($\theta_{(1)}$, $\theta_{(2)}$ . . . $\theta_{(B)}$) and then taking random samples from full conditional distributions $p(\theta_{(1)}| y, \theta_{(2)},...,\theta_{(B)}),\ldots, p(\theta_{(B)}| y, \theta_{(1)},..., \theta_{(B-1)})$ is a possible way. This approach is called as Gibbs sampler and it is a powerful tool for posterior simulation. In this method, first an initial value is chosen and then random draws of θ (i) conditional on previous draws are taken sequentially. It yields a sequence of draws from the posterior. A possible problem in this application is to select the initial value. However, the initial values do not matter in the sense that the Gibbs sampler will give a sequence of draws from the posterior and it is repeated S times. The first S_0 of these replications are called as burn- in replications and the remaining S is used in estimation. Generally, the steps of the Gibbs sampler are as follows:

Step0: choose a starting value, $\theta^{(0)}$, for s = 1. . . S.

Step1: take a random draw, $\theta_{(1)}^{(s)}$ from $p(\theta_{(1)} |y, \theta_{(2)}^{s-1}, \ldots \theta_{(B)}^{s-1})$

Step2: Take a random draw, $\theta_{(2)}^{(s)}$ from $p(\theta_{(2)} | y, \theta_{(1)}^{s-1}, \ldots \theta_{(B)}^{s-1})$

. . .

Step B: Take a random draw, $\theta_{(B)}^{(s)}$ from $p(\theta_{(B)} | y, \theta_{(1)}^{s-1}, \ldots \theta_{(B-1)}^{s-1})$

Where, B is the number of blocks in the parameter set θ; S is the number of replication. The above steps yield S values of θ. To eliminate the effect of initial value the burn-in replications should be dropped and the remaining S draws are used to make inferences. That is, like Monte Carlo integration, the average of S draws converges to E [g (θ) |y] as S goes to infinity. While applying the above procedure, it can be ensured that the effects of the initial value are eliminated. Moreover, unlike the Monte Carlo integration, in Gibbs sampling the draws are not independent from each other. Therefore, Gibbs sampling required more draws than Monte Carlo integration.

The fact that the draws in Gibbs sampling are dependent to each other means that the resulted sequence is a Markov Chain. This kind of simulators is called Markov Chain Monte Carlo (MCMC) algorithms.

3.5.2.2. MCMC for Stochastic Volatility

In a basic SV model which is represented by equation (3.13) the parameters are $\theta = (\varphi, \sigma_\eta 2, \mu)$. The posterior of θ can be written as:

$$\pi(\theta/y) \propto f(y/\theta)\, f(\theta)$$

Where f(y/ θ) is the likelihood function and f (θ) is the prior density for θ. However, since,

$$f(y/\theta) = \int f(y/h, \theta) f(h/\theta) dh$$

(Where $h = (h_1, \ldots, h_T)$), the T volatilities are difficult to find, and so the direct analysis of π(θ/y) is not possible. In such cases, posterior simulators can be used. A possible way to solve this problem is to apply Gibbs sampling which is a MCMC algorithm. In Gibbs sampling, as explained before, the parameter space is divided into blocks and the algorithm proceeds by sampling each block from the full conditional distributions. One cycle of the algorithm is called sweep or a scan, the draws from the sampler will converge to the draws from the density in interest as the number of sweeps or iterations increases.

For the basic SV model, the parameter space is (θ, h) where $\theta = (\varphi, \sigma_\eta^2, \mu)$. The Gibbs sampling algorithm for the SV model is given in Kim, Shephard and Chip (1998) as follows:

1. Initialize h and θ.
2. Sample h_t from h_{-t}, y, θ for t=1, . . ., T. (h_{-t} denotes the rest of the h vector other than h_t)
3. Sample σ_η^2/y, h, φ, μ.
4. Sample φ/y, h, σ_η^2, μ.
5. Sample μ/y, h, φ, σ_η^2.
6. Go to 2.

Cycling from 2 to 5 is a complete sweep of this sampler. Many sweeps should be performed to generate samples from θ, h/y.

The most difficult part of the algorithm is to sample from $h_t|h_{-t}, y_t, \theta$; since this operation has to be done T times for each sweep. However, in SV models it is not possible to sample directly from $f(h_t| h_{-t}, y_t, \theta)$ because

$f(h_t| h_{-t}, y_t, \theta) \propto f(h_t| h_{-t}, \theta)\, f(y_t| h_t, \theta)$, t= 1, . . ., n.

So Metropolis-Hastings procedure is used to draw from $f(h_t| h_{-t}, y_t, \theta)$. The candidate density is taken as normal with parameters μ_t, v_t^2.

Chapter 4: Result and Analysis

This chapter presents the data used in the study, constructs the empirical models and displays the results.

4.1. Data Description and Basic Statistics

The FOREX series for the major currencies were taken from the NBE from February 2, 2001 to March 15, 2013. The FOREX series were a daily NBE records and the series exchange rate was an average of buying and selling rates of NBE spot exchange. First, a time plot of the foreign exchange rate of ETB against major currencies such as EURO, GBP and USD was constructed. The major currencies from the FOREX market of Ethiopia were selected based on the documented sample information for each. That is; the currencies with equal sample observations were selected and analysis was performed for each. Hence, first each currency data series was plotted with the aid of statistical software (R version 3.00) in order to determine the existence or non existence of trend component in the series (displayed on FigureA1 of appendix A).

Accordingly, the plot of each output indicated an upward increasing trend (Figure A1 of Appendix A). Therefore, we can say that each series was including a trend component. Hence, before modeling the series the trend could be removed from the series by transforming the data series.

It is obvious that the FOREX rate series is not stationary since it is increasing upward as time changes. However, increasing pattern of the series is not always an indication of non-stationarity. Hence, a statistical test was made for the identification of the stationarity/or non-stationarity of the data. Before removing the trend first we should decide whether the trend in the series is deterministic or stochastic. In order to do this, ADF and PP unit root tests were applied two times with and without trend test.

4.2. Unit Root Test for Stationarity

Before estimating and developing a model for the series a stationary series must be obtained. The unit root test could help us to determine whether the series is stationary or not. The unit root tests first impose the null hypothesis that the time series has a unit root problem versus the alternative hypothesis that the series is stationary/or have no unit root problem. To do so an Augmented Dickey-Fuller test (ADF) and Phillip Perron (PP) tests are used to check the stationarity of the major currencies foreign exchange rate series per ETB. Initially, we check the stationarity of the original FOREX series for each currency and the result is displayed in Table 4.1 and Table 4.2 below.

In the first case, the ADF equation which does not include the trend component is used to calculate the test statistic; in the second case trend variable is considered in the equation. The trend is deterministic if the series is stationary with trend but non-stationary without trend (Tsay, 2010). ADF statistics are calculated for both cases and the results are as presented in Table 4.1. The hypothesis to be tested by both ADF and PP test is:

***Ho**: Non-stationary ($I_{(1)}$) against **H1**: Stationary ($I_{(0)}$)*

Where $I_{(1)}$ imply there is a unit root problem and $I_{(0)}$ there is no unit root problem.
Hence, based on the R output of the series the following results were obtained.

Table 4.1: ADF test for FOREX data series for the major currencies (with trend)

Currency		Critical values			
	Test statistic	1%	5%	10%	p-value
ETB per USD	-1.3577	-3.96	-3.41	-3.12	0.9409
ETB per EURO	-2.2805				0.4994
ETB per GBP	-2.7283				0.592

Here the test statistics (t=-1.3577, -2.2805 and -2.7283 of USD, EURO and GBP, respectively) for the null hypothesis of *H0* are greater than their corresponding critical values -3.96, -3.41 and -3.12 at 1%, 5% and 10% respectively or less than in absolute value than their corresponding critical values. This result suggests that we cannot reject the null hypothesis of unit root; which implies the series is non-stationary. This output as stated is based on the equation of ADF with trend component considered in the series and in case; the series is found to be non-stationary in all the three major currencies FOREX series.

The following software output again on the assumptions of ADF is made for the equation with only drift and white noise.

Table 4.2: ADF Unit Root test for the Major Currencies FOREX series with drift

Currency		Critical values			p-value
	Test statistic	1%	5%	10%	
ETB per USD	0.8098	-3.43	-2.86	-2.57	0.9009
ETB per EURO	0.1742				0.3902
ETB per GBP	-0.1267				0.8991

In the same manner, for each currency FOREX series the result showed that the series is non-stationary because all the test statistic values were found to be greater than their corresponding critical values at 1%, 5% and 10% level of significance. Hence, according to Table 4.1 and Table 4.2 the series includes a stochastic trend since it is found to be non-stationary in both cases.

Table 4.3: Phillips-Perron (PP) unit test for major currencies per ETB FOREX

PP test with trend					
Currency		Critical values			p-value
	Test statistic	1%	5%	10%	
ETB per EURO	-2.1784	-3.966508	-3.413909	-3.128683	0.6527
ETB per GBP	-2.2493				0.5797
ETB per USD	-1.1426				0.9564
PP test without trend					
	Test statistic	1%	5%	10%	p-value
ETB per EURO	0.0981	-3.435444	-2.862987	-2.567566	0.7521
ETB per GBP	-0.1367				0.4727
ETB per USD	1.2834				0.8567

Table 4.3 presents the result of Phillips-Perron t-statistic for the major currencies exchange rate series. The test statistic in all cases of the data series is greater than its critical values at 1%, 5% and 10% level of significance. The p-value also supports the test statistics, and fails to reject the null hypothesis. The result shows that, the PP t-statistic again fails to reject the unit root problem or the null of non-stationarity. These results

imply again the same indication with that of the ADF; that is, the major currencies foreign exchange rate series per ETB have a unit root problem.

Hence, the exchange rate series of the major currencies data series of each per ETB showed non-stationarity condition that can be stationeries by logarithmic transformation and some additional non-stationarity that can be corrected by differencing the logarithms. After transformation of the series by the logarithmic difference of the major currencies per ETB we also checked the stationarity or unit-root problem of the series.

4.3. Transformation and Unit root test of the transformed series

4.3.1. Transformation of the data series

In order to analyze the volatility models, exchange rate returns are calculated for the daily data series. In financial time series analysis, the first difference of log value of a series is called the return and represents the percentage change in observation from period to period. The return or percentage change is highly applicable if the change is very small. Assume that r_t denotes the exchange rate at time t. Then,

Log-Return= $y_t = log\left(\frac{r_t}{r_{t-1}}\right) = log(r_t) - log(r_{t-1}) = \Delta \log(r_t)$, for t=2, 3. . . T.

Where, Δ stands for the differencing, log (.) stands for natural logarithms and T are the number of observations. To attain the mean stationarity, we took differences and for the variance stationarity, we took the logarithms. Consequently, the series for each currency exchange rate per ETB were made stationery by taking the first difference of the logarithmic series and denoted as y_t. After transformation of the data to stationary process as a return, the return series could be fully used as an input data series for modeling volatility.

4.3.2. Descriptive Statistics and Unit Root Test of the Major Currencies Exchange Rate Return

The plot of the first difference of logarithm of FOREX series for each major currency is displayed under Appendix A of Figure A2 and indicating that the trend is totally removed. In order to support it statistically the above test was applied for the series with trend and the following result was obtained. From the line graphs (Figure A2 of Appendix A), we see that large values are followed by large values and smaller values are followed by smaller ones. That is, an indication for volatility clustering of the data.

Table 4.4: ADF Unit Root Test for the Major Currencies of FOREX Return Series

Currency		Critical values			p-value
	Test statistic	1%	5%	10%	
ETB per USD	-40.2744	-3.96	-3.41	-3.12	<0.001
ETB per EURO	-55.594				<0.001
ETB per GBP	-64.678				<0.001

The computed ADF t-statistic for the first order logarithm difference for the original exchange rate data denoted as $\Delta\log(r_t)$ (or the return) of exchange rate series is smaller than the critical values at 1%, 5% and 10% significance levels respectively for each FOREX rate series. This result leads to the rejection of the null hypothesis of a unit root problem/or non-stationarity of the series. That is, the first difference of the transformed FOREX series does not have a unit-root problem. That is, the series is now stationery. That means the differenced log series is stationary which is denoted as I (0) and fully be used for modeling based on ADF test.

Table 4.5: Phillips-Perron (PP) Unit Root for the FOREX Return Series

Currency		Critical values			p-value
	Test statistic	1%	5%	10%	
ETB per EURO	-58.9658	-3.966508	-3.413909	-3.128683	<0.001
ETB per GBP	-99.0006				<0.001
ETB per USD	-143.6729				<0.001

The PP test also gives a similar inference to ADF test. The computed PP t-statistic for the return series is very small compared to test critical values at 1%, 5% and 10% significance levels. Therefore, we reject the null hypothesis that there is a unit-root problem in the return series. Thus, we make the same inference as in the ADF test where the series does not have a unit root problem in first differenced logarithmic series. That is, exchange rate of the major currencies per ETB series is stationary at first difference of the logarithmic series. Therefore, based on the ADF and PP test we can conclude that the first order difference of the log transformed major currencies series ($\Delta\log(r_t)$) is stationary.

4.4. Test for ARCH Effects

Before estimating volatility models, we must examine the existence of heteroscedasticity in the series for ARCH effects to make sure that this class of models is appropriate for the data. Before modeling; the variance of the square of the residuals of the mean equation, or the return of the series, should be checked whether they include any ARCH effects or not. In order to test for the existence of any ARCH effects or not, the ARCH-LM test is applied to the return series.

4.4.1. Lagrange Multiplier Test for an ARCH effect in the Return Series

For the test of ARCH effect in the return series of the data, there are two methods to use. Either using the series itself or using the square of the residuals extracted from ARMA model of the return (Tsay, 2010). Since both techniques are similar, in this study, direct use of the return series was used first. For implementation, second an ARMA model was built to the EURO return series and ARCH effect was tested from square of the residuals, and both resulted in the same conclusion.

Table 4.6: Test of ARCH effect in the return series

H0: no ARCH effects			
Currency Return	Ch-square value	Df	p-value
EURO return	0.2997	12	1
GBP return	1338.982	12	< 2.2e-16
USD return	1388.902	12	< 2.2e-16

Table 4.6 presents the Lagrange Multiplier test of the return series of foreign exchange rate of the three major currencies. The LM test showed an existence of strong ARCH effect for the GBP and USD return with the test statistic very large and p-value of which is approximately zero. Thus, it is pertinent to assume heteroscedasticity where conditional variance is not constant throughout the time in GBP and USD returns for the FOREX data series per ETB. For the case of EURO return at a default lag twelve (12) in the R software, we can say that; there is no ARCH effect. This situation could be highly evaluated at every lag less than twelve. At lag one; for example, the p-value is found to be 0.4419 and again highly fails to reject the null of no ARCH effect in the series of EURO return so that we can say there is no ARCH effect in the EURO return series.

One may repeat these tests by constructing an ARMA (p, q) model, where the orders are estimated by using PACF and ACF plots for the AR and MA respectively. For the case of EURO series; p and q was found to be two (2) and one (1) respectively. Then, ARMA (2, 1) was modeled to the EURO series and we extracted its residual values. Then, a LM test for the test of conditional heteroscedasticity was applied to the squared residuals extracted from the ARMA (2, 1). The test result also showed that, there is no ARCH effect in the EURO return series. The result is presented in the following table 4.7.

Table 4.7: Test of ARCH effect for the squared residuals of ARMA (2, 1)

Ho: no ARCH effect				
Model	ψ_1	ψ_2	π_1	C
ARMA (2, 1)	0.1847391	-0.0050622	0.2229384	0.0002897
EURO	Chi-squared value	Df or lag value	p-value	
	0.2316	1	0.6303	
	0.2443	2	0.885	

Therefore, the null hypothesis of no ARCH effect could not be rejected for the ARMA model as well, for the case of EURO return series. The ARCH-LM test showed that the ARCH effect is unavailable for the EURO return series. Hence, modeling an ARCH type model or stochastic volatility model for the EURO return series leads to misspecification of modeling its volatility. Therefore, ARCH type model is not appropriate for the EURO return series of FOREX per ETB. Consequently, volatility should be modeled for GBP and USD return series by either an ARCH type or a stochastic volatility model whereas; for the EURO return series it is recommended to model ARMA rather than ARCH type or SV model. For a time being since this theses was aimed to modeling the volatility (ARCH type and SV), EURO return was left in application or in modeling of volatility. Since modeling the volatility of the series without an ARCH effect leads to misspecification of the model.

4.5. Estimation of ARCH type models

Various deterministic volatility models could be fitted to the data, but in practice higher order GARCH models are not preferred because it is known that sGARCH (1, 1) is able to capture the change in variance. Therefore, ARCH type models or deterministic volatility models that are commonly used in the literature such as: ARCH (1), ARCH (2), sGARCH (1, 1), EGARCH (1, 1), and APARCH (1, 1) are fitted to the data return of GBP and USD series. The fitted ARCH type or deterministic volatility models are compared to each other based on the information criterion such as AIC and BIC and the best model were selected. These models were chosen to be modeled since they are common ARCH type models and highly used in the literature.

4.5.1. Model Selection of ARCH type Models

For the return series of GBP and USD, four ARCH type models were initially chosen and a best model for each series was selected based on the selection criterion such as AIC and BIC. The model with the smallest AIC or BIC can better model the deterministic volatility of the return series. Hence, after selecting the best model for each return series, an appropriate methodology was performed for the selected model.

Table 4.8: Summary Statistics for the ARCH Type Models of GBP and USD Return

	USD Return			GBP Return		
Information criteria	sGARCH(1,1)	EGARCH (1, 1)		sGARCH (1, 1)	EGARCH (1, 1)	
AIC	-16.60222	-10.693		-7.418500	-6.7911	
BIC	-16.59245	-10.683		-7.408736	-6.7813	
ARCHLM TR^2 (p-value)	0.005188194 (1)	0.001146 (0.9949)		0.0118136 (1)	0.004961 (0.9975)	
Log Likelihood (normalized)	25663.73 (8.32727)	16531.01		1140.29 (3.7108668)	10500.61	
	APARCH (1, 1)	ARCH (1)	ARCH (2)	APARCH (1, 1)	ARCH (1)	ARCH (2)
AIC	-12.570	-15.1389	-15.1382	-6.8295	-7.4108	-7.4130
BIC	-12.560	-15.1311	-15.1285	-6.8198	-7.4030	-7.4032
ARCHLM Test TR^2 (p-value)	0.0677 (0.9982)	0.02406175 (1)	0.02727194 (1)	0.0405 (0.9778)	0.00959944 (1)	0.009599 (1)
Log Likelihood (Normalized)	19432.03	23401.29 (7.570782)	23401.23 (7.570764)	10560.05	11457.49 (3.70726)	11461.83 (3.708129)

Accordingly, for both cases, GBP and USD return series of the exchange rate; standard GARCH (1, 1) model was found to be the best ARCH type model. That is, the AIC and BIC of sGARCH (1, 1) were found to be the smallest. Hence, we can say that sGARCH (1, 1) model best fits the conditional volatility of exchange rate return of the major currencies, mainly GBP and USD per ETB. For the case of GBP as presented in Table 4.9, the AIC and BIC of sGARCH (1, 1), ARCH (1) and ARCH (2) models are almost similar to each other but still sGARCH (1, 1) is better than the ARCH (1) and ARCH (2) models since its information criterion was a little bit smaller than that of the ARCH (1) and ARCH (2). As a result, sGARCH (1, 1) model was fitted to the return series of GBP and USD. It is also advisable to model the ARCH (1) or ARCH (2) to the USD data return series since the GARCH part of the sGARCH(1, 1) model is not that much significant. For a time being sGARCH (1, 1) is modeled but it is recommended to use ARCH part only if such a situation was happen.

The standard generalized autoregressive conditional hetroscedastic model for both returns was summarized below and all the information about the model was displayed under Table B1 and Table B2 of Appendix B;

$y_t = \sigma_t \varepsilon_t$

Model for GBP return Model

$$\sigma^2_t = 7.433776e\text{-}06 + 0.186055 * y_{t-1}^2 + 0.7379723 * \sigma_{t-1}^2,$$

Model for USD return Model

$$\sigma^2_t = 3.4627e\text{-}10 + 0.92123 * y_{t-1}^2 + 1.0000e\text{-}08 * \sigma_{t-1}^2,$$

The parameter estimation of the model was done by using R software. The parameters of the conditional variance equation were obtained by maximum likelihood estimator method. Using R, the parameter coefficients of the ARCH type models of the first order difference of the logarithm of daily FOREX rate of GBP and USD per ETB return series were obtained. The result of the parameters estimated was summarized under Table 1A and 1B of Appendix B.

The parameters of each model should satisfy the stationarity condition for the conditional hetroscedastic variance. That is, the sum ($\alpha_1 + \beta_1 < 1$; coefficient for the ARCH part and GARCH part, respectively) was satisfied for each model. That is, 0.9240273 and 0.92123001 are the sums of the coefficients of GBP and USD return series sGARCH (1, 1) model and both are less than one. In addition to this; all coefficients of the models are found to be positive that means the restrictions are satisfied for the sGARCH (1, 1) model for ensuring variance to be positive.

4.5.2. Graphical plot of the sGARCH (1, 1) Model of GBP and USD Return Series

A graphical plot for conditional standard deviation is presented in Figure C1 and C4 for GBP and USD under Appendix C, respectively. The values for the conditional standard deviation for both return models were obtained by taking the square root of the conditional variances of the model. From the conditional standard deviation plot in Figure C1 and C4; the long spikes indicates high volatile periods of the series. For the GBP return series the spikes where clearly seen at the beginning, after a 100^{th} index, between 1500^{th} and 2000^{th} index, near 2500^{th} index. These were the high volatility periods of GBP return data series. For the USD return series conditional plot, the spikes were clearly seen twice between 0^{th} and 500^{th} index, four times between the 2000^{th} and 2500^{th} index period. The spikes of the sGARCH (1, 1) model show volatility clustering. The high spikes groups the volatility of the data series of each return. That is the plots point out some of the high and low volatility clustering in the return series. That is, the extraordinary long spikes are the high volatile periods in the data series.

4.5.3. Diagnostic Checking of sGARCH (1, 1) Model of GBP and USD return series

After we have estimated the parameters of the models for both returns, the next step is diagnostic checking the selected sGARCH (1, 1) model of GBP and USD return series. For sGARCH(1, 1) model, the Ljung-Box Q-statistics is very small with p-value very close to one for all Q(10), Q(15) and Q(20) for the standardized residuals and their squared series of GBP and USD return series as presented in table 2A and 2B of appendix B. The Ljung-Box fails to reject the null of no autocorrelation of the residuals and squared residuals of the sGARCH (1, 1) model. Subsequently, we can say that there is no autocorrelation in the residuals/or squared residuals of the sGARCH (1, 1) model. The result indicating that the Ljung–Box statistic of the standardized residuals and their squared series fail to indicate any model inadequacy. That is, the constructed model sGARCH (1, 1) have no serial autocorrelation in it.

In the same manner; the p-value of the Jarque-Bera statistic is very high which fails to reject the null hypothesis of normality, implying that the standardized residuals and their square are normally distributed. These do not conflict with the theory of the model that better fits a certain data series. Hence, we can conclude that sGARCH (1, 1) model is adequate for the GBP and USD return series.

In diagnostic checking stage, a test for assessing of conditional heteroscedasticity in the return series was also conducted using ARCH-LM test on the residuals (Table 2A and 2B of the Appendix B). The ARCH-LM test for one lag difference of standardized residuals squared for GBP and USD is respectively: 0.01181368 and 0.2289299 under $TR^2 \sim \chi 2$ (1). But, the null hypothesis that was stated as; there is no autoregressive conditional heteroscedasticity effect up to order q is not rejected since the p-value for each series is approximately one and is much greater than the significance level of 5%. In general, the fitted model sGARCH (1, 1) was appropriately modeled for the GBP and USD return series.

Therefore, the ARCH-LM test on the residuals of these models indicates that the conditional heteroscedasticity is no longer present in the return series of GBP and USD FOREX return per ETB. This means that we do not need higher order ARCH type model and that the sGARCH (1, 1) model is able to capture appropriately the ARCH effects under the return series.

4.6. Estimation of Stochastic Volatility (SV) Model

In financial returns, the variance is affected by an idiosyncratic noise terms, exactly as returns. When there is an additional noise (with respect to those for returns) which affects the evolution of the variance, we say that we are in a stochastic volatility model. As explained before, the ARCH type models consider the variance of the series as deterministic; conditionally, although it can be stochastic. The SV model introduces an innovation (υ_t) to the conditional variance equation of y_t. The addition of the innovation term increases the flexibility of the model in describing the evolution of σ^2_t, but it also increases difficulty in parameter estimation. The difficulty in estimating a SV model is because for each shock the model uses two innovations ε_t and υ_t. Therefore, the following basic SV model is estimated for both major currencies return series per ETB which models the day-to-day variation of the volatilities as a Markov process. To ensure positiveness of the conditional variance, SV model use ln (σ^2_t) instead of σ^2_t

The mostly used SV model in the application is:

$$y_t = \beta exp\,(h_t/2)\,\varepsilon_t, \text{ for } t \geq 1$$

$$h_t = \mu + \varphi\,(h_{t-1} - \mu) + \sigma_\eta \eta_t$$

Where, $h_1 \sim N\left(\mu, \frac{\sigma^2}{1-\varphi^2}\right)$, ε_t and η_t are uncorrelated white noise process, $\upsilon_t = \sigma_\eta \eta_t$ and $\upsilon_t \sim N\,(0, \tau^2)$, $h_t = \ln(\sigma^2_t)$; which is unobservable or latent volatilities.

In order to estimate the SV model; an MCMC algorithm is used and done jointly by (R version 3.0.0 and a "tsbugs" package called to R from the OpenBUGS software). The MCMC sampler was initialized by setting all the h_t, φ, σ_t^2 and μ on the program/command, that is, for the implementation of Gibbs sampling we used prior distributions which were realized by the "tsbugs" package from the OpenBUGS software. The initial value of μ is the sample mean of the log returns. The algorithm is iterated for both return series 11,000 times. The burn-in period is assumed to be large enough to ensure that the effect of the starting values become insignificant. The results for the posterior draws were summarized in Table 4.9 and Table 4.10 below:

Table 4.9: GBP return Bayesian posterior parameter draws of SV Model

Parameter	μ/y (Psi0)	φ/y(psi1)	β= exp[μ/2]/y	τ^2
Estimate	-10.5189	0.7045	0.005198	14.2504
Standard Error	0.05411	0.03169	0.00021	1.344534
Naive SE	0.001711	0.001002	0.000203	0.0425179
Time-series SE	0.0016121	0.0009511	0.0000054	0.0425179
ESS	1176	69	1176	71
DIC	-23190			

Note: time-series standard error is due to Geweke (1992).

Table 4.10: USD returns Bayesian posterior parameter draws of SV Model

Parameter	μ/y (psi0)	φ/y (psi1)	β= exp[μ/2]/y	τ^2
Estimate	-9.0581	0.9355	0.00192	1.8183
Standard Error	0.084998	0.010337	0.0057	0.20626
Naive SE	0.0026879	0.0003269	0.023	0.006523
Time-series SE	0.0026879	0.0002723	0.018	0.0065225
ESS	7311	742	7311	220
DIC	-22400			

Note: time-series standard error is due to Geweke (1992).

The result presented in Table 4.9 and Table 4.10 above, shows the estimated value of φ is less than one which implies that the log volatility process ln (σ^2_t) is stationary. The estimated φ is close to one (0.7045 and 0.9355, respectively for GBP and USD return series) in both cases, which means the shocks in the log-variance are highly persistent. The naive standard errors of the sample mean deriving from the Monte Carlo simulation are considered as a measure of the accuracy of the estimates. The accuracy could be improved by increasing the number of iterations. The simulation inefficiency factors measure how well the Markov Chains mixes. As a rule of thumb, the simulation should be run until the Naïve SE for each parameter of interest is less than about 5% of the sample standard deviation. It is defined as the ratio of the numerical variance (i.e. square of the Monte Carlo standard error) and the variance of the sample mean that would derive from drawing independent samples, as independent random draws would be the optimal outcome of the simulation procedure, the most desirable inefficiency factor is one that is closest to one. The inefficiency factor can be interpreted as the number of times the algorithm needs to be run to produce the same accuracy in the estimate that would derive from independent draws. The inefficiency factor can generally be reduced by increasing the number of iterations (Geweke, 1992). The time-series standard errors give an estimate of the variation that is expected in computing the mean of the MC replications.

4.6.1. Graphical Presentation of the Parameters of Basic SV model of GBP and USD Return

Figure C7 (B) and C8 (B) of Appendix C; show the trace (or sketch) and corresponding plot density functions of the four coefficient parameters of the basic stochastic volatility model. Figure C7 (A) and C8 (A) of Appendix C shows each at the top, an estimated plot of the volatilities for the GBP and USD return. From the plot of the estimated volatility, we can understand that the volatility of the return series is clearly described by the basic SV model. That is, the small daily volatile of the return series of the GBP and USD return were clearly shown by the plot of the SV model. The plot of Figure C7 (A) and C8 (A) was done by R using "stochvol" package for the purpose of sample volatile description for the return series from the stochastic volatility model.

4.6.2. Diagnostic Checking of SV models

After SV model was fitted to the data the diagnostic checks can be applied to the innovation or the error term.

Table 4.11: Diagnostic Test of Standardized Innovations of SV Model of GBP and USD Return

GBP SV Model				
	Box-Ljung Test	Jarque-Bera	ARCH test	BDS Test
p-value	0.0943	0.4214	0.2288	0.3110
USD SV Model				
	Box-Ljung Test	Jarque-Bera	ARCH test	BDS Test
p-value	0.8721	0.3125	0.9789	0.1234

Table 4.11 shows that the resulting innovation from the SV model fulfils all the necessary assumptions: normality, non existence of serial autocorrelation and test of independence. The Box-Ljung statistic tests the null hypothesis of no serial autocorrelation in the innovations of SV model. Whereas, Jarque-Bera, ARCH test, and BDS test respectively tests the null hypotheses of errors are normally distributed, no ARCH effect and errors are independent and identically distributed. Hence, in all the cases; the null hypothesis could not be rejected. Therefore, we can say that the SV model is adequate for both the GBP and USD return data series. That is, the basic SV passes all tests and we can say that it describes the behavior of the data adequately.

4.7. Discussion of the Result

In researches, discussions could be made in corresponding to the review of the related literature. However, since the review for the modeling of volatility of exchange rate of ETB per major currencies was not done yet, to the knowledge of the researcher, this study tried to discuss the result with those generally done related literatures of exchange rate volatility models.

In this study the volatility of ETB per major currencies as deterministic is using ARCH type models was first modeled. Secondly, the volatility was modeled as stochastic using basic SV model. For the EURO data series of exchange rate per ETB, the data series failed to pass the test for ARCH effect. Due to this, EURO series was not used in the modeling of volatility for the FOREX rate per ETB. Hence, only GBP and USD data return was used for modeling the volatility of exchange rate of ETB.

The study found some evidence that standard GARCH (1, 1) outperforms other models and fits the exchange rate volatility of ETB per GBP and USD, well deterministically. This result coincides with the work of Lunde (2003), who compared 330 ARCH type models and found sGARCH (1, 1) as a best.

In a similar manner, Hsieh (1989) compared different ARCH type models for the daily data series of exchange rate of different currencies and found that sGARCH (1, 1) and EGARCH (1, 1) were efficient for the data but sGARCH (1, 1) proved to be better than EGARCH (1, 1) and the result of this study is consistent again with Hsieh's finding. The study done by Manduca (1991) in Norway also supports the idea that sGARCH (1, 1) models better fit the exchange rate series volatility than any ARCH type implying the same finding with this study.

In general, there are a lot of literatures regarding an ARCH type models and most of them supports the idea behind the performance of sGARCH (1, 1) model as it was the case in this study. For example, Bollerslev (1987) exchange rate GBP; Maana, Mwita, and Odhiambo, (2010) for the exchange rate of Kenyan Shilling; Zakaria (2012) exchange rate of Arab countries, etc and each of them found sGARCH (1, 1) as a best model for the exchange rate volatility. Therefore, this thesis result of the ARCH type model coincide with most others work, that is, it selected sGARCH (1, 1) model as a best model for modeling the exchange rate.

SV is a recently very growing model of the financial time series which was found in many econometrics and statistics as one of the best volatility model that captures the volatility clustering of financial data. For instance, Ronald and Peter (1998) empirically studied the basic SV model among the major currencies (US dollar, British pound sterling, Japanese yen, and German mark); Craine, Lochstoer, and Syrtveit (2000) modeled the SV of the Norwegian exchange rate; Kulikova and Taylor (2013) modeled the SV of the South African exchange rate. All these studies were found to be similar, as their study implied that the basic SV model captures the volatility clustering of the exchange rate. The result of this study is found to be similar to the above findings.

5. Conclusions and Recommendations

5.1. Conclusions

Volatility models cover a wide range of issues in Econometrics and Statistics. There are a lot of volatility models that are assumed to be best for modeling the exchange rate variability or modeling volatility of financial time series data.

The objective of this study was modeling the volatility of exchange rate deterministically and stochastically using ARCH type and SV models, respectively. In this study, models with performance reflected in the literature were used. It is difficult to include all the volatility models and because of this the study considered only few of them and further focused on sGARCH (1, 1) since it was found to be the best for the data return at hand. In general, the models used under the study were ARCH (1), ARCH (2), sGARCH (1, 1), EGARCH (1, 1), APARCH (1, 1) and SV. These models were chosen based on their repeated appearance in the literature of Economics/ or Econometrics and Statistics as best performs and based on their ability to capture the volatility clustering or leverage effect in the financial time series data.

Volatility models are broadly divided into two categories. The first one is, the ARCH type models which models the variance as a deterministic function at time t-1 having all information at hand at time t. ARCH type models include the ARCH (q), GARCH (p, q) and others like EGARCH (p, q), APARCH (p, q) and etc. Each ARCH type model has its own contribution in capturing the volatility of the financial time series data by the conditional variance of being non-constant throughout the time. Among the ARCH type models used in the study, sGARCH (1, 1) outperformed the data in capturing volatility.

The study also considered the basic stochastic volatility model in which the conditional variance is considered as non-deterministic even if all information is available at time t. so the second category is SV model which includes an innovation term to the variance equation. Even though the inclusion of innovation term makes SV model more flexible in modeling the volatility, there are some difficulties in its parameter estimation.

In this study ARCH type models were estimated using the maximum likelihood function. But for the case of basic SV model, a MCMC estimation method was used for its parameter estimation. MCMC estimation method was used for the SV model because MCMC is compared with other methods and better estimates the SV parameter. For example, Geweke (1988) compared GMM, QMLE, and MCMC and found that MCMC is better for the estimation of SV parameters. And in the last decades Statisticians and Econometricians were repeatedly using the MCMC Bayesian methodology of estimation for SV model.

5.2. Recommendations

Based on the findings of this study, the following recommendations can be made:

- ✓ In modeling the volatility of a given currency a careful review of the return series has to be made to check whether it contains an ARCH effect or not. A wrongly diagnosis of ARCH effect may lead to the misspecification of the model. For example, EURO return series considered in this study had no ARCH effect hence rather than modeling its conditional volatility model it is recommended to model its mean function only (or the ARMA)

References:

- Ahmed, K. (2012). Estimation of Exchange Rate Volatility via GARCH Model, Case Study Sudan (1978 – 2009). *International journal of Economics and Finance*, **4**, 160-183.
- Andersen, Torben G., Tim Bollerslev, Francis X, Diebold and Paul, L. (1999). Exchange rate returns standardized by realized volatility are (nearly) Gaussian. New York University, Leonard N. Stern School, Finance Department, and Working Paper Series No: 99-060.
- Andersen, Torben, G. and Sqrensen, B. (1999). GMM estimation of a stochastic volatility model: A Monte Carlo study. Discussion paper, Northwestern University.
- Baillie, R. and Bollerslev, T. (1989). The message in daily exchange rates: A conditional variance tale. *Journal of Business and Economic Statistics*, **7**, 297-305.
- Bodurtha, J. N. and Mark, N. (1991). Testing the CAPM with time varying risks and returns. *Journal of Finance, **46**, 1485-1505.*
- Bollerslev, T. (1986). Generalized Autoregressive Conditional Heteroscedasticity. *Journal of Econometrics, **31**, 307-327.*
- Bollerslev, T., Chou, R. and Kroner, F. (1992). ARCH modeling in finance: A review of the theory and empirical evidence. *Journal of Econometrics, **52**, 5-59.*
- Bollerslev, T., Engle, R. and Nelson, D. (1994). ARCH models. *Handbook of Econometrics,* IV.
- Bollerslev, T., Engle, R. and Wooldridge, J. (1988). A capital asset pricing model with time varying co variances. *Journal of Political Economy, **96**, 116-131.*
- Campbell, J.Y, Lo, A. and Mackinlay, A. (1997). Modeling daily value at risk using realized volatility and ARCH type models. *Journal of empirical finance*, **3**, 379-398.
- Campbell, John Y., Andrew W. Lo and A. Craig MacKinlay (1997). The Econometrics of Financial Markets. Princeton, NJ: Princeton University Press.

- Chib, S., Nardari, F. And Shephard, N. (2001).Markov chain Monte Carlo methods for stochastic volatility models. *Journal of Econometrics, **108**, 281-316.*
- Chirpili, J. (2007). Modeling Exchange rate Volatility in Zambia (Department of Economics, University of Leicester, LE1 7RH, U.K.).
- Chong CW, (2002). Modeling the volatility of currency exchange rate using GARCH model. Pertanika J. Soc. Sci. Hum., **10 (2)**: 85-95.
- Craine, R., Lochstoer, A. and Syrtveit, K. (2000). Estimation of a Stochastic-Volatility Jump-Diffusion Model. University of California, **3**, 203-261.
- Derrese, D. (2008). The parallel foreign exchange market and macroeconomic performance in Ethiopia, African Economic Research Consortium, Nairobi, **107**, 50-61.
- Dickey, D. A. and Fuller, W. A. (1979). Distribution of the estimator's autoregressive time series with a unit root. *Journal of the American Statistical Association, **74**, 427-443.*
- Duffie, D. and Singleton, K. J. (1993). Simulated moment estimation of Markov models of asset prices, *Econometrica, **61**, 929-952.*
- Enders, W. (1995). Applied Econometric Time Series (2nd Ed.). Printed in the United States of America.
- Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrics, **50**, 987-1007.*
- Engle, R. (2002). New frontiers for ARCH models. *Journal of Applied Econometrics, **17**, 425-446.*
- Fama, E. F. (1965). The behavior of stock market prices. *Journal of Business,**38**, 34-105*
- Fernandez, Daniel and Stoddard Vander steel, (1982). Short-run fluctuations in foreign exchange rates: evidence from the data 1973-1979. *Journal of international Economics*, **13**, 171-186.
- Gallant, A. R., Rossi, P. E. and Tauchen, G. (1990). Stock prices and volume. Unpublished manuscript (Department of Economics, Duke University, Durham, NC).

- Glosten, L., Jagannathan, R. and Runkle, D. (1991). Relationship between the expected value and the volatility of the nominal excess return on stocks. Unpublished manuscript (J.L. Kellogg Graduate Scholl, Northwestern University, Evanston, IL).
- Gweke, J. (1988). Exact inference in models with autoregressive conditional heteroscedasticity. *In: E. Berndt, H. White and W. Barnett, eds., Dynamic Econometric Modeling (Cambridge University Press, Cambridge), **4**, 73-104.*
- Hansen, P. R. and Lunde, A. (2003). A forecast comparison of volatility models: Does anything beat a GARCH (1, 1) model. *Unpublished paper. (http://bibliotecadigital.fgv.br/ojs/index.php/rbfin/article/viewFile/1155/246.)*
- Harvey, A., Ruiz, E. and Shephard, N. (1994). Multivariate stochastic variance models. *Review of Economic Studies, **61**, 247-264.*
- Higgings, M. and Bera, A. (1992). A class of nonlinear ARCH models. *International Economic Review, **33**, 137-158.*
- Hol, E. and Koopman, S.J. (2002).Stock index volatility forecasting with high frequency data. Department of Accounting and Finance, University of Birmingham, and Department of Econometrics, Free University Amsterdam, Tinbergen Institute discussion paper.
- Hsieh, A., (1989). The statistical properties of daily foreign exchange rates: 1974-1983. *Journal of International Economics,* **24**, 129-145.
- Hussein, F. Jalil, A. (2007). Effectiveness of Foreign Exchange Intervention: Evidence from Pakistan. SBP Res. Bull., **3(2)**, 99-120
- Jacquier, E., Polson, N. and Rossi, E. (2002). Bayesian analysis of stochastic volatility models with fat tails and correlated errors. *Journal of Business and Economic Statistics, **20**, 69-87.*
- Jithitikulchai, T. (2005). A study on Thai exchange rate volatility model comparison: nonparametric approach. Faculty of Economics, Thammasat University. (http://www.academicjournals.org/AJBM)
- Kemal, M. (2005). Exchange Rate Instability and Trade: The Case of Pakistan. PIDE publications.

- Kamal, Y. (2012). Modeling the exchange rate volatility, using generalized autoregressive conditionally hetroscedastic (GARCH) type models: Evidence from Pakistan. African Journal of Business Management.
- Kim, S., Shephard, N. and Chib, S. (1998). Stochastic volatility: Likelihood inference and comparison with ARCH models. *Review of Economic Studies, **65**, 361-393.*
- Koop, G. (2003). Bayesian Econometrics. John Wiley & Sons, England.
- Koopman, S. and Uspensky, E. (2000). The stochastic volatility in mean model. Tinbergen Institute, *discussion paper.*
- Kuan, C. (2003). Lecture on basic time series models. Institute of Economics, *Academia Sonica, Lecture notes.(http://homepage.ntu.edu.tw/~ckuan/pdf/Lec-DiagTest.pdf)*
- Lambert, P. and Laurent, S. (2000). Modeling skewness dynamics in series of financial data. *Discussion Paper, Institute de Statistique, Louvain-la-Neuve.*
- Li, K. W., Ling, S. and McAleer, M. (2002). Recent theoretical results for time series models with GARCH errors. *Journal of Economic Survey, **16**, 245-269.*
- Liu, S. M. and Brorsen, B. (1995). Maximum likelihood estimation of a GARCH stable model. *Journal of Applied Econometrics, **2**,273-285.*
- M. V. Kulikova and D. R. Taylor (2013). Stochastic volatility models for exchange rates and their estimation using quasi-maximum-likelihood methods: an application to the South African Rand. *Journal of Applied Statistics*, **40(3)**, 495-507
- Maana, I., Mwita, N., and Odhiambo, R. (2010). Modeling the volatility of exchange rates in the Kenyan market. *African Journal of Business Management* ,**4(7)**, 101-140,
- Madura, J. (2006). International financial management (8^{th} Ed.). Mason: Thomson.
- Mandelbrot, B. (1963). The variation of certain speculative prices. *Journal of Business, **36**, 394-416.*
- Melino, A. and Turnbull, M. (1990). Pricing foreign currency options with stochastic volatility. *Journal of Econometrics, **45**, 239-265.*

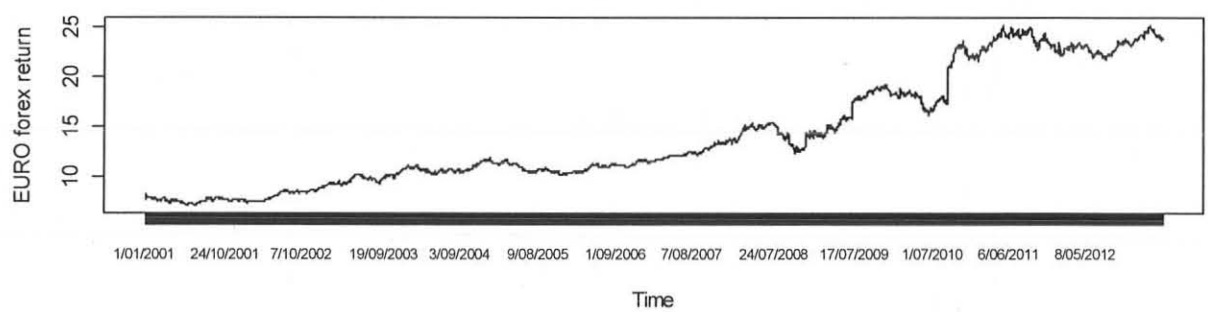
- Milhoj, A., (1987). A conditional variance model for daily observations of an exchange rate. *Journal of Business and Economic Statistics,* **5**, 99-103.
- Mundaca, B. (1991). The volatility of the Norwegian currency basket. Scand. J. Econ., **93(1)**: 53-73.
- National bank of Ethiopia (2009/10). Annual Report Documentation and Publication Service. Addis Ababa, Ethiopia.
- Nelson, D. (1991). Conditional heteroscedasticity in asset returns: a new approach. *Econometrics,* **59**, 349-370.
- Pagan, A. & Ullah, A. (1999). Non-parametric Econometrics. Cambridge University Press: Cambridge, UK.
- Pantula, S. (1985). Estimation of autoregressive models with ARCH errors. *Unpublished manuscript, Department of Statistics, North Carolina State University, Raleigh, N.(http://www.jstor.org/discover/10.2307/25052529)*
- Pederzoli, C. (2003). Stochastic volatility and GARCH: A comparison based on UK stock data. *University of Warwick.*
- Phillips, P. C. B. and Perron, P. (1988). Testing for a Unit Root in Time Series Regression. Biometrika, **75**, 335-346
- Robinson, P. (1987). Testing for strong serial correlation and dynamic conditional heteroscedasticity in multiple regressions. *Journal of Econometrics, 47, 67-84.*
- Ronald, J. and Peter, C. (1998). An empirical Application of Stochastic Volatility models. *Journal of applied Econometrics*, **13**, 333-360.
- Triantafyllopoulos, K. (2003). An introduction to stochastic volatility. *University of Newcastle, Department of Mathematics and Statistics, working paper.*
- Tsay, R. (2002). Analysis of Financial Time Series (2^{nd} Ed.). ISBN 0-471-41544-8.
- *Tsay, R. (2010). Analysis of Financial Time series. University of Chicago, Booth school of Business. (3^{rd} ed.), 30-54.*
- Türker, Ö. (1999). Structural Time Series Modeling of Inflation Behavior in Turkey. *Master Thesis, Department of Statistics, METU.*

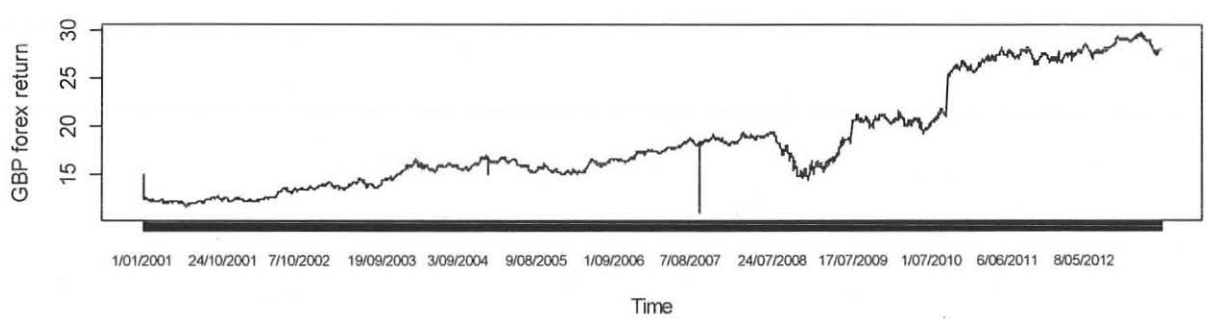
- Vlaar, P. and Palm, F. (1993). The message in weekly exchange rates in the European monetary system: mean reversion, conditional heteroscedasticity and jumps. *Journal of Business and Economic Statistics, **11**, 351-360.*
- Whistler, D. (1988). Non-parametric ARCH estimation of intra daily exchange rate volatility. *Unpublished manuscript (London School of Economics, London).*
- Yohannes, A. (2007). Explaining the Current Sources of Inflation in Ethiopia: A *Macro econometric Model*, NBE and EDRI. Addis Ababa; Ethiopia.
- Yu, J., Yang, Z. and Zhang, X. (2002). A class of nonlinear stochastic volatility models and its implications on pricing currency options. *Discussion paper*, University of Singapore.
- Yu, Jun and Meyer, R. (2006). Multivariate Stochastic Volatility Models: Bayesian Estimation and Model Comparison. *Research Collection School of Economics* (Open Access). Paper 360. (http://ink.library.smu.edu.sg/soe_research/360)
- Yusuf, H. (2009). Modeling exchange rate volatility using SV model. *University of California, Santa Barbara, Economics Department Working Paper*, No: 12-96.
- Zakaria, S. (2012). Modeling Exchange Rate Volatility using GARCH Models: Empirical Evidence from Arab Countries. King Saud University, Riyadh; Kingdom of SaudiArabia.(http://www.ccsenet.org/journal/index.php/ijef/article/download/15163/10280)
- Zivot, E and Wang, R. (2006). Time Series Forecasting with State Space Models. Department of Applied Mathematics, University of Washington, Discussion paper.

Appendix

Appendix A

Figure A1: Plot of the FOREX rate of ETB per USD, EURO and GBP daily exchange rate





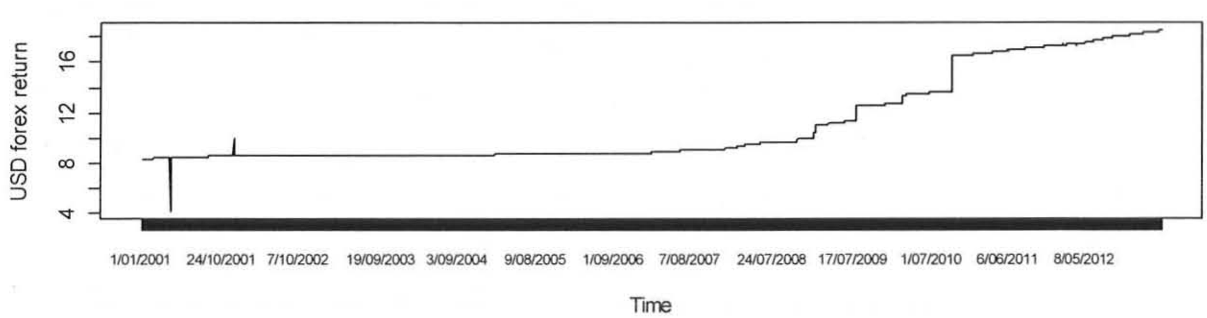


Figure A2: Plot of the first difference of logarithmic series for major currencies of exchange rate

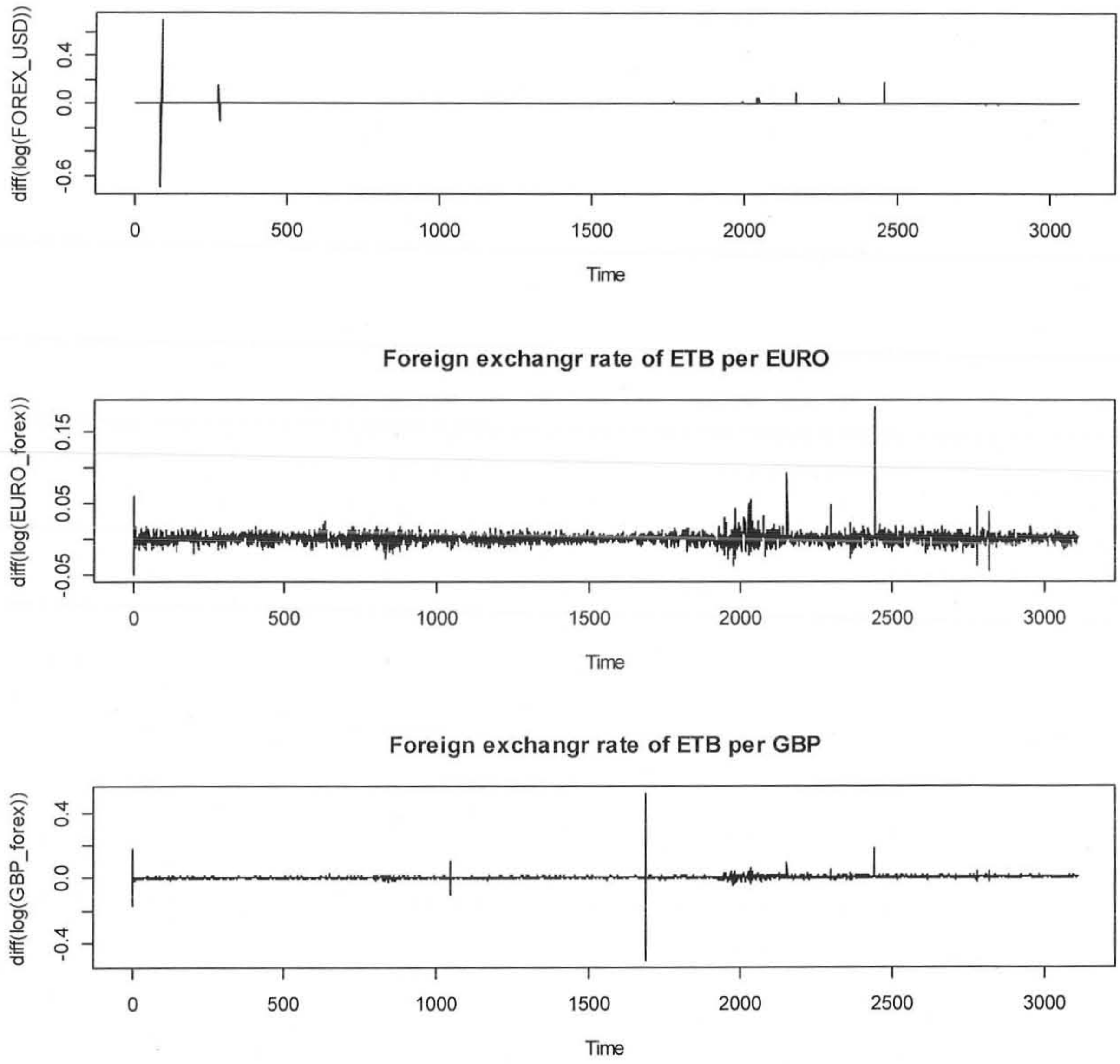


Figure A3: plot of the PACF of a return and squared return of the major currencies series

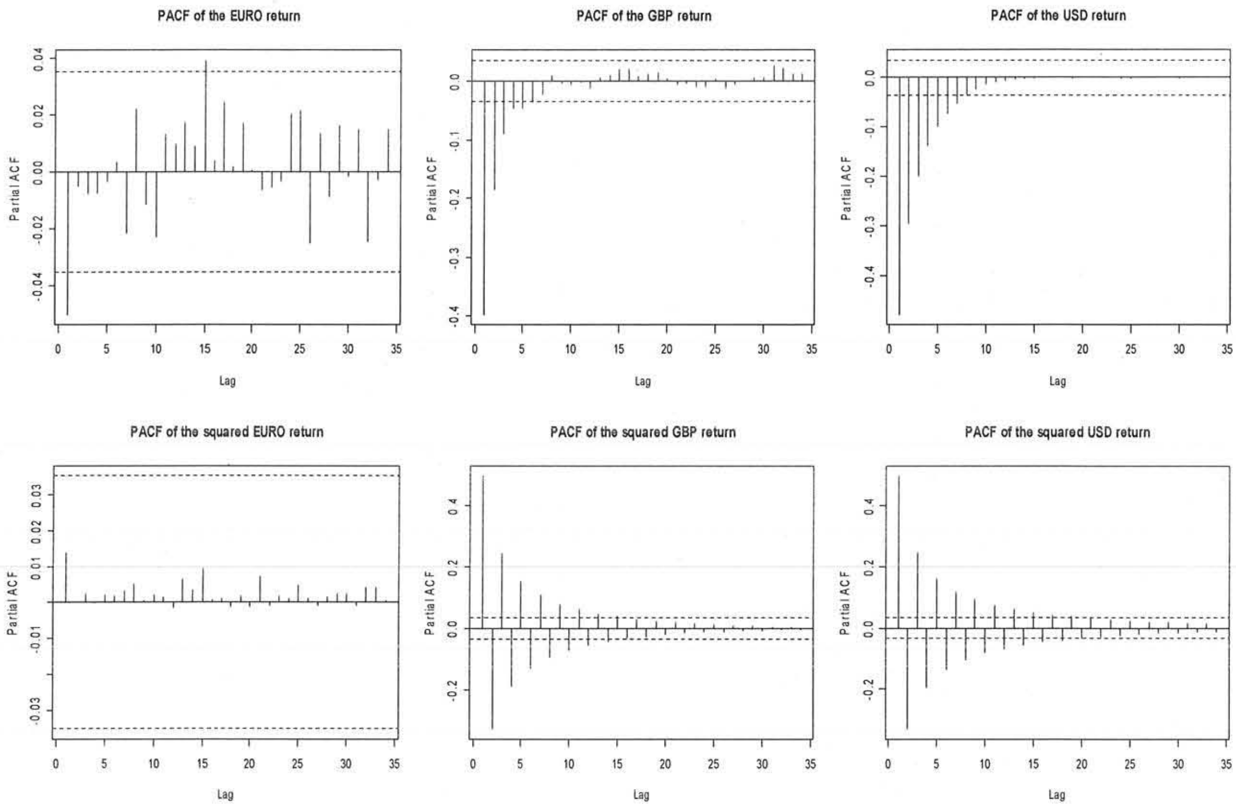


Figure A4: Sample ACF of the Return series of major currencies

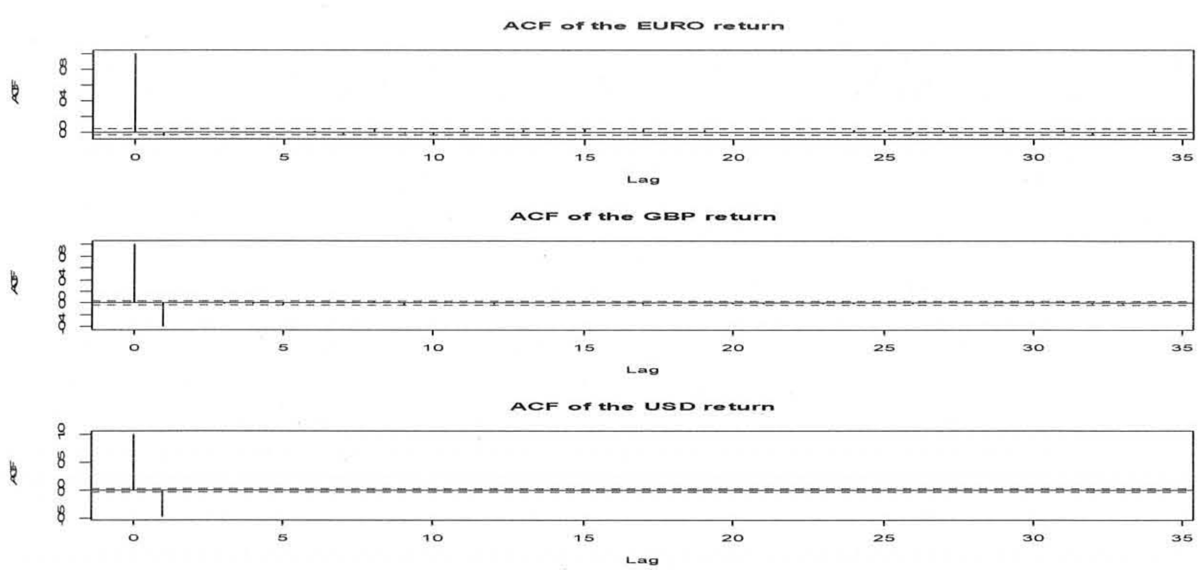
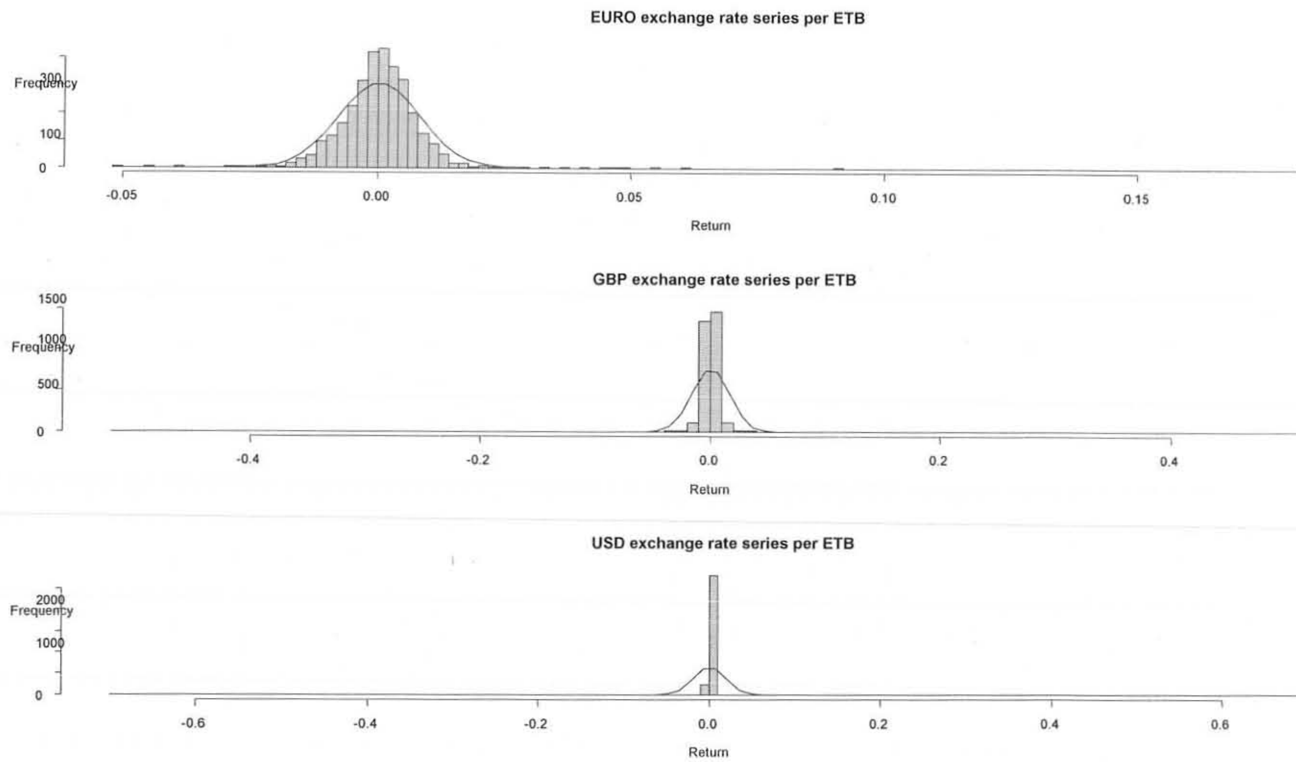


Figure A5: Histogram plot of the Return series of the Major currencies per ETB



Appendix B:

Table 1: Error Analysis for the parameters of sGARCH (1, 1) for each Return series of FOREX

A. GBP return series error analysis

	Estimate	Std. Error	t-value	Pr(>|t|)
Mu	2.837e-04	9.251e-05	3.067	0.00216 **
Omega	1.029e-05	1.812e-06	5.680	1.35e-08 ***
alpha1	1.716e-01	2.772e-02	6.193	5.90e-10 ***
beta1	5.917e-01	4.993e-02	11.849	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood: 11470.29 normalized: 3.710868

B. USD return series error analysis

	Estimate	Std. Error	t-value	Pr(>|t|)
Mu	1.334e-05	5.135e-07	25.987	<2e-16 ***
Omega	3.463e-10	6.548e-09	0.053	0.958
alpha1	1.2000e-1	3.216e-02	31.091	<2e-16 ***
beta1	1.000e-08	1.756e-04	0.000	1.000

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood: 25663.73 normalized: 8.302727

Table 2: Summary Statistics for Standardized Residuals Tests for sGARCH (1, 1)

A. Standardized Residuals Tests of sGARCH (1, 1) of GBP return series

	Statistic	p-value
Jarque-Bera Test R Chi^2	9.3084802	0.254321
Shapiro-Wilk Test R W	0.2937756	0.159083
Ljung-Box Test R Q(10)	2.853246	0.984669
Ljung-Box Test R Q(15)	3.683051	0.998604
Ljung-Box Test R Q(20)	4.904527	0.999761
Ljung-Box Test R^2 Q(10)	0.009653822	0.958345
Ljung-Box Test R^2 Q(15)	0.01417345	0.986925
Ljung-Box Test R^2 Q(20)	0.01746895	0.998726
LM Arch Test	0.01181368	1

B. Standardized Residuals Tests of sGARCH(1, 1) of USD return series

	Statistic	p-value
Jarque-Bera Test R Chi^2	11.5905126	0.12367
Shapiro-Wilk Test R W	0.008994847	0.01258
Ljung-Box Test R Q(10)	0.005550767	0.92857
Ljung-Box Test R Q(15)	0.01074157	0.96257
Ljung-Box Test R Q(20)	0.01523815	0.98876
Ljung-Box Test R^2 Q(10)	0.004316046	0.89278
Ljung-Box Test R^2 Q(15)	0.006487956	0.97272
Ljung-Box Test R^2 Q(20)	0.008670903	0.98527
LM Arch Test	0.005188194	1

Appendix C

Figure C1: plot of conditional Standard deviation of the GBP sGARCH (1, 1) model

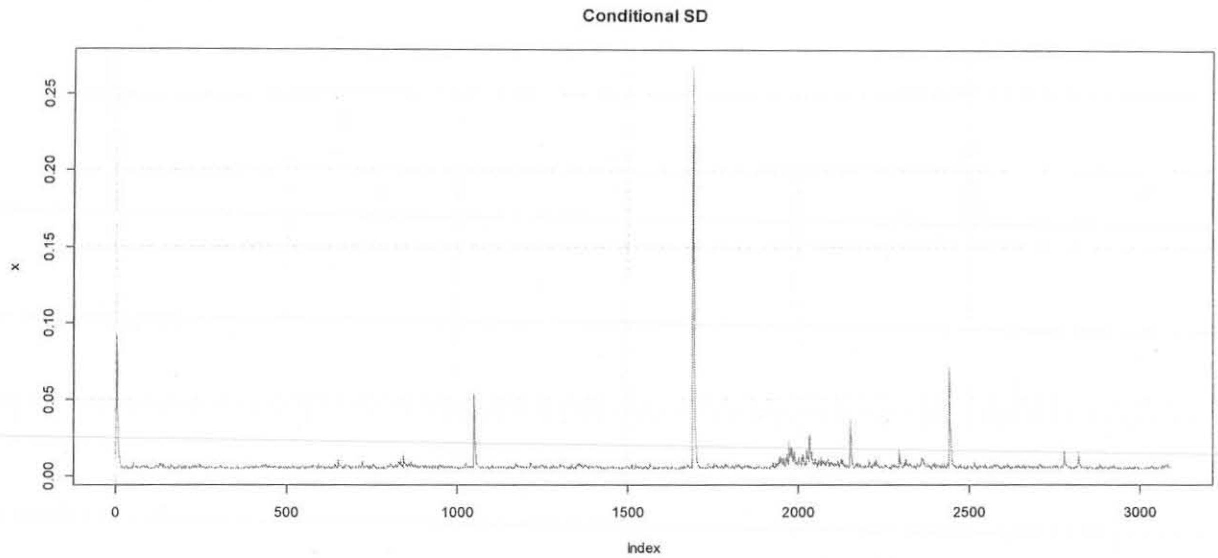


Figure C2: Residual plot of the sGARCH (1, 1) model of GBP return

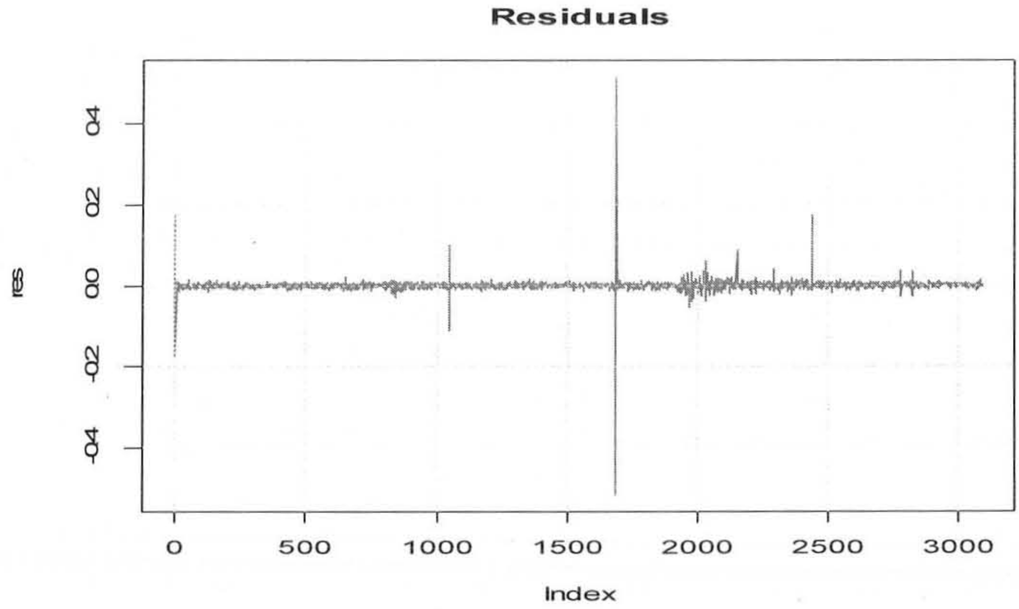


Figure C3: ACF of Standardized residuals and squared standardized residuals of GBP return of sGARCH (1, 1) Model

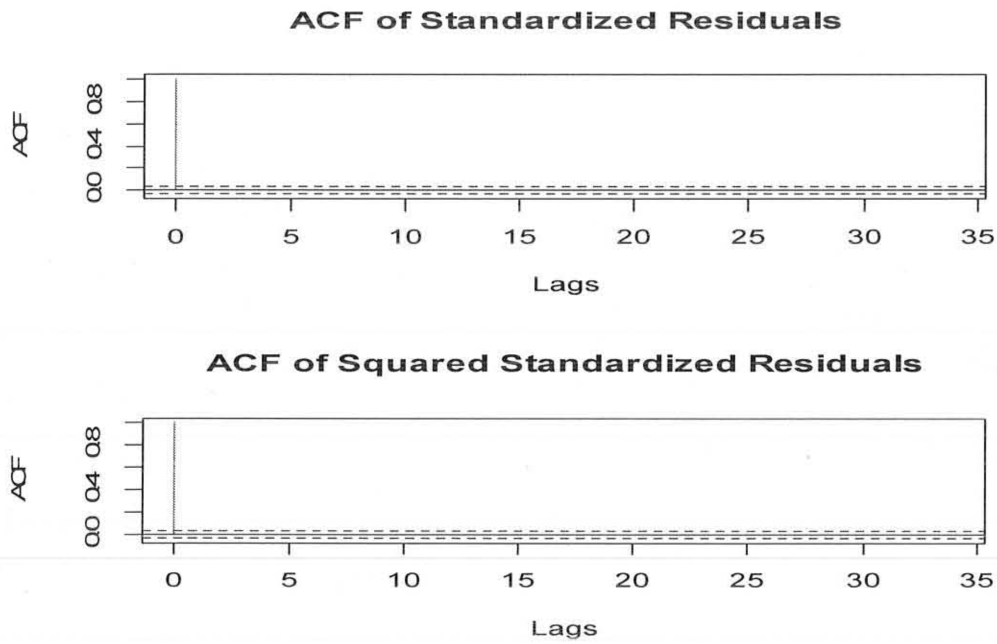


Figure C4: plot of Conditional standard Deviation of USD return series of sGARCH (1, 1) Model

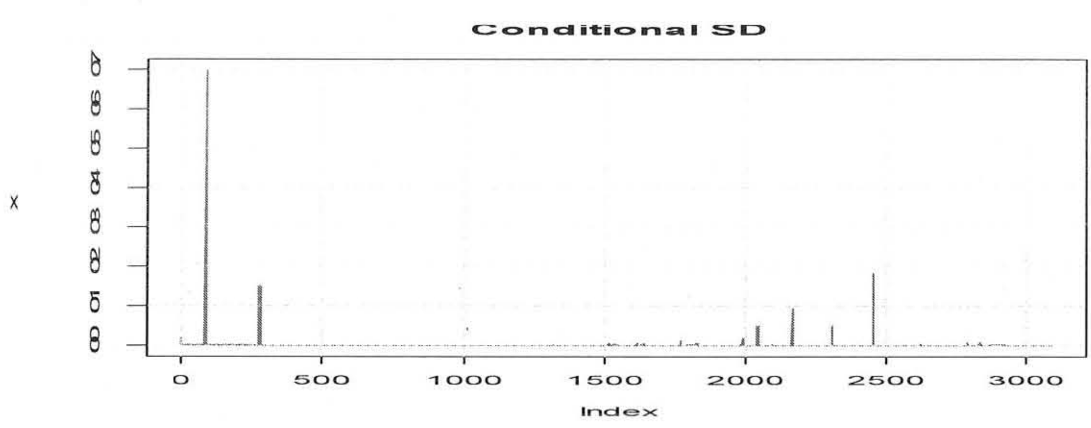


Figure C5: Residual plot of the sGARCH (1, 1) Model of USD Return series

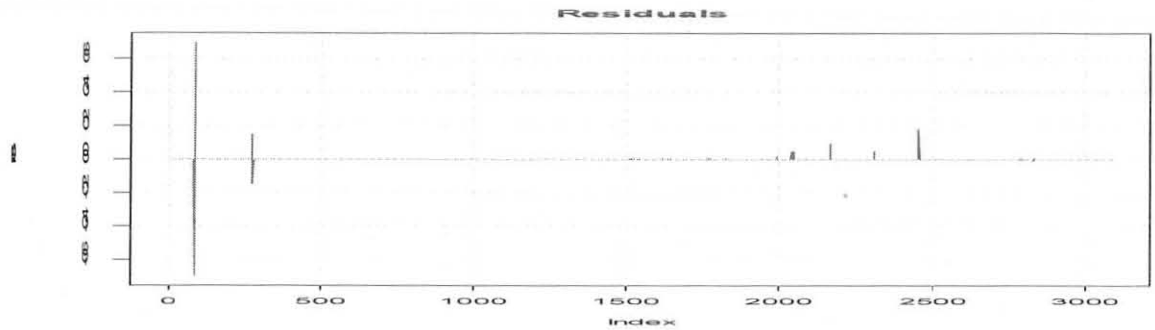


Figure C6: ACF of Standardized residuals and squared standardized residuals of USD return

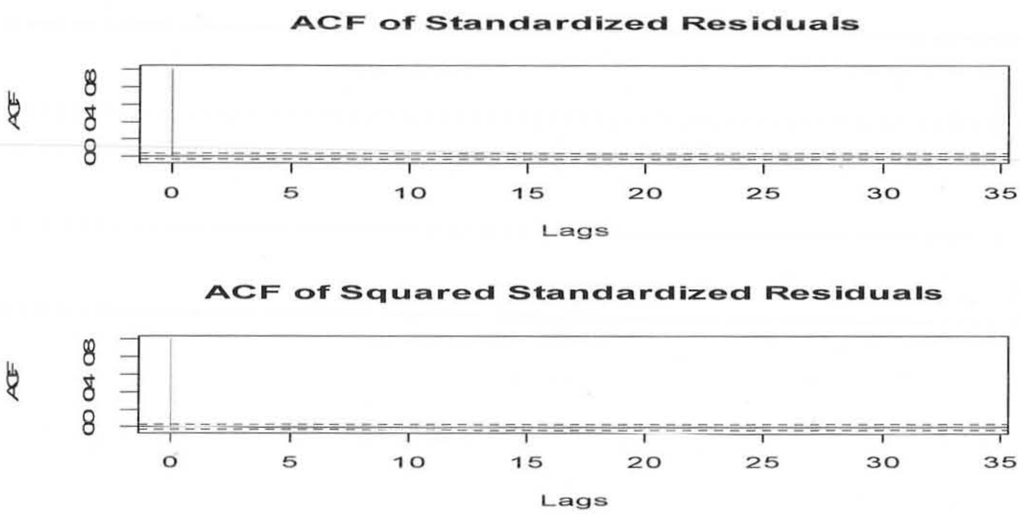
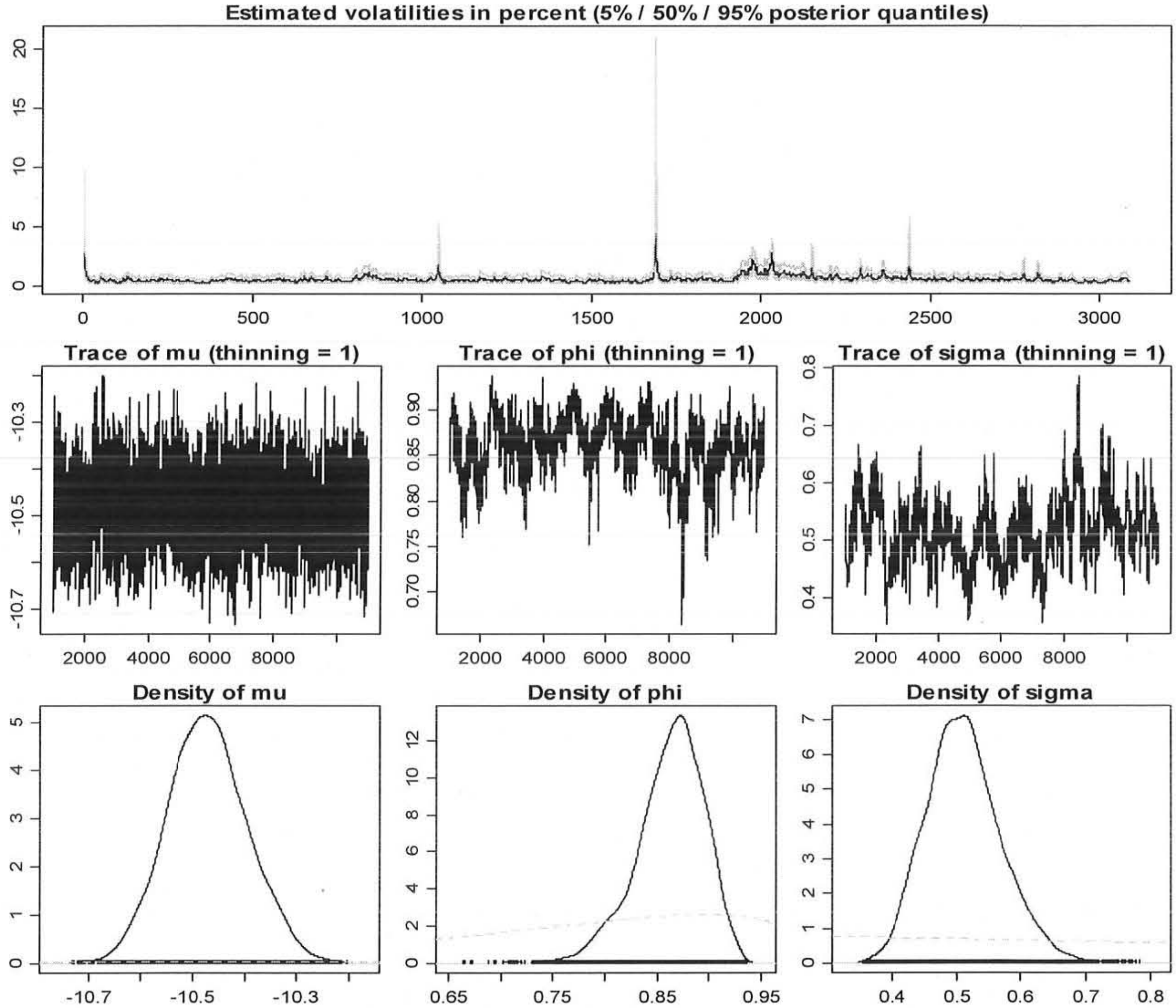


Figure C7: Plot of traces and Density function of the parameters in SV model for the GBP return

A. **Graphical plot of the Estimated volatility, sketch and density plot of the parameters of SV model for GBP return**



B. Plots and Density of the parameters of the SV model for GBP return

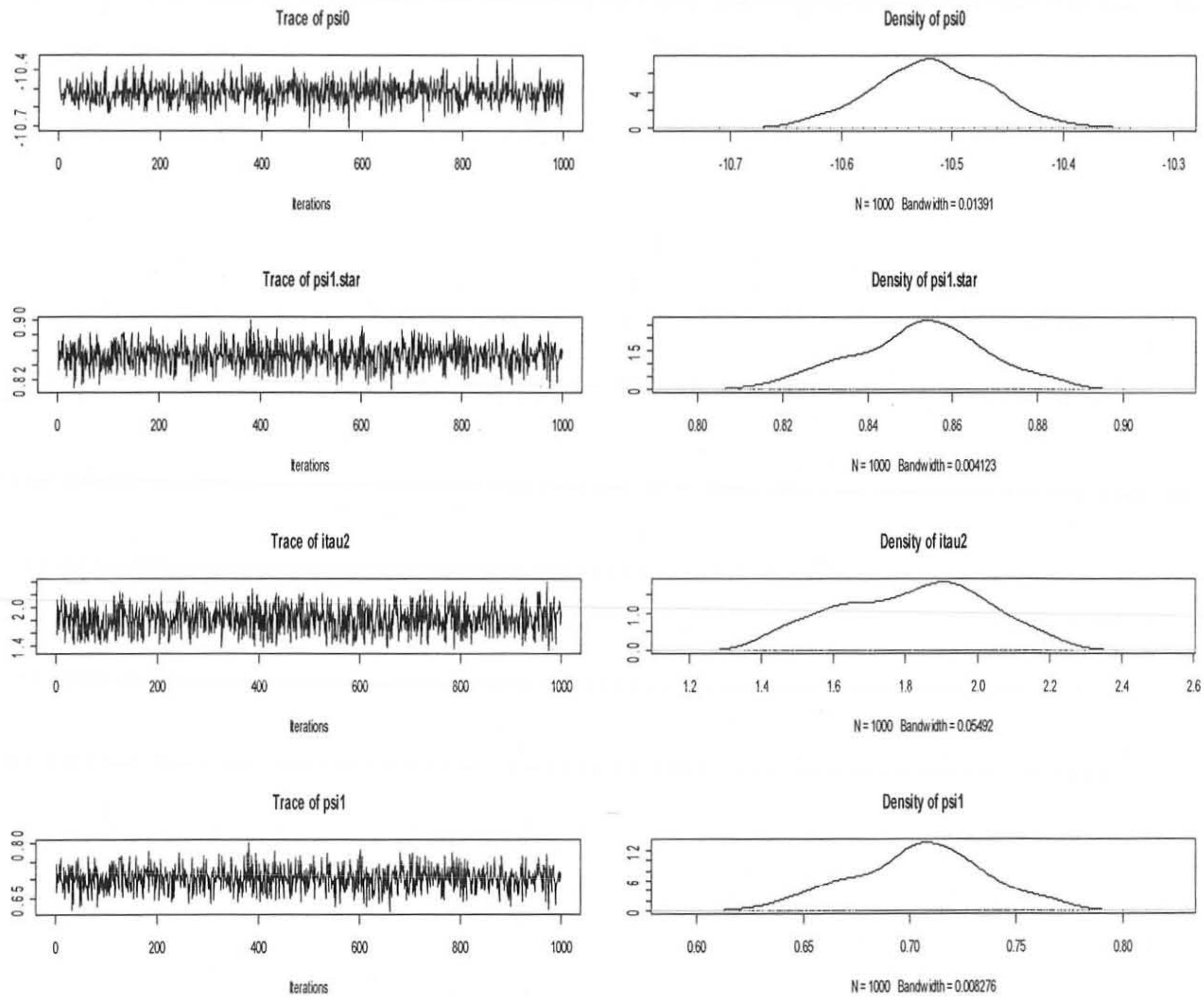
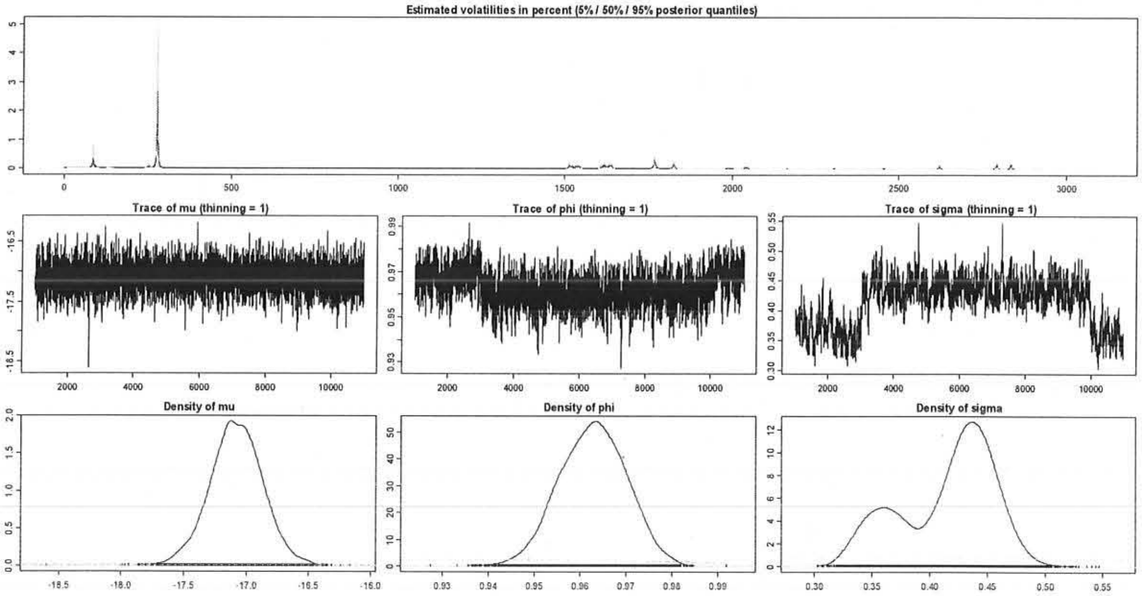
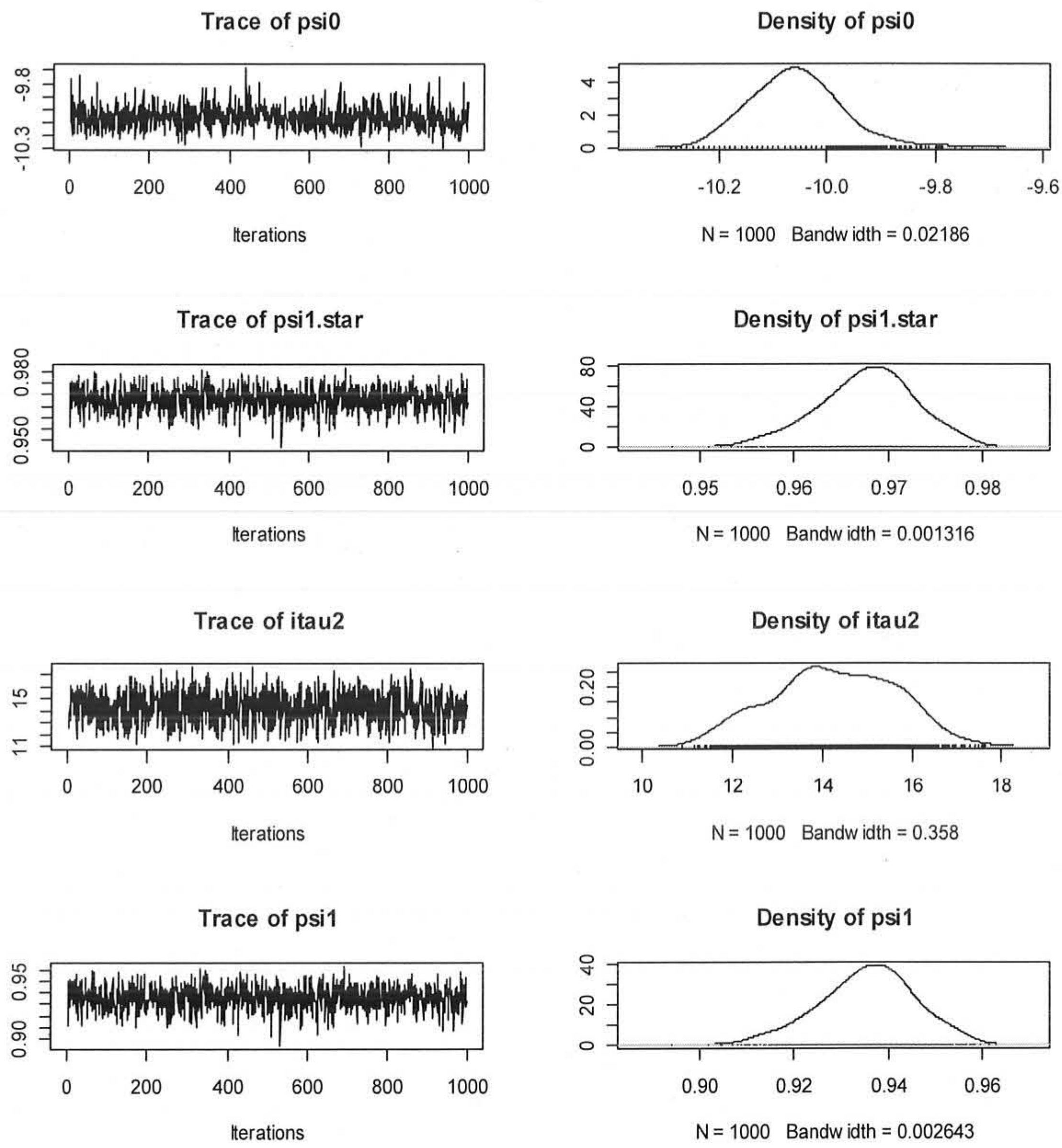


Figure C8: the parameter traces and posterior distributions of USD return series

A. Sample Plot of the Parameters and estimated volatilities for the USD return



B. Sketch and density function of the parameters of SV for the USD return



Appendix D:

Sample Joint Program of R and Open BUGS that was used for Stochastic Volatility Model

```
> library(tsbugs) # time series R package for Bayesian using Gibbs sampler

X1<-diff(log(x)) # first difference of logarithmic series or the return, x is the original data vector

> sv0<-sv.bugs(X1) # calling stochastic volatility function from "tsbugs" package

> print(sv0) #displaying the program for the SV, it could be brought to Open BUGS to run or an
#Open BUGS may be called to an R software to run in R.

model{

#likelihood

for(t in 1:3091){

    y[t] ~ dnorm(y.mean[t], isigma2[t])

    isigma2[t] <- exp(-h[t])

    h[t] ~ dnorm(h.mean[t], itau2)

}

for(t in 1:3091){

    y.mean[t] <- 0

}

for(t in 1:1){

    h.mean[t] <- psi0

}

for(t in 2:3091){

    h.mean[t] <- psi0 + psi1*(h[t-1]-psi0)

}
```

```
#priors

psi0 ~ dnorm(0,0.001)

psi1.star ~ dunif(0,1)

psi1 <- 2*psi1.star-1

itau2 ~ dgamma(0.01,0.01)

tau <- pow(itau2,-0.5)

}

> writeLines(sv0$bug,"sv0.txt")# writing  the printed SV program jointly

> library(R2OpenBUGS) # calling Open BUGS into R software
```

The following program is then defined carefully and it may take time to run; it depends on the RAM of PC

```
> sv0.bug<-bugs(data=sv0$data,

+ inits=list(inits(sv0)),

+ param=c(nodes(sv0,"prior")$name,"h"),

+ model="sv0.txt",

+ n.iter=11000,n.burnin=10000,n.chains=1) # one chain of the MCMC simulation is run for
11000  #iterations, with the first 1000 used for burn in.

> library(coda)

>param.mcmc<-as.mcmc(sv0.bug$sims.matrix[,nodes(sv0,"prior")$name])

> summary(param.mcmc)

> plot(param.mcmc[,1:4]) #plots the parameter traces and posterior distributions using the
coda #package:
>library(fanplot)


# The volatility and time-dependent standard deviations estimates can be derived as
```

```
> h.mcmc <- sv0.bug$sims.list$h

> sigma.mcmc <- sqrt(exp(h.mcmc))

># which allows us to directly view the estimated volatility process or the time-dependent
>#standard deviation using the fanplot package,

>library(stochvol)#(mu=psi0, phi=phi1.star, sigma=itau2)
>res <- svsample(diff(log(Uforex)), priormu = c(0, 10), priorphi = c(10, 1.5), priorsigma
= 1)
>plot(res)# sample plot of the volatility and density of the parameters from the "stochvol"
package
```