Assessment and Mitigation of Voltage Drops on Traction Lines: Case Study of Sebeta-Adama Line

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This is to certify that the thesis prepared by Kassu Bekele, entitled: *Assessment and Mitigation of Voltage Drops on Traction Lines: Case Study of Sebeta-Adama Line* and Submitted in Partial Fulfillment of the requirements for the degree of Master of Sciences (Electrical and Computer Engineering) complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

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Declaration

I, the undersigned, declare that this thesis is my original work, has not been presented for a degree in this or any other university, and all sources of materials used for the thesis have been fully acknowledged.

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This thesis work has been submitted for examination with my approval as a University Advisor.

Dr.Ing Getachew Biru ________________
Advisor’s Name Signature

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Abstract

The transmission line in an electrified railway system must provide power transmission within the voltage limit at safe and quality conditions. Therefore, system analysis within the traction power system is vital to the design and operation of an electrified railway. Loads in traction power system are often characterized by their mobility, wide range of power variations, regeneration and service dependence. One of the main problems in the railway supply stations are the voltage drops due to the changes of the load. Voltage drop in electrified railway have an influence on the location of traction substation (reduces distance between traction transformer and section post length) and operating railway transport in a such condition disturbs transportation system, as a result the reactive power compensator should be installed to have a better voltage at receiving end.

Ethiopian railway cooperation is constructing national railway transport from Sebeta to Meiso with seven feeding traction substation from Sebeta to Adama. In this thesis traction network voltage drop analysis is undertaken for various feeding condition. Taking sample section from Sebeta substation to Indode substation analyses of the traction network voltage drop over the feeding condition has been conducted for different loading cases. The modeling and simulation of the system is done by MATLAB/SIMULINK software.

The systems without Static Synchronous Compensator (STATCOM) have been simulated to investigate the voltage drop for different operating cases. Accordingly, the voltage drop for normal operation case is 5.922kV which is 27.55% voltage regulation. The voltage at the end point of the section is 21.56kV and is within the standard limit (TB10009-2005). However, the voltage drop for over zone (two consecutive sections without section post) is increased to 49.89% and the voltage at end point of the section becomes 18.346kV which is beyond the standard limit.

With the STATCOM, for normal operation case the voltage regulation becomes almost zero and for the over zone operation it is reduced to 0.07%. This result clearly indicate that STATCOM application improve the voltage profile and performance of the railway systems.

Key Words: Voltage Drops, FACTS Devices, STATCOM, Compensator, Voltage Fluctuation

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List of Abbreviations and Symbols

a_{ik}	The mutual distance of conductors in loops
d	The diameter (m) of the round steel bar used as the earthling body
CBPS	Circuit Breaker Paralleling Section
CBS	Section Circuit Breaker
‘e‘	Is axle load in tonne.
E_a	To accelerate the train
E_g	Energy output of driving axle to overcome the gradient
E_r	Energy output of deriving axle to overcome friction
E_{out}	Energy output of deriving axles to accelerate the train
E_{spo}	Specific energy output
E_{con}	Energy consumption
E_{spc}	Specific energy consumption
E_{ov-con}	Over all energy consumption
f	Power supply frequency
F_a	Tractive effort for acceleration
F_g	Tractive effort required to balance the gravitational pull
F_r	Tractive effort to overcome train resistance
F_c	Tractive effort to overcome curve resistance
F_t	Total tractive effort
I_{avg}	Train average current
I_F	Feeder daily average current
I_{FE}	Feeder daily effective current of a double track of unilateral power supply
I_{II}	Feeder current for down track
I_I	Feeder current for up track
I_{rT}	Is the rated current of the transformer on the high-voltage or low-voltage side
l	The length of the upright earthling body
$L^{\cdot}_{ex}$	Is external inductance

MPSS	Middle Point Sub Sectioning
P_{krT}	Is the total loss of the transformer in the windings at rated current
P	Power of traction motor
P_{max}	Power output required from the deriving axle to propel the train is maximum
OCS	Overhead Contact System.
$\dot{R}$	Resistance in Ω/km
R_E	Resistance of the earth returns path and the resistance per unit length are 49.3mΩ/km for 50Hz.
r	Conductor radius
‘r‘	The resistance in kg per ton
R_{grid}	Resistance of transmission line from national grid to Sebeta substation
R	Is the radius of curvature in meters
S_{rT}	Is the rated apparent power of the transformer.
TCSC	Thyrestor Control Series Compensator
U_{rT}	Is the rated voltage of the transformer, on the high-voltage or low-voltage
U_{kr}	Is the short-circuit voltage at rated current in percent
ΔV	Voltage drop is given by
W	Weight of train (hauling capacity of a train)
We	Effective weight of a train
$\dot{X}_{ex}$	External reactance
$\dot{X}_{in}$	Inner reactance of the conductor
X_T	The positive-sequence short-circuits reactance of a two-winding transformer
X_{grid}	Reactance transmission line from national grid to Sebeta substation
Z_m	Mutual impedance between feeders
Z_{grid}	Impedance transmission line from national grid to Sebeta substation
δ_E	Penetration depth of the current in the earth and
ρ_E	Resistivity of soil in Ωm
α	Acceleration of a train
β	Retardation of a train

Chapter One

1. Introduction

1.1 Overview of Electric Railway Systems

The knowledge of railway and steam engines has been around since the sixteenth century. Wagon roads for English coalmines using heavy planks were first designed and built in 1633. Mathew Murray of Leeds in England invented a steam locomotive that could run on timber rails in 1804 and this was probably the first railway engine. Although railway and locomotive technologies were continually developed, the first electrified railway was introduced in the1880s. As a result of this revolution, the traction motor and the power supply system have become important parts of modern electrified railways [1].

Electrified Railway Systems (ERS) are used widely around the world as a significant means of mass and public transportation. They are expanding at great speed throughout the world. Like many other nations, Ethiopia is also working to have the modern railway lines that use the AC power supply system. The Ethiopian Railway Corporation is planned to build more than 5000km during the growth transformation period. The Sebeta-Adama railway project is one, of the project in the first phase. It is a double track and which covers a distance of 138km [2].

Traction power supply system obtains energy from power grid and transits it to electric locomotive after voltage step down. A traction power supply system is composed of traction substation and traction network. Composition of traction power supply system is indicated in Figure1.1. The single phase 50 Hz power for the electric traction is obtained from 132kV extra high voltage 3 phase grid system through step down single phase transformers. For this purpose duplicate feeders comprising of only 2 phases are run from the nearest sub-station of the supply authority to the traction substation.

In general traction power supply begin at power grid and then voltage and frequency conversion at AC traction substation, supply railway train with required electrification, line are electrified today 25kV, 50Hz.

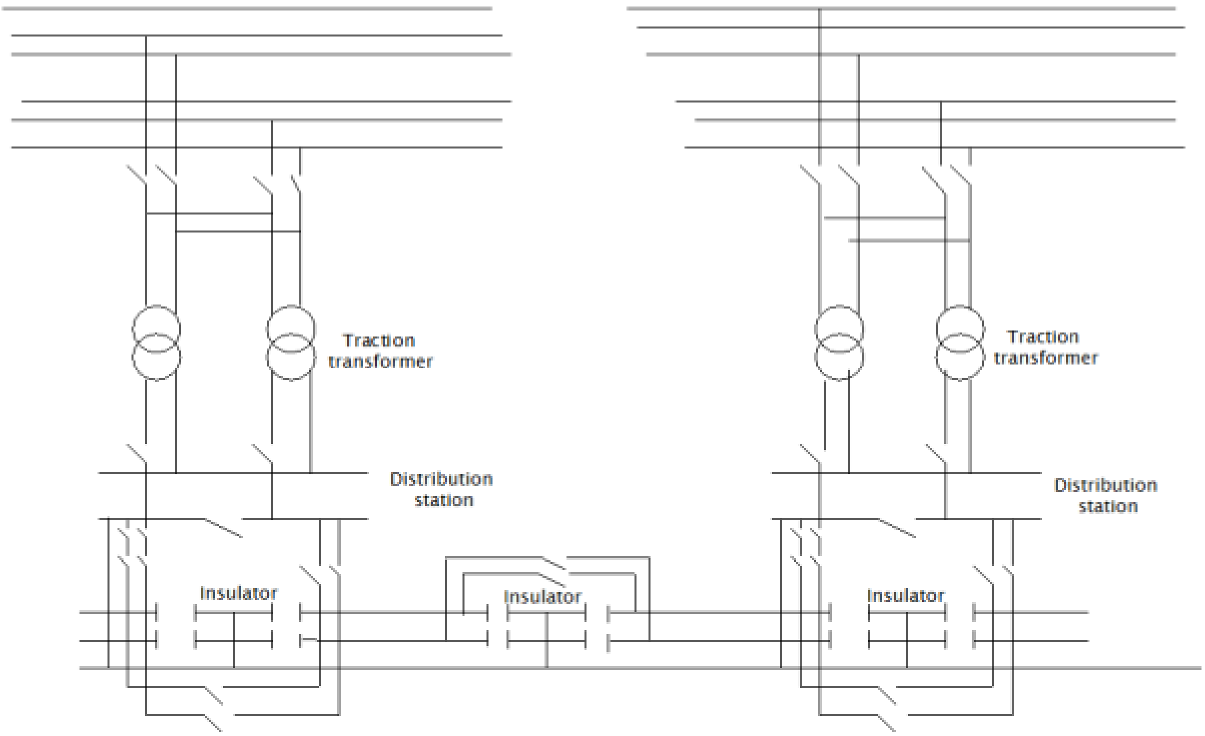

Figure 1.1 Power supply arrangement for 50Hz catenaries arrangement [3] [4]

The single phase catenaries are fed from two phases of the supplying three phase grid; this naturally causes asymmetries in the three phase voltage since the railway supply is a single phase load [5].

Nowadays power system are undergoing changes and becoming more complex from operation, control and stability maintenance standpoints as they have meet ever increasing load demand [6]. Voltage stability is a major concern under such condition.

A system enters a state of voltage instability when increase in load demand or change in system condition causes a progressive and uncontrollable decline in voltage. Power flow in electrical power system can be improved by adjusting reactance parameter of the transmission line. It can also be enhanced by adding a new transmission line in parallel with the existing one [7].

1.2 Statement of the Problem

In railway power transmission lines, the catenary voltage should be within tolerance range so that the train power requirements should met. Voltage along the feeding circuit differs substantially depending up on the train position, train current, number of train in the same power feeding section, track impendence etc.

Currently, traction system of the Sebeta-Adama-Djibouti doesn't have voltage drop compensation system [2]. This voltage drops can cause a decrease in the speed of the train or delay. This can lead to unwanted rescheduling, disturbance in power quality of the feeder side and damage in electrical equipment (malfunctioning). So, it is necessary to have a compensation mechanism for enhancing the operation capability of the system and reducing the voltage drop in a system by flexible AC transmission systems (FACTS) concepts so called STATCOM device.

1.3 Objective

1.3.1 General Objective

The main objective of this thesis is assessing the voltage drops in AC traction system and providing mitigation techniques to make the operation of the system better and reducing its voltage drops, and investigating the advantages of inserting STATCOM to the traction line for enhance power transfer capability and improve traction network voltage drop.

1.3.2 Specific Objective

- Assess the voltage drop in the traction line due to load (passenger and freight flow) variation within the day
- Model of the system with and without STATCOM.
- Simulation of the model using Mat lab software.
- Analysis of the simulation result
- Draw conclusion and recommendation about the use of STATCOM.

1.4 Scope of the Thesis

This thesis assesses the voltage drop due to load (passenger and freight flow) variations within the day and designs the mitigation technique for improving the voltage drops and operation of the system. The performance analysis in this thesis work has been carried out using MATLAB software. In this thesis only the railway line from Sebeta-Adama is considered.

With the objective of enhancing the power flow and improving traction network voltage level in the transmission line using STATCOM, it is essential to know the power flow between two buses and the various parameters involved in the power flow equation.

1.5 Methodology of the Study

In order to achieve the main aim of the study there are various procedural tasks followed by the author. The first method towards processing the work is started with reviewing different literatures where all the theoretical information regarding the traction power system voltage drop analysis is gathered and a comparison of previous similar research is studied. Alongside with literature reviewing, the collection and verification of data for the analysis is performed. This is followed by studying the characteristic and modeling of the traction power system components. Once the model is developed using MATLAB/Simulink, the analysis of the system is performed. Then based on the analysis result mitigation technique is designed. Finally, the performance of the STATCOM is analyzed and a comparison is made. The general block diagram of the methodology is given below.

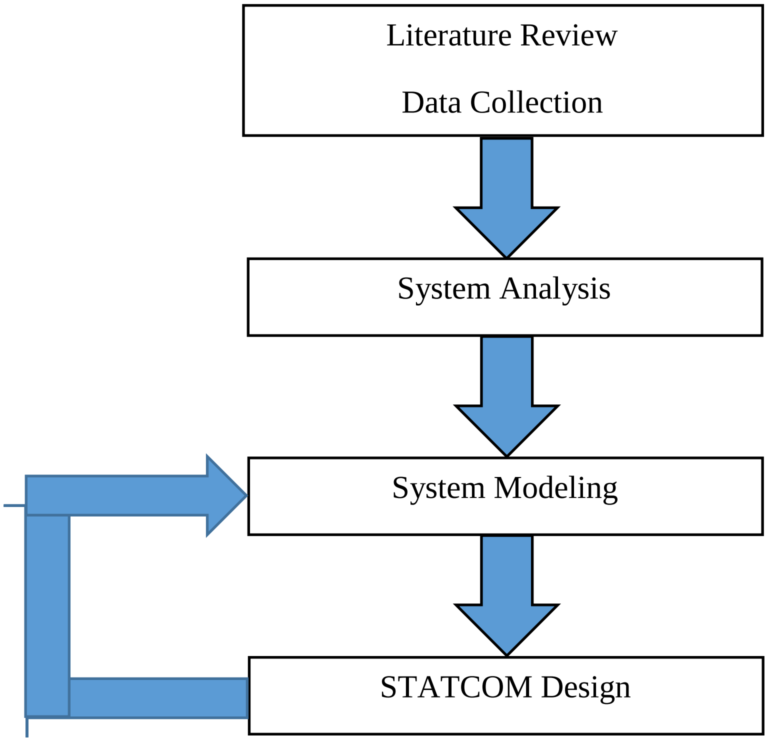

Figure 1.2 Summary of Methodology

1.6 Thesis Organization

This thesis is organized into six chapters as explained here below.

The first chapter provides an introduction of the research and defines the subject of the thesis.

The second chapter covers about the theoretical background and literature review of the research which includes data analysis on traction effort and train energy consumption used to find train average current, train effective RMS current when it is moving a section and introduce currently existing FACTS devices with their detail configuration type, compensation capability and their usage in different system of operation.

The third chapter gives train operation parameters calculation**,** mathematical model for traction substation and feeder network voltage drop is well defined, Analysis for system impedance from its short circuit characteristics, voltage drop of normal section operation, and over zone feeding condition analysis is done.

The fourth chapter covers about system modeling, system simulation and their result discussion without compensator (STATCOM).

The fifth chapter deals with STATCOM design; modeling, simulating and discussing the results obtained by inserting STATCOM into the system.

The sixth chapter puts conclusion depends up on the result, finally it puts recommendation and future works.

Chapter Two

2. Theoretical Background and Literature Review

2.1 Overview of 25kV AC Electrification System

2.1.1 Power Supply

Referring to British railway system a 25 kV, AC 50 Hz single-phase power supply for electric traction is derived from the grid of State Electricity Boards through traction sub-stations located along the route of the electrified sections at distance of 35 to 50 km apart. The distance between adjacent substations may however be even less depending on intensity of traffic and load of train [8].

The security of the incoming supply is of paramount importance to the reliability of the traction distribution system and normally the incoming feeder circuits from the 132 kV supply network to the 25 kV feeder station are duplicated at each supply point. Both incoming circuits are capable individually of carrying the total load at the incoming supply point for normal traffic operating conditions.

Where practicable the high voltage (HV) feeders from the supply system to the 132/25 kV transformers are derived from a source which has itself a level of security at least equivalent to that afforded by the provision of independent, duplicate, fully rated, incoming feeders to the 25 kV railway distribution system. Such levels of security at the supply point may be provided by an HV bus bar, sectionalized by a circuit breaker. With each section of bus bar being fed via an independent circuit from an independent part of the HV network, or by a duplicate HV bus bar with the two bus bars being similarly independently fed, such that failure of supplies to one section of the bus bar does not interrupt supplies to the other. In such a case the two "railway" feeders would be connected one on each section of the HV bus bar but may be "banked" with 132/33 kV or 132/11 kV transformers feeding local distribution networks or other consumers to economize on 132 kV switchgear, the bank being controlled by a single 132 kV circuit breaker. Figure 2.1 shows diagram of a typical **132** kV supply arrangement [8].

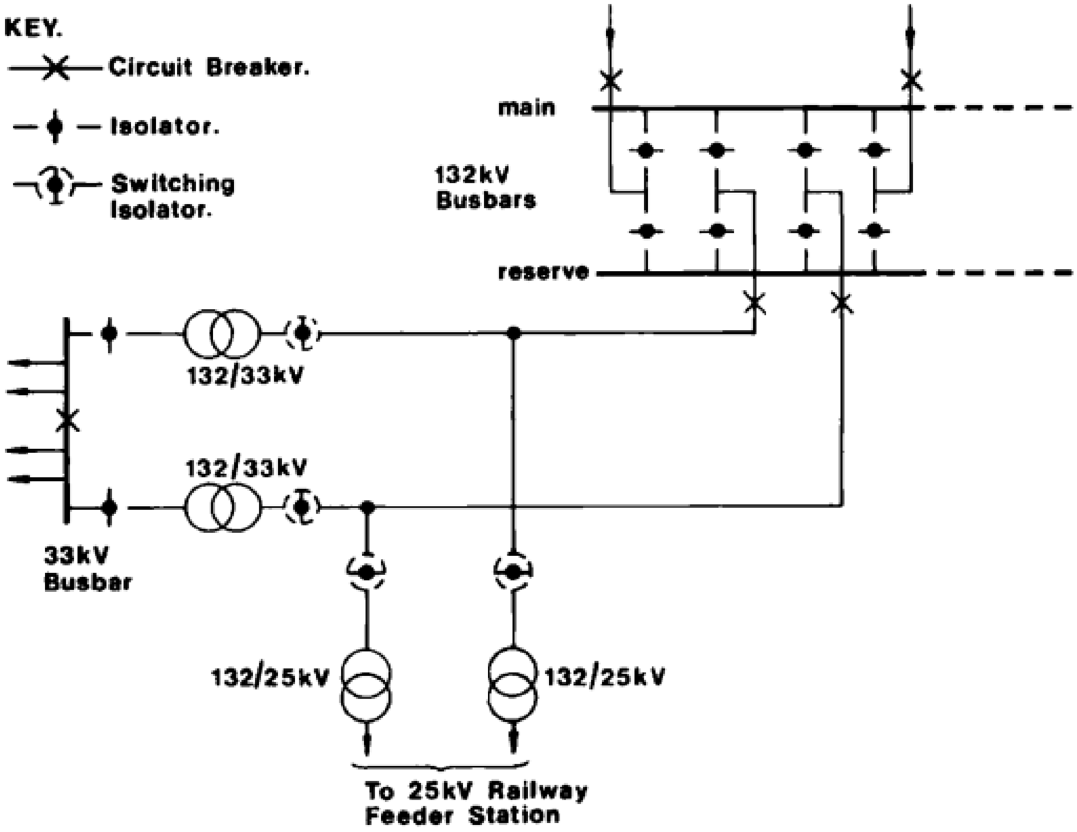

Figure 2.1 Diagram of a Typical 132 kV Power Supply Arrangement of British Railway [8]

2.1.2 AC Railway Overhead Catenary Feeding System Component

The feeding arrangement of the single-phase AC railway power supply requires neutral sections to separate two adjacent feeding networks at the feeder substation, and the mid-point track-sectioning cabin (MPTSC) under normal operation. The MPTSC is located approximately mid-way between feeder substations. Furthermore, there may be other sub-sectioning cabins on the trackside such as an Intermediate track-sectioning cabin (ITSC). When either a temporary fault or a permanent fault occurs on any feeding section, the corresponding sub-sectioning must isolate the fault instantly while other feeding sections continue to operate without any interruption. The ITSC is located between feeder substations and MPTSC. In addition, to provide some conventional equipment (e.g. SVC and/or power factor correction or power filter) there needs to be space at sub-sectioning cabins for installation. A typical feeding arrangement is shown in Figure 2.2 [1].

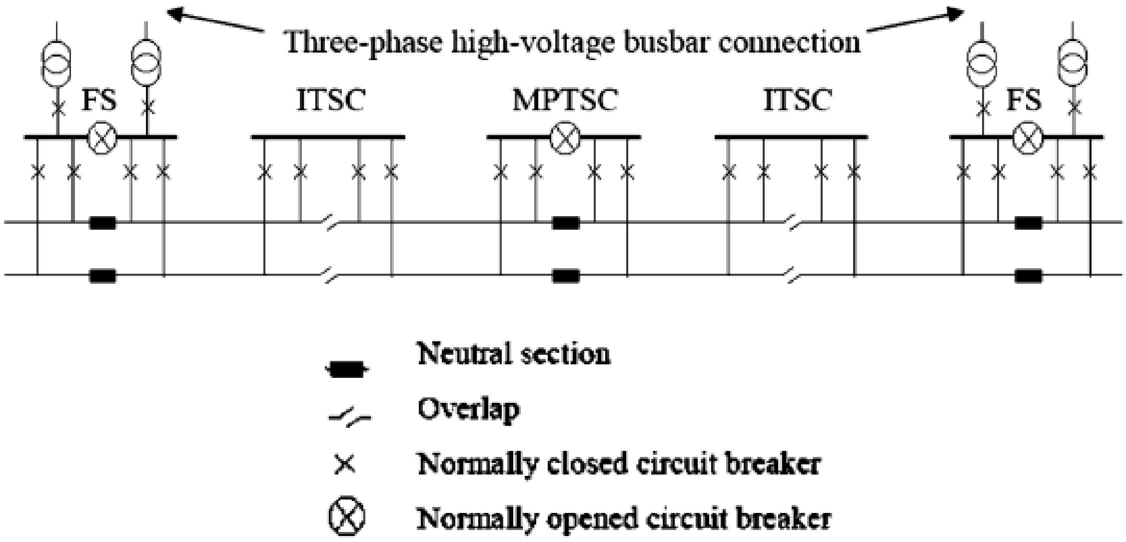

Figure 2.2 Typical feeding diagram of a double-track 25 kV railway in UK [1]

2.1.3 Energy Consumption of the Train

The specific energy consumption of a train running at a given scheduled speed is influenced by distance between stops, acceleration, retardation, maximum speed, type of train equipment and track configuration[9].

✓ **Retardation and acceleration**

Value for a given schedule speed greater the value of acceleration and retarding more will be the period of coasting and therefore, less the period during which power is on, hence D_1 on Fig 2.4 will be small and, therefore, specific energy consumption will be accordingly less.

✓ **Gradient**

Steep gradient will naturally involve more energy consumption even though regenerative breaking is employed.

✓ **Type of train equipment**

Overall efficiency for a given specific energy output at axles, will determine the specific energy consumption. Greater the overall efficiency less will be the specific energy consumption [10].

- ✓ **Speed time curve**

It is the curve showing instantaneous speed of train in kilometer per hour along ordinate and time in second along the abscissa. Area in between the curve and the abscissa give the distance travelled during given time interval. Slope at any point on the curve toward the abscissa give the acceleration or retardation at that instant.

2.2 Traction System

The three kind of traction system used in railway transportation system are steam power, diesel power and electric power. These traction systems can be compared by looking at their efficiency and relationship between weight and output power. This time, electric traction system is the one widely applied in the railway industry. This is because of environmental friendliness, high starting torque, easy speed controllability and many other reasons [11] and [12].

For train the power demand and hence the effects on supply system depend on its operation and location. A relationship between power demand of a train and its mobility is crucial in load flow studies. Train speed and operation mode are the decisive factors of the immediate amount of power required by the train. They are however determined by the traction equipment characteristics, train weight, aerodynamics, and track geometry and drive control. For an inter-station run, a train goes through different speeds and operation modes and the power demand may thus vary significantly within a short period of time. A simple and quick reference linking train speed and operation mode to the power required is essential to load flow calculation [13].

2.2.1 Tractive Effort

2.2.1.1 Tractive Effort for Acceleration

Force is required to give linear acceleration to the train and is given by:

$$F_a = 1000\text{W}\frac{\alpha * 1000}{3600} \rightarrow F_a = 277.8\text{W}\alpha \text{ Newton} \tag{2.1}$$

Where W, is weight in tone, α is train acceleration in kmphps (km/hr/sec).

When the speed of the train is being changed it behaves as a mass greater than its dead weight, this is due to angular speed

$$F_a = 277.8W_e\alpha \text{ Newton assuming } W_e = 1.1 * W$$

$$F_a = \frac{277.8}{9.81} W_e \alpha \text{ Kg} \tag{2.2}$$

Tractive effort to give acceleration of 1kmphps to 1 ton is given as

F_a= 28.3*(1.1W)*1kg =31.1W kg

2.2.1.2 Tractive effort required to balance the gravitational pull

When train is on an up gradient, gravity component of dead weight of a train parallel to the track (W sin Ɵ) will be responsible for the train to come down. In order to prevent this, tractive effort has to be applied in upward direction whose magnitude is given by equation (2.3)

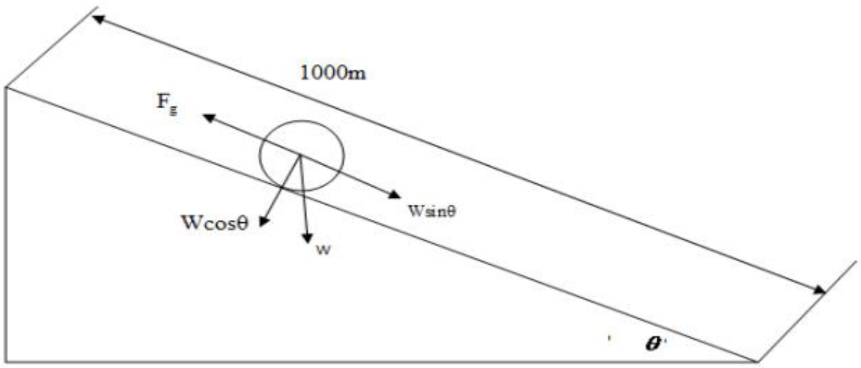

Figure 2.3 Train on up gradient [14]

$$F_g = 1000W \sin\theta * 9.81 \ \mathit{Newton} \tag{2.3}$$

In railway practice, gradient is expressed as rise in meters in a track distance of 1000 meter and is denoted by letter G%, therefore

$$G = \sin\theta * 1000 \tag{2.4}$$

Substitutes (2.4) in (2.3)

$$\mathrm{G} = 1000\mathrm{W} * \frac{\mathrm{G}}{1000} * 9.81$$

$$= 9.81\text{WG Newton} \tag{2.5}$$

Location of gradient has important effect on the over loading that can be permitted on the traction motors. For instance if the short ruling gradient happens to be after a down gradient, it does not create any problem as momentum of the train takes it up the steep gradient. On the other hand if ruling happen to be in the middle of raising gradient, the length of gradient becomes very important in determining the temperatures rise of the motor, another factor which affects the overloading of motor is the location of signal with respect to the gradient to the gradient. If stopping signal is at the foot of rising gradient, speed pick up will be low and motors are likely to be overloaded.

2.2.1.3 Tractive effort to overcome train resistance

Train resistance consists of all the force which opposes the motion the train on level track. Those forces which are internal to the rolling stock such as friction at journals, axle guide bogie pivot buffer etc. And those forces which are external to the rolling stock such as friction between wheels and rails, flange friction resistance as a result of temporary deflection of track and aerodynamic drag. Flange friction increase with oscillation of the coach and affected by side wind pressure. Track resistance depends upon the strength of the track and the nature of the track and the nature of the ballast.

Aerodynamic drag consists of pressure drag and friction drag. Former may be due to end and the latter due to the length of the train. End resistance comprises of head resistance and tail resistance due to suction of air at the rear. Both of the ends depend upon the area perpendicular to the motion. The shape of front and rear and wind velocity, since head resistance is about ten times the suction resistance. Shape of the front carriage is very important in reducing the wind resistance. Wind resistance due to the length of the train is due to air friction on side top and under side of the train. This sometimes termed as skin friction. The pressure drag is found to be far less than the frictional drag for long train. It is also found that endeavor to diminish the pressure drag is not so effective for reduction of total drag.

All the internal and external resistance, excluding wind resistance, is termed as mechanical resistance. Although different component of mechanical resistance behave in a different way with the increase in speed, it is reasonably correct assumption to consider mechanical resistance as a whole to remain constant specially at high speeds and is proportional to the weight of the train .Wind resistance on the other hand is considered to vary with the square of the speed of the train. Total train resistance F_r in the case of electric train hauled by electric locomotive is represented by equation, formula used by S.N.C.F (Society National Des Chemin De France) based on axle loading is given below for comparison sake

$$r = \left(1.5\sqrt{\frac{10}{e}} + \frac{Vm^2}{120e}\right) {}^{kg}/_{tonne} \qquad (2.6)$$

Where ‘e‘ is axle load in tone and ‘r‘ the resistance in kg per ton .This form gives lower value in low speed range and higher value in high speed range than the value obtained by the use of formula adapted by Indian railway ,now total train resistance is given as follows,

$$F_r = 9.81 * \mathrm{r} * \mathrm{W\ Newton} \qquad (2.7)$$

Where W is the weight of train including locomotive and r is the train resistance in ${}^{\mathrm{kg}}/_{\mathrm{tonne}}$.

Starting friction, which is also called “striction “is more than the running friction. This very much depends upon the starting condition of the train. For example starting resistance of the train will be more if inter- vehicle coupler are in tension which happens in the case of up gradient. On the hand inter vehicle coupler will be in compression and starting resistance will be low for train standing on down gradient. It is due to this reason that the signal should be, as possible, not be placed on up gradient. Even on level tracks it is a better practice to move back locomotive by few meters so as to release inter-vehicle tension. This makes the starting friction low as it has been observed that the starting resistance is due to the first few vehicles. Once train starts moving, resistance drops and remaining of vehicle are easily hauled by locomotive. Formulas adopted by S.N.C.F are given below [14].

(i) For gradient up to 7%

r = (gradient per thousand + 4.5) kg up to 10 tone axle load.

= (gradient per thousand + 4.2) kg above 10 tone axle load.

(ii) For gradient above 7%

r= (1.25*gradient per thousand + 2.75) kg up to 10 tone axle load

= (1.25*gradient per thousand +2.45) kg above 10 tone axle load

2.2.1.4 Tractive effort to overcome curve resistance

Curve resistance is due to the friction at a wheel flanges more the radius of the curvatures less will be the curve resistance. Curve resistance is given by the following empirical formula and usually added to the track resistance.

$$F_c = \frac{700}{\mathrm{R}}\mathrm{W\ kg} \tag{2.8}$$

Where R is the radius of curvature in meters, in railway practice sharpness of the curve is expressed more conveniently in degree of curvature than by radius of curvature.1^0 Curve is defined as that which in 100ft. (or 100/327 meters) turns the $1/360^0$ of a complete circle or in a track of 100 meters it turns through 3.27^0 curvature will have 5730/327c meters of radius of curvature substituting this for value of R in equation.

$$F_c = \frac{700 * 327}{5730}\mathrm{C} * \mathrm{W\ kg}$$

$$= 0.4\mathrm{CW\ kg} \tag{2.9}$$

Curve resistance is usually taken to be the part of train resistance and is combined with it .Total tractive effort in that case is given as

$$F_t = F_a + F_g + F_c + F_r \tag{2.10}$$

$$= 28.3W_e\alpha \pm \mathrm{WG} + \mathrm{W}(\mathrm{r} + 0.4\mathrm{C})\mathrm{kg}$$

$$= 277.8W_e\alpha \pm \mathrm{WG} + 9.81\mathrm{W}(\mathrm{r} + 0.4\mathrm{C})\ \mathrm{Newton}$$

$$= 277.8W_e\alpha \pm 9.81\mathrm{WG} + 9.81 * \frac{700}{\mathrm{R}}\mathrm{W} + 9.81\mathrm{Wr\ Newton.}$$

Sometimes in traction mechanics, it is convenient to express curve resistance in equivalent gradient resistance given by equation (2.3) and (2.5) above and added to the actual per thousand gradients

$$F_t = 277.8W_e\alpha + (0.4\mathrm{C} \pm \mathrm{G})\mathrm{W} + \mathrm{W} * \mathrm{r\ kg}$$

$$= 277.8W_e\alpha + 9.81(0.4\mathrm{C} \pm \mathrm{G})\mathrm{W} \pm 9.81\mathrm{W} * \mathrm{r\ Newton} \tag{2.11}$$

Positive sign is to be taken for train movement up gradient and negative sign for train movement down gradient [14].

Table 2.1 Train Specification Parameters HXD1C Freight Electric Locomotive [2]

Startup traction effort	570kN
Continuous traction effort	400kN
Traction power at wheel rim	7200kW
The maximum speed of locomotive	V_m=120kmph

Continuous speed of the locomotive	65kmph
Radius of curvature	800m
Axle load	25ton

Table 2.2 Train acceleration and retardation for various train transportation service [14]

No	Parameter of comparison	Urban service	Sub-urban service	Mainline service
1	Acceleration	1.5 to 4kmphps	1.5 to 4kmphps	0.6 to 0.8kmphps
2	Retardation	3 to 4kmphps	3 to 4kmphps	1.5kmphps
3	Maximum speed	120kmph	120kmph	100kmph
4	Distance between station	1km	2.5-3.5km	More than 10km
5	Special remark if any	Free running period is absent coasting period is small	Free running period is absent coasting period is long	Long free running and coasting Period. Acceleration and breaking period is small

$$\alpha = 2.16\text{kmphps}$$

$$\mathrm{R} = \frac{5370}{327\mathrm{C}} \Rightarrow 800\text{meter}, \quad \mathrm{C} = \frac{5370}{327\mathrm{R}} = \qquad ,$$

$$r = \left(1.5\sqrt{\frac{10}{25}} + \frac{Vm^2}{120*25}\right) = 5.748$$

$$W_e = 1.1$$

$$F_t = W[1.1 * 277.8 * \alpha + 9.81G + \frac{700}{R} + 9.81r]$$

$$W = \frac{F_t}{1.1 * 277.8 * \alpha + 9.81G + \frac{700}{R} + 9.81r} ton$$

$$\mathrm{W} = \frac{570000\mathrm{N}}{1.1 * 277.8 * 0.8 + 9.81 * 0.018 + 9.81 * \frac{700}{800} + 9.81 * 5.748} = 1840.955\mathrm{ton}$$

Tractive effort that is required per ton (hauling capacity of locomotive) is given below

$$\frac{\text{tractive effort}}{\text{weight}} = \frac{570000}{1088.767} = 523.527\ \frac{\mathrm{N}}{\mathrm{ton}}$$

2.2.2 Power of a Traction Motor

Power is the rate of doing work and is given by

$$\mathrm{P} = F_t * \mathrm{V} * \frac{1000}{3600}\mathrm{watt} \tag{2.12}$$

Consider an instant at point D in Fig 2.4 after this point speed remain constant and the tractive effort required is less since no acceleration of train is take place .But before point D, tractive effort required is maximum and the speed is approaching maximum value. Consequently, power output required from the deriving axle to propel is maximum [14]. It is given below

$$P_{max} = F_t * V_{max} * \frac{1000}{3600}\mathrm{watt}$$

$$= 0.278 * F_t * V_{max}$$

$$= 570\mathrm{kN} * 120\,\mathrm{km/hr} * 0.277 = 19\mathrm{MW}$$

If we considered the continuous speed (65km/hr) of the train, it is possible to get continuous power consumption of a train =10.292MW

If η be the efficiency of a transmission gear, maximum power output of the motors will be:

$$P_{max} = \frac{0.278 * F_t * V_{max}}{\eta}\ \mathrm{watt} \tag{2.13}$$

Output power required for continuous motion of a locomotive is calculated using equation (2.12), where the continuous speed is assumed to be 65 km/hr,

$$P_{out} = 400\mathrm{kN} * 65 * 0.277\ \mathrm{watt}$$

$$= 7222\mathrm{kW} \simeq 7200\mathrm{kW}\ \text{is given}$$

When F_t is in Newton

$$\eta = \frac{P_{out}}{P_{max}} = \frac{7200\mathrm{kW}}{19\mathrm{MW}} = 0.379$$

2.2.3 Specific Energy Consumption

It is energy consumed in watt hours per ton kilometer of a train. We will first find out specific energy output of deriving wheels. When this is divided by overall efficiency of transmission gear and motor we will get specific energy consumption.

Total energy output of driving axles is spent as follows:

- To accelerate the train
- To overcome the gradient
- To overcome train resistance
- Energy output of deriving axles to accelerate the train

In this calculation, trapezoidal speed time curve of Figure (2.4) is assumed.

- ✓ **Energy output of driving axle to accelerate the train**

$$E_a = F_a * \text{distance OAD} \tag{2.14}$$

$$= 277.8W_e\alpha\left[\frac{1}{2}\frac{V_{max}*1000}{3600}*\frac{V_{max}}{\alpha}\right]\text{watt} - \text{second}$$

$$= 277.8W_e\alpha\left[\frac{1}{2}\frac{V_{max}*1000}{3600}*\frac{V_{max}}{\alpha}\right]\frac{1}{3600}\text{watt} - \text{hours}$$

$$= 0.01\text{Vmax}^2W_e\text{ watt} - \text{hour} \tag{2.15}$$

- ✓ **Energy output of driving axle to overcome the gradient**

$$E_g = F_g * D_1$$

Where D_1 the distance over which power remains on and its maximum value is equal to the area under OABE.

$$E_g = 9.81 * \text{WG joule or watt} - \text{second}$$

$$= \frac{9.81*1000}{3600} * \text{WG}D_1\ \text{ watt} - \text{hours}$$

$$= 2.725\text{WG}D_1\text{Whrs} \tag{2.16}$$

- ✓ **Energy output of deriving axle to overcome friction**

$$E_r = 1000 * F_r * D_1\ \text{ Joule}$$

$$= \frac{\mathrm{W} * \mathrm{r} * D_1 * 1000}{3600}\ \mathrm{Whrs}$$

$$= 0.277\mathrm{W} * \mathrm{r} * D_1 \tag{2.17}$$

- ✓ **Energy output of deriving axles to accelerate the train**

$$E_{out} = E_a + E_g + E_r \tag{2.18}$$

$$E_{out} = \left[0.01{V_{max}}^2 W_e \pm 2.725WGD_1 + 0.277W * r * D_1\right]Whrs$$

2.2.4 Energy Consumption

Energy consumption of the train is defined as equal to total energy input to traction motor from the supply. It is usually expressed in Watt hour which equals 3600 Joules and can be found by dividing the energy output of the deriving wheels with the combined efficiency of a transmission gear and the motor.

$$E_{con} = \frac{E_{out}}{\eta_{gear} * \eta_{motor}}\ Watt - hour \tag{2.19}$$

2.2.5 Specific Energy Output

$$E_{spo} = \frac{E_{out}}{\mathrm{W} * \mathrm{D}} \tag{2.20}$$

$$E_{spo} = \left[\frac{0.01{V_{max}}^2}{D} * \frac{W_e}{W} \pm 2.725 * G\frac{D_1}{D} + 0.277r\frac{D_1}{D}\right]Whrs/ton - km \tag{2.21}$$

Where D, is run length in kilometer,

D_1 is the total distance over which power remain ON: it is the maximum value equals the distance represented by the area OABE in Fig 2.4 from the start to the end of free running period in case of as per assumption of trapezoidal speed time curve [15].

2.2.6 Specific Energy Consumption

It is energy consumed in watt-hour per ton of mass of a train per kilometer length of the run

$$E_{spc} = \frac{E_{spo}}{\eta} = \frac{E_{con}}{\mathrm{W} * \mathrm{D}}$$

η= Overall efficiency of transmission gear and the motor= $\eta_{gear} * \eta_{motor}$

$$\therefore E_{spc} = [\frac{0.01{V_{max}}^2}{\eta * \mathrm{D}} * \frac{W_e}{\mathrm{W}} \pm 2.725 * \mathrm{G}\frac{D_1}{\eta * \mathrm{D}} + 0.277\mathrm{r}\frac{D_1}{\eta * \mathrm{D}}]\mathrm{Whrs/ton - km} \tag{2.22}$$

If no gradient is involved, specific energy consumption is

$$E_{spc} = [\frac{0.01{V_{max}}^2}{\eta * D} * \frac{W_e}{W} + 0.277r\frac{D_1}{\eta * D}]\text{Whrs/ton} - \text{km} \tag{2.23}$$

$$E_{ov-con} = [E_{spc} * W * D]\text{Whr} \tag{2.24}$$

2.2.7 Simplified Speed/Time Curve

For the purpose of comparative performance for a given service, the actual speed/time curve of Fig. 2.4 is represented by a simplified speed/time curve which does not involve the knowledge of motor characteristics. Such a curve has simple geometric shape so that simple mathematics can be used to find the relation between acceleration, retardation, average speed and distance etc. The simple curve would be fairly accurate provided it

(i) Retains the same acceleration and retardation and

(ii) Has the same area as the actual speed/time curve. The simplified speed/time curve can have either of the two shapes

Trapezoidal shape OABC of Fig. 2.4 Where speed-curve running and coasting periods of the actual speed/time curve have been replaced by a constant speed Period.

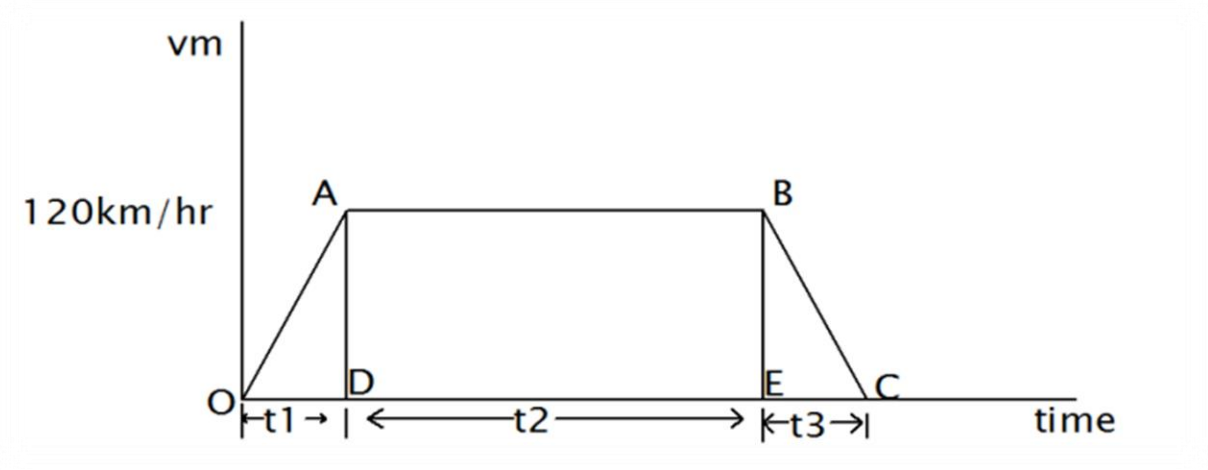

Figure 2.4 Simplified Speed/Time Curve

$$\alpha = \frac{V_{max}}{t_1} \tag{2.25}$$

$$\beta = \frac{V_{max}}{t_3} \tag{2.26}$$

D = OABC

= OAD + ABED + BCE

$$= \frac{1}{2}V_{max} * t_1 + V_{max} * t_2 + \frac{1}{2}V_{max} * t_3, \qquad t = t_1 + t_2 + t_3, \rightarrow t_2 = t - (t_1 + t_3)$$

$$= \frac{1}{2}V_{max} * t_1 + V_{max}[t - (t_1 + t_3)] + \frac{1}{2}V_{max} * t_3$$

$$= \frac{1}{2}V_{max} * t_1 + V_{max} * t - V_{max} * t_1 - V_{max} * t_3 + \frac{1}{2}V_{max} * t_3$$

$$= V_{max}[\frac{1}{2}t_1 + t - t_1 - t_3 + \frac{1}{2}t_3]$$

$$= V_{max}[t - \frac{1}{2}(t_1 + t_3)]$$

$$= V_{max}\left[t - \frac{V_{max}}{2}\left(\frac{1}{\beta} + \frac{1}{\alpha}\right)\right] \qquad K = \frac{1}{2}\left(\frac{1}{\beta} + \frac{1}{\alpha}\right)$$

$$D = V_{max}[t - V_{max}\mathrm{K}]$$

$$t = [\frac{D}{V_{max}} + \mathrm{K}V_{max}] \tag{2.27}$$

Where D = breaking distance +Area OABE, by using equation (2.26)

Breaking distance = BCE = $\frac{1}{2}V_{max} * t_3 \qquad t_3 = \frac{V_{max}}{\beta} = 80\mathrm{s}$

$$= \frac{1}{2}\frac{{V_{max}}^2}{\beta} = \frac{120km/hr^2}{1.5km/hr/s} = 2.6km$$

2.3 Voltage Drops in Traction Network

2.3.1 Introduction

The important consideration in design and operation of a transmission line are determination of voltage drop, line loss and efficiency of a transmission. These values are greatly influenced by the line constants R, L, C of the transmission line for instance the voltage drop in the line depends upon the value of the above three line constants. Similarly the resistance of a transmission line conductor is the most important cause of power loss in the line and determines the transmission efficiency [3].

2.3.2 Classification of Overhead Transmission Lines

A transmission line has three constant R, L, C distributed uniformly along the whole length of the line. These resistance and inductance form series impedance. The capacitance exists between conductor for 1-phase line or from a conductor to a neutral. Therefore, capacitance effect introduces complication in transmission line calculation. Depending upon the manner in which capacitance is taken in to account, the overhead line are classified as described below~~,~~

2.3.3 Medium Transmission Lines

In short transmission line calculations, the effects of the line capacitance are neglected because such lines have smaller lengths and transmit power at relatively low voltages (<20kV). However, as the length and voltage of the line increase, the capacitance gradually becomes of greater importance.

Since medium transmission lines have sufficient length (50-150 km) and usually operate at voltages greater than 20 kV, the effects of capacitance cannot be neglected. Therefore, in order to obtain reasonable accuracy in medium transmission line calculations, the line capacitance must be taken into consideration.

The capacitance is uniformly distributed over the entire length of the line. However, in order to make the calculations simple, the line capacitance is assumed to be lumped or concentrated in the form of capacitors shunted across the line at one or more points. Such a treatment of localizing the line capacitance gives reasonably accurate results. The most commonly used methods known as localized capacitance methods for the solution of medium transmissions lines.

When the length of an overhead transmission line is about 50-150km and the line voltage is moderately high (> 20kV < 100kV), it is considered as a medium transmission line. Due to sufficient length and voltage of the line, the capacitance effect is taken in account. For purpose of calculation the distributed capacitance of the line is divided and lumped in the form of condenser and shunted across the line at one or more point.

Such a treatment of localizing the line capacitance gives reasonably accurate results. The most commonly used methods (known as localized capacitance method) for the solution of medium transmissions lines are transmission circuits that may be represented by an equivalent π or T network using lumped constants as shown in Figure 2.5. Z is the total series impedance (R + jX) L and Y is the total shunt admittance (G + jB) L, where L is the circuit length. The terms inside the brackets in Figure 2.5 are correction factors that allow for the fact that in the actual circuit the parameters are distributed over the whole length of the circuit and not lumped, as in the equivalent circuits.

With short lines it is usually possible to ignore the shunt admittance, which greatly simplifies calculations, but on longer lines it must be included. Another simplification that can be made is that of assuming the conductor configuration to be symmetrical. The self-impedance of each conductor becomes Zp, and the mutual impedance of line-to-line values between conductors becomes Zm. However, for rigorous calculations a detailed treatment is necessary, with account being taken of the spacing of a conductor in relation to its neighbor and earth [16].

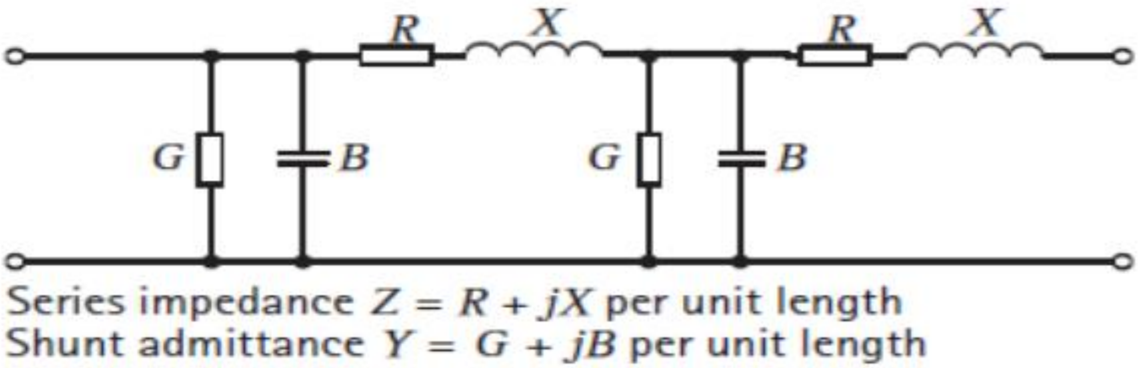

Figure 2.5a For Medium Transmission Line Representation [16][3]

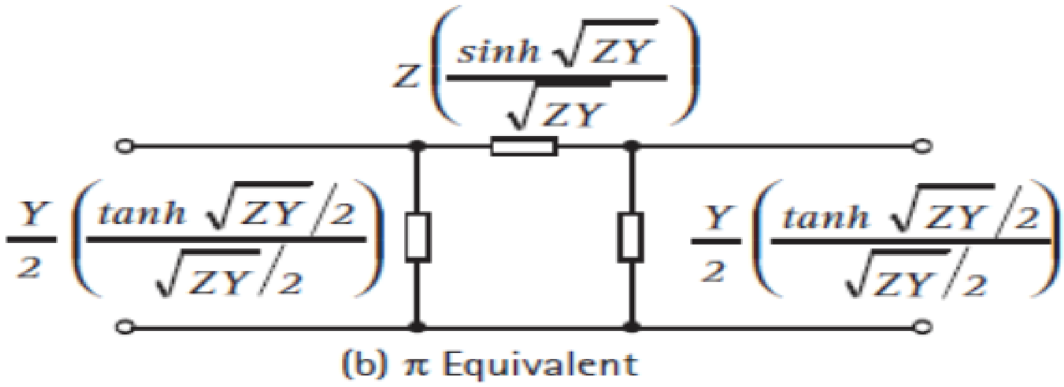

Figure 2.5b Circuit Model for Transmission Line between Sebeta to Indode [16]

Expression obtained from [16]

$$\frac{\sinh\sqrt{ZY}}{\sqrt{ZY}} = 1 + \frac{ZY}{6} + \frac{Z^2Y^2}{120} + \frac{Z^3Y^3}{5040} + \cdots \quad (2.28)$$

$$\frac{\tanh\sqrt{ZY}}{\sqrt{ZY}} = 1 - \frac{ZY}{12} + \frac{Z^2Y^2}{120} + \frac{17Z^3Y^3}{20160} + \cdots \quad (2.29)$$

2.3.4 Mutual Impedance between Phases (Catenaries)

The impedance of a conductor –earth loop consists of its resistance and reactance .The reactance depends on the inductance L and the frequency f .The self-impedance of the conductor –earth loop is composed of the resistance, the inner self-inductance and external inductance. The self-impedance of the loop can be expressed by

$$\dot{Z}_{ii} = \dot{\mathrm{R}} + \dot{R}_E + \mathrm{j}(\dot{X}_{ex} + \dot{X}_{in}) \qquad (2.30)$$

Where $\dot{\mathrm{R}}$ – Resistance

$\dot{R}_E$ - Resistance of the earth return path

$\dot{X}_{ex}$ - The external self-inductance

$\dot{X}_{in}$ - The inner self-inductance

$\dot{R}_E$ is the resistance of the earth return path and the resistance per unit length is 49.3mΩ/km for 50Hz [3].

$$\dot{X}_{ex} = 2\pi * \mathrm{f} * \dot{L}_{ex} = 4\pi * 10^{-7} * \mathrm{f} * \ln(\delta_\mathrm{E}/\mathrm{r})\,\Omega/\mathrm{km} \qquad (2.31)$$

The inner reactance $\dot{X}_{in}$ is obtained from

$$\dot{X}_{in} = 2\pi * \mathrm{f} * \dot{L}_{in} = 4\pi * 10^{-7} * \mathrm{f} * \ln(\mathrm{r}/\mathrm{req})\,\Omega\,/\mathrm{km} \qquad (2.32)$$

The penetration depth δ_E of the current flowing through earth can be calculated as

$$\delta_\mathrm{E} = \frac{0.738}{\sqrt{\mathrm{f} * {\mu_0}/{\rho_\mathrm{E}}}} \qquad \delta_\mathrm{E} \approx 90\sqrt{\rho_\mathrm{E}} \quad \text{for 50HZ [3]}$$

For calculation of the penetration depth, an assumption is made that the earth is homogeneous body having a semi-circle cross section located under the electrified railway line. The inner inductance $\dot{L}_{in}$ per unit length is found to be $\frac{\mu}{8\pi}$ for solid conductors with circular cross section independent of the conductor radius. Therefore

$$\dot{L}_{in} = 2 * 10^{-7} \ln(\frac{\mathrm{r}}{\mathrm{r_{eq}}}) = 4\pi * {10^{-7}}/{8\pi}\ \mathrm{H/m}.$$

Where $\ln(\frac{\mathrm{r}}{\mathrm{r_{eq}}}) = \frac{1}{4}$ and $\mathrm{r_{eq}} = \mathrm{r} * \mathrm{e}^{-0.25} = 0.7788 * \mathrm{r}$

The coupling impedance of two conductor –earth loops i and k can be expressed

$$\dot{Z}_{ik} = \dot{R}_E + \mathrm{j}\dot{X}_{ik} \tag{2.33}$$

The mutual reactance is given by expression defined below [17] [3].

$$\dot{X}_{ik} = 4\pi * 10^{-7} * \mathrm{f} * \ln(\frac{\delta_\mathrm{E}}{a_{ik}}) \tag{2.34}$$

2.3.5 Voltage Regulation

When a transmission line is carrying current, there is a voltage drop in the line due to resistance and inductance of the line. The result is that receiving end voltage (V_r) of the line is generally less than the sending end voltage (V_s). This voltage drop ($V_s - V_r$) in the line is expressed as a percentage of receiving end voltage V_r and is called voltage regulation mathematically.

$$\%\text{ age of voltage regulation} = \frac{V_s - V_r}{V_r} * 100 \tag{2.35}$$

Voltage regulation can be defined as the proportional change in voltage magnitude at the load bus due to change in load current (say from no load to full load). The voltage drop is caused due to feeder impedance carrying the load current as illustrated in Fig. 2.6(a). If the supply voltage is represented by Thevenin’s equivalent, then the voltage regulation (VR) is given by,

$$\mathrm{VR} = \frac{\bar{\mathrm{E}} - \bar{\mathrm{V}}}{\bar{\mathrm{V}}} = \frac{\bar{\mathrm{E}} - \mathrm{V}}{\mathrm{V}} \tag{2.36}$$

$\bar{\mathrm{V}}$, being a reference phasor. In absence of compensator, the source and load currents are same and the voltage drop due to the feeder is given by,

$$\Delta\bar{\mathrm{V}} = \bar{\mathrm{E}} - \bar{\mathrm{V}} = Z_s \bar{I}_1 \tag{2.37}$$

The feeder impedance, $Z_s = R_s + \mathrm{j}X_s$. The relationship between the load apparent powers and its voltage and current is expressed below

$$\bar{S}_1 = \bar{\mathrm{V}} I_1^* = P_1 + \mathrm{j}Q_1 \tag{2.38}$$

Since $\bar{\mathrm{V}} = \mathrm{V}$, the load current is expressed as following

$$I_1 = \frac{P_1 - \mathrm{j}Q_1}{\mathrm{V}} \tag{2.39}$$

Substituting, I_1 from the above equation in (2.35)

$$\Delta\bar{\mathrm{V}} = \bar{\mathrm{E}} - \bar{\mathrm{V}} = (R_s + \mathrm{j}X_s)(\frac{P_1 - \mathrm{j}Q_1}{\mathrm{V}}) \tag{2.40}$$

$$= \frac{R_s P_1 + X_s Q_1}{\mathrm{V}} + \mathrm{j}\,\frac{X_s P_1 - R_s Q_1}{\mathrm{V}} \tag{2.41}$$

$$= \Delta V_R + j\Delta V_x \qquad (2.42)$$

Thus, the voltage drop across the feeder has two components, one in phase ΔVR and another is in phase quadrature ΔV_x with the voltage V as illustrated in Fig. 2.6a.

From the above, it is evident that load bus voltage $\overline{V}$ is dependent on the value of the feeder impedance, magnitude and phase angle of the load current. In other words, voltage change (ΔVx) depends upon the real and reactive power flow of the load and the value of the feeder impedance.

When the compensator is added parallel with the load, the question is: whether it is possible to make $\overline{E} = \overline{V}$ in order to achieve zero voltage regulation irrespective of change in the load, the answer is yes, if the compensator consisting of purely reactive component has enough capacity to supply the required amount of the reactive power. This situation is shown using phasor diagram in Fig. 2.6b.

The net reactive load bus is now $Q_s = Q_x + Q_1$. The compensator reactive power (Q_x) has to be adjusted in such a way as to rotate the phasor ΔV until $\overline{E} = \overline{V}$ [18].

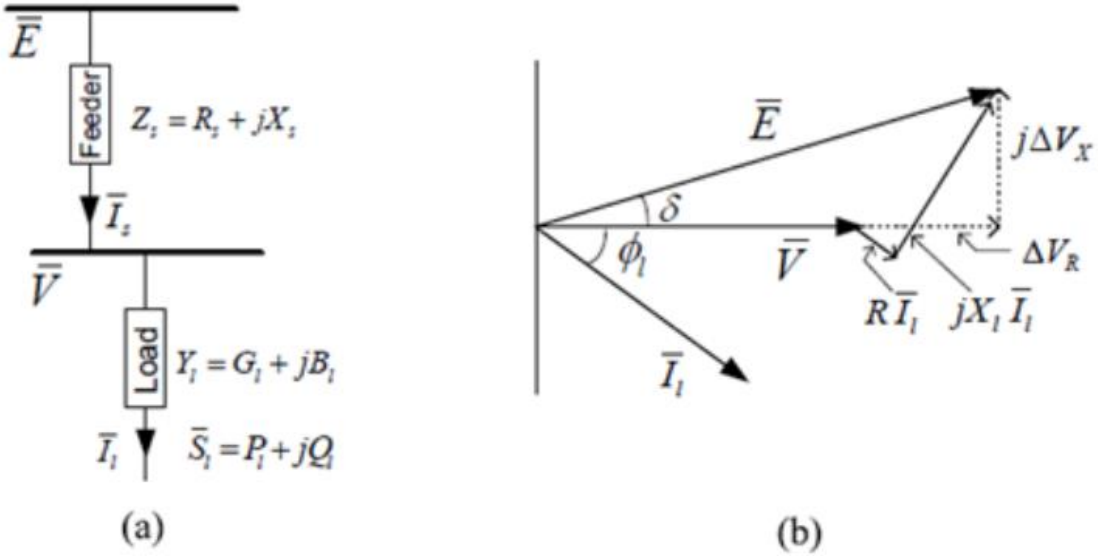

Figure 2.6a Single Phase System with Feeder Impedance, Figure 2.6b Phasor diagram [18].

2.4 FACTS-Devices and Applications

2.4.1 Overview of FACTS Devices

Flexible AC Transmission Systems, called FACTS, got in the recent years a well-known term for higher controllability in power systems by means of power electronic devices. Several FACTS-devices have been introduced for various applications worldwide. A number of new types of devices are in the stage of being introduced in practice. Even more concepts of configurations of FACTS-devices are discussed in research and literature.

In most of the applications the controllability is used to avoid cost intensive or landscape requiring extensions of power systems, for instance like upgrades or additions of substations and power lines. FACTS-devices provide a better adaptation to varying operational conditions and improve the usage of existing installations. The basic applications of FACTS-devices are:

- ✓ Power flow control,
- ✓ Increase of transmission capability,
- ✓ Voltage control,
- ✓ Reactive power compensation,
- ✓ Stability improvement,
- ✓ Power quality improvement,
- ✓ Power conditioning,
- ✓ Flicker mitigation,
- ✓ Interconnection of renewable and distributed generation and storages.

The influence of FACTS-devices is achieved through switched or controlled shunt compensation, series compensation or phase shift control. The devices work electrically as fast current, voltage or impedance controllers. The power electronic allows very short reaction times down to far below one second.

In the following a structured overview on FACTS-devices is given. These devices are mapped to their different fields of applications. The left column in Figure 2.7 contains the conventional devices build out of fixed or mechanically switchable components like resistance, inductance or capacitance together with transformers. The FACTS-devices contain these elements as well but use additional power electronic valves or converters to switch the elements in smaller steps or with switching patterns within a cycle of the alternating current. The left column of FACTS-devices uses Thyristor valves or converters. These valves or converters are well known since several years. They have low losses because of their low switching frequency of once a cycle in the converters or the usage of the Thyristors to simply bridge impedances in the valves.

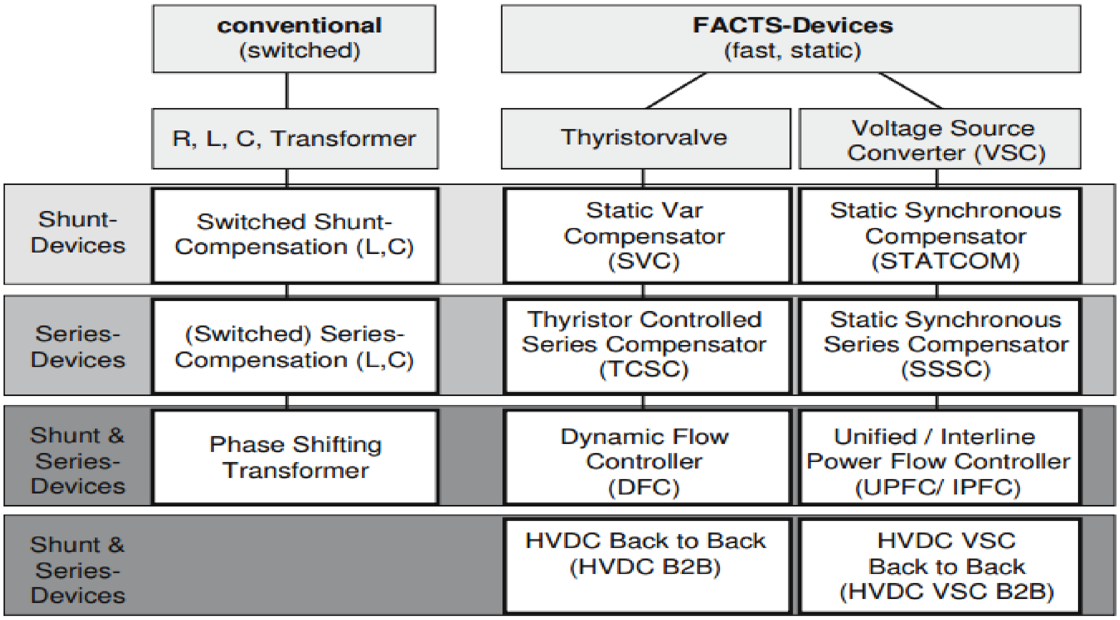

Figure 2.7 Overview of Major FACTS-Devices [19]

The right column of FACTS-devices contains more advanced technology of voltage source converters based today mainly on Insulated Gate Bipolar Transistors (IGBT) or Insulated Gate Commutated Thyristors (IGCT). Voltage Source Converters provide a free controllable voltage in magnitude and phase due to a pulse width modulation of the IGBTs or IGCTs. High modulation frequencies allow to get low harmonics in the output signal and even to compensate disturbances coming from the network. The disadvantage is that with an increasing switching frequency, the losses are increasing as well. Therefore special designs of the converters are required to compensate this.

In each column the elements can be structured according to their connection to the power system. The shunt devices are primarily for reactive power compensation and therefore voltage control. The SVC provides in comparison to the mechanically switched compensation a smoother and more precise control. It improves the stability of the network and it can be adapted instantaneously to new situations. The STATCOM goes one step further and is capable of improving the power quality against even dips and flickers.

The series devices are compensating reactive power. With their influence on the effective impedance on the line they have an influence on stability and power flow. These devices are installed on platforms in series to the line. Most manufacturers count Series Compensation, which is usually used in a fixed configuration, as a FACTS-device. The reason is that most parts and the system setup require the same knowledge as for the other FACTS-devices. In some cases the series compensator is protected with a Thyristor-bridge for the application of control.

The series devices are compensating reactive power. With their influence on the effective impedance on the line they have an influence on stability and power flow. These devices are installed on platforms in series to the line. Most manufacturers count Series Compensation, which is usually used in a fixed configuration, as a FACTS-device. The reason is that most parts and the system setup require the same knowledge as for the other FACTS-devices. In some cases the Series Compensator is protected with a Thyristor-bridge. The application of the TCSC is primarily for damping of inter-area oscillations and therefore stability improvement, but it has as well a certain influence on the power flow [19].

2.4.2 Configurations of FACTS-Devices

2.4.2.1 Shunt Devices

The most used FACTS-device is the SVC or the version with Voltage Source Converter called STATCOM. These shunt devices are operating as reactive power compensators. The main applications in transmission, distribution and industrial networks are:

- Reduction of unwanted reactive power flows and therefore reduced network losses,
- Keeping of contractual power exchanges with balanced reactive power,
- compensation of consumers and improvement of power quality especially with huge demand fluctuations like industrial machines, metal melting plants, railway or underground train systems,
- compensation of Thyristor converters e.g. in conventional HVDC lines,
- Improvement of static or transient stability.

Almost half of the SVC and more than half of the STATCOMs are used for industrial applications. Industry as well as commercial and domestic groups of users require power quality improvement. Flickering lamps are no longer accepted, nor are interruptions of industrial processes due to insufficient power quality. For example demands for increased steel production and rules for network disturbances have, together with increasing cost of energy, made reactive power compensation a requirement in the steel industry. A special attention is given to weak network connections with severe voltage support problems.

A steel melting process demands a stable and steady voltage support for the electric arc furnace. With dynamic reactive power compensation, the random voltage variations characterized by an arc furnace are minimized. The minimized voltage variations are achieved by continuously compensating the reactive power consumption from the arc furnace. The result is an overall improvement of the furnace operation, which leads to better process and production economy.

Railway or underground systems with huge load variations require SVCs or STATCOMs similar to the application above. SVC or STATCOM for even stricter requirements on power quality are used in other kinds of critical factory processes, like electronic or semiconductor productions.

A growing area of application is the renewable or distributed energy sector. Especially offshore wind farms with its production fluctuation have to provide a balanced reactive power level and keep the voltage limitations within the wind farm, but as well on the interconnection point with the main grid. A lot distributed generation devices are interconnected with the grid through a voltage source converter similar to the STATCOM fulfilling all requirements on a stable network operation [19].

2.4.2.1.1 SVC

Electrical loads both generate and absorb reactive power. Since the transmitted load varies considerably from one hour to another, the reactive power balance in a grid varies as well. The result can be unacceptable voltage amplitude variations or even a voltage depression, at the extreme a voltage collapse. A rapidly operating Static Var compensator (SVC) can continuously provide the reactive power required to control dynamic voltage oscillations under various system conditions and thereby improve the power system transmission and distribution stability. Installing an SVC at one or more suitable points in the network can increase transfer capability and reduce losses while maintaining a smooth voltage profile under different network conditions. In addition an SVC can mitigate active power oscillations through voltage amplitude modulation.

SVC installations consist of a number of building blocks. The most important is the Thyristor valve, i.e.Stack assemblies of series connected anti-parallel Thyristors to provide controllability. Air core reactors and high voltage AC capacitors are the reactive power elements used together with the Thyristor valves. The step-up connection of this equipment to the transmission voltage is achieved through a power transformer. The Thyristor valves together with auxiliary systems are located indoors in an SVC building, while the air core reactors and capacitors, together with the power transformer are located outdoors.

In principle the SVC consists of Thyristor Switched Capacitors (TSC) and Thyristor Switched or Controlled Reactors (TSR / TCR).

The coordinated control of a combination of these branches varies the reactive power as shown in Figure 2.8.The first commercial SVC was installed in 1972 for an electric arc furnace. On transmission level the first SVC was used in 1979. Since then it is widely used and the most accepted FACTS-device. A recent installation is shown in Figure 2.9.

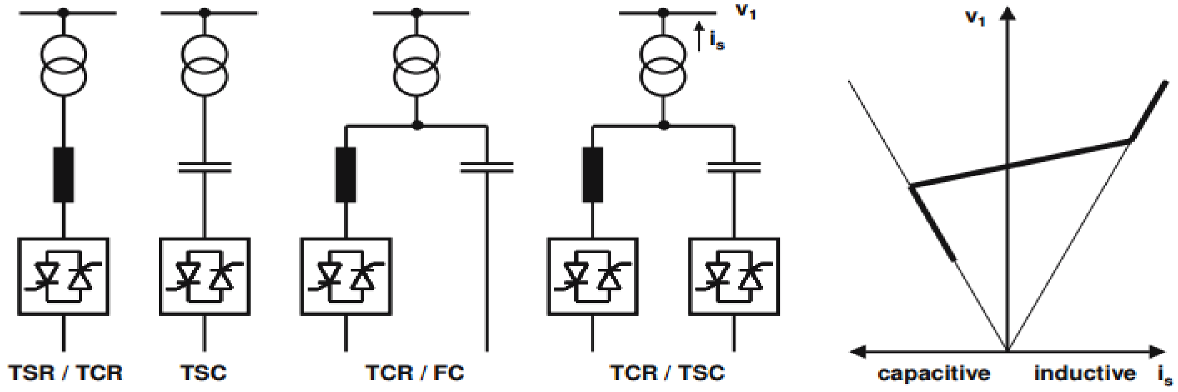

Figure 2.8 SVC building blocks and voltage / current characteristic [19]

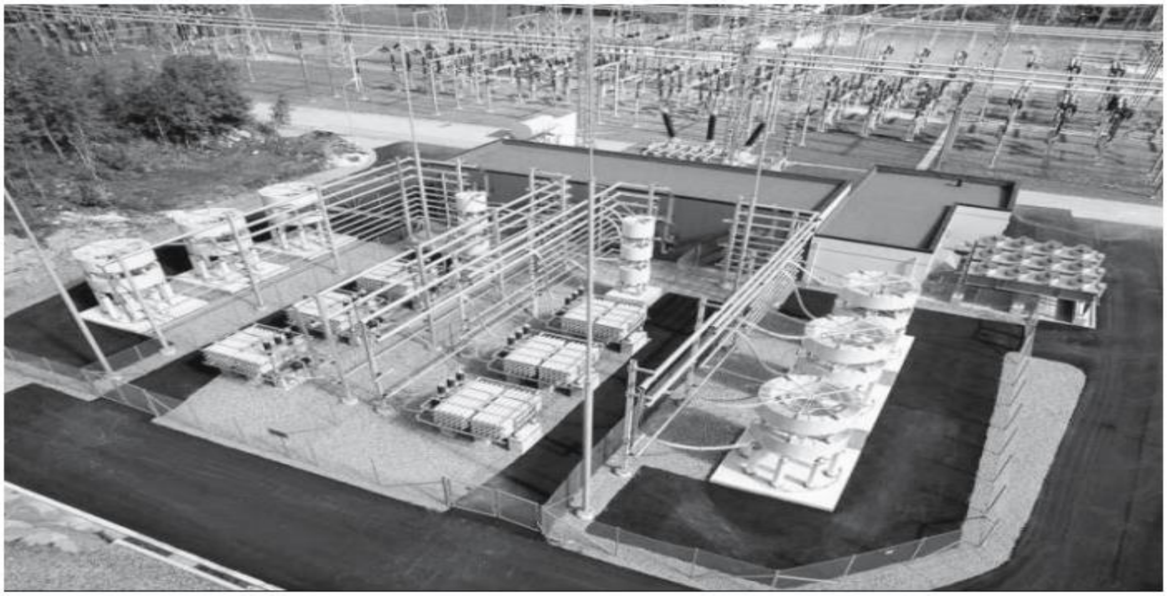

Figure 2.9 SVC (Source: ABB) [19]

2.4.2.1.2 STATCOM

In 1999 the first SVC with Voltage Source Converter called STATCOM (STATic COMpensator) went into operation. The STATCOM has a characteristic similar to the synchronous condenser, but as an electronic device it has no inertia and is superior to the synchronous condenser in several ways, such as better dynamics, a lower investment cost and lower operating and maintenance costs.

A STATCOM is built with Thyristors with turn-off capability like GTO or today IGCT or with more and more IGBTs. The structure and operational characteristic is shown in Figure 2.10. The static line between the current limitations has a certain steepness determining the control characteristic for the voltage. **The advantage of a STATCOM is that the reactive power provision is independent from the actual voltage on the connection point.** This can be seen in the diagram for the maximum currents being independent of the voltage in comparison to the SVC in Figure 2.8. This means, that even during most severe contingencies, the STATCOM keeps its full capability.

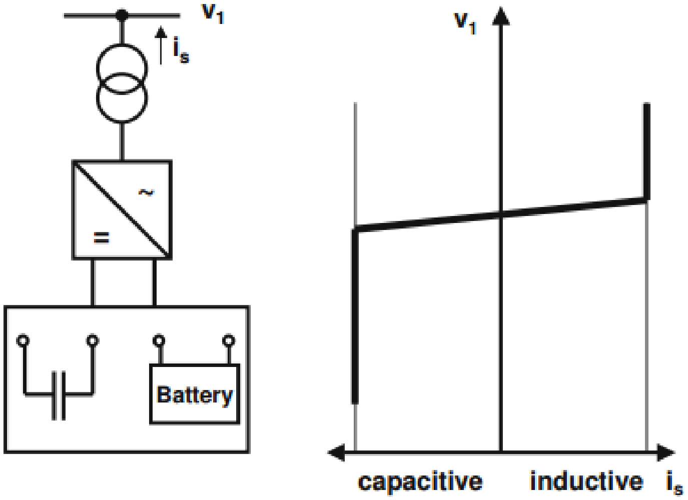

Figure 2.10 STATCOM structure and voltage / current characteristic [19]

In the distributed energy sector the usage of Voltage Source Converters for grid interconnection is common practice today. The next step in STATCOM development is the combination with energy storages on the DC-side. The performance for power quality and balanced network operation can be improved much more with the combination of active and reactive power

Figure 2.11 to Figure 2.13 show a typical STATCOM layout on transmission Level as part of a substation.

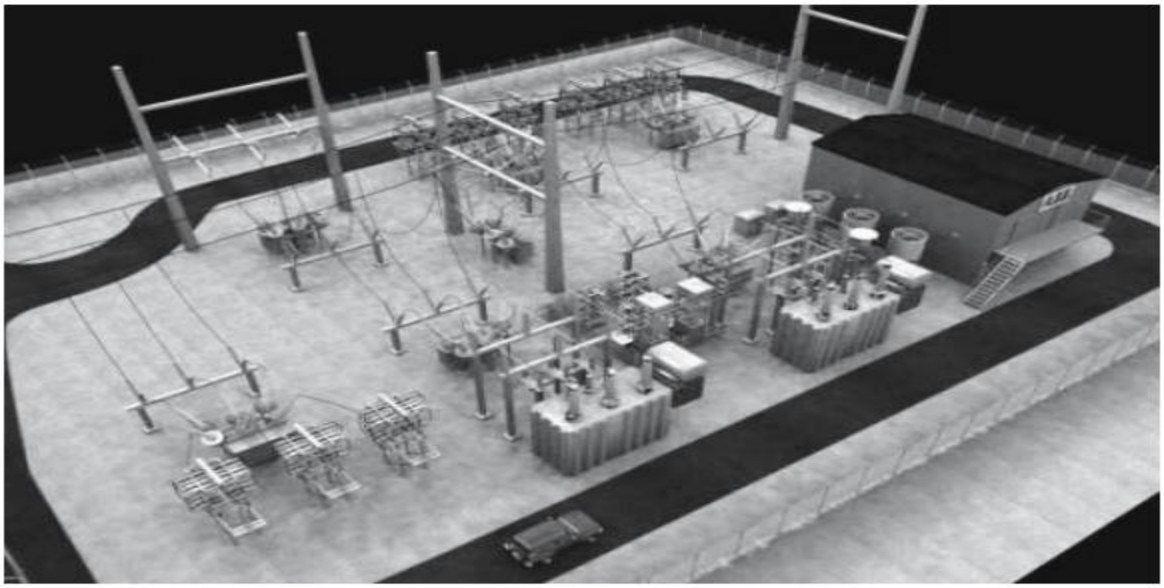

Figure 2.11 Substation with a STATCOM (Source: ABB) [19]

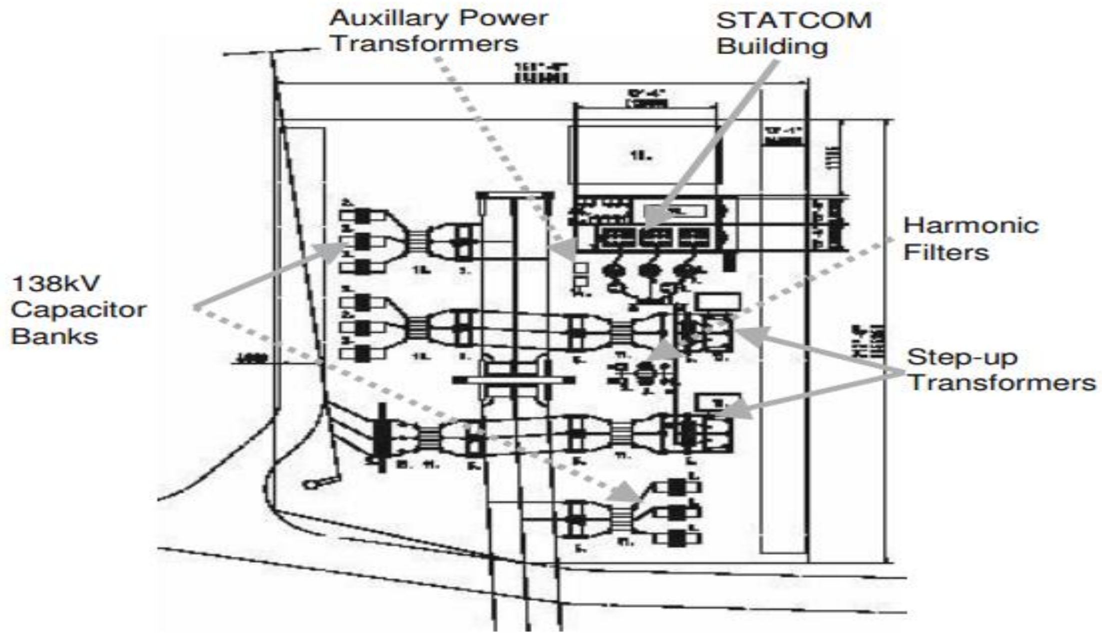

Figure 2.12 Typical substation layout with STATCOM (Source: ABB) [19]

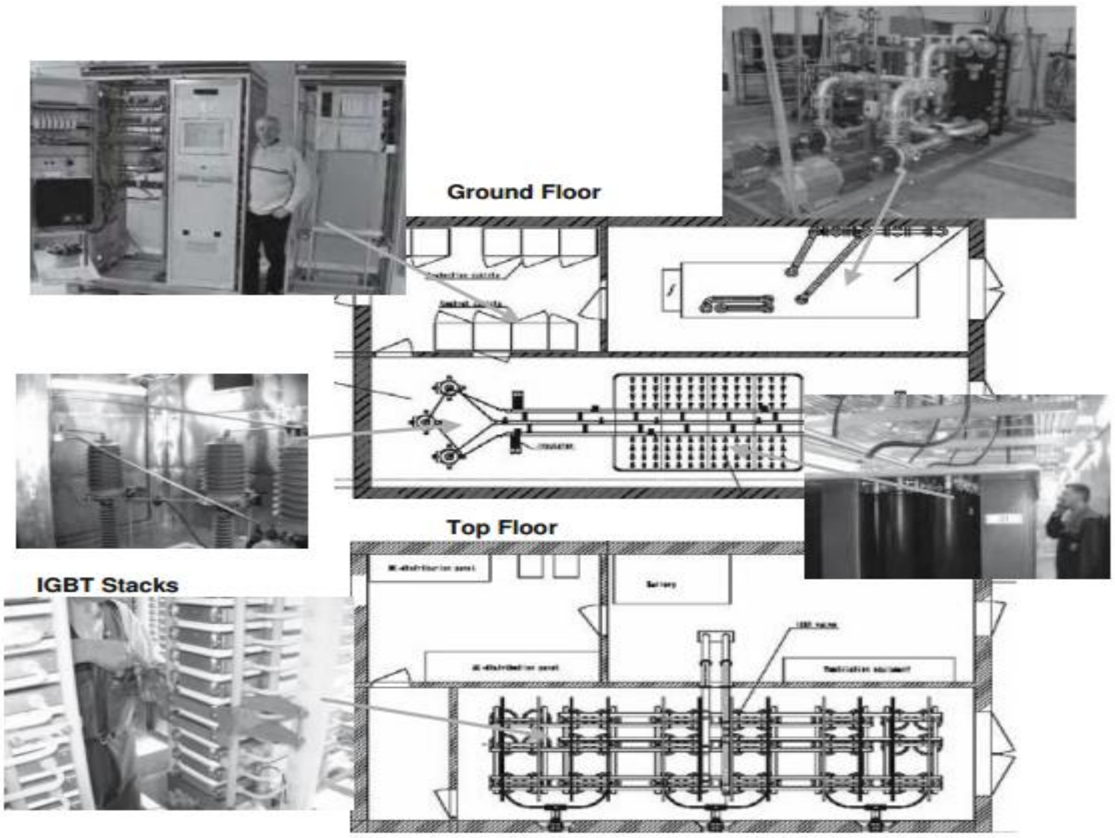

Figure 2.13 Typical layout of a STATCOM-building (Source: ABB) [19]

2.4.2.2 Series Devices

Series devices have been further developed from fixed or mechanically switched compensations to the Thyristor Controlled Series Compensation (TCSC) or even Voltage Source Converter based devices. The main applications are:

- ✓ Reduction of series voltage decline in magnitude and angle over a power line,
- ✓ Reduction of voltage fluctuations within defined limits during changing power transmissions,
- ✓ Improvement of system damping resp. damping of oscillations,
- ✓ Limitation of short circuit currents in networks or substations,
- ✓ Avoidance of loop flows respect of power flow adjustments.

The world's first Series Compensation on transmission level, counted nowadays by the manufacturers as a FACTS-device, went into operation in 1950. Series Compensation is used in order to decrease the transfer reactance of a power line at rated frequency. A series capacitor installation generates reactive power that in a self-regulating manner balances a fraction of the line's transfer reactance. The result is that the line is electrically shortened, which improves angular stability, voltage stability and power sharing between parallel lines.

Series Capacitors are installed in series with a transmission line, which means that all the equipment has to be installed on a fully insulated platform. On this steel platform the main capacitor is located together with the overvoltage protection circuits. The overvoltage protection is a key design factor, as the capacitor bank has to withstand the throughput fault current, even at a severe nearby fault. The primary overvoltage protection typically involves non-linear visitors of metal-oxide type, a spark gap and a fast bypass switch. Secondary protection is achieved with ground mounted electronics acting on signals from optical current transducers in the high voltage circuit.

Even if the device is known since several years, improvements are ongoing. One recent achievement is the usage of dry capacitors with a higher energy density and higher environmental friendliness. As a primary protection Thyristor switches can be used, but cheaper alternatives with almost the same capability based on triggered spark gaps and special breakers without power electronics have recently been developed [19].

Figure 2.14 Series Compensation (Series Capacitor) (Source: ABB) [19]

A special application of Series Compensation can be achieved by combining it with a series reactance to get a fault current limiter. Both components are neutralizing each other in normal operation. In the case of a fault, die Series Compensation is bridged with a fast protection device or a Thyristor bridge. The remaining reactance is limiting the fault current. Pilot installations of such a system configuration are already in use.

2.4.2.2.1 TCSC

Thyristor Controlled Series Capacitors (TCSC) addresses specific dynamical problems in transmission systems. Firstly it increases damping when large electrical systems are interconnected. Secondly it can overcome the problem of Sub Synchronous Resonance (SSR), a phenomenon that involves an interaction between large thermal generating units and series compensated transmission systems. The TCSC's high speed switching capability provides a mechanism for controlling line power flow, which permits increased loading of existing transmission lines, and allows for rapid readjustment of line power flow in response to various contingencies. The TCSC also can regulate steady-state power flow within its rating limits.

From a principal technology point of view, the TCSC resembles the conventional series capacitor. All the power equipment is located on an isolated steel platform, including the Thyristor valve that is used to control the behavior of the main capacitor bank. Likewise the control and protection is located on ground potential together with other auxiliary systems.

Figure 2.15 shows the principle setup of a TCSC and its operational diagram. The firing angle and the thermal limits of the Thyristors determine the boundaries of the operational diagram.

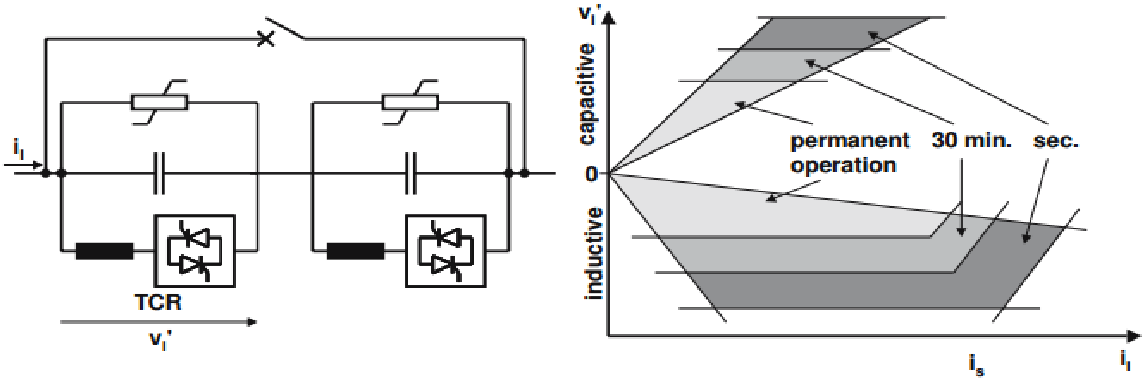

Figure 2.15 Principle setup and operational diagram of a Thyristor Controlled Series Compensation (TCSC) [19]

The main principles of the TCSC concept are two; firstly, to provide electromechanical damping between large electrical systems by changing the reactance of a specific interconnecting power line, i.e. the TCSC will provide a variable capacitive reactance. Secondly, the TCSC shall change its apparent impedance (as seen by the line current) for sub-synchronous frequencies, such that a prospective sub synchronous resonance is avoided. Both objectives are achieved with the TCSC, using control algorithms that work concurrently. The controls will function on the Thyristor circuit in parallel to the main capacitor bank such that controlled charges are added to the main capacitor, making it a variable capacitor at fundamental frequency but a "virtual inductor" at sub-synchronous frequencies. Figure 2.16 shows a TCSC on transmission level. The first TCSC was commissioned in 1996.

Figure 2.16 TCSC (Source: ABB) [19]

2.4.2.2.2 SSSC

While the TCSC can be modeled as series impedance, the SSSC is a series voltage source. The principle configuration is shown in Figure 2.17, which looks basically the same as the STATCOM. But in reality this device is more complicated because of the platform mounting and the protection. A Thyristor protection is absolutely necessary, because of the low overload capacity of the semiconductors, especially when IGBTs are used.

The voltage source converter plus the Thyristor protection makes the device much more costly, while the better performance cannot be used on transmission level. The picture is quite different if we look into power quality applications. This device is then called Dynamic Voltage Restorer (DVR). The DVR is used to keep the voltage level constant, for example in a factory infeed. Voltage dips and flicker can be mitigated. The duration of the action is limited by the energy stored in the DC capacitor. With a charging mechanism or battery on the DC side, the device could work as an uninterruptible power supply. A picture of a modularized installation with 22 MVA is shown on the right in Figure 2.17.

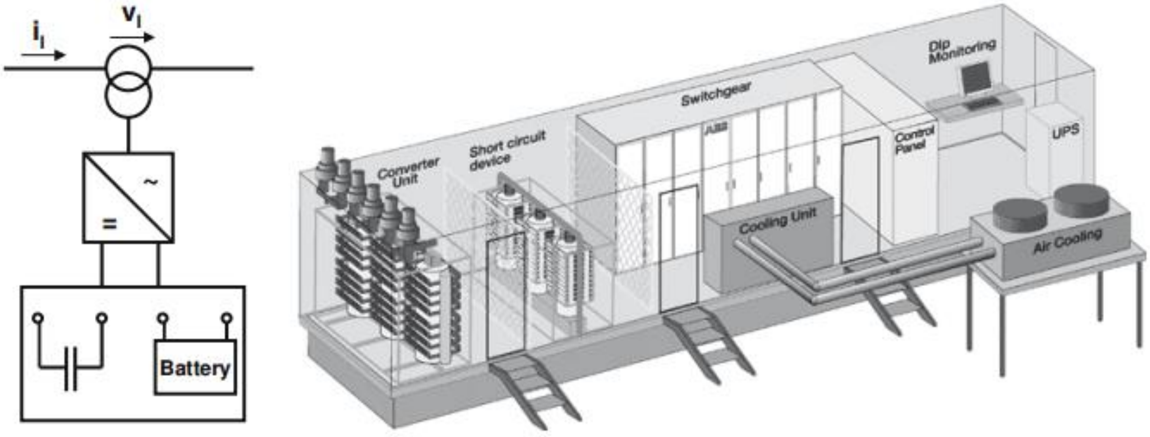

Figure 2.17 Principle setup of SSSC and implementation as DVR for power quality applications (Source: ABB) [19]

2.4.2.3 Shunt and Series Devices

Power flow capability is getting more and more importance with the growing restrictions for new power lines and the more volatile power flow due to the energy market activities.

2.4.2.3.1 Dynamic Flow Controller

A new device in the area of power flow control is the Dynamic Power Flow Controller (DFC). The DFC is a hybrid device between a Phase Shifting Transformer (PST) and switched series compensation.

A functional single line diagram of the Dynamic Flow Controller is shown in Figure 2.18. The Dynamic Flow Controller consists of the following components:

- ✓ a standard phase shifting transformer with tap-changer (PST)
- ✓ series-connected Thyristor Switched Capacitors and Reactors (TSC / TSR)
- ✓ A mechanically switched shunt capacitor (MSC). (This is optional depending on the system reactive power requirements)

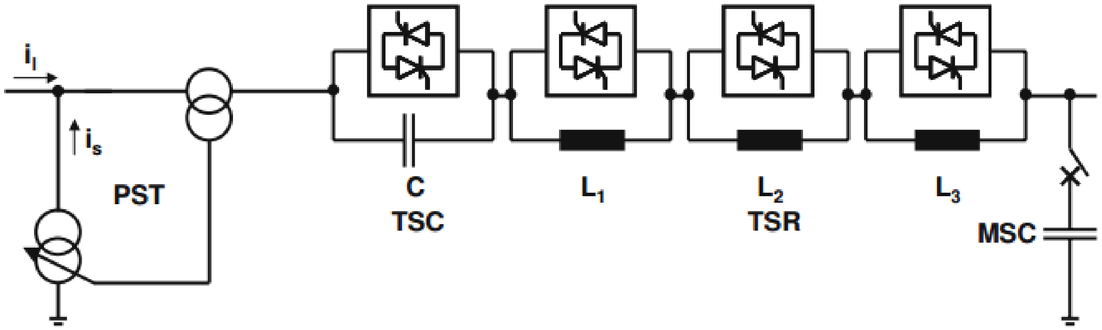

Figure 2.18 Principle configuration of DFC [19]

Based on the system requirements, a DFC might consist of a number of series TSC or TSR. The mechanically switched shunt capacitor (MSC) will provide voltage support in case of overload and other conditions. Normally the reactance of reactors and the capacitors are selected based on a binary basis to result in a desired stepped reactance variation. If a higher power flow resolution is needed, a reactance equivalent to the half of the smallest one can be added.

The switching of series reactors occurs at zero current to avoid any harmonics. However, in general, the principle of phase-angle control used in TCSC can be applied for a continuous control as well. The operation of a DFC is based on the following rules:

TSC / TSR are switched when a fast response is required.

- ✓ The relieve of overload and work in stressed situations is handled by the TSC /TSR.

- ✓ The switching of the PST tap-changer should be minimized particularly for the currents higher than normal loading.
- ✓ The total reactive power consumption of the device can be optimized by the operation of the MSC, tap changer and the switched capacities and reactors.

In order to visualize the steady state operating range of the DFC, we assume an inductance in parallel representing parallel transmission paths. The overall control objective in steady state would be to control the distribution of power flow between the branch with the DFC and the parallel path. This control is accomplished by control of the injected series voltage.

The PST (assuming a quadrature booster) will inject a voltage in quadrature with the node voltage. The controllable reactance will inject a voltage in quadrature with the throughput current. Assuming that the power flow has a load factor close to one, the two parts of the series voltage will be close to collinear. However, in terms of speed of control, influence on reactive power balance and effectiveness at high/low loading the two parts of the series voltage has quite different characteristics. The steady state control range for loadings up to rated current is illustrated in Figure 2.19, where the x-axis corresponds to the throughput current and the y-axis corresponds to the injected series voltage.

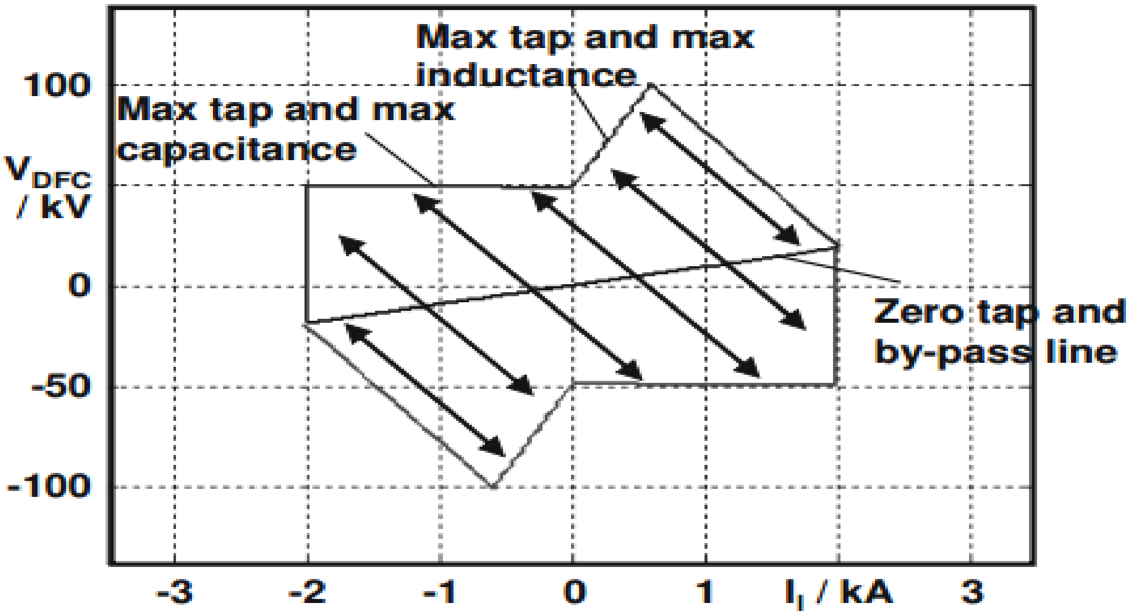

Figure 2.19 Operational diagram of a DFC [19]

Operation in the first and third quadrants corresponds to reduction of power through the DFC, whereas operation in the second and fourth quadrants corresponds to increasing the power flow through the DFC. The slope of the line passing through the origin (at which the tap is at zero and TSC / TSR are bypassed) depends on the short circuit reactance of the PST.

Starting at rated current (2 kA) the short circuit reactance by itself provides an injected voltage (approximately 20 kV in this case). If more inductance is switched in and/or the tap is increased, the series voltage increases and the current through the DFC decreases (and the flow on parallel branches increases). The operating point moves along lines parallel to the arrows in the figure. The slope of these arrows depends on the size of the parallel reactance. The maximum series voltage in the first quadrant is obtained when all inductive steps are switched in and the tap is at its maximum.

Now, assuming maximum tap and inductance, if the throughput current decreases (due e.g. to changing loading of the system) the series voltage will decrease. At zero current, it will not matter whether the TSC / TSR steps are in or out, they will not contribute to the series voltage. Consequently, the series voltage at zero current corresponds to rated PST series voltage. Next, moving into the second quadrant, the operating range will be limited by the line corresponding to maximum tap and the capacitive step being switched in (and the inductive steps by-passed). In this case, the capacitive step is approximately as large as the short circuit reactance of the PST, giving an almost constant maximum voltage in the second quadrant.

2.4.2.3.2 Unified Power Flow Controller

The UPFC is a combination of a static compensator and static series compensation. It acts as a shunt compensating and a phase shifting device simultaneously.

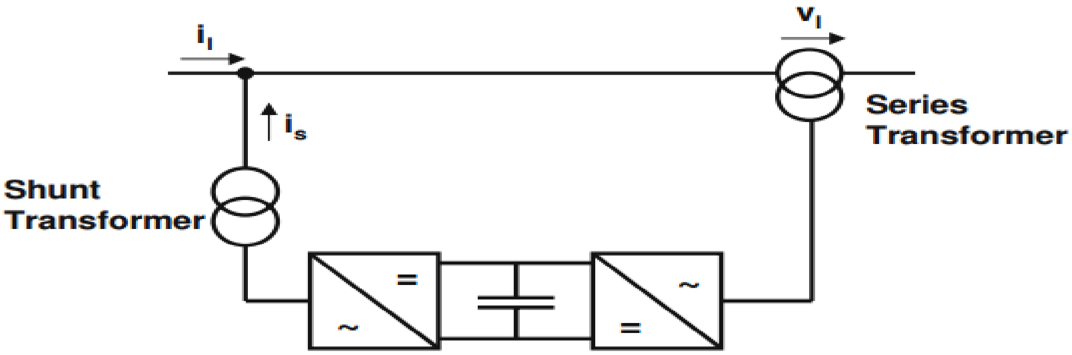

Figure 2.20 Principle configuration of an UPFC [19]

The UPFC consists of a shunt and a series transformer, which are connected via two voltage source converters with a common DC-capacitor. The DC-circuit allows the active power exchange between shunt and series transformer to control the phase shift of the series voltage. This setup, as shown in Figure 2.20, provides the full controllability for voltage and power flow. The series converter needs to be protected with a Thyristor bridge. Due to the high efforts for the Voltage Source Converters and the protection, an UPFC is getting quite expensive, which limits the practical applications where the voltage and power flow control is required simultaneously.

2.4.2.4 Back-to-Back Devices

The Back-to-Back devices provide in general a full power flow controllability and power flow limitation. An overload of these devices is therefore impossible. They can resist cascading outages, which might occur due to line outages when one line after the other is overloaded. This gives a great benefit even if the frequency decoupling characteristic is not needed.

Conventional HVDC Back-to-Back systems with Thyristor converters need space consuming filters to reduce the harmonic distortion. The reactive power is not controllable. These devices are mainly used when two asynchronous networks need to be coupled or in the usual application as power transmission line over long distances.

The HVDC with Voltage Source Converters instead provides benefits as well within synchronous operated networks. It has a much smaller footprint and provides the full voltage controllability to the network on both ends. Therefore it can be operated in addition to the power flow control as two STATCOMS. On both ends a full four quadrant circular operational diagram is provided. This reactive power provision can be used to increase the transmission capability of surrounding transmission lines in addition to balancing the power flow.

Figure 2.21 shows the principle configuration of a HVDC Back-to-Back with voltage source converters. A practical implementation is shown in Figure 2.22, which is based on the design of two STATCOM converters with IGBTs [19].

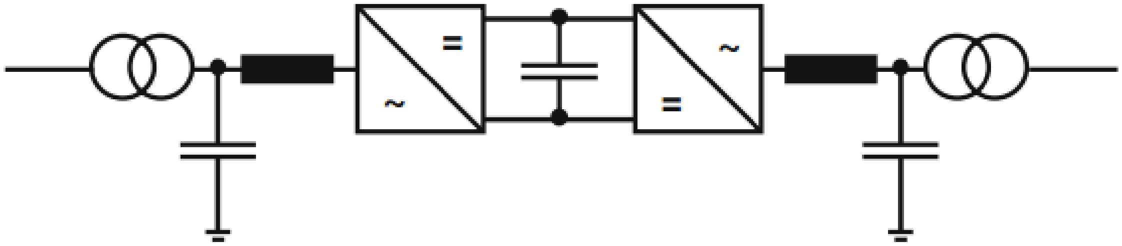

Figure 2.21 Schematic configuration of a HVDC Back-to-Back with Voltage Source Converters [19]

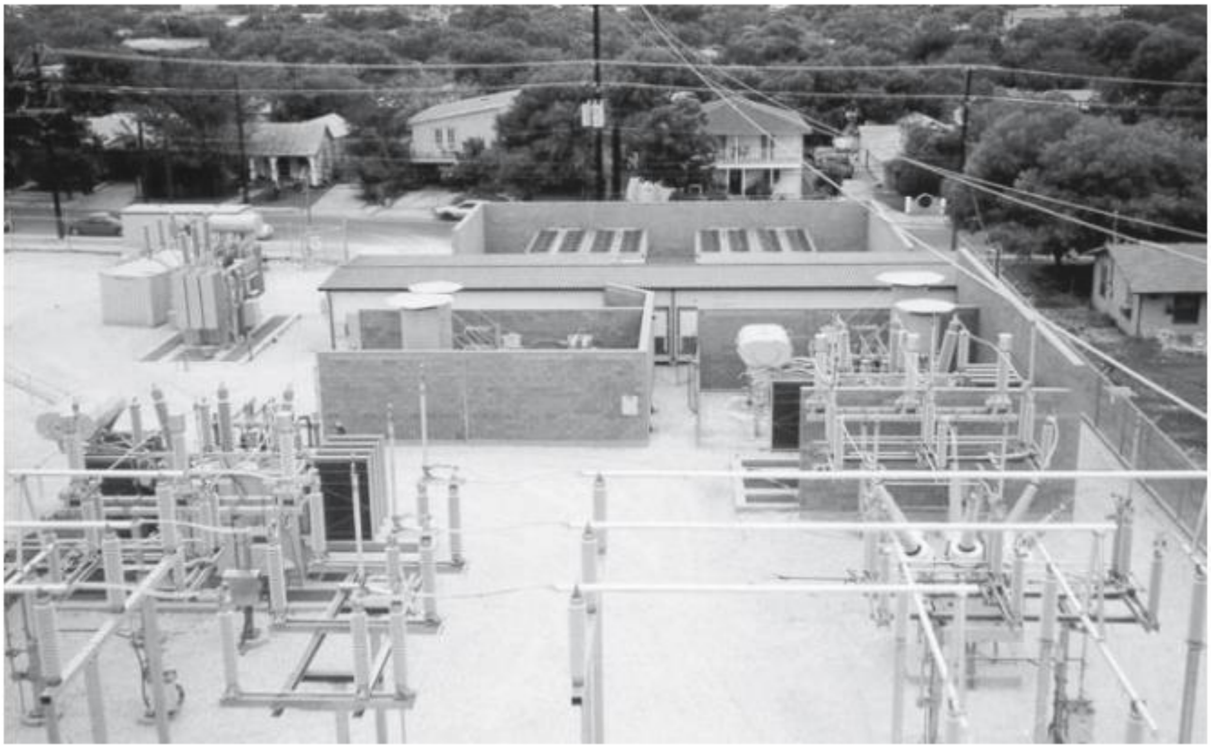

Figure 2.22 HVDC Back-to-Back with Voltage Source Converters, 2x36 MVA (Source: ABB) [19]

2.5 Compensation Technique

2.5.1 Compensation Technique(STATCOM)

Reactive power compensation is an important issue in the control of electric power systems. Reactive power increases the transmission system losses and reduces the power transmission capability of the transmission lines. Moreover, reactive power flow through the transmission lines can cause large amplitude variations in the receiving-end voltage.

A **STATCOM** is used for voltage regulation in a power system. Under lightly loaded conditions, the STATCOM is used to minimize or completely diminish the line overvoltage. On the other hand, it can also be used to maintain certain voltage levels under heavy loading conditions [20]. A STATCOM or Static Synchronous Compensator is a regulating device used on alternating current electricity transmission networks. It is based on a power electronics voltage-source converter and can act as either a source or sink of reactive AC power to an electricity network. If connected to a source of power it can also provide active AC power. It is a member of the FACTS family of devices [21].

STATCOM or Static Synchronous Compensator is a shunt device, which uses force-commutated power electronics (i.e. GTO, IGBT) to control power flow and improve transient stability on electrical power networks. The STATCOM basically performs the same function as the static var compensators but with some advantages [22]. Its advantages of fast speed, great loading rate adaptation, high work efficiency, and small output harmonic content. Especially, adopting two-phase structure can achieve four-phase control of active and reactive power, provide two supply arms of power substation with dynamic reactive compensation, besides, regulate active flow of two supply arms, so as to dynamically balance the loading [23]. The term Static Synchronous Compensator is derived from its capabilities and operating principle, which are similar to those of rotating synchronous compensators (i.e. generators), but with relatively faster operation.

2.5.1.1 Applications

STATCOMs are typically applied in long distance transmission systems, power substations and heavy industries where voltage stability is the primary concern.

In addition, static synchronous compensators are installed in select points in the power system to perform the following:

- Voltage support and control

- Voltage fluctuation and flicker mitigation
- Unsymmetrical load balancing
- Power factor correction
- Active harmonics cancellation
- Improve transient stability of the power system

2.5.1.2 Modeling of STATCOM

A STATCOM is composed of the following components:

A. Voltage-Source Converter (VSC)

The voltage-source converter transforms the DC input voltage to an AC output voltage. Two of the most common VSC types are described below.

1. Square-Wave Inverters using Gate Turn-Off Thyristors

Generally, four three-level inverters are utilized to make a 48-step voltage waveform. Subsequently, it controls reactive power flow by changing the DC capacitor input voltage, simply because the fundamental component of the converter output voltage is proportional to the DC voltage.

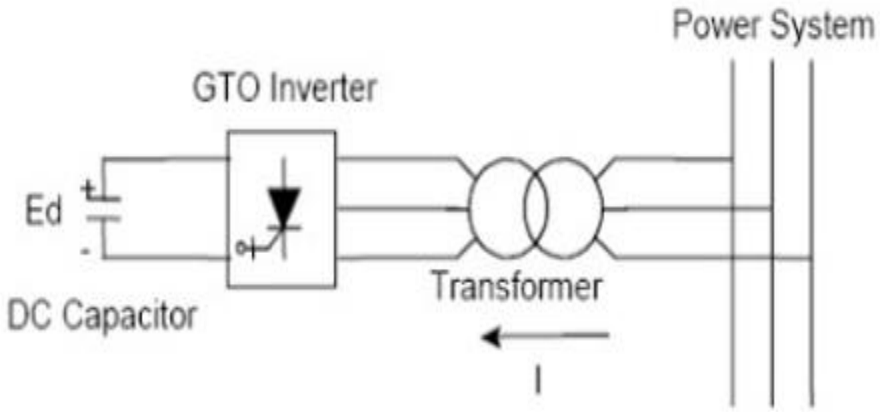

Figure 2.23 GTO-based STATCOM Simple Diagram [22].

In addition, special interconnection transformers are employed to neutralize harmonics contained in the square waves produced by individual inverters.

2. PWM Inverters Using Insulated Gate Bipolar Transistors (IGBT)

It uses Pulse-Width Modulation (PWM) technique to create a sinusoidal waveform from a DC voltage source with a typical chopping frequency of a few kHz. In contrast to the GTO-based type, the IGBT-based VSC utilizes a fixed DC voltage and varies its output AC voltage by changing the modulation index of the PWM modulator.

Moreover, harmonic voltages are mitigated by installing shunt filters at the AC side of the VSC [22].

A STATCOM is a controlled reactive-power source. It provides the desired reactive-power generation and absorption entirely by means of electronic processing of the voltage and current waveforms in a VSC. A STATCOM principle diagram is shown in Figure 2.24.

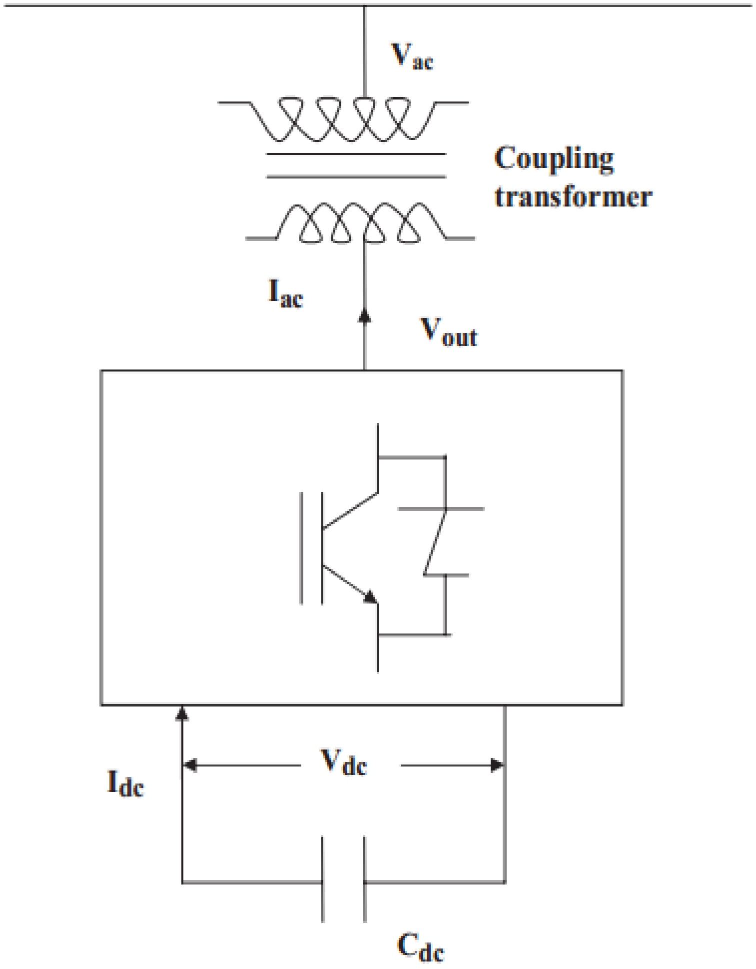

Figure 2.24 Functional Model of STATCOM [20].

The VSC is connected to a utility bus through shunt transformer. V_{ac} is the bus voltage. I_{ac} is STATCOM injected current. V_{out} is the VSC output voltage. V_{dc} and I_{dc} are the DC capacitor side voltage and current. An IGBT with back to back diode denotes the 3 arm IGBT bridge. Top three IGBTs are called as positive group and bottom three IGBTs are called as negative group IGBTs. The inverter operation takes place, when IGBTs conduct and converter operation takes place, when diodes conduct. Figure 2.25 shows the concept of STATCOM power exchange.

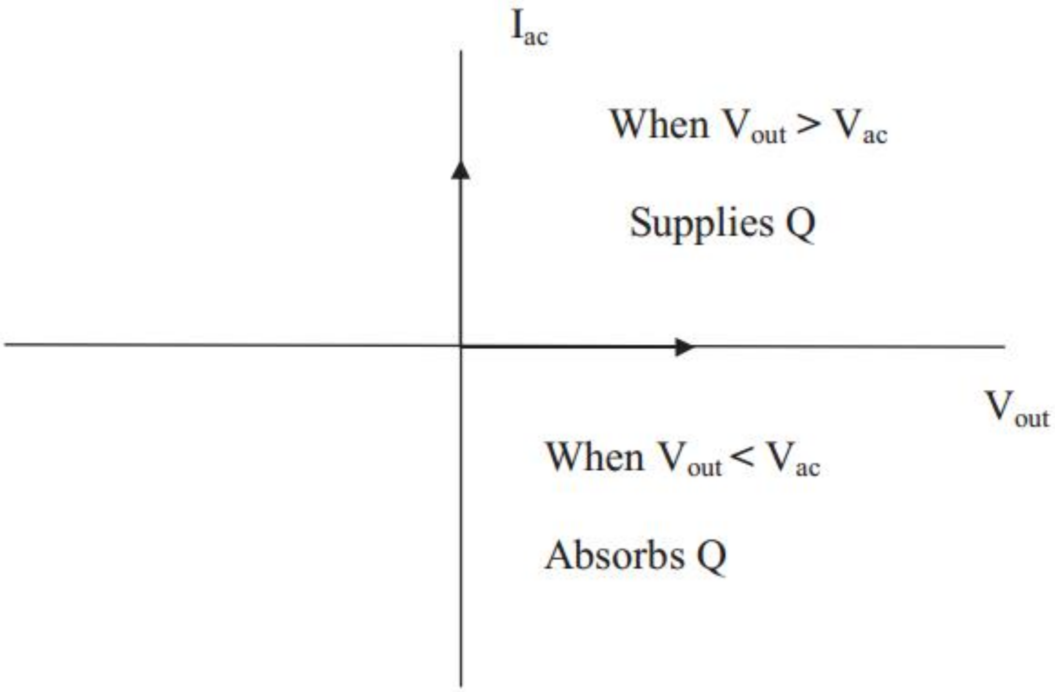

Figure 2.25 STATCOM power exchanges [20].

STATCOM is seen as an adjustable voltage source behind a reactance. It means that the capacitor banks and shunt reactors are not needed for reactive-power generation and absorption; thereby it gives the STATCOM, a compact design. The equivalent circuit of the block diagram of VSC based STATCOM is shown in Figure 2.26.

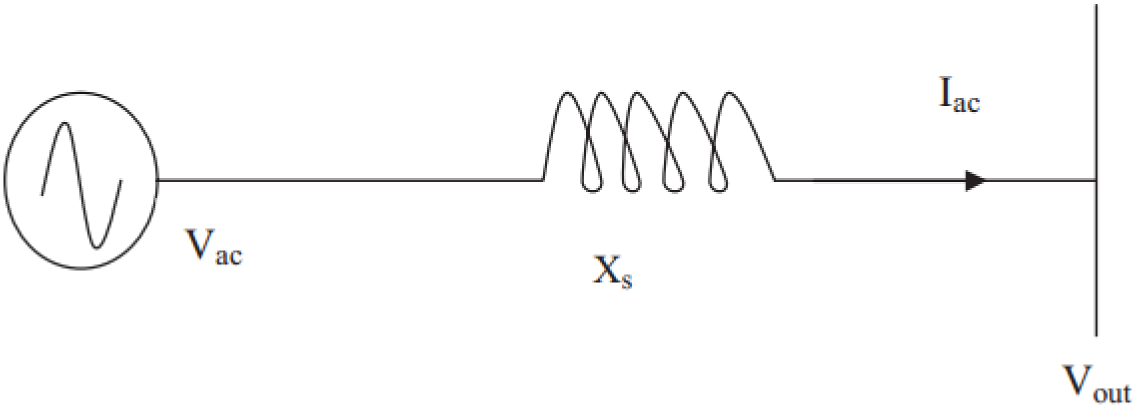

Figure 2.26 Equivalent Circuit of the STATCOM [20].

The exchange of reactive power between the converter and the AC system can be controlled by varying the amplitude of the 3-phase output voltage V_{out} of the converter as illustrated in Figure 2.26, if the amplitude of the V_{out} is increased above that of the utility bus voltage V_{ac}, the current flows through the reactance from the converter to the AC system and the converter generates capacitive-reactive power for the AC system. If the amplitude of V_{out} is decreased below the utility bus voltage, the current flows from the AC system to the converter and the converter absorbs inductive-reactive power from the AC system. The reactive-power exchange becomes zero, if the V_{out} equals the ac system voltage, and in this case the STATCOM is said to be in a floating state.

In the VSC at the DC side, a relatively small DC capacitor is connected. Hence, the STATCOM is capable of only reactive power exchange with the transmission system. If the DC capacitor is replaced by some other DC energy source, the controller can exchange real and reactive power with the transmission system by extending its region of operation from two to four quadrants.

The coupling transformer plays two different roles. First, it connects the converter to the high voltage power system. Secondly, the transformer inductance ensures that DC capacitor is not short-circuited and discharged rapidly.

A STATCOM is used for voltage regulation in a power system. Under lightly loaded conditions, the STATCOM is used to minimize or completely diminish the line overvoltage. On the other hand, it can also be used to maintain certain voltage levels under heavy loading conditions.

The real power flowing into the converter supplies the converter losses due to switching and charges the DC capacitor to a satisfactory DC voltage level. The capacitor is charged and discharged during the course of each switching cycle. But in steady state, the average capacitor voltage remains constant. In steady state, all the power from the AC system is used to provide the losses due to switching. The STATCOM's ability to absorb/supply real power depends on the size of DC capacitor and the real power losses due to switching. Since the DC capacitor and the losses are relatively small, the amount of real power transfer is also relatively small. This implies that the STATCOM's output AC current has to be approximately + 90° with respect to AC system voltage at its line terminals. Depending on the power rating of the STATCOM, different technologies are used for the power converter. High power STATCOMs (several hundreds of Mvars) normally use GTO-based, square-wave voltage-sourced converters

(VSC), while lower power STATCOMs (tens of Mvars) use IGBT-based (or IGCT-based) pulse-width modulation (PWM) VSC [20].

B. DC Capacitor

This component provides the DC voltage for the inverter.

C. Inductive Reactance (X)

It connects the inverter output to the power system. This is usually the leakage inductance of a coupling transformer.

D. Harmonic Filters

Mitigate harmonics and other high frequency components due to the inverters.

2.5.1.3 STATCOM Operation

Basic Principle of Operation:

In the case of two AC sources, which have the same frequency and are connected through a series reactance, the power flows will be:

- Active or Real Power flows from the leading source to the lagging source.
- Reactive Power flows from the higher to the lower voltage magnitude source.

Consequently, the phase angle difference between the sources decides the active power flow, while the voltage magnitude difference between the sources determines the reactive power flow. Based on this principle, a STATCOM can be used to regulate the reactive power flow by changing the output voltage of the voltage-source converter with respect to the system voltage.

Modes of Operation:

The STATCOM can be operated in two different modes:

A. Voltage Regulation

The static synchronous compensator regulates voltage at its connection point by controlling the amount of reactive power that is absorbed from or injected into the power system through a voltage-source converter.

In steady-state operation, the voltage V_{out} generated by the VSC through the DC capacitor is in phase with the system voltage V_{ac} (δ=0), so that only reactive power (Q) is flowing (P=0).

1. When system voltage is high, the STATCOM will absorb reactive power (inductive behavior)

2. When system voltage is low, the STATCOM will generate and inject reactive power into the system (capacitive).

Subsequently, the amount of reactive power flow is given by the equation:

$$Q = \frac{V_{ac}(V_{ac} - V_{out})}{X}$$

B. Var Control

In this mode, the STATCOM reactive power output is kept constant independent of other system parameter.

2.5.1.4 STATCOM Versus SVC

The STATCOM has the ability to provide more capacitive reactive power during faults, or when the system voltage drops abnormally, compared to ordinary static var compensator. This is because the maximum capacitive reactive power generated by a STATCOM decreases linearly with system voltage, while that of the SVC is proportional to the square of the voltage. Also, the STATCOM has a faster response as it has no time delay associated with Thyristor firing. Nevertheless, these advantages come at a higher price (about 20% more) [22].

2.6 Optimum Placement of STATCOM

STATCOM (Static Synchronous Compensator) placement at optimal location will lead to quick recovery of voltage at all buses of interest and improve the uptime of DER units. The SI (sensitivity index) is calculated to know that whether system is inductive if yes then the STATCOM's are placed in those buses which are inductive in nature; this is known by the presence of negative values in SI. The calculations will be performed on the IEEE 16-bus system.

The optimal location of STATCOM is found by sensitivity index. STATCOM ensures fast voltage recovery at all buses of interest. The exposure of DER units to the problem of slow voltage recovery for contingencies like faults is minimized. Simulation results prove that the presence of STATCOM at a bus with highest negative value ensures a fast voltage recovery at all buses of interest [24].

Chapter Three

3. Train Operation Parameters Calculation, Mathematical Modeling for Traction Substation, and Feeder Network Voltage Drops

3.1 Train Operation Parameter Calculations

The train operation parameters include train position, speed, acceleration, operation conditions, current, and energy consumption. The train operating parameters is the core of the traction calculation which includes traction force calculation, basic resistance and the additional resistance, braking force calculation and the calculation of train speed and time parameters. The main outputs are:

- Running time and traction time, excluding coasting (self-service electricity)
- Section up direction and down direction energy consumption
- Speed - distance curve and train current curve, etc.

Energy consumption in the section of Indode substation to Lebu section post at up track using equation (2.22)

$$\therefore E_{spc} = [\frac{0.01V_{max}{}^{2}}{\eta * \mathrm{D}} * \frac{W_e}{\mathrm{W}} \pm 2.72 * \mathrm{G}\frac{D_1}{\eta * \mathrm{D}} + 0.277\mathrm{r}\frac{D_1}{\eta * \mathrm{D}}]\mathrm{Whrs/ton - km}$$

$$\mathrm{E_{spc}} = [0.01 * \frac{\frac{120\mathrm{km}^2}{\mathrm{hr}}}{.378 * 18.808} * 1.1 + 2.72 * 0.018\frac{16.208}{0.378 * 18.808} + 0.27 * 5.7\frac{16.208}{0.378 * 18.808}]$$

$$= 27.598\mathrm{Whr}$$

Using equation (2.24)

$$E_{ov-con} = [E_{spc} * \mathrm{W} * \mathrm{D}]\mathrm{Whr}$$

= 27.598Whr /ton-km * 1840.955ton *18.808 km = 955,572.544Whr

= 955.57kWhr

Energy consumption in the section of Lebu section post to Indode substation at down track using Equation (2.23)

$$E_{spc} = \left[\frac{0.01{V_{max}}^2}{\eta * D} * \frac{W_e}{W} + 0.277r\frac{D_1}{\eta * D}\right] \text{Whrs/ton} - \text{km}$$

$$= [\frac{0.01 * 120\,km/hr^2}{0.378 * 18.808} * 1.1 + 0.277 * 5.748\frac{16.208}{0.378 * 18.808}] \text{Whrs/ton} - \text{km}$$

= 27.525Whr/ton-km

Using equation (2.24)

$$E_{ov-con} = \left[E_{spc} * \mathrm{W} * \mathrm{D}\right]\text{Whr}$$

= 27.525Whr/ton-km * 1840.955ton * 18.808km

= 953,053.402Whr

Train energy consumption will be found by integrating the train current over time and then multiply by the traction network voltage.

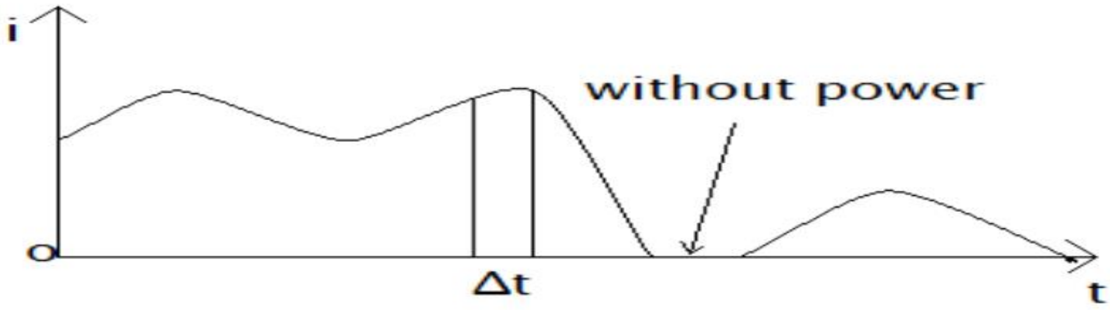

Figure 3.1Train Energy Consumption [5]

$$E_{ov-con} = \mathrm{V}\int_0^{\mathrm{T}} \mathrm{i(t)} * \mathrm{dt}\ \ \mathrm{kVA.h}$$

[0, T] divided into n equal parts Δt (minutes), corresponding current i_o, i_1, i_2, I_n. The section running time T= nΔt. Total energy consumption at train running time

$$E_{ov-con} = \mathrm{V}\int_0^{\mathrm{T}} \mathrm{i(t)} * \mathrm{dt}\ \ \mathrm{kVA.h} = V * \frac{n\Delta t}{60} * \frac{1}{n+1}\sum_{k=o}^{n} i_k$$

$$= \mathrm{V}\frac{\mathrm{n\Delta t}}{60}\mathrm{I}\ \mathrm{kVA}.h\ \ \text{Excluding coasting or braking self-electricity energy consumption}$$

$$= \frac{\mathrm{V*I*t}}{60}$$

Where V is 25kV, is the average of traction network voltage and Δt is in minutes.

3.1.1 Train Average Current

$$I_{avg} = \frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} \mathrm{i} * \mathrm{dt} = \frac{1}{\mathrm{n+1}}\sum_0^{\mathrm{n}} i_k(\mathrm{A})$$

$$I_{avg} = \frac{60 * E_{ov-con}}{\mathrm{V*t}} + 7\mathrm{A(selfelectricity)}$$

By using equation (2.27), $\mathrm{t} = [\frac{\mathrm{D}}{V_{max}} + \mathrm{K}V_{max}]$

$$= \left[\frac{18.808\mathrm{km}}{120\,\mathrm{km/hr}} + \frac{1}{2}\left(\frac{1.5 + 0.8}{1.5 * 0.8}\right) * 120\,\mathrm{km/hr}\right] = (9.404 + 1.916)\mathrm{minutes}$$

$\mathrm{t} = 11.32$ Minute’s section running time

$$I_{avg} = \frac{60 * 1908{,}625.947\mathrm{VAmin}}{25000\mathrm{V} * 2 * 11.3206\mathrm{min}} + 7\mathrm{A}$$

= 209.317A train current per/locomotive

3.1.2 Feeder Current

Traction-load characteristics: The traction load is fluctuating as train change speed along the railway. The magnitude of traction load is variable with time. The higher the transportation density is, the larger the traction loads are, and vice versa. Traction load characteristics make the power supply computation complex.

The calculation of the feeder current is described by the three main values:

- ✓ **Feeder average current**

The average of the feeder current day and night, and is used for estimation on the transformer capacity utilization, determination of the phase split and feeding section of catenary, and negative sequence calculation in primary system caused by traction power supply system [3].

- ✓ **Feeder effective current**

The RMS value of the feeder current day and night, and is used to calculate temperature rise of the electrical equipment, transformer capacity calculation and heat calculation of catenary [3].

- ✓ **Feeder maximum current**

The maximum instantaneous operating current is used for tuning relay protection, and for the choice of electric equipment, such as the choice of transformer capacity [3].

Assumption: train average current in segment equal and traction running is the same

I_1 =……………..I_n= I_{avg} , t_1 =……………t_n= t/n

$$I_{avg} = \frac{60 * E_{ov-con}}{\mathrm{V*t}} + 7\mathrm{A}(\mathrm{selfelectricity})$$

Feeder daily average current of a double track of unilateral power supply is from appendix F [5]

$$I_F = I_{avg}\mathrm{np} = \frac{\mathrm{Nt}}{\mathrm{T}} I_{avg}$$

$$\text{Where P} = \frac{\mathrm{N}t_{g1}}{\mathrm{n*T}}$$

Feeder daily effective current of a double track of unilateral power supply is, appendix F [5]

$$I_{FE} = I_F\sqrt{1 + \frac{{K_{eg}}^2 - P}{n*p}}$$

Where n - Number of a train which appeared at feeding sections up and down direction

N-Train density or number of daily operating train in pairs

t = t_{g1} - Train's running time through feeding section

E_{ov-con} - Train energy consumption in feeding section

P - Traction probability for section

I_{avg} - Average current

I_F - Feeder current

I_{FE} - Feeder effective current

K_{eg} - Traction effective coefficient, and usually K_{eg} = 1.04, ${K_{eg}}^2$ = 1.08 from appendix F [5]

Table 3.1 Sebeta to Adama Train Traffic Condition per Day at Double Track [3] [25]

Year	Interval	Passenger train	Through and sectional trains	Detaching & Attaching train	Subtotal
Short-term	Sebeta (Addis)-Bishoftu	9	8	2	19
Long term	Sebeta(Addis)-Bishoftu	16	17	2	35

Feeder daily average current at double track when only up track or down track is considered for over zone feeding, n = 2 by using the following equation it is possible to calculate the probability of train in segment.

$P = \frac{2Nt_g}{2n*T}$ Where T = $2Nt_g$

$= \frac{19*78.789\text{min}}{2*1440\text{min}} = 0.5$

$I_F = \text{np}I_{avg} = 2 * 0.5 * 231.132\text{A} = 231.16\text{A}$

Considering over zone feeding, double track feeder daily effective current of unilateral (when only up track or down track of double track is considered) power supply will be:

$$I_{FE} = I_F\sqrt{1 + \frac{{K_{eg}}^2 - P}{n*p}}$$

$$= \sqrt{1 + \frac{1.08 - 0.500625}{4*0.500625}} = 231.16 * 1.135 = 262.48\text{A}$$

In the average Section running time + station stopping time =1hr and 42 minutes for one train.

Table 3.2 Analyzed Data of Train Energy Consumption

Station section	E_{ov-con} in double track (in Whr)	Train average current I_{avg} (in A)	Section Distance (in km)	Feeder current I_F (in A)	Feeder effective current I_{FE} (in A)	Section runtime (in minute)
Sebeta-Lebu	1826,473.938	259.572	13.521	259.572	269.755	8.677
Lebu-Indode	1908,625.947	209.317	18.808	209.313	217.524A	11.32
Sebeta-Indode	3735,099.885	231.132	32.329	231.16	262.48A	19.99
Indode-D1k65	1,028,470.753	162.078	28	162.078	184.034A	15.916

3.2 Analysis for System Impedance from its Short Circuit Characteristics

Power system voltage loss of traction substation is calculated according to the Sebeta-Adama line traction network voltage 132kV and minimum short circuit capacity of 400MVA [25].

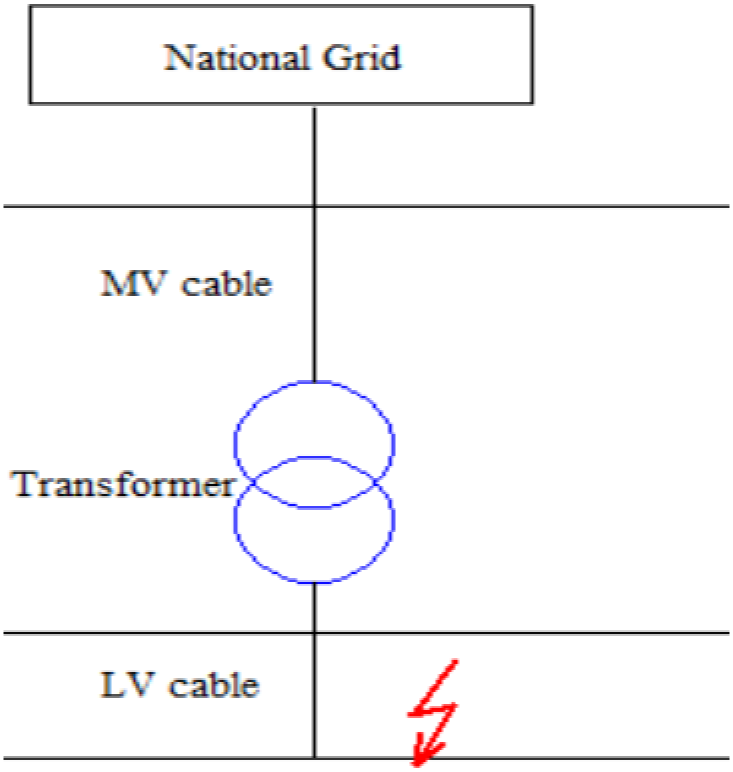

Figure 3.2 Short Circuit Power and Current of the Supply Network [3] [4]

Table 3.3 Calculation in Electrical Parameter of Traction Network [25]

Primary voltage V_{2n}	132 kV
Secondary voltage of the transformer	27.5kV
Minimum short circuit volt-ampere at low voltage side	400MVA
Rated MVA of transformer in Lebu section post to Indode substation	2(20 + 20)MVA
Transformer connection	V-V
Rated percentage reactance U_{kr}	12%
Power factor (cosΘ)	0.95

Short circuit MVA = $\frac{3*V_{phase}}{1000} * I_{phase}$

$$\text{Vphase} = \frac{132000}{\sqrt{3}} \text{ volt}$$

$$\text{Ish} = \frac{400*10^6\text{VA}}{\sqrt{3}*132*10^3} = 1749.54\text{A}$$

As per IEC 60909 which is given in Appendix: D, the equivalent source voltage in RMS is multiplied by correction factor $\mathrm{C_{max}} = 1.1$.

$$Z_{grid} = \frac{C_{max} * 132\text{kV}}{\sqrt{3} * I_{sh}} = \frac{1.1 * 132\text{kV}}{\sqrt{3} * 1749.54} = 47.916\Omega$$

System impedance referred secondary side of the transformer is given by

$$Z_{grid} = \frac{Z_{grid}}{\text{K}^2}$$

$$Z_{grid} = \frac{47.961}{4.8^2} = 2.079\Omega \text{ Where K} = \frac{V_{1n}}{V_{2n}} = \frac{132\text{kV}}{27.5\text{kV}} = 4.8$$

As per IEC 60909 standard (Appendix: A), the resistance and reactance value is multiplied by factor of 0.1 and 0.995 respectively [24]:

$$Z_{grid} = X_{grid} + R_{grid}$$

$$X_{grid} = 0.995 * 2.079\Omega = 2.069\Omega$$

$$R_{grid} = 0.1 * 2.079\Omega = 0.208\Omega$$

$$Z_{grid} = (0.208 + \text{j}2.069)\Omega$$

IEC 60909 standard in Appendix: A given the P_{kr} = 0.8

$$Z_T = \frac{U_{kr}\% * V_{2n}{}^2}{100 * S_{tr}} = \frac{12\% * 27.5\text{kV}^2}{100 * 20\text{MVA}} = 4.54\Omega$$

$$P_{krT} = \frac{P_{kr}*S_{tr}}{100} = \frac{0.8*20*10^6}{100} = 160\text{kW}$$

Where P_{kr} is the loss of the transformer in the windings at rated current.

$$I_{2n} = \frac{S_{tr}}{\sqrt{3}*V_{2n}} = \frac{20*10^6}{\sqrt{3}*27.5kV} = 419.903A$$

$$R_T = \frac{P_{kr}\text{T}}{3*I_{2n}{}^2} = \frac{160\text{kW}}{3*419.903^2\text{A}} = 0.3025\Omega$$

$$X_T = \sqrt{Z_T{}^2 - R_T{}^2} = \sqrt{4.54^2 - 0.3025^2} = 4.527\Omega$$

$$Z_T = (0.3025 + j4.527)\ \Omega$$

Impedance at traction substation is given as the sum of line impedance and impedance of the transformer [4].

$$\sum R_s = \sum R_{grid} + \sum R_T = 0.208\ \Omega + 0.3025\Omega = 0.51\Omega$$

$$\sum X_s = \sum X_{grid} + \sum X_T = j2.069\Omega + j4.527\Omega = j6.6\Omega$$

$$Z_s = \sum R_s + \sum X_s = (0.51 + j6.6)\Omega$$

Table 3.4 Additional Parameter of the Traction Network

Type of operation	Normal condition section operation
No of train per section	1
Current per train (I_{FE}) Lebu to Indode	217.524A
Current per V-V transformer	1259.7A
Section length in kilometer	18.808km
Transformer impedance in Ω	0.4844 + j6.3966 = 2.4575Ω
Line impedance in Ω/km	0.130081 + j0.392381
Distance between contact wires is (midway between track and half length of the gauge. a_{ik}	4m + 0.7175m = 4.7175m
Height of contact wire	5750mm

3.3 Analysis of Mutual Impedance between Catenary Cables

To calculate mutual impedance between lines, the maximum resistance value 30 Ω given in Table 3.5 is used (Appendix C).

$$R = \frac{\rho_E * \ln(\frac{4*l}{d})}{2\pi l} \Rightarrow \rho_E = \frac{2\pi l * R}{\ln(\frac{4*l}{d})} = \frac{2\pi * 0.8 * 30\Omega}{\ln(\frac{4*0.8m}{0.02m})} = \frac{150.796}{5.075}\Omega m = 29.713\Omega m$$

By using equation (2.32)

$$\delta_E = 90\sqrt{\rho_E} = 90 * \sqrt{29.713} = 490.587m$$

$$\mathrm{X_{ik}} = 4\pi * 10^{-7} * \mathrm{f} * \ln(\delta_\mathrm{E}/\mathrm{a_{ik}}) = 4 * \pi * 50 * 10^{-7} * \ln\left(\frac{490.587}{4.7175}\right) = 2.918 * 10^{-4}\Omega/\mathrm{km}$$

$$\mathrm{Z_{ik}} = \dot{R}_E + \mathrm{X_{ik}} = 0.0493\,\Omega/\mathrm{km} + \mathrm{j}2.918 * 10^{-4}\,\Omega/\mathrm{km}$$

Table 3.5 Earthzing Resistance [25].

Category	Earthing resistance in Ω
Switch and arrester	10
Aerial earthing wire	10
Scattered OCS mast	30

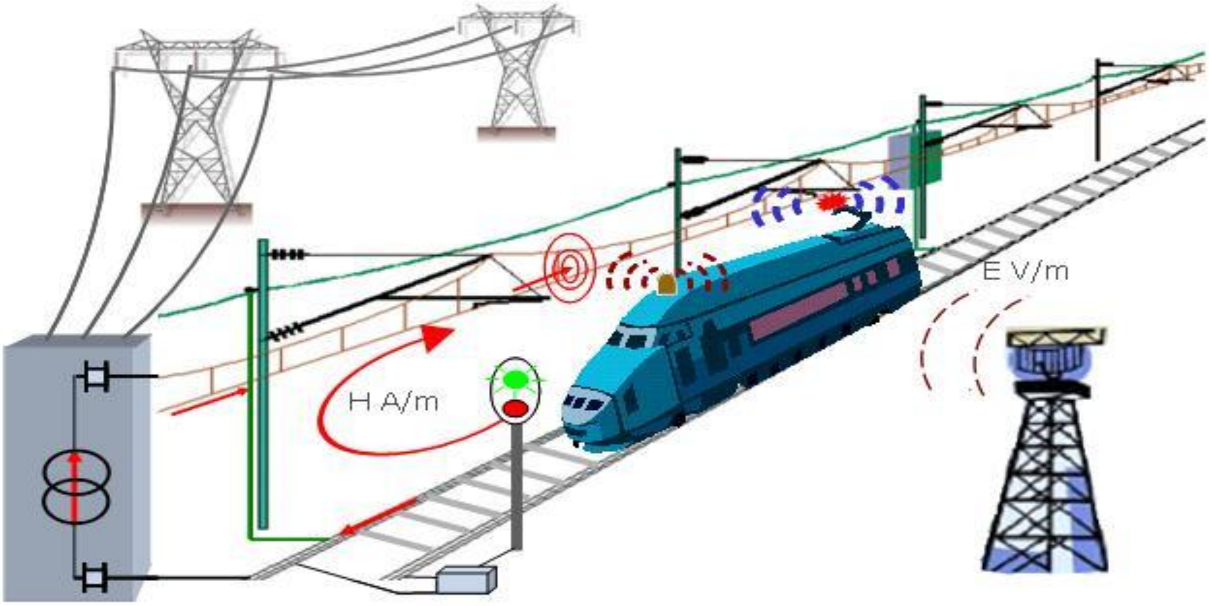

Figure 3.3 Over all Power System of Electrified Railway [5]

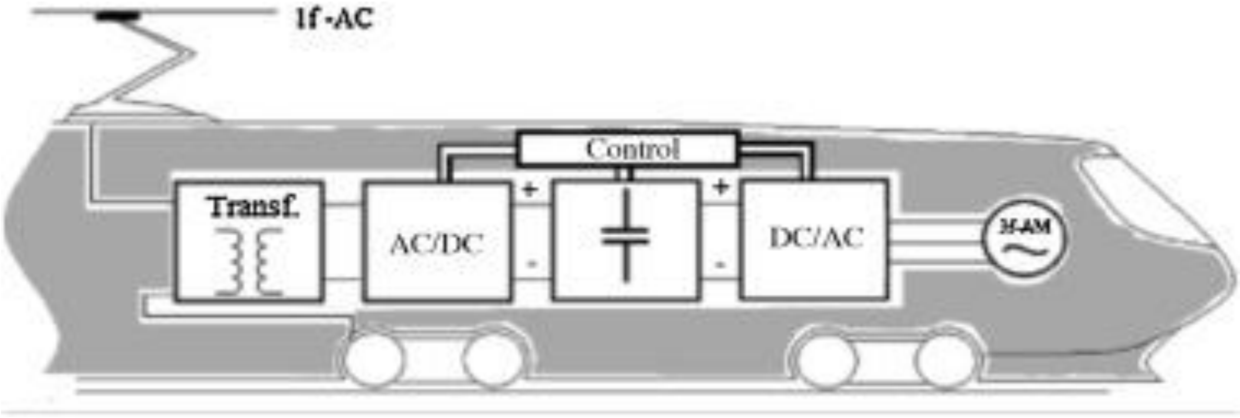

Figure 3.4 AC Electric Locomotive [5]

3.4 Voltage Drop Analysis Normal Section Operation

3.4.1 Voltage Drop at Substation in a Normal Section Operation Indode to Lebu

Voltage drop at substation in a normal section operation Indode to Lebu section post due to up and down track trains

$\Delta \text{V} = I_{FE} * (R\cos\theta + X\sin\theta)$

= 2 * 217.524A (0.4844 * 0.95 + 6.3966 * 0.312) = 1.068kV

Voltage drop at substation in a normal section operation Indode - Lebu section post due to the V-V transformer

= 1259.7A * (0.4844 * 0.95 + 6.3966 * 0.312) = 3.094kV

3.4.2 Voltage Drop at Feeder Network in a Normal Section Operation

The voltage drop for feeder network in a normal section operation is calculated using Eq. (2.42) as follows (Appendix E).

$\Delta \text{V} = (\text{I}\cos\theta - jI\sin\theta) * \text{L}(\text{R} + \text{jX}) + (\text{I}\cos\theta - jI\sin\theta) * \text{L} * Z_m$

= 217.524(0.95-j0.312)*18.808km (0.130081+j0.392381) +217.524(0.95-j0.312)*18.808km

*(0.0493$\Omega/km + j2.918 * 10^{-4}\,\Omega/km$)

= 217.524A*18.808km [(0.246+j0.3321) + (0.046925-j0.0151)Ω/km]

= (1006.433+j1358.7) + (191.98 – j61.777)

= 1198.413 + j1296.923 = $\sqrt{1198.413^{\,2} + 1296.923^{\,2}}$

= 1.76kV

Total voltage drop = 1.76kV + (1.068 + 3.094) kV= 5.922kV

Voltage at the end of feeder section = Supply voltage – Voltage drop at substation - Voltage drop at feeder network

= 27.5kV - 5.922kV = 21.56kV

3.5 Voltage Drop Analysis due to Over zone Feeding Operation(Overload)

The voltage drop of two consecutive sections without section post (over zone) operation case is when two consecutive section is considered as supplied from one substation that is either from Indode or Sebeta substation.

3.5.1 Section Voltage Drops between Indode -Sebeta due to Over zone Feeding and Parallel Power Supply without Employing Section Paralleling

The following figure shows four trains are in operation, two in up track the other two in down track which cause over zone operation since thus four trains are supplied from one substation at a time without applying section post, by disconnecting 25kV isolator and feeder bus tie circuit breaker from Sebeta substation.

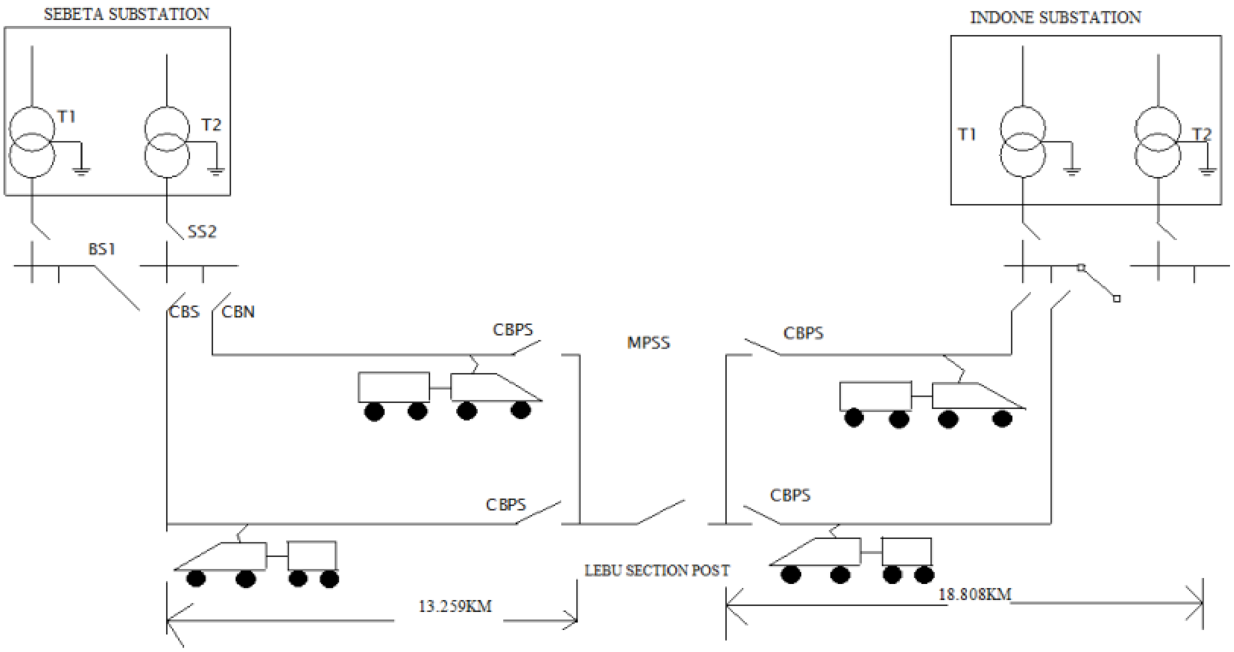

Figure 3.5 Traction Feeder Network Sections between Sebeta to Indode [3]

Table 3.6 Electrical Parameters of Traction Network for Over zone Operation [25]

Type of operation	Over zone feeding operation
Maximum Number of train per two section	4
Section length in kilo meter	32.329km
Transformer impedance Ω	0.4844 + j6.3966
Line impedance Ω/km	0.130081 + j0.392381
Equivalent impedance	$Z' \approx 0.2461\Omega/\text{km}$
Mutual impedance	$0.0493\Omega/\text{km} + j2.913 * 10^{-4}\ \Omega/km$ $Z'_m \approx R\cos\phi + X\sin\phi = 0.04458\,\Omega/km$

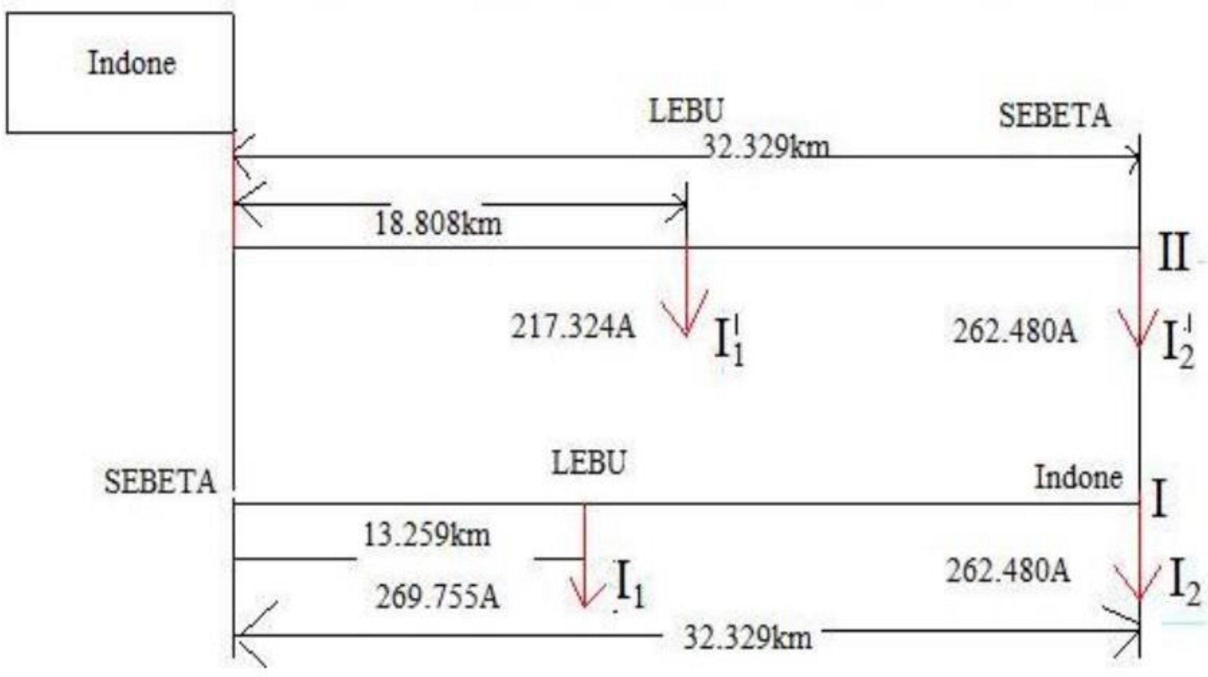

Figure 3.6 Over zone Current Distribution on a Double Track [3].

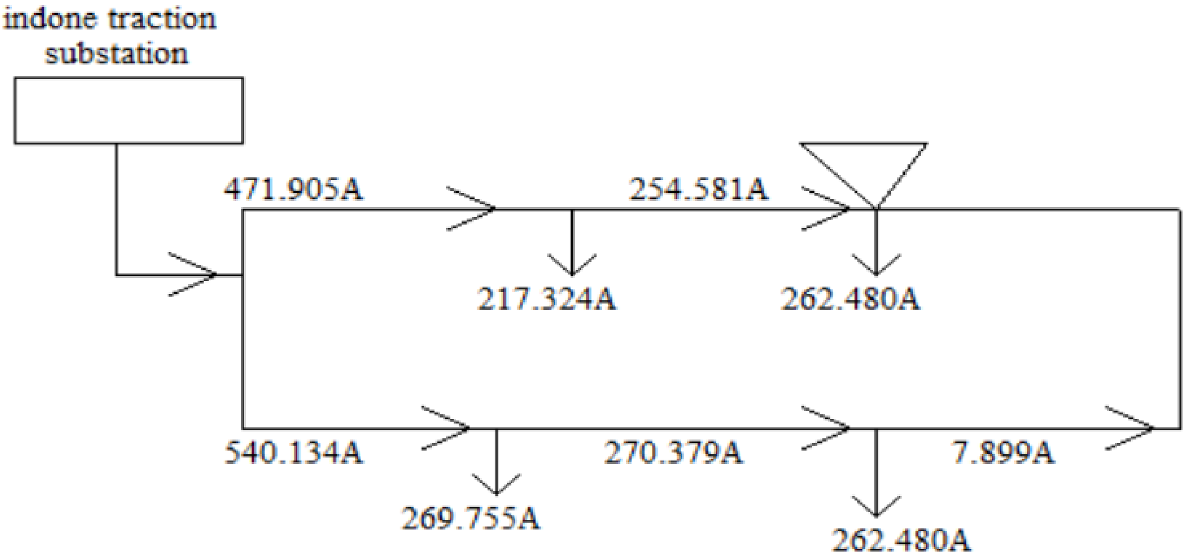

Figure 3.7 Parallel Power Supply (Catenary Breaker is on at the end Section) [3]

Train motion in up direction that is from Sebeta substation to Indode substation. Power is supplied from Indode substation, then feeder current is calculated using equation in Appendix E, as follows:

$$I_I = \sum_{i=1}^{n}\left(\frac{2l-l_i}{2l}\right)\dot{I}_i + \sum_{j=1}^{m}\frac{l'}{2l}\dot{I}_j$$

$$= \left[\frac{64.658km-13.259km}{64.658km} * 269.755A + \frac{64.658km-32.329km}{64.658km} * 262.480A\right] +$$

$$\left[\frac{18.808km}{64.658km} * 217.324A + \frac{32.329km}{64.658km} * 262.480A\right]$$

= 540.134A

For train motion in down direction that is from Indode substation to Sebeta substation, power is supplied from Indode substation, then feeder current is calculated using equation in Appendix E:

$$I_{II} = \sum_{i=1}^{m}\frac{l'}{2l}\dot{I}_i + \sum_{j=1}^{n}\left(\frac{2l-l_i}{2l}\right)\dot{I}_j$$

$$= \left[\frac{13.259km}{64.658km} * 269.755A + \frac{32.329km}{64.658km} * 262.480A\right] +$$

$$\left[\frac{64.658km-18.808km}{64.658km} * 217.324A + \frac{64.658km-32.329km}{64.658km} * 262.480A\right]$$

= 471.905A

3.5.1.1 Voltage Drop on the Feeder Network at the Dividing Point

The voltage drops on the feeder at $\dot{I}_I$ and $\dot{I}_{II}$ points will be calculated as follows.

$$\Delta \mathrm{V} = \mathrm{Z} * \mathrm{L}_1 * \dot{I}_{II} + \mathrm{Z}_\mathrm{m} * \mathrm{L}_1 * \dot{I}_I$$

ΔV = 0.2461Ω/km(18.808km * 471.905A + 254.581A * 13.259km) +

0.04458Ω/km(13.259 * 540.134A + 270.379A * 18.808km + 0.0047175km * 7.899A)

= 3.015kV + 0.545kV = 3.56kV

3.5.1.2 Voltage Drop in the Transformer due to Over zone Feeding

The voltage drop in the V-V connected transformer due to over feeding will be calculated as follows.

$$\Delta \mathrm{V} = I_{FE}(R\cos\theta + X\sin\theta) = (I_I + I_{II}) * (R\cos\theta + X\sin\theta)$$

= (540.134A + 471.905A) * 2.4575Ω = 2.5kV

3.5.1.3 Voltage Drop at Substation in a Normal Section Operation Indode to Lebu Section Post because of V-V Connection of Transformer

The voltage drop in the V-V connected transformer at normal section operation (Indode to Lebu) will be calculated as follows.

$$= 1259.7\mathrm{A} * (0.4844 * 0.95 + 6.3966 * 0.312)$$

= 3.094kV

Total voltage drop = Voltage drop in the transformer + Voltage drop on the feeder network

ΔV = 3.56kV + 2.5kV + 3.094kV = 9.154kV

Voltage at the end of feeder section = Supply voltage – Voltage drop at substation - Voltage drop at feeder network

= 27.5kV – 9.154kV = 18.346kV

3.5.2 Section Voltage Drop between Indode to Sebeta due to Over zone Feeding and Parallel Power Supply with Section Paralleling

Voltage drop on a feeder network with section post due to over zone feeding operation from Indode substation by disconnecting 25kV isolator or feeder bus incoming circuit breaker and feeder bus tie circuit breaker from Sebeta substation. As a result, it is usual to parallel the two catenaries or sub sectioning substations, as illustrated in the Figure 3.7. Load current can then flow in the parallel paths, which reduces the impedance to the load [16] [3].

3.5.2.1 Sebeta to Indode Feeder Network Impedance Model for Section Paralleling at Double Track

The following figure shows modeling of impedance in double track at section post (Lebu station) and at different points of Sebeta – Indode section.

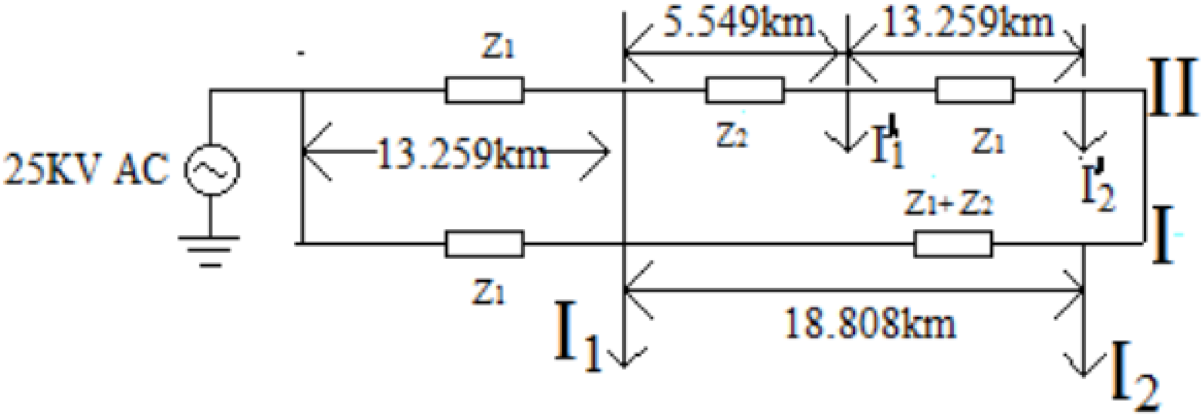

Figure 3.8 Line Impedances Model for Double Track Feeder Network in Section between Sebeta to Indode [3]

$Z_1 = 13.259km * 0.2461\,\Omega/km = 3.263\,\Omega$

$Z_2 = 5.549km * 0.2461\,\Omega/km = 1.366\,\Omega$

$Z_1 + Z_2 = 4.629\,\Omega/km$

3.5.2.2 Voltage Drop on Feeder Network at Dividing Point

The voltage drop at dividing points of Figure 3.8 will be calculated as follows:

$$\Delta \mathrm{V} = \mathrm{Z} * L_1 * \dot{I}_{II} + Z_m * L_1 * \dot{I}_I$$

$$\Delta V = \left[471.905A * \left(\frac{Z_1 * Z_1}{Z_1 + Z_1}\right) + 471.905A * Z_2 + 254.581A * Z_1\right] + 3.654 * 10^{-3}[(540.134A * \frac{Z_1 * Z_1}{Z_1 + Z_1} + 270.379A * (Z_1 + Z_2) + 0.00471475km * 0.7899A * 0.2461\,\Omega/km)]$$

$$= \left[471.905A * \left(\frac{3.263\Omega * 3.263\Omega}{3.263\Omega + 3.263\Omega}\right) + 471.905A * 1.366\Omega + 254.581A * 3.263\Omega\right] + 0.04458\,\Omega/km\,[(540.134A * \left(\frac{3.263\Omega * 3.263\Omega}{3.263\Omega + 3.263\Omega}\right) + 270.379A * 4.629\Omega + 0.00471475km * 0.7899A * 0.2461\,\Omega/km]$$

= 2.34kV

Total voltage drop = voltage drop in the transformer + voltage drop on the feeder network

ΔV = 2.34kV + 2.5kV + 3.094kV = 7.934kV

Voltage at the end of feeder section = Supply voltage – Voltage drop at substation - Voltage drop at feeder network

= 27.5kV – 7.934kV = 19.57kV

Chapter Four

4. Modeling , Simulation and Discussion

This section of the thesis work aims in studying and analyzing how different operating conditions (scenarios) will affect the voltage profile of the system or the voltages on catenary.

4.1 Normal Section Operation Condition

First, a typical AC railway feeder substation is directly connected to the three-phase high-voltage supply grid. Each feeder substation typically consists of two V-V parallel connected power transformers of 132/27.5kV rating at Indode substation. Thus two transformers feed the two lines of catenary which is up track with one train at a time and down track with one train at a time within the same section of the line or Beta side of V-V transformer.

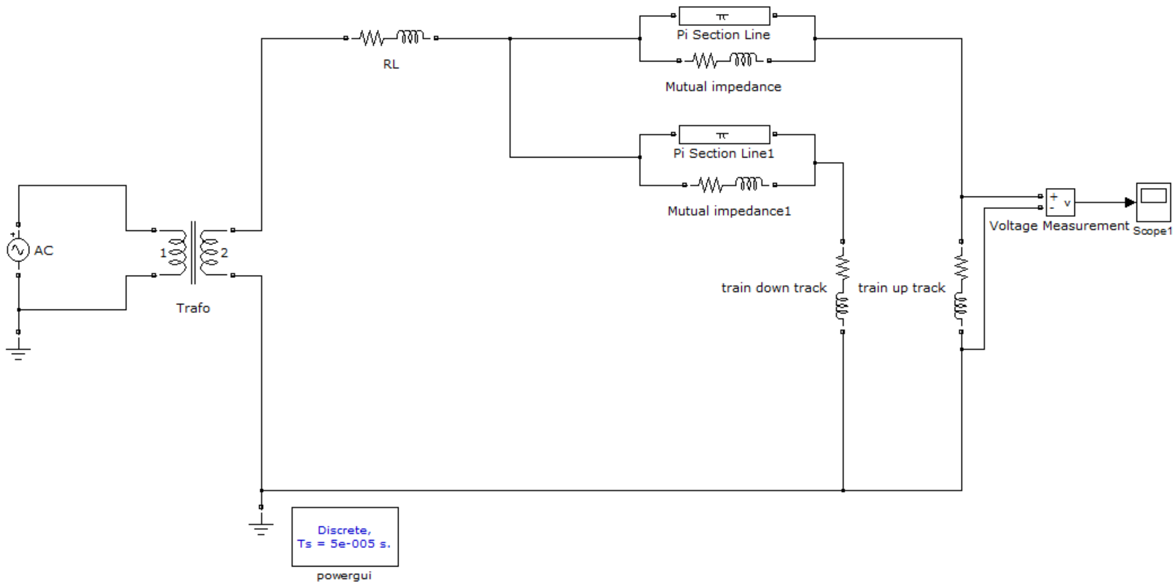

Figure 4.1 Model for Normal Section Operation Indode to Lebu line

The voltage drop of the traction network (including substation and feeder network) at normal section operation of Indode to Lebu become 5.922kV with receiving end voltage of 21.56kV as calculated in section 3.4.2.

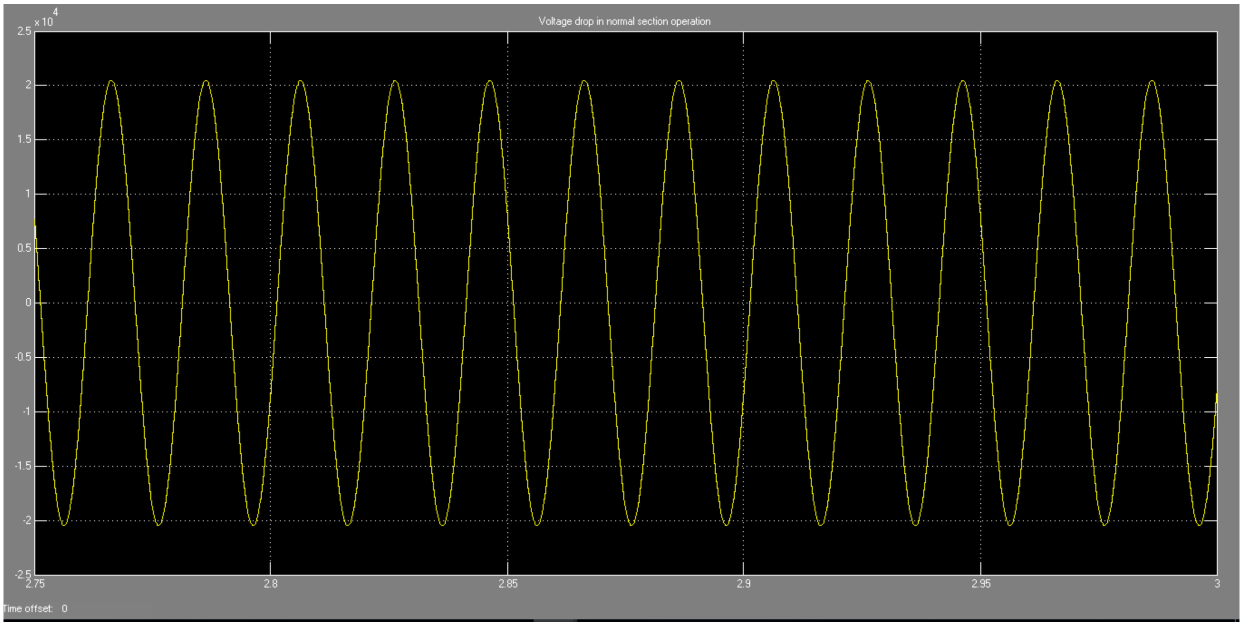

Figure 4.2 Simulation result for normal section operation

Percentage of voltage regulation = $\frac{V_s - V_r}{V_r} * 100$

$$\%\text{VR} = \frac{27.5kV - 21.56kV}{21.56kV} * 100$$

$$= 27.55\%$$

4.2 Over zone (Over Load) Operation Condition

First, a typical AC railway feeder substation is directly connected to the three-phase high-voltage supply grid. Each feeder substation typically consists of two V-V parallel connected power transformers of 132/27.5kV rating at Indode and Sebeta substation. Thus two transformers feed the two lines of catenary which is up track with two trains at a time and down track with two trains at a time within the same section of the line without section post or feeding of two consecutive sections with four trains without section post. That is when the load is twice that of the normal case (over zone scenarios).

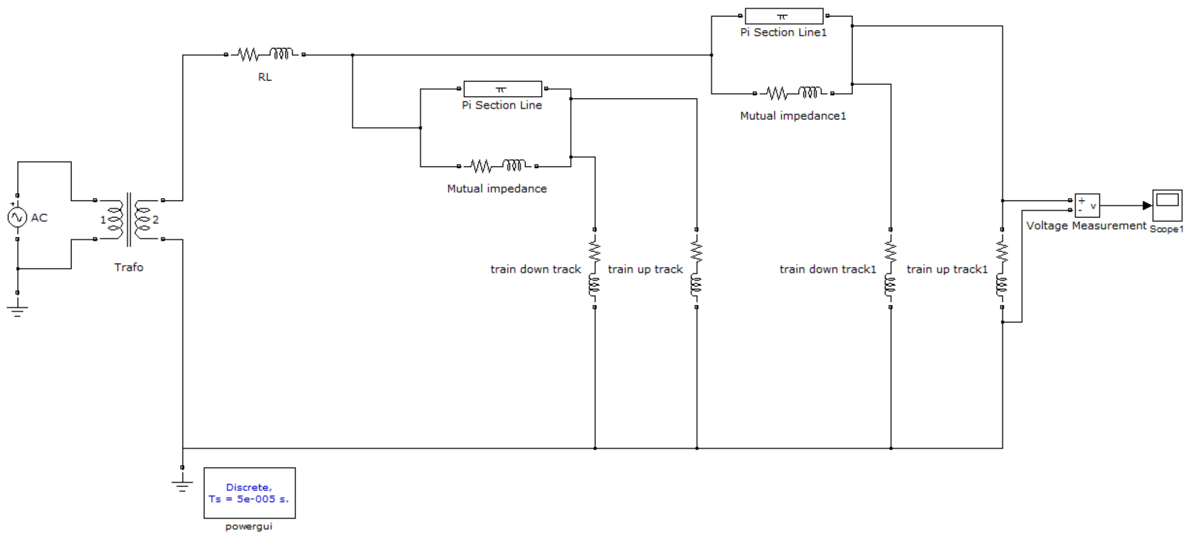

Figure 4.3 Model for Over zone Operation Indode to Sebeta Section

The voltage drop due to complex part of the traction network (including substation and feeder network) and the additional load due to over zone supplying of Indode to Sebeta line become 9.154kV with receiving end voltage of 18.346kV as calculated in section 3.5.1.

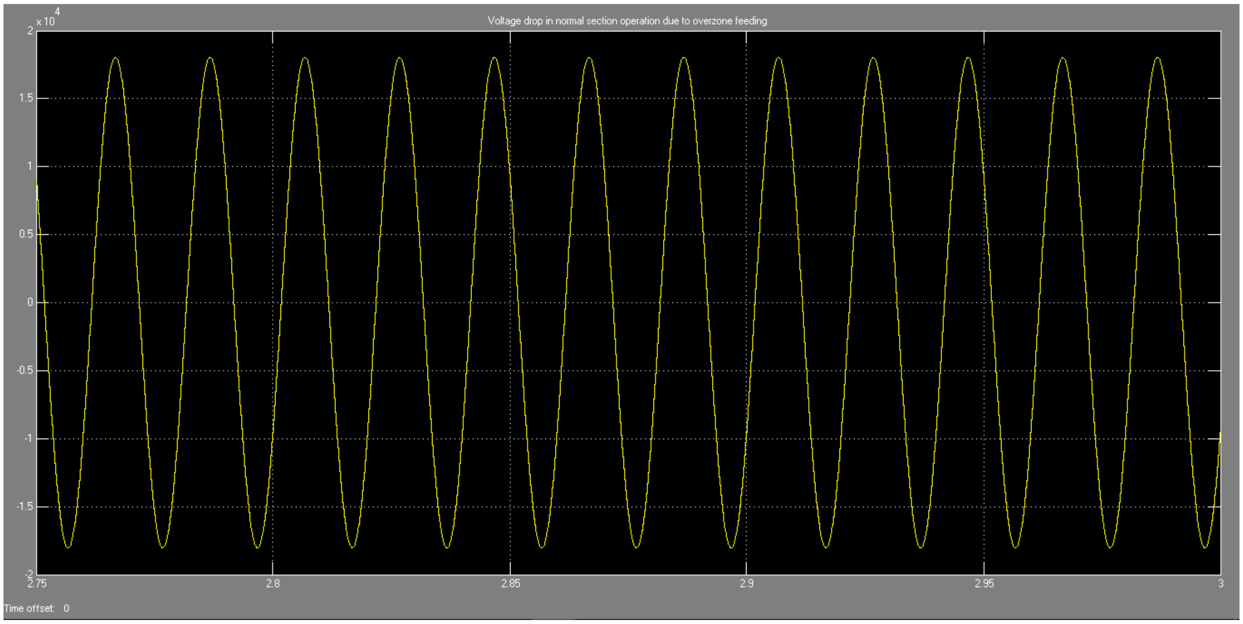

Figure 4.4 Simulation Result for Over zone Feeding Operation

Percentage of voltage regulation = $\frac{V_s - V_r}{V_r} * 100$

$$\%VR = \frac{27.5kV - 18.346kV}{18.346kV} * 100$$

$$= 49.89\%$$

Table 4.1 The Overall Values from Analysis (Calculation)

Operation Condition	Voltage Drop	Voltage at receiving end	%VR
Normal (Indode-Lebu)	5.922kV	21.56kV	27.55%
Over zone (Indode-Sebeta)	9.154kV	18.346kV	49.89%

Table 4.1 shows that the voltage drop increases as the operation condition changes from normal(one train in up track and one train in down track) to over zone(two train in up track and two train in down track). In other word, the receiving end voltage decreases from 21.56kV to 18.346kV. It can also be seen that percentage of voltage regulation varied from 27.55% to 49.89%, this shows that percentage of voltage regulation for over loading condition is 1.8 times that of normal operation condition.

Chapter Five

5. Design of STATCOM and Simulation Results

5.1 Design of STATCOM

5.1.1 Indode to Lebu Design Parameters for STATCOM for Normal Operation

Total reactive power consumption of the section is summation of reactive power consumption of transformer and trains', since the major reactive power is consumed by thus components which are affected by fluctuation of a load with in a day, the line reactive power is not considered compared to them.

Table 5.1 Simulation Parameters for Normal Section Operation

Parameters	
Feeder voltage(V_{2n})	27.5kV
Line impedance(RL)	R= 0.52046Ω, L= 0.023194604H
Catenary Mutual impedance	R= 0.9272344Ω, L= 9.2882828e-7H
Line impedance Indode-Lebu	R= 0.130081Ω, L= 1.2489e-3 H, C = 11e-12F ,l=18.808km
Train down track Active power (in MW)	10.292
Reactive power (Q_{train}) (in Mvar)	1.695127
Feeder current	217.524A
Train up track Active power (in MW)	10.292
Reactive power(Q_{train}) (in Mvar)	1.695127
V-V transformer connection load	
Feeder current	1259.7A
Active power (in MW)	10.292
Reactive power (Q_{V_Vtrafo}) (in Mvar)	1.435496
System nominal frequency	50HZ
System power factor($\cos\varphi$)	0.95

Total reactive power consumption for normal section operation is summation of the reactive power of V-V connected transformer and twice that of trains' reactive power.

$$\mathrm{Q} = Q_{V\_Vtrafo} + 2Q_{train}$$

Where Q_{V_Vtrafo} Reactive power for V-V connected transformer

Q_{train} Reactive power for a train up and down track

$$\mathrm{Q} = 1.4 * 10^6 + 2(1.7 * 10^6)$$

$$\mathrm{Q} = 4.8 * 10^6\ var$$

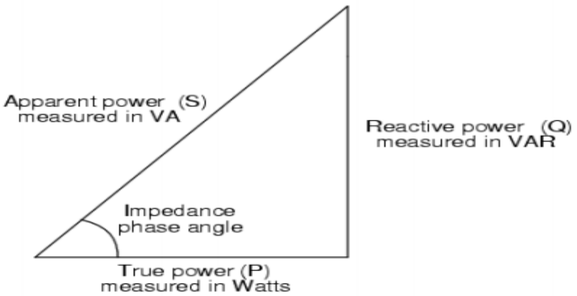

Figure 5.1 Power Triangle Diagram for Power Analysis [26]

$$\sin\varphi = \frac{Q}{S}$$

$$S = \frac{Q}{\sin\varphi},\ S = VI$$

$$V = IX_C,\ I = \frac{V}{X_C}$$

$$\frac{Q}{\sin\varphi} = V * \frac{V}{X_C}$$

$$X_C = \frac{{V_{2n}}^2}{Q}\sin\varphi$$

$$= \frac{(27500)^2 V}{4.8{*}10^6\ var} * 0.312 = 49.156\Omega$$

$$\mathrm{C} = \frac{1}{2\pi \mathrm{f} \mathrm{X_C}} = \frac{1}{2{*}\pi{*}50\mathrm{hz}{*}49.156\Omega} = 6.475 * 10^{-5}\mathrm{F}$$

$\frac{\mathrm{X_L}}{\mathrm{X_C}} = 0.1, 0.2$ and 0.3 [31][32].

For $\frac{\mathrm{X_L}}{\mathrm{X_C}} = 0.1$

$$X_L = 0.1 * X_C = 0.1 * 49.156\Omega = 4.916\ \Omega$$

$$X_L = 2\pi f L$$

$$L = \frac{X_L}{2\pi f} = \frac{4.916\Omega}{2{*}\pi{*}50\mathrm{hz}} = 0.016\ H$$

For $\frac{\mathrm{X_L}}{\mathrm{X_C}} = 0.2$

$$X_L = 0.2 * X_C = 0.2 * 49.156\Omega = 9.83\ \Omega$$

$$X_L = 2\pi f L$$

$$L = \frac{X_L}{2\pi f} = \frac{9.83\ \Omega}{2*\pi*50\text{hz}} = 0.03\ H$$

For $\frac{X_L}{X_C} = 0.3$

$$X_L = 0.3 * X_C = 0.3 * 49.156\Omega = 14.75\Omega$$

$$X_L = 2\pi f L$$

$$L = \frac{X_L}{2\pi f} = \frac{14.75\Omega}{2*\pi*50\text{hz}} = 0.047H$$

Table 5.2 Value of Inductor for Different Ratio of $\frac{X_L}{X_C}$

$\frac{X_L}{X_C}$ ratio	Inductor Values(L)
0.1	0.016H
0.2	0.03H
0.3	0.047H

5.1.2 Indode to Sebeta Design Parameters for STATCOM for Over zone Operation

This is the scenarios in which four trains considered in operation in up and down track of the section without section post in the line.

Table 5.3 Simulation Parameters for Over zone Operation

Parameters	
Feeder voltage (V_{2n})	27.5kV
Line impedance (RL)	R= 0.52046Ω, L= 0.023194604H
Catenary Mutual impedance	R= 0.6536687Ω, L= 1.231533374e-5H
Line impedance Indode-Lebu	R = 0.130081Ω, L = 1.2489e-3 H, C =11e-12F ,l = 18.808km
Lebu-Sebeta	R= 0.130081Ω, L=1.2489e-3 H, C=11e-12F, l = 13.259km

Train down track1 Active power(in MW)	10.292
Reactive power (Q_{train}) (in Mvar)	1.695127
Feeder current	217.524A
Train up track 1 Active power (in MW)	10.292
Reactive power (Q_{train}) (in Mvar)	1.695127
V-V transformer connection load	
Feeder current	1259.7A
Active power (in MW)	10.292
Reactive power (Q_{V_Vtrafo}) (in Mvar)	1.435496
System Nominal frequency	50HZ
System power factor(cos φ)	0.95

$$\mathrm{Q} = Q_{V\_Vtrafo} + 4Q_{train}$$

$$\mathrm{Q} = 1.4 * 10^6 + 4(1.7 * 10^6)$$

$$\mathrm{Q} = 8.2 * 10^6\ var$$

$$X_c = \frac{{V_{2n}}^2}{Q} \sin\varphi$$

$$= \frac{(27500)^2 V}{8.2 * 10^6\ var} * 0.312 = 28.8\Omega$$

$$\mathrm{C} = \frac{1}{2\pi \mathrm{f} \mathrm{X_C}} = \frac{1}{2 * \pi * 50\mathrm{hz} * 28.8\Omega} = 1.105 * 10^{-4}\mathrm{F}$$

For $\frac{\mathrm{X_L}}{\mathrm{X_C}} = 0.1$

$$X_L = 0.1 * X_C = 0.1 * 28.8\Omega = 2.88\ \Omega$$

$$X_L = 2\pi f L$$

$$L = \frac{X_L}{2\pi f} = \frac{2.88\Omega}{2 * \pi * 50\mathrm{hz}} = 0.009H$$

For $\frac{\mathrm{X_L}}{\mathrm{X_C}} = 0.2$

$$X_L = 0.2 * X_C = 0.2 * 28.8\Omega = 5.76\Omega$$

$$X_L = 2\pi f L$$

$$L = \frac{X_L}{2\pi f} = \frac{5.76\Omega}{2 * \pi * 50\mathrm{hz}} = 0.018H$$

For $\frac{\mathrm{X_L}}{\mathrm{X_C}} = 0.3$

$$X_L = 0.3 * X_C = 0.3 * 28.8\Omega = 8.64\ \Omega$$

$$X_L = 2\pi f L$$

$$L = \frac{X_L}{2\pi f} = \frac{8.64\Omega}{2*\pi*50\text{hz}} = 0.028H$$

Table 5.4 Value of Inductor for different ratio of $\frac{X_L}{X_C}$

$\frac{X_L}{X_C}$ ratio	Inductor Values(L)
0.1	0.009H
0.2	0.018H
0.3	0.028H

5.2 Simulation Results with Application of STATCOM and Discussion

5.2.1 Normal Section Operation Indode to Lebu with STATCOM

The two V-V transformers which are connected in parallel to each other is connected with Static synchronous compensator (STATCOM) in parallel to regulate the voltage profile in the up track catenary and down track catenary with one train in each track.

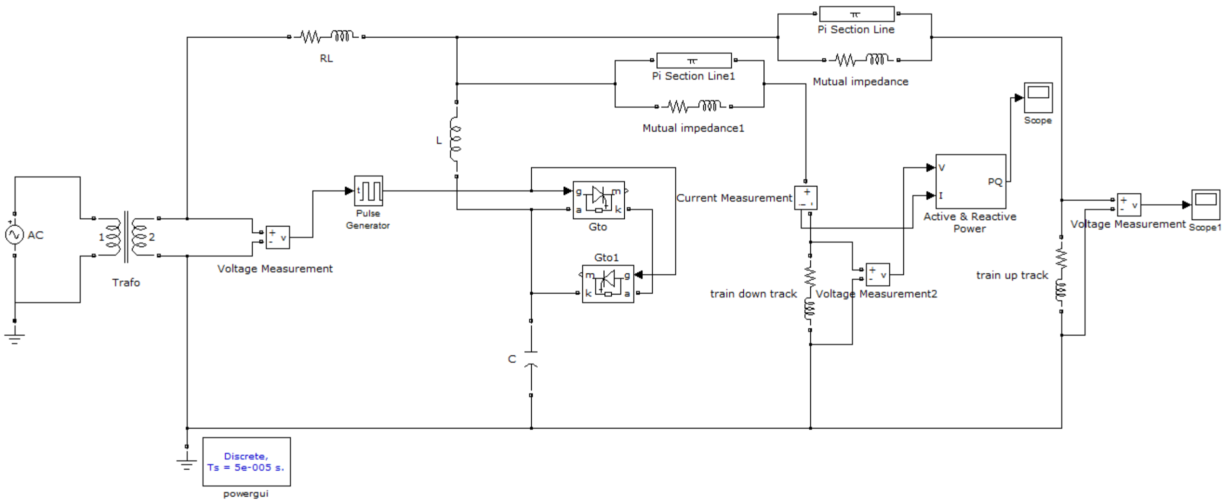

Figure 5.2 Normal Section Operation's Model with STATCOM

Percentage of voltage regulation = $\frac{V_s - V_r}{V_r} * 100$

Since the receiving end voltage is above the rated voltage (27.5kV), the voltage drop is zero which means with compensator this line has not voltage drop.

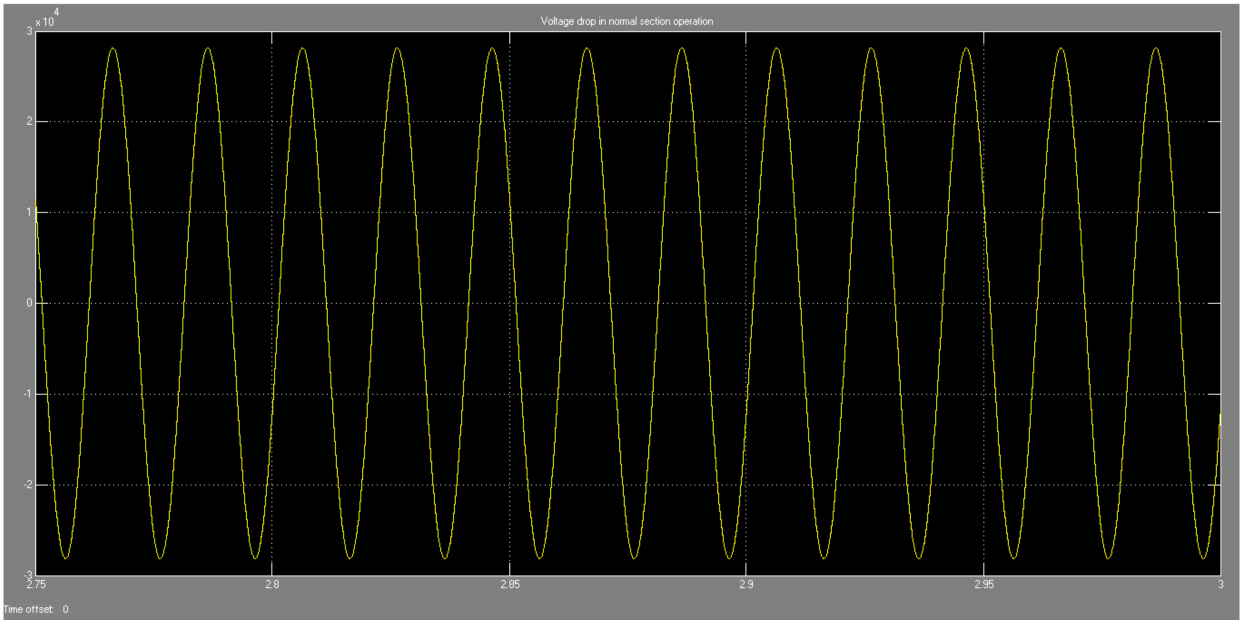

Figure 5.3 Simulation Result of Normal Section Operation with Compensator

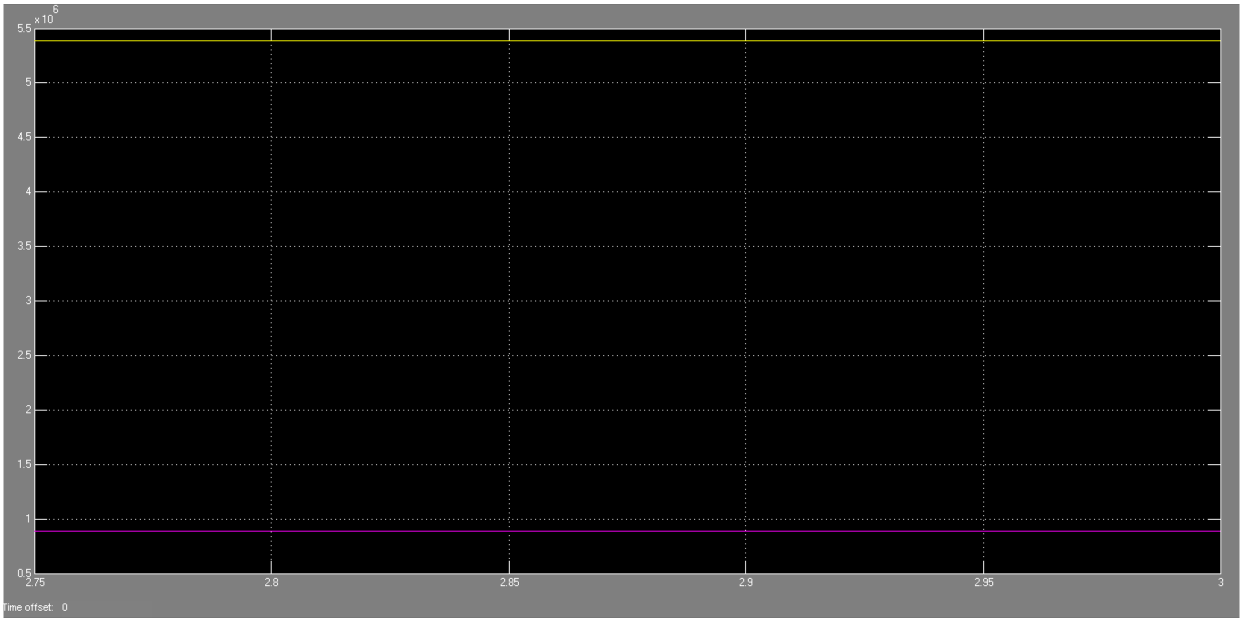

Figure 5.4 Active Powers for Normal Section Operation with Compensator

Table 5.5 The Overall Results from Analysis for Normal Section

$\frac{X_L}{X_C}$ ratio	Receiving end Voltage	Active Power	Voltage Drop
0.1	27kV	4.96MW	0.5
0.2	27.4kV	5.1MW	0.1
0.3	28.1kV	5.4MW	0

5.2.2 Over zone Operation of Indode - Sebeta with STATCOM

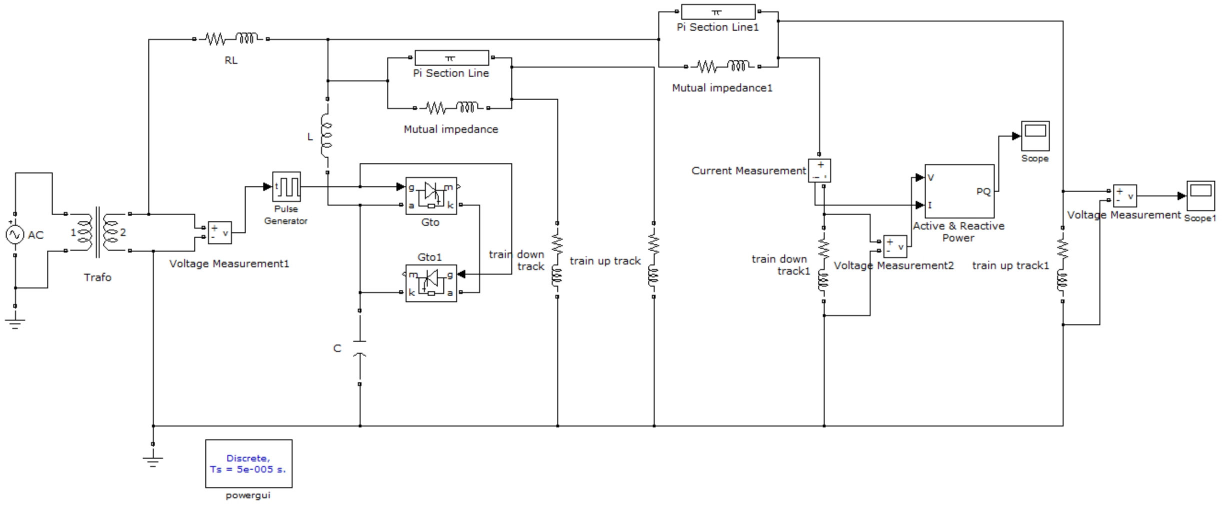

Figure 5.5 Over zone Operation's Model with STATCOM

Percentage of voltage regulation = $\frac{V_s - V_r}{V_r} * 100$

$$\%\text{VR} = \frac{27.5kV - 27.48kV}{27.48KV} * 100$$

$$= 0.07\%$$

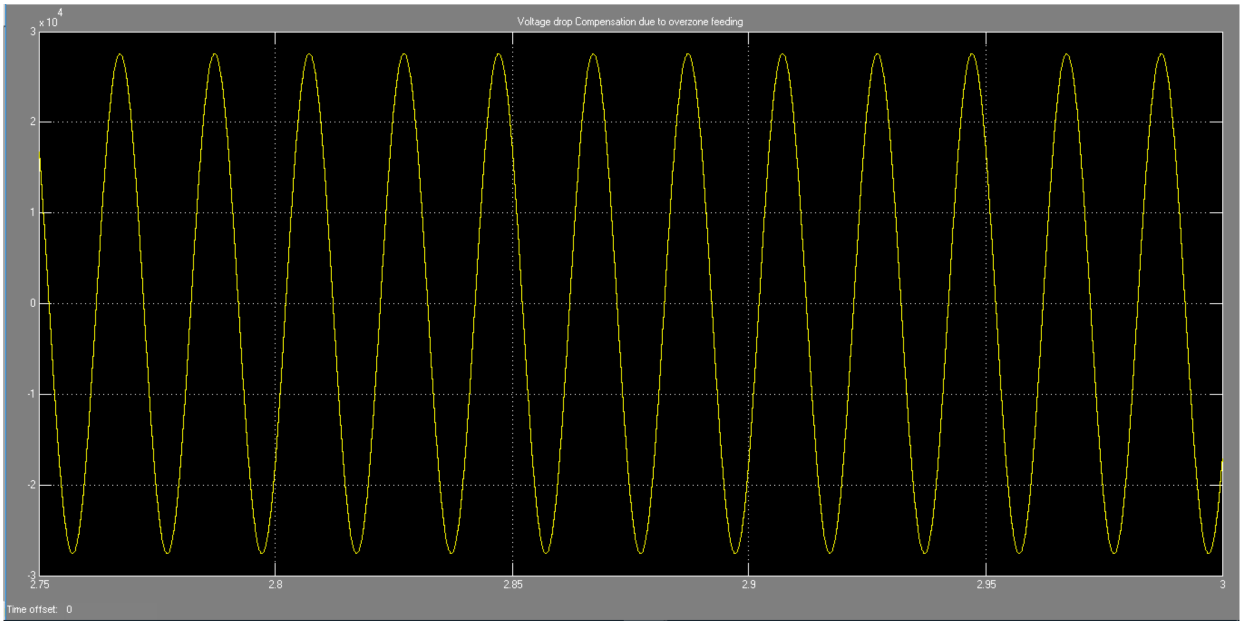

Figure 5.6 Simulation Result of Over zone Operation with Compensator

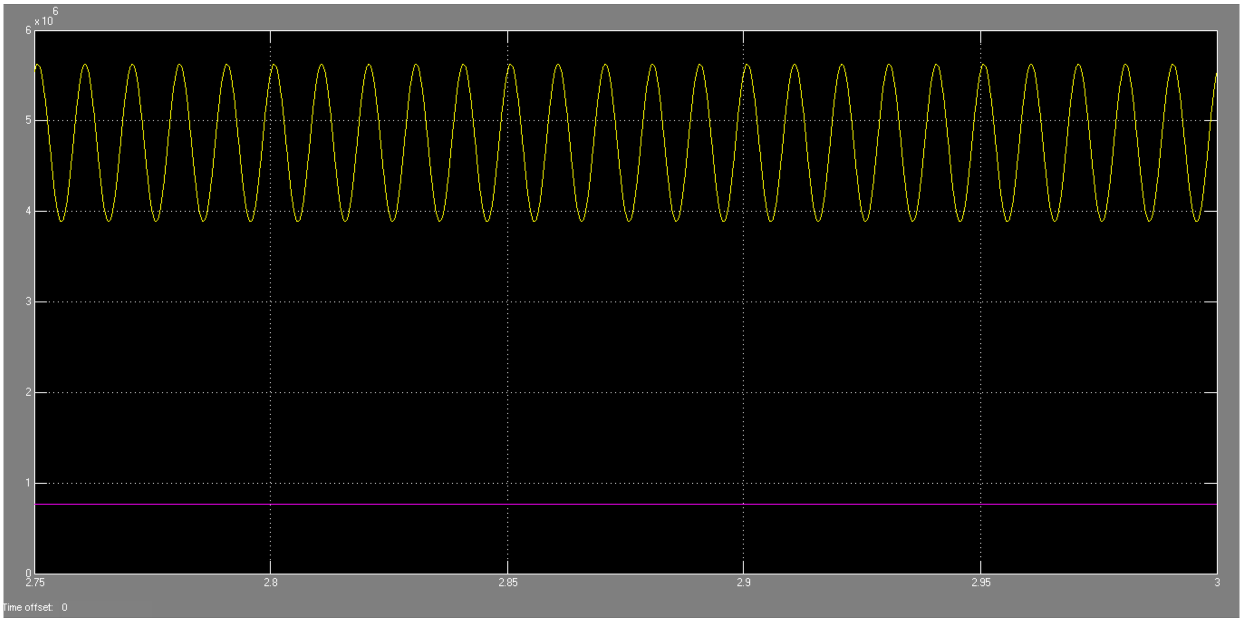

Figure 5.7 Active Power for Over zone Operation with Compensator

Table 5.6 The Overall Results from Analysis for Over zone Section

$\frac{X_L}{X_C}$ ratio	Receiving end Voltage	Active Power	Voltage Drop	%VR
0.1	25.9kV	4.96MW	1.6kV	6.18%
0.2	26.5kV	5.2MW	1kV	3.77%
0.3	27.48kV	5.6MW	0.02kV	0.07%

Table 5.6 shows that the receiving end voltage increases when difference ratio $\frac{X_L}{X_C}$ value of STATCOM is integrated to the system. It can also be seen that the active power varied from 4.96MW to 5.6MW. In addition to this, the voltage drops and percentage of voltage regulation decreases as the receiving end voltage increases.

From **Table 4.1** and **Table 5.6** it is possible to conclude that the receiving end voltage of the section without STATCOM for over zone operation condition is small as compared to with STATCOM. In other word, the receiving end voltage of the section for over zone operation case with STATCOM is 1.5 times that of without STATCOM when 0.3 values is considered for STATCOM.

Chapter Six

6. Conclusion, Recommendation and Future Work

6.1 Conclusion

This thesis presents problems in power quality issues of railway power supply system, especially voltage drop. In order to analyze voltage drops, the proposed Ethiopian single-phase 27.5kV AC electrified system is considered as case study. The scope of this paper is narrowed studying voltage fluctuation in Indode to Sebeta TSS feeding section due to load (Passenger and Freight) variation with in a day. In addition, providing a concrete mitigation technique is part of this work.

A detailed literature survey of 27.5kV AC Electrification system and the voltage drop studies together with related issues such as reactive power compensation, methods of voltage drop analysis and voltage drop monitoring issues are reviewed and presented to summarize the state of the art techniques that are correlated to the methods proposed in this research.

In this research work, railway supply system model has been developed for two scenarios (normal section operation and over zone operation) by considering with and without STATCOM cases.

In this research, a series of deterministic simulations have been performed to investigate the voltage drop level under certain conditions. Thus, voltage drop levels have been quantified for certain operational conditions. Passenger and freight flow variations is the main source of voltage drop. As this load variation affect the power flow within the system, the voltage profile on the catenary also affected which in turn produce voltage drop on the line.

Furthermore, voltage drop analyses have been made based on two different scenarios these are normal section feeding operation and over zone feeding operation. Added to this, plots of waveform for the two scenarios are provided.

The voltage drop analysis with MATLAB/SIMULINK simulation, at normal section operation gives a voltage regulation of 27.55% with receiving end voltage of 21.56kV which is within the limit recommended by TBl 0009- 2005. In worst case scenario which is over zone (over load) operation, the voltage regulation becomes 49.89% with receiving end voltage of 18.346kV which is beyond the limit recommended by TBl 0009- 2005. Generally, in the second scenario for simulation without compensator the voltage profile is beyond the acceptable limit set by TBl 0009- 2005. In account of all this result, it is possible to conclude that the power quality is poor in the absence of compensator (STATCOM) in the system.

The proposed solution for voltage drop mitigation in this work was based on STATCOM (Static Synchronous Compensator). The analysis of STATCOM performances was made by simulation under MATLAB/SIMPOWER environment. Two GTOs installations with inductor and capacitor were taken into consideration in order to reduce the voltage drop under the recommended limits of the TBl 0009- 2005 standard. The results of simulation after application of the proposed STATCOM (Table 5.5 and Table 5.6) shows that the compensation technique employed reduced the voltage regulation to the standard limit (with maximum voltage regulation of 6.18%), even in the worst cases relating to over load feeding.

6.2 Recommendation

To maintain power quality in the proposed Ethiopian single-phase 27.5kV AC electric traction system, one of the major issues to be addressed should be the issue of voltage drop .In this regard, the Ethiopian Railway Corporation (ERC)

- Must look first for preventive solutions aimed to avoid voltage drop and their Consequences or
- Should be in position to take remedial solutions (techniques to overcome the existing Problems)

Thus, it is possible to reduce adverse effects of voltage drop on the traction equipment and to bring power quality improvement. Generally, this thesis recommends that, it is crucial to design voltage regulator (compensator) for reliable and safe operation of railway system.

6.3 Future Works

The following issues might be further studied in the future:

- Mitigation of voltage drop by using thyristors controls series capacitor (TCSC)
- Mitigation of voltage drop by using static var compensator (SVC)
- Mitigation of voltage drop by using unified power flow controller (UPFC)
- Analysis of voltage drop for single track line of Adama - Mieso

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APPENDIX A: Power transformer parameters

The impedance module ZT can be calculated from the rated transformer data as follows:

$$\mathrm{Z_T} = \frac{\mathrm{U_{kr}\%} * \mathrm{V_{2n}}^2}{100 * S_{tr}}$$

Where:

$\mathrm{V_{2n}}$ is the rated voltage of the transformer, on the high-voltage or low-voltage side

S_{tr} is the rated apparent power of the transformer

$\mathrm{U_{kr}}$ is the short-circuit voltage at rated current in percent

The positive-sequence short-circuit resistance R_T of a two-winding transformer is given by the Relationship:

$$R_T = \frac{P_{krT}}{3 * {I_{2n}}^2}$$

Where:

P_{krT} is the total loss of the transformer in the windings at rated current.

I_{2n} is the rated current of the transformer on the high-voltage or low-voltage side.

Note:

The resistance R_T is to be considered if the peak short-circuits current ip or the DC component I_{DC} is to be calculated.

For large transformers, the resistance is so small that the impedance is represented by the reactance only, when calculating short-circuit currents.

The positive-sequence short-circuits reactance X_T of a two-winding transformer results as follows: The positive-sequence short-circuit reactance XT of a two-winding transformer results as follows:

$$X_T = \sqrt{{Z_T}^2 - {R_T}^2}$$

[Short Circuit Analysis Program ANSI/IEC/IEEE &Protective Device Evaluation User's Guide]

APPENDIX B: Voltage drop approximation [5]

Voltage drop is given by ΔV

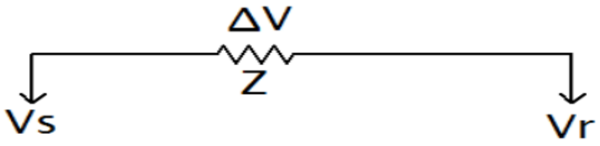


Figure C1 Voltage drop approximation

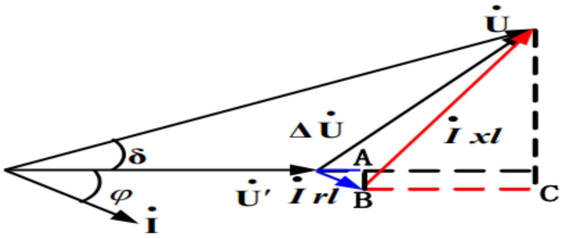


Figure C2 phasor representation of voltage drop

$$\Delta \mathrm{V} = V_s - V_r$$

$$\Delta V \approx V_s \cos\delta - V_r$$

$$\Delta V = \mathrm{Re}(\Delta V)$$

$$\Delta V = \mathrm{RI}\cos\Phi + XI\sin\Phi$$

$$\Delta V = V_r\angle 0^0\ (\mathrm{R} + \mathrm{j}\mathrm{X})\mathrm{LI}\angle-\ \varphi = \mathrm{RLI}\angle-\ \varphi + \mathrm{jXLI}\angle-\ \varphi = \mathrm{RLI}\angle-\Phi +\ \mathrm{XLI}\angle 90-\Phi$$

$$\Delta V = \mathrm{Re}(\Delta V) = (\mathrm{R}\cos\Phi + X\sin\Phi)\mathrm{LI} = Z'LI$$

Appendix C: Earthing used for railway

When l»d, the earthling resistance of upright earthling body can be calculated with the following
formula:

$$R = \frac{\rho}{2\prod l}\ln\frac{4l}{d}$$

Where:

R the earthling resistance of upright grounding body CD);

ρ The soil resistivity

l The length of the upright earthling body

d The diameter (m) of the round steel bar used as the earthling body. The equivalent diameter shall be calculated with the following formula when other profile-type steel is used (Figure C12)

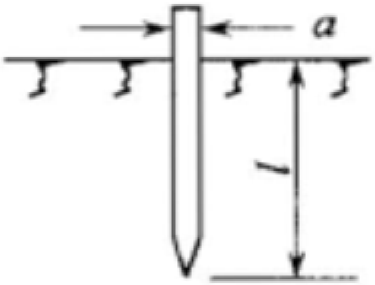


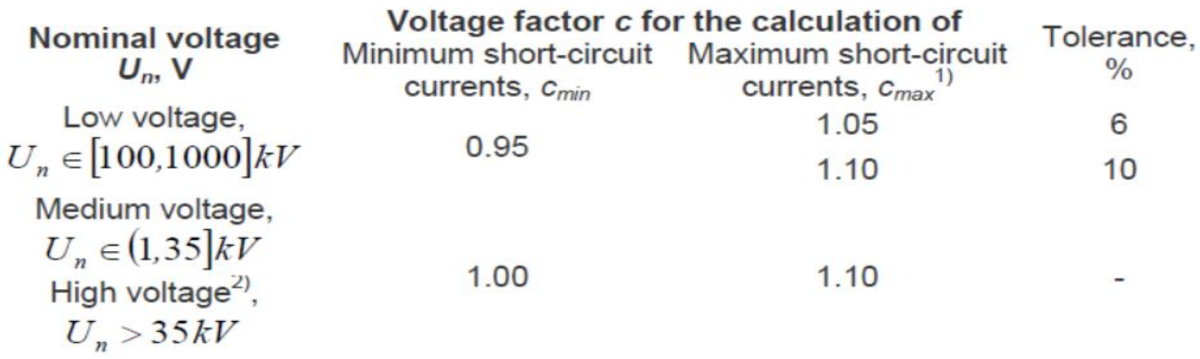
Figure D1 Up right earting body

Appendix D: IEC voltage factor

As per IEC 60909 the equivalent voltage source (rms) is given by the relations

IEC voltage factor

$$U_{es} = \frac{C * U_n}{\sqrt{3}}, V,$$



Nominal voltage U_n, V	**Voltage factor *c* for the calculation of**		Tolerance, %
	Minimum short-circuit currents, c_{min}	Maximum short-circuit currents, c_{max} [1)	
Low voltage, $U_n \in [100,1000] kV$	0.95	1.05 1.10	6 10
Medium voltage, $U_n \in (1,35] kV$ High voltage[2), $U_n > 35kV$	1.00	1.10	-

APPENDIX E: The voltage drop of double-track traction network [5]

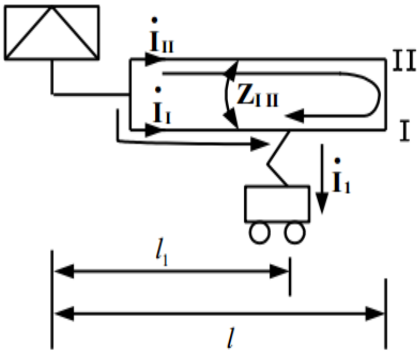
Current distribution principle of a double traction network

Known: double-track traction network

Self-impedance $= Z_I = Z_{II} = Z(\Omega/km)$

Mutual impedance $= Z_I = Z_{II} = Z_M(\Omega/km)$

The voltage drop equation from substation to the locomotive



Short – loop= $\Delta V = Z * L_1 * \dot{I_1} + Z_m * L_1 * \dot{I_{II}}$

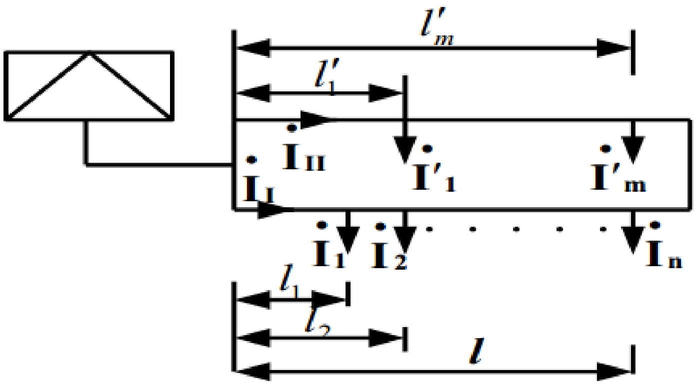
Long loop= $Z(2l - l_1)\dot{I_{II}} + Z_m l_1 \dot{I_I} - 2Z_m(l - l_1)\dot{I_{II}}$

$\dot{I_1} = \dot{I_I} + \dot{I_{II}}$

$$(Z - z_m)l_1\dot{I_I} = (Z - z_m)(2l - l_1)\dot{I_{II}}$$

Solution : $\begin{Bmatrix} \dot{I_I} = \frac{2l-l_1}{2l}\dot{I_1} \\ \dot{I_{II}} = \frac{l_1}{2l}\dot{I_1} \end{Bmatrix}$

Traction network: The current of long loop Current distribution principle of double-track and Short loop is inversely proportional to through path.



$$\dot{I_I} = \sum_{i=1}^{n}\frac{2l-l_i}{2l}\dot{I_i} + \sum_{j=1}^{m}\frac{l_i'}{2l}\dot{I_j}'$$

$$\dot{I_{II}} = \sum_{i=1}^{n}\frac{l_i}{2l}\dot{I_i} + \sum_{j=1}^{m}\frac{2l-l_i'}{2l}\dot{I_j}'$$

Appendix F: Feeder current numerical characteristics [5]

- Traction probability

$$P_F = \frac{T_g}{T}$$

No-load probability

$$P_o = 1 - P_F$$

- Traction average current

$$I_g \triangleq \frac{1}{T_g}\int_0^T i_F\,dt$$

Traction effective current

$$I_{\varepsilon g} \triangleq \sqrt{\frac{1}{T_g}\int_0^T {i_F}^2 dt}$$

- Daily average current

$$I \triangleq \frac{1}{T}\int_0^T i_F\,dt$$

Daily effective current

$$I_\varepsilon \triangleq \sqrt{\frac{1}{T}\int_0^T {i_F}^2 dt}$$

$${I_\varepsilon}^2 T = {I_{\varepsilon g}}^2 T_g = \int_0^T {i_F}^2\,dt$$

$$I_{\varepsilon g} = \frac{I_\varepsilon}{\sqrt{1-P_0}}$$

❖ Daily effective coefficient, traction effective coefficient

$$k_\varepsilon = \frac{I_\varepsilon}{I} \qquad k_{\varepsilon g} = \frac{I_{\varepsilon g}}{I_g} \qquad \Rightarrow \qquad k_\varepsilon = \frac{k_{\varepsilon g}}{\sqrt{1-P_0}}$$

$k_{\varepsilon g}$ = 1.04 ~ 1.08 $k_{\varepsilon g}$ = 1.10 P_0 = 0.2 ~ 0.5 k_ε = 1.23 ~ 1.41

Traction average current in feeding section

$$I_g = \frac{60*A}{t_g U} + 7A$$

Where 7A—Self-electricity for locomotive

A	The total energy consumption when train travel through feeding section kVA.h
V2n	Traction network voltage 25kV
t_g	The traction running time of the train, minutes

❖ Train effective current under traction running time

$$I_{eg} = \sqrt{\frac{1}{t_g}\int_0^{t_g} i^2 dt = k_{eg} I_g}$$

If $I_{eg} = k_{eg} * I_g$, so k_{eg} is train effective current coefficient, Generally k_{eg} = 1.03 ≈ 1.05

Usually k_{eg} = 1.04, ${k_{eg}}^2$ = 1.08

Feeding section segment n

Train density N — daily train operation number (pairs)

t_g -Train traction running time when travelling through a feeding section

A-Train energy consumption when travelling thr

$I_1 = I_2 = \cdots I_n = I_g$ $\qquad$ $t_{g1} = t_{g2} = \cdots t_{gn} = \frac{t_g}{n}$

$I_g = \frac{60*A}{t_g U} + 7A$ $\quad$ i = 1.2…….n

Feeder current daily average value

$$I_F = i_1 + i_2 + i_3 + \cdots i_n = \sum_{k=1}^{n} i_k$$

$$I_F = \frac{1}{T}\int_0^T i_k\, dt$$

$$I_F = \sum_{k=1}^{n}(\frac{2Nt_{gk}}{T}\frac{1}{2Nt_{gk}}\int_0^{2Nt_{gk}} i_k dt)$$

$$I_F = \sum_{k=1}^{n} P_k I_k$$

$P_k = \mathrm{P} = \frac{2Nt_g}{nT}$ traction probability for section

$$I_F = \mathrm{nP}I_g = \frac{2Nt_g}{T} I_g$$

Feeder current effective value

$$I_{FE} = \sqrt{\frac{1}{T}\int_0^T {i_F}^2 dt}$$

$${i_F}^2 = (i_1 + i_1 + i_1 + \cdots i_n)(i_1 + i_1 + i_1 + \cdots i_n)$$

$$\sum_{k=1}^{n} {i_k}^2 + \sum_{k=1}^{n}\sum_{1\neq k}^{n} i_k i_1$$

$$\overline{{i_k}^2} = \frac{1}{T}\int_0^{2Nt_{gk}} {i_k}^2 dt = \frac{2Nt_{gk}}{T}\frac{1}{2Nt_{gk}}\int_0^{2Nt_{gk}} {i_k}^2 dt = P_k {i_{egk}}^2$$

$$\overline{i_k i_1} = \frac{1}{T}\int_0^{2Nt_{gk}} i_k dt \frac{1}{T}\int_0^{2Nt_{gk}} i_1 dt = P_k I_{gk} P_1 I_{g1}$$

$\overline{{i_F}^2} = \sum_{k=1}^{n} {i_k}^2 + \sum_{k=1}^{n-1}\sum_{1\neq k}^{n} i_k i_1$ $\qquad$ $\overline{{i_k}^2} = P_k {I_{egk}}^2$ $\qquad$ $\overline{i_k i_1} = P_k I_{gk} P_1 I_{g1}$

$$\overline{{i_F}^2} = nP{K_{eg}}^2{I_g}^2 + n(n-1)P^2{I_g}^2$$

$= (nPI_g)^2[1 + \frac{{K_{eg}}^2 - P}{nP}]$ $\qquad$ $I_F = nPI_g$

$$= {I_F}^2[1 + \frac{{K_{eg}}^2 - P}{nP}]$$

$I_{FE} = I_F\sqrt{1 + \frac{K_{eg} - P}{nP}}$ $\qquad$ $K_{eF} = \sqrt{1 + \frac{K_{eg} - P}{nP}}$

Feeder of double track unilateral power

Feeder daily average current (same as Single track)

$$I_F = nPI_g = \frac{2Nt_g}{T} I_g$$

Feeder daily effective current Double track feeder effective current

- Feeder daily effective current Double track feeder effective current formula is same as the single track, but not the same value of n.
- At double traction situation, n is represented as the largest number of train (under traction

  operation) which appeared at the feeding sections up and down direction.

$$I_{FE} = I_F\sqrt{1+\frac{{K_{eg}}^2-P}{nP}}$$