ADDIS ABABA UNIVERSITY

ADDIS ABABA INSTITUTE OF TECHNOLOG

SCHOOL OF MECHANICAL & INDUSTRIAL ENGINEERING

SUSPENSION DAMPING INFLUENCE ON RIDE COMFORT OF PASSENGER RAIL VEHICLE

A Thesis Submitted to the Graduate school of Addis Ababa University In partial fulfillment of the Requirements for the degree of Masters of Science

Master of Science

In

Mechanical Engineering

(Under Rail way engineering)

BY

Mohamedamin Gemeda

ADVISOR

Tsegaye Feleke (M. Sc)

May, 2015

DECLARATION

I hereby declare that the work which is being presented in this thesis entitled "Suspension Damping Influence on Ride Comfort of Passenger Railway Vehicles" is original work of my own, has not been presented for a master's degree of any other university and all the resource of materials used for this thesis have been duly acknowledged.

Mohamedamin Gemeda ___________________________

Signature and date

This is to certify that the above declaration made by the candidate is correct to the best of my knowledge.

Tsegaye Feleke (M.Sc) ___________________________

(Advisor) Signature and date

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Approved by Board of Examining:

Birhanu Besheh (Dr.)	______________________	____________________
Railway Center Head	Signature	Date
Tsegaye Feleke (M.Sc)	______________________	____________________
Advisor	Signature	Date
Tolosa Deribe (M.Sc)	______________________	____________________
Internal Evaluator	Signature	Date
Fasil Gesesew (M.Sc)	______________________	____________________
External Evaluator	Signature	Date

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ABSTRACT

This paper analyzes the secondary suspension damping influence of a passenger rail vehicle upon the ride comfort, evaluated via the sterling ride comfort index. The selection of damping ratio is a sensitive issue when a too small damping may compromise the vehicle dynamic performance, and a too high damping leads to an increase in the system dynamic rigidity and, therefore, to an intensification of the vibration behavior.

The complete model of a vehicle with two suspension levels and a flexible car body has been taken into consideration. Upon applying the modal analysis, a new form has been provided for the movement equations that describe the symmetrical and anti-symmetrical movements of the vehicle and their modes of excitation. This work is focusing on optimization of suspension damping coefficient to improve the ride comfort of passenger rail vehicles. In doing rail vehicle body movement equations carried out and performed by MAT-LAB.

An increase in the damping of the secondary suspension may trigger a decrease of the comfort, in dependence with the position along the car body and the velocity. At car body Centre, it will lead to lowering the resonance amplitude for low velocities, whereas the vibration intensifies at higher velocities, thus increasing the Wz index. Above bogie, the increase of ζc has a favorable impact upon comfort, irrespective of speed. At 50 km/h and the same damping ratio ζc = 0.3, there is a lower comfort index Wz by 4%, at car body center and above bogie by about 2 %. The secondary suspension damping greatly influences the ride comfort. This is the reason why the issue of selecting this damping requires a solution of compromise. The primary suspension damping enhances the comfort, but its increase is limited by the wheel/rail dynamic overloads.

Keywords: Passenger rail vehicle, Ride comfort, suspensio damping coefficient, power spectral density of acceleration, sperling index.

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Acronomys & Symbols	**Notations**
A	Factor of track irregularities
a_b	Axle base of bogie
a_c	Axle base of car body
C_{zb}	Vertical damping of primary suspension
C_{zc}	Vertical damping of secondary suspension
EI	Bending module
i_b	Bogie gyration radius
i_c	Car body gyration radius
J_b	Bogie Moment of inertia
J_c	Car body gyration radius
K_{zb}	Vertical rigidity of primary suspension
K_{zc}	Vertical rigidity of secondar suspension
L	Vehicle car body length
η	Track irregularity
Λ	Wave length
ξ	Damping ratio
ξ_m	Modal damping ratio
ω	Damping angular frequency
Ω	Wave number
Ω_c , Ω_r	Truncated wavenumbers
ω_i	Modal angular frequency
ω_o	Natural angular frequency
ω_{te}	Angular frequency induced by the track excitation
RMS	Root Mean Square
PSD	Power Spectral Density
RI	Ride Index
DOF	Degree of Freedom
Wz	Wertungszahl
ERC	Ethiobia railway corporation

CHAPTER ONE

INTRODUCTION

1.1 Background

Railways provide a safe and fast way of transportation. As a matter of higher speeds demands, railway companies are forced to meet more restrictive and severe specifications concerning the dynamics behavior of their railway vehicles. One of the main possibilities to achieve this aim is the improvement of the railway vehicle suspensions [1].

Once dealing with higher values of velocity, important factors like ride comfort and/or safety are forced to suffer from relevant degradation which in critical cases can lead to undesired situations such as a derailment. As a consequence, it is extremely important for the railway companies to consider the ride comfort and safety issues during the operation [1].

Ride comfort of railway vehicles is determined by the combined effects of vibration, noise, temperature, humidity, light, seat texture, height of ceiling, view, ventilation, etc. Accordingly, studies on ride comfort can be classified into the measurement of physical quantities that affect ride comfort, and the measurement of human responses and consciousness. In latter cases, it was very difficult to evaluate ride comfort quantitatively by considering all factors, since the feelings of each passenger according to various factors that affected ride comfort were different. Therefore, vibration accelerations are generally effective in evaluating ride comfort of railway vehicles. Various studies have been conducted to measure vibration accelerations during the running of railway vehicles or roadway vehicles, and to present the requirements and limitations for ride comfort [2].

The ride comfort is a complex concept, representing an essential element in the analysis of the railway vehicles dynamics and needs to be considered when modeling and evaluating their behavior. Besides other factors, the ride comfort primarily depends on the vibration behavior to which the vehicle is subjected [3]. The vibration of railway vehicles, both vertically and horizontally, derives from the track irregularities and need to be mentioned that the two types of vibrations are decoupled due to the construction symmetries [4].

The vertical vibrations – bounce and pitch vibrations –impact the rolling quality, safety, ride comfort and track quality. The rise in the velocity will trigger the increase in the vibration behavior at the car body level, with a negative effect upon the ride comfort; therefore it needs to improve the vehicle construction, by strengthening the car body structural rigidity [5], or the optimization of the suspension parameters [1]. When the suspension system performance is not satisfactory any longer, the solution could be the active suspension [6]. Despite of the favorable results, the active suspension is not a widespread operational solution, due to the fact that the price of implementing and maintenance of this system is too high versus the benefits.

In order to evaluate the ride comfort, there are various international standards, namely ISO 2631 [7], BS 6841 [8], Index Sperling Ride [9], ENV-12999 [10] and UIC 513 [11] – generally speaking, they assess the vibration level in terms of comfort based on the frequency-weighted acceleration. Among them, a simple and widely used method is the Sperling Ride Index [5], which stands out by the fact that its implementation leads in a number with a precise signification that may be easily interpreted, in dependence of the action of different elements in the vehicle vibrant system.

As a starting point to understand the behavior of a rail vehicle and the corresponding effects on the ride comfort, safety and wear it is necessary to investigate different linear and nonlinear system dynamic responses. The critical hunting speed as the origin of the instabilities of a rail vehicle as well as the effects of the wheel conicity, the wheel-rail contact and the track imperfections have been studied thoroughly in [12]. To overcome the negative effects of such parameters on the above mentioned objectives functions, several suspension systems and control strategies have been proposed.

Traditionally, the suspension strategy used in railway vehicles was based on the employment of spring and oil dampers. This type of passive approach is characterized by a high level of simplicity, low-price and the absence of external power supply. Through the careful selection of the suspension design parameters, engineers tried to obtain a compromise between the performances of the vehicle in both straight and curved tracks

[7]. The suspension strategy that takes the advantage of the fully active technology provides the optimal damping response in each time step.

The drawbacks of this Configuration are the high level of complexity concerning the control method and the high level of energy requirements [13]. Because of this, such technology has been applied mainly in the secondary suspension with the aim of improving the ride comfort.

The other type of the new suspension strategies applied on railway vehicles is the so called tilting technology. It is focused on the reduction of the lateral acceleration excess when negotiating a curve [2]. This technology can be based on passive and active actuators and has led to a substantial increase of the velocity on curves. Examples of this technology are the TALGO train (Spain) in the passive case and the ETR-450 “Pendolino” (Italy) or the X2000 (Sweden) in the active one.

As stated in the aforementioned paragraph, most of the semi-active and active suspension systems are more complicated than the corresponding passive techniques and of course need medium to high design and maintenance costs. Passive systems on the other hand can significantly improve the performance and are still a point of interest. However, it is extremely important to formulate and solve several optimization problems to be able to get the best performance out of such systems. In [14] a multi objective optimization with respect to comfort and safety is performed on passive damping elements of both primary and secondary suspension of a railway vehicle obtaining suspension parameters that improve the default performances.

This paper deals with the influence of the suspension damping upon ride comfort evaluated by the Wz index and the maximum level of acceleration. Wz is root mean square value of the frequency-weighted accelerations assessed for definite time limits or definite sections in the track. The Wz index of a vehicle reflects the vehicle ability to maintain the vibrations within the limits that will provide comfort.

1.2. Problem Statement

There are challenges faces when looking for ride comfort as its complexity. For instance the human being interest is different physiologically and biologically. Once the passenger satisfaction is less the competitive of railway transportation is less and then the company loses benefit expected to obtain. The expected benefits may be income as it depends on the number of passengers per trip. The ride comfort can be influenced by various parameters like car body vibrations, stiffness and damping of suspension systems, track irregularity, wheel profile, rail roughness and environmental conditions and so on. The possibility to improve to some extent ride comfort is then studying the above factors and made modifications.

Comfort has both psychological and physiological components, but it involves a sense of subjective well-being and the absence of discomfort, stress or pain. However, comfort is not only defined by the absence of negative attributes. In this thesis, ride comfort, or more precisely ride discomfort, will be used as the technical evaluation of dynamic quantities (motions of the vehicle).

Motion quantities (physical dynamic quantities) are usually: acceleration (lateral, longitudinal and vertical) and angular motions (roll, pitch and yaw). The structural flexibility of the car body can enhance the vibrations at certain frequencies and exert a negative influence on ride comfort. An increase in the damping of the secondary suspension may trigger a decrease of the comfort, in dependence with the position along the car body and the velocity. This is the reason why the issue of selecting this damping requires a solution of compromise. The primary suspension damping enhances the comfort, but its increase is limited by the wheel/rail dynamic overloads.

Presently the ride comfort is to study using Sperling's ride comfort index and the maximum level of acceleration. The influence of some design parameters such as damping and stiffness of the suspension system on the comfort indicators is concerned. The effects of train speed on the comfort indicators are also studied.

The research will also try to answer the following questions.

- Does suspension damping influence on ride comfort?
- Could other factors like vehicle speed affect ride comfort?

1.3. Objective of Study

General Objective

The general objective of study is to analyze the secondary suspension damping influence with speed on ride comfort of passenger railway vehicles using MAT-LAB and analytical computation based on standards.

Specific Objective

This research work specifically aims to:

- Study the influences of the damping of the secondary suspension on the ride comfort and looking for optimal values of the parameters in the practical range.
- Model the train vehicle to analyze the simulations and describe the performance of suspension parameters comparing with the standard specifications.
- Analyze the analytical computations based on frequency weighted acceleration with standard values to predict the level of comfort in passenger trains.

1.4. Scope of Study

As mentioned before there are various options to improve ride comfort of passenger rail vehicle. In this research the influence of suspension damping on ride comfort based on frequency weighted accelerations (sperling comfort index) is discussed. In addition other parameters such as vehicle speed, track irregularity and Power Spectral Density of the trains are taken in account.

1.5. Limitation of Study

Some of the limitations of this study are mentioned as follows:

- Experimental analysis and tests will not be performed in this study due to the absence of laboratory and tools used to perform tests.
- Due to lack of data on human response for ride comfort in Ethiopian context, this paper will not validate the results with experiments.

1.6. Significance of the Study

As a research, the primary merits of the study goes to ERC future research center. Since there are a lot has to be done studies in the area, it will give a comprehensive starting point for more advanced researches. In addition to that if the passenger satisfaction relating to comfort is attractive then the ERC is beneficiary as their income per trip increase with number of passengers and a like. Hence it is better if ERC focus on their customer satisfaction while building their company. It also helps to transport the passengers with comfort and make the railway vehicles competitive with other types of transportation like airplane or passenger cars.

1.7. Organization of thesis

The report consists of six chapters, first being introduction. In second chapter, literature review of various literatures of suspension damping is discussed. The third chapter describes overview of passenger rail vehicle and ride comfort. The fourth chapter deals the analytical and geometry modeling of whole rail vehicle bodies and the numerical application and analysis is described in chapter five. The sixth chapter describes result and discussion. Chapter seven concludes with the conclusion & scope for future work.

CHAPTER TWO

LITERATURE REVIEW

A number of theoretical and experimental investigations on the dynamic response of passive, active and semi-active suspension systems for vehicles have been reported. In these investigations various aspects of suspension system design such as, ride comfort, safety and reliability have been studied. An extensive literature survey has been carried out on the above mentioned and other related topics connected with design of passive, active and semi-active suspension systems by referring various International and National Journals.

Normally in the vibration analysis of vehicle systems, the suspension elements are modeled as linear springs and dampers for simplicity of analysis. In reality, the spring element exhibits a non-linear force versus displacement characteristic; the damper element exhibits a non-linear force versus velocity characteristics. Hence for a realistic vehicle dynamic model, it is necessary to take into account the non-linarites in the passive suspension system.

Gordon Best [15] have presented a process of dynamic optimization of non-linear semi-active suspension controllers. For this, a reasonably realistic suspension ride model has been used including actuator dynamics and damper compliance. A number of semi-active control laws have been analyzed and simulated to compare the optimal performances. It has been seen that a very simple ‘clipped’ linear control law does remarkably well.

Sireteanu Stoia [16] has discussed the method of optimization of passive and semi-active vehicle suspensions by using numerical simulation. The basic idea of this semi-active control strategy is to balance the elastic force by the damping force in order to reduce or even to cancel the forces transmitted through the suspension as long as spring and elastic forces act in opposite direction. The balance logic has been assumed to be achieved with a variable dry friction damper controlled by modulation of normal force applied on the friction plates. The comfort improvement achieved with a semi-active control strategy is compared with the comfort provided by the scheme with the optimum passive linear and quadratic damping.

A SDOF vehicle suspension model with linear elastic characteristic and stationary Gaussian excitation white noise is taken for analysis. It has been shown that the results obtained by numerical simulation are in good agreement with the results obtained by other research workers. Controlled friction is superior to any conventional viscous damper. A variable friction damper can be used to achieve virtually any desired control logic by appropriate adjustment of the force on the friction plates. It has also been shown that by using balance logic the r.m.s. sprung mass acceleration can be reduced by 25-30 %.

Karnoop [17] has presented analytical results for optimum actively damped suspensions under random excitation. A SDOF suspension system subjected to white noise base excitation has been optimized using analytical expressions for i) suspension control force and stroke control and ii) suspension control velocity and stroke control. The results of both the methods have been compared and it has been shown that the analytical results of simple model show trends which may help the results of optimization on more complex models.

Lu et al. [18] have focused on development of a design procedure for the optimization of passive vehicle suspension parameters in a non-linear programming formulation based on statistical analysis of vehicle vibrations and dynamic loads. A 2DOF suspension system has been formulated and results on optimization of parameters of this system have been presented. A numerical example that corresponds to a medium-sized truck travelling on a rough road has been solved using a specially developed algorithm. It has been shown that the simplicity in computations allows fast and inexpensive post-optimal parametric study and development of design charts that give the optimal quantities for any given inputs.

Kumar Rao [19] have analyzed a passenger car with seven degrees of freedom for pitch, bounce and roll motion of the sprung mass and bounce and roll of the front and rear axle . The concept of delayed resonator is adopted for reducing the damping coefficient of the damper. In this method a passive spring–mass–damper system is reconfigured into a real time tunable vibration absorber using an actuator controlled via partial feedback with delay. It has been shown that, by placing delayed resonator on un-sprung mass, one can achieve better passenger ride comfort and car stability.

Majjad [20] has described a quarter car suspension system models and analyzed it with the help of parameter variations. Simulation results of parameter variation studies show that the variation of the damping characteristics has a big influence on resonance frequencies, which means that, the damper parameters have a notable influence on the ride comfort. It has been shown that a way to improve this comfort is to develop a damper controller. The identification of the damper characteristics and the chassis mass will be the base for a good design of suspension controller to improve ride comfort.

Holou et al. [21] have investigated semi-active suspension systems which provide simultaneous real time optimal control of all parameters including roll, pitch and height control. It is claimed that this system provides a tradeoff between the ride comfort and road holding. The novel feature of this system is that, it can also be used for antilock breaking system without any extra cost of the sensors. Two sonar switches are to be used for future development to predict the road variations to prepare the suspension system in advance to respond to road changes in real time. A 2DOF quarter car suspension system has been taken for mathematical analysis and the control force is generated by the damper with variable damping. The body acceleration, tyre deflection and suspension deflection in passive and semi-active suspension systems have been obtained and it has been shown that the r.m.s. values of the these parameters are substantially reduced in the developed semi-active system in comparison with the conventional comparable passive suspension system.

Patil et al. [22] have investigated a half car 4DOF suspension system employing non-linear passive sequential damper. The passive sequential damper is a tunable variable damping unit. The control unit of the damper employs external pressure control valves to modulate, the flow through the orifices, based upon pressure differential across the damper piston. The damper does not require sensors and signal processors as in case of semi-active sequential damper and offers tunable damping characteristics to suit changing applications and road profile. The ride quality improvement potential of a passive sequential damper is investigated. It is shown that the passive sequential damper provides better shock and vibration isolation performance than constant orifice damper.

Sun, S., Deng, H. and Li, W [23] discussed the method of train's vertical vibration suppress. A novel suspension based on Magneto-rheological fluid (MRF) damper and MRF based smart air spring considered. The MRF damper is used to generate variable damping while the smart air spring is used to generate field-dependent stiffness. In the Study the two kind smart devices, MRF dampers and smart air spring, are developed firstly. Then the dynamic performances of these two devices are tested by MTS. Based on the testing results, the two devices are equipped to a high speed train which is built in ADAMS. The skyhook control algorithm is employed to control the novel suspension. In order to compare the vibration suppression capability of the novel suspension with other kind suspensions, three other different suspension systems are also considered and simulated. The other three kind suspensions are variable damping with fixed stiffness suspension, variable stiffness with fixed damping suspension and passive suspension. Generally four different type suspensions are simulated in the study keeping the train runs on random irregular track at the speed of 250km/h. By comparing the passive suspension with damping on-off suspension and stiffness suspension, it conclude that the both variable damping and variable stiffness contribute to further suppress train's vertical vibration and also the suspension with variable stiffness and damping can further attenuate train's vibration. The simulation results indicate that the variable damping and stiffness suspension suppresses the vibration of high speed train better than the other three suspension systems.

Schandl, Lugner, Benatzky, Kozek and Stribersky [24] have designed and simulated a lightweight vehicle model, typically a tram or metro. Less weight of a vehicle car body reduces the structural stiffness and hence has negative impact on the ride comfort, which the researchers have tried to compensate for by using an active vibration reduction system, instead of traditional active suspension. Twelve actuators and twelve sensors were mounted on the car body where structural vibrations could be measured, and a bending moment could be actively applied in order to suppress the first three Eigen-modes. The simulations showed a significant reduction of the structural vibration level of the car body.

In [38] Whole-body vibration transmission influences comfort, performance, and long-term health of the driver is discussed. The study is an objective evaluation of vehicle comfort characteristics based on standard mathematical formulae and frequency analyses. A variety of road types were selected and quantified by using the International Roughness Index (IRI). To assess vibrations transmitted to the passengers, vibration dose values (VDV), kurtosis, frequency response functions (FRF), and power spectral densities (PSD) of the compartment recorded signals were evaluated. SEAT values based on VDV outputs qualified the seat suspension as a vibration isolator, whereas the FRF and PSD quantified that behavior through frequency analyses.

A. Orvnäset al [25]. Discussed a full-scale rail vehicle model used to investigate how lateral ride comfort is influenced by implementing the H∞ and sky-hook damping control strategies. Active lateral secondary suspension with H∞ control to improve ride comfort: simulations on a full-scale model Simulations show that significant ride comfort improvements can be achieved on straight track with both control strategies compared with a passive system.

The study conducted by [38] stated the evaluation of comfort index in rail way vehicles depending on the vertical suspension features. It describes the influence of the vertical suspension features up on the vibrating comfort in terms of velocity. While assuming the idea to minimizing the level of vibration in the critical point, it thus proved the possibility of establishing the best damping of the vertical suspension that leads to the best values of the comfort index.

2.1 Summary of the Reviewed Literatures

In conclusion the exact nature of the suspension system is of great importance to the rail industry and has therefore been the object of many studies. Previous studies have addressed many aspects of the suspension systems and have attempted to define the influence and role of suspension systems in the determination of ride comfort of vehicles reactions. Most of them addressed the optimization and controlling mechanism of suspension systems, and its parameters.

Some of them described the influence of suspension damping and stiffness on ride comfort in terms of velocity and vertical vibrations. In addition the study conducted by [38] focuses on the influence suspension damping on ride comfort of tyre wheel trains and evaluate the level of ride comfort based on sterling ride comfort index. This is related to present study in which the influence of suspension damping on ride comfort in passenger rail vehicles is concerned and the sterling ride comfort index is used to measure the level of comfort. But in this study the variable suspension damping is considered in contrast of the previous study [38] that is based on vertical vibration induced in the train’s car body with suspension damping.

In the light of the above mentioned aspects, suspension system designers are now grappling to develop the suspension systems which can provide an optimum solution to the problem of the conflicting criterion of comfortable ride and better handling, safety and reliability of the rail vehicle. Presently the suspension damping influence on ride comfort is discussed.

CHAPTER THREE

OVERVIEW OF PASSENGER RAIL VEHICLES AND RIDE COMFORT

3.1 Passenger railway vehicle

Passenger Railway Vehicle is one of rail guided vehicle which consist of two main parts:

a) Car body: This part of the vehicle carries the payload, i.e. passengers or goods, and/or the traction equipment.

b) Bogie: It consists of wheel sets, a framework and suspension elements, as shown in Figure 3.1

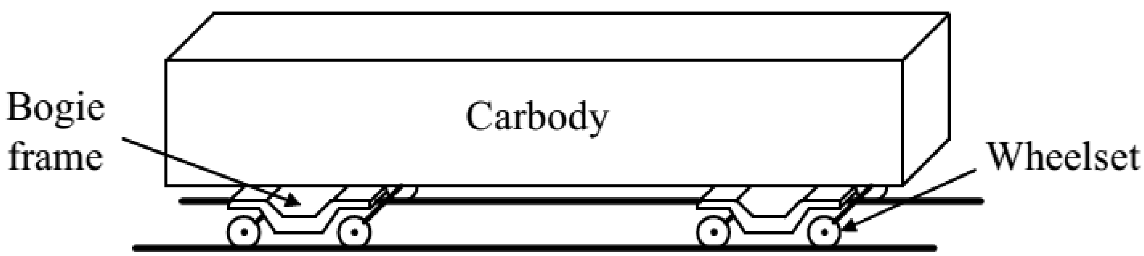


Figure 3.1: Bogie vehicle with two-axle bogies [2]

The bogie vehicles have two levels of suspension, the primary suspension, between the wheel sets and the bogie frame, and the secondary suspension, between the bogie frame and the car body. Though the bogies increase the vehicle weight and costs, they provide isolation for the high frequency contents of the motion due to the inertia of bogie frames. This assemblage has also a geometric advantage since disturbances acting on one wheel set are, in principle, halved at the bogie frame longitudinal midpoint, decreasing their transmission to the car body. The vehicles assembled with bogies have better curving performance and the derailment risk is lower than for rigid frame vehicles. The car body vibrations and the wheel rail contact forces are also reduced as a result of the two levels of suspension. In rail-guided vehicle dynamics, the motion of the vehicle as a whole and the motion of the particular vehicle parts are very important to quantify.

In Table 3.1 the motions corresponding to the six relative degrees of freedom of the rigid bodies that compose a rail vehicle are defined and are represented in Figure 3.2.

Table 3.1 definitions of relative motions [2]

Relative motions	Symbol	Notation
Translation in direction of travel	X	Longitudinal
Translation in transverse direction, parallel to the track plane	Y	Lateral
Translation perpendicular to the track plane	Z	Vertical
Rotation about longitudinal axis	Φ	Roll, Sway
Rotation about a transverse axis, parallel to the track plane	X	Pitch
Rotation about an axis perpendicular to the track plane	Ψ	Yaw

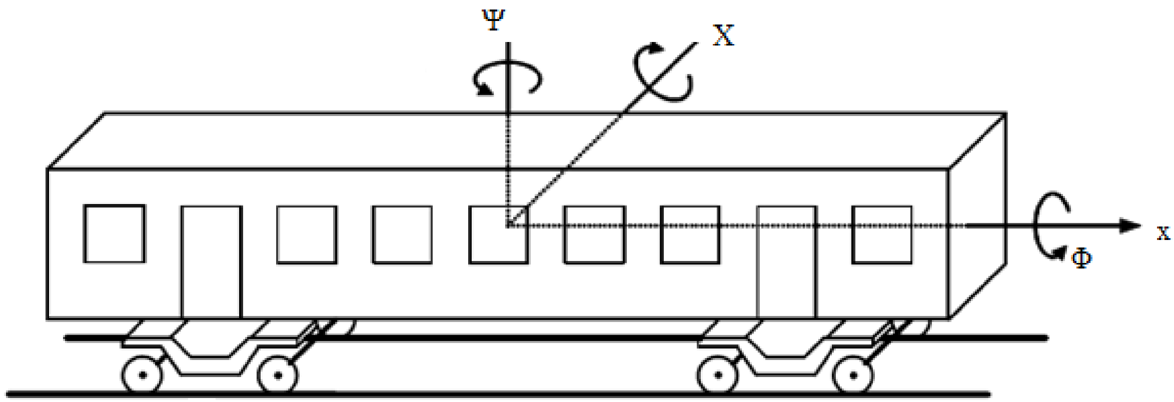


Figure 3.2: Relative rigid body motions of a car body [2]

Passenger rail vehicles classified by operation characteristics as; light rail vehicle, underground or metro vehicle, suburban coach, ordinary passenger car and high speed passenger car.

3.1.1 Passenger Rail Vehicle Composition

Bogie: Guide the vehicle running on track, support car body and mitigate force from the track

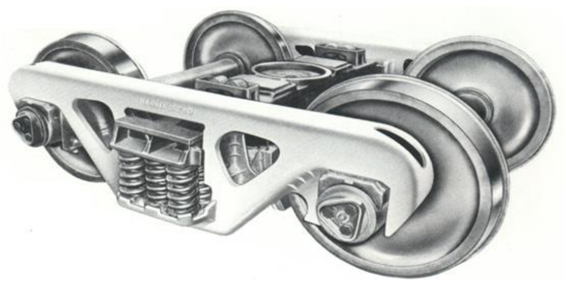


Fig 3.3 passenger rail vehicle bogie [14]

The Rolling motion of wheel set is to forward motion of car and transmit vehicle load and wheel/rail force. Axle box connecting wheel set and bogie frame. Frame, Side frame is the foundation of bogie, connecting all parts, Support and transmits different loads and forces.

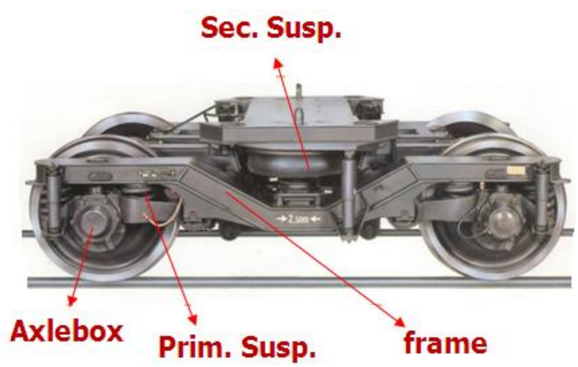


Fig 3.4 bogie frame, axle box and suspensions [14]

In the case studied in this thesis, wheels of steel are rolling on steel rails. This is also by far the most common technique for rail vehicles in the world,[2]. By using this technique, the vehicle is generally automatically steered. (The exception is if the wheel axle is removed and the wheels individually steered.) The steering works in the following way: The wheels are shaped in a way that makes the running circle longer if the wheel is moving outwards on the rails.

At the same time the wheel on the other side moves inwards (assuming the distance between the rails, called gauge, is constant). This way the wheels on the left and the right side travel different far during one revolution, and this way the wheel set is steering towards centering over the rails.

In many cases, including in this thesis, the wheel sets are connected (by a suspension) to a bogie (wheel). The bogies are then connected (by a suspension) to the car body. Using bogies improves the curving performance and decreases the risk of derailment, compared to using a suspension directly between the wheel sets and car body, [26]. Also, using bogies makes it possible to reduce car body vibrations as well as wheel-rail forces, [26].

3.1.1.1 Suspension systems

In passenger railway vehicles, suspension systems are used for isolating vibrations and improving comfort for the passengers. In general, the suspension is the set of elastic elements (springs), dampers and associated components which connect the wheel sets to the car body [27].

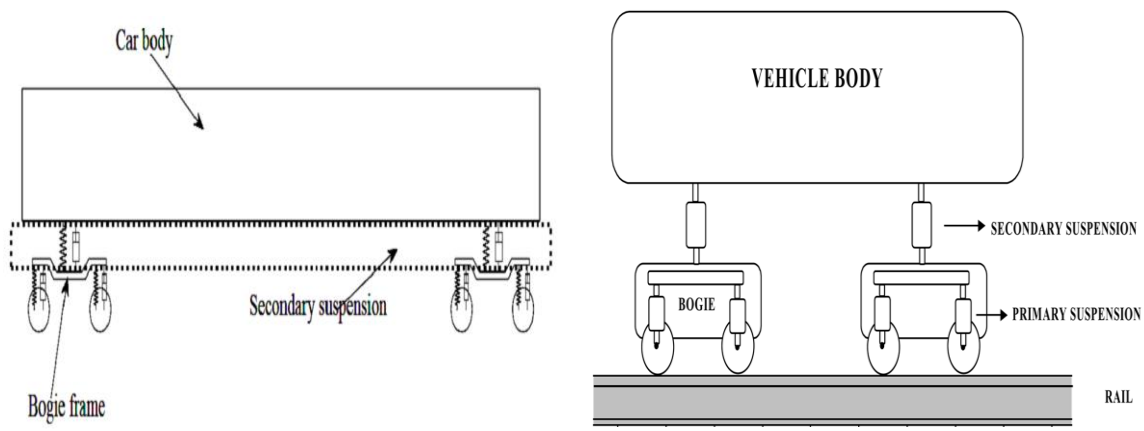


Figure 3.5: Schematic picture of where the secondary suspension is located (It is within the dotted box) [27].

The primary suspension is the suspension between the wheel set and the bogie frame. The secondary suspension is the suspension between the bogie frame and the car body.
The springs are used to equalize the vertical loads between the wheels, stabilize the motion of the vehicle on the track, and to reduce the dynamic forces and accelerations due to track irregularities. Dampers are used to damp the oscillations in the suspension.

3.1.1.1.1 Active suspension system

In the field of rail technology, there are continuously rising requirements concerning riding comfort, running safety, and speed from the side of the railway operators.

These requirements are opposed by the fact that the condition of the tracks is getting worse and maintenance is becoming expensive. In view of this conflict, conventional suspension concepts are quickly at their limits. To meet the very conflicting requirements for introducing active suspension systems, in an economical way, is paramount [28].

An active suspension system mainly comprises of actuators, sensors, and electronic controllers [27]. As a comparison, the conventional, passive, suspension is purely mechanical.

There are three major categories of active suspension that are studied for railway vehicles: active tilting, active secondary suspensions and active primary suspensions.

Tilting of the car body is used to reduce the quasi-static lateral acceleration experienced by the passengers in curves, and by this improving passenger ride comfort. Active tilting is a standard technology for railway vehicles [27].

Secondary active suspension is intended to improve the vehicle dynamic response and provide a better isolation of the vehicle body to the track irregularities, compared to a fully passive suspension. The improved performance could for instance be used to improve the ride comfort for the passengers [27].

Active primary suspension is intended to improve running stability and curving performance. There is a trade-off between those issues, and it is difficult to further improve both simultaneously with passive techniques. When using active primary suspension, the wheel sets can be either independently rotating, or connected by a solid axle [27].

3.2. Ride comfort

Ride quality is a person's reaction to a set of physical conditions in a vehicle environment, such as dynamic, ambient and spatial variables.

Dynamic variables consist of motions, measured as accelerations and changes (jerk) in accelerations in all three axes (lateral, longitudinal and vertical), angular motions about these axes (roll, pitch and yaw) and sudden motions, such as shocks and jolts. Normally, the axes are fixed to the vehicle body (ISO 1999).

The ambient variables may include temperature, pressure, air quality and ventilation, as well as noise and high frequency vibrations, while the spatial variables may include workspace, leg room and other seating variables. Other factors may be convenience of the transport, frequency, etc.

However, many use the term passenger comfort, ride comfort or average ride comfort for ratings on a ride quality scale regarding the influence of dynamic variables. Normally, higher rating on a ride quality scale means better comfort, whereas higher rating on a ride (dis-)comfort scale means less comfort.

In this thesis, **ride comfort**, or more precisely ride discomfort, will be used as the technical evaluation of dynamic quantities (motions of the vehicle). This is in accordance with CEN and in general with ISO (CEN 1996a, CEN 1999, ISO 1996b, ISO 1997, ISO 1999). This technical evaluation is based upon human reactions to these dynamic quantities. However, there is much argument concerning the appearance of these relations, e.g. differences in weighting curves, evaluation formulas and statistical approach.

Psychological variables are important and can modify the severity of the human responses. Modifying human variables include age, gender, posture, alcohol, experience and mental activity [30].

Among these variables are expectation and suggestion, specific conditioning effects of past experiences, habituation effects of past experience, effects of concurrent activity and effects of concurrent emotional state [31].

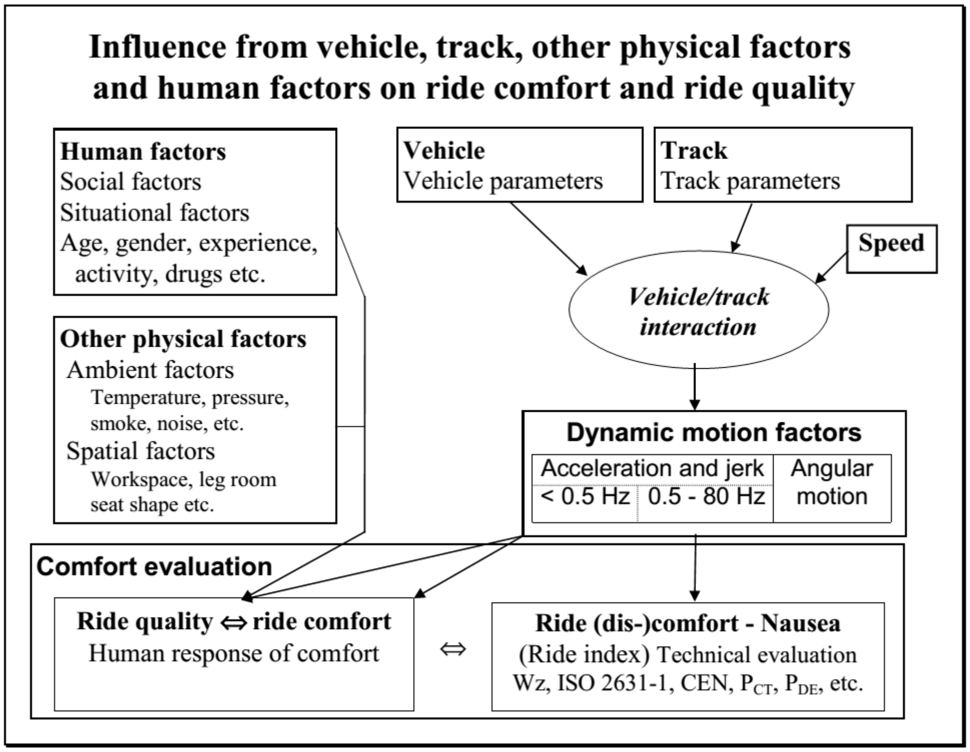


Figure 3.6 Interaction in the field of ride comfort and ride quality evaluation.

The human response (ride quality) involves human variables as well as dynamic motion and other physical variables, but the ride comfort response and technical evaluation of ride (dis-)comfort involves dynamic motion variables only [33].

Motion quantities are also influenced by the type of vehicle, suspension system, tilt system, type of track and the speed of the train. The structural flexibility of the car body can enhance the vibrations at certain frequencies and exert a negative influence on ride comfort [34].

Ride comfort, as human reaction to and technical evaluation of dynamic variables can be divided indifferent categories:

Average ride comfort/discomfort level (ride index), regarding only accelerations (longitudinal, lateral and vertical) in a frequency interval from 0.5 Hz to 80 Hz.
Higher average values show a decreased level of ride comfort and increased level of discomfort. Examples for comfort/discomfort scales are: Wz (Sperling & Betzhold 1956), Ride Index, NMV (CEN 1996), evaluation according to ISO 2631-1 (ISO 1997).

Estimated ride comfort, the subjects rated ride comfort on a 5-point scale from 1 (very poor ride comfort) to 5 (very good ride comfort) [35].

Comfort disturbances due to motions such as high horizontal accelerations, jerks and jolts(Jerk is the rate of change in acceleration. In a train environment, jerk is normally associated with the change of lateral acceleration on a transition curve. Jolt is a sudden motion caused by passing a turn-out or an irregularity in track alignment.).

The first type is a direct comfort disturbance caused by a sudden motion of the vehicle, a so called discrete event [36], resulting in discomfort, such as difficulties in walking, standing, reading or writing. It may have both a high and low frequency content and may be caused by passage of a turnout or some kind of irregularity in track alignment. Discrete events are also comfort disturbances caused by the combined effects of high lateral forces in circular curves and track irregularities.

The second type of disturbance is caused by high lateral accelerations and/or lateral jerks while negotiating transition curves [36]. These accelerations and jerks are results of a changing track geometry (horizontal alignment and cant) together changing tilt angle (tilting train).
The resulting disturbance may also be problems with balance while standing or walking, etc. Works that have addressed the problem of balance include Abernethy, Jacobs, Plank, Stokosa, &Sussman (1980), Graaf de, Bles, &Bos (1997), Nashner (1971).

Harborough (1986a) and CEN (1996) evaluated acceleration and roll velocities up to 2 Hz while in Sweden lateral acceleration is evaluated up 0.5 Hz and jerks up to 0.3 Hz [37].

The third type is **Motion sickness** due to prolonged low-frequency translational (< 0.5 Hz) and/or angular motions. This third type of discomfort results in shakiness, headache or nausea after a shorter or longer period of travel, i.e. motion sickness.

Another problem is the effects of vertical vibrations (0.5 –10 Hz) on writing and drinking for passengers. This has been described by Corbridge and Griffin (1991). It seems like 4 Hz has largest influences on spilling water while drinking and 5 –6 Hz for writing difficulties.

3.2.1 Ride Comfort Evaluation

One of the most important aspects that any type of transportation must ensure is an acceptable level of comfort perceived by the passengers. However, it is a complicated parameter to be measure since it is not defined only by means of physically quantifiable quantities but also by subjective perceptions of each passenger. Among all the possible measureable features that define the ride comfort level in railways, the most widely used in the normative is the value of accelerations inside the car [38].

3.2.1.1 Wertungszahl (Wz)

This comfort index was defined by the German researchers Sperling and Betzhold and it is based on the measurement of the accelerations on the floor of the car body [9]. The frequency weighting functions used in this index are defined so that the passenger is considered to be more affected by frequencies in the range 3 to 7 Hertz.

Good ride comfort is one important issue to aim for with in rail vehicle development. Different conditions require various evaluations methods. Therefore it is difficult to establish universally applicable international standards on ride comfort of rail vehicles. The relation between the comfort index and the sensitivity to vibrations is assessed on a 1 to 4 scale, where Wz = 4 then the vibrations level has a harmful impact upon the human body at a prolonged exposure.

Table 3.2 Show the level of comfort when using the Wz approach

Wz Ride Comfort classification [9]

Ride Index	Comfort Level
1	Just noticeable
2	Clearly noticeable
2.5	More pronounced but not unpleasant
3	Strong, irregular, but still tolerable
3.25	Very irregular
3.5	Extreme irregular, unpleasant
4	Extremely unpleasant. Harmful

This approach is the one selected to compute ride comfort objective function in this thesis.

3.2.1.2 UNE – ENV 12299

Another approach to determine the comfort level in railways is described by The European Committee of Normalization in the document EN-12299. Taking into account the standards UIC-513 and ISO-2631, two hierarchical approaches are defined [CEN 1999].

The first one, characterized by not being compulsory, regards the accelerations inside the vehicle measured not only at the vehicle´s floor but also at the passenger’s seat in the three directions. Moreover, it takes into consideration the effects of the curve transitions and discrete events.

The second one is declared as mandatory in the standard and defined as a simplified method based on measurements of acceleration on the floor of Mean Comfort.

It is calculated using the following equation:

$$N_{MV} = 6 * [(a_{XP\,95}^{\,Wad})^2 + (a_{YP\,95}^{\,Wad})^2 + (a_{ZP\,95}^{\,Wab})^2]^{0.5}$$

Where, $a_{iP\,95}^{\,Waj}$ stands for the 95% of the root-mean-square (r.m.s) value of the frequency weighted accelerations (m/s^2) measured at floor level in the three directions. The r.m.s value must be computed over periods of five seconds in order to take into account the lowest frequencies.

The weighting functions recognize the vibrations at frequencies in the range from 0.5 to 80 Hz as the main interval affecting the passengers. The scale to estimate the ride comfort level is shown in Table as follows.

Table 3.3 shows Ride Comfort classification [CEN 1999].

$N_{MV} < 1$	Very comfortable
$1 \leq N_{MV} < 2$	Comfortable
$2 \leq N_{MV} < 4$	Medium
$4 \leq N_{MV} < 5$	Uncomfortable
$N_{MV} \geq 5$	Very uncomfortable

According to the standard, to properly determine the ride comfort index in a railway vehicle, the value of the mean comfort must be calculated in three points along the railway vehicle, particularly above each bogie and at the center of the vehicle.

3.2.2 Human Sensitivity for Accelerations at Different Frequencies

Human beings are sensitive to shaking and can find that unpleasant. The amount of discomfort experienced varies with the frequency of the acceleration. It is possible to weigh the accelerations for a compound motion together and form a single number that can be used to compare the level of discomfort. This number is called ride index or Wertungszahl (Wz*)*. The higher the number is, the worse is the comfort. How to evaluate comfort is described in this paper since one of the criterions on if the secondary suspension is good, is passenger comfort.

Generally this section described overview of suspension systems and ride comfort. The influence of suspension systems on ride comfort and the evaluation of ride comfort are discussed. Suspension parameter is one of the factors that influence the ride comfort. As a result this thesis is focused on the suspension damping influence on ride comfort of passenger rail vehicles. In doing so the application of ride comfort index is performed by using parameters used in this context.

CHAPTER FOUR

MODELING

4.1. General Rail Vehicle Model

The vehicle body which rests on two bogies each containing two wheel sets (each includes an axle and two wheels) is investigated. A model with 19-DOF is shown in Figure 4.1. The spring and damping elements connecting the wheel set and bogie frame are called the primary suspension. The secondary suspension connects the bogie frame and the vehicle body. The model parameters can be found in the Appendix. They include the vertical, lateral motions and yaw, roll, pitch angles of the car body and the front and rear bogie frames, vertical and lateral motion and yawing angle of front and rear axles.

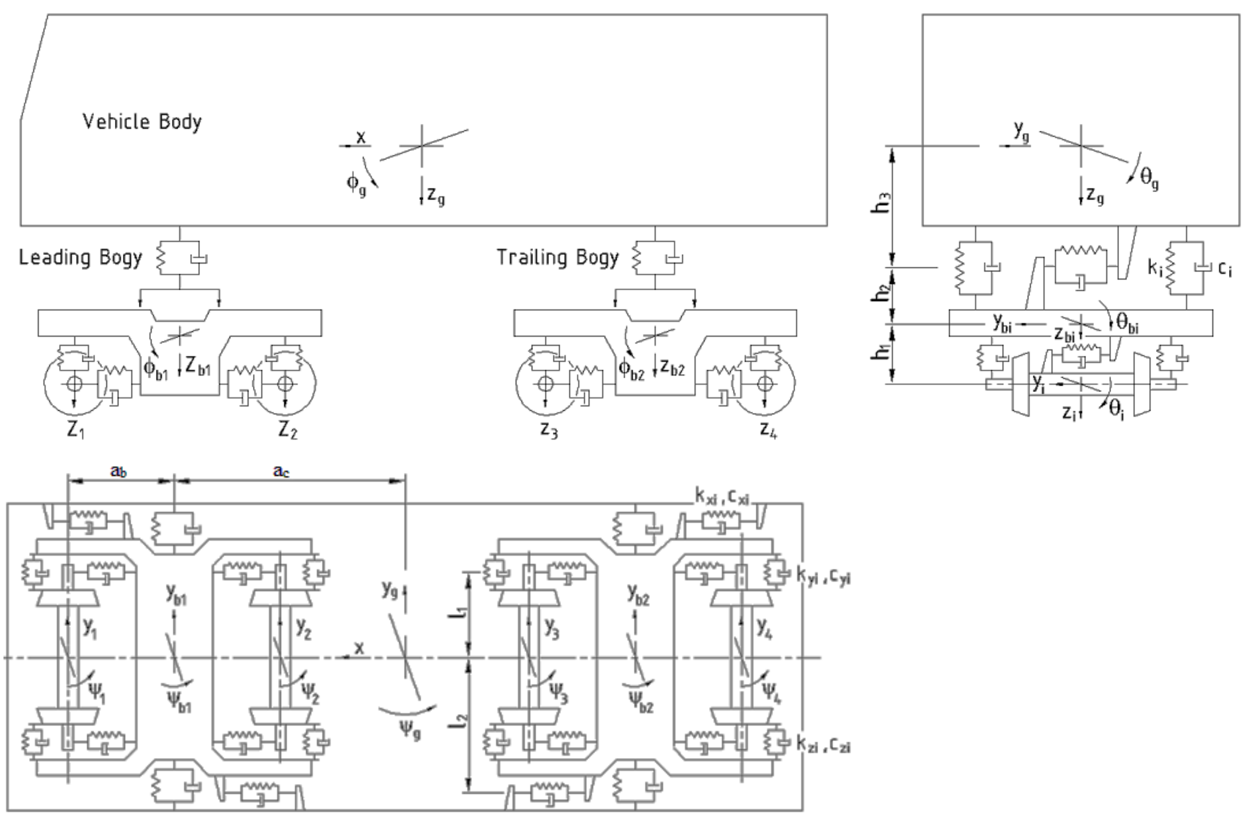


Figure 4.1 A rail vehicle model with 19-DOF [2]

To analyze the dynamic behavior of railway vehicles, it is represented as a multi-body system. A multi-body system consists of rigid bodies, interconnected via mass less force elements and joints. Due to the relative motion of the system's bodies, the force elements generate applied forces and torques. Typical examples of such force elements are springs, dampers, and actuators combined in primary and secondary suspensions of railway vehicles.

4.2 Assumptions

The assumptions made in formulating the model are as follows:

- The springs and dampers of the suspension system elements have linear characteristics.
- The vehicle is moving with constant velocity on rigid and constant gauge.
- An irregularity in vertical direction with same shape for left and right rail is considered in this model.
- Only vertical secondary suspension parameters are considered.

4.3 Model for ride comfort

Track irregularities, vehicle characteristics and vehicle speed generate motion quantities that are perceived by passengers.

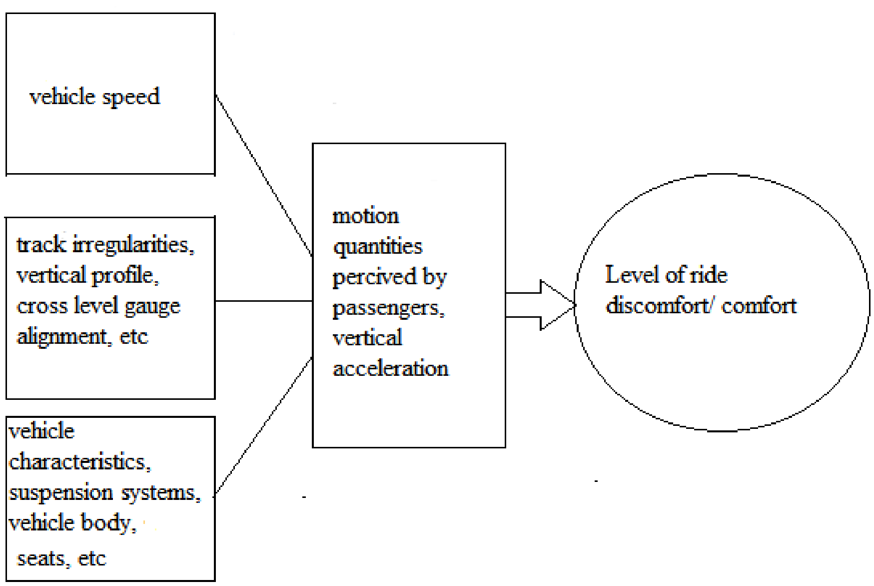


Fig.4.2 Significant track and vehicle parameters for ride comfort [33]

4.4. Mechanical Model and Movement Equations

The case of a two-floor suspension railway vehicle that travels on a constant speed Von a track with random irregularities is considered (Fig. 4.3). The vehicle car body of a length L is modeled via an Euler-Bernoulli beam of constant section and uniformly distributed mass, with the bending module EI, mass on length unit, m and damping coefficient ξ. The displacement of a beam section is w(x, t), where t is time.

Here, only the car body bending modes have been considered for both generality and simplicity. When a specific structure is studied and the car body global and local mode shapes are taken into account, the FEM car body model has to be used.

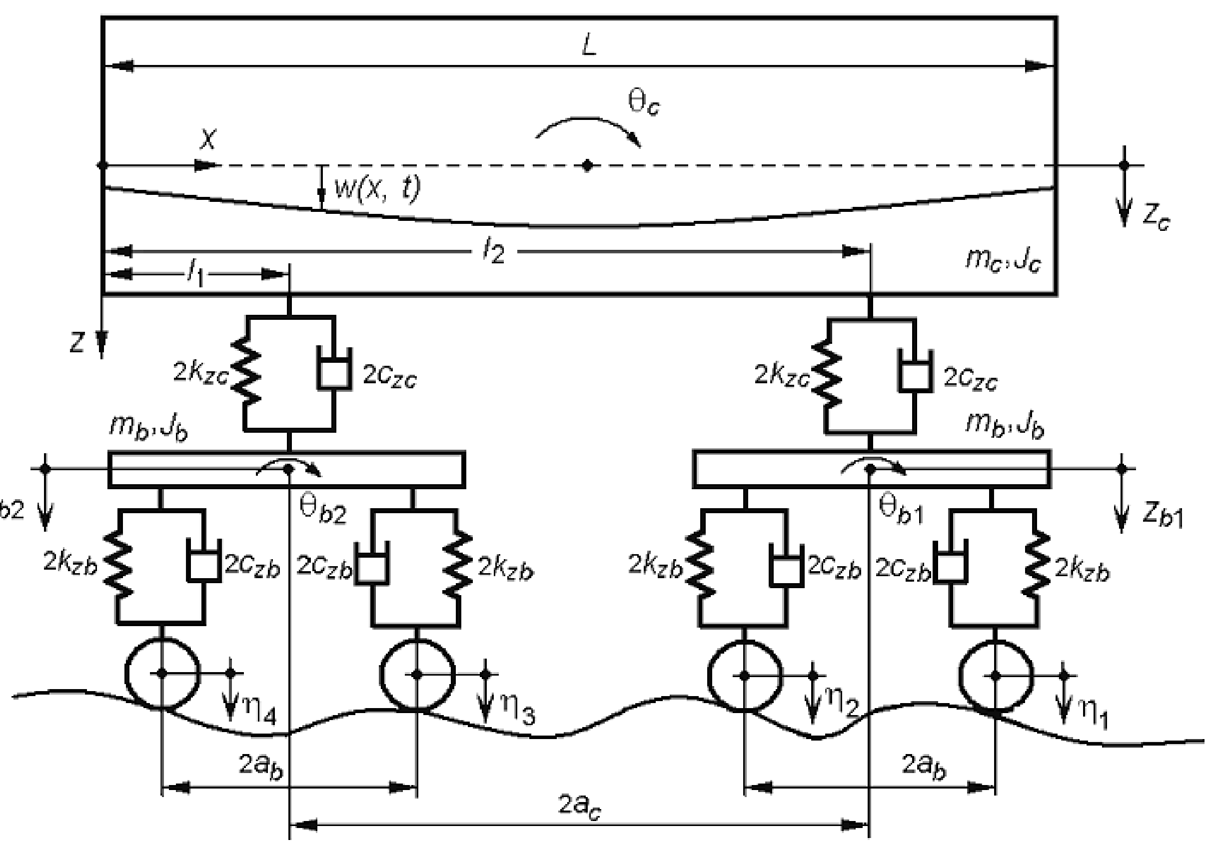


Fig 4.3 the vehicle 2D mechanical model

The hypothesis of rigid track is adopted because the track rigidity is much higher than the one of the vehicle suspension and the frequencies of wheel sets on the track are much higher than the vehicle's. Therefore, the vertical wheel set displacement equals the corresponding irregularity.

The bogies suspended masses are considered two-degree freedom rigid bodies, namely the bounce movement z_{bi} and pitch θ_{bi}, with i = 1, 2. The mass of a bogie is m_b and its inertia moment $2J_b = m_b(i_b)^2$, with i_b– the bogie gyration radius. The suspension levels are modeled via Kelvin-Voigt systems. The elastic constants are k_{zb}, k_{zc}, and the damping ones c_{zb}, c_{zc}.

The movement equations are:

- For bending the car body

$EI\, \partial^4 W(x, t)/ \partial x^4 + I\, \mu\, \partial^5 W(x, t)/ \partial x^4 \partial t + m\, \partial^2 W(x, t)/ \partial t^2 = \sum_{i=1}^{2} Fi\, \delta\, (x - li)$ ----------------(1)

Where δ(.) is Dirac's delta function, and Fi represents the force due to the i secondary bogie suspension that acts at the distance l_i from the car body end.

$F_i = -2C_{zc}\, (\dot{w}(l_i, t) - \dot{z}_{bi}) - 2K_{zc}\, (w(l_i, t) - z_{bi})$--(2)

- For bounce of bogies

$m_b\ddot{z}_{b1,2} + 2c_{zb}(2\dot{z}_{b1,2} - \dot{\eta}_{1,3} - \dot{\eta}_{2,4}) + 2k_{zb}(2z_{b1,2} - \eta_{1,3} - \eta_{2,4}) + 2c_{zc}[\dot{z}_{b1,2} - \dot{w}(l_{1,2}, t)] +$

$2k_{zc}[z_{b1,2} - w(l_{1,2}, t)] = 0$---(3)

- For pitch of bogies

$I_b\ddot{\Theta}_{b1,2} + 2c_{zb}a_b(2a_b\, (\dot{\Theta}_{b1,2} - \dot{\eta}_{1,3} + \dot{\eta}_{2,4}) + 2k_{zb}a_b(2a_b\, \Theta_{b1,2} - \eta_{1,3} + \eta_{2,4}) = 0$-------------------------(4)

Where a_b, a_c are the axle bases of bogies and car body and η_i with i= 1,..,4, the irregularities against the wheel set i. It may be noticed that the pitch movements of bogies are decoupled from the other movements of the vehicle.

The movement equations with partial derivate may be turned into equations with ordinary derivate by implementing the method of modal analysis.

For this, the rigid and bending modes of car body are considered as

$W(x, t) = z_c(t) + (0.5L - x)\theta_c(t) + \sum_{i=2}^{\infty} Xi(x)Ti(t)$ ---(5)

Where $z_c(t)$ and $\theta_c(t)$ represent the car body vibration rigid modes, namely the bounce and pitch. $T_i(t)$ is time-dependent coordinate, and $X_i(x)$ is the Eigen-function of vibration mode i at bending

$X_i(x) = \sin\beta_i x + \sinh\beta_i x - ((\sin\beta_i L - \sinh\beta_i L)/(\cos\beta_i L - \cosh\beta_i L))(\cos\beta_i x - \cosh\beta_i x)$-------------------(6)

With $\beta_i = [\omega_i^2 m / (EI)]^{0.5}$ where ω_i is the natural angular frequency of the vibration mode i.

Looking at the two first bending modes only, symmetrical and anti-symmetrical, the vehicle vibration is described by a set of equations below, among which 6 are coupled, and other two are independent (pitch of bogies).

An appropriate choice of the coordinates and a correct processing of the equations system will give the decomposition of the set with 6 movement equations, coupled in two independent sets of 3 equations each.

The two systems describe the car body symmetrical and anti-symmetrical movements.

If $q_1^+ = z_c$; $q_1^- = \Theta_c$; $q_2^+ = T_2$; $q_2^- = T_3$;

$q_3^+ = 0.5(z_{b1} + z_{b2})$; $q_3^- = 0.5(z_{b1} - z_{b2})$;

$x_2(l_1) = x_2(l_2) = \varepsilon^+$; $x_3(l_1) = -x_3(l_2) = \varepsilon^-$

Then the equations of the symmetrical movements are obtained

$m_c\ddot{q}_1^+ + 4c_{zc}(\dot{q}_1^+ + \varepsilon^+\dot{q}_2^+ - \dot{q}_3^+) + 4k_{zc}(q_1^+ + \varepsilon^+ q_2^+ - q_3^+) = 0$;

$m_{m2}\ddot{q}_2^+ + c_{m2}\dot{q}_2^+ + k_{m2}q_2^+ + 4c_{zc}\,\varepsilon^+ [(\dot{q}_1^+ + \varepsilon^+\dot{q}_2^+ - \dot{q}_3^+)] + 4k_{zc}\,\varepsilon^+[q_1^+ + \varepsilon^+ q_2^+ - q_3^+] = 0$;

$m_b\ddot{q}_3^+ + 4c_{zb}(\dot{q}_3^+ - \dot{\eta}^+) + 4k_{zb}(q_3^+ - \eta^+) + 4c_{zc}(\dot{q}_3^+ - \varepsilon^+\dot{q}_2^+ - \dot{q}_1^+) + 4k_{zc}(q_3^+ - \varepsilon^+ q_2^+ - q_1^+) = 0$;

Along with the anti-symmetrical equations

$J_c\ddot{q}_1^- + 4c_{zc}a_c((a_c\dot{q}_1^- - \varepsilon^-\dot{q}_2^- + \dot{q}_3^-) + 4k_{zc}\,a_c(a_c\,q_1^- - \varepsilon^- q_2^- + q_3^-) = 0$

$m_{m3}\ddot{q}_2^- + c_{m3}\,\dot{q}_2^- + k_{m3}q_2^- + 4c_{zc}\,\varepsilon^- [(-\dot{q}_1^- + \varepsilon^-\dot{q}_2^- - \dot{q}_3^-)] + 4k_{zc}\,\varepsilon^-[-q_1^- + \varepsilon^- q_2^- - q_3^-] = 0$;

$m_b\ddot{q}_3^- + 4c_{zb}(\dot{q}_3^- - \dot{\eta}^-) + 4k_{zb}(q_3^- - \eta^-) + 4c_{zc}(\dot{q}_3^- - \varepsilon^-\dot{q}_2^- - \dot{q}_1^-) + 4k_{zc}(q_3^- - \varepsilon^- q_2^- - a_c\,q_1^-) = 0$;

where m_c and $2J_c = m_c i_c$ are the car body mass and inertia moment with i_c its gyration radius and $m_{m2,3}$, $c_{m2,3}$ and $k_{m2,3}$ are the masses, damping and stiffness of the second and third mode

$m_{m2,3} = m\int_0^L x_{2,3}^2(x)dx$; --(7)

$k_{m2,3} = EI\int_0^L (d^2x_{2,3}(x)/dx^2)^2\,dx$; $c_{m2,3} = \mu I\int_0^L (d^2x_{2,3}(x)/dx^2)^2\,dx$ -------------------------------------(8)

The below equations include the excitation mode of the anti-symmetrical symmetrical movements

$4(\eta_1)^+ = \eta_1 + \eta_2 + \eta_3 + \eta_4$; $4(\eta)^- = \eta_1 + \eta_2 - \eta_3 - \eta_4$ --(9)

The parameters of the car body bending vibration are the modal angular frequency and the damping ratio.

$\omega_{m2,3} = \sqrt{(k_{m2,3} / m_{m2,3})}$, $\xi_{m2,3} = c_{m2,3} / (2\sqrt{(k_{m2,3} m_{m2,3})}$ --(10)

In order to facilitate the analysis of vibrations, the damping ratio of the suspension levels considered uncoupled, as below

$\xi_{b,c} = 4c_{b,c} / (2(\sqrt{4k_{b,c} m_{b,c}}))$ --(11)

In the following section, the movement equations will be used to evaluate the comfort of a passenger vehicle.

To calculate the frequency response, the track irregularities are considered to have a harmonic shape, with the wave length Λ and amplitude

$\eta(x) = \eta_0 \cos(2\pi / \Lambda)x$, where x is the coordinate along the track.

The size of the irregularities against each axle depends on their position. While considering the above equation to correspond to the track irregularity in the middle of the vehicle and that the position of the vehicle reported to the track is given in the relation x =Vt, then the defects against the axles are de-phased by $2\omega_{te}(a_c \pm a_b)/\Lambda$ - against the front bogie and by $2\omega_{te}(-a_c \pm a_b)/\Lambda$ against the axles of the rear bogie, where $2a_c$ is the distance between bogies center and $2a_b$ stands for the bogie wheelbase.

Thus, the car body frequency response is calculated, taking into account the vibration harmonic behavior where the track irregularities have a sinusoidal shape with the wavelength Λ and the amplitude η_0

$\eta_{1,2} = \eta_0 \cos(\omega_{te}(t \pm ((a_b + a_c)/v))$, $\eta_{3,4} = \eta_0 \cos(\omega_{te} (t \pm ((a_b - a_c)/v))$-------------------------------(12)

Where $\omega_{te} = 2\pi v / \Lambda$ is the angular frequency induced by the track excitation.

The vibrations of car body are excited by the track irregularities against the wheel sets, and the plan of bogie axles has both a bounce and pitch movement.

During the steady-state harmonic behavior, all the variables of the system are harmonic, with the angular frequency being induced by the irregularity of the track. Upon introducing the complex variables associated to the real ones,

$z_c = z_c e^{i\omega t}$, $z_c^* = z_c^* e^{i\omega t}$, $z_b^{\pm} = Z_b^{\pm} e^{i\omega t}$, $z_b{}^*_{1,2} = Zb_{1,2}{}^* \, e^{i\omega t}$,---(13)

$\eta_{1,2} = \eta_0 e^{i\omega t} e^{i\,(\omega/v)\,(a2\,\pm a1)t}$, $\eta_{3,4} = \eta_0 e^{i\omega t} e^{i\,(\omega/v)\,(-a2\,\pm a1)t}$

the frequency-response factors may be calculated:- for the bounce movement in the car body center

$H_c = z_c / \eta_0 = [((\omega_c^2 + \alpha_c\omega i)\ (\omega_b^2 + \alpha_b\omega i))/((\ \omega_c^2 - \omega^2 + \alpha_c\omega i)(\mu\ \omega_c^2 + \omega_b^2 - \omega^2 + (\mu\alpha_c + \alpha_b)\omega i) - \mu(\omega_c^2 + \alpha_c\omega i)^2]\ H_v^+$--(14)

- for the effect of car body pitch opposite the bogie

$H_c^- = z_c^- / \eta_0 = [(\varepsilon_c(\omega_c^2 + \alpha_c\omega i)\ (\omega_b^2 + \alpha_b\omega i))/((\varepsilon_c\ \omega_c^2 - \omega^2 + \varepsilon_c\ \alpha_c\omega i)(\mu\ \omega_c^2 + \omega_b^2 - \omega^2 + (\mu\alpha_c + \alpha_b)\omega i) - \mu\varepsilon_c(\omega_c^2 + \alpha_c\omega i)^2]\ H_v^-$ --(15)

- for the symmetrical bounce movement of the bogies

$H_b^+ = z_b^+ / \eta_0 = [((\omega_c^2 - \omega^2 + \alpha_c\omega i)\ (\omega_b^2 + \alpha_b\omega i))/((\ \omega_c^2 - \omega^2 + \alpha_c\omega i)(\mu\ \omega_c^2 + \omega_b^2 - \omega^2 + (\mu\alpha_c + \alpha_b)\omega i) - \mu(\omega_c^2 + \alpha_c\omega i)^2]\ H_v^+$---(16)

- for the anti-symmetrical bounce movement of the bogies

$H_c^- = z_c^- / \eta_0 = [(\varepsilon_c(\omega_c^2 - \omega^2 + \alpha_c\omega i)\ (\omega_b^2 + \alpha_b\omega i))/((\varepsilon_c\ \omega_c^2 - \omega^2 + \varepsilon_c\ \alpha_c\omega i)(\mu\ \omega_c^2 + \omega_b^2 - \omega^2 + (\mu\alpha_c + \alpha_b)\omega i) - \mu\varepsilon_c(\omega_c^2 + \alpha_c\omega i)^2]\ H_v^-$ --(17)

- for the pitch movement of the bogies above the axle

$H_{b1,2}{}^* = z_{b1,2}{}^* / \eta_0 = ((-\ \omega^2 + \varepsilon_b\ \alpha_b\omega i)/(\ \varepsilon_b\ \omega_b^2 - \omega^2 + \varepsilon_b\ \alpha_b\omega i))H_{pb1,2}$-----------------------------------(18)

where we have filtering factors due to the wheelbase and bogie spacing effect

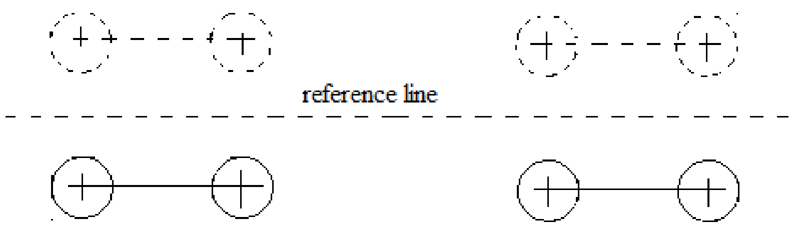
$H_v^+ = \cos(\omega/v)a_1 \cos(\omega/v)a_2$; $H_v^- = i\cos(\omega/v)a_1 \sin(\omega/v)a_2$; $H_{b1,2} = e^{\pm i(\omega/v)a2} i\sin(\omega/v)a_1$-----(19)

With $\omega_c^2 = 4k_2/m_c$, $\alpha_c = 4c_2/m_c$, $\omega_b^2 = 4k_1/m_b$, $\alpha_b = 4c_1/m_b$, $\mu = m_c/2m_b$, $\varepsilon_c = a_2/i_c^2$,$\varepsilon_b = a_1/i_b^2$

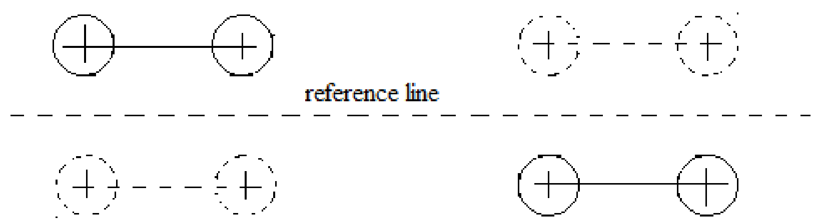
And $\alpha_c = 2\xi_c\omega_c$, $\alpha_b = 2\xi_b\omega_b$

Where $\xi_c = \frac{4c_2}{2\sqrt{4k_2m_c}}$; $\xi_b = \frac{4c_1}{2\sqrt{4k_1m_b}}$ are the damping ratios of the two-levels of suspension.

Since the bogies pitch movement is decoupled from the car body, the symmetrical and anti-symmetrical modes of car body vibration (vehicle) will be determined by the symmetrical and anti-symmetrical axles bounce movements, presented in figure 4.4



a. Symmetric bounce



b. skew-symmetric bounce

Fig.4. 4 Excitation modes of wheel-sets

The frequency features of the two excitation modes depend by the axle base in bogie and car body, as well as by velocity.

Their forms for the symmetrical and anti-symmetrical modes are as follows

$H^+(\omega) = \cos(a_b\,\omega/v)\cos(a_c\,\omega/v)$; $H^-(\omega) = i\cos(a_b\,\omega/v)\sin(a_c\,\omega/v)$ --------------------------------(20)

Where ω is the angular frequency and $i^2 = -1$.

The frequency response in a point *x* of car body is as below

$H_c(x, \omega) = H_{zc}(\omega) + ((L/2) - x)H_{\theta c}(\omega) + \sum_{i=2}^{3} X_i(x)H_{Ti}(\omega)$ --(21)

Where, $H_{zc}(\omega)$, $H_{\theta c}(\omega)$, $H_{T2,3}(\omega)$ are the frequency response corresponding to the vibration modes z_c, θ_c and $T_{2,3}$.

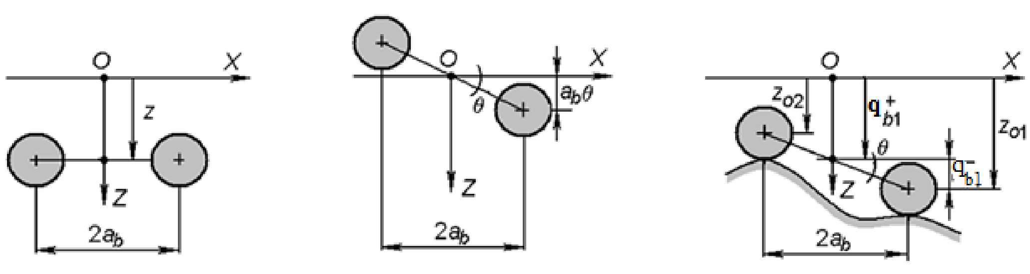
The equation (21) can be customized for various points along the car body. Thus, in the middle of the car body, the response factor is

$H_{cm}(L/2, \omega) = H_{zc}(\omega) + X_2(L/2)H_{T2}(\omega)$ and above the two bogies

$H_{cbi}(l_i, \omega) = H_{zc}(\omega) \pm a_c H_{\theta c}(\omega) + \sum_{i=2}^{3} X_i(l)H_{Ti}(\omega)$---(22)

4.4.1 Mechanism of the Symmetrical and Anti-symmetrical Excitation for the Vehicle Vibration Modes

The irregularities of the rolling track are transmitted to the axles via the elastic contact of the rolling surfaces, thus generating symmetrical and anti-symmetrical movements in the vertical direction of their plans. In Fig. 4.5, the travelling of the front bogie axles plan is presented, as coming from the track longitudinal irregularities. The plan of the axles has a translation movement – bounce z (fig. 4.5, a), and a rotation movement – pitch θ (fig. 4.5, b). For the bounce, it can be noticed that the axles position is symmetrical compared to axis OZ crossing in the middle of distance between them, whereas the axles are in an anti-symmetrical position for the pitch.



(a) (b) (c)

Fig. 4.5 Bounce and pitch of the axles plan for the front bogie.[14]

The position of each bogie axle compared to the reference *OXZ* (fig.4.5, *c*), located on the symmetry axis of the axles plan, is the result of the overlapping of the two movements, bounce and pitch, namely

$z_{o1}= q^{+}_{b1} + q^{-}_{b1}$; $z_{o2} = q^{+}_{b1} – q^{-}_{b1}$,--23

Where q^{+}_{b1}= zis the travelling of axles plan of the front bogie because of the bounce, and

$q^{-}_{b1} = a_b\theta$ is the travelling coming from the pitch. Further on,

$q^{+}_{b1} = ½(z_{o1} + z_{o2})$; $q^{-}_{b1} = ½(z_{o1} - z_{o2})$---24

In a similar way and in dependence on the coordinates z_{o3} and z_{o4} which describe the positions of the axles 3 and 4, $z_{o3}= q^{+}_{b2} + q^{-}_{b2}$; $z_{o4} = q^{+}_{b2} – q^{-}_{b2}$,---25

the travelling of the axles plan corresponding to the rear bogie is obtained

$q^{+}_{b2} = ½(z_{o3} + z_{o4})$; $q^{-}_{b2} = ½(z_{o3} - z_{o4})$---26

4.5 The evaluation of comfort compared to the vertical vibrations

4.5.1 The spectrum of power density of irregularities

In order to evaluate the comfort at vertical vibrations, there will be taken into account the track irregularities as being random and stationary. For the power spectral density of such irregularities, the ORE (Office for Research and Experiments of the International Union of Railways) recommended form is looked at

$S(\Omega) = (A\ \Omega_c^2)/((\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2))$ ---(27)

Where Ω is the wave number, Ω_c= 0.8246 rad/m, Ω_r= 0.0206 rad/m, and A = $9.35*10^{-6}$ rad m

Ω_c, and Ω_r, are truncated wavenumbers (rad/m); A factor of the track irregularities or A = $9.35*10^{-6}$ rad m, depending on the track quality.

This spectrum is based on the measurements made on the railways of the USA [34]. 'Class 6' and 'Class 1' correspond to the best and the worst quality respectively.

Table 4.1: Parameters of different tracks [34]

Track Class	4	5	6
A (m)	2.39X10-5	$9.35\text{X}10^{-6}$	$1.5\text{X}10^{-6}$
Ωr (rad/m)	2.06X10-2	$2.06\ \text{X}10^{-2}$	$2.06\text{X}10^{-2}$
Ωc (rad/m)	0.825	0.825	0.825

The track irregularities become an excitation factor for a vehicle that travels at speed V(m/s) and this is the reason why the power spectral density of the track irregularities need to be expressed as a function of the angular frequency $\omega = V\Omega$, and $G(\omega) = S(\omega/V)/V$ respectively. As a result, eqn. (27) will trigger

$G(\omega) = (A\,V^3\Omega_c^{\,2})/[\,(\omega^2 + (V\Omega_c)^2)(\omega^2 + (V\Omega_r)^2)]$---(28)

Starting from the car body frequency response $H_c\,(x, \omega)$ and the power spectral density of track irregularities, the acceleration power spectral density may be calculated at a point located at distance *x* from the car body referential

$Gc(x,\omega) = \omega^4 G(\omega)\,|H_c\,(x, \omega)|$ --(29)

valid for any point along the car body.

Further, based on the acceleration power spectral density, we will have the Wz comfort index, as seen below. An interest will be taken in the acceleration power spectral density in two points, namely at the center of car body and above bogie.

4.5.2 Ride comfort index Wz

One of the most important aspects that any type of transportation must ensure is an acceptable level of comfort perceived by the passengers. However, it is a complicated parameter to be measured since it is not defined only by means of physically quantifiable quantities but also by subjective perceptions of each passenger. Among all the possible measureable features that define the ride comfort level in railways, the most widely used in the normative is the value of accelerations inside the car [2].

The Wz index of a vehicle reflects the vehicle ability to maintain the vibrations within the limits that will provide comfort or merchandise integrity (the ride quality for goods wagons).

Wz is a root mean square value (r.m.s) of the frequency-weighted accelerations assessed for definite time limits or definite sections in the track. The mathematical expression, introduced by Sperling, is as follows

$W_z = 4.42(a_{wrms})^{0.3}$ ---(30)

Where $a_{wrms}$is the r.m.s value of frequency-weighted acceleration $a_w(t)$ în m/s^2.

Which is: - $a_{wrms} = [1/T\int_0^T (a^w(t))^2 dt]^{0.5}$

Where T is duration of measurement which is 5sec

In order to calculate the total index Wz for a continuous spectrum, at a certain car body point, we have the following formula

$W_z = (2\int_0^T G_c(f)B^2(f)df)^{(1/6.67)}$ ---(31)

Where Gc(f) is the power spectral density of acceleration in cm/s^2/Hz, B(f) is a factor of acceleration evaluation and f is the frequency of vibrations.

For the vertical comfort index, it has to be

$B(f) = 0.588(\sqrt{((1.911f^2 + (0.25f^2)^2)/((1-0.277f^2)^2 + (1.563f-0.0368f^3)^2))}$-------------------------(32)

The relation between the comfort index and the sensitivity to vibrations is assessed on a 1 to 4 scale, where $W_z = 4$ when the vibrations level has a harmful impact upon the human body at a prolonged exposure.

Ride Index (RI) is sometimes used to assess passenger discomfort in trains. The measure originated in between 1940s and 1950s [8, 9] and were recognized by ERRI (ORE) in 1997 [10]. Wz (RI) is a frequency weighted root mean square (r.m.s.) value of accelerations evaluated over defined time intervals or over a defined track section. For an arbitrary acceleration signal that is not necessarily a harmonic signal the frequency weighted root mean square (r.m.s.) value of accelerations should be used and presented in a logarithmic scale.

CHAPTER FIVE

NUMERICAL APPLICATIONS AND ANALYSIS

Table 5.1: - parameter for analysis [source taken in relative to technical specification of ALRT]

Car body mass	mc = 34320 kg
Bending module	$EI = 3.2*10^9\ Nm^2$
Car body length	L = 26.4 m
Car body axle base	$2a_c$= 19 m
Car body gyration radius	i_c = 7.6 m
Car body modal damping ratio	$\zeta_{m2,3}$ = 0.015
Vertical rigidity of secondary suspension	$4k_{zc}$= 2.4 MN/m
Vertical damping of secondary suspension	$4c_{zc}$= 68.88 kNs/m
Bogie mass	m_b= 3200 kg
Bogie gyration radius	i_b = 0.8 m
Bogie axle base	$2a_b$= 2.56 m
Vertical rigidity of primary suspension	$4k_{zb}$= 4.4 MN/m
Vertical damping of primary suspension	$4c_{zb}$= 52.21 kNs/m
Track gauge	1435mm
Speed	V=20, 50 &20 in km/h

Bogie moment of inertia $J_b = 0.5\ m_b \times (\ i_b)^2 = 0.5 \times 3200kg \times (0.8m)^2 = 1024kg\ m^2$

Car body moment of inertia $J_c = 0.5\ m_c \times (i_c)^2 = 0.5 \times 34320\ kg \times (7.6m)^2 = 991161.6kg\ m^2$

Damping ratio of secondary suspension $\zeta_s = 4c_{zc}/\ 2(4k_{zc} \times m_{zc})^{\ 0.5}$

ζ_s = (68.88KNs/m) / 2(2400KN/m x 34.320 KN s^s/m $)^{0.5}$= 0.12

Damping ratio of primary suspension $\zeta_p = 4c_{zc}/\ 2(4k_{zb} \times m_{zb})^{\ 0.5}$

ζ_p = (52.21KNs/m) / 2(4400KN/m x 3.2 KN s^s/m $)^{0.5}$ = 0.23

The Eigen-function of vibration mode i at bending,

$X_i(x) = \sin\beta_i x + \sinh\beta_i x - ((\sin\beta_i L - \sinh\beta_i L)/(\cos\beta_i L - \cosh\beta_i L))(\cos\beta_i x - \cosh\beta_i x$

We have; natural angular frequency of car body $\omega_o = ((2.4 \times 10^6$ Nm)/34320kg$)^{0.5})$ = 8.362rad/s and car body modal damping ratio $\zeta_{m2,3}$= 0.015 and hence $\beta_i = [\omega_i^2\ m\ /\ (EI)]^{0.5}$ computed as follow $\beta_i = [\omega_i^2\ m\ /\ (EI)]^{0.5}$where, ω_i is the natural angular frequency of the vibration mode i which is $8.862(1-0.015^2)^{0.5}$= 8.36rad/s.

Now, $X_i(x)$ can be calculated at car body center (x=L/2 = 13.2) and above bogie (x = 19)

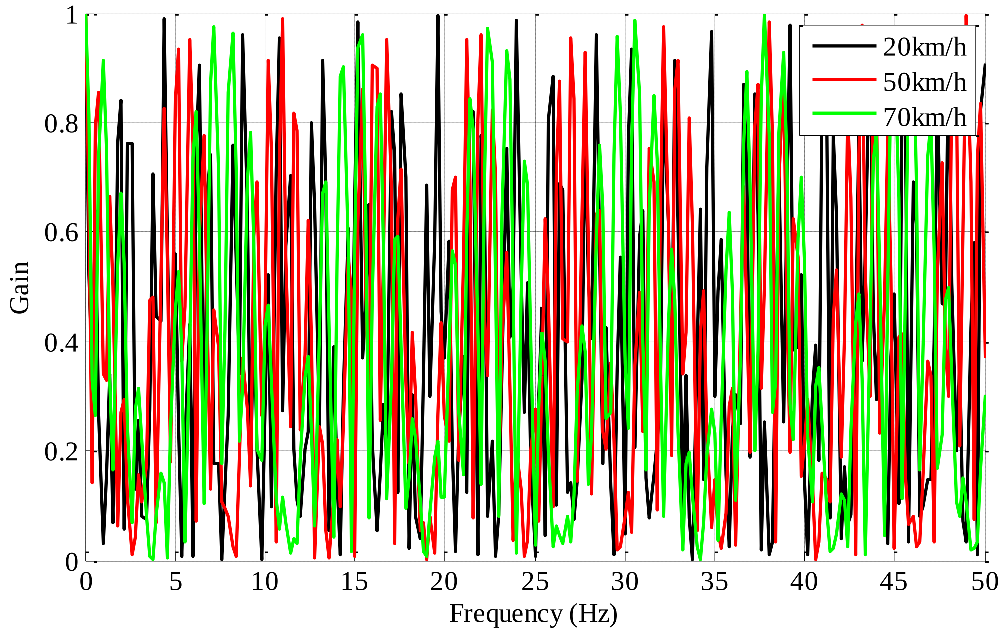
$X_{cm}(13.2) = \sin\beta_i x + \sinh\beta_i x - ((\sin\beta_i L - \sinh\beta_i L)/(\cos\beta_i L - \cosh\beta_i L))(\cos\beta_i x - \cosh\beta_i x$

$X_b(19) = \sin\beta_i x + \sinh\beta_i x - ((\sin\beta_i L - \sinh\beta_i L)/(\cos\beta_i L - \cosh\beta_i L))(\cos\beta_i x - \cosh\beta_i x$

5.1. Analysis

The excitation modes will bring a series of maximum and minimum values depending on the velocity and may attenuate or not the resonance peaks of the car body frequency response. For instance, for the frequencies f = (2n+1)V/4ab with *n* = 0, 1, ..., the wheel-sets' bounce is zero and, thus, the car body bounce and pitch are not excited at these frequencies – the geometric filtering effect of the axle base. The distance between bogies derives the geometric filtering effect for the symmetrical bounce of wheel-sets at frequencies f=(2n+1)V/4ac and for the anti-symmetrical bounce mode at frequencies f = nV/2ac.

Figure 5.1 & 5.2 shows the frequency feature of the two excitation modes at velocities of 20km/h, 50km/h and 70 km/h, for a_c= 19 m and a_b= 2.56 m.



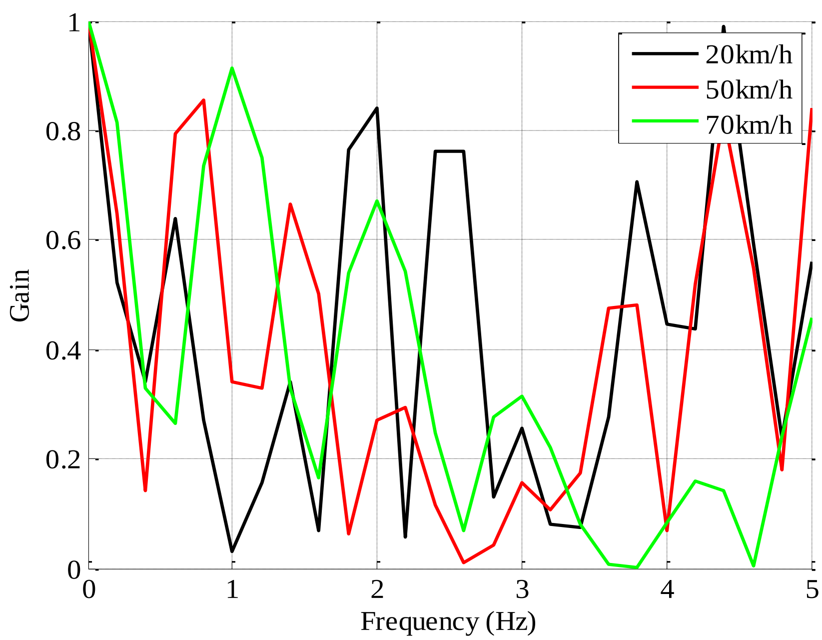
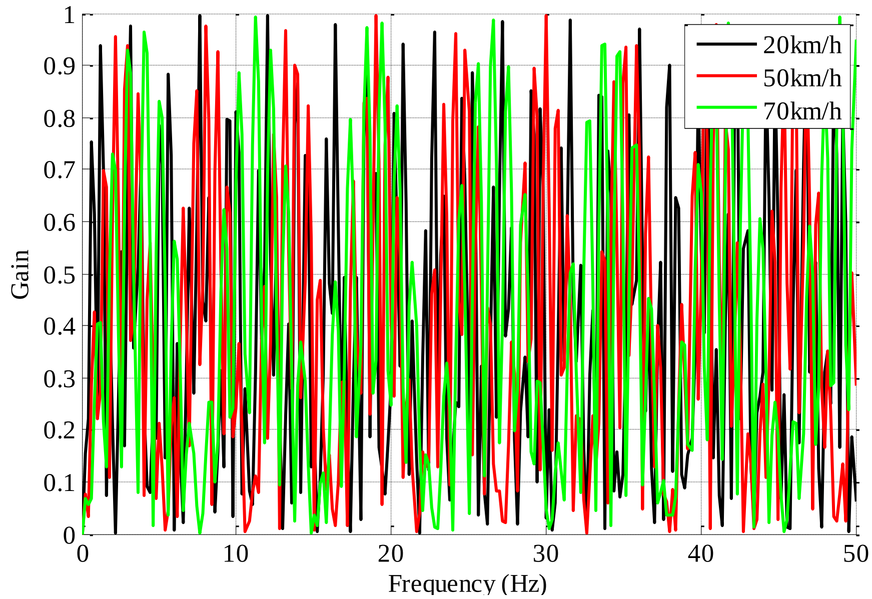


Fig. 5.1 Symmetrical frequency feature of vehicle excitation mode



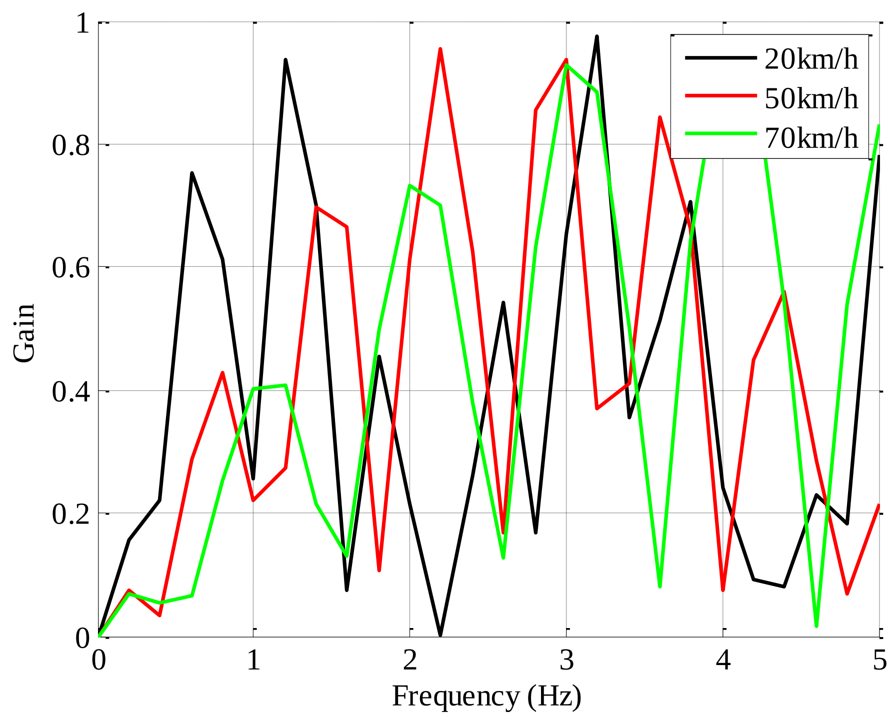


Fig. 5.2 Ant-symmetrical frequency feature of vehicle excitation mode

Since the distance between bogies is much longer than the one between the wheel sets in a bogie, the frequency features will display itself in the shape of a carrying wavelength, corresponding to the geometric filtering effect of the distance between bogies and modulated in amplitude by the effect of geometric filtering of the distance between wheel sets. The frequency of the geometric filtering effect increases along with the velocity of vehicle.

In figures from 5.3 to 5.6 the relation between power spectral density of accelerations and excitation frequency of vehicle vibration, PSD of acceleration and secondary suspension damping coefficient, R.M.S of vertical acceleration and vertical sperling comfort index at car body center and above bogie with speed variation are described. In doing so parameters such as; mass, damping coefficient, track irregularity, angular frequency, pitch movement and others are considered.

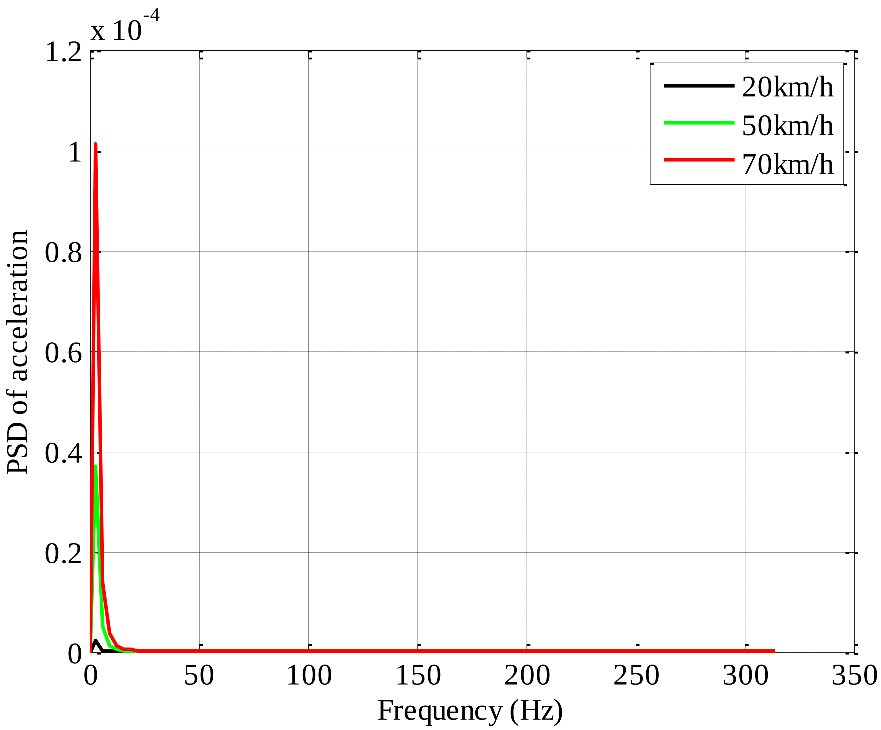


Fig 5.3 Power Spectral Density with excitation frequency

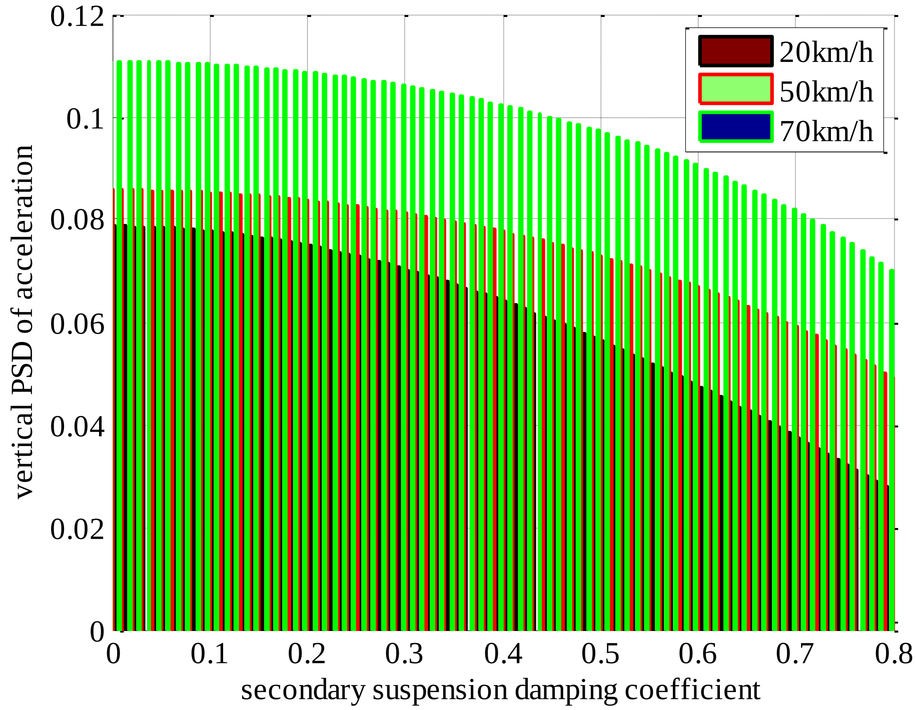


Fig. 5.4 Power spectral density of acceleration

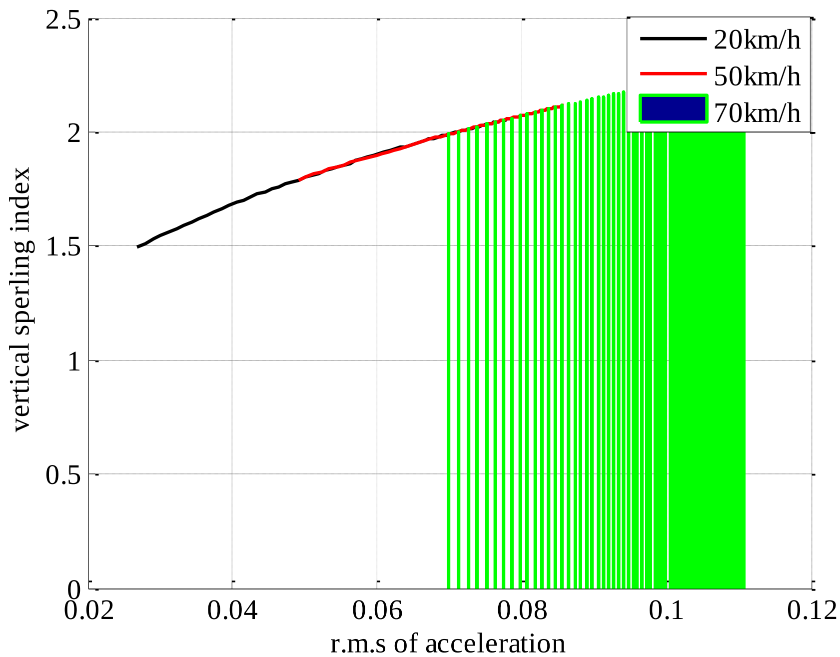


Fig 5.5 r.m.s of acceleration and vertical sperling index at car body center

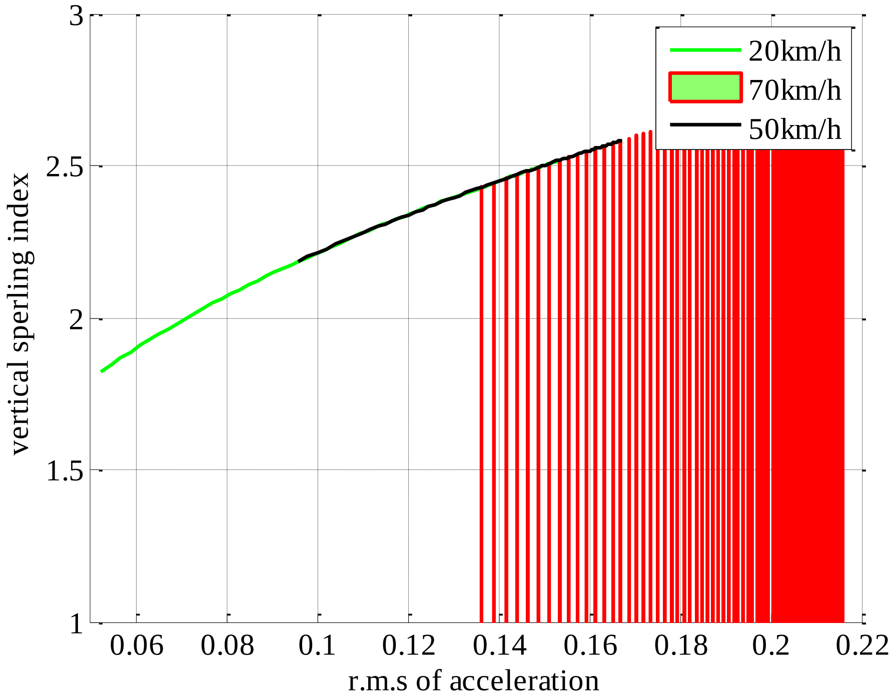


Fig 5.6 r.m.s of acceleration and vertical sperling index above bogie

Figures 5.7 & 5.8 clarifies the relation between vertical sperling comfort index and velocity for damping coefficient variation at car body center and above bogie and fig 5.9 & 5.10 clarifies the relation between secondary suspension damping coefficient and vertical sperling index at car body center and above bogie respectively.

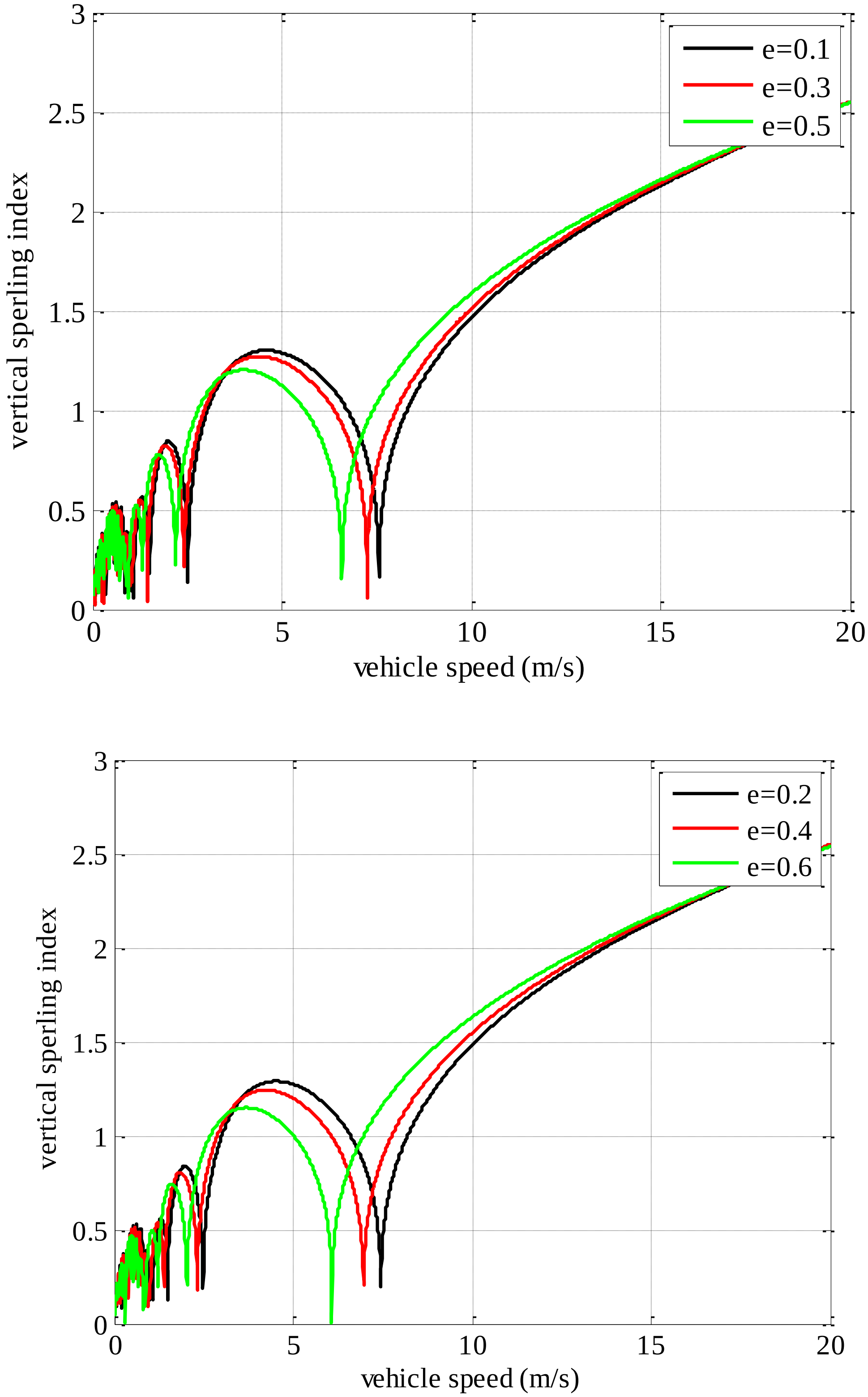
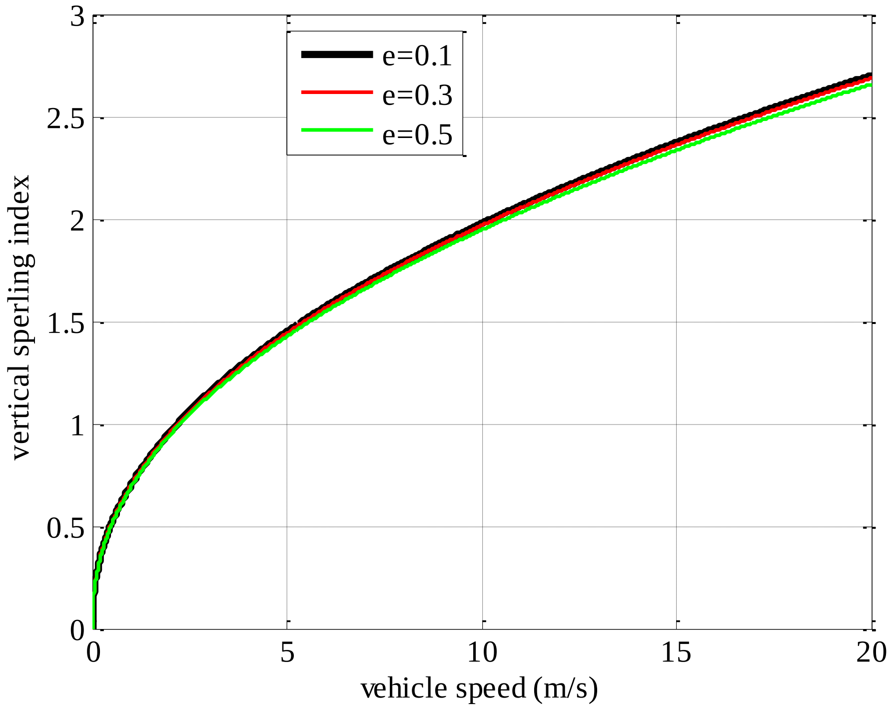


Fig. 5.7 Influence of vehicle speed on ride comfort index at car body center



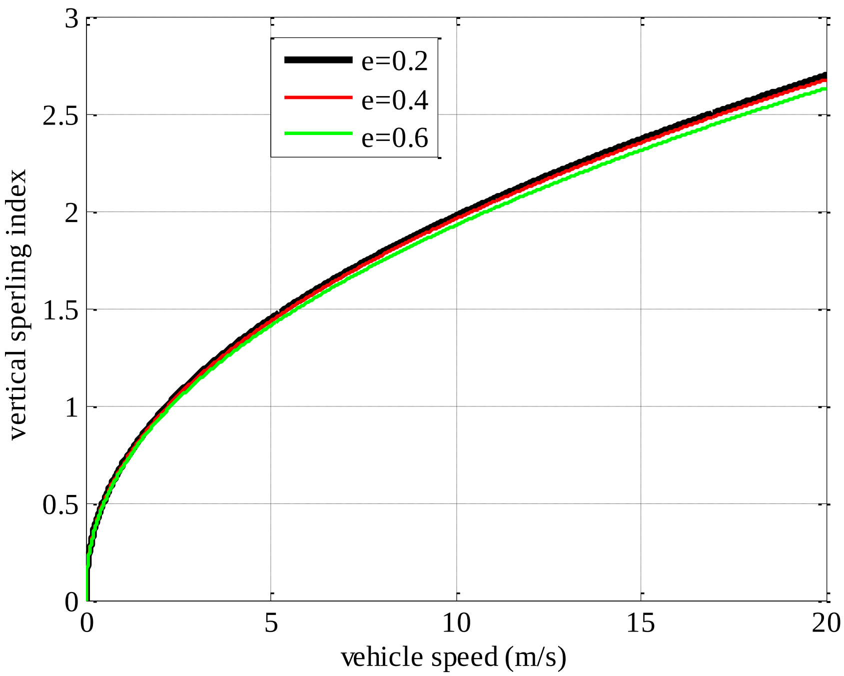


Fig. 5.8 Influence of vehicle speed on ride comfort index above bogie

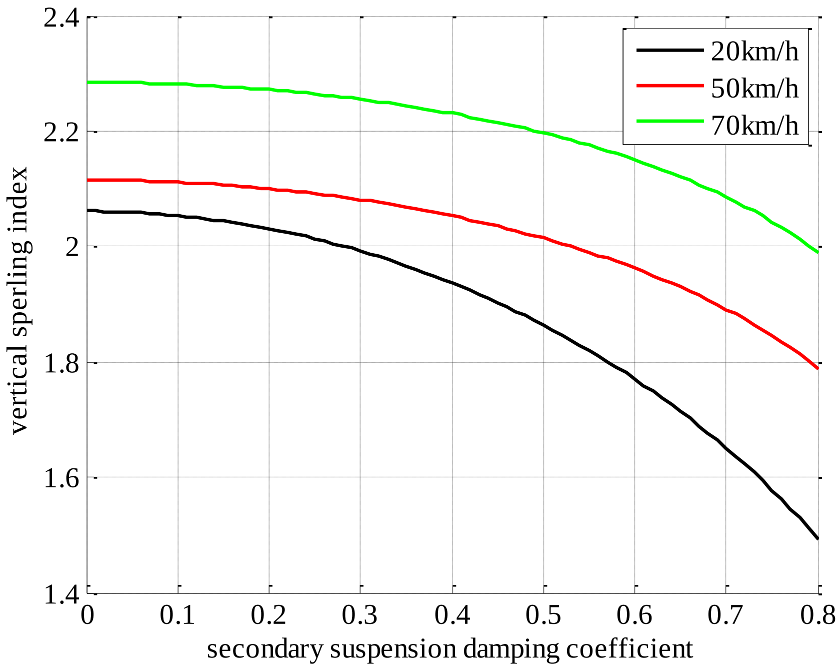


Fig. 5.9 Influence of suspension damping on ride comfort index at car body center

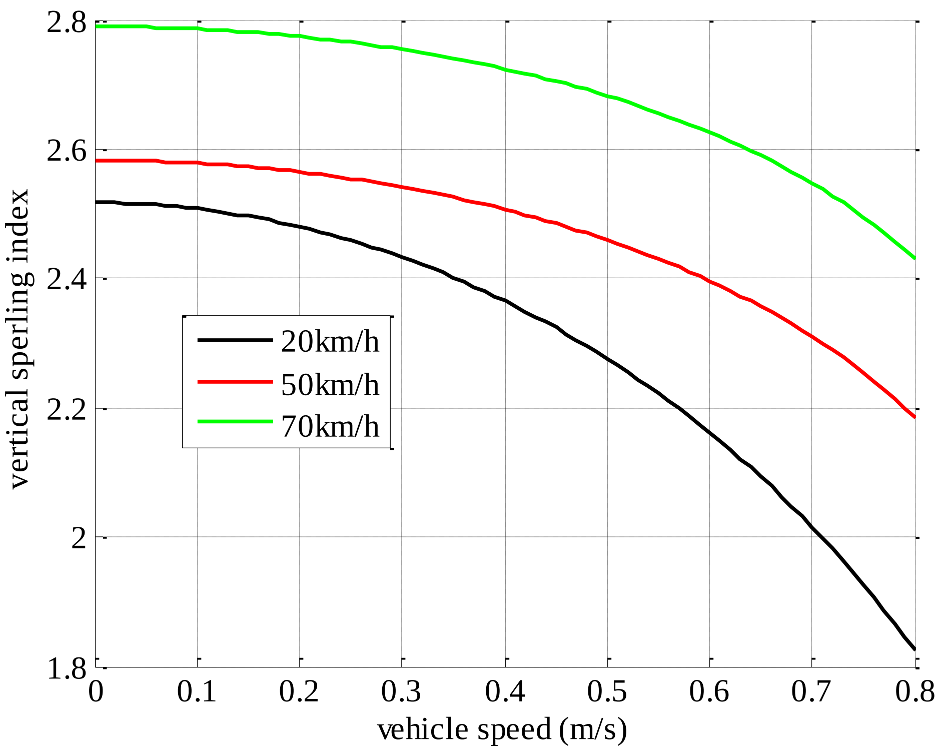


Fig. 5.10 Influence of suspension damping on ride comfort index on bogie

CHAPTER SIX

RESULT AND DISCUSSION

As mentioned above, the frequency feature of two excitation modes and influence of secondary suspension damping at car body center and bogie with different rail vehicle speed is analyzed in this paper

Frequency feature of the two excitation modes (fig 5.1& 5.2) at velocities, at 20 km/h, the frequency of the geometric filtering effect of axles bounce due to the bogie axle base is of 0.36 Hz and 1.28 Hz at 70 km/h for n = 2. The filtering due to the car body axle base introduces the first minimum value at frequency of 0.329Hz for the symmetrical bounce of axles at 20 km/h and of 0.6 Hz for 70 km/h. For the anti-symmetrical bounce, the first minimum shows at 1Hz if velocity is 20 km/h and at 1.6Hz for 70km/h. Should one of the geometric filtering frequencies coincides with one of the natural vehicle frequencies, at a certain velocity, then the intensity of the vibration behavior lowers. Still, at high velocities, the filtering frequency of the axles bounce exceeds the range of the natural vehicle frequencies and this filtering effect exerts a much smaller impact.

The value of the Wz index changes along with velocity increase and its evolution is different compared to the position along car body (fig 5.7 & 5.8). Above bogie, we notice a tendency of Wz index going up with velocity – the car body center is a different story, as a series of maximum and minimum values appears, due to the geometric filtering effect. The maximum values apply to the condition where the geometric filtering effect is close to a natural frequency of the vehicle.

Fig 5.3 Show the relation between power spectral density of acceleration and frequency excitation of vibration for 20km/h, 50km/hand 70km/h. the peak values for each velocity is around 3Hz frequency with 20km/h minimum value and 70km/h maximum value. It verifies that the human being sensitivity to acceleration is affected for frequency 3 to 7Hz.

Fig 5.4 Show the relation between PSD of acceleration and secondary suspension damping coefficient. The PSD of acceleration tends to decrease as suspension damping coefficient increase for constant velocity. For example when the damping coefficient increase from 0.2 to 0.4 at 70km/h the PSD of acceleration decrease by 7% and hence the sperling comfort index become lower. As comfort index decrease the passenger comfort increase.

Fig 5.5 and 5.6 Shows that the root means acceleration relation with vertical comfort index at car body center and above bogie respectively. It verifies that the increase of vehicle speed cause to raise the comfort index. Hence it can be possible to conclude the vehicle speed affect the comfort and as the vehicle speed increase it generate high vertical vibration, that is the R.M.S of acceleration become high and then have negative impact up on ride comfort of passenger rail vehicles.

An increase in the secondary suspension damping ratio has different effects in dependence with the position along the car body and velocity (fig. 5.9 & 5.10). At car body Centre, it will lead to lowering the resonance amplitude for low velocities, whereas the vibration intensifies at higher velocities, thus increasing the Wz index.

Above bogie, the increase of ζc has a favorable impact upon comfort, irrespective of speed. At 50 km/h and the same damping ratio $\zeta c= 0.3$, there is a lower comfort index Wz by about 4%, at car body center and above bogie by about 2 %.

The secondary suspension damping greatly influences the ride comfort, as seen in figure 5.9&5.10. In fact, there is a value for damping that minimizes the car body vibration level in any point of it. The explanation is that, on the one hand, this damping limits the vibration at resonance frequencies at small damping values and, on the other hand, when damping is high, the system dynamic rigidity rises, thus the vibrations behavior is more intense.

The analysis of diagrams shows that index Wz has a minimum value corresponding to a certain damping value, which depends on the position along car body (center or above bogie) and on velocity.

The bogie mass will interfere between the primary suspension and the car body and its inertia reduces the efficiency of the primary suspension damping. Hence, it can be relatively said that the primary suspension damping has a lower influence on ride comfort.

It may be noticed that comfort index Wz becomes minimum at car body center for small values of damping ratio. Increasing damping to a definite value means a general comfort improvement. Beyond this limit, damping has contrary effects, enhances comfort above bogie and worsens it at car body center. The idea is to minimize the Wz value at car body critical point (where vibrations level is higher) to obtain the best damping. Diagrams show that the critical point is above bogie.

CHAPTER SEVEN

CONCLUSION & FUTURE WORK

7.1. Conclusions

Estimating the vibration behavior is a relevant stage in designing the railway vehicles. The suspension, thanks to its features, plays an important role in terms of the vibration behavior at the car body and so in fulfilling the conditions brought by achieving comfort in the passenger vehicles. Relying on the numerical simulations, the paper focuses on the influence of the vertical suspension damping upon the comfort in the railway vehicles, evaluated by the comfort index.

The suspension damping plays an essential role in terms of the car body vibrations behavior and the ride comfort. The selection of damping ratio is a sensitive issue when a too small damping may compromise the vehicle dynamic performance, and a too high damping leads to an increase in the system dynamic rigidity and, therefore, to an intensification of the vibration behavior.

The model of a passenger car with two-level suspension has been considered. Upon applying the modal analysis, the symmetrical and anti-symmetrical decoupled movements and their excitation modes have been pointed out. The analysis of the vibrations behavior due to the track random irregularities has proved that the level of vibrations increases along with the velocity and it is influenced by the geometric filtering effect given by the vehicle axle bases. A solution for improving the vibrations behavior based on the increase in the suspension floors damping has been considered limited.

On the one hand, it is a known fact that the primary suspension damping ratio cannot be increased too much, because of the limitation imposed on by the size of wheel-rail contact dynamic forces. On the other hand, raising the secondary suspension damping ratio has different effects in dependence of the position along the car body and velocity. At car body center, it acts for lowering the amplitude at resonance within the small velocities range, while higher velocities trigger the amplification of vibrations behavior and then to comfort worsening.

The position of the critical point from the vibrations perspective depends on the velocity. Generally speaking, the critical point is above bogie. There is also the possibility that the critical point at high velocities is found at car body center when this one shows great elasticity at bending.

Other solutions that may constitute alternatives for improving the comfort of passenger cars at high velocities refer to using rigid car bodies – difficult to achieve in many circumstances – or to increasing the bogie axle base, provided that the vehicle dynamic behavior is not tampered with during curve travelling.

7.2. Future work

More advanced vehicle models, like using flexible elements instead of rigid bodies could be another point of interest.

Using this result as initial point further assessment of ride comfort of passenger rail vehicles can be performed by experimental tools and observations of passenger response

APPENDIX

1. Some parameters for general rail vehicle model

l_1 = 1m, $l_2 = l_3$ = 1.2m, $h_1 = h_2$ = 0.1m, h_3 = 1m, a_b = 1.28m, a_c = 9.5m

2. The coordinates of symmetrical and ant-symmetrical vibration modes of the vehicles

Vibration modes	Symmetrical vibration modes	Antisymmetrical vibration modes
Carbody bounce	$p_1^+ = z_c$	-
Carbody pitch	-	$p_1^- = \theta_c$
Carbody bending	$p_2^+ = T_2$	$p_2^- = T_3$
Bogies bounce	$p_3^+ = \frac{z_{b1} + z_{b2}}{2}$	$p_3^- = \frac{z_{b1} - z_{b2}}{2}$
Bogies pitch	$p_4^+ = \frac{\theta_{b1} - \theta_{b2}}{2}$	$p_4^- = \frac{\theta_{b1} + \theta_{b2}}{2}$
Bogies rebound	$p_5^+ = \frac{x_{b1} - x_{b2}}{2}$	$p_5^- = \frac{x_{b1} + x_{b2}}{2}$
Bounce of the axles plans	$p_6^+ = \frac{z_{o1} + z_{o2} + z_{o3} + z_{o4}}{4}$	$p_6^- = \frac{z_{o1} + z_{o2} - z_{o3} - z_{o4}}{4}$
Pitch of the axles plans	$p_7^+ = \frac{z_{o1} - z_{o2} - z_{o3} + z_{o4}}{4}$	$p_7^- = \frac{z_{o1} - z_{o2} + z_{o3} - z_{o4}}{4}$
Rebound of the axles plans	$p_8^+ = \frac{x_{o1} + x_{o2} - x_{o3} - x_{o4}}{4}$	$p_8^- = \frac{x_{o1} + x_{o2} + x_{o3} + x_{o4}}{4}$

ANNEX

MAT-LAB COMPUTATION

1 The Eigen-function of vibration mode i at bending

```
EI=3.2.*10.^9;
x= 13.2 & 19;
mc=34320;
mg=3250;
wi=8.36;
L=26.4;
Bi=sqrt((wi.^2.*mg./E));
Xi=sin(Bi.*x)+  sinh(Bi.*x)-((sin(Bi.*L)-  sinh(Bi.*L))./(cos(Bi.*L)-cosh(Bi.*L))).*(cos(Bi.*x)-cosh(Bi.*x));
```

2 Frequency features of vehicle excitation mode

```
f=0:0.2:50
w=2.*pi.*f;
hold on;
H=abs((cos((w.*9.5)./5.56).*cos((w.*1.28)./5.56)));
grid on
plot(f,H, 'black');
B=abs((cos((w.*9.5)./13.9).*cos((w.*1.28)./13.9)));
plot(f,B,'red')
E=abs((cos((w.*9.5)./19.4).*cos((w.*1.28)./19.4)));
plot(f,E,'green')
grid on
f=0:0.2:5
w=2.*pi.*f;
hold on;
H=abs((cos((w.*9.5)./5.56).*cos((w.*1.28)./5.56)));
grid on
```

```
plot(f,H,'black');
B=abs((cos((w.*9.5)./13.9).*cos((w.*1.28)./13.9)));
plot(f,B,'red')
E=abs((cos((w.*9.5)./19.4).*cos((w.*1.28)./19.4)));
plot(f,E,'green')
grid on
f=0:0.2:50
w=2.*pi.*f;
hold on;
H=abs((cos((w.*9.5)./5.56).*sin((w.*1.28)./5.56)));
grid on
plot(f,H, 'black');
B=abs((cos((w.*9.5)./13.9).*sin((w.*1.28)./13.9)));
plot(f,B,'red')
E=abs((cos((w.*9.5)./19.4).*sin((w.*1.28)./19.4)));
plot(f,E,'green')
grid on
f=0:0.2:5
w=2.*pi.*f;
hold on;
H=abs((cos((w.*9.5)./5.56).*sin((w.*1.28)./5.56)));
grid on
plot(f,H,'black');
B=abs((cos((w.*9.5)./13.9).*sin((w.*1.28)./13.9)));
plot(f,B,'red')
 E=abs((cos((w.*9.5)./19.4).*sin((w.*1.28)./19.4)));
plot(f,E,'green')
grid on
```

3 Power spectral density of acceleration with excitation frequency

```
A=9.35.*10.^-6;
f=0:0.5:50;
w=2.*pi.*f;
R=2.06*10.^-2;
r=0.825;
v=5.56;
B=0.588.*sqrt((1.911.*f.^2+(0.25.*f.^2).^2)./((1-0.277.*f.^2).^2+(1.563.*f-0.0368.*f.^3).^2));
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
D=2.*G.*(B.^2)
plot(w,D,'black')
hold on;
v= 13.9
B=0.588.*sqrt((1.911.*f.^2+(0.25.*f.^2).^2)./((1-0.277.*f.^2).^2+(1.563.*f-0.0368.*f.^3).^2));
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
D=2.*G.*(B.^2)
plot(w,D,'green')
v=19.4
B=0.588.*sqrt((1.911.*f.^2+(0.25.*f.^2).^2)./((1-0.277.*f.^2).^2+(1.563.*f-0.0368.*f.^3).^2));
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
D=2.*G.*(B.^2)
plot(w,D,'red')
grid on
```

4 Power spectral density of acceleration with damping coefficient

```
e=0:0.01:0.8;
A=9.35.*10.^-6;
R=2.06*10.^-2;
r=0.825;
hold on;
```

```
v=5.56;
w=0.4.*pi.*sqrt(1-e.^2)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=0.565.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
plot(e,Gc,'black')
v=13.9
w=0.4.*pi.*sqrt(1-e.^2)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=0.565.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
plot(e,Gc,'red')
v=19.4
w=0.4.*pi.*sqrt(1-e.^2)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=0.565.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
plot (e,Gc,'green')
grid on
```

5 R.m.s and vertical sperling index at car body center

```
e=0:0.01:0.8;
A=9.35.*10.^-6;
R=2.06*10.^-2;
r=0.825;
v=5.56;
hold  on;
w=0.4.*pi.*sqrt(1-e.^2)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=0.565.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
```

```
Z=4.42.*(Gc.^0.3)
plot(Gc,Z,'black')
v=13.9
w=0.4.*pi.*sqrt(1-e.^2)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=0.565.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
F=4.42.*(Gc.^0.3)
plot(Gc,F,'red')
v=19.4
w=0.4.*pi.*sqrt(1-e.^2)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=0.565.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
I=4.42.*(Gc.^0.3)
plot (Gc,I,'green')
grid on
```

6 R.m.s and vertical sperling index above bogie

```
e=0:0.01:0.8;
A=9.35.*10.^-6;
R=2.06*10.^-2;
r=0.825;
v=5.56;
hold on;
w=0.4.*pi.*sqrt(1-e.^2)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=1.1.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
Z=4.42.*(Gc.^0.3)
plot(Gc,Z,'green')
```

```
v=13.9
w=0.4.*pi.*sqrt(1-e.^2)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=1.1.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
F=4.42.*(Gc.^0.3)
plot(Gc,F,'black')
v=19.4
w=0.4.*pi.*sqrt(1-e.^2)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=1.1.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
I=4.42.*(Gc.^0.3)
plot (Gc,I,'red')
grid on
```

7. Vehicle speed and ride comfort index at car body center

```
e=0.1;                                  % damping coefficient
v=0:0.01:20;
w=0.4.*pi.*sqrt(1-e.^2)                 % damping angular frequency
A=9.35.*10.^-6;
R=2.06*10.^-2;
r=0.825;
Hv=(cos((w.*9.5)./v).*cos((w.*1.28)./v));
Hc=131591.405./((86.71-w.^2).*(1982.5917-w.^2)-40318.179).*Hv
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
Gc=sqrt(w.^4.*G.*(Hc.^2));
Z=4.42.*(Gc.^0.3)
plot(v,Z, 'black')
hold on
```

```
e=0.3
w=0.4.*pi.*sqrt(1-e.^2)
Hv=(cos((w.*9.5)./v).*cos((w.*1.28)./v));
Hc=131591.405./((86.71-w.^2).*(1982.5917-w.^2)-40318.179).*Hv
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
Gc=sqrt(w.^4.*G.*(Hc.^2));
p=4.42.*(Gc.^0.3)
plot(v,p,'red')
e=0.5
w=0.4.*pi.*sqrt(1-e.^2)
Hv=(cos((w.*9.5)./v).*cos((w.*1.28)./v));
Hc=131591.405./((86.71-w.^2).*(1982.5917-w.^2)-40318.179).*Hv
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
Gc=sqrt(w.^4.*G.*(Hc.^2));
Q=4.42.*(Gc.^0.3)
plot(v,Q,'green')
grid on
e=0.2;                                  % damping coefficient
v=0:0.01:20;
w=0.4.*pi.*sqrt(1-e.^2)                 % damping angular frequency
A=9.35.*10.^-6;
R=2.06*10.^-2;
r=0.825;
Hv=(cos((w.*9.5)./v).*cos((w.*1.28)./v));
Hc=131591.405./((86.71-w.^2).*(1982.5917-w.^2)-40318.179).*Hv
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
Gc=sqrt(w.^4.*G.*(Hc.^2));
Z=4.42.*(Gc.^0.3)
plot(v,Z,'black')
hold on;
```

```
e=0.4
w=0.4.*pi.*sqrt(1-e.^2)
Hv=(cos((w.*9.5)./v).*cos((w.*1.28)./v));
Hc=131591.405./((86.71-w.^2).*(1982.5917-w.^2)-40318.179).*Hv
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
Gc=sqrt(w.^4.*G.*(Hc.^2));
p=4.42.*(Gc.^0.3)
plot(v,p,'red')
e=0.6
w=0.4.*pi.*sqrt(1-e.^2)
Hv=(cos((w.*9.5)./v).*cos((w.*1.28)./v));
Hc=131591.405./((86.71-w.^2).*(1982.5917-w.^2)-40318.179).*Hv
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
Gc=sqrt(w.^4.*G.*(Hc.^2));
Q=4.42.*(Gc.^0.3)
plot(v,Q,'green')
grid on
```

8. Vehicle speed and ride comfort index onbogie

```
e=0.1;
v=0:0.01:20;
w=0.4.*pi.*sqrt(1-e.^2)
A=9.35.*10.^-6;
R=2.06*10.^-2;
r=0.825;
Hv=(cos((w.*9.5)./v).*cos((w.*1.28)./v));
Hc=131591.405./((86.71-w.^2).*(1982.5917-w.^2)-40318.179)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
Gc=sqrt(w.^4.*G.*(Hc.^2));
Z=4.42.*(Gc.^0.3)
plot(v,Z, 'black')
```

```
e=0.3
w=0.4.*pi.*sqrt(1-e.^2)
Hv=(cos((w.*9.5)./v).*cos((w.*1.28)./v));
Hc=131591.405./((86.71-w.^2).*(1982.5917-w.^2)-40318.179)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
Gc=sqrt(w.^4.*G.*(Hc.^2));
p=4.42.*(Gc.^0.3)
plot(v,p,'red')
e=0.5
w=0.4.*pi.*sqrt(1-e.^2)
Hv=(cos((w.*9.5)./v).*cos((w.*1.28)./v));
Hc=131591.405./((86.71-w.^2).*(1982.5917-w.^2)-40318.179)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
Gc=sqrt(w.^4.*G.*(Hc.^2));
Q=4.42.*(Gc.^0.3)
plot(v,Q,'green')
grid on
e=0.2;
v=0:0.01:20;
w=0.4.*pi.*sqrt(1-e.^2)
A=9.35.*10.^-6;
R=2.06*10.^-2;
r=0.825;
Hv=(cos((w.*9.5)./v).*cos((w.*1.28)./v));
Hc=131591.405./((86.71-w.^2).*(1982.5917-w.^2)-40318.179)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
Gc=sqrt(w.^4.*G.*(Hc.^2));
Z=4.42.*(Gc.^0.3)
plot(v,Z,'black')
hold on;
```

```
e=0.4
w=0.4.*pi.*sqrt(1-e.^2)
Hv=(cos((w.*9.5)./v).*cos((w.*1.28)./v));
Hc=131591.405./((86.71-w.^2).*(1982.5917-w.^2)-40318.179)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
Gc=sqrt(w.^4.*G.*(Hc.^2));
p=4.42.*(Gc.^0.3)
plot(v,p,'red')
e=0.6
w=0.4.*pi.*sqrt(1-e.^2)
Hv=(cos((w.*9.5)./v).*cos((w.*1.28)./v));
Hc=131591.405./((86.71-w.^2).*(1982.5917-w.^2)-40318.179)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
Gc=sqrt(w.^4.*G.*(Hc.^2));
Q=4.42.*(Gc.^0.3)
plot(v,Q,'green')
grid on
```

9. Vertical sperling index and damping coefficient at car body center

```
e=0:0.01:0.8;
A=9.35.*10.^-6;
R=2.06*10.^-2;
r=0.825;
v=5.56;
hold on
w=0.4.*pi.*sqrt(1-e.^2)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=0.565.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
Z=4.42.*(Gc.^0.3)
plot(e,Z)
```

```
v=13.9
w=0.4.*pi.*sqrt(1-e.^2)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=0.565.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
F=4.42.*(Gc.^0.3)
plot(e,F,'red')
v=19.4
w=0.4.*pi.*sqrt(1-e.^2)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=0.565.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
I=4.42.*(Gc.^0.3)
plot (e,I,'green')
grid on
```

10. Vertical sperling index and damping coefficient on bogie

```
e=0:0.01:0.8;
A=9.35.*10.^-6;
R=2.06*10.^-2;
r=0.825;
v=5.56;
hold on
w=0.4.*pi.*sqrt(1-e.^2)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=1.1.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
Z=4.42.*(Gc.^0.3)
plot(e,Z)
v=13.9
w=0.4.*pi.*sqrt(1-e.^2)
```

```
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=1.1.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
F=4.42.*(Gc.^0.3)
plot(e,F,'red')
v=19.4
w=0.4.*pi.*sqrt(1-e.^2)
G=(A.*v.^3.*r.^2)./((w.^2+r.^2).*(w.^2+R.^2));
H=1.1.*exp(9.5.*w./v);
Gc=0.588.*sqrt(w.^4.*G.*(H.^2));
I=4.42.*(Gc.^0.3)
plot (e,I,'green')
grid on
```

REFERENCE

[1] D. Younesiana, A. Nankali, Spectral optimization of the suspension system of high-speed trains Int. J. Vehicle Structures & Systems, 1(4), 98-103, 2009.

[2] Andersson, E., Berg, M., Stichel, S. : Rail Vehicle dynamics. RAILWAY GROUP KTH, Centre for Research and Education in Railway Engineering. 2007.

[3] T. Mazilu , Confortul la materialul rulant (Comfort at the rolling stock), Ed. MatrixRom, 2003.

[4] I. Sebe şan, T. Mazilu , Vibra ţiile vehiculelor feroviare (Vibrations of the railway vehicles), Matrixrom, Bucureş ti, 2010. (in Romanian)

[5] J. Zhou, R. Goodall, L Ren, H Zhang, Influences of car body vertical flexibility on ride quality of passenger railway vehicles, Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, 223, pp. 461- 471, 2009.

[6] P. Wu, J. Zeng, H. Dai, Dynamic response analysis of railway passenger car with flexible car body model based on semi-active suspensions, Vehicle System Dynamics, Vol. 41 (Suppl.), pp. 774–783, 2004.

[7] ISO 2631-4, Mechanical vibrations and shock- evaluation of human exposure to whole body vibrations – Part 4: Guidelines for the evaluation of the effects of vibration and rotational motion on passenger and crew comfort of fixed guide way transport systems, 2004.

[8]BS 6841, 1987, Guide to measurement and evaluation of human exposure to whole-body mechanical vibration and repeated shock, BSI, 1987.

[9] Sperling, E., and Betzhold, C.,: Contribution to the evaluation of ride comfort in railway vehicles {Beitrag zur Beurteilung des Fahrkomforts in Schienenfahrzeugen}. Glasers Annalen 80, pp. 314–320, 1956.

[10] ENV12299, Railway applications: Ride comfort for passengers, 1999.

[11] UIC 513R, Guidelines for evaluating passenger comfort in relation to vibration in railway vehicle, International Union of Railways, 1994.

[12] Fa, Y., Wu, W.,: Stability analysis of railway vehicles and its verification through field test data. Journal of the Chinese Institute of Engineers, Vol. 29, No. 3, pp. 493-505, 2006

[13] Orvnäs, A., : On Active Secondary Suspension in Rail Vehicles to Improve Ride Comfort. PhD Thesis. Department of Aeronautical and Vehicle Engineering, KTH Engineering Sciences, 2011

[14] Johnsson, A., Berbyuk, V., Enelund, M. Pareto Optimisation of railway bogie suspension damping to enhance safety and comfort. Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility, 50:9, 1379-1407, DOI:10.1080/00423114.2012.659846. 2012.

[15]. T.J. Gordon and M.C. Best, "Dynamic Optimization of Nonlinear Semi-Active Suspension Controllers", Control'94, 21-24 March 1994, Conference No. 389 © IEE 1994, pp 332-337

[16]. T. Sireteanu and N.Stoia, "Damping Optimization of Passive and Active Vehicle Suspension by Numerical Simulation", Proceedings of Romanian Academy , Series A, Vol. 4 , Number 2/2003

[17]. K. I. Maemori , "Alternate Optimization of Speed Control Hump for Automobiles and Automobile Suspension ", JSME International Journal , Series C , Vol. 38 , 1995 , pp 552-557

[18]. D.Karnoop , "Analytical Results for Optimum Actively Damped Suspensions under Random Excitation", Transactions of ASME , Vol.111 , July 1989, pp 278-282

[19]. X.P.Lu , N.L.Li and P.Papalambros , "A Design Procedure for the Optimization of Vehicle Suspensions ", International journal of Vehicle Design, Vol. 5 No. , 1984, pp 129-143

[20]. P.V.Kumar and P.S.Rao, "Vibration Isolation Performance of a Full Car Suspension System Model using Delayed Resonator Vibration Absorber", Proceedings of The National Conference on Advances in Mechanical Sciences AIMS07 , 22th – 24th March 2007, Sponsored by AICTE , New Delhi , pp 85-90

[21]. R.Majjad, "Estimation of Suspension Parameters", Proceedings of The 1997 IEEE International Conference on Control Applications, Hartford, CT, October 5-7, 1997 , pp 522-527

[22]. N.Al-Holou ,A. Bajwa and D. Joo, "Computer Controlled Individual Semi-Active Suspension System ", Ch 3381-1/93/So1.00 © 1993 , IEEE , pp 208-211

[23]. H.R.O'Neill and G.D.Wale, "Semi-Active Suspension Improves Rail Vehicle Ride", Computing & Control Engineering Journal, August 1994, pp 183-188

[24] Schandl G, Lugner P, BenatzkyC, Kozek M and Stribersky A: Comfort Enhancement by an Active Vibration Reduction System for a Flexible Railway Car Body, Vehicle System Dynamics, Vol. 45, No. 9, pp. 835-847, 2007.

[25]. Anneli Orvnäs – Active Secondary Suspension in Trains a Literature Survey of Concepts and Previous Work, ISBN 978-91-7415-144-2, 2008.

[26]. M.S.Patil , M.K.Hada , S.Y.Bhave and S.G.Joshi , " Vibration Isolation and Transmissibility Characteristics of a Passive Sequential Damper" , Defence Science Journal , Vol. 54 , No.1 January 2004, pp 39-59

[27]. Sun, S., Deng, H. and Li, W. A variable stiffness and damping suspension systems for trains. 2014.

[28]. Jessica Fagerlund - Towards Active Car Body Suspension in Railway Vehicles Chalmers University of Technology Göteborg, Sweden, 2009.

[29]. Dr. Ralph Streiter et al - Active Lateral Suspension for High Speed Trains a Step towards the Mechatronic Bogie, 2001.

[30]. Griffin, M. J. Handbook of Human Vibration, London: Academic Press. 1990.

[31]. Guedry, F. E. Factors influencing susceptibility: Individual differences and human factors, In: Motion sickness: Significance in aerospace operations and prophylaxis. 1991a. (Vol. Lecture series 175.,). Neuilly sur Seine: AGARD.

[32]. Suzuki, H. Research trends on riding comfort evaluation in Japan. Proceedings of the Institution of Mechanical Engineers: Part F Journal of Rail and Rapid Transit., 212(F1), pp. 61-72, 1998.

[33]. Förstberg, J. Comfort disturbances caused by low-frequency motions in modern trains, World Congress on Railway Research (WCRR ´94) (Vol. 2, pp. 1135-1137). Paris: SNCF. 1994b.

[34]. Carlbom, P. Track-induced structural vibrations in rail vehicle car bodies: Modelling, simulation and measurements. TRITA - FKT Report 1998:36. Stockholm: KTH. 1998.

[35]. Förstberg, J. Motion-related comfort levels in trains: A study on human response to differenttilt control strategies for a high speed train. Licentiate Thesis TRITA-FKT report1996:41 (Also publised as VTI Särtryck 274 -1997, Linköping: VTI). Stockholm: KTH: RailwayTechnology. 1996.

[36]. Harborough, P. R. Passenger comfort during high speed curving: Summary report. BRRTR DOS 018. Derby: British Rail Research. 1986b.

[37]. Andersson, E., & Nilstam, N. The development of advanced high speed vehicles in Sweden Proceedings of the Institution of Mechanical Engineers, Part D, 198(15), pp. 229-237. 1984.

[38].Hassan Nahvi Mechanical Eng. Dept., Isfahan University of Tech., Isfahan, Iran

[39]. Department of Railway Vehicles, Romania, University Politehnica of Bucharest, Romania