Anomalous Skin Effect in Layered Conductors

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by

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Abstract

The theory of anomalous skin effect in layered conductors with quasi-two-dimensional energy spectrum under high frequency electromagnetic wave propagation along and perpendicular to the layers is studied in the account of Fermi-liquid effects in the absence of static magnetic field. The surface impedance, the conductivity and the skin depth are calculated explicitly for quadratic energy spectrum and it is observed that the magnitudes differ in different orientations of the sample relative to the direction of propagation of the wave. The dependence of these quantities on the Landau's correlation function is analyzed and they are compared qualitatively with the results in gas approximation theoretical results under the studies of normal metals.

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Content

Introduction

This thesis is devoted to theoretical analysis of anomalous skin effect in layered conductors which have a quasi-two dimensional energy spectrum of charge carriers.

Metals occupy a rather special position in the study of solids. They are excellent conductors of heat and electricity. The challenge of accounting for these metallic features gave the starting impetus to the modern theory of solids.

In order to understand most of the physical properties of metals, one has to determine the shape of the Fermi-surface. There is a class of measurable quantities valued primarily because they contain detailed information about the geometric structure of the Fermi-surface. Such quantities depend only on universal constants, experimentally controlled variables (such as temperature, frequency, magnetic field strength, crystal orientation) and information about the electronic band structure that is entirely determined by the shape of the Fermi-surface.

The importance of determining the Fermi-surface of metals is clear. The shape of the Fermi-surface is involved in the transport coefficients. Experiments on the Fermi-surface can be used to provide data for fitting phenomenological parameters which can then be used to calculate other phenomena. Fermi-surface measurements are of interest also as a further test of the validity of the one-electron semiclassical theory, since there are now many independent ways of extracting Fermi-surface information.

Quantities yielding such Fermi-surface information have a place of special importance in the physics of metals. Their measurement almost always requires single crystals of very pure substances at very low temperatures and is frequently performed in very strong magnetic fields (for example, de Haas-van Alphen effect) or in high frequency electromagnetic fields (Anomalous Skin Effect). These effects are based on the same underlying physical mechanism and their techniques appear to be powerful and simple.

The recently artificially synthesized conductors, generally having complicated composition and structure, serve as a source of new and unique information about the fundamental processes in solids. The significant part of these conducting materials have layered structures, the electric conductivity of which under normal (non superconducting) condition along the layers considerably exceeds electric conductivity along the direction normal to these layers. For verifying the mechanism of formation of superconducting condition, it is important to know the properties of the elementary excitations, which are responsible for the transposition of electrical charges in the crystal. The observation of the de Haas - van Alphen and the Shubnikov - de Haas effects in layered organic conductors justified the supposition that the system of fermions which is similar to conduction electrons in metals but possessing quasi-two-dimensional energy spectrum is responsible for their electronic properties.[1] Consideration of the high frequency electromagnetic waves encourages one to ask how does the Fermi liquid theory affect the results.

The thesis is divided into three chapters. The first and second chapters are preview. The one electron Bloch theory, the distribution function and the Boltzmann kinetic equation were reviewed in chapter one. The Landau Fermi liquid theory is also presented here suited for the purpose to use them in the last chapter.

The second chapter is related to the earlier studies of the normal and anomalous skin effects in normal metals, which have been based on Maxwell's equations and Boltzmann kinetic equation in the Fermi gas approximation, such as, the non-effectiveness qualitative description, derivations of G.E.H. Reuter and E.H. Sondheimer [2]and the description of Abrikosov A.A.[3] who have obtained almost the same result. Several researchers have used measurement of surface resistance to obtain information on the features of the Fermi-surface. In case of suitably averaged surface resistance, in polycrystalline material, one obtains the total area of the Fermi-surface, and measurement on a single crystal material as a function of orientation yield information on its shape[4]. The high frequency electromagnetic wave impedance of metals at low temperatures was first noticed by H. London in 1940 [2,4] and attributed to the mean free path l of conduction electrons being greater than the skin depth δ. It is this effect known as anomalous skin effect. The rise of microwave technology has fostered a fairly extensive study of the phenomenon.

The treatment of the third chapter, which is the main part of the work, differs in the way in which the sample considered is layered conductors, and as well as, the introduction of the Fermi liquid effect. The influence of the Fermi liquid effect on the surface impedance, conductivity and the skin depth of layered conductors with a quasi-two dimensional electron energy spectrum is the main theoretical analysis of the thesis.

The theory for a given simple energy spectrum in the case of specular reflection of electrons from the surface of the sample under perpendicular and parallel propagations of electromagnetic waves to the layers is examined separately. In the extreme anomalous limit, the conductivities for both considered cases of electromagnetic wave propagation directions are seen to be weakly dependent on relaxation time when the Fermi-liquid effect is taken into account, but independent of it in the gas approximation. The conductivity perpendicular to the layers is highly affected by the layered structure while the conductivity along the layers exceeds highly the conductivity perpendicular to the layers at not very low temperatures.

The surface impedance and the penetration depth in layered conductors remains unaffected qualitatively when considering the accounts of the Fermi liquid effects, however the Fermi liquid interaction terms exist in the expressions.

Chapter 1
Electrons in a Crystal Lattice. Boltzmann Kinetic Equation (Preview)

1.1 General Properties of Electrons in Metals

The most obvious property exhibited by metals and conductors having metallic electrical properties is extremely high electrical conductivity. Enormous ratio of metallic to insulator conductivity is observed because there is a free electron gas in metals. The question of why is this gas found in metals but not insulators can be answered by observing that the electrons are tightly bound to the atoms of the lattice in insulators while in metals they are essentially free particles. This is due to considerable over lap between electron states belonging to the outer shells of the constituent atoms. As a result, the valence electrons move easily from one atom to the next, so that one cannot even say to which atom they really belong. This shared nature of the outer electrons is also responsible for the large cohesive energy of metals and explains their specific mechanical properties.[3] In this view, a metal can be thought of as a 'sea' of valence electrons moving in a crystalline lattice of ions. These electrons are often called conduction or 'free' electrons which interact quite strongly among themselves and with the ions and the potential energy of interaction being comparable to the electronic kinetic energy.

An electron moving in a metal can be assumed as if it moves in an external field characterized by a potential energy $U(\mathbf{r})$, which is periodic with period of arbitrary lattice vector $\mathbf{a}_n = n_1\mathbf{a}_1 + n_2\mathbf{a}_2 + n_3\mathbf{a}_3$, where the n_i's are integers running over positive and negative values as well as zero. The energy of the electron depends on the crystal momentum $\mathbf{p}$, which is analogous to the momentum of a free particle. Since $\mathbf{p}$ and $\mathbf{p} + \hbar\mathbf{K}$, where K_i are equal to 2π divided by the reciprocals of the unit cell dimensions a_i, are physically equivalent, the energy $\varepsilon(\mathbf{p})$ must be a periodic function of $\mathbf{p}$ with periods $\hbar\mathbf{K}$. From this it follows that it is sufficient to consider only a single primitive unit cell in the reciprocal space. The volume of such a domain where $\mathbf{p}$ is uniquely defined is equal to $(2\pi\hbar)^3/\mathrm{v}$, $\mathrm{v} = \mathbf{a}_1(\mathbf{a}_2 \times \mathbf{a}_3)$ being the volume of a primitive unit cell of the crystal lattice. The wave function describing the motion of the electron in a periodic field satisfying the condition of periodicity is given by Bloch's function defined as

$$\phi(\mathbf{r}) = e^{i\frac{\mathbf{p}\cdot\mathbf{r}}{\hbar}}u(\mathbf{r}), \tag{1.1}$$

where $u(\mathbf{r})$ is a periodic function with a periodicity of lattice vector $\mathbf{a}_n$ and satisfies Bloch's theorem, i.e., $u(\mathbf{r} + \mathbf{a}_n) = u(\mathbf{r})$. This wave function is like a plane wave, describing the motion of a free particle, but modulated by a periodic function.

In a very large volume successive levels will be infinitesimally close to each other. However, we shall be interested only in the number of states per unit energy range or per unit volume in reciprocal space. This density of states does not depend on the exact nature of the boundary conditions and it is, therefore, easier to determine it using the simplest possible boundary conditions. From this condition it is easy to note that vector $\mathbf{p}$ is thus a discrete variable with components $p_i = \frac{2\pi\hbar n_i}{L_i}$, with $i = x, y, z$ and $V = L_x L_y L_z$ is volume of the sample. If the dimensions of the sample are very large, summations over energy levels can be replaced by integrations. The density of states in p-space is therefore $\frac{V}{(2\pi\hbar)^3}$, and the total number of allowed values of $\mathbf{p}$ is equal to $(V/\mathrm{v}) = N$, where N is the number of primitive unit cells in the sample. The account of the spin of the electron results in a total of $2N$ levels associated with each function $\varepsilon_l(\mathbf{p})$. The functions $\varepsilon_l(\mathbf{p})$ are periodic in reciprocal space and, hence, oscillate between certain maximum and minimum values. For every l we therefore obtain a band of allowed energy values. These bands may be separated by energy gaps in which no electron can have, but can overlap.

The primitive unit cell of the reciprocal lattice is chosen as the natural domain in which p is uniquely defined. This new domain of definition for $\mathbf{p}$ is the Brillouin zone defined by $|\mathbf{p}\mathbf{a}_i| \leq \pi\hbar$, which hold for every basis vector $\mathbf{a}_i$.

Accurate calculations of the energy bands $\varepsilon_l(\mathbf{p})$ require rather complex techniques. In order to illustrate the general properties of the energy bands $\varepsilon_l(\mathbf{p})$ obtained above one can consider two simplest approximations, although they are not suited for an accurate determination of the energy bands $\varepsilon_l(\mathbf{p})$ of real metals: the tight-binding approximation and the nearly-free electron approximation.

In tight-binding approximation it is assumed that electronic shells do not overlap appreciably and that each electron can be assigned to a particular atom as a zero-order approximation. Overlap between shells will be considered as a perturbation.

The potential energy of an electron in the field of all the ions has the form $V(x) = \sum\limits_n U(x - na)$, which is different from zero only in the neighborhood of the n^{th} ion. In one dimensional case one sees that $\varepsilon_l(\mathbf{p})$ is a periodic function of $\mathbf{p}$ with period $\frac{2\pi\hbar}{a}$ and has a form[3]

$$\varepsilon = A + B\cos(pa/\hbar), \tag{1.2}$$

where A and B are constants.

Calculations are more complicated in three dimensions. In the simplest case, corresponding to one atom per unit cell with a single s electron for a bcc lattice if only nearest-neighbors are taken into account one finds that $\varepsilon = C + D\cos(p_x a/2\hbar)\,\cos(p_y a/2\hbar)\,\cos(p_z a/2\hbar)$, where D determines the bandwidth.

Now consider the opposite case in which the conduction electrons are nearly free and the crystal potential is weak so that perturbation theory is applicable. The normalized wave function of a free electron (in one dimension) is $L^{-\frac{1}{2}}e^{i\frac{px}{\hbar}}$, where L is the length of the chain. It's energy is $\varepsilon^{(0)} = \frac{p^2}{2m}$. The next terms are taken by using successive approximations. We again obtain a general picture of energy bands, in which the energy dependences on crystal momentum, rather than on momentum. Ordinary perturbation theory fails for values of $\mathbf{p}$ such that $\varepsilon^{(0)}(\mathbf{p}) = \varepsilon^{(0)}(\mathbf{p} - \hbar\mathbf{K})$ or $p\cos\theta = \frac{1}{2}\hbar K$, where θ is the angle between $\mathbf{p}$ and $\mathbf{K}$. This is the equation of a plane in reciprocal space, perpendicular to the vector $\mathbf{K}$, and intersecting it at a distance $\frac{1}{2}\hbar K$ from the origin. It is the condition for Bragg reflection of the plane waves describing the motion of an electron by crystal planes. If $\mathbf{K}$ is the shortest reciprocal vector in a particular direction, this plane is simply a Brillouin-zone boundary; i.e., the energy discontinuity occurs on it. This is one of the reasons why the Brillouin zone is the most convenient domain of definition for crystal momentum.

1.2 Boltzmann Kinetic Equation and Relaxation time approximation

The combined effect of forces on the electrons, an inhomogeneous electron distribution in the metal, and of scattering processes is best seen by making use of the Boltzmann kinetic equation. It is more commonly used to discuss the properties of conduction electrons which are displaced from equilibrium by a small perturbing force, such as an electric field, magnetic field or a temperature gradient.

In this section we shall first introduce the condition of usage of the semiclassical distribution and then see the form of kinetic equation for electron gas when electron-electron interaction is neglected and then its modification for Fermi liquid, in which electron-electron interaction is taken into consideration.

1.2.1 The Electron as a Wave Packet

Up to now we described electrons by means of stationary solutions of the Schrodinger equation of the Bloch type, $\phi = e^{i\mathrm{pr}/\hbar}u(\mathrm{r})$. These functions are not suited for studying transport phenomena, since they correspond to a well-defined value of the crystal momentum p while the position coordinate remains completely undetermined. This defect can be remedied by constructing a wave packet from Bloch states. One would nevertheless like to retain a reasonably sharply defined crystal momentum. We shall, therefore, take a superposition of Bloch states with a small spread $\Delta\mathrm{p}$. The width of this interval is restricted by the maximum allowed uncertainty in position. According to the uncertainty principle, one has $\Delta p \Delta r \sim \hbar$.

Since electrons are scattered, they have a finite mean free path l, the uncertainty in position must therefore be smaller than l. The maximum allowed value of Δp is clearly the momentum of the particle itself. The restriction $p \gg \Delta p \sim \hbar/\Delta r \gg \hbar/l$ must therefore be obeyed for the wave packet description to make sense. The condition is satisfied provided that $l \gg a$.

Another restriction is related to the damping of quasi-particles. It is $\xi = \varepsilon - \mu >> \gamma$ where γ is the damping coefficient, which appears in the wave function of a quasi-particle $\exp(i\xi t/\hbar - \gamma t/\hbar)$. The coefficient γ is related to the mean lifetime τ, e.g., $\gamma \sim \hbar/\tau$. The second restriction can therefore be written as $\xi \gg \hbar/\tau$ or $k_b T \gg \hbar/\tau$. Thus, the wave packet description is valid under the conditions[3]

$$p \gg \hbar/l, k_b T \gg \hbar/\tau. \tag{1.3}$$

The velocity of an electron described by a wave packet

$$\mathbf{v} = \partial\varepsilon/\partial\mathbf{p}, \tag{1.4}$$

is called the group velocity.

Let us assume that an electron is moving under the influence of an external electric field E. The work done by the applied field is eEv per unit time. It must be equal to the energy gained by the electron. Thus

$$\frac{d\mathbf{p}}{dt} = e\mathbf{E}. \tag{1.5}$$

An increasing **p**, however, does not necessarily imply that the electron is accelerating. In practice, because of scattering, the momentum of the electron cannot undergo a large change under the influence of the external field. It is only accelerated over a mean free path, while the fields that can be set up in metals are never strong enough to effect an appreciable change in electron crystal momentum over a relaxation time. As a result, the electrons effectively remain close to the Fermi-surface.

1.2.2 The Boltzmann Kinetic Equation

Let us introduce the distribution function $n(\mathrm{p}, \mathrm{r}, t)$ in a phase space. The total time rate of change of the distribution function is expressed through its total derivative. This change is due to collisions of electron with other electrons and due to scattering on defects in the crystal. In some circumstances the scattering of electrons by the surfaces of the sample may also be significant.

Statistical mechanics tells us that electrons obey Fermi-Dirac statistics. The probability of occupation of a level of energy ε is

$$n(\varepsilon) = \left(e^{\frac{(\varepsilon-\mu)}{kT}} + 1\right)^{-1} \tag{1.6}$$

where μ is the Fermi energy and at absolute zero it can be seen that the Fermi distribution function becomes simply a step function: $n(\varepsilon) = 1$ for $\varepsilon < \mu$ and $n(\varepsilon) = 0$ for $\varepsilon > \mu$.

If we restrict ourselves to energy conserving collisions, then these may take into account by introducing the collision term $\left(\frac{\partial n}{\partial t}\right)_c$ to obtain generalized Boltzmann equation:

$$\frac{\partial n}{\partial t} + \mathrm{v}\frac{\partial n}{\partial \mathrm{r}} + \mathrm{F}\frac{\partial n}{\partial \mathrm{p}} = \left(\frac{\partial n}{\partial t}\right)_c \tag{1.7}$$

where $\mathrm{v}\frac{\partial n}{\partial \mathrm{r}}$ measures the non uniformity of the carrier distribution and known as a diffusion or drift term since it frequently arises with thermal gradients and $\mathrm{F}\frac{\partial n}{\partial \mathrm{p}}$ is the force term due to an external field.

Usually the use of relaxation time approximation is based on the assumptions on collision processes as elastic scattering and isotropic medium for simplification.

The number of transferred quasi-particles per unit time from state p to state p' due to collision can be given as

$$\frac{\delta (dN)}{\delta t} = \frac{4}{(2\pi\hbar)^6} d^3\mathrm{p} \int d^3\mathrm{p}' W(\mathrm{p},\mathrm{p}')\, \delta(\varepsilon - \varepsilon')(n' - n),$$

$W(\mathrm{p},\mathrm{p}')$ being the probability per unit time that an electron to be occupying the state p be scattered into the state p' known to be unoccupied and have used equal probability of transition from p state to p' and from p' state to p state, i.e., $W(\mathrm{p},\mathrm{p}') = W(\mathrm{p}',\mathrm{p})$ as a common factor.[5] The number of occupied states at any time between p and $\mathrm{p} + d\mathrm{p}$ will be $dN = \frac{2}{(2\pi\hbar)^3} n d^3\mathrm{p}$ and the change in occupied states as a result of collision is $\delta(dN) = \frac{2}{(2\pi\hbar)^3}\left(\frac{\partial n}{\partial t}\right)_c \delta t d^3\mathrm{p}$. By equating the above equations the collision integral will give us[5]

$$\left(\frac{\partial n}{\partial t}\right)_c = \int W(\mathrm{p},\mathrm{p}')\, \delta(\varepsilon - \varepsilon')\, [n' - n] \frac{d^3\mathrm{p}'}{(2\pi\hbar)^3}$$

It is usually permissible in some transport problems to assume that the steady state distribution $n(\mathrm{p},\mathrm{r},t)$ in the field under isothermal conditions does not depart very far from the equilibrium distribution $n_o(\mathrm{p})$. Then the distribution of the excited states:

$$n_1(\mathrm{p},\mathrm{r},t) = n(\mathrm{p},\mathrm{r},t) - n_o(\mathrm{p}).$$

Particularly when interested only in terms of order $n_1(\mathrm{p},\mathrm{r},t)$ in the Boltzmann equation that are linear in the field, then we may linearize (1.7) in the form

$$\frac{\partial n_1}{\partial t} + \mathrm{v}\frac{\partial n_1}{\partial \mathrm{r}} + \mathbf{F}\frac{\partial n_o}{\partial \mathrm{p}} = I(n_1), \tag{1.8}$$

where

$$I(n_1) = \int W(\mathbf{p}, \mathbf{p}') \, \delta(\varepsilon - \varepsilon') \left[n_1(\mathbf{p}) - n_1(\mathbf{p}')\right] \frac{d^3\mathbf{p}'}{(2\pi\hbar)^3}.$$

Finally, to make relaxation time approximation, if $n_1(\mathbf{p}, \mathbf{r}, t)$ is caused by the application of an external field to the equilibrium distribution and, at time $t = 0$, we switch off the external field, then $n_1(\mathbf{p}, \mathbf{r}, t)$ decays exponentially in a characteristic time, τ, called relaxation time: $n_1(\mathbf{p}, \mathbf{r}, t) = n_1(\mathbf{p}, \mathbf{r}, 0) \, e^{\frac{-t}{\tau}}$. This implies that, under isothermal conditions and in a zero external field (1.8) reduces to relaxation time approximation as

$$I(n_1) = \frac{-n_1}{\tau} = -\frac{n(\mathbf{p}, \mathbf{r}, t) - n(\mathbf{p}, \mathbf{r}, 0)}{\tau}.$$

The relaxation time depends only on the relevant collision processes and for most cases depends on the energy ε of electrons $\tau(\varepsilon)$.

Any electron-electron interaction conserves overall momentum and the combined current density of interacting particles and, hence, the collisions that destroy the drift velocity of the electrons are not electron-electron collisions.[6] The collisions effective in changing the current density are primarily interactions between electrons and phonons and between electrons and impurity atoms or lattice imperfections. In pure crystalline materials, electron- phonon collisions are most probable. Since phonons have no charge, momentum conservation no longer conserves current density. Because of the increment of the phonon number as the temperature rises, electron-phonon collisions are most frequent at high temperatures. This means that the average relaxation time τ between electron-phonon collision decreases with rising temperature and accounts for the decrease of electrical conductivity of normal metals. Meanwhile, the mean free path can be defined as the average distance covered by an electron between successive collisions, given as $l = v\tau$, v being the mean velocity. In this work, however, we are not going to analyze using a concrete nature of collision but we use the τ approximation for evaluation purpose only.

1.2.3 Fermi-gas Kinetic Equation

The behavior of free-electron gas is very similar to that of an ideal gas. An electron of a crystal is in the field of the lattice and other electrons. In the absence of external fields and temperature gradient, the distribution of electron gas is the equilibrium one. But if sufficiently small electric field, magnetic field or temperature gradient is applied, there will be small deviation of the distribution function from the equilibrium one and hence one can write

$$n(\mathbf{p}, \mathbf{r}, t) = n_o(\mathbf{p}) + n_1(\mathbf{p}, \mathbf{r}, t), \tag{1.9}$$

where $n_1(\mathbf{p}, \mathbf{r}, t)$ is a small correction to the equilibrium distribution function from $n_o(\mathbf{p})$, i.e., $|n_1| << n_o$. It is seen earlier that this function $n_1(\mathbf{p}, \mathbf{r}, t)$ contributes to the collision integral.

In equation (1.9) the higher order corrections to the equilibrium distribution are neglected based on the assumption that the external fields are weak. In order to obtain a more applicable solution for $n_1(\mathbf{p}, \mathbf{r}, t)$, let us introduce a new variable $\psi(\mathbf{p}, \mathbf{r}, t)$ defined by

$$n_1(\mathbf{p}, \mathbf{r}, t) = -\frac{\partial n_o}{\partial \varepsilon}\psi(\mathbf{p}, \mathbf{r}, t). \tag{1.10}$$

By combining equations (1.8), and (1.10) in relaxation time approximation given above under application of weak external fields one can obtain the linearized equation in an external field as

$$\frac{\partial \psi}{\partial t} + \mathbf{v}\frac{\partial \psi}{\partial r} + \frac{\psi}{\tau} = \mathbf{F}\mathbf{v} \tag{1.11}$$

when $\mathbf{F}$ is the external applied force due to electric or magnetic fields. In case the force applied is the Lorenz force

$$\mathbf{F} = e\mathbf{E} + \frac{e}{c}\mathbf{v} \times \mathbf{B},$$

and if the time dependence is in such a way that $\mathbf{E}, \mathbf{B}$, ψ are proportional to $e^{-i\omega t}$ then equation (1.11) becomes

$$(\nu - i\omega)\psi + \mathbf{v}\frac{\partial \psi}{\partial \mathbf{r}} = e\mathbf{v}\mathbf{E}. \tag{1.12}$$

When there is no temperature gradient and the electric field is very weak, equation (1.8) in steady state can be written in a linearized form as

$$n = n_o - e\tau\mathbf{v}\mathbf{E}\frac{\partial n_o}{\partial \varepsilon}. \tag{1.13}$$

Hence, the variable introduced in equation (1.10) and equation (1.13) can be related as

$$\psi(\mathbf{p}) = e\tau(\mathbf{p})\mathbf{v}\mathbf{E}. \tag{1.14}$$

The expression for the electric current in terms of the distribution function is given in the next section below.

1.3 The Concept of Fermi Liquid Theory

1.3.1 The Quasi-particle concept

In the case when electron-electron interaction is important unlike the behavior of a single electron in the average potential of the lattice and other electrons, the actual case of electron fluid is considered. The states of an interacting system are called quasi-particle states (or elementary excitations), each of which has a definite momentum $\mathbf{p}$. Physically, a quasi-particle may be pictured as a single particle surrounded by a self-consistent distribution of other particles.

The Fermi liquid theory was introduced by Landau[7] to deal with the liquid state of the isotope of helium of mass number 3, but is increasingly extended and being applied to the theory of electron-electron interactions in metals by Silin (1958).[8]

The meaning of this quasi-particle description is clarified when one considers the energy of the vibrating lattice. The energy levels are given by the following expression, which is valid for a system of independent oscillators[3]:

$$E - U_o = \frac{1}{2}\sum_{\mathbf{k},s} \hbar\omega(\mathbf{k},s) + \sum_{\mathbf{k},s} \hbar\omega(\mathbf{k},s) n(\mathbf{k},s).$$

The $n(\mathrm{k}, s)$ are either zero or positive integers. **k** and s correspond to separate oscillators. The first term corresponds to the lowest value of the energy, i.e., the ground state of the system. This is the energy of the so-called zero-point oscillations, that the atoms of a crystal lattice must vibrate even in the ground state is a consequence of the uncertainty principle.

In an excited state, some of the integers $n(\mathrm{k}, s)$ are different from zero. Then the equation describes a system of independent particles with energies $\hbar\omega(\mathrm{k}, s)$. Since the integers $n(\mathbf{k}, s)$ can take arbitrary positive values, any number of phonons can be found in any given state. Therefore, they obey Bose statistics.

The phonon picture remains valid so long as the vibration amplitude remains small compared with the lattice constant. Higher-order terms in the expansion of the potential energy U in powers of the displacements must otherwise be taken into account. The total energy can then no longer be written as the above expression. This happens only in the vicinity of the melting point.

According to Landau, any homogeneous system containing a large number of particles has low-lying states similar to those of a vibrating lattice. The properties of such systems can be described in terms of the quasi-particle model. Quasi-particles can have integer $(n\hbar)$ or half-integer $\left((n+\frac{1}{2})\hbar\right)$ spin, i.e., they can be either bosons or fermions. The statistics of quasi-particles bear no unique relationship to the statistics of real particles in the system. The energy of a quasi particle is a function of its momentum. The corresponding relation $\varepsilon(\mathbf{p})$ is the main characteristic of such low-lying excited states.

For our practical purposes, we are concerned with a system of interacting particles obeying Fermi statistics, by restricting ourselves to the case where the spin of the particles equals $\frac{1}{2}$. Such a system of interacting particles is said to be Fermi liquid.

The excitations in the Fermi liquid are of two types, 'particles' with momenta greater than p_o and 'holes' with momenta less than p_o, which appear and disappear in pairs. For weakly excited states the elementary excitations have momenta in the neighborhood of p_o.

1.3.2 An isotropic Fermi Liquid

Considering the free electron gas of equilibrium distribution function(1.6) where $\varepsilon = \frac{p^2}{2m}$ and in terms of the Fermi momentum p_0, the Fermi energy, $\mu(0) = \frac{p_0^2}{2m}$, one finds that at $T = 0$ all states with momenta inside a sphere of radius $p = p_0$ (the Fermi sphere) are occupied, while those outside are unoccupied. This is a consequence of Pauli's exclusion principle, that only one particle can be in any given state, so that the lowest states are filled at $T = 0$. The occupied volume in phase space, including momenta, coordinates, and spin, divided by $(2\pi\hbar)^3$ must equal the total number of particles. The corresponding volume in momentum space is that of the Fermi sphere. The allowed values of the components of spin along a quantization axis give a factor of 2. We therefore obtain

$$N = \frac{2V}{(2\pi\hbar)^3}\frac{4\pi}{3}p_0^3 \text{ i.e., } p_0 = \hbar\left(\frac{3\pi^2 N}{V}\right)^{1/3}. \tag{1.15}$$

The quasi-particle concept is valid if the damping coefficient $\gamma \ll \xi$ i.e., close to the Fermi level. If one deals with a Fermi liquid in equilibrium at $T \neq 0$, the relevant quasi-particles all have energies $\xi \sim K_b T$. The damping coefficient is then of the order $K_b^2 T^2/\mu$. The description of the liquid in terms of quasi-particles is therefore valid only so long as $K_b T \ll \mu$.

For electrons in metals the quantity $\hbar/p_0$ (the de Broglie wavelength) is of the same order as interatomic distances, i.e., 10^{-8} cm, so that $p_0 \sim 10^{-19}\,\mathrm{g\,cm\,s^{-1}}$. Therefore, one obtains $T \ll T_0 \sim 10^4 - 10^5$ K. This condition shows that the quasi-particle picture certainly applies to solid metals at all temperatures, since T_0 significantly exceeds the melting temperature in any case.

In Landau's theory, it is assumed that a weakly excited state of an isotropic Fermi liquid greatly resembles a weakly excited state of a Fermi gas. Thus, the energy of excitations of the particle type is (for $|p - p_o| << p_o$)

$$\xi_p(p) \approx v(p - p_o),$$

and the energy of excitations of the 'hole' (anti-particle) type is $\xi_a(p) \approx v(p_o - p)$,where $v = \frac{p_o}{m}$, p_o being the quasi-particle momentum at the Fermi surface and m is the effective mass of a quasi-particle. This convention simply means that more energy is required to create an antiparticle deep in side the Fermi distribution than close to the Fermi level.

Despite the great resemblance between excitations in a Fermi liquid and excitations in an ideal Fermi gas, there are also important differences between them, due to the fact that excitations in the liquid interact with each other. As a result of interaction, the idea of elementary excitations makes sense only near the Fermi momentum p_o.

1.3.3 The Anisotropic Fermi Liquid

In order to understand the main features of anisotropic Fermi liquid spectra of electrons in metals, let us first "switch off" the interaction between electrons or, rather, let us consider a gas of non-interacting electrons placed in an average periodic field. The states of a single particle in such a field were analyzed in the preceding section. As shown there, the energy levels form bands separated by forbidden gaps.

The many non-interacting particles present are distributed in some way among these levels. At $T = 0$ (essentially at temperatures below the melting point in metals), all states up to a certain maximum energy (the Fermi level) will be occupied, whereas the higher states will be unoccupied. If the Fermi level coincides with the upper edge of a band, some bands are then completely filled, whereas others are completely empty. In such a case a sufficiently weak electric field cannot give rise to a net current, and the substance will be an insulator rather than a metal.

If the Fermi level falls within the center of a band, it is only partly filled and is called a conduction band. The redistribution of electrons among available states can be achieved at the cost of an infinitesimally small increase in energy. An arbitrarily weak electric field can give rise to current flow. This situation describes a metal. Semiconductors belong to the first category, but the gap between occupied and unoccupied states is then small . Their properties are, therefore, similar to those of metals unless the temperature is very low. We shall limit our attention to actual metals, however.

It is easy to understand why at least one band must be only partly filled if the number of electrons per unit cell is odd .If the number of electrons per unit cell is even, the substance can still be a metal, however, since energy bands can overlap in a real, three-dimensional situation. Several partly filled bands will then exist.

In general the Fermi surface may have a very complex shape. It has a very simple shape in two cases. The first one corresponds to an almost empty band. Only a few electrons are present, and, at $T = 0$, they fill up the lowest available states, and must all be in the vicinity of the minimum of $\varepsilon(\mathrm{p})$. If this minimum occurs at $\mathrm{p}^{(0)}$, we may expand the energy in powers of $\left(\mathrm{p} - \mathrm{p}^{(0)}\right)$. Linear terms will, of course, be absent. For a cubic crystal with $\mathrm{p}^{(0)} = 0$ one obtains $\varepsilon(\mathrm{p}) = \varepsilon_0 + \frac{\mathrm{p}^2}{2m}$ where m is a constant called the band effective mass. In the more general case of arbitrary symmetry, but with $\mathrm{p}^{(0)} = 0$ as before, one obtains a positive definite quadratic form instead of p^2. After a principal axis transformation it has the form $\varepsilon(\mathrm{p}) = \varepsilon_0 + \frac{1}{2}\left(\frac{p_x^2}{m_1} + \frac{p_y^2}{m_2} + \frac{p_z^2}{m_3}\right)$.

A similar situation occurs in the case of an almost filled band. One can then deal with "holes," i.e., empty states in the band, rather than electrons. They will all occur near energy maxima. In the case of a cubic crystal with $\mathrm{p}^{(0)} = 0$ one has $\varepsilon(\mathrm{p}) = \varepsilon_0 - \frac{\mathrm{p}^2}{2m}$. This means that holes behave as particles with a negative mass. Generalizations for arbitrary symmetry and $p^{(0)} \neq 0$ are trivial: In all cases just considered, the Fermi surface is an ellipsoid or consists of a set of ellipsoids.

In this context, it is important to remember that the only relevant properties of the gas model are those involving particles close to the Fermi surface. As a result, our previous discussion referring to electrons "filling" bands may seem of doubtful validity since "deep" states were certainly involved in those arguments.

1.3.4 Energy of a quasiparticle

It is the interaction between quasi-particles which does play an important role in a real Fermi liquid .The interaction between quasi-particles, according to Landau, can be represented as a kind of self-consistent field. This field is due to surrounding quasi-particles acting on a given one. As a result, the energy of a quasi-particle must depend on the state of other quasi-particles, i.e., it must be a functional of the quasi-particle distribution function and the energy of the entire system is not simply the sum of the energies of individual quasi-particle states but depends on the distribution function as well. It is this functional dependence of quasi-particle energy on distribution function which can produce new effects called Fermi liquid effects.

Suppose that $n(\mathrm{p})$ is changed by a small amount $\delta n(\mathrm{p},\mathrm{r})$. The change in the energy of the system to the first order in $\delta n(\mathrm{p},\mathrm{r})$ is[7,8]

$$\delta E = \frac{1}{(2\pi\hbar)^3} \int \varepsilon(\mathrm{p})\, \delta n(\mathrm{p},\mathrm{r})\, d^3\mathrm{p} d^3\mathrm{r} \qquad (1.16)$$

In a situation in which it is necessary to take account of spin explicitly, we consider a separate distribution for each spin state and replace (1.16) by

$$\delta E = \frac{1}{(2\pi\hbar)^3} Tr_\sigma \int \varepsilon_\sigma(\mathrm{p})\, \delta n_\sigma(\mathrm{p},\mathrm{r})\, d^3\mathrm{p} d^3\mathrm{r}, \tag{1.17}$$

where the trace is taken with respect to the spin matrix σ and the quantity $\varepsilon_\sigma(\mathrm{p})$ is the effective energy of the quasi-particle.

1.3.5 The Landau f-Function

The primary effect of electron-electron interactions in a quasi-particle picture is simply to change the excitation energies $\varepsilon(\mathrm{p})$ from their free electron values. This has an important implication for the structure of transport theories.

When electric or thermal currents are carried in a metal, the electronic distribution function $n(\mathrm{p},\mathrm{r},\mathrm{t})$ will differ from its equilibrium form $n_o(\mathrm{p},\mathrm{r})$. For independent electrons this has no effect on the form of $\varepsilon(\mathrm{p})$, that is, on the form of $\varepsilon(\mathrm{p})$ verses p. Since quasi-particle energy takes into account the electron-electron interactions, it is frequently necessary to obtain the change in energy of a system to second order in $\delta n(\mathrm{p},\mathrm{r})$, by making Taylor's expansion in powers of $\delta n(\mathrm{p},\mathrm{r})$. This is written as[5,8]

$$E = E_o + \sum_{p,r,\sigma} \frac{\partial E}{\partial n} \delta n + \frac{1}{2} \sum_{p,r,\sigma} \sum_{p',r',\sigma'} \frac{\partial^2 E}{\partial n \partial n'} \delta n \delta n'.$$

By changing the summation into integration one can have

$$E = E_o + \frac{Tr_\sigma}{(2\pi\hbar)^3} \int \left(\varepsilon_o(\mathrm{p}) + \frac{Tr_{\sigma'}}{(2\pi\hbar)^3} \int f(\mathrm{p},\mathrm{r},\sigma;\mathrm{p}',\mathrm{r}',\sigma')\, \delta n' d^3\mathrm{p}' d^3\mathrm{r}' \right) \delta n d^3\mathrm{r} d^3\mathrm{p},$$

where $f(\mathrm{p},\mathrm{r},\sigma;\mathrm{p}',\mathrm{r}',\sigma') = \frac{\delta^2 E}{\delta n \delta n'}$, $\varepsilon_o(\mathrm{p}) = \frac{\partial E}{\partial n}$. The function $f(\mathrm{p},\mathrm{r},\sigma;\mathrm{p}',\mathrm{r}',\sigma')$, which is the second variational derivative of the total energy with respect to distribution function, is the basic phenomenological quantity characterizing the Fermi liquid and it is symmetric with respect to the interchange of the variables $\mathbf{p}$ and $\mathbf{p}'$, $\mathbf{r}$ and $\mathbf{r}'$,and $\sigma$and σ'. Thus, we have

$$\varepsilon(\mathbf{p}) = \varepsilon_o(\mathbf{p}) + \frac{Tr_{\sigma'}}{(2\pi\hbar)^3}\int\int f(\mathbf{p},\mathbf{r},\sigma;\mathbf{p}',\mathbf{r}',\sigma')\, d^3\mathbf{r}'\delta n' d^3\mathbf{p}'.$$

In non-equilibrium situations, a change in quasi-particle energy due to change of distribution function from equilibrium will have a form of

$$\delta\varepsilon = \frac{Tr_{\sigma'}}{(2\pi\hbar)^3}\int f(\mathbf{p},\mathbf{r},\sigma;\mathbf{p}',\mathbf{r}',\sigma')\,\delta n' d^3\mathbf{p}' d^3\mathbf{r}'. \tag{1.18}$$

Thus, not only $\varepsilon(\mathbf{p})$ for a given distribution, but also the change in $\varepsilon(\mathbf{p})$ produced by a change in distribution function δn, is of essential importance in the theory of Fermi liquid.

In order to calculate the response of the system to slowly varying electromagnetic disturbances, we use the Landau theory of Fermi liquid. By 'slowly varying' means that the frequency ω and the wave vector k of the external field satisfy[9]

$$\hbar\omega \ll \varepsilon_F, \omega \ll \omega_D \text{ and } k \ll k_F,$$

where ω_D is the Debye frequency and k_F is the wave-vector corresponding to Fermi energy. In this limit the response of the fermion system is completely described by a quasi-particle energy-momentum relationship. In equilibrium, the quasi-particle energy

$$\varepsilon_o(\mathbf{p}) = \frac{p^2}{2m}$$

m being the effective mass of electron and the quasi-particle distribution function $n_o(p)$ is given by equation (1.6).

Consider a Fermi liquid under the application of weak electromagnetic field, and let the distribution function changes in such a way that

$$n\left(\mathbf{p},\mathbf{r},t\right)=n_o\left(\varepsilon'\right)-\frac{\partial n_o}{\partial\varepsilon}\psi\left(\mathbf{p},\mathbf{r},t\right)=n_o\left(\varepsilon_o\right)+\delta n\left(\mathbf{p},\mathbf{r},t\right), \tag{1.19}$$

where $n_o\left(\varepsilon'\right)$ is the equilibrium distribution function at temperature different from zero, $\psi\left(\mathbf{p},\mathbf{r},t\right)$ being the variation of energy in the gas approximation, $n_o\left(\varepsilon_o\right)$ is the equilibrium distribution function at temperature $T=0$ and $\delta n\left(\mathbf{p},\mathbf{r},t\right)$ being variation from it. As a result, let's write the energy of a quasi-particle as

$$\varepsilon\left(\mathbf{p},\mathbf{r},t\right)=\varepsilon'\left(\mathbf{p},\mathbf{r}\right)+\delta\varepsilon'\left(\mathbf{p},\mathbf{r},t\right)$$

$\varepsilon'\left(\mathbf{p},\mathbf{r}\right)$ being the sum of energies of a quasi-particle and the self-consistent field and $\delta\varepsilon'\left(\mathbf{p},\mathbf{r},t\right)$ as the variation in the quasi-particle energy as a result of exchange. We can rewrite this as the sum of the equilibrium energy of the quasi-particle in the Fermi gas approximation $\varepsilon_o\left(\mathbf{p}\right)$, the variation of energy from equilibrium due to the self-consistent potential $\delta\varepsilon''\left(\mathbf{r},t\right)$ and the exchange term $\delta\varepsilon'\left(\mathbf{p},\mathbf{r},t\right)$, i.e.,

$$\varepsilon\left(\mathbf{p},\mathbf{r},t\right)=\varepsilon_o\left(\mathbf{p}\right)+\delta\varepsilon'\left(\mathbf{p},\mathbf{r},t\right)+\delta\varepsilon''\left(\mathbf{r},t\right), \tag{1.20}$$

where $\delta\varepsilon'\left(\mathbf{p},\mathbf{r},t\right)=\Psi\left(\mathbf{p},\mathbf{r},t\right)$. Therefore, under these situations equation (1.19) obtains

$$n\left(\mathbf{p},\mathbf{r},t\right)=n_o\left(\varepsilon_o\right)+\frac{\partial n_o}{\partial\varepsilon}\left[\Psi\left(\mathbf{p},\mathbf{r},t\right)-\psi\left(\mathbf{p},\mathbf{r},t\right)\right]. \tag{1.21}$$

Hence, we can get the linearized terms of the Boltzmann kinetic equation by taking partial derivatives of (1.21) with respect to t, $\mathbf{r}$ and $\mathbf{p}$:

$$\begin{aligned}\frac{\partial n}{\partial t} &= \frac{\partial n_o}{\partial\varepsilon}\frac{\partial}{\partial t}\left(\Psi\left(\mathbf{p},\mathbf{r},t\right)-\psi\left(\mathbf{p},\mathbf{r},t\right)\right);\ \frac{\partial n}{\partial\mathbf{r}}=\frac{\partial n_o}{\partial\varepsilon}\frac{\partial}{\partial\mathbf{r}}\left(\Psi\left(\mathbf{p},\mathbf{r},t\right)-\psi\left(\mathbf{p},\mathbf{r},t\right)\right);\\ \frac{\partial n}{\partial\mathbf{p}} &= \frac{\partial n_o}{\partial\varepsilon}\left(\mathbf{v}+\frac{\partial}{\partial\mathbf{p}}\Psi\left(\mathbf{p},\mathbf{r},t\right)-\frac{\partial}{\partial\mathbf{p}}\psi\left(\mathbf{p},\mathbf{r},t\right)\right),\end{aligned}$$

and the partial derivatives of (1.20) with respect to $\mathbf{p}$ and $\mathbf{r}$ gives

$$\frac{\partial \varepsilon}{\partial \mathbf{p}} = \mathbf{v} + \frac{\partial}{\partial \mathbf{p}} \Psi(\mathbf{p}, \mathbf{r}, t) \text{ and } \frac{\partial \varepsilon}{\partial \mathbf{r}} = \frac{\partial}{\partial \mathbf{r}} \left(\Psi(\mathbf{p}, \mathbf{r}, t) + e\phi(\mathbf{r}, t) \right). \tag{1.22}$$

The force acting on a quasi-particle, in this case, is not due to the external field alone, but there is additional effective potential describing the self-consistent field set by other quasi-particles.[3] One must therefore take

$$\frac{d\mathbf{p}}{dt} = e\mathbf{E} + \frac{e}{c} \frac{\partial \varepsilon}{\partial \mathbf{p}} \times \mathbf{H} - \frac{\partial \varepsilon}{\partial \mathbf{r}}. \tag{1.23}$$

After substituting the above linearized terms of the Boltzmann kinetic equation in equation (1.7) one can get

$$\frac{\partial \Psi(\mathbf{p}, \mathbf{r}, t)}{\partial t} - \frac{\partial \psi(\mathbf{p}, \mathbf{r}, t)}{\partial t} - \mathbf{v} \frac{\partial \psi(\mathbf{p}, \mathbf{r}, t)}{\partial r} + e\mathbf{v}\mathbf{E} = \frac{\psi(\mathbf{p}, \mathbf{r}, t)}{\tau},$$

and similar to the consideration made on equation (1.12) the above equation gives

$$(\upsilon - i\omega)\, \psi(\mathbf{p}, \mathbf{r}, t) + \mathbf{v} \frac{\partial \psi(\mathbf{p}, \mathbf{r}, t)}{\partial \mathbf{r}} = e\mathbf{v}\mathbf{E} - i\omega \Psi(\mathbf{p}, \mathbf{r}, t). \tag{1.24}$$

The general expression for the electrical current density is

$$\mathbf{j}(\mathbf{p}, \mathbf{r}, t) = \frac{2e}{(2\pi\hbar)^3} \int \frac{\partial \varepsilon}{\partial \mathbf{p}} n(\mathbf{p}, \mathbf{r}, t)\, d^3p, \tag{1.25}$$

and finally, one can get a more applicable form of the electrical current density

$$\mathbf{j} = \frac{-2e}{(2\pi\hbar)^3} \int \mathbf{v} \frac{\partial n_o}{\partial \varepsilon} \psi(\mathbf{p}, \mathbf{r}, t)\, d^3p, \tag{1.26}$$

in terms of $\psi(\mathbf{p}, \mathbf{r}, t)$ on inserting equations (1.21) and the first part of equation (1.22) in equation (1.25) by considering only the linearized terms and reminding that no current can be produced by equilibrium distribution of quasi-particles.

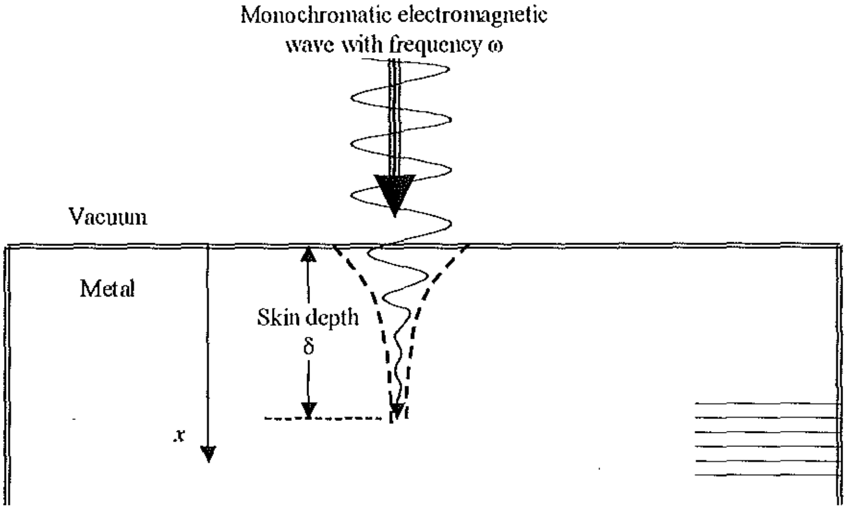
Chapter 2
Theory of anomalous skin effect

The behavior of electromagnetic waves in conducting media is entirely different from that in free space. The difference arises because the propagation in free space (or in a dielectric medium) depends highly on the existence of the displacement current, whereas in conducting media the propagation characteristics are determined by much larger free conduction electrons.

For construction of the theory of anomalous skin effect, we use the Maxwell's equation supplemented by the constitutive law connecting the electric current density with the electric field strength. The expression for the current density obtained with the help of the solutions of the kinetic equation generally takes the form of integral operator, in which the value of the electrical field in all points of the metal appears, and consequently, Maxwell's equation represents an integral-differential equation from which the electrical field should be defined.

We consider a metallic half-space (see fig. 1), on which of a monochromatic electromagnetic wave with a frequency ω is incident from vacuum along the normal to the metal surface .The surface of the metal can coincide with the plane of symmetry of the crystal or can be perpendicular to it. The metal is supposed unmagnetized, so that $\mathbf{B} = \mathbf{H}$ in it.

We first consider the normal skin effect.



1.Geometry of observation of damping effects of electromagnetic wave in layered conductor.

2.1 The Normal Skin Effect

The penetration of electromagnetic wave into a certain depth of a metal in the direction of propagation of the wave is known as skin effect.

In the classical or normal skin effect, the conductivity of a conducting medium is taken to be constant and the material as isotropic for simplicity. This is valid only if the electric field does not vary rapidly in space over the distance of the order of the electron mean free path. The usual relationship between the current density (j) and the electric field (E) (Ohm's law) is considered and the displacement current can be neglected for good conductors for which the frequency of the wave is much less than the electrical conductivity of the metal, that is, $\omega \ll \sigma$.

The appropriate forms of Maxwell's equations used in the calculation of the skin depth (δ) or the penetration of electromagnetic waves into a thin plane slab of metal are

$$\nabla \times \mathrm{H} = -\frac{i\omega}{c}\mathrm{E} + \frac{4\pi}{c}\mathrm{j} \text{ and } \nabla \times \mathrm{E} = -\frac{1}{c}\frac{\partial \mathrm{B}}{\partial t}. \tag{2.1}$$

In metals conductivity is the largest quantity of frequency dimension and one can neglect the term $\frac{i\omega}{c}\mathrm{E}$, which describes displacement current. Furthermore, for nonferromagnetic (normal) metals $\mathbf{B}$ can be replaced by $\mathbf{H}$.

The assumption of classical skin effect is valid when the field does not vary appreciably over a distance of the order of the free path of the conduction electrons. In other words, when the free path is small compared with the effective depth of penetration of the electric field. This condition is satisfied for all values of the frequency if the temperature is sufficiently high.

For monochromatic electromagnetic wave incident normally on the surface ($x - y$ -plane) of a semi-infinite metal $z > 0$ with electric field polarized along y axis and the magnetic field polarized along x axis, Maxwell equations give

$$\frac{d^2 E_\alpha}{dz^2} = \frac{-4\pi i\omega}{c^2} j_\alpha. \tag{2.2}$$

For isotropic material and constant conductivity ($\mathrm{j} = \sigma\mathrm{E}$) the solution of equation (2.2) is proportional to $\exp i\left(kz - \omega t\right)$ where the magnitude of the wave vector k is given as

$$k = \left[\frac{4\pi\omega\sigma}{c^2}\right]^{\frac{1}{2}} e^{\frac{i\pi}{4}}. \tag{2.3}$$

The wave vector has real and imaginary parts and by substituting equitation (2.3) in the solution of the wave equation, we can see that the electromagnetic field decays in side the metal; so that, it only penetrates a distance δ into a surface layer. In other words, electromagnetic field falls off as $\exp\left(-\operatorname{Im}(k)\,z\right)$ in side the metal and the classical skin depth is given by

$$\delta_{cl} = \operatorname{Im}(k)^{-1} = \frac{c}{[2\pi\omega\sigma]^{\frac{1}{2}}} \tag{2.4}$$

and it is this phenomenon which is well known, classically, as a normal skin effect.

For a good metal at the microwave frequency of about 10^{10} Hz and room temperature, the skin depth $\delta \sim 10^{-4}$ cm, while at liquid helium temperature in pure specimens $\delta \sim 10^{-6}$ cm.[3,10]

The surface impedance (Z) is a complex quantity in which the real and imaginary parts are referred to as the surface resistive (R) and the surface reactance (X) respectively. It can be obtained from the ratio of the electric field at the surface of the metal to the total current integrated over the interior of the sample as[3]

$$Z = R - iX = \frac{E_y(o)}{\int\limits_0^\infty j_y(z)\,dz},$$

and it can be transformed into another form by relating equation (2.1) to the above equation as

$$Z = \frac{4\pi E_y(0)}{cH_x(0)} = \frac{4\pi\omega}{c^2 k}, \tag{2.5}$$

which can be rewritten as

$$Z = \frac{4\pi i\omega}{c^2}\frac{E_y(0)}{E_y'(0)} \tag{2.6}$$

using the second pair of Maxwell equation. Thus, the surface impedance Z in the classical limit gets the form

$$Z = R - iX = \left[\frac{4\pi\omega}{\sigma c^2}\right]^{\frac{1}{2}} e^{\frac{-i\pi}{4}}, \tag{2.7}$$

while in this classical limit,

$$R = X = \left[\frac{2\pi\omega}{\sigma c^2}\right]^{\frac{1}{2}}. \tag{2.8}$$

One can get the same result by using the correction to the distribution function in the presence of a weak electric field, i.e., equation (1.13) and the current density[11] which can be given as $j = \frac{-Ne}{m}\frac{\int p_y n(p) g(p) d^3p}{\int n(p) g(p) d^3p}$, where $g(p)$ is the density of states and N being electrons concentration. The integral is evaluated over spherical coordinates (p, θ, ϑ) in momentum space p. After evaluating the angular integrals $j = \frac{Ne^2 E < p^2 \tau >}{3mkT}$ and from the concept of average kinetic energy per particle $\left\langle \frac{p^2}{2m} \right\rangle = \frac{3}{2}kT$, the electrical current density

$$j = \frac{Ne^2 \tau E}{m}, \tag{2.9}$$

from which, according to Ohm's law, the dc conductivity σ is

$$\sigma = \frac{Ne^2 \tau}{m}, \tag{2.10}$$

where N is the number density of charge carries. Then, by combining equations(2.2) and (2.9) one can get for the wave vector

$$k = \frac{(2\pi\omega\sigma)^{\frac{1}{2}}(1+i)}{c} \tag{2.11}$$

and the solution of equation (2.2) for this case will be

$$E(z,t) = E(0)\exp\left(i\frac{(2\pi\omega\sigma)^{\frac{1}{2}}(1+i)z}{c} - i\omega t\right). \tag{2.12}$$

From equation (2.12) it is easy to see that at low frequency ($\omega\tau \ll 1$), the penetration depth is the same as the one obtained with out kinetic equation and distribution function.

2.2 The Anomalous Skin Effect

2.2.1 The noneffectiveness approach (Qualitative)

At sufficiently low temperatures and /or high frequencies the mean free path l of the conduction electrons becomes larger than the skin depth δ. In this situation the relation $\mathbf{j} = \sigma \mathbf{E}$ no longer holds true and the effective field acting on the charge carrier varies rapidly over distance the carrier moves between collisions (l).

The region in which the mean free path of an electron is greater than the skin depth, for which a qualitative description has been given by Pippard, is known as the region of the anomalous skin effect. In this region, the electrons whose direction of motion makes a sufficiently small angle with the surface of the metal are regarded as making an effective contribution to the current density. The density of these "effective" electrons can be approximated as[3,10] $N_{eff} \sim N\left(\frac{\delta}{l}\right)$, where N is the concentration of electrons.

For pure metals at low temperatures and in fields of high frequencies it is possible to examine the expression (2.2) for skin depth as $\delta = \left[\frac{c^2 l}{4\pi\omega a\sigma}\right]^{\frac{1}{3}} \frac{2}{\sqrt{3}}$ when an effective conductivity of the form $\sigma_{eff} = \frac{ia\sigma}{kl}$ is assumed.[3] Here k is the wave vector and a is a constant of order one. In this case it is easy to note that the surface impedance is $Z = \left(\frac{2}{a}\right)^{\frac{1}{3}} \left(\frac{\pi\omega}{c^2}\right)^{\frac{2}{3}} \left(\frac{l}{\sigma}\right)^{\frac{1}{3}} \left(1 - i\sqrt{3}\right)$. From the above expression we can see that the surface impedance is proportional to $\omega^{\frac{2}{3}}$ and the relation between the resistive and the reactive parts is given as $X = \sqrt{3}R$. We can observe also the ratio of the mean free path to the conductivity is independent of temperature and is only determined by the electronic energy spectrum, i.e., $\frac{l}{\sigma} \sim \frac{ml}{Ne^2\tau} = \frac{p_o}{Ne^2}$, where τ is electron relaxation time.

Based on this concept that, it is possible to indicate the importance of microwave surface impedance measurements for obtaining information concerning the electronic energy band structure in metals, and can be rewritten in a more useful way as

$$Z = \left(\frac{2}{a}\right)^{\frac{1}{3}} \left(\frac{\pi\omega}{e^2c^2}\right)^{\frac{2}{3}} \left(\frac{p_o}{N}\right)^{\frac{1}{3}} \left(1 - i\sqrt{3}\right). \tag{2.13}$$

When the above concepts are analyzed, we can see that the frequency requirements necessary for observing anomalous skin effect is greater than 10^7 Hz. To see this result, the condition for $\delta > l$ together with the expression (2.4) are used where the electron concentration N for a good metal is approximately taken as $10^{22}\,\mathrm{cm}^{-3}$ and according to equation (1.15) the Fermi momentum as $p_o \sim 10^{-19}\,\mathrm{g\,cm\,s}^{-1}$ which is a typical value for a good metal and $l \sim 10^{-3}$ cm in the residual resistance range.

2.2.2 The Solution of the Kinetic Equation

G.E.H Reuter and E.H. Sondheimer,[2] have found a better expression for the theory of the anomalous skin effect in metals, which agreed to the qualitative non- effectiveness concept. They have examined the theory based on the electric field distribution in a metal using the Maxwell's equations supplemented with Boltzmann's kinetic equation for the degenerate electron gas distribution.

Considering a semi- infinite metal sample, the Maxwell's equation for the electric field $E(z)\exp(-i\omega t)$ due to a wave incident normally in z-direction is

$$\frac{d^2\mathrm{E}}{dz^2} + \frac{\omega^2}{c^2}\mathrm{E} = -\frac{4\pi i\omega}{c^2}\mathrm{j}(z). \tag{2.14}$$

To obtain the detail motion of the electrons in an arbitrary electric field, the nature of scattering of electrons at the surface of the metal is considered as the determining factor of the distribution of electrons. According to the authors, the fraction p of electrons arriving at the surface is scattered specularlly with reversal of the velocity component v_z ,while the rest $(1-p)$ is scattered diffusely with compete loss of their drift velocity, that is, they would have lost all the memory of their initial direction of their motion.

After calculating the current density for a degenerate electron gas by means of the formula

$$\mathrm{j}\,(z)=\frac{-2em^3}{(2\pi\hbar)^3}\int\int\int v_x n\,(\mathrm{p},\mathrm{r})\,dv_x dv_y dv_z,$$

in the extreme non-classical limit when $\delta \ll l$, one can obtain the surface impedance for specular and diffuse reflection as

$$Z=\frac{8}{9}\left(\frac{3\pi\omega^2 l}{c^4\sigma}\right)^{\frac{1}{3}}\left(1-i\sqrt{3}\right)\ \text{for}\, p=1 \tag{2.15}$$

and

$$Z=\left(\frac{3\pi\omega^2 l}{c^4\sigma}\right)^{\frac{1}{3}}\left(1-i\sqrt{3}\right)\ \text{for}\ p=0. \tag{2.16}$$

It can be seen that these results confirm the qualitative predictions; that is, in the limiting case of extreme departure from classical conditions, surface impedance varies as $\omega^{\frac{2}{3}}$ and is independent of the mean free path l.

Abrikosov A.A.[3], also developed the above idea for normal metals on arbitrary Fermi surface for specular reflection of electrons from sample surface. The result for the surface impedance in this case as this author worked out is

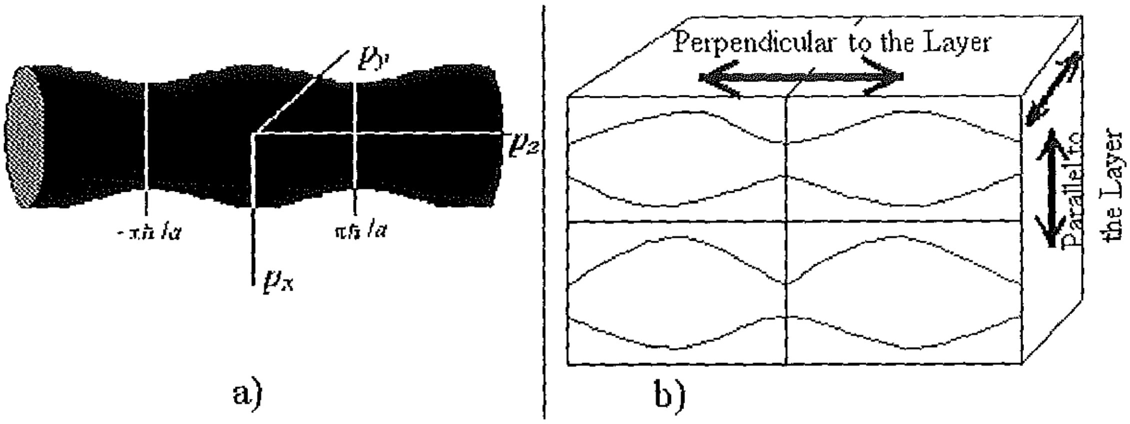
$$Z=\frac{8}{3\sqrt{3}}\pi^{\frac{4}{3}}\omega^{\frac{2}{3}}e^{\frac{-2}{3}}c^{\frac{-4}{3}}\hbar\mathrm{B}_{\beta\gamma}^{\frac{-1}{3}}\left(1-i\sqrt{3}\right)\ \text{for}\, p=1, \tag{2.17}$$

where Z is a tensor whose principal values can be expressed in terms of those of tensor $\mathbf{B}_{\beta\gamma}$: $\mathbf{B}_{\beta\gamma} = \int\limits_0^{2\pi} \frac{n_\beta n_\gamma}{G(\varphi)} d\varphi$ where $n_\beta = \frac{v_\beta}{v}$, $\beta, \gamma = x, y$ and $G(\varphi)$ is the Gaussian curvature of the Fermi surface.

Chapter 3
Theory of Anomalous Skin Effect in Layered Conductors

3.1 Layered Conductors

Many of the conductors synthesized in the recent years have a layered structure. These new artificially synthesized organic conductors have metallic type temperature dependence of electrical conductivity. Their Fermi-surface can be assumed as a poorly corrugated cylinder, i.e., the energy spectrum of quasi-particles weakly depends on the momentum component $p_z = \mathbf{p}\mathbf{n}$ in the direction normal to the plane of the layers. In both normal and superconducting states of these conductors, the electrical conductivity increases with the decreasing temperature above their transition temperature T_c and the superconductivity state is characterized by immeasurably small or actually zero resistance below a critical temperature.[12,13] The most sensitive measurements have shown that the resistance of these materials in the superconducting state is at least 10^{16} times smaller than their room temperature values. So far, 27 elements, numerous alloys, ceramic materials (containing copper oxide), and organic compounds (based, e.g. on selenium or sulfur) have been discovered to possess superconductivity.[12] Their critical temperature T_c have values between 0.01 K and 125 K. Superconductors having a T_c above 77 K (boiling point of liquid nitrogen) are particularly interesting because they do not require liquid helium (boiling point 4 K) or liquid hydrogen (boiling point 20 K) for cooling. Such materials are said to be high temperature



2.A sample with weakly corrugated cylinder Fermi surface open in the direction p_z (a) in electromagnetic field propagated perpendicular to the layers or along the layers (b)

superconductors of which most of them are organic compounds and above their critical temperatures they become normal metallic type conductors.

Interest in this field is gradually shifting from materials which have simple chemical compositions and structures to more complicated ones. Layered conductors are strongly anisotropic in electrical properties measured parallel or perpendicular to the layers. Their electrical conductivity in the normal (nonsuperconducting) state in the direction n perpendicular to the layers is much less than the conductivity along the layers.

The energy spectrum of elementary excitations carrying an electric charge in a layered conductors is a quasi-two-dimensional.[14,15]

We consider a layered conductor whose Fermi surface is a weakly corrugated cylinder. By applying the Fourier series due to the periodicity of the energy $\varepsilon(p)$ over a periodic reciprocal lattice vector in p_z, i.e.,$\varepsilon(p_x,p_y,p_z) = \varepsilon((p_x, p_y,(p_z + b\hbar))$ where $b = \frac{2\pi}{a}$ is the magnitude of reciprocal translational vector along p_z = pn and a being the separation between the layers.

In Fourier series any periodic function in $f(x)$ of periodicity a can be expressed as[16,17]

$$f(x) = \sum_{-\infty}^{\infty} A_n e^{in\beta x},$$

where A_n is a constant. In a similar way the energy of a quasi-particle in layered conductor can be expressed as

$$\varepsilon(p_x, p_y, p_z) = \sum_{-\infty}^{\infty} A_n(p_x, p_y)\, e^{in\beta p_z}$$

where the layer is in $p_x - p_y$ plane. Thus, the energy spectrum of a quasi-two-dimensional conductor by considering only the real constants of the series has a form:

$$\varepsilon(\mathrm{p}) = \sum_{0}^{\infty} \varepsilon_n(p_x, p_y) \cos\left(\frac{anp_z}{\hbar}\right). \tag{3.1}$$

The factor ε_n decreases rapidly with the increase of the number n $(\varepsilon_n >> \varepsilon_{n+1})$. This makes possible to be limited only by the first two terms in the summation over n according to equation (1.2) and gives[18]

$$\varepsilon(\mathrm{p}) = \varepsilon_o(p_x, p_y) + \varepsilon_1 \cos\left(\frac{ap_z}{\hbar}\right), \tag{3.2}$$

where ε_1 is characterizing the energy band width of quasiparticles, independent of $p_z$and weakly dependent on p_x, p_y. Further we assume it as constant in p and (Figure a) shows the geometrical shape of such corrugated cylinders. If we assume that $\varepsilon_o(p_x, p_y)$ is quadratic in p_x and p_y, this dispersion relation can be written as

$$\varepsilon(\mathrm{p}) = \frac{p_x^2 + p_y^2}{2m} + \varepsilon_1 \cos\left(\frac{ap_z}{\hbar}\right). \tag{3.3}$$

The above dispersion law describes an opened Fermi surface in the p_z direction representing a weakly corrugated cylinder.

In the sections 3 and 4 we consider two cases of electromagnetic wave propagation in the layered conductors; one is when the wave propagates perpendicular to the layers and the other will be propagation along the layers. In addition, the account of the Fermi liquid effects in the propagation of monochromatic electromagnetic waves in layered conductors under the semi-classical approximation condition at not very low temperatures, that is, $T > T_c$ where T_c is a transition temperature between normal and superconducting states of the conductor is taken into consideration.

We shall present derivations of some of the electrical properties of the conductor such as the surface impedance, skin depth and electrical conductivity tensor based on the kinetic equation (1.7) with introduction of Fermi liquid effects for specular reflection condition of charge carriers from the sample surface. We first develop the Fermi liquid term to be used later in sections.

3.2 The Landau Correlation Function and the Boltzmann Kinetic Equation

For high frequencies of electromagnetic waves at anomalous skin effect limit the interaction of quasi-particles cannot be neglected. The concept of quasi-particles in the Fermi liquid model has a real sense only at points close to the Fermi surface.

If the Fermi surface can be assumed spherical, the function $\Phi\left(\mathrm{p},\mathrm{p}'\right)$ which characterizes the Fermi liquid appears depending only on the angle θ between p and p'. In relation to this, it is advantageous to take the Legendre polynomial expansion of the function $\Phi(\mathrm{p},\mathrm{p}')^{19}$

$$\Phi(\mathrm{p},\mathrm{p}') = \sum_{0}^{\infty}(2n+1)\,A_n p_n(\cos\theta), \tag{3.4}$$

where

$$p_n(\cos\theta) = \frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n.$$

The factors A_n represent parameters describing the phenomenological theory of the Fermi liquid and is determined from comparison of theory with experiment.

Tbe complete set of equations of the problem consists of the Maxwell's equations given by (2.1) and the kinetic equation in the view of the Fermi liquid effects (1.24) which enables to find the relation between the current density (1.26) and the electrical field strength of the wave.

For normal incidence of electromagnetic wave $\nabla\mathrm{E} = 0$ within the metal and there occur no charge fluctuations in it, since the electric field is always parallel to the surface.

Now, to consider the influence of inter-electron interactions, let's rewrite equation (1.18) as

$$\delta\varepsilon = \frac{2}{(2\pi\hbar)^3}\int f(\mathrm{p},\mathrm{p}';\mathrm{r},\mathrm{r}')\,\delta n' d^3\mathrm{p}' d^3\mathrm{r}' \tag{3.5}$$

in which $f(\mathrm{p},\mathrm{p}';\mathrm{r},\mathrm{r}')$ is the Landau correlation function and the factor 2 is due to the two spin orientations.

According to of Hartree approximation,[20,21] the self-consistent field neglecting the exchange effects of particles with potential energy $U(|\mathrm{r}'-\mathrm{r}|)$ interacting under the central force law, the Landau correlation function has a form

$$f_H\left(\mathrm{p},\mathrm{p}';\mathrm{r},\mathrm{r}'\right) = U\left(|\mathrm{r}'-\mathrm{r}|\right)$$

and this approximation does not take into account the correlation of quasi-particles. But, according to Hartree-Fock approximation,[20,21] the correlation function will have the exchange effects in addition to the self-potential energy,i.e.,

$$f\left(\mathrm{p},\mathrm{r},\sigma;\mathrm{p}',\mathrm{r}',\sigma'\right)=f\delta\left(\mathrm{r}-\mathrm{r}'\right)+U\left(\left|\mathrm{r}'-\mathrm{r}\right|\right).$$

The difference $f-f_H$ is due to the correlation effects in which the simplest is exchange correlation that occurs as the result of quantum identity of particles. The characteristic distance of inter-electron is the order of magnitude; coincides with the average distance a between electrons. In practice, variations of the distribution function over distances much larger than atomic distances will not affect quasiparticle energies, since the relevant electrons have wave lengths comparable to interatomic distances. Based on this concept, the characteristic distance of change of particles distribution function considerably exceeds the radius of correlation and hence we can consider

$$f\left(\mathrm{p},\mathrm{r},\sigma;\mathrm{p}',\mathrm{r}',\sigma'\right)-\ f_H\left(\mathrm{p},\mathrm{p}';\mathrm{r},\mathrm{r}'\right)=f\left(\mathrm{p},\mathrm{p}'\right)\delta\ \left(\mathrm{r}'-\mathrm{r}\right)$$

so that equation (3.5) can be expressed as

$$\delta\varepsilon=\frac{2}{\left(2\pi\hbar\right)^3}\int\left[U\left(\left|\mathrm{r}\prime-\mathrm{r}\right|\right)+f\left(\mathrm{p},\mathrm{p}'\right)\delta\ \left(\mathrm{r}'-\mathrm{r}\right)\right]\delta n'd^3\mathrm{p}'d^3r'. \tag{3.6}$$

It is the first term of the above equation which differs the charged Fermi liquid from the neutral and can be considered as the self-consistent potential energy

$$e\phi=\frac{2}{\left(2\pi\hbar\right)^3}\int d^3\mathrm{p}'d^3r'\frac{e^2}{\left|r-r'\right|}\delta n'.$$

In the second term we denote the correlation function $\int f\left(\mathrm{p},\mathrm{p}'\right)\delta\ \left(\mathrm{r}'-\mathrm{r}\right)d^3r'$ by $F\left(\mathrm{p},\mathrm{p}';\sigma,\sigma'\right)$ having unit of energy times volume. Thus, we can write equation (3.6) based on equation

(1.20) as

$$\varepsilon - \varepsilon_o(\mathrm{p}) = \delta\varepsilon = e\phi(\mathrm{r}, t) + \Psi(\mathrm{p}, \mathrm{r}, t),$$

where

$$\Psi(\mathrm{p}, \mathrm{r}, t) = \frac{Tr_{\sigma'}}{(2\pi\hbar)^3} \int d^3\mathrm{p}' F(\mathrm{p}, \mathrm{p}'; \boldsymbol{\sigma}, \boldsymbol{\sigma}')\, \delta n(\mathrm{p}', \mathrm{r}) \tag{3.7}$$

and $\varepsilon_o(\mathrm{p})$ is the conduction electron energy in the gas approximation. The function $\Psi(\mathrm{p}, \mathrm{r}, t)$ takes into account the correlation effects associated with interelectronic interaction.

If, in addition to electromagnetic wave there is too strong external static magnetic field **B**, then

$$\delta\varepsilon = -2\mu\mathbf{B}\mathbf{s} + e\phi\mathrm{I} + \frac{1}{(2\pi\hbar)^3} Tr_{\sigma'} \int' F(\mathrm{p}, \mathrm{p}'; \boldsymbol{\sigma}, \boldsymbol{\sigma}')\, \delta n(\mathrm{p}', \mathrm{r})\, d^3\mathrm{p}'$$

where $\mathbf{s} = \frac{1}{2}\widehat{\boldsymbol{\sigma}}$, **I** being unit matrix and σ_x, σ_y, σ_z are the familiar Pauli matrices and the spin dependence of the Landau correlation function can be written as[3,22,23]

$$F(\mathrm{p}, \mathrm{p}'; \boldsymbol{\sigma}, \boldsymbol{\sigma}') = \Phi(\mathrm{p}, \mathrm{p}') + \zeta(\mathrm{p}, \mathrm{p}')(\boldsymbol{\sigma}\boldsymbol{\sigma}')$$

for isotropic case and is also approximately valid in real metals. $\Phi(\mathrm{p}, \mathrm{p}')$ represents the exchange interaction between the quasi-particles of parallel spins. But, if the system has no magnetic order and no external magnetic field is present, then, the energies must be independent of spin (σ) but can depend only on the relative orientation of their spins and hence

$$\delta\varepsilon = e\phi + \frac{2}{(2\pi\hbar)^3} \int \Phi(\mathrm{p}, \mathrm{p}')\, \delta n(\mathrm{p}'\mathrm{r}.t)\, d^3p'.$$

For our practical interest let's rewrite the expression (3.7) by making use of equation (1.10) as

$$\Psi(\mathrm{p}, \mathrm{r}, t) = \frac{-2}{(2\pi\hbar)^3} \int \frac{\partial n_o}{\partial \varepsilon'} \psi(\mathrm{p}', \mathrm{r}, t)\, \Phi(\mathrm{p}, \mathrm{p}')\, d^3p'. \tag{3.8}$$

The Landau correlation function $\Phi\left(\mathrm{p},\mathrm{p}'\right)$ can be represented as an expansion in complete set of orthonormalized functions $\varphi_n\left(\mathrm{p}\right)$:

$$\Phi\left(\mathrm{p},\mathrm{p}'\right)=\sum_0^{\infty}A_n\varphi_n\left(\mathrm{p}\right)\varphi_n\left(\mathrm{p}'\right)$$

and equation(3.8) gets

$$\Psi\left(\mathrm{p},\mathrm{r}\right)=\frac{-2}{\left(2\pi\hbar\right)^3}\sum_0^{\infty}A_n\varphi_n\left(\mathrm{p}\right)\int\frac{\partial n_o}{\partial\varepsilon'}d^3\mathrm{p}'\varphi_n\left(\mathrm{p}'\right)\psi\left(\mathrm{p}',\mathrm{r}\right) \qquad (3.9)$$

Assuming that the considered part of the Fermi surface is axially symmetric with respect to a given axis (say z-axis), such as corrugated cylinder given in (figure a), the energy spectrum of (3.3) becomes

$$\varepsilon=\frac{p_\perp^2}{2m}+\varepsilon_1\cos\lambda p_z,\quad \text{where } \lambda=\frac{a}{\hbar} \text{ and } p_\perp^2=p_x^2+p_y^2 \qquad (3.10)$$

Thus, for this case, the function $\Phi\left(\mathrm{p},\mathrm{p}'\right)$ does not change if we change the components $\mathrm{p}_\perp$ and $\mathrm{p}'_\perp$ of the quasi-particle momentum p and p′ simultaneously. They both lie in the p_x-p_y plane. On the Fermi surface (where or near the condition of validity of the Fermi liquid theory is fulfilled) $\Phi\left(\mathrm{p},\mathrm{p}'\right)$ depends on the angle between $\mathrm{p}_\perp$ and $\mathrm{p}'_\perp$ and the components p_z, p'_z ; and $\mathrm{p}_\perp$ and $\mathrm{p}'_\perp$ are expressed in terms of ε, p_z and ε', p'_z respectively. Therefore, the assumed expansion of the Landau correlation function in this case can have the form[1]of

$$\Phi\left(\mathrm{p},\mathrm{p}'\right)=\alpha\frac{\mathrm{p}_\perp\mathrm{p}'_\perp}{p_F^2}+\beta\frac{\mathrm{p}_\perp\mathrm{p}'_\perp p_z p'_z}{p_F^4}$$

where α,β are characteristic value of the Fermi liquid and

$$\mathrm{p}_\perp\mathrm{p}'_\perp=p_\perp p'_\perp\cos\theta=p_\perp p'_\perp\cos\left(\varphi'-\varphi\right)$$

where θ is the angle between $\mathbf{p}_{\perp}$ and $\mathbf{p}'_{\perp}$ and φ, φ' are the angles $\mathbf{p}_{\perp}$, $\mathbf{p}'_{\perp}$ makes with p_x and p'_x axis respectively. Then, the Landau correlation function gets the form

$$\Phi(\mathbf{p},\mathbf{p}') = \frac{p_{\perp}p'_{\perp}}{p_F^2}\left(\alpha+\beta\frac{p_z p'_z}{p_F^2}\right)(\cos\varphi\cos\varphi'+\sin\varphi\sin\varphi') \tag{3.11}$$

To solve for $\Psi(p,r)$ according to (3.8) one has to consider in the integration change of variables from p_x, p_y, p_z to $\varepsilon, p_z, \varphi$ based on the given energy spectrum (3.10). Thus, the following variables are considered independent. Since $p_{\perp} = [2m(\varepsilon-\varepsilon_1\cos\lambda p_z)]^{\frac{1}{2}}$, then $p_x = [2m(\varepsilon-\varepsilon_1\cos\lambda p_z)]^{\frac{1}{2}}\cos\varphi$, $p_y = [2m(\varepsilon-\varepsilon_1\cos\lambda p_z)]^{\frac{1}{2}}\sin\varphi$, and $p_z = p_z$. Therefore, the Jacobian of the transformation in the case of the dispersion equation is equal to the effective mass ($D(\varepsilon,p_z,\varphi) = \frac{\partial(p_x,p_y,p_z)}{\partial(\varepsilon,p_z,\varphi)} = m$), which gives $d^3p = dp_x dp_y dp_z = D(\varepsilon,p_z,\varphi)\,d\varepsilon dp_z d\varphi = m d\varepsilon dp_z d\varphi$.

The Boltzmann kinetic equation is given as in (1.24)

$$(v-i\omega)\psi(\mathbf{p},\mathbf{r},t)+\mathbf{v}\frac{\partial\psi(\mathbf{p},\mathbf{r},t)}{\partial\mathbf{r}} = e\mathbf{v}\mathbf{E}-i\omega\Psi(\mathbf{p},\mathbf{r},t), \tag{3.12}$$

$$\Psi(\mathbf{p},\mathbf{r},t) = \frac{-2}{(2\pi\hbar)^3}\int d^3p'\Phi(\mathbf{p},\mathbf{p}')\frac{\partial n_0}{\partial\varepsilon}(\psi(\mathbf{p}',\mathbf{r},t)-\Psi(\mathbf{p}',\mathbf{r},t)) \tag{3.13}$$

is to be solved, after which this solution is to be inserted in the equation for the current density(1.26)

$$\mathbf{j}(\mathbf{r}) = \frac{-2e}{(2\pi\hbar)^3}\int\mathbf{v}\frac{\partial n_o}{\partial\varepsilon}\psi(\mathbf{p},\mathbf{r},t)\,d^3p$$

which in turn is used in the Maxwell equation(2.14)

$$\frac{d^2\mathbf{E}}{d\mathbf{r}^2}+\frac{\omega^2}{c^2}\mathbf{E} = -\frac{4\pi i\omega}{c^2}\mathbf{j}(\mathbf{r}). \tag{3.14}$$

to solve for E.

In the next sections, where we present the solution of the problem, in approximating the function $\Psi(\mathbf{p},\mathbf{r},t)$ according to (3.13) we neglect the term containing $\Psi(\mathbf{p}',\mathbf{r},t)$ in comparison to $\psi(\mathbf{p}', \mathbf{r}, t)$ in the integrand, and we also use the gas approximation of the function $\psi(\mathbf{p}\prime)$.

To solve explicitly for the function $\Psi(\mathbf{p}, \mathbf{r})$ we consider two cases turn by turn. Propagation of electromagnetic wave in the directions perpendicular (in the direction along z axis) and along the layers (say, in the direction along x). For simplicity, we consider a sample whose face is parallel to the layers.

3.3 Electromagnetic wave propagation perpendicular to the layers

Consider a layered conductor in the absence of external static magnetic field on which a monochromatic wave with frequency ω is incident from vacuum along the normal to the layers (see fig 1 & fig. 2). If the surface of the layers ($z = 0$) coincides with the plane of symmetry of the crystal, the electric and the magnetic fields of the wave lie in the xy plane as seen in the above figure, that is,

$$\mathbf{E}(z) = [E_x(z), E_y(z), 0] \quad \text{and} \quad \mathbf{H}(z) = [H_x(z), H_y(z), 0]$$

and as mentioned earlier the conductor is supposed to be unmagnetized, so that $\mathbf{B} = \mathbf{H}$ in it. For this case the kinetic equation (3.12) has a form

$$(\upsilon - i\omega)\psi(\mathbf{p},z) + v_z \frac{\partial\psi(\mathbf{p},z)}{\partial z} = e\mathbf{v}\mathbf{E} - i\omega\Psi(\mathbf{p},z) \tag{3.15}$$

while the Maxwell equation (2.14) becomes

$$\frac{d^2\mathbf{E}}{dz^2} + \frac{\omega^2}{c^2}\mathbf{E} = -\frac{4\pi i\omega}{c^2}\mathbf{j}(z). \tag{3.16}$$

The general solution of (3.15) will be

$$\psi(z) = \int_{C}^{z} \Omega(z')\, e^{-\gamma(z-z')} dz', \tag{3.17}$$

where

$$\Omega(z') = \frac{e\mathbf{v}\mathbf{E} - i\omega\Psi(\mathrm{p},z)}{v_z},$$

$$\gamma = \frac{\upsilon - i\omega}{v_z}$$

and the constant C is to be defined by boundary condition.

As mentioned above $\mathbf{E}$ has x and y components only, so that

$$\mathbf{E}\mathbf{v} = E_x v_x + E_y v_y = v_\perp \left(E_x \cos\varphi + E_y \sin\varphi\right) \tag{3.18}$$

The electric field of the electromagnetic wave is so weak that the correction to the distribution function must be a scalar proportional to the field strength $\mathbf{E}$ in the first order (we use this gas model approximation for estimating exclusively the form of Ψ). Hence, by using the expression (1.14) we can get

$$\psi(\mathrm{p}) = e\tau(\varepsilon)\, v_\perp \left(E_x \cos\varphi + E_y \sin\varphi\right). \tag{3.19}$$

By combining equation (3.11) of the correlation function and the above one we get

$$\Phi(\mathrm{p},\mathrm{p}')\,\psi(\mathrm{p}') = \frac{e\tau(\varepsilon')\, p_\perp p_\perp'^2}{m p_F^2}\left(\alpha + \beta\frac{p_z p_z'}{p_F^2}\right)$$

$$\times\left(E_x \cos\varphi\cos^2\varphi' + E_x \sin\varphi\sin\varphi'\cos\varphi' + E_y \sin\varphi\sin\varphi'\cos\varphi' + E_y \sin\varphi\sin^2\varphi'\right), \tag{3.20}$$

and equation (3.13) using the assumption that $|\Psi(\mathrm{p}',z)| << |\psi(\mathrm{p}')|$ mentioned in the last section, after transformation of coordinates from $d^3\mathrm{p}$ to $m d\varepsilon dp_z d\varphi$ and substituting equation (3.20) in it and integrating over angle φ' from 0 to 2π we get

$$\Psi\left(\mathbf{p},z\right) = \frac{-2\pi e p_{\perp}}{\left(2\pi\hbar\right)^{3} p_{F}^{2}} \int \tau\left(\varepsilon'\right) p_{\perp}'^{2} \left(\alpha + \frac{\beta p_z p_z'}{p_F^2}\right) \left[E_x\left(z\right)\cos\varphi + E_y\left(z\right)\sin\varphi\right] \frac{\partial n_o}{\partial\varepsilon'} d\varepsilon' dp_z'. \tag{3.21}$$

We have $p_{\perp}'^{2} = 2m\left(\varepsilon' - \varepsilon_1 \cos\lambda p_z\right)$ and, furthermore, we can use the approximate value $\frac{\partial n_o}{\partial\varepsilon} \approx -\delta\left(\varepsilon - \mu\right)$ where μ is Fermi energy and the integration over the energy ε' will give us

$$\Psi\left(\mathbf{p},z\right) = \frac{4\pi m e p_{\perp}\tau\left(\mu\right)}{\left(2\pi\hbar\right)^{3} p_{F}^{2}} \int_{-\frac{\pi}{\lambda}}^{\frac{\pi}{\lambda}} \left(\mu - \varepsilon_1 \cos\lambda p_z\right) \left(\alpha + \frac{\beta p_z p_z'}{p_F^2}\right) \left[E_x\left(z\right)\cos\varphi + E_y\left(z\right)\sin\varphi\right] dp_z'. \tag{3.22}$$

After taking the momentum p_z', integration over a Brillouin zone (from -$\frac{\pi}{\lambda}$ to $\frac{\pi}{\lambda}$, the final expression for $\Psi\left(\mathbf{p},z\right)$ will be $\Psi\left(\mathbf{p},z\right) = \frac{8e\pi^2 m\tau(\mu)\alpha\mu p_{\perp}}{(2\pi\hbar)^3 p_F^2\lambda}\left[E_x\left(z\right)\cos\varphi + E_y\left(z\right)\sin\varphi\right]$, or for simplicity it can be written as

$$\Psi\left(\mathbf{p},z\right) = eA\,p_{\perp}\left[E_x\left(z\right)\cos\varphi + E_y\left(z\right)\sin\varphi\right] = eA\left[p_x E_x + p_y E_y\right],$$

where

$$A = \frac{8\pi^2 m\tau\alpha\mu}{\left(2\pi\hbar\right)^3 p_F^2\lambda}. \tag{3.23}$$

Now, using equation (3.18) and (3.23) the term in equation (3.17) (i.e., $\Omega\left(z'\right)$) gets the form

$$\Omega\left(z'\right) = \frac{e\,p_{\perp}}{v_z}\left[E_x\left(z'\right)\cos\varphi + E_y\left(z'\right)\sin\varphi\right]\left[\frac{1}{m} - i\omega A\right] \tag{3.24}$$

and we had already $v_z = \frac{\partial\varepsilon}{\partial p_z} = -\varepsilon_1\lambda\sin\lambda p_z$.

The constant C in equation (3.17) has to be determined by boundary conditions. It is clear that as $z \longrightarrow \infty$ the function $\psi \longrightarrow 0$, that is, the equilibrium distribution of charged particles far from the surface is not affected by the fields. The potential step at the surface of the sample prevents electrons from escaping out of the conductor, and reflects them back toward the interior of the metal. In this problem, we have considered the conductor surface as perfect and hence electrons undergo specular reflection. In the extreme anomalous limit, the relevant electrons are moving almost parallel to the surface. The boundary condition for specular reflection at $z = 0$ can therefore be written as $\psi(v_x, v_y, v_z) = \psi(v_x, v_y, -v_z)$

For $v_z < 0$, electrons are moving towards the surface from infinity and the boundary condition $\psi \longrightarrow 0$ for $z \longrightarrow \infty$ determines the constant $C = \infty$, since $\operatorname{Re}\gamma < 0$ in the exponent, $\exp(-\gamma z)$, where γ is defined as in (3.17). Therefore,

$$\psi_- = \psi(\mathbf{v}_z < 0) = \int_{\infty}^{z} \Omega(z') e^{-\gamma(z-z')} dz' \tag{3.25}$$

For $v_z > 0$, electrons are moving away from the surface of the conductor, assuming specular reflection at $z = 0$, and also use relations $\Omega(-v_z) = -\Omega(v_z)$, $\gamma(-v_z) = -\gamma(v_z)$, Ω stands for $\Omega(z')$) gets

$$\int_{C}^{0} \Omega(z') e^{\gamma z'} dz' = -\int_{\infty}^{0} \Omega(z') e^{-\gamma z'} dz' \tag{3.26}$$

since if $v_z > 0$ then $-v_z < 0$ and γ is proportional to v_z^{-1}. The general equation (3.17) for $v_z > 0$ and specular reflection, therefore, after relating to (3.26) can be written as

$$\psi_+ = \int_{0}^{z} \Omega(z') e^{-\gamma(z-z')} dz' + \int_{0}^{\infty} \Omega(z') e^{-\gamma(z+z')} dz' \tag{3.27}$$

On inserting equation (3.25) and equation (3.27) into expression for the current density (1.26), we can get the expression for $j_\alpha(z)$, which after combining integrations over the region over $v_z > 0$ and $v_z < 0$ (based on the property that $\varepsilon(-\mathrm{p}) = \varepsilon(\mathrm{p})$ introduced in chap. 1)

$$j_\alpha(z) = \frac{-2e}{(2\pi\hbar)^3} \int_{v_z > 0} v_\alpha \frac{\partial n_o}{\partial \varepsilon} d^3p$$

$$\times\left(\int_0^z \Omega e^{\gamma(z'-z)}dz' + \int_0^\infty \Omega e^{-\gamma(z+z')}dz' + \int_z^\infty \Omega e^{\gamma(z'-z)}dz'\right). \quad (3.28)$$

If the electric filed is formally extended in symmetric fashion out side the conductor, it is possible to change z' into $-z'$ in the second integral so that $\Omega(-z) = \Omega(z)$ and it can be added to the first one, and for the third integral $z > z'$ the exponential part $e^{-\gamma(z-z')} = e^{-\gamma(|z-z|)}$ and hence, after the three integrals are combined, the expression for the current density will be

$$j_\alpha(z) = \frac{-2e}{(2\pi\hbar)^3}\int_{v_z>0} v_\alpha \frac{\partial n_o}{\partial \varepsilon} d^3p \int_{-\infty}^{\infty} \Omega(z') e^{-|\gamma|(|z-z'|)}dz'$$

and the Fourier transform of the above equation, by referring $(A.1)$ and $(A.2)$ of mathematical supplement, will be

$$j_\alpha(k) = \frac{-4em}{(2\pi\hbar)^3}\int_{v_z>0} v_\alpha \Omega(\mathrm{p},k) \frac{|\gamma|}{\gamma^2+k^2}\frac{\partial n_o}{\partial \varepsilon} d\varepsilon dp_z d\varphi, \quad (3.29)$$

where

$$\Omega(\mathrm{p},k) = \frac{ep_\perp}{mv_z}\left[E_x(k)\cos\varphi + E_y(k)\sin\varphi\right]\left[1 - i\omega m A\right]$$

is the Fourier transform of (3.24) and

$$\frac{2\gamma}{\gamma^2+k^2} = \frac{2v_z}{(\nu - i\omega)\left[1 + \frac{k^2v_z^2}{(\nu-i\omega)^2}\right]}$$

is Fourier transform of the exponential term $e^{-\gamma|z|}$

The Fourier transform of electrical current density along x direction by make use of (3.29) in which the details of mathematical analysis is given in $(A.2)$ of mathematical supplement as

$$j_x(k) = \frac{-8e^2Ba_1\tau}{(2\pi\hbar)^3}\int_{v_z>0} \frac{(\varepsilon - \varepsilon_1\cos\lambda p_z)\left[E_x(k)\cos^2\varphi + E_y(k)\sin\varphi\cos\varphi\right]}{a_1^2\left(1 + k^2l_\omega^2\sin^2\lambda p_z\right)}\frac{\partial n_o}{\partial \varepsilon} d\varepsilon d\varphi dp_z \quad (3.30)$$

where $B = (1 - i\omega m A)$, $a_1 = 1 - i\omega\tau$ and $l_\omega = \varepsilon_1 \lambda \tau / a_1^2$ has a dimension of length. When evaluating the φ integration the second term containing $\sin\varphi \cos\varphi$ in the square bracket of the integrand will vanish due to odd function of φ. The remaining term in φ integration multiplies the coefficient by π. Now using $\int F(\varepsilon) \frac{\partial n_o}{\partial \varepsilon} d\varepsilon \approx -F(\mu)$ for ε integration, we obtain

$$j_x(k) = \frac{8e^2 \pi B a_1 \tau E_x(k)}{(2\pi\hbar)^3} I_1(k),$$

where

$$I_1(k) = \int_{\frac{-\pi}{\lambda}}^{0} \frac{(\mu - \varepsilon_1 \cos \lambda p_z)}{a_1^2 \left(1 + k^2 l_\omega^2 \sin^2 \lambda p_z\right)} dp_z = \frac{2}{\lambda} \int_{\frac{-\pi}{2}}^{0} \frac{(\mu - \varepsilon_1 \cos t)}{a_1^2 \left(1 + k^2 l_\omega^2 \sin^2 \lambda p_z\right)} dt.$$

The dp_z integration evaluated for $v_z > 0$, i.e., from $\frac{-\pi}{\lambda}$ to 0 is shown in mathematical supplement $(A.3)$ at the end, which gives after neglecting higher order terms

$$I_1(k) \approx \frac{2}{\lambda a_1^2} \left(\mu \left(\frac{\pi a_1}{2kl} - \frac{\pi a_1^3}{4k^3 l^3} \right) - \varepsilon_1 \left(\frac{\pi a_1}{2kl} + \frac{a_1^2}{k^2 l^2} \right) \right)$$

in the extreme anomalous limit. The electrical current density along x is, therefore,

$$j_x(k) \approx \frac{16e^2 \pi B a_1 \tau}{(2\pi\hbar)^3 \lambda a_1^2} E_x(k) \left(\mu \left(\frac{\pi a_1}{2kl} - \frac{\pi a_1^3}{4k^3 l^3} \right) - \varepsilon_1 \left(\frac{\pi a_1}{2kl} + \frac{a_1^2}{k^2 l^2} \right) \right),$$

and the conductivity along the layer during perpendicular propagation will be

$$\sigma_{xx\perp}(k) \approx \frac{16e^2 \pi B a_1 \tau}{(2\pi\hbar)^3 \lambda a_1^2} \left(\mu \left(\frac{\pi a_1}{2kl} - \frac{\pi a_1^3}{4k^3 l^3} \right) - \varepsilon_1 \left(\frac{\pi a_1}{2kl} + \frac{a_1^2}{k^2 l^2} \right) \right), \quad \sigma_{xy}(k) = 0.$$

Similar calculations show that $\sigma_{yy\perp}(k)$ is the same as $\sigma_{xx\perp}(k)$. It can also be easily seen that the current density $j_z(k) = 0$ and thus $\sigma_{zx}(k) = \sigma_{zy}(k) = 0$. Therefore, we see that for wave propagation in the direction perpendicular to the layers

$$j_x(k) = \sigma_{xx} E_x(k) \quad \text{and} \quad j_y(k) = \sigma_{yy} E_y(k)$$

where Fourier components of the conductivity are

$$\sigma_{xx\perp} = \sigma_{yy\perp} \approx \frac{16e^2\pi B a_1 \tau}{(2\pi\hbar)^3 \lambda a_1^2}\left(\mu\left(\frac{\pi a_1}{2kl} - \frac{\pi a_1^3}{4k^3 l^3}\right) - \varepsilon_1\left(\frac{\pi a_1}{2kl} + \frac{a_1^2}{k^2 l^2}\right)\right) \tag{3.31}$$

and in the extreme anomalous skin effect limit

$$\sigma_{xx\perp} = \sigma_{yy\perp} \approx \frac{e^2 B}{\pi\hbar a^2 k}\left(\frac{1-\eta}{\eta}\right)$$

By Fourier transform of both sides of equation (3.16) neglecting the contribution of displacement current in the expression and knowing that there is a discontinuity in the derivative $\frac{dE_\alpha}{dz}$ at $z = 0$, we have $(\alpha, \beta = x, y)$

$$-k^2 E_\alpha(k) - 2\left(\frac{dE_\alpha}{dz}\right)_{z=0} = -\frac{4\pi i\omega}{c^2}\sigma_{\alpha\beta}(k)\, E_\beta(k) = -\frac{4\pi i\omega}{c^2}\sigma_{xx}(k)\, E_\alpha(k) \tag{3.32}$$

since $\sigma_{\alpha\beta} = \sigma_{xx\perp} = \sigma_{yy\perp}$, as seen above. The above two independent equations have the same form for $E_x(k)$ and $E_y(k)$ due to symmetry of electric field in k along x and y axis and thus we need only consider one of them to find the surface impedance and the skin depth. Thus, equation (3.32) gets the form

$$E(k) = \frac{-2E'(0)}{k^2 - \frac{4\pi i\omega}{c^2}\sigma_{xx}} \approx \frac{-2E'(0)}{k^2 - \frac{4\pi i\omega}{c^2}\frac{16e^2\pi B a_1\tau}{(2\pi\hbar)^3\lambda a_1^2}\left(\mu\left(\frac{\pi a_1}{2kl} - \frac{\pi a_1^3}{4k^3l^3}\right) - \varepsilon_1\left(\frac{\pi a_1}{2kl} + \frac{a_1^2}{k^2l^2}\right)\right)}$$

or

$$E(k) \approx \frac{-2k^3 E'(0)}{k^5 - i\gamma_1 k^2 + i\gamma_2 k + i\gamma_3}, \tag{3.33}$$

where

$$\gamma_1 \equiv \frac{32\omega e^2\pi^3 B\tau}{c^2(2\pi\hbar)^3\lambda l}(\mu - \varepsilon_1), \quad \gamma_2 \equiv \frac{64\omega e^2\pi^2 B a_1\tau\varepsilon_1}{c^2(2\pi\hbar)^3\lambda l^2}, \quad \gamma_3 \equiv \left(\frac{16\omega e^2\pi^3 B a_1^2\tau\mu}{c^2(2\pi\hbar)^3\lambda l^3}\right).$$

In the limit of anomalous skin effect in which $kl >> 1$; l being the mean free path of conduction electrons, further simplifications than in the mathematical supplement can be done neglecting the lower order terms in k in the denominator of (3.33) and it becomes $E_x(k) \approx \frac{-2kE'(0)}{k^3 - i\gamma_1}$. The Fourier transform of the electric field, $E(k)$, is symmetric in k and thus its inverse transform will be $E(z) = \frac{1}{\pi}\int\limits_0^\infty E(k)\cos(kz)\,dk$. Thus, the electric field at the surface, $z = 0$ gets

$$E(0) \approx \frac{-2E'(0)}{\pi}\int\limits_0^\infty \frac{kdk}{k^3 - i\gamma_1}. \tag{3.34}$$

The integral in equation (3.34) can be evaluated using Residue theorem[24] by choosing a proper contour and one finds $\int_0^\infty \frac{kdk}{k^3 - i\gamma_1} = \frac{1}{9}\pi\frac{3+i\sqrt{3}}{\sqrt[3]{\gamma_1}} = \frac{2\pi e^{\frac{i\pi}{6}}}{3\sqrt{3}\gamma_1^{\frac{1}{3}}}$, which gives $\frac{E(0)}{E'(0)} = \frac{-4e^{\frac{i\pi}{6}}}{3\sqrt{3}\gamma_1^{\frac{1}{3}}}$. According to equation (2.6) the surface impedance is thus

$$Z = \frac{4\pi i\omega}{c^2}\frac{E(0)}{E'(0)} \approx \frac{16\pi\omega}{3\sqrt{3}c^2\gamma_1^{\frac{1}{3}}}e^{\frac{5\pi i}{3}}.$$

On inserting the value of γ_1 we find surface impedance of the conductor along the layers during perpendicular propagation of the wave $Z_{A\perp}$ as

$$Z_{A\perp} \approx \frac{16\pi}{3\sqrt{3}}\left(\frac{\omega^2 a^2\hbar\left(\frac{\eta}{1-\eta}\right)}{4c^4e^2\left(1+\omega^2m^2A^2\right)^{\frac{1}{2}}}\right)^{\frac{1}{3}} \exp\frac{i}{3}\left(5\pi - \tan^{-1}(-\omega mA)\right),\ \eta = \frac{\varepsilon_1}{\mu} \tag{3.35}$$

Now, it can be seen that the relaxation time enter into (3.35) through the constant A and the surface impedance depends weakly on it and varies directly as $\omega^{\frac{2}{3}}$ while its resistive and the reactive parts are related in such a way that $X = \sqrt{3}R$ which confirms the noneffectiveness qualitative description (2.13), the Reuter and Sondheimer[2] quantitative expression (2.15) and A.A. Abrikosov's[3] analysis (2.17).

When the quantity α which characterizes the correlation function $\Phi(\mathbf{p}, \mathbf{p}')$ is zero, the above expression will automatically reduce to the surface impedance of electromagnetic wave in Fermi gas approximation, i.e.,

$$Z_{A\perp} \approx \frac{8\pi}{3\sqrt{3}} \left(\frac{2\omega^2 a^2 \hbar \left(\frac{\eta}{1-\eta}\right)}{c^4 e^2} \right)^{\frac{1}{3}} \exp\left(\frac{5\pi i}{3}\right). \tag{3.36}$$

The factor $\left(\frac{\eta}{1-\eta}\right)^{\frac{1}{3}}$ differs qualitatively the surface impedance of electromagnetic wave in normal metals Z_{Nm} from the layered conductors, i.e.,

$$Z_{A\perp} \sim Z_{Nm} \left(\frac{\eta}{1-\eta}\right)^{\frac{1}{3}}.$$

The penetration depth of the electromagnetic wave, i.e. the skin depth δ, in the layered conductors for interacting Fermi liquid in the case of perpendicular propagation of electromagnetic wave to the layers can be approximated with the help of relation (2.5) for comparison with the skin depths during parallel propagation of electromagnetic wave to the layers. Thus,

$$k \approx \frac{3\sqrt{3}}{4} \left(\frac{4\omega e^2 \left(1+\omega^2 m^2 A^2\right)^{\frac{1}{2}} \left(\frac{1-\eta}{\eta}\right)}{c^2 a^2 \hbar} \right)^{\frac{1}{3}} \exp \frac{-i}{3} \left(5\pi - \tan^{-1}\left(-\omega m A\right)\right),$$

and its imaginary part is

$$\mathrm{Im}\left(k\right) \approx \frac{-3\sqrt{3}}{4} \left(\frac{4\omega e^2 \left(1+\omega^2 m^2 A^2\right)^{\frac{1}{2}} \left(\frac{1-\eta}{\eta}\right)}{c^2 a^2 \hbar} \right)^{\frac{1}{3}} \sin \frac{1}{3} \left[5\pi - \tan^{-1}\left(-\omega m A\right)\right]$$

The damping distance of the wave is identified as the inverse imaginary part. Thus, one can get the skin depth perpendicular to the layers during perpendicular propagation $(\delta_{P\perp})$ as

$$\delta_{P\perp} \simeq \frac{-4}{3\sqrt{3}} \left(\frac{c^2 a^2 \hbar}{4\omega e^2 \left(1+\omega^2 m^2 A^2\right)^{\frac{1}{2}}} \left(\frac{\eta}{1-\eta}\right) \right)^{\frac{1}{3}} \csc \frac{1}{3} \left[5\pi - \tan^{-1}\left(-\omega m A\right)\right]. \tag{3.37}$$

This shows us that for electromagnetic wave of fixed frequency, the skin depth varies as $\eta^{\frac{1}{3}}$ and when the exchange term of Landau's correlation function is involved, the surface impedance, conductivity tensor and skin depth are weakly dependent on the relaxation time

τ through A. But in the gas approximation, the quantities are independent of the relaxation time and it can be easily seen that (3.37) reduces to

$$\delta_{P\perp} \approx \frac{8}{9}\left[\frac{c^2a^2\hbar}{4\omega e^2}\left(\frac{\eta}{1-\eta}\right)\right]^{\frac{1}{3}}. \tag{3.38}$$

3.4 Electromagnetic Wave Propagation Along the Layers

An electromagnetic wave similar to the first case, incident from vacuum along the direction parallel to the layers is considered. Let the surface of the conductor $(x=0)$ coincides with the plane of symmetry of the crystal, the electric and the magnetic fields of the wave lie in the $y-z$ plane, i.e., $\mathbf{E}(x) = [0, E_y(x), E_z(x)]$ and $\mathbf{H}(x) = [0, H_y(x), H_z(x)]$; $\psi = \psi(\mathbf{p}, x)$. Then, kinetic equation (1.24)will have a form

$$(\upsilon - i\omega)\psi(\mathbf{p},x) + v_x\frac{\partial\psi(\mathbf{p},x)}{\partial \mathrm{x}} = e\mathbf{v}\mathbf{E} - i\omega\Psi(\mathbf{p},x). \tag{3.39}$$

and the Maxwell's equation (2.14) similar to (3.16) becomes

$$\frac{d^2\mathbf{E}}{dx^2} + \frac{\omega^2}{c^2}\mathbf{E} = -\frac{4\pi i\omega}{c^2}\mathbf{j}(x). \tag{3.40}$$

According to the above assumption $\mathbf{E}\mathbf{v} = E_yv_y + E_zv_z = v_\perp E_y(x)\sin\varphi - E_z(x)\varepsilon_1\lambda\sin\lambda p_z$. The solution of the kinetic equation has the same form as the previous one, but the variable z is changed to x and $\Omega(x') = \frac{e\mathbf{v}.\mathbf{E}-i\omega\Psi(\mathbf{p},x)}{v_x}$, $\gamma = \frac{\upsilon-i\omega}{v_x}$. By using the expression (1.14) similar to (3.19), the non equilibrium distribution function $\psi(\mathbf{p})$ entering the integral equation for ψ is $\psi(\mathbf{p}) = e\tau(\varepsilon)[v_\perp E_y(x)\sin\varphi - \varepsilon_1\lambda E_z(x)\sin\lambda p_z]$.

After following similar steps as in the previous section, the expression for the function $\Psi(\mathbf{p}, x)$, which takes into account the correlation effects associated with the interelectronic interaction will be

$$\Psi\left(\mathbf{p},x\right)=\frac{8e\tau\pi^{2}m\mu\alpha p_{\perp}}{\left(2\pi\hbar\right)^{3}p_{F}^{2}\lambda}E_{y}\left(x\right)\sin\varphi=eAp_{y}E_{y}(x), \qquad (3.41)$$

where A is as defined in (3.23).

After the Fourier transform of $\Omega\left(x'\right)$, i.e.,

$$\Omega\left(k\right)=\frac{e}{v_{x}}\left[p_{y}E_{y}\left(k\right)\left(\frac{1}{m}-i\omega A\right)-\varepsilon_{1}\lambda\sin\lambda p_{z}E_{z}\left(k\right)\right]$$

the transformed form of the solution of Boltzmann kinetic equation gets

$$\psi_{k}\left(\mathbf{p}\right)=2e\frac{\left[\left(\frac{1}{m}-i\omega A\right)p_{y}E_{y}\left(k\right)+v_{z}E_{z}\left(k\right)\right]a_{1}\tau}{a_{1}^{2}+b_{1}^{2}v_{x}^{2}} \qquad (3.42)$$

where a_1 and b_1 have the same meaning as before.

The Fourier transform of the components of current densities $j_y(k)$ and $j_z\left(k\right)$ supplemented by the values of their respective integrals can be written as

$$j_{y}\left(k\right)=\frac{-4e^{2}a_{1}\tau}{\left(2\pi\hbar\right)^{3}}\int p_{\perp}\left[\left(\frac{1}{m}-i\omega A\right)E_{y}\left(k\right)p_{\perp}I_{2}\left(k\right)+v_{z}E_{z}\left(k\right)I_{3}\left(k\right)\right]\frac{\partial n_{o}}{\partial\varepsilon}d\varepsilon dp_{z}; \qquad (3.43)$$

$$I_{2}\left(k\right)=2\int_{0}^{\frac{\pi}{2}}\frac{\sin^{2}\varphi d\varphi}{a_{1}^{2}+b_{1}^{2}\frac{p_{\perp}^{2}}{m^{2}}\cos^{2}\varphi}=\frac{2}{a_{1}^{2}}\int_{0}^{\frac{\pi}{2}}\frac{\sin^{2}xdx}{1+b_{1}^{2}\frac{p_{\perp}^{2}}{a_{1}^{2}m^{2}}\cos^{2}x},$$

The above φ integration is given in mathematical supplement $(A.5)$ and in the extreme anomalous skin effect limit,

$$I_{2}\left(k\right)\approx\frac{1}{2}\pi m\left(\frac{2}{k\tau p_{\perp}a_{1}}-2\frac{m}{k^{2}\tau^{2}p_{\perp}^{2}}+a_{1}\frac{m^{2}}{k^{3}\tau^{3}p_{\perp}^{3}}\right),$$

and similarly, from mathematical supplement of $(A.6)$ we can have

$$I_{3}\left(k\right)=2\int_{0}^{\frac{\pi}{2}}\frac{\sin\varphi d\varphi}{a_{1}^{2}+b_{1}^{2}\frac{p_{\perp}^{2}}{m^{2}}\cos^{2}\varphi}\approx\frac{\pi m}{a_{1}b_{1}p_{\perp}}-\frac{2m^{2}}{b_{1}^{2}p_{\perp}^{2}}+\frac{2a_{1}^{2}m^{4}}{3b_{1}^{4}p_{\perp}^{4}}.$$

On inserting the two integrals, it can be seen that the term containing $I_3(k)$ in the integral (3.43) will vanish due to odd and even functions of $\sin \lambda p_z$ and $\cos \lambda p_z$ and evaluating the energy integration similar to the earlier case one can obtain

$$\begin{aligned} j_y(k) &\approx \frac{4e^2 a_1 \tau}{(2\pi\hbar)^3}\left(\frac{1}{m} - i\omega A\right)\int_{-\frac{\pi}{\lambda}}^{\frac{\pi}{\lambda}} E_y(k)\, dp_z \times \\ &\left[\frac{\pi m (2m\mu)^{\frac{1}{2}}\left(1 - \frac{\varepsilon_1}{2\mu}\cos\lambda p_z\right)}{a_1 k \tau} - \frac{\pi m^2}{k^2\tau^2} + \frac{a_1 m^4\left(1 + \frac{\varepsilon_1}{2\mu}\cos\lambda p_z\right)}{2k^3\tau^3(2m\mu)^{\frac{1}{2}}}\right] \end{aligned}$$

and after taking the p_z integration over a Brillouin zone, the final expression for the transformed current density along y direction is obtained and the conductivity component along the layers during parallel propagation of the wave$\left(\sigma_{yy\|}\right)$ for this case gets

$$\sigma_{yy\|}(k) \approx \frac{8e^2 a_1 \tau\pi}{(2\pi\hbar)^3 \lambda}\left(\frac{\pi (2m\mu)^{\frac{1}{2}}}{k\tau a_1} - \frac{\pi m}{k^2\tau^2} + \frac{a_1 m^2 \pi}{2k^3\tau^3(2m\mu)^{\frac{1}{2}}}\right)(1 - i\omega m A), \sigma_{yz}(k) = 0. \tag{3.44}$$

Employing equation (3.40) similar to the previous case, i.e., $E_y(k) = \frac{-2E'(0)}{k^2 - \frac{4\pi i\omega}{c^2}\sigma_{yy}}$ and on substituting the conductivity (3.44)

$$E_y(k) = \frac{-2k^3 E'(0)}{k^5 - i\gamma_3 k^2 + i\gamma_4 k - i\gamma_5}.$$

Here also $E_y(k)$ is symmetric in k and hence after taking inverse transform we can obtain the ratio of the field at the surface to its first derivative as

$$\frac{E_y(0)}{E_y'(0)} = \frac{-2}{\pi}\int_0^{\infty} \frac{k^3 dk}{k^5 - i\gamma_3 k^2 + i\gamma_4 k - i\gamma_5}, \tag{3.45}$$

where

$$\gamma_3 \equiv \frac{32\pi^3 \omega e^2 (2m\mu)^{\frac{1}{2}}}{c^2 (2\pi\hbar)^3 \lambda}(1 - i\omega m A), \ \gamma_4 \equiv \frac{32\pi^3 \omega e^2 a_1 m}{c^2 (2\pi\hbar)^3 \lambda\tau}(1 - i\omega m A),$$

and

$$\gamma_5 \equiv \frac{16\pi^3 \omega e^2 a_1^2 m^2}{c^2 (2\pi\hbar)^3 \lambda\tau (2m\mu)^{\frac{1}{2}}} (1 - i\omega m A) .$$

In the limit of extreme anomalous skin effect, (3.44) gets

$$\sigma_{yy\|}(k) \approx \frac{e^2 (2m\mu)^{\frac{1}{2}}}{\pi\hbar^2 ak} (1 - i\omega m A) \approx \frac{e^2}{\pi\hbar a^2 k} (1 - i\omega m A) \ . \tag{3.46}$$

This tell us that for a fixed value of k the Fourier transform of conductivity tensor varies inversely as the square of the separation between the layers a and weakly depends on the correlation function. Mean while, the conductivities for the perpendicular and parallel propagations of electromagnetic wave to the layers can be compared, and thus, the ratio of equation (3.31) to equation (3.46) is

$$\frac{\sigma_{yy\perp}}{\sigma_{yy\|}} \simeq \frac{1-\eta}{\eta} \tag{3.47}$$

By neglecting the last two terms in the denominator of (3.44) in comparison to the first, the expression takes on a simple form

$$\frac{E_y(0)}{E_y'(0)} \approx \frac{-2}{\pi} \int_0^{\infty} \frac{kdk}{k^3 - i\gamma_3} = \frac{-4e^{\frac{i\pi}{6}}}{3\sqrt{3}\gamma_3^{\frac{1}{3}}}$$

The contour integral is the same as that of equation (3.34) and hence, the surface impedance along the layers for the electromagnetic wave propagated along the layers (putting $(2m\mu)^{\frac{1}{2}} \approx \frac{\hbar}{a}$) gets

$$Z_{A\|} \approx \frac{8\pi}{3\sqrt{3}} \left(\frac{2\omega^2 a^2 \hbar}{c^4 e^2 (1 + \omega^2 m^2 A^2)^{\frac{1}{2}}} \right)^{\frac{1}{3}} \exp \frac{i}{3} \left[5\pi - \tan^{-1}(-\omega m A) \right] \tag{3.48}$$

Here also the surface impedance varies directly as $a^{\frac{2}{3}}$ and $\omega^{\frac{2}{3}}$ but does not depend on the layered nature of the conductor. Now, it is possible to compare the surface impedances for the two cases of propagations using equations (3.35) and (3.48) to obtain

$$\frac{Z_{A\perp}}{Z_{A\parallel}} \approx \left(\frac{\eta}{1-\eta}\right)^{\frac{1}{3}}, \tag{3.49}$$

and also with the surface impedance of normal metals as $Z_{A\parallel} \sim Z_{Nm}$.

The skin depth of the electric field $E_y(x)$ can be found from equation (3.48) quite similar to the first case as

$$\delta_{A_1\parallel} \simeq \frac{-4}{3\sqrt{3}}\left(\frac{c^2a^2\hbar}{4\omega e^2\left(1+\omega^2m^2A^2\right)^{\frac{1}{2}}}\right)^{\frac{1}{3}} \csc\frac{1}{3}\left[5\pi - \tan^{-1}\left(-\omega mA\right)\right], \tag{3.50}$$

and in the gas approximation (when A=0) equation (3.50) becomes

$$\delta_{A_1\parallel} \simeq \frac{8}{9}\left(\frac{c^2a^2\hbar}{4\omega e^2}\right)^{\frac{1}{3}}.$$

The ratio of the skin depths of (3.38) to (3.50) is

$$\frac{\delta_{P\perp}}{\delta_{A_1\parallel}} \simeq \left(\frac{\eta}{1-\eta}\right)^{\frac{1}{3}} \tag{3.51}$$

This shows us that the skin depth parallel to the layers during parallel propagation of electromagnetic wave to the layers $\left(\delta_{A_1\parallel}\right)$ exceeds the penetration depth perpendicular to the layers $(\delta_{P\perp})$ during perpendicular propagation of the waves by a factor of $\left(\frac{\eta}{1-\eta}\right)^{\frac{1}{3}}$ and it tells us that the parallel propagation of electromagnetic wave is preferable in the sense of transparency.

The Fourier transform of $z-$ component of electrical current density can be given as

$$j_z(k) = \frac{-4e^2a_1\tau m}{(2\pi\hbar)^3}\int \frac{\partial n_o}{\partial \varepsilon}v_z\left[\left(\frac{1}{m} - i\omega A\right)I_4(k)\,E_y(k)\,p_\perp + I_5(k)\,v_zE_z(k)\right]d\varepsilon dp_z \tag{3.52}$$

where $I_4(k) = I_3(k)$ and

$$I_5(k) = 2\int_0^{\frac{\pi}{2}} \frac{d\varphi}{a_1^2 + b_1^2 \frac{p_\perp^2}{m^2}\cos^2\varphi} = \frac{2}{a_1^2}\int_0^{\frac{\pi}{2}} \frac{d\varphi}{1 + b\cos^2\varphi}.$$

Referring to mathematical supplement $(A.7)$,

$$I_5(k) \approx \frac{\pi m}{a_1 b_1 p_\perp} - \frac{a_1 \pi m^3}{2b_1^3 p_\perp^3}.$$

On inserting the values of the above integrals in (3.52), it is clear that the part containing $I_4(k)$ will vanish due to the same reason as the expression(3.43) leaving the result containing no Fermi liquid terms. Integrating the remaining part of the equation over energy of quasi-particle near the Fermi surface (for weak excitations) and the p_z integration over a brillouin zone, gives

$$j_z(k) \approx \frac{4e^2 a_1 \varepsilon_1^2 m^2 \pi^2 \lambda\tau}{(2\pi\hbar)^3}\left[\frac{1}{a_1 b_1 k (2m\mu)^{\frac{1}{2}}} - \frac{a_1 m^2}{2b_1^3 (2m\mu)^{\frac{3}{2}}}\right] E_z(k), \tag{3.53}$$

and the electrical conductivity as

$$\sigma_{zy}(k) = 0\ ,\ \sigma_{zz}(k) \approx \frac{4e^2 \varepsilon_1^2 m^2 \pi^2 \lambda}{(2\pi\hbar)^3}\left[\frac{1}{k(2m\mu)^{\frac{1}{2}}} - \frac{a_1^2 m^2}{2k^3\tau^2 (2m\mu)^{\frac{3}{2}}}\right]. \tag{3.54}$$

In the limit of extreme anomalous skin effect, $kl >> 1$, the second term can be neglected and hence gets

$$\sigma_{zz\|}(k) \approx \frac{4e^2 \varepsilon_1^2 m^2 \pi^2 \lambda}{(2\pi\hbar)^3 k (2m\mu)^{\frac{1}{2}}} \approx \frac{e^2\eta^2}{8\pi\hbar a^2 k},$$

which mainly depends on the layered structure of the conductor. For the same propagation of electromagnetic wave (along the layers), the conductivities along and perpendicular to the layers can be compared as

$$\frac{\sigma_{yy\|}}{\sigma_{zz\|}} \approx \frac{8(1 - i\omega m A)}{\eta^2}.$$

This tells us that the electrical conductivity in the normal state of layered conductors in the direction **n** perpendicular to the layers is much less than the conductivity along the layers.

The Fourier transform of the Maxwell's equation for this component in which the electric field is symmetric in k also gives

$$E_z\left(k\right)=\frac{-2E'\left(0\right)}{k^2-\frac{4\pi i\omega}{c^2}\sigma_{zz}}, \tag{3.55}$$

and on inserting (3.54) in (3.55) gives

$$E_z\left(k\right)=\frac{-2k^3E'\left(0\right)}{k^5-i\gamma_6k^2+i\gamma_7} \tag{3.56}$$

where

$$\gamma_6\equiv\frac{2\omega e^2\varepsilon_1^2m^2\lambda}{c^2\hbar^3\left(2m\mu\right)^{\frac{1}{2}}},\text{ and }\gamma_7\equiv\frac{e^2\omega a_1^2\varepsilon_1^2\lambda m^4}{\hbar^3c^2\tau^2\left(2m\mu\right)^{\frac{3}{2}}}.$$

For the simplicity of the expression, at low temperature in which the theory of anomalous skin effect is appreciable equation (3.56) gets

$$E_z\left(k\right)\approx\frac{-2kE'\left(0\right)}{k^3-i\gamma_6},$$

and upon taking the inverse Fourier transform yields

$$\frac{E_z\left(0\right)}{E_z'\left(0\right)}\approx\frac{-2}{\pi}\int\limits_0^\infty\frac{kdk}{k^3-i\gamma_6}=\frac{-4e^{\frac{i\pi}{6}}}{3\sqrt{3}\gamma_6^{\frac{1}{3}}}$$

and the surface impedance perpendicular to the layers during parallel propagation becomes

$$Z_{P\|}\approx\frac{16\pi}{3\sqrt{3}}\left(\frac{\omega^2\hbar^3\left(2m\mu\right)^{\frac{1}{2}}}{2c^4e^2\varepsilon_1^2m^2\lambda}\right)^{\frac{1}{3}}e^{\frac{5\pi i}{3}}, \tag{3.57}$$

which is the same as

$$Z_{P\|}\approx\frac{16\pi}{3\sqrt{3}}\left(\frac{2\omega^2\hbar a^2}{c^4e^2\eta^2}\right)^{\frac{1}{3}}\left(1-i\sqrt{3}\right),$$

and can be related to the surface impedance of normal metals as $Z_{A\|}\sim Z_{Nm}\eta^{\frac{-2}{3}}$.

The depth of penetration of the electric field $E_z\left(x\right)$ is then

$$\delta_{A_2\|} \approx \frac{8}{9}\left(\frac{2c^2\hbar a^2}{\omega e^2\eta^2}\right)^{\frac{1}{3}} \tag{3.58}$$

The skin depths of the field (for the same propagation) along and perpendicular to the layers can be compared by using (3.50) and (3.58) as

$$\frac{\delta_{A_1\|}}{\delta_{A_2\|}} \sim \frac{\eta^{\frac{2}{3}}}{2\left(1+\omega^2 m^2 A^2\right)}.$$

Therefore, when electromagnetic waves propagate along the layers and the inequality $\eta << 1$ is satisfied, the depth of penetration $\delta_{A_2\|}$ of the electric field E_z = En of a linearly polarized wave exceeds greatly the depth of attenuation $\delta_{A_1\|}$ of the electric field E_y.

One can see the variable dependence of the surface impedance due to the involvement of the correlation function by analyzing equations (3.35) and (3.48) on the assumption , $1 >> \omega^2 m^2 A^2$, as

$$Z \approx \frac{8\pi}{3\sqrt{3}}\left(\frac{2\omega^2 a^2\hbar\left(\frac{\eta}{1-\eta}\right)}{e^2c^4}\right)^{\frac{1}{3}}\left[1-\frac{\omega^2 m^2 A^2}{6}\right]\exp\frac{5\pi i}{3}. \tag{3.59}$$

We see that the surface impedance due to the interaction during perpendicular propagation of the wave can be approximated as

$$Z_{int} \approx \frac{2^{\frac{1}{3}}}{9\sqrt{3}\pi}\left[\frac{\left(\alpha\tau m\right)^2\omega^{\frac{8}{3}}\left(\frac{\eta}{1-\eta}\right)^{\frac{1}{3}}}{e^{\frac{2}{3}}c^{\frac{4}{3}}\hbar^{\frac{11}{3}}a^{\frac{4}{3}}}\right]\exp\frac{8\pi i}{3}, \tag{3.60}$$

and also during parallel propagation of the wave as

$$Z_{int} \approx \frac{2^{\frac{1}{3}}}{9\sqrt{3}\pi}\left[\frac{\left(\alpha\tau m\right)^2\omega^{\frac{8}{3}}}{e^{\frac{2}{3}}c^{\frac{4}{3}}\hbar^{\frac{11}{3}}a^{\frac{4}{3}}}\right]\exp\frac{8\pi i}{3}. \tag{3.61}$$

Thus, during electron-electron interaction through exchange, the surface impedance due to only this interaction varies directly as the square of relaxation time τ and the effective mass m of the interacting quasi-particles and as $\omega^{\frac{8}{3}}$.

A unified description of the phenomena, valid for the high frequency range in the extreme anomalous limit, is obtained by expressing the results of the theory in terms of the surface impedance Z, from which the quantities of experimental interest are then easily derived. The parameter α is experimentally accessible and directly related to the compressibility of the interacting quasi-particle system.[19]

In general, we can see that the involvement of the Landau's correlation function $\Phi\left(\mathrm{p},\mathrm{p}'\right)$ in semi-classical terms does not affect the qualitative features of our results. Its effect, should, of course, be included in calculations (quantitative) based on realistic spectra.

The explicit expressions of the quantities of interest of the effect in layered conductors in the case of quadratic form of energy spectrum (3.10) can be related to the normal metals. The conductivities, the surface impedances and the skin depths are compared in order of magnitude under three categories from the analyses done above. The first of these are those which have the same order of magnitude with that of the normal metals, i.e., the conductivity along the layers during parallel propagation of electromagnetic wave $\left(\sigma_{A\|} \sim \sigma_{Nm}\right)$, the surface impedance along the layers during parallel propagation $\left(Z_{A\|} \sim Z_{Nm}\right)$and the skin depth of electric field along the layers during the same propagation $\left(\delta_{A_1\|} \sim \delta_{Nm}\right)$. The next are those which exceeds that of the normal metals, i.e., the conductivity along the layers during perpendicular propagation of electromagnetic wave

$\left(\sigma_{A\perp} \sim (\frac{1-\eta}{\eta})\sigma_{Nm}\right)$, the surface impedance perpendicular to the layers during parallel propagation $\left(Z_{P\|} \sim Z_{Nm}\eta^{\frac{-2}{3}}\right)$ and the skin depth of electric field perpendicular to the layers during the same case $\left(\delta_{A_2\|} \sim \delta_{Nm}\eta^{\frac{-2}{3}}\right)$. The third are those which have less than that of the normal metals. The conductivity perpendicular to the layers during parallel propagation $\left(\sigma_{P\|} \sim \sigma_{Nm}\eta^{2}\right)$, the skin depth perpendicular to the layers during perpendicular propagation$\left(\delta_{P\perp} \sim \delta_{Nm}(\frac{\eta}{1-\eta})^{\frac{1}{3}}\right)$ and the surface impedance along the layers during perpendicular propagation of electromagnetic wave $\left(Z_{A\perp} \sim Z_{Nm}(\frac{\eta}{1-\eta})^{\frac{1}{3}}\right)$.

Conclusion

The following conclusions are derived in the work:

The surface impedance in both considered cases of electromagnetic wave polarizations is proportional to $\omega^{\frac{2}{3}}$ and weakly depends on the relaxation time τ through the Fermi liquid interaction term, and in the absence of interaction it automatically reduces to the value in the gas approximation. A correction due to the account of Fermi liquid effect is observed in the separate interaction term, although this term is small even at high frequency compared to the main gas term.

The values of the constants ε_1 and α which characterize the bandwidth and Fermi liquid effect respectively can be determined from appropriate experimental data on layered conductors based on expressions derived for conductivity and surface impedance.

The conductivity along the layers is much greater than the conductivity perpendicular to the layers. Likewise, the magnitude of surface impedance of the conductor during parallel propagation of electromagnetic wave highly exceeds the magnitude during the propagation perpendicular to the layers. The explicit expressions of these quantities in layered conductors are related in order of magnitude to that of normal metals:

-The conductivity $(\sigma_{A\|})$, the surface impedance $(Z_{A\|})$ and the skin depth of the electric field along the layers $(\delta_{A_1\|})$ during parallel propagation of electromagnetic wave have the same order of magnitude with that of the normal metals;

-The conductivity along the layers during perpendicular propagation $(\sigma_{A\perp})$, the surface impedance perpendicular to the layers $(Z_{P\|})$ and the skin depth of the electric field perpendicular to the layers ($\delta_{A_2 \parallel}$) during the parallel propagation of the wave exceed the values in normal metals;

-The conductivity perpendicular to the layers during parallel propagation ($\sigma_{P \parallel}$), the skin depth perpendicular to the layers ($\delta_{P \perp}$) and the surface impedance ($Z_{A \perp}$) both during perpendicular propagation of the wave to the layers have less values than in normal metals.

Appendix A
Mathematical Supplement

Although the φ and p_z integrations in chapter 3 are somewhat challenging, one obtains the necessary simple expressions for them in the extreme anomalous limit condition. In solving this one can use computer programs such as the computational engine included in Scientific Workplace (Maple V Release 4), which includes the standard Maple libraries. In our case this Maple program was used to minimize possible human errors and save time. One of the simplest ways is, thus, explicitly presented in this supplement. In addition, some of the details about the way the Fourier transformed components of the electrical current density are obtained are pointed out.

A.1 Fourier transformation of $\psi(z)$

The Fourier transformation of the nonequilibrium part of the distribution function (the solution of the Boltzmann kinetic equation) $\psi(z) = \int\limits_{-\infty}^{\infty} \Omega(z')\, e^{-|\gamma|(|z-z'|)} dz'$ is obtained as follows.

$$\begin{aligned}
\psi_k &= \int\limits_{-\infty}^{\infty} dz \int\limits_{-\infty}^{\infty} \Omega(z')\, e^{-|\gamma|(|z-z'|)} e^{ikz} dz' \\
&= \int\limits_{-\infty}^{\infty} dz'\Omega(z')\, e^{ikz'} \int\limits_{-\infty}^{\infty} dz e^{-|\gamma|(|z-z'|)} e^{ikz} e^{-ikz'} \\
&= \int\limits_{-\infty}^{\infty} dz'\Omega(z')\, e^{ikz'} \int\limits_{-\infty}^{\infty} dz e^{-|\gamma|(z-z')} e^{ik(z-z')}
\end{aligned}$$

$$
\begin{aligned}
&= \int_{-\infty}^{\infty} dz' \Omega\left(z'\right) e^{ikz'} \int_{-\infty}^{\infty} dte^{-|\gamma|t} e^{ikt} \\
&\equiv \Omega\left(k\right) \int_{-\infty}^{\infty} dte^{-|\gamma|t} e^{ikt}.
\end{aligned}
$$

The function ${}^{-|\gamma|t} e^{ikt}$ is an even function of its argument t, so that we can write

$$
\Omega\left(k\right) \int_{-\infty}^{\infty} dte^{-|\gamma|t} e^{ikt} = \Omega\left(k\right) 2 \int_{0}^{\infty} e^{-|\gamma|t} \cos ktdt,
$$

and finally one gets $2 \int_0^\infty e^{-|\gamma|t} \cos ktdt = 2\frac{|\gamma|}{\gamma^2+k^2}, \psi_k = 2\Omega\left(k\right) \frac{|\gamma|}{\gamma^2+k^2}$.

A.2 The current density

The current density is given by

$$
\mathrm{j} = \frac{-2e}{\left(2\pi\hbar\right)^3} \int \mathrm{v} \frac{\partial n_o}{\partial \varepsilon} d^3 p \psi\left(z\right).
$$

Substituting the value found from the integro-differential Boltzmann equation in this expression, one can write the current density as follows:

$$
j_\alpha\left(z\right) = \frac{-2e}{\left(2\pi\hbar\right)^3} \int_{v_z > 0} v_\alpha \frac{\partial n_o}{\partial \varepsilon} d^3 p \int_{-\infty}^{\infty} \Omega\left(z'\right) e^{-\gamma\left(|z-z'|\right)} dz'.
$$

Performing the Fourier transformation,

$$
j_\alpha\left(k\right) = \frac{-4e}{\left(2\pi\hbar\right)^3} \int_{v_z > 0} v_\alpha \frac{\partial n_o}{\partial \varepsilon} d^3 p \Omega\left(k\right) \frac{|\gamma|}{|\gamma|^2 + k^2}
$$

$$
= \frac{-4e}{\left(2\pi\hbar\right)^3} \left[1 - im\omega A\right] \int_{v_z > 0} \frac{|\gamma|}{|\gamma|^2+k^2} v_\alpha \left(\frac{ep_\perp}{v_z} \left[E_x\left(k\right) \cos\varphi + E_y\left(k\right) \sin\varphi\right]\right) \frac{\partial n_o}{\partial \varepsilon} d\varphi dp_z d\varepsilon.
$$

The components of the velocity are

$$
v_x = \frac{p_\perp}{m} \cos\varphi, v_y = \frac{p_\perp}{m} \sin\varphi, v_z = -\varepsilon_1 \lambda \sin \lambda p_z.
$$

Now, for a component (say x component) the current density is

$$j_x(k) = \frac{-4e}{(2\pi\hbar)^3 m}\left[1 - im\omega A\right]\int_{v_z>0} \frac{|\gamma|}{|\gamma|^2+k^2} \times$$
$$\left(\frac{ep_\perp^2}{v_z}\left[E_x(k)\cos^2\varphi + E_y(k)\sin\varphi\cos\varphi\right]\right)\frac{\partial n_o}{\partial\varepsilon}d\varphi dp_z d\varepsilon$$
$$= \frac{-4e\pi E_x(k)}{(2\pi\hbar)^3 m}\left[1 - im\omega A\right]\int_{v_z>0} \frac{|\gamma|}{\gamma^2+k^2}\frac{ep_\perp^2}{v_z}\frac{\partial n_o}{\partial\varepsilon}dp_z d\varepsilon =$$
$$= \frac{-8e^2\pi E_x(k)}{(2\pi\hbar)^3}\left[1 - im\omega A\right]\int_{v_z>0} \frac{|\gamma|}{\gamma^2+k^2}\frac{(\varepsilon - \varepsilon_1\cos\lambda p_z)}{v_z}\frac{\partial n_o}{\partial\varepsilon}dp_z d\varepsilon.$$

The coefficient $\frac{|\gamma|}{\gamma^2+k^2}\frac{1}{v_z}$ can be simplified as follows:

$$\frac{|\gamma|}{\gamma^2+k^2}\frac{1}{v_z} = -\frac{\frac{1-i\tau\omega}{\varepsilon_1\tau\lambda\sin\lambda p_z}}{\left(\frac{1-i\tau\omega}{\varepsilon_1\tau\lambda\sin\lambda p_z}\right)^2+k^2}\frac{1}{\varepsilon_1\lambda\sin\lambda p_z}$$
$$= -\left(-1+i\tau\omega\right)\frac{\tau}{-1+2i\tau\omega+\tau^2\omega^2-k^2\varepsilon_1^2\tau^2\lambda^2+k^2\varepsilon_1^2\tau^2\lambda^2\cos^2\lambda p_z}$$
$$= \frac{(1-i\tau\omega)\tau}{\left((1-i\tau\omega)^2+k^2\varepsilon_1^2\tau^2\lambda^2\left(\sin^2\lambda p_z\right)\right)} = \frac{(1-i\tau\omega)\tau}{\left((1-i\tau\omega)^2+k^2l^2\left(\sin^2\lambda p_z\right)\right)}$$
$$= \frac{\tau}{(1-i\tau\omega)\left(1+k^2l_\omega^2\left(\sin^2\lambda p_z\right)\right)}.$$

Thus

$$j_x(k) = \frac{-8e^2\pi\tau E_x(k)}{(2\pi\hbar)^3}\left[1 - im\omega A\right]\int d\varepsilon\int_{v_z>0}\frac{(\varepsilon - \varepsilon_1\cos\lambda p_z)}{(1-i\tau\omega)\left(1+k^2l_\omega^2\left(\sin^2\lambda p_z\right)\right)}\frac{\partial n_o}{\partial\varepsilon}dp_z$$

or, since $v_z > 0$ is equivalent to $-\frac{\pi}{2\lambda} \leqslant p_z \leqslant 0$,

$$\begin{aligned} j_x(k) &= \frac{8e^2\pi\tau E_x(k)}{(2\pi\hbar)^3 a_1}B\int_{-\frac{\pi}{2\lambda}}^{0}\frac{(\mu - \varepsilon_1\cos\lambda p_z)}{\left(1+k^2l_\omega^2\left(\sin^2\lambda p_z\right)\right)}dp_z \\ &= \frac{8e^2\pi\tau E_x(k)\,a_1}{(2\pi\hbar)^3}B\frac{2}{\lambda a_1^2}\int_{-\frac{1}{2}\pi}^{0}\frac{\mu - \varepsilon_1\cos x}{1+k^2l_\omega^2\sin^2 x}dx = \frac{8e^2\pi\tau E_x(k)\,a_1}{(2\pi\hbar)^3}BI_1(k) \qquad \text{(A.1)}\end{aligned}$$

where the symbols

$$a_1 \equiv (1-i\tau\omega)\,, B \equiv \left[1 - im\omega A\right], l_\omega = \frac{l}{a_1}, l = v_0\tau, v_0 = \varepsilon_1\lambda \qquad \text{(A.2)}$$

are introduced for simplicity. Equation (A.1) is the same as (3.26) after taking the simpler φ and ε integrations.

A.3 The Integral I$_1$

This integral is $I_1\left(k\right) = \frac{2}{\lambda a_1^2}\int_{-\frac{1}{2}\pi}^{0}\frac{\mu-\varepsilon_1\cos x}{1+k^2l_\omega^2\sin^2 x}\,dx$ as stated in the main text (page 46). We start evaluation of the integral as follows, separating it into two terms:

$$\begin{aligned} I_1 &= \frac{2}{\lambda a_1^2}\left(\int_{-\frac{1}{2}\pi}^{0}\frac{\mu}{1+k^2l_\omega^2\sin^2 x}\,dx-\varepsilon_1\int_{-\frac{1}{2}\pi}^{0}\frac{\cos x}{1+k^2l_\omega^2\sin^2 x}\,dx\right)\\ &= I_1^{'}+I_1^{''}. \end{aligned}$$

a) Let's deal first with the simpler $I_1'' = -\frac{2\varepsilon_1}{\lambda a_1^2}\int_{-\frac{1}{2}\pi}^{0}\frac{\cos x}{1+k^2l_\omega^2\sin^2 x}\,dx = -\frac{2\varepsilon_1}{\lambda a_1^2}\frac{\arctan kl_\omega}{kl_\omega}$. We introduce the symbol $b=\left(k^2l_\omega^2\right)^{-1}$ for further evaluations; $\frac{\arctan kl_\omega}{kl_\omega}=\sqrt{b}\arctan\frac{1}{\sqrt{b}}$.

After expanding in power series with respect to $b << 1$(the extreme anomalous skin effect condition $lk >> 1$), and remembering that

$$\operatorname{csgn}(z)=\begin{cases} 1 & \begin{array}{c}\text{if } \operatorname{Re}(z)>0;\\ \text{or } \operatorname{Re}(z)=0 \text{ and } \operatorname{Im}(z)\geq 0\end{array}\\ -1 & \begin{array}{c}\text{if } \operatorname{Re}(z)<0;\\ \text{or } \operatorname{Re}(z)=0 \text{ and } \operatorname{Im}(z)<0\end{array}\end{cases}$$

and

$$\operatorname{signum}(z)=\begin{cases}\frac{z}{|z|} \text{ if } z\neq 0\\ 0 \text{ if } z=0\end{cases},$$

one has

$$\begin{aligned} \sqrt{b}\arctan\frac{1}{\sqrt{b}} &= \frac{1}{2}\operatorname{csgn}\left(\text{conjugate}\left(\sqrt{b}\right)\right)\pi\sqrt{b}-b+\frac{1}{3}b^2-\frac{1}{5}b^3\\ &+\frac{1}{7}b^4+O\left(b^5\right)=\frac{1}{2}\pi\sqrt{b}-b+\frac{1}{3}b^2-\frac{1}{5}b^3+\frac{1}{7}b^4+O\left(b^5\right)\\ &\approx\frac{1}{2}\pi\sqrt{b}-b+\frac{1}{3}b^2-\frac{1}{5}b^3+\frac{1}{7}b^4\\ &=\frac{1}{2}\pi\sqrt{\left(k^2l_\omega^2\right)^{-1}}-\left(k^2l_\omega^2\right)^{-1}+\frac{1}{3}\left(k^2l_\omega^2\right)^{-2}-\frac{1}{5}\left(k^2l_\omega^2\right)^{-3}+\frac{1}{7}\left(k^2l_\omega^2\right)^{-4}\\ &=\frac{1}{210}\frac{105\pi k^7l_\omega^7-210k^6l_\omega^6+70k^4l_\omega^4-42k^2l_\omega^2+30}{k^8l_\omega^8}\\ &=\frac{1}{210}\frac{105\pi k^7\left(\frac{l}{a_1}\right)^7-210k^6\left(\frac{l}{a_1}\right)^6+70k^4\left(\frac{l}{a_1}\right)^4-42k^2\left(\frac{l}{a_1}\right)^2+30}{k^8\left(\frac{l}{a_1}\right)^8} \end{aligned}$$

$$= \frac{a_1}{210k^8l^8}\left(105\pi k^7l^7 - 210k^6l^6a_1 + 70k^4l^4a_1^3 - 42k^2l^2a_1^5 + 30a_1^7\right).$$

Thus, we get

$$I_1'' \approx -\frac{2\varepsilon_1}{\lambda a_1^2}\left(\frac{\pi a_1}{2kl} - \frac{a_1^2}{k^2l^2} + \frac{a_1^4}{3k^4l^4} - \frac{a_1^6}{5k^6l^6} + 30\frac{a_1^8}{7k^8l^8}\right).$$

b) Proceeding with the next integral $I_1' = \frac{\mu}{\lambda a_1^2}\int_{-\frac{1}{2}\pi}^{0}\frac{1}{1+k^2l_\omega^2\sin^2 x}\,dx$, we have

$$\begin{aligned}
&\int_{-\frac{1}{2}\pi}^{0}\tfrac{1}{1+k^2l_\omega^2\sin^2 x}\,dx \\
&= \left\{\left(\arctan\frac{b}{\sqrt{\left(\left(b+2+2\sqrt{(b+1)}\right)b\right)}}\right)\sqrt{(b+1)}\sqrt{\left(\left(b+2-2\sqrt{(b+1)}\right)b\right)}\right. \\
&+ \left(\arctan\frac{b}{\sqrt{\left(\left(b+2+2\sqrt{(b+1)}\right)b\right)}}\right)\sqrt{\left(\left(b+2-2\sqrt{(b+1)}\right)b\right)} \\
&- \left(\arctan\frac{b}{\sqrt{\left(\left(b+2-2\sqrt{(b+1)}\right)b\right)}}\right)\sqrt{\left(\left(b+2+2\sqrt{(b+1)}\right)b\right)} \\
&+ \left.\left(\arctan\frac{b}{\sqrt{\left(\left(b+2-2\sqrt{(b+1)}\right)b\right)}}\right)\sqrt{\left(\left(b+2+2\sqrt{(b+1)}\right)b\right)}\sqrt{(b+1)}\right\}\times \\
&\left(\frac{b}{\sqrt{\left(\left(b+2-2\sqrt{(b+1)}\right)b\right)}\sqrt{(b+1)}\sqrt{\left(\left(b+2+2\sqrt{(b+1)}\right)b\right)}}\right).
\end{aligned}$$

Next, we write expansion of this expression with respect to b

$$\begin{aligned}
&\tfrac{1}{2}\operatorname{csgn}\left(b\operatorname{conjugate}\left(\sqrt{\left(\left(b+2-2\sqrt{(b+1)}\right)b\right)}\right)\right)\pi\sqrt{b} \\
&-\tfrac{1}{4}\operatorname{csgn}\left(b\operatorname{conjugate}\left(\sqrt{\left(\left(b+2-2\sqrt{(b+1)}\right)b\right)}\right)\right)\pi\left(\sqrt{b}\right)^3 \\
&+\tfrac{3}{16}\operatorname{csgn}\left(b\operatorname{conjugate}\left(\sqrt{\left(\left(b+2-2\sqrt{(b+1)}\right)b\right)}\right)\right)\pi\left(\sqrt{b}\right)^5 + O\left(b^3\right) \\
&= \tfrac{1}{2}\pi\sqrt{b} - \tfrac{1}{4}\pi\left(\sqrt{b}\right)^3 + \tfrac{3}{16}\pi\left(\sqrt{b}\right)^5 + O\left(b^3\right) \\
&\approx \tfrac{1}{2}\pi\sqrt{\left(k^2l_\omega^2\right)^{-1}} - \tfrac{1}{4}\pi\left(\sqrt{\left(k^2l_\omega^2\right)^{-1}}\right)^3 + \tfrac{3}{16}\pi\left(\sqrt{\left(k^2l_\omega^2\right)^{-1}}\right)^5 \\
&= \tfrac{1}{16}\pi\tfrac{8k^4l_\omega^4-4k^2l_\omega^2+3}{k^5l_\omega^5}.
\end{aligned}$$

We have, therefore,

$$\begin{aligned} I_1' &\approx \frac{2\mu}{\lambda a_1^2}\left(\frac{1}{16}\pi\frac{8k^4 l_\omega^4}{k^5 l_\omega^5} - \frac{1}{16}\pi\frac{4k^2 l_\omega^2}{k^5 l_\omega^5} + \frac{1}{16}\pi\frac{3}{k^5 l_\omega^5}\right) \\ &= \frac{2\mu}{\lambda a_1^2}\left(\frac{\pi a_1}{2kl} - \frac{\pi a_1^3}{4k^3 l^3} + \frac{3}{16}\frac{\pi a_1^5}{k^5 l^5}\right) \end{aligned}$$

and

$$\begin{aligned} I_1(k) &= I_1' + I_1'' \approx \frac{2}{\lambda a_1^2}\left\{\mu\left(\frac{\pi a_1}{2kl} - \frac{\pi a_1^3}{4k^3 l^3} + \frac{3}{16}\frac{\pi a_1^5}{k^5 l^5}\right)\right. \\ &\left. - \varepsilon_1\left(\frac{\pi a_1}{2kl} + \frac{a_1^2}{k^2 l^2} - \frac{a_1^4}{3k^4 l^4} + \frac{a_1^6}{5k^6 l^6} - 30\frac{a_1^8}{7k^8 l^8}\right)\right\} \end{aligned} \tag{A.3}$$

Now, the current density is

$$j_x(k) \approx \frac{16e^2\pi\tau E_x(k)}{(2\pi\hbar)^3 a_1\lambda} B\times \tag{A.4}$$

$$\left\{\mu\left(\frac{\pi a_1}{2kl} - \frac{\pi a_1^3}{4k^3 l^3} + \frac{3}{16}\frac{\pi a_1^5}{k^5 l^5}\right) - \varepsilon_1\left(\frac{\pi a_1}{2kl} - \frac{a_1^2}{k^2 l^2} + \frac{a_1^4}{3k^4 l^4} - \frac{a_1^6}{5k^6 l^6} + 30\frac{a_1^8}{7k^8 l^8}\right)\right\}.$$

A.4 The Integrals I_2 & I_3

In the following, the integrals $I_2(k)$ and $I_3(k)$ introduced in (3.38) of the main text are analyzed. We start with $I_2(k)$.

A.4.1 a) The Integral I_2

$$I_2(k) = 2\int\limits_0^{\frac{\pi}{2}} \frac{\sin^2\varphi d\varphi}{a_1^2 + b_1^2\frac{p_\perp^2}{m^2}\cos^2\varphi} = \frac{2}{a_1^2}\int_0^{\frac{1}{2}\pi} \frac{\sin^2 x}{1 + b_1^2\frac{p_\perp^2}{a_1^2 m^2}\cos^2 x} dx$$

$$= \frac{\pi}{a_1^2}\frac{1 + b_1^2\frac{p_\perp^2}{a_1^2 m^2} - \sqrt{\left(1 + b_1^2\frac{p_\perp^2}{a_1^2 m^2}\right)}}{\sqrt{\left(1 + b_1^2\frac{p_\perp^2}{a_1^2 m^2}\right)b_1^2\frac{p_\perp^2}{a_1^2 m^2}}}$$

$$= \pi \frac{a_1^2 m^2 + b_1^2 p_\perp^2 - \sqrt{(a_1^2 m^2 + b_1^2 p_\perp^2)} a_1 m}{a_1} \frac{m}{\sqrt{(a_1^2 m^2 + b_1^2 p_\perp^2) b_1^2 p_\perp^2}}$$

$$= \pi \frac{b_1^2 p_\perp^2 \left(\frac{a_1^2 m^2}{b_1^2 p_\perp^2} + 1\right) - b_1 p_\perp \sqrt{\left(\frac{a_1^2 m^2}{b_1^2 p_\perp^2} + 1\right)} a_1 m}{a_1} \frac{m}{\sqrt{\left(\frac{a_1^2 m^2}{b_1^2 p_\perp^2} + 1\right) b_1^3 p_\perp^3}}$$

$$= \pi \frac{\left(\frac{a_1^2 m^2}{b_1^2 p_\perp^2} + 1\right) - \frac{a_1 m}{b_1 p_\perp} \sqrt{\left(\frac{a_1^2 m^2}{b_1^2 p_\perp^2} + 1\right)}}{a_1} \frac{m}{\sqrt{\left(\frac{a_1^2 m^2}{b_1^2 p_\perp^2} + 1\right) b_1 p_\perp}}$$

$$= \pi \frac{(x^2+1) - x\sqrt{(x^2+1)}}{a_1 m a_1} \frac{m}{\sqrt{(x^2+1)}} x.$$

Next, one uses expansion with respect ($x << 1$) to obtain

$$\left(\pi \frac{(x^2+1) - x\sqrt{(x^2+1)}}{a_1 m a_1} \frac{m}{\sqrt{(x^2+1)}} x\right) = \frac{\pi}{a_1^2} x + \left(-\frac{\pi}{a_1^2}\right) x^2 + \frac{1}{2} \frac{\pi}{a_1^2} x^3 + O\left(x^5\right)$$
$$\approx \frac{\pi}{a_1^2} \left(\frac{a_1 m}{b_1 p_\perp}\right) + \left(-\frac{\pi}{a_1^2}\right) \left(\frac{a_1 m}{b_1 p_\perp}\right)^2 + \frac{1}{2} \frac{\pi}{a_1^2} \left(\frac{a_1 m}{b_1 p_\perp}\right)^3$$
$$= \frac{1}{2} \pi m \left(\frac{2 k^2 \tau^2 p_\perp^2}{a_1 k^3 \tau^3 p_\perp^3} - \frac{2 m a_1 k \tau p_\perp}{a_1 k^3 \tau^3 p_\perp^3} + \frac{a_1^2 m^2}{a_1 k^3 \tau^3 p_\perp^3}\right).$$

Finally,

$$I_2(k) \approx \frac{1}{2} \pi m \left(\frac{2}{k \tau p_\perp a_1} - 2 \frac{m}{k^2 \tau^2 p_\perp^2} + a_1 \frac{m^2}{k^3 \tau^3 p_\perp^3}\right). \tag{A.5}$$

A.4.2 b) The Integral I_3

$$I_3(k) = 2 \int_0^{\frac{\pi}{2}} \frac{\sin\varphi d\varphi}{a_1^2 + b_1^2 \frac{p_\perp^2}{m^2} \cos^2\varphi} = 2 \left(\arctan b_1 \frac{p_\perp}{m a_1}\right) \frac{m}{a_1 b_1 p_\perp}$$

$$= 2 \left(\arctan \frac{b_1 p_\perp}{m a_1}\right) \frac{m}{a_1 b_1 p_\perp} = 2 \left(\arctan x^{-1}\right) \frac{1}{a_1^2} x.$$

Performing expansion with respect to $x = \frac{ma_1}{b_1 p_\perp} \ll 1$

$$2\left(\arctan x^{-1}\right)\frac{1}{a_1^2}x = \operatorname{csgn}\left(\operatorname{conjugate}\left(x\right)\right)\frac{\pi}{a_1^2}x + \left(-\frac{2}{a_1^2}\right)x^2 + \frac{2}{3a_1^2}x^4 + O\left(x^6\right)$$

$$\begin{aligned}
&\approx \frac{\pi}{a_1^2}\left(\frac{ma_1}{b_1 p_\perp}\right) + \left(-\frac{2}{a_1^2}\right)\left(\frac{ma_1}{b_1 p_\perp}\right)^2 + \frac{2}{3a_1^2}\left(\frac{ma_1}{b_1 p_\perp}\right)^4 \\
&= \frac{1}{3}m\frac{3\pi b_1^3 p_\perp^3 - 6ma_1 b_1^2 p_\perp^2 + 2a_1^3 m^3}{a_1 b_1^4 p_\perp^4} \\
&= \frac{\pi m}{a_1 b_1 p_\perp} - \frac{2m^2}{b_1^2 p_\perp^2} + \frac{2a_1^2 m^4}{3b_1^4 p_\perp^4}
\end{aligned}$$

Thus,

$$I_3\left(k\right) \approx \frac{\pi m}{a_1 b_1 p_\perp} - \frac{2m^2}{b_1^2 p_\perp^2} + \frac{2a_1^2 m^4}{3b_1^4 p_\perp^4}. \tag{A.6}$$

A.5 The Integral I₅

Now we proceed with the evaluation introduced in (3.47) of the main text

$$I_5\left(k\right) = 2\int\limits_0^{\frac{\pi}{2}} \frac{d\varphi}{a_1^2 + b_1^2 \frac{p_\perp^2}{m^2}\cos^2\varphi} = \frac{2}{a_1^2}\int\limits_0^{\frac{\pi}{2}} \frac{d\varphi}{1 + b\cos^2\varphi}$$

Denotining $\frac{b_1^2 p_\perp^2}{a_1^2 m^2}$ by b, we have

$$\int\limits_0^{\frac{\pi}{2}} \frac{d\varphi}{1 + b\cos^2\varphi} = \int_0^1 \frac{1}{\left(1 + bx^2\right)\sqrt{\left(1 - x^2\right)}}\,dx$$

$$= -\frac{1}{2}\frac{i}{\sqrt{\left(1+b\right)}}\lim_{x\to 1-}\left(-\operatorname{arctanh}\frac{b - ix\sqrt{b}}{\sqrt{\left(1+b\right)b\left(1-x^2\right)}} + \operatorname{arctanh}\frac{b + ix\sqrt{b}}{\sqrt{\left(1+b\right)b\left(1-x^2\right)}}\right).$$

To approximate this result, let's evaluate the limit as follows:

$$\lim_{x\to 0.999999999999999999999999}\left(\operatorname{arctanh}\frac{-b + ix\sqrt{b}}{\sqrt{(1+b)}\sqrt{b}\sqrt{(1-x^2)}} + \operatorname{arctanh}\frac{b + ix\sqrt{b}}{\sqrt{(1+b)}\sqrt{b}\sqrt{(1-x^2)}}\right)$$

$$= \operatorname{arctanh} 7.0711 \times 10^{11}\frac{-1.0b + 1.0i\sqrt{b}}{\sqrt{(1.0+b)}\sqrt{b}} + \operatorname{arctanh} 7.0711 \times 10^{11}\frac{b + 1.0i\sqrt{b}}{\sqrt{(1.0+b)}\sqrt{b}}.$$

Furthermore,

$$\begin{aligned}&\lim_{b\to\infty}\left(\operatorname{arctanh}7.071\,1\times10^{11}\frac{i\sqrt{b}-b}{\sqrt{(1+b)}\sqrt{b}}+\operatorname{arctanh}7.071\,1\times10^{11}\frac{b+i\sqrt{b}}{\sqrt{(1+b)}\sqrt{b}}\right)\\=&\ 3.141\,6i,\end{aligned}$$

from which it follows that

$$\int_0^{\frac{\pi}{2}}\frac{d\varphi}{1+b\cos^2\varphi}\approx-\frac{1}{2}\frac{i}{\sqrt{(1+b)}}(i\pi)=\frac{1}{2\sqrt{(1+b)}}\pi,$$

and

$$\begin{aligned}I_5(k)&\approx\frac{\pi}{a_1^2\sqrt{(1+b)}}=\frac{1}{a_1^2}\frac{\pi}{\sqrt{\left(1+\frac{b_1^2p_\perp^2}{a_1^2m^2}\right)}}\\&=\frac{1}{a_1^2}\frac{a_1m}{b_1p_\perp}\frac{\pi}{\sqrt{\left(1+\frac{a_1^2m^2}{b_1^2p_\perp^2}\right)}}=\frac{1}{a_1^2}x\frac{\pi}{\sqrt{(1+x^2)}}.\end{aligned}$$

Now expanding with respect to $x=\frac{a_1m}{b_1p_\perp}<<1$ one gets

$$\frac{x}{\sqrt{(1+x^2)}}=x+-\frac{1}{2}x^3+O\left(x^5\right),$$

and, after substituting back for x, the approximate value of $I_5(k)$ becomes

$$\begin{aligned}I_5(k)&\approx\frac{1}{a_1^2}\pi\frac{a_1m}{b_1p_\perp}+\left(-\frac{1}{2a_1^2}\pi\right)\left(\frac{a_1m}{b_1p_\perp}\right)^3\\&=\frac{\pi m}{a_1b_1p_\perp}-\frac{a_1\pi m^3}{2b_1^3p_\perp^3}.\end{aligned}\tag{A.7}$$

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