Addis Ababa University

Addis Ababa Institute of Technology

School of Electrical and Computer Engineering

MSc in Telecommunication Engineering

Infrastructure and Spectrum Sharing for Coverage and Capacity Enhancement in Multi-Operator Networks

In Partial Fulfillment of the Requirements for the Degree of Master of Science in Telecommunication Network Engineering

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DECLARATION

I, the undersigned hereby declare that I am the sole author of this thesis. To the best of my knowledge this thesis contains no material previously published by any other person except where due acknowledgement has been made.

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Abstract

The rapid growth of mobile data traffic is putting a strain on wireless networks. Infrastructure and spectrum sharing are two promising ways to alleviate this challenge on the wireless networks. Infrastructure sharing refers to the co-deployment and operation of network infrastructures like base station and other radio access equipment by different mobile network operators (MNOs). Spectrum sharing refers to the use of the same spectrum by multiple MNOs. Full sharing involves the sharing of both infrastructure and spectrum among MNOs.

Sharing between operators is nowadays used as a cost optimization and technology refreshment in developed markets and as coverage and capacity enhancement in emerging markets. In Ethiopia, the two MNOs of the country are not meeting and exceeding the quality of service (QoS) target given by the nation’s communication authority and most customers are not satisfied by the poor QoS specially, mobile data services. Infrastructure and spectrum sharing are effective and efficient ways to fulfill the required QoS.

The research presented in this paper presents an analytical model for infrastructure, spectrum and full sharing scenarios and investigates the performance of the three sharing scenarios in terms of probability of coverage and mean user rate. The results show that infrastructure sharing provides superior coverage as compared to spectrum and full sharing scenarios; whereas, full sharing and spectrum sharing have given the highest mean user rate. Therefore, the findings of this thesis can help MNOs to decide on the best way to share infrastructure and spectrum in order to meet their capacity and coverage requirements.

Key words: infrastructure sharing, spectrum sharing, coverage probability, mean user rate, coverage enhancement, capacity enhancement.

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Lists of Acronyms

- BS – Base Station
- CAPEX - Capital expenses
- ECA – Ethiopian Communication Authority
- HPPP- homogeneous Poisson Point Process
- MNO – mobile network operators
- MOCN – multiple operator core network
- MORAN – multiple operator radio access network
- OPEX - Operational expenses
- OFDMA- Orthogonal Frequency Division Multiple Access
- PPP - Poisson point processes
- PLMN - Public Land Mobile Network
- QoS – Quality of service
- RAN- radio access network.
- RNC - Radio Network Controller
- RSRP - Reference Signals Received Power

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Chapter One

Introduction

1.1 Background

Telecom operators in emerging and mature markets are increasingly considering network sharing. Emerging market operators are looking for economical ways to expand their coverage and capacity, while mature market operators are looking to optimize costs and refresh their technology [6]. As an emerging market, in Ethiopia, the primary target is on the coverage and capacity growth.

Infrastructure sharing is a way for telecom operators to share their physical and electronic infrastructure, such as towers, Radio Access Network (RAN), power, and spectrum. Infrastructure sharing may expand coverage into previously un-served geographic areas and can be used in congested urban centers where new site acquisition is difficult.

Infrastructure sharing can generally be classified as passive and active [6].

- *Passive sharing* refers to the sharing of physical space, such as buildings, sites, masts, and power supply so as to minimize operational expenses (OPEX).
- A*ctive sharing* refers to the sharing of active elements of to the network including antennas, backhaul, base stations, and elements of core network are shared in order to reduced number of physical base station deployments required for coverage enhancement which can reduce capital expenses (CAPEX), increase base station utilization, and reduce overall power consumption.

Spectrum sharing involves operators leasing spectrum from each other. This can help to improve efficiency by making better use of a scarce resource. Spectrum sharing can significantly improve the average data rate of a user but can slightly reduce the network coverage as a result of the increase in interference due to the use of the same frequency band between the mobile network operators (MNOs) [16]. Recently in Ethiopia, ethio telecom has started sharing tower and power and also leasing transmission capacity for, Safaricom, a new entrant operator in Ethiopia telecom market. It is a good opportunity to reduce OPEX and CAPEX, since facilitating sharing provides an additional revenue source and lower CAPEX for the incumbent operator in addition to improving quality of service (QoS).

However, from drive and walk tests in some places in Addis Ababa, it is shown that both MNOs are not satisfying the QoS required by Ethiopian communication authority (ECA) regarding coverage and average datarate[20].(Sample measurements from drive and walk test are provided in the appendix section). Therefore, by a means of best strategic agreement, in addition to the passive infrastructure, both MNOs can share their active infrastructures and spectrum with each other so as to improve their coverage and the average data rate of their users. This will improve their QoS and resource utilization beside cost savings.

Different researchers have considered and modeled different sharing strategies/models using distinct mathematical and machine learning algorithm regarding the CAPEXs and OPEXs [1]-[5]. While others [17] modeled infrastructure sharing from a stochastic geometry perspective and have performed transmitted power and market trade-off analysis. In [18] the authors develop new general models for the multi-cell signal-to-interference-plus-noise ratio (SINR) using stochastic geometry and also derive the mean rate, and then the coverage gains (and mean rate loss) from static frequency reuse.

In this study, it is chosen to make use of stochastic geometry from the general cellular model developed by [18] and from that derive, compute and analyzed the downlink SINR coverage probability and the average data rate of a typical user for three different sharing cases that are: infrastructure sharing, spectrum sharing and full sharing (i.e. a scenario where MNOs share both infrastructure and spectrum). Then evaluate and compare the network performances of the sharing scenarios then the impacts of some parameters like: BS density, user density and bandwidth imbalance on the sharing scenarios are analyzed.

1.2 Problem Statement

Mobile data demand and customers' QoS service requirements are increasing. Beside this, MNOs are not meeting and exceeding the QoS requirements that are requested by the Communication Authority of the Country [20], as observed from the drive and walk tests performed in different places in Addis Ababa, Ethiopia, at different times (sample measurements from drive and walk test are provided in appendix section). The target given by ECA [20] for coverage is RSRP > -100 dBm, but most recorded measurements from the tests are RSRP between -111 dBm and -90 dBm. For average data rate, the target given by ECA is > 1 Mbps, but most measurements are < 500 kbps.

One approach to this problem is new base station (BS) deployment or using other optimization techniques, but this greatly increases MNOs' CAPEX and OPEX. This is not appropriate while the existing infrastructure and spectrum are not being used effectively and efficiently. Another approach for the MNOs is to share their infrastructure and spectrum among themselves. The mobile service coverage signal strength and the average data rate of users can be improved by allowing cooperation between MNOs or merging some of their infrastructure, resources, and spectrum to serve their customers. MNOs can enhance their coverage and capacity by engaging in different sharing strategies.

1.3 Objectives

1.3.1 General Objective

The main objective of the thesis is to show the improvement in network performance of the two MNOs as a result of infrastructure and spectrum sharing by applying stochastic geometry.

1.3.2 Specific Objectives

The specific objectives of the thesis are to:

- Perform a drive-and-walk test to assess the current network performance of the MNOs.
- Build a system model that is closer to the realistic base station distribution of the target area.
- Apply stochastic geometry to derive analytical expressions for the downlink SINR coverage probability and average user rate of a typical user for no sharing case, during infrastructure sharing and spectrum sharing case.
- Perform simulations and verify or validate the closed form or the derived analytical expressions.
- Observe and compare the network performances of the MNOs as a result of the sharing.
- Compare the network performances of the three sharing scenarios
- Perform numerical analysis to see the impacts of some parameters (BS density, user density, and bandwidth) and the throughput for the sharing cases.

1.4 Scope and Limitation of the Thesis

1.4.1 Scope of the Thesis

In this research, the probability of coverage and average rate is computed in downlink transmission assuming that the BSs are distributed according to Poisson distribution. The other thing is that for the system model, it is just assumed that there is a small area in Addis Ababa where the network performance of the MNOs is low since the virtue of infrastructure sharing is significantly seen in an area where one of the MNOs is underperforming. Finally, for the sake of simplicity and avoiding much complexity, the network of both MNOs is assumed to be homogeneous and doesn't consider sharing between small cells.

1.4.2 Limitation

Unlike the analytical expression of the probability of coverage, the analytical expression for average user rate has integral expression which is integrated from 0 to infinity. This makes very difficult to perform the cross validation of the analytical expression for the average user rate. Therefore, the verification of the derived expression for the average user rate is done from the probability of coverage.

1.5 Contribution of the research

This thesis mainly tries to show the improvement in network performance of two MNOs as a result of infrastructure and spectrum sharing between them. The research shows that the MNOs can improve their capacity and coverage without deploying new BSs. Beside this, the performed performance comparison between the sharing scenarios indicates that which sharing scenario is better for coverage and rate enhancement. It is also seen that some parameters such as: BS density, user density and bandwidth has significant impact on the performance of the sharing.

1.6 Literature Review

Many studies are showing possible and beneficial infrastructure sharing strategies and presenting different models using different mathematical and machine learning algorithms. Some research, as in [1], works on energy-efficient infrastructure sharing and presents a framework that shows how multiple MNOs share a portion of their network. Sequential social dilemma and particle swarm algorithms were implemented on the OpenAI Gym-based adaptable framework. The simulation's output has demonstrated that when the appropriate cell allots sufficient resources, users are connected to or linked to adjacent base stations while optimizing the utility or reward function of the MNO (game participants). By creating multiple fictitious cellular network settings with variable numbers of cell sites and mobile users, the performance of the simulation is made visible. The solution is improved by layering on particle swarm optimization. The research wasn't done by collecting actual data for total power consumption as well as the total number of customers registered for each MNO and didn't generate realistic environments. In addition to this, the research is more generic and hasn't considered the impacts of sharing both infrastructure and spectrum on coverage and the average user rate of a user.

Some researchers, as in [14], investigated the current technological solutions and regulatory and technical-economical dimensions in connection with the sharing of mobile telecommunication networks in emerging countries. They evaluated the technological limitations, applicability, and advantages of the network sharing solutions in the context of an emerging market while analyzing the expected savings on capital and operating costs. The research also suggests some general recommendations for infrastructure sharing in emerging countries. However, the quantitative analysis in this paper only focuses on the economic aspects and hasn't analyzed the technical aspects quantitatively.

Cellular networks were usually modelled by placing the base stations on a grid and users randomly or deterministically. Being both highly idealized and not very tractable, complex system-level simulations were used to evaluate coverage and outage probability and rate. A more tractable model is presented in [18], who have developed new generic model for the multi-cell signal-to-interference-plus-noise ratio (SINR) using stochastic geometry. And also derive the mean rate and then the coverage gain (and mean rate loss) from static frequency reuse. Following the proposal of this model, infrastructure sharing has started to be modelled from a stochastic geometry perspective, as in [17], who have modelled and analyzed the infrastructure sharing problem in a large-scale cellular network by exploiting tools and results from stochastic geometry. The researchers have analyzed the tradeoff between the intensity of the BS deployment for a buyer MNO and the transmit power and that between the BS intensity for the seller MNO and the area power consumption are analyzed in an infrastructure sharing scenario. Also, the market competition among MNOs is modelled and analyzed.

[7] Investigated the schemes for active sharing of the radio access among multiple MNOs and analyzed the individual benefits depending on the level of sharing. The proposed model used to assess the sharing strategies is based on multidimensional loss systems, and blocking probability is considered a performance metric. They have studied sharing strategies at individual cells and at an aggregation point in heterogeneous networks. The novelty of this paper is the analysis of the influence of traffic variation for one operator on the performance of the second operator at different levels of sharing.

This research differs from the aforementioned researches in that it mainly aims to show the improvement in coverage and the average user rate of two MNOs as a results of different sharing scenarios and assess the impacts of some parameters like base station density, user density, and bandwidth to give an idea for the MNOs of which sharing scenario with which parameter or constraint more increases their performance.

1.7 Methodology

In order to complete this thesis, various activities and methodologies are used. Generally, the following four research steps are applied:

- Literature Review: literatures related to infrastructure sharing and stochastic geometry are reviewed to aid the work in this research.
- Data collection: drive and walk test measurements are performed to assess the current network performances of the MNOs. Beside this, BS distribution and BS intensity of a MNO in a specific area is taken.
- Data analysis: simulation and numerical analysis are performed for result comparison and to observe some impacts.
- Recommendation: finally, some recommendations are given based on the results that were found. Since the aim of this thesis is to show the significance of sharing to improve coverage area and the average user rate of a user, it is important to first explore areas with low coverage and the average user rate of a user while the services of both MNOs (ethiotelecom and safaricom) do exist. To perform this action, different testing tools can be used to measure the performance of networks. For this thesis, GMoN-PRO and Network Cell Info Lite are used for measuring the RSRP and data rate of both MNOs in some areas. After measuring the network performance and identifying an area with poor performance, the next step is to create a hypothetical environment that is closer to the realistic base station distribution. In a realistic scenario in the country, most of the base stations of the MNOs are co-located. Therefore, the hypothetical environment is created considering this.

Following the method in [18], a stochastic geometry-based analytical model is then used to derive the downlink SINR coverage probability and average user rate of a typical user for the no-sharing case, the infrastructure sharing case, and the spectrum sharing case.

Then numerical analysis is performed to see the impacts of different parameters. MATLAB tool is chosen to show simulations and the impacts of the parameters. Finally, some recommendations are given based on the results that were found.

1.8 Thesis organization

This thesis is organized into six chapters, and the remaining parts are organized as follows: Chapter 2 describes the theoretical background of infrastructure and spectrum sharing, which includes defining the different sharing types and models and the pros and cons of the scenarios and models. Chapter 3 presents a brief introduction to stochastic geometry, the point process, the performance metrics, the derivation of the probability of coverage, and the average user rate for a general stochastic-based cellular model. Chapter 4 summaries the built-system model and the assumptions made. The derived analytical expression for the three sharing cases and the cross-validation of the analytical model are also included. Chapter 5 discusses the results, including the performance comparison of the sharing cases and the impacts of some parameters. Chapter 6 is the last chapter, and in this chapter, the conclusion of the thesis work and recommendations for future works are presented.

Chapter Two

Infrastructure and Spectrum sharing: An Overview and Theoretical Background

2.1 Infrastructure Sharing

Infrastructure sharing in telecommunications refers to the use of an infrastructure or resource by more than one operator through an arrangement. It is a process that offers many options.

The country and internal rules, in general, determine the choice of infrastructure sharing schemes. Active and passive sharing are the two main categories of infrastructure sharing models [19].

2.1.1 Types of infrastructure sharing

2.1.1.1 Passive Sharing

This sharing mechanism is simple and doesn't involve any technical challenges. Passive sharing includes:

- **Site sharing**: This involves the sharing of network sites between the MNOs. Site sharing minimizes the need for leasing, buying, and operating the region. The challenge with this concept, though, is deciding on a fixed spot to deploy a new site that works for everyone. A simple site sharing diagram is shown in Figure 2.1.
- **Tower/mast sharing**: In this form of sharing, the site is shared by several operators, and the towers are also shared. Each MNO is in charge of and deploys its own equipment. The passive sharing agreement can be done between two or more operators and include third-party companies that are

neutral hosts. A neutral host refers to a telecommunications service provider interested in creating additional revenues and sharing sites or masts. In these cases, costs can be significantly reduced when several network operators or service providers share physical assets and transport networks.

Sharing can be managed by the site owner, who acts as a landowner for the operators who lease the site. The owner may be an operator sharing the site or another structure that provides the infrastructure. A simple mast-sharing diagram is shown in Figure 2.2.

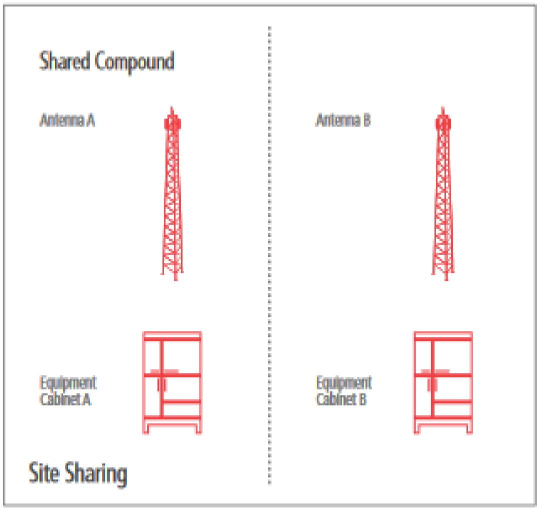

Figure 2.1 : Site sharing [6].

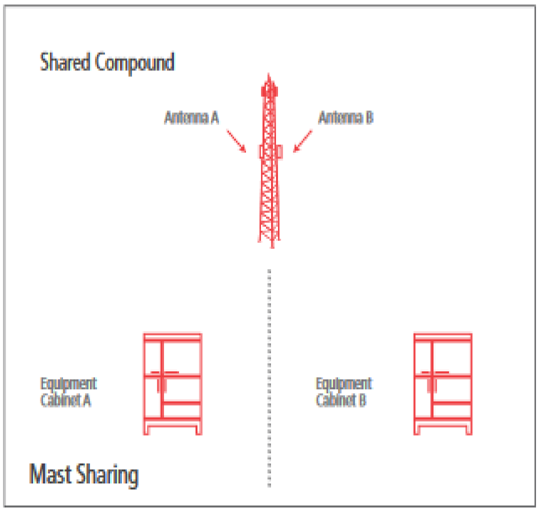

Figure 2.2: Mast Sharing [6].

2.1.1.2 Active Sharing

Sharing extends to the electronic components of the network and the radio spectrum, including the radio access network (which consists of antennas and transceivers, base stations, backhaul networks, and controllers) and the core network (servers and core network functionalities). A distinction is made between:

- **RAN sharing**: The shared equipment includes BTS, NodeB, BSC, RNC, etc. and may extend to feeder cables and antennas. But the transmission network and the core network are independent. Thus, this sharing mode allows operators to control their cells in their core network and have a separate operation. A full RAN sharing diagram is shown in Figure 2.3.

- **Backhaul sharing**: In addition to the RAN infrastructure, operators may decide to share the transmission channel. This is useful to extend their coverage more quickly and to focus on providing quality services to users. For backhaul sharing, several scenarios can be considered: The backhaul can be deployed by a joint venture of the participating mobile operators, or a private entity can deploy and operate the infrastructure and offer it to the operators in a" platform as a service" model.
- **Core network sharing:** Here, even the Home Location Register (HLR), the billing platform, and the value-added system can be shared. The core network sharing diagram is shown in Figure 2.4.

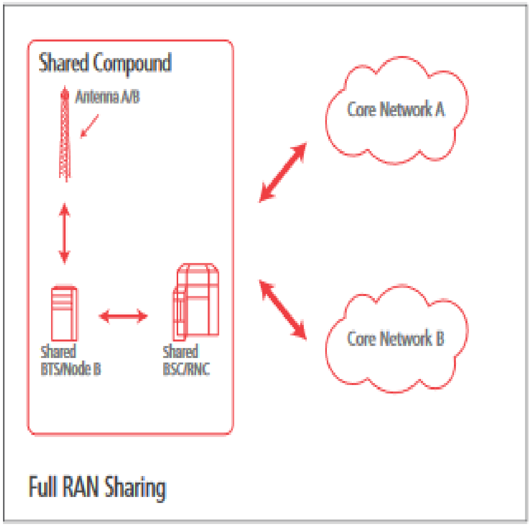

Figure 2.2 : Full RAN sharing [6].

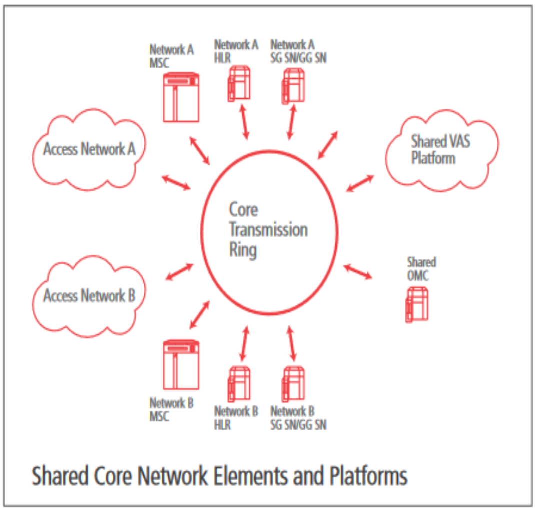

Figure 2.4: Core network sharing [6].

This form can be further classified into Multi-Operator Core Network (MOCN), where radio access networks and spectrum are shared; Multi-Operator Radio Access Network (MORAN), where radio access networks are shared and dedicated spectrum is used by each sharing operator; and core network sharing, where servers and core network functionalities are shared.

2.2 Spectrum Sharing

Spectrum sharing, also known as spectrum trading, is a model that has recently developed in mature, regulated telecom markets. It involves operators leasing their spectrum to other operators on commercial terms. Spectrum sharing is still an option for two or more operators because it is a limited resource that one operator in a specific area may underutilize.

2.3 Full Sharing

In full sharing, operators pool their radio access infrastructure and allow for shared use of spectrum. Interference is enhanced as a result of the spectrum pooling since more transmitters can now operate in the same frequency band, and users can connect to any of the transmitters operated by the sharing operators.

2.4 National Roaming

Roaming-based sharing enables the customers of one operator to flawlessly use another operator's network. This plugs gaps in coverage, for example, to provide a nationwide service. Network roaming can be considered a form of infrastructure sharing, although traffic from one operator's subscriber is actually carried and routed on another operator's network. However, there are no requirements for any common network elements for this type of sharing to occur. As long as a roaming agreement between the two operators exists, roaming can take place. Figure 2.5 shows simple network roaming.

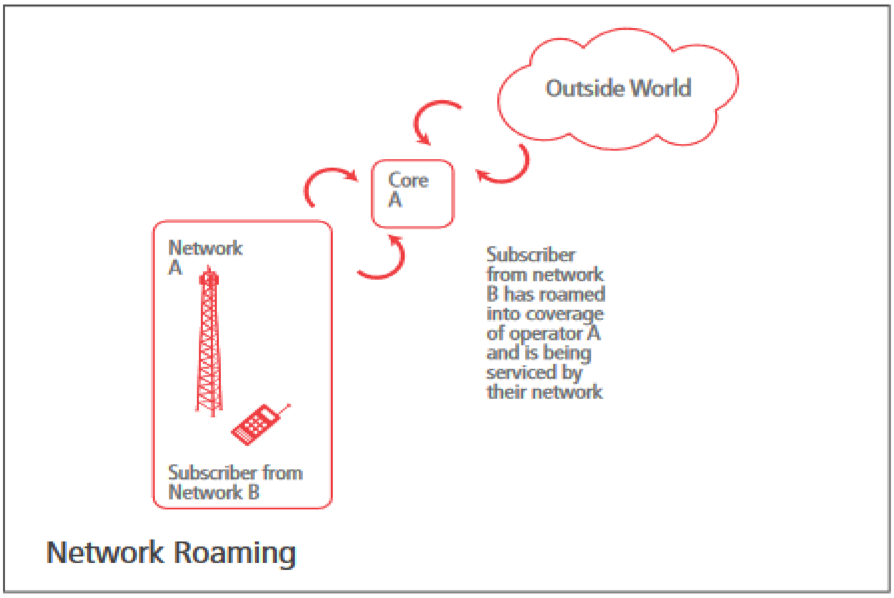

Figure 2.3 : National Roaming [6].

2.5 Pros and Cons of Infrastructure Sharing

The main advantages of infrastructure sharing are the cost benefits. But it is also driven by the migration to new technologies and the deployment of mobile broadband. Despite the advantages, infrastructure sharing can cause market distortion and imbalance and, if enforced from outside, might lessen incentives for developing new infrastructure and using cutting-edge technologies. The table below summarizes the pros and cons of the different infrastructure sharing techniques.

Table 2. 1 Comparison of infrastructure sharing forms (technology) [6].

Sharing form	Pros	Cons
Passive infrastructure sharing	✚ Significant CAPEX/OPEX savings ✚ Lowered risk of site acquisition ✚ Full differentiation and complete control of spectrum ✚ Control over sites to be shared. ✚ No/little regulatory obstacles. ✚ Easy migration to other sharing forms. ✚ Environmental benefits	✚ Availability of free space in existing sites (if existing sites are to be shared) ✚ Similar cell planning may be required
MORAN, MOCN	✚ Limited marginal CAPEX savings compared to site sharing. ✚ Substantial marginal OPEX savings compared to passive infrastructure sharing. ✚ Control over base station be shared. ✚ Reduction of network footprint by sharing operators	✚ Regulatory approval necessary ✚ Complexity of operation ✚ Requires long team commitment between operators. ✚ Difficult to exist from sharing agreements.
Core network sharing	✚ Further CAPEX/OPEX savings compared to MORAN/MOCN ✚ Significant investment can be diverted to services. ✚ Max sharing for operators sharing existing infrastructure.	✚ Regulatory approval necessary ✚ Complexity of operation and tight integration ✚ Challenging to differentiate quality of services
National roaming	✚ Significant CAPEX/OPEX saving. ✚ Clear ownership of equipment ✚ Differentiation based on service layer. ✚ Low risk solution for both incumbent and new entrant.	✚ Regulatory approval necessary ✚ Interconnection required. ✚ Reduce control over the network (eg outage of visited network can be affect home network service). ✚ End to end inter-PLMN QoS and inter -PLMN handover very challenging.

Chapter Three

Stochastic Geometry-based Modelling of Cellular Networks

3.1 Overview

Researchers use stochastic geometry models to study wireless communication networks. These models are based on the idea that the locations of network nodes, such as base stations and users, are random. The models are analyzed to better understand the performance of wireless communication networks. This includes predicting things like the signal strength and data rate that users can expect to receive.

The models require techniques from stochastic geometry and related fields, such as point processes, spatial statistics, geometric probability, and percolation theory. They also use methods from more general mathematical disciplines, such as geometry, probability theory, stochastic processes, queuing theory, information theory, and Fourier analysis.

The use of stochastic geometry models has led to a better understanding of wireless communication networks. This has helped researchers design more efficient and reliable networks [7].

Stochastic geometry models have been used to study wireless networks since the early 1960s. These models are based on the idea that the locations of nodes in a wireless network are random.

The use of stochastic geometry models has allowed researchers to study a variety of wireless network technologies, including mobile ad hoc networks, sensor networks, vehicular ad hoc networks, cognitive radio networks, and cellular networks.

Key performance and quality of service quantities in wireless networks are often based on concepts from information theory, such as the signal-to-interference-plus-noise ratio. This quantity is used to define network connectivity and coverage [4], [5].

The principal idea underlying the research on stochastic geometry models is that it is best to assume that the locations of nodes or the network structure are random in nature. This is because the size and unpredictability of users in wireless networks make it difficult to accurately model their locations deterministically.

The use of stochastic geometry can then allow for the derivation of closed-form or semi-closed-form expressions for key performance and quality of service quantities. This is in contrast to simulation methods, which can be computationally expensive, or deterministic models, which may be inaccurate.

Stochastic geometry is a branch of mathematics that studies random objects in space. In the context of wireless networks, these random objects are usually points or shapes that represent the locations of network nodes, such as receivers and transmitters.

The Euclidean space in which these objects are defined is often the two-dimensional plane, which represents a geographical region. The underlying geometry of the network, such as the relative locations of nodes, plays a fundamental role in wireless networks because it affects the interference between transmitters.

In contrast, the underlying geometry of wired networks, such as the Internet, is less important because the signals in wired networks do not experience interference [7].

Cellular networks are traditionally modelled by assuming the BSs are placed according to a hexagonal layout. However, these models have long suffered from being both intractable and highly idealized. Recently, a general model based on stochastic geometry was proposed in [18], which provides tractable ways to evaluate network performance considering inter-cell interference and fading.

3.2 The Importance of Space in Wireless Networks

The importance of transmit-receive distance in wireless communication has long been known. The challenge of spatial modelling can be illustrated by comparing the spatial resource to time and frequency resources. Wireless transmissions need to be separated in time, frequency, and/or space to avoid excessive interference. Space is by far the most challenging resource to use efficiently, for two reasons:

- In space, transmitters and receivers are not collocated.
- Power from undesired transmitters leaks into space over relatively large distances.

In contrast, when using time or frequency division, transmitters and receivers are collocated (in time or frequency), and spilling can be minimized. [4] [5]

3.2.1 Random Spatial Models

Because spatial configurations may vary widely over an enormous (often infinite) number of possibilities, one cannot design most systems for each specific configuration but must instead consider a *statistical* spatial model for the node locations. The usefulness of recent innovations such as wireless network coding and interference alignment depends critically on the relative positions of transmitters and receivers, but just how likely are configurations where these techniques are effective? An accurate performance assessment is only possible with an accurate probabilistic model for those positions. Just as one uses a fading or shadowing distribution to model the variety of possible propagation environments in a wireless link, one should use a statistical distribution to model the variety of possible network topologies.

3.2.2 Spatial Models and Metrics for Wireless Networks

The effectiveness and efficiency of a wireless network are best characterized by metrics such as connectivity, capacity/throughput, and reliability (e.g., packet error rate or outage probability). Each of these metrics is a complicated function of all the *links* in the network that could potentially connect each pair of nodes. Especially as the network grows in size, a sensible approach is to discuss statistical conditions over the network, such as averages or probability of outage or success.

3.3 Poisson Point Process

A number of point processes have been suggested to model the positioning of wireless network nodes. Among these, the most frequently used is the Poisson process, which gives a Poisson network model. Due to its analytical tractability and practical appeal in situations where transmitters and/or receivers are located or move around randomly over a large area, as shown in Figure 3.1, the (homogeneous) Poisson point process (PPP) has been by far the most popular spatial model. For example, in the 2-D PPP, each node takes up an independent location characterized by a pair of coordinates (xi, yi), the density of nodes in a unit area is λ, and so the average number of nodes in an area A is A. Finally, the probability that there are n nodes in A is given by the Poisson distribution and thus equal to [4][5][7]:

$$P(n) = \frac{(\lambda|A|)^n}{n!} e^{-\lambda|A|} \tag{3.1}$$

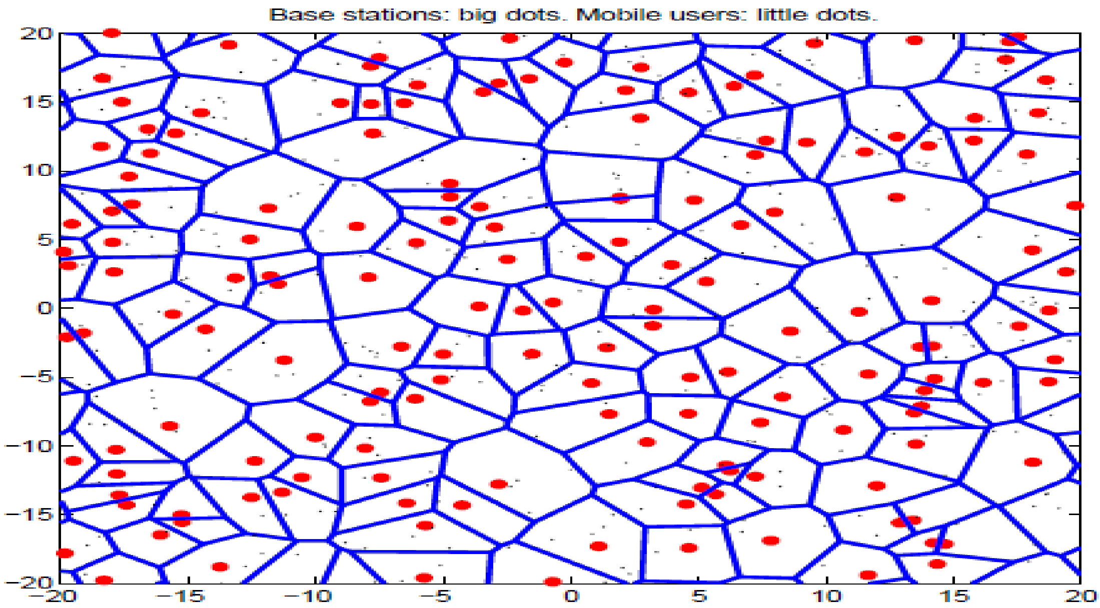

Figure 3.1 : Poisson distributed base stations and mobiles, with each mobile associated with the nearest [18].

Equation (3.1) quickly extends to the $\mathbf{R}^3$ case by replacing the area term with a volume term.

The mathematical tractability or ease of working with Poisson models is mostly because of their 'complete independence', which essentially says that two (or more) disjoint (or non-overlapping) bounded regions contain two (or more) Poisson numbers of points that are independent of each other. This important property characterizes the Poisson process and is often used as its definition.

The complete independence, or `randomness, property of Poisson processes leads to some useful characteristics and results of point process operations, such as the superposition property: The superposition of n Poisson processes with densities $\lambda 1$ to λn is another Poisson process with density

$$\lambda \ = \ \sum_{i=1}^{n} \lambda i \qquad (3.2)$$

Furthermore, randomly thinning a Poisson process (with density λ), where each point is independently removed (or kept) with some probability p (or $1-p$), forms another Poisson process (with density $(1-p)\lambda$) while the kept points also form a Poisson process (with density $p\lambda$) that is independent to the Poisson process of removed points.

These properties and the definition of the homogeneous Poisson process extend to the case of the inhomogeneous (or non-homogeneous) Poisson process, which is a non-stationary stochastic process with a location-dependent density $\lambda(x)$ where x is a point (usually in the plane, $\mathbf{R}^2$). [7]

3.4 Key Performance Metrics in Stochastic Geometry

In stochastic geometry, there are metrics that measures the connectivity of a user, the average rate, and throughputs. Each of the key metrics follows directly from the received signal-to-interference-plus-noise ratio (SINR) on one or more links, so understanding the SINR is essential.

In a wireless communication, when a collection of channels is active at the same time, the interference from the other channels is considered as noise, which motivates the need for the quantity known as the signal-to-interference-plus-noise ratio (SINR). For example, if we have a collection of point-to-point channels, the SINR of the channel of a particular transmitter–receiver pair is defined as: [7]

$$SINR = \frac{S}{I+N} \tag{3.3}$$

Where S is the power, at the receiver, of the incoming signal from said transmitter, I is the combined power of all the other (interfering) transmitters in the network, and N is the power of some thermal noise term. The SINR reduces to SNR when there is no interference (i.e. $I = 0$). In networks where the noise is negligible, also known as "interference limited" networks, we $N = 0$, which gives the signal-to-interference ratio (SIR) [7].

3.4.1 SINR — the Building Block Metric

The SINR is the instantaneous ratio of desired energy to interference and noise energy, and so is a random variable that depends on many factors. The most important factors are as follows [4].

The Distance between the Desired Transmitter and the Desired Receiver — Based on electromagnetic laws, the desired received power falls off with distance and obeys an inverse power-law where the exponent is known as the *path loss exponent*. In free space the power decay is quadratic with distance, but over ground the path loss exponent is usually better modeled by a value between 2.5 and 4 because of scattering and absorption.

i. ***The Set of Active Transmitters*** — there are many potential combinations of active transmitters in even a moderate sized wireless network. The set of active transmitters is often chosen by the MAC protocol. To each receiver, the other active transmitters appear to be interferers.

ii. ***The Sum Interference Power*** — the sum interference power depends on the set of interfering transmitters and their distances from each desired receiver. In networks of moderate to high density the interference power is usually much larger than the noise power. Interference is created as the power from each transmitting cells is increased as shown in Figure 3.2.

iii. ***The Noise Power*** — the impact of the ambient noise power on the SINR depends upon received signal and interference powers: under low transmission power the SINR is noise-limited, while under high transmission power the SINR is interference-limited.

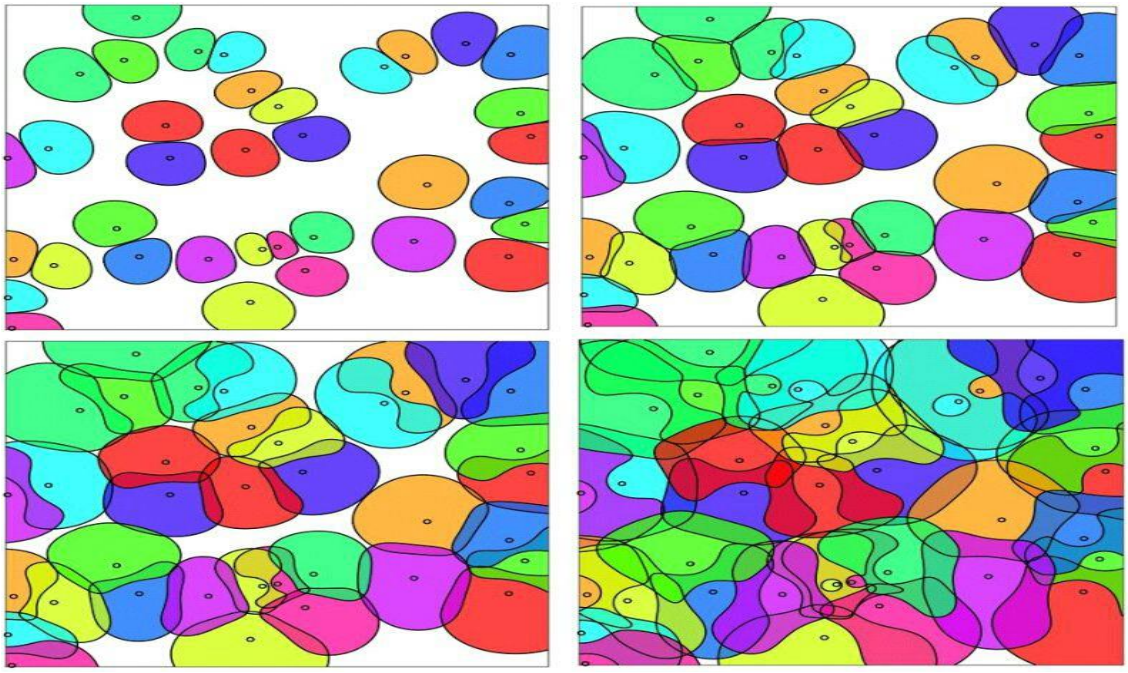

Figure 3.2 : SINR cells of a wireless network model expands as the transmitter powers increase [7].

Many other factors can affect the SINR including random propagation effects (fading and shadowing), specific transceiver design practices (for example, the use of multiple antennas or interference cancellation), and power control. But the spatial interactions are the most fundamental and inescapable — general fading, shadowing, and power control models can be added to the baseline model. A fairly general but simple mathematical description of the SINR at a typical node located at origin o is:

$$SINRo = \frac{h_{oo} P_i r^{-\alpha}}{w + \sum_{i \epsilon \Phi} h_{io} P_i |R_i|^{-\alpha}} \qquad (3.4)$$

Where h_{io} is the (power) fading coefficient of the channel to the desired receiver o from node i, Pi is the transmit power of transmitter i, w is the noise power, and Φ is the set of interfering nodes (Φ is a subset of all possible transmitters). The desired transmitter is a distance r from the desired receiver, while the i^{th} interferer is a distance Ri away. By drawing the distances according to a probabilistic spatial model, the randomness in locations along with many other basic aspects of the network (e.g., path loss) are consolidated into a single random variable, the SINR.

3.4.2 Coverage and Connectivity

A common goal of stochastic geometry wireless network models is to derive expressions for the SINR or for the functions of the SINR which determine coverage (or outage) and connectivity. For example, the concept of the outage probability p_{out}, which is informally the probability of not being able to successfully send a signal on a channel, is made more precise in the point-to-point case by defining it as the probability that the SINR of a channel is less than or equal to some network-dependent threshold.

The coverage probability p_c is then the probability that the SINR is larger than the SINR threshold. In short, given a SINR threshold T, the outage and coverage probabilities are given by:

$$pout = P(SINR \leq T) = \varepsilon \tag{3.5}$$

and

$$pc = P(SINR > T) \tag{3.6}$$

and can be thought of equivalently as (i) the probability that a randomly chosen user can achieve a target SINR T, (ii) the average fraction of users who at any time achieve SINR T, or (iii) the average fraction of the network area that is in "coverage" at any time. The probability of coverage is also exactly the CCDF of SINR over the entire network, since the CDF gives $P[SINR \leq T]$. For example, if this probability is 0.9 for a random selection of a source-destination pair, then one would say that the network is 90 % connected.

The probability of coverage of a typical user is derived from SINR for general cellular model in[18].

The cellular network model consists of base stations (BSs) arranged according to some homogeneous Poisson point process (PPP) of intensity λ in the Euclidean plane. As far as random channel effects such as fading and shadowing, it is assumed that the tagged base station and tagged user experience only Rayleigh fading with mean 1, and constant transmit power of 1/μ. Then the received power at a typical node a distance r from its base station is hr−_ where the random variable h follows an exponential distribution with mean 1/μ, which we denote as h ~ exp(μ).

The interference power follows a general statistical distribution g that could include fading, shadowing, and any other desired random effects. The interference power at the typical receiver Ir is the sum of the received powers from all other base stations other than the home base station and is treated as noise. And the noise power is assumed to be additive and constant with value σ^2. The SNR = $1/\mu\sigma^2$ is defined to be the received SNR at a distance of r = 1.

$$Pc(T,\lambda,\alpha) \triangleq \mathbb{P}[SINR > T], \tag{3.7}$$

$$SINR = \frac{hr^{-\alpha}}{w + I_r} \quad \text{Where,} \tag{3.8}$$

$$Ir = \sum_{i\epsilon\Phi/bo} giRi^{-\alpha} \tag{3.9}$$

Distance to Nearest Base Station: isimportant quantity is the distance r separating a typical user from its tagged base station. Since each user communicates with the closest base station, no other base station can be closer than r. In other words, all interfering base stations must be farther than r. The probability density function (pdf) of r can be derived using the simple fact that the null probability of a 2-D Poisson process in an area A is $exp(-\lambda A)$.

$$P[r > R] = P[No\ BS\ closer\ than\ R]$$

$$= e^{-\lambda\pi R^2} \tag{3.10}$$

Therefore, the CDF is P[r ≤ R] = Fr(R) = 1 − $e^{-\lambda\pi R^2}$ and the pdf can be found as

$$\mathrm{fr(r)} = \frac{dFr(r)}{dr}$$

$$\mathrm{fr(r)} = e^{-\lambda\pi r^2} 2\pi\lambda r \tag{3.11}$$

$$Pc(T, \lambda, \alpha) \triangleq \mathbb{P}[SINR > T],$$

$$Pc(T, \lambda, \alpha) = \mathbb{E}r[\mathbb{P}[SINR > T|r]]$$

$$= \int_{r>0} \mathbb{P}[SINR > T \mid r] fr(r)dr$$

$$= \int_{r>0} \mathbb{P}\left[\frac{hr^{-\alpha}}{\sigma^2 + Ir} > T \middle| r\right] e^{-\pi\lambda r^2}\, 2\pi\lambda r dr$$

The final derived expression for Pc is therefore:

$$Pc\,(T, \lambda, \alpha) = \int_{r>0} e^{-\pi\lambda r^2} e^{-\mu T r^{\alpha}\sigma^2} \mathcal{L}_{\mathrm{Ir}}\,(\mu T r^{\alpha})\, 2\pi\lambda r dr \tag{3.12}$$

Where $\mathcal{L}_{\mathrm{Ir}}(s)$ is the Laplace transform of random variable I_r evaluated at **S** conditioned on the distance to the closest BS from the origin. Significant simplification is done in [18] assuming the interference power follows an exponential distribution with unit mean, $\mu = 1$; i.e. interference experiences Rayleigh fading and shadowing is neglected.

$$\mathcal{L}_{\mathrm{Ir}}\,(r^{\alpha}) = \exp\left(-\pi r^2 \lambda \sqrt{T}\left(\frac{\pi}{2} - arctan\frac{1}{\sqrt{T}}\right)\right) \tag{3.13}$$

Therefore, the probability of coverage of a typical randomly located mobile user experiencing exponential interference assuming the system is interference limited(no noise power i.e. $\sigma^2 = 0$) can be generalized as

$$\mathrm{Pc} = \int_{r>0} \mathcal{L}_{\mathrm{Ir}}\,(r^{\alpha})\ fr(r)dr = \int_{r>0} e^{-\pi\lambda r^2} \mathcal{L}_{\mathrm{Ir}}\,(r^{\alpha})\ 2\pi\lambda r dr \qquad (3.14)$$

3.4.3 Channel Capacity and Average User Rate

One aim of the stochastic geometry models is to derive the probability laws of the Shannon channel capacity or rate of a typical channel when taking into account the interference created by all other channels.

In the point-to-point channel case, the interference created by other transmitters is considered as noise, and when this noise is Gaussian, the law of the typical Shannon channel capacity is then determined by that of the SINR through Shannon's formula (in bits per second):

$$C = B\log_2(1 + SINR) \qquad (3.15)$$

Where B is the bandwidth of the channel in hertz. In other words, there is a direct relationship between the coverage or outage probability and the Shannon channel capacity. The problem of determining the probability distribution of C under such a random setting has been studied in several types of wireless network architectures or types.

The average ergodic rate of a typical mobile user and its associated base station in the downlink is also derived in [18] in a similar fashion and is given by:

$$\tau(\lambda, \alpha) \triangleq \mathbb{E}[\ln(1 + SINR))] \qquad (3.16)$$

$$= \int_{r>0} e^{-\pi\lambda r^2} \int_{T>0} e^{-\sigma^2 \mu r^{\alpha}(e^T - 1)}\, \mathcal{L}_{\mathrm{Ir}}(\mu r^{\alpha}(e^T - 1))\mathrm{dT}2\pi\lambda r dr \qquad (3.17)$$

Assuming $\sigma^2 = 0, \alpha = 4$ and $\mu = 1$.

Where,

$$\mathcal{L}_{\mathrm{Ir}}\left(r^{\alpha}(e^{T}-1)\right) = \exp\left(-\pi r^{2}\lambda\sqrt{(e^{T}-1)}\left(\frac{\pi}{2} - arctan\frac{1}{\sqrt{(e^{T}-1)}}\right)\right) \qquad 3.18$$

Similar to a general expression of Pc as in (16), we can also obtain a general expression for τ

$$\tau = Bi\int_{r>0} fr(r)\int_{T>0}\mathcal{L}\mathrm{Ir}\ \left(r^{\alpha}(e^{T}-1)\right)\,\mathrm{d}T dr \qquad 3.19$$

3.4.4 Throughput

Throughput is one of the most important performance metrics for wireless networks, and a number of different notions of throughput exist.

Link Throughput — Spatial models lend themselves to an analytical characterization of the per link throughput, which is a critical determinant of end-to-end rate in multi-hop networks and is the quantity of interest for single-hop networks. Per link throughput is dictated by the SINR and can be defined in different ways.

The average per link throughput is $R_{avg} = \mathbb{E}\,[log(1 + SINR/\Gamma)]$,

Where the average is with respect to the sources of randomness encapsulated in the random variable SINR (e.g., locations and fading). This metric can be appropriate for settings in which the transmitted rate is adjusted to the instantaneous SINR, whereas outage-based metrics are more appropriate when dynamic rate adjustment is not performed.

For the network as a whole, it is then necessary to determine the outage Pr[SINR < T]. However, this depends on the Tx-Rx distance, and the locations of the active transmitters. Clearly, if fewer transmitters are active, then the SINR and hence the outage capacity can be increased, but the overall network throughput would also decrease. It is necessary to balance these two effects with a different metric. One such metric is the transmission capacity, first defined as $\tau\varepsilon = (1 - \varepsilon)\lambda b$ where λ is the maximum average number of active transmitters sending a rate of b b/s/Hz per unit area for which the outage probability is less than ε. In order words, the transmission capacity is the average number of successful active links of a certain rate that can be supported per square meter in the network has units of area spectral efficiency, for example b/s/Hz/m2.

Chapter Four

System Model and Assumptions

4.1 System Model

Most assumptions in this research are taken from [18] since they have proposed a general stochastic model for a cellular network. In the real-world scenario, most BS distributions are random, and the movement of the mobile user is obviously unpredictable. In Figure 4.1 below, it can be seen that the BS of the MNO is located randomly and doesn't have regular pattern (i.e. unpredictable). Hence, it is convenient to model the system based on the concepts of stochastic geometry particularly homogeneous Poisson Point Process (hPPP).

BS deployment is modeled as a point process by defining the network nodes or BSs as points that are randomly placed similar to the realistic BS distribution in the country.

Figure 4.1: BS distribution of a portion of North Addis Ababa zone (ethiotelecom)

It is assumed that there are two MNOs (MNO1 – refers to ET (ethiotelecom) and MNO2 – SAF (safaricom)). Let 'S' be the set of mobile operators, S = {MNO1, MNO2}. The locations of BSs of ET and SAF follow independent PPP distributions Φ1 and Φ2 with BS densities $\lambda 1$ and λ 2, respectively.

$\Phi = \{\Phi 1, \Phi 2\} \subset R2$ Where, $\Phi = \sum_{i \in S} \Phi i$;

$\lambda = \lambda 1 + \lambda 2;$ Where, λ is the sum of the BS densities when the two MNOs merge.

The bandwidth of operator i is denoted as Bi, and when spectrum is shared between the two operators, the aggregate bandwidth is then $B = \sum_{i \in S} Bi$ (the individual spectrum bands being aggregated are non-overlapping).

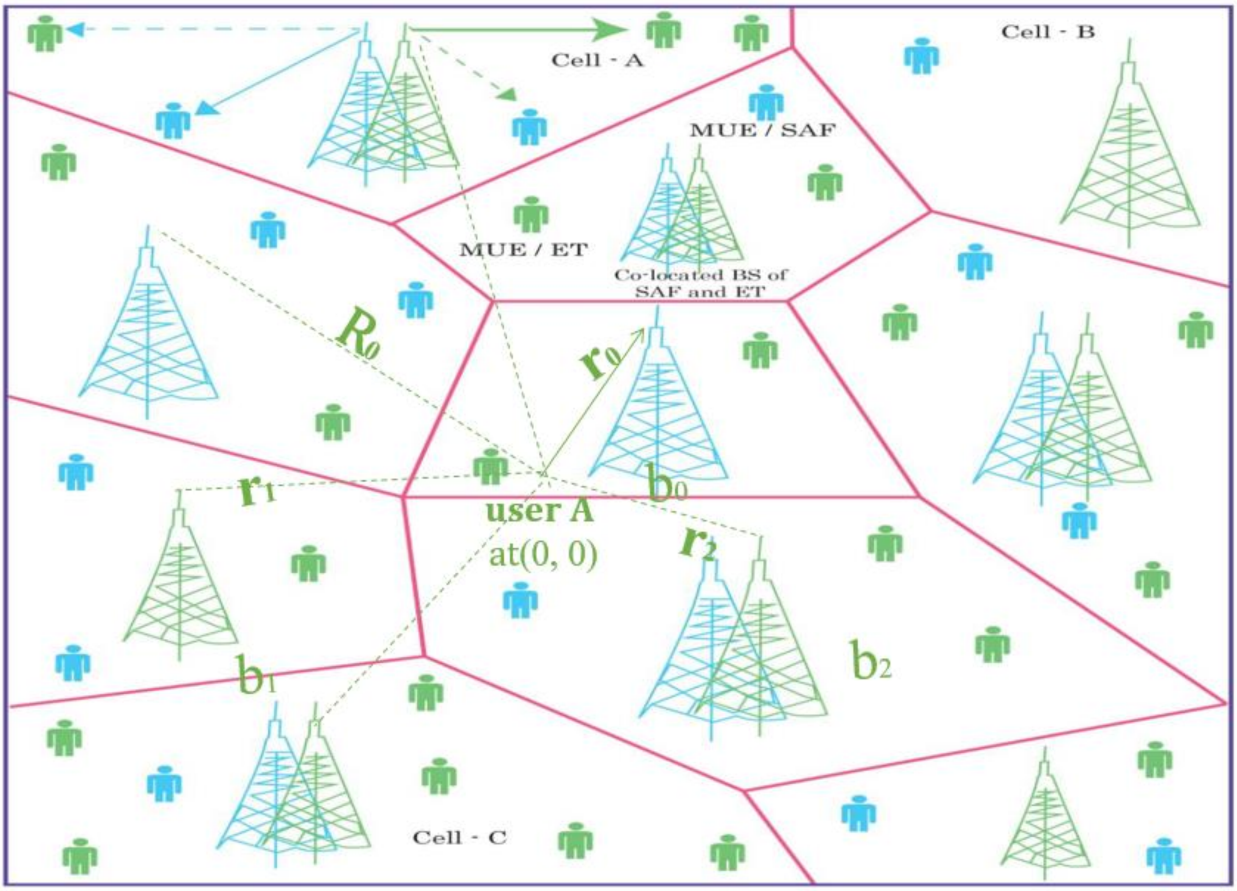

Figure 4.2: System model of Poisson distributed base station distribution in Addis Ababa.

4.2 Assumptions

In the model, it is assumed that some of the BSs of the two MNOs are collocated or share the same tower in some sites/places and deploy their site independently in some other places as illustrated in Figure 4.1. Mobile users, located according to some independent stationary point process are considered and each mobile user to be connected to the nearest base station. But for the sake of simplicity, a typical user located at the origin in Euclidean plane is considered for the derivation of the expressions. Therefore, the derived expressions are for a typical user and not for the aggregated users.

Similar to [18] the system is considered as interference limited network (an assumption where the noise power is 0; i.e. w=0). The standard power loss propagation model is the path loss model $r_o^{-\alpha}$ with path loss exponent α = 4. It is assumed that the connected base station and user experience only Rayleigh fading with mean 1, and constant transmit power of 1/μ. Then power fading of the message signal h_o; at a typical node a distance r from its base station is exponentially distributed with unit mean (ho ~ exp(1)). The interference power at the typical receiver Ir is the sum of the received powers from all other base stations other than the home base station at distances Ri from the typical user and similarly for the sake of simplicity; the interfering signal h_i, follows an exponential distribution with unit mean (hi ~ exp(1)). The SINR threshold T is the target SINR that a user should achieve to say in it's a network coverage.

4.3 Performance Metrics

The two important metrics, probability of coverage and average user rate, from a general stochastic geometry based cellular model are derived in [18] and stated in Sections 3.4.2 and 3.4.3 of this paper. From these two general expressions, an analytical expression is derived for the three sharing scenarios based on the assumptions and the system model.

4.4 Analytical Expression for Probability of Coverage

Analytical derivations of expressions for Pc under non shared network and three different sharing cases is done to evaluate and compare network performances of the sharing cases.

From Section 3.4.2 in (3.11), (3.13) and (3.14) we have:

- $fr(r) = e^{-\lambda \pi r^2} 2\pi\lambda\, r$
- $\mathcal{L}_{Ir}(Tr^{\alpha}) = \exp\left(-\pi r^2 \lambda \sqrt{T}(\frac{\pi}{2} - arctan\frac{1}{\sqrt{T}})\right)$

$\text{Pc(T)} = \int_{r>0}^{\infty} \mathcal{L}_{Ir}(Tr^{\alpha}) fr(r) dr$

4.4.1 Probability of coverage and sharing scenarios

i. No sharing between MNOs (ns)

For the non-shared network, i.e. if there is no sharing agreement between MNOs, there is no merging of infrastructures and resources between the MNOs and the MNOs serve their customers independently. The Pc of a typical user of MNOi whose nearest transmitter is at a distance r and BS intensity λi is then::

$$Pci(T) = \frac{1}{1+\sqrt{T}(\frac{\pi}{2} - arctan(\frac{1}{\sqrt{T}}))} \tag{4.1}$$

The expression for probability of coverage for non-shared network of MNOi which is totally independent on the BS intensity λ. Proof is provided in appendix A.

ii. Infrastructure Sharing Between MNOs (IS)

In infrastructure sharing case, since MNOs share their radio access infrastructure, the BS distribution and BS intensity is changed and it is the sum of the two independently distributed networks. $\Phi = \Phi1 + \Phi2$; $\lambda = \lambda1 + \lambda2$. The user can connect to any near-by transmitter that belongs to its home MNO or BS of the MNO that has entered a sharing agreement with the home MNO. For the derivation of an expression for Pc only intra-operator (self-interference) is considered and there is no inter-operator (between MNOs) interference because operators didn't share their frequencies in this scenario.

Therefore, the probability of coverage of a typical user of MNOi is

$$\mathrm{Pci}(T) = \frac{(\lambda 1+\lambda 2)}{(\lambda 1+\lambda 2)+\lambda i\sqrt{T}(\frac{\pi}{2}-\arctan(\frac{1}{\sqrt{T}}))} \tag{4.2}$$

Proof is provided in appendix B.

iii. Spectrum Sharing between MNOs (SS)

In this sharing case, the MNOs pool their spectrum and the bandwidth of the MNOs become the sum of the spectrum bands of the individual MNOs; i.e. B = B1 + B2. Unlike infrastructure sharing case, the user can only connect to its nearest home BS but both intra-operator and inter-operator interference are considered for the derivation of and expression for Pc.

Therefore, the probability of coverage of a typical user of MNOi is

$$\mathrm{Pci}(T) = \frac{\lambda i}{(\lambda 1+\lambda 2)\sqrt{T}(\frac{\pi}{2}-\arctan(\frac{1}{\sqrt{T}}))+\lambda i} \tag{4.3}$$

iv. Full sharing between MNOs(FS)

In this sharing case, MNOs share both the radio access network infrastructure and spectrum. The BS distribution and BS intensity is changed and it is the sum of the two independently distributed networks. $\Phi = \Phi 1 + \Phi 2$; $\lambda = \lambda 1 + \lambda 2$. Beside this, the bandwidth of the MNOs is now the sum of the individual MNOs bandwidth B= B1 + B2. User can connect to the nearest transmitter and gets better signal strength. Both intra-operator and inter-operator interferences are considered in this case because it is considered that all the BS now operate in the same frequency band and users may connect to any of the BSs of the sharing MNOs. The derived expression for Pc for full sharing case is:

Therefore, the probability of coverage of a typical user of MNOi is

$$\mathrm{Pci(T)} = \frac{1}{\sqrt{T}(\frac{\pi}{2} - \mathrm{arctan}(\frac{1}{\sqrt{T}}))+1} \qquad (4.4)$$

Proof is provided in appendix D.

The above expression is the same as the Pc of non-shared network (no sharing) case and it is independent of the BS intensityλ.

4.5 Analytical Expression for Average User Rate (τ)

Analytical derivations of expressions for average user rate (τ) under non shared network and three different sharing cases is done to evaluate and compare network performances of the sharing cases.

From section 3.4.3 in (11), (20) and (21):

$$fr(r) = e^{-\lambda i \pi r^2} 2\pi\lambda\mathrm{i}\, r$$

$$\mathcal{L}_{\mathrm{Ir}}(r^{\alpha}(e^{T} - 1)) = \exp\left(-\pi r^2 \lambda \sqrt{(e^{T} - 1)}\left(\frac{\pi}{2} - arctan\frac{1}{\sqrt{(e^{T}-1)}}\right)\right)$$

$$\tau = Bi \int_{r>0} fr(r) \int_{T>0} \mathcal{L}\mathrm{Ir}\ (r^{\alpha}(e^{T} - 1))\ \mathrm{dT}dr$$

4.5.1 Average user rate and sharing scenarios

i. No sharing between MNOs (ns)

For no sharing case, the average user rate τ of a typical user of MNOi is:

$$\tau i = Bi \int_0^{\infty} \frac{1}{1+\sqrt{(e^{T}-1)}(\frac{\pi}{2} - \mathrm{arctan}(\frac{1}{\sqrt{(e^{T}-1)}}))}\ \mathrm{dT} \qquad (4.5)$$

Proof is provided in Appendix A.

ii. Infrastructure sharing between MNOs(IS)

In infrastructure sharing case, only radio access infrastructures are shared and since there is no spectrum sharing, only intra-operator interference is considered for the derivation of the expression.

$$\tau i = Bi \int_{r>0} fr(r) \int_{T>0} \mathcal{L}\mathrm{Ir}\ (r^{\alpha}(e^{T}-1))\ \mathrm{dT}dr$$

$$= Bi \int_{r>0} e^{-(\lambda 1+\lambda 2)\pi r^2} 2\pi(\lambda 1+\lambda 2)r \int_{T>0} \exp\left(-\pi r^2 \lambda i \sqrt{(e^T-1)}(\tfrac{\pi}{2} - arctan\frac{1}{\sqrt{(e^T-1)}})\right) \quad \mathrm{dT}dr$$

$$= Bi(2\pi(\lambda 1+\lambda 2)) \int_{T>0}\int_{r>0} re^{-(\lambda 1+\lambda 2)\pi r^2} \exp\left(-\pi r^2 \lambda i \sqrt{(e^T-1)}(\tfrac{\pi}{2} -$$
$$arctan\frac{1}{\sqrt{(e^T-1)}})\right) \quad \mathrm{dr}dT$$

$$= Bi(2\pi(\lambda 1+\lambda 2)) \int_{T>0}\int_{r>0} r\exp\left(-r^2((\lambda 1+\lambda 2)\pi + \pi\lambda i\sqrt{(e^T-1)}\left(\tfrac{\pi}{2} - arctan\frac{1}{\sqrt{(e^T-1)}}\right))\right) \ \mathrm{dr}dT$$

Using the integral $\int_0^{\infty} xe^{-cx^2}\ \mathrm{dx} = -\frac{1}{2c}\ e^{-cx^2}$ and performing some simplification the average user rate τi of a typical user of MNOi who has agreed to share radio access infrastructure is:

$$\tau i = Bi \int_0^{\infty} \frac{(\lambda 1+\lambda 2)}{(\lambda 1+\lambda 2) + \lambda i\sqrt{(e^T-1)}(\frac{\pi}{2} - \arctan(\frac{1}{\sqrt{(e^T-1)}}))}\ \mathrm{dT} \tag{4.6}$$

Proof is provided in Appendix B.

i. Spectrum sharing between MNOs(SS)

In this scenario, the Bandwidth of the two MNOs is added. Only inter-operator interference, as a result of the sharing of the same frequency between the operators, is considered for the derivation of the expression.

$$\tau i = Bi \int_{r>0} fr(r) \int_{T>0} \mathcal{L}\mathrm{Ir}\ (r^{\alpha}(e^{T}-1))\ \mathrm{dT}dr$$

By Performing some simplification the average user rate τi of a typical user of MNOi who has agreed to share spectrum is:

$$\tau i = (B1 + B2) \int_0^\infty \frac{\lambda i}{\lambda i + (\lambda 1 + \lambda 2)\sqrt{(e^T - 1)}\left(\frac{\pi}{2} - \arctan\left(\frac{1}{\sqrt{(e^T - 1)}}\right)\right)} \, \mathrm{dT} \qquad (4.7)$$

Proof is provided in Appendix C.

ii. Full sharing between MNOs(FS)

The bandwidth of the operators are added and both inter and intra operator interferences are considered for the derivation of the expression for τ.

The average user rate τi of a typical user of MNOi who has agreed to share both radio access infrastructure and spectrum is:

$$\tau i = (B1 + B2) \int_0^\infty \frac{1}{1 + \sqrt{(e^T - 1)}\left(\frac{\pi}{2} - \arctan\left(\frac{1}{\sqrt{(e^T - 1)}}\right)\right)} \, \mathrm{dT} \qquad (4.8)$$

Proof is provided in Appendix D.

4.6 Simulation and Verification of the derived expressions

4.6.1 Simulation of the derived expressions

After all the expressions are derived and before going to evaluation of the network performances, simulation is performed to verify the correctness of the derived expressions. To do this six important steps are followed.

Step1: Fix a user location at the origin (0, 0) and set a target area of 1Km^2.

Step2: Generate Poisson distributed random BS locations for both MNOs with BS intensities $\lambda 1$=20 network nodes/Km^{2}and $\lambda 2$=15 network nodes/Km^2.

Step3: Calculate the distance between a user and each BSs to get the shortest distance.

Step4: Calculate the path loss, the received power and the sum of the interferences.

Step5: Finally, calculate the Probability that the SINR of the user is greater than the threshold. (i.e. Pc). The general flow diagram of the simulation is shown below in Figures 4.2 and 4.3.

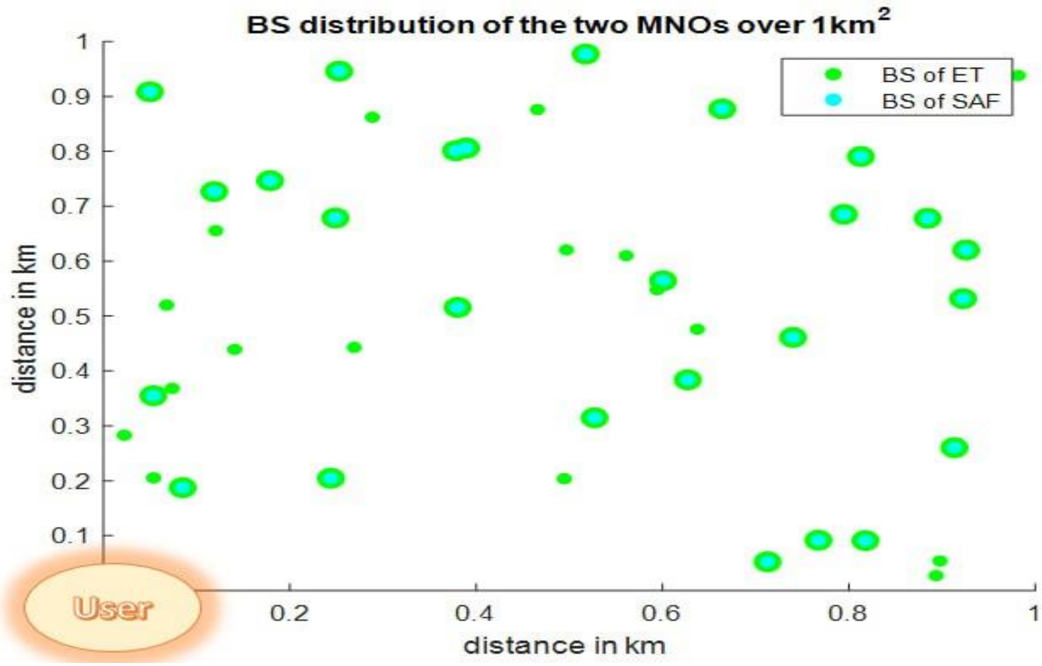

Figure 4.2: Poisson distributed generated BS location of the two MNOs.

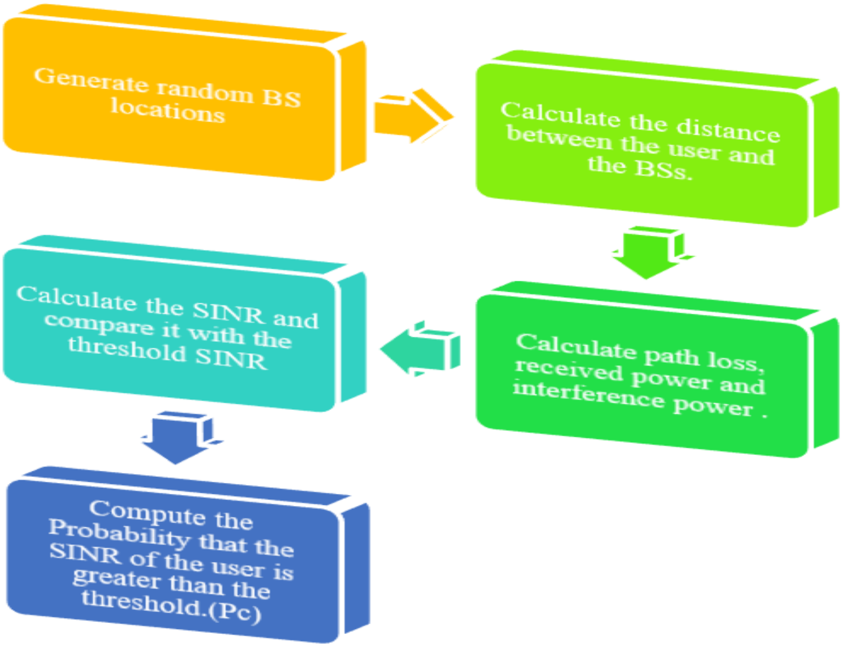

Figure 4.3: General flow diagram of the simulation.

The simulation is performed by first computing the SINR of a typical user.

$$Pc \triangleq \mathbb{P}[\mathrm{SINR} > \mathrm{T}],$$

$$SINR = \frac{hor^{-\alpha}}{w + I_r} \quad \text{Where, } I_r = \textstyle\sum_{i\epsilon\Phi/bo} h_i R^{-\alpha}.$$

For no sharing case, Target area is 1km^2

BS: BS distribution Φ1 (hPPP with BS intensity, $\lambda 1$ =20 network nodes per km^2)

Signal power: S = ho$r^{-\alpha}$, where ho is exponentially fading power and r is the minimum distance from the user to the BS of its MNO.

Total interference is the sum of the signal powers coming from all the BSs of an operator, that are found under the target area, except the signal power coming from the nearest BS to the user. $I_r = \sum_{i\epsilon\Phi 1/bo} h_i R^{-\alpha}$

For infrastructure sharing case, Target area is 1km^2.

BS: BS distribution Φ=Φ1+ Φ2 (hPPP with BS intensity, $\lambda = (\lambda 1 + \lambda 2)$ =(20+15) network nodes per km^2).

Signal power: S = ho$r^{-\alpha}$, where ho is exponentially fading power and r is the minimum distance from the user to the BS of one of the two MNOs.

Total interference is the sum of the signal powers coming from all the BSs of an operator, that are found under the target area, except the signal power coming from the nearest BS to the user. $I_r = \sum_{i\epsilon\Phi 1/bo} h_i R^{-\alpha}$

For spectrum sharing case, Target area is 1km^2.

BS: BS distribution Φ1 (hPPP with BS intensity, $\lambda 1$ =20 network nodes per km^2).

Signal power: S = ho$r^{-\alpha}$, where ho is exponentially fading power and r is the minimum distance from the user to the BS of its MNO.

Total interference is the sum of the signal powers coming from all the BSs of the two MNOs that are found under the target area, except the signal power coming from the nearest BS to the user. $I_r = \sum_{i\epsilon\Phi/bo} h_i R^{-\alpha}$.

For Full sharing case, Target area is 1km^2.

BS: BS distribution $\Phi=\Phi1+\ \Phi2$ (hPPP with BS intensity, $\lambda = (\lambda1 +\ \lambda2)$ =(20+15) network nodes per km^2).

Signal power: $S = ho r^{-\alpha}$, where ho is exponentially fading power and r is the minimum distance from the user to the BS of one of the two MNOs.

Total interference is the sum of the signal powers coming from all the BSs of the two MNOs that are found under the target area, except the signal power coming from the nearest BS to the user. $I_r = \sum_{i\epsilon\Phi/bo} h_i R^{-\alpha}$.

4.6.2 Cross-validation of the derived expressions

The simulation is performed to verify the correctness of the derived expressions. The plot in Figure 4.4 show that the simulation result strongly matched the analytical results.

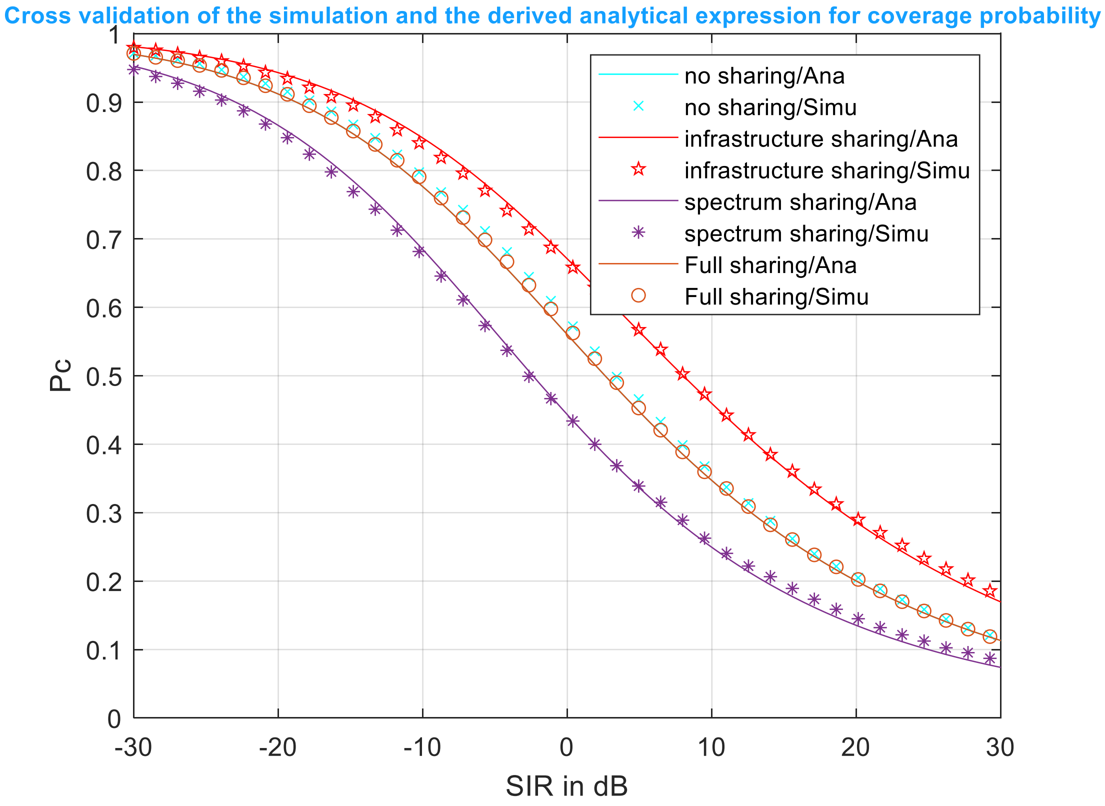

Figure 4.4: Verification of the derived expressions.

Chapter Five

Results and Discussion

Once the verification is completed. The next task is to evaluate and compare the performances of the three sharing scenarios. The performances metrics are probability of coverage, average user rate and throughput. The impacts of different parameters on the sharing scenarios are also observed in this thesis.

5.1 Performance comparison of the sharing scenarios

5.1.1 Probability of Coverage for different sharing scenarios

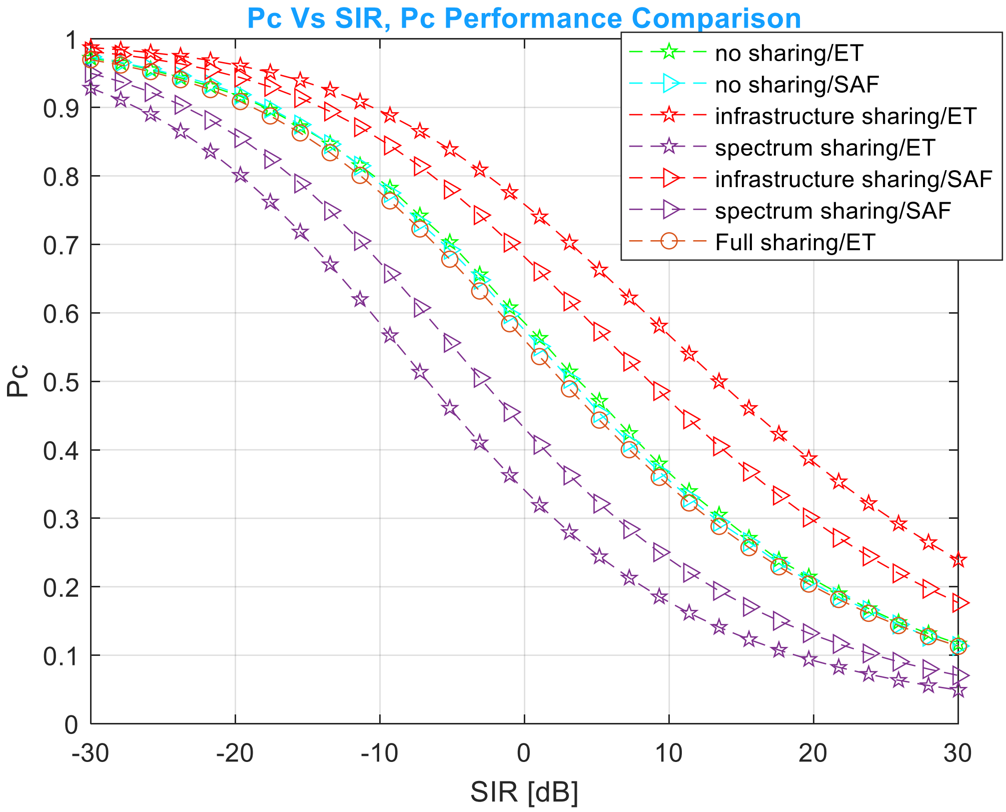

Figure 5.1 comparison of probability of coverage for different sharing scenario

The probability of coverage versus SIR threshold plot in figure 5.1 shows that the performance of the three sharing cases in terms of coverage. From the plot we can observe that infrastructure sharing gives superior coverage as compared to full sharing and spectrum sharing cases. This is because with increase in radio access network as a result of infrastructure sharing shorten the distance to the nearest transmitter and gives better signal strength to the user. When the MNOs are evenly sized, it is observed that infrastructure sharing provides highest coverage over the others. This comes from the fact that infrastructure sharing increases the number of transmitters that a user can attach to without affecting the interference in the network. From the derived analytical expression and the plot in figure 5.1, Full sharing case doesn't depend on the BS intensity λ and has the same coverage probability as no sharing case. Therefore, sharing both infrastructure and a spectrum band at the same time doesn't bring significant performance increase as compared to infrastructure sharing case.

Since both interference with in an operator (intra-operator) and between operators (inter-operator) is considered while analytical expression is derived, spectrum sharing gives the least coverage probability.

5.1.1 Mean user rate for different sharing scenarios

From the plot in figure 5.1, it is clearly shown that full sharing and spectrum sharing scenarios provide the highest mean user rate as a result of the sharing of bandwidth between the MNOs since bandwidth has a direct relationship with the user rate. The value of the mean user rate in the absence of sharing is doubled, when the MNOs share both infrastructure and spectrum. i.e. when the MNOs engaged in full sharing agreement. Therefore, if MNOs want to enter in to a sharing agreement to boost their users' rate, full sharing and spectrum sharing are the best options.

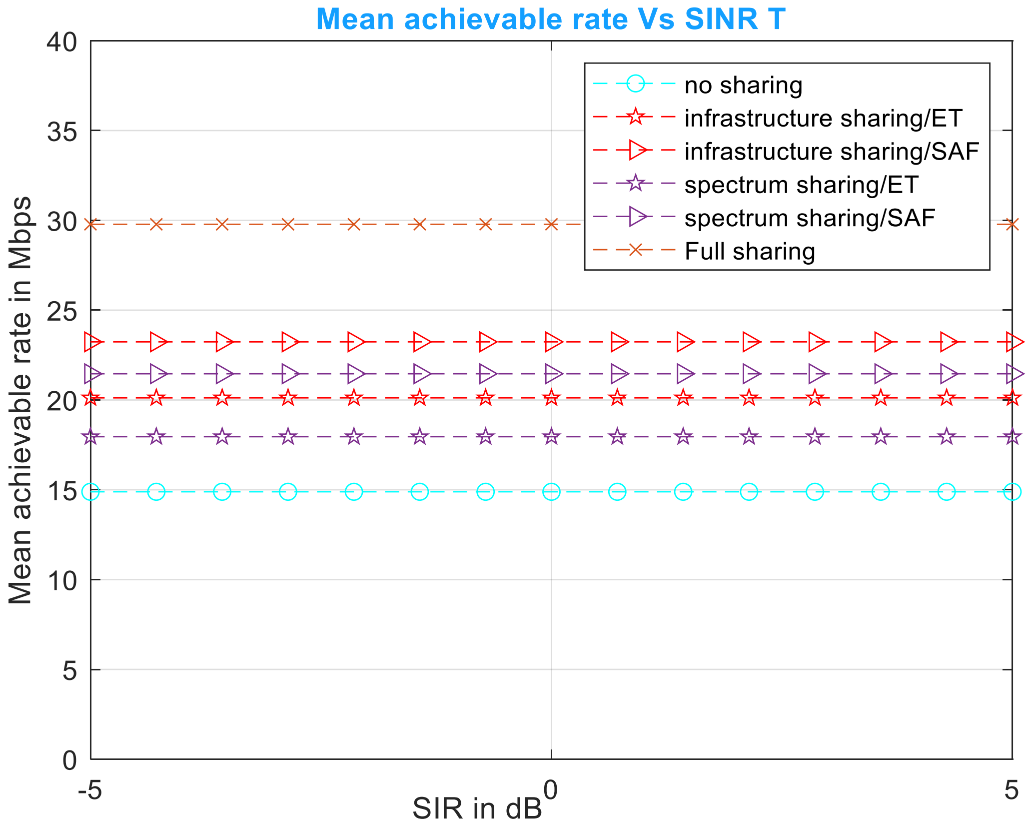

Figure 5.2: Mean user rate for different sharing scenarios

5.2 The Impact of BS density on the sharing scenarios

To see the impacts of BS density on the sharing cases, a ratio of the BS densities of the two MNOs is taken and the result is seen for various values of the ratio. i.e. $\frac{\lambda 1}{\lambda 2} = x$, for values 0.1, 0.5, 1, 2, 3.

5.2.1 The Impact of BS density on Infrastructure Sharing

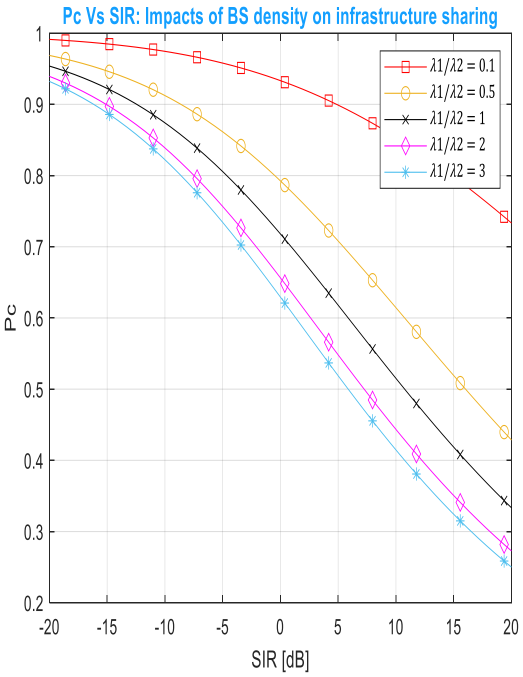

Figure 5.3: Pc Vs SIR: the impact of BS density on infrastructure sharing

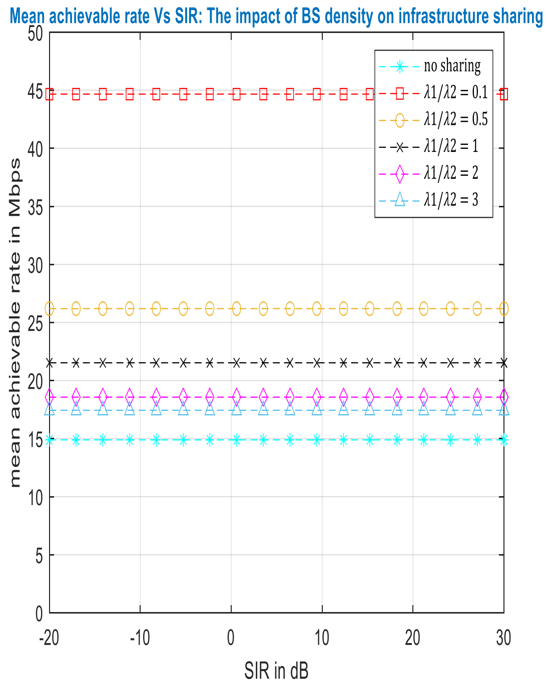

Figure 5.4: Rate Vs SIR the impact of BS density on infrastructure sharing

From the plots in figure 5.3 and 5.4 it is observed that, the user of the smaller (small BS density) size MNO benefits greatly in terms of coverage, from infrastructure sharing than the user of the larger (large BS density) sized MNO. This is because, BSs of the smaller MNO are relatively less dense and the distance to the nearer transmitter is shorten as a result of the sharing and the user can get better signal strength but the user of larger sized MNO can benefit more if the intra-operator interference is not considered or OFDMA is applied to avoid self-interference). Similarly, the user of the smaller size operator benefits greatly, in terms of mean rate, from infrastructure sharing than the larger operator.

5.2.2 The Impact of BS density on Spectrum Sharing

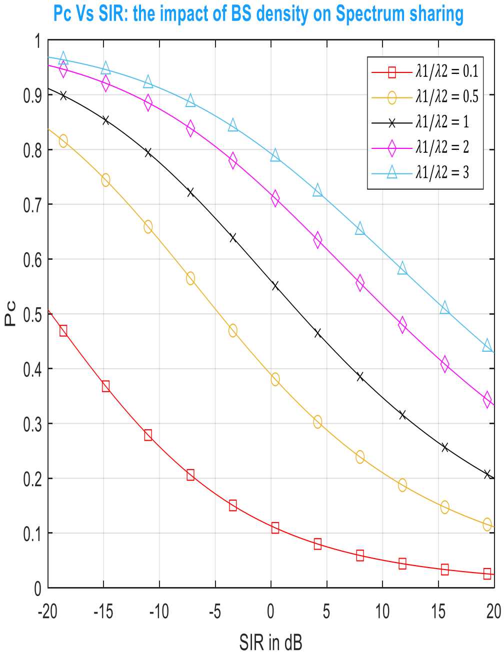

Figure 5.5: Pc Vs SIR: the impact of BS density on infrastructure sharing

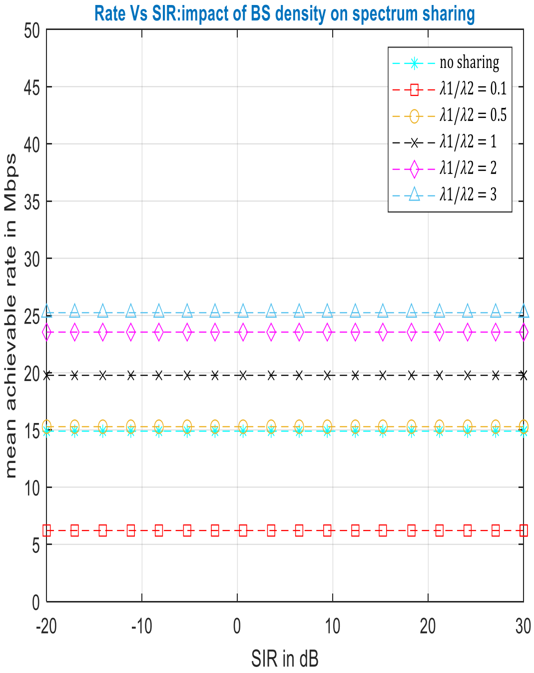

Figure 5.6: Rate Vs SIR: the impact of BS density on infrastructure sharing

From the plots in figure 5.5 and 5.6 it is observed that, the user of a larger size MNO benefits greatly in terms of coverage probability, from spectrum sharing than the user of the smaller MNO. This is because there is less interference coming from the smaller MNO. In spectrum sharing, MNOs use the same frequency bands and the user of smaller MNO suffers more from the interference coming from the larger MNO. Higher interference results lower SINR and lower coverage. For the same reason, the user of a larger operator benefits greatly in terms of mean rate from spectrum sharing.

5.3 Impact of BS density, User density and Bandwidth on the Throughput

To calculate the average throughput of a typical user located inside a target area of A, let's assume that the MNO employs proportional fairness scheduling decisions on its BSs and for proportional fairness the average throughput of the user is the rate divided by the number of users in that cell. The number of users in each cell of a MNO is approximately the user density ‘n’ divided by the BS density ‘λ ’. Therefore, if there is no sharing between the MNOs the average throughput is:

Average throughput (no sharing) $= \frac{\tau}{n/\lambda} = \frac{\lambda}{n}\tau(\text{ns})$

If the MNOs share infrastructure, the average throughput is:

Average throughput (infrastructure sharing) $= \frac{\tau}{n/\lambda} = \frac{\lambda 1+\lambda 2}{n1+n2}\tau(\text{IS})$

If the MNOs share spectrum, the average throughput is:

Average throughput (spectrum sharing) $= \frac{\tau}{n/\lambda} = \frac{\lambda 1+\lambda 2}{n1+n2}\tau(\text{SS})$

If the MNOs share both infrastructure and spectrum sharing, the average throughput is:

Average throughput (Full sharing) $= \frac{\tau}{n/\lambda} = \frac{\lambda 1+\lambda 2}{n1+n2}\tau(\text{FS})$

5.3.1 The Impact of BS density on the Throughput

By fixing the number of users of the two MNOs n1=n2=100 and their bandwidth B1=B2=10MHz and varying their BS density $\frac{\lambda 1}{\lambda 2} = x$, from 0 to 3 we can clearly see the impact of BS density under the three sharing strategies.

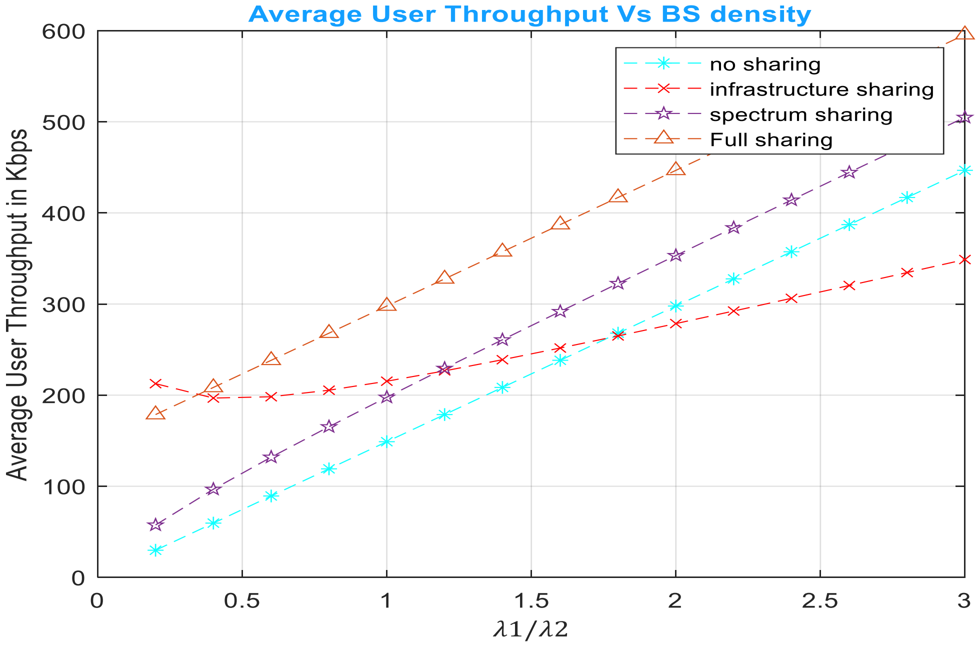

Figure 5.7: The impact of BS density on the throughput

From figure 5.7 we can observe that, the average throughput of a user increases as the size (BS density) of the MNO increases. Full sharing provides the highest average throughput because in full sharing, in addition to the sharing in bandwidth which has a direct relationship with the rate, an increase in BS increases the SINR and the increase in SINR increases the rate. The average throughput of the user increases by three times as the BS density increases from 0 to 3.

5.3.2 The Impact of user density on the Throughput

By fixing the BS density of the two MNOs $\lambda 1=20$ network nodes/km^2 and $\lambda 2=15$ network nodes/km^2 and their bandwidth B1=B2=10MHz then varying their user density, $\frac{n1}{n2} = x$, from 0 to 3, we can clearly see the impact of user density under the three sharing strategies.

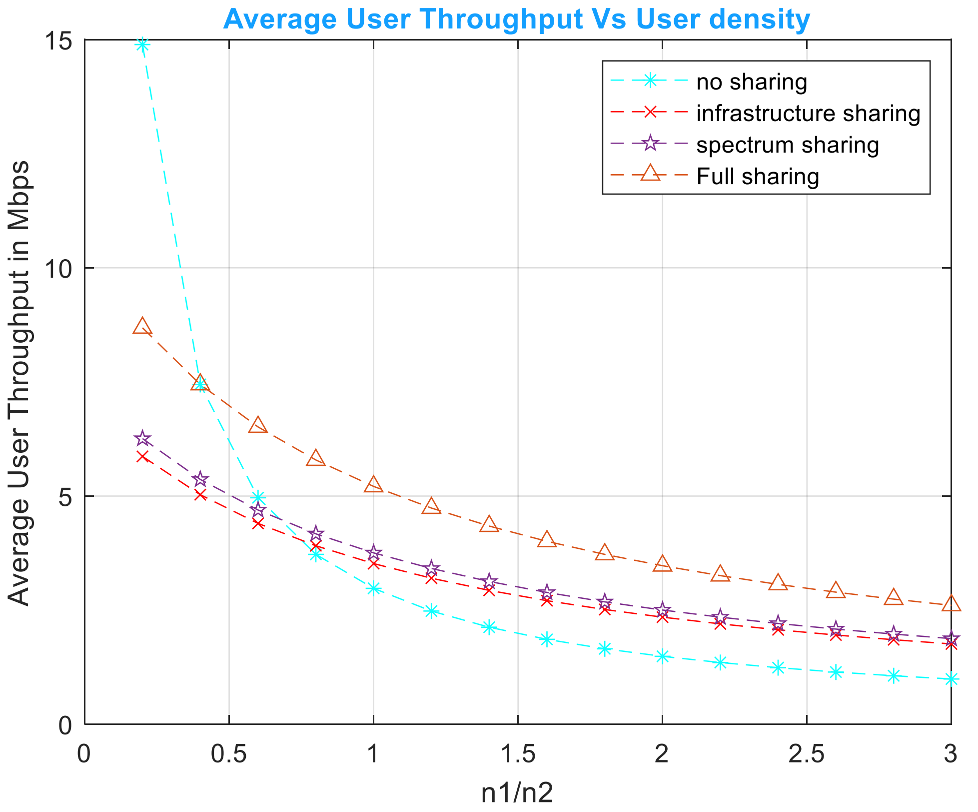

Figure 5.8: The impact of user density on the throughput

From figure 5.8 we can observe, that the average throughput drops as the number of users increases. This because more users are sharing the fixed radio resource (bandwidth). Full sharing provides highest throughput however; the average throughput drops from around 900kbps to 300kbs. Interestingly, the average user throughput for no sharing case has got the highest value i.e. 1500kbps for smaller user density. Therefore, for smaller user density no sharing is the best option.

5.3.3 The Impact of bandwidth imbalance on the Throughput

By fixing the BS density of the two MNOs $\lambda 1=20$ network nodes/km^2 and $\lambda 2=15$ network nodes/km^2 and the number of users of the two MNOs n1=n2=100 then varying their bandwidth, $\frac{B1}{B2} = x$, from 0 to 3, we can clearly see the impact of bandwidth under the three sharing strategies.

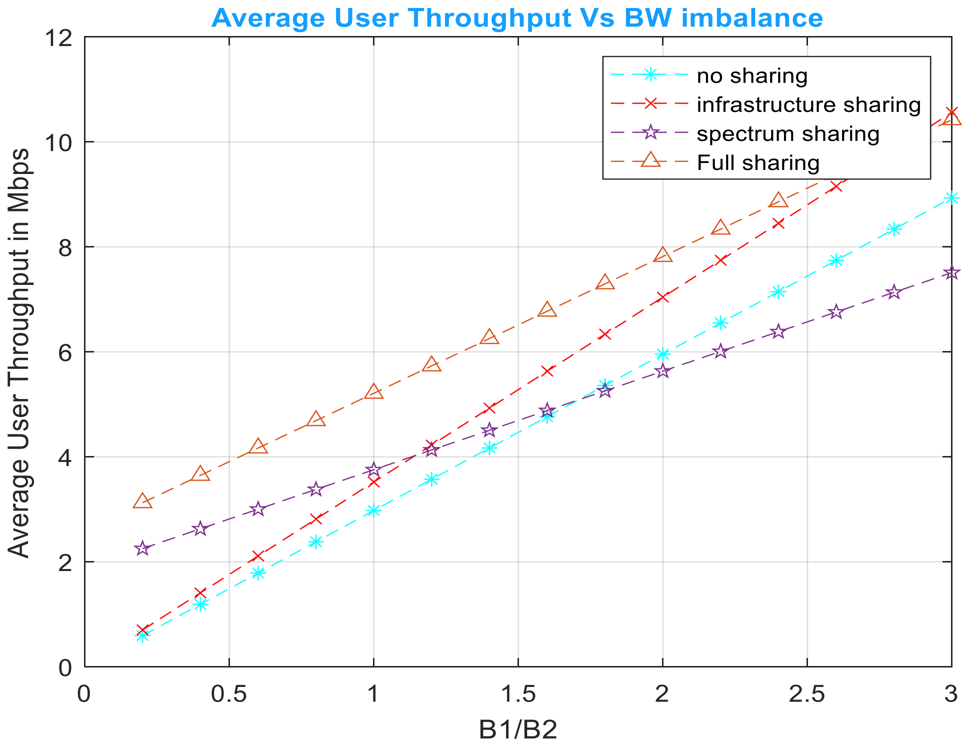

Figure 5.9: The impact of bandwidth imbalance on the throughput

From figure 5.9 we can observe that, the average throughput increases almost linearly as the total bandwidth increases. The linearity witnesses the fact that the bandwidth has direct relationship with the rate and the throughput. Again full sharing case provides highest throughput than the others. The throughput of the user has increased almost five times as the bandwidth ratio increases from 0 to 3.

Chapter Six

Conclusion and Recommendation

6.1 Conclusion

The closed form expression of coverage probability and mean achievable rate for full sharing scenario doesn't depend on the BS density. i.e. not affected by BS imbalance.

Infrastructure sharing gives superior coverage than the other sharing cases. Mean rate of the user is doubled when MNOs shares both infrastructure and spectrum (Full sharing).

Operator with smaller size network benefits more from infrastructure sharing than the larger operator in both coverage and capacity enhancement. Operator with larger size network benefits more from spectrum sharing than the larger operator in both coverage and capacity enhancement. Therefore, MNO, whose network size is much smaller, can choose and enter an infrastructure sharing agreement to boost their coverage and capacity. MNO with larger sized networks can choose and enter a spectrum sharing agreement to boost their coverage capacity. But this doesn't mean that infrastructure sharing has no use for larger size operator rather, it has significant network performance improvement.

6.2 Recommendation

In this thesis, in the system model, it is assumed that most BSs of the two MNO are collocated and are distributed according to homogeneous Poisson Point Process (hPPP). Therefore, for future work I recommend to change the perspective and observe the impact that clustering of infrastructure has on different sharing scenarios by clustering the infrastructures of the MNOs with a variable cluster radius.

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Appendix

Appendix A

For non-shared network:

Pci(T)=$\int_{r>0}^{\infty} \mathcal{L}_{\mathrm{Ir}}\,(Tr^{\alpha}) fr(r) dr$

$fr(r)$= $e^{-\lambda\pi r^{2}}2\pi\lambda\, r$

$\mathcal{L}_{\mathrm{Ir}}\,(Tr^{\alpha})$ = $\exp\left(-\pi r^{2}\lambda\sqrt{T}(\frac{\pi}{2}\; - arctan\frac{1}{\sqrt{T}})\right)$

Pci(T)=$\int_{r>0}^{\infty} e^{-\lambda\pi r^{2}}2\pi\lambda\mathrm{i}\; r(\exp\left(-\pi r^{2}\lambda i\sqrt{T}(\frac{\pi}{2}\; - arctan\frac{1}{\sqrt{T}})\right))$ dr

$= 2\pi\lambda\mathrm{i} \int_{r>0}^{\infty} e^{-\lambda i\pi r^{2}}\; (\exp\left(-\pi r^{2}\lambda i\sqrt{T}(\frac{\pi}{2}\; - arctan\frac{1}{\sqrt{T}})\right))\; r$ dr

$= 2\pi\lambda\mathrm{i} \int_{r>0}^{\infty}\; r\; (\exp\left(-r^{2}(\;\pi\lambda i\; +\; \pi\lambda i\sqrt{T}(\frac{\pi}{2}\; - arctan\frac{1}{\sqrt{T}})\right)))$ dr

Using the integral$\int_{0}^{\infty} xe^{-cx^{2}}$ dx $= -\frac{1}{2c}\; e^{-cx^{2}}$, we can get an expression for probability of coverage for non-shared network of MNOi which is totally independent on the BS intensity λ.

- $$\boldsymbol{Pci(T)}\; = \;\frac{\mathbf{1}}{\mathbf{1}+\sqrt{\boldsymbol{T}}(\frac{\boldsymbol{\pi}}{\mathbf{2}}-\boldsymbol{arctan}(\frac{\mathbf{1}}{\sqrt{\boldsymbol{T}}}))}\boldsymbol{\tau}$$

$\tau i = Bi \int_{r>0} fr(r) \int_{T>0} \mathcal{L}\mathrm{Ir}\; (r^{\alpha}(e^{T} - 1))\; \mathrm{dT}dr$

$= Bi \int_{r>0} e^{-\lambda i\pi r^{2}}2\pi\lambda\mathrm{i}\; r \int_{T>0} \exp\left(-\pi r^{2}\lambda i\sqrt{(e^{T} - 1)}(\frac{\pi}{2}\; - arctan\frac{1}{\sqrt{(e^{T}-1)}})\right)\quad \mathrm{dT}dr$

$= Bi(2\pi\lambda\mathrm{i}) \int_{T>0}\int_{r>0}\; re^{-\lambda i\pi r^{2}}\; \exp\left(-\pi r^{2}\lambda i\sqrt{(e^{T} - 1)}(\frac{\pi}{2}\; - arctan\frac{1}{\sqrt{(e^{T}-1)}})\right)\quad \mathrm{dr}dT$

$= Bi(2\pi\lambda\mathrm{i}) \int_{T>0}\int_{r>0}\; r\exp\left(-r^{2}(\lambda i\pi + \pi\lambda i\sqrt{(e^{T} - 1)}\left(\frac{\pi}{2}\; - arctan\frac{1}{\sqrt{(e^{T}-1)}}\right))\right)\quad \mathrm{dr}dT$

Using the integral$\int_{0}^{\infty} xe^{-cx^{2}}$ dx $= -\frac{1}{2c}\; e^{-cx^{2}}$ and performing some simplification the average user rate τi of a typical user of MNOi for non-shared network is

$$\tau = Bi\int_0^{\infty} \frac{1}{1+\sqrt{(e^T-1)}(\frac{\pi}{2} - \arctan(\frac{1}{\sqrt{(e^T-1)}}))}\,\mathrm{dT}$$

Appendix B

For infrastructure sharing:

- Pc:

Pci(T)=$\int_{r>0}^{\infty} \mathcal{L}_{\mathrm{Ir}}(Tr^{\alpha}) fr(r)dr$

$fr(r)$= $e^{-(\lambda 1+\lambda 2)\pi r^2}$ 2π($\lambda 1 + \lambda 2$) r

$\mathcal{L}_{\mathrm{Ir}}(Tr^{\alpha})$ = exp $\left(-\pi r^2 \lambda i\sqrt{T}(\frac{\pi}{2} - arctan\frac{1}{\sqrt{T}})\right)$

Pci(T)=$\int_{r>0}^{\infty} e^{-(\lambda 1+\lambda 2)\pi r^2}$2π($\lambda 1 + \lambda 2$) r(exp$\left(-\pi r^2\lambda i\sqrt{T}(\frac{\pi}{2} - arctan\frac{1}{\sqrt{T}})\right)$) dr

= 2π($\lambda 1 + \lambda 2$) $\int_{r>0}^{\infty} e^{-(\lambda 1+\lambda 2)\pi r^2}$ (exp$\left(-\pi r^2\lambda i\sqrt{T}(\frac{\pi}{2} - arctan\frac{1}{\sqrt{T}})\right)$) r dr

= 2πλi $\int_{r>0}^{\infty}$ r (exp$\left(-r^2(\pi(\lambda 1 + \lambda 2) + \pi\lambda i\sqrt{T}(\frac{\pi}{2} - arctan\frac{1}{\sqrt{T}})\right)$)) dr

Using the integral$\int_0^{\infty} xe^{-cx^2}$ dx = $-\frac{1}{2c}$ e^{-cx^2}

Therefore, the probability of coverage of a typical user of MNOi is

Pci(T) = $\frac{(\lambda 1+\lambda 2)}{(\lambda 1+\lambda 2)+\lambda \mathrm{i}\sqrt{T}(\frac{\pi}{2} - \arctan(\frac{1}{\sqrt{T}}))}$

- τ

$\tau i = Bi\int_{r>0} fr(r)\int_{T>0} \mathcal{L}\mathrm{Ir}\ (r^{\alpha}(e^T-1))\ \mathrm{dT}dr$

$= Bi\int_{r>0} e^{-(\lambda 1+\lambda 2)\pi r^2}$2π($\lambda 1 + \lambda 2$)$r\int_{T>0}exp\left(-\pi r^2\lambda i\sqrt{(e^T-1)}(\frac{\pi}{2} - arctan\frac{1}{\sqrt{(e^T-1)}})\right)$ dTdr

$= Bi(2\pi(\lambda 1 + \lambda 2)) \int_{T>0} \int_{r>0} r e^{-(\lambda 1 + \lambda 2)\pi r^2} \exp\left(-\pi r^2 \lambda i \sqrt{(e^T - 1)}(\frac{\pi}{2} - arctan \frac{1}{\sqrt{(e^T - 1)}})\right)$ d$r dT$ $= Bi(2\pi(\lambda 1 + \lambda 2)) \int_{T>0} \int_{r>0} r \exp\left(-r^2((\lambda 1 + \lambda 2)\pi + \pi \lambda i \sqrt{(e^T - 1)}\left(\frac{\pi}{2} - arctan \frac{1}{\sqrt{(e^T - 1)}}\right))\right)$ d$r dT$ Using the integral $\int_0^\infty x e^{-cx^2}$ dx $= -\frac{1}{2c} e^{-cx^2}$ and performing some simplification the average user rate τi of a typical user of MNOi who has agreed to share radio access infrastructure is:

$$\tau i = Bi \int_0^\infty \frac{(\lambda 1 + \lambda 2)}{(\lambda 1 + \lambda 2) + \lambda i \sqrt{(e^T - 1)}(\frac{\pi}{2} - \arctan(\frac{1}{\sqrt{(e^T - 1)}}))} \mathrm{dT}$$

Appendix C

For spectrum sharing:

- Pc:

Pci(T)=$\int_{r>0}^{\infty} \mathcal{L}_{\mathrm{Ir}} (Tr^\alpha) fr(r) dr$

$fr(r) = e^{-\lambda i \pi r^2} 2\pi\lambda\mathrm{i}\, r$ $\mathcal{L}_{\mathrm{Ir}} (Tr^\alpha) = \exp\left(-\pi r^2 (\lambda 1 + \lambda 2) \sqrt{T}(\frac{\pi}{2} - arctan \frac{1}{\sqrt{T}})\right)$

Pci(T)=$\int_{r>0}^{\infty} e^{-\lambda i \pi r^2} 2\pi\lambda\mathrm{i}\, r(\exp\left(-\pi r^2 (\lambda 1 + \lambda 2)\sqrt{T}(\frac{\pi}{2} - arctan \frac{1}{\sqrt{T}})\right))$ dr

$= 2\pi\lambda\mathrm{i} \int_{r>0}^{\infty} e^{-\lambda\mathrm{i}\pi r^2} (\exp\left(-\pi r^2 (\lambda 1 + \lambda 2) \sqrt{T}(\frac{\pi}{2} - arctan \frac{1}{\sqrt{T}})\right)) r$ dr

$= 2\pi\lambda\mathrm{i} \int_{r>0}^{\infty} r (\exp\left(-r^2 (\pi\lambda\mathrm{i} + \pi(\lambda 1 + \lambda 2) \sqrt{T}(\frac{\pi}{2} - arctan \frac{1}{\sqrt{T}}))\right))$ dr

Using the integral $\int_0^\infty x e^{-cx^2}$ dx $= -\frac{1}{2c} e^{-cx^2}$

Therefore, the probability of coverage of a typical user of MNOi is

$\boldsymbol{Pci(T)} = \frac{\boldsymbol{\lambda\mathrm{i}}}{\boldsymbol{(\lambda 1 + \lambda 2)\sqrt{T}(\frac{\pi}{2} - \mathrm{arctan}(\frac{1}{\sqrt{T}})) + \lambda\mathrm{i}}}$

- τ

$$\tau i = Bi\int_{r>0} fr(r)\int_{T>0} \mathcal{L}\mathrm{Ir}\,(r^{\alpha}(e^{T}-1))\,\mathrm{dT}dr$$

$$= (B1+B2)\int_{r>0} e^{-\lambda i\pi r^2}2\pi\lambda ir\int_{T>0}\exp\left(-\pi r^2(\lambda1+\lambda2)\sqrt{(e^T-1)}(\frac{\pi}{2} - arctan\frac{1}{\sqrt{(e^T-1)}})\right)\ \mathrm{dT}dr$$

$$= (B1+B2)(2\pi\lambda \mathrm{i})\int_{T>0}\int_{r>0} re^{-\lambda i\pi r^2}\exp\left(-\pi r^2(\lambda1+\lambda2)\sqrt{(e^T-1)}(\frac{\pi}{2} - arctan\frac{1}{\sqrt{(e^T-1)}})\right)\ \mathrm{dr}dT$$

$$= (B1+B2)(2\pi\lambda i)\int_{T>0}\int_{r>0} r\exp\left(-r^2(\lambda i\pi + \pi(\lambda1+\lambda2)\sqrt{(e^T-1)}\left(\frac{\pi}{2} - arctan\frac{1}{\sqrt{(e^T-1)}}\right))\right)\ \mathrm{dr}dT$$

Using the integral $\int_0^\infty xe^{-cx^2}\,\mathrm{dx} = -\frac{1}{2c}\,e^{-cx^2}$ and performing some simplification the average user rate τi of a typical user of MNOi who has agreed to share spectrum is:

$$\tau i = (B1+B2)\int_0^\infty \frac{\lambda i}{\lambda i + (\lambda1+\lambda2)\sqrt{(e^T-1)}(\frac{\pi}{2} - \arctan(\frac{1}{\sqrt{(e^T-1)}}))}\,\mathrm{dT}$$

Appendix D

For full network sharing:

- Pc:

$$\mathrm{Pci(T)}=\int_{r>0}^{\infty}\mathcal{L}_{\mathrm{Ir}}\,(Tr^{\alpha})fr(r)dr$$

$$fr(r)= e^{-(\lambda1+\lambda2)\,\pi r^2}2\pi(\lambda1+\lambda2)\ r$$

$$\mathcal{L}_{\mathrm{Ir}}\,(Tr^{\alpha}) = \exp\left(-\pi r^2(\lambda1+\lambda2)\sqrt{T}(\frac{\pi}{2} - arctan\frac{1}{\sqrt{T}})\right)$$

$$\mathrm{Pci(T)}=\int_{r>0}^{\infty}e^{-(\lambda1+\lambda2)\,\pi r^2}2\pi(\lambda1+\lambda2)\,r(\exp\left(-\pi r^2(\lambda1+\lambda2)\sqrt{T}(\frac{\pi}{2} - arctan\frac{1}{\sqrt{T}})\right))\ \mathrm{dr}$$

$$= 2\pi(\lambda1+\lambda2)\int_{r>0}^{\infty}e^{-(\lambda1+\lambda2)\,\pi r^2}(\exp\left(-\pi r^2(\lambda1+\lambda2)\sqrt{T}(\frac{\pi}{2} - arctan\frac{1}{\sqrt{T}})\right))\ r\ \mathrm{dr}$$

$$= 2\pi(\lambda1+\lambda2)\int_{r>0}^{\infty} r\ (\exp\left(-r^2(\ \pi(\lambda1+\lambda2) + \pi(\lambda1+\lambda2)\sqrt{T}(\frac{\pi}{2} - arctan\frac{1}{\sqrt{T}})\right)))\ \mathrm{dr}$$

Using the integral $\int_0^\infty xe^{-cx^2}\,\mathrm{dx} = -\frac{1}{2c}\,e^{-cx^2}$

Therefore, the probability of coverage of a typical user of MNOi is

$$\mathrm{Pci(T)} = \frac{1}{\sqrt{T}(\frac{\pi}{2} - \arctan(\frac{1}{\sqrt{T}})) + 1}$$

- τ

The bandwidth of the operators are added and both inter and intra operator interferences are considered for the derivation of the expression for τ.

$$\tau i = Bi \int_{r>0} fr(r) \int_{T>0} \mathcal{L}Ir\,(r^{\alpha}(e^T - 1))\; dTdr$$

$$= (B1 + B2) \int_{r>0} e^{-(\lambda 1+\lambda 2)\pi r^2} 2\pi(\lambda 1 + \lambda 2) r \int_{T>0} exp\Big(-\pi r^2(\lambda 1 + \lambda 2)\sqrt{(e^T - 1)}(\frac{\pi}{2} - arctan\frac{1}{\sqrt{(e^T - 1)}})\Big)\; dTdr$$

$$= (B1 + B2)(2\pi(\lambda 1 + \lambda 2)) \int_{T>0} \int_{r>0} re^{-(\lambda 1+\lambda 2)\pi r^2} \exp\Big(-\pi r^2(\lambda 1 + \lambda 2)\sqrt{(e^T - 1)}(\frac{\pi}{2} - arctan\frac{1}{\sqrt{(e^T - 1)}})\Big)\; \mathrm{dr}dT$$

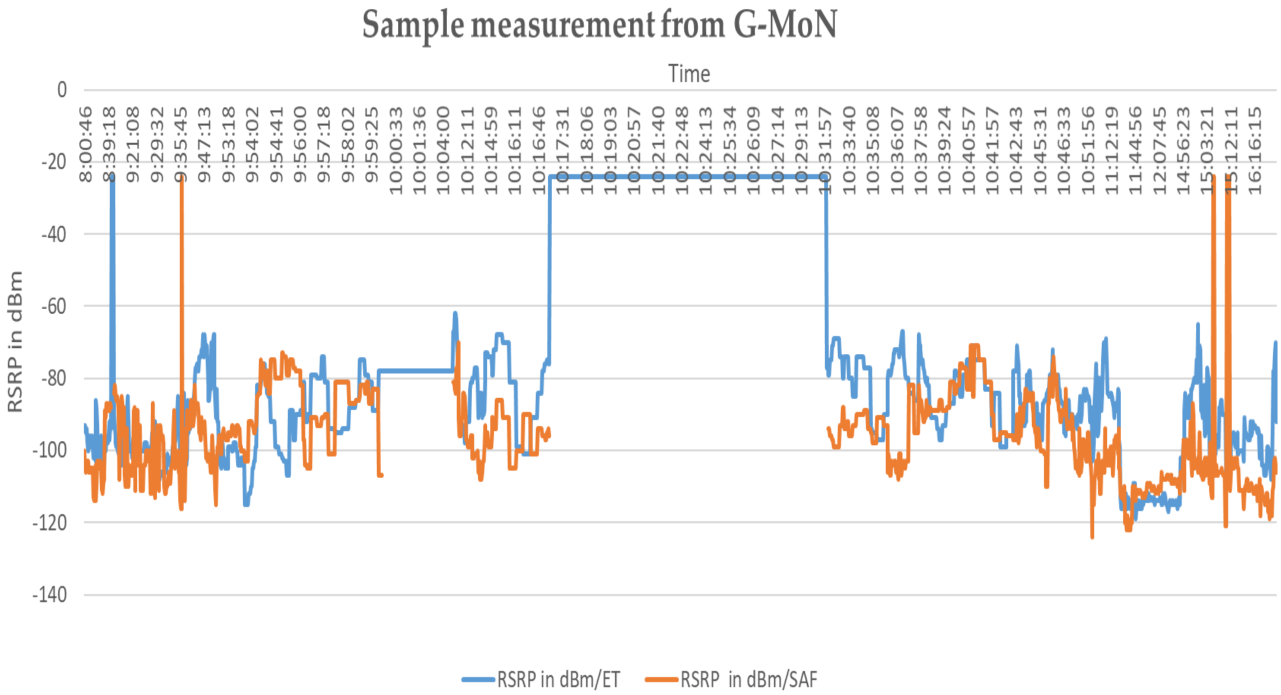
$$= (B1 + B2)(2\pi(\lambda 1 + \lambda 2)) \int_{T>0} \int_{r>0} r \exp\Big(-r^2((\lambda 1 + \lambda 2)\pi + \pi(\lambda 1 + \lambda 2)\sqrt{(e^T - 1)}\Big(\frac{\pi}{2} - arctan\frac{1}{\sqrt{(e^T - 1)}}\Big))\Big)\; \mathrm{dr}dT$$

Using the integral $\int_0^\infty xe^{-cx^2}\,\mathrm{dx} = -\frac{1}{2c}\,e^{-cx^2}$ and performing some simplification the average user rate τi of a typical user of MNOi who has agreed to share both radio access infrastructure and spectrum is:

$$\tau i = (B1 + B2) \int_0^\infty \frac{1}{1 + \sqrt{(e^T - 1)}(\frac{\pi}{2} - \arctan(\frac{1}{\sqrt{(e^T - 1)}}))}\; \mathrm{dT}$$

Appendix E

Sample measurements from drive and walk test that are taken using G-MoN pro application.



SIM2
LTE - Ethio telecom
TAC: 11206, ECI: 28734980 | PCI: 421
nPCI: 420

SIM2
LTE - Safaricom
TAC: 14000, ECI: 37646336 | PCI: 183
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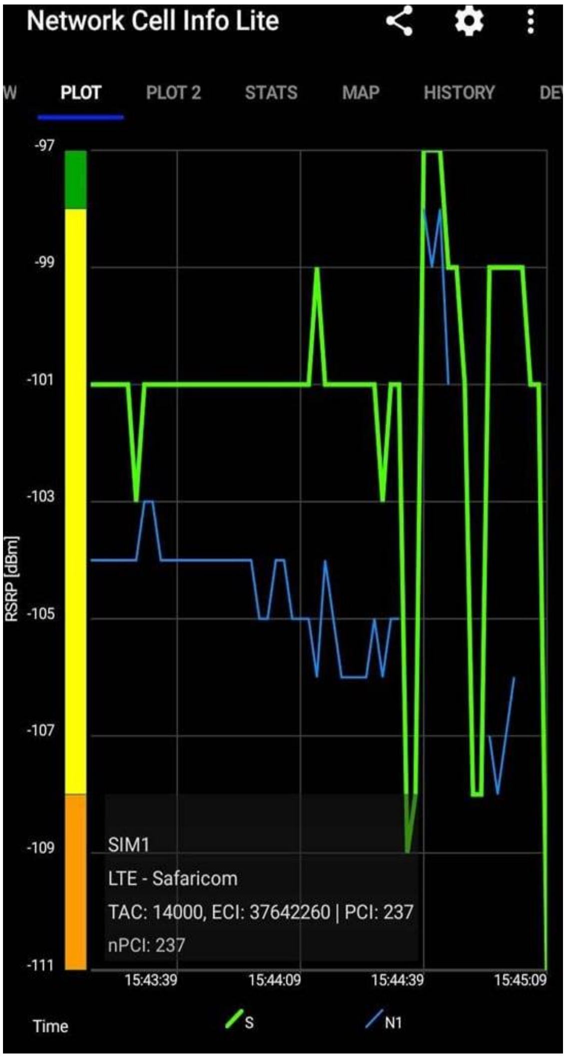

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Infrastructure and Spectrum Sharing for Coverage and Capacity Enhancement in Multi-Operator Networks

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Abstract— the rapid growth of mobile data traffic is putting a strain on wireless networks. Infrastructure and spectrum sharing are two promising ways to alleviate this challenge on the wireless networks. The research presented in this paper presents an analytical model for infrastructure, spectrum and full sharing scenarios and investigates the performance of the three sharing scenarios and the performances of the two MNOs as a result of sharing in terms of probability of coverage and mean user rate. The results show that infrastructure sharing provides highest coverage as compared to spectrum and full sharing cases; whereas, full sharing and spectrum sharing have given the highest mean user rate. The MNO with smaller network size has got better coverage and rate from infrastructure sharing, while the MNO with larger network size has got better coverage and rate from spectrum sharing. Beside this, increasing the base station density increases the throughput but increasing the user density drops the throughput. The throughput also increases as the sharing bandwidth increases.

Keywords— infrastructure sharing, spectrum sharing, coverage probability, mean user rate, coverage enhancement, capacity enhancement

I. INTRODUCTION

Telecom operators in emerging and mature markets are increasingly considering network sharing. Emerging market operators are looking for economical ways to expand their coverage and capacity, while mature market operators are looking to optimize costs and refresh their technology [6]. As an emerging market, in Ethiopia, the primary target is on the coverage and capacity growth. Infrastructure and spectrum sharing can be used to improve capacity of the MNOs.

Infrastructure sharing refers to the co-deployment and operation of network infrastructures like base station and other radio access equipment by different mobile network operators (MNOs). Infrastructure sharing may expand coverage into previously un-served geographic areas and can be used in congested urban centers where new site acquisition is difficult. [6] Spectrum sharing refers to the use of the same spectrum by multiple MNOs. Full sharing involves the sharing of both infrastructure and spectrum among MNOs. Spectrum sharing can significantly improve the average data rate of a user but can slightly reduce the network coverage as a result of the increase in interference due to the use of the same frequency band between the mobile network operators (MNOs) [16].

Recently in Ethiopia, the two MNOs have started sharing same site, tower and power to minimize their OPEXs.

Figure1: BS distribution of a small portion of Addis Ababa

The base stations are located randomly and the location to the next base station is unpredictable. The above Figure1shows a sample base station distribution of an area in the city of Addis Ababa.

A number of point processes are included in stochastic geometry for the positioning of wireless network nodes. Due to its analytical tractability and practical appeal in situations where transmitters and/or receivers are located or move around randomly over a large area, the (homogeneous) Poisson point process (PPP) has been by far the most popular spatial model. [4][5][7]

Different researchers have considered and modeled different sharing strategies/models using distinct mathematical and machine learning algorithm regarding the CAPEXs and OPEXs [1]-[5]. While others [17] modeled infrastructure sharing from a stochastic geometry perspective and have performed transmitted power and market trade-off analysis. In [18] the authors develop new general models for the multi-cell signal-to-interference-plus-noise ratio (SINR) using stochastic geometry and also derive the mean rate, and then the coverage gains (and mean rate loss) from static frequency reuse.

In this research, it is chosen to make use of stochastic geometry from the general cellular model developed by [18] and from that derive, compute and analyzed the downlink SINR coverage probability and the average data rate of a typical user for three different sharing cases that are: infrastructure sharing, spectrum sharing and full sharing (i.e. a scenario where MNOs share both infrastructure and spectrum). Then evaluate and compare the network performances of the sharing scenarios as well as the network performances of the MNOs as a result of sharing then the impacts of some parameters like: BS density, user density and bandwidth imbalance on the throughput are analyzed.

A. Problem Statement

Mobile data demand and customers' QoS service requirements are increasing. Beside this, MNOs are not meeting and exceeding the QoS requirements that are requested by the Communication Authority of the Country [20], as observed from the drive and walk tests performed in different places in Addis Ababa, Ethiopia, at different times (sample measurements from drive and walk test are provided in appendix section).

One approach to this problem is new base station (BS) deployment or using other optimization techniques, but this greatly increases MNOs' CAPEX and OPEX. This is not appropriate while the existing infrastructure and spectrum are not being used effectively and efficiently. Another approach for the MNOs is to share their infrastructure and spectrum among themselves. The mobile service coverage signal strength and the average data rate of users can be improved by allowing cooperation between MNOs or merging some of their infrastructure, resources, and spectrum to serve their customers. MNOs can enhance their coverage and capacity by engaging in different sharing strategies.

B. Contribution of the research

In this thesis, we mainly tries to show the improvement in network performance of two MNOs as a result of infrastructure and spectrum sharing between them. We have also showed that the MNOs can improve their capacity and coverage without deploying new BSs. Beside this, the performed performance comparison between the sharing scenarios indicates that which sharing scenario is better for coverage and rate enhancement. It is also seen that some parameters such as: BS density, user density and bandwidth has significant impact on the performance of the sharing.

II. System Model and Assumptions

Most assumptions in this research are taken from [18] since they have proposed a general stochastic model for a cellular network. In the real-world scenario, most BS distributions are random, and the movement of the mobile user is obviously unpredictable. Hence, it is convenient to model the system based on the concepts of stochastic geometry particularly homogeneous Poisson Point Process (hPPP).

BS deployment is modeled as a point process by defining the network nodes or BSs as points that are randomly placed similar to the realistic BS distribution in the country.

It is assumed that there are two MNOs (MNO1 – refers to ET (ethiotelecom) and MNO2 – SAF (safaricom)). Let 'S' be the set of mobile operators, S = {MNO1, MNO2}. The locations of BSs of ET and SAF follow independent PPP distributions $\Phi 1$ and $\Phi 2$ with BS densities $\lambda 1$ and λ 2, respectively.

$\Phi = \{\Phi 1, \Phi 2\} \subset R2$ Where, $\Phi = \sum_{i\in S} \Phi i$;

$\lambda = \lambda 1 + \lambda 2$; Where, λ is the sum of the BS densities when the two MNOs merge.

The bandwidth of operator i is denoted as Bi, and when spectrum is shared between the two operators, the aggregate bandwidth is then $B = \sum_{i\in S} Bi$ (the individual spectrum bands being aggregated are non-overlapping).

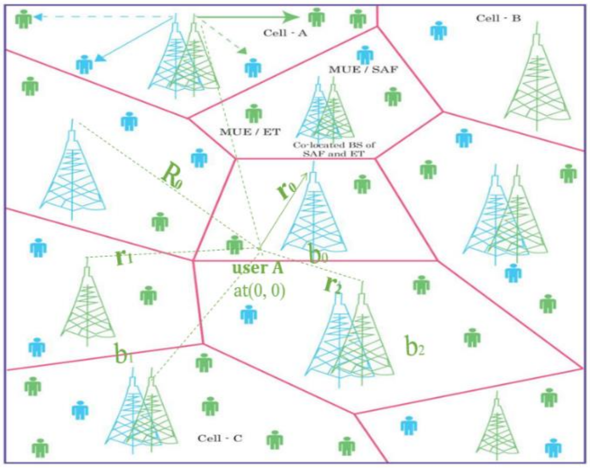


Figure 2: System model of Poisson distributed base station distribution in Addis Ababa.

We have considered that the base stations of the two MNOs are deployed independently where in some places the base stations are co-located as shown in Figure 2. Mobile users are located according to some independent stationary point process are considered and each mobile user to be connected to the nearest base station. But for the sake of simplicity, a typical user located at the origin (0,0) in Euclidean plane is considered for the derivation of the expressions. Therefore, the derived expressions are for a typical user and not for the aggregated users.

Similar to [18] the system is considered as interference limited network (an assumption where the noise power is 0; i.e. w=0). The standard power loss propagation model is the path loss model $r_o^{-\alpha}$ with path loss exponent α = 4. It is assumed that the connected base station and user experience only Rayleigh fading with mean 1. Then power fading of the message signal h_o; at a typical node a distance r from its base station is exponentially distributed with unit mean (ho ~ exp(1)). The interference power at the typical receiver Ir is the sum of the received powers from all other base stations other than the home base station at distances Ri from the typical user and similarly for the sake of simplicity; the interfering signal h_i, follows an exponential distribution with unit mean (hi ~ exp(1)). The SINR threshold T is the target SINR that a user should achieve to say it's in a network coverage.

A. Performance Metrics

The two important metrics, probability of coverage and average user rate, from a general stochastic geometry based cellular model are derived in [18]. From these two general expressions, we derived an analytical expression for the three sharing cases based on the assumptions and the system model

i. Probability of Coverage

The downlink SINR probability of coverage of a typical user is derived in [18].

Starting from an expression $Pc(T) \triangleq P[SINR > T]$

Where $SINR = \frac{h_o r^{-\alpha}}{I_r + W}$;

They have derived an expression for Pc:

$$\mathrm{Pc(T)} = \int_{r>0}^{\infty} \mathcal{L}_{\mathrm{Ir}}(Tr^{\alpha}) fr(r) dr \qquad (1)$$

$$fr(r) = e^{-\lambda\pi r^2} 2\pi\lambda\, r \qquad (2)$$

$$\mathcal{L}_{\mathrm{Ir}}(Tr^{\alpha}) = \exp\left(-\pi r^2 \lambda \sqrt{T}\left(\frac{\pi}{2} - arctan \frac{1}{\sqrt{T}}\right)\right) \qquad (3)$$

ii. Mean User Rate

The mean user rate of a typical user can be derived as in [18]:

Starting from an expression $\tau \triangleq \mathrm{Bi}\mathbb{E}[\log(1 + SINR)]$

They have derived an expression for τ:

$$fr(r) = e^{-\lambda i\pi r^2} 2\pi\lambda \mathrm{i}\, r$$

$$\mathcal{L}_{\mathrm{Ir}}(r^{\alpha}(e^{T} - 1)) = \exp\left(-\pi r^2 \lambda \sqrt{(e^{T} - 1)}\left(\frac{\pi}{2} - arctan \frac{1}{\sqrt{(e^{T}-1)}}\right)\right) \qquad (4)$$

$$\tau = Bi \int_{r>0} fr(r) \int_{T>0} \mathcal{L}\text{Ir}\ (r^{\alpha}(e^{T}-1))\ \text{d}T dr$$
(5)

B. Sharing Cases

i. No Sharing case

If there is no sharing agreement between MNOs, there is no merging of infrastructures and resources between the MNOs and the MNOs serve their customers independently. The Pc and the mean user rate of a typical user of MNOi whose nearest transmitter is at a distance r and BS intensity λi is then:

$$Pci(T) = \frac{1}{1+\sqrt{T}(\frac{\pi}{2} - arctan(\frac{1}{\sqrt{T}}))} \quad (6)$$

and the mean user rate is

$$\tau i = Bi \int_0^{\infty} \frac{1}{1+\sqrt{(e^{T}-1)}(\frac{\pi}{2} - \text{arctan}(\frac{1}{\sqrt{(e^{T}-1)}}))}\ \text{dT} \quad (7)$$

ii. Infrastructure sharing case

In infrastructure sharing case, since MNOs share their radio access infrastructure, the BS distribution and BS intensity is changed and it is the sum of the two independently distributed networks. $\Phi = \Phi1 + \Phi2$; $\lambda = \lambda1 + \lambda2$. The user can connect to any near-by transmitter that belongs to its home MNO or BS of the MNO that has entered a sharing agreement with the home MNO. For the derivation of an expression for Pc only intra-operator (self-interference) is considered and there is no inter-operator (between MNOs) interference because operators didn't share their frequencies in this scenario.

Therefore, the probability of coverage of a typical user of MNOi is

$$\text{Pci(T)} = \frac{(\lambda1+\lambda2)}{(\lambda1+\lambda2)+\lambda\text{i}\sqrt{T}(\frac{\pi}{2} - \text{arctan}(\frac{1}{\sqrt{T}}))} \quad (8)$$

and the mean user rate is

$$\tau i = Bi(2\pi(\lambda1 + \lambda2)) \int_{T>0}\int_{r>0} r \exp\left(-r^{2}((\lambda1 + \lambda2)\pi + \pi\lambda i\sqrt{(e^{T}-1)}\left(\frac{\pi}{2} - arctan \frac{1}{\sqrt{(e^{T}-1)}}\right))\right) \text{d}r dT \quad (9)$$

iii. Spectrum sharing between MNOs

In this sharing case, the MNOs pool their spectrum and the bandwidth of the MNOs become the sum of the spectrum bands of the individual MNOs; i.e. B = B1 + B2. Unlike infrastructure sharing case, the user can only connect to its nearest home BS but both intra-operator and inter-operator interference are considered for the derivation of and expression for Pc. Therefore, the probability of coverage of a typical user of MNOi is

$$\text{Pci(T)} = \frac{\lambda\text{i}}{(\lambda1+\lambda2)\sqrt{T}(\frac{\pi}{2} - \text{arctan}(\frac{1}{\sqrt{T}}))+\lambda\text{i}} \quad (10)$$

and the mean user rate is

$$\tau i = (B1 + B2) \int_0^{\infty} \frac{\lambda i}{\lambda i + (\lambda1+\lambda2)\sqrt{(e^{T}-1)}(\frac{\pi}{2} - \text{arctan}(\frac{1}{\sqrt{(e^{T}-1)}}))}\ \text{dT} \quad (11)$$

iv. Full sharing between MNOs

In this sharing case, MNOs share both the radio access network infrastructure and spectrum. The BS distribution and BS intensity is changed and it is the sum of the two independently distributed networks. $\Phi = \Phi1 + \Phi2$; $\lambda = \lambda1 + \lambda2$. Beside this, the bandwidth of the MNOs is now the sum of the individual MNOs bandwidth B= B1 + B2. User can connect to the nearest transmitter and gets better signal strength. Both intra-operator and inter-operator interferences are considered in this case because it is considered that all the BS now operate in the same frequency band and users may connect to any of the BSs of the sharing MNOs. The derived expression for Pc for full sharing case is:

$$Pci(T) = \frac{1}{1+\sqrt{T}(\frac{\pi}{2} - arctan(\frac{1}{\sqrt{T}}))}$$

and the mean user rate is

$$\tau i = (B1 + B2) \int_0^{\infty} \frac{1}{1+\sqrt{(e^{T}-1)}(\frac{\pi}{2} - \text{arctan}(\frac{1}{\sqrt{(e^{T}-1)}}))}\ \text{dT} \quad (12)$$

C. Cross-validation of the derived expressions

After we derived the expressions for the sharing cases, we have performed a simulation to verify the correctness of the derived expressions. We have followed general steps. First we have set a target area and then generated a Poisson distribution based random points as BS locations. Then fixed a location of a user and then found Euclidean distances from the user to the BSs. Next to these we have found the path loss, the received power and the total interferences. Finally, we calculated the SINR of the user and the probability of coverage for each iteration. The below Figure 3: shows the cross-validation of the derived analytical expressions.

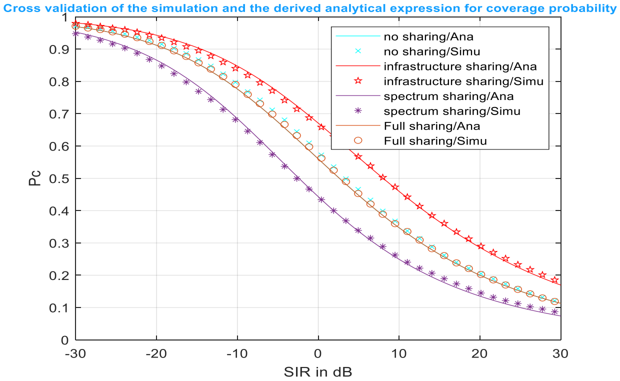


Figure 3: The verification of the derived expressions

III. NUMERICAL RESULTS AND DISCUSSION

After the verification we have evaluated and compared the performances of the three sharing scenarios and the performances of the two MNOs. The performances metrics are probability of coverage, average user rate and throughput.

A. Coverage

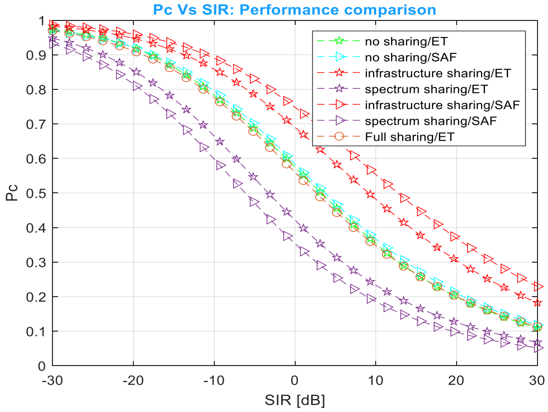


Figure 4: Performance Comparison three sharing cases in terms of coverage.

From Figure 4, we can observe that infrastructure sharing gives superior coverage as compared to full sharing and spectrum sharing cases. This is because with increase in radio access network as a result of infrastructure sharing shorten the distance to the nearest transmitter and gives better signal strength to the user.

From the derived analytical expression and the plot in Figure 4, we can observe that, Full sharing case doesn't depend on the BS intensity λ and has the same coverage probability as no sharing case. Therefore, sharing both infrastructure and a spectrum band at the same time doesn't bring significant coverage increment as compared to infrastructure sharing case.

Since both interference with in an operator (intra-operator) and between operators (inter-operator) is considered while analytical expression is derived, spectrum sharing gives the least coverage probability.

On the other hand, from the plot we can observe that the coverage of the two MNOs has improved as a result of infrastructure sharing. But the most benefited is the one with smaller number of network nodes (BSs).

B. Mean User Rate

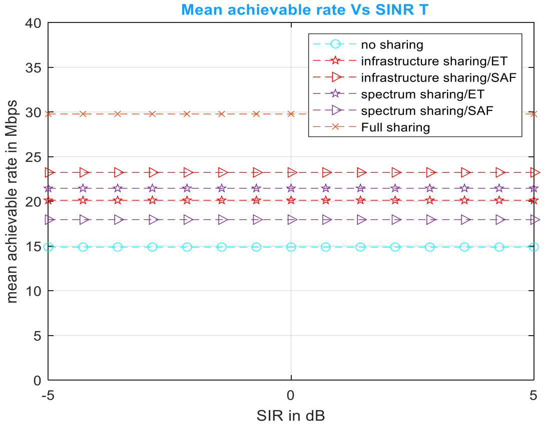


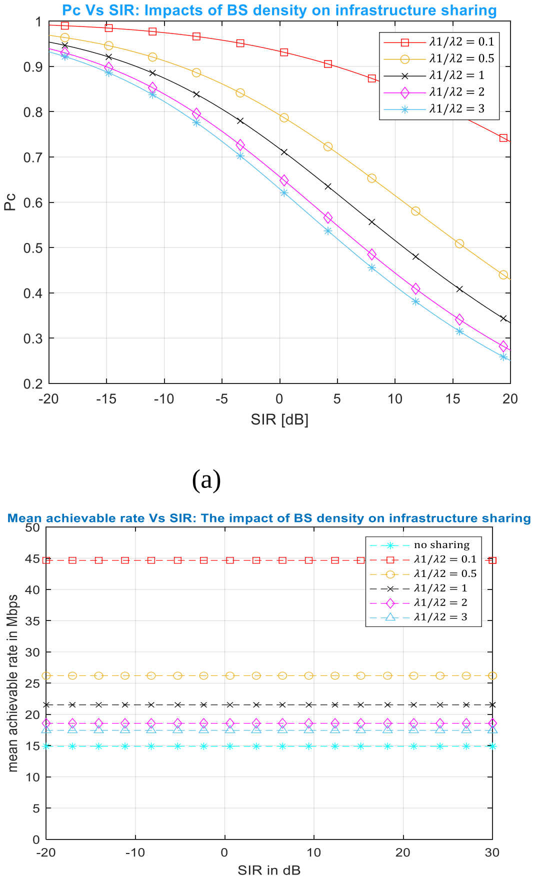
Figure 5: Mean user rate for different sharing scenarios

From the plot in Figure 5, it is clearly shown that full sharing and spectrum sharing scenarios provide the highest mean user rate as a result of the sharing of bandwidth between the MNOs since bandwidth has a direct relationship with the user rate. The value of the mean user rate in the absence of sharing is doubled, when the MNOs share both infrastructure and spectrum. i.e. when the MNOs engaged in full sharing agreement. Therefore, if MNOs want to enter in to a sharing agreement to boost their users' rate, full sharing and spectrum sharing are the best options.

C. The Impact of BS density on the sharing scenarios

To see the impacts of BS density on the sharing cases, a ratio of the BS densities of the two MNOs is taken and the result is seen for various values of the ratio. i.e. $\frac{\lambda 1}{\lambda 2} = x$ **,** for values 0.1, 0.5, 1, 2, 3.

i. The Impact of BS density on Infrastructure Sharing



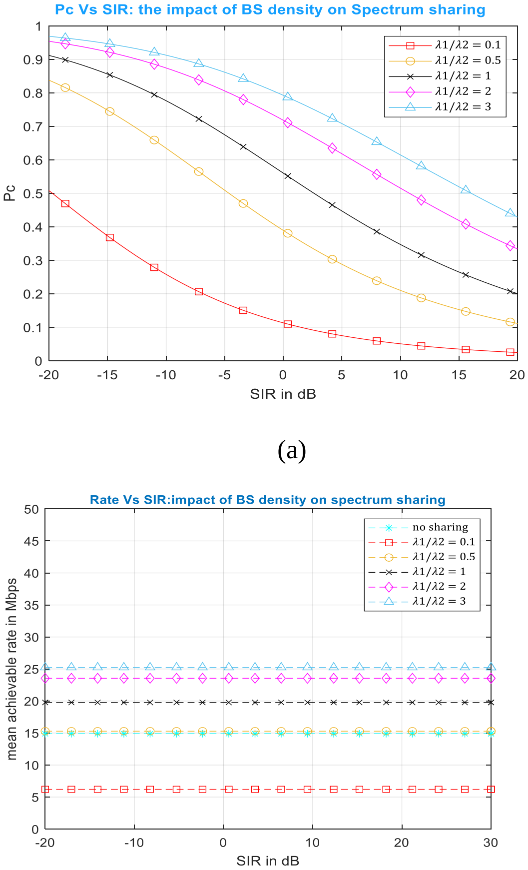
(a)

(b)

Figure 6: The impact of BS density on infrastructure sharing in terms of (a) probability of coverage, and (b) mean user rate.

From Figure 6a and 6b, we can observe that the user of the smaller (small BS density) size MNO benefits greatly in terms of coverage, from infrastructure sharing than the user of the larger (large BS density) sized MNO. This is because, BSs of the smaller MNO are relatively less dense and the distance to the nearer transmitter is shorten as a result of the sharing and the user can get better signal strength. Similarly, the user of the smaller size operator benefits greatly, in terms of mean rate, from infrastructure sharing than the larger operator.

ii. The Impact of BS density on Spectrum Sharing



(a)

(b)

Figure 7: The impact of BS density on spectrum sharing in terms of (a) probability of coverage, and (b) mean user rate.

From Figure 7a and 7b, we can see that, the user of a larger size MNO benefits greatly in terms of coverage probability, from spectrum sharing than the user of the smaller MNO. This is because there is less interference coming from the smaller MNO. In spectrum sharing, MNOs use the same frequency bands and the user of smaller MNO suffers more from the interference coming from the larger MNO. Higher interference results lower SINR and lower coverage. For the same reason, the user of a larger operator benefits greatly in terms of mean rate from spectrum sharing.

D. Impact of BS density, User density and Bandwidth on the Throughput

To see the impacts of some parameters on the throughput, we can calculate the throughput from the rate.

i. The Impact of BS density on the Throughput

By fixing the number of users of the two MNOs n1=n2=100 and their bandwidth B1=B2=10MHz and varying their BS density $\frac{\lambda1}{\lambda2} = x$, from 0 to 3 .we can clearly see the impact of BS density under the three sharing strategies.

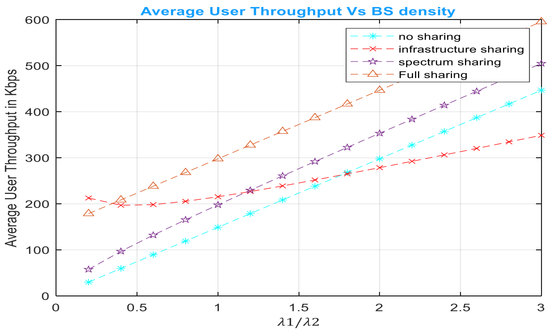


Figure 8: The impact of BS density on the throughput

Figure 8 shows that the average throughput of a user increases as the size (BS density) of the MNO increases. Full sharing provides the highest average throughput because in full sharing, in addition to the sharing in bandwidth which has a direct relationship with the rate, an increase in BS increases the SINR and the increase in SINR increases the rate. The average throughput of the user increases by three times as the BS density increases from 0 to 3.

ii. The Impact of user density on the Throughput

By fixing the BS density of the two MNOs $\lambda1$=20 network nodes/km^2 and $\lambda2$=15 network nodes/km^2 and their bandwidth B1=B2=10MHz then varying their user density, $\frac{n1}{n2} = x$, from 0 to 3, we can clearly see the impact of user density under the three sharing strategies.

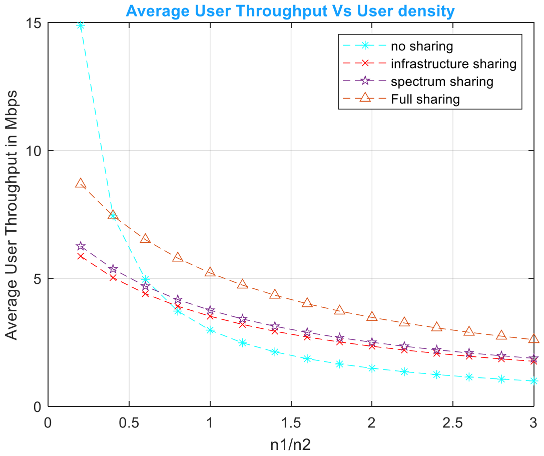


Figure 9: The impact of BS density on the throughput.

From Figure 9, we can observe, that the average throughput drops as the number of users increases. This because more users are sharing the fixed radio resource (bandwidth). Full sharing provides highest throughput however; the average throughput drops from around 900kbps to 300kbs. Interestingly, the average user throughput for no sharing case has got the highest value i.e. 1500kbps for smaller user density. Therefore, for smaller user density no sharing case is the best option.

iii. The Impact of bandwidth on the Throughput

By fixing the BS density of the two MNOs $\lambda1$=20 network nodes/km^2 and $\lambda2$=15 network nodes/km^2 and the number of users of the two MNOs n1=n2=100 then varying their bandwidth, $\frac{B1}{B2} = x$, from 0 to 3, we can clearly see the impact of bandwidth under the three sharing strategies.

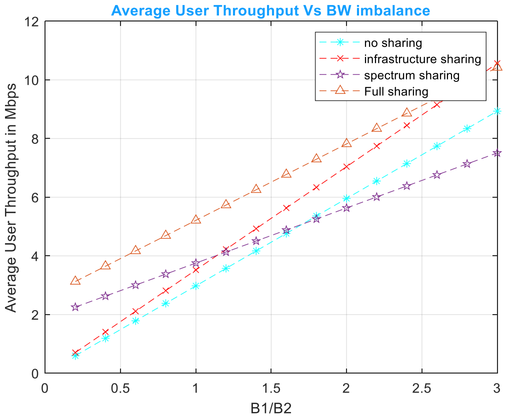


Figure10: The impact of bandwidth on the throughput

From Figure 10, we can observe that, the average throughput increases almost linearly as the total bandwidth increases. The linearity witnesses the fact that the bandwidth has direct relationship with the rate and the throughput. Again full sharing case provides highest throughput than the others. The throughput of the user has increased almost five times as the bandwidth ratio increases from 0 to 3.

IV. CONCLUSION

Infrastructure sharing gives superior coverage than the other sharing cases. Mean rate of the user is doubled when MNOs shares both infrastructure and spectrum (Full sharing).

Operator with smaller size network benefits more from infrastructure sharing than the larger operator in both coverage and capacity enhancement. Operator with larger size network benefits more from spectrum sharing than the larger operator in both coverage and capacity enhancement. Therefore, MNO, whose network size is much smaller, can choose and enter an infrastructure sharing agreement to boost their coverage and capacity. MNO with larger sized networks can choose and enter a spectrum sharing agreement to boost their coverage capacity. But this doesn’t mean that infrastructure sharing has no use for larger size operator rather, it has significant network performance improvement.

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