Addis Ababa University

Addis Ababa Institute of Technology

School of Civil and Environmental Engineering

Developing New Expression for Dynamic Impact Factor of Railway Bridges

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A Thesis for MSc. in Railway Engineering

(Civil Infrastructure)

By Milki Feyissa

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A Thesis

Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Science

The undersigned have examined the thesis titled '**Developing New Expression for Dynamic Impact Factor of Railway Bridges"** presented by **Milki Feyissa Lemma**, a candidate for the degree of **Master of Science** and hereby certify that it is worthy of acceptance.

Dr. Abrham Gebre

Advisor	Signature	Date

Dr. Adil Zekaria

Internal Examiner	Signature	Date

Dr. Asnake Adamu

External Examiner	Signature	Date

Chairperson	Signature	Date

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I certify that research work titled "Developing New Expression for Dynamic Impact Factor of Railway Bridges" is my own work. The work has not been presented elsewhere for assessment. Where material has been used from other sources it has been properly acknowledged/referred.

Milki Feyissa Lemma

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Signature

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Date

ABSTRACT

There is a significant demand worldwide, that is ever-increasing on understanding the dynamic performance of railway bridges. One of these demands is directed towards finding a comprehensive expression for determining the Dynamic Amplification Factor (DAF), or simply called impact factor. A bridge's dynamic response is affected by many bridge, track and train characteristics. However, the Dynamic Amplification Factor as given by different codes is generally a function of bridge span only. In addition, there is a significant variation in the codes and no provision exists that considers other influencing parameters in combination with the bridge's span. Therefore, the dynamic amplification factor, given it being an important parameter in the design of bridges, to this day has not found a general expression that represents its true value. Some disagreement exists among various bridge codes as well. In this research, a new Dynamic Amplification Factor expression for railway bridges is developed by performing a parametric study considering track stiffness, bridge span length, concrete compressive strength, train speed and length of transition zone. After 32 combinations of these parameters were generated using the Latin Hypercube Sampling technique, 32 bridges were modelled in ABAQUS using MATLAB. Static and Dynamic analysis were performed for these bridges, and the dynamic amplification values were determined from the displacement results of the two analysis. Finally, multivariate regression was performed to develop the best fitting Dynamic Amplification Factor expression. The developed expression is also compared with the Dynamic Amplification Factor values from codes.

Keywords*: bridges, impact factor, bridge design codes, dynamic amplification factor, dynamic analysis, Abaqus2Matlab.*

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NOTATION

AALRT	Addis Ababa Light Rail Transit
AASHTO	American Association of State Highway and Transportation Officials
BOEF	Beam on Elastic Foundation
BS	British Standard
CAN	Canada
CSA	Canadian Standards Association
DA	Dynamic Allowance
DAF	Dynamic Amplification Factor
DLA	Dynamic Load Amplification
ERRI	European Rail Research Institute
FE	Finite Element
HSLM	High-Speed Load Model
HSR	High-Speed Railway
LHS	Latin Hypercube Sampling
OHBDC	Ontario Highway Bridge Design Code
SIA	Swiss Society of Engineers and Architects
SPSS	Statistical Package for Social Sciences
VIS	Variance Inflation Factors

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1. INTRODUCTION

1.1 General Background

In the past few years, railway development has emerged anew in Ethiopia. It was just when the country had thought would never hear talks of rail transport, with the century-old legacy railway merely left as a memory, plans arose for the Light Rail Transit and the National Railway Network of Ethiopia. In the year 2010, the Ethiopian Government proposed 8 railway routes running across the country, with the highest impact route being the 759 km Addis Ababa to Djibouti railway line which is now operational as of January 01, 2018. The Awash-Woldiya and Woldiya-Mekelle railway routes are under construction. The capital city, Addis Ababa, has also welcomed three years ago, a light rail transit running in the N-S and E-W corridors of the city.

In this age of rapid rail transportation, bridges have an important role to play. They allow for railroads to cross over otherwise impassable obstacles. Especially in a mountainous country like Ethiopia, and the need to have more gentle gradients for railways than highways, bridges are the answer at the expense of cost. Currently, the demand for bridges is growing with the railway industry. There are 65 bridges in the route from Awash to Woldiya, and another 76 from Woldiya to Mekelle. Overall, a good number of bridges are being constructed in the country.

With the advancement of railway bridge design and construction and the demand for high-speed trains, there is a strong need to understand the dynamic performance of railway bridges. Bridges' response to dynamic loads is one of the most important factors in safety and durability analysis [1]. Since dynamic loads are imposed on the bridge structure in various forms, the study of these loads, their specifications and their effects on bridges, improve the methods of design and increases the safety and efficiency. One determinant in understanding bridge dynamics is the Dynamic Amplification Factor or simply called an Impact Factor.

A moving train generates deflections and stresses in the structure that are generally greater than those caused by load of the same train applied statically. In most codes, for design purpose, the dynamic effects of all vehicles on all bridges are taken into account by multiplying the static live loads by a dynamic load allowance (DLA) greater than 1.

The dynamic amplification (DA) resulting from the passage of one train on a specific bridge is calculated by:

$$DA = \frac{R_{dyn} - R_{sta}}{R_{sta}} \tag{1}$$

where R_{dyn} is the maximum dynamic response of the bridge and R_{sta} is the maximum static response. Therefore,

$$R_{dyn} = R_{sta}(1 + DA) \tag{2}$$

where *(1+DA)* is the Dynamic Amplification Factor (DAF) for the structure.

DAF, in short, is a measure of dynamic response with respect to static response for a moving load. There are three available ways of estimating DAF. One is by taking the DLA values as given by codes, secondly, by performing complete dynamic analysis, and lastly through tests under controlled train loads.

It is a challenging task to estimate the real DAF of railway bridges due to the various parameters that influence the response, such as speed, weight and dynamic characteristics of the train, type of bridge and state of the structure, damping characteristics, irregularities of rail surface and expansion joints, bridge supports, soil-structure interaction, and so on [2], which even the codes do not refute. The DLA for railway bridges as given by codes, however, is almost always a function of length only.

This raises the interest to determine the significance of the other influencing parameters and better yet, develop a more comprehensive expression that attempts to consider some significant parameters in combination. Also, the equation to be developed helps as a means of verification of the adequacy of provisions provided on codes.

1.2 Statement of the Problem

High-speed railways are becoming one of the major indicators of a country’s economic performance. The faster you deliver, whether goods or people, the more you attract customers, and the greater your revenue. However, with increased speed of the train, increases the need to comprehend bridge dynamics. But, one disagreement exists among researchers and various design codes as to the expression of the dynamic amplification factor. This fact has brought structural problems which relate to the design of railway bridges.

Several studies have concluded, at times, the impact factor as provided on codes give underestimated values. Building codes, such as Eurocode and AREMA have determined expressions for the DLA by considering the studies carried out by Timoshenko, Fryba, and others, whose main emphasis was on the dynamic response of a simple beam to a single moving load. This simplification has resulted in an insufficient provision as shown by some researchers.

In conclusion, the dynamic amplification factor, as important parameter as it is in the design of bridges, has not found an agreeable expression. Moreover, a substantial difference exists in the values among various bridge codes, which in some cases give an underestimated value when compared with model results [1]. The DAF depends, in addition to the maximum span, on many other bridge, train and track parameters. And these influencing parameters are not considered in the provisions.

1.3 Significance of the Study

Competition for a more resource-efficient transport system is a major driver behind the modal shift from road to rail. This indicates it becomes inevitable to upgrade the capacity of rail network infrastructure. Therefore, proving the existing infrastructure can be upgraded to future demands with sufficient safety margins and finding more cost and time efficient methods of designing railway bridges are topics of interest. One input to this is having a better assessment and verification of DAF, which is used to determine the load carrying capacity of bridges.

A realistic prediction of the total train load effect may prove that more bridges can be upgraded to higher loads without expensive strengthening or replacement or that some bridges are more susceptible to damage and failure than the expected, leading to serious safety concerns. Either of which is a significant finding in the performance assessment of bridges. A good prediction of DAF ensures a realistic estimation of the dynamic response of bridges.

The results from this practical-design-oriented study would enable bridge engineers to design new reinforced concrete railway bridges more reliably and economically. Furthermore, the results can be used to evaluate the load-carrying capacity of existing railway bridges, as even small increase in strength for the live load can make a difference between closing a bridge, retrofitting or leaving it open. This research aims to fill this gap by developing a comprehensive expression for DAF.

1.4 Aim and Scope of the Study

The ***general objective*** of this research is to develop DAF expression for railway bridges that will consider influencing parameters simultaneously.

The ***specific objectives*** are:

- Identifying and investigating track, bridge and train parameters that have previously been studied, and are considered to influence the DAF.
- Prioritizing and selecting parameters for the study.
- Assigning values for the parameters and systematically generating combinations of them for modelling using Latin Hypercube Sampling Technique.
- Modelling in ABAQUS environment in 2D for both static and dynamic cases, and analyzing the model outputs for the determination of DAF.
- Conducting a parametric study and sensitivity analysis to come up with the best fitting DAF expression.
- Verifying the adequacy/inadequacy of code provisions.

This research is limited to reinforced concrete railway bridges. Since simple span railway bridges are common due to their simple construction and suitability in minimizing train interruptions by allowing replacement of each individual span, they are used in this research. The bridge is taken to be well-constructed straight track, and the modelling is performed in 2D.

1.5 Methodology and Assumptions

Prior to modelling, a study is made to determine the parameters that have an impact on bridge dynamics. Priority is given to those parameters that have been identified as significant from literature and previous works. Parameters of interest are then selected and ranges of values assigned for each. To reduce the number of simulations needed, and have well distributed representative samples, combinations of the parameters are generated using Latin Hypercube Sampling (LHS) technique. The generated bridge prototypes are modelled in Abaqus/CAE 6.13-1 [3] by using MATLAB R2016a [4] scripting where a dos function is introduced to call

ABAQUS to perform the analysis. The modelling results are post-processed using Abaqus2Matlab plug-in application [5].

The bridge has one straight track with two identical girders making it cross-sectionally symmetrical. Since the length of the bridge is taken to be much greater than twice its width, a two-dimensional model is assumed to sufficiently reflect the bridge's behavior. Therefore, centerline modelling is adopted and is taken to give reasonable results. This plane model considers the two rails as one and the whole bridge as one beam.

Timoshenko beam elements resting on simple supports are used to model the bridge structure. Both concrete and steel are modelled using the elastic constitutive model. The effects of rail irregularities are ignored due to their insignificance in bridge dynamics as compared to passenger comfort analysis, as explained in various literature.

Eurocode is adopted throughout this study. Accordingly, the damping ratio for each span length is selected as specified by the code. The stiffness of the track (i.e. the stiffness of the ballast, sleepers, and the connections between the rails and sleepers) is taken as aggregated stiffness value taken as per the ERRI guidelines and previous works. Lateral motion of the train is neglected and the track is taken to be symmetric in the longitudinal direction.

The train is considered to be accurately simulated using the Sprung Mass Model by considering mass inertia. The vehicle is considered as separate mass at each axle. In this model, the rotational effect of the vehicle body is neglected.

Based on the convergence study made by [6] to determine the maximum time step for bridge dynamic analysis using an interaction model, a very fine time step is needed. Due to the longer analysis time that would be required, the finest possible time step of 0.0005 sec has been used for this research.

Based on a study by Yang [7], the impact factor based on deflection gives greater value than those based on other responses like acceleration and moment. Therefore, the displacement model outputs of static and dynamic analysis are used to develop the DAF expression.

The model is validated by using experimental data found on previously performed research.

1.6 Structure of the Thesis

The general content of this thesis is the determination of Dynamic Amplification Factor equation for reinforced concrete railway bridges. The paper starts with an introductory section which gives the relevant basis for the Dynamic effects of trains on bridges and how it is accounted for in design by a factor called Dynamic Amplification Factor as specified on various bridge design codes. The objectives of the study, the methodology used and the assumptions taken to conduct this research are also summarized in this first chapter.

The introduction is followed by a brief Literature Review. Eurocode's provision for dynamic analysis is also summarized in the second chapter.

In Chapter 3, an extensive parametric study is made on influencing parameters of railway bridge dynamics. The parameters are first identified and selected according to their significance as determined from literature. Based on that, the desired parameters for analysis were selected. The bridge, track and train characteristics that will be used in the model are then discussed and the parameters systematically combined to give 32 bridge prototypes for modelling.

Chapter 4 deals with modelling. This chapter defines how the bridge, train, and their interaction are modelled in ABAQUS. It also explains the steps of using MATLAB to perform simulations on ABAQUS, which was the method adopted for this research.

In Chapter 5, the dynamic amplification factor is calculated for each model from the results of the static and dynamic analysis. Regression is also performed to develop DAF expression. Towards the end of this chapter, the developed equation for DAF is compared with the provisions from AREAM and Eurocode.

The Final Chapter concludes the thesis and summarizes the findings of the research and recommends the areas for further research.

2. LITERATURE REVIEW

In common methods of bridge design, in order to account for dynamic effects of vehicle load, the traffic load is assumed as a static load increased by the impact factor [1]. The impact factor (IM) is defined based on the maximum value of dynamic and static responses as shown previously on equation (1).

The problem of dynamic amplification was recognized in the 19th century, following the collapse of some railway bridges in Great Britain [2]. Ever since, several studies have been made on the determination of dynamic amplification. In most current design codes span length (L) has been recognized as the only effective parameter for the determination of dynamic amplification factor. For instance, Eurocode suggests equation (3) for impact factor 'ϕ' which is based on determinant length L_ϕ; a parameter that was originally derived from cases of simply supported beams, but is adapted for a variety of structural components. From [8], the equation is given as:

a) for carefully maintained track:

$$\phi_2 = \frac{1.44}{\sqrt{L_\phi} - 0.2} + 0.82 \quad \text{with: } 1.0 \le \phi_2 \le 1.67$$

b) for track with standard maintenance:

$$\phi_2 = \frac{2.16}{\sqrt{L_\phi} - 0.2} + 0.73 \quad \text{with: } 1.0 \le \phi_2 \le 2.0 \qquad (3)$$

American Railway Bridges manual (AREMA) has proposed equation (4) for impact coefficient in steel railway bridges [9]. For concrete bridges equation (5) is provided.

$$I = \begin{cases} 40 - \dfrac{3L^2}{148.6} & \text{L} \le 24\text{m} \\ 16 + \dfrac{182.9}{L - 9.1} & \text{L} \ge 24\text{m} \end{cases} \quad \text{Steel Bridges} \qquad (4)$$

$$\begin{aligned} &\textit{For } \text{L} \le 4 \text{ m} && I = 60\% \\ &\textit{For } 4 \text{ m} \le \text{L} \le 39 \text{ m} && I = \frac{125}{\sqrt{L}}\% \quad \text{Concrete Bridges} \\ &\textit{For } \text{L} > 39 \text{ m} && I = 20\% \end{aligned} \qquad (5)$$

Iranian code for loading on bridges suggests relation (6) for impact factor based on bridge span length [10].

$$I = \begin{cases} \dfrac{1.44}{\sqrt{L}-0.2} + 0.82 & \text{Good maintenance} \\ \dfrac{2.16}{\sqrt{L}-0.2} + 0.73 & \text{Other situations} \end{cases} \tag{6}$$

An important study was made by Paultre et al [2], to assess the DLA, as given by various codes, i.e. American (AASHTO 1989), Swiss (SIA 160E 1988), British (BS 5400 1978), Ontarian (OHBDC 1983a, 1983b), and Canadian (CAN/CSA-S6 1988) standards, as well as India, Germany and France DLAs which showed a significant variation of the DLA vs. fundamental frequency of the bridge as shown in Figure 2-1. Although specified live loads are different in most countries, it does not explain the large disagreement on the DLA for different national codes. In plotting this graph, the codes that give DLA values in terms of maximum span have been converted using the relationship of frequency and maximum span in equation (7). This correlation was suggested by RILEM Committee 65 MDB and is based on testing performed on more than 200 European bridges.

$$f_o = 82L_{\max}{}^{-0.9} \tag{7}$$

where f_o is frequency and L_{max} is the maximum span.

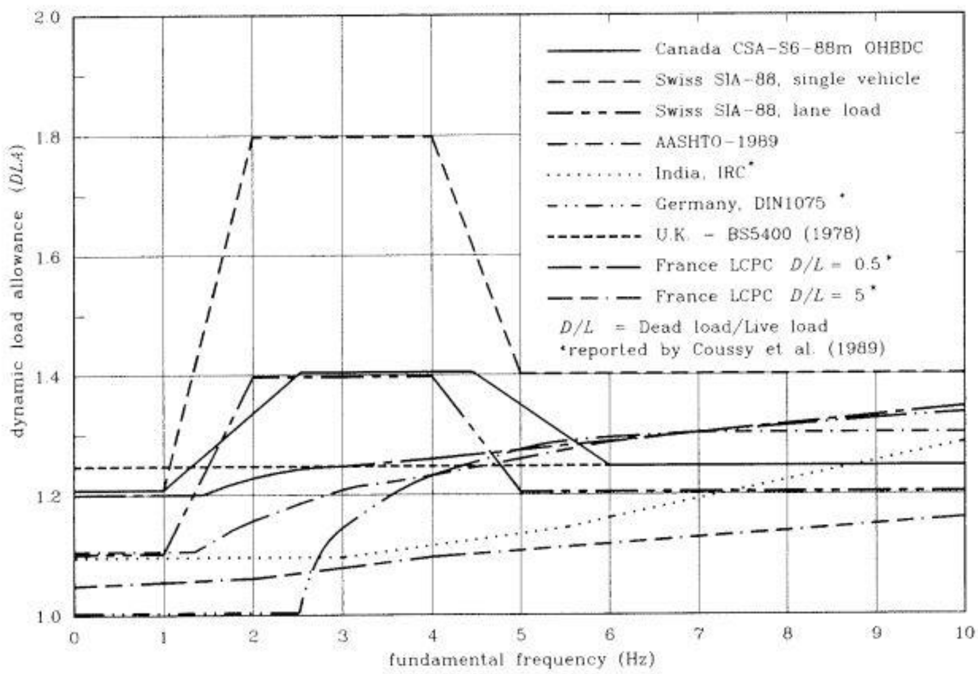

Figure 2-1– Dynamic load allowance (DLA) versus fundamental frequency for different national codes [2].

Similarly, researchers Moghimi and Ronagh [10], on their study on impact forces for a composite steel bridge, assessed several codes and noted the variation. For instance, on the OHBDC (Canada) and Australian manual, impact factor is measured based on the first vibration frequency of the bridge. OHBDC specifies

$$I = \begin{cases} 0.2 & \omega_1 < 1\text{Hz} \\ 0.4 & 2.5\text{Hz} \le \omega_1 \le 4.5\text{Hz} \\ 0.25 & \omega_1 > 6\text{Hz} \end{cases} \quad (8)$$

Where ω_1 is the first natural frequency. For points between 1–2.5 Hz and 4.5–6 Hz the value of impact factor varies linearly.

The impact factor relations in Figure 2-2 are drawn based on bridge span length according to different codes. Their research also noted that there was no distinctive correlation between Dynamic Load Allowance and the initial bounce of the vehicle at the time of entrance to span.

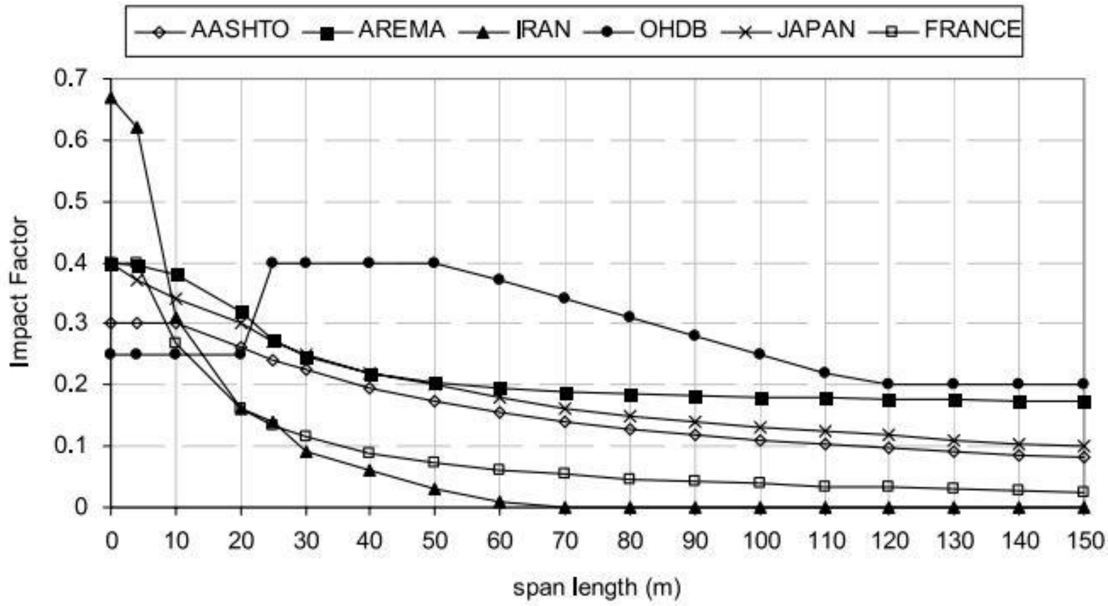

Figure 2-2 – Impact factor based on various codes [1]

The researchers concluded from Figure 2-2 that impact factor values calculated from various codes show that in span ranges of 25 to 50 m OHDBC suggests higher values for *I* among others, while for smaller spans, AREMA and Japan manuals are more conservative.

A recent and relevant work was done [1], to come up with a relation of impact factor for steel railway bridges considering simultaneous effects of vehicle speed and axle distance to span length ratio. From their study, they concluded the dynamic analysis results show that in most cases the calculated impact factor values are higher than those recommended by the relevant codes and are thus underestimated. It has also been shown that the value of impact factor rises incredibly with train velocity. Their research resulted in an expression for dynamic amplification of railway steel bridges considering parameters of velocity, train axle distance, and the bridge span length which are shown in equation (9).

$$For\ V \le 180km/h \Rightarrow I = \begin{cases} -0.533\left(\dfrac{d}{L}\right) + 0.845 & d \le L \\ -0.119\left(\dfrac{d}{L}\right) + 0.411 & d \ge L \end{cases} \quad (9)$$

$$For\ 180 \le V \le 300km/h \Rightarrow I = -0.285\left(\dfrac{d}{L}\right) + 1.177$$

where d is the train axle distance, V is the train velocity, and L is the span length.

Yang et al. [7] had studied Impact Factor for vehicles moving over simple and continuous beams and showed impact factors for different bridge responses (moment, support reactions and deflections) are not the same and suggested different formulas for the Impact Factor.

Zhang [11] conducted research to determine the impact factor for concrete-steel composite bridges. He analyzed 120 bridges considering different parameters including span length, the number of main beams and the number of traffic lines in ABAQUS environment. To simulate the traffic load, some concentrated moving loads were implemented. As the vehicle mass was not very relative to bridge mass, the vehicle bridge interaction was disregarded. According to this research, the Impact Factor for composite bridges based on AASHTO formula is over-estimated for moment and deflection and is under-estimated for support reaction.

Fryba [12] studied the resonance condition caused by the train movement on bridges and introduced two parameters as the main causes of resonance vibration: the first is the exerting of consecutive loads due to train axles and the other is the high speed of modern trains. In this research, simple equations for impact factor are proposed. He also showed that the magnitude of vibration amplitude in a resonance condition is in direct relation with bridge span length and squared value of velocity and adversely relates to damping, train length, and bridge stiffness.

Cheng [13] has demonstrated that rail's conditions do not have a noticeable effect on bridge vibrations by studying railway bridge vibration with consideration of the rail's conditions.

Lin [14], after studying the resonance condition in the dynamic response of railway bridges to train movement, showed that the bridge vibration frequencies must be different from train frequency.

Lou [15] evaluated railway bridge and train responses with the finite element model and studied the effect of rail smoothness on the reduction of bridge dynamic response. The results have shown that the rail conditions have serious effects on vertical displacement and acceleration of the train but not on train body rotation and vertical displacement and acceleration of the bridge. Therefore, the rail condition is only important for passengers' comfort.

Goicolea [16], considering the resonance phenomenon in bridges as a result of consecutive moving loads exerted by train passage, emphasized the inadequacy of the European design manual's methods. According to his research, the dynamic response of bridges designed based on European Rail Research Institute (ERRI) recommended specifications resulted in responses higher than expected values in some velocities and specific axle distances.

Yang et al. [17] studied the dynamic response of bridge girders with elastic bearings to moving train loads. The results indicate that the insertion of elastic bearings at the supports of the beam for the purpose of isolating the earthquake forces may adversely amplify the dynamic response of the beam to moving train loads.

The accuracy of moving load model was studied by Museros et.al, [18], in which they explained the inaccuracy of adopting a moving load model for short span bridges (L≤ 20-25 m) as stated on Eurocode due to the model's conservative results of acceleration and displacement as compared to experiments. In their research, they studied two factors; the distribution of load through sleepers and ballast layer and the interaction between the bridge and train model; the two factors that are fully ignored in the moving load model. After running several numerical simulations, they have found that the distribution of load through sleepers and ballast does not have any influence on the results, while the interaction between bridge and train cause a considerable reduction in acceleration and displacement of the short span bridges.

Varandas et.al [19], have made a contribution by studying the dynamic behavior of railway track on transition zones. In their study, they have presented a numerical solution for dynamic loads on the ballast by a train passing over the transition zone. Their model was validated with field measurement data taken on two transition zones, which gave very similar results. This indicates that the model describes the dynamic behavior of the track on the transition zone quite well during train passage. Their research also took in to account the long-term track deformation, the non-constant stiffness of the support and the possibility of voids under the sleepers.

The effect of ballast model in the dynamic response of railway viaducts was studied by Rigueiro et.al. [20]. Three types of track models and two types of vehicle models (moving load and train-structure interaction model) have been assessed. They tested their models against three real structures whose modal parameters and acceleration response under real traffic was available. Acceleration response of their models was computed with time domain. Results from a model having a track were compared with the no track model. Their conclusion was that the track model did not affect the frequency content when the frequency is between 10-15 Hz. However, the track model acted as a filter for higher frequencies.

Another insight on the train-interaction model was made by Liu et.al, [21], on which they investigated in their research which conditions to consider in the dynamic analysis of a bridge by passing train. In their study, they found the effects of several parameters related to bridge and train model, such as, the ratio of the mass of the vehicle and the bridge, the ratio of the natural frequency of the vehicle and the bridge, the train speed and the damping ratio of the bridge to be significant in the dynamic response of a bridge. From their results, they were able to conclude that at resonance speed the train-bridge interaction model gives lesser values for acceleration as compared to moving load model. This reduction is large for acceleration at mid-span as compared to corresponding displacement results.

Causse et.al. [22], studied the influence of the track stiffness and damping parameters on the train dynamic behavior. They performed a multi-body simulation, where the track consists of a combination of vertical and lateral stiffness and damping elements connecting the rails, the sleepers, and the ballast. Their aim was to analyze the influence of the track parameter values on the dynamic response of a passenger multiple unit trains. The stiffness values were defined according to field measurements of the quasi-static track stiffness. The damping values were varied in a range of ± 50% of the nominal values. The results of the simulation showed a significant influence of the track parameters on the dynamic response, especially for vertical wheel/rail contact forces and lateral accelerations. These results confirmed the significance of setting correctly the stiffness and the damping parameters according to the characteristics of the track.

Eurocode addresses the dynamic analysis of railway bridges due to rail traffic in EN 1991-2:2002 Section 6.4 where requirements are placed for the analysis which is briefly summarized in the following paragraphs.

Eurocode states as "Principle", in performing the dynamic analysis, loading should be taken using characteristic values from both the Real Train specified for the particular project and also using High-Speed Load Model (HSLM) as given by the code. Load Model HSLM comprises of two separate Universal Trains with variable coach lengths, HSLM-A and HSLM-B. Either of the two trains should be applied in accordance with the requirements as shown in Table 2-1.

Table 2-1 Application of HSLM-A and HSLM-B [8]

Structural Configuration	Span	
	L < 7m	L ≥ 7m
Simply supported span[a]	HSLM-B[b]	HSLM-A[c]
Continuous structure Or Complex structure	HSLM-A Trains A1 to A10 inclusive[d]	HSLM-A Trains A1 to A10 inclusive[d]

[a] Valid for bridges with only longitudinal line beam or simple plate behavior with negligible skew effects on rigid supports

[b] for simply supported spans with span up to 7 m, a single critical Universal Train from HSLM-B may be sued for the analysis

[c] for simply supported spans with a span of 7 m or greater a single critical Universal Train HSLM-A may be used for the dynamic analysis (Alternatively Universal trains A1 to A10 inclusive may be used).

[d] All trains A1 to A10 inclusive should be used in the design.

[e] Any structure that is not simple (skew structure, bridge with significant torsional behavior, half through structure with significant floor and main girder vibration modes etc.)

HSLM-A is defined in Figure 2-3 and Table 2-2.

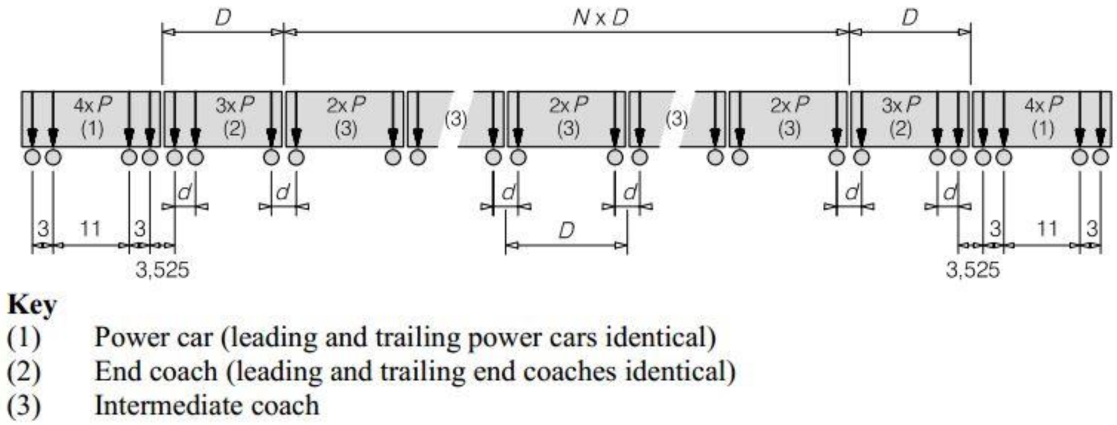

Figure 2-3 HSLM-A Train Model [8]

Table 2-2 Universal Trains Specification for HSLM-A load model [8]

Universal Train	Number of intermediate coaches N	Coach Length D [m]	Bogie axle spacing d [m]	Point force [kN]
A1	18	18	2.0	170
A2	17	19	3.5	200
A3	16	20	2.0	180
A4	15	21	3.0	190
A5	14	22	2.0	170
A6	13	23	2.0	180
A7	13	24	2.0	190
A8	12	25	2.5	190
A9	11	26	2.0	210
A10	11	27	2.0	210

For simply supported spans with a span of 7 m or greater a single critical Universal Train HSLM-A may be used for the dynamic analysis or alternatively Universal trains A1 to A10 inclusive may be used.

The critical universal train is selected based on the critical wavelength of excitation λ_C as defined in EN 1991-2 Annex E.2(4) shown here on APPENDIX C, where it is a function of the maximum value of aggressivity $A_{(L/\lambda)}G_{(\lambda)}$ for the span length, L in the range of excitation wavelength from 4.5 m to λ_v.

The wavelength of excitation at the Maximum Design Speed λ_v (m) is given as:

$$\lambda_v = \frac{v_{DS}}{n_o} \tag{10}$$

Where:

n_o is the first natural frequency of the simply supported span (Hz)

v_{DS} is the maximum Design Speed

Based on λ_c, the characteristics of the critical universal train can be determined from Figure 2-4.

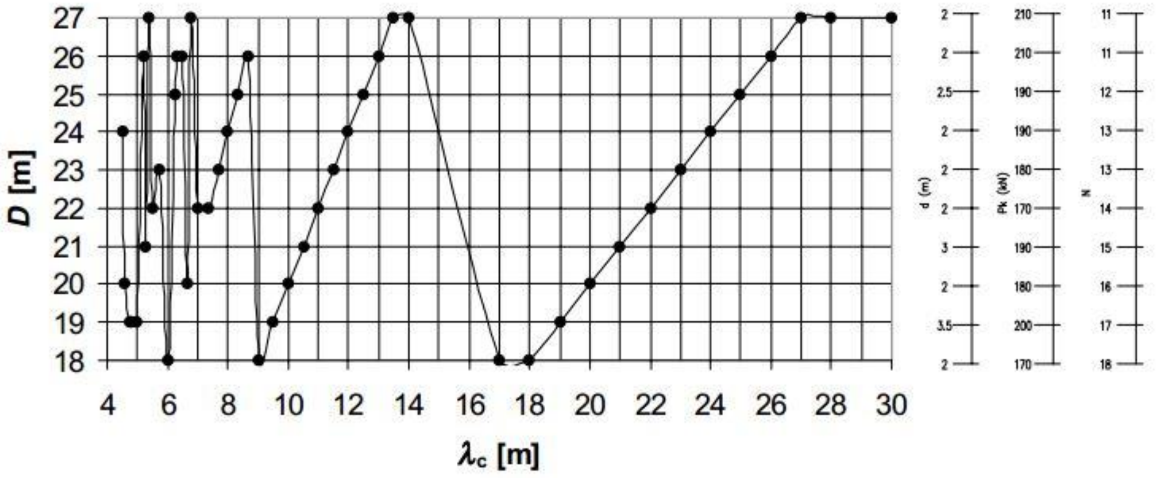

Figure 2-4 - Parameters defining critical Universal Train in HSLM-A as a function of critical wavelength of excitation

Where on the figure:

D	Length of intermediate and end coaches defined in Figure 2-3
d	Spacing of bogie axles for intermediate and end coaches defined as in Figure 2-3
N	Number of intermediate coaches defined as in Figure 2-3
P_k	Point force at each as defined in Figure 2-3
λ_c	Critical wavelength of excitation

Another requirement is to consider frequencies up to 30Hz in the dynamic analysis or 1.5 times the value of the first mode of vibration, including at least three modes of vibration [8].

3. PARAMETRIC STUDY

3.1 Identification of Key Influencing Parameters

As stated on EN 1991-2:2002: Traffic loads on Bridges [8], the static stress and deformations (and associated bridge deck acceleration) induced in a bridge are increased and decreased under the effects of moving traffic by the following:

- the rapid rate of loading due to the speed of traffic crossing the structure and the inertial response (mass, stiffness, and damping) of the structure,
- the passage of successive loads with an approximately uniform spacing which can excite the structure and under certain circumstances create resonance (where the frequency of excitation matches a natural frequency of the structure, there is a possibility that the vibrations caused by successive axles running onto the structure will be excessive),
- variations in wheel loads resulting from track or vehicle imperfections (including wheel irregularities).

The dynamic behavior of a bridge is known to be influenced by several bridge, train, and track parameters. The principal factors which influence the dynamic behavior, as listed on Eurocode are:

i. the speed of traffic across the bridge,
ii. the span length of the element and the influence line length for deflection of the element being considered
iii. the mass of the structure
iv. the natural frequencies of the whole structure and relevant elements of the structure and the associated mode shapes (eigenforms) along the line of the track,
v. the number of axles, axle loads and the spacing of axles,
vi. the damping of the structure,
vii. vertical irregularities in the track,
viii. the un-sprung/sprung mass and suspension characteristics of the vehicle,
ix. the presence of regularly spaced supports of the deck slab and/or track (cross girders, sleepers, etc.),
x. vehicle imperfections (wheel flats, out of round wheels, suspension defects, etc.)

xi. the dynamic characteristics of the track (ballast, sleepers, track components, etc.).

The key parameters affecting the dynamics can be classified into four categories:

a. Train characteristics

- ✓ Variation in the magnitude of the axle loads
- ✓ Axle spacing
- ✓ The spacing of regularly occurring loads
- ✓ Number of regularly occurring loads
- ✓ Train Speed

b. Structure (bridge) characteristics

- ✓ Span or the influence length (for simply supported structures the two are equivalent)
- ✓ Natural frequency (which is itself a function of span, stiffness, mass and support conditions)
- ✓ Damping
- ✓ Mass per meter of the bridge

c. Track irregularities

- ✓ The profile of the irregularity (shape and size)
- ✓ The presence of regularly spaced defects (e.g. poorly compacted ballast under several sleepers or alternatively the presence of regularly spaced stiff components such as cross-beams)
- ✓ The size of the unsprung axle masses. An increase in unsprung mass causing an increased effective axle force
- ✓ Stiffness variation at transition zones

d. Others

- ✓ Out of run wheels
- ✓ Suspension defects, etc.

The train and structure characteristics play a more significant role on the bridge dynamics, whereas the rail irregularities and other defects related to the vehicle mostly influence the vehicle dynamics and are more significant in passenger comfort analysis.

Bridges constructed for high-speed lines should take care of the dynamic loads and resonance effects. As described in section 1, one way to consider the dynamic behavior of bridge is by multiplying the static responses by an amplification factor, i.e. the DAF. But it is important to note here that Quasi-static methods, i.e. the use of DAF multiplied by static load response, does not take into account the effects of resonance which may occur due to e.g. repeatedly moving axle loads. To include the resonance effect in the calculations detailed dynamic analysis of the bridge is required.

In the following table, the parameters that influence the DAF are identified based on previous studies. It is desired to limit the study to a maximum of 5 parameters due to the large set of simulations that would be required. Those parameters that have previously been identified to have an influence on the DAF are selected. The interdependence of the parameters is verified to ensure only those that are independent are considered for modelling. The parameters are summarized in Table 3-1.

From the table, parameters such as the train speed, the bridge span length, vehicle-structure mass interaction, concrete grade, track stiffness and length of transition are selected as parameters of interest.

Table 3-1 – Summary of identified and selected parameters

No.	Parameter	Consideration/limitation	Remark	Reference/ Indicating Research
1	Train speed	PoI[1]	Incredible rises of IM with increasing velocity	[1]
2	Train Axle distance	Dependent on the load model considered and real train specification as per Eurocode	Selection of load model dependent on speed and span length	[8]
3	Train Axle load	Ditto	Ditto	[8]
4	Train number of axles	Ditto	Affects IM only during resonance conditions	[1]
5	Bridge span length	PoI	Taken to be an effective parameter to determine DAF	[1], [8]
6	Support Condition	The study limited to simply supported bridges.	DAF of hogging moments can be as significant as sagging moments.	[23]
7	Bridge Damping	Adopted from Eurocode	Dependent on span length	[8]
8	Bridge cross section	Dependent on span length	A relationship developed between the second moment of area and bridge length is adopted.	[6]
9	Bridge skew angle	Straight sections are considered		
10	Vehicle/structure mass interaction	PoI	Sprung mass interaction model is adopted.	[6]
11	Concrete Grade	PoI	Researcher's interest	
13	Bridge mass	Dependent on bridge cross-section and span length	Chart indicating the relationship from previously done research is used	[6]
14	Track stiffness	PoI	Studied to have an impact on the bridge's dynamics	[22]
15	No of parallel tracks	Single track is considered		
16	Length of transition zone	PoI	Change in stiffness when entering the bridge causes dynamic impact	[8]
17	Vehicle damping and suspension	Not varied in this study		

[1] PoI – Parameter of Interest

3.2 Model Characteristics and Values

In this section, the bridge, track and train characteristics that will later be used in the model are explained. The selected parameters from the end of Section 3.1, (i.e. bridge span length, track stiffness, transition length, train speed, and concrete grade) are assigned values, which will be combined in a systematic way, in Section 0, to have samples for modelling.

3.2.1 Bridge Characteristics

3.2.1.1 Bridge Span Length

Most common simply supported railway bridges range from 15 to 45 m spans. Span lengths within that range are considered in this study. Shorter span bridges (i.e. culverts) are not susceptible to dynamic effects from trains and are thus not considered in this study.

3.2.1.2 Bridge Section Properties

The section properties are taken from [24], where design charts have been developed from a study made on reinforced concrete railway bridges for their frequency and mass properties with varying span length. The graphs are shown in Figure 3-1 and Figure 3-2. The regression line is used to determine the section properties for each bridge span length. These graphs are developed by incorporating the mass of the ballast having a width of 6.2 m, thickness of 0.6 m and a density of 2000 kg/m^3. From the frequency and mass values, the section property (i.e. second moment of area of the bridge) is determined.

After reading the values of the fundamental frequency and mass from the developed charts of Figure 3-1 and Figure 3-2 for each span length L, the value of the corresponding second moment of area, I can be determined from the relationship shown in equation (11) by rearranging for I. The graph was developed for $E = 25$ *GPa*. Since the concrete compressive strength will be varied for each span length, the Modulus of Elasticity, E_{cm} can be determined for each combination of span length and concrete compressive strength from equation (12) according to [25].

The nth critical frequency in Hz of a simply supported beam is calculated using equation (11).

$$\omega_n = \frac{1}{2\pi}(n\pi)^2\sqrt{\frac{EI}{mL^4}} \tag{11}$$

Where,

$\overline{m}$ is the mass in kg/m

E is the Elastic modulus in N/m^2

I is the second moment of area in m^4

L is the bridge span length in m

ω_n is the nth critical frequency in Hz

Modulus of elasticity of concrete is determined from:

$$E_{cm}(GPa) = 22\left[\left(\frac{f_{ck}+8}{10}\right)\right]^{0.3} \tag{12}$$

Where f_{ck} is the characteristics strength of concrete in MPa.

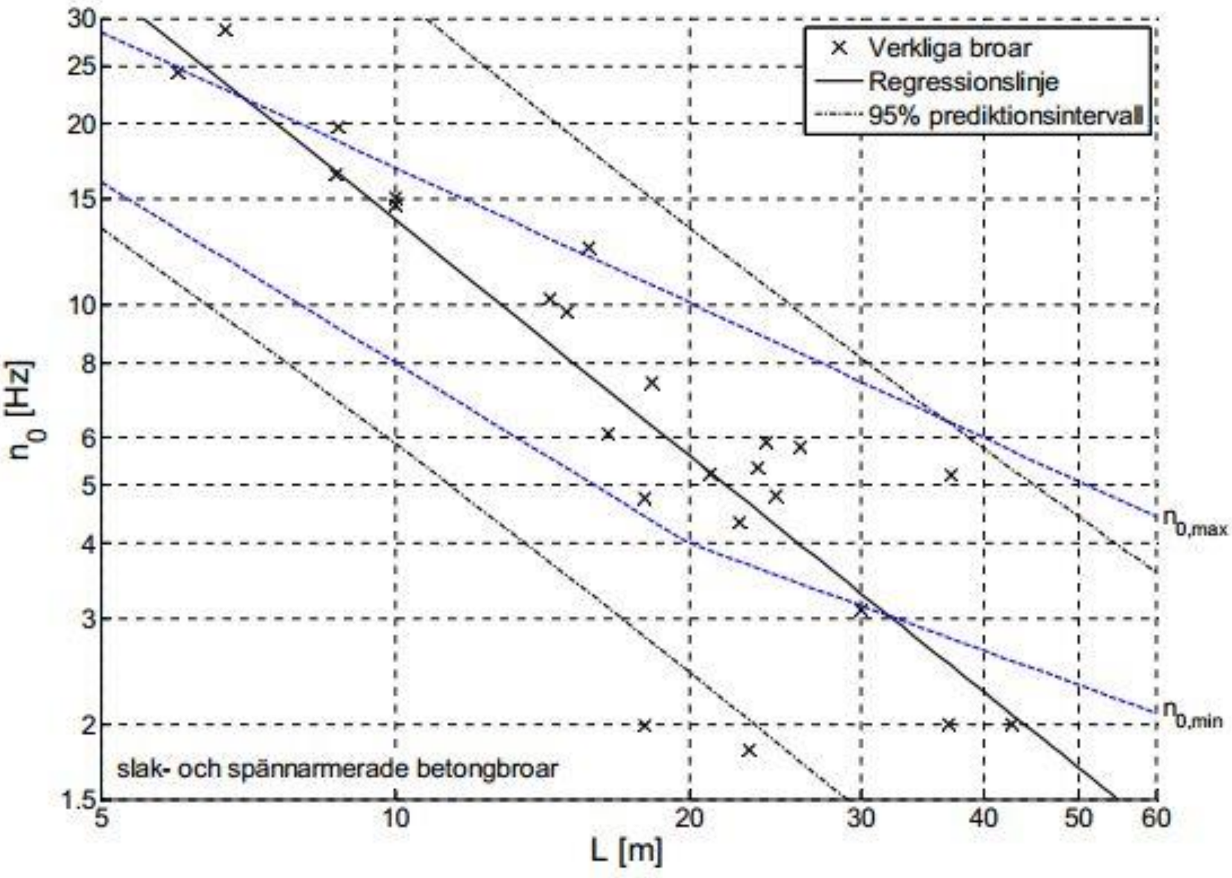

Figure 3-1 Frequency for different spans of Reinforced Concrete bridges [24]

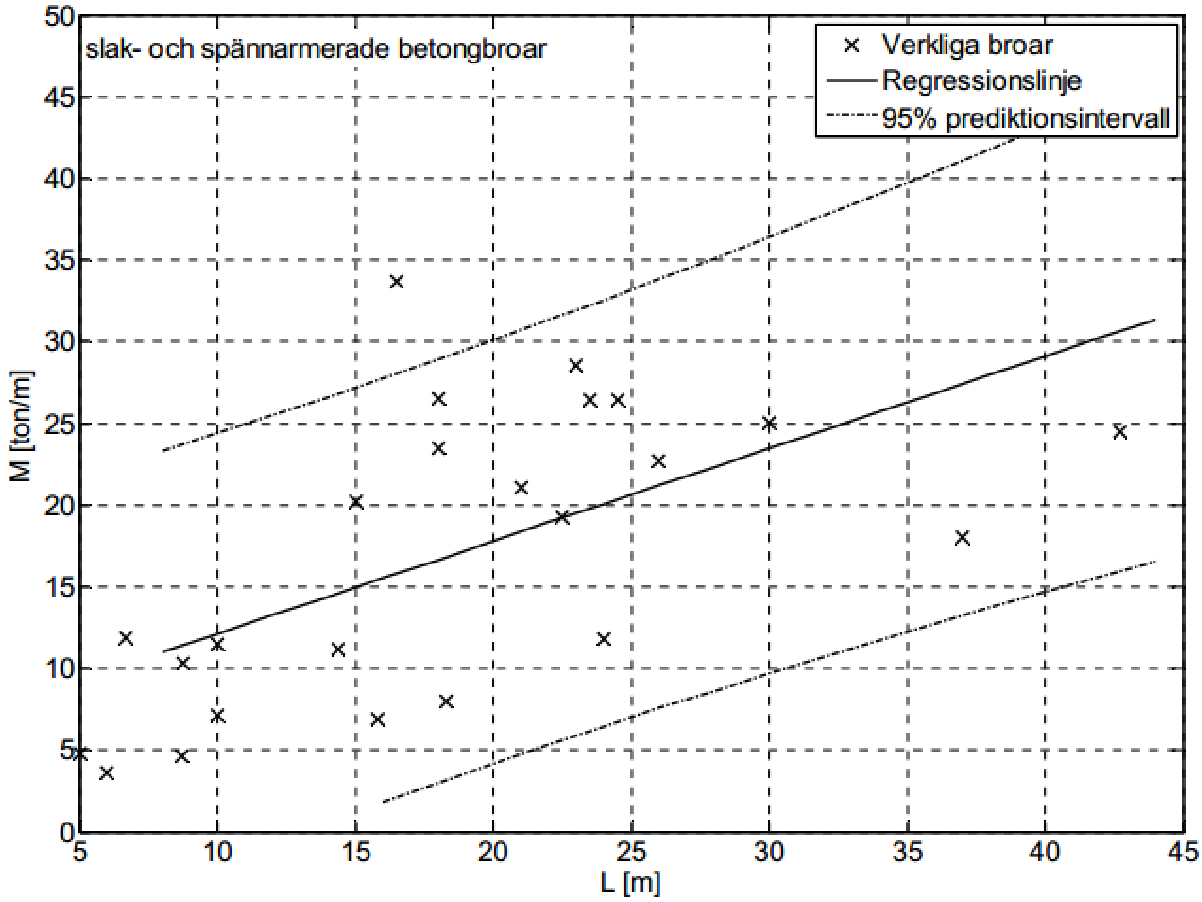

Figure 3-2 Mass for different spans of Reinforced Concrete bridges [24]

3.2.1.3 Bridge Damping

Bridge damping is the property of the structure, which reduces the dynamic response helping the bridge reach a state of equilibrium after a train's passage. In bridge structures, damping can either be from internal means arising from internal friction, cracks and non-homogeneous properties of building material, etc. In addition, there is an external source of damping in bridges arising from friction between supports and bearing, friction in the ballast, friction in the joints of the structure, viscoelastic properties of soil, foundation and abutment, etc. This indicates that damping is a result of several factors; exact values of which can only be determined from measurements.

Generally, damping depends on the material and the condition of the bridge, as well as the amplitude and frequency of vibrations. However, the frequency has a rather insignificant influence on damping in the range up to 50 Hz, which usually covers the most common natural frequencies of railway bridges. Considering this point, damping in bridges is well described by structural damping theory in the range of linear material deformations.

The maximum response of a bridge is dependent upon the rate at which dynamic motions dissipate through damping. The less the damping, the greater the maximum dynamic effects. Therefore, it is important that lower bound values of damping are used in calculations to ensure that safe estimates of peak dynamic effects at resonance are obtained. The design codes, such as Eurocode, are based on empirical damping ratios obtained from measurements on different bridges. From the measurements, it was found that the structural material and span length show a strong correlation with the damping. Accordingly, lower bound estimates are provided on Eurocode for different bridge materials as shown in Table 3-2.

Table 3-2 - Eurocode recommendation of structural damping (EN 1991-2) [8]

Bridge Type	ζ Lower limit of percentage of critical damping [%]	
	Span L $<$ 20 m	Span $L \geq$ 20 m
Steel and Composite	$\zeta = 0.5 + 0.125\ (20 - L)$	$\zeta = 0.5$
Prestressed Concrete	$\zeta = 1.0 + 0.07\ (20 - L)$	$\zeta = 1.0$
Filler beam and reinforced concrete	$\zeta = 1.5 + 0.07\ (20 - L)$	$\zeta = 1.5$

The bridge structural damping for reinforced concrete bridge is determined from Table 3-2 using equation (13).

$$\xi = \begin{cases} 1.5 + 0.07x(20 - L) & \text{for L < 20m} \\ 1.5 & \text{for L} \geq \text{20m} \end{cases} \tag{13}$$

3.2.2 Track Characteristics (Stiffness and Damping)

In a typical ballasted railway track system, the rails are supported by a number of elements mentioned in descending order as the rail-pads, sleepers, ballast, sub-ballast and subgrade or formation, as illustrated schematically in Figure 3-3. In a well-constructed and well-maintained track, these elements all deform essentially elastically (in the sense that the majority of their deflection is not permanent) when loaded and unloaded during train passage.

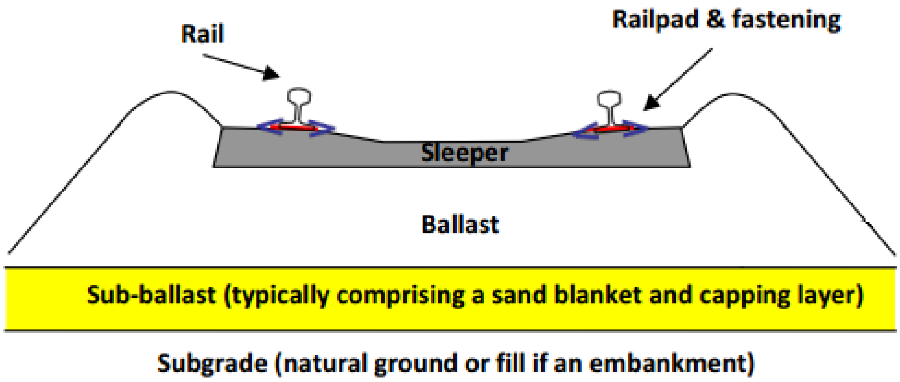

Figure 3-3 Schematic cross-section of a ballasted track system [28]

In its most general form, the term “track stiffness” may be taken to mean the (point) load required to produce a unit deflection of the rail at the location where the load is applied. This is a simple and intuitive definition, which has units of force divided by deflection (e.g. kN/mm). Its value depends on the effective stiffnesses of all of the individual elements of the track system combined. It may, therefore, be considered a composite or global stiffness [28]. Therefore, the stiffness of the track is an aggregate of the stiffness of the ballast, sleepers and the connections between the rails and sleepers, and when a train passes, these elements behave like a complete track.

The track is commonly modelled as a beam (the rails) on an elastic foundation (the support) generally abbreviated as “BOEF” – Beam on Elastic Foundation. This indicates the convenience of separating the flexural rigidity of the rails (which for a standard rail type is well defined) from the direct stiffness of the support system (rail-pads, ballast, sub-ballast and subgrade), which is known to vary widely.

The effective or combined stiffness of the system supporting the rails may be characterized by means of a modulus. This is defined as the stiffness of the equivalent springs supporting the rails in terms of the load per unit length needed to produce a unit displacement at the point of measurement on the rail. This type of modulus has units of force per unit length divided by displacement, MN/m^2.

In order to study the effects of track stiffness on the bridge dynamic response, the track stiffness over the bridge is varied from 200 to 600 MN/m. Even though no criteria are placed for track stiffness on Eurocode, the selected range of values is compliant with the guidelines for Track Stiffness from [28].

Table 3-3 Track Stiffness and Damping values considered in the study

	Over bridge	**Over approach track**
Spring Stiffness, *S* (*MN/m*)	200 - 600	100
Vertical damping (*kNs/m*)	100	100

3.2.3 Transition Length

As the bridge section is stiffer than the approach track, a sudden bump is felt on the train as it enters the bridge section due to the sudden change in track stiffness. This is said to cause a dynamic impact on the bridge structure. In order to reduce this effect, a transition zone is provided between the track and the bridge. Going from soft to stiff track is worse than going from stiff to soft track [6]. To determine the effect of the size of transition zones, a transition length of bridge span length multiplier of 0.25, 0.5, 0.75, and 1 are taken. An average value of the springs over the track and bridge is used for springs over the transition length, which is determined from equation (14).

$$S_{approach} = \frac{S_{bridge} + S_{extra\_l}}{2} \qquad (14)$$

Table 3-4 Transition length factor values considered in the study

Transition length factor, *f*	0.25	0.5	0.75	1.00

3.2.4 Train Characteristics

The train parameters to be considered in the models are train type, train speed and the vehicle damping and suspension.

3.2.4.1 Train Speed

It is intended to study how significantly train speed influences the dynamic response of a bridge on high-speed lines, and thus its significance to the DAF value. High-speed ranges from 150 km/h to 300 km/h are used in this study in accordance with the minimum and maximum values specified by Eurocode.

3.2.4.2 Train type and Load Model

As described in Section 3 Code Summary, according to Eurocode [8], the dynamic analysis should be performed using characteristic values of the loading from the Real Train as specified for the particular project and from the Load Model HSLM (High-Speed Load Model).

Real Train: Euro Train X2000

The train's technical information is provided in Table 3-5.

Table 3-5 X2000 Real Train Specifications

No of power car	1
No of passenger coaches	4
Length *m*	140
Maximum speed *Km/h*	275
Unsprung mass for each coach (kg)	2050; 1650; 1650; 1800
Sprung mass for each coach (kg)	16,200; 10,100; 9,850; 14,200

HSLM

For simply supported span bridges with span length greater than 7 m HSLM-A is to be used with a single critical universal train or all trains A1 to A10 inclusive. The critical universal train is determined from train speed and natural frequency of the bridge, as explained in the Eurocode summary provided at the end of Chapter 2: Literature Review. Accordingly, the critical HSLM-A train has to be determined for each combination of train speed and bridge span length.

From [29], The wheel mass of HSLM trains is 2000 kg, and the bogie mass is 3000 kg. From this, the body mass can be determined using equation (15).

$$m_3(kg) = \frac{Px1000}{g} - m_1 - m_2 \tag{15}$$

Where $m_1+m_2+m_3 = P\ kN$ and g is the acceleration due to gravity.

3.2.4.3 Vehicle Suspension and Damping

Vehicle suspension and damping values vary from one vehicle to another. X2000 train has a damping coefficient of *30 kNs/m* and stiffness value that alternate at the front and rear axle as *1.45 MN/m* and *1.05MN/m* [6]. No specific information is given on Eurocode on the spring stiffness and damping coefficients of HSLM or general guidelines of such values. Thus, similar values of the X2000 train are adopted for HSLM-A.

3.2.5 Rail Characteristics

The rail used is 60E1 (UIC60) Steel Rail and is shown in Figure 3-4. UIC60, manufactured according to European standard EN13674-1, has the specifications listed in Table 3-6.

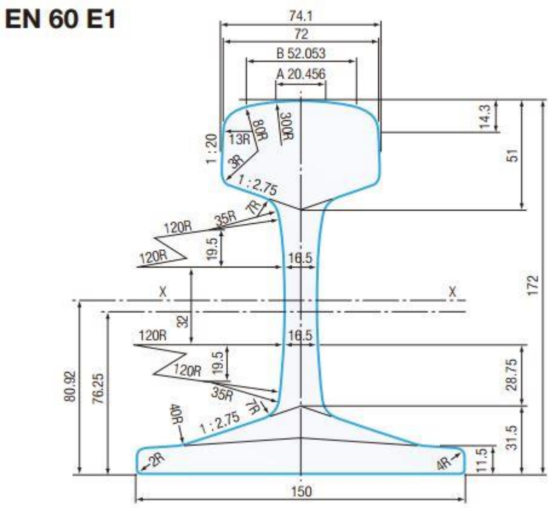

Figure 3-4 UIC60 EN13674-1:2002

Table 3-6 UIC60 specifications for a single Rail

Young's Modulus	*210 x 10⁹ N/m²* → $210 \times 10^9\ N/m^2$	**Moment of inertia, I_x**	3038 cm^4
Poisson's ratio	*0.3*	**Mass**	60.21 *kg/m*
Steel Density	$7850\ kg/m^3$	**Head Section modulus, Z_{xh}**	334 cm^3
Cross-sectional area, *S*	$76.70\ cm^2$	**Base Section modulus, Z_{xb}**	376 cm^3

3.2.6 Concrete Characteristics

It is intended to investigate the effect of concrete grade on the dynamic properties of railway bridges. Concrete having characteristics compressive strength of *25, 30, 35, 40,* and *45 MPa* will be studied.

Shear modulus of concrete will be determined from equation (16).

$$G = \frac{E}{2(1+\nu)} \tag{16}$$

Where E is the Elastic Modulus of concrete in *GPa* and ν is Poisson's ratio of concrete.

Table 3-7 Concrete Compressive Strength values used in this study

Characteristics compressive strength, f_{ck}	25	30	35	40	45
Modulus of Elasticity, E_{cm} (*GPa*)[2]	31	33	34	35	36
Poisson's ratio, ν	0.2	0.2	0.2	0.2	0.2
Shear Modulus (*GPa*)	12.9	13.8	14.2	14.6	15.0

3.3 Combination of Parameters for Modelling

There are several methods of generating samples of combined random variables, such as random sampling, Monte Carlo sampling, and Latin hypercube sampling. The latter is adopted for this study as it is a good approach to combining variables effectively when the sample size is reasonably small. It generates samples that represent well the full coverage of combined variables.

3.3.1 Latin Hypercube Sampling Theory

This method of sampling is used to generate combinations of variables for modelling. In this section, the theory behind it is explained to give the reader some background as to what it is.

Latin hypercube sampling was developed to address the need for uncertainty assessment for a particular class of problems. Consider a variable Y that is a function of other variables X_1, $X_2, \ldots, X_k$. This function may be very complicated, for example, a computer model. A question

[2] As calculated from equation (12).

to be investigated is "How does Y vary when the X_s vary according to some assumed joint probability distribution?"

A conventional approach to these questions is to apply Monte Carlo Sampling. By sampling repeatedly from the assumed joint probability density function of the X_s and evaluating Y for each sample, the distribution of Y, along with its mean and other characteristics, can be estimated. The LHS software supports this approach for generating samples of the X_s. The program output, for n Monte Carlo repetitions, is a set of n vectors of input variables (each such vector is *k*-dimensional). Each input vector can then be evaluated by the function or program to generate n values of the result Y (Y may be a scalar or a vector whose dimensionality is determined by the function or program of interest). This approach yields reasonable estimates for the distribution of Y if the value of n is quite large. However, since a large value of n requires a large number of computations from the function of the program of interest, which is potentially a very large computational expense, additional methods of uncertainty estimation were sought.

An alternative approach, which can yield more precise estimates, is to use a constrained Monte Carlo sampling scheme. One such scheme, developed by McKay, Conover and Beckman is Latin hypercube sampling. Latin hypercube sampling selects n different values from each of k variables $X_1, X_2, \ldots, X_k$ in the following manner. The range of each variable is divided into n non-overlapping intervals on the basis of equal probability. One value from each interval is selected at random with respect to the probability density in the interval. The *n* values thus obtained for X_1 are paired in a random manner (equally likely combinations) with n values of X_2. These n pairs are combined in a random manner with the *n* values of X_3 to form n triplets, and so on until *n k*-tuplets are formed. These *n k*-tuplets are the same as the *n k*-dimensional input vectors described in the previous paragraph. This is the Latin hypercube sample. It is convenient to think of this sample (or any random sample of size n) as forming an (n x k) matrix of input where the i^{th} row contains specific values of each of the k input variables to be used on the i^{th} run of the computer model [30].

The theory is further explained in Figure 3-5. On the figure, if one is interested to select 5 samples from random variables having either normal or uniform distribution using LHS method, the samples would be randomly picked from the 5 non-overlapping intervals created in terms of the Density Function and Cumulative Distribution Function. As it can be seen on the figure, for the normal distribution the intervals near the mean value are closely spaced and the size of the interval increases as we go to the minimum and maximum values in accordance to the normal distribution behavior, whereas for the uniform distribution the intervals have a uniform size.

The Latin hypercube sampling technique has been applied to many different computer models since 1975. Unlike simple random sampling, the LHS method ensures full coverage of the range of each variable by maximally stratifying each marginal distribution.

The LHS can be summarized as:

- divide the cumulative distribution of each variable into N equiprobable intervals;
- from each interval select a value randomly, for the ith interval, the sampled cumulative probability can be written as:

$$\text{Prob}_i = (1/N)ru + (i-1)/N \tag{17}$$

where ru is uniformly distributed random number ranging from 0 to 1;

- transform the probability values sampled into the value x using the inverse of the distribution function F-1:

$$x = F-1(\text{Prob}) \tag{18}$$

- the N values obtained for each variable x are paired randomly (equally likely combinations) with the ns values of the other variables.

The method is based on the assumption that the variables are independent of each other, but in reality, most of the input variables are correlated to some extent.

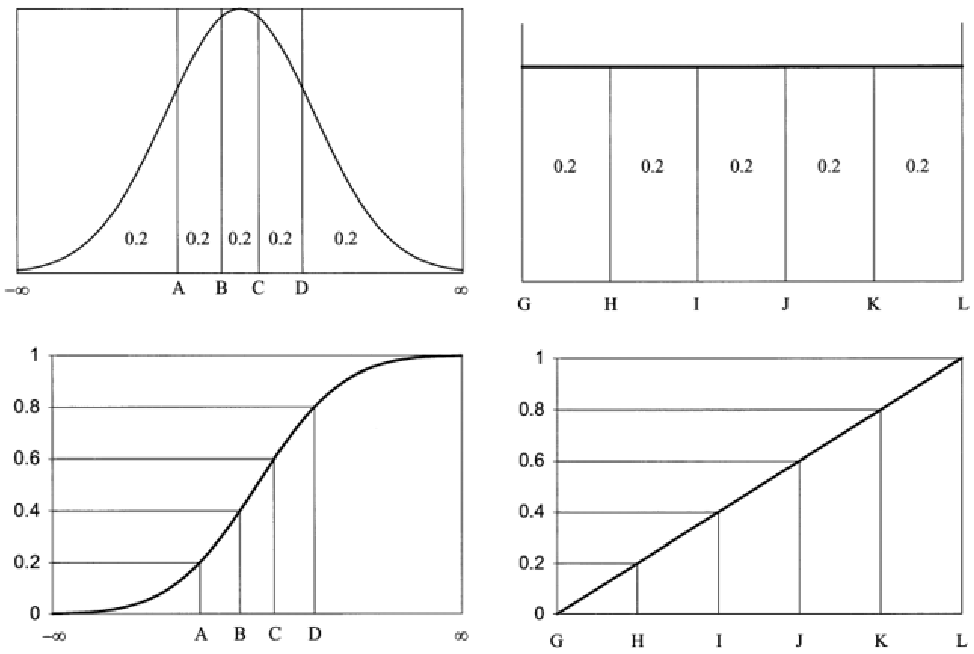

Figure 3-5 Intervals Used with a Latin Hypercube Sample of Size n= 5 for a Normal Random Variable (left) and Uniform Random Variable (right) [30]

3.3.2 Generating Combinations

Based on the above steps, a MATLAB code is written to produce samples for modelling using LHS. Once ranges of values are given for the parameters as explained in section 3.2, samples were generated and combined in a way that would give each value equal validity across the whole parameter range.

Table 3-8 Maximum and Minimum values of parameters

		Range Considered for this study		
Parameter No.	**Bridge specifications**	**Minimum**	**Maximum**	**Standard Deviation**
1	Span Length (*L*) in m	15	45	8.83
2	Track Stiffness (*MN/m*)	200	600	122.14
3	Transition Length factor (*f*)	0.25	1.0	0.25
4	Train Speed (*km/h*)	150	300	47.43
5	Concrete characteristics compressive strength, f_{ck} (MPa)	25	45	6.6

This will reduce the number of simulations from thousands (representing a full combination of all parameters) to the minimum required samples for *n* variables, which can be determined by the rule of thumb 2^n. In this case, for 5 variables, a minimum of 32 samples are required. The generated combinations of parameters are shown in Table 3-9.

Table 3-9 Combined parameters using LHS for modelling

Model No.	Span Length, *L* *(m)*	*Track Stiffness (MN/m)*	Transition length factor (*f*)	Train Speed, *V* *(km/h)*	Concrete compressive strength, f_{ck} *(MPa)*
1	35	500	0.75	210	35
2	33	450	0.75	179	40
3	40	300	0.5	285	25
4	43	520	1	160	25
5	40	250	0.75	220	30
6	43	470	0.25	246	45
7	20	200	0.25	260	35
8	18	495	0.5	160	35
9	23	295	0.25	170	25
10	38	330	0.25	279	30
11	25	250	0.5	235	35
12	35	350	0.25	265	30
13	25	400	0.5	200	30
14	28	445	0.75	289	40
15	45	550	0.75	185	40
16	15	400	1	295	25
17	33	215	0.5	267	30
18	23	520	0.75	195	45
19	43	310	0.75	297	35
20	18	210	1	215	25
21	35	450	1	275	45
22	20	300	0.75	175	30
23	40	550	1	250	40
24	25	450	0.75	225	45
25	33	255	0.5	284	40
26	38	235	0.5	199	35
27	30	350	0.25	155	40
28	28	245	0.25	242	35
29	38	500	0.75	272	30
30	30	500	0.5	190	35
31	28	550	0.5	299	45
32	15	600	0.5	165	40

Each combined sample is then modelled twice; one for Euro Train X2000 and one for High-Speed Load Model – A in both dynamic and static analysis as explained in section 3.2.4.2. A total of 128 simulations are made for those 32 model prototypes.

3.3.3 Calculation for model characteristics

The bridge characteristics (i.e. frequencies, modulus of elasticity, moment of inertia, and damping coefficients) as explained in section 3.2.1 are determined for each model. The results are shown in Table 3-10.

Table 3-10 Bridge characteristics for each span length

Model No.	M (ton/m) (from graph)	ω_1 (Hz) (from graph)	I (m^4) (as per the graph)	E_{cm} (G Pa)	ω_1 (Hz)	ω_2 (Hz)	ζ (%)
1	25.880	2.570	4.16	34	3.000	11.990	1.5
2	24.759	2.848	3.63	35	3.370	13.474	1.5
3	28.630	2.170	5.59	31	2.420	9.660	1.5
4	30.404	1.975	6.27	31	2.200	8.795	1.5
5	28.630	2.170	5.59	33	2.490	9.970	1.5
6	30.404	1.975	6.27	36	2.370	9.478	1.5
7	17.777	5.600	1.45	34	6.540	26.160	1.5
8	16.290	6.614	1.08	34	7.703	30.801	1.675
9	19.190	4.707	1.77	31	5.248	20.985	1.5
10	27.582	3.323	9.76	33	3.818	15.268	1.5
11	20.555	4.300	2.41	34	5.020	20.070	1.5
12	25.880	2.570	4.16	33	2.950	11.810	1.5
13	20.555	4.300	2.41	33	4.920	19.770	1.5
14	21.936	3.567	2.59	35	4.223	16.887	1.5
15	31.530	1.840	7.09	35	2.180	8.700	1.5
16	15.000	8.900	0.97	31	9.890	39.540	1.85
17	24.759	2.848	3.63	33	3.272	13.083	1.5
18	19.190	4.707	1.77	36	5.655	22.615	1.5
19	30.404	1.975	6.27	34	2.304	9.211	1.5
20	16.290	6.614	1.08	31	7.355	29.411	1.675
21	25.880	2.570	4.16	36	3.090	12.340	1.5
22	17.777	5.600	1.45	33	6.440	25.770	1.5
23	28.630	2.170	5.59	35	2.570	10.270	1.5
24	20.555	4.300	2.41	36	5.160	20.650	1.5
25	24.759	2.848	3.63	35	3.370	13.474	1.5
26	27.582	3.323	9.76	34	3.876	15.497	1.5
27	23.200	3.210	3.14	35	3.800	15.190	1.5
28	21.936	3.567	2.59	34	4.162	16.644	1.5
29	27.582	3.323	9.76	33	3.818	15.268	1.5
30	23.200	3.210	3.14	34	3.740	14.980	1.5
31	21.936	3.567	2.59	36	4.283	17.127	1.5
32	15.000	8.900	0.97	35	10.510	42.010	1.85

Similarly, the train characteristics (critical train and its specification) for each combination of span length and vehicle speed, are determined as explained in section 3.2.4.2. The results are shown in Table 3-11.

Table 3-11 HSLM-A critical train characteristics for each model

Model No.	L (m)	V (km/hr)	f_{ck} (MPa)	ω_1 (Hz)	λ_V (m)	λ_c (m)	Coach Length, D (m)	Bogie axle spacing, d(m)	Point force, P_k (kN)	No of intermediate coaches, N	HSLM-A Critical Train
1	35	210	35	3.000	70	27	27	2	210	11	10
2	33	179	40	3.370	53	27	27	2	210	11	10
3	40	285	25	2.420	118	22	22	2	170	14	5
4	43	160	25	2.200	73	21	21	3	190	15	4
5	40	220	30	2.490	88	22	22	2	170	14	5
6	43	246	45	2.370	104	21	21	3	190	15	4
7	20	260	35	6.540	40	27	27	2	210	11	10
8	18	160	35	7.703	21	21	21	3	190	15	4
9	23	170	25	5.248	32	27	27	2	210	11	10
10	38	279	30	3.818	73	20	20	2	180	16	3
11	25	235	35	5.020	47	27	27	2	210	11	10
12	35	265	30	2.950	90	27	27	2	210	11	10
13	25	200	30	4.920	41	27	27	2	210	11	10
14	28	289	40	4.223	68	27	27	2	210	11	10
15	45	185	40	2.180	85	27	27	2	210	11	10
16	15	295	25	9.890	30	27	27	2	210	11	10
17	33	267	30	3.272	82	27	27	2	210	11	10
18	23	195	45	5.655	34	27	27	2	210	11	10
19	43	297	35	2.304	129	21	21	3	190	15	4
20	18	215	25	7.355	29	27	27	2	210	11	10
21	35	275	45	3.090	89	27	27	2	210	11	10
22	20	175	30	6.440	27	27	27	2	210	11	10
23	40	250	40	2.570	97	22	22	2	170	14	5
24	25	225	45	5.160	44	27	27	2	210	11	10
25	33	284	40	3.370	84	27	27	2	210	11	10
26	38	199	35	3.876	51	20	20	2	180	16	3
27	30	155	40	3.800	41	27	27	2	210	11	10
28	28	242	35	4.162	58	27	27	2	210	11	10
29	38	272	30	3.818	71	20	20	2	180	16	3
30	30	190	35	3.740	51	27	27	2	210	11	10
31	28	299	45	4.283	70	27	27	2	210	11	10
32	15	165	40	10.510	16	14	27	2	210	11	10

4. MODELLING

The choice of a structural model is of great importance to enable realistic prediction of the real dynamic bridge response. Even though the level of detail in the model increases the accuracy of results, there is necessarily a trade-off between accuracy and computational time due to the large set of simulations that are normally required.

Dynamic analysis of bridges is assumed as a two-dimensional problem in several studies. This is because the bridge responses, such as deflection are important in the vertical direction. One dimension is along the bridge length which is along the passing of moving loads, and the other is vertical or in direction of load application.

4.1 Bridge Modelling

Two types of 2D beam elements are available in ABAQUS; the Timoshenko Beam (i.e. B21 element) and the Euler-Bernoulli Beam (i.e. B23 element). The difference between the two beams is the consideration of shear deformation in the Timoshenko beam, where it is neglected in the Euler-Bernoulli Beam element. B21 beam elements are considered in this study. The bridge is thus modelled in two-dimensions considering the two rails as one and the whole bridge as one Timoshenko beam element. The model takes into account shear deformation and rotational bending effects, making it suitable for describing the behavior of thick beams or beams subject to high-frequency excitation when the wavelength approaches the thickness of the beam. The beam element is resting on elastic supports.

4.2 Train Modelling

Various models from the simple model such as concentrated loads, or complicated ones such as the sprung mass model are available for modelling trains. Figure 4-1 shows common models of trains. In model (a) of Figure 4-1 each vehicle load is considered in the axle as a concentrated load and mass inertia is neglected. If the vehicle mass is not considerable in comparison to the bridge mass, this method will be precise and valid for calculating bridge dynamic responses. In cases where the mass of the vehicle is notable, moving mass method can be used as (d) of Figure 4-1. Increased speed and calculation power of computers have facilitated the application of more complicated models in analysis. In model (c) of Figure 4-1 the vehicle is considered as separate masses at each axle. In this model, the rotational effect of the vehicle body is neglected. And lastly, on Figure 4-1 (b) each train wagon is modelled as an integrated model [1].

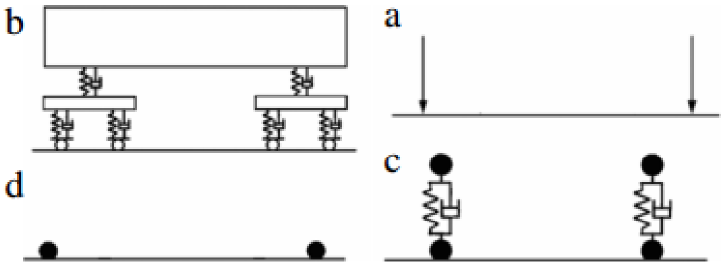

Figure 4-1 Various methods of vehicle modeling [1]

It has been determined from studies [11] and [31] that train modelling details mostly increase the precision of vehicle response calculations, and are less significant for bridge analysis as they are for passenger comfort assessment. The moving load model is simpler; however, it will only give accurate results when the ratio of the natural frequency of the vehicle and the bridge is much smaller than one and when the mass of the vehicle to mass of bridge ratio is small. The moving load model also neglects the inertia effect of the train mass and dynamic interaction between the train and the track.

From [32], we know that for span less than 30 m, simplified interaction model and detailed interaction model produce almost similar results. Therefore, the train is modelled with a simplified interaction model of the sprung mass as shown in Figure 4-1 (c). This is a reasonable simplification making it easier to construct and consume less analysis time for nearly similar accuracy.

Each axle of the simplified interaction model, as shown in Figure 4-2, consist of two masses connected by a spring and a damper. Upper mass represents the suspended mass of the bogie and the vehicle body mass, and the lower mass represents the unsprung mass of the wheelset, while spring and damper represent the primary suspension system. X2000 train and HSLM-A train are modelled as described here.

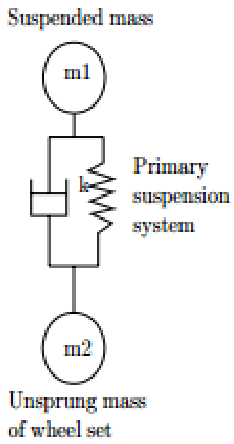

Figure 4-2 Sprung mass axle

4.3 Bridge-Track-Train Modelling

The track system lying on the bridge is modelled with an infinite length of rail supported by a continuous and homogenous viscoelastic foundation of springs and dampers. The bridge track model incorporates an extra track structure of length L_x and transition zone with length L_t and the bridge span length L_b. The track structure and the transition zone are modelled on both sides of the bridge making the model longitudinally symmetrical. The sprung mass model of the train is placed just before entering the transition length as shown in Figure 4-3.

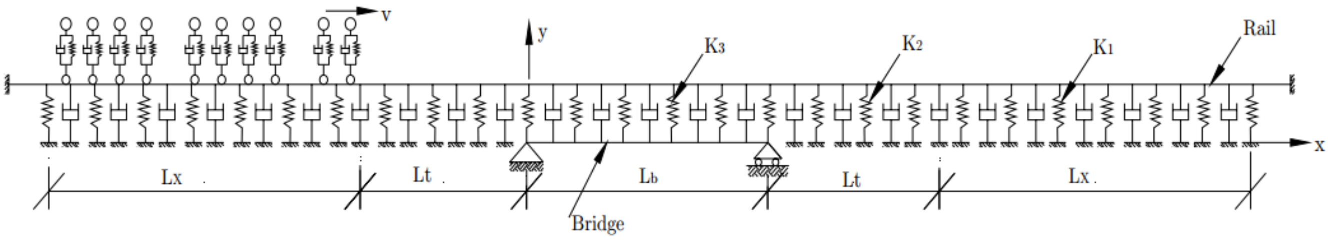

Figure 4-3 Bridge-track-train model

4.4 Summary of Assumed Model

The model considerations as explained in the previous sections are summarized in this section based on Figure 4-4.

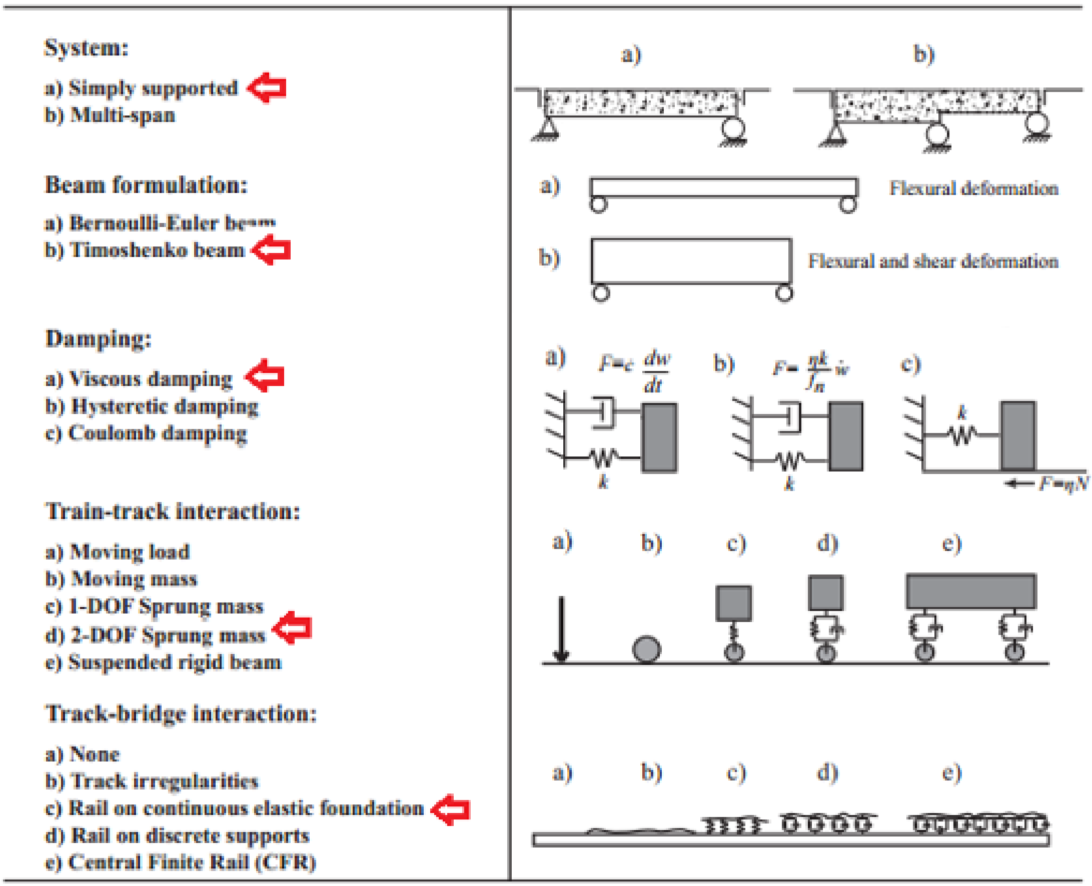

Figure 4-4 Summary of the assumed model (arrow indicating the model consideration)

System Definition: Bridges could either be simply supported or multiple span continuous bridges. Since a simply supported bridge is considered in this study, the system is taken as simply supported as shown in a) on Figure 4-4.

Beam formulation: The difference between Euler Bernoulli Beam and Timoshenko beam are in the assumptions of both theories. In the Euler-Bernoulli the cross-section is perpendicular to the bending line. In a Timoshenko beam, a rotation between the cross section and the bending line is allowed. This rotation comes from a shear deformation, which is not included in a Bernoulli beam. Timoshenko beam is mostly used to represent thick beams as shown in Figure 4-4 and it is considered in this study to model the whole bridge as one beam.

Damping definition: Viscous damping is a common form of damping which is found in many engineering systems. The viscous damping force is proportional to the first power of the velocity across the damper, and it always opposes the motion, so that the damping force is a linear continuous function of the velocity. Coulomb damping is a type of constant mechanical damping in which energy is absorbed via sliding friction. The friction generated by the relative motion of the two surfaces that press against each other is a source of energy dissipation. Hysteresis damping is the damping caused by the friction between the internal planes that slip or slide as the material deforms. Viscous damping is considered in this study with a spring-damper system represented with spring stiffness and damping coefficient values.

Train – Track interaction: There are several methods of modelling a train on a track. Moving load model considers no interaction between the train and track and the train's axles are simply represented by concentrated loads. Moving Mass model is used when the mass of the train vehicle is large and it is considered as a mass rolling on a bridge surface. The sprung mass system could either have one or two degrees of freedom depending on the consideration of the damping at the primary suspension. The full interaction model considers both the primary and secondary suspensions and takes the vehicle as a rigid beam. For this study as explained in section 4.2 the sprung mass model with a spring-damper system of the train's primary suspension is considered.

Track – Bridge interaction: The track is modelled as a rail resting on viscoelastic supports representing the stiffness of the track. The track stiffness is the aggregate stiffness of the ballast, sleepers and the connections between the rail and sleepers.

4.5 ABAQUS Modelling

4.5.1 Abaqus2Matlab

Abaqus2Matlab serves to provide a connection between ABAQUS and MATLAB software. The first is a sophisticated finite element package and the second is the programming language of technical computing. And Abaqus2Matlab offers a connection between simulation and programming. Simulation offers advanced modelling, which would be a laborious task if only programming was used to make it. On the other hand, simulation loses flexibility in handling its various results or data, to adjust on a case-dependent basis. Abaqus2Matlab is a first attempt to combine and reconcile these two aspects.

The model is developed on a MATLAB script which serves as a template to develop the model by adjusting the desired parameters and writing an ABAQUS input file. ABAQUS is called with a dos function to perform the analysis or by importing the generated input file manually from ABAQUS and creating the job from there.

The versions of software used are Abaqus/CAE 6.13-1 [3] and MATLAB R2016a [4]. The analysis flow chart is shown in Figure 4-5, as used by [6] and the steps are briefly explained here. A sample code from Model 1 having a bridge span length of 35 m, track stiffness of 500 MN/m, transition length 75% of bridge span length, train Speed of 210 km/h and Concrete compressive strength of 35 MPa is provided in APPENDIX D.

Step 1 - Input Variables: input variables are provided in MATLAB, which remain constant for one type of the bridge. These input variables include damping ratio, sleeper spacing, stiffness of ballast, stiffness of bridge and rail structure, span length and element size, etc.

Step 2 - Creating the bridge structure: the bridge structure is created from the inputs during the analysis.

Step 3 – Creating the sprung mass system: each axle is created with two masses connected by a spring and a damper. Complete train is modeled with these axles and move over the rail called master surface.

Step 4 – Writing ABAQUS input file: the bridge structure and loads are then copied in the main input file, which is then used for analysis in the ABAQUS.

Step 5 – Finite Element Analysis: direct integration method is used for the analysis.

Step 6 - Filtering the data: Butterworth filter is used to exclude the higher frequencies from the results which will be explained in section 4.5.2. Higher frequency accelerations or displacements do not have any significant effect on the ballast and need to be filtered out. Eurocode [8] recommends that all modes with frequencies higher than 30 Hz or 1.5 times the natural frequency, including at least three modes of vibration, should be excluded from the results.

Step 7 – Read maximum displacement.

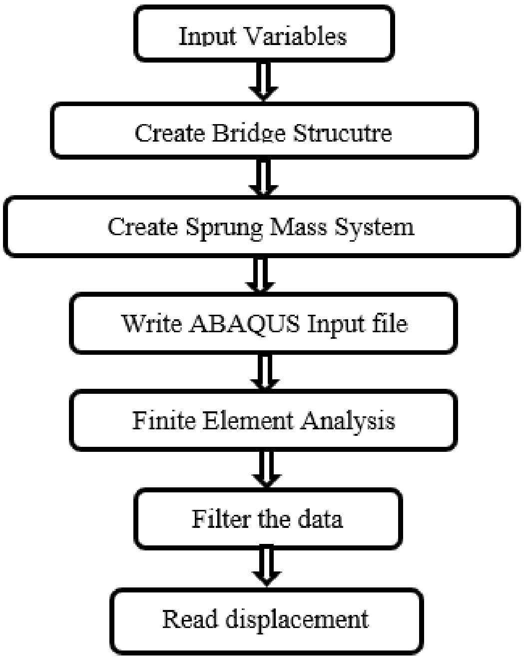

Figure 4-5 Schematic diagram for Sprung mass MATLAB-ABAQUS model

4.5.2 Filtering history data [33]

Filters can be created in ABAQUS and can be applied to history output requests. ABAQUS filters history data when the analysis is running; filtering during the analysis, which is called as "real-time" filtering can reduce the size of the output database by excluding high-frequency data before it is saved. ABAQUS offers 3 Filter toolsets; Butterworth, Type I Chebyshev, and Type II Chebyshev. The different filter types are distinguished by their capabilities to transform from the acceptance of low-frequency data to the rejection of the data above the filter's cut off frequency.

An ideal filter would stop all data above the cutoff frequency and have a flat response (no effect on accepted data); "real" filters include a transition band around the cutoff where some data are accepted and they usually have some effect on the accepted data. The Butterworth filter provides a maximally flat response magnitude with a wider transition band (slower transition) than the Chebyshev filters.

The Chebyshev filters introduce an oscillation—a ripple—in the response magnitude, but they have a narrower transition band than the Butterworth filter. The two types of Chebyshev filters differ in where in their response ripples occur; the Ripple factor indicates how much oscillation you will allow in exchange for an improved filter response.

Butterworth filters are specified by two parameters, the cutoff frequency and the filter order [33]. The filter order determines the size of the filter's transition band. The higher the order, the narrower the transition band, the higher the computational cost. The filter order must be a positive, even integer no greater than 20. The filter order is the delay in samples used in creating each output samples. Higher order filters do better in approximating the "brick-wall" as shown in Figure 4-6.

Due to its simplicity and ability to provide a more linear phase response in the passband, the ***Butterworth filter is selected for this research. The cut-off frequency is taken as 50 Hz with order 6.*** A MATLAB syntax for Butterworth filter is available on MATLAB R2016a documentation [34] which has been used in this research.

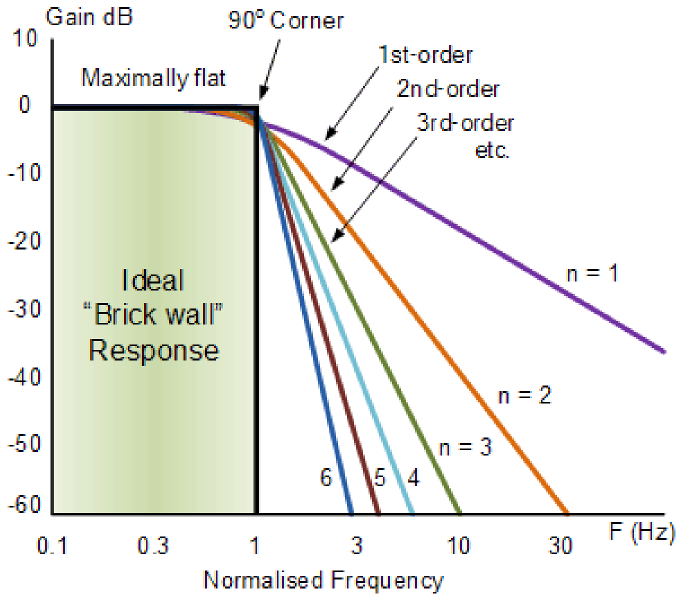

Figure 4-6 Effect of increasing filter order in idealizing the brick wall response of Butterworth filter [35]

Note: The higher the Butterworth filter order, the higher the number of cascaded stages there are within the filter design, and the closer the filter becomes to the ideal "brick wall" response. However, in practice, this "ideal" frequency response is unattainable as it produces excessive passband ripple.

4.5.3 Model validation

Experimental data serve as valuable input in understanding the real structural behavior of a bridge and for updating or revising the proposed structural models [36]. The developed model in this research was validated using bridge data from Sweden. The Banafjäl bridge is high-speed railway line in Northern Sweden, which has the characteristics shown in Table 4-1. This bridge was mainly selected due to the several research and experiments done for it, and thus the results are reliable.

Table 4-1 Bridge data for model validation

Name	Banafjäl bridge
Support	*Simple*
Material	*Reinforced concrete*
Length	*40 m*
Design Speed	*250 km/h*
Operating Speed	*200 km/h*
Mass	*18140 kg/m*
EI	$1.31x10^{-11}$ Nm^2
P	170 kN

Both concrete and steel (rail) were modelled as Timoshenko beam elements in the elastic range satisfying Hooke's material law, where the material behavior is assumed to be completely defined by two physical constants, the Elastic modulus, and Poisson's ratio. The deformations are considered to be small that do not significantly affect the action of external forces.

Table 4-2 Concrete and Steel material behaviors

	Concrete	**Steel**
Elastic Modulus, *E (GPa)*	31	210
Poisson's ratio, *v*	0.2	0.3

The results from the exact data and the ABAQUS-MATLAB program are shown in Figure 4-7 and Figure 4-8, respectively. These graphs show a very good resemblance.

Table 4-3 Comparison of displacement from exact measurement and ABAQUS-MATLAB for concentrated load

	Exact measurement	**ABAQUS-MATLAB**
Midspan displacement	3 mm	2.91 mm

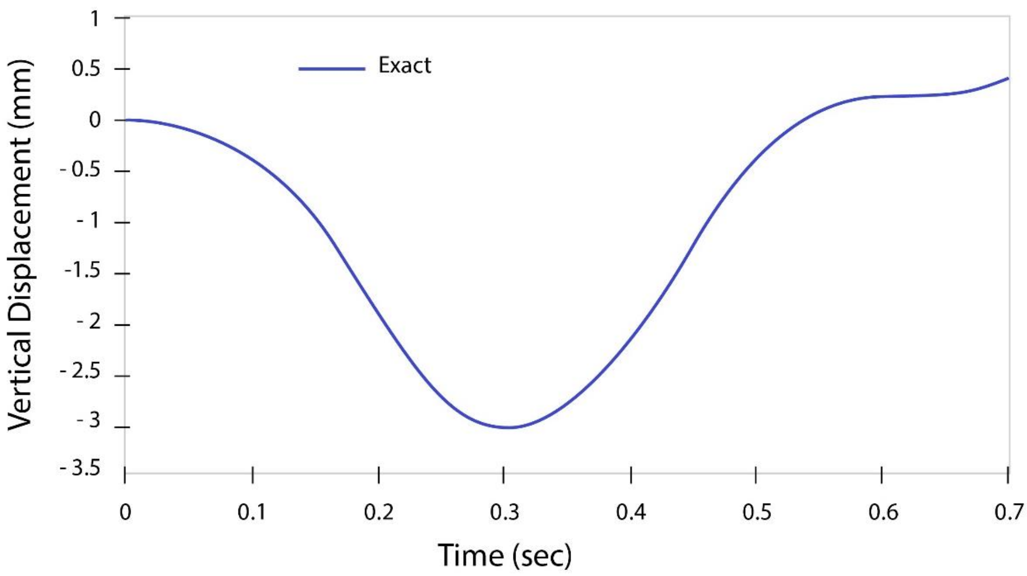

Figure 4-7 Results of exact mid-span displacement of Banafjäl bridge

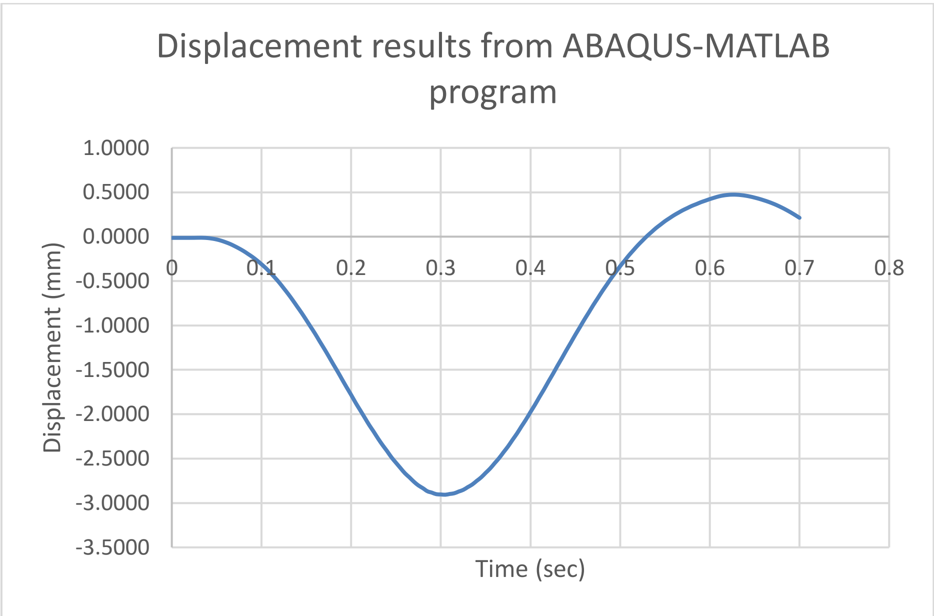

Figure 4-8 Results of the mid-span displacement of Banafjäl bridge from ABAQUS-MATLAB program

The same model was also checked with HSLM-A1 against a previously done work [6] as given in Figure 4-10. Results are compared in Table 4-4.

Table 4-4 Comparison of displacement from previous work and ABAQUS-MATLAB model for HSLM-A1 load

	Previous work	ABAQUS-MATLAB
Mid span displacement @ V = 250 km/h	0.014 m	0.011 m
Mid span displacement @ V = 120 km/h	0.0096 m	0.0094 m

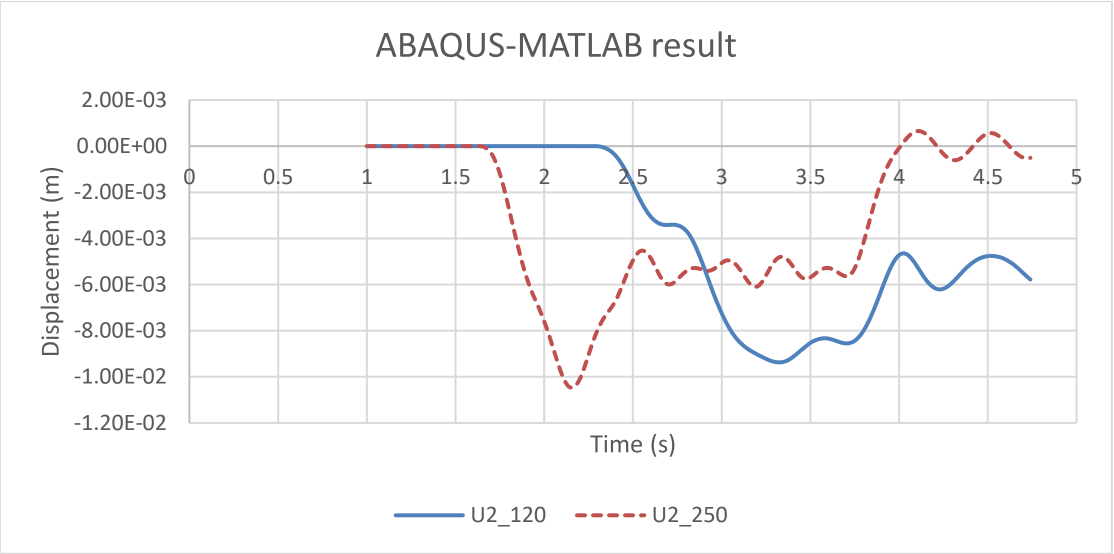

Figure 4-9 Displacement for HSLM-A1with speed 120 and 250 km/h from ABAQUS-MATLAB

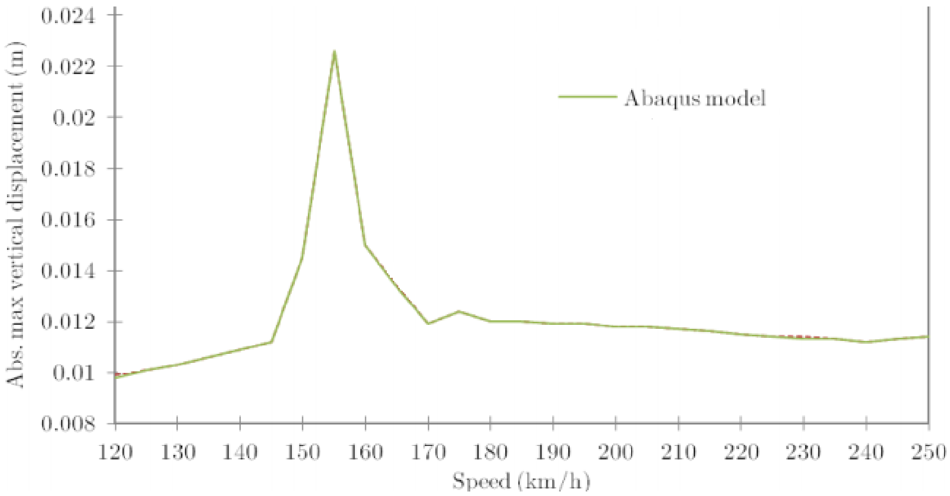

Figure 4-10 Speed vs. displacement data taken from previous research for Banafjäl bridge [6].

5. RESULTS AND DISCUSSION

5.1 Dynamic Analysis

After validation of the model, dynamic analysis was performed and results were produced for the 32 prototype models with both the Real Train (X2000) and HSLM-A . Sprung mass model for both types of trains was used. Maximum displacement has been determined for both X2000 and HSLM, and the absolute maximum values are taken for further investigation as specified by Eurocode. The results can be seen in Table 5-1.

From Table 5-1 col. 2 and 3, the dynamic displacement results of the two train models, it can be noted that there are indeed cases – Bridge Model 3, 10, 19, 23, and 29 – where the response from Real Train was larger than the response from the High-Speed Load Model. This supports Eurocode's recommendation on performing the analysis using both Real Trains and HSLM to determine the maximum dynamic response. In this case, the differences between the results of the train models were in the range of < 2mm. This implies, despite the efforts made to determine the worst possible trainload from the codes, prior knowledge of the train expected to use the bridge track is useful in the determination of the actual effects of the moving train on the bridge.

5.2 Static Analysis

For the static analysis to be performed, determination of location and arrangement of the train that will yield maximum displacement at mid-span is necessary. This is not an easy task and requires several trials, especially for Real Train which has varying loads on each axle. It is found by determining the location of loads that will yield the heaviest resultant load at the center of the bridge resulting in maximum displacement at that location. For HSLM, since all the axles carry similar load, the section of the train that has smaller axle spacing is loaded on the bridge that will give the maximum resultant at the center. A Microsoft Excel template was prepared to determine the location and arrangement of the loads for the 32 bridge spans for both X2000 train and the HSLM-A load model.

The results from the static analysis are as shown in Table 5-1 col. 5 and 6 for both X2000 Train and HSLM. The absolute maximum displacement values are taken to determine the DAF.

Table 5-1 Displacement results from dynamic analysis for each model

Model No.	Dynamic Analysis			Static Analysis		
	Max Displacement (mm) X2000	Max Displacement (mm) HSLM	Abs. max. Displacement (mm)	Max Displacement (mm) X2000	Max Displacement (mm) HSLM	Abs. max. Displacement (mm)
1	5.606	7.721	7.721	5.224	6.025	6.025
2	4.967	6.142	6.142	4.490	5.129	5.129
3	9.094	8.077	9.094	6.451	6.571	6.571
4	8.128	9.763	9.763	7.350	7.975	7.975
5	7.188	7.293	7.293	6.061	6.174	6.174
6	7.822	8.032	8.032	6.331	6.869	6.869
7	2.134	3.058	3.058	1.832	1.289	1.832
8	1.511	2.037	2.037	1.470	0.715	1.470
9	3.035	4.013	4.013	2.629	2.136	2.629
10	7.611	7.040	7.611	5.951	6.043	6.043
11	2.978	3.851	3.851	2.637	2.263	2.637
12	6.363	7.978	7.978	5.384	6.209	6.209
13	3.160	3.798	3.798	2.717	2.830	2.830
14	3.650	4.925	4.925	3.416	3.650	3.650
15	7.607	10.269	10.269	6.145	8.527	8.527
16	1.114	1.747	1.747	0.931	1.019	1.019
17	5.398	6.396	6.396	4.764	5.441	5.441
18	2.639	3.585	3.585	2.264	1.840	2.264
19	9.334	9.259	9.334	6.704	7.274	7.274
20	1.673	2.699	2.699	1.613	0.874	1.613
21	5.808	7.218	7.218	4.935	5.691	5.691
22	2.141	3.089	3.089	1.887	1.330	1.887
23	7.222	6.790	7.222	5.714	5.820	5.820
24	2.867	3.295	3.295	2.491	2.595	2.595
25	5.183	6.828	6.828	4.492	5.130	5.130
26	6.173	7.295	7.295	5.778	5.866	5.866
27	4.154	4.513	4.513	3.898	4.373	4.373
28	3.717	5.021	5.021	3.518	3.758	3.758
29	7.233	6.811	7.233	5.775	5.864	5.864
30	4.498	5.370	5.370	4.012	4.500	4.500
31	3.599	4.764	4.764	3.321	3.549	3.549
32	0.946	1.573	1.573	0.825	0.902	0.902

5.3 Dynamic Amplification Factor

Dynamic amplification is calculated from the dynamic and static responses as described in section 1.1 using equation (1), repeated here for convenience.

$$DA = \frac{R_{dyn} - R_{sta}}{R_{sta}}$$

where R_{dyn} is the maximum dynamic response of the bridge and R_{sta} is the maximum static response. Therefore, the dynamic amplification factor will be, *DAF=(1+DA).* Accordingly, the values for each bridges prototype are determined in Table 5-2 from the absolute static and dynamic displacements.

Table 5-2 Dynamic Amplification Factor for each model

Model No.	Displacement dynamic (*mm*)	Displacement static (*mm*)	Dynamic Amplification, DA	Dynamic Amplification Factor, DAF
1	5.224	6.025	0.281	1.281
2	4.490	5.129	0.198	1.198
3	6.451	6.571	0.384	1.384
4	7.350	7.975	0.224	1.224
5	6.061	6.174	0.181	1.181
6	6.331	6.869	0.169	1.169
7	1.832	1.289	0.669	1.669
8	1.470	0.715	0.386	1.386
9	2.629	2.136	0.526	1.526
10	5.951	6.043	0.259	1.259
11	2.637	2.263	0.460	1.460
12	5.384	6.209	0.285	1.285
13	2.717	2.830	0.342	1.342
14	3.416	3.650	0.349	1.349
15	6.145	8.527	0.204	1.204
16	0.931	1.019	0.714	1.714
17	4.764	5.441	0.176	1.176
18	2.264	1.840	0.583	1.583
19	6.704	7.274	0.283	1.283
20	1.613	0.874	0.673	1.673
21	4.935	5.691	0.268	1.268
22	1.887	1.330	0.637	1.637
23	5.714	5.820	0.241	1.241
24	2.491	2.595	0.270	1.270
25	4.492	5.130	0.331	1.331
26	5.778	5.866	0.244	1.244
27	3.898	4.373	0.032	1.032
28	3.518	3.758	0.336	1.336
29	5.775	5.864	0.233	1.233
30	4.012	4.500	0.193	1.193
31	3.321	3.549	0.342	1.342
32	0.825	0.902	0.744	1.744

5.3.1 Correlation and Significance test

The Pearson's correlation coefficient, r can range in value from −1 to +1; 0 indicating no correlation and +1 indicating perfect linear relationship. It can be determined from:

$$Corr(x,y) = \frac{Cov(x,y)}{\sigma_x \sigma_y}$$
$$Cov(x,y) = \frac{\sum((x-\bar{x})(y-\bar{y}))}{n-1} \tag{19}$$

Where Cov(x,y) is the covariance of parameters x and y, and σ is the standard deviation.

The significance of a correlation can also be determined by comparing the p-value to the significance level, α, which is usually taken as 0.05. A value of 0.05 indicates that the risk of concluding that a correlation exists—when, actually, no correlation exists—is 5%. The p-value generally tells whether the correlation coefficient is significantly different from 0.

It should be noted that if the P-value ≤ 0.05, indicates that the correlation is statistically significant and the correlation coefficient is different from zero, but the reverse does not necessarily mean that there is no correlation. It only tells that we cannot conclude that the correlation is different from 0. The plus or minus sign of Pearson's coefficient indicates the direction of the relationship. If both variables tend to increase or decrease together, the coefficient is positive, and the line that represents the correlation slopes upward. If one variable tends to increase as the other decreases, the coefficient is negative, and the line that represents the correlation slopes downward.

The correlation of the 5 parameters, namely bridge span length, concrete compressive strength, transition length, vehicle speed, and track Stiffness were studied. Test for correlation and significance were made among the parameters using IBM SPSS Statistics 20.0.0 and the output is described in the following paragraphs. The statistical descriptions of the parameters with their correlation coefficient are shown in Table 5-3.

The correlation between bridge span length and DAF has been flagged as highly correlated with Pearson's r-value of -0.777. This is no surprise as several DAF equations from codes are explicitly defined in terms of bridge span length. The negative value indicates that DAF decreases with an increase in length. This parameter has scored highly significant with a P-value of 0.000.

The next correlated parameter is concrete compressive strength with r- value of -0.241. The negative here also indicates that DAF decreases as we go from low-grade concrete to a higher-grade concrete.

Track stiffness is correlated with DAF with r-value of 0.127. There is a positive correlation between transition length factor and DAF, with r-value of 0.11. The vehicle speed is correlated to DAF, only very slightly with r-value of -0.018.

However, care should be taken in interpreting correlation and significance values determined from the regression coefficients because these numbers do not take under consideration the scales of these parameters and the covariance of each parameter. A parameter with small correlation coefficient would have a small regression coefficient, but should not be disregarded and left out of the regression equation because the value of that parameter could be large enough, even when multiplied with its smaller regression coefficient could outweigh the other parameters. Therefore, sensitivity analysis needs to be performed after a regression equation is developed inclusive of all parameters.

Table 5-3 Statistical Description of the parametric values

	N	Range	Minimum	Maximum	Std. Deviation	Variance	Correlation Coefficient
	Statistic	Statistic	Statistic	Statistic	Statistic	Statistic	
L in meters	32	30.00	15.00	45.00	8.83170	77.999	-0.777
f_{ck} in MPa	32	20.00	25.00	45.00	6.59912	43.548	-0.241
f	32	.75	.25	1.00	.25288	.064	0.11
V in km/hr	32	144.00	155.00	299.00	47.42871	2249.483	-0.018
S in MN/m	32	400.00	200.00	600.00	122.13645	14917.314	0.127
DAF	32	.71	1.03	1.74	.18193	.033	
Valid N (listwise)	32						

5.3.2 Developing DAF Formula

Multiple regression is an extension of simple regression. It is used when we want to predict the value of a variable based on the value of two or more other variables. The variable we want to predict is called the dependent variable, in this case, the DAF. The variables we are using to predict the value of the dependent variable are called the independent variables, which are *L*, f_{ck}, *f, V, and S.*

Multiple regression also allows the determination of the overall fit (variance explained) of the model and the relative contribution of each of the predictors to the total variance explained. For example, we might want to know how much of the variation in dynamic amplification factor can be explained by bridge span length, concrete compressive strength, transition length factor, train speed, and track stiffness "as a whole", but also the "relative contribution" of each of these independent variables in explaining the variance.

An equation is developed using; the linear and polynomial regressions. The linear regression is performed in 3 methods: level-level, log-level, and log-log linear regression. The polynomial regression was also performed in the second order, with and without cross terms. And the best fitting equations is selected out of these 4 equations based on the regression model having the highest R Squared value. The regression results are summarized in Table 5-4.

Table 5-4 DAF equations from different regression models

Equation No.	Regression Model	DAF equation	R^2	Remark
1	Level - Level	$DAF = 1.830 – 0.017L – 0.005f_{ck} + 0.1f + 0.001V + 3.5x10^{-5}S \geq 1.0$	0.83	DAF statically predicted with F(5,26) = 11.34, P=0.000. Reasonable fit.
2	Log - Level	$DAF = e^{0.617 – 0.012 L – 0.004 fck + 0.071 f + 0.001 V – 3.23x10-5 S} \geq 1.0$	0.82	DAF statically predicted with F(5,26) = 10.8, P=0.000
3	Log - Log	$DAF = 3.4L^{-0.34} f_{ck}^{-0.108} f^{0.03} V^{0.123} S^{-0.008} \geq 1.0$	0.85	DAF statically predicted with F(5,26) = 14.07, P=0.000. Good fit.
4	Level - Log	$DAF = 2.792 – 0.482 \ln(L) – 0.147 \ln(f_{ck}) + 0.041 \ln(f) + 0.151 \ln(V) – 0.015 \ln(S) \geq 1.0$	0.86	DAF statically predicted with F(5,26) = 15.135, P=0.000. This model fits the data very well.
5	Polynomial with no cross terms (2nd order)	$DAF = 2.967 – 0.068L – 0.04f_{ck} + 0.206 f + 0.005V – 0.002S + 0.001L^2 + 0.01 f_{ck}^2 - 0.156 f^2 – 8.28x10^{-6} V^2 + 2.21x10^{-6} S^2 \geq 1.0$	0.83	
6	Polynomial with cross terms (2nd order)	$DAF = 4.501 - 0.098L - 0.047f_{ck} + 0.258f - 0.002V - 0.003S + 0.001L^2 + 0.001f_{ck}^2 - 0.322f^2 - 8.5x10^{-7}V^2 + 2.57x10^{-6}S^2 - 9.45x10^{-5}Lf_{ck} – 0.013Lf + 5.54x10^{-5}LV + 8.9x10^{-6}LS + 0.019f_{ck}f + 3.53x10^{-5}fckV – 2.11x10^{-5}f_{ck}S – 0.002fV + 0.001fS + 3.3x10^{-6}VS$	0.90	Best fit but equation is too complicated.

Therefore, the best fitting equation for DAF is determined from the 2nd order polynomial with cross terms regression model as shown on equation 6 having the highest R^2 value of 90%. However, for simplicity, the level-level linear regression equation is adopted for this study in which the parameters explain 83% of the variability of DAF.

$$DAF = 1.83 - 0.017(L) - 0.005\left(f_{ck}\right) + 0.1\left(f\right) + 0.001\left(V\right) + 3.5x10^{-5}\left(S\right) \qquad (20)$$

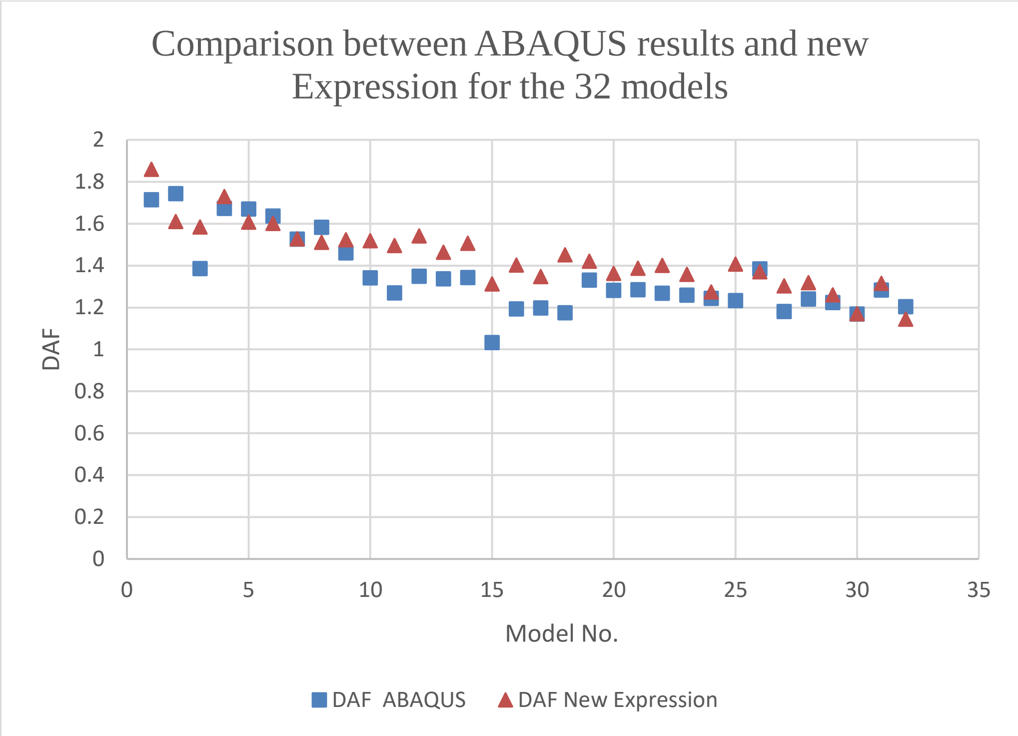

Figure 5-1 Graph showing the DAF from ABAQUS result and the new expression for the 32 bridge prototypes

5.4 Sensitivity analysis

Sensitivity analysis is the technique used to determine how the variation (uncertainty) in the output of a statistical model can be attributed to different variation in the inputs of the model. It is used to select the order of importance of random variables. The sensitivity of each random variable is represented by the squared value of the partial coefficient of correlation. It can be calculated as:

$$\alpha_i = \frac{\partial F}{\partial x_i} \frac{\overline{x_i}}{\overline{\overline{F}}} \qquad (i = 1, 2, 3, ..., n) \qquad (21)$$

Where α_i is the sensitivity factor of random variable i

F is the function with statistical variation in this case

$$F = DAF = 1.83 - 0.017(L) - 0.005\left(f_{ck}\right) + 0.1\left(f\right) + 0.001\left(V\right) + 3.5x10^{-5}\left(S\right)$$

$\overline{F}$ *and* $\overline{x_i}$ are the means of F and the random variable i

x_i is the random variable

Table 5-5 Sensitivity Analysis for DAF

x_i	$\overline{x_i}$	$\overline{F}$	$\frac{\partial F}{\partial x_i}$	α_i	Cov_i	$U_i = \alpha_i \, Cov_i$	Order of significance
L	30.5	1.3505	0.017	0.384	1.247	0.478	1
f_{ck}	35.0	1.3505	0.005	0.130	0.289	0.037	2
S	389.9	1.3505	0.000035	0.010	2.828	0.029	3
V	229.5	1.3505	0.001	0.170	0.151	0.026	4
f	0.6	1.3505	0.1	0.045	0.005	0.000	5

Therefore, the most sensitive parameter is indeed bridge span length. However, the transition length factor has nearly no influence on the DAF. Therefore, the *f* term can be omitted from the equation.

$$DAF = 1.83 - 0.017(L) - 0.005\left(f_{ck}\right) + 0.001\left(V\right) + 3.5x10^{-5}\left(S\right) \qquad (22)$$

5.5 Comparison with Codes

In Figure 5-2, a comparison is made among the new developed expression for DAF with those provided on various Railway Bridge Design codes, such as Eurocode, AREMA and Iranian Code. In developing the graph, the mean values were used for the parameters other than span length (i.e. *V=230 km/h, fck = 35 GPa, S = 390 MN/m*).

As it can be seen on the graph, there is a significant underestimation of DAF as provided from Codes for short span bridges than long span. The Iranian Code estimate of DAF gives the lowest values. The values from the newly developed expression are everywhere above the provision from Eurocode, whereas AREMA code gives higher DAF value for spans greater than 40 m. The developed expression provides a higher estimate of DAF for railway bridges.

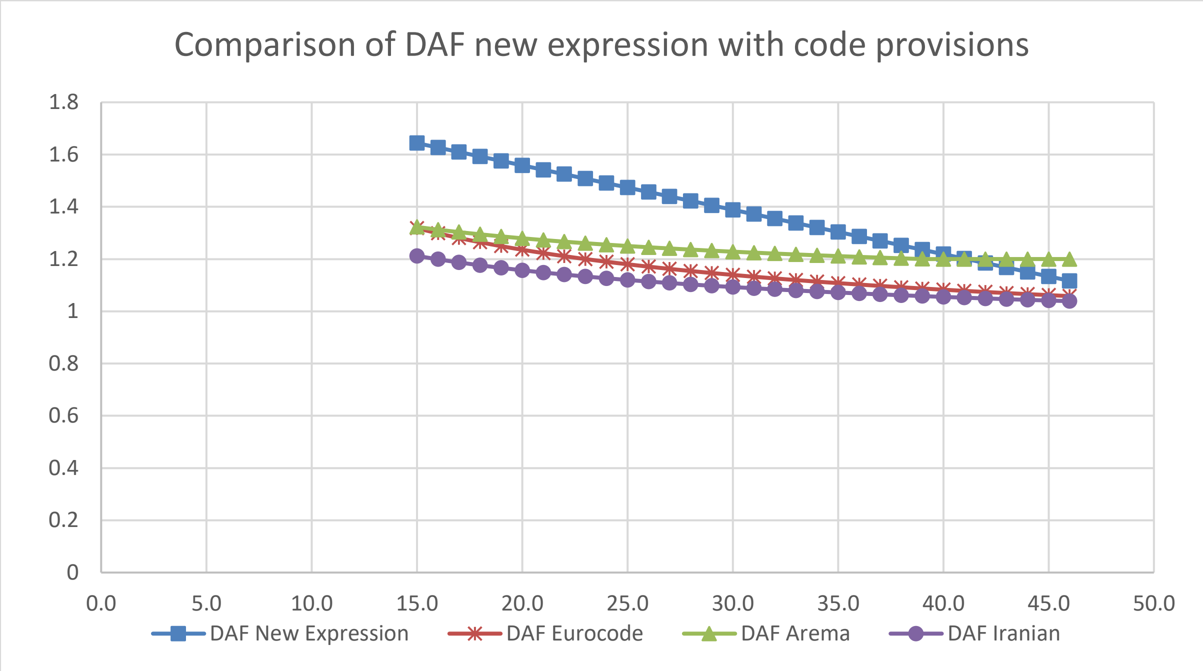

Figure 5-2 Code Comparison for DAF expression

6. CONCLUSION AND RECOMMENDATION

6.1 Conclusion

In this research, a parametric study has been conducted to specify the effects of various parameters including bridge span length, track stiffness, train speed, concrete compressive strength and length of transition on reinforced concrete railway bridge responses and dynamic amplification factor. DAF has been calculated for 32 bridge prototypes whose parametric values have been systematically combined to give representative samples using LHS method.

Bridge span length from *15 m to 45 m* were studied, to cover both short span and medium span bridges. Two Types of trains, a Euro Train called X2000 for the Real Train model and HSLM-A from Eurocode have been used to idealize the vehicle load running with speeds ranging from *150 to 300 km/h.* In order to determine the effect of track stiffness on the dynamic response, spring stiffness ranging from 200 to 600 MN/m have been used. To also analyze the effect of using higher concrete grades on DAF of bridges, five different types of grades of concrete were used having characteristics compressive strength value of *25, 30,35, 40, 45 MPa.* To determine the effect of transition zones on DAF, lengths of transitions of 0.25L, 0.5L, 0.75L, and L were used.

Considering the research results and comparing them to bridge design codes show that in many cases the dynamic amplification factor values proposed by several bridge design codes are underestimated, and this is even more true for short span bridges as compared to long span bridges.

Dynamic amplification factor relations offered in the design codes have been defined based on the span length only, where there is a decrease in the amplification factor with an increase in span length. The results from this research also indicate a similar inverse relationship between the two parameters. However, in addition to bridge span length, concrete grade, track stiffness, and vehicle speed have been found to be important parameters in the determination of Dynamic amplification factor. Another finding is that the transition length factor has no significant effect on the DAF.

The results show that with an increase in speed and track stiffness, the DAF value increases whereas with an increase in length and concrete compressive strength, the DAF value decreases.

From the results of the analysis, a formula is proposed for DAF as a function of span length, concrete compressive strength, vehicle speed, and track stiffness.

In summary, the following conclusions can be drawn from this research.

1. The bridge span length is indeed the most significant parameter in the determination of DAF value. A 1% increase in span length will bring a 1.6% decrease in DAF value.
2. With an increase in train speed, there is a significant increase in the dynamic response of a bridge. A 1% increase in train speed brings a 1.4% increase in DAF value.
3. Using higher grade concrete can lower the dynamic response of a bridge. A 1.1% decrease in DAF can be expected from a 1% increase in concrete compressive strength.
4. Track stiffness slightly affects the dynamic response of a bridge. A 1% increase in track stiffness brings 0.1% increases in DAF.
5. Increase in the area of the transition zone does not have any impact on the dynamic response of the bridge. Thus, the minimum length of transition as required for other requirements such as passenger comfort should be provided.
6. Dynamic response from Real Train specific to the project could indeed result in maximum effects as compared with a response due to specified live loads from codes. This supports Eurocode’s recommendation on performing the analysis not just with the High-Speed Load Model but also with the real train expected to use the bridge to determine the maximum effect for designing the railway bridge. This indicates knowledge of the specifications of the train expected to use the bridge during the design phase is vital.

Finally, this research recommends the determination of DAF as a function of span length, concrete characteristics compressive strength, train speed and track stiffness within their ranges of applicability using the following relationship:

$$DAF = 1.83 - 0.017(L) - 0.005\left(f_{ck}\right) + 0.001\left(V\right) + 3.5x10^{-5}\left(S\right)$$

where:

L is Bridge Span Length in m

fck is the Characteristics Compressive Strength of Concrete in MPa

V is the Train Speed in km/h, and

S is the Track Stiffness in MN/m

6.2 Recommendation for future works

1. The same research could be performed by increasing the number of samples to increase the precision of the coefficients for the DAF equation.
2. Performing the analysis with the 4 parameters identified from this study plus 2 additional parameters (axle spacing and axle load) would result in an even more comprehensive DAF equation.
3. The same research could be performed in a 3D model for comparison of results by taking into account the effects of other bridge parameters such as the bridge cross section (no and size of girders, the thickness of deck, etc.), presence of bearings, presence of cross girders, etc.
4. The same research could be performed by extending the ranges of parameter values considered in this study, such as spans greater than 45 m and lower speeds.
5. A study to reduce the analysis time of the sprung mass model could go a long way in enabling the assessment of more parameters and their combinations.
6. Even though track stiffness has a significant influence on DAF, there is no provision for track stiffness in Eurocode. A study could be made to come up with optimized stiffness and damping values for railway bridges.

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APPENDIX A

MATLAB code: To generate LHS combinations (From Budiman)

```
%   Budiman (2004)  budiman@acss.usyd.edu.au    Revised: Nov 2004
function s=lhsu(xmin,xmax,nsample)
% s=lhsu(xmin,xmax,nsample)
% LHS from uniform distribution
% Input:
%   xmin    : min of data (1,nvar)
%   xmax    : max of data (1,nvar)
%   nsample : no. of samples
% Output:
%   s       : random sample (nsample,nvar)
%   Budiman (2003)

nvar=length(xmin);
ran=rand(nsample,nvar);
s=zeros(nsample,nvar);
for j=1: nvar
   idx=randperm(nsample);
   P =(idx'-ran(:,j))/nsample;
   s(:,j) = xmin(j) + P.* (xmax(j)-xmin(j));
end
```

APPENDIX B

MATLAB code: To generate HSLM-A (From Raid Karoumi)

```
% *********************************************
% * HSLM-A Trains *
% * type 1-10 *
% * prEN 1991-2:2002 final draft (p.83) *
% * Raid Karoumi 2003-12-14 *
% *********************************************
function [TRAINLOAD,TRAINDIST]=HSLMA(type)
N=[18 17 16 15 14 13 13 12 11 11]; % number of intermediate
coaches
D=[18 19 20 21 22 23 24 25 26 27]; % coach length (m)
d=[2 3.5 2 3 2 2 2 2.5 2 2]; % bogi-axle spacing (m)
P=[170 200 180 190 170 180 190 190 210 210]; % point force (KN)
N=N(type);D=D(type);d=d(type);P=P(type);
da=D-d;
% ---> Power car, end coach and intermediate coach <---
l1=0; % total length of car (m)
m1=2000; % wheel mass (kg)
m2=3000; % bogi mass (kg)vagn
m3=P*1000/9.81-m1-m2; % body mass (kg)
m1+m2+m3=P kN
ax=[P]; % axle data vector
TRAINLOAD=ones(7+N*2+7,1)*ax; % all axles of the train
% calculate and store the position of each axels (m)
TRAINDIST=[];TRAINDIST2=[]; TRAINDIST3=[];
TRAINDIST1=[0; 3; 14; 17;
20.525; 20.525+d; 20.525+d+D-d/2-d-3.525/2;];
for vagn=1:N
 if vagn==1
 a=TRAINDIST1(7);
 else
 a=TRAINDIST2(size(TRAINDIST2,1));
 end
 TRAINDIST22=[a+d; a+d+da;];
 TRAINDIST2=[TRAINDIST2; TRAINDIST22];
end
a=TRAINDIST2(size(TRAINDIST2,1));
TRAINDIST3=[a+d; a+d+D-d/2-d-3.525/2; a+d+D-d/2-d-3.525/2+d;];
a=TRAINDIST3(size(TRAINDIST3,1));
TRAINDIST3=[TRAINDIST3; a+3.525; a+3.525+3; a+3.525+3+11; a+3.525+3+11+3;];
TRAINDIST=[TRAINDIST1; TRAINDIST2; TRAINDIST3];
TRAINDIST=TRAINDIST.*(1); % All cars are placed before the bridge left support
```

APPENDIX C

The critical universal train is selected based on the critical wavelength of excitation λ_C as defined in EN 1991-2 Annex E.2(4) shown here on Figures E.7 to E.17, where it is a function of the maximum value of aggressivity A(L/λ)G(λ) for the span length, L in the range of excitation wavelength from 4.5 m to λv.

The wavelength of excitation at the Maximum Design Speed λ_v (m) is given by equation (10) as $\lambda_v = \frac{v_{DS}}{n_o}$ where n_o is the first natural frequency of the simply supported span (Hz) and v_{DS} is the maximum Design Speed.

After reading the critical wavelength of excitation λ_C, the characteristics of the critical universal train can be determined from Figure 2-4.

The following Charts were used for determining Critical Wavelength of Excitation, λc (From EUROCODE) to find Critical Universal Train of HSLM-A.

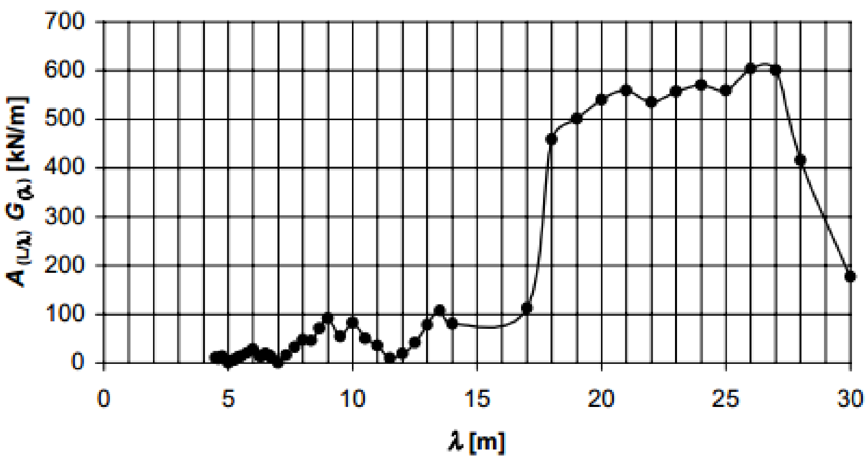

Figure E.7 - Aggressivity $A_{(L/\lambda)}G_{(\lambda)}$ as a function of excitation wavelength λ for a simply supported span of L = 15,0 m and damping ratio ζ = 0.01

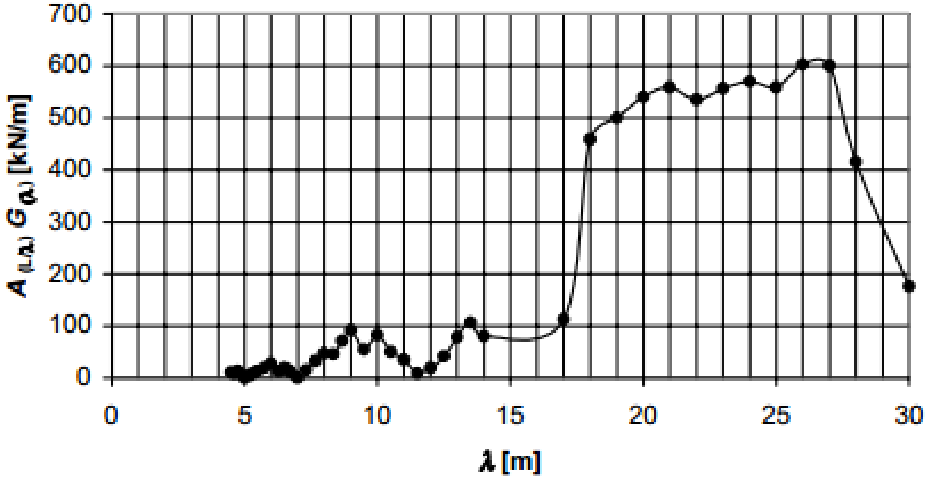

Figure E.8 - Aggressivity $A_{(L/\lambda)}G_{(\lambda)}$ as a function of excitation wavelength λ for a simply supported span of L = 17,5 m and damping ratio ζ = 0.01

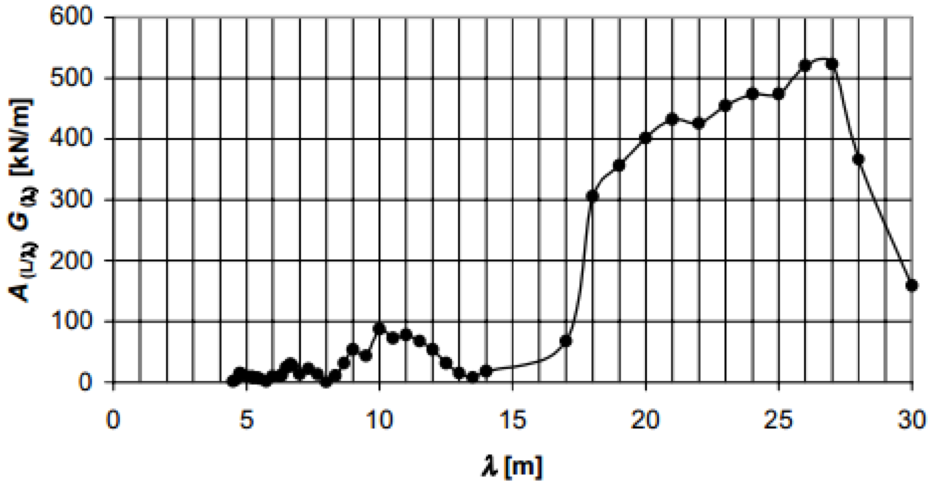

Figure E.9 - Aggressivity $A_{(L/\lambda)}G_{(\lambda)}$ as a function of excitation wavelength λ for a simply supported span of L = 20,0 m and damping ratio ζ = 0.01

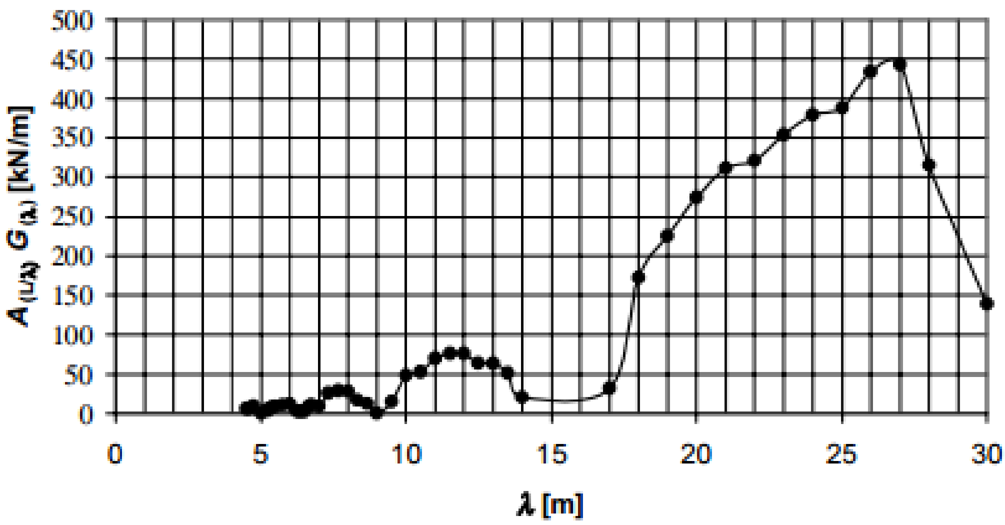

Figure E.10 - Aggressivity $A_{(L/\lambda)}G_{(\lambda)}$ as a function of excitation wavelength λ for a simply supported span of L = 22,5 m and damping ratio ζ = 0.01

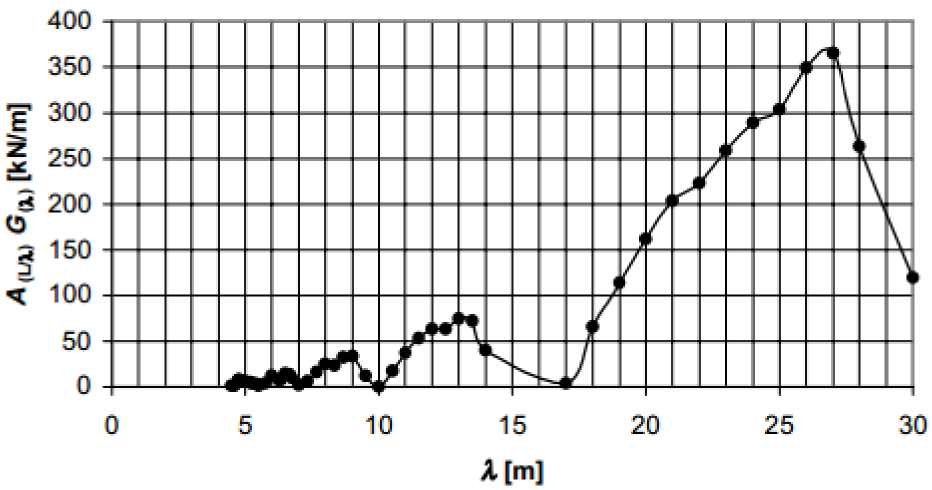

Figure E.11 - Aggressivity $A_{(L/\lambda)}G_{(\lambda)}$ as a function of excitation wavelength λ for a simply supported span of L = 25,0 m and damping ratio ζ = 0.01

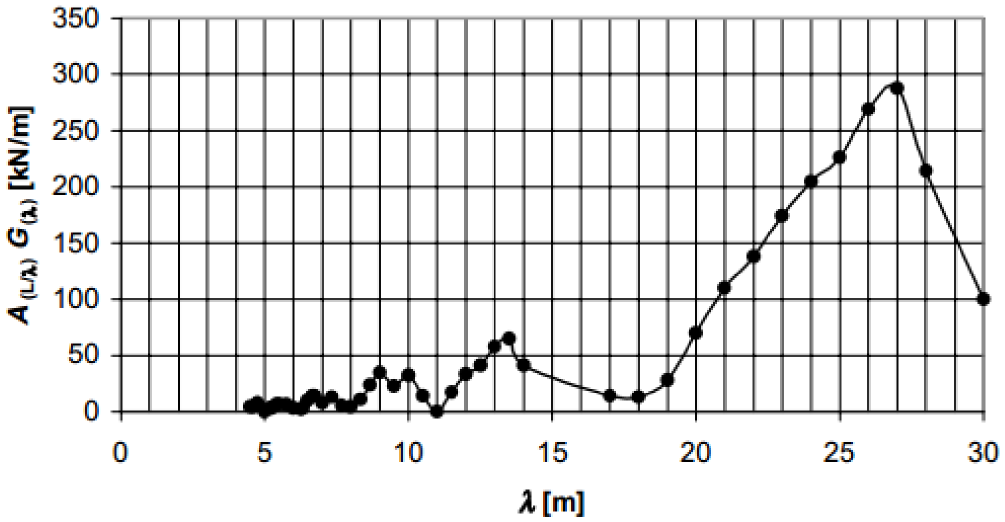

Figure E.12 - Aggressivity $A_{(L/\lambda)}G_{(\lambda)}$ as a function of excitation wavelength λ for a simply supported span of L = 27,5 m and damping ratio ζ = 0.01

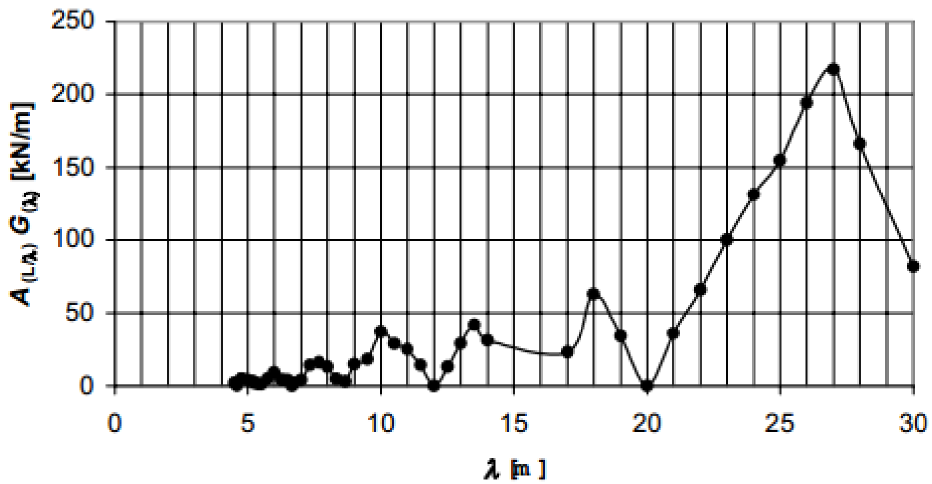

Figure E.13 - Aggressivity $A_{(L/\lambda)}G_{(\lambda)}$ as a function of excitation wavelength λ for a simply supported span of L = 30,0 m and damping ratio ζ = 0.01

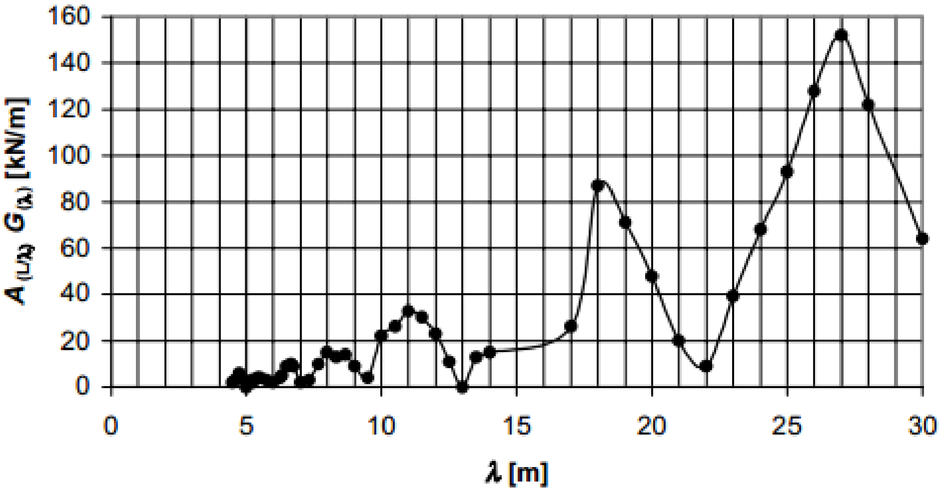

Figure E.14 - Aggressivity $A_{(L/\lambda)}G_{(\lambda)}$ as a function of excitation wavelength λ for a simply supported span of L = 32,5 m and damping ratio ζ= 0.01

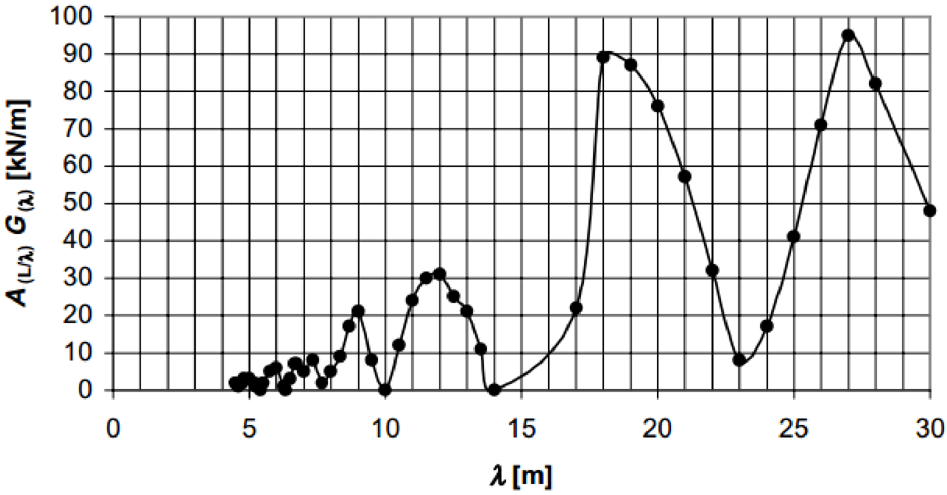

Figure E.15 - Aggressivity $A_{(L/\lambda)}G_{(\lambda)}$ as a function of excitation wavelength λ for a simply supported span of L = 35,0 m and damping ratio ζ= 0.01

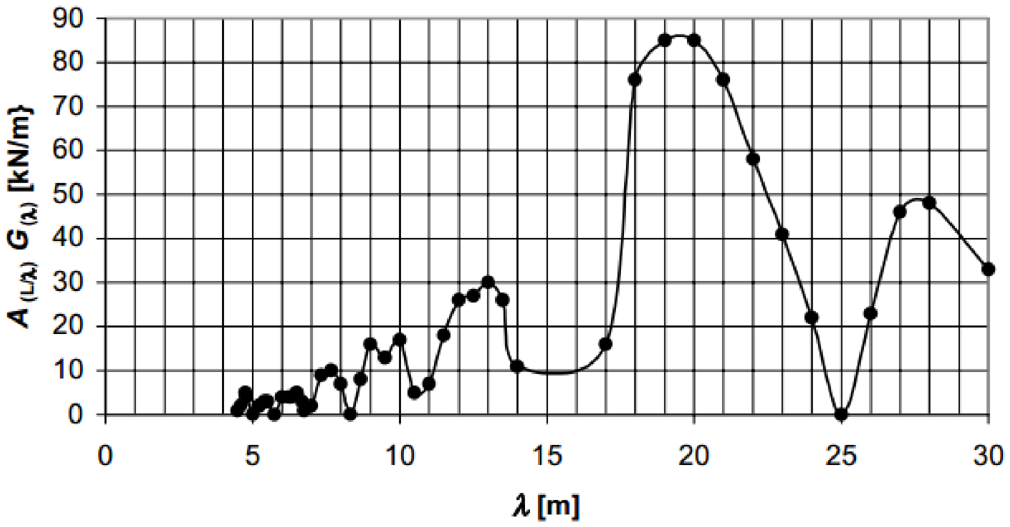

Figure E.16 - Aggressivity $A_{(L/\lambda)}G_{(\lambda)}$ as a function of excitation wavelength λ for a simply supported span of L = 37,5 m and damping ratio ζ = 0.01

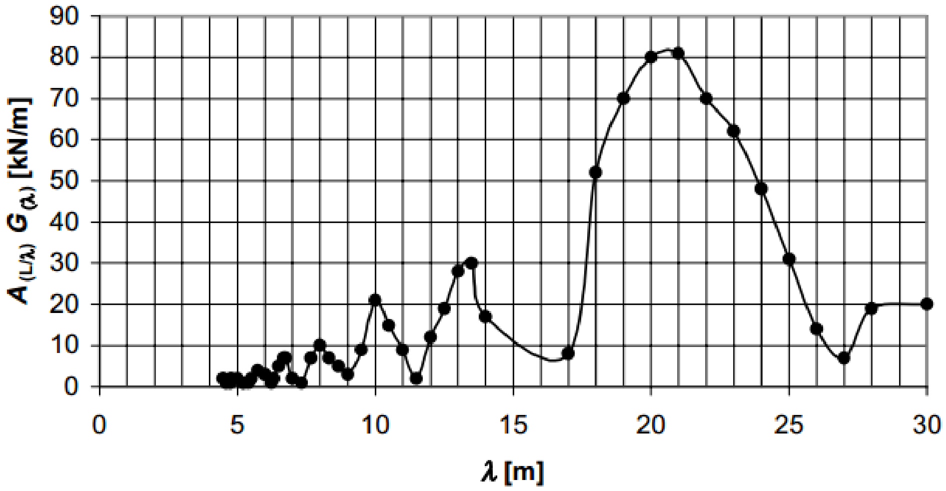

Figure E.17 - Aggressivity $A_{(L/\lambda)}G_{(\lambda)}$ as a function of excitation wavelength λ for a simply supported span of L = 40,0 m and damping ratio ζ = 0.01

APPENDIX D

Sample code from Model 1 having a bridge span length of 35 m, track stiffness of 500 MN/m, transition length 75% of bridge span length, train Speed of 210 km/h and Concrete compressive strength of 35 MPa is shown here.

Step 1 - Input Variables: input variables are provided in MATLAB, which remain constant for one type of the bridge. These input variables include damping ratio, sleeper spacing, stiffness of ballast, stiffness of bridge and rail structure, span length and element size, etc.

```
L=35; %Span Length (m)
Xi=0.015; %Damping Ratio
Frqmax=50; %Maximum Frequency of Request(HZ)
T_Inc=0.0005; %Dynamic Time Increment(Sec)
E=34e9; %Uncracked Concrete Young's Modulus(N/m2
vi=0.2; %Poisson's Ratio (concrete)
Con_D=25880; %Concrete Density including ballast(Kg/m3)
Es=210e9; %Steel Young's Modulus(N/m2)
vis=0.3; %Poisson's Ratio (steel)
Con_S=7850; %Steel Density(Kg/m3)
Ball_T=0.6; %Ballast Thickness(m)
Ball_w=6.2; %Ballast width(m)
Ball_D=2000; %Ballast Density(Kg/m3)
K_ex=100e6; %Vertical Spring Stiffness of extra track(N/m)
K_t=300e6; %Vertical Spring Stiffness of approach track(N/m)
K_b=500e6; %Vertical Spring Stiffness of bridge(N/m)
C_b=100e3; %Vertical Damping for ballast and approach track(Ns/m)

%% ABAQUS INPUT FILE
Sleeper_s=0.625; %Sleeper spacing or element size
Lt=0.75*L; %transition zone length
ExTrack_L=190+(L-Lt); %Track length
GroundNodes_Left=101+(ExTrack_L/Sleeper_s);
%Ground nodes before transition zone on left side
AppGroundNodes_Left=GroundNodes_Left+(Lt/Sleeper_s);
%Ground nodes for transition zone on left side
BridgeNodes=AppGroundNodes_Left+(L/Sleeper_s);
%Bridge Nodes
AppGroundNodes_Right=BridgeNodes+(Lt/Sleeper_s);
%Ground nodes for transition zone on Right side
GroundNodes_Right=AppGroundNodes_Right+(ExTrack_L/Sleeper_s);
%Ground nodes after transition zone on Right side
ExRailNodes_Left=GroundNodes_Right+1+(ExTrack_L/Sleeper_s);
%Rail Nodes on track Left side
AppRailNodes_Left=ExRailNodes_Left+(Lt/Sleeper_s);
%Rail Nodes on Transition zone Left side
RailNodes=AppRailNodes_Left+(L/Sleeper_s);
%Bridge Rail Nodes
AppRailNodes_Right=RailNodes+(Lt/Sleeper_s);
%Rail Nodes on Transition zone Right side
ExRailNodes_Right=AppRailNodes_Right+(ExTrack_L/Sleeper_s);
%Rail Nodes on track Right side
```

```
%%ELEMENTS%
%***************************************************************************
BridgeEl=BridgeNodes-AppGroundNodes_Left; %Bridge Elements
ExRailEl_Left=ExRailNodes_Left-(GroundNodes_Right+1);
%Track Rail Elements left side
AppRailEl_Left=AppRailNodes_Left-ExRailNodes_Left;
%Transition zone Rail Elements left side
RailEl=RailNodes-AppRailNodes_Left; %Bridge Rail Elements
AppRailEl_Right=AppRailNodes_Right-RailNodes;
%Transition zone Rail Elements Right side
ExRailEl_Right=ExRailNodes_Right-AppRailNodes_Right;
%Track Rail Elements Right side
GroundSpringEl_Left=(ExTrack_L/Sleeper_s);
AppGroundSpringEl_Left=(Lt/Sleeper_s);
BridgeSpringEl=(L/Sleeper_s)+1;
AppGroundSpringEl_Right=(Lt/Sleeper_s);
GroundSpringEl_Right=(ExTrack_L/Sleeper_s);
GroundDashpotEl_Left=(ExTrack_L/Sleeper_s);
AppGroundDashpotEl_Left=(Lt/Sleeper_s);
BridgeDashpotEl=(L/Sleeper_s)+1;
AppGroundDashpotEl_Right=(Lt/Sleeper_s);
GroundDashpotEl_Right=(ExTrack_L/Sleeper_s);

%GENERAL SECTIONS
%***************************************************************************
G=E/(2*(1+vi)); %Shear modulus for Bridge
I11=4.16;I12=0;I22=0; %General Section definition for Bridge
A=1;
Gr=Es/(2*(1+vis)); %Shear modulus for Rail
Ir11=0.00006120;Ir12=0;Ir22=0; %stiffness for two Rails
Ar=0.015374; %cross section area for two Rails
```

Step 2 - Creating the bridge structure: the bridge structure is created from the inputs during the analysis.

```
%HEADING
%.......................................................................
 Text=[ '*HEADING\n' ...
'** Job name: Comb1_HSLMA10 Model name: Comb1_HSLMA10\n' ...
'**\n'
 ];
fprintf(fid,Text);
%ground NODES Left side
%.......................................................................
 Text=['*Node\n' '101,-' num2str(Lt+ExTrack_L) ',-1\n'
num2str(GroundNodes_Left-1) ',-'  num2str(Lt+Sleeper_s) ',-1\n' '*NGEN,
Nset=GroundNodes_Left\n' '101,' num2str(GroundNodes_Left-1) ',1\n' ];
fprintf(fid,Text);
%APPROACH NODES LEFT SIDE
%.......................................................................
 Text=[ '*Node\n' ...
 num2str(GroundNodes_Left) ',-'  num2str(Lt) ',-1\n' ...
 num2str(AppGroundNodes_Left-1) ',-'  num2str(Sleeper_s) ',-1\n' ...
'*NGEN, Nset=AppGroundNodes_Left\n' ...
num2str(GroundNodes_Left) ','  num2str(AppGroundNodes_Left-1) ',1\n' ...
 ];
fprintf(fid,Text);
```

```
%BRIDGE NODES
%......................................................................
 Text=[ '*Node\n' ...
 num2str(AppGroundNodes_Left) ',0,-1\n' ...
num2str(BridgeNodes) ',' num2str(L) ',-1\n' ...
'*NGEN, Nset=BridgeNodes\n' ...
num2str(AppGroundNodes_Left) ',' num2str(BridgeNodes) ',1\n' ...
 ];
fprintf(fid,Text);
%BRIDGE ELEMENTS
%.....................................................................
 Text=[ '*Element, type=B21\n' '1,' num2str(AppGroundNodes_Left) ','
num2str(AppGroundNodes_Left+1) '\n' '*ELGEN, ELSET=Bridgebeam\n' '1,'
num2str(BridgeEl) '\n' ];
fprintf(fid,Text);
% Bridge beam SECTION
%.....................................................................
 Text=['*Beam General Section, Density=' num2str(Con_D)
',ELSET=Bridgebeam\n' ... % (ELSET:defines to which elements the bloc is
assigned)
num2str(A) ',' num2str(I11) ',' num2str(I12) ',' num2str(I22) ', 0,
0,0\n' ...  % A, I11, I12, I22, J, (gamma)0,(gamma)w
'0, 0, -1\n' ... % beamsectionorientation. for planar beams: 0,0,-1
num2str(E) ',' num2str(G) '\n' ... % Young´s modulus E, shear modulus G
'**\n' ...
'*DAMPING,Composite=0.015\n' ...
%!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
'**\n' ...
 ];
fprintf(fid,Text);
%APPROACH NODES Right SIDE
%......................................................................
 Text=[ '*Node\n' ...
 num2str(BridgeNodes+1) ',' num2str(L+Sleeper_s) ',-1\n' ...
 num2str(AppGroundNodes_Right) ',' num2str(L+Lt) ',-1\n' ...
'*NGEN, Nset=AppGroundNodes_Right\n' ...
 num2str(BridgeNodes+1) ',' num2str(AppGroundNodes_Right) ',1\n' ...
 ];
fprintf(fid,Text);
%ground NODES Right side
%......................................................................
 Text=[ '*Node\n' ...
 num2str(AppGroundNodes_Right+1) ',' num2str(L+Lt+Sleeper_s) ',-1\n' ...
 num2str(GroundNodes_Right) ',' num2str(L+Lt+ExTrack_L) ',-1\n' ...
'*NGEN, Nset=GroundNodes_Right\n' ...
 num2str(AppGroundNodes_Right+1) ',' num2str(GroundNodes_Right) ',1\n' ...
 ];
fprintf(fid,Text);
%Extra Rail NODES left side
%......................................................................
 Text=[ '*Node\n' ...
 num2str(GroundNodes_Right+1) ',-' num2str(Lt+ExTrack_L) ',0\n' ...
 num2str(ExRailNodes_Left-1) ',-' num2str(Lt+Sleeper_s) ',0\n' ...
'*NGEN, Nset=ExRailNodes_Left\n' ...
 num2str(GroundNodes_Right+1) ',' num2str(ExRailNodes_Left-1) ',1\n' ...
 ];
fprintf(fid,Text);
% Extra Rail ELEMENTS left side
%.....................................................................
```

```
 Text=[ '*Element, type=B21\n' ...
 num2str(BridgeEl+1) ',' num2str(GroundNodes_Right+1) ','
num2str(GroundNodes_Right+2) '\n' ...
'*ELGEN, ELSET=ExRail_Left\n' ...
 num2str(BridgeEl+1) ',' num2str(ExRailEl_Left) '\n' ...
 ];
fprintf(fid,Text);
% Extra Rail SECTION Left side
%.......................................................................
 Text=[
'*Beam General Section, Density=' num2str(Con_S) ',ELSET=ExRail_Left\n'
...
num2str(Ar) ',' num2str(Ir11) ',' num2str(Ir12) ',' num2str(Ir22) ', 0,
0,0\n' ...
'0, 0, -1\n' ...
 num2str(Es) ',' num2str(Gr) '\n' ...
'**\n' ...
 ];
fprintf(fid,Text);
%Approach Rail NODES Left side
%........................................................................
 Text=[ '*Node\n' ...
 num2str(ExRailNodes_Left) ',-' num2str(Lt) ',0\n' ...
 num2str(AppRailNodes_Left-1) ',-' num2str(Sleeper_s) ',0\n' ...
'*NGEN, Nset=AppRailNodes_Left\n' ...
 num2str(ExRailNodes_Left) ',' num2str(AppRailNodes_Left-1) ',1\n' ...
 ];
fprintf(fid,Text);
% Approach Rail ELEMENTS Left side
%.......................................................................
 Text=[ '*Element, type=B21\n' ...
 num2str(ExRailEl_Left+BridgeEl+1) ',' num2str(ExRailNodes_Left) ','
num2str(ExRailNodes_Left+1) '\n' ...
'*ELGEN, ELSET=AppRail_Left\n' ...
num2str(ExRailEl_Left+BridgeEl+1) ',' num2str(AppRailEl_Left) '\n' ...
 ];
fprintf(fid,Text);
% Approach Rail SECTION Left side
%.......................................................................
 Text=[
'*Beam General Section, Density=' num2str(Con_S) ',ELSET=AppRail_Left\n'
...
num2str(Ar) ',' num2str(Ir11) ',' num2str(Ir12) ',' num2str(Ir22) ', 0,
0,0\n' ...
'0, 0, -1\n' ...
 num2str(Es) ',' num2str(Gr) '\n' ...
'**\n' ...
 ];
fprintf(fid,Text);
%Rail NODES
%.......................................................................
 Text=[ '*Node\n' ...
 num2str(AppRailNodes_Left) ',0,0\n' ...
 num2str(RailNodes) ',' num2str(L) ',0\n' ...
'*NGEN, Nset=RailNodes\n' ...
 num2str(AppRailNodes_Left) ',' num2str(RailNodes) ',1\n' ...
 ];
fprintf(fid,Text);
% Rail ELEMENTS
```

```
%..................................................................
 Text=[ '*Element, type=B21\n' ...
 num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+1) ','
num2str(AppRailNodes_Left) ',' num2str(AppRailNodes_Left+1) '\n' ...
'*ELGEN, ELSET=Rail\n' ...
 num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+1) ',' num2str(RailEl) '\n'
...
 ];
fprintf(fid,Text);
% Rail SECTION
%..................................................................
 Text=[
'*Beam General Section, Density=' num2str(Con_S) ',ELSET=Rail\n' ...
num2str(Ar) ',' num2str(Ir11) ',' num2str(Ir12) ',' num2str(Ir22) ', 0,
0,0\n' ...
'0, 0, -1\n' ...
 num2str(Es) ',' num2str(Gr) '\n' ...
'**\n' ...
 ];
fprintf(fid,Text);
%Approach Rail NODES Right side
%...................................................................
 Text=[ '*Node\n' ...
 num2str(RailNodes+1) ',' num2str(L+Sleeper_s) ',0\n' ...
 num2str(AppRailNodes_Right) ',' num2str(L+Lt) ',0\n' ...
'*NGEN, Nset=AppRailNodes_Right\n' ...
 num2str(RailNodes+1) ',' num2str(AppRailNodes_Right) ',1\n' ...
 ];
fprintf(fid,Text);
% Approach Rail ELEMENTS Right side
%..................................................................
 Text=[ '*Element, type=B21\n' ...
num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+1) ','
num2str(RailNodes) ',' num2str(RailNodes+1) '\n' ...
'*ELGEN, ELSET=AppRail_Right\n' ...
num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+1) ','
num2str(AppRailEl_Right) '\n' ...
 ];
fprintf(fid,Text);
% Approach Rail SECTION Right side
%..................................................................
 Text=['*Beam General Section, Density=' num2str(Con_S)
',ELSET=AppRail_Right\n' ...
 num2str(Ar) ',' num2str(Ir11) ',' num2str(Ir12) ',' num2str(Ir22) ', 0,
0, 0\n' ...
'0, 0, -1\n' ...
 num2str(Es) ',' num2str(Gr) '\n' ...
'**\n' ...
 ];
fprintf(fid,Text);
%Extra Rail NODES Right side
%...................................................................
 Text=[ '*Node\n' ...
 num2str(AppRailNodes_Right+1) ',' num2str(L+Lt+Sleeper_s) ',0\n' ...
 num2str(ExRailNodes_Right) ',' num2str(ExTrack_L+L+Lt) ',0\n' ...
'*NGEN, Nset=ExRailNodes_Right\n' ...
 num2str(AppRailNodes_Right+1) ',' num2str(ExRailNodes_Right) ',1\n' ...
 ];
fprintf(fid,Text);
```

```
% Extra Rail ELEMENTS Right side
%...................................................................
 Text=[ '*Element, type=B21\n' ...
num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+1) ','
num2str(AppRailNodes_Right) ',' num2str(AppRailNodes_Right+1) '\n' ...
'*ELGEN, ELSET=ExRail_Right\n' ...
num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+1) ','
num2str(ExRailEl_Right) '\n' ...
 ];
fprintf(fid,Text);
% Extra Rail SECTION Right side
%...................................................................
 Text=['*Beam General Section, Density=' num2str(Con_S)
',ELSET=ExRail_Right\n' ...
num2str(Ar) ',' num2str(Ir11) ',' num2str(Ir12) ',' num2str(Ir22) ', 0,
0, 0\n' ...
'0, 0, -1\n' ...
 num2str(Es) ',' num2str(Gr) '\n' ...
'**\n' ...
 ];
fprintf(fid,Text);
% Spring ELEMENTS for connection to ground left side
%....................................................................
 Text=[ '*Element, type=SpringA\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+1) ',101,' num2str(GroundNodes_Right+1) '\n' ...
'*ELGEN, ELSET=GroundSpring_Left\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+1) ',' num2str(GroundSpringEl_Left) '\n' ...
'*Spring, elset=GroundSpring_Left\n' ...
'2,2\n' ...
'100e+06\n' ...
 ];
fprintf(fid,Text);
% Dashpot ELEMENTS for connection to ground left side
%...................................................................
 Text=[ '*Element, type=DashpotA\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+1) ',101,' num2str(GroundNodes_Right+1) '\n'
...
'*ELGEN, ELSET=GroundDashpot_Left\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+1) ',' num2str(GroundDashpotEl_Left) '\n'
...
'*Dashpot, elset=GroundDashpot_Left\n' ...
'2,2\n' ...
'100000\n' ...
 ];
fprintf(fid,Text);
% Spring ELEMENTS for connection to ground left side approach track
%...................................................................
 Text=[ '*Element, type=SpringA\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+1) ','
num2str(GroundNodes_Left) ',' num2str(ExRailNodes_Left) '\n' ...
'*ELGEN, ELSET=AppGroundSpring_Left\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+1) ','
num2str(AppGroundSpringEl_Left) '\n' ...
```

```
'*Spring, elset=AppGroundSpring_Left\n' ...
'2,2\n' ...
'300e+06\n' ...
 ];
fprintf(fid,Text);
% Dashpot ELEMENTS for connection to ground left side approach track
%.....................................................................
 Text=[ '*Element, type=DashpotA\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+AppGroundSpringEl_Left+1
) ',' num2str(GroundNodes_Left) ',' num2str(ExRailNodes_Left) '\n' ... %
nb of the element, 1st node forming the element, 2nde node
'*ELGEN, ELSET=AppGroundDashpot_Left\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+AppGroundSpringEl_Left+1
) ',' num2str(AppGroundDashpotEl_Left) '\n' ...
'*Dashpot, elset=AppGroundDashpot_Left\n' ...
'2,2\n' ...
'100000\n' ...
 ];
fprintf(fid,Text);
% Spring ELEMENTS for bridge beam
%.....................................................................
 Text=[ '*Element, type=SpringA\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+AppGroundSpringEl_Left+A
ppGroundDashpotEl_Left+1) ',' num2str(AppGroundNodes_Left) ','
num2str(AppRailNodes_Left) '\n' ...
'*ELGEN, ELSET=BridgeSprings\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+AppGroundSpringEl_Left+A
ppGroundDashpotEl_Left+1) ',' num2str(BridgeSpringEl) '\n' ...
'*Spring, elset=BridgeSprings\n' ...
'2,2\n' ...
'600e+06\n' ...
 ];
fprintf(fid,Text);
% Dashpot ELEMENTS for bridge beam
%.....................................................................
 Text=[ '*Element, type=DashpotA\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+AppGroundSpringEl_Left+A
ppGroundDashpotEl_Left+BridgeSpringEl+1) ',' num2str(AppGroundNodes_Left)
',' num2str(AppRailNodes_Left) '\n' ...
'*ELGEN, ELSET=BridgeDashpots\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+AppGroundSpringEl_Left+A
ppGroundDashpotEl_Left+BridgeSpringEl+1) ',' num2str(BridgeDashpotEl) '\n'
...
'*Dashpot, elset=BridgeDashpots\n' ...
'2,2\n' ...
'100000\n' ...
 ];
fprintf(fid,Text);
% Spring ELEMENTS for connection to ground right side approach track
%.....................................................................
 Text=[ '*Element, type=SpringA\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+AppGroundSpringEl_Left+A
```

```
ppGroundDashpotEl_Left+BridgeSpringEl+BridgeDashpotEl+1) ','
num2str(BridgeNodes+1) ',' num2str(RailNodes+1) '\n' ...
'*ELGEN, ELSET=AppGroundSpring_Right\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+AppGroundSpringEl_Left+A
ppGroundDashpotEl_Left+BridgeSpringEl+BridgeDashpotEl+1) ','
num2str(AppGroundSpringEl_Right) '\n' ...
'*Spring, elset=AppGroundSpring_Right\n' ...
'2,2\n' ...
'300e+06\n' ...
 ];
fprintf(fid,Text);
% Dashpot ELEMENTS for connection to ground right side approach track
%.......................................................................
 Text=[ '*Element, type=DashpotA\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+AppGroundSpringEl_Left+A
ppGroundDashpotEl_Left+BridgeSpringEl+BridgeDashpotEl+AppGroundSpringEl_Rig
ht+1) ',' num2str(BridgeNodes+1) ',' num2str(RailNodes+1) '\n' ...
'*ELGEN, ELSET=AppGroundDashpot_Right\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+AppGroundSpringEl_Left+A
ppGroundDashpotEl_Left+BridgeSpringEl+BridgeDashpotEl+AppGroundSpringEl_Rig
ht+1) ',' num2str(AppGroundDashpotEl_Right) '\n' ...
'*Dashpot, elset=AppGroundDashpot_Right\n' ...
'2,2\n' ...
'100000\n' ...
 ];
fprintf(fid,Text);
% Spring ELEMENTS for connection to ground Right side
%.......................................................................
 Text=[ '*Element, type=SpringA\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+AppGroundSpringEl_Left+A
ppGroundDashpotEl_Left+BridgeSpringEl+BridgeDashpotEl+AppGroundSpringEl_Rig
ht+AppGroundDashpotEl_Right+1) ',' num2str(AppGroundNodes_Right+1) ','
num2str(AppRailNodes_Right+1) '\n' ...
'*ELGEN, ELSET=GroundSpring_Right\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+AppGroundSpringEl_Left+A
ppGroundDashpotEl_Left+BridgeSpringEl+BridgeDashpotEl+AppGroundSpringEl_Rig
ht+AppGroundDashpotEl_Right+1) ',' num2str(GroundSpringEl_Right) '\n' ...
'*Spring, elset=GroundSpring_Right\n' ...
'2,2\n' ...
'100e+06\n' ...
 ];
fprintf(fid,Text);
% Dashpot ELEMENTS for connection to ground Right side
%.......................................................................
 Text=[ '*Element, type=DashpotA\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+AppGroundSpringEl_Left+A
ppGroundDashpotEl_Left+BridgeSpringEl+BridgeDashpotEl+AppGroundSpringEl_Rig
ht+AppGroundDashpotEl_Right+GroundSpringEl_Right+1) ','
num2str(AppGroundNodes_Right+1) ',' num2str(AppRailNodes_Right+1) '\n' ...
'*ELGEN, ELSET=GroundDashpot_Right\n' ...
,num2str(BridgeEl+ExRailEl_Left+AppRailEl_Left+RailEl+AppRailEl_Right+ExRai
lEl_Right+GroundSpringEl_Left+GroundDashpotEl_Left+AppGroundSpringEl_Left+A
ppGroundDashpotEl_Left+BridgeSpringEl+BridgeDashpotEl+AppGroundSpringEl_Rig
```

```
ht+AppGroundDashpotEl_Right+GroundSpringEl_Right+1) ','
num2str(GroundDashpotEl_Right) '\n' ...
'*Dashpot, elset=GroundDashpot_Right\n' ...
'2,2\n' ...
'100000\n' ...
 ];
fprintf(fid,Text);

%BOUNDARY CONDITIONS
Text=[ '*BOUNDARY\n' ...
'GroundNodes_Left,Encastre\n' ... %node nb, type of boundary
'GroundNodes_Right,Encastre\n' ...
'AppGroundNodes_Left,Encastre\n' ...
'AppGroundNodes_Right,Encastre\n' ...
 num2str(AppGroundNodes_Left) ',2\n' ...
 num2str(BridgeNodes) ',PINNED\n' ...
 num2str(GroundNodes_Right+1) ',1\n' ...
 num2str(ExRailNodes_Right) ',1\n' ...
 'wsn,1,1\n' ...
'csn,1,1\n' ...
'** Interaction: ContactCondition\n' ...
'*Contact Pair, interaction=ContactProp\n' ...
'slave,master\n' ...
 ];
fprintf(fid,Text);
```

Step 3 – Creating the sprung mass system: each axle is created with two masses connected by a spring and a damper. Complete train is modeled with these axles and move over the rail called master surface.

```
%Sprung mass model of HSLMA-10 train
%.......................................................................
 Text=[ '*Node, Nset=wsn\n' ...
'8001,-'  num2str(L+2) ',0\n' ...
'8002,-'  num2str(L+5) ',0\n' ...
'8003,-'  num2str(L+16) ',0\n' ...
'8004,-'  num2str(L+19) ',0\n' ...
'8005,-'  num2str(L+22.53) ',0\n' ...
'8006,-'  num2str(L+24.53) ',0\n' ...
'8007,-'  num2str(L+46.76) ',0\n' ...
'8008,-'  num2str(L+48.76) ',0\n' ...
'8009,-'  num2str(L+73.76) ',0\n' ...
'8010,-'  num2str(L+75.76) ',0\n' ...
'8011,-'  num2str(L+100.76) ',0\n' ...
'8012,-'  num2str(L+102.76) ',0\n' ...
'8013,-'  num2str(L+127.76) ',0\n' ...
'8014,-'  num2str(L+129.76) ',0\n' ...
'8015,-'  num2str(L+154.76) ',0\n' ...
'8016,-'  num2str(L+156.76) ',0\n' ...
'8017,-'  num2str(L+181.76) ',0\n' ...
'8018,-'  num2str(L+183.76) ',0\n' ...
'*Node, Nset=csn\n' ...
'9001,-'  num2str(L+2) ',1\n' ...
'9002,-'  num2str(L+5) ',1\n' ...
'9003,-'  num2str(L+16) ',1\n' ...
'9004,-'  num2str(L+19) ',1\n' ...
```

```
'9005,-'  num2str(L+22.53) ',1\n' ...
'9006,-'  num2str(L+24.53) ',1\n' ...
'9007,-'  num2str(L+46.76) ',1\n' ...
'9008,-'  num2str(L+48.76) ',1\n' ...
'9009,-'  num2str(L+73.76) ',1\n' ...
'9010,-'  num2str(L+75.76) ',1\n' ...
'9011,-'  num2str(L+100.76) ',1\n' ...
'9012,-'  num2str(L+102.76) ',1\n' ...
'9013,-'  num2str(L+127.76) ',1\n' ...
'9014,-'  num2str(L+129.76) ',1\n' ...
'9015,-'  num2str(L+154.76) ',1\n' ...
'9016,-'  num2str(L+156.76) ',1\n' ...
'9017,-'  num2str(L+181.76) ',1\n' ...
'9018,-'  num2str(L+183.76) ',1\n' ...
 ];
fprintf(fid,Text);
% ELEMENTS
%......................................................................
 Text=[
'*Surface, type=ELEMENT, name=master\n' ...
'ExRail_Left,SPOS\n' ...
'AppRail_Left,SPOS\n' ...
'Rail,SPOS\n' ...
'AppRail_Right,SPOS\n' ...
'ExRail_Right,SPOS\n' ...
'*Surface, type=NODE, name=slave\n' ...
'wsn, 1.\n' ...
'** INTERACTION PROPERTIES\n' ...
'**\n' ...
'*Surface Interaction, name=ContactProp\n' ...
'1.\n' ...
'*Friction\n' ...
'0.1\n' ...
'*Surface Behavior, pressure-overclosure=HARD\n' ...
'**\n' ...
 ];
fprintf(fid,Text);
%......................................................................
 Text=[ '*Element, type=MASS, elset=m1_wheel\n' ...
'5001,8001\n' ...
'5002,8002\n' ...
'5003,8003\n' ...
'5004,8004\n' ...
'5005,8005\n' ...
'5006,8006\n' ...
'5007,8007\n' ...
'5008,8008\n' ...
'5009,8009\n' ...
'5010,8010\n' ...
'5011,8011\n' ...
'5012,8012\n' ...
'5013,8013\n' ...
'5014,8014\n' ...
'5015,8015\n' ...
'5016,8016\n' ...
'5017,8017\n' ...
'5018,8018\n' ...
'*Mass, elset=m1_wheel\n' ...
'2000., \n' ...
```

```
'*Element, type=MASS, elset=m1_body\n' ...
'6001,9001\n' ...
'6002,9002\n' ...
'6003,9003\n' ...
'6004,9004\n' ...
'6005,9005\n' ...
'6006,9006\n' ...
'6007,9007\n' ...
'6008,9008\n' ...
'6009,9009\n' ...
'6010,9010\n' ...
'6011,9011\n' ...
'6012,9012\n' ...
'6013,9013\n' ...
'6014,9014\n' ...
'6015,9015\n' ...
'6016,9016\n' ...
'6017,9017\n' ...
'6018,9018\n' ...
'*Mass, elset=m1_body\n' ...
'19000., \n' ...
'*Element, type=dashpotA, Elset=dashpot\n' ...
'7001,8001,9001\n' ...
'7002,8002,9002\n' ...
'7003,8003,9003\n' ...
'7004,8004,9004\n' ...
'7005,8005,9005\n' ...
'7006,8006,9006\n' ...
'7007,8007,9007\n' ...
'7008,8008,9008\n' ...
'7009,8009,9009\n' ...
'7010,8010,9010\n' ...
'7011,8011,9011\n' ...
'7012,8012,9012\n' ...
'7013,8013,9013\n' ...
'7014,8014,9014\n' ...
'7015,8015,9015\n' ...
'7016,8016,9016\n' ...
'7017,8017,9017\n' ...
'7018,8018,9018\n' ...
'*Dashpot, elset=dashpot\n' ...
'2 ,2\n' ...
'30000.\n' ...
'*Element, type=springA, ELSET=spring-1\n' ...
'4001,8001,9001\n' ...
'4002,8002,9002\n' ...
'4003,8003,9003\n' ...
'4004,8004,9004\n' ...
'4005,8005,9005\n' ...
'4006,8006,9006\n' ...
'4007,8007,9007\n' ...
'4008,8008,9008\n' ...
'4009,8009,9009\n' ...
'4010,8010,9010\n' ...
'4011,8011,9011\n' ...
'4012,8012,9012\n' ...
'4013,8013,9013\n' ...
'4014,8014,9014\n' ...
'4015,8015,9015\n' ...
```

```
'4016,8016,9016\n' ...
'4017,8017,9017\n' ...
'4018,8018,9018\n' ...
'*Spring, elset=spring-1\n' ...
'2 ,2\n' ...
'1.45e+06\n' ...
 ];
fprintf(fid,Text);
```

Step 4 – Writing ABAQUS input file: the bridge structure and loads are then copied in the main input file, which is then used for analysis in the ABAQUS.

```
%% ABAQUS input file
 A2=[];
 U2=[];
v=210; %Velocity in Km/h
v_msec=v/3.6;
L_totload=Lt+(2*L)+ExTrack_L;
T_tot=(L_totload/(v_msec));
t_inc_dyn=0.0005;
inc=ceil(T_tot/t_inc_dyn)+500;
fid=fopen([ 'Comb1_HSLMA10' , '.inp' ] , 'w' );
```

Step 5 – Finite Element Analysis: direct integration method is used for the analysis.

```
Text=[ '**STEP:gravity\n' ...
'**\n' ...
'*Step, name=gravity\n' ...
'*Static\n' ...
'**\n' ...
'** LOADS\n' ...
'**\n' ...
'** Name: GRAVITY-1 Type: Gravity\n' ...
'*Dload\n' ...
'm1_wheel, GRAV, 10., 0., -1.\n' ...
'm1_body, GRAV, 10., 0., -1.\n' ...
'**\n' ...
'** OUTPUT REQUESTS\n' ...
'**\n' ...
'** FIELD OUTPUT: F-Output-1\n' ...
'**\n' ...
'*Output, field, variable=PRESELECT\n' ...
'**\n' ...
'** HISTORY OUTPUT: H-Output-1\n' ...
'**\n' ...
'*Output, history, variable=PRESELECT\n' ...
'*End Step\n' ...
'** ----------------------------------------------------------------\n' ...
'** \n' ...
'** STEP: Step-2-dynamic\n' ...
'**\n' ...
'*Step, name=Step-2-dynamic, inc='  num2str(inc) '\n' ...
'*Dynamic,alpha=-0.05,direct\n' ...
 num2str(t_inc_dyn) ','  num2str(T_tot) ',\n' ...
'** \n' ...
'** BOUNDARY CONDITIONS\n' ...
```

```
'*Boundary\n' ...
'wsn, 1, 1,'  num2str(L_totload) '\n' ...
'csn, 1, 1,'  num2str(L_totload) '\n' ...
'**\n' ...
'** OUTPUT REQUESTS\n' ...
'**\n' ...
'**\n' ...
'** FIELD OUTPUT: F-Output-2\n' ...
'**\n' ...
'*Output, field, variable=PRESELECT\n' ...
'**\n' ...
'** HISTORY OUTPUT: H-Output-1\n' ...
'**\n' ...
'*Output, history, variable=PRESELECT\n' ...
'*Node output, nset=BridgeNodes\n' ...
'U2\n' ...
'*NODE PRINT, nset=BridgeNodes\n' ...
'U2\n' ...
'*END STEP\n' ];
fprintf(fid,Text);
        dos('abaqus job=Comb1_HSLMA10.inp interactive' );
fclose('all' );
```

Step 6 - Filtering the data: Butterworth filter is used to exclude the higher frequencies from the results which will be explained in section 4.5.2. Higher frequency accelerations or displacements do not have any significant effect on the ballast and need to be filtered out. Eurocode [8] recommends that all modes with frequencies higher than 30 Hz or 1.5 times the natural frequency, including at least three modes of vibration, should be excluded from the results.

```
Amax=[];
Umax=[];
for n=334:340
AValues=[];
UValues=[];
fid=fopen([ 'SM5mLa=0,5L' , '.dat' ], 'r' );
LineText=fgetl(fid);
 rate=1000;
Wn=50;
 N=6;
Wnr=(Wn*2)/rate; % relative cut-off frequency
 [b,a]=butter(N,Wnr);
av=filter(b,a,AValues);
uv=filter(b,a,UValues);
 Amax=[Amax;max(abs(av))];
Umax=[Umax;max(abs(uv))];
end
```

***Step 7 – Read maximum displacement*.**

```
A2=[A2;max(Amax)];
U2=[U2;max(Umax)];
fclose('all' );
```