Uniaxial Interaction Charts for Fully Encased Composite Columns

By

Selamawit Dege

A THESIS SUBMITED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR MASTER OF SCIENCE (STRUCTURAL ENGINEERING)

ADDIS ABABA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

INSTITUTE OF TECHNOLOGY

SCHOOL OF CIVIL AND ENVIROMENTAL ENGINEERING

December 2015

Addis Ababa, Ethiopia

STATEMENT OF PERMISSION TO USE

ABSTRACT

Nowadays, steel-concrete composite construction is used to meet performance and functional requirements of mid to high-rise structures as well as large span structures. These structures acquire the structural and constructional advantages of both the concrete and steel. Among composite members is a composite column. A steel-concrete composite column is a compression member, comprising either a concrete encased steel section or a concrete filled tubular steel section.

The resistance of a composite column to combined compression and bending is determined using an interaction curve of its cross-section. Developing interaction curves requires rigorous section analysis. For this reason, different practice codes incorporated simplified analysis and design procedures. Among which, the Eurcode 4 provides a simplified method for composite sections and columns satisfying certain requirements. According to this method, the axial force-moment interaction curve of a composite cross-section is obtained by assuming full plastic stress distribution. Furthermore, the Eurocde 4 approximates the entire interaction curve by a polygon made up of four or five points on the interaction curve. Despite these simplifications and approximations, the analysis of a composite section is yet computationally demanding. In addition to this, for composite sections that violate the code requirements a more rigorous analysis is mandatory. This has been a major disincentive for using composite frames. Despite their advantages, the topic of composite columns is given few attentions in the Ethiopian construction industry. Moreover, neither design aids nor analysis tool haven't been developed yet to assist structural engineers in analysis and design of composite columns.

In this thesis, more accurate uniaxial interaction curves are developed for "I" and "H" steel sections that are fully encased in concrete. The stress resultants were evaluated starting with strain distributions in the ultimate limit states that were adopted from the Eurocode 2. The stress-strain laws of materials were taken from Eurocode 2 and Eurocode 3. The stress resultants of the concrete and structural steel involved double integrals of the stress over the compressed regions of the concrete and structural steel section as well as over the tensioned regions of the steel section. These integrals were then transformed into line integrals by using Green's theorem. Finally, the line integrals were solved using Gauss Quadrature which is numerically exact method with the adopted material laws. Double counting of the concrete area in compression zone which is replaced by the structural steel section and the reinforcement was avoided.

The uniaxial curves developed were verified against the outputs of the software called MASQUE. The comparisons indicate that the developed interaction curve is almost identical with that of MASQUE output.

To increase the applicability of this study, especially in the Ethiopian market, a computer program UICISEC with a friendly graphical user interface is incorporated.

Finally, a design example for a column length subjected to biaxial bending was carried out according to the Eurocode 4 simplified method. Here, the uniaxial interaction curves of this study were utilized.

Key Words: composite column, fully encased composite section, strain distribution in the ULS, section analysis, stress resultants, column length

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ACRONYMS

ACI	American concrete institute
AISC	American institute of steel construction
FEA	Finite element analysis
FEM	Finite element model
GUI	Graphical user interface
LRFD	Load and resistance factor design
ULS	Ultimate limit state

LIST OF SYMBOLS

Latin upper case letters

A_a	Cross-sectional area of the structural steel section
A_c	Cross-sectional area of concrete in the compression zone
A_s	Cross-sectional area of reinforcement bars
A_v	Shear area of a structural steel section
C_y	Thickness of concrete cover to the structural steel in the y-direction
C_z	Thickness of concrete cover to the structural steel in the z-direction
E_a	Modulus of elasticity of structural steel
E_{cm}	Secant modulus of elasticity of concrete
$E_{c,eff}$	Effective modulus of elasticity for concrete
E_s	Modulus of elasticity of reinforcing steel
$(EI)_{eff}$	Effective flexural stiffness for calculation of relative slenderness
I_a	Second moment of area of the structural steel section
I_c	Second moment of area of the un-cracked concrete section
I_s	Second moment of area of the reinforcement
K_e	Correction factor to account for a reduced stiffness of concrete due to Cracking
L	Buckling length of the column
M_{Ed}	Design bending moment
$M_{max.Rd}$	Maximum design value of the resistance moment in the presence of a compressive normal force
$M_{pl,a,Rd}$	Design value of the plastic resistance moment of the structural steel section
$M_{pl,N,Rd}$	Design value of the plastic resistance moment of the composite section taking into account the compressive normal force
$M_{pl.Rd}$	Design value of the plastic resistance moment of the composite section with full shear connection

$M_{pl,y,Rd}$	Design value of the plastic resistance moment about the y-y axis of the composite section with full shear connection
$M_{pl,z,Rd}$	Design value of the plastic resistance moment about the z-z axis of the composite section with full shear connection
M_y	Resultant moment resisted by the section for bending about the y-y axis
M_{ya}	Resultant moment resisted by the structural steel for bending about the y-y axis
M_{yc}	Resultant moment resisted by the concrete for bending about the y-y axis
M_{ys}	Resultant moment resisted by the reinforcement for bending about the y-y axis
M_z	Resultant moment resisted by the section for bending about the z-z axis
M_{za}	Resultant moment resisted by the structural steel for bending about the z-z axis
M_{zc}	Resultant moment resisted by the concrete for bending about the z-z axis
M_{zs}	Resultant moment resisted by the reinforcement for bending about the z-z axis
N_a	Resultant axial force carried by the structural steel
N_c	Resultant axial force carried by the concrete
N_{cr}	Elastic critical normal force
$N_{cr,eff}$	Critical normal force for the relevant axis and corresponding to the effective flexural stiffness with the effective length taken as the column length
N_{Ed}	Design value of the applied axial force
$N_{G,Ed}$	Design value of the part of the compressive normal force that is permanent
$N_{pl.Rd}$	Design value of the plastic resistance of the composite section to compressive normal force

$N_{pl.Rk}$ Characteristic value of the plastic resistance of the composite section to compressive normal force

$N_{pm,Rd}$ Design value of the resistance of the concrete to compressive normal force

N_s Resultant axial force carried by the reinforcements

R_a Stress resultants of structural steel

R_c Stress resultants of concrete

$V_{a,Ed}$ Design value of the shear force acting on the structural steel section

$V_{c,Ed}$ Design value of the shear force acting on the reinforced concrete web Encasement

V_{Ed} Design value of the shear force acting on the composite section

$V_{pl,a,Rd}$ Design value of the plastic resistance of the structural steel section to vertical shear

W_{pa} Plastic modulus of steel section

W_{pc} Plastic modulus of concrete in compression zone

W_{ps} Plastic modulus of reinforcement steel

Z_{max} Maximum distance from the geometrical center to the outer most fiber of a cross-section

Latin lower case letters

b Width of the flange of a steel section

c Neutral axis depth from top most fiber of the cross-section

d Overall diameter of circular hollow steel section

f_{cd} Design value of the cylinder compressive strength of concrete

e_y Concrete cover to the reinforcement in the y-direction

e_z Concrete cover to the reinforcement in the z-direction

f_{ck} Characteristic value of the cylinder compressive strength of concrete at 28 days

f_{sd} Design value for the yield strength of reinforcement

f_{sk}	Characteristic value of the yield strength of reinforcing steel
f_y	Nominal value of the yield strength of structural steel
f_{yd}	Design value for the yield strength of structural steel
h	Depth of the structural steel section
h_n	Depth of the plastic neutral axis from the geometrical center
k	Amplification factor for second-order effects
n	Exponent in stress-strain relation for concrete
r	Ratio of the smaller to the larger end moments; radius of fillet between the flange and web of structural steel section; nonnegative integer
s	nonnegative integer
t	Thickness of circular hollow steel section
t_f	Thickness of a flange of the structural steel section
t_w	Thickness of the web of the structural steel section
w_i	Gauss weights
x_i	Gauss points
y	Distance from the geometrical center of a cross-section; lever arm
z	Distance from the geometrical center of a cross-section; lever arm

Greek lower case letters

α_{cr}	Factor by which the design loading would have to be increased to cause elastic instability
α_M	Coefficient related to bending of a composite column
β	Equivalent moment factor
γ_a	Partial factor for structural steel
γ_c	Partial factor for concrete
γ_s	Partial factor for reinforcing steel
δ	Structural steel contribution ratio
ε_a	Strain in the structural steel

ε_c	Compressive strain in the concrete
ε_{c2}	Compressive strain in the concrete at reaching the maximum strength
ε_{cu2}	Ultimate compressive strain in the concrete
ε_s	Strain in the reinforcing steel
ε_{sd}	Design yield strain in the reinforcing steel
ε_{yd}	Design yield strain in the structural steel
ε_{ud}	Ultimate design tensile strain in steel
$\overline{\lambda}$	Relative slenderness
μ	Related moment factor
μ_d	Moment factor related to design for compression and uniaxial bending
μ_{dy}	Factor μ_d for bending about the y-y axis
μ_{dz}	Factor μ_d for bending about the z-z axis
ρ	Parameter related to reduced design bending resistance accounting for vertical shear
σ_a	Stress in the structural steel
σ_c	Compressive stress in the concrete
σ_s	Stress in the reinforcement
ϕ	Value to determine the reduction factor χ
Φ_t	Creep coefficient
χ	Reduction factor for flexural buckling
ψ	Distance from the neutral axis

ACKNOWLEDGEMENTS

This thesis owes its existence to the kind assistance I received from my professors, colleagues, friends and family. First and foremost, I would like to express my sincere appreciation and gratitude to my advisor Dr. Ing Girma Zerayohannes. You have been a tremendous mentor to me. Your advices in both my research as well as career have been priceless.

I am greatly indebted to Dr. Esayas G.youhannes, who has been kind enough with his time to advise me and provided me with the necessary journals. I am also thankful to lecturer Melaku Seyoum, in the department of civil and environmental engineering, for his guidance in development of the program. I would like to extend my gratitude to lecturer Natan Tadesse, in the department of electrical engineering. He took his time in designing the graphical user interface and deploying the program. I wish to thank lecturer Mekdes Tadesse, in the department of civil and environmental engineering, for kindly sharing her office while I was working on my thesis.

I would like to thank Afro-European Engineers for their kind support by letting me use office tools unlimitedly.

I am greatly indebted to Haimanot Tadesse and Natneal Tsehay, who reviewed and edited my paper. Your comments definitely improved my work. I would also like to thank Professor Nader Elhajj for taking his time and reviewing my thesis.

I would like to express my sincere gratitude to my family for investing in my education, making me a better person and supporting me unconditionally. I am very grateful to my friends for encouraging me during tough times. Last but not least, I wish to thank all who have helped me during my study.

1 INTRODUCTION

1.1 Background

Nowadays, steel-concrete composite construction is used to meet performance and functional requirements of structures. Composite structures acquire the structural and constructional advantages of both concrete and steel. Concrete has low material costs, good fire resistance and easy to place. Steel has high ductility, high strength-to-weight and stiffness-to-weight ratios. When properly combined, composite construction can yield saving in initial and life-cycle costs. [1]

A steel-concrete composite column is a compression member, comprising either a concrete encased steel section or a concrete filled tubular steel section. In a composite column, both the steel and concrete resist the external loading by interacting together through bond and friction. Supplementary reinforcement in the concrete encasement prevents excessive spalling of the concrete both under normal load and fire conditions.

The resistance of a composite column to combined compression and bending is determined using an interaction curve of its cross-section. In a typical interaction curve of a column with steel section only the moment resistance undergoes a continuous reduction with an increase in the axial load. However, a short composite column will often exhibit an increase in the moment resistance beyond plastic moment under relatively lower values of axial load. This is because the compressive axial load prevents concrete cracking and makes the composite cross-section of a short column more effective in resisting moment.

Most codes of practice such as, the widely used AISC-LRFD, ACI-318 and Eurocode 4 have incorporated simplified methods for analysis and design of composite columns. These provisions are generally extrapolated from either reinforced concrete column or steel column design codes. [2]

1.2 Statement of the Problem

The design of a composite column commences by assuming a certain cross-section with all the dimensions and material properties defined. Then, an interaction curve has to be developed for the assumed section. In EN 1994-1-1:2004 [3], a simplified method is provided for the analysis and design of composite column. But, this method is applicable for symmetrical sections fulfilling certain requirements. In this method, the moment-axial force interaction curve of a cross-section is determined by assuming full plastic stress distributions. The evaluation of points on the interaction curve this way is computationally difficult. For this reason, the interaction curve is further approximated with a polygon made by computing four or five points of the curve. Despite these approximations, the determination of the design resistance of a section is yet computationally demanding. The capacity of the column length is then determined using the interaction curve of its cross-section by accounting for the slenderness and member imperfection effects. Finally, the column length is checked whether it is capable of resisting the design action effects or not. If the capacity of the column length turned out to be inadequate or over designed, another section will have to be assumed and the above process is repeated all over again. In addition to the above, a rigorous section analysis is mandatory for all other sections that fail to satisfy the code preconditions.

Although high-rise construction is being introduced in Ethiopia, the topic of composite frames is often given only a limited consideration up to now. However, they can provide optimal solution in medium to high-rise buildings or large span structures. It is a customary practice for a structural engineer to design reinforced concrete columns regardless of the nature of structures. Together with the multistory, larger span for functional space of halls or parking areas may be required. As a result, an individual column may be subjected to high axial load and moments. Such cases are going to require axial members with high strength and yet with reasonable dimensions to serve the architectural purposes. In this scenario, the use of steel-concrete composite frames may provide the best solution. However, neither design aid nor analyses tool haven't been developed according to the revised Ethiopian building codes yet.

1.3 Objectives

This thesis attempts to develop uniaxial interaction curves for "I" or "H" section fully encased in concrete according to the Eurocodes.

In addition, the following are believed to be addressed:

- To show the design procedure of composite columns according to EN 1994-1-1:2004 [3] simplified method.
- To assist structural engineers with the analysis and design of composite column.

1.4 Methodology

Section analysis for a fully encased composite section that is subjected to a uniaxial bending as well as axial force is carried out based on the possible strain distributions in the ULS. These strain distributions are taken from EN 1992-1-1:2004 [5], which were initial provided for reinforced-concrete cross-sections. Full interaction among the concrete, reinforcement and structural steel until failure is assumed. Hence, the stress resultants are obtained by summing the contributions of the concrete, reinforcement and structural steel. Moreover, the stress-strain relations of the concrete and reinforcement steel are adopted from EN 1992-1-1:2004 [5] and that of the structural steel from EN 1993-1-1:2005 [6]. The constitutive laws are expressed with piecewise defined polynomials. Solutions of the stress resultants of the concrete in compression and structural steel which is either in compression or tension involve double integrals over their corresponding regions. These double integrals are transformed to line integrals along their closed boundaries using Green's theorem. For ease of computation, the line integrals are solved using a numerically exact method of Gauss Quadrature.

The output of this study is verified using a section analysis program known as MASQUE, which is developed by Busjaeger and Quast [7].

A computer program with a friendly user interface is also developed to enhance the applicability of this study.

Finally, design example of a composite column length under biaxial bending is carried out using the simplified design method of EN 1994-1-1:2004 [3]. The uniaxial curves of this study in the stronger and weaker axis are utilized.

1.5 Scope

This thesis only addresses uniaxial interaction chart for “I” or “H” sections fully encased in concrete. Furthermore, this study is also limited to the following:

- Only normal strength concrete i.e., from C12/15 to C50/60 is considered.
- The reinforcement steel grade is limited to S500.
- The structural steel grade is limited to S275.

1.6 Application of Results

This thesis is going to present solution method of concrete encased composite sections subjected to axial load and major or minor axis bending in particular for “I” or “H” structural steel sections and normal strength concrete. Furthermore, a program with friendly user interface is developed to assist structural engineers in analyzing composite columns. Evaluation of the capacity of the column length under biaxial bending using the uniaxial curves produced in this thesis is shown with example. This research is believed to assist the Ethiopian market by providing an analysis tool called UICISEC.

1.7 Content Organization and Description

This thesis is organized into six chapters. Chapter 2 provides detailed review of the current specification for analysis and design of composite columns according to the Eurocode 4, EN 1994-1-1:2004. Additionally, analytical and experimental investigation of fully encased composite columns is also reviewed. Chapter 3 presents a solution method to a composite section subjected to normal force and uniaxial moments in either of the principal axis. Moreover, development of the program UICISEC together with its basic features is discussed. Finally, verification of the interaction curve developed is included. In Chapter 4, one application of the output of this study is demonstrated with a design example. Chapter 5 summarizes this thesis and Chapter 6 presents the recommendations out of this study.

In the Annexes, the material properties for concrete and the database for reinforcement and structural steel class that are used in the program UICISEC are listed in tables. Reinforcement bar sizes and structural steel section types are also described.

2 LITERATURE REVIEW

2.1 Background

The necessities for large span and high-rise construction with reasonable construction time and cost have made composite structures of steel and concrete construction popular. Among the composite members is a composite column. Until the 1950s, structural steel sections were encased in light weight concrete for fire protection. The steel columns were analyzed and designed as if uncased. It was not until later on that was learned the encasement reduces the buckling length of the steel column and hence the buckling load increases. As a result, empirical methods were developed for calculating the reduced slenderness. This simple approach was not reasonable enough as the encasement also carries its share of axial load and bending moments. [4] After many tests that have been carried out, current practices include the contribution of concrete and reinforcement bars to resistance. [4] In the case of encased sections, the concrete reduces the buckling length of the steel sections and this is also taken into account in EN 1994-1-1:2004 [3].

In EN 1994-1-1:2004 [3], composite columns are divided into three categories in general (Figure 2.1):

- Concrete encased sections (a)
- Partially concrete-encased sections (b and c)
- Concrete-filled hollow sections (d, e, and f).

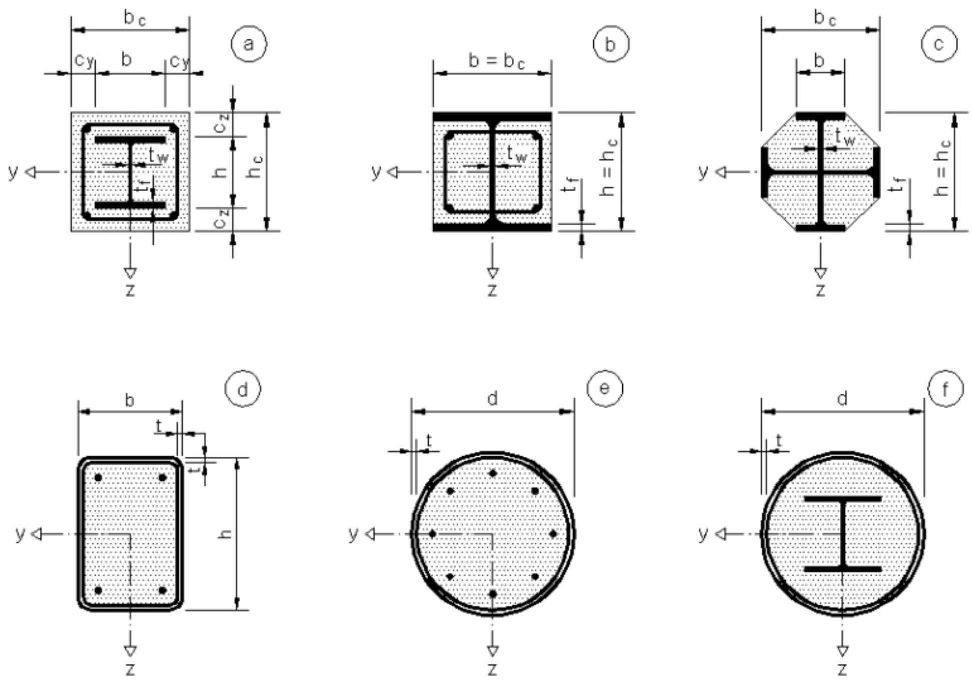


Figure 2.1: Typical cross-sections of composite columns [1]

Composite columns have the following advantages over reinforced concrete columns and steel columns:

- High load capacity with small cross-section and economical material use
- Increased stiffness, leading to reduced slenderness and increased buckling resistance
- Possibility of plastic deformation and enhanced ductile behavior
- High resistance to compressive stresses
- Reduced risk of local buckling of the steel section
- Good fire resistance for the case of encased columns
- Erection of high rise building in an efficient manner.

[1] *EN 1994-1-1:2004, 62*

2.2 Analysis and Design of Composite Columns According to the Eurocode 4

The Eurocode 4, EN 1994-1-1:2004 [3] provision for design of composite columns is limited to:

- Composite columns that is part of steel frame or composite frame;
- Design of composite columns with cross-sections as shown in Figure 2.1;
- Limited material strength class of steel grades S235 to S460 and normal weight concrete of strength class C20/25 to C50/60;
- Limited structural steel contribution ratio, δ :

$$0.2 \leq \delta \leq 0.9 \tag{2.1}$$

$$\delta = \frac{A_a f_{yd}}{N_{pl,Rd}} \tag{2.2}$$

Where:

A_a is the cross-sectional area of the structural steel section

f_{yd} is the design value for the yield strength of structural steel

$N_{pl.Rd}$ is design value of the plastic resistance of the composite section to compressive normal force

If δ is less than *0.2*, the column should be treated as reinforced concrete and if δ is greater than *0.9* the column should be treated as steel column. [4]

EN 1994-1-1:2004 [3] provides two methods of design of composite columns. These are “a general method’’ and “a simplified method’’.

The general method can be applied for all types of composite columns including columns of non-symmetrical or non-uniform cross-sections over the column length.

The simplified method is applicable for columns of doubly symmetric and uniform cross section over the member length.

Both methods of design assumed full interaction among the concrete, reinforcement steel and structural steel and hence plane sections remain plane while the column deforms up to failure.

2.3 The Simplified Method

2.3.1 Scope

To use the simplified procedure of EN 1994-1-1:2004 [3], the composite member has to meet the following requirements:

- The member has to be doubly symmetrical and uniform cross section along the length.
- The relative slenderness $\bar{\lambda}$ of the column should be less than *2.0*.
- If the longitudinal reinforcement is considered in design, then it should not exceed *6%* of the concrete area.

  A minimum reinforcement ratio of 0.3% is required to be considered in the contribution of resistance of concrete encased sections. [3]
- For a fully encased steel section (Figure 2.1), limits to the maximum thickness of concrete cover that may be used in calculation are:

  $$C_z \leq 0.3h \quad \text{and} \quad C_y \leq 0.4b \tag{2.3}$$

  Furthermore, to maintain the safe transmission of bond forces, for the protection of steel against corrosion and to prevent spalling of concrete, a minimum cover to the structural steel is required. According to EN 1994-1-1:2004 [3], this cover should be the maximum of 40mm or one-sixth of the flange width of the structural steel section.
- The ratio of depth *(hc)* to width *(bc)* of the cross-section (Figure 2.1), should be within the limits $0.2 \leq hc / bc \leq 5.0$.

2.3.2 Resistance of Cross-sections and Members in Axial Compression

The plastic resistance of encased cross-sections subjected to axial load $N_{pl,Rd}$ is given by equation 2.4. This equation superposes the contribution of the structural steel, the concrete and the reinforcement.

$$N_{pl,Rd} = A_a f_{yd} + 0.85 A_c f_{cd} + A_s f_{sd}$$

(2.4)

Where:

A_c is the cross-sectional area of concrete in the compression zone

A_s is the cross-sectional area of reinforcement bars

f_{cd} is design value of the cylinder compressive strength of concrete

f_{sd} is design value of the yield strength of reinforcing steel

A short ideal compression member, which is perfectly straight and loaded centrically, can carry as much load as its section capacity. However, columns in reality have imperfections. In addition to the axial load, the column must resist the associated imperfection moment. Thus, the axial load capacity of the column is reduced than its full section capacity due to member imperfections together with the slenderness effect. In EN 1994-1-1:2004 [3], the design action effect should satisfy the following to meet the stability requirement:

$$\frac{N_{Ed}}{\chi N_{pl,Rd}} \leq 1.0 \tag{2.5}$$

Where:

N_{Ed} is the design value of the applied axial force

χ is the reduction factor for flexural buckling

In EN 1994-1-1: 2004 [3], the verification of a composite column under axial compression is based on the European buckling curves that were initially developed for structural steel columns. The initial member imperfections are accounted in these buckling curves. Out of the five European buckling curves in EN 1993-1-1:2005 [6], only curve *a,b and c* are adopted for composite columns (Figure 2.2). This is due to the fact that concrete reduces the effective buckling length of the steel section.

The relevant buckling curves for different cross-sections of composite columns are given in Table 2.4. For concrete encased sections buckling curve b and c are adopted for bending about the major and minor axis respectively.

The value of χ is based on the relevant buckling mode given in EN 1993-1-1:2005 [6] in terms of the relevant relative slenderness $\bar{\lambda}$.

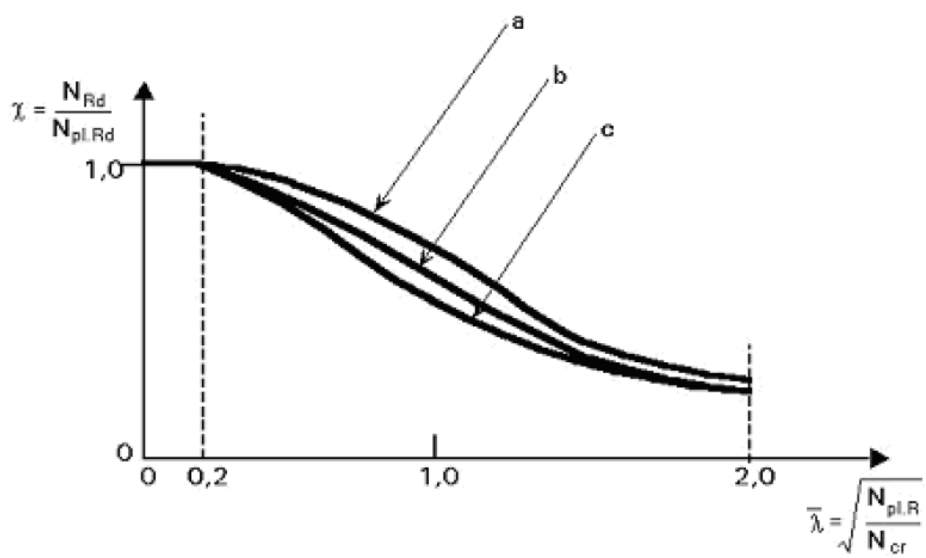


Figure 2.2: European buckling curves for composite columns

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \leq 1.0 \tag{2.6}$$

$$\phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right] \tag{2.7}$$

The imperfection factor α is given in Table 2.1 corresponding to the appropriate buckling curve.

Table 2.1: Imperfection factor

European bulking curve	a	b	c
Imperfection factor α	0.21	0.34	0.49

The relative slenderness for the plane of bending is given by:

$$\bar{\lambda} = \sqrt{\frac{N_{pl,Rk}}{N_{cr}}} \tag{2.8}$$

Where:

$N_{pl,Rk}$ is the characteristic value of plastic resistance to compression

N_{cr} is the elastic critical force for the relevant buckling mode

The characteristic squash load $N_{pl,Rk}$ is evaluated as in stated by (2.4) but instead of the design strength, the characteristic values of strength of materials are used.

$$N_{pl,Rk} = 0.85 A_c f_{ck} + A_s f_{sk} + A_a f_y \tag{2.9}$$

Where:

f_{ck} is characteristic value of the cylinder compressive strength of concrete at 28 days

f_{sk} is characteristic value of the yield strength of reinforcing steel

f_y is nominal value of the yield strength of structural steel

The critical buckling load N_{cr} is evaluated by Euler's formula, in which the flexural rigidity of the column is computed taking in account of the reduced stiffness of the concrete due to cracking.

$$N_{cr} = \frac{\pi^2 (EI)_{eff}}{L^2} \tag{2.10}$$

Where:

L is the buckling length of the column

$(EI)_{eff}$ is the effective flexural stiffness of the composite cross-section

By reducing the stiffness of the concrete, the characteristic effective flexural stiffness is calculated based on the gross cross-section of the concrete.

$$(EI)_{eff} = E_a I_a + E_s I_s + K_e E_{cm} I_c \tag{2.11}$$

Where:

I_a is second moment of area of the structural steel section

I_s is second moment of area of the steel reinforcement

I_c is second moment of area of the un-cracked concrete section

E_{cm} is secant modulus of elasticity of concrete

K_e is a correction factor to account for a reduced stiffness of concrete due to cracking and in EN 1994-1-1:2004 [3] the recommended value is *0.60*.

The long term effects of sustained loads should also be accounted on the effective flexural rigidity of the column. In EN 1994-1-1:2004 [3] the modulus of the concrete is further reduced to $E_{c,eff}$ and the effective flexural rigidity is evaluated by (2.11).

$$E_{c,eff} = \frac{E_{cm}}{\left[1 + \left(\frac{N_{G,Ed}}{N_{Ed}} \right) \phi_t \right]} \tag{2.12}$$

Where:

Φt is the creep coefficient according to EN 1994-1-1:2004, 5.4.2.2

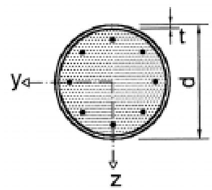
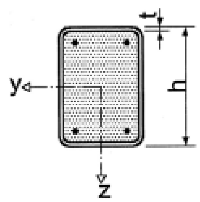
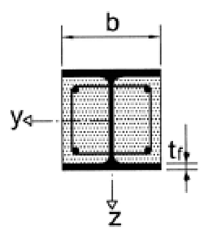
$N_{G,Ed}$ is part of N_{Ed} that is permanent

If the increase in the first order bending moments as a result of creep deformations is not more than 10%, creep and shrinkage effects can be neglected.

2.3.3 Local Buckling of Steel Members, Second Order Effects and Member Imperfections

For compression members, local buckling of the structural steel has to be checked first. According to EN 1994-1-1:2004 [3], the effects of local buckling may be ignored for composite sections fulfilling a specified depth to thickness ratios as stated in Table 2.2. Otherwise allowance should be made to account for the reduction of the ultimate capacity as a result of local buckling. For sections completely encased in concrete satisfying at least the minimum concrete cover code requirements, local buckling need not be checked.

Table 2.2: Maximum values (d/t), (h/t) and (b/tf) with fy in N/mm^2 [2]

Cross-section		max (d/t), max (h/t) and max (b/t)
Circular hollow steel sections		$\max\ (d/t) = 90\dfrac{235}{f_y}$
Rectangular hollow steel sections		$\max\ (h/t) = 52\sqrt{\dfrac{235}{f_y}}$
Partially encased I-sections		$\max\ (b/t_f) = 44\sqrt{\dfrac{235}{f_y}}$

For member verification, second order effects due to both the global ($P-\Delta$) and local ($P-\delta$) deformations have to be considered while determining the action effects. According to EN 1994-1-1:2004 [3], the evaluation of internal forces should be based on the design effective flexural stiffness $(EI)_{eff,II}$.

$$(EI)_{eff,II} = K_o\left(E_a I_a + E_s I_s + K_{e,II} E_{cm} I_c\right) \tag{2.13}$$

[2] *EN 1994-1-1:2004, 63*

Where:

$K_{e,II}$ is a correction factor which should be taken as 0.5

K_o is a calibration factor which should be taken as 0.9

The influence of second order effects may be neglected for braced and non-sway frames:

- If the relevant action effects increase by less than 10% of the first-order analysis results due to deformations of a member. This condition is assumed to be fulfilled if the following criterion is satisfied:

$$\alpha_{cr} = \frac{N_{Ed}}{N_{cr,eff}} \leq 10 \tag{2.14}$$

  Where:

  α_{cr} is the factor by which the design loading would have to be increased to cause elastic instability

  $N_{cr,eff}$ is the critical normal force for the relevant axis and corresponding to the effective flexural stiffness with the effective length taken as the column length

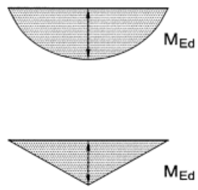
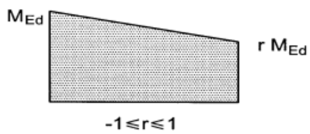
- If the elastic critical load is determined with $(EI)_{eff,II}$.

If the above conditions are not satisfied, second-order analysis shall be carried out. Second order effects may also be allowed approximately by amplifying the maximum first order design moment within the column length with factor k:

$$k = \frac{\beta}{1 - \frac{N_{Ed}}{N_{cr,eff}}} \geq 1 \tag{2.15}$$

The equivalent moment factor, β, is given in Table 2.3 for different types of moment distributions along the column length. In Table 2.3, r is the ratio of smaller to the larger end moments.

Table 2.3: Factors β for the determination of moments to second order theory [3]

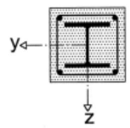
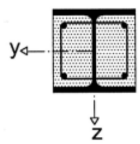
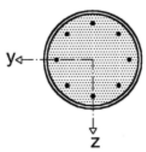
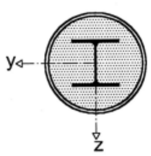
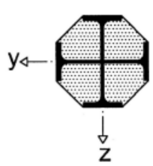
Moment distribution	Moment factors β	Comment
	First-order bending moments from member imperfection or lateral load: $\beta = 1.0$	M_{Ed} is the maximum bending moment within the column length ignoring second-order effects
	End moments: $\beta = 0.66 + 0,44r$ but $\beta \geq 0.44$	M_{Ed} and $r\ M_{Ed}$ are the end moments from first-order or second-order global analysis

Imperfections in composite frame may arise due to lack of verticality of columns, lack of fit between members, effect of residual stresses in steel section and temperature gradient within the structure. [4]

In EN 1994-1-1:2004 [3], the geometrical and structural imperfections are considered by equivalent geometrical imperfections (initial bows) as given in Table 2.4, where L is the column length. This eccentricity is allowed locally in verification of members only. The initial bow is assumed to occur at the mid length of the member. Hence, multiplying the geometrical imperfection by the design axial force gives the imperfection moment. Furthermore, the imperfection moment is amplified to account for the second order effects if the frame is susceptible to secondary effects. Therefore, the column length must have sufficient capacity to resist the design moment plus the imperfection moment for a given design axial load.

[3] *EN 1994-1-1:2004, 69*

Table 2.4: Buckling curves and member imperfections for composite columns [4]

Cross-section	Limits	Axis of buckling	Buckling curve	Member imperfection
concrete encased section 		y-y	b	*L*/200
		z-z	c	*L*/150
partially concrete encased section 		y-y	b	*L*/200
		z-z	c	*L*/150
circular and rectangular hollow steel section 	$\rho_s \leq 3\%$	any	a	*L*/300
	$3\% < \rho_s \leq 6\%$	any	b	*L*/200
circular hollow steel sections with additional I-section 		y-y	b	*L*/200
		z-z	b	*L*/200
partially concrete encased section with crossed I-sections 		any	b	*L*/200

[4] *EN 1994-1-1:2004, 70*

2.3.4 Resistance of Cross-sections and Members under Axial Load and Uniaxial Bending

The ultimate capacity of a cross-section subjected to axial load and bending moment is determined from the moment-axial force interaction curve of a particular section. In a steel beam-column interaction curve, the moment resistance reduces with increasing axial load. However, in a composite beam-column, the moment resistance increases up to the balanced point for a lower value of axial compression. This is due to the pre-stressing effect of the compressive force which prevents excessive cracking of concrete.

According to the simplified method of EN 1994-1-1:2004 [3], the points of the interaction curve can be determined by assuming a full plastic stress distribution known as rigid-plastic approximation. The structural steel section and the reinforcing steel bars are assumed fully plasticized either in tension or compression with the stress ordinates equal to their design yield strengths. For the concrete, a rectangular compressive stress of $0.85f_{cd}$ that is distributed uniformly between the most compressed face and the plastic neutral axis is assumed.

Initially several plastic neutral axis positions in the direction of bending are assumed. Then, the corresponding values of moment and axial load are evaluated from the resulting stress blocks. Finally, the equilibrium conditions are checked.

In the simplified method, the interaction curve is further approximated by a polygon made by connecting four or five points of the interaction curve. For bending about the major axis, it is sufficient to know four of these points shown in Figure 2.3. For weaker axis bending however, additional point between A and C shall be determined as the polygon bulges from the curve significantly.

Point A (Figure 2.3) represents the plastic axial load capacity of a composite section as given by equation (2.4).

$$N_A = N_{pl,Rd} \quad and \quad M_A = 0 \tag{2.17}$$

Point B is the plastic moment resistance of a section.

$$N_B = 0 \quad and \quad M_B = M_{pl,Rd} \tag{2.18}$$

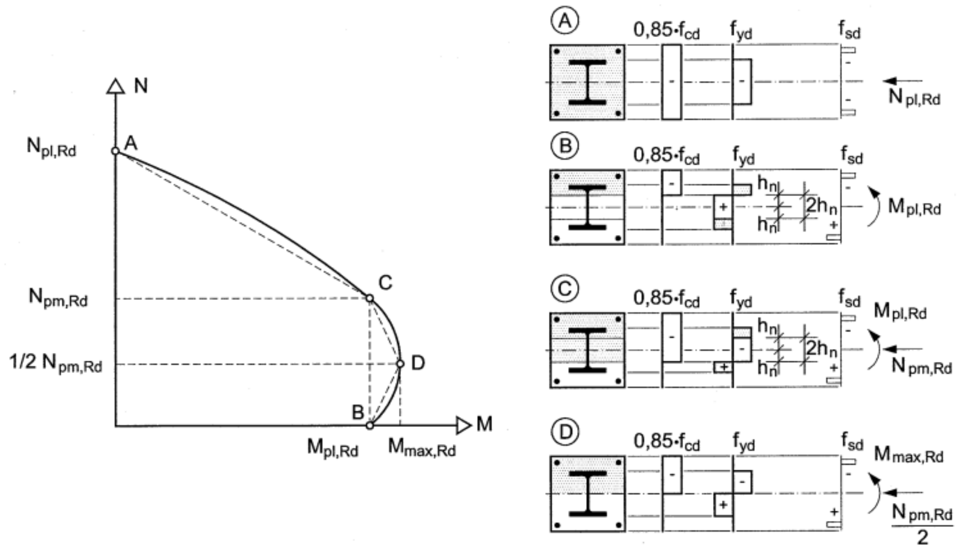


Figure 2.3: Simplified interaction curve and corresponding stress distributions[5]

Point C corresponds to the same plastic moment resistance as point B but with resultant axial compression force. For concrete encased sections:

$$N_C = N_{pm,Rd} = 0.85 * A_c * f_{cd} \quad and \quad M_C = M_{pl,Rd} \tag{2.19}$$

Point D is a balanced point representing the maximum moment carrying capacity of a composite section. For concrete encased sections:

$$N_D = 0.5 * N_{pm,Rd} \quad and \quad M_D = M_{max,Rd} \tag{2.20}$$

The depth h_n (Figure 2.3) is proportioned in a way that the stress distribution of type *C* provides the same value of moment as type *B*. It is assumed that the resulting resistance to axial force $N_{pm.Rd}$ is due to the concrete only. This can be seen by adding up the stress distributions in *B* and *C*, with regard to the equilibrium of forces, i.e. the resulting axial force.

[5] EN 1994-1-1:2004, 66

Member resistance that is subjected to axial load and bending moments is determined from its cross-section capacity curve including the influence of Imperfections.

According to EN 1994-1-1:2004 [3], a member must satisfy the following:

$$\frac{M_{Ed}}{M_{pl,N,Rd}} = \frac{M_{Ed}}{\mu_d M_{pl,Rd}} \leq \alpha_M \qquad (2.24)$$

Where:

M_{Ed} is the greatest of the end moments and the maximum bending moment within the column length, calculated including imperfections and second order effects if necessary

$M_{pl,N,Rd}$ is the plastic bending resistance taking into account the normal force N_{Ed}, given by $\mu_d M_{pl,Rd.;}$

$M_{pl,Rd}$ is the plastic bending resistance, given by point B in Figure 2.3.

The factor α_M is a correction factor for the unconservative assumption that the rectangular stress block for concrete extends to the plastic neutral axis. For steel grades up to S355, α_M is 0.9 and for steel grades S420 and S460 α_M is 0.8.

In EN 1994-1-1:2004 [3] the value of μ_d greater than one is recommended only if the bending moment M_{Ed} depends directly on the action of the design normal force $N_{Ed.}$ This is the case for example, if the moment is resulted from eccentrically applied design normal force. Otherwise an additional verification shall be performed according to EN 1994-1-1:2004, 6.7.1(7) [3]. This clause states that for composite compression members subjected to bending moments and normal forces resulting from independent actions, the partial factor γ_F for those internal forces that lead to an increase of resistance should be reduced by 20%.

2.3.5 Resistance of Members under Axial Load and Biaxial Bending

The verification of composite column subjected to biaxial bending is based on separate check in each of the principal axis as shown in Figure 2.4. Imperfections are accounted only in the plane on which failure is likely to occur. If this plane is not apparent, then separate checks is required in each of the planes. [3]

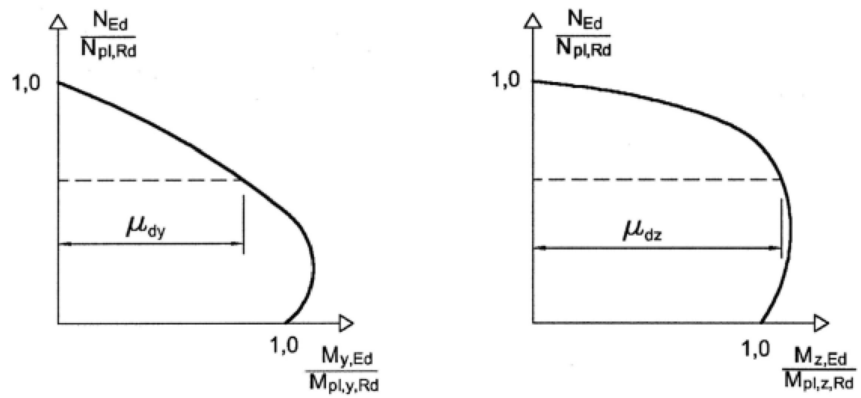


Figure 2.4: Design of column length under axial load and biaxial bending [6]

The column must satisfy the following conditions for the stability within the column length and ends:

$$\frac{M_{y,Ed}}{\mu_{dy} M_{pl,y,Rd}} \leq \alpha_{M,y} \quad and \quad \frac{M_{z,Ed}}{\mu_{dz} M_{pl,z,Rd}} \leq \alpha_{M,z} \tag{2.25}$$

$$\frac{M_{y,Ed}}{\mu_{dy} M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} M_{pl,z,Rd}} \leq 1 \tag{2.26}$$

Where:

$M_{pl,y,Rd}$ and $M_{pl,z,Rd}$ are the plastic bending resistances for bending about the y-y and z-z plane respectively

$M_{y,Ed}$ and $M_{z,Ed}$ are the design bending moments including second-order effects and imperfections about the y-y and z-z plane respectively

[6] EN 1994-1-1:2004, 71

As mentioned above, it may not be obvious in which plane failure is anticipated. In this case, equations (2.25) and (2.26) are checked twice by taking the imperfections one at a time in both planes. First it is checked whether the moment capacity $\mu_{dy} M_{pl,y,Rd}$ is sufficient to resist the design moment $M_{y,Ed}$ about the y-axis plus the imperfection moment. But on the other axis $\mu_{dz} M_{pl,z,Rd}$ is checked if it is capable of resisting the design moment $M_{z,Ed}$ about the z-axis. Finally, the capacity of the column length is verified if it can resist the biaxial moment with equation (26). The reverse is repeated, taking the member imperfection about the z-direction only. The column length must satisfy all the conditions in both the cases.

2.3.6 The Influence of Transverse Shear Force

If the design shear force $V_{a,Ed}$ of the steel section exceeds 50% of the design shear resistance $V_{pl,a,Rd}$ of the steel section, then the influence of this shear force on the resistance to axial force and bending should be considered. According to EN 1994-1-1:2004 [3], the influence of shear is accounted by using reduced steel strength for the web by $(1-\rho)f_{yd}$ in the shear area Av.

For simplification the design shear force V_{Ed} may be assumed to act on the structural steel section alone. However, the design shear force on the steel section should not exceed the shear resistance of the steel section. If this is not the case and V_{Ed} is greater than $V_{pl,a,Rd}$, then the design shear force may be distributed between the structural steel $V_{a,Ed}$ and the reinforced concrete $V_{c,Ed}$. The design shear force on the reinforced concrete is verified with the same approach as for reinforced concrete members according to EN 1992-1-1:2004 [5].

Unless there is a more accurate analysis to determine the shear distributed between the concrete and steel, in EN 1994-1-1:2004 [3] the following is recommended:

$$V_{a,Ed} = V_{Ed} \frac{M_{pl,a,Rd}}{M_{pl,Rd}} \tag{2.27}$$

$$V_{c,Ed} = V_{Ed} - V_{a,Ed} \tag{2.28}$$

Where:

$M_{pl,a,Rd}$ is the plastic moment resistance of the steel section

The design plastic shear resistance $V_{pl,a,Rd}$ of the structural steel section is computed in the same way as for steel sections given in EN 1993-1-1:2005, 6.2.6, [6].

$$V_{pl,a,Rd} = A_v \frac{f_{yd}}{\sqrt{3}} \tag{2.29}$$

The shear area, Av, for rolled I or H sections is given by:

$$A_v = A_a - 2bt_f + (t_w + 2r)t_f \tag{2.30}$$

Where:

b is the overall breadth

h is the overall depth

h_w is the depth of web

t_f is flange thickness

For class 1 or 2 steel cross-sections, the reduction factor, ρ, is evaluated by:

$$\rho = \left(\frac{2V_{Ed}}{V_{pl,a,Rd}} - 1 \right)^2 \tag{2.31}$$

After reducing the yield strength of the web, the moment-axial load interaction curve can be evaluated in the same way using the simplified method.

2.4 Analytical and Experimental Investigation of Concrete Encased Composite Columns

As reviewed, EN 1994-1-1:2004 [3] is suitable only for the analysis and design of composite columns fulfilling certain requirements. The simplified method is thus based on certain assumptions. Aside from aforementioned simplifications, different properties of the composite columns are not sufficiently considered.

E. Ellobody et al. [8] carried out analytical study on eccentrically loaded concrete encased steel composite columns along their major plane. They developed a nonlinear 3-D finite element model (FEM) that considered the inelastic behavior of constituent materials of the composite column. The effect of confinement to the concrete by the steel flanges and the transverse reinforcement was also accounted. To reveal the bond behavior, the interface between the structural steel and concrete, the concrete and transverse reinforcement, the concrete and longitudinal reinforcement and the longitudinal reinforcement and the transverse reinforcements were modeled. The initial geometrical imperfection was also comprised in the model.

E. Ellobody et al. [8] adopted the work of Sheikh and Uzumeri [9] and Mander et al. [10] for reinforced concrete column to model the confined concrete. The composite columns were divided into highly confined concrete, partially confined concrete and the unconfined concrete zones (Figure 2.5). Chen and Lin [11] evaluated the confinement factors for the highly and partially confined concrete zones.

Mander et al. [10] expressed the confined concrete compressive strength and the corresponding confined strain in terms of the lateral confining pressure. This pressure was determined approximately having known the confinement factors for the highly and partially confined concrete as given by Chen and Lin [11]. Depending on the steel section shape and the spacing between the transverse reinforcements, the confinement factor varied from 1.10 to 1.97 for highly confined concrete and 1.09 to 1.50 for the partially confined concrete.

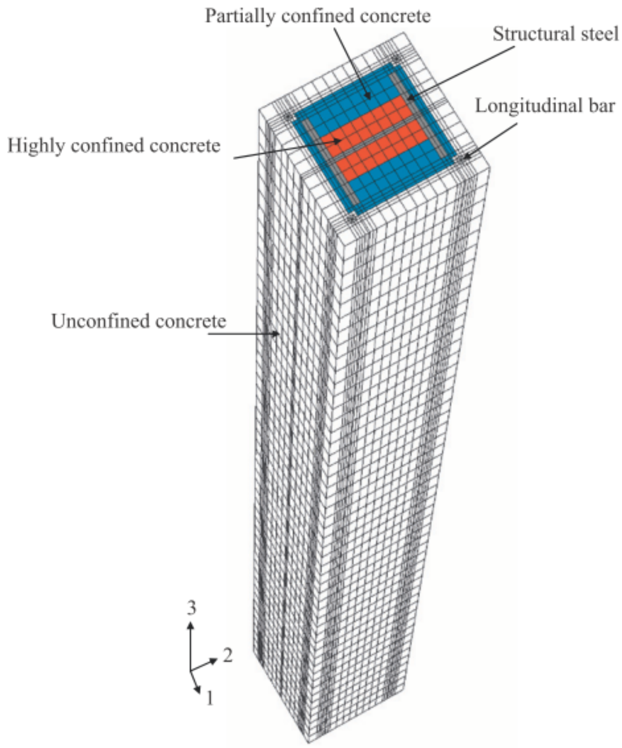


Figure 2.5: Finite element model of concrete encased steel composite column [7]

E. Ellobody et al. [8] verified the developed FEM against the test results by Al-Shahari et al. [12], SSRC Task Group 20 [13], and Morino et al. [14]. Good agreement was obtained with the test results with mean value of axial capacity from the finite element analysis (FEA) to the axial capacity from test ratio was 0.95.

E. Ellobody et al. [8] extended their work to parametric study of columns having different eccentricities, overall cross-section dimensions, structural steel sections, concrete strength and structural steel yield strength. Square and rectangular columns were considered in the parametric studies and the details of the studied columns can be found in the journal. From this study, it was observed that increase in the structural steel strength was significant for the columns with small eccentricities. For the

[7] *Ehab Ellobody, Ben Young, Dennis Lam, Eccentrically loaded concrete encased steel composite columns, 57*

columns with higher eccentricity, the increase of the yield strength of the steel was significant when the columns were encased in normal strength concrete.

The axial load and moment obtained from the finite element analyses (FEA) in the parametric study were compared with the unfactored design axial force and moment according to the Eurocode 4 simplified design method. The axial loads obtained from FEA and Eurocode 4 were almost the identical with mean values of axial load of FEA to axial load of Eurocode 4 ratio were 1.02 and 0.99 for the square and rectangular columns respectively. On the other hand, the calculated design moments were considerably higher than the FEA with the mean values of ratio were 0.73 and 0.75 for the square and rectangular columns respectively. However, unlike the FEA, it was not clearly realized that the effects of yield strength of the structural steel in relation to the concrete grade using the Eurocode 4.

Mirza S.A. and Lacroix E.A. [15] compared the strength determined from 150 physical tests of rectangular concrete encased steel composite columns available in the published literature with the strength calculated from selected computational procedures, the ACI 318-02, AISC-LRFD and Eurocode 4 (CEN 1994). The tested columns were encased in normal density- normal strength concrete reinforced with longitudinal bars and transverse ties. The columns were pin ended and subjected to short term loads that produced pure axial force, axial force combined with equal and opposite end moments either in the major or minor axis, or pure bending moments.

The computed strengths according to the codes were unfactored. The Eurocode 4 simplified method was used to compute the axial load-bending moment interaction of the cross-sections and the columns capacity.

The tested strengths were divided by the computed unfactored capacities to obtain normalized strength ratio. The strength ratios were taken as bending moment ratios for columns that were subjected to pure flexure and for all others were axial load strength ratios.

With the Eurocode 4 simplified method, the column strengths were computed more accurately with an average strength ratio of 1.04 and a coefficient of variation 0.15.

The different end eccentricities, slenderness ratio (the column length to depth ratio), structural steel contribution ratio, longitudinal steel contribution ratio and characteristic compressive cylindrical strength of the concrete that were used in the tests didn't affect the strength ratios pronouncedly. However, the various hoop volumetric ratio of the tested columns resulted in increased strength ratio. This was because the Eurocode 4 do not account for the increase in strength of the concrete due to the confinement by the transverse reinforcement.

3 UNIAXIAL INTERACTION CHART FOR I-SECTION FULLEY ENCASED IN CONCRETE

3.1 Background

Fafitis [16] computed the interaction surface of an arbitrary reinforced concrete section under axial load and biaxial bending. Analytically exact method based on Green's theorem was applied to transform the double integrals of the concrete stress fields to line integrals along the boundaries of the concrete in the compression zone.

Sousa and Muniz [2] proposed a procedure for the analysis of polygonal composite cross-sections that employs Green's theorem for evaluation of cross-sectional properties.

In this thesis, a uniaxial interaction curve is developed for fully encased rectangular composite cross-sections. The section analysis for a given cross-section starts by assigning a particular strain distribution in the ULS. The stress fields are then determined using constitutive laws of materials. The stress-strain laws of materials are defined with piecewise polynomial functions. The stress resultants of a section are derived by summing the stress resultants for each of the materials i.e., the concrete, reinforcement and structural steel. Green's theorem is used to transform the area integrals of the stress fields of the concrete in compression and structural steel either in tension or compression to line integrals along the corresponding boundaries of materials. The transformed line integrals are solved numerically by Gauss Quadrature.

3.2 Basic Assumptions

The major assumptions made in this study are:

- There is full interaction among the concrete, reinforcement and structural steel until failure occurs. The strain in bonded reinforcement and structural steel, whether in tension or compression is the same as the surrounding concrete. Hence, plane sections remain plane after deformation.
- The tensile strength of the concrete is neglected.

3.3 Constitutive Laws of Materials

The stress-strain laws of the materials are described with piecewise defined polynomial functions.

For design of a cross-section, parabolic-rectangular stress-strain relation is recommended in EN 1992-1-1:2004 [5]. The parabolic part is a second degree polynomial while the rectangular portion is a constant function (Figure 3.1).

$$\sigma_c = f_{cd}\left[1-\left(1-\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] \quad \text{for} \quad 0 \le \varepsilon_c \le \varepsilon_{c2} \tag{3.1a}$$

$$\sigma_c = f_{cd} \quad \text{for} \quad \varepsilon_{c2} \le \varepsilon_c \le \varepsilon_{cu2} \tag{3.1b}$$

Where:

n is the exponent according to EN 1992-1-1:2004

ε_c is compressive strain in the concrete

ε_{c2} is the strain at reaching the maximum strength

ε_{cu2} is the ultimate compressive strain in the concrete

For normal strength concrete, the values of n, ε_{c2} and ε_{cu2} can be found in Annex A.

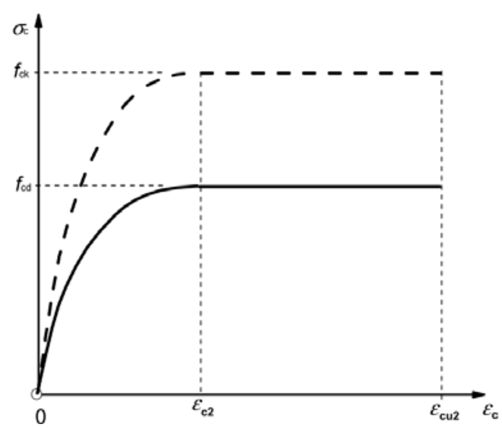


Figure 3.1: Parabolic-rectangular stress-strain relation of concrete under compression [8]

[8] EN 1992-1-1:2004, 35

EN 1992-1-1:2004 [5] provides two types of stress-strain relations for reinforcing steel as shown in Figure 3.2. In this study the bilinear relation, which doesn't limit the strain after yielding, is adopted and it is expressed as:

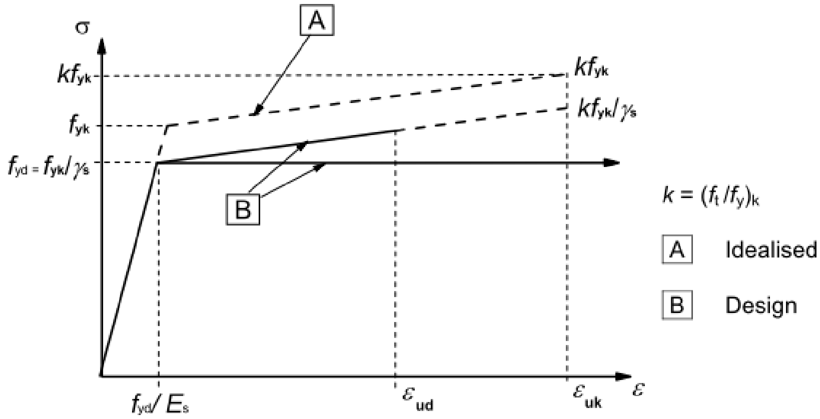


Figure 3.2: Idealized and design stress-strain diagrams for reinforcing steel [9]

$$\sigma_s = E_s \varepsilon_s \text{ for } 0 \le \varepsilon_s \le \varepsilon_{sd} \quad (3.2a)$$

$$\sigma_s = f_{sd} \quad (3.2b)$$

Where:

ε_s is the strain in the reinforcing steel

ε_{sd} is the yield strain in the reinforcing steel

The stress-strain relation for the structural steel is taken from EN 1993-1-1: 2005 [6] (Figure 3.3) and is given by:

$$\sigma_a = E_a \varepsilon_a \text{ for } 0 \le \varepsilon_a \le \varepsilon_{yd} \quad (3.3a)$$

$$\sigma_a = f_{yd} \quad (3.3b)$$

[9] EN 1992-1-1:2004, 40

Where:

ε_a is the strain in the structural steel

ε_{yd} is the yield strain in the structural steel

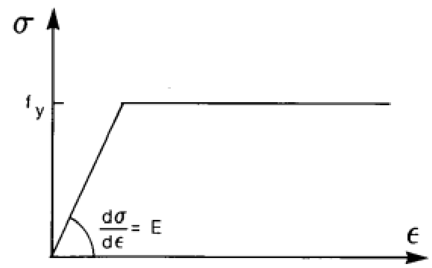


Figure 3.3: Bi-linear stress-strain relationship for structural steel [10]

3.4 Strain Distributions in the Ultimate Limit State

Section analysis of a cross-section that is subjected to axial load and bending moments starts with assumed strain distribution in the ULS. Then, using stress-strain relationships of the constituent materials, the stress fields are determined. The stress fields are integrated over the respective regions of the materials in the cross-section to obtain their resultants.

The possible strain distributions in the ULS are shown in Figure 3.4, which is adopted from EN 1992-1-1:2004 [5]. A section that has reached its ultimate capacity fails by either crushing of concrete or rupture of the steel in tension. Hence, a strain distribution in the ULS has to pass either through the concrete strain limit ε_{cu2} or the concrete pure compression limit ε_{c2} or the reinforcing steel tension strain limit ε_{ud}.

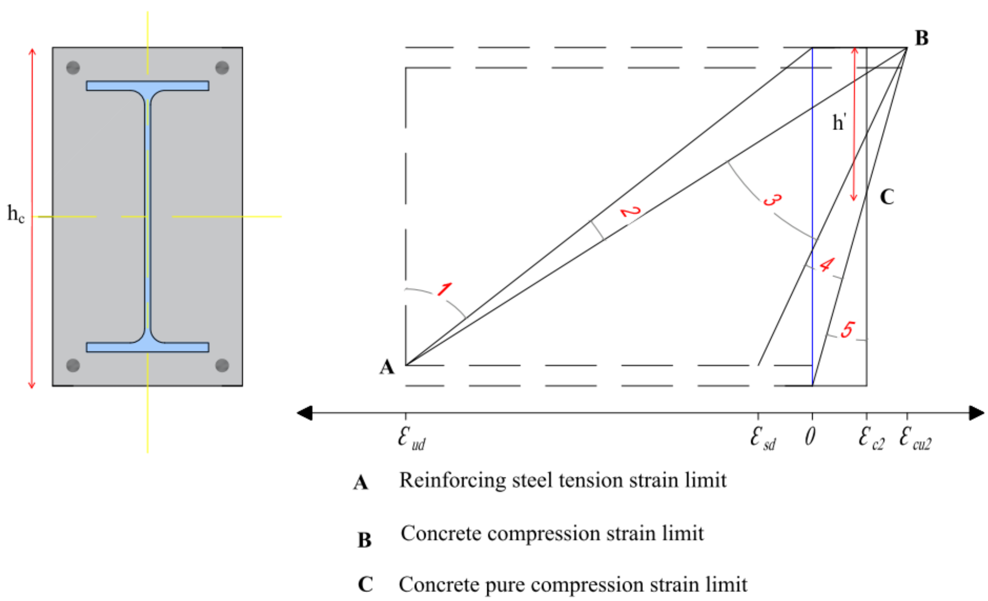
The limiting values of strain in compression or tension at the ultimate load have to be known. In EN 1992-1-1:2004 [5] the crushing strain ε_{cu2} of the most compressed concrete fiber is kept to *3.5‰* for normal strength concrete. The characteristic rupture strain ε_{uk} of the most tensioned steel fiber depends on the ductility class of the

[10] EN 1993-1-1:2005, 39

reinforcement or the structural steel. According to EN 1992-1-1:2004 [5] *25‰, 50‰* and *75‰* may be used for Class *A*, Class *B* and Class *C* reinforcing steel respectively. In this thesis the ultimate strain in the steel is held to *25‰* and for determining capacity of a section, the factored strain ε_{ud} is kept $0.8\varepsilon_{uk}$ which is equal to *20‰.*

The strain distributions in the ULS can be divided into five regions as shown Figure 3.4.

The first region (1) represents when the section is under tension. The section is subjected to axial tension and small bending. The neutral axis lies outside the section in all the cases. The most tensioned steel fiber has reached ε_{ud}. In this region, a section fails by rupture of steel.



$$h' = (1 - \varepsilon_{c2} / \varepsilon_{cu2})h_c$$

Figure 3.4: Possible strain distributions in the ultimate limit state

The second region (2) embodies when some portion of the section is under compression, but the concrete has not yet reached its crushing strain. The section in this case is subjected to axial tension or compression and bending. The most tensioned steel fiber has reached ε_{ud}. Failure mode is still by the rupture of most strained steel.

The third region (3) represents when the most compressed fiber reaches ε_{cu2} and the strain in the most strained steel in tension is less than ε_{ud} but greater or equal to its yield strain ε_{sd}. The section is subjected to axial compression and bending. A section in this region fails by crushing of concrete.

The fourth region (4) depicts when the strain in the most strained steel in tension is below its yield strain. The section fails by crushing of concrete.

Finally the fifth region (5) represents when the section is under compression. All the strain profiles pass through point *C* (Figure 3.4).

3.5 Stress Resultants

After the material stress-strain laws defined and a particular strain distribution in the ULS is known, the stress resultants can be determined. These resultants, i.e. the axial load and moments, are determined by integrating the stresses over the corresponding regions of materials in the cross-section. Solving the integrals involves evaluation of double integrals of the stresses of the concrete and the structural steel. The reinforcements can be assumed as discrete points. Therefore, their contribution can be directly determined by multiplying the stress by the area of the reinforcement along the same fiber.

3.5.1 Cross-Sections Subjected to Axial load and Major axis Bending

The stress in the concrete or structural steel is a function of the normal strain ε_{ψ}.

$$\sigma_c = f(\varepsilon_{\Psi}) \tag{3.4}$$

$$\sigma_a = f'(\varepsilon_{\Psi}) \tag{3.5}$$

All the possible strain distributions in ULS have to pass through one of the points *A, B or C* (Figure 3.4). Having known which of these points a certain strain distribution crosses and from strain compatibility, the entire strain profile can be described in terms of $\varepsilon_{cu2}, \varepsilon_{c2}$ or ε_{ud}.

For a particular strain distribution shown in Figure 3.5, the strain ε_{ψ} at a distance ψ from the neutral axis can be expressed as follows:

$$\varepsilon_{\Psi} = \frac{\Psi}{c} \varepsilon_{cu2} \tag{3.6}$$

Where:

Ψ is the distance from the neutral axis

c is the depth from the most compressed fiber to the neutral axis

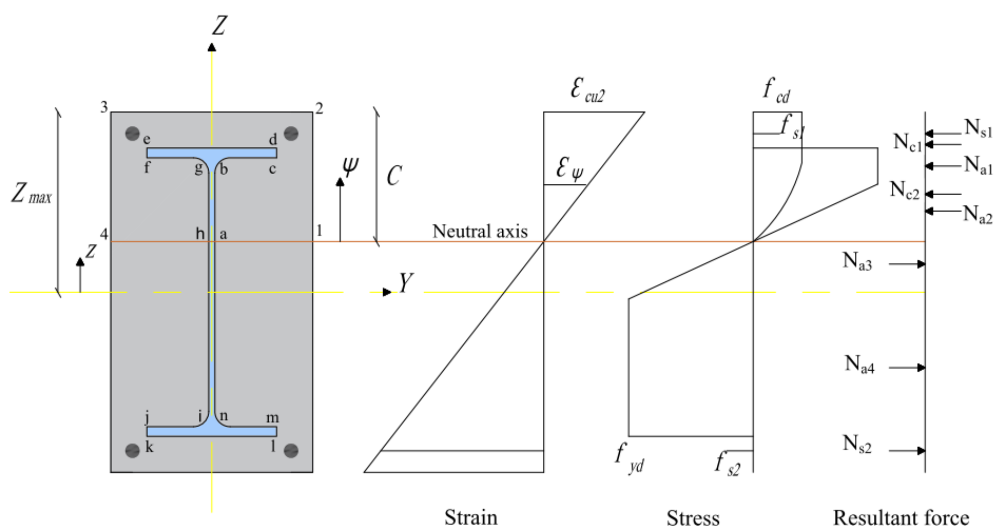


Figure 3.5: Arbitrary cross-section under axial load and major axis bending

In finite element analysis, columns are modeled as a line element represented at its geometrical center. Therefore, the action effects obtained from structural analysis of a system refer to centroids of columns. For this reason, Ψ is described in terms of a coordinate system referring to the centroid of a section. As can be seen from Figure 3.5:

$$\Psi = z + c - Z_{max} \tag{3.7}$$

And replacing Ψ in equation (3.6) with (3.7), the normal strain can be expressed in terms of z as:

$$\varepsilon_{\Psi} = \frac{\left(z + c - Z_{max}\right)}{c}\varepsilon_{cu2} \tag{3.8}$$

Hence, the stress in the concrete or structural steel is also a function of *z*:

$$\sigma_c = f\left(\Psi\right) = f\left(z + c - Z_{max}\right) = g\left(z\right) \tag{3.9}$$

$$\sigma_a = f'\left(\Psi\right) = f\left(z + c - Z_{max}\right) = h\left(z\right) \tag{3.10}$$

Equation (3.10) describes the stress in structural steel either in tension or compression.

The functions g (z) and h (z) are described with piecewise defined functions that are expressed in equations (3.1a) and (3.1b) for the concrete and for that of structural steel with equations (3.3a) and (3.3b). Substituting the expression for the normal strain in the above equations with equation (3.8), the stress in the concrete can be expressed as:

$$g(z) = f_{cd}\left[1 - \left(1 - \frac{(z + c - Z_{max})\varepsilon_{cu2}}{c * \varepsilon_{c2}}\right)^n\right] \text{ for } 0 \le \frac{\left(z + c - Z_{max}\right)}{c}\varepsilon_{cu2} \le \varepsilon_{c2} \tag{3.11a}$$

$$g(z) = f_{cd} \qquad \text{for } \varepsilon_{c2} \le \frac{\left(z + c - Z_{max}\right)}{c}\varepsilon_{cu2} \le \varepsilon_{cu2} \tag{3.11b}$$

And the stress in the structural steel is given by:

$$h(z) = E_a \frac{\left(z + c - Z_{max}\right)}{c}\varepsilon_{cu2} \text{ for } 0 \le \frac{\left(z + c - Z_{max}\right)}{c}\varepsilon_{cu2} \le \varepsilon_{yd} \tag{3.12a}$$

$$h(z) = f_{yd} \tag{3.12b}$$

The stress resultants of the concrete in the compression zone of area, *A,* are given by:

$$N_c = \iint_A g\left(z\right)dA \tag{3.13}$$

$$M_{yc} = \iint_A zg\left(z\right)dA \tag{3.14}$$

Where:

N_c is the axial force carried by the concrete

M_{yc} is the moment resisted by the concrete about the y-y plane

z is the lever arm of the concrete to the geometrical center

The structural steel contributions are evaluated as:

$$N_a = \iint_{A'} h(z) dA' \tag{3.15}$$

$$M_{ya} = \iint_{A'} z' h(z) dA' \tag{3.16}$$

Where:

N_a is the axial force carried by the structural steel

M_{ya} is the moment resisted by the structural steel about the y-y plane

A' is the area of structural steel either in tension or compression

z' is the lever arm of the structural steel to the geometrical center

The reinforcement bars are assumed as discrete points and their contributions are:

$$N_s = \sum_{j=1}^{N_b} A_{sj} \sigma_{sj} \tag{3.17}$$

$$M_{ys} = \sum_{j=1}^{N_b} Z_{sj} A_{sj} \sigma_{sj} \tag{3.18}$$

Where:

N_s is the axial force carried by the reinforcements

M_{ys} is the moment resisted by the reinforcements about the y-y plane

A_{sj} is the area of reinforcements at coordinates Y_{sj} and Z_{sj}

σ_{sj} is the stress in the steel at coordinate Y_{sj} and Z_{sj}

Z_{sj} is the lever arm of the reinforcements to geometrical center

Therefore, the stress resultants of the cross-section can be determined by summing the contribution of concrete, reinforcement and structural steel.

$$N_x = N_c + N_s + N_a \tag{3.19}$$

$$M_y = M_{yc} + M_{ys} + M_{ya} \tag{3.20}$$

Where:

N_x is the axial force carried by the cross-section

M_z is the moment resisted by the cross-section for bending about the y-y plane

3.5.2 Cross-Sections Subjected to Axial load and Minor axis Bending

A similar procedure used for major axis bending is applied to evaluate the stress resultants of a section subjected to axial load and minor axis bending.

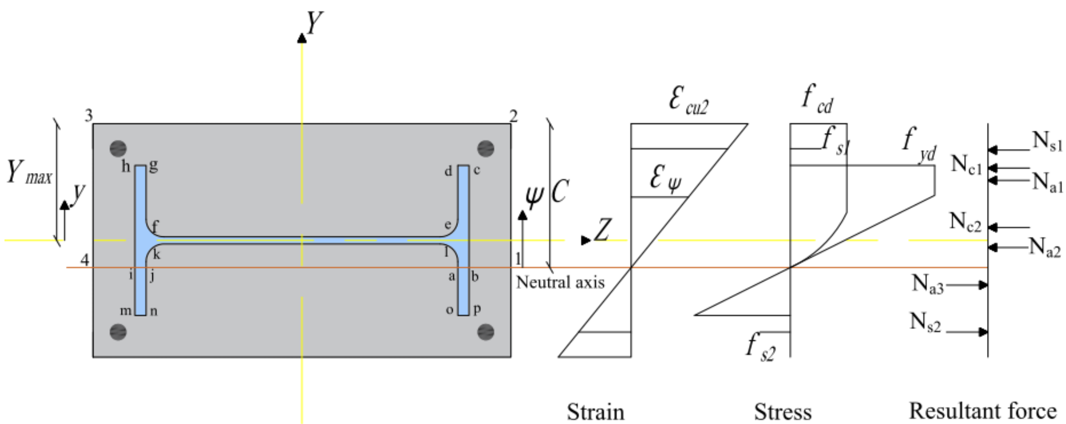


Figure 3.6: Arbitrary cross-section under axial load and minor axis bending

From Figure 3.6, the strain at distance ψ is:

$$\varepsilon_{\Psi} = \frac{\Psi}{c}\varepsilon_{cu2} = \frac{\left(y + c - Y_{max}\right)}{c}\varepsilon_{cu2} \tag{3.21}$$

The stress in the concrete or structural steel is a function of the normal strain ε_{ψ} and hence it is also a function of y.

$$\sigma_c = f\left(\Psi\right) = f\left(y + c - Y_{max}\right) = g\left(y\right) \tag{3.22}$$

$$\sigma_a = f'\left(\Psi\right) = f\left(y + c - Y_{max}\right) = h\left(y\right) \tag{3.23}$$

The functions g(y) and h(y) are piecewise defined material laws for the concrete and structural steel respectively.

$$g(y) = f_{cd}\left[1-\left(1-\frac{(y+c-Y_{max})\varepsilon_{cu2}}{c*\varepsilon_{c2}}\right)^n\right] \quad \text{for} \quad 0 \le \frac{(y+c-Y_{max})}{c}\varepsilon_{cu2} \le \varepsilon_{c2} \qquad (3.24a)$$

$$g(y) = f_{cd} \qquad \text{for} \quad \varepsilon_{c2} \le \frac{(y+c-Y_{max})}{c}\varepsilon_{cu2} \le \varepsilon_{cu2} \qquad (3.24b)$$

And the stress in the structural steel is given by:

$$h(y) = E_a\frac{(y+c-Y_{max})}{c}\varepsilon_{cu2} \quad \text{for} \quad 0 \le \frac{(y+c-Y_{max})}{c}\varepsilon_{cu2} \le \varepsilon_{yd} \qquad (3.25a)$$

$$h(y) = f_{yd} \qquad (3.25b)$$

The force and moment carried by the concrete under compression are:

$$N_c = \iint_A g(y)dA \qquad (3.26)$$

$$M_{zc} = \iint_A yg(y)dA \qquad (3.27)$$

Where:

N_c is the axial force carried by the concrete

M_{zc} is the moment resisted by the concrete about the z-z plane

y is the lever arm of the concrete to the geometrical center

The contributions of the structural steel are:

$$N_a = \iint_{A'} h(y)dA' \qquad (3.28)$$

$$M_{za} = \iint_{A'} y'h(y)dA' \qquad (3.29)$$

Where:

N_a is the axial force carried by the structural steel

M_{za} is the moment resisted by the structural steel about the z-z plane

y' is the lever arm of the structural steel to the geometrical center

And the contributions of reinforcements are:

$$N_s = \sum_{j=1}^{N_b} A_{sj}\sigma_{sj} \tag{3.30}$$

$$M_{zs} = \sum_{j=1}^{N_b} Y_{sj}A_{sj}\sigma_{sj} \tag{3.31}$$

Where:

N_s is the axial force carried by the reinforcements

M_{ys} is the moment resisted by the reinforcements about the z-z plane

Y_{sj} is the lever arm of the reinforcements to geometrical center

Thus, the capacity of the cross-section is:

$$N_x = N_c + N_s + N_a \tag{3.32}$$

$$M_z = M_{zc} + M_{zs} + M_{za} \tag{3.33}$$

Where:

N_x is the axial force carried by the cross-section

M_z is the moment resisted by the cross-section for bending about the z-z plane

3.6 Solution of the Stress Resultants of the Concrete and Structural Steel

As realized in section 3.5, the stress resultants of the concrete and structural steel necessitate solving double integrals over their respective regions in compression and in tension. The evaluation of these integrals is computationally difficult. By applying Green's theorem, a particular double integral over an area A can be transformed to line integrals along several lines L that enclose the area A.

Let A be a closed bounded region in the yz-plane whose boundaries L consists of finitely many smooth curves. If P and Q are functions that are continuous and have continuous partial derivatives $\partial p / \partial z$ and $\partial Q / \partial y$ everywhere in a domain containing A, Then: [17]

$$\iint_A \left(\partial Q / \partial y - \partial P / \partial z\right) dydz = \oint_L Pdy + \oint_L Qdz \tag{3.34}$$

From the Figure 3.5, the compressed region *A* of the concrete is bounded by *"1-2-3-4-1"* and that of the structural steel is bounded by *"a-b-c-d-e-f-g-h-a"*. The structural steel in tension is *"h-i-j-k-l-m-n-a-h"*. The stresses of the concrete and the structural steel are reducible with respect to the y and z. Hence, Green's theorem is applicable for solving the stress resultants.

Fafitis [16] tailored Green's theorem to be applicable for reinforced concrete sections subjected to axial load and biaxial bending. His work can be extended to serve analysis of composite sections in this study.

Let $P(y, z)$ is equal to zero, let "r" and "s" be nonnegative integers and Q be:

$$Q = \frac{1}{r+1}\int y^{r+1} z^{s} f\left(z\right) dy \tag{3.35}$$

In equations (3.35), $f\,(z)$ is the stress either in concrete or structural steel as has been defined in section 3.5.

With the above definitions of P and Q, equation (3.34) becomes:

$$\iint_A y^r z^s f\left(z\right) dydz = \frac{1}{r+1}\oint_L y^{r+1} z^s f\left(z\right) dz \tag{3.36}$$

If $f(z)$ is equal to $g(z)$, the stress resultant of the concrete R_c over the compressive zone is reduced to the line integrals:

$$R_c = \frac{1}{r+1}\oint_L y^{r+1} z^s g\left(z\right) dz \tag{3.37}$$

For r=0 and s=0, the left-hand side of equation (3.37) represents the axial load N_c.

$$N_c = \oint_L y g\left(z\right) dz \tag{3.38}$$

For r=0 and s=1, the left-hand side of equation (3.37) represents the bending moment M_{yc}.

$$M_{yc} = \oint_L yzg\left(z\right) dz \tag{3.39}$$

If $f(z)$ is equal to $h(z)$, the stress resultant of the structural steel R_a either in tension or compression zone is reduced to the line integrals:

$$R_a = \frac{1}{r+1}\oint_{L'} y^{r+1} z^s h(z)\,dz \tag{3.40}$$

For r=0 and s=0, the left-hand side of equation (3.40) represents the axial load N_a.

$$N_a = \oint_{L'} y h(z)\,dz \tag{3.41}$$

For r=0 and s=1, the left-hand side of equation (3.40) represents the bending moment M_{ya}.

$$M_{ya} = \oint_{L'} yz h(z)\,dz \tag{3.42}$$

In a similar way the stress resultants of a section subjected to minor axis bending and axial load can be transformed to line integrals along the boundaries of the compressed concrete, compressed structural steel and tensioned structural steel. Thus the axial load and moment carried by the concrete are:

$$N_c = \oint_{L} z g(y)\,dy \tag{3.43}$$

$$M_{zc} = \oint_{L} zy g(y)\,dy \tag{3.44}$$

And that of the structural steel are:

$$N_a = \oint_{L'} z h(y)\,dy \tag{3.45}$$

$$M_{za} = \oint_{L'} zy h(y)\,dy \tag{3.46}$$

The stresses in the materials are expressed with polynomial functions. Therefore, the line integrals from equation (3.37) to (3.46) are definite integrals. Even though such integrals are explicit, it is preferred to solve them numerically to facilitate computation. Among the many numerical methods is Gaussian Quadrature, which yields exact solution of the definite integration. In the above line integrations, the highest degree of the integrand is third order. Consequently, it is sufficient to use three points Gaussian quadrature to solve the line integrals exactly.

The sides of the compressive zones of the concrete or structural steel and the sides of the tensile region of structural steel can be defined with Y-Z coordinates whose origin is at the geometrical center of the section. For example from Figure 3.4 the integral along the side "*1-2"* of the concrete is:

$$R_{c1} = \frac{1}{r+1}\int_{z_1}^{z_2} y^{r+1} z^s g(z)\,dz \tag{3.47}$$

The domain of integration in equation (3.47) is from [z_1, z_2]. But in three-point Gaussian Quadrature, the domain of integration is [-1, 1] in the form as below:

$$\int_{-1}^{1} f(x)\,dx = \sum_{i=1}^{3} w_i f(x_i) \tag{3.48}$$

Where:

f(x) is a polynomial function

w_i is gauss weights

x_i is gauss points

All the line integrals from equation (3.37) to (3.46) are out of the domain [-1, 1]. Therefore, it is necessary to transform these integrals onto this range using change of variables. If we let the following be:

$$z = \frac{z_2 + z_1}{2} + \frac{z_2 - z_1}{2}u \quad \text{and} \tag{3.49}$$

$$dz = \frac{z_2 - z_1}{2}du \tag{3.50}$$

Thus, equation (3.47) can be rewritten as:

$$R_{c1} = \frac{1}{r+1}\frac{z_2 - z_1}{2}\int_{-1}^{1} y^{r+1} z^s g\left(\frac{z_2 + z_1}{2} + \frac{z_2 - z_1}{2}u\right)du \tag{3.51}$$

Likewise, all the line integrals can be restated to fit in the Gauss Quadrature domain. Once this is done, the solutions of the integrals can be determined directly with simple arithmetic. Thus, for a section subjected to major axis bending, the stress resultants of the concrete are:

$$N_c = \iint_L yg(z)dz = y\sum_{i=1}^{3} w_i g(z_i) \tag{3.52}$$

$$M_{yc} = \iint_L yzg(z)dz = y\sum_{i=1}^{3} w_i G(z_i) \tag{3.53}$$

$$G(z) = zg(z) \tag{3.54}$$

And the stress resultants of the structural steel are:

$$N_a = \iint_{L'} yh(z)dz = y\sum_{i=1}^{3} w_i h(z_i) \tag{3.55}$$

$$M_{yc} = \iint_{L'} yzh(z)dz = y\sum_{i=1}^{3} w_i H(z_i) \tag{3.56}$$

$$H(z) = zh(z) \tag{3.57}$$

Similarly for a section under minor axis bending and axial force, the stress resultants of the concrete are:

$$N_c = \iint_L zg(y)dy = z\sum_{i=1}^{3} w_i g(y_i) \tag{3.58}$$

$$M_{zc} = \iint_L zyg(y)dy = z\sum_{i=1}^{3} w_i G(y_i) \tag{3.59}$$

$$G(z) = yg(y) \tag{3.60}$$

And that of the structural steel are:

$$N_a = \iint_{L'} zh(y)dy = z\sum_{i=1}^{3} w_i h(y_i) \tag{3.61}$$

$$M_{zc} = \iint_{L'} zyh(y)dy = z\sum_{i=1}^{3} w_i H(y_i) \tag{3.62}$$

$$H(y) = yh(y) \tag{3.63}$$

For three points Gauss Quadrature the gauss number and gauss weights are as follow:

Table 3.1: Gauss numbers and weights for three points Gaussian Quadrature

Gauss Number			Gauss Weight		
z_1/y_1	z_2/y_2	z_3/y_3	w_1	w_2	w_3
$-\sqrt{3/5}$	0	$\sqrt{3/5}$	$8/9$	$5/9$	$8/9$

The concrete area that is replaced by the reinforcements or structural steel section in the compression zone is accounted by applying force and moment reduction. For the area replaced by the reinforcements, these are obtained by directly multiplying the area of the reinforcements at a particular fiber by the stress in the concrete in that same fiber. Integrating the stress in the concrete over the regions of structural steel in the compression zone provides the reduction for the area displaced by steel section.

3.7 Development of the Program

As have been discussed in section 3.4, all the possible strain distributions in the ULS are classified into five regions for ease of calculation and programming. Having a particular strain distribution, the corresponding stress resultants yield a point on the interaction curve. To facilitate evaluation of many more of such points on the interaction curve, an algorithm is developed. Furthermore, to increase the applicability of this study a Matlab program with a friendly graphical user interface called UICISEC is integrated.

3.7.1 Qualitative Flowchart

For a particular neutral axis located at c distance from the most compressed fiber of a cross-section (Figure 3.5), the corresponding strain distribution category can be known. The limiting neutral axis depth for a particular region can be described in terms of the ultimate strains in steel and concrete as well as the cross-sectional dimensions. For different values of c, the entire curve is evaluated. A general representation of this process is shown on Figure 3.7 for the case of normal force major axis bending interaction. In the flowchart, the subscripts “1-5” indicate the stress resultants in which regions of strain distribution they are computed. Similar procedure is adopted for developing minor axis bending interaction curves.

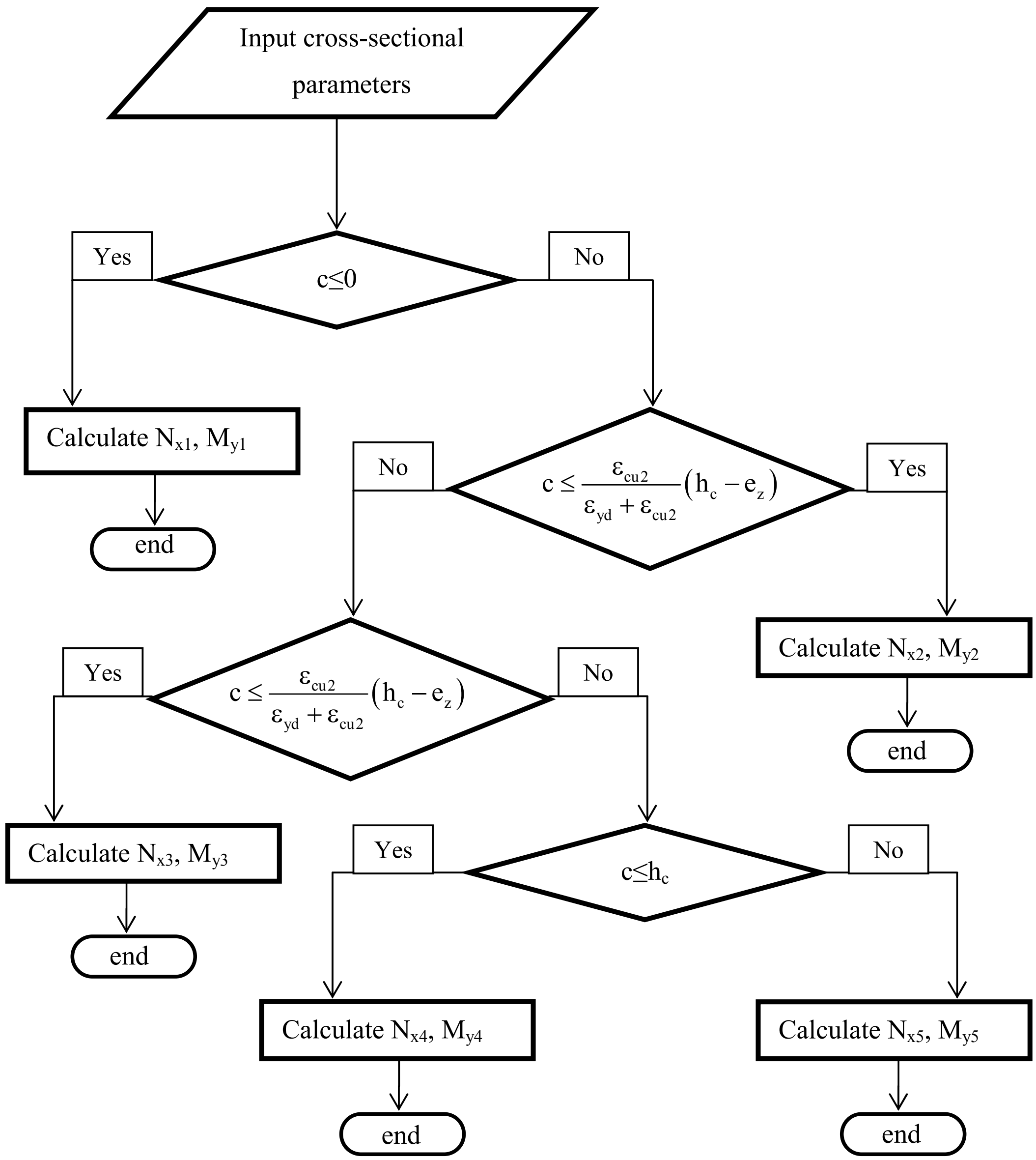


Figure 3.8: Qualitative flowchart

3.7.2 Limitations

The developed program has the following limitations:

- It is developed for uniaxial charts.
- Normal strength concrete is considered according to EN 1992-1-1:2004 [5].
- The grade of the reinforcement is limited to S500.
- The grade of structural steel is limited to S275.
- The database for the size of reinforcement is as stated in Annex D.
- The types of structural steel that are included in the database are all groups of *HEA*, *HEB*, *HEM* and *IPE* sections according to the European standardization.
- The web reinforcements are ignored in the calculation of major axis bending and axial load interaction. However, they are accounted in evaluation of the section capacity for minor axis bending. This is reasonable enough as these reinforcements lie on the farthest side from the geometrical center and they have higher lever arms for bending about the weaker plane.
- The same concrete covers to the structural steel on the top and bottom sides or on the left and right sides are assigned.
- The same concrete covers to the reinforcement on the top and bottom layers or on the left and right side layers are assigned.
- Material properties and partial safety factors for materials are adopted according to EN 1994-1-1:2004 [3].
- The effect of confinement to the concrete by the flanges of the structural steel section or the transverse reinforcement is not considered.
- The influence of transverse shear on the strength of the web of the structural steel section is ignored.

3.7.3 Material Properties and Partial Safety Factors

The material properties for concrete, reinforcement and structural steel are taken according to EN 1994-1-1:2004 [3]. In EN 1994-1-1:2004, 3 [3], unless specified exclusively, the material properties of concrete and reinforcement are adopted from EN 1992-1-1 [5] and that of structural steel from EN 1993-1-1 [6].

The modulus of elasticity of the structural steel E_a is taken as 210Gpa. EN 1994-1-1:2004, 3.2 [3] allows the modulus of elasticity of the reinforcement to be taken equal to the value of structural steel. However, in this study it is kept 200Gpa.

The partial safety factors of materials used are:

$\alpha_{cc}=0.85$ is factor to account for long term effects on the compressive strength and of unfavorable effects resulting from the way the load is applied.

$\gamma_c = 1.50$ is the partial safety factor for the concrete.

$\gamma_s = 1.15$ is the partial safety factor for the reinforcement.

$\gamma_a = 1.10$ is the partial safety factor for the structural steel.

Therefore, the design values of material strength can be determined by dividing their characteristics strength by the corresponding safety factors.

3.7.4 Sign Convention

Axial compression is taken as positive and axial tension is taken as negative. Counterclockwise moment is positive while clockwise moment is taken as negative.

3.7.5 Data Base for Strength Class and Size of Materials

In this study, only encased sections in normal strength concrete are considered. The strength class of concrete with their respective properties can be found in Annex A.

The strength class and size of the reinforcement are listed in Annex B and Annex D respectively.

In Annex C and Annex E, the strength class and structural steel section types together with their geometrical properties are listed respectively. The structural steel types are taken according to the European standardization [18-21] for groups of *HEA*, *HEB*, *HEM* and *IPE* sections. There are ninety sections in total.

3.7.6 The Graphical User Interface

To advance the practice of this program, a friendly graphical user interface is incorporated. The material strength class, size of reinforcement bars, type of structural steel sections and cross-sectional dimensions are inputted by the user of UICISEC. To assist in understanding of what the variables of the dropdown boxes or static boxes are referring to, a side view of a typical composite cross-section with description of these variables is integrated.

As can be seen in Figure 3.8, the interaction curve is plotted in the same window where the cross-sectional parameters and material types are defined. The kind of curve that is plotted is activated by the push buttons, which are lying on the left bottom corner of the window. The" N_x-M_y" push button activates the plotting of major axis bending and normal force interaction curve. On the other hand, the" N_x-M_z" push button initiates the plotting of minor axis bending plus normal force interaction curve.

A data cursor tool is available, which lies on the right top corner of the window, to enable reading of points on the curve accurately. However, only points that have been calculated by the program can be read. Intermediate values between points can be obtained by interpolating.

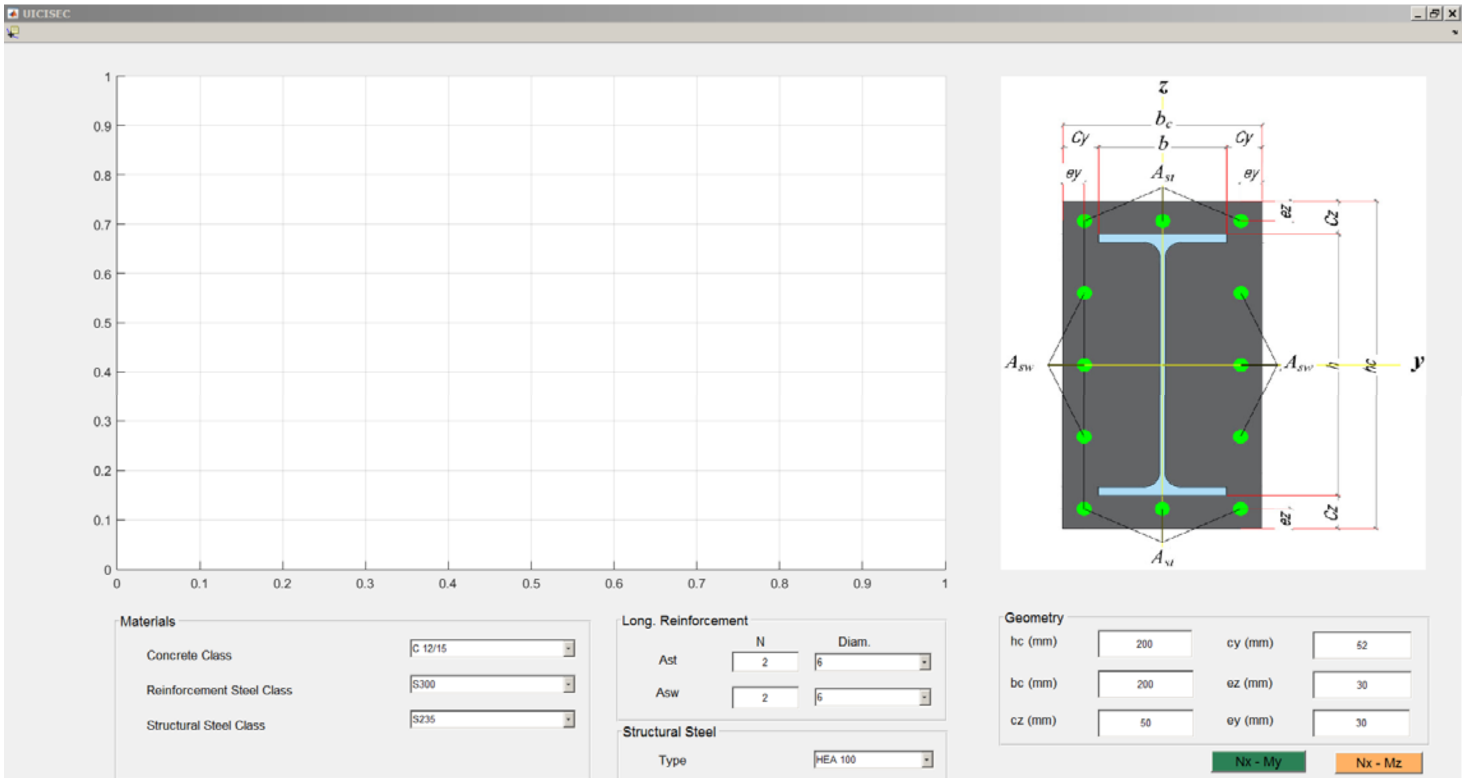


Figure 3.8: The graphical user interface of UICISEC

3.8 Verification

Points on the interaction curve are validated with the output of section analysis software called MASQUE, which was developed by Busjäger and Quast [7]. The verified cross-section is shown in Figure 3.9. Several points on the interaction curves are checked.

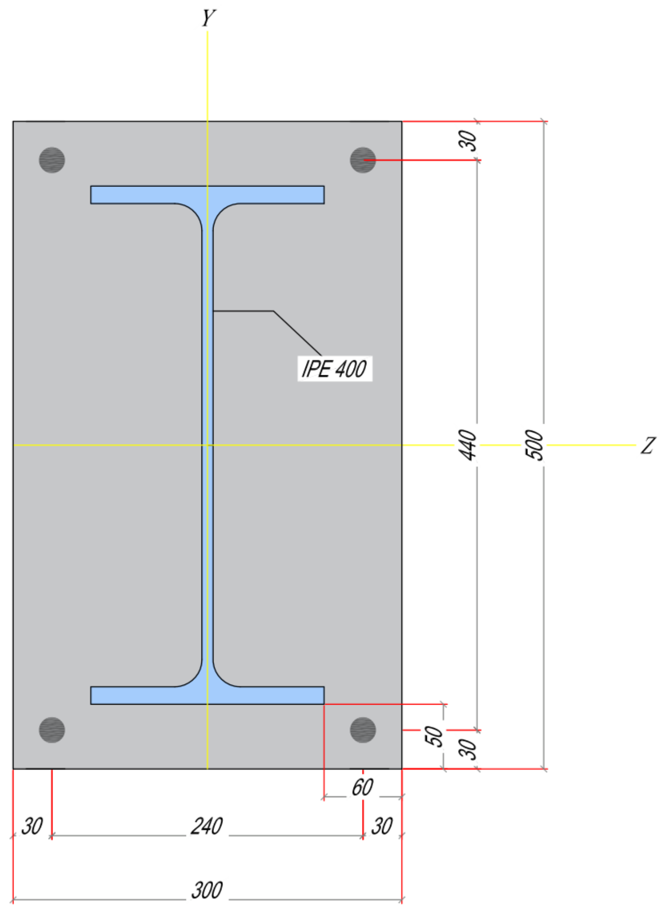


Figure 3.9: Verified section

The section is made of strength class of concrete *C30/37*, reinforcement *S500* and structural steel *S235*.

As shown in Figure 3.9, there are four corner reinforcement of *Ø20*.

For a given cross-section and material properties the ultimate resistance R_u (N_u, M_{yu}, M_{zu}) on the interaction surface is determined that is associated with a given initial force vectors R_i (N_i, M_{yi}, M_{zi}). The ultimate resistance is obtained iteratively until the convergence criteria given in the program is meet. [19]

Comparison of the output of this study and MASQUE are summarized in Table 3.2 and Table 3.3 for the minor axis bending and major axis bending interactions with the normal force respectively. Values of the ratio of R to R_u that are very close to 1 are obtained for both the cases.

Table 3.2: Comparison for minor axis bending

From Thesis (UICISEC)		MASQUE		Ratio
N_i [kN]	M_{zi} [kNm]	N_u [kN]	M_{zu} [kNm]	$\frac{R}{R_u}$
4737.00	0.00	4638.78	0.0007	0.9793
3972.00	80.27	3960.91	80.46	0.9972
3544.00	117.10	3537.09	116.872	0.9981
3138.00	143.60	3135.55	143.488	0.9992
2643.00	165.80	2648.3	166.109	1.0019
2197.00	179.50	2203.91	180.065	1.0031
1520.00	195.90	1528.18	196.919	1.0052
1204.00	196.70	1211.68	197.937	1.0063
688.70	194.60	693.295	195.848	1.0064
0.00	186.74	0.00096	188.182	1.0077
-478.20	178.50	-471.089	175.86	0.9852
-934.90	153.00	-912.583	149.351	0.9761
-1399.00	114.50	-1348.13	110.335	0.9636
-1795.00	70.51	-1729.81	67.9513	0.9637
-2352.00	0.00	-2269.64	0.0000	0.9650

Table 3.3: Comparison for major axis bending

From Thesis (UICISEC)		MASQUE		Ratio
N_i [kN]	M_{yi} [kNm]	N_u [kN]	M_{yu} [kNm]	$\frac{R}{R_u}$
4737.00	0.00	4638.78	0.0007	0.9793
3612.00	201.10	3629.27	202.062	1.0048
3247.00	263.40	3263.08	264.724	1.005
2823.00	330.00	2837.71	331.659	1.0051
2457.00	384.10	2469.98	386.098	1.0052
2042.00	442.70	2052.92	445.060	1.0053
1661.00	489.20	1669.58	491.694	1.0051
1317.00	516.20	1323.58	518.761	1.0050
1109.00	521.00	1113.99	523.319	1.0045
795.00	514.90	797.764	516.891	1.0039
323.40	483.40	324.314	484.746	1.0028
0.00	446.30	0.0938	447.1666	0.0938
-305.00	400.90	-301.958	396.846	0.9899
-681.60	346.30	-650.185	330.278	0.9538
-1062.00	267.80	-1020.83	257.417	0.9612
-1425.00	196.60	-1364.96	188.322	0.9579
-1601.00	161.10	-1532.81	154.243	0.9574
-1903.00	99.61	-1821.17	95.3385	0.9570
-2352.00	0.00	-2269.64	0.0000	0.9650

4 DESIGN EXAMPLE

One application of this study is demonstrated with a design example for a column length subjected to biaxial bending using the EN 1994-1-1:2004 [3] simplified method. The uniaxial interaction curves of UICISEC are utilized.

The cross-section, which has been verified in Chapter 3, 3.8, is used in this example. This section is proportioned to meet the requirements of EN 1994-1-1:2004 [3] for using the simplified method as reviewed in Chapter 2, 2.3.1. First, the capacity of the column length is examined independently for each axis of bending. Then, the capacity of column length is verified for the biaxial bending.

For simplification, the long term effects of creep and shrinkage are not considered.

The sectional parameters and material type of the cross-section are:

Cross-section parameters:

The variables describing the cross-sectional parameters are shown in Figure 4.1 labeled:

$hc=500mm$, $b_c=300mm$, $C_z=50mm$, $C_y=60mm$, $e_z=30mm$, $e_y=30mm$, $A_{st}=2\phi20$, $A_{sb}=2\phi20$.

The structural steel type is *IPE 400*: The section properties can be found in Annex E.

Materials

Strength class of concrete *C30/37*, reinforcement *S500*, structural steel *S235*

Column length (Buckling length) =5.0m

Design values of action effects from first order analysis

$N_{Ed}=1500.0kN$

The maximum moment about the y-y plane is $M_{y,max}=150.0kNm$

The maximum moment about the z-z plane is $M_{z,max}=50.0kNm$

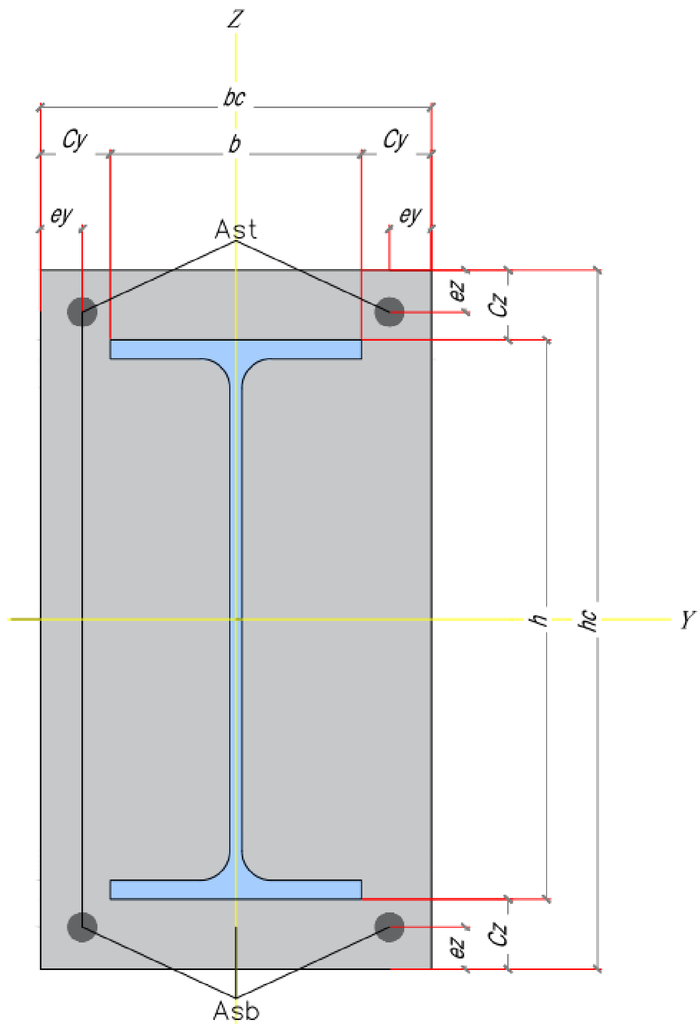


Figure 4.1: Cross-sectional parameters

4.1 Resistance of the Column Length for Major Axis Bending

Cross-section capacity

All the cross-sectional parameters are inserted in the GUI of UICISEC as shown in Figure 4.2.

Then, the M_y-N_x interaction is plotted (Figure 4.3).

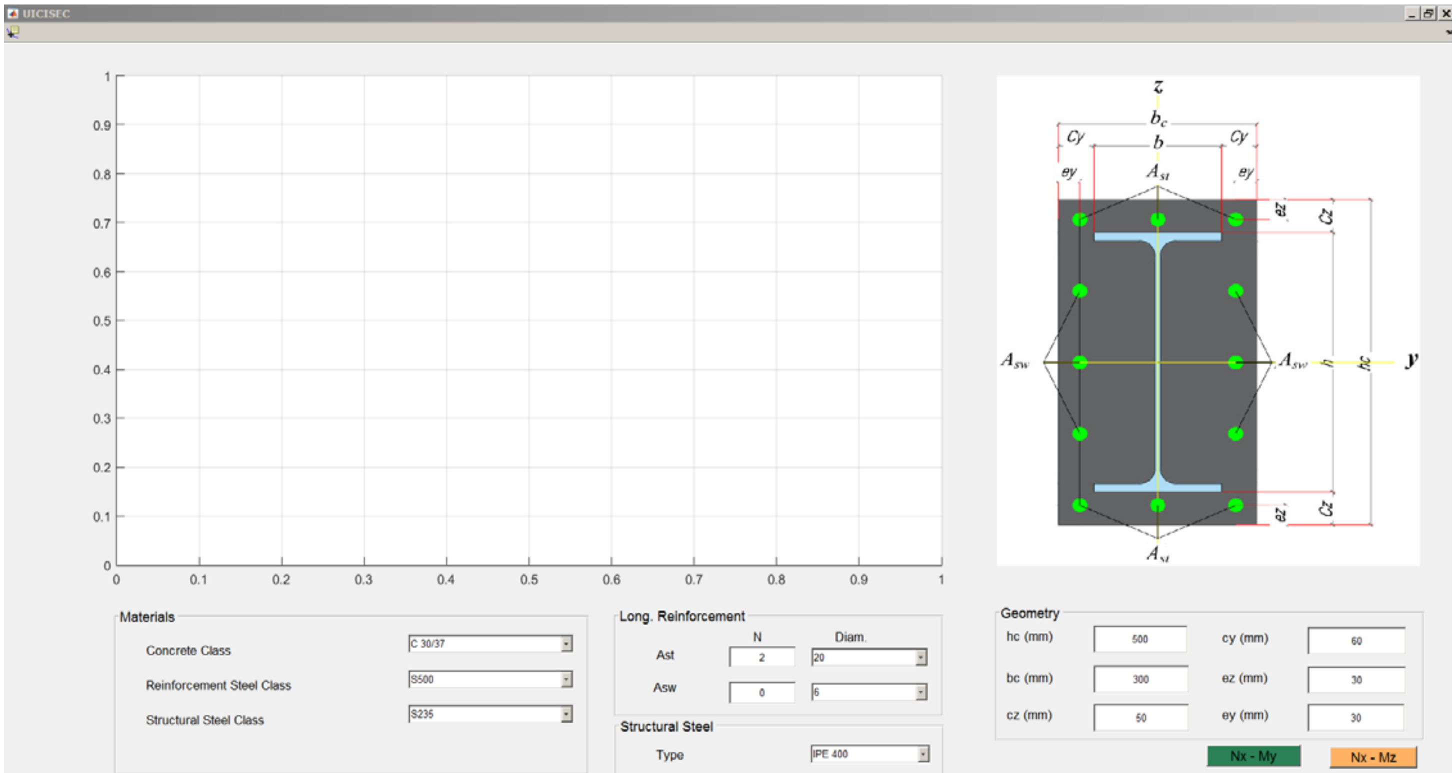


Figure 4.2: Inputted parameters

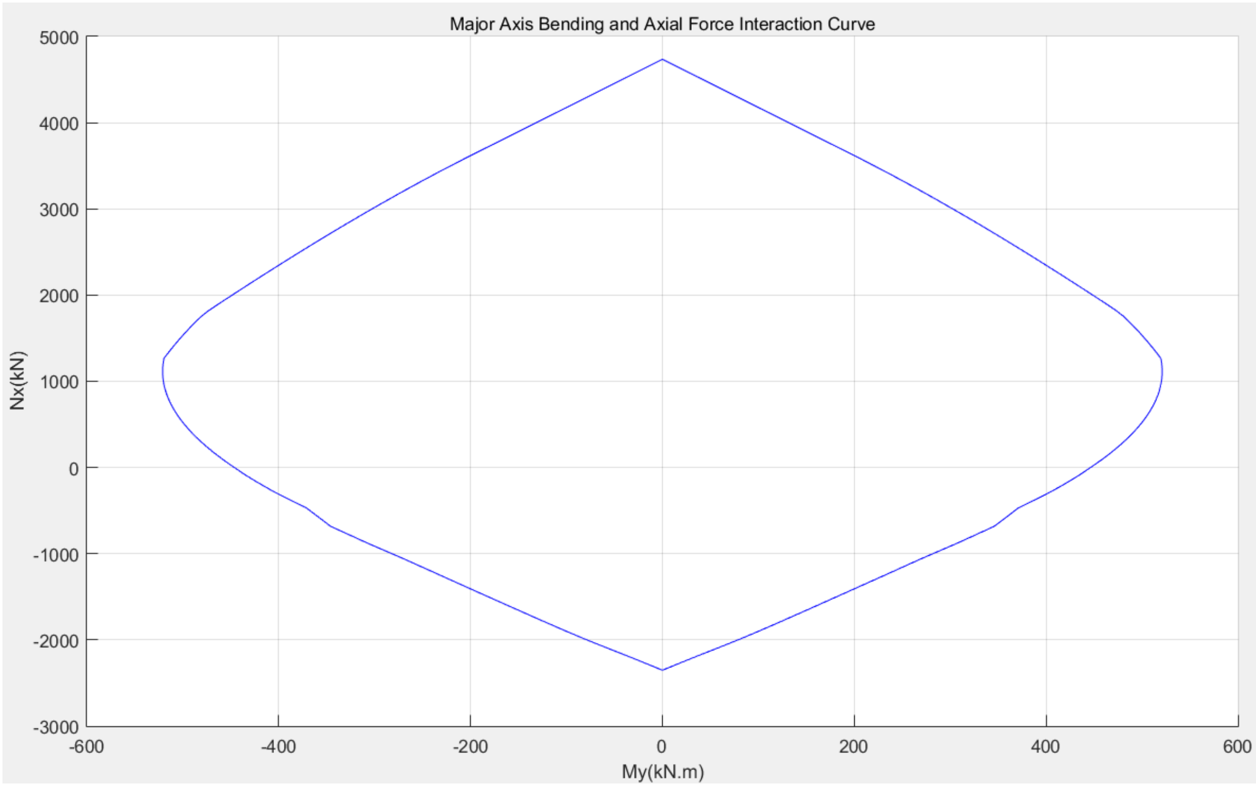


Figure 4.3: M_y-N_x interaction curve

Design of the column according to the simplified method of EN 1994-1-1:2004

Once the section capacity is determined, the design of the column length is done by using the EN 1994-1-1:2004 simplified method.

- The effective flexural stiffness

As mentioned at the beginning of this chapter, the long term loading effects on the flexural rigidity of the column is not considered in this example. Hence, the effective stiffness is computed considering the short term loading.

The effective flexural rigidity for bending about y-y axis is:

$$(EI)_{y,eff} = E_a I_{y,a} + E_s I_{y,s} + 0.6 E_{cm} I_c$$

$$I_{y,a} = 231,300 * 10^3 mm^4$$

$$I_{y,s} = 2 * 628mm^2 * (220mm)^2 = 60,790.4 * 10^3 mm^4$$

$$I_c = \frac{300mm * (500mm)^3}{12} - \left(231,300 * 10^3 - 60,790.4 * 10^3\right) mm^4 = 2,832,909.6 * 10^3 mm^4$$

$$(EI)_{y,eff} = \left(210Gpa * 231,300 * 10^3 mm^4\right) + \left(200Gpa * 60,790.4 * 10^3 mm^4\right)$$

$$+ \left(0.6 * 33Gpa * 2,832,909.6 * 10^3 mm^4\right)$$

$$(EI)_{y,eff} = \left(48.573 * 10^9 + 12.158 * 10^9 + 56.092 * 10^9\right) kNmm^2 = 116.823 * 10^9 kNmm^2$$

The second moment of area of structural steel is obtained from Annex E, E.4.

- The critical buckling loads

$$N_{y,cr,eff} = \frac{\Pi^2 (EI)_{y,eff}}{L^2} = \frac{\Pi^2 * 116.823 * 10^9 kNmm^2}{(5000mm)^2} = 46,120kN$$

- The imperfection moment

For buckling about the y-y axis, the member imperfection is given in Table 2.4,

$$e_o = \frac{L}{200} = \frac{5.0m}{200} = 0.025m$$

And the imperfection moment is:

$$M_{y,Imp} = e_o * N_{Ed} = 0.025m * 1500kN = 37.5kNm$$

- Second Order moments

As stated in Chapter 2, 2.3.3, second order effects can be neglected if the following is satisfied:

$$N_{y,cr,eff} \geq 10N_{Ed}$$

$$46,120kN \geq 10 * 1500kN = 15,000kN$$

Hence, the second order moment can be ignored.

The total design moment is:

$$M_{y,Ed} = M_{y,max} + M_{y,Imp} = 150.0kNm + 37.5kNm = 187.5kNm$$

- Check for the capacity of the column length

In addition to the design moment from first order analysis, the column length must have a sufficient capacity for the imperfection moment.

The available resistance to flexure is limited to the plastic moment resistance $M_{pl,Rd}$ unless it is justified that the design bending moment M_{Ed} depends directly on the action of the axial force N_{Ed}. In this example the maximum resistance for moment is limited to $M_{pl,,Rd}$ (Figure 4.4). For the design axial load, the corresponding moment capacity is taken from the interaction curve as shown in Figure 4.4.

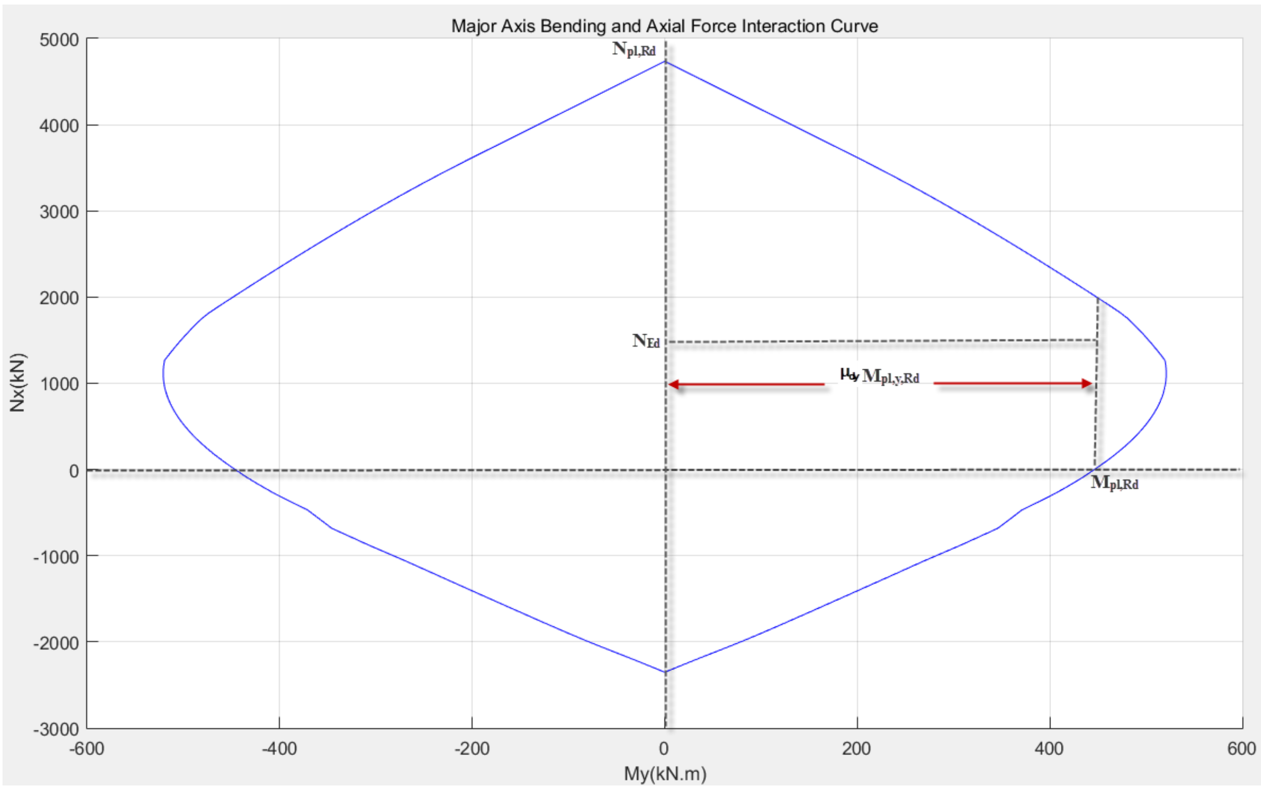


Figure 4.4: Design for axial force and major axis bending

$M_{pl,y,Rd}$ is obtained by interpolating between the nearby points on the interaction curve. Thus:

$$\mu_{dy} M_{pl,y,Rd} = 446.3 kNm$$

For stability check, the column length must satisfy the following:

$$M_{y,Ed} \leq \alpha_m \mu_{dy} M_{pl,y,Rd}$$

In EN 1994-1-1:2004 [3], the factor α_M is needed to account for the unconservative assumption made while evaluating the interaction curve by rigid plastic assumption. The concrete in the compression zone is assumed to resist a stress of $0.85f_{cd}$ constant over the depth from the plastic neutral axis to the most compressed concrete fiber. However, in this study the points on the interaction curve are computed beginning from the actual strain distributions through the cross-section. As a result, this factor can be omitted. Therefore:

$$M_{y,Ed} \leq \mu_{dy} M_{pl,y,Rd}$$

$$187.5 kNm < 446.3 kNm$$

Hence, the column length has sufficient capacity with the assumed initial cross-section to resist the design action effects.

4.2 Resistance of the Column Length for Minor Axis Bending

Cross-section capacity

The cross-sectional inputs are as shown in figure 4.2.

Then, the M_z-N_x interaction is plotted and it shown in Figure 4.5.

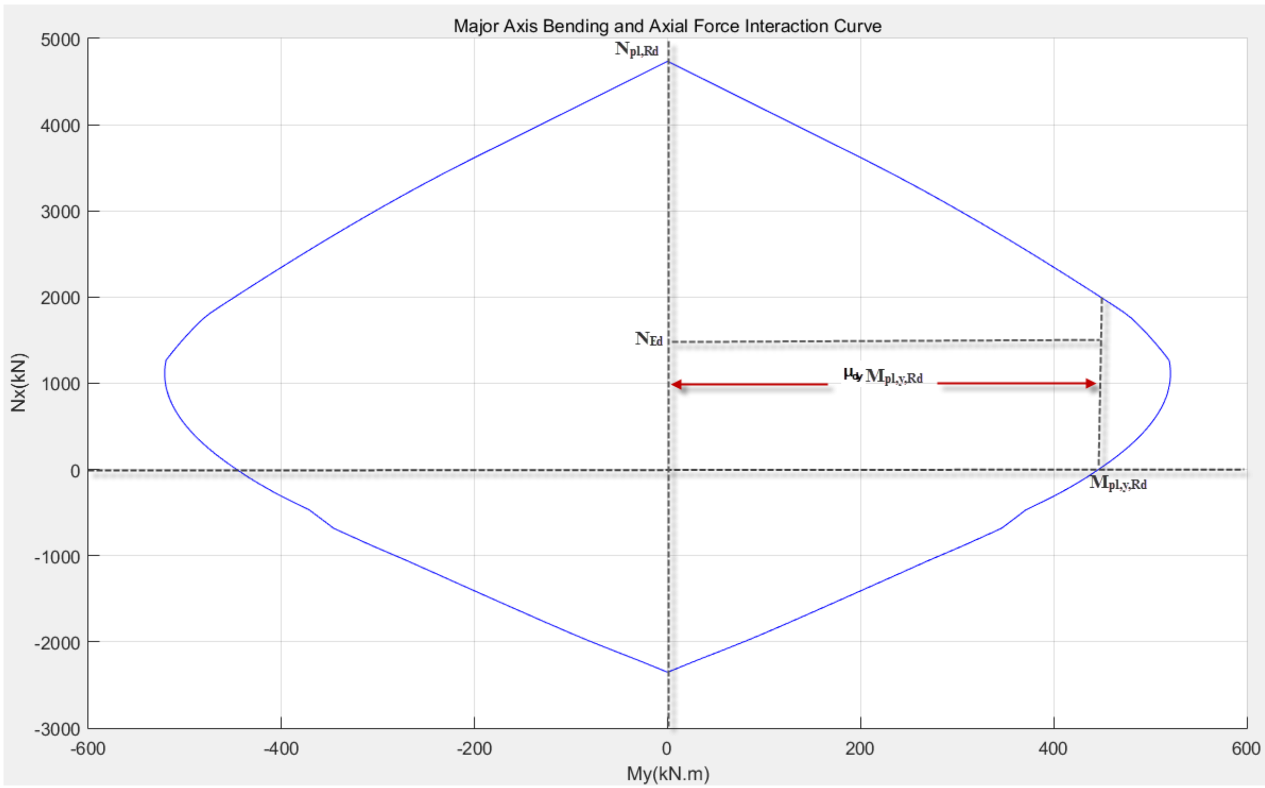


Figure 4.5: M_z-N_x interaction curve

Design of the column according to the simplified method of EN 1994-1-1:2004

- The effective flexural stiffness

The effective flexural rigidity for bending about z-z axis is:

$$(EI)_{z,eff} = E_a I_{z,a} + E_s I_{z,s} + 0.6 E_{cm} I_c$$

$$I_{z,a} = 13{,}180 * 10^3 mm^4$$

$$I_{z,s} = 2 * 628mm * (120mm)^2 = 18{,}086.4 * 10^3 mm^4$$

$$I_c = \frac{500mm * (300mm)^3}{12} - \left(13{,}180 * 10^3 - 18{,}086.4 * 10^3\right) mm^4 = 1{,}093{,}733.6 * 10^3 mm^4$$

$$(EI)_{z,eff} = \left(210Gpa * 13{,}180 * 10^3 mm^4\right) + \left(200Gpa * 18{,}086.4 * 10^3 mm^4\right)$$

$$+ \left(0.6 * 33Gpa\right) * 1{,}093{,}733.6 * 10^3 mm^4$$

$$(EI)_{z,eff} = \left(2.768 * 10^9 + 3.617 * 10^9 + 21.656 * 10^9\right) kNmm^2 = 28.041 * 10^9 kNmm^2$$

The second moment of area of structural steel is obtained from Annex E, E.4.

- The critical buckling loads

$$N_{cr,z} = \frac{\Pi^2 (EI)_{eff}}{L^2} = \frac{\Pi^2 * 28.041 * 10^9 kNmm^2}{(5000mm)^2} = 11{,}070kN$$

- The imperfection moment

For buckling about the z-axis, the member imperfection is given in Table 2.4,

$$e_o = \frac{L}{150} = \frac{5.0m}{150} = 0.0333m$$

And the imperfection moment is:

$$M_{z,Imp} = e_o * N_{Ed} = 0.0333m * 1500kN = 50.0kNm$$

- Second Order moments

Second order effects can be neglected if the following is satisfied:

$N_{y,cr,eff} \geq 10N_{Ed}$

$11{,}070\text{kN} \leq 10 * 1500\text{kN} = 15{,}000\text{kN}$

Hence, the second order moments have to be considered. As reviewed in chapter 2, 2.3.3, the second order moments can be accounted by amplifying the first order results.

The amplification factor *k* is evaluated as: $k = \dfrac{\beta}{1 - \dfrac{N_{Ed}}{N_{cr,eff}}} \geq 1$

The moment factor β is obtained from Table 2.3. For both the first order moment and the imperfection moment $\beta = 1.0$.

Therefore: $k_{imp} = \dfrac{1}{1 - \dfrac{1500}{11{,}070}} = 1.16 \geq 1$

The total design moment is:

$M_{z,Ed} = k(M_{z,max} + M_{z,Imp}) = 1.16(50.0\text{kNm} + 50.0\text{kNm}) = 116.0\text{kNm}$

- Check for the capacity of the column length

Limiting the maximum resistance for moment to $M_{pl,z,Rd}$, the capacity of the column to minor axis bending is checked.

For the design axial load, the corresponding moment capacity is taken from the interaction curve of Figure 4.6. $M_{pl,z,Rd}$ is obtained by interpolating between the nearby points on the interaction curve. Thus:

$\mu_{dz} M_{pl,z,Rd} = 186.6\text{kNm}$

The column length must also satisfy the following:

$M_{Ed,z} \leq \mu_{dz} M_{pl,Rd,z}$

$116.0\text{kNm} < 186.6\text{kNm}$

Hence, the column length has sufficient capacity with the assumed initial cross-section to resist the design action effects.

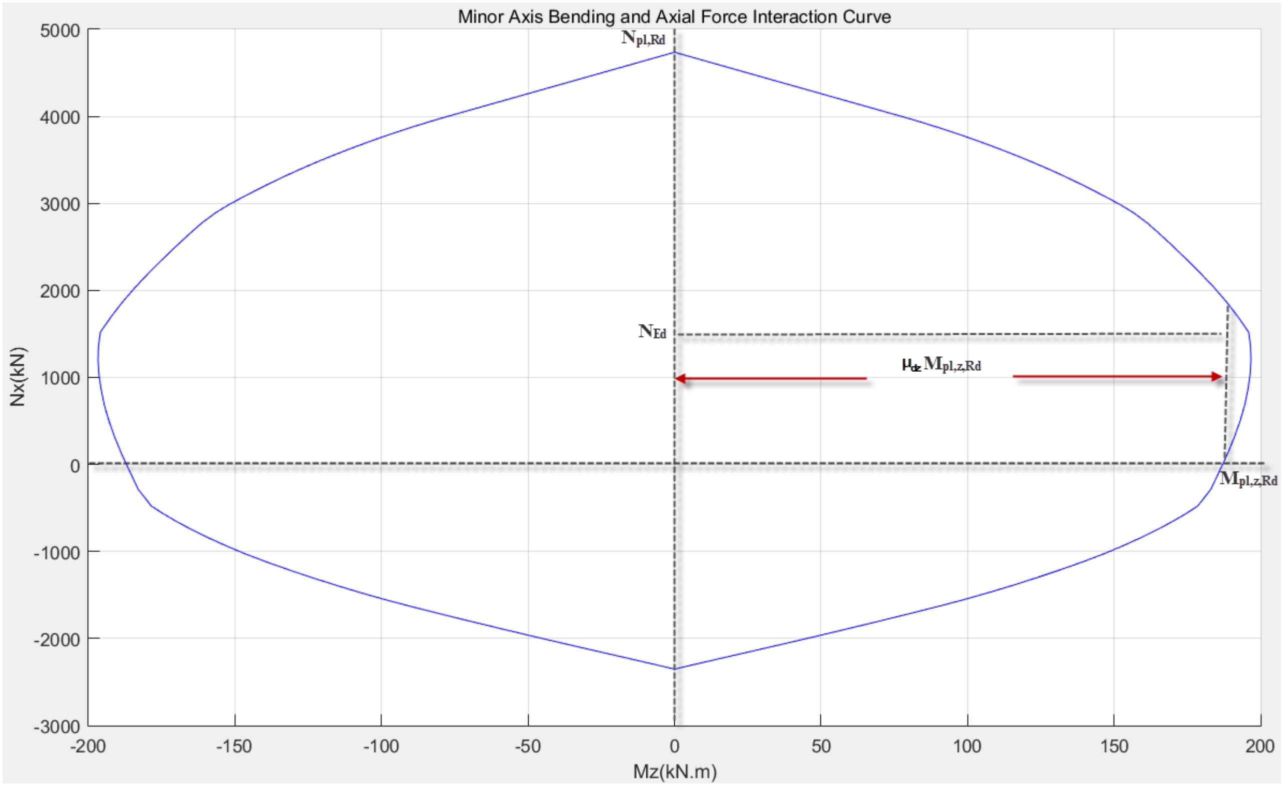


Figure 4.6: Design for axial force and minor axis bending

4.3 Resistance of the Column Length for Biaxial Bending

For biaxial bending, the column length must satisfy the following conditions other than the separate check for uniaxial bending carried out in section 4.1 and 4.2:

$$\frac{M_{y,Ed}}{\mu_{dy} M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} M_{pl,z,Rd}} \leq 1$$

As reviewed in Chapter 2, 2.3.5, the member imperfection is taken one at a time in each plane of bending and checked.

Taking the initial bow in the y-y axis:

$$\frac{187.5\text{kNm}}{446.3\text{kNm}} + \frac{58.0\text{kNm}}{186.6\text{kNm}} \leq 1$$

$$0.73 < 1$$

Taking the initial bow in the z-z axis:

$$\frac{150\text{kNm}}{446.3\text{kNm}} + \frac{116.0\text{kNm}}{186.6\text{kNm}} \leq 1$$

$$0.96 < 1$$

Hence, the column length has sufficient capacity to resist the axial load and biaxial moment.

5 SUMMARY AND DISSCUSSION

The EN 1994-1-1:2004 [3] provides simplified procedure to evaluate the interaction curve of a composite section fulfilling certain requirements. This method is based on some assumptions and approximations. Regardless of these simplifications, the evaluation of the capacity of a cross-section is yet demanding and this has been a constraint to use composite frame systems. High-rise construction is being introduced in Ethiopia in recent times and many more are expected to be built. Composite systems may provide optimal solution with regard to such constructions. However, neither design aid nor analysis tool is developed until now according to the revised building codes. To this end, this thesis proposed a solution method for computing the uniaxial interaction curves for “I” or “H” sections fully encased in concrete.

In this study full interaction among the component materials of the cross-sections and plane sections remain plane after deformation were assumed.

The section analysis commences from a particular strain distribution in the ULS. The possible strain distributions in the ULS were adopted from EN 1992-1-1:2004 [5]. Having a particular strain distribution in the ULS, the stress in the materials was found using the stress-strain relationships. The constitutive laws of the materials were described with piecewise defined polynomial functions. For the concrete, a parabolic-rectangular stress-strain relation was taken according to EN 1992-1-1:2004 [5]. For the reinforcement steel and structural steel, bi-linear stress-strain relations were adopted according to EN 1992-1-1:2004[5] and EN 1993-1-1:2005 [6] respectively.

The stress resultants of the cross-section were obtained by summing the contributions of the concrete, reinforcement and structural steel. Evaluation of the stress resultants of the concrete and structural steel required evaluation of double integrals over their respective regions. These integrals were transformed into line integrals along the boundaries of the concrete in compression and the structural steel either in compression or tension. For ease of computation, the line integrals were solved using three-points Gauss Quadrature, which is an exact method for the assumed stress-strain relationships of the materials.

The reinforcements are assumed as discrete points and thus, their stress resultants were obtained by directly multiplying the stress along a fiber by the corresponding area of the reinforcements.

Double counting of the concrete area that is replaced by the reinforcements and structural steel was avoided.

The developed interaction curves were verified with the output of MASQUE [7]. A sample cross-section was used for both plane of bending interactions with the normal force. Almost exact results were obtained.

To facilitate the evaluation of points on the interaction curve for different types of composite cross-sections, an algorithm was developed. To advance the applicability of this study, a Matlab program with a graphical user interface called UICISEC was incorporated. The database of UICISEC is limited to normal strength concrete, reinforcement grade up to S500 and that of structural steel up to S275. The database for the reinforcement sizes and structural steel sections are listed in the Annexes.

A design example for a column length that was subjected to a biaxial bending was carried out according to the EN 1994-1-1:2004 [3] simplified method. This method is based on the uniaxial interaction curves in both planes of bending. Hence, the uniaxial plots of UICISEC were utilized. In EN 1994-1-1:2004 [3], the factor α_M is included in the stability check of a member to accounts for the unconservative assumption during the evaluation of the interaction curves with the simplified method. This assumption was that the concrete in the compression zone resists a stress of $0.85f_{cd}$ which is constant over the entire depth up to the plastic neutral axis. Depending on the structural steel grade, α_M has a value of 0.9 or 0.8. This factor is multiplied with the available resistance for flexure. However, in this study the points on the interaction curves were computed starting with the actual strain distributions in the ULS throughout the cross-section. As a result, this factor can be omitted in the verification of a column length using the curves of UICISEC.

An analysis tool like UICISEC can be very appealing to structural engineer as it facilitates the effort of providing safety and economy in structural systems. With proportioning and designing structural systems, come the associated iterative design procedures. For steel-concrete composite columns, these are very time taking and inflexible progression. Furthermore, as the code analysis procedure is for sections proportioned in a certain way only, this also puts another constraint to the designers. All together with unsuitable analysis tools, it left the use of composite section under question. Hence, it is believed that for encased composite section at least, this study alleviates these problems in some way.

6 RECOMMENDATION

This thesis addressed the section analysis problem for fully encased rectangular composite sections by providing a solution method and software that enables to plot the uniaxial curves. Furthermore, a biaxial design example is done to show the one application of this study. The design was carried out according to the Eurocode 4, which is adopted by the new Ethiopian building code for the design of composite structures. Designers can use the software in analyzing and designing composite columns.

A generic program for the analysis of arbitrary composite sections that are subjected to biaxial bending and axial load can be developed. Furthermore, the confinement effects by the flanges of steel sections and by the transverse reinforcements on the concrete ultimate crushing strain as well as strength can be incorporated. The influence of the transverse shear on the strength of the web of steel section can also be integrated.

ANNEX

Annex A Strength and Deformation Characteristics for Normal Strength Concrete

Strength classes for concrete									
Class	12/15	16/20	20/25	25/30	30/37	35/45	40/50	45/55	50/60
f_{ck} (Mpa)	12	16	20	25	30	35	40	45	50
$f_{ck,cube}$ (Mpa)	15	20	25	30	37	45	50	55	60
f_{ctm} (Mpa)	1.6	1.9	2.2	2.6	2.9	3.2	3.5	3.8	4.1
E_{cm} (Gpa)	27	29	30	31	33	34	35	36	37
ε_{c2} (‰)	2.0								
ε_{cu2} (‰)	3.5								
n	2.0								

Annex B Strength Class of Reinforcement

Class	S300	S400	S500
f_{sk} (Mpa)	300	400	500

Annex C Strength Class of Structural Steel

Class	S300	S400	S500
Fyk (Mpa), t≤40mm	300	400	500

Annex D Size of Reinforcement bar

Diameter of bar (mm)	Area (mm^2)
6	28.3
8	50.3
10	78.5
12	113.0
14	154.0
16	201.0
18	254.0
20	314.0
22	380.0
24	452.4
26	531.0
30	707.0
32	804.2
34	908.0
40	1257.0

Annex E Type of Structural Steel Section

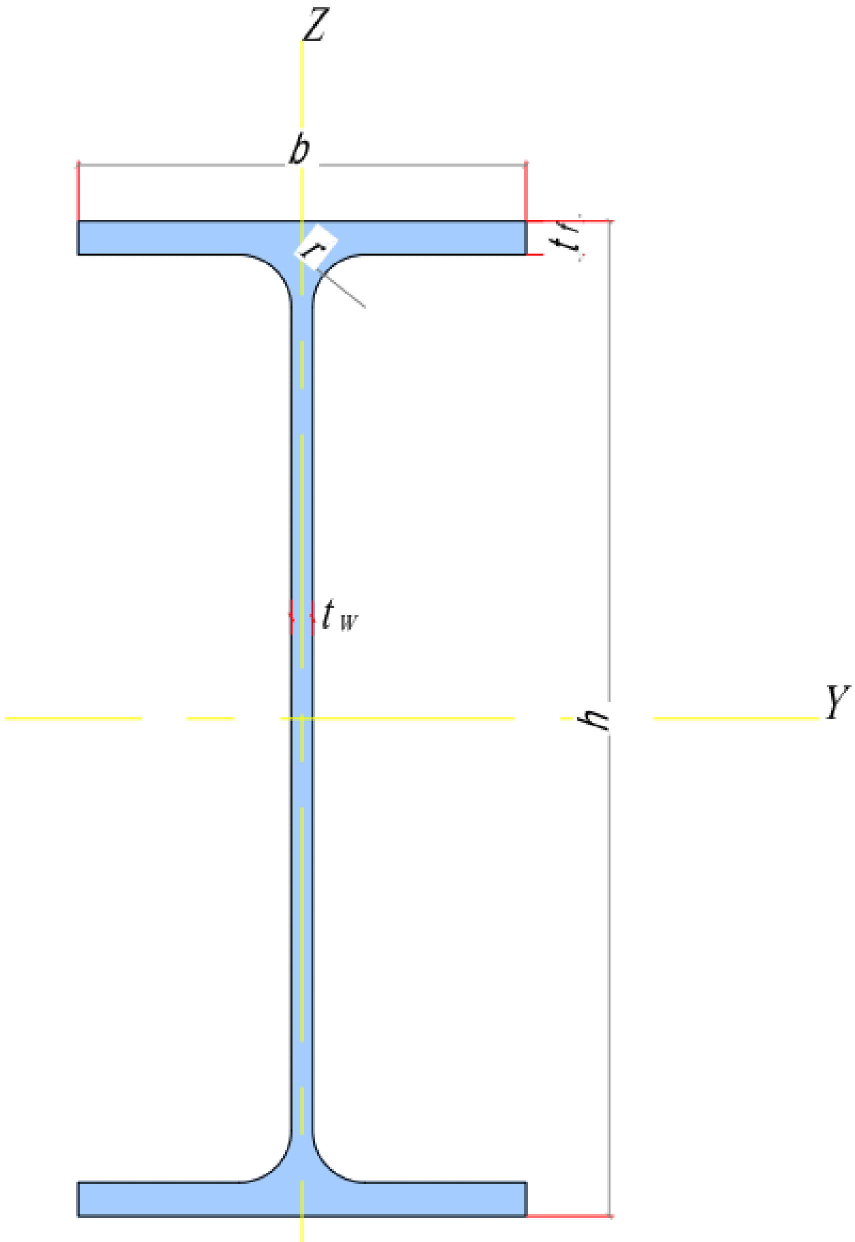


Figure E.1: Cross-sectional designation of structural steel

E.1 HEA Groups

Type	Nominal dimensions					Area	Section properties	
HEA	*b* (mm)	*h* (mm)	t_w (mm)	t_f (mm)	*r* (mm)	A_a (cm^2)	I_y ($cm^{4)}$)	I_z (cm^4)
HEA 100	100	96	5.0	8.0	12	21.2	349.2	133.8
HEA 120	120	114	5.0	8.0	12	25.3	606.2	230.9
HEA 140	140	133	5.5	8.5	12	31.4	1,033	389.3
HEA 160	160	152	6.0	9.0	15	38.8	1,673	615.6
HEA 180	180	171	6.0	9.5	15	45.3	2,510	924.6
HEA 200	200	190	6.5	10.0	18	53.8	3,692	1,336
HEA 220	220	210	7.0	11.0	18	64.3	5,410	1,955
HEA 240	240	230	7.5	12.0	21	76.8	7,763	2,769
HEA 260	260	250	7.5	12.5	24	86.8	10,450	3,668
HEA 280	280	270	8.0	13.0	24	97.3	13,670	4,769
HEA 300	300	290	8.5	14.0	27	112.5	18,260	6,310
HEA 320	300	310	9.0	15.5	27	124.4	22,930	6,985
HEA 340	300	330	9.5	16.5	27	133.5	27,690	7,436
HEA 360	300	350	10	17.5	27	142.8	33,090	7,887
HEA 400	300	390	11	19.0	27	159.0	45,070	8,564
HEA 450	140	440	11.5	21.0	27	178.0	63,720	9,465
HEA 500	300	490	12.0	23.0	27	197.5	86,970	10,370
HEA 550	300	540	12.5	24.0	27	211.8	111,900	10,820
HEA 600	300	590	13	25.0	27	226.5	141,200	11,270
HEA 650	300	640	13.5	26.0	27	241.6	175,200	11,720
HEA 700	300	690	14.5	27.0	27	260.5	215,300	12,180
HEA 800	300	790	15	28.0	30	285.8	303,400	12,640
HEA 900	300	890	16	30.0	30	320.5	422,100	13,550
HEA 1000	300	990	16.5	31.0	30	346.8	553,800	14,000

E.2 HEB Groups

Type	Nominal dimensions					Area	Section properties	
HEB	b (mm)	h (mm)	t_w (mm)	t_f (mm)	r (mm)	A_a (cm^2)	I_y ($cm^{4)}$	I_z (cm^4)
HEB 100	100	100	6	10	12	26.0	449.5	167.3
HEB 120	120	120	6.5	11.0	12	34.0	864.4	317.5
HEB 140	140	140	7.0	12.0	12	43.0	1,509	549.7
HEB 160	160	160	8.0	13.0	15	54.3	2,492	889.2
HEB 180	180	180	8.5	14.0	15	65.3	3,831	1,363
HEB 200	200	200	9.0	15.0	18	78.1	5,696	2,003
HEB 220	220	220	9.5	16.0	18	91.0	8,091	2,843
HEB 240	240	240	10.0	17.0	21	106.0	11,260	3,923
HEB 260	260	260	10.0	17.5	24	118.4	14,920	5,135
HEB 280	280	280	10.5	18.0	24	131.4	19,270	6,595
HEB 300	300	300	11.0	19.0	27	149.1	25,170	8,563
HEB 320	300	320	11.5	20.5	27	161.3	30,820	9,239
HEB 340	300	340	12.0	21.5	27	170.9	36,660	9,690
HEB 360	300	360	12.5	22.5	27	180.6	43,190	10,140
HEB 400	300	400	13.5	24.0	27	197.8	57,680	10,820
HEB 450	300	450	14.0	26.0	27	218.0	79,890	11,720
HEB 500	300	500	14.5	28.0	27	238.6	107,200	12,620
HEB 550	300	550	15.0	29.0	27	254.1	136,700	13,060
HEB 600	300	600	15.5	30.0	27	270.0	171,000	13,530
HEB 650	300	650	16.0	31.0	27	286.3	210600	13,980
HEB 700	300	700	17.0	32.0	27	306.4	256,900	14,440
HEB 800	300	800	17.5	33.0	30	334.2	359,100	14,900
HEB 900	300	900	18.5	35.0	30	371.3	494,100	15,820
HEB 1000	300	1000	19.0	36.0	30	400.0	644,700	16,280

E.3 HEM Groups

Type	Nominal dimensions					Area	Section properties	
HEM	b (mm)	h (mm)	t_w (mm)	t_f (mm)	r (mm)	A_a (cm^2)	I_y (cm^4)	I_z (cm^4)
HEM 100	106	120	12	20	12	53.2	1,143	399.2
HEM 120	126	140	12.5	21	12	66.4	2,018	702.8
HEM 140	146	160	13	22	12	80.6	3,291	1,144
HEM 160	166	180	14	23	15	97.1	5098	1,759
HEM 180	186	200	14.5	24	15	113.3	7,483	2,580
HEM 200	206	220	15	25	18	131.3	10,640	3,651
HEM 220	226	240	15.5	26	18	149.4	14,600	5,012
HEM 240	248	270	18	32	21	199.6	24,290	8,153
HEM 260	268	290	18	32.5	24	219.6	31,310	10,450
HEM 280	288	310	18.5	33	24	240.2	39,550	13,160
HEM 300	310	340	21	39	27	303.1	59,200	19,400
HEM 320	309	359	21	40	27	312.0	68,130	19,710
HEM 340	308	395	21	40	27	315.8	76,370	19,710
HEM 360	307	432	21	40	27	318.8	84,870	19,520
HEM 400	307	478	21	40	27	325.8	104,100	19,340
HEM 450	306	524	21	40	27	335.4	131,500	19,340
HEM 500	306	572	21	40	27	344.3	161,900	19,150
HEM 550	305	620	21	40	27	354.4	198,000	19,160
HEM 600	305	620	21	40	27	363.7	237,400	18,980
HEM 650	305	668	21	40	27	373.7	281,700	18,980
HEM 700	304	716	21	40	27	383.0	329,300	18,800
HEM 800	303	814	21	40	30	404.3	442,600	18,660
HEM 900	302	910	21	40	30	423.6	570,400	18,450
HEM1000	302	1008	21	40	30	444.2	722,300	18,460

E.4 IPE Groups

Type	Nominal dimensions					Area	Section properties	
IPE	b (mm)	h (mm)	t_w (mm)	t_f (mm)	r (mm)	A_a (cm^2)	I_y ($cm^{4)}$	I_z (cm^4)
IPE 80	46	80	3.8	5.2	5.0	7.64	80.1	8.49
IPE 100	55	100	4.1	5.7	7.0	10.3	171	15.9
IPE 120	64	120	4.4	6.3	7.0	13.2	318	27.7
IPE 140	73	140	4.7	6.9	7.0	16.4	541	44.9
IPE 160	82	160	5.0	7.4	9.0	21.9	869	68.3
IPE 180	91	180	5.3	8.0	9.0	23.9	1,317	101.0
IPE 200	100	200	5.6	8.5	12.0	28.5	1,943	142.0
IPE 220	110	220	5.9	9.2	12.0	33.4	2,772	205.0
IPE 240	120	240	6.2	9.8	15.0	39.1	3,892	284.0
IPE 270	135	270	6.6	10.2	15.0	45.9	5,790	420.0
IPE 300	150	300	7.1	10.7	15.0	53.8	8,356	604.0
IPE 330	160	330	7.5	11.5	18.0	62.6	11,770	788.0
IPE 360	170	360	8.0	12.7	18.0	72.7	16,270	1,043.0
IPE 400	180	400	8.6	13.5	21.0	84.50	23,130	1,318.0
IPE 450	190	450	9.4	14.6	21.0	98.82	33,740	1,676.0
IPE 500	200.	500	10.2	16.0	21.0	115.5	48,200	2,142.0
IPE 550	210	550	11.1	17.2	24.0	134.4	67,120	2,668.0
IPE 600	220	600	12.0	19.0	24.0	156.0	92,080	3,387.0

REFERENCES

1. Dong Keon Kim, “A Database for composite columns”, 1.
2. João Batista Marques de Sousa Jr.and Rodrigo Barreto Caldas, “Numerical Analysis of Composite Steel-Concrete Columns of Arbitrary Cross Section”, Engineering Structures 29 (2007) 618–625.
3. EN 1994-1-1:2004, “Eurocode 4: Design of composite steel and concrete structures-Part 1-1: General rules and rules for buildings”, 27, 62-72, 76.
4. Johnson R.P., “Composite structures of steel and concrete 3rd Ed.”, 22-24, 172-174.
5. EN 1992-1-1:2004, “Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings, 27-29, 34-35, 37-41, 83-84.
6. EN 1993-1-1:2005, “Eurocode 3: Design of steel structures - Part 1-1: General rules and rules for buildings”, 25-28, 39, 50-51, 56-57.
7. Busjäger,D., and Quast, U., “Programmesteuerte Berechnung beliebiger Massivbauquerschnitte unter zweiachsiger Biegung mit Längskraft’’, Deutscher Ausschuss für Stahllbeton, Heft 415, Beuth Verlag, Berlin, Germany, 1990.
8. Ehab Ellobody, Lam Dennis and Young Ben, “Eccentrically loaded concrete encased steel composite columns”, Thin-Walled Structures 49 (2011) 53–65.
9. Sheikh SA, Uzumeri SM., “Analytical model for concrete confinement in tied columns.” Journal of Structural Division—ASCE 1982; 108(12):2703–22.
10. Mander JB, Priestley MJN, Park R., “Theoretical stress–strain model for confined concrete. Journal of Structural Engineering ASCE 1988; 114(8):1804–26.
11. Chen CC, Lin NJ. “Analytical model for predicting axial capacity and behavior of concrete encased steel composite stub columns.” Journal of Constructional Steel Research 2006; 62:424–33.

12. Al-Shahari AM, Hunaiti YM, Abu Ghazaleh A., "Behaviour of lightweight aggregate concrete-encased composite columns." Steel and Composite Structures 2003; 3(2):97–110.

13. SSRC Task Group 20. "A specification for the design of steel–concrete composite columns." AISC Engineering Journal Fourth Quarter 1979:101–15.

14. Morino S, Matsui C, Watanabe H., "Strength of biaxially loaded SRC columns." In: Proceedings of the US/Japan joint seminar on composite and mixed construction. New York: ASCE; 1984. p. 185–94.

15. Mirza S.A., Lacroix E.A. Comparative strength analyses of concrete-encased steel composite columns. Journal of Structural Engineering—ASCE 2004; 130 (12):1941–53.

16. Fafitis A., "Interaction surfaces of reinforced-concrete sections in biaxial bending." ASCE J Struct Eng 2001; 127 (7):840–6.

17. Erwin Creyszig, "Advanced Engineering Mathematics, 10th Ed.", 433.

18. HEA (IPBL) beams, European standard wide flange H beams, dimensions, specifications. HEA beams in accordance with former standard Euronorm 53-62, http://www.b2bmetal.eu/hea-sections-specification.

19. HEB beams, European standard wide flange H beams, dimensions, specifications. HEB beams in accordance with former standard Euronorm 53-62, http://www.b2bmetal.eu/heb-sections-specification.

20. HEM beams, European standard wide flange H beams dimensions, specifications. HEM beams in accordance with former standard Euronorm 53-62, http://www.b2bmetal.eu/en/pages/index/index/id/71/.

21. IPE beams. European standard universal I beams (I section) with parallel flanges. Dimensions, specifications, accordance with former standard EU 19-57, http://www.b2bmetal.eu/i-sections-ipe-specification.

22. Zerayohannes G., "Bemessungsdiagramme für schiefe Biegung mit Längskraft nach DIN 1045-1:2001-07", 6-7.