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Optimization of Hydropower Plant Expansion

Case Study: Nile Basin, Ethiopia

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June 2008

Acknowledgments

I would like to thank my advisor, Dr. Zelalem Hailu, Head of the Civil Engineering Department at the University of Addis Ababa, Technology Faculty, for his continual encouragement, guidance and support. Without his help I would never have completed this thesis.

I am very grateful for all of the assistance from the office staff at ENTRO, EEPCO and Ethio-Sudan Ethio-Kenya power interconnection system project.

Finally, a very special thank you is given to my husband, Andenet, my mother, Alemtsehaye, my father, Geletu, their love, constant support and encouragement has allowed me to successfully complete this thesis.

Abstract

This research is of a sequencing expansion problem in which capacity can be added only at discrete points in time. There is a forecast of demand in each period, and five expansion projects each with a given capacity and cost.

Dynamic programming is used to determine the sequence of expansions necessary to provide sufficient capacity to meet the demand in all periods at minimum discounted cost. The alternative scenarios are represented with a tree structure.

This research also provides preliminary results for a discounted cash flow as well as a dynamic programming solution.

KEYWORDS: Capacity expansion, dynamic programming, sensitivity analysis

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1 Introduction

1.1 Background

Ethiopia has a huge hydropower potential, which has hardly been exploited. Studies put the gross hydro potential of Ethiopia at 650TWh/year. Although there are substantial hydropower resources, Ethiopia has one of the lowest levels of per capital electrical consumption in the world. The annual consumption of electricity in 1995 was 1670 Gwh, equivalent to 30kwh/capital. The installed capacity was about 417.75 MW (377.75 MW + 40MW) of which 90% was provided by hydropower. The present capacity deficit was estimated to be about 300MW.

The first large hydroelectric facilities were constructed in the Awash river basin from 1960 on – Koka (Awash I) with 43 MW capacity and Awash II and Awash III both with 32 MW capacity. These were followed with the Finchaa 100 MW facility in 1974 and the Melka Wakana 153 MW facility in 1988, while the first geothermal plant, at Aluto Langano with 7.3 MW capacity, came on-stream in 1999.

These plants, along with the smaller Tis Abay 11.4 MW plant (completed 1964) and 13 small diesel plants with a combined capacity of 21.8 MW – of which the largest are Dire Dawa 4.5 MW, Adwa 3 MW and Nekempt 2.3 MW – comprise the country's major power supply system, the Inter-Connected System (ICS). These have a total capacity on paper of 400.7 MW, although due to ageing plant the dependable capacity is currently only 389.5 MW. The ICS supplies 404 of the 457 electrified towns (out of the 962 towns in the country). The remaining 53 towns are supplied by a secondary system, the Self-Contained System (SCS), which consists of three mini-hydro plants with a combined capacity of 6.2 MW and a number of diesel plants with a combined capacity of 13.7 MW.

The Nile basin is one of the strategic basins and has several favorable sites where hydroelectric energy can be economically generated in Ethiopia. Ethiopia can also produce energy at relatively cheaper rates for export to countries like Egypt and Sudan in the basin. The hydropower potential in the Nile basin in Ethiopia can be harnessed optimally:

- To generate and distribute electricity in the sub-basins by co-operation.
- To supply cheap and renewable energy to the sub-basin countries and there by save fuel from the large thermal generation.
- To encourage the development of large hydropower schemes in Ethiopia for the benefit of the whole sub-basin countries by creating a market for the surplus energy by means of facilitating financial resources.
- To improve the water use efficiency of the basin by regulating and storing in the highlands and deep gorges of Ethiopia, where evaporation is minimum.
- To protect the environment by providing environment friendly energy resource.

To achieve these objectives four dams are in plan to be constructed on the Nile basin. These dams are known as Karadobi, Mandaya and Border in Ethiopia and Dal in Sudan. Mabil is left out because when the height of Mandaya was increased the site of Mabil flooded. [Pre-feasibility study of Mandaya hydropower project (2007)]

1.2 Problem Statement

Considering the social, economic and environmental dimensions of hydropower and its potential contribution to achieving sustainable development goals, there is a need to develop hydropower that is economically, socially, and environmentally sustainable.

One of the problems faced by planners when designing an expansion plan for water resources systems is selecting the sequence of projects that satisfies a given demand at minimum cost, some prescribed time in the future. Optimal scheduling maximizes basin wide operating efficiency while satisfying power load demands, water demands, reliability constraints, operational restrictions and security requirements. At the same time it allows each electrical utility to make savings on power plant investment as a result of the improved use of the system. It also contributes to the quality of electricity supplied as well as reduces environmental damage.

Due to absence of a decision making model, most decisions on the inclusion of new hydropower plants in the system (sequencing decision) is made based on factors that does not consider the optimality of the sequences into consideration. One of the ways to address this problem is to use a dynamic programming model to determine an optimal expansion plan for the generating capacity of an electric power system. The optimization model will be so designed that it determines the least-cost mix of capacity, the size of the plants to add to the system, and the timings of these additions. The problem is simultaneously optimized over all time steps and scenarios.

1.3 Thesis Objectives

The main objective of this thesis is to identify the best sequence for construction of the hydropower plants to lead to a foundation for maximizing the economic and socially compatible productivity and reliability. But in addition this thesis is also dedicated to the developing a hydropower system based on the existing network of power supply in Ethiopia and also including plants, which are studied and planned for future development.

1.4 Scope

Information is available on the capacity of existing; committed plants as well as that are under study. This research involves determining the sequence of construction of hydropower plants that are under study in Ethiopia. Efforts were made to include other Nile Basin Countries in the system. But, due to insufficient data gathered on the power systems of other Nile Basin countries, the scope of the research has been limited to determine the sequencing of power plant systems in Ethiopia.

1.5 Thesis Layout

This thesis is divided into seven parts: chapter one offers an introduction to the thesis followed by literature review about dynamic programming which is explained in chapter two. The data for current and future generation of Eastern Nile countries is given in chapter three. On chapter four the power expansion plan for Ethiopia is described. The application of DP on the capacity expansion problem is presented in chapter five while sensitivity analysis is discussed on chapter six. Finally, conclusions and recommendations follow in chapter seven.

2 Literature Review

2.1 Planning Models and Solution Procedures

There are two approaches for solving planning models: Simulation and optimization. Simulation relies on trial and error to identify near-optimal solutions. The value of each decision variables is set, and the resulting objective values are evaluated. The difficulty with the simulation approach is that there is often a frustratingly large number of feasible solutions or plans. Even when combined with efficient techniques for selecting the values of each decision variable, an enormous computational effort may lead to a solution that is still far from the best possible.

To their credit, simulation methods are able to solve water resource systems planning model with highly nonlinear relationships and constraints. Constrained optimization procedures are seldom able to deal with all the complexities and nonlinearities which are easily incorporated in a simulation model. Still, when an optimization procedure can be constructed to efficiently solve an adequate approximation to the real problem, they can greatly narrow down the search with simulation for a global optimum by identifying plans that may be close to the optimum.

Constrained optimization algorithms include a diverse set of techniques that use calculus and matrix algebra. Optimization techniques include Lagrange multipliers, linear programming, dynamic programming, quadratic programming and geometric programming. Each of these and other solution procedures are highly dependent on mathematical structure of the management model.

Typical planning model generally include at least one objective function that is either to be maximized or minimized and which serves to rank the alternative solutions or plans. In addition to an objective, planning problems incorporates a number of requirements which are formulated as constraints. It is important to distinguish the different roles played by the objective function and the constraints. The optimal solution of the planning problem is a plan that achieves the largest (or smallest) value of the objective while satisfying all the constraints. [Daniel P. Loucks (1981)]

2.2 Dynamic Programming

Dynamic programming is ideally suited for sequential decision problems. Sequential decision problems are those in which decisions are made sequentially, one after another, based on the state of the system. Unlike linear programming problems, dynamic programming problems are not amendable to a single, standard algebraic formulation. Different types of sequential decision problems may need to be formulated differently considering specific features of the problem and therefore, dynamic programming problem is said to be as much an art as it is a mathematical technique.

In many practical situations a net-benefit function may not be so continuous, or so conveniently concave for maximization or convex for minimization, making calculus-based methods for their solution difficult. A possible solution method for constrained optimization problems containing continuous and/or discontinuous functions of any shape is called discrete dynamic programming. Each decision-variable value can assume one of a set of discrete values. For continuous valued objective functions, the solution derived from discrete dynamic programming may therefore be only an approximation of the best one. For all practical purposes this is not a significant limitation, especially if the intervals between the discrete values of the decision-variables are not too large and if simulation modeling is used to refine the solutions identified using dynamic programming.

Dynamic programming is an approach that divides the original optimization problem, with all of its variables, into a set of smaller optimization problems, each of which needs to be solved before the overall optimum solution to the original problem can be identified. The water supply allocation problem, for example, needs to be solved for a range of water supplies available to each firm. Once this is done the particular allocations that maximize the total net benefit can be determined.

Dynamic programming models can be applied to design problems, such as the capacity-expansion problem or a reservoir storage capacity–yield, or to operating problems, such as the water allocation and reservoir operation problems, but rarely to problems having both unknown design and operating policy decision-variables. While there are some tricks that may allow dynamic programming to be used to find the best solutions to both design and operating problems encountered in water resources planning and management studies, other optimization methods, perhaps combined with dynamic programming where appropriate, are often more useful. [Daniel P. Loucks (1981)]

As capacity expansion is usually a strategic issue that involves substantial capital investment, the methodologies and models used to aid capacity planning are of vital importance. Capacity expansion planning consists primarily of determining future expansion times, sizes, and locations, as well as the types of facilities. The combination of these factors together with sometimes the dynamic nature of product demands makes expansion decisions extremely complex. In this thesis, we look at a typical capacity expansion problem and develop a dynamic programming approach to solve the problem.

Considerable research has been done on developing different approaches to solve expansion sequencing problems. Among them dynamic programming is the one that has been used most. Butcher, Haimes, and Hall (1969) formulated the problem by using a dynamic programming approach similar to that for sequencing jobs on a single facility, but their algorithm did not guarantee an optimal solution in general. Erlenkotter (1973) has developed a backward dynamic programming formulation to find an optimal sequence. The algorithm is quite efficient in terms of the number of calculations required to obtain an optimal solution. The computational applicability of the algorithm however is limited by the exponential growth of the number of possible states, as is common to most dynamic programming methods. Hence the dynamic programming approach is conveniently applied for problems with relatively small number of projects (up to about 20).

3 Nile Power Profile

3.1 Introduction

The Nile Basin encompasses ten countries: Burundi, Democratic Republic of Congo, Egypt, Eritrea, Ethiopia, Kenya, Rwanda, Sudan, Tanzania, and Uganda. In 1997, the Nile riparian countries initiated dialogue on a long term Cooperative Framework and in 1999 the Council of Ministers of Water Affairs of the Nile Basin States (Nile-COM) formally launched the Nile Basin Initiative (NBI)-an interim financing mechanism for development of regional projects. The NBI is represented by its executive arm, the Nile Secretariat (Nile-SEC) based in Entebbe, Uganda.

Then following the establishment of the Eastern Nile Council of Ministers (ENCOM), Ethiopia, Egypt, and Sudan have jointly adopted a strategy to develop, utilize, and manage water resources of the Eastern Nile basin in an integrated, equitable, and sustainable manner.
In defining the cooperative development paradigm, the countries identified sixty-four potential regional projects. Out of these seven priority projects, known as the Integrated Development of the Eastern Nile (IDEN) were selected. These are: Ethiopia-Sudan Interconnection Project, Eastern Nile Power Trade Investment Program Study, Eastern Nile Multi-sector Planning Model, Baro-Akobo Multipurpose Hydro Power Project, Flood Preparedness and Early Warning, Irrigation & Drainage Development, and Watershed Management. [Pre-feasibility study of Border hydropower project (2007)]

Ethiopia and the neighbouring countries power systems are quite complementary (e.g Thermal source Kenya, Djibouti and Sudan hydropower from Ethiopia) and a tie line between these countries power systems could bring clear benefits in the intermediate and long term future.

Regional power trade has several benefits such as

a) Energy benefits
    - Reduced energy cost
    - Improvement in energy utilization
    - More economical daily, weekly, monthly and seasonally load dispatch.

b) Investment benefits
  - Reduction in investment requirements
  - Reduction in stand by reserves
  - Economies of scale

c) Operating benefits
  - Maintenance scheduling
  - Emergency support

d) Political benefits
  - Base for economic interaction
  - Assisting/ supporting purchase stability

Since November 2003 the idea of establishing the East African Power Pool has been worked on. Similarities in some of the objectives of the East African Power Pool and the Nile Basin Power Trade Project relating to the Regional Master Plan and the formation of institutional frame work for power trade seemed to have been causing the same effort and misuse of resource. An optimum use of resource was discussed in March 2006 and a joint action plan was agreed on.

The studies done on the electric power supply and demand for the Equatorial Lakes Region (Kenya, Uganda, Rwanda, Burundi, Eastern DRC and Tanzania) indicated that in the coming two decades the available electrical energy resource within the region shall not be able to satisfy the growing demand. Therefore, they would need to use the resource outside the region. This can be illustrated by forecast figures compared to the available resources. By the end of 2020, the total demand of the region is estimated at 3423 MW for base forecast, 4968 MW for high scenario and 12600 MW for transformation scenario. The available resource however is only 2349 MW of best evaluated option, plus 2129 MW of other options. Any amount of load greater than the available resource needs to be covered either from electricity import outside the region or from power plants running on imported fuel which are currently suffering from power shedding.

Considering plants running on imported fuel it is not encouraging at all due to the fast rising oil prices. Therefore, for the region to benefit from power trade as well as to achieve development of the available resources in the region there should be facilities which allow power generation options to develop in the region and power flow from outside the region.

3.2 Power Profile for Eastern Nile Countries

3.2.1 Sudan

The electrification ratio of the Sudan (percentage of households with electricity supply) is one of the lowest in the world, estimated at about 19 per cent (made up from about 16.3% metered NEC connections, 2.3% connections to private supply companies and 0.2% un metered connections).

Sudan utility

The National Electricity Corporation (NEC) is the governmental entity responsible for generation, transmission and distribution of electric power in the Sudan. NEC's power system comprises mainly the national grid (NG) and a number of isolated diesel power stations. The electricity system within Sudan consists of the main National Grid, a number of isolated off-grid systems and some existing private generation companies. NEC’s main grid system is divided into the Khartoum, Central, Eastern and Northern areas.

The towns of Atbara and Shendi in River Nile state, which were previously supplied by local off-grid generation, were connected to the National Grid as part of the Merowe transmission reinforcement scheme in the second half of 2005.

Current generation supply

Over the period 1997 – 2005, the energy generated increased from 2 150 GWh in 1997 to 3 768 GWh in 2005, an annual compound growth of 7.3 per cent per year. Before the year 2003 demand exceeded supply at a certain time of the year (April – August), as a result the consumers subjected to long periods of power cuts which resulted in high economic losses especially in industrial and agricultural sectors. At the times of capacity shortages NEC was forced to carry programmed and un- programmed power cuts. These cuts mainly carried in the summer season when the demand is at peak load and the hydro output is low. In 2003, Gerri I and Gerri II combined cycle power generating facilities were commissioned adding to the grid about 386 MW generating capacity. The supply exceeds the demand and the power cuts are mainly limited to failures in transmission and distribution. At the time being the total capacity available for dispatch on the National Grid is about 826 MW, of which some 59% is conventional thermal plant and the remaining 41% is hydroelectric plant.

The table here below, sets out the generation mix on the National Grid as at July 2006 and provide a summary of installed and available capacities from the existing on-grid power plants.

Table 3.1– Sudan's current generation supply

Power Plant	**Plant Type**	**Fuel Type**	**Net Capacity (MW)**
Khartoum North ST's	Thermal	HFO	157.0
Khartoum North GT's	Thermal	Gas Oil	50.4
Garri 1 CCGT's	Thermal	Gas Oil	164.0
Garri 2 OCGT's	Thermal	Gas Oil	84.0
El Fau Diesel	Thermal	Gas Oil	10.0
Kassala Diesel's	Thermal	Gas Oil	7.9
Girba Diesel's	Thermal	Diesel	4.0
Kuku GT's	Thermal	Gas Oil	19.0
Total Thermal Plant			**496.3**
Roseires	Hydro		280.0
Sennar	Hydro		15.0
Kashm El Girba	Hydro		18.1
Jebel Aulia	Hydro		28.1
Total Hydro Plant			**341.2**
Net Installed Capacity			**837.5**
Thermal Capacity Part			59%
Hydro Capacity Part			41%

Committed projects

According to NEC master plan, the following power plants have been identified as committed contributors to the Sudan generation expansion plan.

- Khartoum North Units 5 and 6 (100 MW each – 2008)

Conversion of Garri 2 power station to combined cycle operation	(200 MW – 2008)
Kilo X GT	(80 MW - 2007)
Garri (3) steam plant	(540 MW - 2010)
Garri (4) steam plant	(100MW – 2007)
Port Sudan steam plant	(405 MW – 2009)
Kosti steam plant	(500 MW – 2010)
El Bagair steam plant	(540 MW – 2010)
Kassala diesel plant	(50 MW – 2007)
Al Fula steam plant	(540 MW – 2010)
Merowe hydroelectric plant	(1250 MW – 2008)
Sennar extension hydroelectric plant	(1 250 MW – 2008)
The heightening of the Rosieres hydroelectric plant, with Dinder	(135 MW – 2012)

Transmission

The Sudanese system consists mainly in 110 and 220 kV lines. The system includes a 800 km 220 kV double circuit line from Roseires HPP, located in the south close to Ethiopia border, to Khartoum along to the Blue Nile River. A 110 kV double circuit ring supplies Khartoum that represents 50% of the total load. This 110 kV ring is connected to the 220 kV system with two 220/110 kV substations at Eid Babiker and Kilo X. In year 2007, the network was reinforced with a 500 kV double circuit line from Merowe HPP (installed capacity 1 250 MW) to Khartoum and a 500 kV single circuit line between Merowe and Atbara located on the Nile, 300 km north east of Khartoum. In the next years, NEC intends to extend its 220 kV system by about 2 000 km of new lines. [Analysis of the network expansion plan, Final report, Vol3-3- Sudan (2007)]

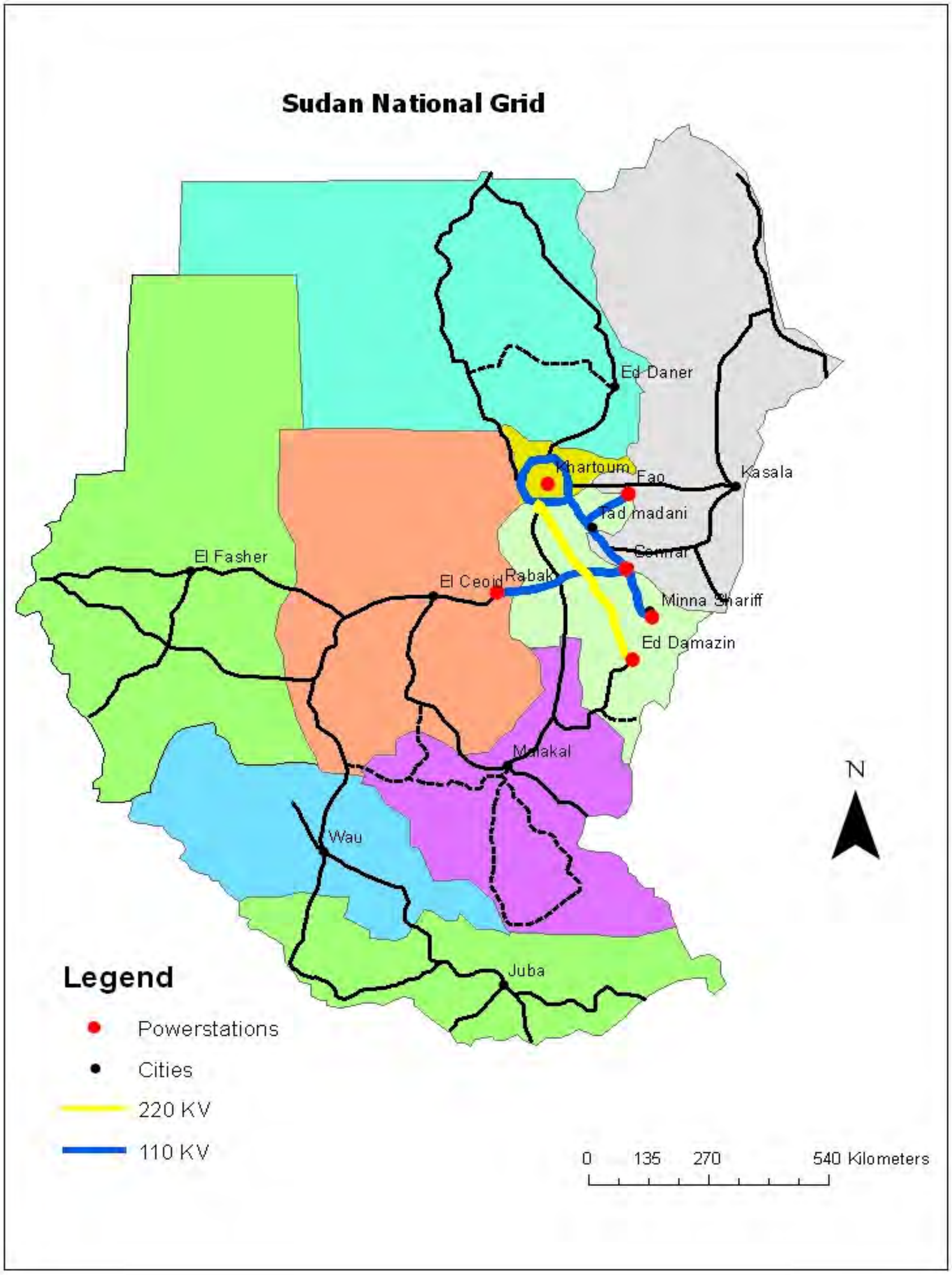


Fig 3.1 Sudan National Grid

3.2.2 Egypt

Egypt occupies the north-eastern corner of the continent of Africa, including the Sinai Peninsula, with a population of about 69.997 millions (2005), 43% in urban areas and 57% in rural areas. The growth rate of population is currently 1.96% (2006). The economy of the country has developed in the last years with an annual GDP rate of 5%, pushed up by a significant production of petroleum products, electricity developments, and industrialization. Egypt has a per capita electric energy consumption of 1 350 kWh (2001/2002). Access to electricity is high, around 98%, with negligible isolated systems. Environmental improvements can be noticed by the reduction of rate of CO_2 production in Egypt. It has been reduced from about 2.8 tons of CO_2 per Toe in 1981/1982 to about 2.5 in the year 2001/2002. This is because of the increase of the use of natural gas in the electric energy production.

Egypt Utility

Egyptian electric company is currently comprised of nine regional electricity distribution companies, five regional electricity generation companies, one electricity transmission companies. All these companies are blended in a Holding company, the Egyptian Electricity Holding Company (EEHC). Different authorities, such as New & Renewable Energy and Hydro Power, are directly linked to the Ministry of Electricity & Energy.

Current demand and generation supply

In Egypt, peak demand increased from 5 400 MW (1985/1986) to 17 300 MW (2005/2006). In the same period, energy generated increased from 32 TWh to 108 TWh, with a growth rate of 7% in the last ten years. The total installed capacity in 2006 is 20 508 MW, with 17 543 MW of thermal plants, 225 MW of wind farms, and 2 740 MW of hydropower (4 plants).

Table 3.2 – Egypt's total installed capacity in 2006

Installed capacity (MW)	ST	CCGT	OCGT	WIND	HYDRO	Total
Cairo	2270	1485	600			4355
West Delta	3330	1224	835			5391
East Delta	3991	1409	453	225		6078
Upper Egypt	1944					1944
Hydro					2740	2740
Total	11535	4118	1890	225	2740	20508
Installed Capacity	56%	20%	9%	1%	13%	

Committed Project

One hydro plant, two thermal plants, and two wind farms are committed:

- The New Naga-Hammadi 64 MW and 460 GWh/year is planned to operate in 2008/2009,
- Talkha 750 MW CC (NG/HFO) in East Delta is planned to operate in 2007/2008,
- Kurimat (2) 750 MW CC (NG/HFO) in Upper Egypt is planned to operate in 2007/2008,
- Zafarana / Gabal El-Zait 55 MW is planned to operate in 2006/2007,
- Zafarana / Gabal El-Zait 150 MW is planned to operate in 2007/2008.

In the Egyptian hydro system, irrigation is the priority, the power production is only a by-product. The Ministry of Water and Irrigation defines the daily discharges in power plants and send this information to NECC every week.

Existing transmission system and power trade

Egypt is interconnected with Libya and Jordan. These interconnections are used for emergency situations and for power trade between Egypt and Jordan. Exports and imports measured from 2003 to 2005 represented less than 1% of total Egyptian electrical generation, but 20% of Jordanian generation. An export balance of 20 GWh to Lybia and of 680 GWh to

Jordan was measured in 2004/2005. The existing transmission system is equipped with a double circuit 500 kV backbone along the Nile river, from High Dam (2 100 MW) to Cairo (main load centre), and a single circuit (500 KV) from Cairo to the interconnection with Jordan. A 132 kV and 220 kV circuit follows the 500 kV backbone along the Nile river. The delta zone is supplied with a meshed 220 kV network, and extends towards west to Libya with a double circuit interconnection. An extension of the 500 kV network is currently under construction from Cairo 500 to Sidi Krir in West Delta. It is also the first milestone to reinforcement of the interconnection with Libya in 500/400 kV. [Analysis of the network expansion plan, Final report, Vol3-1 Egypt (2007)]

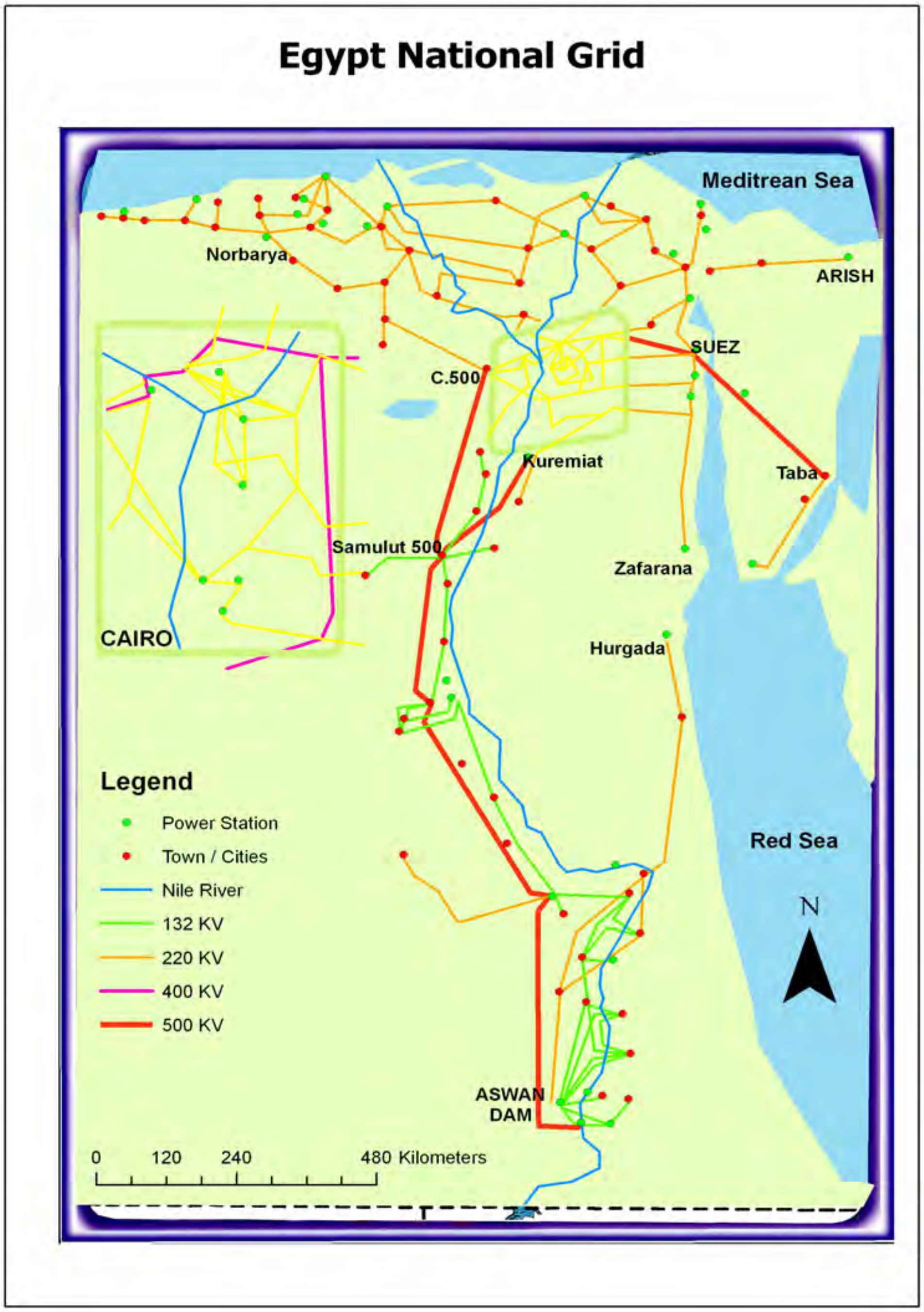


Fig 3.2 Current Egypt National Grid

3.2.3 Ethiopia

Ethiopia has one of the lowest levels of energy consumption per capita in the world at 28 kWh. Access to electricity is considered to be 17% of population at present.

Ethiopia utility

Ethiopian Electric Power Corporation (EEPCO) is responsible for generation, transmission and distribution of the interconnected system (ISC) as well as some isolated or self contained system (SCS). The current trend for development of generation using IPP (Independent Power Producers) which entitles EEPCO to buy generated power at negotiated rates is also encouraged.

Current generation supply

The ICS has a total installed capacity of 766,9 MW (end of 2006) including 96,3 MW of Diesel plants at Dire Dawa, Awash, and Kaliti, and a Geothermal plant at Aluto-Langano.

The existing thermal plants in the ICS (end of 2006) are as follows:

Table 3.3 – Ethiopia's Existing Thermal Plants

ICS Diesel and Geothermal Plants	**Installed capacity (MW)**	**Dependable capacity (MW)**
Dire Dawa Diesel	38	30
Awash 7 Kilo Diesel	27	12
Kaliti Diesel	12	6
Others	12	0
Aluto Geothermal*	7,3	0
Total Thermal Power Plant (ICS)	**96,3 MW**	**48 MW**

The existing hydropower plants in the ICS (end of 2006) are as follows:

Table 3.4 – Ethiopia's Existing Hydropower Plants in the ICS

Plants	**Installed Capacity (MW)**	**Dependable capacity (MW)**	**Average & firm energy capacity (GWh)**
Gilgel Gibe I (in 2004)	192(with 3 units)	184	840 /620
Maleka Wakana (in 1988)	153	153	550 /450
Finchaa (1973-2003)	134 (with 4 units)	128	640 /615
Tis Abay I (in 1964)	11,4	11,4	80 / 65
Tis Abay II (in 2001)	73	68	280 /145
Koka (in 1960)	43,2	38,4	110 /80
Awash II (in 1966)	32	32	165 /125
Awash III (in 1971)	32	32	165 /125
Total Hydro Power Plant (ICS)	**670,6 MW**	**646,8 MW**	**2.8 / 2.2 TWh**

Committed projects

Five hydropower projects are committed by EEPCO in 2006 and under construction: Gilgel Gibe II and Gibe III, Tekeze, Upper Beles and Neshe whose main characteristic and commissioning dates are shown in the following table:

Table 3.5 – Ethiopia's Committed Hydropower Projects

Hydro Power Plant (Commissioning date)	**Installed capacity (MW)**	**Average energy capacity (GWh)**
Gigel Gibe II - 2008	420	1,600
Gibe III - 2012	1870	6,240
Tekeze - 2008	300	960
Beles - 2009 (w/o Tis Abay I & II)	460 (376)	2,000 (1,630)
Neshe HPP - 2010	97	225
Total new capacity (w/o Tis Abay I & II)	3,147 MW **(3 063 MW)**	11,025 GWh **(10,655 GWh)**

Transmission system

The Ethiopian transmission system consists mainly in 230 and 132 kV lines. The 230 kV network extends from Addis Ababa about 400 km eastward to Dire Dawa, about 300 km southward to Shashemene and about 1000 km northward to Tekeze and Gonder. Three 230 kV substations supply Addis Ababa, that represents 60% of the total demand. A 400 kV network will be soon erected to evacuate the generation of Gilgel Gibe II HPP until Addis Ababa. Ethiopia will be interconnected with Sudan with a 230 kV double circuit line between Gonder and Gedaref in Sudan. The commissioning is expected in year 2008. [Analysis of the network expansion plan, Final report, Vol3-2- Ethiopia (2007)]

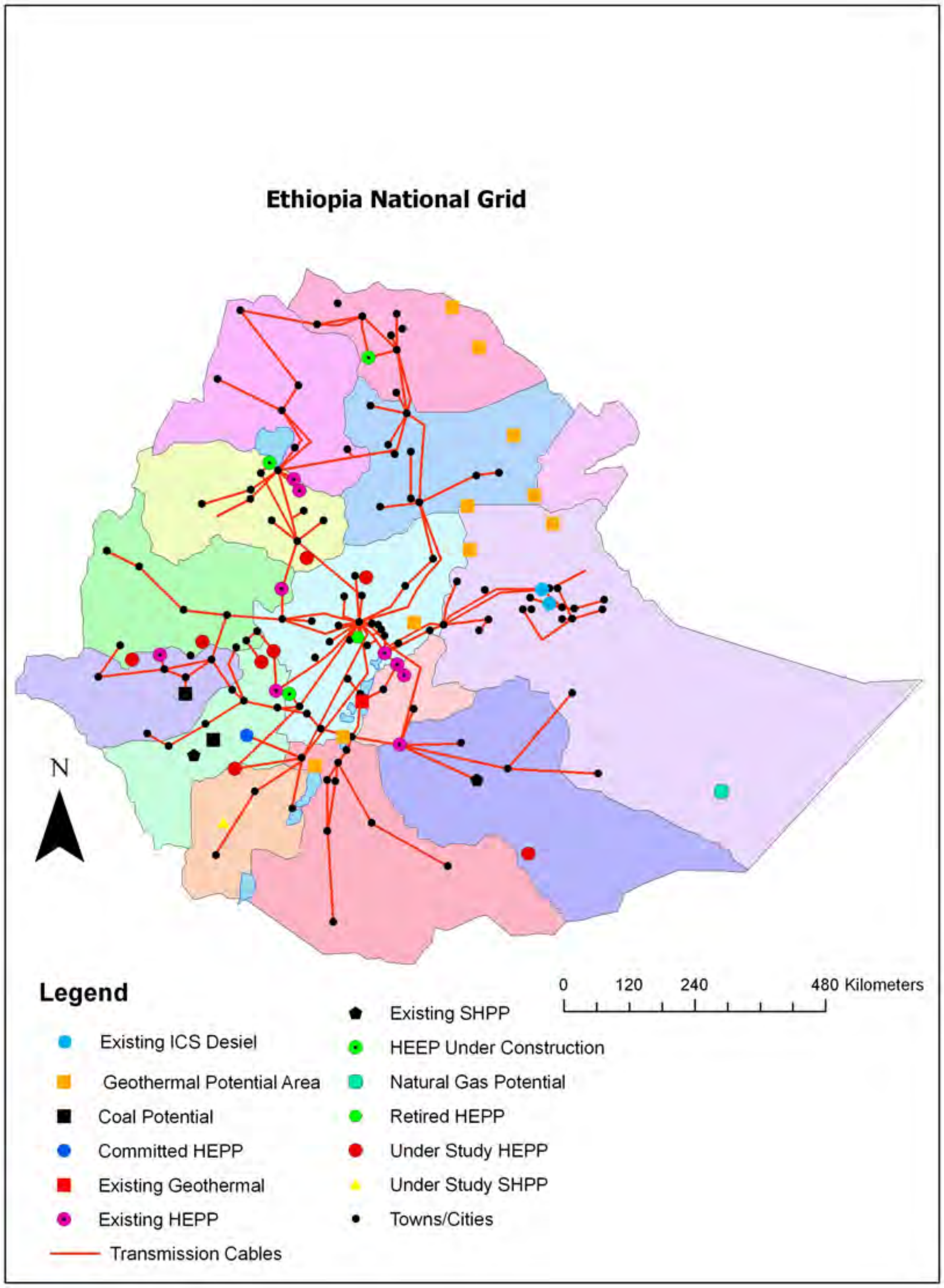


Fig 3.3 Ethiopian National Grid

3.2.4 Kenya

The largest share of Kenya's electricity supply comes from hydroelectric stations at dams along the upper Tana River, as well as the Turkwel Gorge Dam in the west. A petroleum-fired plant on the coast, geothermal facilities at Olkaria (near Nairobi), and electricity imported from Uganda make up the rest of the supply. Kenya's installed capacity stood at 1,142 megawatts a year between 2001 and 2003. The state-owned Kenya Electricity Generating Company (KenGen), established in 1997 under the name of Kenya Power Company, handles the generation of electricity, while the Kenya Power and Lighting Company (KPLC), which is slated for privatization, handles transmission and distribution. Shortfalls of electricity occur periodically, when drought reduces water flow. In 1997 and 2000, for example, drought prompted severe power rationing, with economically damaging 12-hour blackouts. Tax and other concessions are planned to encourage investment in hydroelectricity and in geothermal energy, in which Kenya is a pioneer. The government plans to open two new power stations in 2008, Sondu Miriu (hydroelectric) and Olkaria IV (geothermal), but power demand growth is strong, and demand is still expected to outpace supply during periods of drought.

On the current power supply situation in Kenya, the effective generation capacity was 1,054 MW comprising: Hydro. 654 MW (62%), geothermal 57 MW (5.4%) and thermal 343 MW (32.6%). The demand for power of 760 MW, a surplus generation capacity of 39% existed. The higher unit cost per KWh was as a result of forced costs incurred for excess capacity. KPLC had a favorable energy supply mix. On the recent power rationing experience, out of their 501 MW planned net generation capacity additions between 1991 and 2000 only 347 MW was actually developed, this resulted in a capacity short fall of 154MW. The reserve capacity margin fell to an all time low of only 9% above peak demand by 1998/99. The worst drought in 50 years was experienced in 1999 and 2000 when hydro generation contribution fell 75% of total before 1999 to 32% in 2000/2001. During that period the estimated cost of power rationing to the economy was US$65 million per month. The major challenges and constraints facing the country's power industry as: Making electricity available to the majority of Kenyans as only 10% of the total population of 30 million are connected to utility supplied electricity; Restructuring the power industry so that it could provide the service more efficiently; Lack of capital which had led to increased technical losses due to low investment in transmission and distribution network; Withdrawal of concessionary funding delayed building of generation capacity; The substitute private sector funded power generation was expensive due to the perceived high political and commercial risks; Tariff charges– were largely constant with no adjustment for inflation or for meeting the high cost of energy generation and transmission; Low sales growth rate due to distribution and customer constraints; Sources of Power – Hydro, the cheapest power source was almost fully exploited. Hydro Power the main source was very susceptible to vagaries of weather while geothermal development entails high capital and resource risk and thermal generation relied on imported fuel; and Cash flow constraints resulting from lowered sales, this has led to erosion of the financial base.

Fig 3.4 Kenya Power Plants

4 Power Expansion Plan

4.1 Introduction

At river basin levels integrated approaches amongst the various sectors is important for hydropower development. The main source of power outside the region is expected to be hydropower from Ethiopia, whose hydropower potential is estimated to be 30,000 MW. This thesis focuses on the power expansion plan for Ethiopia.

The following tables show the existing and future hydropower capacity of Ethiopia from EEPCO.

Table 4.1 Ethiopian existing and future hydropower capacity

Year on line	Addition	Capacity, (MW)	Cost $ (MUSD)	Plant Factor	Firm Energy (GWh)
2006	Existing	726			3088
2009	Gilgel Gibe II	420	451.3		1505
2009	Tekeze**	300			957
2010	Beles	460	468.7		1479
2010	Amerti Neshe	97	129		209
2011	GGIII (Phase 1)	935	957.3		220
2012	GGIII (Phase 2)	935	410.3		2734
?	Hallele Worabessa	422	474		1907
?	Chemoga Yeda	280	391.2		954
?	Karadobi	1600	2467	67%	9390.72
?	Mandaya	2000	2471.7	69%	12088.8
?	Border	1200	1480.6	57%	5991.84

EEPCO arranged the year online of GGII, Tekeze, Beles, Amerti Neshe, GGIII (Phase 1) and GGIII (Phase 2) which are already underway to be constructed by taking into account the stages of development of the government. Yayu Coal is left out because of environmental problems and no finance is getting ready for it.

The economic studies proved that the exportation of hydro generation from Ethiopia to Sudan and Egypt was profitable for the three countries. In Ethiopia, Mandaya, Karadobi and Border, whose proposed installed capacities were respectively 2 000 MW, 1 600 MW and 1 200 MW were mainly devoted to power exchanges and provided sufficient power for these exchanges. Chemoga Yeda and Hallele Worabessa with proposed installed capacities of 280 MW and 422 MW are also to be constructed along with these dams.

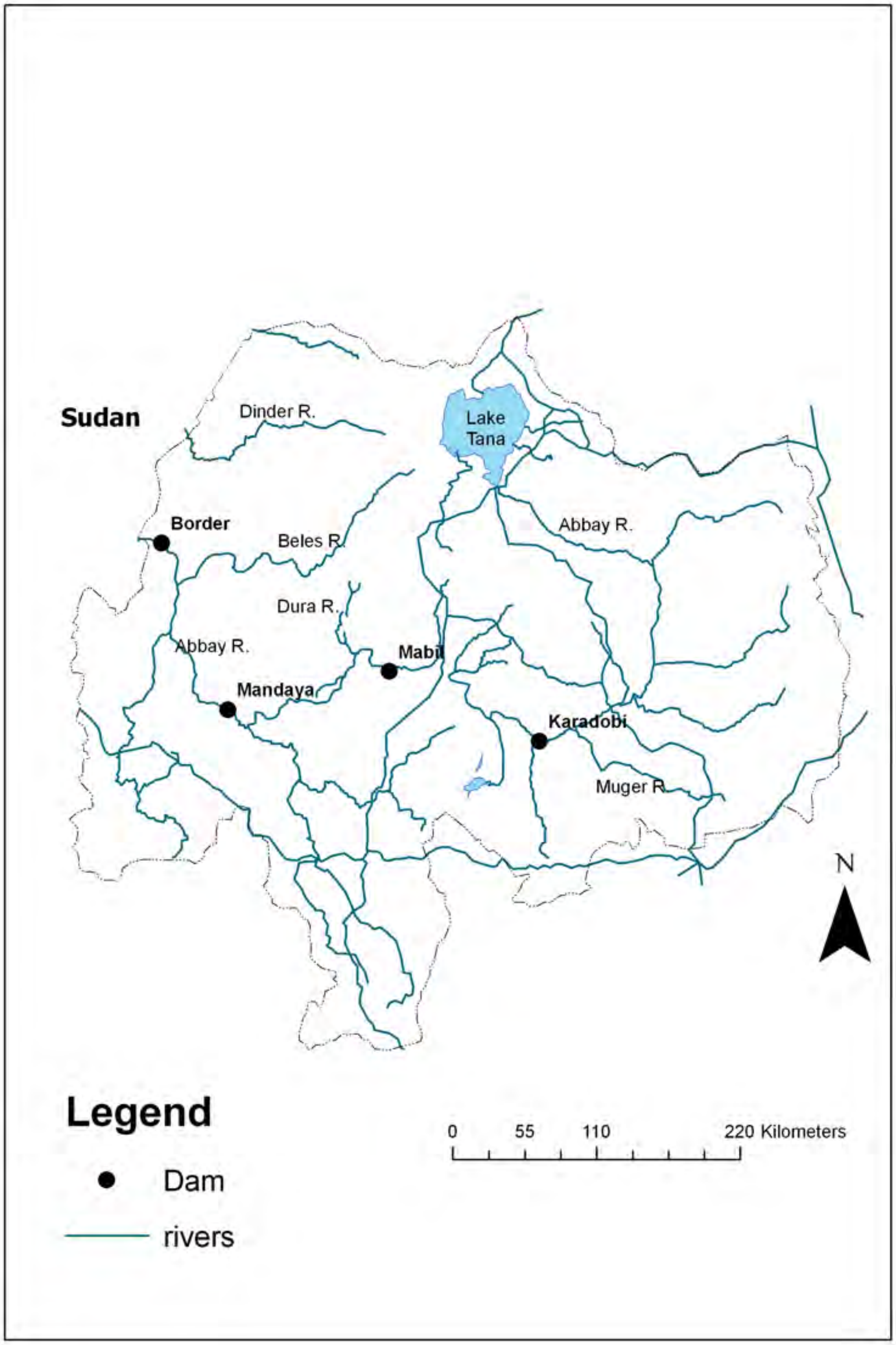


Figure 4.1 Location of dams to be constructed on the Nile Basin

4.2 Discounted cash flow

The construction cost of each of the hydropower plants is in MUSD and is shown on table 4.1. The costs are obtained from feasibility studies by EEPCO, power system planning unit. The number of sequences for the five hydropower plants is 5! = 120. There are thirteen sequences all together because the others are not included in the feasible policy space. The discounted cost for the different sequences is calculated in the following tables.

Table 4.2 Discounted cash flow for the different sequences

1	Year	Additions	F	$P=F/(1+i)^n$		2	Year	Additions	F	$P=F/(1+i)^n$
A	2009	Chemoga Yeda	391.2	355.6363636		A	2009	Chemoga Yeda	391.2	355.636364
B	2013	Hallele Worabessa	474	323.7483778		B	2013	Hallele Worabessa	474	323.748378
C	2017	Border	1481	690.7108271		C	2017	Border	1480.6	690.710827
D	2021	Karadobi	2467	786.0622273		E	2021	Mandaya	2471.7	787.559792
E	2025	Mandaya	2472	537.9139349		D	2025	Karadobi	2467	536.891078
3						4				
A	2009	Chemoga Yeda	391.2	355.6363636		A	2009	Chemoga Yeda	391.2	355.636364
B	2013	Hallele Worabessa	474	323.7483778		C	2013	Border	1480.6	1011.26972
D	2017	Karadobi	2467	1150.873707		B	2017	Hallele Worabessa	474	221.124498
C	2021	Border	1481	471.7647887		D	2021	Karadobi	2467	786.062227
E	2025	Mandaya	2472	537.9139349		E	2025	Mandaya	2471.7	537.913935
5						6				
A	2009	Chemoga Yeda	391.2	355.6363636		C	2009	Border	1480.6	1346
B	2013	Hallele Worabessa	474	323.7483778		B	2013	Hallele Worabessa	474	323.748378
E	2017	Mandaya	2472	1153.066292		A	2017	Chemoga Yeda	391.2	182.497687
C	2021	Border	1481	471.7647887		D	2021	Karadobi	2467	786.062227
D	2025	Karadobi	2467	536.891078		E	2025	Mandaya	2471.7	537.913935
7						8				
A	2009	Chemoga Yeda	391.2	355.6363636		E	2009	Mandaya	2471.7	2247
C	2013	Border	1481	1011.269722		D	2013	Karadobi	2467	1684.99419
E	2017	Mandaya	2472	1153.066292		C	2017	Border	1480.6	690.710827
B	2021	Hallele Worabessa	474	151.0310076		B	2021	Hallele Worabessa	474	151.031008
D	2025	Karadobi	2467	536.891078		A	2025	Chemoga Yeda	391.2	85.1365179

9	Year	Additions	F	$P=F/(1+i)^n$		10	Year	Additions	F	$P=F/(1+i)^n$
A	2009	Chemoga Yeda	391.2	355.6363636		A	2009	Chemoga Yeda	391.2	355.636364
D	2013	Karadobi	2467	1684.994194		E	2013	Mandaya	2471.7	1688.20436
B	2017	Hallele Worabessa	474	221.1244982		B	2017	Hallele Worabessa	474	221.124498
C	2021	Border	1481	471.7647887		C	2021	Border	1480.6	471.764789
E	2025	Mandaya	2472	537.9139349		D	2025	Karadobi	2467	536.891078
11						12				
A	2009	Chemoga Yeda	391.2	355.6363636		B	2009	Hallele Worabessa	474	430.909091
D	2013	Karadobi	2467	1684.994194		A	2013	Chemoga Yeda	391.2	267.194864
E	2017	Mandaya	2472	1153.066292		C	2017	Border	1480.6	690.710827
B	2021	Hallele Worabessa	474	151.0310076		D	2021	Karadobi	2467	786.062227
C	2025	Border	1481	322.2216985		E	2025	Mandaya	2471.7	537.913935
13										
D	2009	Karadobi	2467	2242.727273						
C	2013	Border	1481	1011.269722						
B	2017	Hallele Worabessa	474	221.1244982						
A	2021	Chemoga Yeda	391.2	124.6483759						
E	2025	Mandaya	2472	537.9139349						

4.3 Demand Analysis

In this section the 13 different sequences are checked so that the demand given by Ethiopian Electricity and Power Corporation (EEPCO) is met.

Table 4.3 Ethiopian power demand

Year EFY	Total Sales (MWh)	Load Factor
2005	2,028,938	55.6%
2006	2,541,669	57.0%
2007	3,043,711	57.0%
2008	3,705,601	57.0%
2009	4,468,195	57.0%
2010	5,359,181	57.0%
2011	6,223,192	57.0%
2012	7,188,274	57.0%
2013	8,264,363	57.0%
2014	9,463,101	57.0%
2015	10,798,116	57.0%
2016	12,285,279	57.0%
2017	13,882,222	57.0%
2018	15,596,034	57.0%
2019	17,474,282	57.0%
2020	19,534,643	57.0%
2021	21,797,172	57.0%
2022	24,338,380	57.0%
2023	27,196,815	57.0%
2024	30,416,898	57.0%
2025	34,049,863	57.0%
2026	38,154,848	57.0%
2027	42,800,238	57.0%
2028	48,065,142	57.0%
2029	54,041,218	57.0%
2030	60,834,735	57.0%

Demand analysis is done assuming a time interval of three, four and five years for the construction period. The stage of the optimization is referred to as the construction period. Starting from the year 2009 we have three different time periods with different intervals and the corresponding demand as shown in the following tables.

Table 4.4 Demand for construction periods of 3, 4 and 5 years

n=3		**n=4**		**n=5**	
Year	Demand, GWh	Year	Demand, GWh	Year	Demand, GWh
2009	4468	2009	4468	2009	4468
2012	7188	2013	8264	2014	9463
2015	10798	2017	13882	2019	17474
2018	15596	2021	21797	2024	30416
2021	21797	2025	34049	2029	54041

The firm energy for n=4 for one sequence is given below:

Table 4.5 Firm energy for sequence 1, n=4

1	Year	Dams	Installed Capacity, MW	Firm Energy (GWh)
A	2009	Chemoga Yeda	280	6504
B	2013	Hallele Worabessa	422	13053
E	2017	Border	1200	19044.84
C	2021	Karadobi	1600	28435.56
D	2025	Mandaya	2000	40524.36

Comparing firm energy of table 4.5 (the sequence starting with the minimum capacity and in increasing order) and demand of table 4.4 for n=4, all sequences satisfy the demand.

Table 4.6 Firm energy for sequence 1, n=3

1	Year	Dams	Installed Capacity, MW	Firm Energy (GWh)
A	2009	Chemoga Yeda	280	6504
B	2012	Hallele Worabessa	422	13053
E	2015	Border	1200	19044.84
C	2018	Karadobi	1600	28435.56
D	2021	Mandaya	2000	40524.36

Comparing firm energy of table 4.6 (the sequence starting with the minimum capacity and in increasing order) and demand of Table 4.4 for n=3, all sequences satisfy the demand.

Table 4.7 Firm energy for sequence 1, n=5

1	Year	Dams	Installed Capacity, MW	Firm Energy (GWh)
A	2009	Chemoga Yeda	280	6504
B	2014	Hallele Worabessa	422	13053
E	2019	Border	1200	19044.84
C	2024	Karadobi	1600	28435.56
D	2029	Mandaya	2000	40524.36

Comparing firm energy of table 4.7 (the sequence starting with the minimum capacity and increasing order) and demand of table 4.4 for n=5, the demand at 2029 (=54041 GWh) is greater than the firm energy at 2029 (=40524.36 GWh), meaning that it does not satisfy the demand. Since the sequence starting with the minimum capacity and increasing order doesn't satisfy the demand, the rest won't. Therefore all the sequences don't satisfy the demand.

Since the demand is satisfied when n=3 and n=4 only, these two values are used in our analysis.

5 Dynamic Programming

5.1 Introduction

Water resource decision makers often face problems related to investments on expansion of existing capacities of either an entire system or individual components of a system. The decisions deal with investments at different times, starting with the present time, on capacity expansion needed over the years in future. A typical problem is to decide in what steps the expansion over the next n years should be carried out so that the present worth of investment is a minimum.

If one considers an electric utility planning to increase the capacity of its infrastructure (ex: generating capacity of hydropower plants) in future. The increments are to be made sequentially in specified time intervals. Let the capacity at the beginning of time period *t* be S_t (existing capacity) and the required capacity at the end of that time period be K_t. Let x_t be the added capacity in each time period. The cost of expansion at each time period can be expressed as a function of S_t and x_t, i.e. $C_t(S_t, x_t)$. The problem is to plan the time sequence of capacity expansions which minimizes the present value of the total future costs subjected to meet the capacity demand requirements at each time period. Hence, the objective function of the optimization model can be written as,

$$\text{Minimize } \sum_{t=1}^{T} C_t(S_t, x_t)$$

Where $C_t(S_t, x_t)$ the present value of the cost of adding an additional capacity x_t in the time period *t* with an initial capacity S_t. Each period's final capacity or next period's initial capacity should be equal to the sum of initial capacity and the added capacity. Also at the end of each time period, the required capacity is fixed. Thus, for a time period *t*, the constraints can be expressed as

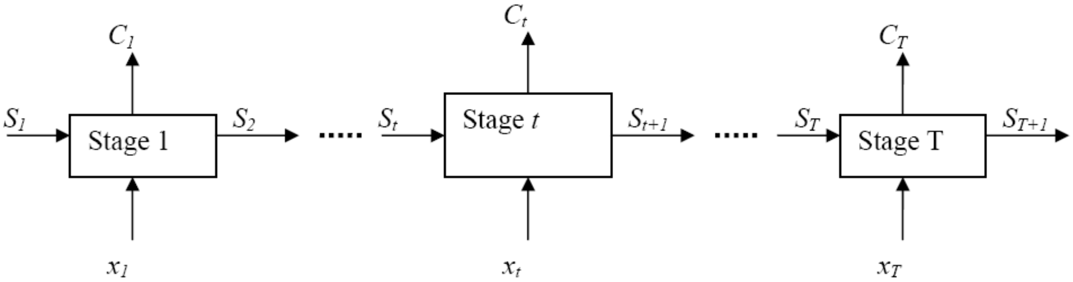
$S_{t+1} = S_t + x_t$ for t=1, 2, …. T

$S_{t+1} \geq K_t$ for t=1, 2, …T

The capacity expansion problem defined above can be solved in a sequential manner using dynamic programming. The solution procedure using forward recursion and backward recursion are explained below.

5.1.1 Forward Recursion

Consider the stages of the model to be the time periods in which capacity expansion is to be made and the state to be the capacity at the end of each time period *t*, S_{t+1}. Let S_1 be the present capacity before expansion and $f_t(S_{t+1})$ be the minimum present value of total cost of capacity expansion from present to the time *t*.



Considering the first stage, the objective function can be written as,

$$f_1(S_2) = \min\ C_1(S_1, x_1)$$

$$f_2(S_3) = \min\left[C_2(S_2, x_2) + f_1(S_2)\right] = \min\left[C_2(S_3 - x_2, x_2) + f_1(S_3 - x_2)\right]$$

Hence, in general for a time period *t*, the optimization function can be represented as

$$f_t(S_{t+1}) = \min\left[C_t(S_{t+1} - x_t, x_t) + f_{t-1}(S_{t+1} - x_t)\right]$$

5.1.2 Backward Recursion

The expansion problem can also be solved using a backward recursion approach with some modifications. Consider the state S_t be the capacity at the beginning of each time period *t*. Let $f_T(S_T)$ be the minimum present value of total cost of capacity expansion in periods *t* through *T*.

For the last period *T*, the final capacity should reach K_T after doing the capacity expansions. Thus, the objective function can be written as,

$$f_T(S_T) = \min\left[C_T(S_T - x_T)\right]$$

This is solved for all values S_T ranging from K_{T-1} to K_T.
In general, for a time period *t*, the function $f_t(S_t)$ can be expressed as

$$f_t(S_{t+1}) = \min\left[C_t(S_t, x_t) + f_{t+1}(S_t + x_t)\right]$$

Which should be solved for all discrete values of S_T ranging from K_{T-1} to K_T.

5.2 Hydropower capacity expansion problem

In our problem we have a five stage capacity expansion problem. The capacity to be added at the end of each time period is given below.

$$Firm\_Energy = Power * Time * PlantFactor$$

t=1, K_t
P1=726(existing)+280(Chemoga Yeda)+300(Tekeze)+420(Gilgel Gibe II)=**1726MW**

t=2, K_t
P2=1726(P1)+460(Beles)+97(Amerti Neshe)+935(GGIII Phase 1)+935(GGIII Phase 2)+422(Hallele Worabessa)=**4575MW**

t=3, K_t

P3=4575(P2)+1200(Border)=**5775MW**

t=4, K_t

P4=5775(P3)+1600(Karadobi)**=7375MW**

t=5, K_t

P5=7375(P4)+2000(Mandaya)=**9375MW**

Table 5.1 Minimum Capacity

t	K_t
1	1726
2	4575
3	5775
4	7375
5	9375

Construction period of 4 years and interest rate i=10%

The expansion costs for each combination of expansion for each stage are shown in the corresponding links in the form of a figure below.

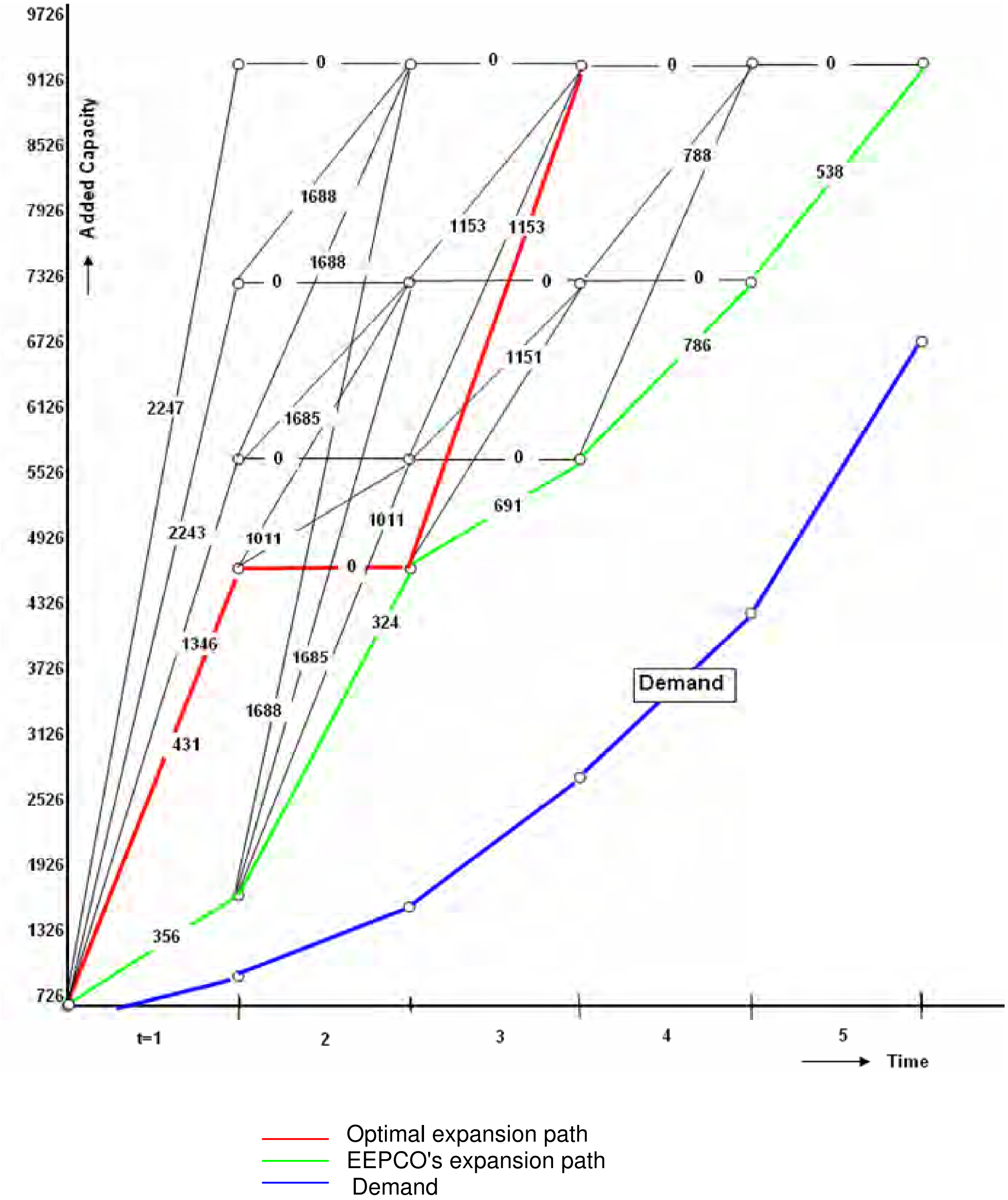


Figure 5.1 Network of discrete capacity-expansion decisions (links) that meet the projected demand for n=4 years and i=10%.

Solution Using Forward Recursion

Table 5.2 Forward recursion stage1, n=4 years and interest rate=10%

State Variables, S_2	Added Capacity, $X_1=S_2 - S_1$	$C_1(S_2)$	$f_1^*(S_2)$
1726	1000	356	356
4575	3849	431	431
5775	5049	1346	1346
7375	6649	2243	2243
9375	8649	2247	2247

The capacity at the initial stage is given as S_1=726. Considering the first and second stages together, the state variable S_3 can take values from K_2 to K_5. Thus, the objective function for the second sub problem is

$$f_2(S_3) = \min[C_2(S_2, x_2) + f_1(S_2)] = \min[C_2(S_3 - x_2, x_2) + f_1(S_3 - x_2)]$$

The value of x_2 should be taken in such a way that the minimum capacity at the end of stage 2 should be 10, i.e. S=4575.

The computations for stage 2 are given in the table below.

Table 5.3 Forward recursion stage2, n=4 years and interest rate=10%

State Variable, S_3	Added Capacity, x_2	$C_2(S_3)$	$S_2=S_3-x_2$	$f_1^*(S_2)$	$f_2^*(S_3)= C_2(S_3)+ f_1^*(S_2)$	$f_2^*(S_3)$
4575	726	0	3849	431	431	431
	1726	324	2849	356	680	
5775	726	0	5049	1346	1346	1346
	1726	1011	4049	431	1442	
	4575	1011	1200	356	1367	
7375	726	0	6649	2243	2243	2041
	1726	1685	5649	1346	3031	
	4575	1685	2800	431	2116	
	5775	1685	1600	356	2041	
9375	726	0	8649	2247	2247	2044
	1726	1688	7649	2243	3931	
	4575	1688	4800	1346	3034	
	5775	1688	3600	431	2119	
	7375	1688	2000	356	2044	

The computations for stages 3 to 5 are shown in tables below.

Table 5.4 Forward recursion stage3, n=4 years and interest rate=10%

State Variable, S_4	Added Capacity, x_3	$C_3(S_4)$	$S_3=S_4-x_3$	$f_2^*(S_3)$	$f_3^*(S_4)= C_3(S_4)+ f_2^*(S_3)$	$f_3^*(S_4)$
5775	726	0	5049	1346	1346	1122
	1726	691	4049	431	1122	
7375	726	0	6649	2041	2041	1582
	1726	1151	5649	1346	2497	
	4575	1151	2800	431	1582	
9375	726	0	8649	2044	2044	1584
	1726	1153	7649	2041	3194	
	4575	1153	4800	1346	2499	
	5775	1153	3600	431	1584	

Table 5.5 Forward recursion stage4, n=4 years and interest rate=10%

State Variable, S_5	Added Capacity, x_4	$C_4(S_5)$	$S_4=S_5-x_4$	$f_3^*(S_4)$	$f_4^*(S_5)= C_4(S_5)+ f_3^*(S_4)$	$f_4^*(S_5)$
7375	726	0	6649	1582	1582	1582
	1726	786	5649	1122	1908	
9375	726	0	8649	1584	1584	1584
	1726	788	7649	1582	2370	
	4575	788	4800	1122	1910	

Table 5.6 Forward recursion stage 5, n=4 years and interest rate=10%

State Variable, S_6	Added Capacity, x_5	$C_5(S_6)$	$S_5=S_6-x_5$	$f_4^*(S_5)$	$f_5^*(S_6)= C_5(S_6)+ f_4^*(S_5)$	$f_5^*(S_6)$
9375	726	0	8649	1584	1584	1584
	1726	538	7649	1582	2120	

The figure below shows the solutions with the cost of each addition along the links and the minimum total cost at each node.

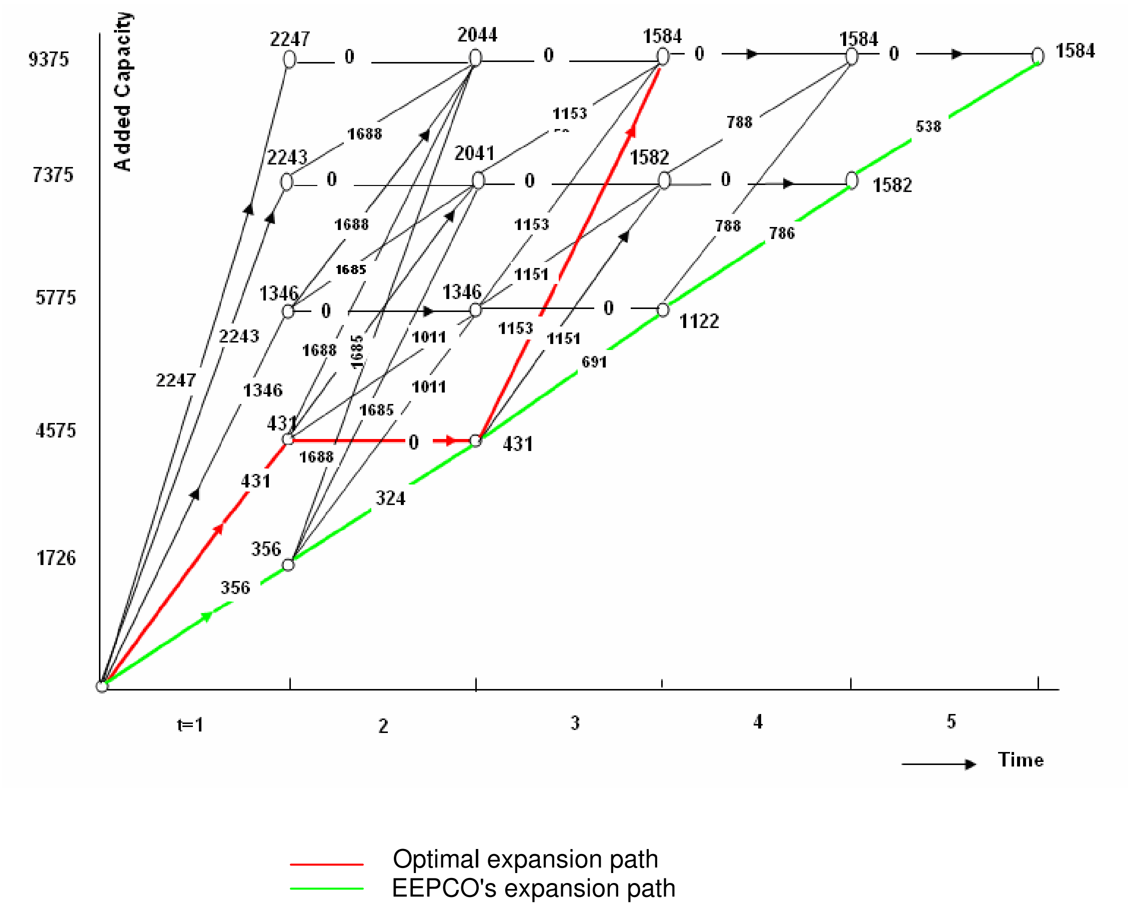


Figure 5.2 A capacity expansion, showing the results of a forward-moving dynamic programming algorithm for n=4 years and i=10%.

From the figure, the optimal cost of expansion is 1584 MUSD. By doing backtracking from the last stage (farthest right node) to the initial stage, the optimal expansion to be done at first stage = 3849MW, third stage = 4800MW and the rest of the stages = 0 MW.

Solution Using Backward Recursion

The capacity at the final stage is given as S_6 = 6819. Consider the last stage, t =5. The initial capacity for stage 5, S_5 can take values between K_4 to K_5. The objective function for first sub problem with state variable as S_5 can be expressed as

$$f_5(S_5) = \min\left[f_5(S_5, x_5)\right]$$

Table 5.7 Backward recursion stage5, n=4 years and interest rate=10%

State Variable, S_5	Added Capacity, x_5	$C_5(S_5)$	$f_5^*(S_5)$
7375	1726	538	538
9375	726	0	0

Following the same procedure for all the remaining stages, the optimal cost of expansion is achieved.

Table 5.8 Backward recursion stage4, n=4 years and interest rate=10%

State Variable, S_4	Added Capacity, x_4	$C_4(S_4)$	$S_5=S_4+x_4$	$f_3^*(S_5)$	$f_4^*(S_4)= C_4(S_4)+ f_5^*(S_5)$	$f_4^*(S_4)$
5775	726	786	6501	538	1324	788
	1726	788	7501	0	788	
7375	726	0	8101	538	538	538
	1726	788	9101	0	788	
9375	726	0	10,101	0	0	0

Table 5.9 Backward recursion stage3, n=4 years and interest rate=10%

State Variable, S_3	Added Capacity, x_3	$C_3(S_3)$	$S_4=S_3+x_3$	$f_4^*(S_4)$	$f_3^*(S_3)= C_3(S_3)+ f_4^*(S_4)$	$f_3^*(S_3)$
4575	726	691	5301	788	1479	1153
	1726	1151	6301	538	1689	
	4575	1153	9150	0	1153	
5775	726	0	6501	788	788	1584
	1726	1151	7501	538	1689	
	4575	1153	10,350	0	1153	
7375	726	0	8101	538	538	538
	1726	1153	9101	0	1153	
9375	726	0	10,101	0	0	0

Table 5.10 Backward recursion stage2, n=4 years and interest rate=10%

State Variable, S_2	Added Capacity, x_2	$C_2(S_2)$	$S_3=S_2+x_2$	$f_1^*(S_3)$	$f_2^*(S_2)= C_2(S_2)+ f_3^*(S_3)$	$f_2^*(S_2)$
1726	1726	324	3452	1153	1477	1477
	4575	1011	6301	788	1799	
	5775	1685	7501	538	2223	
	7375	1688	9101	0	1688	
4575	726	0	5301	1153	1153	1153
	1726	1011	6301	788	1799	
	4575	1685	9150	538	2223	
	5775	1688	10,350	0	1688	
5775	726	0	6501	788	788	788
	1726	1685	7501	538	2223	
	4575	1688	10,350	0	1688	
7375	726	0	8101	538	538	538
	1726	1688	9101	0	1688	
9375	726	0	10,101	0	0	0

Table 5.11 Backward recursion stage1, n=4 years and interest rate=10%

State Variable, S_1	Added Capacity, x_1	$C_1(S_1)$	$S_2=S_1+x_1$	$f_2^*(S_2)$	$f_1^*(S_1)= C_1(S_1)+ f_2^*(S_2)$	$f_1^*(S_2)$
726	1726	356	2452	1477	1833	1584
	4575	431	5301	1153	1584	
	5775	1346	6501	788	2134	
	7375	2243	8101	538	2781	
	9375	2247	10,101	0	2247	

The solution is given by the figure below with the minimum total cost of expansion at the nodes.

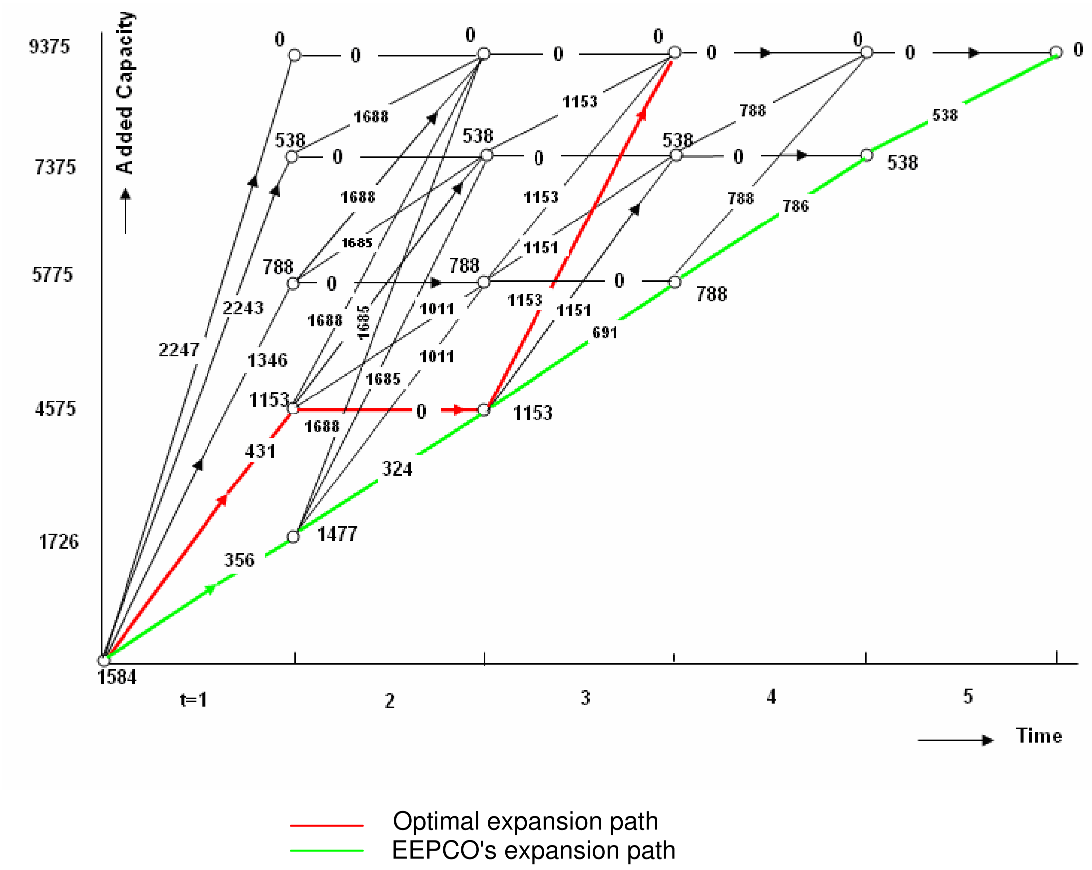


Figure 5.3 A capacity expansion, showing the results of a backward-moving dynamic programming algorithm for n=4 years and i=10%.

The optimal cost of expansion is obtained from the node value at the first node i.e. 1584 MUSD which is the same as obtained from forward recursion. The optimal expansions at each time period can be obtained by moving forward from the first node to the last node. Thus, the optimal expansions to be made are 3849MW at the first stage and 4800MW at the last stage. Hence the final requirement of 8649MW is achieved.

In this chapter the use of both forward and backward recursion are applied to yield the same result. In actual practice, use of any one of the two approaches can be applied. Both approaches are used in this thesis to cross-check results.

6 Sensitivity Analysis

6.1 Introduction

Sensitivity analysis is an exercise of obtaining the new solution corresponding to a change in the data of the original Dynamic programming problem, given the original problem and the final simplex table, without solving afresh the new problem with changed data. The starting point is the final simplex table of the original problem, followed by an investigation of the extent to which the current optimal solution will be affected because of a change in the data of the original problem. The new solution is determined by further iterations. In many cases, the number of these iterations will be far less than those required to solve the problem afresh from its new starting solution.

A change in the data of the original problem may affect optimality or feasibility, or both optimality and feasibility, of the current solution. Three cases are possible

1. Changes that affect only optimality
2. Changes that affect only feasibility
3. Changes that affect both optimality and feasibility

6.2 Sensitivity Analysis

Sensitivity analysis is done for the following cases described in the table below.

Table 6.1 Cases

No	Interest Rate, %	Year interval	Name of Combination
1	10	4	Original Problem
2	8	3	Case I
3	10	3	Case II
4	12	3	Case III
5	8	4	Case IV
6	12	4	Case V
7	8	5	Case VI
8	10	5	Case VII
9	12	5	Case VIII

Case VI, VII and VIII are left out since they **dont** meet the demand.

Case I (i=8%, n=3)

The construction period, n is three years and the interest rate, i is 8%.

Solution Using Forward Recursion

Table 6.2 Forward recursion stage1, n=3 years and interest rate=8%

State Variables, S_2	Added Capacity, $X_1 = S_2 - S_1$	$C_1(S_2)$	$f_1^*(S_2)$
1726	1000	362	362
4575	3849	439	439
5775	5049	1371	1371
7375	6649	2284	2284
9375	8649	2289	2289

Table 6.3 Forward recursion stage2, n=3 years and interest rate=8%

State Variable, S_3	Added Capacity, x_2	$C_2(S_3)$	$S_2=S_3-x_2$	$f_1^*(S_2)$	$f_2^*(S_3)= C_2(S_3)+ f_1^*(S_2)$	$f_2^*(S_3)$
4575	726	0	3849	439	439	439
	1726	376	2849	362	738	
5775	726	0	5049	1371	1371	1371
	1726	1175	4049	439	1614	
	4575	1175	1200	362	1537	
7375	726	0	6649	2284	2284	2284
	1726	1958	5649	1371	3329	
	4575	1958	2800	439	2397	
	5775	1958	1600	362	2320	
9375	726	0	8649	2289	2289	2289
	1726	1962	7649	2284	4246	
	4575	1962	4800	1371	3333	
	5775	1962	3600	439	2401	
	7375	1962	2000	362	2324	

Table 6.4 Forward recursion stage3, n=3 years and interest rate=8%

State Variable, S_4	Added Capacity, x_3	$C_3(S_4)$	$S_3=S_4-x_3$	$f_2^*(S_3)$	$f_3^*(S_4)= C_3(S_4)+ f_2^*(S_3)$	$f_3^*(S_4)$
5775	726	0	5049	1371	1371	1021
	1726	933	4049	439	1372	
7375	726	0	6649	2284	2284	1994
	1726	1555	5649	1371	2926	
	4575	1555	2800	439	1994	
9375	726	0	8649	2289	2289	1997
	1726	1558	7649	2284	3842	
	4575	1558	4800	1371	2929	
	5775	1558	3600	439	1997	

Table 6.5 Forward recursion stage4, n=3 years and interest rate=8%

State Variable, S_5	Added Capacity, x_4	$C_4(S_5)$	$S_4=S_5-x_4$	$f_3^*(S_4)$	$f_4^*(S_5)= C_4(S_5)+ f_3^*(S_4)$	$f_4^*(S_5)$
7375	726	0	6649	1994	1994	1994
	1726	1234	5649	1371	2605	
9375	726	0	8649	1997	1997	1997
	1726	1236	7649	1994	3230	
	4575	1236	4800	1371	2607	

Table 6.6 Forward recursion stage5, n=3 years and interest rate=8%

State Variable, S_6	Added Capacity, x_5	$C_5(S_6)$	$S_5=S_6-x_5$	$f_4^*(S_5)$	$f_5^*(S_6)= C_5(S_6)+ f_4^*(S_5)$	$f_5^*(S_6)$
9375	726	0	8649	1997	1997	1997
	1726	982	7649	1994	2976	

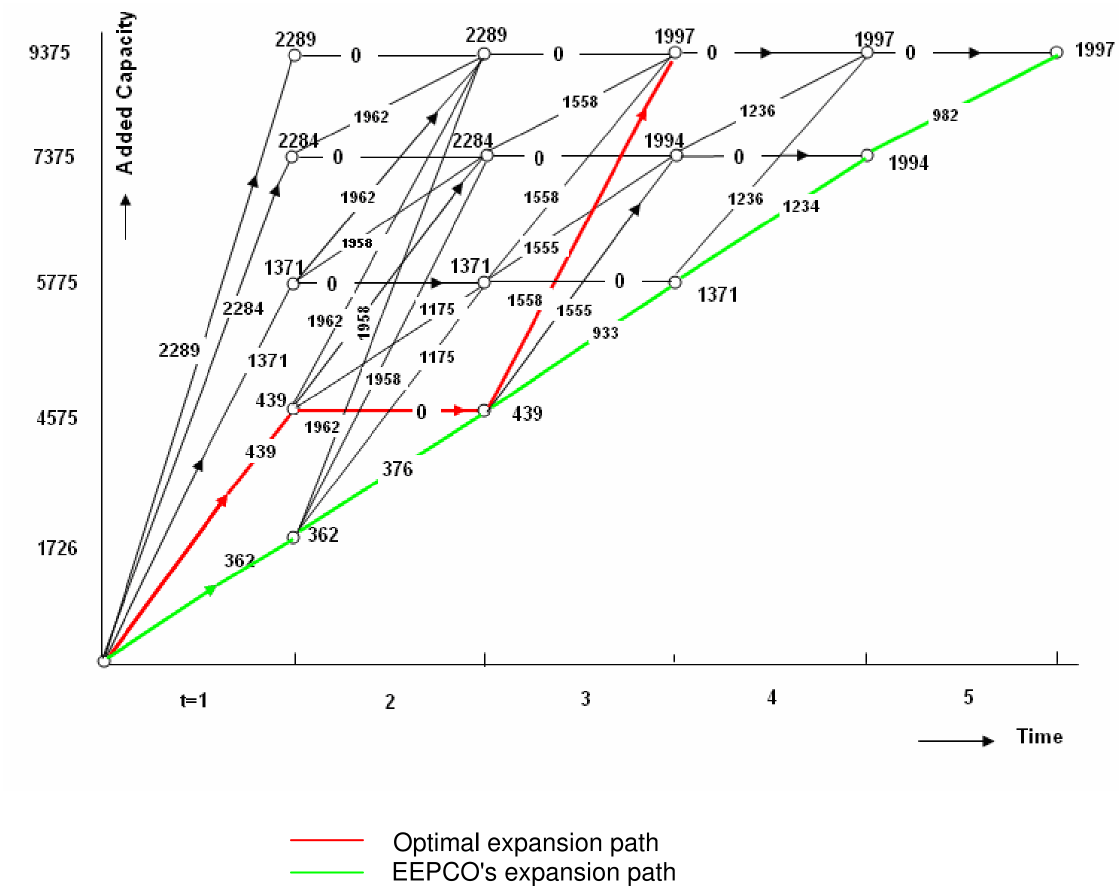


Figure 6.1 A capacity expansion, showing the results of a forward-moving dynamic programming algorithm for n=3 years and i=8%.

Solution Using Backward Recursion

Table 6.7 Backward recursion stage5, n=3 years and interest rate=8%

State Variable, S_5	Added Capacity, x_5	$C_5(S_5)$	$f_5^*(S_5)$
7375	1726	982	982
9375	726	0	0

Table 6.8 Backward recursion stage4, n=3 years and interest rate=8%

State Variable, S_4	Added Capacity, x_4	$C_4(S_4)$	$S_5=S_4+x_4$	$f_3^*(S_5)$	$f_4^*(S_4)= C_4(S_4)+ f_5^*(S_5)$	$f_4^*(S_4)$
5775	726	1234	6501	982	2216	1236
	1726	1236	7501	0	1236	
7375	726	0	8101	982	982	982
	1726	1236	9101	0	1236	
9375	726	0	10,101	0	0	0

Table 6.9 Backward recursion stage3, n=3 years and interest rate=8%

State Variable, S_3	Added Capacity, x_3	$C_3(S_3)$	$S_4=S_3+x_3$	$f_4^*(S_4)$	$f_3^*(S_3)= C_3(S_3)+ f_4^*(S_4)$	$f_3^*(S_3)$
4575	726	933	5301	1236	2169	1558
	1726	1555	6301	982	2537	
	4575	1558	9150	0	1558	
5775	726	0	6501	1236	1236	1236
	1726	1555	7501	982	2537	
	4575	1558	10,350	0	1558	
7375	726	0	8101	982	982	982
	1726	1558	9101	0	1558	
9375	726	0	10,101	0	0	0

Table 6.10 Backward recursion stage2, n=3 years and interest rate=8%

State Variable, S_2	Added Capacity, x_2	$C_2(S_2)$	$S_3=S_2+x_2$	$f_1^*(S_3)$	$f_2^*(S_2)= C_2(S_2)+ f_3^*(S_3)$	$f_2^*(S_2)$
1726	1726	376	3452	1558	1934	1934
	4575	1175	6301	1236	2411	
	5775	1958	7501	982	2940	
	7375	1962	9101	0	1962	
4575	726	0	5301	1558	1558	1558
	1726	1175	6301	1236	2411	
	4575	1958	9150	982	2940	
	5775	1962	10,350	0	1962	
5775	726	0	6501	1236	1236	1236
	1726	1958	7501	982	2940	
	4575	1962	10,350	0	1962	
7375	726	0	8101	982	982	982
	1726	1962	9101	0	1962	
9375	726	0	10,101	0	0	0

Table 6.11 Backward recursion stage1, n=3 years and interest rate=8%

State Variable, S_1	Added Capacity, x_1	$C_1(S_1)$	$S_2=S_1+x_1$	$f_2^*(S_2)$	$f_1^*(S_1)= C_1(S_1)+ f_2^*(S_2)$	$f_1^*(S_2)$
726	1726	362	2452	1934	2296	1997
	4575	439	5301	1558	1997	
	5775	1371	6501	1236	2607	
	7375	2284	8101	982	3266	
	9375	2289	10,101	0	2289	

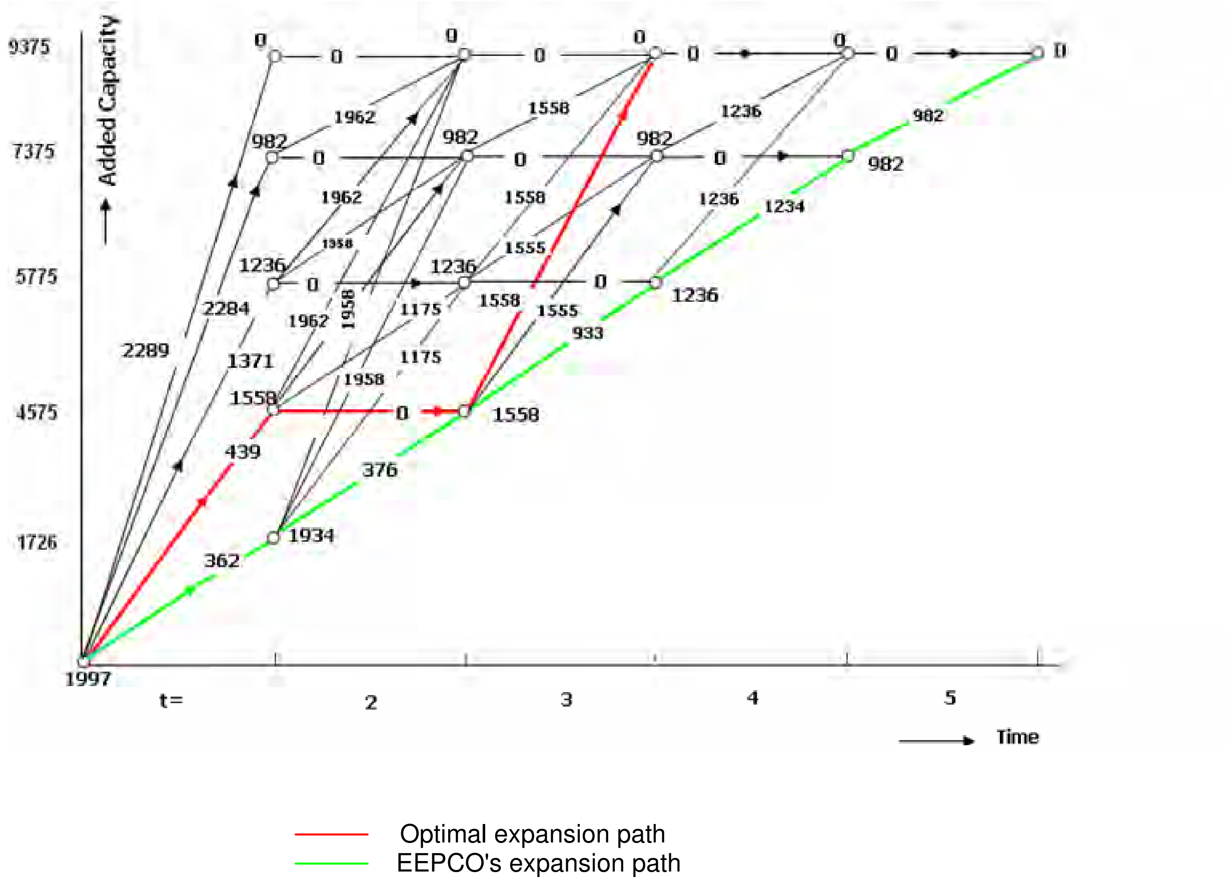


Figure 6.2 A capacity expansion, showing the results of a backward-moving dynamic programming algorithm for n=3 years and i=8%.

Case II (i=10%, n=3)

The construction period, n is three years and the interest rate, i is 10%.

Solution Using Forward Recursion

Table 6.12 Forward recursion stage1, n=3 years and interest rate=10%

State Variables, S_2	Added Capacity, $X_1=S_2 - S_1$	$C_1(S_2)$	$f_1^*(S_2)$
1726	1000	356	356
4575	3849	431	431
5775	5049	1346	1346
7375	6649	2243	2243
9375	8649	2247	2247

Table 6.13 Forward recursion stage2, n=3 years and interest rate=10%

State Variable, S_3	Added Capacity, x_2	$C_2(S_3)$	$S_2=S_3-x_2$	$f_1^*(S_2)$	$f_2^*(S_3)= C_2(S_3)+ f_1^*(S_2)$	$f_2^*(S_3)$
4575	726	0	3849	431	431	431
	1726	356	2849	356	712	
5775	726	0	5049	1346	1346	1346
	1726	1112	4049	431	1543	
	4575	1112	1200	356	1468	
7375	726	0	6649	2243	2243	2210
	1726	1854	5649	1346	3200	
	4575	1854	2800	431	2285	
	5775	1854	1600	356	2210	
9375	726	0	8649	2247	2247	2213
	1726	1857	7649	2243	4100	
	4575	1857	4800	1346	3203	
	5775	1857	3600	431	2288	
	7375	1857	2000	356	2213	

Table 6.14 Forward recursion stage3, n=3 years and interest rate=10%

State Variable, S_4	Added Capacity, x_3	$C_3(S_4)$	$S_3=S_4-x_3$	$f_2^*(S_3)$	$f_3^*(S_4)= C_3(S_4)+ f_2^*(S_3)$	$f_3^*(S_4)$
5775	726	0	5049	1346	1346	1267
	1726	836	4049	431	1267	
7375	726	0	6649	2210	2210	1824
	1726	1393	5649	1346	2739	
	4575	1393	2800	431	1824	
9375	726	0	8649	2213	2213	1826
	1726	1395	7649	2210	3605	
	4575	1395	4800	1346	2741	
	5775	1395	3600	431	1826	

Table 6.15 Forward recursion stage4, n=3 years and interest rate=10%

State Variable, S_5	Added Capacity, x_4	$C_4(S_5)$	$S_4=S_5-x_4$	$f_3^*(S_4)$	$f_4^*(S_5)= C_4(S_5)+ f_3^*(S_4)$	$f_4^*(S_5)$
7375	726	0	6649	1824	1824	1824
	1726	1046	5649	1267	2313	
9375	726	0	8649	1826	1826	1826
	1726	1048	7649	1824	2872	
	4575	1048	4800	1267	2315	

Table 6.16 Forward recursion stage5, n=3 years and interest rate=10%

State Variable, S_6	Added Capacity, x_5	$C_5(S_6)$	$S_5=S_6-x_5$	$f_4^*(S_5)$	$f_5^*(S_6)= C_5(S_6)+ f_4^*(S_5)$	$f_5^*(S_6)$
9375	726	0	8649	1826	1826	1826
	1726	788	7649	1824	2612	

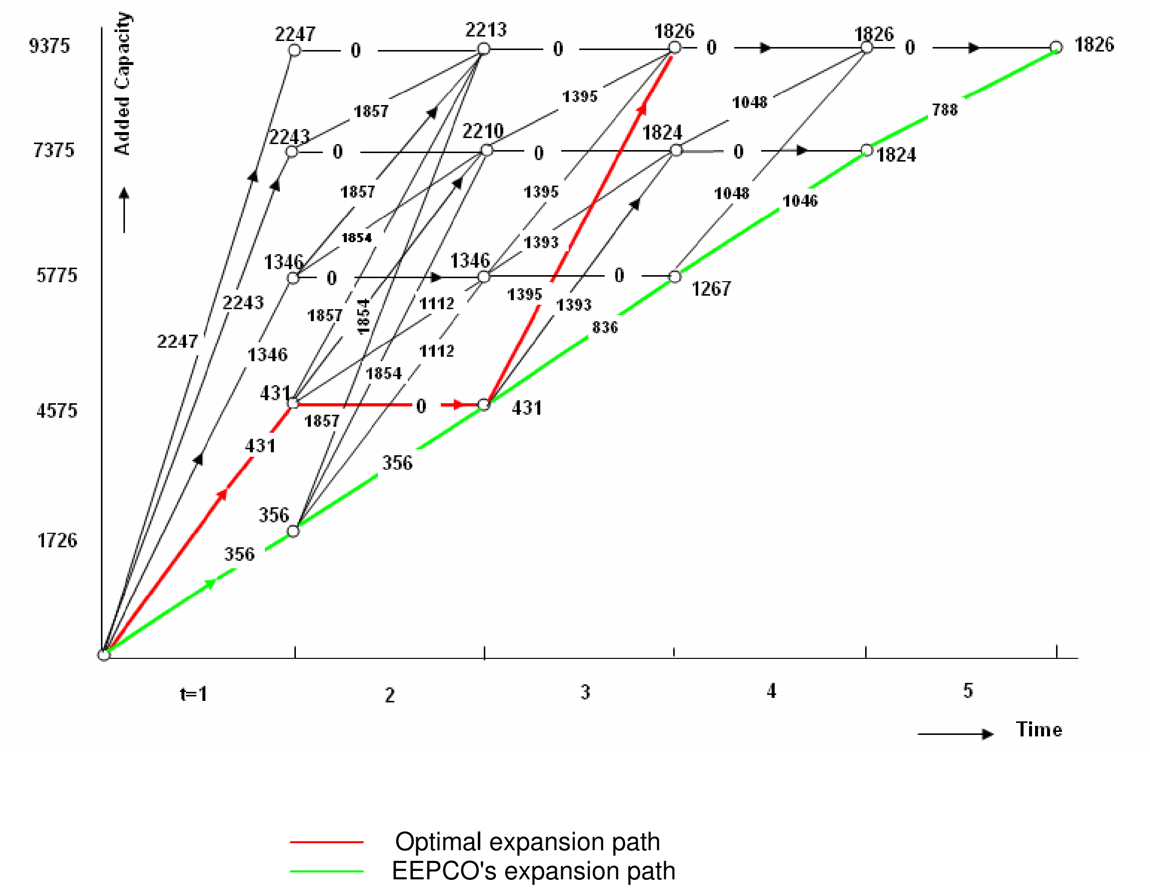


Figure 6.3 A capacity expansion, showing the results of a forward-moving dynamic programming algorithm for n=3 years and i=10%.

Solution Using Backward Recursion

Table 6.17 Backward recursion stage5, n=3 years and interest rate=10%

State Variable, S_5	Added Capacity, x_5	$C_5(S_5)$	$f_5^*(S_5)$
7375	1726	788	788
9375	726	0	0

Table 6.18 Backward recursion stage4, n=3 years and interest rate=10%

State Variable, S_4	Added Capacity, x_4	$C_4(S_4)$	$S_5=S_4+x_4$	$f_3^*(S_5)$	$f_4^*(S_4)= C_4(S_4)+ f_5^*(S_5)$	$f_4^*(S_4)$
5775	726	1046	6501	788	1834	1048
	1726	1048	7501	0	1048	
7375	726	0	8101	788	788	788
	1726	1048	9101	0	1048	
9375	726	0	10,101	0	0	0

Table 6.19 Backward recursion stage3, n=3 years and interest rate=10%

State Variable, S_3	Added Capacity, x_3	$C_3(S_3)$	$S_4=S_3+x_3$	$f_4^*(S_4)$	$f_3^*(S_3)= C_3(S_3)+ f_4^*(S_4)$	$f_3^*(S_3)$
4575	726	836	5301	1048	2932	1395
	1726	1393	6301	788	2181	
	4575	1395	9150	0	1395	
5775	726	0	6501	1048	1048	1048
	1726	1393	7501	788	2181	
	4575	1395	10,350	0	1395	
7375	726	0	8101	788	788	788
	1726	1395	9101	0	1395	
9375	726	0	10,101	0	0	0

Table 6.20 Backward recursion stage2, n=3 years and interest rate=10%

State Variable, S_2	Added Capacity, x_2	$C_2(S_2)$	$S_3=S_2+x_2$	$f_1^*(S_3)$	$f_2^*(S_2)= C_2(S_2)+ f_3^*(S_3)$	$f_2^*(S_2)$
1726	1726	356	3452	1395	1751	1751
	4575	1112	6301	1048	2160	
	5775	1854	7501	788	2642	
	7375	1857	9101	0	1857	
4575	726	0	5301	1395	1395	1395
	1726	1112	6301	1048	2160	
	4575	1854	9150	788	2642	
	5775	1857	10,350	0	1857	
5775	726	0	6501	1048	1048	1048
	1726	1854		788	2642	
	4575	1857		0	1857	
7375	726	0		788	788	788
	1726	1857		0	1857	
9375	726	0		0	0	0

Table 6.21 Backward recursion stage1, n=3 years and interest rate=10%

State Variable, S_1	Added Capacity, x_1	$C_1(S_1)$	$S_2=S_1+x_1$	$f_2^*(S_2)$	$f_1^*(S_1)= C_1(S_1)+ f_2^*(S_2)$	$f_1^*(S_2)$
726	1726	356	2452	1751	2107	1826
	4575	431	5301	1395	1826	
	5775	1346	6501	1048	2394	
	7375	2243	8101	788	3031	
	9375	2247	10,101	0	2247	

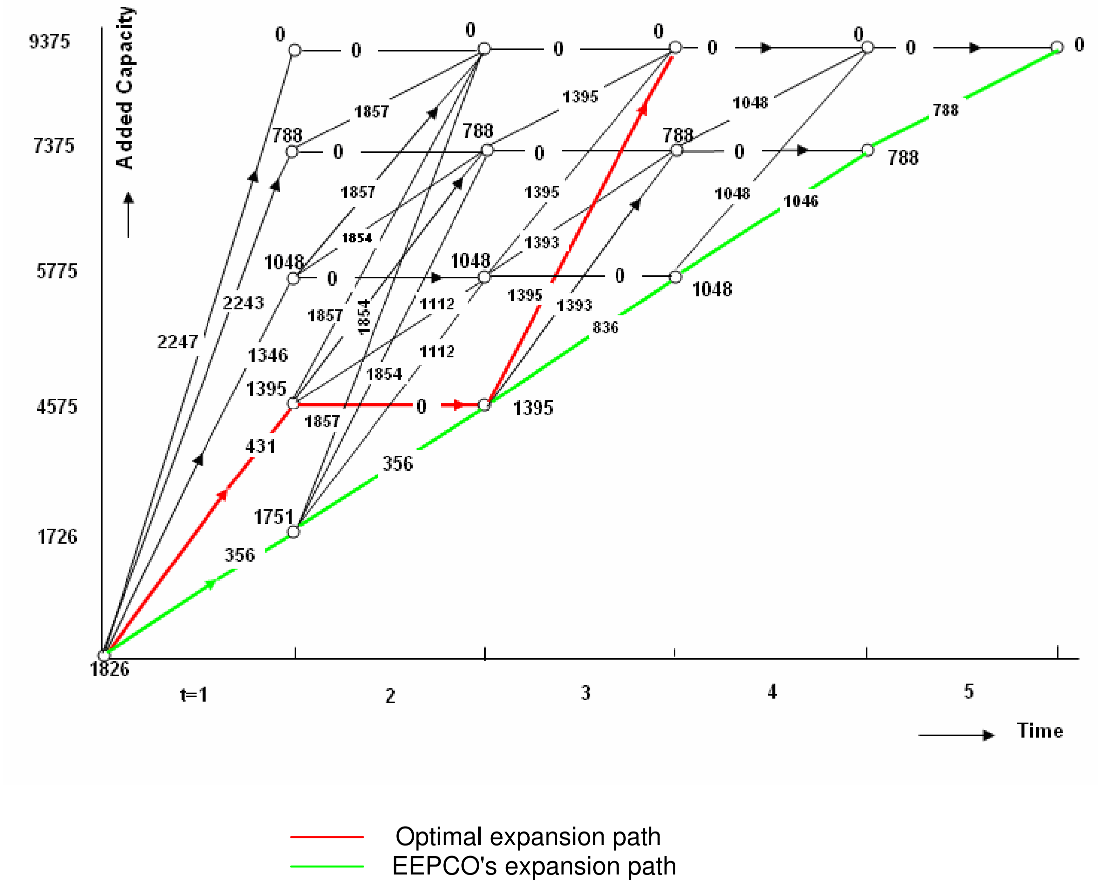


Figure 6.4 A capacity expansion, showing the results of a backward-moving dynamic programming algorithm for n=3 years and i=10%.

Case III (i=12%, n=3)

The construction period, n is three years and the interest rate, i is 12%.

Solution Using Forward Recursion

Table 6.22 Forward recursion stage1, n=3 years and interest rate=12%

State Variables, S_2	Added Capacity, $X_1=S_2 - S_1$	$C_1(S_2)$	$f_1^*(S_2)$
1726	1000	349	349
4575	3849	423	423
5775	5049	1321	1321
7375	6649	2203	2203
9375	8649	2207	2207

Table 6.23 Forward recursion stage2, n=3 years and interest rate=12%

State Variable, S_3	Added Capacity, x_2	$C_2(S_3)$	$S_2=S_3-x_2$	$f_1^*(S_2)$	$f_2^*(S_3)= C_2(S_3)+ f_1^*(S_2)$	$f_2^*(S_3)$
4575	726	0	3849	423	423	423
	1726	337	2849	349	686	
5775	726	0	5049	1321	1321	1321
	1726	1054	4049	423	1477	
	4575	1054	1200	349	1403	
7375	726	0	6649	2203	2203	2105
	1726	1756	5649	1321	3077	
	4575	1756	2800	423	2179	
	5775	1756	1600	349	2105	
9375	726	0	8649	2207	2207	2108
	1726	1759	7649	2203	3962	
	4575	1759	4800	1321	3080	
	5775	1759	3600	423	2182	
	7375	1759	2000	349	2108	

Table 6.24 Forward recursion stage3, n=3 years and interest rate=12%

State Variable, S_4	Added Capacity, x_3	$C_3(S_4)$	$S_3=S_4-x_3$	$f_2^*(S_3)$	$f_3^*(S_4)= C_3(S_4)+ f_2^*(S_3)$	$f_3^*(S_4)$
5775	726	0	5049	1346	1346	1171
	1726	750	4049	421	1171	
7375	726	0	6649	2105	2105	1671
	1726	1250	5649	1346	2596	
	4575	1250	2800	421	1671	
9375	726	0	8649	2108	2108	1673
	1726	1252	7649	2105	3357	
	4575	1252	4800	1346	2598	
	5775	1252	3600	421	1673	

Table 6.25 Forward recursion stage4, n=3 years and interest rate=12%

State Variable, S_5	Added Capacity, x_4	$C_4(S_5)$	$S_4=S_5-x_4$	$f_3^*(S_4)$	$f_4^*(S_5)= C_4(S_5)+ f_3^*(S_4)$	$f_4^*(S_5)$
7375	726	0	6649	1671	1671	1671
	1726	890	5649	1171	2061	
9375	726	0	8649	1673	1673	1673
	1726	891	7649	1671	2562	
	4575	891	4800	1171	2062	

Table 6.26 Forward recursion stage5, n=3 years and interest rate=12%

State Variable, S_6	Added Capacity, x_5	$C_5(S_6)$	$S_5=S_6-x_5$	$f_4^*(S_5)$	$f_5^*(S_6)= C_5(S_6)+ f_4^*(S_5)$	$f_5^*(S_6)$
9375	726	0	8649	1673	1673	1673
	1726	634	7649	1671	2305	

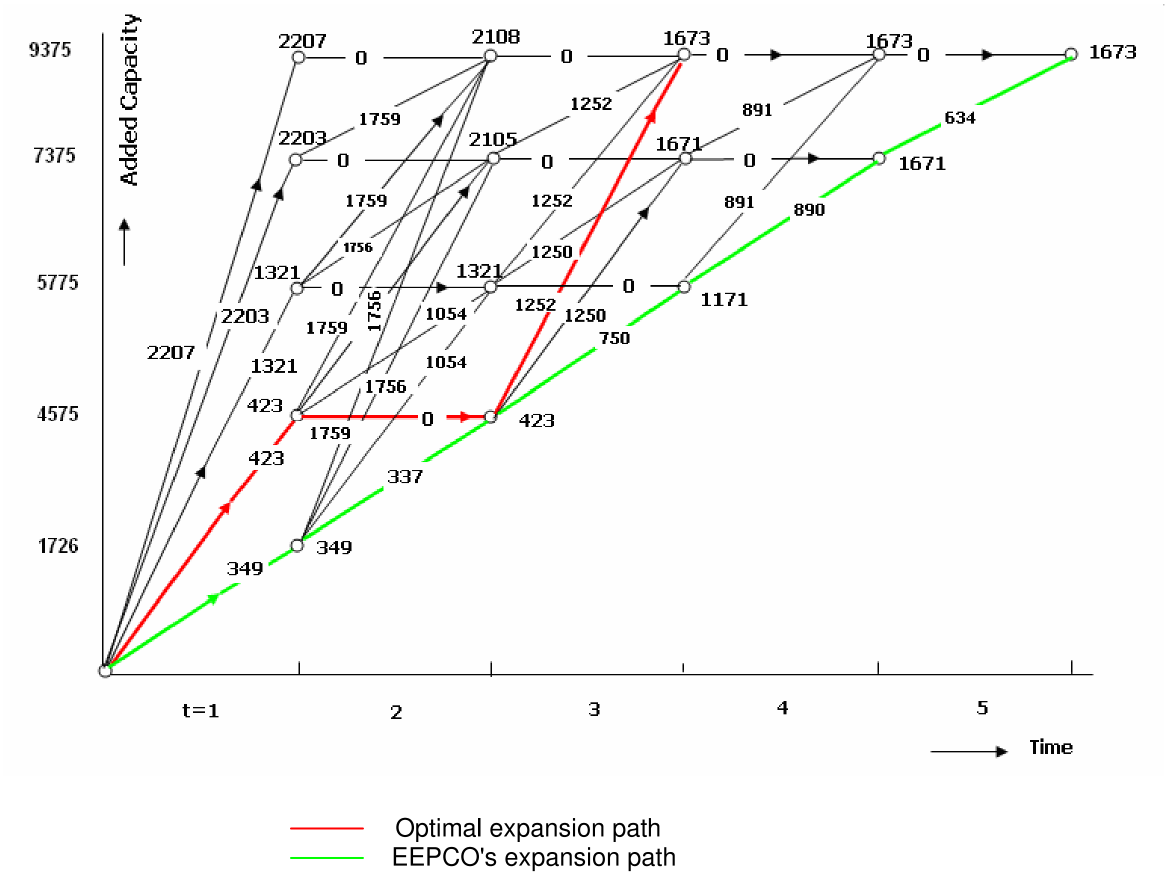


Figure 6.5 A capacity expansion, showing the results of a forward-moving dynamic programming algorithm for n=3 years and i=12%.

Solution Using Backward Recursion

Table 6.27 Backward recursion stage5, n=3 years and interest rate=12%

State Variable, S_5	Added Capacity, x_5	$C_5(S_5)$	$f_5^*(S_5)$
7375	1726	634	634
9375	726	0	0

Table 6.28 Backward recursion stage4, n=3 years and interest rate=12%

State Variable, S_4	Added Capacity, x_4	$C_4(S_4)$	$S_5=S_4+x_4$	$f_3^*(S_5)$	$f_4^*(S_4)= C_4(S_4)+ f_5^*(S_5)$	$f_4^*(S_4)$
5775	726	890	6501	634	1524	891
	1726	891	7501	0	891	
7375	726	0	8101	634	634	634
	1726	891	9101	0	891	
9375	726	0	10,101	0	0	0

Table 6.29 Backward recursion stage3, n=3 years and interest rate=12%

State Variable, S_3	Added Capacity, x_3	$C_3(S_3)$	$S_4=S_3+x_3$	$f_4^*(S_4)$	$f_3^*(S_3)= C_3(S_3)+ f_4^*(S_4)$	$f_3^*(S_3)$
4575	726	750	5301	891	1641	1252
	1726	1250	6301	634	1884	
	4575	1252	9150	0	1252	
5775	726	0	6501	891	891	891
	1726	1250	7501	634	1884	
	4575	1252	10,350	0	1252	
7375	726	0	8101	634	634	634
	1726	1252	9101	0	1252	
9375	726	0	10,101	0	0	0

Table 6.30 Backward recursion stage2, n=3 years and interest rate=12%

State Variable, S_2	Added Capacity, x_2	$C_2(S_2)$	$S_3=S_2+x_2$	$f_1^*(S_3)$	$f_2^*(S_2)=C_2(S_2)+f_3^*(S_3)$	$f_2^*(S_2)$
1726	1726	337	3452	1252	1589	1589
	4575	1054	6301	891	1945	
	5775	1756	7501	634	2390	
	7375	1759	9101	0	1759	
4575	726	0	5301	1252	1252	1252
	1726	1054	6301	891	1945	
	4575	1756	9150	634	2390	
	5775	1759	10,350	0	1759	
5775	726	0	6501	891	891	891
	1726	1756	7501	634	2390	
	4575	1759	10,350	0	1759	
7375	726	0	8101	634	634	634
	1726	1759	9101	0	1759	
9375	726	0	10,101	0	0	0

Table 6.31 Backward recursion stage1, n=3 years and interest rate=12%

State Variable, S_1	Added Capacity, x_1	$C_1(S_1)$	$S_2=S_1+x_1$	$f_2^*(S_2)$	$f_1^*(S_1)=C_1(S_1)+f_2^*(S_2)$	$f_1^*(S_2)$
726	1726	349	2452	1589	1938	1675
	4575	423	5301	1252	1675	
	5775	1321	6501	891	2212	
	7375	2203	8101	634	2837	
	9375	2207	10,101	0	2207	

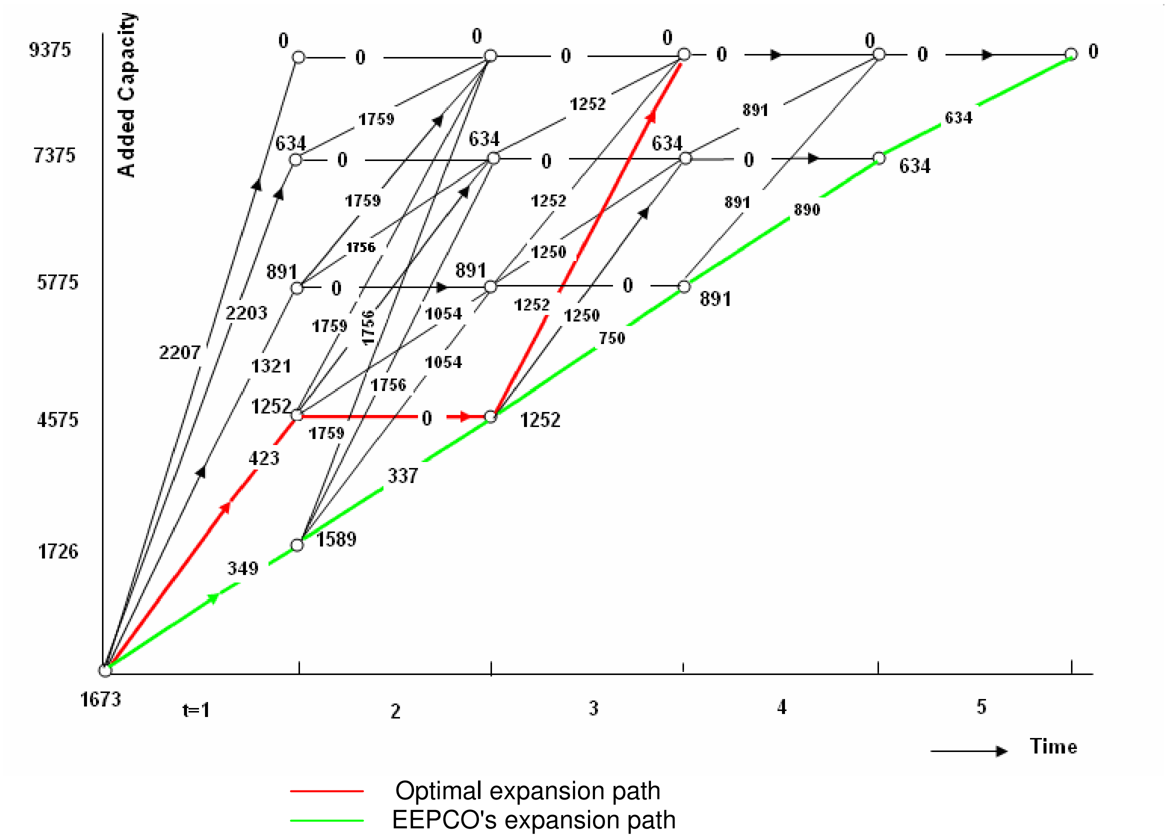


Figure 6.6 A capacity expansion, showing the results of a backward-moving dynamic programming algorithm for n=3 years and i=12%.

Case IV (i=8%, n=4)

The construction period, n is four years and the interest rate, i is 8%.

Solution Using Forward Recursion

Table 6.32 Forward recursion stage1, n=4 years and interest rate=8%

State Variables, S_2	Added Capacity, $X_1=S_2 - S_1$	$C_1(S_2)$	$f_1^*(S_2)$
1726	1000	362	362
4575	3849	439	439
5775	5049	1371	1371
7375	6649	2284	2284
9375	8649	2289	2289

Table 6.33 Forward recursion stage2, n=4 years and interest rate=8%

State Variable, S_3	Added Capacity, x_2	$C_2(S_3)$	$S_2=S_3-x_2$	$f_1^*(S_2)$	$f_2^*(S_3)= C_2(S_3)+ f_1^*(S_2)$	$f_2^*(S_3)$
4575	726	0	3849	439	439	439
	1726	348	2849	362	710	
5775	726	0	5049	1371	1371	1371
	1726	1088	4049	439	1527	
	4575	1088	1200	362	1450	
7375	726	0	6649	2284	2284	2175
	1726	1813	5649	1371	3184	
	4575	1813	2800	439	2252	
	5775	1813	1600	362	2175	
9375	726	0	8649	2289	2289	2179
	1726	1817	7649	2284	4101	
	4575	1817	4800	1371	3188	
	5775	1817	3600	439	2256	
	7375	1817	2000	362	2179	

Table 6.34 Forward recursion stage3, n=4 years and interest rate=8%

State Variable, S_4	Added Capacity, x_3	$C_3(S_4)$	$S_3=S_4-x_3$	$f_2^*(S_3)$	$f_3^*(S_4)= C_3(S_4)+ f_2^*(S_3)$	$f_3^*(S_4)$
5775	726	0	5049	1371	1371	1239
	1726	800	4049	439	1239	
7375	726	0	6649	2175	2175	1772
	1726	1333	5649	1371	2704	
	4575	1333	2800	439	1772	
9375	726	0	8649	2179	2179	1774
	1726	1335	7649	2175	3510	
	4575	1335	4800	1371	2706	
	5775	1335	3600	439	1774	

Table 6.35 Forward recursion stage4, n=4 years and interest rate=8%

State Variable, S_5	Added Capacity, x_4	$C_4(S_5)$	$S_4=S_5-x_4$	$f_3^*(S_4)$	$f_4^*(S_5)= C_4(S_5)+ f_3^*(S_4)$	$f_4^*(S_5)$
7375	726	0	6649	1772	1772	1772
	1726	980	5649	1239	2219	
9375	726	0	8649	1774	1774	1774
	1726	982	7649	1772	2754	
	4575	982	4800	1239	2221	

Table 6.36 Forward recursion stage5, n=4 years and interest rate=8%

State Variable, S_6	Added Capacity, x_5	$C_5(S_6)$	$S_5=S_6-x_5$	$f_4^*(S_5)$	$f_5^*(S_6)= C_5(S_6)+ f_4^*(S_5)$	$f_5^*(S_6)$
9375	726	0	8649	1774	1774	1774
	1726	722	7649	1772	2494	

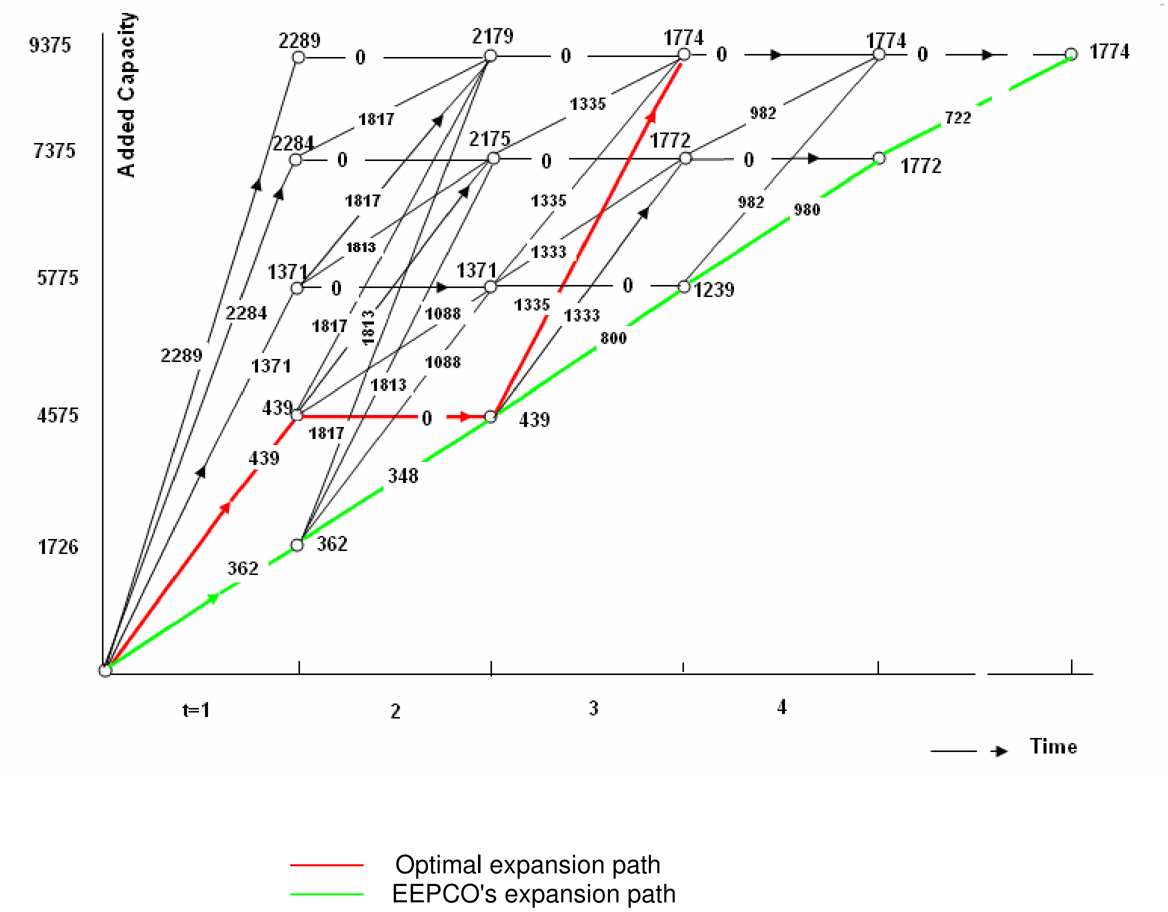


Figure 6.7 A capacity expansion, showing the results of a forward-moving dynamic programming algorithm for n=4 years and i=8%.

Solution Using Backward Recursion

Table 6.37 Backward recursion stage5, n=4 years and interest rate=8%

State Variable, S_5	Added Capacity, x_5	$C_5(S_5)$	$f_5^*(S_5)$
7375	1726	722	722
9375	726	0	0

Table 6.38 Backward recursion stage4, n=4 years and interest rate=8%

State Variable, S_4	Added Capacity, x_4	$C_4(S_4)$	$S_5=S_4+x_4$	$f_3^*(S_5)$	$f_4^*(S_4)= C_4(S_4)+ f_5^*(S_5)$	$f_4^*(S_4)$
5775	726	980	6501	722	1702	982
	1726	982	7501	0	982	
7375	726	0	8101	722	722	722
	1726	982	9101	0	982	
9375	726	0	10,101	0	0	0

Table 6.39 Backward recursion stage3, n=4 years and interest rate=8%

State Variable, S_3	Added Capacity, x_3	$C_3(S_3)$	$S_4=S_3+x_3$	$f_4^*(S_4)$	$f_3^*(S_3)= C_3(S_3)+ f_4^*(S_4)$	$f_3^*(S_3)$
4575	726	800	5301	982	1782	1335
	1726	1333	6301	722	2055	
	4575	1335	9150	0	1335	
5775	726	0	6501	982	982	982
	1726	1333	7501	722	2055	
	4575	1335	10,350	0	1335	
7375	726	0	8101	722	722	722
	1726	1335	9101	0	1335	
9375	726	0	10,101	0	0	0

Table 6.40 Backward recursion stage2, n=4 years and interest rate=8%

State Variable, S_2	Added Capacity, x_2	$C_2(S_2)$	$S_3=S_2+x_2$	$f_1^*(S_3)$	$f_2^*(S_2)= C_2(S_2)+ f_3^*(S_3)$	$f_2^*(S_2)$
1726	1726	348	3452	1335	1683	1683
	4575	1088	6301	982	2070	
	5775	1813	7501	722	2535	
	7375	1817	9101	0	1817	
4575	726	0	5301	1335	1335	1335
	1726	1088	6301	982	2070	
	4575	1813	9150	722	2535	
	5775	1817	10,350	0	1817	
5775	726	0	6501	982	982	982
	1726	1813	7501	722	2535	
	4575	1817	10,350	0	1817	
7375	726	0	8101	722	722	722
	1726	1817	9101	0	1817	
9375	726	0	10,101	0	0	0

Table 6.41 Backward recursion stage1, n=4 years and interest rate=8%

State Variable, S_1	Added Capacity, x_1	$C_1(S_1)$	$S_2=S_1+x_1$	$f_2^*(S_2)$	$f_1^*(S_1)= C_1(S_1)+ f_2^*(S_2)$	$f_1^*(S_2)$
726	1726	362	2452	1683	2045	1774
	4575	439	5301	1335	1774	
	5775	1371	6501	982	2353	
	7375	2284	8101	722	3006	
	9375	2289	10,101	0	2289	

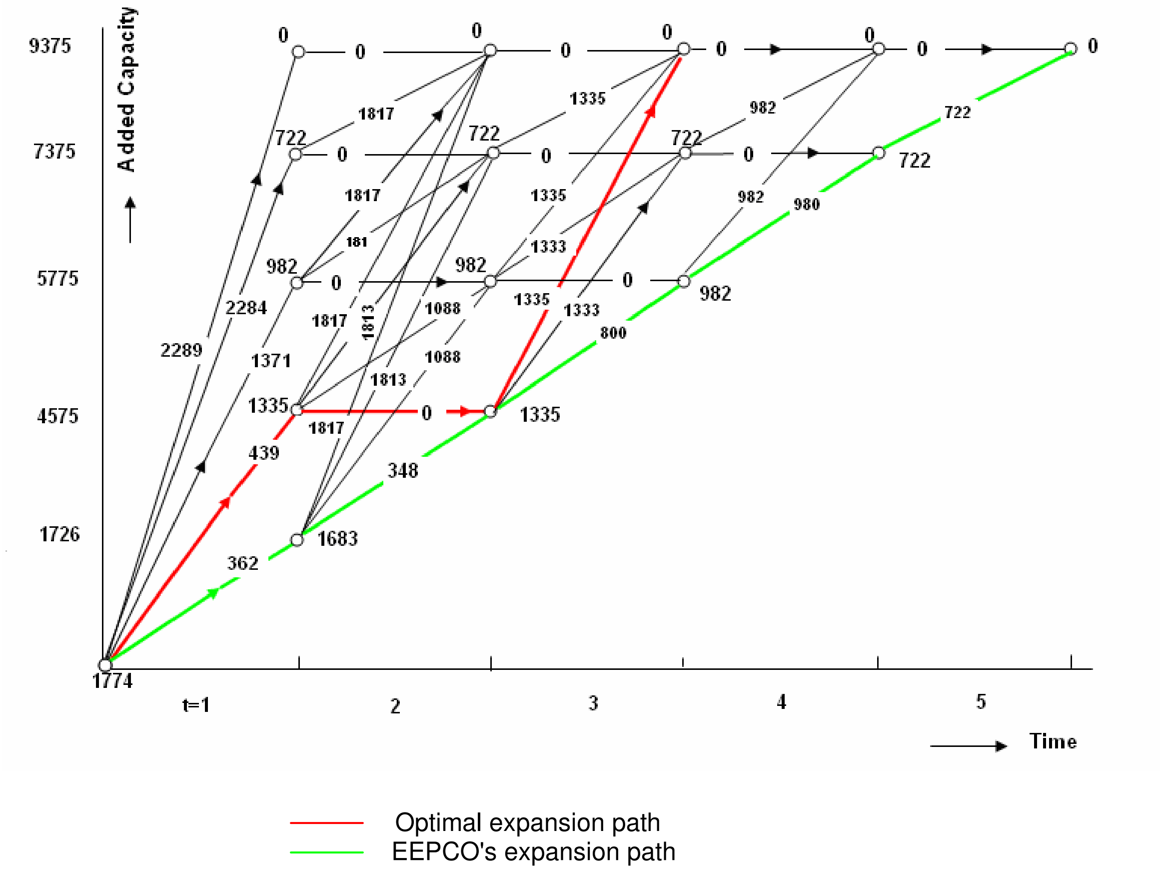


Figure 6.8 A capacity expansion, showing the results of a backward-moving dynamic programming algorithm for n=4 years and i=10%.

Case V (i=12%, n=4)

The construction period, n is four years and the interest rate, i is 12%.

Solution Using Forward Recursion

Table 6.42 Forward recursion stage1, n=4 years and interest rate=12%

State Variables, S_2	Added Capacity, $X_1=S_2 - S_1$	$C_1(S_2)$	$f_1^*(S_2)$
1726	1000	349	349
4575	3849	423	423
5775	5049	1322	1322
7375	6649	2203	2203
9375	8649	2207	2207

Table 6.43 Forward recursion stage2, n=4 years and interest rate=12%

State Variable, S_3	Added Capacity, x_2	$C_2(S_3)$	$S_2=S_3-x_2$	$f_1^*(S_2)$	$f_2^*(S_3)= C_2(S_3)+ f_1^*(S_2)$	$f_2^*(S_3)$
4575	726	0	3849	423	423	423
	1726	301	2849	349	650	
5775	726	0	5049	1322	1322	1290
	1726	941	4049	423	1364	
	4575	941	1200	349	1290	
7375	726	0	6649	2203	2203	1917
	1726	1568	5649	1322	2890	
	4575	1568	2800	423	1991	
	5775	1568	1600	349	1917	
9375	726	0	8649	2207	2207	1920
	1726	1571	7649	2203	3774	
	4575	1571	4800	1322	2893	
	5775	1571	3600	423	1994	
	7375	1571	2000	349	1920	

Table 6.44 Forward recursion stage3, n=4 years and interest rate=12%

State Variable, S_4	Added Capacity, x_3	$C_3(S_4)$	$S_3=S_4-x_3$	$f_2^*(S_3)$	$f_3^*(S_4)= C_3(S_4)+ f_2^*(S_3)$	$f_3^*(S_4)$
5775	726	0	5049	1290	1290	1021
	1726	598	4049	423	1021	
7375	726	0	6649	1917	1917	1419
	1726	996	5649	1290	2286	
	4575	996	2800	423	1419	
9375	726	0	8649	1920	1920	1421
	1726	998	7649	1917	2915	
	4575	998	4800	1290	2288	
	5775	998	3600	423	1421	

Table 6.45 Forward recursion stage4, n=4 years and interest rate=12%

State Variable, S_5	Added Capacity, x_4	$C_4(S_5)$	$S_4=S_5-x_4$	$f_3^*(S_4)$	$f_4^*(S_5)= C_4(S_5)+ f_3^*(S_4)$	$f_4^*(S_5)$
7375	726	0	6649	1419	1419	1419
	1726	633	5649	1021	1654	
9375	726	0	8649	1421	1421	1421
	1726	634	7649	1419	2053	
	4575	634	4800	1021	1655	

Table 6.46 Forward recursion stage5, n=4 years and interest rate=12%

State Variable, S_6	Added Capacity, x_5	$C_5(S_6)$	$S_5=S_6-x_5$	$f_4^*(S_5)$	$f_5^*(S_6)= C_5(S_6)+ f_4^*(S_5)$	$f_5^*(S_6)$
9375	726	0	8649	1421	1421	1421
	1726	403	7649	1419	1822	

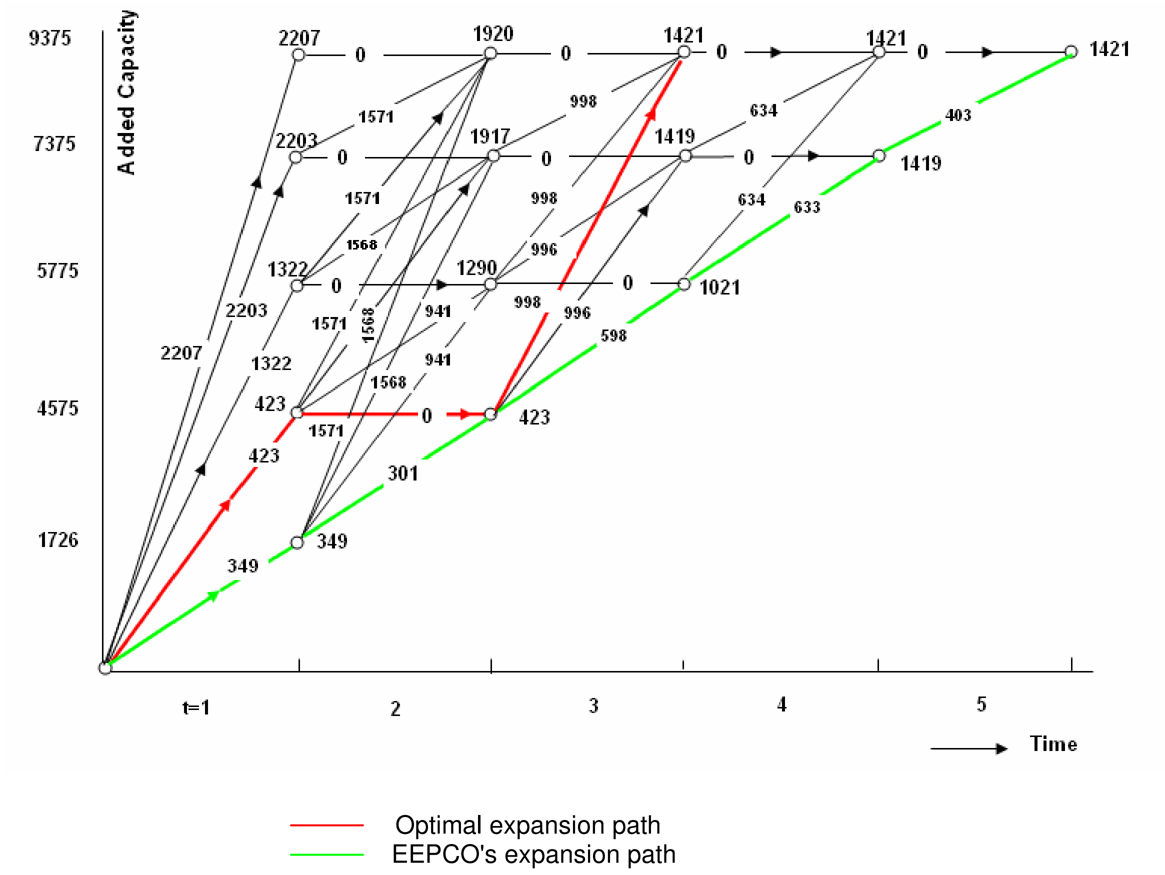


Figure 6.9 A capacity expansion, showing the results of a forward-moving dynamic programming algorithm for n=4 years and i=12%.

Solution Using Backward Recursion

Table 6.47 Backward recursion stage5, n=4 years and interest rate=12%

State Variable, S_5	Added Capacity, x_5	$C_5(S_5)$	$f_5^*(S_5)$
7375	1726	403	403
9375	726	0	0

Table 6.48 Backward recursion stage4, n=4 years and interest rate=12%

State Variable, S_4	Added Capacity, x_4	$C_4(S_4)$	$S_5=S_4+x_4$	$f_3^*(S_5)$	$f_4^*(S_4)= C_4(S_4)+ f_5^*(S_5)$	$f_4^*(S_4)$
5775	726	633	6501	403	1036	634
	1726	634	7501	0	634	
7375	726	0	8101	403	403	403
	1726	634	9101	0	634	
9375	726	0	10,101	0	0	0

Table 6.49 Backward recursion stage3, n=4 years and interest rate=12%

State Variable, S_3	Added Capacity, x_3	$C_3(S_3)$	$S_4=S_3+x_3$	$f_4^*(S_4)$	$f_3^*(S_3)= C_3(S_3)+ f_4^*(S_4)$	$f_3^*(S_3)$
4575	726	598	5301	634	1232	998
	1726	996	6301	403	1399	
	4575	998	9150	0	998	
5775	726	0	6501	634	634	634
	1726	996	7501	403	1399	
	4575	998	10,350	0	998	
7375	726	0	8101	403	403	403
	1726	998	9101	0	998	
9375	726	0	10,101	0	0	0

Table 6.50 Backward recursion stage2, n=4 years and interest rate=12%

State Variable, S_2	Added Capacity, x_2	$C_2(S_2)$	$S_3=S_2+x_2$	$f_1^*(S_3)$	$f_2^*(S_2)= C_2(S_2)+ f_3^*(S_3)$	$f_2^*(S_2)$
1726	1726	301	3452	998	1299	1299
	4575	941	6301	634	1575	
	5775	1568	7501	403	1971	
	7375	1571	9101	0	1571	
4575	726	0	5301	998	998	998
	1726	941	6301	634	1575	
	4575	1568	9150	403	1971	
	5775	1571	10,350	0	1571	
5775	726	0	6501	634	634	634
	1726	1568	7501	403	1971	
	4575	1571	10,350	0	1571	
7375	726	0	8101	403	403	403
	1726	1571	9101	0	1571	
9375	726	0	10,101	0	0	0

Table 6.51 Backward recursion stage1, n=4 years and interest rate=12%

State Variable, S_1	Added Capacity, x_1	$C_1(S_1)$	$S_2=S_1+x_1$	$f_2^*(S_2)$	$f_1^*(S_1)= C_1(S_1)+ f_2^*(S_2)$	$f_1^*(S_2)$
726	1726	349	2452	1299	1648	1421
	4575	423	5301	998	1421	
	5775	1322	6501	634	1956	
	7375	2203	8101	403	2606	
	9375	2207	10,101	0	2207	

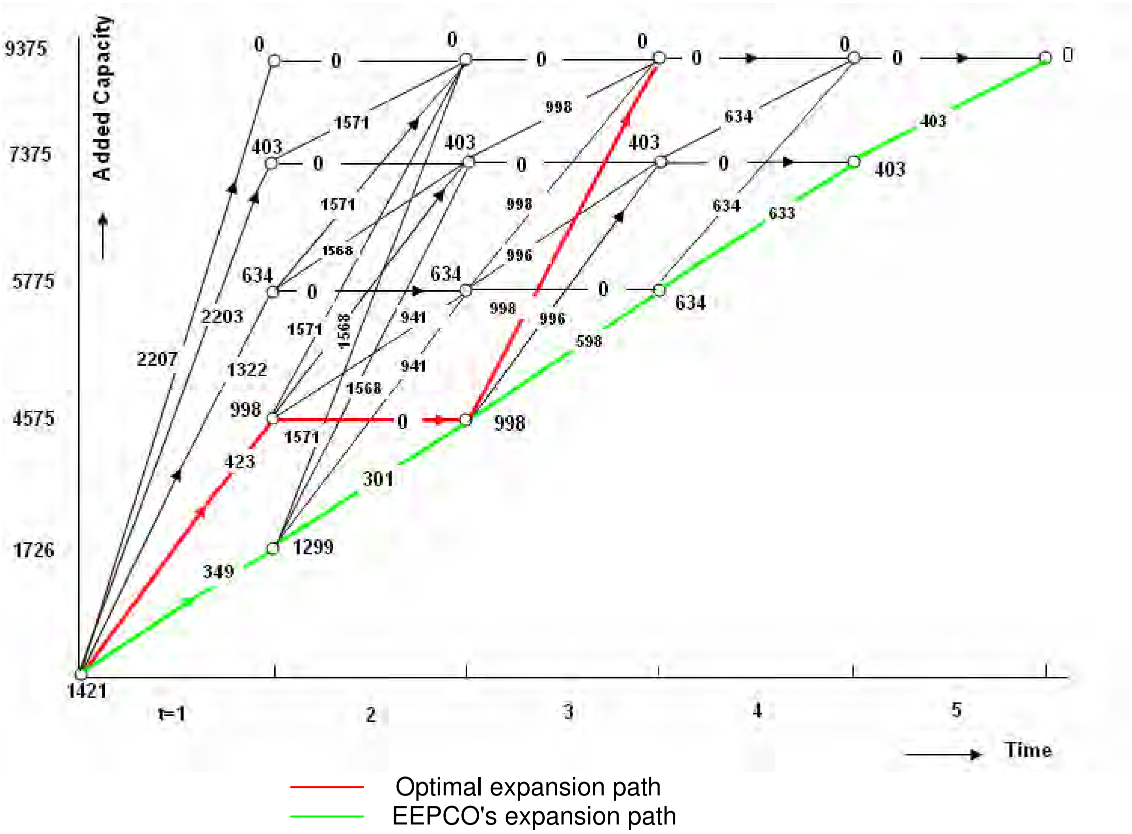


Figure 6.10 A capacity expansion, showing the results of a backward-moving dynamic programming algorithm for n=4 years and i=12%.

From the analysis case V (n=4, i=12%) gives us the optimal cost of expansion, 1421 MUSD which is the same for both the forward and backward recursion. Therefore the optimum sequence for Case V is the best sequence. This sequence is obtained by backtracking from the last stage (farthest right node) to the initial stage, the optimal expansion to be done at first stage = 3849MW, third stage = 4800MW and the rest all stages = 0 MW. Hence the final requirement of 8649MW is achieved at the end of stage three.

From the analysis it is observed that all the cases end up with the same path. To check the sensitivity of the problem additional cases are considered. Since n which describes the construction period cannot be above five the interest rate is the only parameter to be changed. Applying the same principle for n=4 and i=14, 16, 18, 20, 24 there was no change in the path. A different path was achieved for i=26% with a slight difference of two MUSD in the discounted cost. When using an interest rate lower than 8% there was no change in the path as well. Checking the same thing for n=3 the path was the same up to i=26%. To achieve another path for n=3 the analysis should go beyond i=26% in which case will give a different path for n=4. When using an interest rate lower than 8% there would be no change for n=3 because the discounted cost would increase.

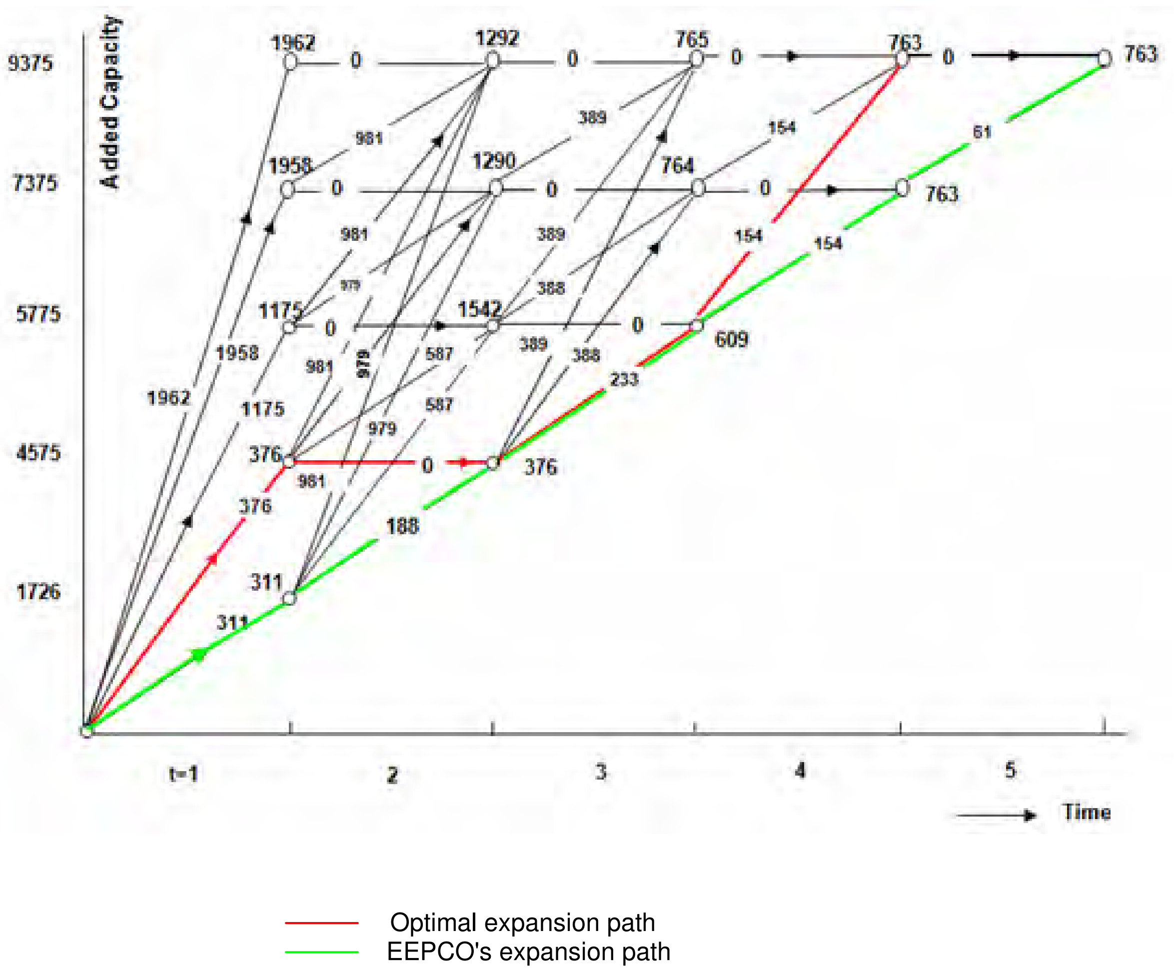


Figure 6.11 A capacity expansion, showing the results of a forward-moving dynamic programming algorithm for n=4 years and i=26%.

7 Conclusions and Recommendations

In this thesis work the best sequence for scheduling the hydroplants is obtained with minimum discounted cost. This path has Chemoga Yeda, Tekeze, Gilgel Gibe II, Beles, Amerti Neshe, GGIII Phase 1, GGIII Phase 2 and Hallele Worabessa added into the system in period one. During period two it has no additions and finally adds Border, Karadobi and Mandaya until the end of period three. This result is not exactly similar to the policy presented by EEPCO.

The DP model does not take many other external factors like environmental, availability of finances etc. It only considers the minimum net present cost which is purely an economical criteria. In future researches, these are recommended to be included.

The case that gives a different path is when the interest rate is 26% and the construction period is four which is a large value. Therefore we can conclude that the problem is not sensitive for change in interest rate and n! The reason for this is because we have two hydro plants included in our analysis with lower costs compared to the other three plants.

It may be only of academic value if plants are considered which are actually not practically implementable. There are plants for which decision to go ahead with construction has already started with the plants underway to be constructed such as GGIII.

This thesis has made a basis to follow this decision-making systems for plants to be constructed in the future.

Future work of a similar type of exercise should be conducted taking more precise data and also additional plants from Sudan, Egypt and Kenya using the demand forecast of these countries. With more refined data, better and reliable results could be obtained. A more comprehensive level thorough investigation is also required with more precise data for all the Nile basin countries.

Future work of a similar type of exercise with the objective function as maximization of net benefit should be done.

It was a very enlightening exercise although the optimal rule developed can't be used practically because of prior decisions made taking other factors into consideration.

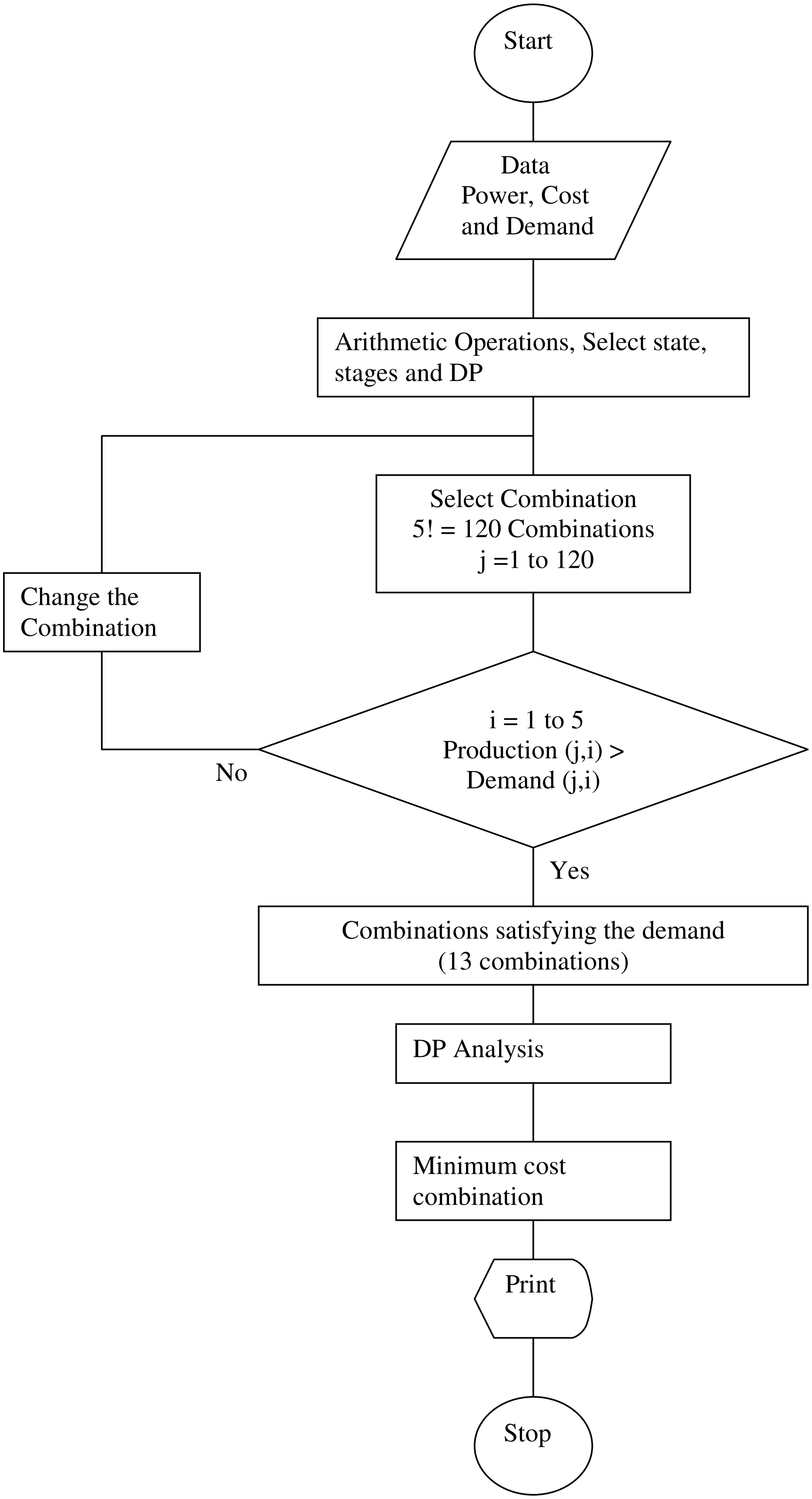
DP is unique for each problem and so a model specific to that particular problem should be developed, so this thesis has developed a unique model. The same approach can be adapted to develop a DP model covering other regions in the Nile Basin. One of the important outputs of a research is to find new areas of research. Thus, this thesis has clearly indicated new areas of intervention.

Appendix

Algorithm

- Collect the data on the proposed hydropower stations
- Select the order satisfying the power demand
- Apply the DP algorithm to determine the sequence with the minimum present worth cost
- Select the sequence and recommend for execution on following the minimum cost sequence.

Flow Chart


Start
Data
Power, Cost
and Demand
Arithmetic Operations, Select state,
stages and DP
Select Combination
5! = 120 Combinations
j =1 to 120
Change the
Combination
i = 1 to 5
Production (j,i) >
Demand (j,i)
No
Yes
Combinations satisfying the demand
(13 combinations)
DP Analysis
Minimum cost
combination
Print
Stop

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Declaration

This thesis is my original work and has not been presented for a degree in any other university, and all sources of material used for this thesis have been dually acknowledged.

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Signature: