STUDIES ON ELECTRON-POSITRON PAIR PRODUCTION IN COLLISION OF VIRTUAL PHOTONS AND PAIR CONVERSION OF VIRTUAL BREMSSTRAHLUNG PHOTON

A THESIS SUBITTED TO

THE SCHOOL OF GRADUATE STUDIES OF

ADDIS ABABA UNIVERSTY

IN PARTIAL FULFIFFMENT OF THE REQUIREMENTS

FOR THE DEGREE OF MASTER OF SCIENCE IN PHYSICS

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ADDIS ABABA

MARCH 2007

ADDIS ABABA UNIVERSTY

SCHOOL OF GRADUATE STUDIES

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Acknowledgment

Above all, I want to give thanks and acknowledgment of my heart, to God for the extent to which he has enabled me to realize my goals and for the wisdom and strength, he has given me.

It is with great pleasure that I thank and acknowledge my advisor, Dr. Shashank Bhatnagar, for his comments, suggestions and genuine advises he offered me during the progress of this work. His dedication, knowledge and skill as a qualified instructor and advisor are an inspiration. I am very grateful to my instructors Dr. Sib. Krishna Ghoshal, Dr. Abraham Amaha, Dr. Ramanand Jha, and Prof. A.K. Chaubey.

To my father Dinegde and mother Beletech I owe more than words can describe for their lifelong devotion and guidance.

Dedication

I wish to dedicate this thesis to those I treasure above all else on this earth, and who have given me meaning to my life –to my wife Tigist, and our children Hiwot, Worku, Feben, and Eden.

Abstract

This thesis deals with the physics of two photon processes. We first introduce covariant perturbation theory and apply it to some quantum electrodynamics processes. We start with study of electron-positron pair production in the field of a static nucleus. We then deal with applications to electron-positron pair production through collision of virtual photons in high energy electron-electron scattering. Next we study the pair conversion of bremsstrahlung photon in high energy proton-proton scattering. Detailed evaluations of cross sections are given.

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Chapter 1

Introduction

In quantum electrodynamics (QED), charged particles interact through exchange of virtual photons[1, 2, 3]. However by colliding charged particles at high energies, the collision of two virtual photons can be studied even though no intense colliding beams of real photons are available to the experimentalists.

The study of two virtual photon collision processes also opens a completely new region of investigation in high energy physics, for instance the study of interaction such as $\gamma\gamma \longrightarrow$hadrons was earlier done in Ref[4, 5]. In these thesis we are primarily interested in the study of processes involving pair creation of a lepton pair in $\gamma\gamma$ collisions as well as in the the process of pair conversion of a virtual bremsstrahlung photon into a pair leptons.

Electron-positron pair creation is a process in which an electron and positron are simultaneously created in the vicinity of a nucleus or subatomic particle. Electron-positron pair creation[6, 7] is an example of conversion of energy into matter as predicted by special relativity and is accurately described by quantum electrodynamics. Pair production usually refers to external pair production, in which the electron and positron are created from a high energy gamma rays as it passes through the matter. Electron-positron pairs are also produced from internal pair conversions in nuclei, decays of unstable subatomic particles, and collisions between charged particles. In external collision, the energy of the incoming gamma ray(a high energy electromagnetic photon) is directly converted in to the mass energy of electron positron pair. The photon energy must therefore exceed twice the rest mass of electron $2m_0c^2$ (equal to 1.002 MeV where m_0 is the electron mass), the energy value that corresponds to soft gamma ray photons. The creation of much more massive pair, like a proton and antiproton, requires photons with energy more than 1.88 GeV(hard gamma ray photons). Since the momentum of initial photon must be absorbed by something, pair production can not occur in empty space out of a single photon; the nucleus is needed to conserve both momentum and energy.

Among different electromagnetic processes of particle production,we are interested in the process of two photon production of leptons. In this process, two colliding particles emit virtual photons which collide to produce a pair of leptons. Two photon particle production is connected directly with the process of light-light interaction[5].

Through this process as stated earlier, we can also study the interactions such as $\gamma\gamma \longrightarrow$h(hadrons). The study of such interaction depends not only on the energy, but also on the masses of both the colliding particles.

These processes did not attract attention for a long time since they are for the 4^{th} order in electromagnetic coupling $\alpha = \frac{1}{137}$ and thus at a first glance their cross section seems to be very small. However in a number of cases these processes are dominant.

In this thesis we have calculated the cross section for these processes in quantum electrodynamics with no strong interaction effects taken into account. It is seen that the two photon production cross section increases logarithmically with energy[5].

.In this thesis we will first study the electron-electron scattering process at high energies which gives rise to collision of two virtual photons which in turn produce electron positron pair i.e. the interaction, $e^-+e^- \longrightarrow e^-+e^-+e^-+e^+$, which is described in Fig.1

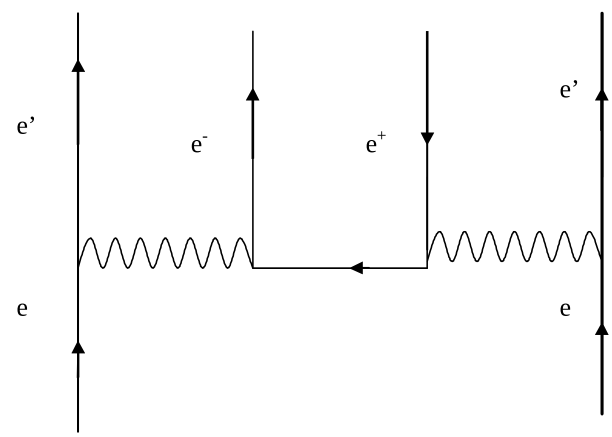


Fig 1: Lowest order Feynman diagram for e^+e^-pair production through collision of two virtual photons in high energy electron-electron scattering.

We also study the process involving the lepton pair conversion of a bremsstrahlung photon in detail which is also a QED process of $O(\alpha^4)$ in high energy proton-proton collision. This is very interesting process. Here the bremsstrahlung photon produced in proton-proton collision of sufficiently high energy and its momentum is off shell and produce electron-positron pair. We study this process in detail and calculate the cross section for electron-positron pair production. This process is described in Fig. 2.

. Bremsstrahlung[2,3] is the electromagnetic radiation that occurs when charged particles with energies large compared to their rest energies are decelerated over very short distances. Bremsstrahlung has a continuous spectrum. Bremsstrahlung may also be referred to as free radiation. This refers to the radiation that arises as a result of a charged particle that is free both before and after deflection(acceleration) that causes the emission. Strictly speaking, bremsstrahlung[8-11] refers to any radiation due to acceleration of a charged particle, which includes synchrotron radiation.

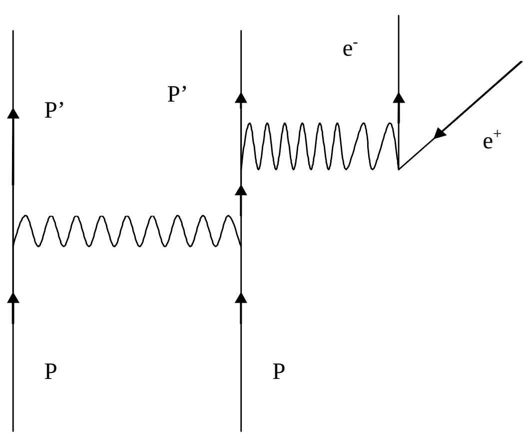


Fig. 2: e^+e^- pair production by bremsstrahlung photon.

The cross section for both of these processes are found to increase logarithmically with energy[5]. This is shown clearly in our calculations. This thesis is organized as follows:

In chapter 2, we introduced the covariant perturbation theory with application to Quantum Electrodynamics(QED). Preliminary application is given to the process of e^+e^- pair creation in the field of an atomic nucleus. In chapter 3 the process of e^+e^- pair production through collision of two virtual photons produced in high energy electron-electron scattering is studied. The cross section is calculated for this process. This is followed by the study the cross section calculation for the process of e^+e^- pair creation of a virtual bremsstrahlung photon in high energy proton-proton collision in chapter 4. This is followed by summery in chapter 5.

Chapter 2

Covariant perturbation theory with preliminary application to electron-positron pair production process in a Coulomb field of a nucleus

2.1 Interaction picture

We first recall the interaction picture of the time evolution of a physical state ket. Let the state ket of a physical system coincide with $|\alpha\rangle$ at t_0, where t_0 is often taken to be zero. At later time, we denote the same state ket in Shchrödinger picture by $|\alpha, t_{0;}t\rangle_s$. We now define the corresponding state in the interaction picture as[1]:

$$|\alpha, t_{0;}t\rangle_I = e^{iH_0 t}|\alpha, t_{0;}t\rangle_s. \tag{2.1}$$

We define observable in the interaction picture as :-

$$\theta_I = e^{iH_0 t}\theta_s e^{-iH_0 t}. \tag{2.2}$$

In particular

$$H_I = e^{iH_0 t}H_s e^{-iH_0 t}, \tag{2.3}$$

where H$_s$ is the time dependent Hamiltonian in Schrödinger picture

$$H_s = H_I^{(s)} + H_0^{(s)}. \tag{2.4}$$

Here H$_0$ is unperturbed Hamiltonian. $H_I = \int d^3x H_I'$, where $H_I' = -ie\overline{\Psi}\gamma_\mu \Psi A_\mu(x)$. The fundamental differential equation that characterizes the time evolution of a state ket in the interaction picture can be obtained differentiating (2.1) with respect to time can be written as,

$$i\frac{\partial}{\partial t}|\alpha, t_o; t\rangle_I = i\frac{\partial}{\partial t}\left(e^{iH_0 t}|\alpha, t_0; t\rangle_s\right) = -H_0 e^{iH_0 t}|\alpha, t_0; t\rangle_s + e^{iH_0 t}\left(H_I + H_0\right)|\alpha, t_0; t\rangle_s.$$
$$= e^{iH_0 t} H_I e^{-iH_0 t} e^{iH_0 t}|\alpha, t_0; t\rangle_s = H_I|\alpha, t_0; t\rangle_I.$$
$$= H_I|\alpha, t_0; t\rangle_I. \tag{2.5}$$

Let $\mathrm{U}_I\left(t, t_0\right)$ be time evolution operator in interaction picture such that

$$|\alpha, t_0; t\rangle_I = U_I\left(t, t_0\right)|\alpha, t_0\rangle_I. \tag{2.6}$$

Substituting (2.6) in (2.5) we obtain,

$$i\frac{\partial}{\partial t}U_I\left(t, t_0\right) = H_I U_I\left(t, t_0\right). \tag{2.7}$$

Imposing the initial condition $U_I\left(t, t_0\right)|_{t=t_0} = 1$, the solution of (2.7) becomes,

$$U_I\left(t, t_0\right) = 1 - i\int_{t_0}^{t} H_I\left(t_1\right) U_I\left(t_1, t_0\right) dt_1. \tag{2.8}$$

We can obtain an approximate solution to this equation by iteration[1] as:

$$U_I\left(t, t_0\right) = 1 - i\int_{t_0}^{t} H_I\left(t_1\right) dt_1 + (-i)^2\int_{t_0}^{t} dt_1\int_{t_0}^{t_1} dt_2 H_I\left(t_1\right) H_I\left(t_2\right) ...+$$
$$(-i)^n\int_{t_0}^{t} dt_1\int_{t_0}^{t_1} dt_2 ...\int_{t_0}^{t_{n-1}} dt_n H_I\left(t_1\right) H_I\left(t_2\right) ... H_I\left(t_n\right). \tag{2.9}$$

If | i ⟩ is the initial state of the system at time t_0 and | f ⟩ is the final state of the same system at time, t then $|\langle f|U_I\left(t,t_0\right)|i\rangle|^2 = |U_{fi}\left(t,t_0\right)|^2$ is the transition probability of the system in state | f ⟩ in time t if it was in state| i ⟩ at initial time t_0. We can define S-matrix by S=U($+\infty, -\infty$). It connects state vector from t_0 =-∞ $to\ t = +\infty$ according to $|\alpha,\infty\rangle = S(|\alpha,-\infty\rangle)$.The explicit form of S-matrix can easily be read from (2.9) as,

$$
\begin{aligned}
S &= S^{(0)} + S^{(1)} + S^{(2)} + S^{(3)} + ... \\
&= 1 - i\int_{-\infty}^{\infty} dt_1 H_I\left(t_1\right) + (-i)^2 \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 H_I\left(t_1\right) H_I\left(t_2\right) + ... \\
&(-i)^n \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 ... \int_{-\infty}^{t_{n-1}} dt_n H_I\left(t_1\right) H_I\left(t_2\right) ... H_I\left(t_n\right). \qquad (2.10)
\end{aligned}
$$

In QED, the S-matrix expansion converges rapidly so that we obtain results that agree extremely with observations just by considering the first few terms; this, of course, is because the dimensionless coupling constant $\alpha = \frac{1}{137}$ is small compared to unity.

2.2 Feynman's space-time approach to the electron propagator

The relativistic equation of motion of a free Dirac particle is[1]

$$
\left(\gamma_\mu \partial_\mu + m\right)\Psi = 0, \qquad (2.11)
$$

where Ψ is 4-component wave function of a particle and γ_μ are Dirac matrices. When a relativistic particle of charge e and mass m is placed in electromagnetic field A_μ the Dirac equation becomes

$$\left(\gamma_\mu \partial_\mu + m\right)\Psi = ie\gamma_\mu A_\mu(x)\Psi. \tag{2.12}$$

Let us considering the simpler unit source problem

$$\left(\gamma_\mu \partial_\mu + m\right) K\left(x, x'\right) = -i\delta^{(4)}\left(x, x'\right), \tag{2.13}$$

where $\delta^{(4)}\left(x, x'\right)$ is the 4-dimension delta function and K(x-x') is the Green's function for the free particle Dirac equation. The solution of (2.12) can be written as

$$\Psi\left(x\right) = -\int d^4x' K\left(x, x'\right) e\gamma_\mu A_\mu\left(x'\right)\Psi\left(x'\right). \tag{2.14}$$

To be more explicit let us consider

$$\Psi\left(x\right) = \Psi_0\left(x\right) - \int d^4x' K\left(x, x'\right) e\gamma_\mu A_\mu\left(x'\right)\Psi\left(x'\right), \tag{2.15}$$

where $\Psi_0\left(x\right)$ is a solution to the Dirac equation when e=0. In scattering problem ,$\Psi_0\left(x\right)$ may represent an incident plane wave which would be present even if there is no interaction. If a perturbation expansion in a power of e is applied, then using iteration method (2.15) becomes,

$$\Psi(x) = \Psi_0(x) + \int d^4x' K(x,x')\,[-e\gamma_\mu A_\mu(x')]\Psi(x') +$$

$$\int d^4x' \int d^4x'' K(x,x')\,[-e\gamma_\mu A_\mu(x')]K(x',x'')\,[-e\gamma_\nu A_\nu(x^{''})\Psi_0(x^{''}) +$$

$$\int d^4x' \int d^4x'' \int d^4x''' K(x,x')\,[-e\gamma_\mu A_\mu(x')]K(x,x'')\,[-e\gamma_\nu A_\nu(x^{''})]$$

$$\times\, K(x'',x''')\,[-e\gamma_\lambda A_\lambda(x''')]\Psi_0(x''') + ..., \tag{2.16}$$

where the integral representation of electron propagator is,

$$K(x,x') = \frac{-i}{(2\pi)^4}\int d^4p \frac{(-i\gamma.p + m)}{p^2+m^2} e^{ip.(x-x')}. \tag{2.17}$$

This can be expressed after carrying out the integrations over dp_0 as;

$$K(x,x') = \lim_{V\longrightarrow\infty}\sum_p\sum_{s=1}^{2}\frac{m}{EV}u^{(s)}(\overrightarrow{p})\,\overline{u}^{(s)}(\overrightarrow{p})\,e^{ip.(x-x')}, for\ t > t' \tag{2.18}$$

and

$$K(x,x') = -\lim_{V\longrightarrow\infty}\sum_p\sum_{s=1}^{2}\frac{m}{EV}v^{(s)}(\overrightarrow{p})\,\overline{v}^{(s)}(\overrightarrow{p})\,e^{-ip.(x-x')}, for\ t < t', \tag{2.19}$$

where u and v are electron and positron spinors respectively. Let's consider a potential problem assuming the validity of the second order perturbation theory. The perturbation expansion can be written as:

$$\begin{aligned}
\Psi\left(x\right) &\cong \Psi_0\left(x\right)+\int d^4x'\int d^4x''K\left(x,x'\right)\left[-e\gamma_\mu A_\mu\left(x'\right)\right]K(x',x^{''})[-e\gamma_\nu A_\nu(x^{''})]\Psi_0\left(x''\right),\\
&\cong \Psi_0\left(x\right)+\sum_{p'}\sum_{s=1}^{2}C^{+}_{p',s'}\left(t\right)\sqrt{\frac{m}{EV}}u^{(s)}(\overrightarrow{p'})e^{ip'.x}\\
&+\sum_{p'}\sum_{s=1}^{2}C^{-}_{p',s'}\left(t\right)\sqrt{\frac{m}{EV}}v^{(s)}\left(\overrightarrow{p}'\right)e^{-ip'.x'},
\end{aligned} \tag{2.20}$$

where

$$C^{+}_{p',s'}\left(t\right)=\left(-e\right)^2\int d^4x'\int d^4x''\sqrt{\frac{m}{EV}}\overline{u}^{(s)}(\overrightarrow{p'})e^{-ip'.x'}[\gamma_\mu A_\mu\left(x'\right)]K(x,x')[\gamma_\nu A_\nu\left(x''\right)\Psi_0(x'')], \tag{2.21}$$

and

$$C^{-}_{p',s'}\left(t\right)=\left(-e\right)^2\int d^4x'\int d^4x''\sqrt{\frac{m}{EV}}\overline{v}^{(s)}\left(\overrightarrow{p}\right)e^{ip.x'}[e\gamma_\mu A_\mu\left(x'\right)](Kx',x^{''})[e\gamma_\nu A_\nu(x^{''})]\Psi_0(x''), \tag{2.22}$$

where $\Psi_0\left(x''\right)=\sqrt{\frac{m}{EV}}u^{(s)}\left(\overrightarrow{p}\right)e^{ip.x''}$ is the normalized wave solution characterized by p and s. We see that $C^{+}_{p',s'}\left(\infty\right)$ is precisely the $S^{(2)}$ matrix element for the transition of an electron initially in (p, s) in to another electron state (p', s'). According to the usual interpretation of wave mechanics, $|C^{+}_{p',s'}\left(t\right)|^2$ gives the probability for finding the electron at time t in a positive energy state characterized by(p', s') (assumed to be different from p, s) where the electron is known with certainty to be in state (p, s) in the remote past. This is reasonable because a positive energy electron is known to be able make a transition into some other positive energy state in the presence of an external potential. $|C^{+}_{p',s'}(t)|^2$ is the probability for finding a positron in a state (p', s') which will certainly be annihilated at time t=∞. $|C^{-}_{p',s'}(-\infty)|$ is the probability amplitude for a particle initially in positive energy state characterized by (p, s) to be scattered to a negative energy state characterized by (p', s') propagating backward in time. As a preliminary application of the above formulation, we first consider the electron-positron pair production in Coulomb field of a nucleus in the next section.

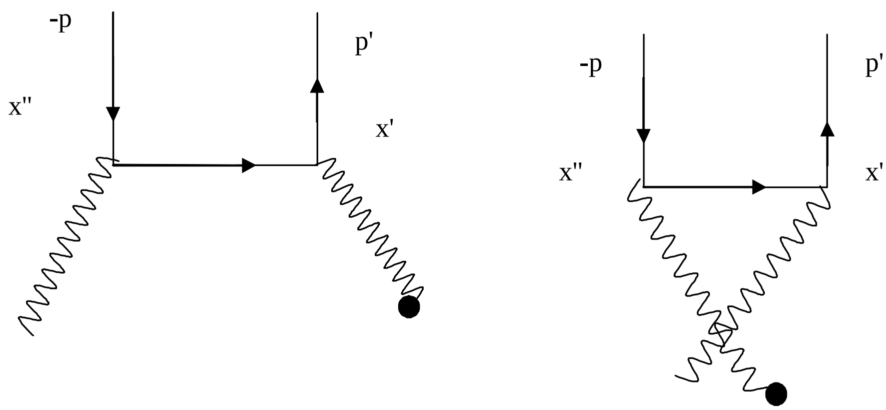
2.3 Electron-positron pair production process on Coulomb field of a nucleus

Pair production refers to the creation of elementary particle and its antiparticle, usually from photon[7]. This is allowed, provided the energy of the photon is greater than twice the rest mass energy of the particles. Energy and momentum are both conserved in this process.

In nuclear physics, this occurs when a high-energy photon interacts with an atomic nucleus, allowing it to produce an electron and positron with out violating conservation of momentum. Since the momentum of the initial photon must be absorbed by something, pair production can not occur in empty space out of a single photon;the nucleus is needed to conserve both momentum and energy.

In this section we employ covariant perturbation theory derived in section 2.2 to study electron-positorn pair production in the Coulomb field of a nucleus. In this process only the quantum electrodynamic effects have been incorporated. For this process there are two topologically inequivalent graphs shown in Fig. 2-1.

In this section we are going to calculate the relativistic transition amplitude and differential cross section for the process given in Fig. 2-1. i.e for electron-positron pair production process in Coulomb field of a nucleus.



. (a) (b)

Fig.2-1 (a) direct and (b) crossed Feynman's diagram for pair production in Coulomb field of a nucleus. p and p' are 4-momenta of positron and electron respectively.

The overall transition amplitude for this process is:

$$S_{fi}^{(2)} = S_{fi}^{(2)}|_A + S_{fi}^{(2)}|_B, \tag{2.23}$$

where $S_{fi}^{(2)}|_A$ and $S_{fi}^{(2)}|_B$ are transition amplitudes for direct and exchange processes respectively. The transition amplitude for a direct process is

$$S_{fi}^{(2)}|_A = e^2 \int d^4x' \int d^4x'' \sqrt{\frac{m}{EV}} \overline{u}^{(s')}\left(\overrightarrow{p'}\right) e^{-ip'.x'} [\gamma_\mu A_\mu(x')] K(x', x'') [\gamma_\nu A_\nu(x'')]$$
$$\times \sqrt{\frac{m}{EV}} v^{(s)}(\overrightarrow{p}) e^{-ip.x''}, \tag{2.24}$$

where

$$\gamma_\mu A_\mu(x') = \gamma_4 A_4(x') = \frac{\gamma_4 ez}{4\pi x'}, \tag{2.25}$$

and A_4 is the static Coulomb potential of the nucleus. The electron propagator is given as:

$$K\left(x', x''\right) = \frac{-i}{(2\pi)^4} \int \frac{d^4q \left(-i\gamma.q + m\right)}{q^2 + m^2} e^{iq.\left(x' - x''\right)}, \tag{2.26}$$

and

$$\gamma_\nu A_\nu(x') = \frac{\gamma.\epsilon}{\sqrt{2\omega V}} e^{ik.x'}, \tag{2.27}$$

where $A_\nu(x') = \frac{\epsilon_\nu}{\sqrt{2\omega V}} e^{ik.x'}$ is the normalized photon wave function of the incident photon. Substituting (2.25), (2.26), (2.27) in (2.24) we obtain

$$S_{fi}^{(2)}|_A = \frac{e^3 z}{(2\pi)^4} \sqrt{\frac{m^2}{2EE'\omega V^3}^{2}}^{2} \int d^4x' \int d^4x'' \overline{u}^{(s')}\left(\overrightarrow{p}'\right) \gamma_4 \frac{(-i\gamma.q + m)}{q^2 + m^2}$$
$$\times \frac{\gamma.\epsilon v^{(s)}\left(\overrightarrow{p}\right) e^{iq.(q-p')} e^{ix''.(k-p-q)}}{4\pi x'}. \tag{2.28}$$

Expressing integrals over d^4x' and d^4x'' in terms of the corresponding Delta functions as,

$$\int \frac{d^4x' e^{ix'.(k-p-p')}}{4\pi x'} = \frac{(2\pi)\,\delta\,(\omega - E - E')}{|k-p-p'|^2}, \tag{2.29}$$

$$\int d^4x'' e^{ix''.(k-p-q)} = (2\pi)^4\,\delta^{(4)}\,(k-p-q)\,, \tag{2.30}$$

and

$$\int \frac{d^4q\,(-i\gamma.q + m)\,\delta^{(4)}\,(k-p-q)}{q^2+m^2} = \frac{-i\gamma.\,(k-p)+m}{-2k.p}, \tag{2.31}$$

and substituting (2, 29), (2, 30), and (2, 31) in (2, 28) we obtain $S_{fi}^{(2)}$ for direct process as

$$S_{fi}^{(2)}|_A = e^3 z\sqrt{\frac{m^2}{2EE'\omega V^3}}\frac{(2\pi)\,\delta\,(\omega - E - E')\,\overline{u}^{(s')}(\overrightarrow{p})}{|k-p-p'|^2}\frac{(\gamma_4[-i\gamma.(k-p)+m])\gamma.\epsilon v^{(s)}\,(\overrightarrow{p})}{-2k.p}. \tag{2.32}$$

Applying similar techniques used above we obtain the transition probability for the exchange process as,

$$S_{fi}^{(2)}|_B = e^3 \int d^4x' \int d^4x'' \int d^4q\overline{u}^{(s')}\,\sqrt{\frac{m}{E'V}}\overline{u}^{(s')}(\overrightarrow{p}')e^{-ip'.x'}\frac{\gamma.\epsilon}{\sqrt{2\omega V}}e^{ik.x'}\frac{(-i\gamma.q+m)}{q^2+m^2}$$
$$\times e^{iq.(x-x')}\frac{i\gamma_4 ez}{4\pi x''}\sqrt{\frac{m}{EV}}v^{(s)}\,(\overrightarrow{p})\,e^{-ip.x''}. \tag{2.33}$$

Thus we can express $S_{fi}^{(2)}|_B$ as

$$S_{fi}^{(2)}|_B = e^3 z\sqrt{\frac{m^2}{2EE'\omega V^3}}\frac{(2\pi)\,\delta\,(\omega - E - E')\,\overline{u}^{(s')}\,(\overrightarrow{p}')\gamma.\epsilon}{|k-p-p'|^2}\frac{[-i\gamma.(p'-k)+m])\gamma_4 v^{(s)}\,(\overrightarrow{p})}{-2k.p'}. \tag{2.34}$$

The transion amplitude for the complete process is

$$S_{fi}^{(2)} = e^3 z\sqrt{\frac{m^2}{2EE'V^3}}\frac{(2\pi)\,\delta\,(\omega - E - E')}{|k-p-p'|^2}$$
$$\left[\frac{\overline{u}^{(s')}(\overrightarrow{p})}{}\frac{(\gamma_4[-i\gamma.(k-p)+m])\gamma.\epsilon}{-2k.p} + \frac{[-i\gamma.(p'-k)+m])\gamma_4 v^{(s)}\,(\overrightarrow{p})}{-2k.p'}\right]$$

The matrix element for this compete process is thus,

$$M_{fi} = \overline{u}^{(s')}\,(\overrightarrow{p'})\frac{\gamma_4[-i\gamma.(k-p)+m]\gamma.\epsilon}{2k.p} + \frac{\gamma.\epsilon}{}\frac{[-i\gamma.(p'-k)+m])\gamma_4}{2k.p'}v^{(s)}\,(\overrightarrow{p})\,. \tag{2.35}$$

From free particle Dirac equation

$$\gamma_4(i\gamma.p+m)\gamma.\epsilon v^{(s)}\,(\overrightarrow{p}) = \gamma_4\gamma.\epsilon\,\,(-i\gamma.p+m)v^{(s)}\,(\overrightarrow{p}) = 0\ , \tag{2.36}$$

and

$$\overline{u}^{(s')}\,\,(\overrightarrow{p}')\gamma.\epsilon(-i\gamma.p'+m])\gamma_4 = \overline{u}^{(s')}\,\,(\overrightarrow{p}')(i\gamma.p'+m\ )\gamma.\epsilon\gamma_4 = 0. \tag{2.37}$$

Substituting (2.36) and (2.37) in (2.35) M_{fi} becomes

$$M_{fi} = \overline{u}^{(s')}(\overrightarrow{p}')[\frac{-i\gamma_4(\gamma.k)(\ \gamma.\epsilon)}{2k.p} + \frac{i(\ \gamma.\epsilon)(\gamma.k)\gamma_4}{2k.p'}]v^{(s)}\,(\overrightarrow{p})\,. \tag{2.38}$$

Thus M$_{fi}$ can be written as

$$M_{fi} = \overline{u}^{(s')} \, (\overrightarrow{p'})\Gamma v^{(s)} \, (\overrightarrow{p}) \, , \tag{2.39}$$

where Γ which is the sum over the product of γ- matrices is

$$\Gamma = \frac{-i\gamma_4(\gamma.k)(\, \gamma.\epsilon)}{2k.p} + \frac{i(\, \gamma.\epsilon)(\gamma.k)\gamma_4}{2k.p'}. \tag{2.40}$$

Now,

$$|M_{fi}|^2 = |\overline{u}^{(s')}(\overrightarrow{p}')\Gamma v^{(s)} \, (\overrightarrow{p}) \, |^2. \tag{2.41}$$

In this process electron and positron produced in final state will exist in all polarization states. Thus, we sum the all polarizations of electron and positron in the final state. Hence we can express $|M_{fi}|^2$ as

$$|M_{fi}|^2 = \sum_{s,s'} [v^{(s)} \, (\overrightarrow{p}) \, \overline{\Gamma} u^{(s')}(\overrightarrow{p'})][\overline{u}^{(s')}(\overrightarrow{p'})\Gamma v^{(s)} \, (\overrightarrow{p})], \tag{2.42}$$

where

$$\overline{\Gamma} = \gamma_4 \Gamma^{\dagger} \gamma_4 = \frac{i(\gamma.\epsilon)(\gamma.k)\gamma_4}{2k.p} - \frac{i\gamma_4(\gamma.k)(\gamma.\epsilon)}{2k.p'}. \tag{2.43}$$

Making use of the relation $\sum\limits_{s} u^{(s)} \, (\overrightarrow{p}) \, \overline{u}^{(s)} \, (\overrightarrow{p}) = (\frac{-i\gamma.p+m}{2m})$ and

$\sum_s v^{(s)}(\overrightarrow{p})\,\overline{v}^{(s)}(\overrightarrow{p}) = -(\frac{i\gamma.p+m}{2m})$ (2.42) can be expressed as

$$|M_{fi}|^2 = \sum_{s,} \overline{v}^{(s)}(\overrightarrow{p})\,\overline{\Gamma}\frac{(-i\gamma.p'+m)}{2m}v^{(s)}(\overrightarrow{p})\,. \tag{2.44}$$

Thus

$$|M_{fi}|^2 = \frac{1}{4m^2}Tr\left[\overline{\Gamma}\frac{(-i\gamma.p'+m)}{2m}\Gamma\frac{(-i\gamma.p+m)}{2m}\right] \tag{2.45}$$

where Tr is trace which is the sum of the diagonal elements of a matrix. This expression simplifies considerably under ultra relativistic limit (where p>>m). Simplifying (2.45) we get:

$$\begin{aligned} |M_{fi}|^2 = \frac{1}{4m^2}Tr[&\frac{(\gamma.\epsilon)(\gamma.k)(\gamma_4)(\gamma.p')(\gamma_4)(\gamma.k)(\gamma.\epsilon)(\gamma.p)}{4\,(k.p)^2} \\ &- \frac{(\gamma.\epsilon)(\gamma.k)(\gamma_4)(\gamma.p')(\gamma.\epsilon)(\gamma.k)(\gamma_4)(\gamma.p)}{4(k.p)(k.p')} \\ &- \frac{(\gamma_4)(\gamma.k)(\gamma.\epsilon)(\gamma.p')(\gamma_4)(\gamma.k)(\gamma.\epsilon)(\gamma.p)}{4\,(k.p)\,(k.p')} \\ &+ \frac{(\gamma_4)(\gamma.k)(\gamma.\epsilon))(\gamma.p')(\gamma.\epsilon)(\gamma.k)(\gamma_4)(\gamma.p)}{4\,(k.p')^2}]. \end{aligned} \tag{2.46}$$

Doing this trace calculation and simplifying it follows that,

$$|M_{fi}|^2 = \frac{1}{4m^2}\left\{\frac{4\omega E + 2k.p' + 4\omega E'}{k.p} + \frac{3\omega E' + 4\omega E + 2k.p}{k.p'} - \frac{4\,(p.p')\,\omega^2}{(k.p)\,(k.p')} + 4\right\}, \tag{2.47}$$

where ω is energy of photon. The differential cross section, dσ , is defined as:

$$d\sigma = \frac{|S_{fi}|^2}{T\ |\overrightarrow{j}_{in}|\frac{1}{V}} \left\{ \frac{V d^3 \overrightarrow{p'} V d^3 \overrightarrow{p}}{(2\pi)^3\,(2\pi)^3} \right\}, \tag{2.48}$$

where $|\overrightarrow{j}_{in}|$ is the incident flux density which is given by,

$$|\overrightarrow{j}_{in}| = \frac{|\overrightarrow{k}|}{\omega V} = \frac{1}{V} \quad , \tag{2.49}$$

and$\left\{ \frac{V d^3 \overrightarrow{p'} V d^3 \overrightarrow{p}}{(2\pi)^3 (2\pi)^3} \right\}$ is the number of final states in the momentum intervals d$^3\overrightarrow{p}$ and d$^3\overrightarrow{p'}$, where phase space volumes can be expressed as,

$$d^3\overrightarrow{p'} = |\overrightarrow{p'^2}|d\overrightarrow{p'}d\Omega' = |\overrightarrow{p'}|E'dE'\ d\Omega', \tag{2.50}$$

and

$$d^3\overrightarrow{p} = |\overrightarrow{p^2}|d\overrightarrow{p}d\Omega = |\overrightarrow{p}|EdEd\Omega. \tag{2.51}$$

Substituting (2.51), (2.50), and (2.49) in (2.48) and integrating over the final states we obtain,

$$\begin{aligned} \frac{d^2\sigma}{d\Omega d\Omega'} &= \frac{e^6 z^2 m^2}{8m^2\omega V^3} \int dE \int dE' \frac{(2\pi)\,\delta\,(\omega - E - E')\,|\overrightarrow{p'}||\overrightarrow{p}|}{|k-p-p'|^4} [\frac{4\omega E' + 2k.p' + 4\omega E}{k.p} \\ &- \frac{4(p.p')\omega^2}{(k.p)\,(k.p')} + \frac{3\omega E' + 4\omega E + 2k.p}{k.p'} + 4]. \end{aligned} \tag{2.52}$$

Calculation of the integration over E' becomes

$$\int dE' \frac{\delta\left(\omega - E - E'\right)|\overrightarrow{p'}||\overrightarrow{p}|}{|k-p-p'|^4}[\frac{4\omega E' + 2k.p' + 4\omega E}{k.p} - \frac{4(p.p')\omega^2}{(k.p)\,(k.p')} + \frac{3\omega E' + 4\omega E + 2k.p}{k.p} + 4]$$
$$= \frac{|\overrightarrow{p'}||\overrightarrow{p}|}{|k-p-p'|^4}[\frac{4\omega^2 + 2k.p'}{k.p} - \frac{4\omega^2\,(p.p')}{(k.p)\,(k.p')} + \frac{3\omega^2 - E\omega + 2k.p}{k.p'} + 4]. \tag{2.53}$$

Substituting (2.53) in (2.52) we obtain

$$\frac{d^2\sigma}{d\Omega d\Omega'} = \frac{e^6 z^2}{\omega\,(2\pi)^5} \int \frac{dE|\overrightarrow{p'}||\overrightarrow{p}|}{8|k-p-p'|^4}[\frac{4\omega^2 + 2(k.p')}{k.p} - \frac{4\omega^2\,(p.p')}{(k.p)\,(k.p')} + \frac{3\omega^2 - E\omega + 2k.p}{k.p'} + 4]. \tag{2.54}$$

Integrating (2.54) and simplifying we obtain

$$\frac{d^2\sigma}{d\Omega d\Omega'} = \frac{|\overrightarrow{p'}||\overrightarrow{p}|e^6 z^2}{8\omega\,(2\pi)^5\,|k-p-p'|^4}[\frac{4\omega^2 E + 2E\,(k.p')}{k.p} - \frac{4\omega^2 E\,(p.p')}{(k.p)\,(k.p')}$$
$$+ \frac{3\omega^2 E - 0.5\omega E^2 + 2E\,(k.p)}{k.p'} + 4E]. \tag{2.55}$$

Thus we can express $\frac{d^2\sigma}{d\Omega d\Omega'}$ as

$$\frac{d^2\sigma}{d\Omega d\Omega'} = \frac{|\overrightarrow{p'}||\overrightarrow{p}|e^6 z^2}{8\omega\,(2\pi)^5\,(p^2 + p'^2 - 2k.p - 2k.p' + 2p.p')^2}[\frac{4\omega^2 E + 2E\,(k.p')}{k.p} - \frac{4\omega^2 E\,(p.p')}{(k.p)\,(k.p')}$$
$$+ \frac{3\omega^2 E - 0.5\omega E^2 + 2E\,(k.p)}{k.p'} + 4E]. \tag{2.56}$$

Now lets consider the kinematical description of this process in Fig 2-2 where the incident momentum of real photon k, and momentum of electron and positron is p' and p respectively.

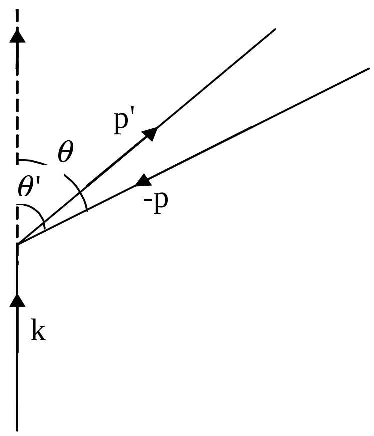


Fig. 2-2 kinematical description of the said process where momentum of real photon is k, and momenta of electron and positron are p' and p respectively.

In Fig 2-2, p' is momentum of created electron, p momentum of created positron, $\beta \equiv (\theta - \theta')$ is angle between p' and p, $\gamma \equiv (180 - \theta)$ is angle between k and p, and $\alpha \equiv (180 - \theta')$ is angle between k and p'.

Making use of kinematics of the process described in Fig. 2-2 and employing ultra relativistic limit where $E^2 >> m_e^2$ and $p^2 >> m_e^2$, $|p'| \cong E'$ and $|k| \cong \omega$,we get differential cross section for the above process which is given as,

$$\frac{d^2\sigma}{d\Omega d\Omega'} = \frac{EE'e^6z^2}{8\omega\,(2\pi)^5\,[2\omega ECos\gamma - 2\omega E'Cos\alpha + 2EE'Cos\beta + E^2 + E'^2]^2} \times \left\{ \frac{4E'Cos\alpha + 2\omega}{Cos\gamma} - \frac{4ECos\beta}{Cos\gamma Cos\alpha} + \frac{-0.5E^2 + 3\omega E + 2E^2Cos\gamma}{E'Cos\alpha} + 4E \right\}. \quad (2.57)$$

It is observed that the differential cross section for this process depends linearly on the energies[1] of the created electron and positron and also is a function of the kinematical parameters β, γ, and α We now study the more involved processes:

a) electron-positron pair production by collision of virtual photons in high energy electron-electron scattering in chapter 3

b) electron-positron pair production by Bremsstrahlung photon in high energy proton-proton scattering in and chapter 4.

Chapter 3

Electron-positron pair production by collision of two virtual photons in electron-electron scattering at high energies

In this section we employ covariant perturbation theory derived in chapter- 2 to study electron positron pair production by collision of two virtual photons in electron-electron scattering at high energies[12, 13, 14, 15]. Now let's consider two electrons of momenta p_1 and p_2 which interacted and exchange two virtual photons of momenta q_1 and q_2 and these photons annihilate to produce electron-proton pair as shown in Fig. 3-1. This is the 4^{th}-order $\mathrm{O}(\alpha^4)$ process. We calculate the cross section for this process as indicated below.

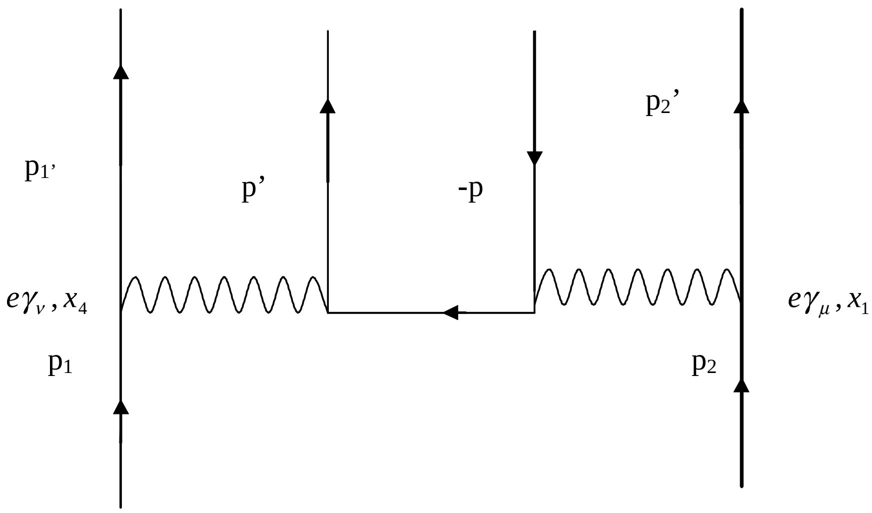


Fig. 3-1 Feynman diagram for electron positron pair production by collision of two virtual photons in electron electron scattering: p_1 and p_2 are 4-momenta of the two incident electrons, p and p' are 4-momenta of the created positron and electron respectively. p_1' and p_2' are 4-momenta of the scattered electrons .

We know that the photon propagator is $D_F(x, x')$ is obtained from solutions of Maxwell's equation $\Box^2 A_\mu(x) = j_\mu(x)$ where $A_\mu(x)$ is electromagnetic field and $j_\mu(x)$ is the 4-current density and $\Box^2 = \nabla^2 - \frac{\partial^2}{c^2 \partial t^2}$. By considering a unit source problem $\Box^2 D_F(x, x') = \delta^{(4)}(x - x')$,it is seen that $D_F(x, x')$ has the same value for t>t' and t<t', since photon is its own anti- particle. Thus the positive frequency photon propagating forward in time and a negative frequency photon propagating backward in time contributes equally to the $A_\mu(x)$ field. We apply Feynman's rules to Fig.3-1. We represent the incident fermion's wave function of momentum p and spin s entering the space

time point x by $\Psi(x) = \sqrt{\frac{m}{EV}} u^{(s)}(\overrightarrow{p})\, e^{ip.x}$, and the scattered fermion of momentum p' and spin s' leaving space time point x by $\Psi(x) = \sqrt{\frac{m}{E'V}} \overline{u}^{(s')}(\overrightarrow{p'}) e^{-ip'.x}$. We represent the photon propagator by $D_F(x,x') = \frac{1}{(2\pi)^4}\int d^4q(\frac{-1}{q^2})e^{iq.(x'-x)}$ and the electron that goes from point x to point x' by electron propagator $K(x,x') = \frac{1}{(2\pi)^4}\int d^4q(\frac{-i\gamma.q+m}{m^2+q^2})e^{iq.(x'-x)}$ .We can then write the S- matrix element for this process as :

$$\begin{aligned} S_{fi}^{(4)} &= (-ie)^4 \int d^4x_1 \int d^4x_2 \int d^4x_3 \int d^4x_4 \sqrt{\frac{m_e}{E_1'V}} \overline{u}^{(s_1')}(\overrightarrow{p_1'}) e^{-ip_1'.x_4}\, \gamma_\nu \sqrt{\frac{m_e}{E_1V}}\, u^{(s_1)}(\overrightarrow{p_1}) \\ & e^{ip_1.x_4} \frac{1}{(2\pi)^4} \int d^4q_2(\frac{-i}{q_2^2}) e^{iq_2.(x_3-x_4)} \sqrt{\frac{m_e}{E'V}} \overline{u}^{(s')}(\overrightarrow{p'}) e^{-ip'.x_3} \gamma_\nu \frac{1}{(2\pi)^4} \int d^4q_3 \left(\frac{-i\gamma.q_3+m_e}{q_3^2+m_e^2}\right) \\ & \times e^{iq_3.(x_3-x_2)} \gamma_\mu \sqrt{\frac{m_e}{EV}} v^{(s)}(\overrightarrow{p})\, e^{-ip.x_2} \frac{1}{(2\pi)^4} \int d^4q_1(\frac{-i}{q_1^2}) e^{iq_1.(x_2-x_1)} \sqrt{\frac{m_e}{E_2'V}} \overline{u}^{(s_2')}(\overrightarrow{p_2'}) \\ & e^{-ip_2'.x_1} \times \gamma_\mu \sqrt{\frac{m_e}{E_2V}} u^{(s_2)}(\overrightarrow{p_2}) e^{ip_2.x_1}. \end{aligned} \tag{3.1}$$

This can in turn be expressed as,

$$\begin{aligned} S_{fi}^{(4)} &= \frac{e^4 m_e^3}{V^3[(2\pi)^4]^3 \sqrt{E_2' E_2 E_1 E_1' E E'}} \\ & \times \int d^4x_1 \int d^4x_2 \int d^4x_3 \int d^4x_4 \int d^4q_1 \int d^4q_2 \int d^4q_3 \\ & \times \overline{u}^{(s_2')}(\overrightarrow{p_2'})\gamma_\nu u^{(s_2')}(\overrightarrow{p_2'})(\frac{-i}{q_2^2})\overline{u}^{(s')}(\overrightarrow{p'})\gamma_\nu \left(\frac{-i\gamma.q_3+m_e}{q_3^2+m_e^2}\right)\gamma_\mu \\ & \times v^{(s)}(\overrightarrow{p})\,(\frac{-i}{q_1^2})\overline{u}^{(s_1')}(\overrightarrow{p_1'})\gamma_\mu \overline{u}^{(s_1')}(\overrightarrow{p_1'}) e^{ix_1.(p_1-p_1'-q_1')} \\ & \times e^{ix_2.(q_1-p'-q_3')} e^{ix_3.(q_2-p-q_3)} e^{ix_4.(p_2-p_2'-q_2)}. \end{aligned} \tag{3.2}$$

We express the integrals over $d^4x_1,\ d^4x_2,\ d^4x_3,\ and\ d^4x_4$ in terms of the corresponding

Delta functions as,

$$\int d^4x_1\ e^{ix_1.(p_1-p_1'-q_1')} = (2\pi)^4\,\delta^{(4)}\left(p_1-p_1'-q_{1'}\right); \tag{3.3}$$

$$\int d^4x_2 e^{ix_2.(q_1-p-q_3')} = (2\pi)^4\,\delta^{(4)}(q_1-p-q_3'), \tag{3.4}$$

$$\int d^4x_3 e^{ix_3.(q_2-p'-q_3)} = (2\pi)^4\delta^{(4)}(q_2-p'-q_3), \tag{3.5}$$

$$\int d^4x_4 e^{ix_4.(p_2-p_2'-q_2)} = (2\pi)^4\delta^{(4)}(p_2-p_2'-q_2), \tag{3.6}$$

$$\frac{-1}{q_1^2}\delta^{(4)}\left(p_1-p_1'-q_1\right) = \frac{-1}{|p_1-p_1'|^2}\delta^{(4)}\left(p_1-p_1'-q_1\right), \tag{3.7}$$

$$\frac{-1}{q_2^2}\delta^{(4)}(p_2-p_2'-q_2) = \frac{-1}{|p_2-p_2'|^2}\delta^{(4)}(p_2-p_2'-q_2), \tag{3.8}$$

$$\int d^4q_1\delta^{(4)}(q_1-q_3-p)\delta^{(4)}\left(p_1-p_1'-q_1\right) = \delta^{(4)}(p_1-p_1'-p-q_3), \tag{3.9}$$

$$\int d^4q_2\delta^{(4)}(p_2-p_2'-q_2)\delta^{(4)}(q_2-p'-q_3) = \delta^{(4)}(p_2-p_2'-p'-q_3), \tag{3.10}$$

$$(\frac{-i\gamma.q_3+m_e}{q_3^2+m_e^2})\delta^{(4)}(p_1-p_1'-p-q_3)$$
$$=\frac{-i\gamma.(p_1-p_1'-p)+m_e}{|p_1-p_1'-p|^2+m_e^2}\delta^{(4)}(p_1-p_1'-p-q_3), \tag{3.11}$$

and

$$\int d^4q_3\delta^{(4)}(p_1-p_1'-p-q_3)\delta^{(4)}(p_2-p_2'-p'-q_3)$$
$$=\delta^{(4)}(p_1+p_2-p_1'-p_2'-p'-p). \tag{3.12}$$

Substituting (3.3), (3.4), (3.5), (3.6), (3.7), (3.8), (3.9), (3.10), (3.11), and (3.12) into (3.2) we get

$$S_{fi}^{(4)}=\frac{e^4m_e^3}{V^3\sqrt{E_2'E_2E_1E_1'EE'}}\,(2\pi)^4\,\delta^{(4)}(p_1+p_2-p_1'-p_2'-p'-p)\overline{u}^{(s_2')}(\overrightarrow{p_2'})\gamma_\nu u^{(s_2')}(\overrightarrow{p_2'})$$
$$(\frac{-1}{|p_2-p_2'|^2})\overline{u}^{(s')}(\overrightarrow{p'})\gamma_\nu(\frac{-i\gamma.(p_1-p_1'-p)+m_e}{|p_1-p_1'-p|^2+m_e^2})\gamma_\mu v^{(s)}\,(\overrightarrow{p})\,\frac{-1}{|p_1-p_1'|^2}\overline{u}^{(s_1')}(\overrightarrow{p_1'})\gamma_\mu u^{(s_1)}(\overrightarrow{p_1}), \tag{3.13}$$

where $\delta^{(4)}(p_1+p_2-p_1'-p_2'-p'-p)$ corresponds to the over all energy-momentum conservation.

The M-matrix element for this process is:

$$M_{fi} = \left[\begin{array}{c} \overline{u}^{(s_2')}(\overrightarrow{p_2'})\gamma_\nu u^{(s_2)}(\overrightarrow{p_2})\}(\frac{-1}{|p_2-p_2'|^2})\{\overline{u}^{(s')}(\overrightarrow{p'})\gamma_\nu(\frac{-i\gamma.(p_1-p_1'-p)+m_e}{|p_1-p_1'-p|^2+m_e^2})\gamma_\mu v^{(s)}\,(\overrightarrow{p}) \\ \times(\frac{-1}{|p_1-p_1'|^2})\{\overline{u}^{(s_1')}(\overrightarrow{p_1'})\gamma_\mu u^{(s_1)}(\overrightarrow{p_1}) \end{array} \right] \tag{3.14}$$

To evaluate differential cross section for this process,we need to evaluate $|S_{fi}^{(4)}|^2$ which is expressed as

$$|S_{fi}^{(4)}|^2 = \frac{e^8 m_e^6 (2\pi)^{(4)}\delta^{(4)}(p_1+p_2-p_1'-p_2'-p'-p)VT|M_{fi}|^2}{V^6 E_2' E_2 E_1 E_1' E E'}, \tag{3.15}$$

where V is interaction volume and T is interaction time. The differential cross section for this process dσ is:

$$d\sigma = \frac{|S_{fi}^{(4)}|^2}{VT|\overrightarrow{j}_{in}|\frac{1}{v}} \left\{ \frac{(Vd^3\overrightarrow{p'})(Vd^3\overrightarrow{p})(Vd^3\overrightarrow{p_2'})(Vd^3\overrightarrow{p_1'})}{(2\pi)^3\,(2\pi)^3\,(2\pi)^3\,(2\pi)^3} \right\}. \tag{3.16}$$

Here $|\overrightarrow{j}_{in}|$ is the incident flux density which is

$$|\overrightarrow{j}_{in}| = \frac{2|\overrightarrow{\beta}|}{V}, \tag{3.17}$$

and $\left\{ \frac{(Vd^3\overrightarrow{p'})(Vd^3\overrightarrow{p})(Vd^3\overrightarrow{p_2'})(Vd^3\overrightarrow{p_1'})}{(2\pi)^3(2\pi)^3(2\pi)^3(2\pi)^3} \right\}$ is density of final state for this process. The phase space volume can be expressed as:

$$d^3\overrightarrow{p'} = |\overrightarrow{p'}|E'dE'd\Omega', \tag{3.18}$$

$$d^3\overrightarrow{p'_1} = |\overrightarrow{p'_1}|E'_1 dE'_1 d\Omega'_1, \tag{3.19}$$

$$d^3\overrightarrow{p} = |\overrightarrow{p}|EdEd\Omega \tag{3.20}$$

Expressing phase space volume $d^3\overrightarrow{p'_2}$ as

$$d^3\overrightarrow{p'_2} = 2E'_2 \int_{-\infty}^{\infty} d^4p'_2 \delta\left(p'^2_2 + m_e^2\right)\theta\left(p'_{20}\right). \tag{3.21}$$

and substituting (3.17), (3.18), (3.19), (3.20)and (3.21) in (3.16) we get

$$\begin{aligned}\frac{d^3\sigma}{d\Omega d\Omega' d\Omega'_1} &= \frac{e^8 m_e^4 m_p^2}{(2\pi)^8\beta}\int dE \int dE'_1 \int dE'_2 \frac{\delta^{(4)}(p_1+p_2-p'_1-p'_2-p'-p)|\overrightarrow{p}|||\overrightarrow{p'_1}||\overrightarrow{p'}|}{E_1E_2} \\ &\times |M_{fi}|^2\delta\left(p'^2_2+m_e^2\right)\theta\left(p'_{20}\right),\end{aligned} \tag{3.22}$$

where

$$|M_{fi}|^2\delta^{(4)}(p_1+p_2-p'_1-p'_2-p'-p) = |M_{fi}|^2_{p'_2=p_1+p_2-p'_1-p'-p}\delta^{(4)}(p_1+p_2-p'_1-p'_2-p'-p), \tag{3.23}$$

and

$$\int_{-\infty}^{\infty} d^4p_2'\delta^{(4)}(p_1+p_2-p_1'-p_2'-p'-p)\delta\left(p_2'^2+m_e^2\right)\theta\left(p_{20}'\right)$$

$$=\delta[(p_1+p_2-p_1'-p'-p)^2+m_e^2]\theta(p_{10}+p_{20}-p_{10}'-p_0'-p_0). \tag{3.24}$$

The calculation simplifies considerably in the ultra relativistic limit where

$$\mathrm{E}^2>>\mathrm{m}_e^2,\ \mathrm{p}^2>>\mathrm{m}_e^2,\ \mathrm{E}\cong|\overrightarrow{p}|, \tag{3.25}$$

and considering center of mass frame of reference($\overrightarrow{p_1}+\overrightarrow{p_2}=0$) and taking E_1=E_2=E, E_1'=E_2'=E$'$, we get.

$$\theta(\mathrm{p}_{10}+\mathrm{p}_{20}-\mathrm{p}_{10}'-\mathrm{p}_0'-\mathrm{p}_0)=\theta(\mathrm{E}-2\mathrm{E}'). \tag{3.26}$$

Also,

$$\begin{aligned}|p_1+p_2-p_1'-p'-p|^2 &= (\overrightarrow{p_1}+\overrightarrow{p_2}-\overrightarrow{p'}_1-\overrightarrow{p'}-\overrightarrow{p})^2-(E-2E')^2.\\ &=|\overrightarrow{p_1'}|^2+|\overrightarrow{p'}|^2+|\overrightarrow{p}|^2+2\overrightarrow{p_{1'}}.\overrightarrow{p}+2\overrightarrow{p_1'}.\overrightarrow{p'}+2\overrightarrow{p}.\overrightarrow{p'}\\ &-(E^2-4EE'+4E'^2).\end{aligned} \tag{3.27}$$

Now lets consider the kinematical description of this process where the momenta of incident and scattered particles as shown in Fig. 3-2.

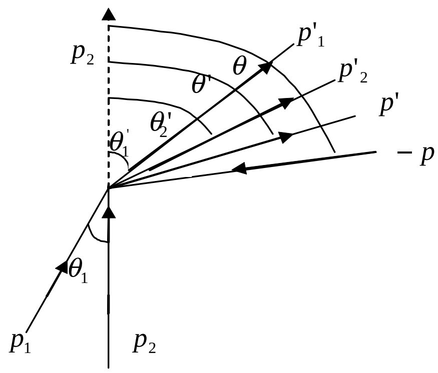


Fig.3-2 Kinematics of the $e^-+e^+ \longrightarrow e^-+e^-+e^-+e^+$ through collision two virtual photons is described in this figure.

In Fig. 3-2:

(i) p_1 is 4-momenta of the first incident electron.

(ii) p_2 is 4-momenta of the second incident electron.

(iii) $p'_1$is 4-momenta of the first scattered electron.

(iv) p'_2 is 4-momenta of the second scattered electron.

(v) -p is 4-momenta of created positron.

(vi) p' is 4-momenta of created electron.

(vii) $\theta_1 \equiv(\theta - \theta')$ is angle between p and p'.

(viii) $\theta_2 \equiv(\theta - \theta'_1)$ is angle between p and p'_1.

(ix) $\theta_3 \equiv(\theta - \theta'_2)$ is angle between p_1 and p'_2.

(x) $\theta_4 \equiv(180-\theta)$ is angle between p and p_2.

(xi) $\theta_5 \equiv[180+(\theta - \theta_1)]$ is angle between p and p_1.

(xii) $\alpha_1 \equiv$[180+$(\theta_2' - \theta_1)$] is angle between p_1 and p_2'.

(xiii) $\alpha_2 \equiv(180 - \theta_2')$ is angle between p_2 and p_2'.

(xiv) $\alpha_3 \equiv \theta_1$ is angle between p_1 and p_2.

(xv) $\gamma_1 \equiv(\theta_1' - \theta_2')$ is angle between p_1' and p_2'.

(xvi) $\gamma_2 \equiv$[180+$(\theta_1' - \theta_1)$] is angle between $p_1'$and p_1'.

(xvii) $\gamma_3 \equiv$(180-θ_1') is angle between p_1' and p_2.

(xviii) $\beta_1 \equiv(\theta' - \theta_1')$ is angle between p' and p_1'.

(xix) $\beta_2 \equiv(\theta' - \theta_2')$ is angle between p' and p_2'.

(xx) $\beta_3 \equiv$(180-θ') is angle between p' and p_2.

(xxi) $\beta_4 \equiv$[180+$(\theta' - \theta_1)$ is angle between $p_1$and p'.

Making use of the kinematics of the process described in Fig. 3-2, for p_1'=p$'$=E$'$and p_1=p=E we get

$$
\begin{aligned}
|p_1 + p_2 - p_1' - p' - p|^2 &= E^2 + 2E'^2 + 2EE'Cos\theta_2 + 2E'^2 Cos\beta_1 + 2EE'Cos\theta_1 \\
&- \left(E^2 - 4E'E + 4E'^2\right) \\
&= 2E'^2\left(Cos\beta_1 - 1\right) + 2EE'\left(Cos\theta_2 + Cos\theta_1 + 2\right). \qquad (3.28)
\end{aligned}
$$

Substituting (3.23), (3.24), (3.25) ,(3.26), and (3.27)in (3.22) we get

$$\frac{d^3\sigma}{d\Omega d\Omega' d\Omega'_1} = \frac{e^8 m_e^6 |\overrightarrow{p_{1'}}||\overrightarrow{p'}||\overrightarrow{p}|}{(2\pi)^8 \overrightarrow{\beta} E_1 E_2} \int dE \int dE'_1 \int_0^{\frac{E}{2}} dE |M_{fi}|^2_{p'_2 = p_1 + p_2 - p'_1 - p' - p} [\delta \left(2E'^2 Cos\beta_1 - 1\right)$$

$$+ 2EE' \left(Cos\theta_2 + Cos\theta_1 + 2\right)]. \tag{3.29}$$

Thus after carrying out the integrals we will obtain,the differential cross section for this process as

$$\frac{d^3\sigma}{d\Omega d\Omega' d\Omega'_1} = \frac{e^8 m_e^6 E \ln E \left(Cos\beta_1 - 1\right) |M_{fi}|^2_{p'_2 = p_1 + p_2 - p'_1 - p' - p}}{2 (2\pi)^8 \left(Cos\beta_1 + Cos\theta_2 + Cos\theta_1 + 1\right) \left(Cos\theta_2 + Cos\theta_1 + 2\right)}. \tag{3.30}$$

Finally, (3.30) can be written as

$$\frac{d^3\sigma}{d\Omega d\Omega' d\Omega'_1} = \frac{e^8 m_e^6 E \ln E}{2 (2\pi)^8} \left\{ \frac{(Cos\beta_1 - 1)|M_{fi}|^2_{p'_2 = p_1 + p_2 - p'_1 - p' - p}}{\left(Cos\beta_1 + Cos\theta_2 + Cos\theta_1 + 1\right) \left(Cos\theta_2 + Cos\theta_1 + 2\right)} \right\}, \tag{3.31}$$

where from (3.14) we can express $|\mathrm{M}_{fi}|^2$ as

$$|M_{fi}|^2 = \left\{ \frac{\begin{array}{c} \{[\overline{u}^{(s'_2)}(\overrightarrow{p'_2})\gamma_\nu u^{(s_2)}(\overrightarrow{p_2})]^\dagger [\overline{u}^{(s'_2)}(\overrightarrow{p'_2})\gamma_\nu u^{(s_2)}(\overrightarrow{p_2})]\} \\ \{[\overline{u}^{(s')}(\overrightarrow{p'})\Gamma v^{(s)}(\overrightarrow{p})]^\dagger [\overline{u}^{(s')}(\overrightarrow{p'})\Gamma v^{(s)}(\overrightarrow{p})]\} \\ \{[\overline{u}^{(s'_1)}(\overrightarrow{p'_1})\gamma_\mu u^{(s)}(\overrightarrow{p})]^\dagger [\overline{u}^{(s'_1)}(\overrightarrow{p'_1})\gamma_\mu u^{(s)}(\overrightarrow{p})]\} \end{array}}{|p_2 - p'_2|^4 |p_1 - p'_1|^4 (|p_1 - p'_1 - p|^2 + m_e^2)^2} \right\}, \tag{3.32}$$

where $\Gamma=\gamma_\nu[-i\gamma.(p_1 - p_1'-p)+m_e]\gamma_\mu$. If the incident electron and proton are un polarized, then we average over the initial spin states and sum over the final spin states. Thus we get,

$$|M_{fi}|^2 = \frac{1}{2}\sum_{s_2',s_2}\sum_{s_1',s_1}\sum_{s',s}\left\{\begin{array}{c} \{[u^{(s_2)}(\overrightarrow{p_2})\overline{\gamma_\nu}u^{(s_2')}(\overrightarrow{p_2'})][\overline{u}^{(s_2')}(\overrightarrow{p_2'})\gamma_\nu u^{(s_2)}(\overrightarrow{p_2})]\} \\ \{[\overline{v}^{(s)}(\overrightarrow{p})\overline{\Gamma}u^{(s')}(\overrightarrow{p'})][\overline{u}^{(s')}(\overrightarrow{p'})\Gamma v^{(s)}(\overrightarrow{p})]\} \\ \dfrac{\{[\overline{u}^{(s_1)}(\overrightarrow{p_1})\overline{\gamma_\mu}u^{(s_1')}(\overrightarrow{p_1'})][\overline{u}^{(s_1')}(\overrightarrow{p_1'})\gamma_\mu u^{(s_1)}(\overrightarrow{p_1})]\}}{|p_2 - p_2'|^4|p_1 - p_1'|^4(|p_1 - p_1' - p|^2+m_e^2)^2} \end{array}\right\}, \qquad (3.33)$$

where $\overline{\gamma_\nu}=.\gamma_4\gamma_\nu\gamma_4,\ \overline{\Gamma}=\gamma_4\Gamma^\dagger\gamma_4,$ *and* $\overline{\gamma_\mu}=\gamma_4\gamma_\mu\gamma_4$. Thus,making use of the relation $\sum_s u^{(s)}(\overrightarrow{p})\overline{u}^{(s)}(\overrightarrow{p}) = \frac{(-i\gamma.p+m)}{2m}$ and $\sum_s v^{(s)}(\overrightarrow{p})\overline{v}^{(s)}(\overrightarrow{p}) = -\frac{(i\gamma.p+m)}{2m}$, (3.33) can be expressed as,

$$|M_{fi}|^2 = \frac{1}{16m_e^2 m_p}\sum_{s_2}\sum_{s_1}\sum_{s}\left\{\begin{array}{c} \{u^{(s_2)}(\overrightarrow{p_2})\overline{\gamma_\nu}(-i\gamma.p_2'+m_e)\gamma_\nu u^{(s_2)}(\overrightarrow{p_2})\} \\ \{v^{(s)}(\overrightarrow{p})\overline{\Gamma}(-i\gamma.p'+m_e)\Gamma\overline{v}^{(s)}(\overrightarrow{p})\}\{[\overline{u}^{(s_1)}(\overrightarrow{p_1})\overline{\gamma_\mu} \\ \dfrac{(-i\gamma.p_1'+m_e)\gamma_\mu u^{(s_1)}(\overrightarrow{p_1})]\}}{|p_2 - p_2'|^4|p_1 - p_1'|^4(|p_1 - p_1' - p|^2+m_e^2)^2} \end{array}\right\}. \qquad (3.34)$$

We can thus express $|M_{fi}|^2$ in above equation as product of trace over $\gamma-$matrices as

$$|M_{fi}|^2 = \left\{ \frac{\begin{array}{c} Tr[\overline{\gamma_\nu}(-i\gamma.p_2' + m_e)\gamma_\nu(-i\gamma.p_2 + m_e)]Tr[\overline{\Gamma}(-i\gamma.p' - m_e)\Gamma(-i\gamma.p - m_e)] \\ \times Tr[\overline{\gamma_\mu}(-i\gamma.p_1' + m_e)\gamma_\mu(-i\gamma.p_1 + m_e)] \end{array}}{128m_e^6|p_{2-}p_2'|^4|p_{1-}p_1'|^4(|p_{1-}p_1' - p|^2 + m_e^2)^2} \right\}. \tag{3.35}$$

After doing this long trace calculation we get,

$$|M_{fi}|^2 = \left\{ \frac{\begin{array}{c} \{2(p_2'.p_1)(p_2.p_1) + 2(p_2'.p_1)\,(p_2.p_1')\}\{-32\,(p_1.p')\,(p_1.p) + 16\,(p.p') \\ +32\,(p_1.p')\,(p_1.p) - 32\,(p_1.p_1')\,(p.p) + 32\,(p_1'.p)\,(p'.p_1) + 32\,(p.p)\,(p'.p_1) \\ -32\,(p_1.p')\,(p_1'.p) + 16\,(p_1.p_1')\,(p'.p) - 32\,(p.p)\,(p'.p_1) - 16\,(p.p')\,(p.p)\} \end{array}}{128m_e^6\,(p_2^2 + p_2^{2\prime} + p_2.p_2')^2\,[p_1^2 + p_1^{2\prime} - p_1.p_1']^2\,(p_1^2 + p_1^{2\prime} + 2p_1.p_1' + 2p_1'.p)^2} \right\}. \tag{3.36}$$

Making use of kinematics of the process described in fig. 3-2 (3.35) becomes,

$$|M_{fi}|^2 = \left[\frac{\begin{array}{c} -(32p_1p_2p'p'_1((p^2-p_1^2)Cos\theta_1+2p_1(p+p_1Cos\theta_5) \\ Cos\beta_4+2((p+p_1Cos\theta_5)Cos\beta_1 \\ +p_1(Cos\theta_2Cos\beta_4-Cos\theta_1Cos\gamma_2))p'_1-(Cos\theta_1-2Cos\theta_2Cos\beta_1)p_1'^2) \\ p'_2(p_1Cos\gamma_3Cos\alpha_1+Cos\gamma_1Cos\alpha_3p'_2) \end{array}}{\begin{array}{c} 128m_e^6\{((p_1^2+2p_1p'_1Cos\gamma_2+p_1'^2)^2(p_1(p_1+2pCos\theta_5) \\ +2(pCos\theta_2+p_1Cos\gamma_2)p'_1+p_1'^2)^2(p_2^2+2p_2Cos\alpha_2p'_2+p_2'^2)^2\} \end{array}}\right]. \tag{3.37}$$

Substituting (3.37) in (3.32) and simplifying we obtain the differential cross section for the above process as;

$$\frac{d^3\sigma}{d\Omega d\Omega' d\Omega'_1} = \frac{e^8 E\ln E}{256\,(2\pi)^5}\left[\frac{\begin{array}{c} (32pp_2(Cos\beta_1-1)p'p'_1((p^2-p_1^2)Cos\theta_1+2p_1(p+p_1Cos\theta_5) \\ \times Cos\beta_4+2((p+p_1Cos\theta_5)Cos\beta_1(p_1(Cos\theta_2Cos\beta_4 \\ -Cos\theta_1Cos\gamma_2))p'_1-(Cos\theta_1-2Cos\theta_2Cos\beta_1)p_1'^2)p'_2 \\ (p_1Cos\gamma_3Cos\alpha_1+Cos\gamma_1Cos\alpha_3p'_2)) \end{array}}{\begin{array}{c} (Cos\beta_1+Cos\theta_2+Cos\theta_1+1)\,(Cos\theta_2+Cos\theta_1+2) \\ (p_1^2+2p_1p'_1Cos\gamma_2+p_1'^2)^2(p_1(p_1+2pCos\theta_5 \\ +2(pCos\theta_2+p_1Cos\gamma_2)p'_1+p_1'^2)^2(p_2^2+2p_2Cos\alpha_2p'_2+p_2'^2)^2 \end{array}}\right]. \tag{3.38}$$

It is thus seen that the cross section varies as ElnE[4, 5], where E is the energy of the incident electrons. Also cross section depends on the kinematical varaiables described in Fig. 3-2 as well as on the momenta of the particles involved in the process. A similar feature of cross section is also obtained in the related process of $\gamma + \gamma \longrightarrow$hadron in Ref. [4, 5].We now study the electron-positron pair production by Bremsstrahlung photon in high energy proton-proton scattering in chapter-4.

Chapter 4

Electron-positron pair production by Bremsstrahlung photon in high energy proton-proton scattering

When an electron passes through the Coulomb field of a nucleus it is deflected. Associated with this deflection is an acceleration which, according to Classical theory, results in radiation. According to quantum electrodynamics[3], there is a certain probability that the incident electron will make a transition to a different electron state with a photon emitted, while in the field of a nucleus. Interaction with a field of the nucleus is necessary to satisfy conservation of energy and momentum. that is, the electron can not emit a photon and make a transition to a different electron state while traveling along in a vacuum. Fast electrons and positrons lose energy by collision with other electrons and may be scattered through some angle in such collisions. They lose very little energy to nuclei, because of the disparity in mass, but can suffer larger deflections. In all collisions in which accelerations are experienced, electrons and positrons also lose energy by the radiative processes known as bremsstrahlung . Heavy particles do not radiate appreciably in this way because their accelerations are much smaller[9-10].

In this chapter we return to the general discussion of scattering formulation responsible for emission of photon from an accelerated proton moving in the electromagnetic field produced by another proton where this photon is annihilated to produce e^+e^- pair as shown in Fig. 4-1 In this process also QED effects have been incorporated.

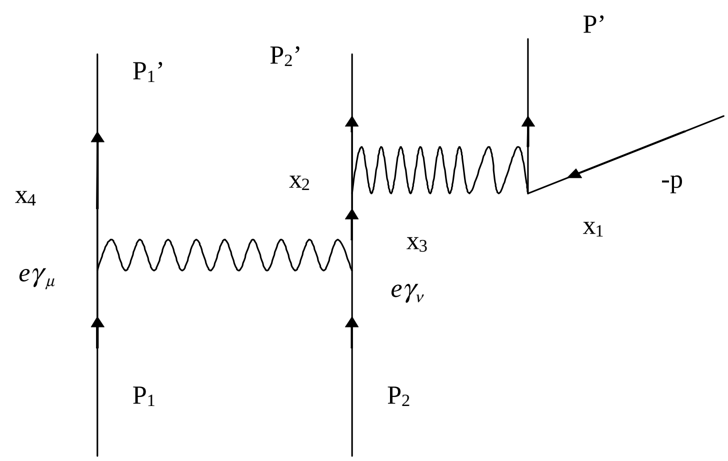


Fig. 4-1 Feynman diagram for e^+e^- pair production by Bremsstrahlung photon; where are p_1 and p_2 are 4-momenta of incident protons, p_1' and p_2' are 4- momenta of scattered protons, p, p' are 4-momenta of e^+e^- pair, m_p and m_e are masses of proton and electron respectively.

Having studied e^+e^- pair production by collision of two virtual photons in electron-

electron scattering at high energies in detail, we can readily obtain the transition amplitude for the processes of Fig 4-1. Applying Feynman's rules to Fig. 4-1 we can write the 4-th order S-matrix element for this process as follows:

$$
\begin{aligned}
S_{fi}^{(4)} = (-ie)^4 \int d^4x_1 \int d^4x_2 \int d^4x_3 \int d^4x_4 \sqrt{\frac{m_p}{E_2'V}}\overline{u}^{(s_2')}(\overrightarrow{p_2'})e^{-ip_2'.x_4}\gamma_\nu \sqrt{\frac{m_p}{E_2V}}u^{(s_2)}(\overrightarrow{p_2'})e^{ip_2.x_4} \\
\times \frac{1}{(2\pi)^4}\int d^4q_2(\frac{-1}{q_2^2})e^{iq_2.(x_3-x_4)}\sqrt{\frac{m_p}{E_1'V}}\overline{u}^{(s_1')}(\overrightarrow{p_1'})e^{-ip_1'.x_2}\gamma_\mu(\frac{-i\gamma.q_3+m_p}{q_3^2+m_p^2})e^{iq_3.(x_2-x_3)}\gamma_\nu \\
\times \sqrt{\frac{m_p}{E_1V}}u^{(s_1)}(\overrightarrow{p_1})e^{ip_1.x_2}\frac{1}{(2\pi)^4}\int d^4q_1(\frac{-1}{q_1^2})e^{iq_1.(x_2-x_1)}\sqrt{\frac{m_e}{E'V}}\overline{u}^{(s')}(\overrightarrow{p'})e^{-ip.x_1}\gamma_\mu \\
\times \sqrt{\frac{m_e}{EV}}v^{(s)}(\overrightarrow{p})e^{ip.x_1}. \qquad (4.1)
\end{aligned}
$$

This in turn can be expressed as:

$$
\begin{aligned}
S_{fi}^{(4)} &= \frac{(-ie)^4\, m_p^2 m_e}{V^3(\frac{1}{(2\pi)^4})^3\sqrt{E_2E_2'E_1E_1'EE'}} \\
&\times \int d^4x_1 \int d^4x_2 \int d^4x_3 \int d^4x_4 \int d^4q_1 \int d^4q_2 \int d^4q_3 \\
&\times [\overline{u}^{(s_2')}(\overrightarrow{p_2'})\gamma_\nu u^{(s_2)}(\overrightarrow{p_2})(\frac{-1}{q_2^2})[\overline{u}^{(s_1')}(\overrightarrow{p_1'})\gamma_\mu(\frac{-i\gamma.q_3+m_p}{q_3^2+m_p^2}) \\
&\times \gamma_\nu u^{(s_1)}(\overrightarrow{p_1})](\frac{-1}{q_1^2})\overline{u}^{(s')}(\overrightarrow{p'})\gamma_\mu v^{(s)}(\overrightarrow{p}) \\
\times & e^{ix_1.(-p-p'-q_1)}e^{ix_2.(q_3-p_1'+q_1)}e^{ix_3.(q_2+p_1-q_3)}e^{ix_4.(p_2-p_2'-+q_2)}. \qquad (4.2)
\end{aligned}
$$

We express integrals over x_1, x_2, x_3,and x_4 in terms of Delta functions as

$$\int d^4x_1 e^{ix_1.(-p-p'-q_1)} = (2\pi)^4\,\delta^{(4)}(-p-p'-q_1), \tag{4.3}$$

$$\int d^4x_2 e^{ix_2.(q_3-p_1'+q_1)} = (2\pi)^4\,\delta^{(4)}(q_3-p_1'+q_1), \tag{4.4}$$

$$\int d^4x_3 e^{ix_3.(q_2+p_1-q_3)} = (2\pi)^4\,\delta^{(4)}(q_2+p_1-q_3), \tag{4.5}$$

$$\int d^4x_4 e^{ix_4.(p_2-p_2'+q_2)} = (2\pi)^4\,\delta^{(4)}(p_2-p_2'+q_2), \tag{4.6}$$

and making use of the results

$$\int d^4q_1 \delta^{(4)}(q_3-p_1'+q_1)\delta^{(4)}(-p-p'-q_1) = \delta^{(4)}(-p-p'-q_1+q_3), \tag{4.7}$$

$$\int d^4q_2 \delta^{(4)}(q_2+p_1-q_3)\delta^{(4)}(p_2-p_2'-+q_2) = \delta^{(4)}(p_2-p_2'-q_3+p_1), \tag{4.8}$$

$$\left(\frac{-i\gamma.q_3+m_p}{q_3^2+m_p^2}\right)\delta^{(4)}(-p-p'-p_1'+q_3) = \frac{-i\gamma.(p+p_1'+p')+m_p}{|p+p_1'+p'|^2+m_p^2} \tag{4.9}$$

$$\delta^{(4)}(-p-p'-p_1'+q_3),$$

$$\int d^4q_3 \delta^{(4)}(-p-p'-q_1+q_3)\delta^{(4)}(p_2-p_2'-q_3+p_1) = \delta^{(4)}(p_2+-p_2'-p-p'+p_1-p_1'), \tag{4.10}$$

substituting equations from(4.3) to (4.10) in(4.2) we can express $S^{(4)}_{fi}$ as,

$$S^{(4)}_{fi} = \frac{(-ie)^4\, m_p^2 m_e\, (2\pi)^4\, \delta^{(4)}(p_2 + -p_2' - p - p' + p_1 - p_1')}{V^3\sqrt{E_2E_2'E_1E_1'EE'}}[\overline{u}^{(s_2')}(\overrightarrow{p_2'})\gamma_\nu u^{(s_2)}(\overrightarrow{p_2})](\frac{-1}{|p_2 - p_2'|^2})$$
$$[\overline{u}^{(s_1')}(\overrightarrow{p_1'})\gamma_\mu(\frac{-i\gamma.(p_1 + p_1' + p') + m_p}{|p_1 + p_1' + p|^2 + m_p^2})\gamma_\nu u^{(s_1)}(\overrightarrow{p_1})](\frac{-1}{|p + p'|^2})[\overline{u}^{(s')}(\overrightarrow{p'})\gamma_\mu v^{(s)}(\overrightarrow{p})], \tag{4.11}$$

where $\delta^{(4)}(p_2 + -p_2' - p - p' + p_1 - p_1')$ corresponds to the overall energy momentum conservation for this process. The M_{fi} matrix element for this process is,

$$M_{fi} = \{[\overline{u}^{(s_2')}(\overrightarrow{p_2'})\gamma_\nu u^{(s_2)}(\overrightarrow{p_2})](\frac{-1}{|p_2 - p_2'|^2})[\overline{u}^{(s_1')}(\overrightarrow{p_1'})\gamma_\mu(\frac{-i\gamma.(p_1 + p_1' + p') + m_p}{|p_1 + p_1' + p|^2 + m_p^2})$$
$$\gamma_\nu u^{(s_1)}(\overrightarrow{p_1})](\frac{-1}{|p + p'|^2})[\overline{u}^{(s')}(\overrightarrow{p'})\gamma_\mu v^{(s)}(\overrightarrow{p})]\}. \tag{4.12}$$

To evaluate the differential cross section for this process, we need to evaluate $|S^{(4)}_{fi}|^2$ which is expressed as

$$|S^{(4)}_{fi}|^2 = \frac{e^8 m_e^4 m_p^2 \delta^{(4)}(2\pi)^{(4)}(p_1 + p_2 - p_1' - p_2' - p' - p)VT|M_{fi}|^2}{V^6 E_2' E_2 E_1 E_1' E E'}, \tag{4.13}$$

where V is interaction volume and T is the interaction time. The differential cross section $d\sigma$ for this process is thus

$$d\sigma = \frac{|S^{(4)}_{fi}|^2}{VT\ |\overrightarrow{j}_{in}|\frac{1}{V}}\left\{\frac{(Vd^3\overrightarrow{p'})(Vd^3\overrightarrow{p})(Vd^3\overrightarrow{p_2'})(Vd^3\overrightarrow{p_1'})}{(2\pi)^3\,(2\pi)^3\,(2\pi)^3\,(2\pi)^3}\right\}, \tag{4.14}$$

where $|\overrightarrow{j}_{in}|$ is the incident flux density which is $|\overrightarrow{j}_{in}|=\frac{2\overrightarrow{\beta}}{V}$ in the c.m frame, and $\left\{\frac{(Vd^3\overrightarrow{p'})(Vd^3\overrightarrow{p})(Vd^3\overrightarrow{p'_2})(Vd^3\overrightarrow{p'_1})}{(2\pi)^3(2\pi)^3(2\pi)^3(2\pi)^3}\right\}$ is density of final states for this process. The phase space volume can be expressed as

$$
\begin{aligned}
d^3\overrightarrow{p'} &= |\overrightarrow{p'}|E'dE'd\Omega', \\
d^3\overrightarrow{p'_1} &= |\overrightarrow{p'_1}|E'_1dE'_1d\Omega'_1, \\
d^3\overrightarrow{p} &= |\overrightarrow{p}|EdEd\Omega, \\
\text{and} \quad d^3\overrightarrow{p'_2} &= 2E'_2\int_{-\infty}^{\infty} d^4p'_2\delta\left(p'^2_2+m_e^2\right)\theta\left(p'_{20}\right),
\end{aligned} \tag{4.15}
$$

where we express the 3-dimensinal volume element $d^3\overrightarrow{p'_2}$ in terms of a 4-D integral $d^3\overrightarrow{p'_2}$ $\theta\left(p'_{20}\right)$ is the unit step function. Substituting (4.15) in (4.14) we get,

$$
\begin{aligned}
\frac{d^3\sigma}{d\Omega d\Omega' d\Omega'_1} &= \frac{e^8m_e^4m_p^2}{(2\pi)^8\,\overrightarrow{\beta}}\int dE\int dE'_1\int dE'_2\frac{\delta^{(4)}(p_1+p_2-p'_1-p'_2-p'-p)|\overrightarrow{p}|||\overrightarrow{p'_1}||\overrightarrow{p'}|}{E_1E_2} \\
&\quad\times|M_{fi}|^2\delta\left(p'^2_2+m_e^2\right)\theta\left(p'_{20}\right),
\end{aligned} \tag{4.16}
$$

where

$$
|M_{fi}|^2\delta^{(4)}(p_1+p_2-p'_1-p'_2-p'-p) = |M_{fi}|^2_{p'_2=p_1+p_2-p'_1-p'-p}\delta^{(4)}(p_1+p_2-p'_1-p'_2-p'-p), \tag{4.17}
$$

and

$$\int_{-\infty}^{\infty} d^4p'_2\delta^{(4)}(p_1+p_2-p'_1-p'_2-p'-p)\delta\left(p'^2_2+m_e^2\right)\theta\left(p'_{20}\right)$$

$$=\delta[(p_1+p_2-p'_1-p'-p)^2+m_e^2]\theta(p_{10}+p_{20}-p'_{10}-p'_0-p_0). \tag{4.18}$$

The calculation simplifies considerably in the ultra relativistic limit where $E^2>>m_e^2$, $p^2>>m_e^2$, $E\cong|\overrightarrow{p}|$ and considering center of mass frame of reference($\overrightarrow{p_1}+\overrightarrow{p_2}=0$) and taking $E_1=E_2=E$, $E'_1=E'_2=E'$, we get.

$$\theta(p_{10}+p_{20}-p'_{10}-p'_0-p_0)=\theta(E-2E'), \tag{4.19}$$

and

$$\begin{aligned}|p_1+p_2-p'_1-p'-p|^2 &= (\overrightarrow{p_1}+\overrightarrow{p_2}-\overrightarrow{p'}_1-\overrightarrow{p'}-\overrightarrow{p})^2-(E-2E')^2. \\ &= |\overrightarrow{p'_1}|^2+|\overrightarrow{p'}|^2+|\overrightarrow{p}|^2+2\overrightarrow{p_{1'}}.\overrightarrow{p}+2\overrightarrow{p'_1}.\overrightarrow{p'}+2\overrightarrow{p}\,\overrightarrow{p'} \\ &\quad-(E^2-4EE'+4E'^2).\end{aligned} \tag{4.20}$$

Now lets consider the kinematical description of this process where the incident momenta of particles and scattered momenta of particles and anti-particle as shown by Fig.4-2.

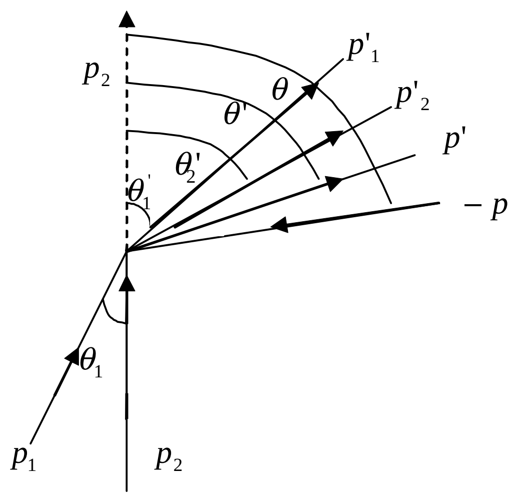


Fig.4-2 Kinematics of the process, p+p⟶p+p+e^-+e^+ through pair conversion of a bremsstrahlung photon.

The above figure describes the orientation of the incident particles, and orientation of scattered particles.

Here:

(i) p_1 is 4-momenta of the first incident proton.

(ii) p_2 is 4-momenta of the second incident proton.

(iii) $p'_1$is 4-momenta of the first scattered proton.

(iii) p'_2 is 4-momenta of the second scattered proton.

(v) -p is 4-momenta of created positron.

(vi) p' is 4-momenta of created electron.

(vii) $\theta_1 \equiv(\theta - \theta')$ is angle between p and p'.

(viii) $\theta_2 \equiv(\theta - \theta_1')$ is angle between p and p_1'.

(ix) $\theta_3 \equiv(\theta - \theta_2')$ is angle between p_1 and p_2'.

(x) $\theta_4 \equiv(180-\theta)$ is angle between p and p_2.

(xi) $\theta_5 \equiv[180+(\theta - \theta_1)]$ is angle between p and p_1.

(xii) $\alpha_1 \equiv[180+(\theta_2' - \theta_1)]$ is angle between p_1 and p_2'.

(xiii) $\alpha_2 \equiv(180 - \theta_2')$ is angle between p_2 and p_2'.

(xiv) $\alpha_3 \equiv \theta_1$ is angle between p_1 and p_2.

(xv) $\gamma_1 \equiv(\theta_1' - \theta_2')$ is angle between p_1' and p_2'.

(xvi) $\gamma_2 \equiv[180+(\theta_1' - \theta_1)]$ is angle between $p_1'$and p_1'.

(xvii) $\gamma_3 \equiv(180\text{-}\theta_1')$ is angle between p_1' and p_2.

(xviii) $\beta_1 \equiv(\theta' - \theta_1')$ is angle between p' and p_1'.

(xix) $\beta_2 \equiv(\theta' - \theta_2')$ is angle between p' and p_2'.

(xx) $\beta_3 \equiv(180\text{-}\theta')$ is angle between p' and p_2.

(xxi) $\beta_4 \equiv[180+(\theta' - \theta_1)$ is angle between $p_1$and p'.

Making use of the kinematics of the process described in fig. 4-2, where $p_1'=p'=E'$ and $p_1=p=E$, we can express (4.21) as

$$(p_1 + p_2 - p_1' - p' - p)^2 = 2E'^2\left(Cos\beta_1 - 1\right) + 2EE'\left(Cos\theta_2 + Cos\theta_1 + 2\right). \qquad (4.21)$$

Substituting (4.17), (4.18), (4.19),(4.20) and (4.21) in (4.16) we get

$$\frac{d^3\sigma}{d\Omega d\Omega' d\Omega'_1} = \frac{e^8 m_p^4 m_e^2 |\overrightarrow{p_{1'}}||\overrightarrow{p'}||\overrightarrow{p}|}{(2\pi)^8 \overrightarrow{\beta} E_1 E_2} \int dE \int dE'_1 \int_0^{\frac{E}{2}} dE |M_{fi}|^2_{p'_2=p_1+p_2-p'_1-p'-p} \delta[\left(2E'^2 Cos\beta_1 - 1\right)$$

$$+ 2EE'\left(Cos\theta_2 + Cos\theta_1 + 2\right)]. \tag{4.22}$$

Thus after carrying out the integrals in (4.22) we obtain

$$\frac{d^3\sigma}{d\Omega d\Omega' d\Omega'_1} = \frac{e^8 m_p^4 m_e^2 E'_1 |\overrightarrow{p'_1}||\overrightarrow{p'}||\overrightarrow{p}| \ln E \left(Cos\beta_1 - 1\right) |M_{fi}|^2_{p'_2=p_1+p_2-p'_1-p'-p}}{2E_1 E_2 (2\pi)^8 \overrightarrow{\beta} \left(Cos\beta_1 + Cos\theta_2 + Cos\theta_1 + 1\right)\left(Cos\theta_2 + Cos\theta_1 + 2\right)}. \tag{4.23}$$

For $|\overrightarrow{p'_1}|$=$|\overrightarrow{p'}|$=$|\overrightarrow{p}|$=E$'$=E$'_1$=E$_2$=E, (4.23) becomes

$$\frac{d^3\sigma}{d\Omega d\Omega' d\Omega'_1} = \frac{e^8 m_p^4 m_e^2 E \ln E (Cos\beta_1 - 1)|M_{fi}|^2_{p'_2=p_1+p_2-p'_1-p'-p}}{2(2\pi)^8 \left(Cos\beta_1 + Cos\theta_2 + Cos\theta_1 + 1\right)\left(Cos\theta_2 + Cos\theta_1 + 2\right)}. \tag{4.24}$$

Finally, $\frac{d^3\sigma}{d\Omega d\Omega' d\Omega'_1}$ can be express as

$$\frac{d^3\sigma}{d\Omega d\Omega' d\Omega'_1} = \frac{e^8 m_e^4 m_p^2 E \ln E}{2(2\pi)^8} \left\{ \frac{(Cos\beta_1 - 1)|M_{fi}|^2_{p'_2=p_1+p_2-p'_1-p'-p}}{\left(Cos\beta_1 + Cos\theta_2 + Cos\theta_1 + 1\right)\left(Cos\theta_2 + Cos\theta_1 + 2\right)} \right\}. \tag{4.25}$$

Further from equation (4.12) we can write the matrix element M$_{fi}$ for this process as

$$|M_{fi}| = \{[\overline{u}^{(s'_2)}(\overrightarrow{p'_2})\gamma_\nu u^{(s_2)}(\overrightarrow{p_2})](\frac{-1}{|p_2 - p'_2|^2})[\overline{u}^{(s'_1)}(\overrightarrow{p'_1})\gamma_\mu(\frac{-i\gamma.(p_1 + p'_1 + p') + m_p}{|p_1 + p'_1 + p|^2 + m_p^2})$$

$$\gamma_\nu u^{(s_1)}(\overrightarrow{p_1})](\frac{-1}{|p + p'|^2})[\overline{u}^{(s')}(\overrightarrow{p'})\gamma_\mu v^{(s)}(\overrightarrow{p})]\}. \tag{4.26}$$

Since the quantities like $\overline{u'}\gamma_v u$ are 1x1 matrices,we can express $|M_{fi}|^2$ as

$$|M_{fi}|^2 = \left\{ \frac{\begin{array}{c}[\overline{u}^{(s_2')}(\overrightarrow{p_2'})\gamma_\nu u^{(s_2)}(\overrightarrow{p_2})]^\dagger[\overline{u}^{(s_2')}(\overrightarrow{p_2'})\gamma_\nu u^{(s_2)}(\overrightarrow{p_2})][\overline{u}^{(s_1')}(\overrightarrow{p_1'})\Gamma u^{(s_1)}(\overrightarrow{p_1})]^\dagger \\ [\overline{u}^{(s_1')}(\overrightarrow{p_1'})\Gamma u^{(s_1)}(\overrightarrow{p_1})][\overline{u}^{(s')}(\overrightarrow{p'})\gamma_\mu[v^{(s)}(\overrightarrow{p})]^\dagger[[\overline{u}^{(s')}(\overrightarrow{p'})\gamma_\mu v^{(s)}(\overrightarrow{p})]\}\end{array}}{|p+p'|^4|p_2-p_2'|^4(|p_1+p_1'+p|^2+m_p^2)^2} \right\}, \quad (4.27)$$

where $\Gamma = \gamma_\mu(-i\gamma.(p_1+p_1'+p')+m_p)\gamma_\nu$. If the incident protons are un polarized, then we average over the initial spin states and sum over the final spin states. Thus $|M_{fi}|^2$ can be expressed as,

$$|M_{fi}|^2 = \frac{1}{2}\sum_{s_2',s_2}\sum_{s_1',s_1}\sum_{s,s} \left\{ \frac{\begin{array}{c}\{[\overline{u}^{(s_2)}(\overrightarrow{p_2})\overline{\gamma_\nu}u^{(s_2')}(\overrightarrow{p_2'})][\overline{u}^{(s_2')}(\overrightarrow{p_2'})\gamma_\nu u^{(s_2)}(\overrightarrow{p_2})]\} \\ \{[\overline{u}^{(s_1)}(\overrightarrow{p_1})\overline{\Gamma}u^{(s_1')}(\overrightarrow{p_1'})][\overline{u}^{(s_1')}(\overrightarrow{p_1'})\Gamma u^{(s_1)}(\overrightarrow{p_1})]\} \\ \{[\overline{v}^{(s)}(\overrightarrow{p})\overline{\gamma_\mu}u^{(s')}(\overrightarrow{p'})][\overline{u}^{(s')}(\overrightarrow{p'})\gamma_\mu v^{(s)}(\overrightarrow{p})]\}\end{array}}{|p+p'|^4|p_2-p_2'|^4(|p_1+p_1'+p|^2+m_p^2)^2} \right\}, \quad (4.28)$$

where $\overline{\gamma_\nu} = \gamma_4\gamma_\nu\gamma_4,\ \overline{\gamma_\mu} = \gamma_4\gamma_\mu\gamma_4,\ \overline{\Gamma} = \gamma_4\Gamma^\dagger\gamma_4$.Thus, making use of the relation. $\sum_s u^{(s)}(\overrightarrow{p})\,\overline{u}^{(s)}(\overrightarrow{p}) = \frac{(-i\gamma.p+m)}{2m}$ and $\sum_s v^{(s)}(\overrightarrow{p})\,\overline{v}^{(s)}(\overrightarrow{p}) = -\frac{(i\gamma.p+m)}{2m}$, (4.28) can be expressed as

$$|M_{fi}|^2 = \frac{1}{16m_p^2 m_e}\sum_{s_2}\sum_{s_1}\sum_{s}\left\{\frac{\begin{array}{c}\{[\overline{u}^{(s_2)}(\overrightarrow{p_2})\overline{\gamma_\nu}(-i\gamma.p_2' + m_p)\gamma_\nu u^{(s_2)}(\overrightarrow{p_2})]\} \\ \{[\overline{u}^{(s_1)}(\overrightarrow{p_1})\overline{\Gamma}(-i\gamma.p_1' + m_p)\Gamma u^{(s_1)}(\overrightarrow{p_1})]\} \\ \{[\overline{v}^{(s)}(\overrightarrow{p})\overline{\gamma_\mu}((-i\gamma.p' + m_e)\gamma_\mu v^{(s)}(\overrightarrow{p})]\}\end{array}}{|p+p'|^4|p_2 - p_2'|^4(|p_1 + p_1' + p|^2 + m_p^2)^2}\right\}. \quad (4.29)$$

We can in turn express (4.29) as a product of traces over $\gamma-$matrices as,

$$|M_{fi}|^2 = Tr\left\{\frac{\begin{array}{c}Tr\{\overline{\gamma_\nu}(-i\gamma.p_2' + m_p)\gamma_\nu(-i\gamma.p_2 + m_p)\} \\ \times Tr\{\overline{\Gamma}(-i\gamma.p_1' + m_p)\Gamma(-i\gamma.p_1 + m_p)\} \\ \times Tr\{\overline{\gamma_\mu}(-i\gamma.p' + m_e)\gamma_\mu(-i\gamma.p' - m_e)\}\end{array}}{128m_p^4 m_e^2|p+p'|^4|p_2 - p_2'|^4(|p_1 + p_1' + p|^2 + m_p^2)^2}\right\}. \quad (4.30)$$

Evaluating traces over the product of $\gamma-$matrices in the above expression and using the ultra relativistic limit (p^2>m^2, E^2>m^2, $|\overrightarrow{p}|\simeq$E we obtain

$$|M_{fi}|^2 = \left\{ \frac{\begin{array}{c} \{2(p_2'.p')(p_2.p) + 2(p_2'.p)(p.p')\}\{p_1'.p_1)(p_1'.p_1') \\ +32(p_1'.p_1')(p.p_1) + 32(p.p_1')(p.p_1) - 16(p.p)(p_1'.p_1) \\ +32(p'.p_1')(p.p_1) - 32(p'.p)(p_1'.p_1) \\ +32(p'.p_1)(p.p_1) + 32(p_1'.p_1')(p'.p_1) - 16(p'.p')(p_1'.p_1)\} \end{array}}{128m_p^4m_e^2|p+p'|^4|p_2-p_2'|^4(|p_1+p_1'+p|^2+m_p^2)^2} \right\}. \tag{4.31}$$

Making use of kinematics of the process described in fig. 4-2, (4.31) can in turn be expressed as,

$$|M_{fi}|^2 = \left\{ \frac{\begin{array}{c} (32p_1p_2p'(p^2Cos\theta_1Cos\theta_4 + p_1p'Cos\beta_2Cos\beta_4)p_1' \\ (p^2(^2Cos\theta_2Cos\theta_5 - Cos\gamma_2) + 2Cos\beta_1Cos\beta_4 - Cos\gamma_2)p'^2 \\ +2pp_1'Cos\theta_5 + p_1^2Cos\gamma_2 + 2p'(p(Cos\theta_5Cos\beta_1 \\ +Cos\theta_2Cos\beta_4 - Cos\theta_1Cos\gamma_2) + p_1'Cos\beta_4))) \end{array}}{\begin{array}{c} 128m_p^4m_e^2((p^2+2pp'Cos\theta_1+p'^2)^2 + p_1'(2pCos\theta_2+p_1') \\ +2p'(Cos\theta_1+Cos\beta_1p_1'))^2(p_2^2+2p_1p_2'Cos\alpha_2+p_2'^2)^2 \end{array}} \right\}. \tag{4.32}$$

Finally by substituting (4.32) in (4.25) we get the differential cross section for the above process as

$$\frac{d^3\sigma}{d\Omega d\Omega' d\Omega_1'} = \frac{e^8 E \ln E}{256\,(2\pi)^8} \left\{ \frac{\begin{array}{c} (32p_2p_1(Cos\beta_1 - 1)p'(p^2Cos\theta_1Cos\theta_4 + pCos\beta_2)p_1' \\ (p^2(2Cos\theta_2Cos\theta_5 - Cos\gamma_2) + 2Cos\beta_1Cos\beta_4 - Cos\gamma_2)p'^2 \\ +2pp_1'Cos\theta_5 2p'^2Cos\gamma_2 + 2p'(p(Cos\theta_5Cos\beta_1 \\ +Cos\theta_2Cos\beta_4 - Cos\theta_1Cos\gamma_2) + p_1'Cos\beta_4))) \end{array}}{\begin{array}{c} (Cos\beta_1 + Cos\theta_2 + Cos\theta_1 + 1)\,(Cos\theta_2 + Cos\theta_1 + 2) \\ (p^2 + 2p_2Cos\theta_1p_2 + p'^2)^2(p'^2 + p_1(2p(Cos\theta_2 + p_1') \\ +2p(pCos\theta_1 + 2pCos\beta_1p_1'))^2(p_2^2 + 2p_2Cos\alpha_2p_2' + p_2'^2)^2 \end{array}} \right\}. \tag{4.33}$$

We again notice the variation of cross section as ElnE [5]where E is the incident energy of the incident electron. Also the cross section depends on the kinematical variables described in Fig. 4-2 as well as on the momenta of the particles involved in the process.

Chapter 5

Conclusion

In this thesis we first study the solutions of the Dirac equation for an electron of charge e and mass m interacting with electromagnetic field $A_\mu(x)$, where transition amplitudes S_{fi} to any desired in the interaction strength $\alpha = \frac{1}{137}$ that an electron in an initial state $|i\rangle$ is transferred to a final state $|f\rangle$ as a result of interacting with $A_\mu(x)$ field can be expressed as:

$$
\begin{aligned}
&S_{fi} = \langle f|S|i\rangle; \\
&S_{fi} = \delta_{fi} - i \int_{-\infty}^{\infty} dt_1 \langle f|H_I(t_1)|i\rangle + (-i)^2 \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \langle f|H_I(t_1)H_I(t_2)|i\rangle + ... \qquad (5.1)
\end{aligned}
$$

We first discussed the propagator formulation of Dirac particles (electrons and photon) in interaction with electromagnetic field $A_\mu(x)$. As a preliminary application of the above formulation we calculated the transition amplitude and the corresponding differential cross section for e^+e^- pair production in a Coulomb field of a nucleus in chapter 2. We obtained the approximate differential cross section for this process as,

$$\frac{d^2\sigma}{d\Omega d\Omega'} = \frac{EE'e^6z^2}{8\omega\left(2\pi\right)^5\left[2\omega ECos\gamma - 2\omega E'Cos\alpha + 2EE'Cos\beta + E^2 + E'^2\right]^2} \tag{5.2}$$

$$\times\left\{\frac{4E'Cos\alpha + 2\omega}{Cos\gamma} - \frac{4ECos\beta}{Cos\gamma Cos\alpha} + \frac{-0.5E^2 + 3\omega E + 2E^2Cos\gamma}{E'Cos\alpha} + 4E\right\} \tag{5.3}$$

The differential cross section for this process depends on the energy of incident photon and the energies of electron and positron in the final state.

We then calculated the transition amplitude and the differential cross section for e^+e^-pair production by collision of two virtual photons in electron-electron scattering at high energies in chapter 3. However in these processes only the Quantum Electrodynamics(QED) effects have been incorporated. Even though calculation of the matrix element M_{fi} and the corresponding traces of product of γ-matrices are very long and complicated, we explicitly calculated them. In the calculation of the cross section we considered the c.m. frame of reference since colliding particles were of same mass. It was observed that the calculation simplifies considerably in the ultra-relativistic limit. The differential cross section for this process, $\frac{d^3\sigma}{d\Omega d\Omega' d\Omega'_1} \sim E\ln E$, where E is the energy of the incident particles. This is similar to the behavieor of the cross section formula for a related process of $\gamma + \gamma \longrightarrow$hadrons studied in Ref.[5].

We also studied the process of internal conversion of a bremsstrahlung photon in to electron-positron pair. We calculated the differential cross section process of e^+e^- pair production from a virtual bremsstrahlung photon produced in high energy proton-proton collision in chapter 4. The differential cross section for this process again goes as, $\frac{d^3\sigma}{d\Omega d\Omega' d\Omega'_1} \sim E \ln E$. The difference between the differential cross sections for the two processes lies in the M-matrix part which contains the essential physics of the process concerned.

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Declaration

I declare that this thesis is my original work, has not been presented for a degree in any other university and that all sources of materials used for thesis have been duly acknowledged.

Name Urga Dinegde Yadete

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This thesis has been submitted for examination with my approval as university advisor

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March, 2007.