STATISTICAL METHODS IN ESTIMATING THE DEMAND FOR AND SUPPLY OF MONEY IN ETHIOPIA

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ABSTRACT

During the past four decades, a body of econometric techniques designed to deal with the problem of single equation estimation as well as simultaneous-equations bias has been developed and applied to a variety of economic relationships. Such methods had wide application in present day developed countries. However, there has been very little attempt to apply these techniques to empirical relationships describing the monetary sector of developing countries in general and Ethiopia in particular.

The major purpose of the study is, therefore,

1. to formulate a mathematical model describing the various sectors of the economy with more emphasis on the role of money supply;
2. to apply various econometric methods and compare their relative efficiency in explaining the empirical relations;
3. to identify the significant explanatory variables that are important in explaining variations in supply of money and demand for money in Ethiopia;
4. to test a structural model of the monetary sector by first specifying demand for and supply of money and the interrelation with other economic variables.

The relevant time series data is collected from the Quarterly Bulletin published by the National Bank of Ethiopia and from the Statistical Abstract of the Central Statistical Authority have been used to estimate the demand and supply functions for money in single and simultaneous equations models format. The results indicate that the important determinants of the demand functions for real money balance are real income, rate of change of price and lagged real money balance variables. The monetary base and total domestic credit are important explanatory variables in the supply function for nominal money balance in Ethiopia.

To my father, mother,

sisters, brothers and friends

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CHAPTER 1

INTRODUCTION

1.1 General

Because money has an important influence on economic activities of nations has been used by governments as one instrument to increase or improve the welfare of their people. Appropriate monetary policy, could be an economic instrument available to governments so as to be able to influence output, employment, price and the balance of payment. In other words, governments can apply monetary policy so as

a) to create the most favourable conditions for increased production and distribution, i.e. to stimulate economic growth and thereby increase national income of its people;

b) to stimulate employment generating activities;

c) to maintain a reasonably stable price level, and

d) to sustain a favourable balance of payment and to stabilize external value of its currency.

In order to fulfill these and possibly other additional objectives relevant to the social system and stage of economic growth the monetary system that is aimed to accomplish the objectives should necessarily be put into practice through the utilization of a modern banking system or other government instruments through which the country's money supply, foreign exchange and reserves are manipulated, controlled or administered. The degree of effectiveness depends on the availability of necessary and sufficient financial infrastructures.

In Ethiopia at present, the National Bank of Ethiopia (NBE) is empowered to regulate and administer the monetary affairs of the nation consistent with the major economic objectives as any central bank does. In addition, it performs the standard functions in the area of currency issue, control of banks and other financial institutions, regulation of credit, administration and control of foreign assets and the like.

Despite the fact that Ethiopia has a small monetised sector knowledge about the parameters of demand for and supply of money are important in order to formulate monetary policy. However, the examination of the monetary sector has not been a favourite topic of research in Ethiopia. The present paper seeks to throw some light on the parameters of demand for and supply of money functions in the case of Ethiopia by applying single and simultaneous equations method.

1.2 Objectives of the Study

The main objective of the study is to introduce the concept of the system of single and simultaneous equations techniques and show its applications on finding consistent estimates under conditions in which the classical least-squares method fails. In addition to the above objective, the purpose of the study is:

1. to build a mathematical model describing the various sectors of the economy with more emphasis on the role of money supply;
2. to identify the significant explanatory variables that are important in explaining variations in supply of money and demand for money in Ethiopia;
3. to test a structural model of the monetary sector by first specifying demand for and supply of money and the interrelation with other economic variables;
4. to apply various econometric techniques and identify the ones that are more robust with high explanatory and descriptive power.

1.3 The Structure of the Study

The study is organized in four parts. An introduction is included in the first part. Part two discusses various econometric techniques such as ordinary least-squares, two-stage least squares, etc., applied in the estimation of the demand for and supply of money functions. Various types of money supply and demand for money models are specified in part three. Finally, part four deals with the estimation of the demand and supply functions for money, interpretation of results and conclusions.

CHAPTER 2

METHODOLOGY

There are various econometric methods with which we may obtain estimates of the parameters of the demand for and supply of money functions. However, we will consider only the most appropriate estimation methods.

2.1 Single-Equation Regression Methods

For the econometrics of estimating the demand for and supply of money functions, the most important statistical tool is regression analysis. In this section, we shall takeup the statistical theory of single equation regression methods to form a basis for equations system estimations in the next sections.

2.1.1 Ordinary Least-Squares (OLS)

The method of least squares is probabily one of the most widely used tools of quantitative economic research. Consider the linear regression model:

$$y_t = \beta_o+\beta_1X_{t1}+\beta_2X_{t2}+\ldots+\beta_kX_{tk}+e_t \qquad (1)$$

where y_t is the t^{th} observation on the dependent variable in the regression, X_{ti} is the t^{th} observation on the i^{th} independent variable (regressor), β_i is the regression coefficient corresponding to the i^{th} regressor, and e_t is the t^{th} observation on the disturbance (error) term and $t=1,2,\ldots,T$. Note that T is the number of observations and k is the number of regressor.

The above model can be written in matrix form as

$$\underline{Y} = \underline{X}\underline{B} + \underline{\varepsilon} \qquad (2)$$

Here $\underline{Y}$ is of dimension Tx1, $\underline{X}$ is TxK, $\underline{B}$ is Kx1 and $\underline{\varepsilon}$ is Tx1.

It should be clear now that the full specification of the regression model includes not only the form of the regression as given in (1) but also a specification of the probability distribution of the disturbance and a statement indicating how the values of the explanatory variable are determined. This information is given by the basic assumptions. These assumptions, which are taken to apply to all observations, are as follows:

i) zero mean: $E(^{e}t) = 0$.

ii) Homoscedasticity: The variance of each e_t is the same for all the X_t values, i.e. $E(e_t{}^2) = \delta^2$ constant.

iii) Non-autocorrelation: The values of e_t (corresponding to X_t) are independent from the values of any other $e_t{}'$.

$$E(e_t e_t{}') = 0 \text{ for } t \neq t'.$$

iv) Independence of e_t and X_t: Every disturbance term e_t is independent of the explanatory variables

$$E(e_t X_{1t}) = E(e_t X_{2t}) = \ldots = E(e_t X_{kt}) = 0.$$

v) The X_t's are a set of fixed values in the hypothetical process of repeated sampling which underlies the linear regression model.

vi) Normality: e_t is normally distributed.

The assumptions underlying the classical normal linear regression model are used in deriving estimators of the regression parameters. Let $\underline{b}_*$ denote any arbitrary k-element vector. This in turn serves to define a vector of errors, or residuals,

$$\underline{e}_* = \underline{Y} - \underline{X}\underline{b}_* \tag{3}$$

The least-squares principle for choosing $\underline{b}_*$ is to minimize the sum of the squared residuals $\underline{e}_*' \underline{e}_*$. From Eq(3)

$$\underline{e}_*'\underline{e}_* = (\underline{Y}-\underline{X}\underline{b}_*)' \ (\underline{Y}-\underline{X}\underline{b}_*)$$

$$= \underline{Y}'\underline{Y}-2\underline{b}_*'\underline{X}'\underline{Y}+\underline{b}_*'\underline{X}'\underline{X}\underline{b}_*.$$

Thus
$$\frac{\partial(\underline{e}_*'\underline{e}_*)}{\partial \underline{b}_*} = -2\underline{X}'\underline{Y}+2\underline{X}'\underline{X}\underline{b}_* \tag{4}$$

Equating Eq(4) to zero, we get OLS solution for $\underline{b}_*$,

$$\underline{X}'\underline{X}\underline{b}_* = \underline{X}'\underline{Y} \tag{5}$$

Thus an equivalent expression for b_* is

$$\underline{b}_* = (\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Y} \qquad (6)$$

These parameter estimates have some optimal properties. These are: (1) unbiasedness; (2) Least-variance; (3) efficiency; (4) best, linear, unbiasdness (BLU); (5) least-mean-square error (MSE); (6) sufficiency.

In setting up the regression equation in Eq(1) or Eq(2) way we are, infact, assuming that the current value of Y_t may depend on the current value of X_t but not on any of the past values of X_t. A more general formulation, which would allow for the current as well as the past values of X_t to affect Y_t, would be written as

$$Y_t=\alpha+\beta_o X_t+\beta_1 X_{t-1}+\beta_2 X_{t-2}+\ldots=\alpha+\sum_{s=o}^{\infty}\beta_s X_{t-s}+e_t \qquad (7)$$

Unless we state otherwise, we shall assume that the error term is normally distributed, independent of X, and neither serially correlated nor heteroscedastic.

The two major problems with this approach are: (1) There are many regressors that eat up a large number of degrees of freedom; and (2) These regressors may be closely related, making multicollinearity a serious problem. In these circumstances, it is natural to look for a simplifying assumption to make this problem more manageable.

2.1.1.1 The Koyck Distributed Lag (Geometric Lag)

This is one of the most popular distributed lag. models in applied research. The geometric lag assumes that the weights of the lagged explanatory variables are all positive and decline geometrically with time:

$$\beta_t = \beta_o\lambda^t \quad (8)$$

with λ being a positive fraction less than one (i.e. $0<\lambda<1$).

While maintaining a high degree of realism, the Koyck model achieves a substantial simplification: There are only two parameters to estimate, λ and β_o, as follows:

1. Substitute (8) into (7):

$$Y_t=\alpha+\beta_oX_t+\beta_o\lambda X_{t-1}+\beta_o\lambda^2X_{t-2}+\ldots+e_t \quad (9)$$

and for the previous t, of course,

$$Y_{t-1}=\alpha+\beta_oX_{t-1}+\beta_o\lambda X_{t-2}+\beta_o\lambda^2X_{t-3}+\ldots+e_{t-1} \quad (10)$$

If Eq (10) is multiplied by λ, and **subtracted** from Eq (9), most of the terms dropout and we obtain

$$Y_t=\partial+\beta_oX_t+\lambda Y_{t-1}+e^*_t \quad (11)$$

where $e^*_t = e_t - \lambda e_{t-1}$ (12)

2. Now regress Y_t on Y_{t-1} and X_t as in Eq (11) to obtain the estimates of λ and β_o.

3. Substituting these estimates into Eq(8) to generate the estimates of β_t.

Unfortunately several problems remain. The error term e_t^* in Eq(12) probabily is serially correlated, which makes the OLS regression in step 2 less efficient. Worse yet, the error term e_t^* is likely to be correlated with the regressor Y_{t-1} in Eq(11) (since both depend on e_{t-1}^*) and this makes the OLS regression in step 2 biased for small samples.

A similar form to Koyck's transformation ($Y_t = f(X_t, Y_{t-1})$) may be established by applying other behavioural assumptions, different from Koyck's. Two such models are discussed below. They are Cagan's 'adaptive expectations' model and Nerlove's 'partial adjustment' model.

2.1.1.2 Cagan's Adaptive Expectations

The adaptive expectations model postulates that changes in Y_t are related to changes in the "expected" level of the explanatory variable X_t. We may write this model as

$$Y_t = \partial^* + \beta_o^* X_t^* + e_t^* \tag{13}$$

where X_t^* represents the desired or expected level of X_t. The expected level of X_t is defined by a second relationship, in which expectations are assumed to be altered every time period as an adjustment between the current observed value of X_t and the previous expected value of X_t. The relationship is

$$X_t^* = X_t + (1-\theta) X_{t-1}^*, \text{ where } 0<\theta\leq 1. \quad (14)$$

It is sometimes more useful to rewrite (14) as

$$X_t^* = \theta X_t + (1-\theta) X_{t-1}^* \quad (15)$$

This suggests that the expected level of X_t is a weighted average of the present level of X_t and the previous expected level of X_t. Expected levels of X_t are adjusted period by period by taking into account present levels of X_t. Rewriting (15) by lagging the model period by period while at the same time multiplying by $(1-\theta)^s$, where s is the number of periods involved in the lag process:

$$(1-\theta) X_{t-1}^* = \theta (1-\theta) X_{t-1} + (1-\theta)^2 X_{t-2}^* \quad (16)$$

$$(1-\theta)^2 X_{t-2} = \theta (1-\theta)^2 X_{t-2} + (1-\theta)^3 X_{t-3}^*$$

Now, substitute (16) into (15) and combine terms:

$$X_t^* = \theta [X_t + (1-\theta) X_{t-1} + (1-\theta)^2 X_{t-2} + \ldots] = \theta \sum_{s=0}^{\infty} (1-\theta)^s X_{t-s} \quad (17)$$

One should note that the desired level of X_t is a weighted average of all present and previous values of X_t, since the weights sum to unity. Substituting (17) into (13) we get

$$Y_t = \partial^* + \beta_0^* \theta \sum_{s=0}^{\infty} (1-\theta)^s X_{t-s} + e_t^* \quad (18)$$

The equivalence of this model to the original geometric lag model (19) can be seen by letting:

$\partial = \partial^*$, $\beta_o = \beta_o^* \theta$, $\lambda = (1-\theta)$, and $e_t = e_t^*$.

Thus, estimation of the economic specification associated with the adaptive expectations model is identical to the problem of estimating the Koyck geometric lag, with Eq(11) now becoming

$$Y_t = \partial^* \theta + \beta_o^* \theta X_t + (1-\theta) Y_{t-1} + U_t^* \qquad (19)$$

where $U_t = e_t^* - (1-\theta) e_{t-1}^*$.

2.1.1.3 Nerlove's Partial Adjustment

Another process which can generate lagged dependent variables among the regressors is that of partial adjustment. The partial adjustment model assums that the desired level of Y_t is dependent upon the current level of X_t, that is,

$$Y_t^* = \partial' + \beta' X_t + e_t' \qquad (20)$$

In any given period, the actual value of Y_t may not adjust completely to obtain the desired level. Lack of knowledge, technical constraints, and other items might be responsible for this partial adjustment. We can represent the adjustment process as

$$Y_t - Y_{t-1} = \gamma (Y_t^* - Y_{t-1}), \quad 0<r<1. \qquad (21)$$

The equation specifies that the change in Y_t will respond only partially to the difference between the desired stock of Y_t and the past value of Y_t, the rate of response being a function of the adjustment coefficient r. Substituting for Y_t^* in Eq(21) and solving for Y_t yields.

$$Y_t = \partial'\gamma + \gamma\beta' X_t + (1-\gamma) Y_{t-1} + e_t' \tag{22}$$

2.2 Simultaneous-Equations Methods

Many alternative techniques of estimating coefficients appearing in the structural representation of econometric models have been developed. In the estimation of structural coefficients it is well known that two-stage least-squares (2SLS), limited-information maximum likelihood (LIML) and three-stage least-squares (3SLS) estimators are consistent and asymptotically efficient for large samples. Before proceeding with the discussion of these techniques, it is necessary to develop further some definitions.

2.2.1 Structural and Reduced Forms

A structure is a model with the numerical values of the parameters specified and a structural model is a complete system of equations which describe the structure of the relationships of the economic variables. Structural equations express the endogenous variables (variables whose values are to be explained by the model) as functions of other endogenous variables, predetermined variables (combinations of lagged endogenous variables and exogenous (independent) variables) and disturbances.

Using the conventional notation we write the structural model of G linear simultaneous stochastic equations with K predetermined variables, in matrix form, as

$$\underset{\sim}{\beta}\underset{\sim}{Y}_t+\underset{\sim}{\gamma}\underset{\sim}{X}_t = \underset{\sim}{U}_t, \ldots. \quad (23)$$

where $\underset{\sim}{Y}_t = \begin{bmatrix} Y_{1t} \\ Y_{2t} \\ \vdots \\ Y_{Gt} \end{bmatrix}_{(G\times 1)}$, $\underset{\sim}{X}_t = \begin{bmatrix} X_{1t} \\ X_{2t} \\ \vdots \\ X_{Kt} \end{bmatrix}_{(K\times 1)}$, $\underset{\sim}{U}_t = \begin{bmatrix} U_{1t} \\ U_{2t} \\ \vdots \\ U_{Gt} \end{bmatrix}_{(G\times 1)}$

$$\underset{\sim}{\beta} = \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1G} \\ \beta_{21} & \beta_{22} & \cdots & \beta_{2G} \\ \vdots & \vdots & & \vdots \\ \beta_{G1} & \beta_{G2} & \cdots & \beta_{GG} \end{bmatrix}_{(G\times G)}, \quad \underset{\sim}{\gamma} = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1K} \\ \gamma_{21} & \gamma_{22} & \cdots & \gamma_{2K} \\ \vdots & \vdots & & \vdots \\ \gamma_{G1} & \gamma_{G2} & \cdots & \gamma_{GK} \end{bmatrix}_{(G\times K)}$$

The $\underset{\sim}{Y}$'s are endogenous varibales, the $\underset{\sim}{X}$'s are predetermined variables, the $\underset{\sim}{U}$'s (stochastic disturbances) are assumed to be serially independent and G-Variate normal with mean zero and covariance matrix $\underset{\sim}{\Sigma} = (\delta_{ij})$. The $\underset{\sim}{\beta}$'s and the $\underset{\sim}{\gamma}$'s are known as the structural coefficients. In each equation one of the $\underset{\sim}{\beta}$'s is taken to be unity, thus indicating that one of the endogenous variables serves as the "dependent" variable when the equation is written out as a standard regression equation. It should also be noted that some of the equations may actually be identies, which means that all their coefficients are known and that they contain no stochastic disturbances.

The reduced form of a structural model is the model in which the endogenous variables are expressed as a function of the predetermined variables and disturbances only. To bring out the explicit dependence of the dependent variables on the predetermined variables and the desturbances, we should solve the etructural form into the reduced form. Using matrix notation, the result may be written as

$$\underline{Y}_t = \underline{\Pi}\underline{X}_t + \underline{V}_t \tag{24}$$

where

$$\underline{\Pi} = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \cdots \Pi_{1K} \\ \Pi_{21} & \Pi_{22} & \cdots \Pi_{2K} \\ \vdots & \vdots & \vdots \\ \Pi_{G1} & \Pi_{G2} & \cdots \Pi_{GK} \end{bmatrix}_{(G\times k)}, \quad \underline{V}_t = \begin{bmatrix} V_{1t} \\ V_{2t} \\ \vdots \\ V_{Gt} \end{bmatrix}_{(G\times 1)}$$

The $\underline{\Pi}$'s represent the reduced form coefficients and the $\underline{V}$'s the reduced form disturbances. In general, each reduced form disturbance is a linear function of all structural disturbances. The relation between the structural form and the reduced form can be derived explicitly by solving Eq(23) for $\underline{Y}_t$. This gives

$$\underline{Y}_t = -\underline{\beta}^{-1}\underline{\gamma}\underline{X}_t + \underline{\beta}^{-1}\underline{U}_t \tag{25}$$

Comparing this result with the reduced form Eq(24), we can see that

$$\underline{\Pi} = -\underline{\beta}^{-1}\,\underline{\gamma} \qquad (26)$$

and $$\underline{V}_t = \underline{\beta}^{-1}\,\underline{U}_t \qquad (27)$$

The distinctive feature of the reduced-form is that in each of its equations only one dependent variable appears. The reduced-form parameters measure the total effect, direct and indirect, of a change in the predetermined variable on the endogenous variables, after taking account of the interdependences among the jointly dependent endogenous variables (the current endogenous variables), while a structural parameter indicates only the direct effect within a single sector of the economy.

2.2.2 The Identification Problem

For an econometrician wishing to use the model of simultaneous equations there are two major problems to be resolved. First is the problem of identification. After a model has been specified in the form of the structural equations (23), it is capable of explaining the relations between the endogenous variables and the predetermined variables through the reduced-form (24) which can be observed. Let there be a sample of T observations $(\underline{Y}_t, \underline{X}_t)$ $(t = 1, 2, \ldots, T)$, which can be used to estimate the parameter $\underline{\Pi}$ of the reduced-form. It is possible to infer from $(\underline{\Pi})$ the structural parameters

(β, γ). This is the problem of identification. We discuss the identification problem in this section. Second, given that a certain subset of the structural parameters can be identified, how should they be estimated using the observations available? This is the problem of estimation and should be discussed in the next sections. Returning to a discussion of the identification problem, we consider conditions for the identification of the parameters of one structural equation. One basic idea is that for a set of structural parameters to be identifiable they must be uniquely determined if the reduced-form parameters are known. The parameters of the reduced-form (24) can be consistently estimated by the method of least-squares.

To investigate the conditions for the identifiability of the parameters of one structural equation, one may ask under what conditions the parameters of this equation can be inferred uniquely from the parameters of the reduced form through the relation $\beta\Pi = -\gamma$. We consider the first structural equation since by rearrangement any equation can be renamed the first. The coefficients of the first equation are the elements of the first row of $[\beta\ \gamma]$. Let $\beta_{11} = -1$, and let G_1 endogenous variables and K_1 predetermined variables be included in the first equation. The coefficients of the first equation can be written as

$(-1\ \underset{\sim}{\beta}'\ \underset{\sim}{0}\ \underset{\sim}{\gamma}'\ \underset{\sim}{0})$, where the two zeros are respectively the coefficients of the $G-G_1$ endogenous variables and the $K-K_1$ predetermined variables excluded from the first structural equation. The excluded variables can always be arranged to be the last.

The relation between the coefficients of the first structural equation and the reduced-form coefficients $\underset{\sim}{\Pi}$ is given by the first row of $\underset{\sim}{\beta}\,\underset{\sim}{\Pi} = -\underset{\sim}{\gamma}$,

$$(-1\ \underset{\sim}{\beta}'\underset{\sim}{0})\begin{bmatrix}\underset{\sim}{\Pi}_{11} & \underset{\sim}{\Pi}_{12}\\ \underset{\sim}{\Pi}_{21} & \underset{\sim}{\Pi}_{22}\end{bmatrix} = (-\underset{\sim}{\gamma}'\underset{\sim}{0}) \qquad (28)$$

where the matrix $\underset{\sim}{\Pi}$ has been partitioned, $\underset{\sim}{\Pi}_{11}$ and $\underset{\sim}{\Pi}_{12}$ having G_1 rows, $\underset{\sim}{\Pi}_2$ and $\underset{\sim}{\Pi}_{22}$ having $G-G_1$ rows, $\underset{\sim}{\Pi}_{11}$ and $\underset{\sim}{\Pi}_{21}$ having K_1 columns, and $\underset{\sim}{\Pi}_{12}$ and $\underset{\sim}{\Pi}_{22}$ having $K-K_1$ columns. Expression (28) consists of two sets of equations,

$$(-1\ \underset{\sim}{\beta}')\ \underset{\sim}{\Pi}_{11} = -\underset{\sim}{\gamma}' \qquad (29)$$

$$(-1\ \underset{\sim}{\beta}')\ \underset{\sim}{\Pi}_{12} = \underset{\sim}{0} \qquad (30)$$

If $\underset{\sim}{\beta}$ can be identified, γ can always be identified by using (29). The question then is under what conditions $\underset{\sim}{\beta}$ can be identified by relation (30) given $\underset{\sim}{\Pi}_{12}$.

Equation (30) is a set of $K-K_1$ linear equations in the G_1-1 unknowns which are elements of the vector $\underset{\sim}{\beta}$. A necessary and sufficient condition for a unique solution for $\underset{\sim}{\beta}$ (and thus for the identification of the first structural equation) is that the rank of the matrix $\underline{\Pi}_{12}$ be $G_1 - 1$, that is, be equal to the number of unknown coefficients of the endogenous variables included in the first structural equation. This is known as the rank condition for identification of a structural equation.

A necessary condition for a unique solution for $\underline{\beta}$, and thus for the identification of the first equation, is that the number $K-K_1$ of equations be no smaller than the number G_1-1 of unknown in $\underline{\beta}$. This is known as the order condition for identification of a structural equation

$$K - K_1 \geq G_1 - 1 \qquad (31)$$

If the necessary condition (31) is not met, the structural equation is underidentified or not identifiable. If the equality sign holds in (31), the structural equation is just identified or exactly identified. There are exactly as many linear equations in (30) as unknown, the elements of $\underline{\Pi}_{12}$ being the given coefficients for determining the unknown elements of $\underline{\beta}$. When the inequality sign holds in (31), the number of equations or the number of columns of $\underline{\Pi}_{12}$, $K-K_1$, is larger than the number of unknowns in $\underline{\beta}$, G_1-1. The structural equation is said to be overidentified. A structural equation is identified if it is either just identified or over identified.

2.2.3 Two-Stage Least-Squares (2SLS)

The two-stage least-squares method provides estimators of the parameters of a single justidentified or overidentified equation containing two or more jointly dependent variables was put forward by Theil (1958 and 1961) and also by Basmann (1957). The method of 2SLS assumes knowledge of all the predetermined variables of the complete system of simultaneous equations. If the specification of these variables is not correct, the estimates of the parameters will not have the optimal properties mentioned previously. The method is fairly simple in conception and in computations. It has yielded more satisfactory results than any other econometric methods and has become the most important technique for the estimation of overidentified functions.

The steps that need to be followed in the estimation of 2SLS are

a) From among the joint dependent variables in the equation to be estimated, select the one that is to be used as the dependent variable in the second stage of the process described below.

b) Compute the least-squares estimates of the reduced-form equations for the remaining jointly dependent variables in the equation (all but the one selected in step (a)), using all the predetermined variables in the model (the step is the "first stage" of the two-stage process).

c) Replace the observed data for these remaining jointly dependent variables (all but one of those in the equation) by their calculated values from the reduced-form as estimated in step (b).

d) Compute the least-squares regression of the selected dependent variable on the set of variables consisting of the calculated values of the jointly dependent variables obtained in step (c) and the observed values of the predetermined variables in the equation (this step is the "second stage" of the process). The resulting estimates are the 2SLS estimates of the parameters of the equation to be estimated.

To give an explicit mathematical account of the 2SLS method, consider the estimation of single equation in a set of simultaneous equations.

Let $$\underline{y}_1 = \underline{Y}_1 \underline{\beta}_1 + \underline{X}_1 \underline{\gamma}_1 + \underline{U} \qquad (32)$$

be the first structural equation from the system of structural equations,

$$\underline{Y}\underline{\beta} + \underline{X}\underline{\gamma} + \underline{U} = \underline{O},$$

Here $\underline{Y}$ is of dimension TxG, $\underline{X}$ is TxK, $\underline{y}_1$ is Tx1, $\underline{Y}_1$ is $T\underline{x}(G_1-1)$ included endogenous variables, $\underline{X}_1$ is $Tx(K_1)$ included predetermined variables, $\underline{\beta}$ is $(G_1-1)x1$, $\underline{\gamma}_1$ is K_1x1; and $\underline{U}$ is Tx1. The application of OLS to Eq(32) will yield inconsistent parameter estimates due to the fact that $\underline{Y}_1$ and $\underline{U}$ are (asymptotically) correlated.

However, 2SLS yields consistent estimates by purging $\underset{\sim}{Y}_1$ of the component which is correlated with $\underset{\sim}{V}$ and then rerunning the new regression using OLS. These considerations suggest the following procedure.

1. Obtain the classical least-squares estimator of each of the right-hand endogenous variables by regressing $\underset{\sim}{Y}_1$ on $\underset{\sim}{X}$. This is equivalent to estimating the reduced-form equations associated with the G_1-1 right-hand endogenous variables. We might represent this as

$$\underset{\sim}{Y}_1 = \underset{\sim}{X}_1\underset{\sim}{\Pi}_1+X_2\underset{\sim}{\Pi}_2+\underset{\sim}{V} \tag{33}$$

or

$$\underset{\sim}{Y}_1 = \underset{\sim}{X}\underset{\sim}{\Pi} + \underset{\sim}{V} \tag{34}$$

where $\underset{\sim}{X}_2$ is $Tx(K-K_1)$ excluded predetermined variables from first equation, $\underset{\sim}{\Pi}_1$ is of demension $K_1x(G_1-1)$, $\underset{\sim}{\Pi}_2$ is $(K-K_1)x(G_1-1)$ and $\underset{\sim}{V}$ is $Tx(G_1-1)$. The resulting first-stage estimator is

$$\hat{\underset{\sim}{\Pi}} = (\underset{\sim}{X}'\underset{\sim}{X})^{-1}\underset{\sim}{X}'\underset{\sim}{Y}_1 \tag{35}$$

and obtain the calculated values in these regression:

$$\hat{\underset{\sim}{Y}}_1 = \underset{\sim}{X}\hat{\underset{\sim}{\Pi}} \tag{36}$$

2. Take the classical least-squares regression of y_1 on $\hat{\underset{\sim}{Y}}_1$ and $\underset{\sim}{X}_1$. The resulting coefficients are the 2SLS of $\underset{\sim}{\beta}_1$ and γ_1. Thus the two-stage least-squares are

$$\begin{bmatrix} \hat{\beta}_1 \\ \hat{\gamma}_1 \end{bmatrix} = [(\hat{Y} \;\; X_1)' \; (\hat{Y}_1 \;\; X_1)]^{-1} \; (\hat{Y}_1 \;\; X_1)' \; y_1 \qquad (37)$$

or
$$\begin{bmatrix} \hat{\beta}_1 \\ \hat{\gamma}_1 \end{bmatrix} = \begin{bmatrix} \hat{Y}_1'\hat{Y}_1 & \hat{Y}_1'X_1 \\ X_1'\hat{Y}_1 & X_1'X_1 \end{bmatrix}^{-1} \begin{bmatrix} \hat{Y}_1'y_1 \\ X_1'y_1 \end{bmatrix} \qquad (38)$$

We can rewrite the 2SLS estimators in a more-useful form by taking into account the fact that the residuals of the first-stage regression are orthogonal to the fitted value of the dependent variable and to each explanatory variables, that is

$$\hat{V}'X = 0 = X'\hat{V} \qquad (39)$$

Also,
$$\hat{Y}_1'\hat{V} = 0 \qquad (40$$

since $\hat{Y}_1$ is a linear combination of predetermined variables. Thus

$$\hat{Y}_1'\hat{Y}_1 = \hat{Y}_1'(Y_1-V) = \hat{Y}_1'Y_1 = Y_1'X(X'X)^{-1}X'Y_1$$

and $\hat{Y}'X_1 = (Y_1-V)'X_1 = Y_1'X_1$.

Therefore, we can rewrite Eq(38) as

$$\begin{bmatrix} \hat{\beta}_1 \\ \hat{\gamma}_1 \end{bmatrix} = \begin{bmatrix} Y_1'X(X'X)^{-1}X'Y & Y_1'X_1 \\ X_1'Y_1 & X_1'X_1 \end{bmatrix}^{-1} \begin{bmatrix} Y_1'X \;\; (X'X)^{-1}X'y_1 \\ X_1'y_1 \end{bmatrix}$$

(41)

Its asymptotic variance is,

$$\text{asy Var}\begin{bmatrix} \hat{\beta}_1 \\ \hat{\gamma}_1 \end{bmatrix} = s^2 \begin{bmatrix} Y_1 \; X(X'X)^{-1}X'Y_1 & Y_1' \; X_1 \\ X_1' \; Y_1 & X_1' \; X_1 \end{bmatrix}^{-1}$$

where $s^2 = T^{-1}(y_1 - Y_1\hat{\beta}_1 - X_1\hat{\gamma}_1)' \; (y_1 - Y_1\hat{\beta}_1 - X_1\hat{\gamma}_1)$

which is a consistent estimator of δ_u^2 .

The method of 2SLS requires rather a large number of observations, especially if the model includes many predetermined variables, which will be used in the first stage for obtaining the estimated values ($\hat{Y}$'s) of the endogenous variables.

2.2.4 Limited-Information Maximum Likelihood (LIML)

The LIML method, which was originally derived by Anderson and Rubin (1949), provides estimators of the parameters of a single justidentified or overidentified structural equation on the maximum-likelihood principle, the likelihood maximizing being done subject only to the a priori identifying restrictions imposed on the equation being estimated. In other words, Anderson and Rubin arrived at these estimators by application of the maximum-likelihood principle under the specification that the structural disturbances are normally distributed and utilizing only the restrictions on the structural equation being estimated. For this reason the method is generally known as the "Limited-information maximum-likelihood" method.

It is based on the same idea (like 2SLS) of purging the endogenous variables, which appear as explanatory in the particular equation, from their random component, so that they become non-stochastic and hence independent of the random term U of the particular structural equation.

The mathematical development of the limited-information maximum-likelihood estimator is complicated and lengthy. However, the so-called least-variance ratio (LVR) principle gives the same estimators in this situation as the maximum-likelihood principle. We shall present a relatively simple non-rigorous justification of the LVR principle based on the fact that the least-squares method yields maximum-likelihood estimators for equations having only one dependent variable (all the others being predetermined). After having thus justified the LVR principle, we shall follow the rigorous derivation of the limited information estimators from it. The LVR estimates are derived as follows.

Consider again the structural equation

$$\underline{y}_1 = \underline{Y}_1\underline{\beta}_1 + \underline{X}_1\underline{\gamma}_1 + \underline{U}_1$$

and rewrite it as

$$\tilde{\underline{y}}_1 = \underline{X}_1\underline{\gamma}_1 + \underline{U}_1 \qquad (43)$$

where $\tilde{\underline{y}}_1 = \underline{y}_1 - Y_1\underline{\beta}_1$

so that the $\tilde{\underline{Y}}_1$ vector is a linear combination of the endogenous variables appearing in the equation. If $\tilde{\underline{Y}}_1$ is regressed on $\underline{X}_1$, then

$$\hat{Y}_1 = (\underline{X}_1' \underline{X}_1)^{-1} \underline{X}_1' \tilde{\underline{Y}}_1 ,$$

and the residual sum of squares is

$$SSE_1 = (\tilde{\underline{Y}}_1 - \underline{X}_1 \tilde{Y}_1)' (\tilde{\underline{Y}}_1 - \underline{X}_1 \tilde{Y}_1)$$

$$= \tilde{\underline{Y}}_1' \tilde{\underline{Y}}_1 - \tilde{\underline{Y}}_1' \underline{X}_1 (\underline{X}_1' \underline{X}_1)^{-1} \underline{X}_1' \tilde{\underline{Y}}_1 .$$

Similarly, if $\tilde{\underline{Y}}_1$ is regressed on all the predetermined variables, $\underline{X} = (\underline{X}_1 \; \underline{X}_2)$, we have

$$\tilde{Y} = (\underline{X}' \underline{X})^{-1} \underline{X}_1' \tilde{\underline{Y}}_1$$

and the residual sum of squares would be

$$SSE = (\tilde{\underline{Y}}_1 - \underline{X} \tilde{\underline{Y}})' (\tilde{\underline{Y}}_1 - \underline{X} \tilde{\underline{Y}})$$

$$= \tilde{\underline{Y}}_1' \tilde{\underline{Y}} - \tilde{\underline{Y}}_1 - \underline{X} (\underline{X}' \underline{X})^{-1} \underline{X}' \tilde{\underline{Y}}_1 .$$

The second residual sum of squares will be no greater than the first since the second regression includes all the explanatory variables in the first regression $\underline{X}_1$ plus the set $\underline{X}_2$. Thus, the ratio

$$\ell = \frac{SSE_1}{SSE} = \frac{\tilde{\underline{Y}}_1' \tilde{\underline{Y}}_1 - \tilde{\underline{Y}}_1' \underline{X}_1 (\underline{X}_1' \underline{X}_1)^{-1} \underline{X}_1' \tilde{\underline{Y}}_1}{\tilde{\underline{Y}}_1' \tilde{\underline{Y}}_1 - \tilde{\underline{Y}}_1' \underline{X} (\underline{X}' \underline{X})^{-1} \underline{X}' \tilde{\underline{Y}}_1} \qquad (44)$$

can never be smaller than unity.

The next step is the evaluation of β_1. Rewrite Eq(43) as

$$\tilde{y}_1 = Y_{1\Delta}\beta_{1\Delta}$$

where $\quad Y_{1\Delta} = (y_1 Y_2 \cdots y_{G1})$ and $\beta_{1\Delta} = \begin{bmatrix} 1 \\ \beta_{12} \\ \vdots \\ \beta_{1G1} \end{bmatrix}$

Rewrite ℓ as

$$\ell = \frac{\beta_{1\Delta}' W_{1*} \beta_{1\Delta}}{\beta_{1\Delta}' W_1 \beta_{1\Delta}} \tag{45}$$

where $\quad W_{1*} = Y_{1\Delta}' Y_{1\Delta} - (Y_{1\Delta}' X_1)(X_1' X_1)^{-1} X_1' Y_{1\Delta}$

$$W_1 = Y_{1\Delta}' Y_{1\Delta} - (Y_{1\Delta}' X)(X'X)^{-1} X' Y_{1\Delta}.$$

Least-variance ratio is obtained by estimating β_{1i} $(i=1,\ldots,G_1)$ elements of $\beta_{1\Delta}$ which minimize ℓ; and which is the same criterion as that for the LIML estimator. Differentiating ℓ with respect to $\beta_{1\Delta}$, we obtain:

$$\frac{\partial \ell}{\partial \beta_{1\Delta}} = \frac{2(W_{1*}\beta_{1\Delta})(\beta_{1\Delta}' W_1 \beta_{1\Delta}) - 2(\beta_{1\Delta}' W_{1*}\beta_{1\Delta})(W_1\beta_{1\Delta})}{(\beta_{1\Delta}' W_1 \beta_{1\Delta})^2} \tag{46}$$

Setting the result equal to zero vector gives

$$W_{1*}\tilde{\tilde{\beta}}_{1\Delta} - \frac{\beta_{1\Delta}' W_{1*} \beta_{1\Delta}}{\beta_{1\Delta}' W_1 \beta_{1\Delta}} W_1 \tilde{\tilde{\beta}}_{1\Delta} = 0$$

or $$(W_{1*} - \ell W_1)\tilde{\tilde{\beta}}_{1\Delta} = 0 \tag{47}$$

i) The estimates obtained from the application of LIML are biased for small samples. However, the estimates are consistent, that is, their bias tends to zero and their distribution collapses on the true value of the parameters as the size of the sample grows infinitely large.

ii) If the disturbances of the structural model are normally distributed the LIML estimates are asymptotically efficient.

iii) The computational procedure of LIML is cumbersome. Certainly it is more complicated than 2SLS. This is one of the reasons for the perference of econometricians for 2SLS in actual econometric research. Two stage least-squares may also have an advantage over LIML in small samples.

2.2.5 Three-Stage Least-Squares (3SLS)

The simultaneous equations to be considered in this section are the same as those in sections (2.2.3) and (2.2.4). However, sections (2.2.3) and (2.2.4) discussed "single-equation" methods of estimation, in the sense that the estimators there operated on each equation separately. This section will discuss, 3SLS, "systems" methods of estimation, which estimate all equations jointly.

This set of equations will only have a nontrivial solution if the determinantal equation

$$|W_{1*} - \ell W_1| = 0 , \tag{48}$$

is satisfied. This gives a polynomial in ℓ , which must be solved for smallest root $\hat{\ell}$. That is, clearly we choose the smallest $\hat{\ell}$ values since we aim at the minimum variance ratio.

Having estimated $\hat{\ell}$ and using our assumption that the first element of $\tilde{\beta}_{1\Delta}$ equal to unity, we substitute in the system of equation (47) and the estimator $\tilde{\tilde{\beta}}_{1\Delta}$ obtained from

$$(W_{1*} - \hat{\ell} W_1) \tilde{\tilde{\beta}}_{1\Delta} = 0.$$

In order to obtain estimates of the γ's we substitute the LVR estimates of $\tilde{\tilde{\beta}}_{1\Delta}$'s in the structural equation of the form

$$\hat{\tilde{Y}}_1 = Y_{1\Delta} \tilde{\tilde{\beta}}_{1\Delta}$$

and regressing $\hat{\tilde{Y}}_1$ on X_1 gives

$$\tilde{\gamma}_1 = (X_1' X_1)^{-1} X_1' \hat{\tilde{Y}}_1 = (X_1 X_1)^{-1} X_1' Y_{1\Delta} \tilde{\tilde{\beta}}_{1\Delta}$$

These are the LIML estimates. The LIML estimators have the same asymptotic variance-covariance matrix as 2SLS. The estimates of the asymptotic variances, however, will differ since s^2 is computed from the estimated structural coefficients, which will be different in the two cases. One should note the following:

The motivation for considering joint estimation procedures, of course, is that they are generally more (asymptotically) efficient than the single-equation procedures. The 3SLS was developed by Theil and Zellner (1962) as a logical extension of Theil's 2SLS. 3SLS method involves the application of least-squares in three stages. The first two stages are the same as 2SLS except that we deal with reduced-form of all the equations of the system. The third stage involves the application of generalized least-squares, that is, the application of least-squares to a set of transformed equations, in which the transformation required is obtained from the reduced-form residuals of the previous stage.

Again, let us consider the simultaneous equation model containing G jointly dependent variables and K predetermined variables. The i^{th} equation may be written

$$\underline{y}_i = \underline{Y}_i \underline{\beta}_i + \underline{X}_i \underline{\gamma}_i + \underline{U}_i$$

$$= \underline{Z}_i \underline{\delta}_i + \underline{U}_i \qquad (49)$$

where $\underline{y}_i$ is a Tx1 vector of sample observations on the dependent variable in the i^{th} equation, $\underline{X}_i$ is a TxK_i matrix of observations on the predetermined variables in the equation, $\underline{Y}_i$ is a TxG_i matrix of observations on the other endogenous variables in the equation, $\underline{\beta}_i$

and γ_i are vectors of structural parameters, and U_i is the column vector of T structural desturbances; and

$$Z_i = (Y_i \; X_i) \; , \; \delta_i = \begin{bmatrix} \beta_i \\ \gamma_i \end{bmatrix}$$

Further, we write X for the TxK matrix of values taken by all (K) predetermined variables, and we shall suppose that its rank is K. Our objective is to estimate the parameter vectors δ_i, and for this purpose it will be supposed that all equations are identifiable. If Eq(49) is premultiplied by X', we obtain

$$X'y_i = X'Z_i\delta_i + X'U_i \tag{50}$$

Assuming that the predetermined variables are all "fixed" variables, we find for the variance-covariance matrix of the disturbance vector $X'U_i$

$$V(X'U_i) = E(X'U_i'U_iX) = \delta_{ii'}X'X \, . \tag{51}$$

where $\delta_{ii'}$ is the variance of each of T disturbances of the i^{th} structural equation. Then, applying Aitken's method of generalized least-squares (GLS) to Eq(50), we obtain

$$Z_i'X \;(\delta_{ii'}X'X)^{-1}X'Y_i = Z_iX(\delta_{ii'}X'X)^{-1}X'Z_i \, d_i \tag{52}$$

from which we derive the 2SLS estimator

$$d_i = \left[Z_i'X\,(X'X)^{-1}X'Z_i\right]^{-1}Z_i'X(X'X)^{-1}X'Y_i \tag{53}$$

Equation (53) is simply another way of writing the 2SLS estimator of Eq(49), as may be verified by substituting for $\underline{Z}_i$, multiplying out, and comparing with the original expression for the 2SLS estimator in Eq(41).

The crucial idea of the 3SLS is that Eq(50) can be written in the following form for all equations combined:

$$\begin{bmatrix} \underline{X}'\underline{y}_1 \\ \underline{X}'\underline{y}_2 \\ \vdots \\ \underline{X}'\underline{y}_G \end{bmatrix} = \begin{bmatrix} \underline{X}'\underline{Z}_1 & 0 & \cdots & 0 \\ 0 & \underline{X}'\underline{Z}_2 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \underline{X}'\underline{Z}_G \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \\ \delta_G \end{bmatrix} + \begin{bmatrix} \underline{X}'\underline{U}_1 \\ \underline{X}'\underline{U}_2 \\ \vdots \\ \underline{X}'\underline{U}_G \end{bmatrix} \qquad (54)$$

which is a system of KG equations involving

$$n = \sum_{i=1}^{G} n_i \qquad (55)$$

parameters. Let us write $\underline{\delta}$ for the n-element column vector of parameters on the right of Eq(54). Then we can apply generalized least-squares to Eq(54) to estimate all elements of $\underline{\delta}$ simultaneously. For this purpose we need the covariance matrix of the disturbance vector of Eq(54).

$$\underline{V} \begin{bmatrix} \underline{X}'\underline{U}_1 \\ \underline{X}'\underline{U}_2 \\ \vdots \\ \underline{X}'\underline{U}_G \end{bmatrix} = \begin{bmatrix} \delta_{11}\underline{X}'\underline{X} & \delta_{12}\underline{X}'\underline{X} & \cdots & \delta_{1G}\underline{X}'\underline{X} \\ \delta_{21}\underline{X}'\underline{X} & \delta_{22}\underline{X}'\underline{X} & \cdots & \delta_{2G}\underline{X}'\underline{X} \\ \vdots & \vdots & & \vdots \\ \delta_{G1}\underline{X}'\underline{X} & \delta_{G2}\underline{X}'\underline{X} & \cdots & \delta_{GG}\underline{X}'\underline{X} \end{bmatrix} = \Sigma \otimes X'X, \qquad (56)$$

where $\delta_{ii'}$ is the contemporaneous covariance of the structural disturbances of the i^{th} and i'^{th} equation. Direct computation shows that the inverse of covariance matrix of Eq(56) is

$$\underline{V}^{-1}\begin{bmatrix}\underline{X}'\underline{U}_1\\ \vdots\\ \underline{X}'\underline{U}_G\end{bmatrix}=\begin{bmatrix}\delta^{11}(\underline{X}'\underline{X})^{-1} & \delta^{12}(\underline{X}'\underline{X})^{-1} & \ldots\delta^{1G}(\underline{X}'\underline{X})^{-1}\\ \delta^{21}(\underline{X}'\underline{X})^{-1} & \delta^{22}(\underline{X}'\underline{X})^{-1} & \ldots\delta^{2G}(\underline{X}'\underline{X})^{-1}\\ \vdots & & \vdots\\ \delta^{G1}(\underline{X}'\underline{X})^{-1} & \delta^{G2}(\underline{X}'\underline{X})^{-1} & \ldots\delta^{GG}(\underline{X}'\underline{X})^{-1}\end{bmatrix} \tag{57}$$

where $\delta^{ii'}$ is an element of the inverse of the contemporaneous covariance matrix of the structural disturbances. Straightforward application of GLS of Eq(52) gives then the following results:

$$\begin{bmatrix}\delta^{11}\underline{Z}_1'\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{y}_1+\ldots+\delta^{1G}\underline{Z}_1'\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{y}_G\\ \vdots\\ \delta^{G1}\underline{Z}_G'\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{y}_1+\ldots+\delta^{1G}\underline{Z}_1'\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{y}_G\end{bmatrix}$$

$$=\begin{bmatrix}\delta^{11}\underline{Z}_1\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Z}_1 & \ldots & \delta^{1G}\underline{Z}_1'\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Z}_G\\ \vdots & & \vdots\\ \delta^{G1}\underline{Z}_G'\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Z}_1 & \ldots & \delta^{GG}\underline{Z}_G'\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Z}_G\end{bmatrix}\begin{bmatrix}d_1\\ \vdots\\ d_G\end{bmatrix} \qquad \ldots(58)$$

These matrices involve δ's which are generally unknown. The Zellner-Theil suggestion is to estimate first each structural equation by 2SLS, giving the residual vectors

$$\hat{\underline{U}}_i = \underline{y}_i - \underline{Z}_i\underline{d}_i \quad , \; i = 1,\ldots,G.$$

where $\underline{d}_i$ is the 2SLS estimator of $\underline{\delta}_i$. The elements of Σ are then estimated by

$$S_{ii'} = \frac{\hat{\underline{U}}_i'\hat{\underline{U}}_{i'}}{n} \text{ for all } i,\, i'$$

giving $\hat{V} = \hat{\Sigma} \otimes I$.

$$\underline{d}_{3SLS} = \begin{bmatrix} s^{11}\underline{Z}'_1\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Z}_1 & \cdots & s^{1G}\underline{Z}'_1\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Z}_G \\ \vdots & & \\ s^{G1}\underline{Z}'_G\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Z}_1 & \cdots & s^{GG}\underline{Z}'_G\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Z}_G \end{bmatrix} x$$

$$\begin{bmatrix} \sum_{i=1}^{G} s^{1i}\underline{Z}'_1\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'y_i \\ \vdots \\ \sum_{i=1}^{G} s^{Gi}\underline{Z}'_G\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'y_i \end{bmatrix} \qquad (59)$$

where the $s^{ii'}$ denote the elements in $\hat{\Sigma}^{-1}$. The moment matrix of d_{3SLS} is

$$\underline{V}(d_{3SLS}) = \begin{bmatrix} s^{11}\underline{Z}'_1\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Z}_1 & \cdots & s^{1G}\underline{Z}'_1\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Z}_G \\ \vdots & & \\ s^{G1}\underline{Z}'_G\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Z}_G & \cdots & s^{GG}\underline{Z}'_G\underline{X}(\underline{X}'\underline{X})^{-1}\underline{X}'\underline{Z}_G \end{bmatrix} \qquad (60)$$

It will be observed that there is a gain in asymptotic efficiency compared with 2SLS only if $(\delta_{ii'})$ is not diagonal, if it is diagonal, two and three-stage least-squares are identical, because the nondiagonal blocks of Eq(60) are then zero and $\delta^{ii} = \frac{1}{\delta_{ii}}$. Further, Zellner and Theil suggest, however, that the equation system be divided into two groups, one consisting of all the overidentified equations and the other consisting of all the exactly identified equations and they recommend that for the estimation of the parameters in the group containing overidentified equations, the 3SLS method be applied for this group, ignoring all the exactly identified equations in the system. They point out that this procedure does not affect the efficiency of the estimates, that is, the large sample variance-covariance matrix of the estimates of the parameters in the group of overidentified equations is the same whether the 3SLS method is applied taking the exactly identified equations into account or if these latter equations are completely disregarded, and then the 3SLS estimators of the justidentified equations are obtained by adding to the relevant 2SLS estimates a linear combination of the 3SLS estimates of the overidentified equations.

CHAPTER 3

MODEL SPECIFICATIONS

In constructing econometric models of demand for and [su]pply of money in Ethiopia, the significant variables may be [de]fined as follows:

M_{1t} (narrow nominal money balance) = currency held by the public plus net demand deposits;

M_{2t} (broad nominal money balance) = M_{1t} + time deposits + savings deposits;

Y_t = Gross domestic product at current factor cost;

P_t = General retail price index (1963 = 100).

MB_t (monetary base) = currency held by the public + bank reserves;

Cash reserves = currency held by the NBE + Bank reserves;

DC_t = Total domestic credit;

$m_{1t} = (\frac{M_{1t}}{P_t})$ = Narrow real money balance;

$m_{2t} = (\frac{M_{2t}}{P_t})$ = Broad real money balance;

$y_t = (\frac{Y_t}{P_t})$ = real income.

$\dot{p}_t = (\frac{dP_t}{d_t})/P_{t-1}$ = the rate of change of price;

q_t (cash reserve ratio) = cash reserves divided by bank deposits.

k_t = dummy variable, to Ethiopia's pre- and post-revolutions period.

t = time.

.1 Supply of Money in Ethiopia

The term "money supply" includes the aggregate ock of domestic money owned by the public in a country. It efers to ownership by private individuals and business firms erating in the economy. It excludes from itself the cash alances held by the Central Government, the Central Bank, e Treasury and the Cash Reserves owned by the Commercial anks. the supply of money at any particular moment of time s then the total amount of money in circulation.

Theoretically money supply is fixed by monetary authoities. But once it is issued it is affected by a number of conomic variables. In other words, change in money supply rom one period to another is explained by many interrelated conomic factors. In what follows we will consider only four f the explanatory variables which may explain variation in money upply of Ethiopia.

One of the most important determinants of money supply of thiopia in any given year is the monetarybase (high-powered oney). Total domestic credit and cash reserve ratio are also n important explanatory variables that explain Variation in oney supply. Other determinant of money supply is assumed to e rate of change of price, as suggested in recent studies by eigen (1964), Smith (1967) and Modigliani, Rasche and Cooper 1970). Although foreign assets are theoretically believed to ffect money supply, their role is minimal in Ethiopia's case nd hence are disregarded.

With the above theoretical premise, an attempt will be ade to study the effects of monetary base, domestic credit, sh reserve ratio, and the rate of change of price on upply of money in Ethiopia. The narrow and broad money re defined as

$$M_{1t} = C + DD \quad (61)$$

$$M_{2t} = M_{1t} + TD + SD \quad (62)$$

here M_{1t} is narrow money, M_{2t} is broad money, C is currency eld by the public other than commercial banks, DD is net emand deposits, TD is time deposits, and SD is savings eposits. Attension is now directed to single-equation odels of money stock determination in the case of Ethiopia.

The single-equation model for the money supply in Ethiopia can be written as:

$$M_{it} = a_0 + + a_1 \dot{P}_t + a_2 MB_t + a_3 \sigma_t + a_4 DC_t + U_t \quad (63)$$

here M_{it} = nominal money balances, i=1,2.

$\dot{P}_t$ = rate of change of price

MB_t = monetary base

σ_t = cash reserve ratio

DC_t = total domestic credit

U_t = error term.

We should expect the following signs of the coefficients to hold true $a_1 > 0$, $a_2 > 0$, $a_3 > 0$, $a_4 > 0$. Recall the narrow money and broad money definitions:

$$M_1 = C + DD \quad (61)$$

$$M_2 = M_{1t} + TD + SD \quad (62)$$

nd monetary base (MB) is defined as:

$$MB = C + BR \qquad (64)$$

BR = bank reserves

$D = DD + TD + SD$

or the narrow money and broad money definitions, divide hrough equations (61) and (62) respectively by (64) and earrange terms to get:

$$M_1 = MB.\frac{b(1+p)}{b+p} \qquad (65)$$

$$M_2 = MB.\frac{b'(1+p')}{b'+p'} \qquad (66)$$

here $b = \frac{DD}{BR}$, $p \frac{DD}{C}$, $b' = \frac{D}{BR}$, $p' = \frac{D}{C}$

he stock money is thus seen to be linked arithmetically to he monetary base, to the ratio of commercial bank reserves o deposits, and to the ratio of the public's currency oldings to commercial bank deposits. When these ratios are reated as constants then the coefficients

$$\frac{b(1+p)}{b+p} \text{ and } \frac{b'(1+p')}{b'+p'}$$

an be defined to be money-multipliers(m). Since the multi- lier is affected by the behaviour of commercial banks and e public, they are not, in general, expected to be constant. us it is argued that the money multipliers cannot be used determine the stock of money. Moreover, multiplier is ot unique at a point of time.

The objection that multipliers are static, not useful . prediction, may be overcome by introducing dynamic ele- its in the model. The dynamic model is given as,

$$M_{it} = m_o + mMB^*_t \ , \ i = 1,2 \qquad (67)$$

ere MB^*_t is the desired or expected value of monetarybase timeperiod "t". MB^*_t is not directly observable and hence partial adjustment mechanism relating actual to desired lue is postulated as,

$$MB^*_t = (1-b)\ MB_t + bMB^*_{t-1} \ ; \ 0<b\leq 1 \qquad (68)$$

substituting (68) into (67), it can be shown that

$$M_{it} = a_o + a_1 MB_t + a_2 M_{it-1} + U_t \qquad (69)$$

ere $a_0 = m_o(1-b)$ $(i = 1,2)$

$a_1 = m(1-b)$

d $a_2 = b$

e signs for all coefficients of Eq(63) and (69) expected to positive. Relations (63) and (69) will be fitted by the dinary least-squares method.

.2 Demand for Money in Ethiopia

Since money is primarily held, in Ethiopia, fpr the pur- ose of financing transactions, and since transactions undertaken e related to income, one expects the amount of money demanded depend on income. The number of amount of goods a given iantity of money can finance depends on the price of level.

Therefore, the demand for money in Ethiopia will depend on income and the rate of change of price. Adding dummy variable to Ethiopia's pre- and post-revolution years, one can then hypothesize this simple form of demand for money function as,

$$M_{it} = b_o + b_1\dot{p}_t + b_2Y_t + b_3K_t + V_t \quad (i=1,2) \qquad (70)$$

where Y_t = nominal income

$$K_t = \begin{cases} 1, \text{ for post-revolution years} \\ 0, \text{ for pre-revolution years} \end{cases}$$

V_t = error term,

and

$$m_{it} = b_o + b_1\dot{p}_t + b_2y_t + b_3k_t + V_t \quad (i=1,2) \qquad (71)$$

where $m_{it} = \frac{M_{it}}{p_t}$ = real money balance

$y_t = \frac{Y_t}{p_t}$ = real income.

If the annual money demand model assumes an instantaneous adjustment of the dependent variable in the adjustment of income expectations, a slightly different specification is called for. The model permits some degree of lag, which in a developing economy, would not expect to be large because of a low level of per capital income. In other words, we assume that a year is a long enough period for wealth holders to bring their actual holdings of real money balances into conformity with their desired. To fix ideas, begin with a simple linear relation where demand for money depends only on expected or desired income, or $$M_{it} = B_0 + B_1 Y_t^*$$ (72)

Suppose that permanent income is a distributed lag of current and past income levels, then,

$$Y_t^* = \sum_{T=0}^{\infty} \lambda_T Y_{t-T}$$ (73)

where the $_T$'s are weights, $\lambda_T > 0$ and $\sum_{T=0}^{\infty} \lambda_T = 1$. If, following Koyck, we assume that the weights decline exponentially, then (73) reduces to

$$y_t^* = (1-\lambda) \sum_{T=0}^{\infty} \lambda^T Y_{t-T}$$ (74)

(72) becomes

$$M_{it} = C_0 + C_1 Y_t + C_2 M_{it-1}$$ (75)

where $C_0 = B_0(1-\lambda)$; $C_1 = B_1(1-\lambda)$; $C_2 = \lambda$.

For generality, rewrite (75) in compact form, switch to dummy variable, and error term to obtain,

$$M_{it} = b_0 + b_1 \dot{p}_t + b_2 Y_t + b_3 M_{it-1} + b_4 K_t + V_t$$
(for i = 1,2) (76)

where M_{it-1} = one period lag nominal money balance,

and $$m_{it} = b_0 + b_1 \dot{p}_t + b_2 Y_t + b_3 m_{it-1} + b_4 k_t + V_t$$
(for i = 1,2) (77)

where $m_{it} = (\frac{M_{it}}{p_t})$ = real money balance

$y_t = = (\frac{Y_t}{P_t}) =$ real income

$m_{it-1} = (\frac{M_{it-1}}{P_t})$ = lagged real money balance.

Relations (70), (71), (76) and (77) will be fitted by using ordinary least squares method and Eq(76) and Eq(77) used to test the applicability of the permanent income hypothesis.

3.3 A Structural Model for Money Demand and Money Supply Functions

Since both demand and supply functions for money are sections (3.1) and (3.2), the parameters of the demand for and supply of money functions for Ethiopia will be estimated by systems of simultaneous equation techniques.

Thus, the demand for money function may be specified as,

$$M_{it} = b_0 + b_1 \dot{P}_t + b_2 Y_t + b_3 M_{it-1} + b_4 k_t + V_t \qquad (\text{for } i = 1,2) \qquad (76')$$

on a priori grounds the expected signs of b_2, b_3 and b_4 are positive and the expected sign of b_1 is negative.

Specification of the supply function completes the model. The money supply function is specified as follows:

$$M_{it} = a_0 + a_1 \dot{P}_t + a_2 MB_t + a_3 q_t + a_4 DC_t + U_t \qquad (\text{for } i = 1,2) \qquad (63')$$

In the supply function (63') the expected signs of a_1, a_2, a_3 and a_4 are positive.

Thus the model contains two structural equations in which the money balances and the rate of change of price are endogenous and income, one-period lagged money balances, monetary bases, cash reserve ratio, total domestic credit and dummy variable are exogenous.

CHAPTER 4

ESTIMATION OF THE MODELS AND INTERPRETATION OF RESULTS

This section considers the OLS, the 2SLS, the LIML and 3SLS estimates of the money demand and supply functions. Data for this time-series analysis were obtained fromvarious issues of the Quarterly Bulletin of the National Bank of Ethiopia and from the Statistical Abstruct of the Central Statistical Authority. All data represent January-December callender year and annual time-series extending from 1964 to 1985, giving a total of twenty-two observations and thereby provide empirical results to various models formulated in part three. The basic data appear in Table I of the appendix.

4.1 Estimates of Money Supply Functions

The regression results, which are estimated using the method of ordinary least-squares are given in Table 1 below.

Table 1

Estimates of Money Supply Functions

Equations	Dependent Variables	Explanatory Variables	Coefficients	S.E.	R^2	F	D.W.
		$\dot{P}_t$	3.180	4.567	0.982	231.791	2.241
		MB_t	0.646	0.302			
(63)	M_{1t}	q_t	128.221	144.269			
		DC_t	0.260	0.101			
		Constant	-169.518	328.746			
		$\dot{P}_t$	9.049	5.885	0.986	294.089	1.723
		MB_t	0.612	0.389			
(63)	M_{2t}	q_t	43.767	185.908			
		DC_t	0.481	0.130			
		Constant	75.564	423.631			

Table 1 (contd.)

Equations	Dependent Variables	Explanatory Variables	Coefficients	S.E.	R^2	F	D.W.
		MB_t	0.585	0.216	0.975	373.222	2.194
(69)	M_{1t}	M_{1t-1}	0.584	0.196			
		Constant	-6.784	46.605			
		MB_t	0.467	.272	0.985	642.00	2.140
(69)	M_{2t}	M_{2t-1}	0.827	0.171			
		Constant	0.806	52.573			

The results appear to be satisfactory, when one considers the a priori restrictions imposed on estimation. All of the coefficients have their expected signs and their magnitudes are plausible values. Standard error (s.e.) of the estimates are reasonably low and Durbin-Watson (D.W.) statistics indicate no serial correlation of the residuals. A brief discussion of the results will follow:

Equation (63) explains 98.2 per cent and 98.6 per cent of the variation in M_{1t} and M_{2t}, respectively, by means of r_t, MB_t, q_t and DC_t. A high significant linear association exists between these variables as the F-ratios of (63) show. It is seen that MB_t and DC_t are significant in the case of M_{1t}, and only DC_t is significant in M_{2t} case. These suggest that monetary base and total domestic credit are dependable and useful in explaining variation in the supply of money variable (for M_{1t}) and only total domestic credit in the supply of money variable (for M_{2t}). Equation (69) shows also better fit. signs of all the variables came out right. The two independent variables (for M_{1t}) and one independent variable, M_{2t-1}, (for M_{2t}) are significant. These show that monetary base and lagged nominal money balance are reliable predictors of money supply for M_{1t} and only lagged nominal money balance is best predictor of money supply for M_{2t}.

It can be verified that money supply and money demand functions (Eq (63') and Eq (76')) for both definitions of money are overidentified. Thus econometric techniques such as 2SLS, LIML, and 3SLS were employed. The results of money supply functions are depicted in Table 2 below.

Table 2

Estimates of Money Supply Functions

Equations	Dependent Variables	Explanatory Variables	Coefficients	S.E.	R^2	F	D.W.
		$\dot{P}_t$	8.297	6.856	0.982	245.017	2.14
		MB_t	0.452	0.354			
(63a')	M_{1t}	σ_t	131.160	140.136			
		DC_t	0.343	0.125			
		Constant	-151.094	319.584			
		$\dot{P}_t$	25.859	6.217	0.992	524.211	2.022
		MB_t	-0.150	0.357			
(63a')	M_{2t}	σ_t	-37.272	141.434			
		DC_t	0.751	0.122			
		Constant	349.834	327.187			

Table 2 (contd.)

Equations	Dependent Variables	Explanatory Variables	Coefficients	S.E.	R^2	F	D.W.
(63b')	M_{1t}	$\dot{p}_t$	7.758	6.120	0.980	302.05	2.457
		MB_t	0.443	0.234			
		q_t	119.221	140.853			
		DC_t	0.338	0.076			
		Constant	-113.279	318.717			
(63b')	M_{2t}	$\dot{p}_t$	30.303	6.714	0.975	237.821	2.429
		MB_t	-0.319	0.364			
		σ_t	-2.324	219.372			
		DC_t	0.806	0.118			
		Constant	328.023	496.380			
(63c')	M_{1t}	$\dot{p}_t$	7.211	5.375	0.98	241.04	2.45
		MB_t	1.204	0.066			
		q_t	165.604	109.131			
		DC_t	0.021	0.004			
		Constant	271.244	216.658			
(63c')	M_{2t}	$\dot{p}_t$	19.394	6.900	0.986	431.06	2.23
		MB_t	1.417	0.078			
		σ_t	19.954	83.165			
		DC_t	0.011	0.003			
		Constant	212.823	201.408			

Equation (63a'), Eq (63b'), and Eq (63c') represent 2SLS, LIML and 3SLS estimates respectively. In these estimates, the degree of explanation achieved in all equations is quite high. The

D.-W. statistics all indicate that no serial correlation exists in the residuals of any equation. For M_{1t}, all of the signs agree with expectations, and for M_{2t}, only the coefficients of DC_t and $\dot{p}_t$ agree with expectations in the 2SLS and LIML cases.

In narrow money definition equations (M_{1t}) which give the best results in terms of the R^2 and the statistical significance of the regression coefficients, we see that only the coefficients of the total domestic credit and the monetary base variables are statistically significant. The cash reserve ratio and the rate of change of price coefficients are not significant. In other words, the monetary base and the total domestic credit for money supply, for M_{1t}, are instantaneous.

As for 'narrow' money, the results of the supply of money equations (for M_{2t}) show a reasonably high R^2. The Durbin-Watson statistics are in the acceptable region. Estimates of money supply functions (for M_{2t}) seem to be more sensitive to the rate of change of price than 'narrow' money (M_{1t}). This is reflected by the significance of the coefficients of $\dot{p}_t$ in M_{2t}, and the nonsignificance of it in M_{1t}. Therefore, the supply of money in the economy is not wholly determined by the amount of the monetarybase and the total domestic credit; it also depends on the rate of change of price.

The major results of the money supply function can be summarized briefly:

i) The expected regression equations for nominal money balances, as shown in Tables 1 and 2, indicate that the overall goodness fit of the model is excellent and that the individual variables contribute significantly to the explanation of behaviour of nominal money balances.

ii) In Table 1, the signs of all explanatory variables are consistent with a priori specifications. The coefficient of the lagged dependent variable implies that there is generally a short adjustment period for actual nominal money balances to adjust to their desired values.

iii) The coefficients of the rate of change of price, $\dot{p}_t$, as presented in Tables 1 and 2, have the correct positive sign, but are statistically insignificant in the OLS estimates (shown in Table 1) and are statistically significant in the 2SLS, LIML and 3SLS estimates (shown in Table 2). This may be so because the OLS estimate suffers from simultaneous equation bias.

iv) Similar gain or loss of information in the equations can be seen when the dependent variables are "M_{1t}" and "M_{2t}"; this supports the view that for Ethiopia, both narrow and broad money definitions must be applied for money supply functions.

v) Finally, the OLS estimates indicate that monetarybase, lagged nominal balance and total domestic credit are appropriate explanatory variables of the money supply. While 2SLS, LIML and 3SLS estimates show that total domestic credit, monetary base, and the rate of change of price are important explanatory variables in explaining the money supply functions.

4.2 Estimates of Money Demand Function

In this section we present an empirical results of the demand for money for the two definitions of money. The results with the highest multiple coefficient of correlation and generally significant regression coefficients, though not necessarily significant t-values, are reported in Tables 3 and 4. We have singled out specific estimates for discussion in this section. Equation (70) is estimated first.

The coefficients of '$\dot{p}_t$' and 'Y_t', of Eq (70), are of the expected sign. The nominal income coefficients are significant and the rate of change of price coefficients, however, are not significant. Although the over-all regression is significant with one percent probability error, nominal income, the rate of change of price and dummy variable can explain 92.5 percent (for M_{1t}) and 93.6 percent (for M_{2t}) of the total variation in the demand for nominal money balances.

The corresponding model, Eq (71), is in the real form and estimated. The coefficients are of the expected sign and the real income coefficients in both cases are significant. Real income, rate of change of price and dummy variable account for about 67 percent variation in the demand for real money balances in both narrow and broad money definitions.

Lagged values of real money balance have often figured as explanatory variables in demand functions for money. Eq (76) and (77) in Table 3 incorporate a one-period lagged money balance variable in addition to $\dot{p}_t$, y_t, k_t.

The results of Eq (76) show that a very high proportion of the observed variance in money balances is explained by the two arguments of the demand function. The lagged money balance coefficient is high and differs substantially depending on the definition of money used. The rate of change of price coefficient is also high and substantially different for the two definitions of money. Assuming that the residuals are free of autocorrelation, the tests indicate that only one-period lagged nominal money balance is statistically significant for both definitions of money. A significant lagged nominal balance coefficient suggests that the total effect of a change in nominal income on the demand for money is not realised in the same year but extends over future years. Alternatively, it may be interpreted as an indication, that a 'permanent income variable' may have better explanatory power than current income.

Now, we consider estimates of the money demand functions, of Eq (77), with respect to one-period lagged real money balances and three independent variables ($\dot{p}_t$, y_t and k_t) for two definitions of money.

The results are fairly satisfactory since we are able to explain onver 79 percent (in m_{1t}) and 86 percent (in m_{2t}) of the variation in the demand for real money balances. The Durbin-Watson statistics are much better although they should be discounted on account of the presence of a lagged endogenous variable in the fitted equations.

All coefficients are of the expected sign and the rate of change of price coefficient, in both cases, is significant. One period lagged real money balance is also statistically significant for both definitions of money and k_t is significant for m_{1t} but not significant for m_{2t}. The t-value of the real income variable is not significant in both cases.

The meaning of this result is that the role of 'permanent' real income effects are the primary influence and the rate of change of price effects are the secondary.

Next, we consider estimates of the money demand functions which were fitted by means of 2SLS, LIML, and 3SLS to the data of the same time periods as we fitted the linear form of equations, shown in Table 3, by means of ordinary least squares. The results are presented in Table 4. Eq (76a'), Eq(76b') and Eq(76c') represent two stage least-

Table 3

Estimates of Money Demand Functions (using OLS)

Equations	Dependent Variables	Explanatory Variables	Coefficients	S.E.	R^2	F	D.W.
(70)	M_{1t}	$\dot{p}_t$	-5.623	7.409	0.925	73.497	0.729
		Y_t	0.338	0.39			
		K_t	-100.848	187.768			
		Constant	-922.941	166.369			
(70)	M_{2t}	$\dot{p}_t$	-5.247	9.832	0.936	88.73	0.677
		Y_t	0.497	0.050			
		K_t	-189.975	249.198			
		Constant	-1387.877	220.794			
(71)	m_{1t}	$\dot{p}_t$	-0.023	0.020	0.671	12.259	1.059
		y_t	0.030	0.033			
		K_t	1.872	0.368			
		Constant	2.198	0.998			
(71)	m_{2t}	$\dot{p}_t$	-0.022	0.032	0.668	12.048	0.72
		y_t	0.089	0.053			
		K_t	2.977	0.585			
		Constant	1.640	1.588			
(76)	M_{1t}	$\dot{p}_t$	-3.358	4.958	0.968	130.096	2.257
		Y_t	0.057	0.063			
		M_{1t-1}	0.914	0.188			
		K_t	27.71	127.86			
		Constant	-130.942	197.254			

Table 3 (contd.)

Equations	Dependent Variables	Explanatory Variables	Coefficients	S.E.	R^2	F	D.W.
(76)	M_{2t}	$\dot{p}_t$	-0.335	4.958	0.984	253.671	2.206
		Y_t	0.039	0.071			
		M_{2t-1}	1.047	0.151			
		K_t	-54.3	132.23			
		Constant	-83.987	220.447			
(77)	m_{1t}	$\dot{p}_t$	-0.037	0.017	0.788	15.84	1.825
		y_t	0.005	0.029			
		m_{1t-1}	0.45	0.147			
		K_t	1.08	0.39			
		Constant	1.746	0.837			
(77)	m_{2t}	$\dot{p}_t$	-0.044	0.022	0.863	26.879	1.548
		y_t	0.723	0.039			
		m_{2t-1}	0.695	0.141			
		K_t	0.778	0.589			
		Constant	1.789	1.047			

Table 4

Estimates of Money Demand Functions

(76a')	M_{1t}	$\dot{p}_t$	-6.751	6.585	0.969	134.66	2.330
		Y_t	0.072	0.064			
		M_{1t-1}	0.85	0.198			
		K_t	59.503	132.075			
		Constant	-163.837	196.274			

Table 4 (contd.)

Equation	Dependent Variables	Explanatory Variables	Coefficients	S.E	R^2	F	D.W.
		$\dot{p}_t$	1.218	5.867	0.984	254.26	2.176
		Y_t	0.39	0.070			
(76a')	M_{2t}	M_{2t-1}	1.05	0.149			
		K_t	-71.198	135.799			
		Constant	-86.896	220.685			
		$\dot{p}_t$	-16.73	12.52	0.934	85.65	2.177
		Y_t	0.052	0.083			
(76b')	M_{1t}	M_{1t-1}	0.969	0.246			
		K_t	-217.7	152.39			
		Constant	-149.407	259.069			
		$\dot{p}_t$	10.41	14.30	0.98	308.526	2.131
		Y_t	0.046	0.076			
(76b')	M_{2t}	M_{2t-1}	1.017	0.162			
		K_t	71.01	129.87			
		Constant	-89.257	238.777			
		$\dot{p}_t$	-32.529	3.364	0.932	123.0	2.356
		Y_t	0.102	0.031			
(76c')	M_{1t}	M_{1t-1}	0.432	0.082			
		K_t	435.802	9.971			
		Constant	-52.896	93.335			
		$\dot{p}_t$	-39.56	5.265	0.978	230.12	2.218
		Y_t	0.138	0.029			
(76c')	M_{2t}	M_{2t-1}	0.381	0.052			
		K_t	432.952	32.985			
		Constant	26.536	100.914			

squares, limited-information maximum likelihood and three stage least-squares estimates respectively. The results are quite satisfactory since, the degree of explanation achieved, in each case, is very high; the Durbin-Watson statistics are much better, and the F-observed is signidicantly large.

As expected, the coefficient of the lagged endogenous variable in the demand function is highly significant by the t-test, supporting the hypothesis that a lagged response exists on the demand side. We observe that in all but two of the regressions the coefficient for $\dot{p}_t$ has the wrong sign. The coefficient for nominal income and one-period lagged nominal money balances have the correct sign in all the regressions. We also note that the t-value of the nominal income variable is not significant in the 2SLS and LIML cases.

For the money demand function on the narrow money definition, the four independent variables $\dot{p}_t$, Y_t, M_{1t-1} and K_t together explained over 93 percent of the variations in M_{1t}. However, as all equations reveal, the lagged nominal money balance was the most important of the four. The demand for money is seen to vary inversely with the rate of change of price and directly with respect to nominal income and lagged nominal money balance. The effect of the rate of change of price is not very clear. Its coefficients were statistically nonsignificant. Note also that the coefficients for nominal income, rate of change of price and lagged nominal money balance have very similar values when the dependent variable is M_{1t} (in the 2SLS and LIML cases).

For the broad money definition, the major explanatory variables to be $\dot{p}_t$, Y_t, M_{2t-1} and K_t which explain, in each case, 98 percent of the variation in the total demand for money. The coefficients for nominal income and lagged nominal money balance have the correct sign in all of the regressions. The rate of change of price is positively signed, for two equations, thereby contradicting the standard hypothesis that the demand for money relates inversely with the rate of change of price.

We now summarize several features of the results reported in the Tables 3 and 4.

i) The models provide a good fit for the data, that is, the models form a generally good approach for "explaining" variations in money demand since the R^2 values are uniformly high;

ii) The introduction of lagged money balances improves the explanatory powers of the model since R^2 values for regression equations with lagged term are higher than those of regression equations without lagged term;

iii) The signs obtained on the estimated parameters of the models, using OLS, are generally consistent with expectations and are supportive of the money demand functions;

iv) The demand functions for real money balances are fairly similar for both narrow and broad money definitions. Real income is important determinant of aggregate demand for money when lagged real money balance is absent from the model;

v) When the lagged real money balance is introduced in the model, only lagged real money balance and rate of change of price do affect the demand for real money balance;

vi) Regarding the estimated demand equations for nominal money balance, one would be led, incorrectly, to accept a nul hypothesis of no relationship between demand for money and the rate of change of price in contrast to what is suggested by the demand functions for real money balances.

vii) The coefficients of lagged nominal money balance have the correct positive sign and are statistically significant in each regression.

4.3 Conclusion

From the results and discussions, we can draw the following conclusions:

1. The supply of money in Ethiopia over the period 1964-1985 can be best explained by monetary-base and total domestic credit.
2. The study also shows that money supply, among other variables, depends on the rate of change of price. This is revealed in the 2SLS, LIML and 3SLS estimates and not in

the OLS estimate. This could be due to simultaneous equations bias in the latter.

3. There exist a substantial degree of supply adjustment and expectation lags in money supply function as is shown by the significant values taken by lagged nominal money balance.

4. Cash reserve ratio appears to have no significant influence on the money supply in Ethiopia.

5. The nominal income and lagged nominal money balances coefficients, of the money demand function are positive and are statistically significant. These results clearly suggest that nominal income and lagged nominal money balances are the determinants of the money demand function.

6. The rate of change of price and lagged real money balances are important explanatory variables of the demand function for real money balance of Ethiopia. In other words, it seems that permanent real income specification is superior to that of current real income in Ethiopia.

7. It is obvious, from the results, that the introduction of lagged money balance adds the explanatory powers of the model.

8. The results do not conclusively indicate the superiority of either the narrow (M_{1t}) or broad (M_{2t}) definition of money in both the demand for and supply of money cases.

Both definitions perform well in the regression. However, the R^2 and t-values on the M_{2t} definition are by and large higher and the Durbin-Watson statistics also falls in the acceptable range in most of the regressions. This result would seem to indicate that the M_{2t} definition is the more appropriate one.

APPENDIX

Table I: DATA FOR THE ESTIMATION OF THE MONEY DEMAND AND SUPPLY FUNCTIONS

End of Period	M_{1t}	M_{1t-1}	M_{2t}	M_{2t-1}	MB_t	$\dot{p}_t$	q_t
1964	303.3	.0	371.5	.0	232.3	.00	2.31
1965	350.1	303.3	421.3	371.5	266.9	.00	2.18
1966	363.7	350.1	449.4	421.3	276.5	.40	2.30
1967	356.0	363.7	456.3	449.4	272.6	.79	1.68
1968	383.7	356.0	508.9	456.3	297.0	.16	1.81
1969	431.8	383.7	581.8	508.9	345.2	1.41	2.02
1970	427.9	431.8	614.5	581.8	353.2	-.17	1.94
1971	408.6	427.9	623.2	614.5	331.6	.21	1.92
1972	454.3	408.6	719.5	623.2	372.0	-5.69	1.72
1973	582.9	454.3	946.2	719.5	446.4	8.84	1.66
1974	694.4	582.9	1075.1	946.2	572.6	8.57	1.82
1975	883.0	694.4	1184.9	1075.1	813.3	6.51	1.34
1976	809.7	883.0	1259.6	1184.9	948.7	28.57	1.18
1977	1038.1	809.7	1511.3	1259.6	1036.1	16.55	1.35
1978	1226.2	1038.1	1709.4	1511.3	1041.7	14.40	1.36
1979	1327.1	1226.2	1863.1	1709.4	1141.6	16.05	1.48
1980	1229.7	1327.1	1857.9	1863.1	1275.2	4.46	1.31
1981	1719.8	1229.7	2438.2	1857.9	1482.3	6.14	1.16
1982	1860.6	1719.8	2653.1	2438.2	1621.1	4.37	1.15
1983	2171.4	1860.6	3092.4	2653.1	1612.3	.77	1.22
1984	2329.7	2171.4	3387.3	3092.4	1943.9	8.41	1.13
1985	2743.8	2329.7	3965.1	3387.3	2028.3	19.07	1.16

Table I (contd.)

End of Period	DC_t	m_{1t}	m_{1t-1}	m_{2t}	m_{2t-1}	Y_t	y_t
1964	212.4	2.41	.00	2.95	.00	2801.6	22.27
1965	240.0	2.76	2.41	3.34	2.95	3167.6	25.08
1966	269.9	2.87	2.76	3.54	3.34	3376.7	26.63
1967	319.8	2.79	2.87	3.57	3.54	3591.6	28.10
1968	383.9	3.00	2.79	3.98	3.57	3805.0	29.73
1969	444.2	3.33	3.00	4.48	3.98	4173.0	32.15
1970	523.7	2.99	3.33	4.30	4.48	4408.9	30.83
1971	557.0	2.85	2.99	4.35	4.30	4416.6	30.82
1972	593.8	3.36	2.85	5.32	4.35	4642.7	34.35
1973	603.5	3.96	3.36	6.43	5.32	5180.0	35.21
1974	634.6	4.35	3.96	6.73	6.43	5103.2	31.95
1975	754.0	5.19	4.35	6.97	6.73	5530.0	32.51
1976	942.1	3.70	5.19	5.76	6.97	6146.3	28.10
1977	1177.6	4.07	3.70	5.93	5.76	6487.4	25.45
1978	1544.0	4.21	4.07	5.86	5.76	7086.5	24.30
1979	1846.9	3.92	4.21	5.51	5.86	7624.7	22.53
1980	2668.3	3.48	3.92	5.26	5.51	8156.4	23.07
1981	2801.1	4.58	3.48	6.501	5.26	8296.6	22.11
1982	3314.2	4.75	4.58	6.78	6.50	9082.7	23.19
1983	3873.9	5.50	4.75	7.84	6.78	8942.8	22.66
1984	4267.2	5.421	5.50	7.92	7.84	3913.5	20.84
1985	4638.3	5.39	5.42	7.78	7.92	10468.6	20.55

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