SINGULAR FERMI LIQUID LOW ENERGY COLLECTIVE FLUCTUATION

A thesis submitted to the
School of Graduate Studies
Addis Ababa University

In partial Fulfilment of
the Requirements for the Degree
of Master of Science in Physics

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Addis Ababa, Ethiopia
July 5, 2005.

ADDIS ABABA UNIVERSITY
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DEPARTMENT OF PHYSICS

The undersigned hereby certify that they have read and recommend to the Faculty of Science School of Graduate Studies for acceptance a thesis entitled "**Singular Fermi Liquid Low Energy Collective Fluctuation**" by **Seid Yimer Ebrie** in partial fulfillment of the requirements for the degree of **Master of Science in Physics**.

Dated: June 2005

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Acknowledgements

I am very grateful for my advisor and Instructor Dr.Tesgera Bedassa not only for his enthusiasm and constructive criticism, but also for his direct involvement almost in all aspect of the work along with his full patience and friendly approach and willingness to deal with me at any time under any situation.

I am greatly indebted to all Instructors in my graduate school years who were helped me in the evolution of this thesis. I would like to appreciate Lecturer Yitagesu Elfag, among my friends and Lecturer Aragaw Yimer, my brother for their willingness to help me to improve my attitude towards solid state physics by supplying books and important suggestion.

I can not forget Dr.Mulugeta Bekele, Dr.Meberatu G/M and Dr.Genene Tessema for their outstanding encouragement during my study of graduate school in all aspects.

I would also like to thank the Somali Regional state Teacher-Training institute for sponsoring me during my study.
Finally, I would also like to thank all my families and my graduate school friends for their constructive advises.

Abstract

Landau's Fermi liquid theory is analyzed theoretically. The explanation for singularity which arise due to break-down of the theory in one dimension is considered. In particular , the low energy collective fluctuations is emphasized by using the method of static and dynamic screening. The transverse conductivity is deduced by making the use of macroscopic transport equation taking in-to account electron-electron interaction.

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INTRODUCTION

Some of the Recently discovered metals are characterized by thermodynamic and transport properties which at low temperature are different from that of usual metals described by Landau Fermi liquid theory. The low energy properties of these metals are dominated by singularities in their phase diagram. Such system are referred to as singular Fermi liquid (or non Fermi liquid).

In this thesis the low energy collective fluctuations effect is analytically considered. The objective of the study is to give a unified account of the effect of electron-electron interaction in metals at low temperature for a system having an energy very close to the fermi energy.

A considerable difference of a system of conduction electrons from ideal Fermi gas is that actually electrons in metals interact intensively among themselves, and the average energy of their coulomb interaction appears of the same order with their average kinetic energy. Therefore, the most acceptable conception of conduction electrons not their representation as a gas consisting of the Fermi particles but as a liquid and this liquid should be a quantum as for electron system in real metals is strongly degenerated.

Theoretical attempts to consider the influence of inter-electron interactions on the

properties of electron system were set about during the beginning of the formation of quantum mechanics[1,2] However, determination of the system energy based on theories of perturbations has shown that their disagreeing terms appear already in the Boron approximation. Therefore, the sequential theory, which takes into account the inter-electron interactions in real metals, can only be a phenomenological type theory containing arbitrary constants that should be defined from the comparison of theoretical results with experimental data. Such a theory was obtained in 1957 by V. P. Silin [3] on the basis of analysis of the quantum liquid theory for derivation of properties of helium-3, which was offered by L. A. Landau [4] a year earlier.

Some results of the Landau - Silin electron liquid theory are actually reasonable possibility of the usage of the Fermi gas model in a wide range of kinetic characteristics of electron transportation in metals. The qualitative differences in the properties of the electron Fermi liquid from properties of electron gas occur in considerably high-frequency processes.

The coulomb interaction between the conduction electrons in a metal is reduced to the screened short range potential $\exp(-\lambda/r)$, because the fourier components of the coulomb potential in the range of short wave vector are separated as collective excitations called the plasmons. This effect is reflected in the energy dispersion relation of the conduction electrons, which varies inversely with the the effective mass m^* of the quasi-particle or quasi-electrons and may be slightly different from that of the mass of non interacting free electrons.

These quasi-particles are shown to be stable at energy close to the Fermi level but dissipate with time due to transitions to other states at energies far from the

Fermi level. Thus the Fermi liquid at absolute zero possesses a well defined fermi surface, it provides a firm basis for electron transport theories and electronic properties dominated by electrons near the fermi level. The Fermi liquid theory can be applied successfully to a system forming an extended band in ordinary metals and alloys. The electronic specific heat in the interacting electron system is linearly proportional to the absolute temperature, the proportionality constant being proportional to the ratio m^*/m, which in turn is related to correlation term.

The thesis consists of Four chapters. The first chapter is the Landau Fermi liquid theory, the general description of interacting electrons and the microscopic description of many body systems are discussed by using the assumption that there is a one to one correspondence between interacting and non interacting systems. The Landau correlation function which is the heart of Fermi liquid theory will be derived. Also the break down of Fermi liquid theory in one dimension with some significant consequence is explained in detail. In the second chapter the basic idea of the singular Fermi liquid theory with the long range coulomb interaction between the electrons is presented. Some of the effects like the Kondo problem, the screening of coulomb interaction, the divergence of interaction energy are stated.

Chapter three is devoted to the Singular Fermi liquid and collective fluctuation of charged Fermion systems. In this chapter the low energy collective fluctuation of charged Fermion system is analytically expressed. Dynamic screening of the electrostatic field of the electrons at large distance for neutral and charged Fermions, the failurity of higher order perturbation theory as the main cause of the logarithmic divergence are discussed.

At the end of the thesis over all theoretical conclusion and summary of important

points are set. The original material and sources in formulation of the thesis are listed in the Reference section.

Chapter 1

LANDAU THEORY OF FERMI LIQUID

1.1 Introduction

It is a remarkable fact of nature that a single particle description is applicable at least in qualitative sense to real solids. A general description of interacting electrons can be studied based on the Landau theory of Fermi liquid (Landau, 1957) [1], (Landau, 1958) [2], (Landau, 1959) [3] which was extended to charged systems by Silin , (1958). The Landau theory of Fermi liquid is an ingenious simplification of an extremely complex and almost incalculable many body systems in an elegant manner such that a relatively good approximation to all order of perturbation may be obtained. It still remains to be discussed how accurate this method actually is and the approximation used in this theory are discussed.

The Hartree-Fock methods fails to properly include the correlation between the particles. Thus the two particle Green functions are included only partially with out the vertex part. In the microscopic description of many body systems, Green's functions are normally used in stead of the wave functions. This does not give a complete description of the system but allows us to concentrate on most essential features of the system for finding the states and macroscopic properties.

Landau theory of Fermi liquid has a wide range of application to matter of short range two body force, with some modification is then considered and the relative advantage of this method with respect to others are stated. The phenomenological theory of Landau based on his original ideas (Lan, 57) are then explored more thoroughly using the method of quantum field theory (GM, 58) [4], (AGD, 67) [5] , (NOZ, 64) [6].

The basic assumption of the theory is that the weakly excited state of a Fermi liquid greatly resembled to that of a weekly excited Fermi gas. These can be described with a set of elementary excitation with a spin half and momenta close to the Fermi surface. It is then assumed that there is a one to one correspondence between the number of state of a perfect gas to that of a normal Fermi liquid. These may be physically realized by adiabatically switching on the two particle interaction , the concept of quasi-particle or elementary excitation is thus introduced. In this manner how ever the important low lying collective state of the liquid are lost which are necessary for the description of superconductivity.

The elementary excitation obtained are not the exact stationary state of the system but a superposition of a large number of exact stationary state of the system with a narrow spread of energy which leads to the damping of the states. The damping may be explained as the interaction between the quasi-particles with conserved law of momenta and energy. This can occur through processes where the excitation decay in to several others or where the quasi-particles are scattered by each others. The decay of excitation plays a role only at high temperature where the system tends to behave like a non interacting systems. When the temperature is sufficiently low there are only few low energy quasi-particles which rarely scatter and thus the interaction between the quasi-particle is weak.

1.2 Quasi-particles

The essential idea of Landau theory is that the energy levels of the interacting systems can be classified according to the same quantum numbers used for non interacting particles. In other words we start from some definite state in the absence of interaction, then turn on the interaction slowly, the interaction representation matrix element of the potential can be regarded as containing a factor $\exp(\alpha t)$ for $t < 0$ in which α is small positive quantity.

Under the influence of interactions our states evolves in some way but remains characterized by the same wave vector. Wave vector is conserved at each vertex when

the two body interaction is transitionally invariant and there is a series of calculating the full single particle Green's function for a state of wave vector k , starting from the particle Greens functions the state of the interacting systems are called quasi-particles state or elementary excitations.

Physically a quasi-particle may be pictured as a single particle surrounded by a self consistent distribution of other particles. Quasi particles state includes quasi electrons and quasi holes, the states are occupied according to Fermi statistics. The energy of the entire systems is not simply the sum of the energies of individual quasi particle states. A single particle state is not in general an eigen state of the Hamilton for interacting system. Such a state has a life time, which is determined from the imaginary part of the self energy functions. How ever, close to the fermi energy, the life time becomes very long (it is infinite exactly at the Fermi surface). Thus the notation of the quasi particle is most useful for states close to the Fermi energy.

The distribution of the quasi particle is described by a function $n(k)$, if this function were specified it would be possible in principle to compute the total energy, thus the total energy of the system is a function of $n(k)$, which denotes as $E[n(k)]$. For an interesting extension of this ideas to position space , (Hohenberg and Kohn ,1964) [7]. In general, we do not know $n(k)$ explicitly, suppose that $n(k)$ is changed by a small amount $\delta n(k)$, the change in the energy of the system is δE to first order in δn. This is

$$\delta E = \frac{V}{(2\pi)^3} \int \varepsilon(k) \delta n(k) d^3k \tag{1.2.1}$$

where V is the volume of the system. In a situation in which it is necessary to take

account of spin explicitly, we consider a distribution spin state $n_{\sigma(k)}$

$$\delta E = \frac{V}{(2\pi)^3} \sum_{\sigma} \int \varepsilon_{\sigma}(k) \delta n_{\sigma}(k) d^3k \tag{1.2.2}$$

The quantity $\varepsilon_{\sigma}(\vec{k})$ is the effective energy of the quasi particles. (position variable are neglected since variation of distribution function over distance much greater than inter atomic dimensions will not affect quasi particle energy.)

If k is on the Fermi surface (we denote this by k_F with out implying that the Fermi surface is spherical), the addition of one particle in k_F, with the original systems in its ground state of energy ε_g, gives as an (N+1) particle system in its ground state thus

$$\varepsilon(k_F) = \varepsilon_g(N+1) - \varepsilon_g(N) = \mu \tag{1.2.3}$$

in which μ is the chemical potential. It is frequently necessary to obtain the change in energy of the system to second order in δn. This is written as

$$\delta E = \frac{V}{(2\pi)^3} \sum_{\sigma} \int \varepsilon_{\sigma}(k) \delta n_{\sigma}(k) d^3k + \frac{1}{2} \frac{V}{(2\pi)^3} \sum_{\sigma,\sigma'} \int f_{\sigma,\sigma'}(k,q) \delta n_{\sigma}(k) \delta n_{\sigma'}(q) d^3k d^3q \tag{1.2.4}$$

The function $f_{\sigma,\sigma'}(k,q)$ is the second functional derivative of the energy with respect to the distribution functions. This function is a fundamental quantity which characterize the Fermi liquid theory, it is symmetric

$$f_{\sigma,\sigma'}(k,q) = f_{\sigma',\sigma}(k,q) \tag{1.2.5}$$

It can be shown that f is the forward amplitude for quasi-electrons, quasi-holes scattering when both k and q are on the Fermi surface. We can interpret $\varepsilon_{\sigma}(k)$ as the

energy of an isolated quasi particles, if there are no other quasi-particles present. In the presence of other particles, we have the energy $E_\delta(k)$ which is according to the above equation as

$$E_\sigma(k) = \varepsilon_\sigma(k) + \frac{V}{(2\pi)^3} \sum_{\sigma,\sigma'} \int f_{\sigma,\sigma'} \delta n_{\sigma'}(q) d^3q \tag{1.2.6}$$

If the system has no magnetic order and no external magnetic field is present, the energy must be independent of δ and $f_{\sigma,\sigma'}$ can depend only on $\vec{\sigma}.\vec{\sigma'}$. It is then convenient to express $f_{\sigma,\sigma'}$ as a sum of direct and exchange terms f_d and f_{ex} respectively. Hence

$$f_{\sigma,\sigma'}(k,q) = f_d(k,q) + f_{ex}(k,q)\delta_{\sigma,\sigma'} \tag{1.2.7}$$

Quasi particles are fermions , it will be assumed that the expression for the entropy of non interacting fermions is applied here

$$S = -\frac{V}{(2\pi)^3} \sum_{\sigma} \int n_\sigma(k) \ln n_\sigma(k) + [1 - n_\sigma(k)] \ln[1 - n_\sigma(k)] d^3k \tag{1.2.8}$$

The entropy is to be maximized subject to the constraints that the total number of particles N and the total energy E are fixed. The number is

$$N = \frac{V}{(2\pi)^3} \sum_{\sigma} \int n_\sigma(k) d^3k \tag{1.2.9}$$

It is required that

$$\delta N = 0$$

and

$$\delta E = 0$$

It follows that from 1.2.6 and 1.2.7 that the energies of the quasi particles state depends on temperature .

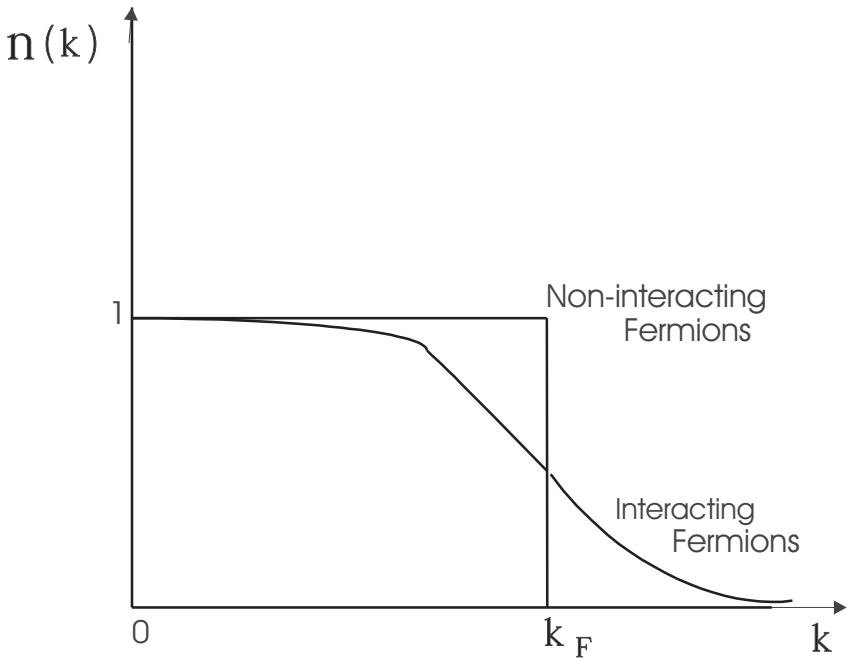

Figure 1.1: Graphical representation of interacting and non interacting fermions

1.2.1 Quasi-particles in an isotropic Fermi liquid

According to the hypothesis, the quasi-particles spectrum of an isotropic Fermi-liquid of strongly interacting identical particles is similar to that of the ideal gas. This means that there is a certain value P_0 which according to Landau's theory is still real to the particle density by the ideal gas formula. Two kind of quasi-particles exist. Particle like with $P > P_0$ and anti-particle like with $P < P_0$, so long as $P - P_0 \ll P_o$ their energy is given by

$$\xi_p(p) = v(P - P_o) \tag{1.2.10}$$

$$\xi_a(p) = v(P_0 - P) \tag{1.2.11}$$

In the general case v is un known constant having the dimension of velocity. Instead of v one can introduce another constant via the relation

$$v = \frac{P_0}{m*} \tag{1.2.12}$$

where the quantity $m*$ has the dimension of mass and is called the effective mass. The quasi-particle concept is valid if the damping coefficient $\gamma \ll \xi$ is close to the Fermi level. If one deal with a Fermi liquid in equilibrium at T is not equal to zero the relevant quasi-particle all have energies ξ propertional to T. The damping coefficient is then of the order of $(\frac{K_B T}{\mu})^2$ the description of the liquid in terms of quasi-particles is there fore valid only so long as $K_B T \ll \mu$. For electrons in metals the quantity $\frac{\hbar}{P_0}$ (the de brogle wave length) is of the same order as inter atomic distance i.e, $10^{-8} cm$, so that $P_0 = 10^{-19} \frac{gcm}{s}$. There fore one obtains for metals $T \ll To$ as 10^4 to 10^5 k.

These condition shows that the quasi-particle picture certainly applies to solid metals at all temperature, since T_0 significantly exceeds the melting temperature in any case.

1.2.2 Quasi-particle in the anisotropic Fermi liquid

In order to understand the main feature of electrons in metals let first switch off the interaction between electrons or rather let us consider a gas of non interacting electrons placed in an average periodic field. The state of a single particles in such a field were analyzed in the above situation. As shown there, the energy levels from bands separated by forbidden gaps each bands contains 2N states , where N is the number of unit cell in the sample. If the Fermi level coincides with the upper edge of a band, some band are then completely filled, where are others are completely empty. In such a case a sufficiently weak electric field can not give rise to a net current and the substance will be an insulator rather than a metal.

If the Fermi level falls within the center of a band the band is called a conduction band. An arbitrarily weak electric field can give rise to current flow. This situation describes a metal, semiconductors belong to the first category, but the gap between occupied and unoccupied state is then small. Their properties are,there fore,similar to that of metals unless the temperature is very low. We shall limit our attention to actual metals. The position of the Fermi level for the electron gas in a crystal is fixed by the condition $\varepsilon(p) = \mu$. This equation describes a surface in momentum space called the Fermi surface. Its symmetry is determined from that of the lattice, in this

case it is also possible to define particle like and anti-particle like quasi-particles with momenta out side and in side the Fermi surface respectively.

In general the Fermi surface may have a very complex shape. It has a very simple shape in two cases. The first one corresponds to an almost empty band. Only few electrons are present and at $T = 0$, they fill up the lowest available states,and must all be in the vicinity of the minimum of $\varepsilon(p)$. If this minimum occurs at $P^{(0)}$, we may expand the energy in power of $\left(P - P^{(0)}\right)$. For a cubic crystal, with $P^{(0)} = 0$ one obtains

$$\varepsilon(p) = \varepsilon_0 + \frac{P^2}{2m*} \tag{1.2.13}$$

Where $m*$ is a constant called the band effective mass. In the more general case of arbitrary symmetry, but with $P^{(0)} = 0$ as before, one obtains a positive axis transformation it has the form as follows.

$$\varepsilon_p = \varepsilon_0 + \frac{1}{2}\left(\frac{P_x^2}{m1} + \frac{P_y^2}{m2} + \frac{P_z^2}{m3}\right) \tag{1.2.14}$$

A similar situation occurs in the case of almost filled bands, one can then deal with hole i.e, empty states in the band, rather than electrons. They will occur near energy maxima, but in the case of a cubic crystal with $P^{(0)} = 0$ one has

$$\varepsilon(p) = \varepsilon_0 - \frac{P^2}{2m*} \tag{1.2.15}$$

This means that holes behaves as particles with a negative mass. Generalization for arbitrary symmetry and $P^{(0)}$ is not equal to zero are trivial. In all cases considered, the fermi surface is an ellipsoid or consists of a set of ellipsoids. If the number of

electrons per unit cell is odd, some bands may be completely filled, but as pointed out earlier, at least one band will be partly filled.

If there is only one such band, it must contains N electrons. Since the whole brilloun zone contains $2N$ states, the volume enclosed by the Fermi surface and the brilloun zone faces (if the number makes contact with this face) must be half that of the brilloun zone, the substance is then metal. If the number of electrons per unit cell is even it does not mean the substance is necessarily an insulator in order to have small number of electrons and holes , both an even number of electrons and per unit cell and a small overlap between the highest two relevant bands are required. If this is the case some of the electrons from the highest occupied band (the so called valence band) will spill over in the lowest un occupied (the conduction band) so that it will contain a small number of electrons , while the other band will contain an equal number of holes. This situations occurs semi metals, in addition a metal might have several partly filled bands one of which contain a small number of electrons or holes.

The non interacting gas model actually describes the properties of quasi-particles in a real metal just as a free electron gas can be used to describe the quasi-particle properties of an isotropic Fermi liquid. In this context it is important to remember that the only relevant properties of the gas model are those involving particles close to the fermi surface. As a result the previous discussion referring to electrons "filling" bands may seem of doubtful validity since "deep" states were certainly involved in these arguments.

1.3 Basic Assumptions

On defining Fermi liquid, we are not going to use the Green's function and the mathematical apparatus of the many body perturbation theory, but we adopt an alternative approach. This consists in comparing the interacting real liquids with the non interacting ideal gas, we establish a one to one correspondence between the eigne state of the two systems, such approach will provide us with a qualitative understanding of the excitation spectrum of an interacting systems. Consider an eigne state of the ideal systems, characterized by the distribution function n_p.

In order to establish a connection with the real system, we imagine that the interaction between the particles is switched on infinitely slowly Under such adiabatic condition, the ideal eigne state will progressively transform in to certain eigne states of the real interacting systems. We there fore assume that the real ground state may be adiabatically generated staring from some ideal eigne states with a distribution n_p^0. This statement may be considered as the definition of the normal Fermion systems. For reasons of symmetry, the distribution n_p^0 of an isotropic system is spherical. As a result the spherical Fermi surface is not changed when the interaction between the particles is switched on. In such case the real ground state is generated from the ideal ground state,under this circumstance the Fermi surface will certainly be deformed when the interaction is switched on. In such case the real ground may be shown to follow adiabatically from some excited state of the non interacting system. This situation will not cause any major difficulty with the theory. Let us now add a particle with momentum P to the ideal distribution n_p^0 and again turn on the

interaction adiabatically, we generate an excited state of the real liquid, which has momentum P conserved in particle collision.

As the interaction is increased the bare picture are slowly perturbing the particles in its vicinity. Let S_F be the Fermi surface characterized the un perturbed distribution n_p^o from which the real ground state is built up. Because of the exclusion principle quasi-particle excitation can be generated only if their momentum p lies out side S_F. Using the same adiabatic switching procedure we can define a quasi-hole, with momentum p lying in side the Fermi surface. The quasi particles and quasi holes thus appears as elementary excitation of the real systems, when combined gives rise to a large class of excited states. We have established our desired one to one correspondence between ideal and real eigne states.

Any definitions of elementary excitation is some what impressive since damping occurs due to real collusion of quasi- particle life time becomes sufficiently long on time in the immediate vicinity of the Fermi surface, that the quasi-particle concept makes sense in that region. In pure systems at zero temperature, the life time varies as the inverse of energy separation from ε_F. As one gets closer and closer to the fermi surface, S_F remains sharply defined. We shall now examine an actual system of interacting electrons, i.e, an electron fluid. The behavior of such a system can be understood on the base of Landau's idea (2) on the energy spectrum of condensed quantum systems and his own theory of Fermi liquid.

The simplest illustration of Landau's general approach is provided by the vibration

of the crystal lattice. If the oscillation are small, the potential energy of interactions between atoms in the crystals can be expanded in power of the atomic displacement u. First order terms are absent since the equilibrium configuration corresponds to a potential energy minimum. For a second order terms we have

$$U = U_0 + \frac{1}{2}\sum A^{\alpha}_{nj} A^{\alpha'}_{n'j'} u^{\alpha}_{nj} u^{\alpha'}_{n'j'} \tag{1.3.1}$$

The lattice translation vectors are a_n. The index j labels particular atoms in a unit cells. The meaning of this quasi-particles description is clarified when one considers the energy of the vibrating lattice. The energy levels are given by the following expressions which is valid for a system of independent oscillations.

$$E - U_0 = \sum_{ks} \hbar\omega(k,s)\left[n(k,s) + \frac{1}{2}\right] \tag{1.3.2}$$

The $n(k,s)$ are either zero or positive integers. It can be written as a sum of two terms

$$E - U_0 = \frac{1}{2}\sum_{k,s} \hbar\omega(k,s) + \sum (k,s)\hbar\omega n(k,s) \tag{1.3.3}$$

The first one corresponds to the lowest value of the energy i.e. the ground state of the system. This energy is the so called zero point oscillations. The atom of the crystal must vibrate even in the ground state is a consequence of the uncertainty principles. According to this principles a particle, can not be at rest in equilibrium since it would otherwise have simultaneously well defined value of position and momentum. In an excited state, some of the energies $n(k,s)$ are different from zero. Then equation 1.3.3 describes a system of independent particles with energies $\hbar\omega(k,s)$. Since the integers $n(k,s)$ can take on arbitrary positive values any number of phonon can be

found in any given states, there fore they obey Bose statistics. According to Landau, any homogeneous systems containing a large number of particles has low lying states similar to those of a vibrating lattice. The properties of any such systems can be described in terms of the quasi-particle model. Quasi-particles can have integer ($n\hbar$) or half- integer $(n + \frac{1}{2})\hbar$ spin i.e. they can be either bosons or fermions. The energy of a quasi-particle is a function of its momentum , the corresponding relation $\varepsilon(p)$ is the main characteristic of such low lying excited states.

1.4 Phenomenological derivation

Consider a system of N identical Fermions in a volume V assumed to be large at zero temperature, in case of a simple gas of non interacting particles i.e, a perfect gas, the eigne states are antisymmetric combination of a single particle states to be taken as plane waves. The planes are characterized by their wave vector k , to define the eigne state of the whole system it is sufficient to indicate which plane waves are occupied with the distribution function $n(k)$. Let the ground state of the system corresponds to an isotropic distribution $n_o k$. If the distribution function is changed by an infinitesimal quantity $\sigma n(k)$, the total energy of the system changes by an amount

$$\sigma E = \sum_k \hbar^2 \frac{k^2}{2m} \sigma n(k) \tag{1.4.1}$$

The functional derivative of the energy with respect to the distribution function

is the kinetic energy of the particles with wave vector k.

$$\frac{\sigma E(n)}{\sigma n(k)} = \sum_{k'} \hbar^2 \frac{k'^2}{2m} \sigma k k' \tag{1.4.2}$$

where $\sigma k k' = \frac{\sigma n(k')}{\sigma n(k)}$. For zero temperature, $\sigma n(k)$ is necessarily positive for $k > k_F$ and negative for $k < k_F$. When an additional particle of wave vector k with $k > k_F$ is added to a perfect gas in the ground state and then the interaction is turned on an eigne state of the real gas is obtained. The life time of particle defined in this manner is only long near the Fermi surface. Thus the concept of quasi particles is only valid in the neighborhood of $k = k_F$. Similarly a quasi hole is the removal of a particles of wave vector k with $k < k_F$. Though the same distribution function $n(k)$ now characterize the real states , it gives the distribution of quasi particles and not of real particles. The excitation of the system is measured by

$$\sigma n(k) = n(k) - n_o(k) \tag{1.4.3}$$

As $\sigma n(k)$ is appreciable only near the Fermi surface in which the quasi particle is well defined. The variation of energy to first order will be given as

$$\sigma E = \sum_k \varepsilon_k \sigma n(k) \tag{1.4.4}$$

with $\varepsilon(k) = \frac{\sigma E}{\sigma n(k)}$ being interpreted as the energy of the quasi-particles. This relation is only valid when the number of quasi particles added or removed is small as compared to the total number N of particles in the systems. If the second order effect is

considered the variation of the total energy is given by

$$\sigma E = \sum_k \varepsilon_0 \sigma n(k) + \sum_k \sum_{k'} f(k,k')\sigma n(k)\sigma n(k') \tag{1.4.5}$$

and thus $f(k,k')$ is the second functional derivative of E with respect to the distribution functions.

The relation between effective mass m^* and the interaction term f will now be derived using the concept , the momentum of a unit volume of the Fermi liquid is the same as the momentum of the quasi particles in this volume, the current of the particles in the Fermi liquid is equal to the current of quasi particles.

$$\int Kn(k)\frac{dk}{(2\pi)^3} = m\int dk\frac{vn(k)}{(2\pi)^3} \tag{1.4.6}$$

substituting $v = \nabla_k \varepsilon(k)$, where $E = E[n(k)]$, we obtain

$$\int \frac{dk}{(2\pi)^3}\frac{k}{m}\sigma n = \int \frac{dk}{(2\pi)^3}\sigma n\nabla\varepsilon_k + \frac{1}{2}\int \frac{dkdk'}{(2\pi)^6}\nabla_k f(k,k')n\delta n' \tag{1.4.7}$$

which on integration by parts and permuting k to k' is as follows

$$\int \frac{dk}{(2\pi)^3}\frac{k}{m}\sigma n = \int \frac{dk}{(2\pi)3}\sigma n\nabla_k\varepsilon(k) - 1/2\int \frac{dkdk'}{(2\pi)^6}\nabla_{k'} f(k,k')n'\sigma n \tag{1.4.8}$$

The average spin indices is taken since n and ε do not depend up on spin here, since σn is arbitrary , it follows that

$$\frac{k}{m'} = \nabla_k\varepsilon(k) - \frac{1}{2(2\pi)^3}\int dk' f(k,k')\nabla_{k'}n' \tag{1.4.9}$$

an estimate for $\nabla_{k'}$ at $k' = k_F$ is given by the expression

$$\nabla_{k'}n' = \frac{-k'}{k}\delta(k'-k_F) \tag{1.4.10}$$

since f depends up on the angle ϕ between k and k' we obtain the following relations

$$\frac{1}{m*} = \frac{1}{m} + \frac{k_F}{2(2\pi)^3} \int f(\phi) cos(\phi) d\Omega \tag{1.4.11}$$

Where $f(\phi)$ is the value of $f(k, k')$ at $k = k' = k_F$ and $d\Omega$ is the infinitesimal solid angle.

1.5 Electronic Heat capacity

The electronic fluid can be described in terms of a gas of quasi-particles behaving like independent electrons in a periodic field. We use the name " electrons " for these quasi-particles, one must of course, remember that these electrons are different from that make up the Fermi liquid. An expression for the electrons heat capacity of metals will be derived as follows. The energy of such a Fermi gas is given by

$$E = \frac{2V}{(2\pi\hbar)^3} \int \varepsilon(p) f d^3p \tag{1.5.1}$$

Where $\frac{2V}{(2\pi\hbar)^3}$ is the density of states in crystal momentum space (the factor two corresponds to the two possible spin orientations) and f is the Fermi Dirac distribution functions.

$$f = \frac{1}{\exp(\epsilon - \mu)\beta + 1} \tag{1.5.2}$$

$\beta = \frac{1}{K_B T}$

The crystal momentum integration extends over the Brillouin zone. A summation over partially filled band is required if several of them exist. Differentiating equation

1.4.1 with respect to T, we obtain the specific heat per unit volume

$$C = \frac{1}{V}\left(\frac{\partial E}{\partial T}\right)_V = \frac{2}{(2\pi\hbar)^3}\int \varepsilon(p)\frac{\partial f}{\partial T}d^3p \tag{1.5.3}$$

Where

$$\frac{\partial f}{\partial T} = \frac{\exp(\varepsilon-\mu)\beta}{T\Big(\exp(\varepsilon-\mu)\beta+1\Big)^2}\left(\frac{\varepsilon-\mu}{T}+\frac{d\mu}{dT}\right) \tag{1.5.4}$$

Since

$$\frac{\partial f}{\partial \varepsilon} = -\frac{\exp(\varepsilon-\mu)\beta}{T\Big(\exp(\varepsilon-\mu)\beta+1\Big)^2} \tag{1.5.5}$$

We can write

$$\frac{\partial f}{\partial T} = -\frac{\partial f}{\partial \varepsilon}\left(\frac{\varepsilon-\mu}{T}+\frac{d\mu}{dT}\right) \tag{1.5.6}$$

The chemical potential μ is determined by requiring conservation of quasi-particle number (the number of quasi particles in the gas model is equal to the actual number of particles).Thus

$$\frac{\partial N}{\partial TV} = \frac{2}{(2\pi\hbar)^3}\int \frac{df}{dT}d^3p = 0 \tag{1.5.7}$$

Substituting this in above , we obtain

$$C = \frac{2}{(2\pi\hbar)^3}\int \varepsilon\frac{\partial f}{\partial \varepsilon}\left(\frac{\varepsilon-\mu}{T}+\frac{d\mu}{dT}\right)d^3p \tag{1.5.8}$$

$$\int \frac{\partial f}{\partial \varepsilon}\left(\frac{\varepsilon-\mu}{T}+\frac{d\mu}{dT}\right)\frac{d^3p}{(2\pi\hbar)^3} = 0 \tag{1.5.9}$$

Both of this equation contains $\frac{\partial f}{\partial \varepsilon}$. If $T \ll \mu(0)$, as is the case in metals ,$\frac{\partial f}{\partial \varepsilon}$ is appreciable only in a narrow energy interval of T about the Fermi level $\mu(0)$. The momentum space integral can be transformed as follows. Consider a constant energy surface $\varepsilon(p) = constant$ in momentum space. The integration over d^3P can be performed separately over this surface and then over $d\varepsilon$. If dS is an infinitesimal element of that constant energy $d^3P = dSdp_n$, where dp_n is associated with integration along the normal to the surface element dS.

$$dp_n = \frac{d\varepsilon}{\nabla_p \varepsilon} \tag{1.5.10}$$

and

$$\int d^3p = \int d\varepsilon \int \frac{dS}{V} \tag{1.5.11}$$

We define the density of states $D(\varepsilon) = \frac{2}{(2\pi\hbar)^3} \int \frac{dS}{V}$. Since $\frac{\partial f}{\partial \varepsilon}$ is different from zero under near $\varepsilon = \mu$, we may expand in power of $\varepsilon - \mu$. Representing

$$F(\varepsilon) = F(\mu) + (\varepsilon - \mu)F'(\mu) + \frac{1}{2}(\varepsilon - \mu)^2 F''(\mu) + - - - \tag{1.5.12}$$

Then $\frac{\partial f}{\partial \varepsilon} = -\left(4T \cosh^2 \frac{\varepsilon - \mu}{2T}\right)$ in view of the rapid decay of this function away from $\varepsilon - \mu$,the limit of integration on $z = \varepsilon - \mu$, can be expanded from minus infinity to positive infinity.

$$\int (\varepsilon - \mu)^2 \frac{\partial f}{\partial \varepsilon} d\varepsilon = \frac{-1}{4T} \int \frac{z^2 dz}{\cosh^2 z(2T)^{-1}} = \frac{-\pi^2 T^2}{3} \tag{1.5.13}$$

So that

$$\int F(\varepsilon) \frac{\partial f}{\partial \varepsilon} = -F(\mu) - \frac{\pi^2 T^2}{6} F"(\mu) \tag{1.5.14}$$

Then we have

$$C = \mu \frac{d\mu}{dT} D(\mu) + \frac{\pi^2 T}{3} \frac{d}{d\mu} [\mu D(\mu)] \tag{1.5.15}$$

In which

$$\frac{d\mu}{dT} = \frac{-\pi^2 T}{3} \frac{D'(\mu)}{D(\mu)} \tag{1.5.16}$$

Substituting we get

$$C = \frac{\pi^2 T}{3} D(\mu). \tag{1.5.17}$$

This formula has a simple physical interpretations. Only electrons close to the Fermi level are involved in the thermal excitations of the system. Their number is of the order of T times the density of states $D(\mu)$. Using the isotropic model of a metal,

$$D(\mu) = \frac{2}{(2\pi\hbar)^3} \int \frac{dS}{V} = \frac{p_0 m^*}{\pi^2 \hbar^3} \tag{1.5.18}$$

Where m^* is the effective mass which depends on the Landau correlation function $f(k, k')$. Thus

$$C = \frac{P_0 m^* T}{3\hbar^3} \tag{1.5.19}$$

Experimentally one measures the total heat capacity, rather than its electronic components. It is well known how ever that the contribution from the lattice has a cubic dependance at low temperature. The total heat capacity can there be represented as

$$C = AT + BT^3 \tag{1.5.20}$$

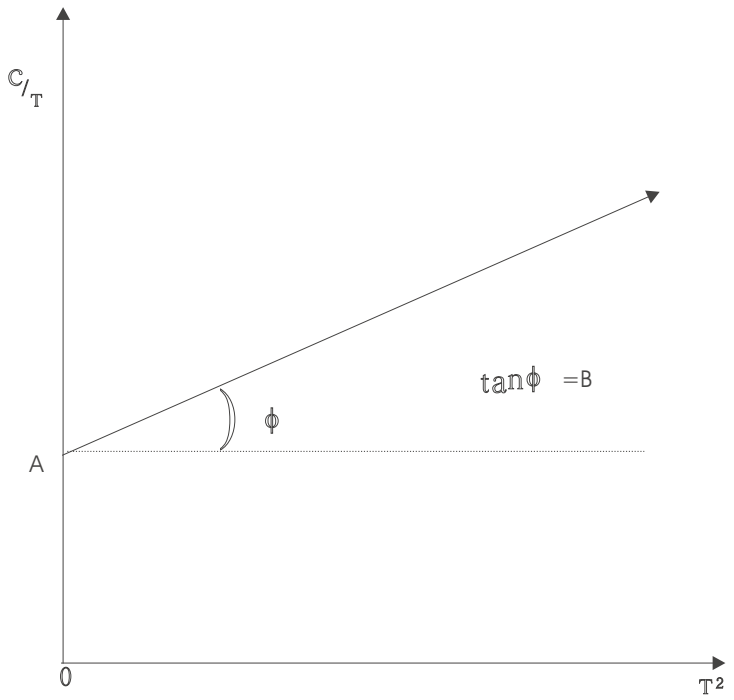

Figure 1.2: Determination of Electronic and Lattice Components of Heat capacity C from experimental data , Note that $\frac{c}{T}$ versus square of the absolute temperature T.

1.6 Break down of Landau Fermi-liquid theory in one dimension

The familiar Fermi liquid picture in three dimension is based on the existence of long lived quasi particles or quasi holes excitations as the energy of excitations approach to zero or as the energy approaches to the Fermi energy. These quasi- particles or quasi-holes can be traced back to bare electrons (holes) with a one to one correspondence, starting from the interacting situation followed by adiabatic turning on the electron electron interaction. Phenomenologically supposing the existence of such low energy excitations, Landau was able to account for a rich variety of physical phenomena exhibited by conventional metals in the presence of non negligible interactions.

The fermi liquid picture finds more rigorous justification in many body perturbation theory (Abrikosov , 1963) [8] (Nozieres, 1964) through in depth analysis of the quasi particle life time, which varies as $(k-k_F)^{-2}$ when k approaches k_F or $(E-E_F)^{-2}$ when E approach the Fermi energy. (luttinger, 1960) [9] and the interaction operator Υ and the reduced operator γ leading to the well known identities reflecting under lying conservation lows i.e, continuity equation reflecting the conservation of charge and enabling us to relate vertex functions and interaction operators to the Landau parameters.

The Fermi liquid pictures breaks in one dimension. This is a direct consequence of the unique phase space structure in one dimension, notably the fact that the Fermi

surface consists of two discrete points plus or minus k_F rather than a line or surface and that for each branch of momentum energy dispersion left or right moving the one dimensional wave vector uniquely determine the energy. Some of the most significant interrelated consequences includes the following.

1. The reduced one dimensional phase space leads to a quasi particle scattering rate $Im \sum(k, w)$ is proportional to $(k - k_F)$ and proportional to ω rather than $(k - k_F)^{-2}$. $Re(\sum)$ at $E = E_F(w = 0)$ (Metzer, 1998) [10]. Here $\sum$ denotes the self energy so that the real and imaginary part of $\sum$ are related by the Kramers-Kroing relations.

2. Logarithmic divergence appear in two particle interaction operator commencing in second order operator perturbation theory related to (1) above.

3. A logarithmic divergence at E_F directly implies that the quasi particle renormalization or quasi particle weight vanishes. Hence the one to one correspondence of the un perturbed k electrons state to the elementary excitation of the interacting systems is lost.

4. Low energy electron hole excitations arising from interaction can occur about the two discrete fermi points at plus or minus k_F.

5. Spin-charge separation occurs ie the spin and charge elementary excitations propagates at different velocities. The presence of low energy, un gapped bosons

charge and spin modes is a direct consequence of the finite spin and charge density responses at very low q and w.

All of these well known behavior points to the incapable fact that the fermi liquid picture is not appropriate in 1D once electron electron interaction is turned on. The Fermi liquid can not serve as an adequate starting point for understanding certain aspect of this strongly correlated systems, particularly those processes involving the so called charge and spin sector where injection or removal of bare electrons takes place. In contrast, thermodynamics properties, which depend on the neutral excitations (electron hole pairs, collective modes,plasmon etc)at low w and $q(w \ll E_F, q \ll k_F)$, can still adequately be described by the Fermi liquid.

Historically there have been two major parallel and complementary approaches to the theoretical investigation of interaction metallic 1D systems,with their focus centered on the idealized tomonaga luttinger luttinger models (Tomonaga, 956) [11] (Luttinger, 1963) [12]. One approach is based on the bosonization techniques (Matthis and Lieb, 1965) [13] and the other on the so called g-ology model in connection with renormalization group treatment. A general discussion of the topic of interaction in one dimensional encompasses a rather diverse range of systems and phenomena. In addition to metallic systems other related systems such as the one dimensional Hubbard models with strong back scattering are of great interest in their own right. Here we confine our attention to metallic systems with weak back scattering to avoid complication introduced by insulating tendencies or by gapped behavior in the spectrum of low energy excitations.

The bosonization approach first introduced by Tomonaga (1950) and later expand up on by Luther and (peschel , 1974) [14] and (Haldane, 1981) [15], indicates that a bosons description is appropriate in one dimension for interacting systems dominated by forward scattering. The renormalization group approach is basically a method for going beyond perturbation theory and in many instance has the effect of summing up the most relevant (Logarithmically divergent) diagram in a systematic and controlled way. Since its implementation is invariably founded on many body perturbation theory, the usual many body technique play a natural and useful role. These includes using the ward identities arising out of conservation lows to connect the vertex function with physically measurable quantities, equation of motion methods and others, as a consequence the analysis has proven helpful in elucidating the relation between a Fermi liquid and a tomonaga luttinger liquid and at the same time allowing clear cut differentiation of the two systems.

Chapter 2

SINGULAR FERMI-LIQUIDS

2.1 Breakdown of Landau Theory

From Landau's phenomenological theory, one can only say that the theory breaks down when the physical properties - specific heat divided by temperature, compressibility, or the magnetic susceptibility - diverge or when the collective modes representing oscillations of the Fermi-surface in any harmonic and singlet or triplet spin combinations become unstable. The latter, called the Landau-Pomeranchuk singularities, are indeed one route to the breakdown of Landau theory and occur when the Landau parameters $f_l^{s,a}$ reach the critical value $-(2l+1)$. A phase transition to a state of lower symmetry is then indicated. The new phase can again be described in Landau theory by defining distribution functions consistent with the symmetry of the new ground state.

Landau theory breaks down when the quasiparticle amplitude $Z_{\mathbf{k}}$ given by

$$Z_{\mathbf{k}}^{1/2} = \left\langle \psi_{\mathbf{k}}^{N+1} \left| a_{\mathbf{k}}^{+} \right| \psi_{\mathbf{k}}^{N} \right\rangle$$

becomes zero. This requires that the single-particle self-energy be singular as a function of energy ε at $k \simeq k_F$. This in turn means that the Green's functions of singular Feremi liquid contain branch cuts rather than the poles unlike Landau Fermi-liquids. If a divergent number of low-energy particle-hole pairs is created upon addition of a bare particle, it means that the low-energy response functions (which all involve creating particle-hole pairs) of singular Feremi liquid are also divergent. Actually the single-particle self-energy can be written in terms of integrals over the complete particle-hole interaction vertex. The implication is that the interaction vertices are actually more divergent than the single-particle self-energy.

Another route to singular Feremi liquid is the case in which the interactions generate new quantum numbers which are not descriptive of the non-interacting problem. This happens most famously in the Quantum Hall problems and in one-dimensional problems as well as problems of impurity scattering with special symmetries. In such cases the new quantum numbers characterize new low-energy topological excitations.

In the final analysis all breakdowns of Landau theory are due to degeneracies leading to singular low-energy fluctuations. If the characteristic energy of the fluctuations is lower than the temperature, a quasi-classical statistical mechanical problem results. We note the following routes to breakdown of Landau theory.

Landau-Pomeranchuk Singularities: Landau theory points to the possibility of its breakdown through the instability of the collective modes of the Fermi- surface. These collective modes can be characterized by the angular momentum l of oscillation of the Fermi-surface and whether the oscillation is symmetric "s" or anti-symmetric

"a" in spin. [14]

Critical regions of Large Q-Singularities: Landau theory concerns itself only with long wavelength response and correlations. A Fermi-liquid may have instabilities at a nonzero wave-vector, for example a charge-density wave (CDW) or spin-density wave (SDW) instability. An important point to note is that they arise perturbatively from repeated scattering between the quasiparticle parts of the Green's function while the scattering vertices are regular. The superconductive instability for any angular momentum is also an instability of this kind. [15,16]

Singular Fermi-liquid behavior is generally expected to occur only in the critical regime of such instabilities. [17] If the transition temperature T_c is finite then there is usually a stable low temperature phase in which unstable modes are condensed to an order parameter, translational symmetry is broken, and gaps arise in part or all of the Fermi-surface. For excitations on the surviving part of the Fermi-surface, Fermi-liquid theory is usually again valid. The fluctuations in the critical regime are classical, i.e. with characteristics frequency $\omega << k_B T_c/\hbar$. If the transition is tuned by some external parameter so that it occurs at zero temperature, one obtains a Quantum Critical Point (QCP). If the transition is approached at $T = 0$ as a function of the external parameter, the fluctuations are quantum-mechanical, while if it is approached as a function of temperature for the external parameter at its critical value, the fluctuations have a characteristic energy proportional to the temperature.

Long-Range Interactions: Breakdown of Landau Fermi-liquid may come about through long-range interactions, either in the bare Hamiltonian through the irreducible interaction or through a generated effective interaction. The latter, of course, happens in the critical regime of phase transitions. Coulomb interactions will not

do for the former because of screening of charge fluctuations. In other words, the longitudinal electromagnetic mode acquires mass in a metal. The latter is not true for current fluctuations or transverse electromagnetic modes which due to gauge invariance must remain massless. This will be discussed in the next section, where it is shown that no metal at low enough temperature is a Fermi-liquid. However, the cross-over temperature is too low to be of experimental interest.

An off-shoot of singular Fermi liquid through current fluctuations is the search for extra (induced) conservation laws for some quantities to keep their fluctuations massless. This line of investigation may be referred to generically as gauge theories. Extra conservation laws imply extra quantum numbers and associated orthogonality. The one-dimensional interacting electron problem and the Quantum Hall effect problems may be usefully thought of in these terms.

In the following subsections we discuss a particular simple form of Fermi-liquid formed by electrons interacting with a dilute concentration of magnetic impurity. Many of the concepts of Fermi-liquid theory are revisited in this problem. Variants of the problem provide an interesting array of soluble problems of singular Fermi-liquid behavior.

2.2 The Kondo Problem

The Kondo problem is one of the simplest and one of the most subtle examples of the effects of strong correlation effects in electronic systems. Experiments concern metals with a dilute concentration of magnetic impurities. In the Kondo model one

considers only a single impurity; the Hamiltonian then is

$$H = t \sum_{i,j} a^+_{i\alpha} a_{j\alpha} + J\mathbf{S} \cdot a^+_0 \sigma a_0$$

where $a^+_{i\alpha}$ ($a_{j\alpha}$) denote the creation (and annihilation) operators of a conduction electron at site i with projection a in the z-direction of spin s. The second term is the exchange interaction between a single magnetic impurity at the origin (with spin $S = 1/2$) and a conduction electron spin. When the exchange constant $J > 0$ the system is a Fermi-liquid. The ferromagnetic ($J < 0$) variant of this problem is one of the simplest examples of a singular Fermi-liquid.

There are two seemingly simple starting points for the problem: (i) $J = 0$: This turns out to describe the unstable high temperature fixed point. The term proportional to J is a marginal operator about the high temperature fixed point because as discovered by Kondo [18] in a third order perturbation calculation, the effective interaction acquires a singularity. (ii) $t = 0$: The perturbative expansion about this point is well behaved. This turns out to describe the low temperature Fermi-liquid fixed point. One might be surprised by this, considering that typically the bare t/J is of order 10^{+3}. But such is the power of singular renormalizations.

The interaction between conduction electrons and the localized electronic level is not a direct spin interaction. It originates from quantum-mechanical charge fluctuations that (through the Pauli principle) depend on the relative spin orientation. To see this explicitly it is more instructive to consider the Anderson model [19] in which

$$H = t \sum_{i,j} a^+_{i\alpha} a_{j\alpha} + \epsilon_d \sum_{\sigma} a^+_{0\sigma} a_{0\sigma} + U a^+_{0\uparrow} a_{0\downarrow} a^+_{0\downarrow} a_{0\uparrow} + \sum_{k,\sigma} \left(V_k a^+_{0\sigma} a_{0\sigma} + hc \right)$$

The last term in this Hamiltonian is the hybridization between the localized impurity state and the conduction electrons, in which spin is conserved. In the particle-hole

symmetric case, $\epsilon_d = -U/2$ is the one-hole state on the impurity site in the Hartree-Fock approximation and the one-particle state has the energy $U/2$. Following a perturbative treatment in the limit t/V , $U/V >> 1$ the Anderson model reduces to the Kondo Hamiltonian with an effective exchange constant $J_{eff} \sim (V^2/t)^2/U$.

The Anderson model has two simple limits. $V = 0$ (high temperature regime) describes a local moment with Curie susceptibility $\chi \sim \mu^2 B/T$. This limit is the correct point of departure for an investigation for the high temperature regime. As noted, one soon encounters the Kondo divergences. In the limit $U = 0$ (low temperature regime) the impurity forms a resonance of width $\Gamma \sim V^2/t$ at the chemical potential which in the particle-hole symmetric case is half- occupied. The ground state is a spin singlet. This limit is the correct starting point for an examination of the low temperature properties ($T << T_K$). A temperature independent contribution to the susceptibility and a linear contribution to the specific heat ($\sim N(0)T/\Gamma$) are contributed by the resonant state.

The passage from the high-temperature regime to the low-temperature regime was first done correctly by Wilson [19] through the invention of the Numerical Renormalization Group by Anderson and Yuval [20,21] by analytic methods). The analysis showed that under Renormalization Group scaling transformations the ratio J/t increases monotonically continuous RG flows are observed from the high temperature extreme to the low temperature extreme and a smooth crossover between the two regimes occurs at the Kondo temperature

$$T_K \sim te^{-t/2J}$$

Because all flow is towards the strong-coupling fixed point, universal forms for the

thermodynamic functions are found. For example, the specific heat C_v and the susceptibility χ are

$$C_v = T f_c\left(T/T_K\right), \qquad \chi = \mu^2 B f_\chi\left(T/T_K\right)$$

where the f's are universal scaling functions. An important theoretical result is that compared to a non-interacting resonant level at the chemical potential, the ratio of the magnetic susceptibility to the specific heat changes,

$$R = \frac{\delta\chi/\chi}{\delta C_v/C_v}$$

for spin 1/2 impurities at $T << T_K$ is precisely 2 [22,22]. In a noninteracting model, this ratio, nowadays called the Wilson ratio, is equal to 1, since both χ and C_v are proportional to the density of states $N(0)$. Thus the Wilson ratio is a measure of the importance of correlation effects. It is in fact the analogue of the Landau parameter f_0^a of Eq. ().

As we saw in earlier section, a Fermi-liquid description is appropriate as long as the spectrum retains a coherent single particle piece of finite weight $Z > 0$. If the evaluation of Z reduces to an overlap integral between two orthogonal wave functions then the system is a singular Fermi-liquid.

2.3 Orthogonality catastrophe

In the thermodynamic ($N \to \infty$) limit, an orthogonality catastrophe arises if the injection of an infinitely massive particle in more than one dimension produces

an effective finite range scattering potential for the remaining N electrons. Such orthogonality is exact only in the thermodynamic limit: The single particle wave functions are not orthogonal. It is only the overlap between the ground state formed by their Slater determinants which vanishes as N tends to infinity.

More quantitatively, if the injection of the additional particle produces an s-wave phase shift δ_0 for the single particle wave functions,

$$\psi(kr) = \frac{\sin(kr)}{kr} \rightarrow \frac{\sin(kr + \delta_0)}{kr}$$

then an explicit computation of the Slater determinants reveals that their overlap diminishes as

$$\langle \psi_N \, | \psi'_N \rangle \sim N^{-\delta_0^2/\pi^2}$$

Here $| \psi_N \rangle$ is the determinant Fermi sea wave function for N particles and $| \psi'_N \rangle$ is the wavefunction of the system after undergoing a phase shift by the local perturbation produced by the injected electron.

Generally, such an orthogonality ($Z = 0$) arises also if two N particle states of a system possess different quantum numbers and almost the same energy. These new quantum numbers might be associated with novel topological excitations. This is indeed the case in the Quantum Hall Liquid where new quantum numbers are associated with fractional charge excitations. The singular Fermi liquid properties of the interacting one-dimensional fermions may also be looked on as due to orthogonality. Often orthogonality has the effect of making a quantum many-body problem approach the behavior of a classical problem.

2.4 X-ray Edge singularities

The term X-ray edge singularity is used for the line shape for absorption in metals by creating a hole in an atomic core-level and a particle in the conduction band above the chemical potential. In the non-interacting particle description of this process, the absorption starts at the threshold frequency ω_D. In this case, a Fermi edge reflecting the density of unoccupied states in the conduction band is expected to be visible the spectrum.

However, when a hole is generated in the lower level, the potential that the conduction electrons see is different. The relevant Hamiltonian is now

$$H = \varepsilon_d \left(d^+ d - 1/2\right) + \sum_k \varepsilon_k a_k^+ a_k + \frac{1}{L} \sum_{k,k'} V\left(k, k'\right) \left(a_k^+ a_{k'} - 1/2\right) \left(d^+ d - 1/2\right),$$

where the operators (d^+, d) annihilate or create holes in the core level, which is taken to be dispersionless. The first two terms in the Hamiltonian represent the unperturbed energies of the core hole and the free electrons. The last term depicts the screened Coulomb interaction between the conduction electrons and the hole in the core level.

As a consequence of the interactions, the line shape is quite different. There are two kinds of effects,

(a) Excitonic - the particle and the hole attract, leading to a shift of the edge and a sharpening of the edge singularity - and (b) An orthogonality effect of the type just discussed above, which smoothens the edge irrespective of the sign of the interaction. This changes the absorption spectrum in the presence of interactions. The form of the singularity is

$$A\left(\omega\right) \sim \left(\omega - \omega_D\right)^{-2\delta_0/\pi + \delta_0^2/\pi^2}$$

The exponent δ_0^2/π^2 is a consequence of the orthogonality catastrophe overlap integral; the exponent $(-2\delta_0/\pi)$ is due to the excitonic particle hole interactions. If the hole has finite mass we have a problem with recoil which is not exactly solvable, but we know the essential features of the solution. The recoil removes the singularity in two and three dimensions and the absorption edge acquires a characteristic width of the order of the dispersion of the hole band. If the hole moves only in one dimension, the singularity is not removed.

2.5 A Spinless Model with Finite Range Interactions

A model, which is a generalization of the Ferromagnetic Kondo problem and in which the low-energy physics is dominated by the orthogonality catastrophe, is given by the following Hamiltonian:

$$H = \sum_{k,l} \varepsilon_k \gamma_{kl}^+ \gamma_{kl} + \frac{t}{\sqrt{L}} \sum_{k,k'} \left(\gamma_{k0}^+ d + hc \right) + \frac{1}{L} \sum_{k,k'} V_l \left(\gamma_{kl}^+ \gamma_{k'l} - 1/2 \right) \left(d^+ d - 1/2 \right).$$

The operators (γ^+, γ) are the annihilation and creation operators of spinless conduction electrons with kinetic energy ε_k. The local chemical potential has been set to zero ($\varepsilon_d = 0$) and the Hamiltonian is particle-hole symmetric. The new index l is an orbital angular momentum index (or a channel index). Hybridization conserves point-group symmetry, so the localized orbital hybridizes with only one channel ($l = 0$). By contrast, the impurity couples to all channels via the interaction V_l. As we are summing over all moments (k, k') this interaction is local.

This problem may be mapped onto the anisotropic Kondo model. Indeed the transformation

$$\begin{aligned} d^+ &\to S^+, \qquad\qquad d^+d - 1/2 \to S_z, \\ t &\to \tfrac{J_{\perp 0}}{\sqrt{2\pi a}}, \quad 2V_l \to \sqrt{2}J_{zl} - 2\pi v_F \left(\sqrt{2}-1\right)\delta_{l0} \end{aligned}$$

produces

$$H = \sum_{k,l,\sigma} \varepsilon_k a^+_{kl\sigma} a_{kl\sigma} + \frac{1}{2}\sum_{k,k'} J_{\perp 0}\left(S^+ s^-_l + hc\right) + \sum_l J_{zl} S_{zl} s_{zl}.$$

Here a is short distance cutoff. In the resulting (anisotropic multi-channel) Kondo Hamiltonian the spin operators $\mathbf{S}$ and $\mathbf{s}_l$ portray charge excitations of the local orbital and conduction band; the spin index in the resulting Kondo Hamiltonian should now be regarded as a charge label. Physically, this map- ping is quite natural. The impurity may or may not have an electron, this is similar in character to having spin up or spin down. Similarly, the kinetic hybridization term transforms into a spin flip interaction term of the form $(S^+d^- + hc)$. As V_l couples to the occupancy of the impurity site, we might anticipate J_z to scale with V_l. The additional correction $\left(-2\pi v_F\left(\sqrt{2}-1\right)\delta_{l0}\right)$ originates from the subtle transformation taking the original fermionic system into an effective spin model.

This problem has been solved by renormalization group methods. But simple arguments based on the x-ray edge singularity, orthogonality and recoil give the correct qualitative physics. When $t = 0$, the problem is that of the x-ray edge Hamiltonian (with $\varepsilon_d = 0$). When t is finite, the charge at the impurity orbital fluctuates (the impurity site alternately empties and fills). This generates, in turn, a fluctuating potential. The x-ray absorption spectrum is the Fourier transform of the particle-hole pair correlator

$$\Delta(\omega) \sim \left\langle \gamma^+(t)\, d(t)\, d^+(0)\, \gamma(0)\right\rangle_\omega .$$

This quantity should display the x-ray edge characteristics for large frequencies ($\omega > \Delta_{eff}$) where the effect of recoil is unimportant:

$$\Delta(\omega) \approx \Delta_0 (\omega/W)^{\gamma}; \quad \gamma = -\frac{2\delta_0}{\pi} + \sum_l \frac{\delta_l^2}{\pi^2}.$$

The threshold frequency Δ_{eff} is determined by the recoil energy. W is the bandwidth. The bare hybridization width Δ_0 is $\sim t^2/W$. The exponent in the singularity contains an exitonic shift ($-\frac{2\delta_0}{\pi}$) as well as an orthogonality contribution ($\sum_l \delta_l^2/\pi^2$). The recoil is cut off by Δ_{eff}. For $\omega < \Delta_{eff}$ the electron gas becomes insensitive to the change in the potential. As the x-ray edge singularity is cut off at $\omega = O(\Delta_{eff})$, self-consistency implies that $\Delta_{eff} = \Delta(\omega = \Delta_{eff})$. This leads to the identification

$$\Delta_{eff} = W (\Delta_0/W)^{\frac{1}{1-\gamma}}$$

so that for

$$\gamma < 1, \Delta_{eff} \to 0 as W \to \infty.$$

For $\gamma < 1$, a singular Fermi-liquid emerges in which the hybridization of the localized d-orbital with the electron gas scales to zero at zero frequency. The actual value of γ determines the singular properties at low energy or temperature. In the single channel problem such a scenario occurs if the potential V_0 is sufficiently attractive. On mapping to the spin problem we find that this region corresponds to the singular Fermi-liquid Ferromagnetic Kondo problem. The scaling of the hybridization to zero corresponds, in the spin-model, to $J_{\pm} = 0$.

Chapter 3

SINGULAR FERMI LIQUID THEORY AND COLLECTIVE FLUCTUATION

For a Homogeneous systems of interacting fermions in order to have a guarantee of its stability, we assume that the electrons are immersed in a uniform back ground of positive charge of density equal to the average electron density. For the high electron densities and low temperature in which we shall be interested, the non interacting electrons obey Fermi dirac statistics. The system may be regarded as a quantum plasma, in analogy to the usual classical plasma, for which the non interacting electrons are described by a Maxwell-Boltzman distribution.

The quantum plasma is like wise the natural analog, for charged particles of the neutral Fermi liquid. We have there- fore considered it may equally well be regarded as a charged Fermi liquid. Quantum plasma serve as a useful model for the behavior of electrons in the conduction band of simple metals. For many purpose the effect of the periodic array of ions in the system may be well approximated by the uniform

charge back ground. The coulomb interaction between the pair of electrons falls of slowly with distance it is a long range interaction. As a result a charged Fermi liquid differs appreciably from its neutral counter part.

3.1 Static and Dynamic Screening

The first theoretical treatments of screening dealt with the response of classical and quantum plasmas to a fixed external charges [Debye and Huckel , (1923) Mott and Jones, (1936)]. Let us consider a charge z introduced at the origin of a quantum plasma. The electrostatic potential $\Phi(\vec{r})$ felt by an electrons far from the origin will not be simply that due z alone. The external charge acts to polarize the electrons in its immediate vicinity. A distant electrons there fore respond both to the external charge and the induced polarization charge, $e\rho(\vec{r})$. The effective potential and the polarization charge are related by poisson's equations

$$\nabla^2\Phi(\vec{r}) = -4\pi[z\delta(\vec{r}) - e\rho(\vec{r})] \tag{3.1.1}$$

The induced charge density $e\rho(\vec{r})$ may be calculated with the idea of the Fermi Thomas approximation [Mott and Jones (1936)]. In that approximation, the chemical potential of the electrons is regarded as the sum of a potential energy $e\Phi(\vec{r})$ and of a kinetic energy ε_F^0 given by the relation appropriate to a system of non interacting electrons.

$$N = \frac{(2m\varepsilon_F^0)^{\frac{3}{2}}}{3\pi^2} \tag{3.1.2}$$

At equilibrium, the chemical potential must be constant. Hence ε_F^0 is a function of position. At a given point $\vec{r}$, we have

$$\varepsilon_F^0(\vec{r}) = \varepsilon_F^0 - e\Phi(\vec{r}) \tag{3.1.3}$$

The fluctuation in $\varepsilon_F^0(\vec{r})$ implies a fluctuation in the particle density N

$$\rho(\vec{r}) = \frac{3}{2}\frac{Ne\Phi(\vec{r})}{\varepsilon_F^0} \tag{3.1.4}$$

Taking the fourier transform of the resulting equations one then obtains,

$$(q^2 + q_{FT}^2)\phi_q = 4\pi z \tag{3.1.5}$$

And $\Phi(\vec{r}) = \frac{z}{r}\exp(-\vec{q}_{FT}.\vec{r})$, where the so called Fermi Thomas screening wave vector is given by

$$q_{FT} = \left(\frac{6\pi Ne^2}{\varepsilon_F^0}\right)^{\frac{1}{2}} \tag{3.1.6}$$

The field of the external charge is effectively screened with in a distance of the order of $\lambda_{FT} = \frac{1}{q_{FT}}$. We further remark that the screening length is determined by the computations between the influence of the potential energy and kinetic energy on the motion of the electrons.

The static screening methods with the response of classical and quantum plasmas to a fixed external charge [Debye and Huckel 1923, Mott and Jones, 1936] can not be applied directly to the screening of electron electron interaction in plasmas because a moving electron does not in general give, rise to a nearly static charge distribution in the plasma. To be specific suppose, the charge z which is introduced in -to the

plasma moves with some velocity $\vec{v}_e$ in that case, the charge it produces is given by $z\rho_e(\vec{r},t) = z\delta(\vec{r} - \vec{v}_e t)$.

If the charge is assumed to be at the origin at time $t = 0$, it is expected that the external charge will polarize the electron in the neighborhood and the resultant electronic polarization charge $e\rho(\vec{r},t)$ is now time dependent. The effective potential and polarization charge are related by a time dependent poisson's equation.

$$\nabla^2\Phi(\vec{r},t) = -4\pi[z\delta(\vec{r} - \vec{v}_e t) + e\rho(\vec{r},t)] \tag{3.1.7}$$

Taking the fourier transform in space and time of equation 3.1.1 we fined

$$\Phi(q,\omega) = \frac{4\pi}{q^2}[2\pi z\delta(\omega - \vec{q}.\vec{v}_e) + e\rho(\vec{q},\omega)] \tag{3.1.8}$$

Then it will reduce to that of determining the frequency and wave vector dependent polarization density $\rho(\vec{q},\omega)$. This quantity offers a direct measure of the dynamic screening in the electron gas.

The above problem serves for a consideration of the screening of electron-electron interaction. Qualitatively, each electron in the system behaves like a moving test charge. It acts to polarize its surroundings, another electron sees the electron plus its accompanying time dependent polarization cloud. The effective interaction between the electron is dynamically screened. By using such concept of dynamic screening it is not difficult to reformulate the transport equation of the Landau theory of Fermi liquid, So that it can be applied to quantum plasmas with out difficulty. In this fashion, one can obtain a rigorous expression for the response of a quantum plasma to external electric field that varies slowly in space and time(such being the limitation

of the Landau theory). More- over one finds quite generally that inclusion of dynamic screening serves to remove the divergence in the ground state energy and quasi-particle velocity.

3.2 Collective Fluctuations and Plasma oscillations

Collective mode represent a second kind of possible elementary excitations for the Fermi liquid. Physically they involve a coherent motion of the system as a whole. As we have seen any given quasi particle is subject to the field of the surrounding particles in the medium. In equilibrium under the homogenous conditions, the corresponding force average to zero. If however the distortion of the quasi particle distribution takes place as a result of some internal fluctuation of the system, the average interaction force no longer vanish, instead it acts to return the distribution towards equilibrium and there by serve as a restoring force for a collective oscillation about the equilibrium states. In other words a collective modes involves a cooperative motion of the system governed by the global interaction between the particles. We may picture the system as moving in its own self consistent field, in such a mode the individual particles have lost their meaning.

In the present case we assume the Fermi liquid to be in its ground state. This case is some what special in that one automatically in a non collision regime, in which collisions between thermally excited quasi particle play no role. At finite temperature

such collision acts to disrupt the self consistent fields responsible for collective modes and acts to damp the wave with a damping frequency ν, if ν characterize the collision frequency for the quasi-particles then as long as $\omega \gg \nu$, one is effectively in a non collision regime for which the consideration of the present section are valid.

The early theoretical treatment of quantum plasmas were haunted by the appearance of divergence. For example, one finds a logarithmic divergence by carrying out a second order perturbation theoretic calculation of the ground state energy of a quantum plasma. A similar divergence appears in the Hartree-Fock approximation calculation of the group velocity of a particle on the Fermi surface. Again one finds out the interaction energy of the quasi-particles in an inhomogeneous charged fermi liquid is divergent then the transport equation developed by the system with short range particle interaction is clearly inapplicable.

The origin of the divergent is the long range of coulomb interaction between electrons. A given electrons interact not just with a few near neighbors, but with a very large number of other electrons, so that its motion can not easily be decoupled from that of its neighbors. The fourier transform of the coulomb interaction is

$$V_q = \int d^3\vec{r} exp[-i\vec{q}.\vec{r}]\frac{e^2}{r} = \frac{4\pi e^2}{q^2} \tag{3.2.1}$$

in any low order perturbation. But a pilling up of a factor $1/q^2$ give rise to logarithmic divergence for higher order perturbation.

A plasma oscillation in a metal is a collective longitudinal excitation of the conduction electron gas. A plasmon is a quantum of plasma oscillation. We may excite a

plasmon by passing an electron through a thin metallic film or by reflecting an electron or a photon from a film. The charge of the electrons couples with the electrostatic field fluctuations of the plasma oscillations. The reflected or transmitted electrons will show an energy loss equal to integral multiples of the plasmon energy. It is equally possible to excite collective plasma oscillation in dielectric films. Results for several dielectrics are included. The collective plasma energies of silicon, germanium are based on four valence electrons per atoms. In a dielectrics the plasma oscillation is physically the same as in a metal. The entire valence electrons sea oscillates back and fourth with respect to the ion cores.

3.3 Screening and plasma oscillations

One of the important physical phenomena which is characteristics of homogeneous electron systems is a collective oscillation of the electrons as a whole, called the plasma oscillations. The exitance of organized oscillation in the plasma is complementary to the existence of screening. When the electrons moves to screen a charge distribution in the plasma, they will in general tend to over shot the mark some what. They are consequently pulled back toward that region, overshot again etc, in such a way that an oscillation is set up about the state of charge neutrality. The restoring force responsible for the oscillation is simply the average self consistent field of all the electrons, which is the same mechanism as was operative for zero sound. Because of the long range of the coulomb force, the frequency of oscillation is very high. For long

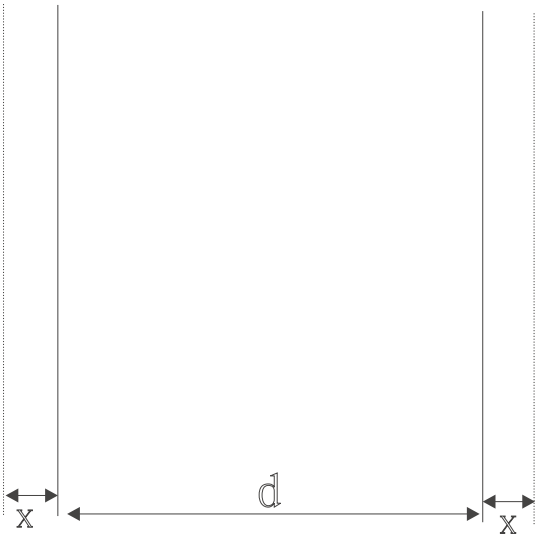

Figure 3.1: Displacement of a slab of charge.

wave length it is very nearly constant and is equal to

$$\omega_p = \sqrt{\frac{4\pi N e^2}{m}} \tag{3.3.1}$$

An elementary derivation of the above equation can be given by the following. Suppose that a charge imbalance is established in the plasma by the displacement of a slab of charge of thickness d by small x , $x \ll d$. In this case the slab behaves like a condenser, a constant electric field $\vec{E}$ is set up which acts to restore charge neutrality. The magnitude of the field is

$$\vec{E} = -4\pi N e x \tag{3.3.2}$$

Since the surface charge on either of the condenser is Nex, an electrons in side

the slab obeys the equation of motions.

$$m\frac{d^2\vec{x}}{dt^2} = eE = -4\pi Ne^2x \tag{3.3.3}$$

With the result that the slab will oscillate at frequency ω_p. Such a simple derivation neglects the random motion of the electrons, an approximation which turns out to be justified in the limit of a very long wave length oscillations. Plasma oscillation like screening represents a typical polarization phenomena in the plasma. Indeed both effects can be easily described with the frame work of a general treatment of polarization or dielectric processes in an electron system. By following in detail the response of the plasma to an external longitudinal field,the formalism so developed will enable us to expand the Landau theory to the case of a charged Fermi liquid.

The dielectric response $\epsilon(\vec{q},\omega)$ furnishes a natural description of the linear response of the plasma to an external field. For a static charge of strength z located at the origin, the fourier transform of the net potential is given by

$$\Phi(\vec{q}) = \frac{4\pi z}{q^2\epsilon(\vec{q},0)} \tag{3.3.4}$$

The static wave vector dependent dielectric constant $\epsilon(\vec{q},0)$ describes the screening action of the electron systems. In the Fermi-Thomas approximation one finds

$$\varepsilon_{FT}(\vec{q},0) = 1 + \left(\frac{q_{FT}^2}{q^2}\right) \tag{3.3.5}$$

If we now suppose the charge z to move at some velocity $\vec{v}_e$ we would write

$$\Phi(\vec{q},\omega) = \frac{4\pi z 2\pi\delta(\omega - \vec{q}.\vec{v}_e)}{q^2\epsilon(\vec{q},\omega)} \tag{3.3.6}$$

As the knowledge of $\epsilon(\vec{q},\omega)$ is also permits us to write down the dispersion relation for the plasma oscillations, in the absence of an external charge we may write this as $\epsilon(\vec{q},\omega)\vec{q}.\vec{\varepsilon}(\vec{q},\omega) = 0$ and $i\vec{q}.\vec{E}(\vec{q},\omega) = 4\pi e\rho(\vec{q},\omega)$, the usual solution for this is $\rho(\vec{q},\omega) = \vec{q}.\vec{E}(\vec{q},\omega) = 0$, which corresponds to no net charge density or electric field present in the plasma. How ever, for frequencies ω_q, such that $\epsilon(\vec{q},\omega_q) = 0$, means that the free oscillation of the charge density there being with no external field.

The above condition is the condition for the exitance of plasma oscillation at frequency ω_q, in the limit of very long wave length as $\vec{q}$ goes to zero, the dielectric constant is equal to

$$\epsilon(0,\omega) = 1 - \left(\frac{\omega_p^2}{\omega^2}\right) \tag{3.3.7}$$

the corresponding frequency of plasma oscillation is thus seen to be ω_p

3.4 Macroscopic Transport Equation

In order to show how the concept of dynamic screening enables to extend the Landau transport equation to electron systems, we consider a state characterized by the distribution function

$$n_p(\vec{r},t) = n_p^0 + \delta n_p(\vec{r},t) \tag{3.4.1}$$

The departure, δn_p, from the ground state is assumed to be small, to contain only long wave length of fluctuations, and to be restricted to the vicinity of the Fermi surface, where quasi-particle are well defined. Since the total energy is as a function of the distribution function and if we then attempt an expansion in power of δn_p,

we get the expression for the interaction energy of the two quasi-particles, $f(pr, p'r')$ which is divergent for the long range of the coulomb interaction between the electrons.

As Silin (1957) has first shown, these difficulties are removed if we allow for the dynamic screening of the particle motion in a self consistent fashion. The necessary things in the construction of the theory is that one take in to account the formation of the screening cloud. These can be done by the following methods, consider the electrostatic interaction between the averaged charge distribution of the excited quasi-particles, the average density fluctuation from the equilibrium is

$$\rho(\vec{r}, t) = \sum_{p} \delta n_p(\vec{r}, t) \tag{3.4.2}$$

and hence to a space charge electrostatic field, $\vec{\varepsilon}_p(\vec{r}, t)$ which is given by

$$\nabla . \vec{\varepsilon}_p(\vec{r}, t) = 4\pi e \sum_{p} \delta n_p(\vec{r}, t) \tag{3.4.3}$$

This part of interaction between quasi-particle can be accounted for in the transport equation by regarding $\vec{\varepsilon}_p(\vec{r}, t)$ as an additional applied field, which acts to screen the field produced by any given quasi particles. In addition, when a given quasi particles approach another it acts to alter the polarization cloud of the particles giving raise to a departure from the plain coulomb interaction.

By the introduction of the Landau interaction energy $f_{pp'}$, it led to express the local excitation energy of a quasi-particle as

$$\varepsilon_p(\vec{r}) = \varepsilon_p + \sum_{p'} f_{pp'} \delta n_{p'}(\vec{r}) \tag{3.4.4}$$

If we consider a periodic disturbance, with wave vector $\vec{q}$ and frequency ω it takes the following form

$$i(\vec{q}.\vec{v}_p - \omega)\delta n_p + i\vec{q}.\vec{v}_p\delta(\varepsilon_p - \mu)\sum_{p'} f_{pp'}\delta n_{p'} - e\vec{\varepsilon}_p.\vec{v}_P\delta(\varepsilon_p - \mu) = 0 \qquad (3.4.5)$$

Where the polarization field $\vec{\varepsilon}_p$ is equal to

$$\vec{\varepsilon}_p = -i\vec{q}\frac{4\pi e}{q^2}\sum_p \delta n_p \qquad (3.4.6)$$

We may write the transport equation in the absence of an external field as an integral equation Homogeneous in δn_p

$$(\vec{q}.\vec{v}_p - \omega)\delta n_p + \vec{q}.\vec{v}_p\delta(\varepsilon_p - \mu)\sum_{p'}\left(f_{pp'} + \frac{4\pi e^2}{q^2}\right) = 0 (3.4.7)$$

This result is quite striking and shows that the total interaction between charged quasi-particle is characterized by the fourier coefficients $\frac{4\pi e^2}{q^2} + f_{pp'}$. It shows that for a neutral systems, $f_{pp'}$, represents the total interaction energy between quasi particles, which remained regular in the limit $\vec{q}$ goes to 0. For a charged systems on the other hand the total interaction is singular as a consequence of the long range of coulomb interaction. In order to avoid this difficulty, we split away the singular term $\frac{4\pi e^2}{q^2}$ and treat it in terms of an average polarization field acting on the electrons. What remains is the screened interaction $f_{pp'}$, which is regular when $\vec{q}$ goes to zero, the transport equation

$$\frac{\partial\delta n_p}{\partial t} + \vec{v}_p.\vec{\nabla}_r\delta n_p + \vec{v}_p\delta(\varepsilon_P - \mu).\sum_P f_{pp'}\nabla_r\delta n_{p'} - e\vec{\varepsilon}_p.\vec{v}_p\delta(\varepsilon_p - \mu) = 0 \qquad (3.4.8)$$

is easily extended to cover the case of a longitudinal external electric field $\vec{D}_{ext}(\vec{r},t)$. Because the charge on the quasi-particle is e (charge being conserved when the interaction between particles is switched on), the force produced by $\vec{D}_{ext}(\vec{r},t)$ on a

quasi-particle is simply $\vec{F} = e\vec{D}_{ext}(\vec{r},t)$ adding this to above equation , an additional forcing term $-e\vec{D}_{ext}(\vec{r},t).\vec{v}_p\delta(\varepsilon_p - \mu)$, we may distinguish two case of interest depending on the character of the external field. For inhomogeneous external field, in this case $\vec{D}_{ext}(\vec{r},t) = \vec{D}(\vec{r},t)$ and using the effective electric field $\vec{\varepsilon}(\vec{r},t)$ to which the electrons respond then

$$\vec{D}(\vec{q},\omega) = \varepsilon(\vec{q},\omega)\vec{\varepsilon}(\vec{q},\omega) \tag{3.4.9}$$

The dynamic screening of the electron plays an essential role in determining the response of the electrons and indeed in making possible consistent calculation of the response. For Homogeneous external field ($\vec{q} \equiv 0$) , in this case there is no screening. A polarization , $\vec{\varepsilon}_p$, could only result from surface charges located at the boundary of the system, such an effect has been ruled by the boundary conditions. We there fore have $\vec{\varepsilon}(0,\omega) = \vec{D}_{ext}(0,\omega)$, in both case it is appropriate to use as our transport equations

$$\frac{\partial\delta n_p}{\partial t} + \vec{v}_p.\vec{\nabla}_r\delta n_p + \vec{v}_p.\left(\sum_{p'} f_{pp'}\nabla_r\delta n_{p'} - e\vec{\varepsilon}\right)\delta(\varepsilon_p - \mu) = 0 \tag{3.4.10}$$

3.5 Macroscopic Transverse Response and Collective Modes

We now apply the above transport equation to the evaluation of the macroscopic conductivity tensor $\delta_{\alpha\beta}(\vec{q},\omega)$, in the limit in which $\vec{q}$ and ω are small. Consider the

limit $\vec{q}$ goes to zero. The departure from equilibrium δn_p is then given by

$$\delta n_p(\vec{q},\omega) = \frac{-ie}{\omega}\vec{\varepsilon}.\vec{\nabla}_p n_p^0 \tag{3.5.1}$$

Hence δn_p does not depend on the direction of $\vec{q}$.

The current is the same whether the field $\vec{\varepsilon}$ is transverse or longitudinal. For the case $\omega = 0$, we wish to calculate the response to a quasi-static transverse field. Using the the transport equation we have

$$\tilde{\delta n_p} = \frac{-ie\vec{\varepsilon}.\vec{v}_p}{\vec{q}.\vec{v}_p - i\varsigma}\delta(\varepsilon_p - \mu) \tag{3.5.2}$$

(Where the term $i\varsigma$ express the adiabatic switching on the field). In the present case, $\vec{\varepsilon}$ is perpendicular to $\vec{q}$. So that equation 3.5.2 does not reduce to the simple form. The current $\vec{J}$ is given by

$$\vec{J} = \sum_P \tilde{\delta n_p}\vec{v}_p = -ie\sum_p \delta(\varepsilon_p - \mu)(\vec{\varepsilon} - \vec{v}_p)\left(p(\frac{1}{\vec{q}.\vec{v}_p}) + i\pi\delta(\vec{q}.\vec{v}_p)\right) \tag{3.5.3}$$

For reasons of symmetry, the only contribution arises from the δ- functions. One thus finds the following transverse conductivity.

$$\sigma_\perp(\vec{q},0) = \frac{3\pi N e^2}{4qp_F} \tag{3.5.4}$$

We note that $\sigma_\perp$ is real, corresponding to a dissipative processes, the electromagnetic wave transfers energy to those electrons that moves in a direction perpendicular to $\vec{q}$. The result 3.5.4 depends only on the Fermi momentum $\vec{p}_F$ but does not depend on either m^* or $f_{pp'}$. The static transverse conductivity is thus the same as that for a non

interacting systems. This conclusion remains valid for non isotropic systems , such as electrons in a real metals. A measurement of $\delta\perp(\vec{q},0)$ may be used to determine the shape of the real Fermi surface. When ω is finite, the calculation of $\delta\perp$ is much more difficult. The dispersion relation for the coupled " electromagnetic collective modes " can be obtained directly once we now the transverse conductivity.

By combining equations below

$$curl\vec{E} = \frac{-1}{C}\frac{\partial\vec{H}}{\partial t} \tag{3.5.5}$$

and

$$curl\vec{H} = \frac{1}{C}\frac{\partial\vec{E}}{\partial t} + \frac{4\pi e}{c}\vec{J} \tag{3.5.6}$$

The frequency of a transverse wave satisfies the equation

$$c^2q^2 = \omega^2\varepsilon\perp(\vec{q},\omega) = \omega^2 + 4\pi i\omega\sigma\perp(\vec{q},\omega) \tag{3.5.7}$$

One of the high frequency root of equation 3.5.7 such that $\omega \gg qv_F$. In this case the solution yields

$$\omega^2 = \omega_p^2 + c^2q^2 \tag{3.5.8}$$

The solution 3.5.7 corresponds to the usual electromagnetic wave propagation. There are two regions of interest.

1. The short wavelength regime ($cq \gg \omega_p$), in which the frequency ω is very close to cq. The electromagnetic field is then weakly coupled to the system.

2. The long wave length regime ($cq \ll \omega_p$) , in which $\omega \cong \omega_p$, the electromagnetic field is then strongly coupled to the electrons, as witnessed by the enormous modification in its natural frequency cq .

The frequency 3.5.8 is finite and large. In principles the systems motion can not be obtained from the Landau theory, which only applies to very low frequencies. In addition to the solution 3.5.8 corresponding to photon propagating, equation 3.5.7 exhibits another low frequency roots such that, $\omega \sim qv_F$, such roots corresponds to genuine transverse collective modes.

Chapter 4

SUMMARY AND CONCLUSION

The correspondence between a dense fluid of fermions ie a Fermi liquid and an ideal Fermi gas resembles, when we deliberately ignored an important complications. The energy spectrum of a Fermi gas is completely determined by the energy of a single free particle, the interaction between quasi-particle, which is by no means weak in general, does play an important role in a real Fermi liquid.

Due to the break down of the Landau's Fermi liquid theory, Some of the physical properties like specific heat per temperature, compressibility or magnetic susceptibility diverges, the quasi-particle amplitude Z_k vanishes due to orthogonality of wave vectors, the single particle self energy is singular as a function of energy ε at $k \simeq k_F$, the generation of new quantum number characterize new low energy fluctuations which does not describe the non interacting systems.

There appears instabilities of the collective modes of Fermi surface, charge density waves (CDW) and spin density waves (SDW). Singular Fermi liquid occurs more

famously in the quantum Hall problems, one dimensional problems and problems of impurity scattering with special symmetries.

The Landau correlation function of the interacting fermion systems which is the heart of the Fermi liquid theory has a great significant for transport and thermodynamic properties. The long range coulomb interaction which is the main cause of logarithmic divergence and singularity can be avoided by using the screening principles so that the long lived excitation of quasi-particles will exist. The low energy fluctuation of spin and charge densities has a great role on macroscopic transport phenomena. Such collective fluctuation of excitation are called plasmon.

Collective modes represents a second kind of possible elementary excitation of the Fermi liquid. Physically they involve as a coherent motion of the system as a whole. Since a distortion of the quasi-particle distribution takes place as a result of some internal fluctuation of the system , the average interaction force no longer vanishes. Instead , it acts to serve as a restoring force for a collective oscillation about the equilibrium states , in other words a collective mode involves a cooperative motion of the system governed by the global interaction between the particles.

The following theoretical conclusions are drawn from the work.

-When a given quasi particle approaches another, it acts to alter the polarization cloud of that quasi-particle giving rise to a departure from the plain coulomb interaction. Both effects appear as a fluctuating field associated with each quasi-particle.

-Applying the higher order perturbation theory for the long range coulomb interaction between the electrons, there appears the weight of the quasi-particles to be vanish and hence the singularity of Fermi liquid theory will occurs, such singularity can be removed by using the so called screening effects of electrostatic field.

-The transverse conductivity does not depend on the effective mass m^* and Landau correlation function $f_{pp'}$, but depends on the Fermi momentum p_F.

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Declaration

I declare that the thesis is my original work , has not been presented for a Degree in any other University and that all sources of material the thesis have been duly acknowledged.

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Place and Date of Submission

Addis Ababa University

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June 2005.

This thesis has been submitted for examination with my approval as University advisor.

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Signature ——————

Date ——————