SPIN DEPENDENT 2D ELECTRON SCATTERING ON NANOMAGNETS

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Abstract

This thesis briefly uses 2D scattering theory and Born approximation of scattering in 2D with account of the interaction of magnetic moment of an electron and the nanomagnet. Since the electrons are fermions it has half integral spin either up or down.Due to this,the scattering amplitudes in this problem are the component spinors. The electrons scattered by the nanomagnet splits into two parts with opposite direction of spins. They are calculated as functions of the electron spin orientation,the electron energy and the scattering angle. In general this is the problem of elastic scattering i.e we don't consider when the interacting particles are accompanied by a change of internal states.If we consider the problem as inelastic again it will be another area of research. It is known that the great mystery in electronics is spin controlling method. So,this thesis considers 2D scattering of an electron by a magnetized nanoparticle as one of controlling method of spin current. Finally theory and review literature parts are also carefully referred from different sources as described in reference part.

Acknowledgements

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Chapter 1

Introduction

1.1 Introduction to spintronics

Spintronics , known as magneto electronics, is an emerging technology that exploits both the intrinsic spin of the electron and its associated magnetic moment, in addition to its fundamental electronic charge, in solid-state devices.

Spintronics, or spin electronics, refers to the study of the role played by electron (and more generally nuclear) spin in solid state physics, and possible devices that specifically exploit spin properties instead of or in addition to charge degrees. For example, spin relaxation and spin transport in metals and semiconductors are of fundamental research interest not only for being basic solid state physics issues, but also for the already demonstrated potential these phenomena have in electronic technology.

The prototype device that is already in use in industry as a read head and a memory storage cell is the giant magnetoresistive (GMR) sandwich structure which consists of alternating ferromagnetic and nonmagnetic metal layers. Depending on the relative orientation of the magnetizations in the magnetic layers, the device resistance changes from small (parallel magnetizations) to large (anti parallel magnetizations). This change in resistance (also called magneto resistance) is used to sense changes in magnetic fields. Recent efforts in GMR technology have also involved magnetic tunnel junction devices where the tunneling current depends on spin orientations of the electrodes.[1]

Current efforts in designing and manufacturing spintronic devices involve two different approaches. The first is perfecting the existing GMR-based technology by either developing new materials with larger spin polarization of electrons or making improvements or variations in the existing devices that allow for better spin filtering. The second effort, which is more radical, focuses on finding novel ways of both generation and utilization of spin-polarized currents. These include investigation of spin transport in semiconductors and looking for ways in which semiconductors can function as spin polarizers and spin valves. The importance of this effort lies in the fact that the existing metal-based devices do not amplify signals (although they are successful switches or valves), where as semiconductor based spintronic devices could in principle provide amplification and serve, in general, as multi functional devices. Perhaps even more importantly, it would be much easier for semiconductor-based devices to be integrated with traditional semiconductor technology.[1] While there are clear advantages for introducing semiconductors in novel spintronic applications, many basic questions pertaining to combining semiconductors with other materials to produce a viable spintronic technology remain open. For example, whether placing a semiconductor in contact with another material would impede spin transport across the interface is far from well-understood. In the past, one of the strategies to advance understanding of spin transport in hybrid semiconductor structures was to directly borrow knowledge obtained from studies of more traditional magnetic materials. However, there is also an alternative approach involving the direct investigation of spin transport in all-semiconductor device geometries. In such a scenario a combination of optical manipulation (for example, shining circularly polarized light to create net spin polarization) and material inhomogeneities (e.g. by suitable doping as in the recently discovered $Ga_{1-x}Mn_xAs$ type ferromagnetic materials where Mn impurities act as dopants) could be employed to tailor spin transport properties. In addition to the near-term studies of various spin transistors and spin transport properties of semiconductors, a long-term and ambitious subfield of spintronics is the application of electron and nuclear spins to quantum information processing and quantum computation . It has long been pointed out that quantum mechanics may provide great advantages over classical physics in physical computation. Among the many quantum computer hardwares that were proposed are the ones based on electron and nuclear spins.

Electrons are spin-1/2 fermions and therefore constitute a two-state system with spin "up" and spin "down". To make a spintronic device, the primary requirements are, first, a system that can generate a current of spin-polarized electrons comprising more of one spin species up or down than the other (called a spin injector), and, secondly, a separate system sensitive to the spin polarization of the electrons (spin detector). Manipulation of the electron spin during transport between injector and detector (especially in semiconductors) via spin precession (a change in the orientation of the rotation axis of a rotating body) can be accomplished using real external magnetic fields or effective fields caused by spin-orbit interaction(an any interaction of a particle's spin with its motion).

Spin polarization in non-magnetic materials can be achieved either through the Zeeman effect in large magnetic fields and low temperatures, or by non-equilibrium methods. In the latter case, the non-equilibrium polarization will decay over a time scale called the "spin lifetime". Spin lifetimes of conduction electrons in metals are relatively short (typically less than 1 nanosecond). However in semiconductors the lifetimes can be very long (microseconds at low temperatures), especially when the electrons are isolated in local trapping potentials (for instance, at impurities, where lifetimes can be milliseconds).

1.2 Basic concept in spintronics

The birth of spin electronics heralds a new era of integration in semiconductor electronics, in which the electron charge, the electron spin and the photon helicity are controlled, leading to the prospect of a vastly expanded range of design possibilities for electronic devices.In particular, significant improvements in data storage, processing, and communications are already in sight. Giant magnetoresistance (GMR) sensors are a now familiar example of the integration of electron charge and electron spin manipulation; on the other hand the integration of spin polarized electron transport with polarized photon manipulation is still in development. For the electron spin to be successfully employed in a spin electronic device, the spin degree of freedom must be handled most delicately, because the spin will rapidly depolarize in the device: the spin lifetime is $10ns$ at most in $GaAs$[2] . Spin polarized electrons/holes are injected from a source electrode and detected electrically (or optically) at a drain electrode. The device functionality is based on the manipulation of the electron spin during propagation from the source to the drain electrode.The device relies on three different key processes:

(1) spin injection

(2) spin manipulation (control) and

(3) spin detection

. The spin field effect transistor (FET) as proposed by Datta and Das is an example of a spin electronic device based on this concept. For the spin FET, electron (or hole) spins are injected from a source, modulated by the gate bias and finally detected electrically at a drain. In order to successfully implement such a device, however, there are still obstacles to overcome. Since the spin-orbit interaction is relatively weak, a long device length would be needed to manipulate the spin via the Rashba effect, as was originally proposed . To achieve efficient spin control in the transistor base, therefore a more effective mechanism, such as Larmor precession, may be necessary. Furthermore, spin injection and detection efficiencies at room temperature reported to date are too low to be practical. However, while the concept of the spin FET has yet to be successfully realized, it has nevertheless been very stimulating to the field.

1.3 Future prospects of spintronics

Much remains to be understood about the behavior of electron spins in materials for technological applications, but radio frequency much has been accomplished. A number of novel spin-based microelectronic devices have been proposed, and the giant magnetoresistive sandwich structure is a proven commercial success, being a part of every computer coming off the production line [3]. In addition, spintronic-based nonvolatile memory elements may very well become available in the near future. But before we can move forward into broad application of spin-based multi-functional and novel technologies, we face the fundamental challenges of creating and measuring spin, understanding better the transport of spin at interfaces, particularly at ferromagnetic/semiconductor : The Quantum Theory of Non relativistic Collisions interfaces, and clarifying the types of errors in spin-based computational systems.Tackling these will require that we develop new experimental tools and broaden considerably our theoretical understanding of quantum spin, learning in the process how to actively control and manipulate spins in ultra small structures. If we can do this the payoff will be an entirely new world of spin technology with new capabilities and opportunities. In particular, the tantalizing possibility of building a spintronics quantum computer will keep researchers busy for quite some time.If spintronic devices are ever to be practical, we need to understand how spins move through materials and how to create large quantities of aligned spins.

Today, the range of materials we can study has significantly increased, including novel ferromagnetic semiconductors, high-temperature superconductors and carbon nanotubes. But several questions such as the role of the interface separating different materials and how to create and measure spin polarization still remain open and are of fundamental importance to novel spintronic applications. As devices decrease in size, the scattering from interfaces plays a dominant role. In these hybrid structures the presence of magnetically active interfaces can lead to spin-dependent transmission (spin filtering) and strongly influence operation of spintronic devices by modifying the degree of spin polarization. One way to test these ideas is by directly injecting spins from a ferromagnet, where the spins start out in alignment, into a nonmagnetic semiconductor. Understanding this kind of spin injection is also required for hybrid semiconductor devices, such as the Datta-Das spin transistor. But this situation is very complicated, and a complete picture of transport across the ferromagnetic-semiconductor interface is not yet available. In its absence, researches have been studying a simpler case of normal metal - semiconductor contacts. Unfortunately, experiments on spin injection into a semiconductor indicate that the obtained spin polarization is substantially smaller than in the ferromagnetic spin injector, spelling trouble for spintronic devices. In this case,where spins diffuse across the interface, there is a large mismatch in conductivities, and this presents a basic obstacle to achieving higher semiconductor spin polarization with injection.An interesting solution has been proposed to circumvent this limitation. By inserting tunnel contacts a special kind of express lane for carriers investigators found that they could eliminate the conductivity mismatch. Moreover, to reduce significant material differences between ferromagnets and semiconductors, one can use a magnetic semiconductor as the injector. While it was shown that this approach could lead to a high degree of spin polarization in a nonmagnetic semiconductor, it only worked at low temperature. For successful spintronic applications, future efforts will have to concentrate on fabricating ferromagnetic semiconductors in which ferromagnetism will persist at higher temperatures. The issues involving spin injection in semiconductors, as well as efforts to fabricate hybrid structures, point toward a need to develop methods to study fundamental aspects of spin-polarized transport in semiconductors.Recently, It is suggested that studying hybrid semiconductor-superconductor structures for understanding spin transmission properties, where the presence of the superconducting region can serve as a tool to investigate interfacial transparency and spin-polarization.

Chapter 2

Review Literature

2.1 Review literature of spintronics

Spintronics emerged from discoveries in the 1980s concerning spin-dependent electron transport phenomena in solid-state devices. This includes the observation of spin-polarized electron injection from a ferromagnetic metal to a normal metal by Johnson and Silsbee (1985) and the discovery of giant magnetoresistance independently by Albert Fert et al and Peter Grnberg et al. (1988). The origins of spintronics can be traced back even further to the ferromagnet/superconductor tunneling experiments pioneered by Meservey and Tedrow, and initial experiments on magnetic tunnel junctions by Julliere in the 1970s. The use of semiconductors for spintronics can be traced back at least as far as the theoretical proposal of a spin field-effect-transistor by Datta and Das in 1990.

The last half of the 20th century, it has been argued with considerable justification, could be called the micro- electronics era. During that 50-year period, the world witnessed a revolution based on a digital logic of electrons. From the earliest transistor to the remarkably powerful microprocessor in desktop computer, most electronic devices have employed circuits that express data as binary digits, or bits ones and zeros represented by the existence or absence of electric charge. Furthermore, the communication between microelectronic devices occurs by the binary flow of electric charges.The technologies that emerged from this simple logic have created a multi trillion dollar per year global industry whose products are ubiquitous. Indeed, the relentless growth of microelectronics is often popularly summarized in Moores Law, which holds that microprocessors will double in power every 18 months as electronic devices shrink and more logic is packed into every chip.Yet even Moores Law will run out of momentum one day as the size of individual bits approaches the dimension of atoms this has been called the end of the silicon road map. For this reason and also to enhance the multi functionality of devices (for example,carrying out processing and data storage on the same chip), investigators have been eager to exploit another property of the electron a characteristic known as spin [3]. Spin is a purely quantum phenomenon roughly akin to the spinning of a childs top or the directional behavior of a compass needle. The top could spin in the clockwise or counterclockwise direction; electrons have spin of a sort in which their compass needles can point either up or down in relation to a magnetic field.Spin therefore lends itself elegantly to a new kind of binary logic of ones and zeros. The movement of spin, like the flow of charge, can also carry information among devices. One advantage of spin over charge is that spin can be easily manipulated by externally applied magnetic fields, a property already in use in magnetic storage technology. Another more subtle (but potentially significant) property of spin is its long coherence, or relaxation, time once created it tends to stay that way for a long time, unlike charge states, which are easily destroyed by scattering or collision with defects, impurities or other charges. These characteristics open the possibility of developing devices that could be much smaller, consume less electricity and be more powerful for certain types of computations than is possible with electron-charge-based systems. Those of us in the spintronics (short for spin electronics) community hope that by understanding the behavior of electron spin in materials we can learn something fundamentally new about solid state physics that will lead to a new generation of electronic devices based on the flow of spin in addition to the flow of charge. In fact, the spin of electronic, optoelectronic and magneto electronic multi-functionality on a single device that can perform much more than is possible with todays microelectronic devices.

2.2 Spin dependent scattering of electrons in magnetic multilayers

There are different types of scattering carriers may experience in magnetic multilayers.We are mainly concerned with elastic (energy conserving) scattering. In each scattering act, only the direction of propagation of carriers changes. It is essential to distinguish between spin-dependent scattering,which causes the GMR, and spin-flip scattering,which is detrimental to the GMR.The two types of scatterings.In the case of spin-dependent scattering,the orientation of the carrier spin is conserved in each scattering event,but the probabilities of scattering for carriers with ↑ and ↓ spin projections are different.However,when a carrier undergoes a spin-flip scattering, its spin orientation changes from $\uparrow$ $(s_z = \frac{\hbar}{2})$ to $\downarrow$ $(s_z = -\frac{\hbar}{2})$or vice versa and,at the same time,the spin of the scattering center changes by $\Delta = \hbar$ so that the total spin is conserved [1]. There are several sources of spin-flip scattering.When magnetic multilayers are prepared, some of the magnetic atoms may enter the nonmagnetic spacer layer to form magnetic impurities.When a carrier is scattered off a magnetic impurity the spins of the carrier and that of the impurity can interchange provided the impurity spin is free to rotate.This is the case when the impurity spin is not strongly coupled to the spins of the ferromagnetic layers (i.e., when the impurity is not near the ferromagnet/spacer interface) [4]. Carriers can also be scattered from spin waves in the ferromagnetic layers.Spin waves are quasiparticles with spin one, and, therefore, creation (annihilation) of a spin wave in a collision with a carrier leads to a flip of the carrier spin.Since creation (annihilation) of spin waves involves the spin wave energy, this is an inelastic process that is only important at elevated temperatures. Finally, when impurities with a strong spin-orbit interaction, such as gold, are present in the multilayer, the spin of a carrier incident on such an impurity may be reversed due to the spin-orbit interaction [5]. Since all these processes mix ↑ and ↓ spin channels, they are detrimental to the GMR. In what follows, we shall assume that spin flip scattering is weak so that no mixing of the ↑ and ↓ spin channels takes place. This assumption may break down for relatively thick multilayers and the implications of spin-flip scattering for GMR . We now

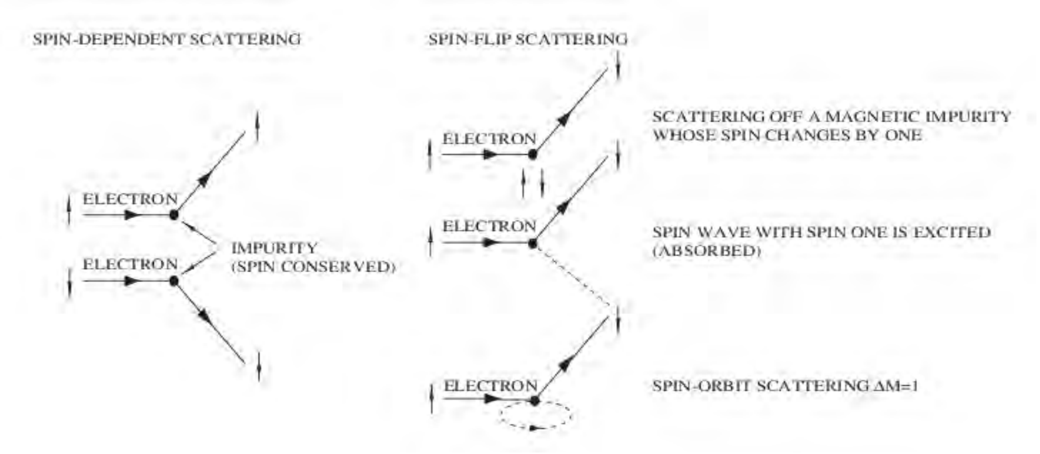

Figure 2.1: Different types of scattering in magnetic multilayers.

turn to the spin-dependent scattering that conserves the carrier spin. The key feature here is that carriers with different spin orientations (↑↓) are scattered at different rates when they enter the ferromagnetic layers. Given that carriers obey the Pauli exclusion principle, a carrier can be scattered from an impurity only to quantum states that are not occupied by other carriers. At zero (low) temperatures, all the states with energies E below the Fermi energy E_F are occupied and those with $E > E_F$ are empty. Since scattering from impurities is elastic, carriers at the Fermi level (which carry the current) can be scattered only to states in the immediate vicinity of the Fermi level. It follows that the scattering probability is proportional to the number of states available for scattering at E_F i.e., to the density of states $D(E_F)$.

2.3 Giant magnetoresistance

The era of spintronics begun with the discovery that the resistance of a multilayer consisting of a sequence of thin magnetic layers separated by equally thin nonmagnetic metallic layers is low when the magnetizations of the neighboring magnetic layers are parallel but becomes much higher when they are ordered antiparallel [1].

Assuming that the thickness of the nonmagnetic spacer layer is chosen so that the spontaneous orientation of the adjacent magnetic layers is antiparallel,change of the magnetic configuration from antiferromagnetic to ferromagnetic can be effected by an applied magnetic field. The relative change of the resistance can be larger than 200 percent.That is the reason why the effect is called the giant magnetoresistance(GMR) [6].

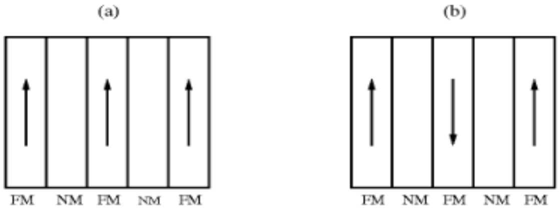

Figure 2.2: Ferromagnetic (a) and antiferromagnetic (b) configurations of a magnetic multilayer.

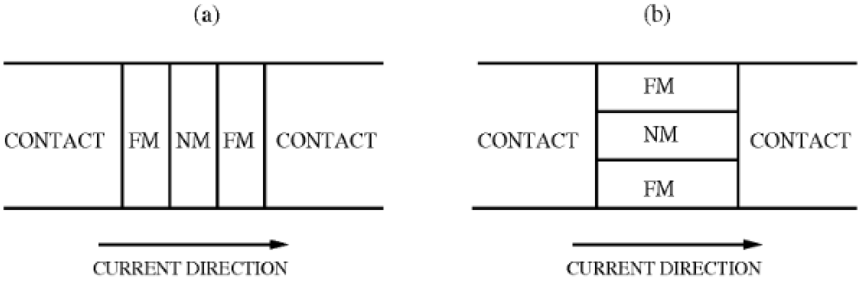

Figure 2.3: Current perpendicular to plane (a) and current in plane (b) GMR geometries.

There are two principal geometries of the GMR effect.In the first case, the current flows perpendicular to the layers (CPP geometry) and the more usual geometry when the current flows in plane of the layers (CIP)[1].

Chapter 3

Quantum Description of Spin

3.1 Meaning of spin

Spin is phenomena that occurred in quantum mechanics and has no analogy classical physics.The electron was the first elementary particle whose spin is detected. Several experiments which could not be classically interpreted ;motivated Goudismit and Uhlenbeck in 1925 to hyphotesize: Every electron has an intrinsic angular momentum (spin) of $\frac{1}{2}\hbar$ which crossponds to a magnetic moment of one Bohr magneton $\mu_B = \frac{|e|\hbar}{2mc}$ [7]. Spin relaxation (how spins reach the thermal equilibrium) and spin transport (how spins move in metals and semiconductors) are fundamentally important not only as basic physics questions but also because of their demonstrated value as phenomena in electronic technology.

Physicists and engineers are creating an entirely new generation of microelectronic devices that operate on a quantum mechanical property of electrons called **spin** rather than on the electrons electrical charge.These investigators are racing to use spin effects to create transistors and other circuit elements,including quantum computers,in a field known as **spintronics**.

Researchers and developers of spintronic devices currently take two different approaches.In the first,they seek to perfect the existing GMR-based technology either by developing new materials with larger populations of oriented spins (called spin polarization) or by making improvements in existing devices to provide better spin filtering.The second effort,which is more radical,focuses on finding novel ways both to generate and to utilize spin-polarized current that is,to actively control spin dynamics.

The intent is to thoroughly investigate spin transport in semiconductors and search for ways in which semiconductors can function as spin polarizers and spin valves.This is crucial because, unlike semiconductor transistors, existing metal-based devices do not amplify signals(although they are successful switches or valves). If spintronic devices could be made from semiconductors,however,then in principle they would provide amplification and serve,in general, as multifunctional devices.Perhaps even more importantly,semiconductor-based devices could much more easily be integrated with traditional semiconductor technology. Although semiconductors offer clear advantages for use in novel spintronic applications, many basic questions pertaining to combining semiconductors with other spintronic technology remain unknown materials to produce viable sewered. For example,it is far from well understood whether or how placing a semiconductor in contact with another material would impede spin transport across the interface. In the past, our strategy for understanding spin trans port in hybrid semiconductor structures was to

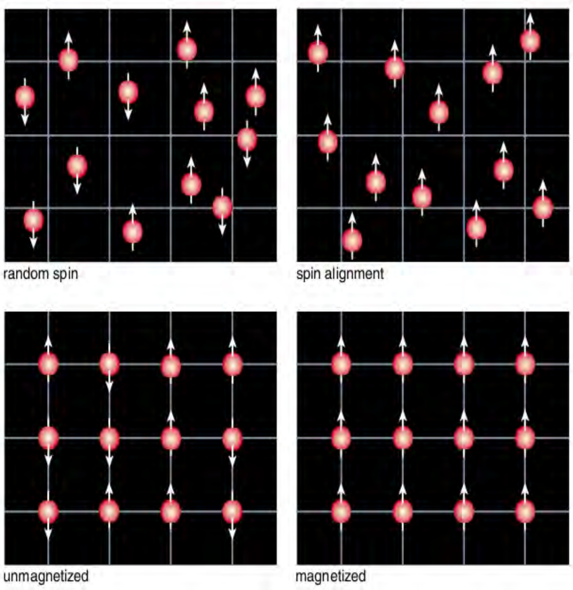

Figure 3.1: Diffrent types of spin alignment

borrow knowledge obtained from studies of more traditional magnetic materials. More recently,however, investigators have begun direct investigation of spin transport across interfaces in all-semiconductor devices. In such a scenario a combination of optical manipulation (for example, shining circularly polarized light on a material to create net spin polarization) and material inhomogeneities (by suitable doping as in a recently discovered class of gallium-manganese arsenide ferromagnetic materials) can be employed to tailor spin transport properties.In addition to the near-term studies of various spin transistors and spin transport properties of semiconductors, a long-term and ambitious sub- field of spintronics is the application of electron and nuclear spins to quantum information processing and quantum computation.

The late Richard Feynman and others have pointed out that quantum mechanics may provide great advantages over classical physics in computation. However, the real boom started after Peter Shor of Bell Labs devised a quantum algorithm that would factor very large numbers into primes, an immensely difficult task for conventional computers and the basis for modern encryption. It turns out that spin devices may be well suited to such tasks, since spin is an intrinsically quantum property.

3.2 Spin injection

Electrons possess both charge (-e) and spin angular momentum ($\pm\frac{\hbar}{2}$). Therefore, when an electron moves in a material, it carries not only a charge but also an angular momentum. Figure.A shows the schematic of electron transfer from a ferromagnetic material (FM) to a non-magnetic material (NM) through an interface [4]. In the ferromagnetic material, since electron spins are polarized, an electric current accompanies the net flow of spins, that is, spin current. The spins traveling as the spin current in the ferromagnet are then injected (spin injection) into the non-magnetic material through the interface. The injected spins are subjected to spin relaxation because of spin orbit interaction, and

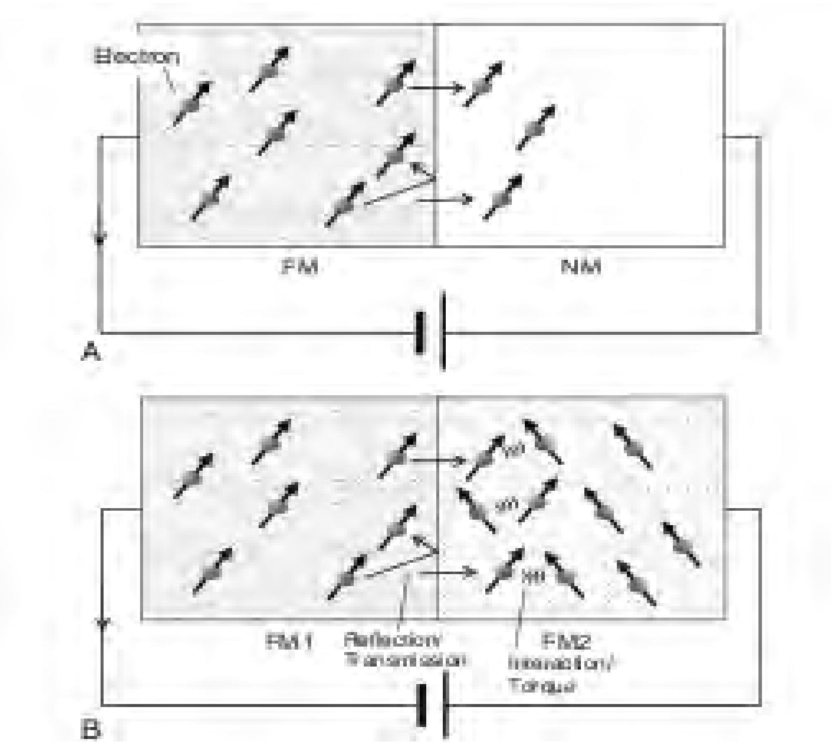

Figure 3.2: Spin injection across an interface. (A) Spin injection from a ferromagnetic material (FM) to a non-magnetic material (NM). Application of an electric charge current through the interface results in injection of a spin-polarized current into NM. (B) Spin injection from a ferromagnetic material ($FM1$) to another ferromagnetic material with different magnetization direction ($FM2$). Spin-dependent reflection and transmission at the interface give rise to the magneto resistance effect. Injected spins interact with spins of the host material and exert torque on it.

they lose their spin orientations as they move away from the interface. Spin injection phenomena have been observed by detecting the emission of circularly polarized light from injected electrons in semiconductors such as $GaAs$, and by observing the spin dependent electrochemical potentials in so-called nonlocal magneto resistance geometries or using the magneto resistive/filtering effect. What happens if we pass a current through two ferromagnetic materials with different magnetization orientations. It is well known that the magneto resistance effect occurs.

In addition, the electron spins injected from the ferromagnetic material on the left-hand side ($FM1$) into the ferromagnetic material on the right-hand side ($FM2$) interact with the electron spins in $FM2$ through exchange interaction. As a result,a precession can be excited and the magnetization can be switched in $FM2$.

3.3 Operator of the rotation of spin

Let us now return to the transformation of the spinors and show how the coefficient of this transformation can in fact be expressed interims of the angles of rotation of the coordinate axes [6].By the definition of the spin operator $1 + i\delta\phi.\vec{n}.\hat{s}$ is the operator of the rotation through an angle $\delta\phi$ about a direction specified by the unit vector $\vec{n}$: for application to the wave function of the particle with spin $\frac{1}{2}$ i.e spinor of rank one we must take $\hat{s} = \frac{1}{2}\hat{\sigma}$ in this operator.The operator of the rotation through a finite angle ϕ about the same direction will be correspondingly given by,

$$\hat{U}_n = e^{\frac{1}{2}i\phi\vec{n}i.\hat{\sigma}}. \tag{3.3.1}$$

Like any function of Pauli matrices this expression reduces to one that is linear in this matrices.

$$\hat{U}_n = l\cos\frac{1}{2}\phi + i\vec{n}.\hat{\sigma}\sin\frac{1}{2}\phi, \tag{3.3.2}$$

where l is the 2×2 unit matrix and $\hat{\sigma}$ is Pauli matrices.

For example,with a rotation about the z-axis,

$$\hat{U}_z(\phi) = \cos\frac{1}{2}\phi + i\vec{n}.\hat{\sigma}_z \sin\frac{1}{2}\phi = \begin{pmatrix} e^{i\frac{1}{2}\phi} & 0 \\ 0 & e^{-i\frac{1}{2}\phi} \end{pmatrix}$$

This means that the component of the spinor are transformed in such a rotation according to,

$$\begin{aligned} \psi_1' &= \psi_1 e^{i\frac{1}{2}\phi} \\ \psi_2' &= \psi_2 e^{-i\frac{1}{2}\phi} \end{aligned} \tag{3.3.3}$$

In particular,in a rotation through an angle 2π the spinor components change sign; spinors of any odd rank must there fore have the same property. Similarly we can find the matrices of transformation consisting of a rotation through angle ϕ about the x-axis or y-axis.

$$\begin{aligned} \hat{U}_x(\phi) &= \begin{pmatrix} \cos\frac{1}{2}\phi & i\sin\frac{1}{2}\phi \\ i\sin\frac{1}{2}\phi & \cos\frac{1}{2}\phi \end{pmatrix} \\ \hat{U}_y(\phi) &= \begin{pmatrix} \cos\frac{1}{2}\phi & \sin\frac{1}{2}\phi \\ -\sin\frac{1}{2}\phi & \cos\frac{1}{2}\phi \end{pmatrix} \end{aligned} \tag{3.3.4}$$

Now in this prepare we use the transformation of the rotation about the y-axis in the x-z plane by an angle $\theta = \frac{\phi}{2}$.

3.4 Spinors

When the spin is zero,the wave function has only one component ,$\psi(0)$.The effect of the spin operator is to reduce it to zero: $\hat{s}\psi = 0$.The relation between $\hat{s}$and the operator of the infinitesimal rotation implies that the wave function of a particle with zero spin is invariant under the rotation of the coordinate system.i.e it is a scalar [4].The wave function of the particle with spin $\frac{1}{2}$ has two components $\psi(\frac{1}{2})$ and $\psi(-\frac{1}{2})$ for convenience we distinguish these components by the subscripts 1 and 2 respectively. These two component quantity,

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \psi(\frac{1}{2}) \\ \psi(-\frac{1}{2}) \end{pmatrix}, \tag{3.4.1}$$

is called spinor.

In any rotation of the coordinate system, the component of the spinor undergoes a linear transformation:

$$\begin{aligned} \psi_1' &= a\psi_1 + b\psi_2 \\ \psi_2' &= c\psi_1 + d\psi_2 \end{aligned} \tag{3.4.2}$$

This may be written as,

$$\begin{gathered} \psi_{\lambda'} = (\hat{U}\psi)_\lambda, \lambda = 1, 2, 3... \\ \hat{U} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \end{gathered} \tag{3.4.3}$$

Where $\hat{U}$ is the transformation matrix.Its elements are in general complex function of the angle of rotation of the coordinate axes.

3.5 Wave function with spins

By taking the spin in to account,we assign the degree of freedom of particle to describe this additional degree of freedom we introduce the component of the spin in the z- direction as an argument of wave function .The component S_z can only take two values, namely $\pm\frac{\hbar}{2}$.Therefore,the wave function has the following coordinate representation [7].

$$\psi = \psi(\vec{r}, S_z) \tag{3.5.1}$$

While the complete wave function is,

$$\psi = \begin{pmatrix} \psi_1(\vec{r}) \\ \psi_2(\vec{r}) \end{pmatrix}.$$

$$\begin{aligned} &= \psi_1(\vec{r})\chi_+ + \psi_2(\vec{r})\chi_- \\ &= \psi_1(\vec{r}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \psi_2(\vec{r}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{aligned} \tag{3.5.2}$$

The introduction of product functions for both components i.e $\psi_1(\vec{r})\chi_+ = \psi_1(\vec{r}) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and$\psi_2(\vec{r})\chi_- = \psi_2(\vec{r}) \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ is particularly convenient. The functions

$\chi_{\pm}$ indicate only the state of the "spin up" or "spin down" $|\psi_1|$ is obviously the probability of finding an electron with spin up at location r. $|\psi_2|$ is the probability of finding an electron with spin down at location r.The total probability of finding the electron independently is of its spin direction must be 1.Thus

$$\int |\psi_1(\vec{r})|^2 + |\psi_2(\vec{r})|^2 dv = 1 \tag{3.5.3}$$

How the spin operates written as Pauli matrices act on spinors. The eigenstate of the operator σ_Z are,

$$\begin{aligned} \sigma_Z \begin{pmatrix} \psi_1 \\ 0 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \psi_1 \\ 0 \end{pmatrix} = (+1) \begin{pmatrix} \psi_1 \\ 0 \end{pmatrix} \\ \sigma_Z \begin{pmatrix} 0 \\ \psi_2 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ \psi_2 \end{pmatrix} = (-1) \begin{pmatrix} 0 \\ \psi_2 \end{pmatrix} \end{aligned} \tag{3.5.4}$$

where $\chi_{\pm}$ are unit spinors and $\chi_+ \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\chi_- \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Therefore;

$$\begin{aligned} \hat{\sigma}_z \chi_+ &= (+1)\chi_+ \\ \hat{\sigma}_z \chi_- &= (-1)\chi_+ \end{aligned} \tag{3.5.5}$$

Internal magnetic field produces a physical effect equivalent to applying an external voltage and could effectively tailor the width of the junction. (At the same time, this affects spin-up and spin down electrons differently: A spin-polarized current results as well). Such a device could find use in magnetic sensor technology such as magnetic read heads or magnetic memory cells.

3.6 Pauli equation

The Pauli equation is also known as the Schrodinger-Pauli equation,is a formulation of Schrodinger equation for spin $\frac{1}{2}$ particle which takes in to account the interaction of particles spin with the electromagnetic field.It is hence non- relativistic limit of Dirac equation and can be used where particles are slow enough that relativistic effect can be neglected [9] .It was formulated by Wolfgang Pauli in 1927.

We are going to describe the quantum dynamics of an electron in a magnetic field.Assume that there are no external forces rather than pure magnetic field. The Hamiltonian of an electron in a pure magnetic field is obtained by adding the potential energy of magnetic moment int reaction to the usual expression for the kinetic energy in the magnetic field. The Hamiltonian for the motion of an electron (charge) in an electro-magnetic field in the absence of spin is given by,

$$\hat{H}_o = \frac{1}{2m_e}(\hat{P} + \frac{e}{c}\vec{A})^2 + eV, \qquad (3.6.1)$$

where $\hat{P} = -i\hbar\nabla$,e is charge of electron, $\vec{A}$ is vector potential,V is electrostatic potential and $\vec{\mu}$ is magnetic moment.

But in this thesis it is not important to consider V and above equation becomes,
$\hat{H}_o = \frac{1}{2m_e}(\hat{P} + \frac{e}{c}\vec{A})^2$.

Since the spin interacts with the magnetic field the electron gains additional potential energy.The magnetic moment reads,

$$\vec{\mu} = g\frac{-|e|}{2mc}\hat{s}. \qquad (3.6.2)$$

But g=2 for electron and
$\hat{s} = \frac{\hbar}{2}\hat{\sigma}$,therefore the above equation becomes,

$$\vec{\mu} = -\mu_B.\hat{\sigma}, \qquad (3.6.3)$$

where $\hat{\sigma} = \hat{\sigma}_x, \hat{\sigma}_y$ and $\hat{\sigma}_z$ Pauli matrices ,

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and $\mu_B = \frac{|e|\hbar}{2mc}$

Therefore potential energy in the magnetic field becomes,

$$U = -\vec{\mu}.\vec{B}. \qquad (3.6.4)$$

The Hamiltonian of an electron with spin takes the form,

$$\hat{H} = \hat{H}_o + U = \frac{1}{2m_e}(\hat{P} + \frac{e}{c}\vec{A})^2 + \mu_B \hat{\sigma}.\vec{B}. \tag{3.6.5}$$

Now finally we get the Schrdinger equation of particle with spin known as Pauli equation.

$$\left[\frac{1}{2m_e}(\hat{P} + \frac{e}{c}\vec{A})^2 + \mu_B \hat{\sigma}.\vec{B}\right]\vec{\psi} = E\vec{\psi}, \tag{3.6.6}$$

where $\vec{\psi} = \begin{pmatrix}\psi_1 \\ \psi_2\end{pmatrix}$
When this written separately,

$$\begin{aligned}\hat{H}_1\psi_1 &= \left[\frac{1}{2m_e}(\hat{P} + \frac{e}{c}\vec{A})^2 + \mu_B \hat{\sigma}.\vec{B}\right]\psi_1 = E\psi_1, \\ \hat{H}_2\psi_2 &= \left[\frac{1}{2m_e}(\hat{P} + \frac{e}{c}\vec{A})^2 - \mu_B \hat{\sigma}.\vec{B}\right]\psi_2 = E\psi_2,\end{aligned}$$

Where $-\mu_B$ is due to spin down$(-\frac{\hbar}{2})$ andμ_B is due to spin up $(\frac{\hbar}{2})$.

Thus Pauli equation is the system of two coupled differential equations for ψ_1 and ψ_2 describing the electrons with the z-component of their spin up or down,respectively .Because of form of the Pauli spin matrices we can easily see thus the system in equation is decoupled for $\hat{\sigma}_z$ and only coupled for $\hat{\sigma}_x$ and $\hat{\sigma}_y$.The magnetic field should be homogeneous and possess only a z-component.

Chapter 4

Scattering Theory

4.1 Introduction

In classical mechanics,collision of two particles are entirely determined by their velocities the concept of the path is meaningless,and therefore so for the impact parameter.The purpose of the scattering is here only to calculate the probability that,as a result of the collision, the particle will deviate (scattered) through any given angle ϕ.So, we are speaking here of what are called elastic collisions in which the particle, or the internal state of the colliding particles (electron and nanomagnet in our case) if this are complex,are unchanged.But we doesn't consider the inelastic collisions,when they are accompanied by as change internal state of the colliding particles ,forexample,the change may consist of excitation or ionization of atoms or disintegration of nuclei and so on. Where a collision (e.g a nuclei reaction) may be accompanied by a various physical process, these are referred to as various channels of the reaction.

Q.how can we study such properties? So,when the fast electron elastically collide with nanomagnet we treat the probability by using Born approximation and we also take account about the dipole-dipole moment int reaction of electron and scattering center (nanomagnet).Again we consider electron moment and the magnetic moment $\vec{\mu}$ of the magnetic nanoparticle are in the same plane to bring the problem in to two dimensional.

4.2 Born approximation in three dimension

Particularly good approach for calculating the scattering amplitude when the energy of the incident beam is large in comparison with the magnitude of the potential energy in Born approximation [8].

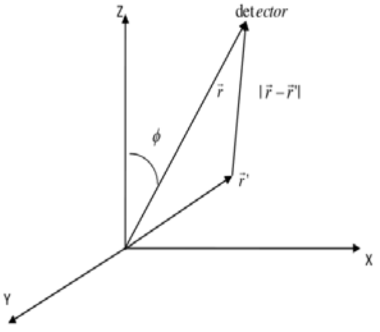

Figure 4.1: Diagrammatically representation of three dimensional scattering.

$$\{\frac{-\hbar^2}{2m}\nabla^2 + V(r)\}\psi(r) = E(r). \tag{4.2.1}$$

In the form,

$$\{\nabla^2 + \kappa^2\}\psi(r) = \frac{2m}{\hbar^2}V(r)\psi(r), \tag{4.2.2}$$

where $\kappa = \sqrt{\frac{2mE}{\hbar^2}}$.

Incoming plane wave $Ae^{i\kappa z}$ is a solution to equation,

$$\{\nabla^2 + \kappa^2\}\psi(r) = 0. \tag{4.2.3}$$

It is convenient to express the formal solution to equation (4.2.2) in the form ,

$$\psi(r) = Ae^{i\kappa z} + \int d^3r' G(\vec{r}, \vec{r'})\frac{2m}{\hbar^2}V(r')\psi(r'), \tag{4.2.4}$$

where $G(\vec{r}, \vec{r})$ is the green function and satisfies the differential equation,

$$\{\nabla^2 + \kappa^2\}G(\vec{r}, \vec{r'}) = \delta^3(\vec{r} - \vec{r'}). \tag{4.2.5}$$

The iterative procedure for determining the wave function in equation (4.2.4) is known as Born approximation. And

$$G(\vec{r}, \vec{r'}) = -\frac{e^{i\kappa|\vec{r}-\vec{r'}|}}{4\pi|\vec{r} - \vec{r'}|}. \tag{4.2.6}$$

Therefore,

$$\psi(r) = Ae^{i\kappa z} - \frac{m}{2\pi\hbar^2}\int d^3r' \frac{e^{i\kappa|\vec{r}-\vec{r'}|}}{4\pi|\vec{r}-\vec{r'}|}V(r')\psi(r). \tag{4.2.7}$$

In this equation,since r $\gg |r'|$ we can approximate the following,

$$\frac{1}{|\vec{r}-\vec{r'}|}^{\prime} \approx \frac{1}{r}. \tag{4.2.8}$$

Again we have,

$$e^{i\kappa|\vec{r}-\vec{r'}|} = e^{i\kappa r}.e^{-i\vec{\kappa}_f.\vec{r'}}, \tag{4.2.9}$$

where $\vec{\kappa}_f = \kappa\hat{U}_r$,with this two approximations asymptotic limit becomes,

$$\psi(r) \to Ae^{i\kappa z} - \frac{me^{i\kappa r}}{2\pi\hbar^2 r}\int d^3r' e^{-i\vec{\kappa}_f.\vec{r'}}V(r')\psi(r'). \tag{4.2.10}$$

We are now ready for Born approximation.If the potential V=o the solution for $\psi(r')$, it would be simply,

$$\psi(r') \approx Ae^{i\kappa z} - \frac{me^{i\kappa r}}{2\pi\hbar^2 r}\int d^3r' e^{-i\vec{\kappa}_f.\vec{r'}}V(r')e^{i\kappa z'}. \tag{4.2.11}$$

We know that at $\frac{r'}{r} \ll 1$,we have the wave function $\psi = Ae^{i\kappa z} + f(\theta,\phi)\frac{e^{i\kappa r}}{r}$ So,comparing to this,the scattering amplitude is given in the Born approximation by,

$$f(\theta,\phi) = -\frac{m}{2\pi\hbar^2}\int d^3r' V(r')e^{i\vec{q}.\vec{r'}}, \tag{4.2.12}$$

where $\vec{q} = \vec{\kappa}_i - \vec{\kappa}_f$. In our main problem we will drive the int reaction potential $V(r')$ and use it to solve $f(\theta,\phi)$.

4.3 Rutherford scattering formula in three dimension

We have scattering amplitude from Born approximation,

$$f(\theta,\phi) = -\frac{m}{2\pi\hbar^2}\int d^3r' V(r')e^{i\vec{q}.\vec{r'}}, \tag{4.3.1}$$

where $d^3r' = r'^2 \sin\theta' dr d\theta' d\phi$ and

$$V(r') = \frac{1}{4\pi\epsilon_0}\frac{z_1 z_2 e^2}{r'}.$$

$$f(\theta,\phi) = \frac{-m}{2\pi\hbar^2}(\frac{1}{4\pi\epsilon_0}\frac{z_1z_2e^2}{r'})\int_0^\infty\int_0^\pi\int_0^{2\pi} r'e^{i\vec{q}.\vec{r'}}\sin\theta' d\phi' d\theta' dr'.$$
$$= \frac{mi}{2\pi\hbar^2 q}\frac{z_1z_2e^2}{2\epsilon_0}\int_o^\infty (e^{iqr'-e^{-}iqr'})dr'.$$

Finally we have,

$$f(\theta,\phi) = \frac{mz_1z_2e^2}{4\pi\hbar^2\epsilon_0 q^2}. \tag{4.3.2}$$

But we have $\vec{q} = \vec{\kappa}_i - \vec{\kappa}_f$,

where $\vec{\kappa}_i$ is the wave vector of the incident wave,$\vec{\kappa}_f$ is the wave vector of the scattered wave.

Again $q^2 = (\vec{\kappa}_i - \vec{\kappa}_f)^2 = 4\kappa^2\sin^2\frac{\theta}{2}$. Using this relation,

$$f(\theta) = \frac{mz_1z_2e^2}{4\pi\hbar^2\epsilon_0(4\kappa^2\sin^2\frac{\theta}{2})}. \tag{4.3.3}$$

Now to find the differential cross section,

$$\frac{d\delta}{d\Omega} = |f(\theta)|^2 = [\frac{mz_1z_2e^2}{4\pi\hbar^2\epsilon_0(4\kappa^2\sin^2\frac{\theta}{2})}]^2 = \frac{[mz_1z_2e^2]^2}{16^2\pi^2\hbar^4\epsilon_0^2\kappa^4\sin^4\frac{\theta}{2}}.$$
$$\frac{d\delta}{d\Omega} = \frac{[z_1z_2e^2]}{64\pi^2E^2\sin^4\frac{\theta}{2}}, \tag{4.3.4}$$

where $E = \frac{\hbar^2\kappa^2}{2m}$.

This is the famous result for Rutherford scattering.

4.4 Scattering in two dimension

Now consider the incident wave $\psi = Ae^{i\kappa z}$ is along the positive z-axis and outgoing wave is in x-z plane with scattering angle ϕ from the direction of incident wave. In two dimension, the wave function far from the target (scatterer) is the superposition of plane wave (incident wave) and an outgoing cylindrical wave [8]. Outside the range potential where the particle detectors are located,the outgoing cylindrical wave must be solution to

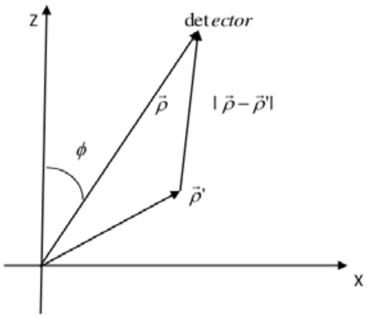

Figure 4.2: Diagrammatically representation of two dimensional scattering.

Schrodinger equation i.e $\psi \approx Ae^{i\kappa z}$ + (outgoing cylindrical wave).

The Schrodinger equation here is,

$$\{\frac{-\hbar^2}{2m}\nabla^2 + V(\rho)\}\psi(\rho,\phi) = E\psi(\rho,\phi) \tag{4.4.1}$$

where $\rho = \sqrt{x^2+z^2}$.

In the region where the potential is zero (i.e $\rho' \ll \rho$),

$$\frac{-\hbar^2}{2m}\nabla^2\psi(\rho,\phi) = E\psi(\rho,\phi), \tag{4.4.2}$$

where ∇^2,the Laplace in ρ, ϕ coordinates,is

$$\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial}{\partial\rho}) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} \tag{4.4.3}$$

Inserting this in to the main equation to get,

$$\frac{-\hbar^2}{2m}\{\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial}{\partial\rho}\psi(\rho,\phi)) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}\psi(\rho,\phi)\} - E\psi(\rho,\phi) = 0$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial}{\partial\rho}\psi(\rho,\phi)) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}\psi(\rho,\phi) + \frac{2mE}{\hbar^2}\psi(\rho,\phi) = 0$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial}{\partial\rho}\psi(\rho,\phi)) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}\psi(\rho,\phi) + \kappa^2\psi(\rho,\phi) = 0. \tag{4.4.4}$$

where $\kappa^2 = \frac{2mE}{\hbar^2}$ and $\rho = \sqrt{x^{2+z^2}}$.

Multiply both side by ρ^2 to get,

$$\rho\frac{\partial}{\partial\rho}(\rho\frac{\partial}{\partial\rho}\psi(\rho,\phi)) + \kappa^2\psi(\rho,\phi) + \frac{\partial^2}{\partial\phi^2}\psi(\rho,\phi) = 0. \tag{4.4.5}$$

Note that we have the part of expansion depends only on ρ and other exclusively on ϕ.Assume a form of wave function that is $\psi(\rho, \phi) = f(\phi)U(\rho)$.As short hand just denote $\psi = fU$.Let us consider the following expressions[10].

- $\frac{\partial\psi}{\partial\rho} = fU'$.
- $\rho\frac{\partial\psi}{\partial\rho} = f\rho U'$.
- $\frac{\partial}{\partial\rho}(\rho\frac{\partial\psi}{\partial\rho}) = f\rho U'' + fU'$.
- $\rho\frac{\partial}{\partial\rho}(\rho\frac{\partial\psi}{\partial\rho}) = f\rho^2 U'' + f\rho U'$.
- $\frac{\partial^2\psi}{\partial\phi^2} = f''U$.

Replacing last two items in to above main equation to get,

$$f\rho^2 U'' + f\rho U' + (k\rho)^2 Uf + Uf'' = 0. \tag{4.4.6}$$

Dividing both side by fU to get,

$$\rho^2\frac{U''}{U} + \rho\frac{U'}{U} + (k\rho)^2 + \frac{f''}{f} = 0. \tag{4.4.7}$$

Solving for f first.

$$\frac{f''}{f} = -(\rho^2\frac{U''}{U} + f\rho\frac{U'}{U} + (k\rho)^2). \tag{4.4.8}$$

Letting $\frac{f''}{f} = -m^2$,we have,

$$\rho^2\frac{U''}{U} + f\rho\frac{U'}{U} + (k\rho)^2 - m^2 = 0. \tag{4.4.9}$$

This because ultimately what we want is $U(\rho)$.And now divide both side by ρ^2 and rearranging U in the equation,it leads to,

$$U'' + \frac{U'}{\rho} + U(k^2 - \frac{m^2}{\rho^2}) = 0. \tag{4.4.10}$$

Which is the normal Bessel equation [3].But now we demand for large ρ.This assumes for simplicity,that there is no azimuthal dependence ,that is no angular momentum ,or m=0.And again let $p = k\rho$,this results in Bessel equation,

$$p^2U'' + pU' + p^2U = 0. \tag{4.4.11}$$

The solution to this equation is,

$$U(P) = AJ_0(p) + BY_0(p). \tag{4.4.12}$$

Which is linear combination of $J_0(p)$ called the first kind zero order Bessel function and $Y_0(p)$ called the second kind zero order Bessel function.Note that Bessel function of the first kind are diverge at infinity but Bessel function of second kind are well behave at $\rho \gg \rho'$ [12].

Therefore to obtain physically meaningful solutions drop the Bessel function of the first kind and the only consider the second kind.

$$U(k\rho) = BY_0(k\rho).$$

$$H_0^{(1)}(\kappa\rho) = J_0(\kappa\rho) + iY_0(\kappa\rho) \tag{4.4.13}$$

but in our case $H_0^{(1)}(\kappa\rho) = iY_0(\kappa\rho)$ this implies,

$$Y_0(k\rho) = -iH_0^{(1)}(\kappa\rho) \tag{4.4.14}$$

But at $(\frac{\rho'}{\rho} \ll 1)$ the Hankel function $H_0^{(1)}(\kappa\rho)$ the asymptotic [13] behavior of ,

$$H_0^{(1)}(\kappa\rho) \simeq \frac{1-i}{\sqrt{\pi\rho\kappa}}e^{ik\rho} \tag{4.4.15}$$

Now this implies,

$$U(k\rho) \simeq B(\frac{1-i}{\sqrt{\pi\kappa}})\frac{e^{ik\rho}}{\sqrt{\rho}} \simeq A\frac{e^{ik\rho}}{\sqrt{\rho}} \tag{4.4.16}$$

So,we find the correction to wave function at large distance ρ from the field axis and the over all physical meaningful solution of the wave function is,

$$\psi(\rho,\phi) \approx Ae^{i\kappa z} + f(\phi)\frac{e^{i\kappa\rho}}{\sqrt{\rho}}. \tag{4.4.17}$$

Or it can be written as,

$$\psi(\rho,\phi) \approx Ae^{i\kappa z} + f(\phi)H_0^{(1)}(\kappa\rho), \tag{4.4.18}$$

where f(ϕ) is scattering amplitude,ϕ is scattering angle between z-axis and direction of scattering, ${H_0}^1(\kappa\rho)$ is the first kind zero order Hankel function.

4.5 Born approximation in two dimension

A particularly good approach for calculating the scattering amplitude when the energy of the incident beam is large in comparison with the magnitude of the potential energy in Born approximation [8],i.e $E \ll V(\rho)$.

$$\{\frac{-\hbar^2}{2m}\nabla^2 + V(\rho)\}\psi(\rho) = E\psi(\rho), \tag{4.5.1}$$

where $\rho = \rho(x,z), \psi(\rho) = \psi(x,z)$ and $V(\rho) = V(x,z)$

Rearranging equation (4.5.1)we have,

$$\{(\nabla^2 + \kappa^2)\}\psi(\rho) = \frac{2m}{\hbar^2}V(\rho)\psi(\rho), \tag{4.5.2}$$

where $\kappa^2 = \frac{2m}{\hbar^2 E}$ and $\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial z^2}$ (two dimensional Laplace operator)

The incoming (incident wave) plane wave $\psi = Ae^{i\kappa z}$ is the solution to the equation,

$$\{{\nabla_1}^2 + \kappa^2\}\psi(r) = 0, \tag{4.5.3}$$

where $r = r(z), \psi(r) = \psi(z), \nabla_1^2 = \frac{\partial}{\partial z^2}$ and ∇_1^2 is one dimensional Laplace operator.

It is convenient to express the formal solution to equation (4.5.2) as,

$$\psi = Ae^{i\kappa z} + \int d^2\rho' G(\vec{\rho},\vec{\rho'})\frac{2m}{\hbar^2}V(\rho')\psi(\rho'), \tag{4.5.4}$$

where $G(\vec{\rho},\vec{\rho})$ is the Green function with cylindrical symmetry and it satisfy the differential equation,

$$\{\nabla^2 + \kappa^2\}G(\vec{\rho},\vec{\rho'}) = \delta(\vec{\rho} - \vec{\rho'}) \tag{4.5.5}$$

where $\nabla^2 = \frac{\partial}{\partial x^2} + \frac{\partial}{\partial z^2}$ (two dimensional Laplace operator)

A Green function in a cylindrical coordinate that satisfy the 2D Helmholtz equation is given by,

$$G(\overrightarrow{\rho}, \overrightarrow{\rho'}) = \frac{i}{4} {H_0}^{(1)}(\kappa|\vec{\rho} - \vec{\rho'}|), \tag{4.5.6}$$

where ${H_0}^{(1)}(\kappa|\vec{\rho} - \vec{\rho'}|)$ is a first kind zero order Hankel function. And again when $\frac{\rho'}{\rho} \ll 1$ Hankel function converges to,

$$H_0^{(1)}(\kappa|\vec{\rho} - \vec{\rho'}|) = \frac{1-i}{\sqrt{\pi\rho\kappa}} e^{i\kappa(\rho - \vec{\rho'}.\vec{n})}, \tag{4.5.7}$$

where$\vec{n} = \frac{\vec{\rho}}{\rho}$ unit vector along $\vec{\rho}$,

Again we can substitute the wave function under integral by incident wave $\psi = Ae^{i\kappa z}$ when V=o or out of the range of potential.

Finally substituting equation (4.5.6) and (4.5.7)we have,

$$\psi(\rho) \approx Ae^{i\kappa z} + \frac{2m}{\hbar^2} \int d^2\rho' (\frac{i}{4}) \frac{(1-i)}{\sqrt{\pi\rho\kappa}} e^{i\kappa(\rho - \vec{\rho}.\vec{n})} V(\rho') e^{i\kappa\rho'}, \tag{4.5.8}$$

Now finally to write Born approximation we should single out the $f(\phi)$ comparing this equation with equation (4.4.17),

$$f(\phi) = \frac{m}{\hbar^2\sqrt{2\pi\kappa}} \int e^{-i\vec{q}.\vec{\rho'}} V(\rho') d^2\rho' \tag{4.5.9}$$

For the spin independent scattering potential $V(\rho')$,the interaction potential becomes a scalar and considers with known formula for 2D scattering in Born approximation . In the next section this will discussed in detail.

4.6 Spin dependent 2D scattering by nanomagnets

4.6.1 Introduction

The main object of the spintronics is finding reliable methods of controlling the spin orientations and the arrangement the so called spin currents.One of these methods is usage of the spin orbit interaction coupling the electron the electron moment and its spin.It is known,that the spin orbit coupling appears as the term proportional to $(\frac{v}{c})^2 \ll 1$ in the non relativistic limit of Dirac equation and must be relatively small [1]. To exploit the spin degree of freedom there should be mechanism to control their polarization.Many research recently conducted in these area use the spin orbit interaction terms based on Rashba or Dresslhaus gate voltage that makes it convenient in spintronics applications. In this section,we analyze the scattering of electrons by magnetized nanoparticles or nanomagnets.The modern nanotechnology can develop nanomagnets of typical size of $5-100nm$.It has been reported that ferrite particles of 24-27 nm with Co doping exhibits superparamagnetism even at room temperatures.These magnetic nanoparticles can be treated as gigantic molecules with anomalous magnetic moments. Forexample,the specific magnetic moments(number of Bohr magneton per impurity atom) of CeO ^{2}doped with Co is $8.2\mu_B$. The high temperature ferromagnetism with gigantic magnetic moments posses SnO_2 doped with Co($7.5 \pm 5\mu_B$), Mn($8.0\mu_B$) Co($10.8\mu_B$), and Fe($12.6\mu_B$).Anomalous magnetic moments ($21.5\mu_B$ per Fe atom and $22.9\mu_B$ per Co atom) for diluted magnetic semiconductor (TiO) .It worth noting that the interaction of magnetic moments of the electron and of nanomagnet appears in the Pauli equation in the zeroth order of small parameter $\frac{v}{c}$ and can be considerable. Below we consider two dimensional (2D) scattering of electron by the magnetic moment and show how the scattering amplitude depends on mutual orientation of the magnetic moment of the nanomagnet and the electron ,energy of electron and scattering angle.

4.6.2 Definition of the problem

The Pauli equation of an electron in a uniform magnetic field $\vec{B}$ has,

$$\hat{H} = \frac{1}{2m}[\hat{p} + \frac{e}{c}\vec{A}]^2 + \mu_B \hat{\sigma}.\vec{B} \tag{4.6.1}$$

Further, we consider the nanomagent particles as a point like dipole $\vec{\mu}$ built in a sphere of radius a, which specifies a radius of the nanoparticle. The magnetic moment of nanoparticle is taken from the experimental data on gigantic magnetic moments.The vector potential of the dipole $\vec{\mu}$ and its magnetic field are given by the expression,

$$\vec{A} = \frac{\vec{\mu} \times \vec{r}}{r^3}, \vec{B} = \frac{3(\vec{\mu}.\vec{n})\vec{n} - \vec{\mu}}{r^3} \tag{4.6.2}$$

where $\vec{n}$ is a unit vector along the radius vector $\vec{r}$.Substituting(4.6.2) in to 4.6.1) we obtain,

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{1}{2m}(\frac{e}{c}A)^2 + \frac{e}{mc}\frac{\vec{\mu}.\hat{L}}{r^3} + \frac{\mu_B}{r^3}(3(\vec{\mu}.\vec{n})\vec{n}.\hat{\sigma} - \vec{\mu}.\hat{\sigma})$$

Here the large value in the denominator we cancel the second term and again we have,

$$\hat{H} = \frac{1}{2m}\hat{p}^2 + \frac{e}{mc}\frac{\vec{\mu}.\hat{L}}{r^3} + \frac{\mu_B}{r^3}(3(\vec{\mu}.\vec{n})\vec{n}.\hat{\sigma} - \vec{\mu}.\hat{\sigma}) \tag{4.6.3}$$

where,$\hat{L} = \vec{r} \times \hat{p}$ is the angular momentum operator. Let the magnetic moment of the nanoparticle $\vec{\mu}$ and the electron momentum $\vec{p}$ be in the x-z plane.In this 2D geometry (4.6.3) takes the form,

$$\hat{H} = \frac{1}{2m}\hat{p_2}^2 + \frac{\mu_B}{\rho^3}(3(\vec{\mu}.\vec{n})\vec{n}.\hat{\sigma} - \vec{\mu}.\hat{\sigma}) \tag{4.6.4}$$

Here $\hat{p_2}^2$ is the two dimensional momentum operator and ρ is the two dimensional radius vector in the x-z plane and we took in to account that the term $\sim \vec{\mu}.\hat{L}$ in our geometry vanishes become $\vec{\mu}$ is perpendicular to the angular momentum $\hat{L}$. The Schrodinger equation corresponding to Hamiltonian(4.6.4)takes the form,

$$(\nabla^2 + \kappa^2)\vec{\psi} = \frac{2m}{\hbar^2}\hat{V}\vec{\psi} \tag{4.6.5}$$

Where,$\kappa^2 = \frac{2mE}{\hbar^2}$,E is the energy of the scattered electron,∇_2^2 is 2D Laplace operator and $\vec{\psi} = \begin{vmatrix} \psi_1 \\ \psi_2 \end{vmatrix}$ is a two component spinor.

Equation (4.6.5) is a typical equation from the scattering theory with the interaction potential $\hat{V}$ depending on spin variables.Basically (4.6.5) is a system of two equations for unknown functions of ψ_1 and ψ_2. Comparing the equation (4.6.4) and (4.6.5) the interaction potential is given by,

$$\hat{V} = \frac{\mu_B}{\rho^3}(3(\vec{\mu}.\vec{n})\vec{n}.\hat{\sigma} - \vec{\mu}.\hat{\sigma}) \tag{4.6.6}$$

The solution to the non-homogeneous differential equation (4.6.5) can be formally presented as the following integral equation.

$$\vec{\psi}(\vec{\rho}) = \vec{\psi}_0(\vec{\rho}) + \frac{2m}{\hbar^2}\int d^2\vec{\rho'}G(\vec{\rho},\vec{\rho'})\hat{V}(\vec{\rho'})\vec{\psi}(\vec{\rho'}) \tag{4.6.7}$$

with $\vec{\psi}_0(\vec{\rho})$ is the solution of homogeneous equation,

$$(\nabla^2 + \kappa^2)\vec{\psi}_0 = 0. \tag{4.6.8}$$

and $G(\vec{\rho},\vec{\rho'})$ is the Green function of this equation.

Below,we consider the case when the interaction $\hat{V}$ can be treated as a small perturbation. In this case, equation (4.6.7) is presented as,

$$\vec{\psi}(\vec{\rho}) = \vec{\psi}_0(\vec{\rho}) + \vec{\psi}_s(\vec{\rho}), \tag{4.6.9}$$

where $\vec{\psi}_0(\vec{\rho})$ is solution of homogeneous equation(4..6.8)(incident wave) and $\vec{\psi}_s(\vec{\rho})$ is a scattered wave that is a small correction to $\vec{\psi}_0$.

Keeping this in mind, we can present the correction $\vec{\psi}_s$ in the form,

$$\vec{\psi}_s(\vec{\rho}) = \frac{2m}{\hbar^2}\int d^2\vec{\rho'}G(\vec{\rho},\vec{\rho'})\hat{V}(\vec{\rho'})\vec{\psi}_0(\vec{\rho'}). \tag{4.6.10}$$

Let the incident wave be the plane wave propagating along the z-axis.

$$\vec{\psi}_0(\vec{\rho}) = exp(i\kappa z)\vec{\chi}_0(s) \tag{4.6.11}$$

with the spin function $\hat{\delta}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \vec{\chi}_0(s)$.At large distance $(\rho'/\rho \ll 1)$ from the scattering center (nanomagnet),the scattered wave in our 2D geometry is a cylindercal wave.

$$\vec{\psi}_s(\rho) = \vec{f}(\phi)\frac{e^{i\kappa\rho}}{\sqrt{\rho}} \tag{4.6.12}$$

with the "scattering amplitude " $\vec{f}(\phi) = \begin{vmatrix} f_1(\phi) \\ f_1(\phi) \end{vmatrix}$, that is a two component spinor and has dimension of the square root of length,ϕ is the scattering angle between the incident wave vector $\overrightarrow{\kappa}_0$(along z-axis) and the scattered wave vector $\vec{\kappa}$ (along $\vec{\rho}$). The Green function in (4.6.10) in the cylindrical geometry is the first kind zeroth order Hankel function.

$$G(\vec{\rho}, \vec{\rho'}) = \frac{i}{4} H_0^{(1)}(\kappa|\vec{\rho} - \vec{\rho'}|) \tag{4.6.13}$$

At $(\rho'/\rho \ll 1)$, it has the asymptotic behavior,

$$H_0^{(1)}(\kappa|\vec{\rho}, \vec{\rho'}|) \simeq \frac{1-i}{\sqrt{\pi\rho\kappa}} e^{ik(\rho - \vec{\rho'}.\vec{n})}, \tag{4.6.14}$$

where ,$\vec{n} = \frac{\vec{\rho}}{\rho}$. Substituting (4.6.13) and (4.6.14) into (4.6.10) we can single out amplitude of cylindrical wave.

$$f(\phi) = \frac{m}{\hbar^2\sqrt{2\pi\kappa}} \int e^{-i\vec{q}.\vec{\rho'}} \hat{V}(\vec{\rho'}) d^2\rho' \vec{\chi}(s) \tag{4.6.15}$$

where $\vec{q} = \vec{\kappa} - \vec{\kappa}_0, \kappa_0^2 = \kappa^2 = \frac{2mE}{\hbar^2}, q = 2\kappa_0 \sin\frac{\phi}{2}$(here we consider the elastic collision). For the spin independent scattering potential $\hat{V}$, the scattering potential becomes a scalar and considers with known formula for 2D scattering in Born approximation.In the next section we use to calculate the scattering amplitude of electrons by magnetic nanoparticles.

4.6.3 Scattering amplitude

A.Magnetic moment of nanoparticles $\|$ to the velocity of incident electron.

Consider the scattering of electrons (coming from infinity parallel to the z-axis) by a nanomagnet oriented along the z-axis.In the first one (x,z) the magnetic moment of nanoparticle

$\vec{\mu}$ is along the z-axis. Now in this case, we expected to show four situations [14].These are:

- $f^{\parallel}_{\uparrow\uparrow}(\phi)$ (the scattering amplitude when direction of spin electron and magnetic moment are initially in up state).
- $f^{\parallel}_{\downarrow\downarrow}(\phi)$ (the scattering amplitude when direction of spin electron and magnetic moment are initially in down state).
- $f^{\parallel}_{\uparrow\downarrow}(\phi)$ (the scattering amplitude when direction of spin electron is initially up and magnetic moment is initially down state).
- $f^{\parallel}_{\downarrow\uparrow}(\phi)$ (the scattering amplitude when direction of spin electron is initially down and magnetic moment is initially up state).

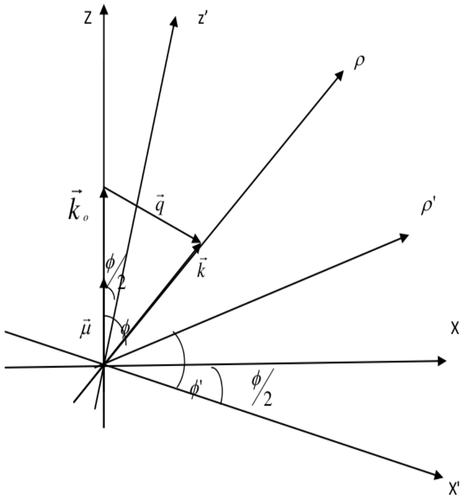

Figure 4.3: Coordinate system when magnetic moment and incident wave vector are parallel and both are along the z-axis and $\vec{q} \parallel x'$.

To evaluate the integral one has to choose appropriate coordinate system. But the calculation of the integral in (4.6.15) is convenient to carry out in another coordinate system with x'-axis directed along q', in which the dot product in the exponent (4.6.15) has he simplest form $\vec{q}.\vec{\rho'} = q\rho' \cos\phi'$($\rho'$ and ϕ' are variables of integration). Now to find interaction potential in more simplified form we use the equation (4.6.6) and we get,

$$\hat{V} = \frac{\mu_B}{\rho'^3}((\mu_{x'}\hat{i} + \mu_{z'}\hat{k}).(n_{x'}\hat{i} + n_{z'}\hat{k}))(n_{x'}\hat{i} + n_{z'}\hat{k}).(\sigma_{x'}\hat{i} + \sigma_{z'}\hat{k}) - (\mu_{x'}\hat{i} + \mu_{z'}\hat{k}).(\sigma_{x'}\hat{i} + \sigma_{z'}\hat{k}) \tag{4.6.16}$$

The substituting $n_{x'} = \cos\phi'$ and $n_{z'} = \sin\phi'$,the operator of potential energy (4.6.6) in this coordinate system takes the form,

$$\hat{V} = \frac{\mu_B}{\rho'^3}\{[\mu_{x'}(3\cos^2\phi' - 1) + \frac{3}{2}\mu_{z'}\sin 2\phi']\hat{\sigma_{x'}} + [\mu_{z'}(3\sin^2\phi' - 1) + \frac{3}{2}\mu_{x'}\sin 2\phi']\hat{\sigma_{z'}}\}. \tag{4.6.17}$$

In this coordinate system, the magnetic moment $\vec{\mu}$ has two components $\mu_{z'} = \mu\cos\frac{\phi}{2}$ and $\mu_{x'} = -\mu\sin\frac{\phi}{2}$. In the x-z coordinate system $\vec{\chi}_0(s) = \vec{\chi}_\uparrow(s) = \begin{vmatrix}1\\0\end{vmatrix}$, for the electron spin parallel to $\vec{\mu}$.But the calculation of (4.6.15) is carried out in the coordinate system $x' - z'$, we have to transform the spinor wave functions $\vec{\chi}_0(s)$ to this coordinate system.The operator of rotation of a two component spinors through a finite angle θ about the given axis is given by,

$$\hat{U_n} = \mathbf{1}\cos\frac{\theta}{2} + i\hat{n}.\hat{\sigma}_n\sin\frac{\theta}{2}, \tag{4.6.18}$$

where $\mathbf{1} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$ and $\hat{n}$ is a unit vector along the rotation axis.From above figure one can see that the required transformation is rotation about the y-axis in the x-z plane by an angle $\theta = \frac{\phi}{2}$.

The operator of this rotation is,

$$\hat{U_y} = \begin{pmatrix}\cos\frac{\phi}{4} & \sin\frac{\phi}{4}\\ -\sin\frac{\phi}{4} & \cos\frac{\phi}{4}\end{pmatrix} \tag{4.6.19}$$

Hence the spinor $\vec{\chi}_{\uparrow 0}(s)$ in the $x' - z'$ coordinate system expressed as,

$$\vec{\chi}_\uparrow(s) = \hat{U_y}\begin{vmatrix}1\\0\end{vmatrix} = \begin{pmatrix}\cos\frac{\phi}{4}\\ -\sin\frac{\phi}{4}\end{pmatrix} \tag{4.6.20}$$

Inserting (4.6.17) and (4.6.20) in to (4.6.15) and with account of results $\hat{\sigma_x}\vec{\chi}_{\uparrow}(s)$ and $\hat{\sigma_z}\vec{\chi}_{\uparrow}(s)(\hat{\sigma_x} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\hat{\sigma_z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix})$and we obtain,

$$\begin{aligned} \vec{f'}(\phi) = \quad & \gamma \int_0^{2\pi} \int_a^{\infty} \frac{e^{-i\rho' q \cos\phi'}}{\rho'} \Bigg[[-\sin\frac{\phi}{2}(3\cos^2\frac{\phi'}{2} - 1) + \frac{3}{2}\cos\frac{\phi}{2}\sin 2\phi'] \begin{pmatrix} -\sin\frac{\phi}{4} \\ \cos\frac{\phi}{4} \end{pmatrix} + \\ & [\cos\frac{\phi}{2}(3\sin^2\frac{\phi'}{2} - 1) - \frac{3}{2}\sin\frac{\phi}{2}\sin 2\phi'] \begin{pmatrix} \cos\frac{\phi}{4} \\ \sin\frac{\phi}{4} \end{pmatrix} \Bigg] d\rho' d\phi', \end{aligned} \tag{4.6.21}$$

where $\gamma = \frac{m\mu\mu_B}{\hbar^2\sqrt{2\pi\kappa}}$.It is necessary to note that the two component spinor $\vec{f'}(\phi)$ relates to the $x' - z'$ coordinate system that is denoted by the sign $(')$ now it is convenient first to carry out integration over ϕ' in (4.6.21) with the help of the known presentation of the Bessel function:

$$\int_0^{2\pi} e^{-i\upsilon\cos\phi'} d\phi' = 2\pi J_0(\upsilon),$$

$$\int_0^{2\pi} \cos^2\phi' e^{-i\upsilon\cos\phi'} d\phi' = -\frac{d^2}{d\upsilon^2}\int_0^{2\pi} e^{i\upsilon\cos\phi'} d\phi'$$

$$= -2\pi\frac{d^2}{d\upsilon^2} J_0(\upsilon) = 2\pi\frac{J_1(\upsilon)}{\upsilon} - 2\pi J_2(\upsilon)$$

This is obtained with the help of recurrence relation $\upsilon J_n'(\upsilon) = nJ_n(\upsilon) - \upsilon J_{n+1}(\upsilon)$. Again we another important Bessel identities,

$$J_{n+1}(\upsilon) = \tfrac{n}{\upsilon}J_n(\upsilon) - J_n'(\upsilon)$$
$$J_{n-1}(\upsilon) = \tfrac{n}{\upsilon}J_n(\upsilon) + J_n'(\upsilon)$$

From the first equation $J_n'(\upsilon) = \frac{n}{\upsilon}J_n(\upsilon) - J_{n+1}(\upsilon)$ and substituting this in to the second, we have $J_{n-1}(\upsilon) = 2\frac{n}{\upsilon}J_n(\upsilon) - J_{n+1}(\upsilon)$,letting n=1,we have,

$$J_1(\upsilon) = \frac{\upsilon}{2}J_0(\upsilon) + \frac{\upsilon}{2}J_2(\upsilon)$$

Here $J_0(\upsilon), J_1(\upsilon)$ and $J_2(\upsilon)$ are the Bessel functions of zeroth,and second order respectively $(\upsilon = qp')$.we also need one more integral,

$$\int_0^{2\pi} (\sin\phi'\cos\phi') e^{-i\upsilon\cos\phi'} d\phi' = 0$$

Employing the obtained result in (4.6.21) we find,

$$\vec{f}(\phi)=\frac{6\gamma\pi}{a}\left[-I_1\sin\frac{\phi}{2}\begin{pmatrix}-\sin\frac{\phi}{4}\\ \cos\frac{\phi}{4}\end{pmatrix}+I_2\cos\frac{\phi}{4}\begin{pmatrix}\cos\frac{\phi}{4}\\ \sin\frac{\phi}{4}\end{pmatrix}\right] \tag{4.6.22}$$

where,

$$I_1=\frac{qa}{6}\int_{qa}^{\infty}\frac{1}{\upsilon^2}\left[J_o(\upsilon)-3J_2(\upsilon)\right]d\upsilon \tag{4.6.23}$$

$$I_2=\frac{qa}{6}\int_{qa}^{\infty}\frac{1}{\upsilon^2}\left[J_o(\upsilon)+3J_2(\upsilon)\right]d\upsilon \tag{4.6.24}$$

The dimensionless factor qa in I_1 and I_2 compensates divergence of these integrals at $qa\to 0$. To obtain the spin dependent scattering amplitudes in the original coordinate system,we have to rotate the spinors in (4.6.22) by -$\frac{\phi}{2}$

$$\vec{f}(\phi)=\frac{6\gamma\pi}{a}\left[-I_1\sin\frac{\phi}{2}\hat{U}_y(-\frac{\phi}{4})\begin{pmatrix}-\sin\frac{\phi}{4}\\ \cos\frac{\phi}{4}\end{pmatrix}+I_2\cos\frac{\phi}{4}\hat{U}_y(-\frac{\phi}{4})\begin{pmatrix}\cos\frac{\phi}{4}\\ \sin\frac{\phi}{4}\end{pmatrix}\right] \tag{4.6.25}$$

with account(4.6.19),this gives,

$$\begin{vmatrix}f^{\shortparallel}_{\uparrow\uparrow}(\phi)\\ f^{\shortparallel}_{\uparrow\downarrow}(\phi)\end{vmatrix}=\frac{6\gamma\pi}{a}\left[I_1\sin^2\frac{\phi}{2}+I_2\cos^2\frac{\phi}{2}\right]\begin{pmatrix}1\\ 0\end{pmatrix}+[\frac{1}{2}\left[I_2-I_1\right]\sin\phi]\begin{pmatrix}0\\ 1\end{pmatrix} \tag{4.6.26}$$

Here we introduce the scattering amplitude with no spin flipping $f^{\shortparallel}_{\uparrow\uparrow}(\phi)$ and scattering amplitude with spin flipping $f^{\shortparallel}_{\downarrow\uparrow}(\phi)$.

Now,we present the scattering amplitude as,

$$f^{\shortparallel}_{\uparrow\uparrow}(\phi)=-\sqrt{A_0}\frac{I_1\sin^2\frac{\phi}{2}+I_2\cos^2\frac{\phi}{2}}{\sqrt{\kappa a}} \tag{4.6.27}$$

$$f^{\shortparallel}_{\downarrow\uparrow}(\phi)=-\sqrt{A_0}\frac{\left[I_2-I_1\right]\sin\phi}{2\sqrt{\kappa a}} \tag{4.6.28}$$

where $A_o=18a(\frac{m\mu\mu_B}{a\hbar^2})^2$ is the typical length of our problem. The dimensionless combination in brackets can be presented as the ratio of $\frac{\mu\mu_B}{a^3}$ and $\frac{\hbar^2}{ma^2}$ energy of the dipole-dipole interaction of the magnetic moments of electron and nanoparticle and the energy of electron localized on the typical length a.If this ratio is small the accepted Born approximation is applicable for all velocities of the incident particles. The scattering of particle with arbitrary energy the relation ,$|V|\ll\frac{\hbar^2}{m\rho'^2}$ holds true,where ρ' is the range of action of potential.

For calculation we set $|V| \approx \frac{\mu\mu_B}{\rho'^3}$,and for fast dipole-dipole interaction $\rho' = 2a$[14].Using all these we can presented as, the following inequality

$$\frac{2\pi\nu\mu_B^2 n_a m a^2}{3\hbar^2} \ll 1 \tag{4.6.29}$$

where for an evaluation of the magnetic moment of nanoparticle we used the formula $\mu = \frac{4\pi}{3}\nu\mu_B n_a m a^3$, ν is a number of Bohr magnetons carried by the ferromagnetic atom ,and n_a is the density of atoms of the nanomagnets.For the fast particles $\kappa a \gg 1$ the application of the Born approximation gets wider because the right hand side of inequality multiplied by κa.Due to the relative weakness of the spin nanomagnet interaction ,inequality can be satisfied only for nanoparticles of the following size $a \ll \sqrt{\frac{1}{\nu}}100nm$.This condition is obtained by substituting the numerical values of the Parameter entering into with $n_a = 10^{22}cm^{-3}$

Now,we consider the scattering of electrons moving along the z- axis with the spins aligned opposite to the magnetic moment $\vec{\mu}$.The spin wave function $\vec{\chi}_{\downarrow o}(s) = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$ in the coordinate system with the z-axis parallel to $\vec{\mu}$ in the coordinate system $x' - z'$ is,

$$\vec{\chi}_{\downarrow}(s) = \hat{U}_y \vec{\chi}_{\downarrow o}(s) = \begin{pmatrix} \cos\frac{\phi}{4} & \sin\frac{\phi}{4} \\ -\sin\frac{\phi}{4} & \cos\frac{\phi}{4} \end{pmatrix} \begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{pmatrix} \sin\frac{\phi}{4} \\ \cos\frac{\phi}{4} \end{pmatrix}$$

Substituting this spinor and interaction potential (4.6.17) in to (4.6.15) and following the same procedure as above ,we will find $f^{''}_{\downarrow\downarrow}$ and $f^{''}_{\downarrow\uparrow}$.

$$\begin{aligned} \vec{f'}(\phi) = \quad & \gamma \int_0^{2\pi}\int_a^{\infty} \frac{e^{-ip'q\cos\phi'}}{\rho'} \Bigg[[-\sin\frac{\phi}{2}(3\cos^2\frac{\phi'}{2} - 1) + \frac{3}{2}\cos\frac{\phi}{2}\sin 2\phi'] \begin{pmatrix} \cos\frac{\phi}{4} \\ \sin\frac{\phi}{4} \end{pmatrix} + \\ & [\cos\frac{\phi}{2}(3\sin^2)\frac{\phi'}{2} - 1) - \frac{3}{2}\sin\frac{\phi}{2}\sin 2\phi'] \begin{pmatrix} \sin\frac{\phi}{4} \\ -\cos\frac{\phi}{4} \end{pmatrix} \Bigg] d\rho' d\phi' \end{aligned} \tag{4.6.30}$$

where $\gamma = \frac{m\mu\mu_B}{\hbar^2\sqrt{2\pi\kappa}}$.Letting $\rho' q = \upsilon$ and integrate with respect to ϕ' in $x' - z'$ coordinate system.it becomes,

$$\vec{f}(\phi) = \frac{6\gamma\pi}{a}\left[-I_1 \sin\frac{\phi}{2} \begin{pmatrix} \cos\frac{\phi}{4} \\ \sin\frac{\phi}{4} \end{pmatrix} + I_2 \cos\frac{\phi}{4} \begin{pmatrix} \sin\frac{\phi}{4} \\ -\cos\frac{\phi}{4} \end{pmatrix} \right] \tag{4.6.31}$$

Again to find the spinor in the original coordinate we rotate $\vec{f'}(\phi)$ by $-\frac{\phi}{2}$.

$$\vec{f}(\phi) = \frac{6\gamma\pi}{a}\left[-I_1 \sin\frac{\phi}{2}\hat{U}_y(-\frac{\phi}{4})\begin{pmatrix}\cos\frac{\phi}{4}\\ \sin\frac{\phi}{4}\end{pmatrix} + I_2\cos\frac{\phi}{4}\hat{U}_y(-\frac{\phi}{4})\begin{pmatrix}\sin\frac{\phi}{4}\\ -\cos\frac{\phi}{4}\end{pmatrix}\right] \tag{4.6.32}$$

Using $\hat{U}_y(-\frac{\phi}{4}) = \begin{pmatrix}\cos\frac{\phi}{4} & -\sin\frac{\phi}{4}\\ \sin\frac{\phi}{4} & \cos\frac{\phi}{4}\end{pmatrix}$,we have,

$$\vec{f}(\phi) = \frac{6\gamma\pi}{a}\left[-I_1 \sin\frac{\phi}{2}\begin{pmatrix}\cos\frac{\phi}{2}\\ \sin\frac{\phi}{2}\end{pmatrix} + I_2\cos\frac{\phi}{4}\begin{pmatrix}\sin\frac{\phi}{2}\\ -\cos\frac{\phi}{2}\end{pmatrix}\right] \tag{4.6.33}$$

$$\begin{vmatrix}f^{\parallel}_{\downarrow\downarrow}(\phi)\\ f^{\parallel}_{\downarrow\uparrow}(\phi)\end{vmatrix} = \frac{6\gamma\pi}{a}\left[-I_1\sin^2\frac{\phi}{2} - I_2\cos^2\frac{\phi}{2}\right]\begin{pmatrix}1\\ 0\end{pmatrix} + [\frac{1}{2}\left[I_2 - I_1\right]\sin\phi]\begin{pmatrix}0\\ 1\end{pmatrix} \tag{4.6.34}$$

This implies that,

$$f^{\parallel}_{\downarrow\downarrow}(\phi) = \sqrt{A_0}\frac{I_1\sin^2\frac{\phi}{2} + I_2\cos^2\frac{\phi}{2}}{\sqrt{\kappa a}} \tag{4.6.35}$$

$$f^{\parallel}_{\downarrow\uparrow}(\phi) = -\sqrt{A_0}\frac{[I_2 - I_1]\sin\phi}{2\sqrt{\kappa a}} \tag{4.6.36}$$

Generally from this part, we find the following relations.

$$f^{\parallel}_{\downarrow\uparrow}(\kappa a, \phi) = f^{\parallel}_{\uparrow\downarrow}(\kappa a, \phi) = -\sqrt{A_0}\frac{[I_2 - I_1]\sin\phi}{2\sqrt{\kappa a}} \tag{4.6.37}$$

$$f^{\parallel}_{\downarrow\downarrow}(\kappa a, \phi) = -f^{\parallel}_{\uparrow\uparrow}(\kappa a, \phi) = \sqrt{A_0}\frac{I_1\sin^2\frac{\phi}{2} + I_2\cos^2\frac{\phi}{2}}{\sqrt{\kappa a}} \tag{4.6.38}$$

Here the scattering amplitude $f^{\parallel}_{\downarrow\uparrow}(\phi)$ and $f^{\parallel}_{\downarrow\downarrow}(\phi)$ are the scattering amplitudes with spin flipping and no spin flipping after scattering respectively. This means that the probability of spin flipping which is proportional to square of the scattering amplitudes for the down state is same as for the up state case.

B.Magnetic moment of nanoparticles $\perp$ to velocity of incident electron.

Let us consider the case when the magnetic moment $\vec{\mu}$ is transverse to the incident wave.We keep the previous direction for incident wave along z- axis. But now $\vec{\mu}$ is along positive x-axis. And like the section above we expected to show four situations.These are:

- $f^{\perp}_{\uparrow\uparrow}(\phi)$ (the scattering amplitude when direction of spin electron and magnetic moment are initially up state).

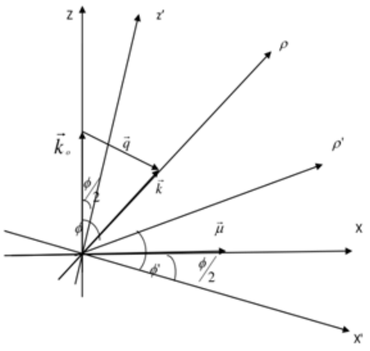

Figure 4.4: Coordinate system when magnetic moment and incident wave vector are prpendicularl and magnetic moment is along the positive x-axis and $\vec{q} \parallel x'$.

- $f^{\perp}_{\downarrow\downarrow}(\phi)$ (the scattering amplitude when direction of spin electron and magnetic moment are initially down state).
- $f^{\perp}_{\uparrow\downarrow}(\phi)$ (the scattering amplitude when direction of spin electron is initially up and magnetic moment is initially down state).
- $f^{\perp}_{\downarrow\uparrow}(\phi)$ (the scattering amplitude when direction of spin electron is initially down and magnetic moment is initially up state).

The interaction potential (4.6.17) preserves its form.Acting as above and taking in to account that the components of the magnetic moment of the nanoparticle $\mu_{x'} = \mu \cos\frac{\phi}{2}$ and $\mu_{z'} = \sin\frac{\phi}{2}$.We obtain,

$$f(\phi) = \frac{-m}{\hbar^2\sqrt{2\pi\kappa}} \int e^{-i\vec{q}.\vec{\rho'}} \hat{V}(\vec{\rho'}) d^2\rho' \vec{\chi}(s) \tag{4.6.39}$$

where $\hat{V} = \frac{\mu_B}{\rho'^3}\{[\mu_{x'}(3\cos^2\phi' - 1) + \frac{3}{2}\mu_{z'}\sin 2\phi']\hat{\sigma_{x'}} + [\mu_{z'}(3\sin^2\phi' - 1) + \frac{3}{2}\mu_{x'}\sin 2\phi']\hat{\sigma_{z'}}\}$

$$\vec{f}'(\phi) = \gamma \int_0^{2\pi} \int_a^{\infty} \frac{e^{-ip'q\cos\phi'}}{\rho'} \{[\cos\frac{\phi}{2}(3\cos^2\frac{\phi'}{2} - 1) + \frac{3}{2}\sin\frac{\phi}{2}\sin 2\phi'] \begin{pmatrix} -\sin\frac{\phi}{4} \\ \cos\frac{\phi}{4} \end{pmatrix}$$

$$+[\sin\frac{\phi}{2}(3\sin^2\phi - 1) - \frac{3}{2}\cos\frac{\phi}{2}\sin 2\phi'] \begin{pmatrix} \cos\frac{\phi}{4} \\ \sin\frac{\phi}{4} \end{pmatrix}\} d\rho' d\phi' \tag{4.6.40}$$

$$\vec{f}(\phi) = \frac{6\gamma\pi}{a}\left[I_1\cos\frac{\phi}{2}\begin{pmatrix}-\sin\frac{\phi}{4}\\ \cos\frac{\phi}{4}\end{pmatrix} + I_2\sin\frac{\phi}{2}\begin{pmatrix}-\cos\frac{\phi}{4}\\ \sin\frac{\phi}{4}\end{pmatrix}\right] \tag{4.6.41}$$

The scattering amplitude $f^{\perp}_{\uparrow\uparrow}(\phi)$ and $f^{\perp}_{\uparrow\downarrow}(\phi)$ describe scattering of the electron by the transverse magnetic moment with no spin change and spin change respectively.The spinors in (4.6.41) are related to the $x' - z'$ coordinate system.To observe the effect of spin dependent electron scattering amplitudes ,we have to rotate the spinors in (4.6.41) back to the original coordinate system (x-z) by $\frac{-\phi}{2}$ along the y-axis. Doing this rotation and following the same procedure that we have done for case of magnetic moment parallel to velocity of electron ,the scattering amplitudes of the electron whose spin is transverse to the magnetic moment ($\vec{\mu}$), we obtain,

$$\begin{vmatrix}f^{\perp}_{\uparrow\uparrow}(\phi)\\ f^{\perp}_{\uparrow\downarrow}(\phi)\end{vmatrix} = \frac{6\gamma\pi}{a}\left[I_1\cos\frac{\phi}{2}\begin{pmatrix}-\sin\frac{\phi}{2}\\ \cos\frac{\phi}{2}\end{pmatrix}\begin{pmatrix}0\\ 1\end{pmatrix} + I_2\sin\frac{\phi}{2}\begin{pmatrix}\cos\frac{\phi}{2}\\ \sin\frac{\phi}{2}\end{pmatrix}\begin{pmatrix}1\\ 0\end{pmatrix}\right] \tag{4.6.42}$$

Now this implies,

$$f^{\perp}_{\uparrow\uparrow}(\phi) = -\sqrt{A_0}\frac{[I_2 - I_1]\sin\phi}{2\sqrt{\kappa a}}, \tag{4.6.43}$$

$$f^{\perp}_{\uparrow\downarrow}(\phi) = -\sqrt{A_0}\frac{I_1\cos^2\frac{\phi}{2} + I_2\sin^2\frac{\phi}{2}}{\sqrt{\kappa a}}, \tag{4.6.44}$$

$$f^{\perp}_{\uparrow\uparrow}(ka,\phi) = f^{\parallel}_{\downarrow\uparrow}(ka,\phi). \tag{4.6.45}$$

For the electron with spin anti parallel to direction of propagation ($\vec{\chi}_{0\downarrow}(s) = \begin{vmatrix}0\\ 1\end{vmatrix}$ for down state) before scattering and perpendicular to the magnetic moment $\vec{\mu}$. Inserting the spinor and interaction potential to $f(\phi) = \frac{-m}{\hbar^2\sqrt{2\pi\kappa}}\int e^{-i\vec{q}.\vec{\rho'}}\hat{V}(\vec{\rho'})d^2\rho'\vec{\chi}(s)$,we solve $f^{\perp}_{\downarrow\downarrow}$ and $f^{\perp}_{\downarrow\uparrow}$.

$$\vec{f}(\phi) = \frac{6\gamma\pi}{a}\left[I_1\cos\frac{\phi}{2}\begin{pmatrix}\cos\frac{\phi}{4}\\ \sin\frac{\phi}{4}\end{pmatrix} + I_2\sin\frac{\phi}{2}\begin{pmatrix}\sin\frac{\phi}{4}\\ -\cos\frac{\phi}{4}\end{pmatrix}\right] \tag{4.6.46}$$

Doing rotation and following the same procedure that we have done and to obtain,

$$f^{\perp}_{\downarrow\downarrow}(\phi) = -\sqrt{A_0}\frac{[I_2 - I_1]\sin\phi}{2\sqrt{\kappa a}} \tag{4.6.47}$$

$$f^{\perp}_{\downarrow\uparrow}(\phi) = -\sqrt{A_0}\frac{I_1\cos^2\frac{\phi}{2} + I_2\sin^2\frac{\phi}{2}}{\sqrt{\kappa a}} \tag{4.6.48}$$

Generally we find the following from this part.

$$f^{\perp}_{\downarrow\downarrow}(ka,\phi) = -f^{\perp}_{\uparrow\uparrow}(ka,\phi) \tag{4.6.49}$$

$$f^{\perp}_{\downarrow\uparrow}(ka,\phi) = -f^{\perp}_{\uparrow\downarrow}(ka,\phi) \tag{4.6.50}$$

In 2D spin independent scattering problem,the measurable quantities are the differential length $dL = |f(\phi)|^2 d\phi$ (function of energy of incident particle and the scattering angle ϕ) and the total scattering length $L = \int |f(\phi)|^2 d\phi$. In 2D spin dependent scattering problem , the scattering amplitudes are two component spinors,which describe the process with different orientation of spins after scattering in general case depend on a mutual orientation of spin of scattering electrons and magnetic moment of nanoparticle.In this thesis,we consider only two spin orientation $\parallel$ and $\perp$to $\vec{\mu}$.In the next section,we analyze the scattering lengths of the considered processes and discuss their peculiarities for typical cases of 2D spin dependent electron scattering by nanomagnets.

4.6.4 Numerical analysis and graphical solutions.

For more understandable form let us square equations in (4.6.37),(4.6.38),(4.6.49) and (4.6.50) and collecting like equations to obtain in the following form,

$$L_1 = |f^{\parallel}_{\uparrow\uparrow}(ka,\phi)|^2 = |f^{\parallel}_{\downarrow\downarrow}(ka,\phi)|^2 = \left(\sqrt{A_0}\frac{I_1\sin^2\frac{\phi}{2} + I_2\cos^2\frac{\phi}{2}}{\sqrt{\kappa a}}\right)^2, \tag{4.6.51}$$

$$L_2 = |f^{\perp}_{\uparrow\downarrow}(ka,\phi)|^2 = |f^{\perp}_{\downarrow\uparrow}(ka,\phi)|^2 = \left(-\sqrt{A_0}\frac{I_1\cos^2\frac{\phi}{2} + I_2\sin^2\frac{\phi}{2}}{\sqrt{\kappa a}}\right)^2, \tag{4.6.52}$$

$$\begin{aligned} L_3 = \quad & |f^{\parallel}_{\uparrow\downarrow}(ka,\phi)|^2 = |f^{\parallel}_{\downarrow\uparrow}(ka,\phi)|^2 = |f^{\perp}_{\uparrow\uparrow}(ka,\phi)|^2 = |f^{\perp}_{\downarrow\downarrow}(ka,\phi)|^2 \\ & = \left(-\sqrt{A_0}\frac{[I_2 - I_1]\sin\phi}{2\sqrt{\kappa a}}\right)^2. \end{aligned} \tag{4.6.53}$$

As in three dimensional scattering theory we can made a general conclusions concerning the scattering amplitudes in the limit of slow particles ($ka \ll 1$) and fast particles ($ka \gg 1$). In the first case, $I_1 = I_2 = \frac{1}{6}$. With this in mind we get,

$$|f^{\parallel}_{\uparrow\uparrow}(ka,\phi)|^2 = |f^{\parallel}_{\downarrow\downarrow}(ka,\phi)|^2 = |f^{\perp}_{\uparrow\downarrow}(ka,\phi)|^2 = |f^{\perp}_{\downarrow\uparrow}(ka,\phi)|^2 = \frac{A_0}{36ka} \tag{4.6.54}$$

$$|f^{\parallel}_{\uparrow\downarrow}(ka,\phi)|^2 = |f^{\parallel}_{\downarrow\uparrow}(ka,\phi)|^2 = |f^{\perp}_{\uparrow\uparrow}(ka,\phi)|^2 = |f^{\perp}_{\downarrow\downarrow}(ka,\phi)|^2 = 0 \tag{4.6.55}$$

The total scattering lengths of slow particles for the process denoted by (4.6.54) is obtained by integration over all ϕ and equals,

$$A(ka) = \frac{\pi A_0}{18ka}. \tag{4.6.56}$$

From (4.6.54) we see that spin dependent electron scattering is isotropic for slow particles ($ka \ll 1$). Relation (4.6.55) shows the probability of scattering with spin flipping by the magnetic moment parallel to the velocity of the electron and the scattering with no spin flipping by the transversal magnetic moment are close to zero.

For the fast particles ($ka \gg 1$),the the exponent in (4.6.15) is the fast oscillating and the result of integration is nonzero only for small scattering angles ($2ka\sin\frac{\phi}{2} \sim 1$ or $\sim \frac{1}{ka} \ll 1$). The scattering length is vary in this case because it suppressed by a large factor $ka \gg 1$.These conclusions are in agreement with general scattering theory and our numerical calculations given below . Equalities (4.6.51),(4.6.52) and (4.6.53) allow us to introduce three normalized dimensionless scattering lengths.

$$A_1 = \frac{1}{A_0}\frac{dL_1}{d\phi} = \frac{1}{ka}[I_1\sin^2\frac{\phi}{2} + I_2\cos^2\frac{\phi}{2}]^2, \tag{4.6.57}$$

$$A_2 = \frac{1}{L_0}\frac{dL_2}{d\phi} = \frac{1}{ka}[I_1\cos^2\frac{\phi}{2} + I_2\sin^2\frac{\phi}{2}]^2, \tag{4.6.58}$$

$$A_3 = \frac{1}{L_0}\frac{dL_3}{d\phi} = \frac{1}{4ka}[I_2 - I_1]^2\sin^2\phi \tag{4.6.59}$$

The analytical expressions of I_1 and I_2 are solved by using the" Mathematica 5.2" and are given below.

$$\begin{array}{l} I_1 = \frac{qa}{6}(-\frac{1}{\upsilon}H_1 - \frac{3}{8}\upsilon H_2)|_{qa}^{\infty} \\ I_2 = \frac{qa}{6}(-\frac{1}{\upsilon}H_1 + \frac{3}{8}\upsilon H_2)|_{qa}^{\infty} \end{array} \tag{4.6.60}$$

where $H_1 = HPFQ[\{-\frac{1}{2}\},\{\frac{1}{2},2\},-\frac{\upsilon^2}{4}]$ and $H_2 = HPFQ[\{-\frac{1}{2}\},\{\frac{3}{2},3\},-\frac{\upsilon^2}{4}]$ are hypergeometric functions and HPFQ stands for generalized hypergeometric function,$\upsilon = \rho' q$ and $qa = 2ka\sin\frac{\phi}{2}$. At $qa \to \infty$ we can not find the reliable expression for the above integrals.As result $I_1(qa \geq 10^3) = \frac{-2}{3}ka\sin\frac{\phi}{2}$ and $I_2(qa \geq 10^3 = 0)$. Using these approximation,

$$
\begin{aligned}
I_1 &= \tfrac{-2}{3}ka\sin\tfrac{\phi}{2} - \tfrac{2ka\sin\frac{\phi}{2}}{6}\left(-\tfrac{1}{2ka\sin\frac{\phi}{2}}H_1 - \tfrac{3}{8}(2ka\sin\tfrac{\phi}{2})H_2\right) \\
I_2 &= -\tfrac{2ka\sin\frac{\phi}{2}}{6}\left(-\tfrac{1}{2ka\sin\frac{\phi}{2}}H_1 + \tfrac{3}{8}(2ka\sin\tfrac{\phi}{2})H_2\right)
\end{aligned}
$$

when we inserting all the values I_1, I_2, H_1 and H_2 the final equations are presented below.

$$
\begin{aligned}
A_1 \quad &= \frac{1}{36ka}\Bigg[-2ka\sin\frac{\phi}{2} + 2ka\sin\frac{\phi}{2}\cos\phi + \\
&HypergeometricPFQ[\{-\frac{1}{2}\},\{\frac{1}{2},1\},-(ka)^2\sin^2\frac{\phi}{2}] - \\
&\frac{3}{2}(ka)^2\sin^2\frac{\phi}{2}\cos\phi HypergeometricPFQ[\{\frac{1}{2}\},\{\frac{3}{2},3\},-(ka)^2\sin^2\frac{\phi}{2}]\Bigg]^2
\end{aligned}
\tag{4.6.61}
$$

$$
\begin{aligned}
A_2 \quad &= \frac{1}{36ka}\Bigg[[-2ka\sin\frac{\phi}{2} - 2ka\sin\frac{\phi}{2}\cos\phi + \\
&HypergeometricPFQ[\{-\frac{1}{2}\},\{\frac{1}{2},1\},-(ka)^2\sin^2\frac{\phi}{2}] + \\
&\frac{3}{2}(ka)^2\sin^2\frac{\phi}{2}\cos\phi HypergeometricPFQ[\{\frac{1}{2}\},\{\frac{3}{2},3\},-(ka)^2\sin^2\frac{\phi}{2}]\Bigg]^2
\end{aligned}
\tag{4.6.62}
$$

$$
A_3 = ka\left(\sin\frac{\phi}{2}\sin\phi(\frac{1}{3} - \frac{ka}{4}\sin\frac{\phi}{2}HypergeometricPFQ[\{\frac{1}{2}\},\{\frac{3}{2},3\},-(ka)^2\sin^2\frac{\phi}{2}])\right)^2
\tag{4.6.63}
$$

The dimensionless scattering lengths A_1 and A_2 relate to the scattering with spin no flipping of electrons with initial velocities parallel and perpendicular to the magnetic moment of nanoparticle respectively. The quantity A_3 relates to the scattering with spin flipping in both cases. Formulas (4.6.57),(4.6.58) and(4.6.59) show that all these functions are periodical with a periodical of 2π.This means that small angles in the vicinity of $\phi = 0$ and $\phi = 2\pi$ corresponds to the forward scattering.The small angles in in the vicinity of $\phi = \pi$ corresponds to the backward scattering. Below in figure (4.5),(4.6),(4.7) we present $3D$ graphs of A_1, A_2 and A_3 as function of ka and ϕ built with the help of formulas in equation (4.6.61),(4.6.62) and (4.6.63) in the range of ka where spin dependent scattering is anisotropic.

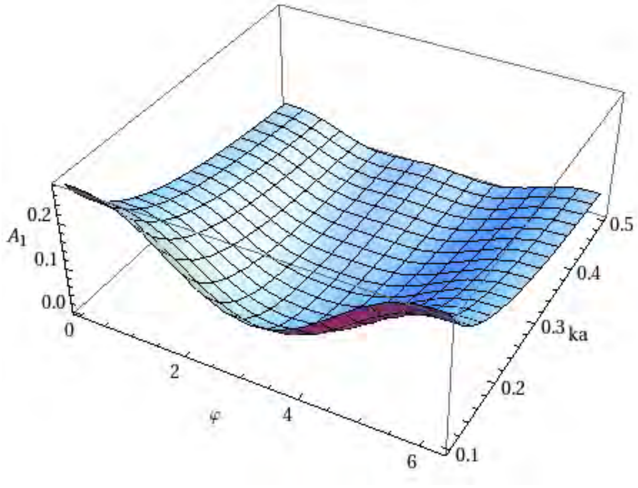

Figure 4.5: The dimensionless differential scattering length A_1 versus ka andϕ(in rad).

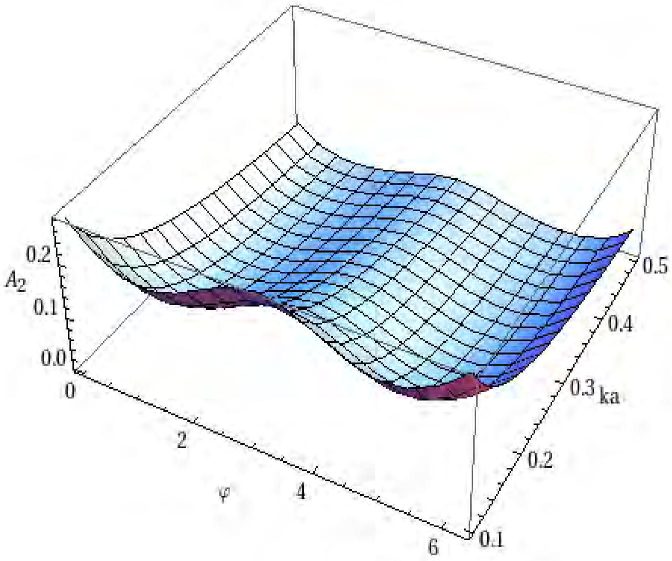

Figure 4.6: The dimensionless differential scattering length A_2 versus ka and ϕ (in rad).

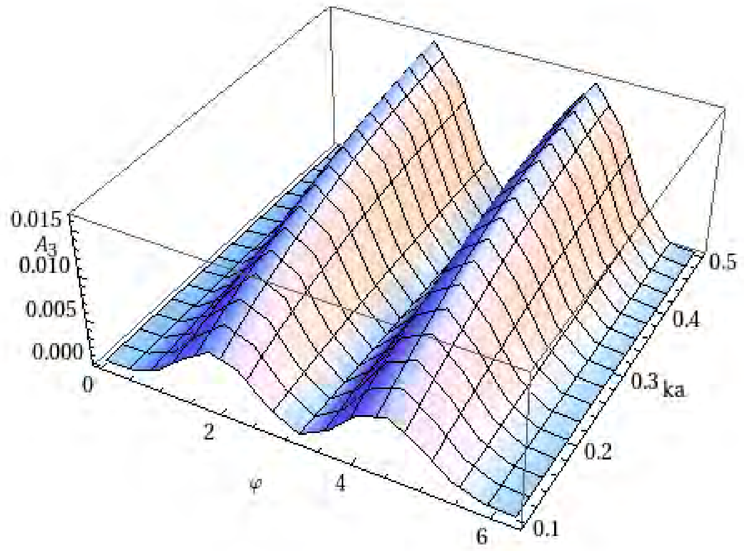

Figure 4.7: The dimensionless differential scattering length A_3 versus ka and ϕ(in rad).

Graphical solutions of A_1, A_2 and A_3 for different values of ka.

Now we have, the graph of differential scattering lengths as function of ϕ and $ka = 0.1, 0.20.5$ and 1.At ka=0.2 the wave length of incident electron $\lambda = 10\pi a$. From figure (4.5)and (4.6) one can see that the scattering with no spin flipping dominate for small scattering angles. Figure (4.7) depicts $A_3(\phi)$ and relates to the scattering with spin flipping. It shows that the position maxima of graph(4.6) corresponds to the position of minima of figure(4.7).The maxima of A_3 located near $\phi = \frac{\phi}{2}$ and $\phi = \frac{3\phi}{2}$. For example the scattered electrons with flipped spin initially moving along $\vec{\mu}$ are concentrated in the direction transversal to their initial velocity. The same is true for the spin flipped scattered electrons initially perpendicular to $\vec{\mu}$. This feature of 2D spin dependent electron scattering by the nanomagnet allows to preform experiments on separation of electrons with different orientation of spins. In particular, launching a polarized beam of electrons along the magnetic moment of a nanoparticle, it is possible to obtain scattered electrons with opposite oriented spin in the direction transversal to their velocity. It would be interesting to compare the parameter A_0,which specifies the scattering length with the typical size a of the nanomagnet . The quantity A_0 can be presented in the form

$$\frac{A_0}{a} \approx 10^3 \nu^2 (\frac{m\mu_B^2}{\hbar^2} n_a a^2)^2 \tag{4.6.64}$$

The dimensionless combination $\frac{A_0}{a}$ basically determine the th scale of spin dependent scattering. It would be describe that it be not very small comparing to unit.Taking $\nu = 10$ that is in agreement with [14], we obtain $\frac{A_0}{a} = 0.1$ for $a = 10nm$ and $\frac{A_0}{a} = 1.6$ for a=20nm.

1.Graphical solutions of A_1 for $ka = 0.197, 0.2, 0.5$ and 1.

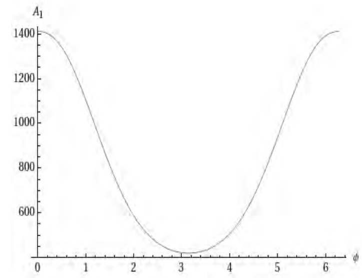

Figure 4.8: The dimensionless differential scattering length A_1 versus ϕ (in rad) and ka=0.197.

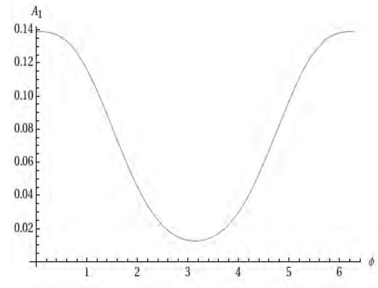

Figure 4.9: The dimensionless differential scattering length A_1 versus ϕ(in rad) and ka=0.2.

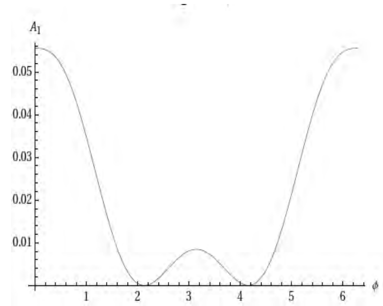

Figure 4.10: The dimensionless differential scattering length A_1 versus ϕ (in rad) and ka=0.5.

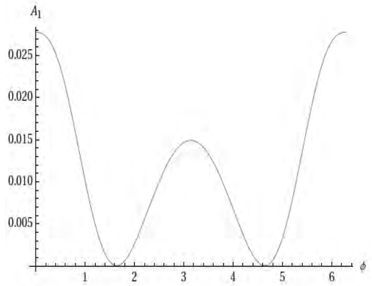

Figure 4.11: The dimensionless differential scattering length A_1 versus ϕ (in rad.) and ka=1.

When we merge these above four graphs on a one plane it look like the following.

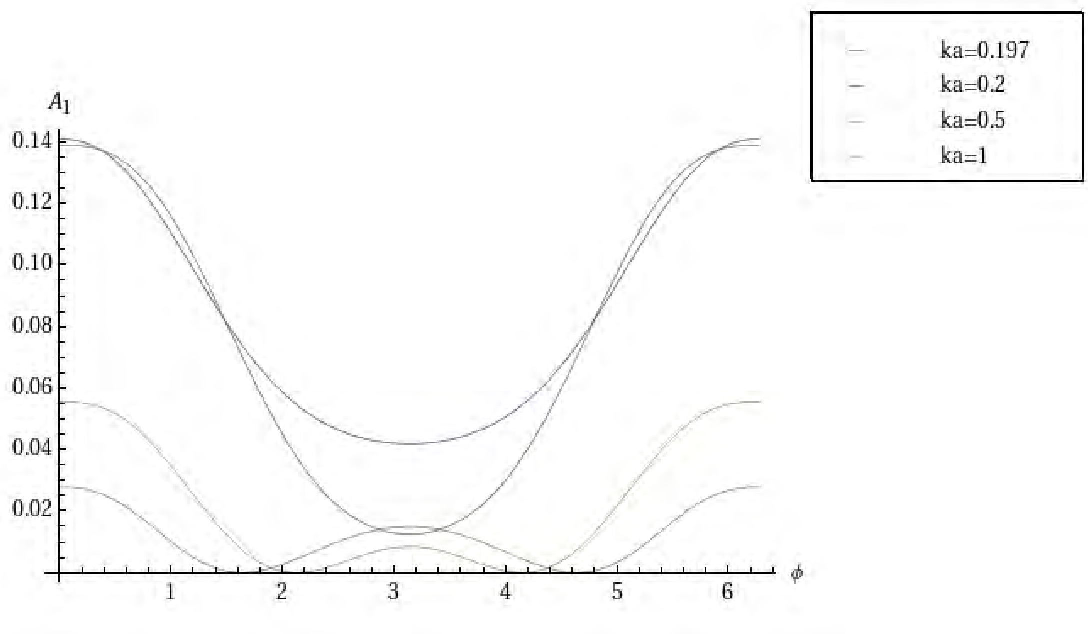

Figure 4.12: The dimensionless differential scattering length A_1 versus ϕ (in rad) and ka=0.197,0.2,0.5,and 1.

2.Graphical solutions of A_2 for $ka = 0.1, 0.2, 0.5$ and 1.

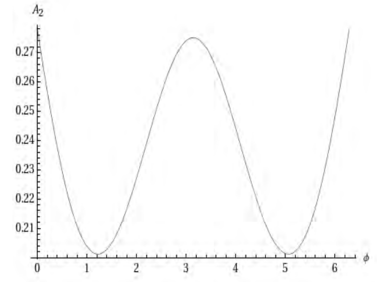

Figure 4.13: The dimensionless differential scattering length A_2 versus ϕ (in rad) and ka=0.1.

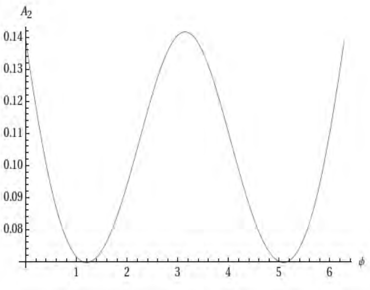

Figure 4.14: The dimensionless differential scattering length A_2 versus ϕ (in rad) and ka=0.2.

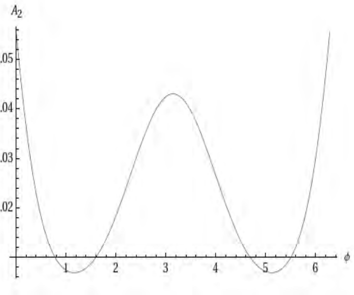

Figure 4.15: The dimensionless differential scattering length A_2 versus ϕ (in rad) and ka=0.5.

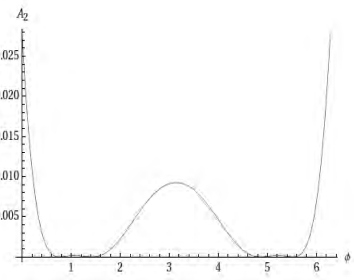

Figure 4.16: The dimensionless differential scattering length A_2 versus ϕ (in rad) and ka=1.

Now when we put this four graphs in a one plane it look like,

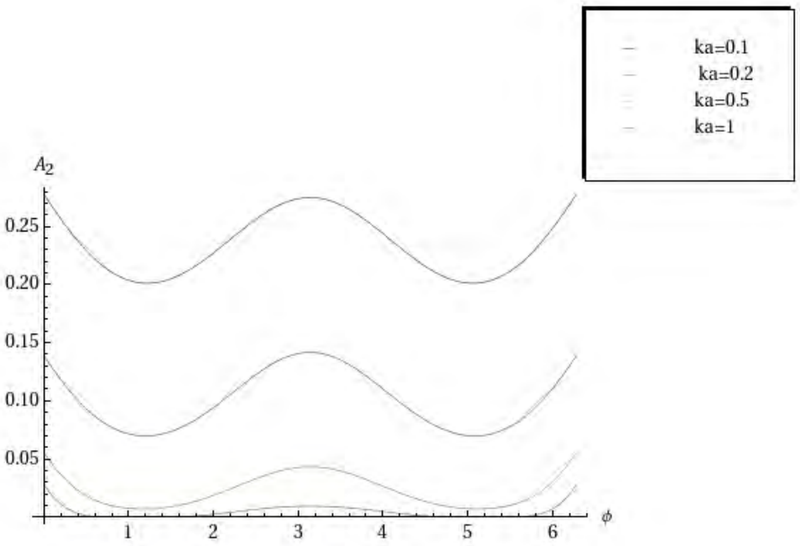

Figure 4.17: The dimensionless differential scattering length A_2 versus ϕ (in rad) and ka=0.1,0.2,0.5,and 1.

3.Graphical solutions of A_3 for $ka = 0.1, 0.2, 0.5$ and 1.

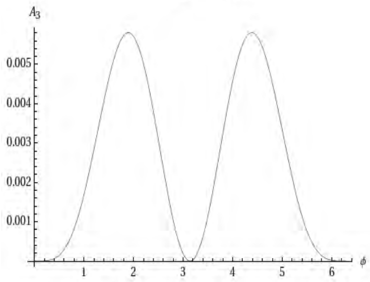

Figure 4.18: The dimensionless differential scattering length A_3 versus ϕ (in rad) and ka=0.1.

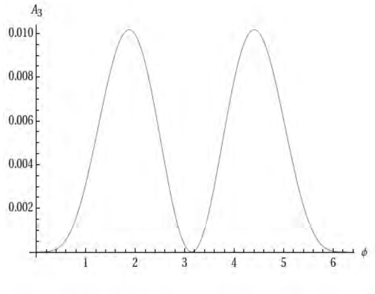

Figure 4.19: The dimensionless differential scattering length A_3 versus ϕ (in rad) and ka=0.2.

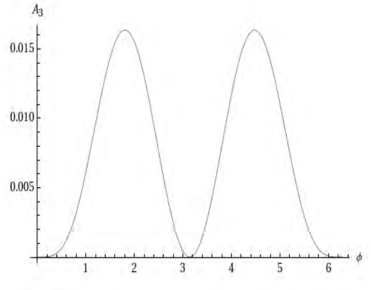

Figure 4.20: The dimensionless differential scattering length A_3 versus ϕ (in rad) and ka=0.5.

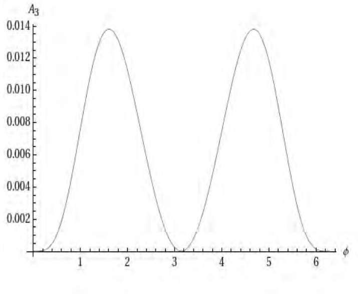

Figure 4.21: The dimensionless differential scattering length A_3 versus ϕ (in rad) and ka=1.

To see and compare the relation of graphs of A_3 for different values of ka=0.1,0.2,0.5 and 1, let us put four of them in a one plane as shown below.

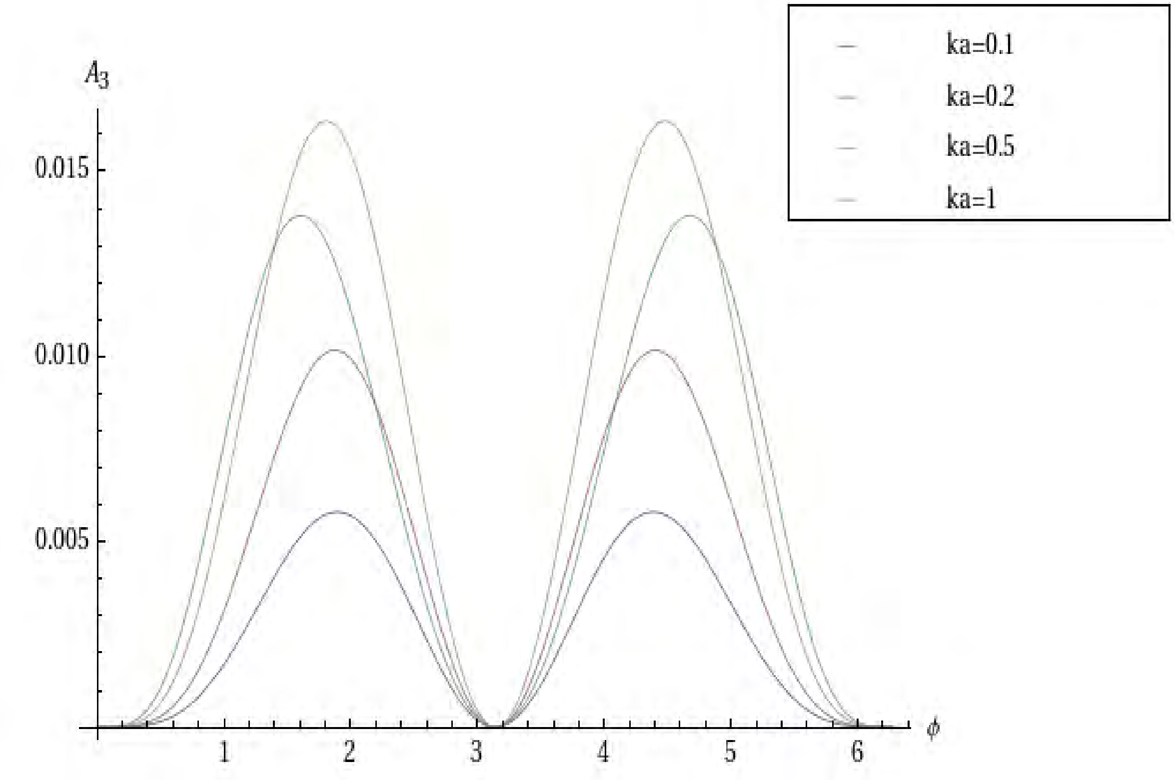

Figure 4.22: The dimensionless differential scattering length A_3 versus ϕ (in rad) and ka=0.1,0.2,0.5,and 1.

Chapter 5

Summary and Conclusion

5.1 Summary

In this thesis,we seen:

- The basic concept of spintronics,
- The spin independent three dimensional scattering theory,
- The spin dependent two dimensional scattering theory.

And we focused more on two dimensional spin dependent scattering by one nanonagnet.

5.2 Conclusion

In this work we have shown;

- Two dimensional spin dependent scattering manifests strong dependence on the scattering angle ϕ and energy of particles ka.
- For applicability of Born approximation the typical size of particle must satisfy the following inequality,
$$a \ll \frac{100nm}{\sqrt{\nu}},$$
where ν is the number of Born magneton per atom of the nanomagnet.For which $\nu \approx 10$,it gives a $\ll$ 30nm

- The effect of the scattering under consideration is comparatively small because of the weakness of the magnetic interaction.
- The enhancements of peculiarities of 2D spin dependent electron scattering can be done by considering a large number of scatterers.
- This type of scattering may be considered as one more way of controlling the polarized spin currents.

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Declaration

This thesis is my original work, has not been presented for a degree in any other University and that all the sources of material used for the thesis have been dully acknowledged.

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Signature:– – – – – – – – – – – –

Place and time of submission: Addis Ababa University, June 2011

This thesis has been submitted for examination with my approval as University advisor.

Name: Prof.V.N.Mal'nev

Signature:– – – – – – – – – – –