

**ADDIS ABABA UNIVERSITY**  
**ADDIS ABABA INSTITUTE OF TECHNOLOGY**  
**AFRICAN RAILWAY CENTRE OF EXCELLENCE**



**ADAPTIVE TERMINAL SLIDING-MODE CONTROL  
FOR SPEED TRACKING OF MULTIPLE HIGH SPEED  
TRAIN**

*A Thesis Submitted to the School of Graduate Studies of Addis Ababa University in Partial  
Fulfillment of the Requirements for the Degree of  
Master of Science in Railway Engineering (Rolling Stock)*

**By**

**Liya Koshana**

**Advisor: Celestin Nkundineza (Ph.D.)**

**April, 2022**

Addis Ababa, Ethiopia

## APPROVAL SHEET

The undersigned member of Addis Ababa Institute of Technology under the department of Railway Engineering have examined the thesis entitled “**ADAPTIVE TERMINAL SLIDING-MODE CONTROL FOR SPEED TRACKING OF MULTIPLE HIGH SPEED TRAIN**” presented by **Liya Koshana**, a candidate for the degree of **Master of science** in **Rolling Stocks Design** and hereby certify that it is worth of acceptance.

### Approved By Board of Examiners

Dr. Celestin Nkundineza

|                            |           |      |
|----------------------------|-----------|------|
| Advisor                    | Signature | Date |
| External Examiner          | Signature | Date |
| Internal Examiner          | Signature | Date |
| Chair Person of Department | Signature | Date |
| Graduate Committee (DGC)   | Signature | Date |

## DECLARATION

I declare that research work titled “**ADAPTIVE TERMINAL SLIDING-MODE CONTROL FOR SPEED TRACKING OF MULTIPLE HIGH SPEED TRAIN**” is my own work. The work has not been presented elsewhere for assessment. Where material has been used from other sources it has been properly acknowledged / referred.

---

Liya Koshana

## ABSTRACT

In today's world, the railway transportation entered into the era of high speed. Which means the concern becomes more of to travel faster and safer. High speed trains are a train with a maximum operating speed of more than 200 km/h. Automatic train operation system is an operational safety enhancement device used to help automate the operation of the high speed trains. This system does its controlling function based on controllers which is embedded on it. But, due to some influencing factors which reduce the performance of the controllers which embedded on ATO like; parameter uncertainties, unknown model parameters, actuator effectiveness loss, coupler effect and external disturbance, until now there is a limitation in controllers to attain finite time convergence of the tracking error to the equilibrium point rather than achieving asymptotic convergence, that means stability within finite time is not guaranteed.

In real time operation of high speed trains (HSTs), stability within finite time interval is desired instead of the asymptotic stability, as it is more physically realizable than concerning infinite time. In this paper, adaptive terminal sliding-mode control scheme was developed for finite time stable speed tracking control for high speed trains with unknown parameters of a model, parameters uncertainties, actuator effectiveness losses, coupler effect and external disturbance. In this paper the train system is modeled as a multi-point mass model and considered the in-train forces between the adjacent cars which are the effect of couplers. To attain stability within in a finite time terminal sliding-mode control is proposed and introduced for multiple high speed train system that drives the dynamical system of a tracking error to the predesigned sliding surface within defined time and thereafter sustain a sliding motion on it. For online estimation of unknown parameters of the model and the actuator fault compensation term adaptive technique is designed and cooperates with a terminal sliding-mode control. Saturation function is used to eliminate the chattering problem caused due to the terminal sliding mode control. Lyapunov theory is used to demonstrate the stability of a closed-loop train system. Lastly, to demonstrate a performance of the controller Matlab/Simulink environment is used. Finally, the system response settles within 3.134ms and zero steady state error is attained. So, the main objective of the paper, stability in finite time is achieved with good tracking accuracy. To realize the strong robustness of the proposed controller, its performance was demonstrated for different scenarios and obtained good results.

**Keyword:** high-speed trains; multi-point mass model; tracking control; finite time stability; adaptive control; terminal sliding mode surface; actuator faults.

## ACKNOWLEDGEMENT

First and foremost, I would like to thank God Almighty for giving me the strength, faith and opportunity and through him I could accomplish this thesis and all other achievements in my life.

I would like to express my genuine gratitude to my advisor **Dr. CELESTIN NKUNDINEZA** for the continuous support of my research, for his patience, motivation and immense knowledge. His guidance helped me in all the time of research.

## CONTENTS

|                                                                           |     |
|---------------------------------------------------------------------------|-----|
| APPROVAL SHEET .....                                                      | ii  |
| DECLARATION .....                                                         | iii |
| ABSTRACT .....                                                            | iv  |
| ACKNOWLEDGEMENT .....                                                     | v   |
| ABBREVIATIONS AND ACRONYMS .....                                          | ix  |
| LIST OF TABLE .....                                                       | x   |
| LIST OF FIGURES .....                                                     | xi  |
| CHAPTER ONE .....                                                         | 1   |
| 1. INTRODUCTION .....                                                     | 1   |
| 1.1. Background of the study .....                                        | 1   |
| 1.1.1. Automatic Train Control System .....                               | 2   |
| 1.1.2. Working principle of Automatic Train Control (ATC) system.....     | 3   |
| 1.1.3. Train speed control by Automatic Train Operation (ATO) System..... | 4   |
| 1.1.3.1. Automatic Train Operation (ATO) Speed controller Algorithm ..... | 5   |
| 1.2. Statement of the problem .....                                       | 5   |
| 1.3. Objective .....                                                      | 6   |
| 1.3.1. General objective .....                                            | 6   |
| 1.3.2. Specific objective .....                                           | 6   |
| 1.5. Significance of the study .....                                      | 6   |
| 1.6. Scope of the Study.....                                              | 6   |
| 1.7. Organization of the Paper.....                                       | 6   |
| CHAPTER TWO .....                                                         | 7   |
| 2. LITERATURE REVIEW .....                                                | 7   |
| 2.1. Introduction .....                                                   | 7   |
| 2.1.1. Adaptive sliding-mode control approach .....                       | 7   |
| 2.1.2. Adaptive Fuzzy Logic Control approach .....                        | 8   |
| 2.1.3. Adaptive Iterative Learning Control approach.....                  | 8   |
| 2.2. Literature Summary.....                                              | 9   |
| CHAPTER THREE .....                                                       | 11  |
| 3. METHODOLOGY .....                                                      | 11  |
| 3.1. Data collection and literature survey.....                           | 11  |
| 3.2. Mathematical Modeling .....                                          | 11  |

|                                                                                                     |    |
|-----------------------------------------------------------------------------------------------------|----|
| 3.3. Controller Designing .....                                                                     | 11 |
| 3.4. Material Used .....                                                                            | 11 |
| CHAPTER FOUR.....                                                                                   | 13 |
| 4. MATHEMATICAL MODEL OF RAILWAY HIGH SPEED TRAIN SYSTEM.....                                       | 13 |
| 4.1. High Speed Train dynamic model.....                                                            | 13 |
| 4.2. Description of Train Dynamical Model with Actuator Faults .....                                | 18 |
| CHAPTER FIVE .....                                                                                  | 20 |
| 5. CONTROLLER DESIGN .....                                                                          | 20 |
| 5.1. Control System .....                                                                           | 20 |
| 5.2. Sliding Mode Control Design .....                                                              | 20 |
| 5.2.1. Definitions and Preliminaries .....                                                          | 20 |
| 5.2.2. Sliding Surface Design .....                                                                 | 22 |
| 5.2.2.1. Conventional sliding mode surface.....                                                     | 22 |
| 5.2.2.2. Terminal sliding mode surface.....                                                         | 23 |
| 5.2.2.3. Non-singular Terminal Sliding Mode surface .....                                           | 23 |
| 5.2.3. Control Law Design.....                                                                      | 26 |
| 5.2.4. Chattering problem .....                                                                     | 29 |
| 5.3. Adaptive Control Design.....                                                                   | 30 |
| 5.3.1. Definitions and Preliminaries .....                                                          | 30 |
| 5.3.2. Adaptive Control Law design .....                                                            | 31 |
| 5.3.3. Adaptive Updating parameters selection .....                                                 | 33 |
| 5.5. Reference Speed trajectory generation.....                                                     | 37 |
| CHAPTER SIX.....                                                                                    | 38 |
| 6. SIMULATION RESULT AND DISCUSSION .....                                                           | 38 |
| 6.1. Result and discussion for actuator fault free case .....                                       | 41 |
| 6.2. Result and discussion for the actuator fault case.....                                         | 44 |
| 6.3. Result and discussion for the increased external disturbance and angle of curvature case ..... | 47 |
| 6.4. Comparative analysis of the result with the state of art .....                                 | 49 |
| CHAPTER SEVEN .....                                                                                 | 51 |
| 7. CONCLUSION, RECOMMENDATION AND FUTURE WORKS.....                                                 | 51 |
| 7.1. Conclusion.....                                                                                | 51 |
| 7.2. Recommendation.....                                                                            | 51 |

|                        |    |
|------------------------|----|
| 7.3. Future Work ..... | 51 |
| REFERENCES .....       | 52 |
| APPENDIX.....          | 55 |

## **ABBREVIATIONS AND ACRONYMS**

|      |                                    |
|------|------------------------------------|
| HSR  | High speed railway                 |
| ATC  | Automatic train control            |
| ATP  | Automatic train protection         |
| ATS  | Automatic train supervision        |
| ATO  | Automatic train operation          |
| VSC  | Variable structure control         |
| SMC  | Sliding mode control               |
| TSM  | Terminal sliding mode              |
| NTSM | Non-singular terminal sliding mode |
| PID  | Proportional integral derivative   |
| ILC  | Iterative learning control         |
| PPD  | Push Pull driving                  |
| DD   | Distributed Driving                |

## LIST OF TABLE

|                                                |    |
|------------------------------------------------|----|
| Table 1.1: Automation level of ATO system..... | 4  |
| Table 6.1: Parameters of the train .....       | 38 |
| Table 6.2: Controller parameters .....         | 40 |

## LIST OF FIGURES

|                                                                                                                                                                 |    |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| Figure 1.1: Train operation mode with manual labor[1].....                                                                                                      | 1  |
| Figure 1.2: Train operation mode with ATC[1].....                                                                                                               | 2  |
| Figure 1.3: Configuration of automatic train control system[13].....                                                                                            | 3  |
| Figure 3.1: Methodology chart .....                                                                                                                             | 12 |
| Figure 4.1: Force diagram of the train .....                                                                                                                    | 14 |
| Figure 4.2: Multi-point model of HSTs .....                                                                                                                     | 15 |
| Figure 4.3: Description of the actual actuator output .....                                                                                                     | 18 |
| Figure 5.1: The idea of sliding mode .....                                                                                                                      | 27 |
| Figure 5.2: Sliding surface with chattering[18] .....                                                                                                           | 29 |
| Figure 5.3: adaptive block diagram for online estimated parameters .....                                                                                        | 33 |
| Figure 6.1: Adaptive terminal sliding mode control block diagram .....                                                                                          | 39 |
| Figure 6.2: A reference trajectory for train operations between two stations.....                                                                               | 40 |
| Figure 6.3: Disturbance applied for (a) train 1, (b) train 2 and (c) train 3 respectively.....                                                                  | 41 |
| Figure 6.4: Controller output velocity of the trains .....                                                                                                      | 42 |
| Figure 6.5: Speed tracking error of the trains .....                                                                                                            | 43 |
| Figure 6.6: Actual control force.....                                                                                                                           | 44 |
| Figure 6.7: Speed tracking error of the trains if the actuator losses 10% of its effectiveness .....                                                            | 44 |
| Figure 6.8: Speed tracking error if the actuator losses 30% of its effectiveness .....                                                                          | 45 |
| Figure 6.9: Speed tracking error if the actuator losses 45% actuator effectiveness loss .....                                                                   | 46 |
| Figure 6.10: The actual control force for case 3. ....                                                                                                          | 46 |
| Figure 6.11: 50% increased external disturbance with 150 degree curve angle disturbance applied for (a) train 1, (b) train 2 and (c) train 3 respectively. .... | 47 |
| Figure 6.12: Speed tracking error for external disturbance with 150 degree curve angle .....                                                                    | 48 |
| Figure 6.13: Speed tracking error for 100% increased external disturbance with 150 degree curve angle.....                                                      | 48 |
| Figure 6.14: 100% increased external disturbance with 150 degree curve angle disturbance applied for (a) train1, (b) train 2 and (c) train3 respectively. ....  | 49 |

# CHAPTER ONE

## 1. INTRODUCTION

With a rapid evolution of economy and society, railroad transportation due to its huge capacity, high speed, high reliability and low energy consumption plays a more and more important role in mass transit. Railway transportation, the primary line railroad, metropolitan rail travel (metro, tram, underground framework) and the new high velocity rail route (HSR), an effective methods for public vehicle mode via vehicles running on rail line tracks (rail)[1][2].

In today's world, the railway transportation entered into the era of high speed. Which means the concern becomes more of to travel faster and safer. The increasing demand for speed, transport capacity and safety require stable and large railway vehicles. This issues leads to the development of new high speed trains with new control mechanisms[3][4][5].

High speed trains is a train with a maximum operating speed of more than 200 km/h [6]. Compared to the traditional rail traffic, high-speed train is a type of rail transport that operates significantly faster and more eco-friendly, in which the automatic train control (ATC) system is used to supervise, control and adjust the train operations to guarantee safety, punctuality (according to the schedule) and comfort with such a high speed.

### 1.1. Background of the study

In recent years, rail transportation technology shows a significant progress. Trains are expected to run at high speeds to decrease transportation time with the minimal cost. As the train operational speed increases, safety and comfort are of paramount concern[7]. Therefore, to address the issue of passenger safety, comfort and punctually train speed control is mandatory. The speed control problem is one of the demanding control problems for high-speed train, which is used for automatically controlling the train speed to follow a desired speed to guarantee the safety with such a high speed.

In the soonest long stretches of railroads, the protected development of the train is refined via physical work by means of assistance wayside visual signs. For this situation, a train is completely constrained through the driver in the lodge, who gets rail condition data from wayside hardware progressively, and ceaselessly takes relating train control orders as per the train dispatching orders from a rail route traffic signal focus[1][8][9].

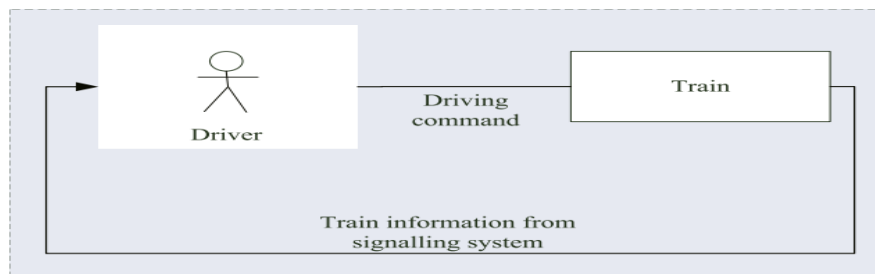


Figure 1.1: Train operation mode with manual labor[1]

This type of rail vehicle control methods has two key problems. One is, utilizing visual signals makes the driver to look over the outside, misread or even ignore the signal and proceed without permissions[10]. In these situations, it may cause severe catastrophes. Then again, manual driving is essentially founded on decisions of drivers, manual train driving activities can't accomplish great exhibitions[1][9].

However, with the improvement of railroad frameworks, automatic train control framework which utilizes electromechanical hardware's for consequently caution a driver by a horn in a lodge when moving toward the signal. Likewise, if the driver didn't recognize a horn inside 2-3 s, a train will consequently stop to a stand-still was created as an arising innovation to control the activity of the train[1].

### 1.1.1. Automatic Train Control System

Applications of control theory in intelligent transportation systems have gone through significant growth in recent years. One of the demanding problems in this field is development of autonomous vehicles. In this context, a speed control system, which enables the vehicle to automatically adjust its velocity, becomes more important and requires new technologies to achieve higher performance[11].

In recent times, rapid rails is emerging with great accomplishments around the world and how to control the running operation of thus high speed trains to achieve safe and effective operation becomes the main objectives of the studies. And as, control and computer technologies is advanced in last decades, automatic control system advanced in its operation from the so-called “ATC” system which is more likely train automatic warning system. The modern ATC system come to be integrated signaling system which associates railway train control, supervision and management (to guide drivers or totally substituting drivers) to automatically control the train operation[1][12][13][14].

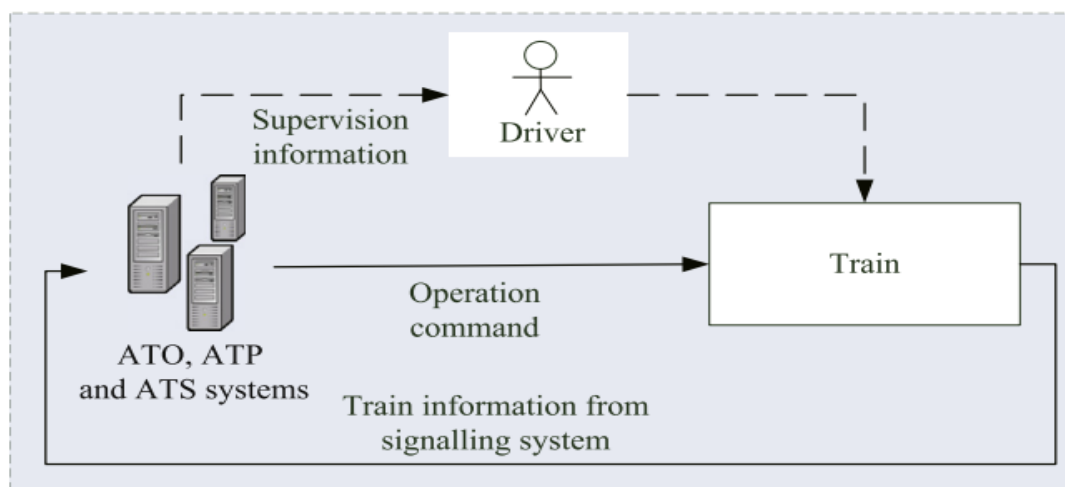


Figure 1.2: Train operation mode with ATC[1]

For automated railway train systems ATC particularly consists three subsystems;

**(i). Automatic train supervision (ATS) system:** it is accountable for monitoring the train motion to guarantee trains adjusted to the expecting plan and traffic pattern. Specifically, ATS framework interfaces monitors train stop and train activity in metropolitan rail frameworks which helps diminish harm coming about because of framework irregularities and gear glitches by playing out the accompanying undertakings, observation of train conditions, routing determination, automatic scheduling and automatic system monitoring monitoring[1][15].

(ii). **Automatic train protection (ATP) system:** is a fail-safe scheme, responsible for a protected development of each trains. Putting maximum speed restrictions on a running train is also responsible of ATP. When the train surpasses as far as possible, ATP will consequently execute slowing down (or crisis slowing down) request to stop the train to keep security[1][16].

**(iii). Automatic train operation (ATO) system:** ATO framework plays out the on-board works emplace of the driver to guarantee a smooth speed increment of a train to the running velocity, speed guideline and train halting at the objective stage definitely. In conventional circumstances, ATO is obligated for all the train footing and slowing down control orders, and accordingly, it may be a key to the working productivity and benefit of train activity frameworks.

### 1.1.2. Working principle of Automatic Train Control (ATC) system

Whether or not a railway system can be operated safely and efficiently depends essentially on the performance of its Automatic Train Control (ATC) system[13]. Thus three subsystems such as ATS, ATP and ATO of ATC system works together to guarantee the secure and productive running of automated railway trains. Specifically, ATS provides train steering and timetable adherence guidelines to ATO as per the train condition and timetable. ATO will accumulate the applicable data, for example, train running velocity, modified stop and staying time, afterward choose the brake or speeding up paces of a train. In general, ATP ensures safe train departure and over speed protection, ATS preserves trains to be in the programmed schedule, and ATO is alarmed on train functioning strategies which is related to operation efficiency[1].

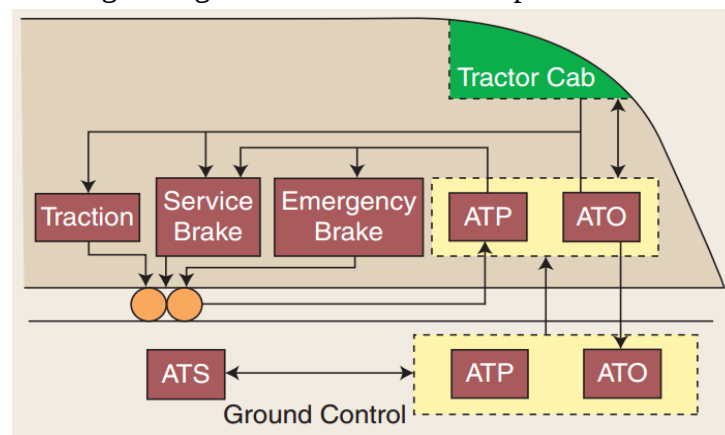


Figure 1.3: Configuration of automatic train control system[13]

### 1.1.3. Train speed control by Automatic Train Operation (ATO) System

Automatic train speed control process is the center innovation of automatic train control system, and it adjusts the train's acceleration and braking through actual operation states to guarantee train operation safety, stability and energy-consumption to the lowest level. Because of the qualities of the programmed train control measure, like nonlinear, enormous hysteresis and high constant, the capacity of programmed speed control is a most complex module of the programmed train control framework. Furthermore, it mirrors the knowledge and control level of the programmed train control innovation[1][17]. Unlike manual driving where a train is fully controlled by the driver, ATO system utilizes integrated logic between the wayside system where the train speeds are controlled without interference of the operator[1]

The recommended speed is sated (generated) by a ground station or directly by the train driver then after generating the optimal recommended speed profile by ground station or derives the speed controller of ATO receives the command signal and also ATO accepts the real time feedback data and track information through on-board speed sensors and wayside positioning devices at the same time the speed controller of ATO compares the feedback information  $v$  with the recommended speed  $v_{\text{recommended}}$  in position  $x$  to calculate the control command  $U$ . After that, the control force  $U$  drives the train motors in order to produce traction force  $F^+$  or braking force  $B^-$  to retain the speed tracking errors of a train to zero so, that the train accurately follow the desired speed profile as much as possible. As well, the train status are measured and updated in real-time in the control loop until train reaches and stops at the next station.

ATO system is an operational safety enhancement device used to help automate the operation of trains. The degree of automation is indicated by the Grade of Automation (GoA), starting from GoA level 0 up to GoA level 4. ATO is primarily used on automated guideway transit and rapid transit systems where it is easier to ensure the safety of humans. On most systems, there is a driver present to mitigate risks associated with failures or emergencies.

| Grade of automation | Train operation                  | Description                                                                                                                                                |
|---------------------|----------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------|
| GoA 0               | On-sight                         | No automation                                                                                                                                              |
| GoA 1               | Manual                           | A train driver controls starting and stopping, operation of doors and handling of emergencies or sudden diversions.                                        |
| GoA 2               | Semi-automatic                   | Starting and stopping are automated, but a driver operates the doors, drives the train if needed and handles emergencies.<br>➤ Many ATO systems are GoA 2. |
| GoA 3               | Driverless (DTO)                 | Starting and stopping are automated, but a train attendant operates the doors and drives the train in case of emergencies.                                 |
| GoA 4               | Unattended train operation (UTO) | Starting and stopping, operation of doors are all fully automated without any on-train staff.                                                              |

Table 1.1: Automation level of ATO system

#### **1.1.3.1. Automatic Train Operation (ATO) Speed controller Algorithm**

ATO based speed controlling is basically based on control algorithms which is embedded on the on-board computers[11]. Under the ATC system, the movement of high-speed trains relies on continuous bi-directional communication link between trains and controllers, and the distances between two trains are continuously adjusted according to their actual speeds and positions, which ensure that the trains are operating safely at all times

Until now, different types of controllers were embedded on ATO for the controlling of the speed like, PID, Fuzzy logic control, Adaptive control and so on. But, the control methods which is used for train speed tracking were still now are not effective due to, a variety of affecting factors, such as input nonlinearities, parameter uncertainties in the train dynamic model, aerodynamic resistances, actuator failures and actuator saturation.

### **1.2. Statement of the problem**

In railway high speed train transportation system the target operating speed of the train between stations at the stages of acceleration, coasting and braking is calculated by ground station system based on the line speed restriction and grade conditions designated by ATP system. Then controlling of the train to track this generated speed profile curve in its running and braking operation is the responsibility of ATO system by means of controllers which is embedded on it.

In running operation of a high speed trains, there are some influencing factors which reduce the performance of controllers which embedded on ATO; such as parameter uncertainties, unknown model parameters, actuator effectiveness loss, coupler effect and external disturbance. Until now several researchers were tried to develop a tracking controller which can tolerate the effect of these listed affecting factors and effectively control the train to track the specified speed curves. But, due to an increase in speed and complex external environment for the cause of high speed trains, these factors become more and more and make the speed tracking control difficult. Due to this factor, until know there is a limitation in controllers to attain finite time convergence of the tracking error to the equilibrium point rather than achieving asymptotic convergence, that means stability within finite time is not guaranteed. Due to this, the operating speed is lower than the allowable speed of ATP, resulting in speed reduction of high-speed trains or the interval between train departures becomes extended, that affects the operating efficiency of the trains, which in turn affects the entire railway network utilization rate, and cannot give full play to the transportation capacity of the entire railway network. Although, in noticing this problem many of the researchers exclude the couplers effect by modeling the train as a single-point mass model and ignore the in-train force between the adjacent cars.

Therefore, to solve this problem an adaptive terminal sliding-mode control method for finite time stable speed tracking of multiple high speed trains is designed by considering all the above influencing factors including the coupler effect by modeling the HST system as a multi-point mass model and demonstrated the effectiveness of the proposed controller on MATLAB/SIMULINK software.

### **1.3. Objective**

#### **1.3.1. General objective**

The general objective of this research is to develop adaptive terminal sliding-mode control for speed tracking of a multiple high speed trains.

#### **1.3.2. Specific objective**

The specific objective of this research is;

- ✓ To develop a dynamic model for the railway multiple high speed train system
- ✓ To generate the reference speed trajectory of the train
- ✓ To design adaptive terminal sliding-mode control for the railway multiple high speed train to track the generated speed trajectory effectively
- ✓ To demonstrate the effectiveness of proposed controller through simulation
- ✓ To validate the result

### **1.5. Significance of the study**

The main contributions of this paper is first in considering the coupler effect, this paper primary constructs a multi-point mass dynamical model of high-speed trains. For the second, in order to increase the accuracy of the tracking control actuator faults also considered.

### **1.6. Scope of the Study**

In this thesis, the main of the high speed train problem unknown model parameters, parameter uncertainties, external disturbance, actuator faults and the coupler effect between the adjacent trains were considered. But due to the complexity of the system, this thesis limited to consider actuator saturation effect.

### **1.7. Organization of the Paper**

The whole thesis work may encompass up to seven main chapters, which will be presented as following described bellows. The first chapter briefly gives detailed information about the general introduction, background, problem of the thesis, objectives to be achieved and significance of this research work. The second chapter presents Literature Reviews of related works on speed tracking controllers for the railway train in recent years. The third chapter presents the methodology; how this research paper was conducted. The fourth chapter was concerned with the dynamic modeling of a multiple high speed train system. The fifth chapter presents design of the proposed controller which is adaptive terminal sliding mode control in detail. The sixth chapter dealt with explained results and discussions with simulation in detail. In this chapter, the performance of the adaptive terminal sliding mode controller was presented. The last chapter, chapter seven presented about conclusions drawn which are done in this thesis, further works and recommendations.

## CHAPTER TWO

### 2. LITERATURE REVIEW

#### 2.1. Introduction

The need for automatic control of railroad trains shows a quick advancement and enhancement in continuously upgrading the train speeds and regularly escalating the railway networks across the country and around the world. The vital qualities of safety and effectiveness throughout operation of the trains could be improved via by the automatic train control (ATC) technology and speed control is directly the main tasks of its main subsystem which is automatic train operation system. ATO has been broadly investigated within the final two decades. A current research attention is in simulation conformation based on the established train dynamic models via selecting appropriate control schemes to enhance its performance qualities.

Recently, the tracking control of high-speed train has attracted lots of attention from the control community and many interesting research results have been obtained. Numerous researchers had developed different sorts of control strategies to solve the speed tracking issues for the trains by considering different parameters, from those different research methods some are listed below.

##### 2.1.1. Adaptive sliding-mode control approach

M. Ganesan *et al.* [7], proposed “Hybrid model reference adaptive second order sliding mode controller for automatic train operation”. This study presents decplacement and speed tracking control for high speed train by making consideration to the rolling stock and track environment. The control law encompasses model reference adaptive control and adaptive second order sliding mode control to make the control signal continuous and to reduce chattering problem. The adaptive gains of the proposed control law is driven based on Lyapunov method. The proposed method modeled the train as a multiple trains but assumed the locomotive and the coaches are connected through the linear coupler-draft gear by excluding non-linear coupler-draft gear spring and damping coefficients. In addition, the resistance force is not included due to its non-linearity and parameter uncertainty is not considered.

Y. Zhou and Z. Zhang [18], have proposed “High-speed train control based on multiple-model adaptive control with second-level adaptation”. This paper presents position and speed tracking control strategy for nonlinear high speed trains by using an input decomposition technique. The papere modeled the high speed train as a multi-point system and concederd the intrain forces between the cars. The simulation outcomes proves that the proposed controller is attains small tracking errors. But in this paper there was a gup of considering the unmeasrable external distrubances, parameter uncertainties in the model and the coupler effect between the wagons.

To solve position and velocity tracking control problem of ATO system of high speed trains subject to unknown parameters, model uncertainty and external disturbances P. Wu *et al.* [19], proposed “Automatic train operation based on adaptive sliding mode control” and Q. Song and T. Sun [20], developed “Neuroadaptive PID-like Fault-Tolerant Control of High Speed Trains with

Uncertain Model and Unknown Tracking / Braking”. The simulation results of both papers shows that the proposed methodologies achieves small tracking error. But, the train modeled as a single-point mass modeled and ignored in-train forces between the trains.

Z. Mao *et al.* [21], developed “Adaptive Fault-Tolerant Sliding-Mode Control for High-Speed Trains with Actuator Faults and Uncertainties”. With in the proposed strategy, longitudinal model dynamics is concentrated, and the unsettling influence conjointly with actuator issues is reflected. In arrange to solve actuator uncertainties and unidentified faults concurrently, an adaptive technique joint with sliding-mode control to ensure the asymptotical convergence of the tracking errors is exploited. The dynamic model of the train is based on a single-point mass model which doesn’t characterize the full performance of the train dynamics.

Center on the position and velocity tracking control issue of high speed train with uncertain system model and external disturbance as well as unknown traction/braking actuation features Y. D. Song *et al.*[22], have proposed “Fault-tolerant adaptive control of high-speed trains under Traction/Braking failures, a virtual parameter-based approach” and Q. Song and T. Sun [20] developed “Neuroadaptive PID-like Fault-Tolerant Control of High Speed Trains with Uncertain Model and Unknown Tracking / Braking” and both achieved small tracking error with asymptotic convergency. Both ignored the coupler effect between the trains and modeled the train as a single mass model.

### **2.1.2. Adaptive Fuzzy Logic Control approach**

X. G. Guo *et al.* [23], have developed “Adaptive fuzzy fault-tolerant control for multiple high-speed trains with proportional and integral-based sliding mode”. In the proposed method velocity tracking control with actuator failures, uncertain parameters, unknown non-linear dynamics, and external disturbances were examined for HSTs and good tracking execution were accomplished without considering the effect of the coupler. But, this type of control methods often has complex structure and high computational burden.

### **2.1.3. Adaptive Iterative Learning Control approach**

The other proposed method is “Iterative learning control for train trajectory tracking under speed constraints with iteration-varying parameter” by Z. Li *et al.*[24]. This paper presents iterative learning control-based ATO algorithm strategy that seems preserve the train tracking the required speed flawlessly through the impact of weather conditions whereas still fulfilling speed constraints. Speed limitations that means tracking error is constantly in a predefined border are attained by means of Barrier Composite Energy Function (BCEF) method. On the other hand, this paper limited to consider parameter uncertainties and the coupler effect in achieving asymptotic convergence of the tracking error.

H. Ji. *et al.* [25] and D. Huang *et al.*[6], formulates an adaptive iterating control technique for velocity and displacement curve tracking control issues for HSTs with unknown time-varying parameters and lumped uncertainties additionally the first one includes unknown speed delays

and input saturations. To address the issue of train movement dynamics having nonlinearities the HSTs is framed as a nonlinearly parameterized framework additionally rather than estimation of train delays in the second method, an unknown time-varying delay term is coordinated into the speed on delay analysis by means of Lyapunov–Krasovskii function. In the first scheme to analyze the stability of closed-loop system, composite energy function method is utilized. Result of both papers shows that, the tracking performance of the train is meaningfully enhanced along with the increase of the number of operations.

Y. Chen *et al.* [26], “Tracking control via iterative learning for high-speed trains with distributed input constraints”. This document presents, tracking control issue of HSTs in occurrence of an input constraints initiated by distribution and output capacity of power systems. The repetitive pattern of train operation with the backstepping technique adaptive ILC approach is proposed, where the unknown time-varying parameters are adjusted online between successive operations and an input-dependent auxiliary system is designed to compensate the influence of input constraints. During the design of the controller, the Lyapunov function and composite energy function are constructed to guarantee the stability of the closed-loop system.

However, those iterative based controllers had drawbacks of requiring zero initial values for the target trajectory throughout the repeated iterations, all of which may restrict the application domains in practical applications.

C. Xu *et al.* [27], have proposed “Model-Independent Adaptive Fault-Tolerant Tracking Control for High-Speed Trains with Actuator Saturation”. This paper presents a fault-tolerant tracking control issue of HSTs subject to unidentified model parameters, un-measurable external disturbance, and random actuator faults constrained by actuator saturation. As stated in the paper to overcome the actuator saturation, an auxiliary composition term is designed. A simulation result shows that, the proposed method has good tracking ability, effective anti-actuator saturation capability, and strong robustness to the lumped uncertainty. But, the paper limited to consider the couplers effect and the presented method guarantee only asymptotic stability.

## 2.2. Literature Summary

While observing the literatures, all the papers presented in above considers unknown parameters of the model and external disturbances and some of them considered parameters uncertainties in their proposed controller. Only a limited literature models a train as cascaded cars (as a multiple cars) and deals with in-train forces. Many literature regards a train as a single-point mass, if in-train forces for the preceding car are the resistances, then for the rear car, they will become tractive forces. Therefore, moderate in-train forces actually play a crucial role in the dynamics of train movements. When the effects of in-train forces on the dynamics of train movements are explored, the couplers effect will have to be considered.

Although in looking over the literatures, there is a gap of considering actuator effectiveness faults and actuator saturation conditions in the researches except [21] and [27]. In paper [21], only

the actuator effectiveness loss was considered but in paper [27] both actuator effectiveness faults and actuator saturation are considered simultaneously and achieved good tracking ability but, there is a gap of attaining finite time convergence of the system rather they attune only asymptotic convergence and also there is a gap in modeling the train as a single-point mass model and ignores the effect of couplers rather than multi-point mass model.

Therefore, this research will fill the gap from literatures and given as future working areas in review articles, by designing adaptive terminal sliding-mode controller for railway multiple high speed trains speed tracking by considering unknown parameters of the model, parameters uncertainties, actuator effectiveness losses, the couplers effect and external disturbance.

## **CHAPTER THREE**

### **3. METHODOLOGY**

#### **3.1. Data collection and literature survey**

In conducting a research the first step is identification of the problem therefore, in this research the first thing was diagnose the situation to identify and focus on the problem. Then the second step was data collection and literature review for the identified research problem and in this proposed research different literatures was reviewed and a lot of data's like; the train type, mass of the train, time table data's, and the train resistance data's (i.e, wind, gradient and curve at specific sections) were taken from standard and some of them collected from literatures for the train system.

#### **3.2. Mathematical Modeling**

Mathematical modeling of the train system is the next step in conducting this research paper. In the proposed research multi-point train model will be used because this model is viable in characterizing the dynamic performance of trains because it considers the in-train dynamics between the wagons. In a multi-point train modeling, the train is defined in a nonlinear frame work.

#### **3.3. Controller Designing**

In this proposed research two type of control algorithm is used by integration for the train target speed tracking control which is adaptive control and sliding-mode control. Then the fourth step, was designing Adaptive terminal sliding mode control for speed tracking of the train system. The proposed controller uses sliding mode control for its robustness characteristics. Based on the terminal sliding mode control primarily, an adaptive thequnice is utilized to estimate the unknown parameters of high speed train online. Finally, the result has been analyzied and interpreted by obtaining the proper feedback gain from the controller to compare and observe the train is tracking the required speed.

#### **3.4. Material Used**

For the simulation of the designed speed tracking adaptive terminal sliding mode controller Computer simulation was carried out using MATLABR/SIMULINK environment to investigate its performance.

### Methodology chart

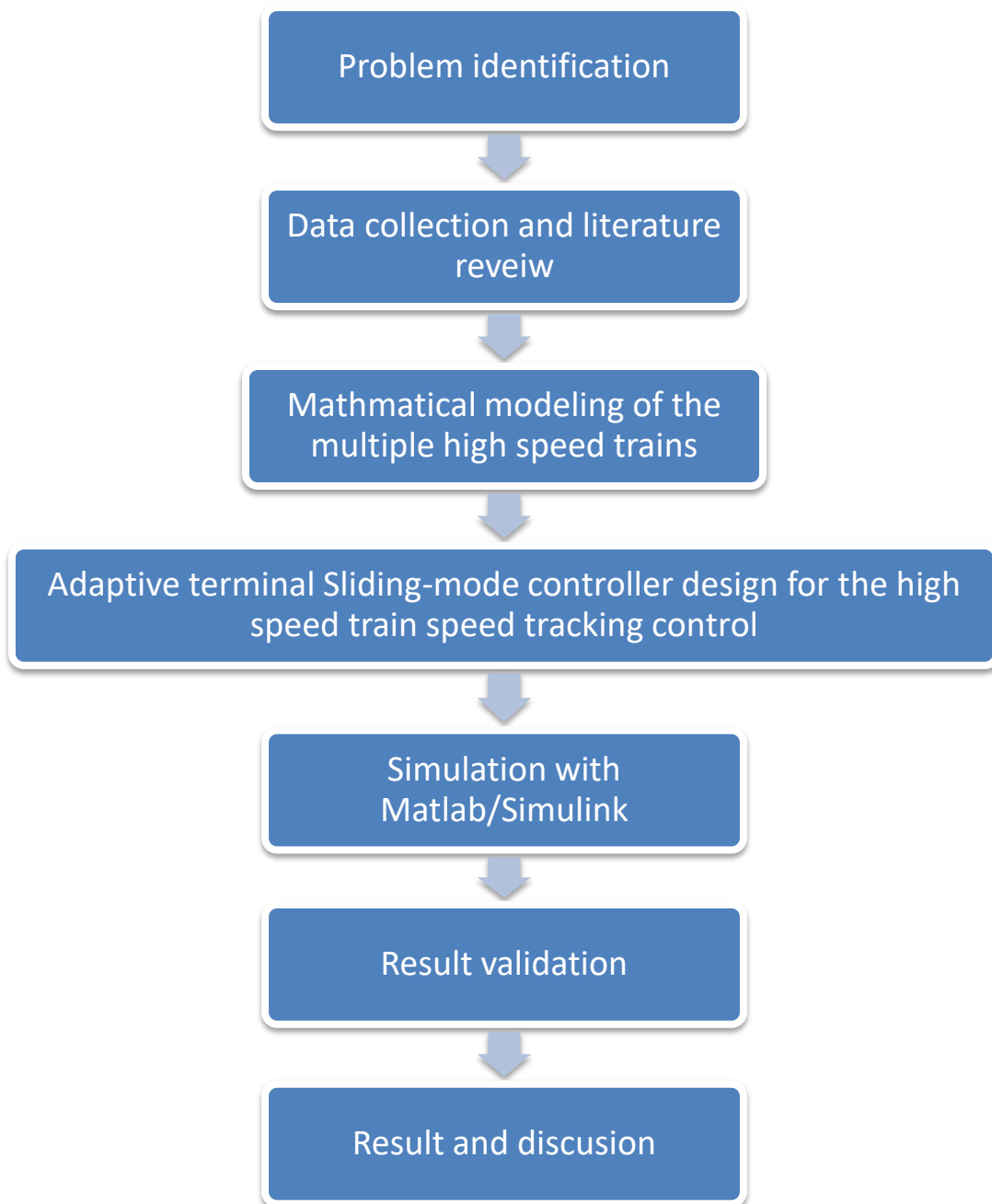


Figure 3.1: Methodology chart

## CHAPTER FOUR

### 4. MATHEMATICAL MODEL OF RAILWAY HIGH SPEED TRAIN SYSTEM

Mathematical models of physical forms are the basics of control theory. The existing investigation and synthesis tools are all grounded on certain sorts of mathematical portrays of the systems to be controlled, too called plants. Mathematical methods for analyzing and designing control systems drop into two divers' classes:

- **Frequency domain:** for frequency domain modeling of system, values of the state factors, which is the numerical variables expressing system's input, output and feedbacks are represented as functions of frequency. To transform input signal and a system's transfer\_function from time function to frequency function a Fourier transform, Laplace transform, or Z transform is utilized. The advantage of this strategy is that it comes almost in a disentanglement of the arithmetic; the differential equations that symbolize the system are substitute by algebraic equations in the frequency domain which is much simpler to solve. Yet, frequency domain technique can only be utilized for linear systems.
- **Time-domain state space representation:** In this sort the values of the state variables are denoted as functions of time. By this model, a system being analyzed is nominated by one or more differential equations. Since recurrence area strategy is restricted to direct frameworks, time space is broadly utilized to break down certifiable nonlinear frameworks.

#### 4.1. High Speed Train dynamic model

Railway trains are articulated vehicles means, a cascade of cars (which is called wagons) are connected by flexible couplers. Therefore, there are two methods in modeling of trains for designing of nonlinear controllers. Thus are;

1. **The first is a single-point mass model of a HST:** in which the entire train or the train consisting of multiple cars is considered as a single rigid body and its longitudinal motion is characterized approximately by a single-point mass equation. Therefore, the dynamics within the train is ignored. The controller design and stability examination are easier in this case. Be that as it may, the model has the drawback of overlooking the dynamics inside wagons[28].
2. **In the second is multi-point mass model of a HST:** n number of HST wagons are represent as a system of n point masses (multi point) which are associated by couplers. In this model, the influence between wagons are taken into consideration. however, the control design and stability investigation is moderately complicated[11].

The single-point-mass model, which treats the train as a single unit, make the train modeling easier and consumes less computational time, so it has been extensively utilized in the tracking control of trains [29][27]-[30]. But, as described in above it have a major draw backs in ignoring the in train internal dynamics of the trains. Having this issue, the multi-point mass model is selected to consider the coupler effects in the simulation.

The existing train's designs drop into two diverse classes based on the composition of traction forces. The two main sorts are;

- Push Pull Driving (PPD) design and
- Distributed Driving (DD) design.

The difference is that in DD sort each car is equipped with motor and for PPD sort it has two motorized car found at both ends of a train and the trailers are among the motorized cars[11][28].

In different papers a relative study is done among these two types of designs and it is discovered that the DD trains are predominate in terms of velocity tracking and disturbance rejection and it is also one of the foremost common commercial train arrangement, which gives a few focal point over other sort of trains[11]. On the other hand, the PPD trains had benefit of energy saving and minimum upkeep cost. Velocity tracking issue of PPD trains is more complex in contrast with the DD trains since it has a higher internal dynamics and one has to confirm its stability to get a significant tracking[11][28].

In this paper the case where, every car to be fitted with a traction motor was considered. This matches to distributed drive configuration of trains, because of its superiority in relation to velocity tracking and disturbance rejection and also commercial availability. A multi- point mass model is utilized to deal with the mathematical modeling of trains because of its real-time good performance and consideration of the internal dynamics of the couplers with in the trains. In view of a train dynamics in a longitudinal direction, the force diagram of the multi-point-mass model of the trains is represented as follows;

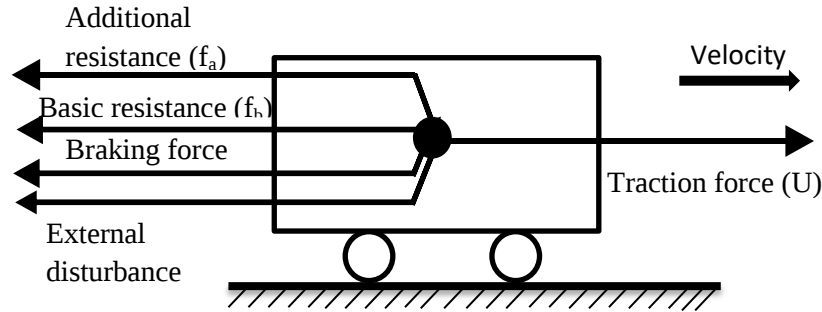


Figure 4.1: Force diagram of the train

In this work, a dynamic model of an ordered set of high-speed train system framework is defined and develops the speed control problem of multiple highspeed train movements to ensure that each train tracks the desired speed and meanwhile guarantee the safety.

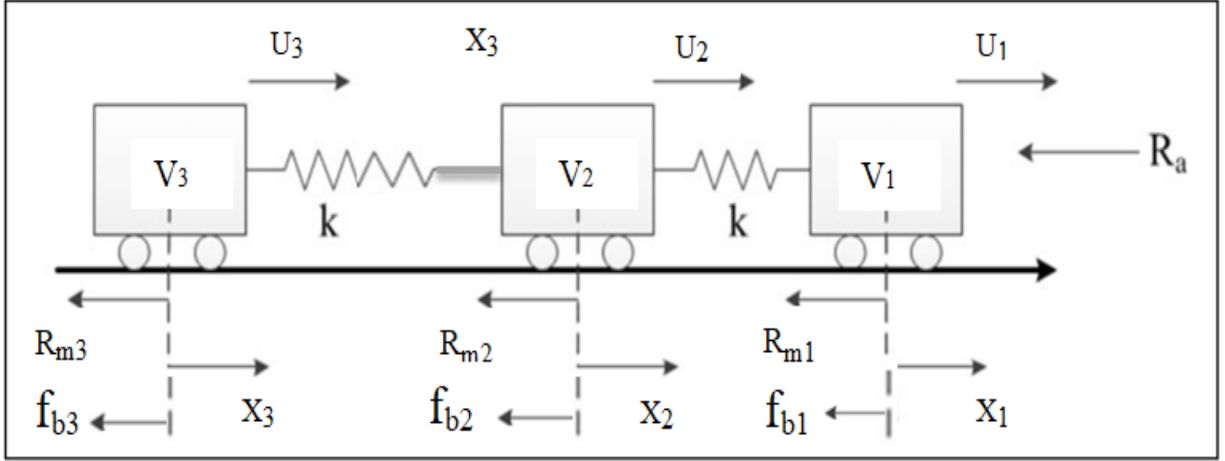


Figure 4.2: Multi-point model of HSTs

Train movements have been modeled for simulation study as the interactions between cars through couplers. Figure 1 demonstrates the coupling relationship between cars of a train.

In this paper, a high-speed train which have three number of vehicle's connected by coupler is considered and based on Newton's second law of motion, model of the train dynamics is established. Which, is broadly utilized for application and simulation in the area of train control.

$$\sum F = Ma$$

$$M_1 \ddot{x}_1(t) = F_1(t) - k(\Delta x_{1,2}) - f_{b1}(t) - f_{a1}(t) - d(t)$$

$$M_2 \ddot{x}_2(t) = F_2(t) - k(\Delta x_{2,1} + \Delta x_{2,3}) - f_{b2}(t) - f_{a2}(t) - d(t)$$

$$M_3 \ddot{x}_3(t) = F_3(t) - k(\Delta x_{3,2}) - f_{b3}(t) - f_{a3}(t) - d(t) \quad (1)$$

Where,  $M$  is the total mass of the trains which is unknown but constant at every run of the train,  $\ddot{x}(t)$  is the actual acceleration of the trains,  $F(t)$  combines traction force of a train (during traction phase) and braking forces of a train (during braking phase, when it becomes negative) generated by the train itself as the control input of the high-speed train,  $f_b(t)$  is basic resistance,  $f_a(t)$  is additional resistance,  $\Delta x_{i,j}$  is the change in displacement between the adjacent cars,  $k$  is the stiffness coefficient and  $d(t)$  is external disturbance and the same for all trains.

The behavior of couplers is described by a spring model such that the restoring force of a coupler is a function of the relative displacement between two adjacent cars as  $k(\Delta x_{i,j})$ .

In this paper, we consider the basic resistance separately for the individual car because the driving motors are distributed in several cars of a train.

In real time operation there are inescapable perturbations arising from parametric uncertainties and external disturbances that is why a bounded perturbation term ( $d$ ) were added in a system equations to deal with the effect of these disturbances.

**Remark 1:** the total equivalent mass in each car is distinctive due to the number of travelers getting on and off the train at each run. The overall equivalent mass may change around a fixed value. In practice, numerous sensors which are capable to measure the total equivalent mass are installed in a train. Thus, the time shifting total equivalent mass must to be utilized within the controller[31].

The basic resistance  $f_b(t)$  in (1) can be described as;

$$f_b(t) = R_m(t) + R_a(t) \quad (2)$$

$$R_m(t) = a(t) + b(t) * \dot{x}(t) \quad (3)$$

$$R_a(t) = c(t) * \dot{x}^2(t) \quad (4)$$

Where,  $a(t)$ ,  $b(t)$  and  $c(t)$  are coefficients of Davis equation.

The basic resistance force  $f_b(t)$ , that consists mechanical resistance force ( $R_m(t)$ ) and air resistance (aerodynamic drag) ( $R_a(t)$ ) forces, exists obstinately throughout the train running operation. The mechanical resistance force, proportionate to the train speed, consists the sliding and rotating resistance, which can be labeled as  $R_m(t)$ . The air resistance force ( $R_a(t)$ ) is proportionate to the square of the speed, as the term  $c\dot{x}^2$ . It is understandable that, the percentage of the air resistance out to be higher when the train speed increases, comparing with mechanical resistance.

Notice that  $R_a(t)$  is a nonlinear function in it's  $\dot{x}^2(t)$  and comes to be the main resistance force as the train runs at high speed. When the train speed is exceptionally low it can be taken as a linear force, which mean the train dynamics come to be almost linear. As the train speed is increased up in modern railway transportation system, it is necessary to give more consideration to the effect of the air resistance as it accounts for a large percentage of the basic resistance[31].

The additional resistance force  $f_a(t)$  in (1) makes impact only as the train were in certain special segments of the railway such as ramps and expressed as:

$$f_a(t) = Mg \sin \theta \quad (5)$$

Where,  $M$  is a total equivalent mass,  $g$  is gravitational acceleration and  $\theta$  is the ramp angle.

**Assumption 1:** The longitudinal movement of a train is continuously within the direction of a horizontal axis.

**Assumption 2:** A high-speed train was modeled as a cascade of mass connected by coupler, where the basic resistance was considered, and the aerodynamic resistance was assumed to act on the leading vehicle only.

Substituting (2) and (5) in to (1) yields;

$$\begin{aligned} M_1 \ddot{x}_1(t) &= F_1(t) - k(\Delta x_{1,2}) - a_1(t) - b_1(t) * \dot{x}_1(t) - c_1(t) * \dot{x}_1^2(t) - M_1 g \sin \theta(t) - d(t) \\ M_2 \ddot{x}_2(t) &= F_2(t) - k(\Delta x_{2,1} + \Delta x_{2,3}) - a_2(t) - b_2(t) * \dot{x}_2(t) - M_2 g \sin \theta(t) - d(t) \\ M_3 \ddot{x}_3(t) &= F_3(t) - k(\Delta x_{3,2}) - a_3(t) - b_3(t) * \dot{x}_3(t) - M_3 g \sin \theta(t) - d(t) \end{aligned} \quad (6)$$

Note that the parameters  $M$ ,  $a(t)$ ,  $b(t)$ , and  $c(t)$  in equation (6) are indefinite and vary in certain range due to various influences such as, changes of travelers and load, line conditions, climate, speed of train etc. they cannot be attained accurately so,  $f_b(t)$  is actually an unknown bounded functions[2].

Considering parametric uncertainties, the trains dynamic model described in (6) can be modified as;

$$\begin{aligned} (M_1 + \Delta M_1) \ddot{x}_1(t) &= F_1(t) - k(\Delta x_{1,2}) - (a_1 + \Delta a_1)(t) - (b_1 + \Delta b_1)(t) * \dot{x}_1(t) - (C_1 + \Delta C_1)(t) * \dot{x}_1^2(t) - (M_1 + \Delta M_1) g \sin \theta - d(t) \\ (M_2 + \Delta M_2) \ddot{x}_2(t) &= F_2(t) - k(\Delta x_{2,1} + \Delta x_{2,3}) - (a_2 + \Delta a_2)(t) - (b_2 + \Delta b_2)(t) * \dot{x}_2(t) - (M_2 + \Delta M_2) g \sin \theta - d(t) \\ (M_3 + \Delta M_3) \ddot{x}_3(t) &= F_3(t) - k(\Delta x_{3,2}) - (a_3 + \Delta a_3)(t) - (b_3 + \Delta b_3)(t) * \dot{x}_3(t) - (M_3 + \Delta M_3) g \sin \theta - d(t) \end{aligned} \quad (7)$$

Where  $M, a(t), b(t)$  and  $c(t)$  represents the nominal values and  $\Delta M, \Delta a(t), \Delta b(t)$  and  $\Delta c$  are the modeling uncertainties of  $M, a(t), b(t)$  and  $c(t)$ .

Rewriting equation (7) yields;

$$\begin{aligned} M_1 \ddot{x}_1(t) &= F_1(t) - k(\Delta x_{1,2}) - a_1(t) - b_1(t) * \dot{x}_1(t) - c_1(t) * \dot{x}_1^2(t) - M_1 g \sin \theta + P_1 \\ M_2 \ddot{x}_2(t) &= F_2(t) - k(\Delta x_{2,1} + \Delta x_{2,3}) - a_2(t) - b_2(t) * \dot{x}_2(t) - M_2 g \sin \theta + P_2 \\ M_3 \ddot{x}_3(t) &= F_3(t) - k(\Delta x_{3,2}) - a_3(t) - b_3(t) * \dot{x}_3(t) - M_3 g \sin \theta + P_3 \end{aligned} \quad (8)$$

Where,  $P_1 = -\Delta a_1 - \Delta b_1 \dot{x}_1(t) - \Delta C_1 \dot{x}_1^2(t) - \Delta M_1 g \sin \theta - \Delta M_1 \ddot{x}_1(t) - d(t)$

$$P_2 = -\Delta a_2 - \Delta b_2 \dot{x}_2(t) - \Delta M_2 g \sin \theta - \Delta M_2 \ddot{x}_2(t) - d(t)$$

$$P_3 = -\Delta a_3 - \Delta b_3 \dot{x}_3(t) - \Delta M_3 g \sin \theta - \Delta M_3 \ddot{x}_3(t) - d(t)$$

**Assumption 3:** assume that  $P$  satisfying the following condition;

$$|P| < b_0 + b_1|\dot{x}| + b_2|\dot{x}^2| + b_3|\ddot{x}| \quad (9)$$

Where,  $b_0 > 0, b_1 > 0, b_2 > 0, b_3 > 0$  and  $b_0, b_1, b_2, b_3$  are unknown.

**Remark 2:** To guarantee traffic safety, a speed of trains is continuously below the speed standards given by the train manufacturer. Passenger's consolation is the interminable interest throughout the train operation process, and travelers would feel more comfortable in case the acceleration is zero or exceptionally minimum. Therefore, the acceleration  $\ddot{x}$  of the train is also limited. In addition, the external disturbances  $d$ , the mass uncertainty  $\Delta M$ , and uncertainties  $\Delta a$ ,  $\Delta b$  and  $\Delta c$  of David's equation (2) are bounded. Thus, assumption 3 is practical.

#### 4.2. Description of Train Dynamical Model with Actuator Faults

In practical train systems when a train runs on the line, unanticipated actuator failures may occur unexpected actuator faults such as faults initiated by overheating or turn-to-turn short circuits of engines, faults from overcurrent in the traction converter, faults from wear of mechanical drivers and so on, are inescapable[27][32][33]. A train dynamical model, considering the loss of actuator effectiveness faults can be traditionally is described as;

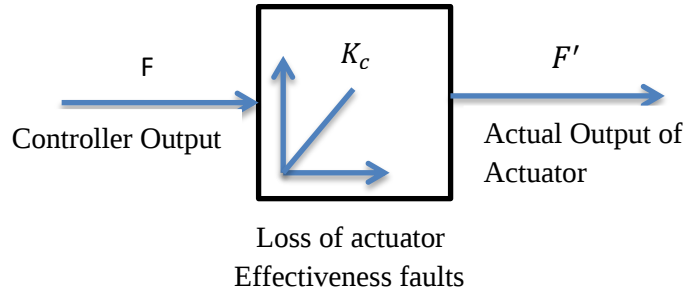


Figure 4.3: Description of the actual actuator output

So, the actual output of actuator subject to actuator effectiveness faults can be defined as follows:

$$F' = K_c * F \quad (10)$$

Where  $F'$  is the actual actuators output and  $F$  represents actuator output of a controller without faults.  $K_c$  Stands for actuator's effectiveness and satisfies  $0 \leq K_c \leq 1$ .

Where,

$K_c = 1$  Indicates the actuator is healthy;

$0 < K_c < 1$  Indicates the actuator occurs partial loss of effectiveness fault and

$K_c = 0$  Indicates the actuator totally failed.

For convenience of analysis, we define  $K_c$  as;

$$K_c = 1 - \tau_c \quad (11)$$

Therefore, by substituting equation (11) in to equation (10) gives;

$$F' = (1 - \tau_c)F(t) \quad (12)$$

Substituting equation (12) in to the train dynamic equation (8) the final equation of the train dynamic model which considers actuator effectiveness loss becomes;

$$\begin{aligned} M_1 \ddot{x}_1(t) &= (1 - \tau_c)F_1(t) - k(\Delta x_{1,2}) - a_1(t) - b_1(t) * \dot{x}_1(t) - C_1(t) * \dot{x}_1^2(t) - M_1 g \sin \theta + P_1 \\ M_2 \ddot{x}_2(t) &= (1 - \tau_c)F_2(t) - k(\Delta x_{2,1} + \Delta x_{2,3}) - a_2(t) - b_2(t) * \dot{x}_2(t) - M_2 g \sin \theta + P_2 \\ M_3 \ddot{x}_3(t) &= (1 - \tau_c)F_3(t) - k(\Delta x_{3,2}) - a_3(t) - b_3(t) * \dot{x}_3(t) - M_3 g \sin \theta + P_3 \end{aligned} \quad (13)$$

## CHAPTER FIVE

### 5. CONTROLLER DESIGN

#### 5.1. Control System

A control system is a system that offers a required outcome by controlling the output. A control system is an organization of physical components that directed to adjust a different physical system so that this system shows particular preferred characteristics.

In order to advance the efficiency of a control scheme various kinds of controllers could be utilized. A controller is the mechanism which tries to find way to reduce the variation among the real value and the required value of the system. Controllers are the central part of control engineering and totally utilized in every complex control systems.

A high-speed railway system is a typical complex dynamical system with large time-delays, high nonlinearity and multi-objectives, which is typically working under time-varying and uncertain conditions. Therefore, many researchers investigated different types of control techniques and strategies to upgrade the performance of high speed train speed tracking control system. PID controller, Fuzzy Logic controller, Fuzzy-PID controller, sliding mode controller and adaptive controller were some of the frequently used. In this thesis, adaptive terminal sliding mode controller is presented to solve a stability problem of the train speed tracking control.

#### 5.2. Sliding Mode Control Design

##### 5.2.1. Definitions and Preliminaries

Variable structure control (VSC) is the type of intermittent nonlinear control. This control system utilizes high-frequency switching control which means the state feedback control law is not a continuous function of time instead it changes from one smooth state to another in order to adjust the dynamics of a nonlinear system. Therefore, the arrangement of a control law fluctuates based on the position of a state path and the process changes from one smooth control law to another in a very fast speeds (e.g., for a countable infinite number of times in a finite time interval)[34].

The main type of variable structure control approach is sliding mode control (SMC). The rudimentary plan of a SMC is founded on imposing a system to move towards a steady state regime, which is known as sliding switching surface, by outlining an appropriate control force. In SMC, the arrangement of a controller is changed on purpose by means of a switching feedback commandment to drive the trajectories of a controlled system to the stated sliding surface, known as reaching phase, and enforce them to stay on the surface and slides towards a equilibrium point. Such condition is known as sliding mode[35][36].

The variable structure behavior of a SMC offers switching ability between diverse control laws. Insensitive to changes and external excitation, makes SMC the first choice from other types of control approaches[35]. The system is controlled in such a way that the state continually moves

to the sliding surface and hits it. The sign of a control value should have to change at the intersection among the state trajectory and sliding surface.

A most eminent feature of SMC is that it is absolutely insensitive to parametric uncertainty and external disturbances during sliding mode, significantly reduced order plant dynamics modeling and finite-time convergence for some types of SMCs.

**A switching surface:** take a common type of system characterized in a state equation,

$$\dot{x} = f(x, u, t), x \in R^n, u \in R^m \quad (14)$$

The control  $u(x, t)$  with its respective entry  $u_i(x, t)$  has the form:

$$u_i(x, t) = \begin{cases} u_i^+(x, t), & \text{when } s_i(x) > 0 \\ u_i^-(x, t), & \text{when } s_i(x) < 0 \end{cases} \quad (15)$$

Where,  $u_i^+(x, t)$ ,  $u_i^-(x, t)$  and  $s_i(x)$  are continuous functions and  $i = 1, \dots, m$ .  $s_i(x)$  is an  $(n - 1)$  dimensional switching function. As,  $u_i(x, t)$  experiences discontinuity on a surface  $s_i(x) = 0$ ,  $s_i(x) = 0$  is known as a switching surface or switching hyper-plane.

**Sliding mode:** Let  $S = \{x | s(x) = 0\}$  is the switching surface which contains the origin  $x = 0$ . If, for every  $x_0$  in, we have  $x(t)$  in  $S$  for all  $t > t_0$ , then  $x(t)$  is the sliding mode of the system.

In order to sliding mode occur, the velocity vector of a state trajectory is continuously moved to the switching surface in a neighborhood of a switching surface  $S$ .

**Sliding surface:** If sliding mode occurs on  $S = \{x | s(x) = 0\}$ , which means, if for every point in the surface there are trajectories reaching it from both sides of the surface, then the switching surface  $S$  is termed as a sliding surface or sliding manifold.

**Reaching condition and region of attraction:** Existence of a sliding mode needs stability of a state trajectory towards the sliding surface  $S = \{x | s(x) = 0\}$  at least in a vicinity of  $S$ . This sufficient condition for sliding mode is called reaching condition. The principal vicinity of  $S$  for which the reaching condition is satisfied is called the region of attraction.

Having defined all the fundamental theories, a broadly utilized design procedure of SMC is explained here. Generally, the design of SMC contains two parts: First, the sliding surface, which is typically lower order than the specified practice, should be constructed such that the system performance during sliding mode satisfies the design purposes, in relation to stability, performance index minimization, linearization of nonlinearities, order reduction, etc. Second, the switched feedback control is designed such that it satisfies the reaching condition and thus drives the state trajectory to the sliding surface in finite time and preserves it there afterwards.

### 5.2.2. Sliding Surface Design

The thought of conspiring linear switching surface for linear dynamic framework has been set up in incredible profundity and fullness whereas the design of sliding surface for nonlinear frameworks stays in a great extent open issue. A few common strategies for characterizing the differential equation of sliding mode is précised here.

#### 5.2.2.1. Conventional sliding mode surface

The frame of a sliding surface decides if the dynamics execution of the corresponding sliding mode control system is good or not. In 1996G.C, researchers proposed a linear sliding surface centered controller to stabilize a course of frameworks with asymptotic convergence rate in infinite time[29]. In this conventional sliding mode control the sliding surface has been outlined as a linear dynamic equation as,

$$s(t) = \left( \frac{d}{dt} + c \right)^{n-1} e(t) \quad (16)$$

However, due to asymptotic convergence behavior of the linear switching manifolds in this type of linear sliding surfaces, it can ensures only the asymptotic convergence of the error in a sliding mode, i.e. the output error cannot merge to zero in limited time[37][34].

Asymptotic stability means;

- Asymptotically stable, which means tracking errors converge to the equilibrium points as time goes to infinity.
- For asymptotic stable systems, the transient responses may occur during any time in the system response and this will cause undesired phenomena like high frequency oscillation, rapid change and high magnitude of the output to be occurring.
- For tracking control problem it is not required.

In numerous real time conditions, stability within finite time interval is desired instead of the classic Lyapunov asymptotic stability, as it is more physically practicable than in relation to infinite time. The concept of finite time stability means that for any initial values a system converges to zero within a finite time interval.

Finite time stability means;

- Finite-time stability means that, for any initial values the system converges to zero within a finite time interval.
- In other words, the exact estimates of the settling time for the system response are known.
- For systems that are known to operate only over a finite interval of time, as, the systems' variables must lie within specific bounds finite time stability is a must.

Due to this stability over finite time condition problem terminal sliding mode control was developed.

### 5.2.2.2. Terminal sliding mode surface

To solve the disadvantages in a conventional linear sliding mode manifold, nonlinear manifolds were offered. In recent years, terminal sliding mode (TSM) control with a nonlinear sliding surface, which can guarantees states of the closed-loop system response to converge to the equilibrium point within limited time, has been acquired incredible attention[29][36].

Terminal Sliding Mode concept is related to a finite time control. Compared with the conventional linear hyper-plane based sliding modes, TSM provides certain predominant properties such as quick, defined time convergence. The conventional TSM is designated by following first-order terminal sliding variable as;

$$s = \dot{x} + \beta x^{\frac{q}{p}} \quad (17)$$

Where,  $\beta > 0$  is a design constant,  $x$  and  $\dot{x}$  are state of the system and  $p$  and  $q$  are positive odd integers, which satisfy the following condition;

$$q > p \quad (18)$$

So, that the error's converged to zero in finite time. This controller is especially valuable for maximum accuracy control because it increases the rate of convergence close to the equilibrium point. But, in the first derivative of the TSM function (17) which is;

$$\dot{s} = \ddot{x} + \beta \frac{q}{p} x^{\frac{q}{p}-1} \dot{x} \quad (19)$$

TSM function which is stated in (19), it is seen that the second term having  $x^{\frac{q}{p}-1} \dot{x}$  could cause a singularity problem if  $\dot{x} \neq 0$  when  $x = 0$ . The singularity problem may take place in a reaching phase when there is inadequate control to guarantee that  $\dot{x} \neq 0$  while  $x = 0$ .

For the case when,  $x = 0$  before the system states reach the TSM  $s = 0$ , the TSM controller cannot ensure a bounded control signal. Moreover, a singularity may too happen indeed after the sliding mode  $s = 0$  is reached as, due to computation errors and uncertain factors, the system states cannot be sure to always remain in the sliding mode especially near the equilibrium point ( $x = 0$ ;  $\dot{x} = 0$ ), and the case of  $\dot{x} \neq 0$  while  $x = 0$  may occur from time to time. This emphasizes the importance of tending the singularity issue in conventional TSM framework.

### 5.2.2.3. Non-singular Terminal Sliding Mode surface

The underline within significance of addressing the singularity issue in conventional TSM system leads to the development of non-singular terminal sliding mode control. In arrangement to overcome this singularity issue in the conventional TSM systems, numerous approaches have been designed. One approach is developed by Man and Yu in 1997G.C, which is to switch the sliding mode between TSM and linear hyper-plane based sliding mode. Another approach is proposed by Wu et al. in 1998G.C, which is to handover a path to a pre-specified open region

where TSM control is not singular. This approach is adopting indirect methods to remove the singularity. The third one, which is simple and capable to escape the singularity problem completely, is developed by Fenget al.[34], in 2002G.C with the NTSM model are described as follows;

$$s = x + \frac{1}{\beta} \dot{x}^{\frac{q}{p}} \quad (20)$$

The sufficient condition for the existence of TSM is;

$$\frac{1}{2} \frac{d}{dt} s^2 < -\eta |s| \quad (21)$$

Where,  $\eta > 0$  is constant

### Proof

It is obvious that if  $s(0) \neq 0$ , the system states will reach the sliding mode  $s = 0$  in a finite time  $t_r$ , which satisfies;

$$t_r \leq \frac{|s(0)|}{\eta} \quad (22)$$

After the sliding mode  $s = 0$  is reached, the nonlinear differential equation labeled below will govern the system dynamics;

$$x_2 + \beta x_1^{\frac{q}{p}} = \dot{x}_1 + \beta x_1^{\frac{q}{p}} = 0 \quad (23)$$

Where,  $x_1 = 0$  is the terminal attractor of the system (20). The finite time  $t_s$  that is taken to travel from  $x_1(t_r) \neq 0$  to  $x_1(t_s + t_r) = 0$  be given by;

$$t_s = -\beta^{-1} \int_{x_1(t_r)}^0 \frac{dx_1}{x_1^{\frac{q}{p}}} = \frac{p}{\beta(p-q)} |x_1(t_r)|^{1-q/p} \quad (24)$$

This implies that, within TSM manifold (20), both the system states  $x_1$  and  $x_2$  converges to zero in finite time.

To make time varying surface for tracking control of a system, one method is to characterize the sliding surface concurring to the specified control bandwidth. But, as summarized in the existing literatures the speed trajectory tracking control of high speed train the accurate speed tracking is realized in the control but the stability performances of the control algorithm is not achieved. Therefore, on this paper a non-singular TSM control manifold in the form of (20) is selected as;

$$s(t) = e + \left(\frac{1}{\beta} * \dot{e}^{\frac{q}{p}}\right) \quad (25)$$

Where,  $e$  is the error of the system;  $\beta > 0$  is a strictly positive constant which determines the closed-loop bandwidth and  $p$  and  $q$  are positive odd integers satisfying  $1 < (q/p) < 2$ ;  $\beta$ ,  $p$ , and  $q$  are three parameters of the sliding surface (25) to be designed.

It is noticeable that  $s$  depends only on the tracking error  $e$ . Therefore, defying position tracking error and its first derivative (velocity) and second derivatives (acceleration) tracking errors similarly for all cars as follows;

$$e(t) = x(t) - x_d(t) \quad (26)$$

$$\dot{e}(t) = \dot{x}(t) - \dot{x}_d(t) \quad (27)$$

$$\ddot{e}(t) = \ddot{x}(t) - \ddot{x}_d(t) \quad (28)$$

Where  $x_d$ ,  $\dot{x}_d$  and  $\ddot{x}_d$  is the desired position, velocity and acceleration, correspondingly.

Then the time derivative of (25) is;

$$\dot{s}(t) = \dot{e} + \left(\frac{q}{\beta * p} * \dot{e}^{\frac{q-p}{p}} * \ddot{e}\right) \quad (29)$$

By, substituting (28) in to (29), yields;

$$\dot{s}(t) = \dot{e} + \left(\frac{q}{\beta * p} * \dot{e}^{\frac{q-p}{p}}\right) * (\ddot{x}(t) - \ddot{x}_d(t)) \quad (30)$$

As, the nominated train model is a multi-point mass model the TSM control manifold proposed in (30) can be designated for each cars. Therefore, substituting equation (13) in to (30) we get;

$$\begin{aligned} \dot{s}_1(t) &= \dot{e}_1 + \left(\frac{q}{\beta * p} * \dot{e}_1^{\frac{q-p}{p}}\right) * \left(\frac{1}{M_1} [(1 - \tau_c)F_1(t) - k(\Delta x_{1,2}) - a_1(t) - b_1(t) * \dot{x}_1(t) - C_1(t) * \dot{x}_1^2(t) \right. \\ &\quad \left. - M_1 g \sin \theta + P_1] - \ddot{x}_d(t)\right) \\ \dot{s}_2(t) &= \dot{e}_2 + \left(\frac{q}{\beta * p} * \dot{e}_2^{\frac{q-p}{p}}\right) * \left(\frac{1}{M_2} [(1 - \tau_c)F_2(t) - k(\Delta x_{2,1} + \Delta x_{2,3}) - a_2(t) - b_2(t) * \dot{x}_2(t) \right. \\ &\quad \left. - M_2 g \sin \theta + P_2] - \ddot{x}_d(t)\right) \\ \dot{s}_3(t) &= \dot{e}_3 + \left(\frac{q}{\beta * p} * \dot{e}_3^{\frac{q-p}{p}}\right) * \left(\frac{1}{M_3} [(1 - \tau_c)F_3(t) - k(\Delta x_{3,2}) - a_3(t) - b_3(t) * \dot{x}_3(t) - M_3 g \sin \theta + P_3] - \right. \\ &\quad \left. \ddot{x}_d(t)\right) \end{aligned} \quad (31)$$

To eliminate the coefficient multiply both sides by  $M$ ;

$$\begin{aligned}
M_1\dot{S}_1(t) &= M_1\dot{e}_1 + \left(\frac{q}{\beta * p} * \dot{e}_1^{\frac{q-p}{p}}\right) * ((1 - \tau_c)F_1(t) - k(\Delta x_{1,2}) - a_1(t) - b_1(t) * \dot{x}_1(t) - c_1(t) * \dot{x}_1^2(t) \\
&\quad - M_1 g \sin \theta + P_1 - M_1 \ddot{x}_d(t)) \\
M_2\dot{S}_2(t) &= M_2\dot{e}_2 + \left(\frac{q}{\beta * p} * \dot{e}_2^{\frac{q-p}{p}}\right) * ((1 - \tau_c)F_2(t) - k(\Delta x_{2,1} + \Delta x_{2,3}) - a_2(t) - b_2(t) * \dot{x}_2(t) \\
&\quad - M_2 g \sin \theta + P_2 - M_2 \ddot{x}_d(t)) \\
M_3\dot{S}_3(t) &= M_3\dot{e}_3 + \left(\frac{q}{\beta * p} * \dot{e}_3^{\frac{q-p}{p}}\right) * ((1 - \tau_c)F_3(t) - k(\Delta x_{3,2}) - a_3(t) - b_3(t) * \dot{x}_3(t) - M_3 g \sin \theta + P_3 - \\
&\quad M_3 \ddot{x}_d(t))
\end{aligned} \tag{32}$$

### 5.2.3. Control Law Design

After, the sliding surfaces have been chosen properly, finding the control effort  $u$  which can guarantee the control action is the next step. The equivalent control in sliding mode control is attained by recognizing  $\dot{s}(t) = 0$  that is the necessary condition for a state trajectory to remain on the sliding surface  $s(x) = 0$ .

So that, setting  $(\dot{S}_1(t), \dot{S}_2(t) \text{ and } \dot{S}_3(t) = 0)$  in (32) calculate the equivalent control.

$$\begin{aligned}
U_{eq1}(t) &= (1 - \tau_c)^{-1} (k(\Delta x_{1,2}) + a_1(t) + b_1(t) * \dot{x}_1(t) + c_1(t) * \dot{x}_1^2(t) + M_1 g \sin \theta - P_1 \\
&\quad + M_1 \ddot{x}_d(t) - \left(\frac{\beta * p}{q} * \dot{e}_1^{\frac{p-q}{p}} * M_1 * \dot{e}_1\right)) \\
U_{eq2}(t) &= (1 - \tau_c)^{-1} (k(\Delta x_{2,1} + \Delta x_{2,3}) + a_2(t) + b_2(t) * \dot{x}_2(t) + M_2 g \sin \theta - P_2 + M_2 \ddot{x}_d(t) \\
&\quad - \left(\frac{\beta * p}{q} * \dot{e}_2^{\frac{p-q}{p}} * M_2 * \dot{e}_2\right)) \\
U_{eq3}(t) &= (1 - \tau_c)^{-1} (k(\Delta x_{3,2}) + a_3(t) + b_3(t) * \dot{x}_3(t) + M_3 g \sin \theta - P_3 + M_3 \ddot{x}_d(t) - \\
&\quad \left(\frac{\beta * p}{q} * \dot{e}_3^{\frac{p-q}{p}} * M_3 * \dot{e}_3\right))
\end{aligned} \tag{33}$$

During sliding mode control, one can attain the equivalent control  $U_{eq}$  but, merely using  $U_{eq}$  does not force the state to the sliding surface  $S$  if the initial conditions of the system are not on  $S$ . Consideration is need be given to solve reachability problem. In other words, the controlled system must satisfy the reaching conditions. One common strategy is to combine the equivalent control with a switched part.

Sliding mode based on reaching law comprises reaching phase and sliding phase. The reaching phase drive system to uphold a stable manifold and the sliding phase drive system guarantees slide to equilibrium.

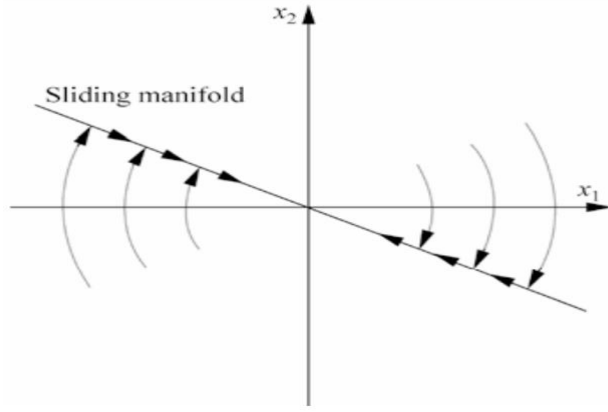


Figure 5.1: The idea of sliding mode

This contains selection of the state feedback control function  $u$  that can enforce the state  $x$  to the surface and keeps it on the surface afterwards.

**Constant Rate Reaching Law:**

$$U_{sw} = \dot{s} = -k_s \text{sign}(s) \quad (34)$$

Where,  $k_s$  is a switching control gain involved with the upper bound of uncertainties, and  $\text{sign}(s)$  is a sign function.

The law in (34) drives the switching variable  $s(x)$  to reach the switching manifold  $S$  at a constant rate of  $k_s$ .

Therefore, the total control force becomes;

$$U = U_{eq} + U_{sw} \quad (35)$$

The high speed tracking control law, for the closed loop system (13) proposed as follows;

For the first vehicle the control force becomes;

$$F_{01}(t) = k(\Delta x_{1,2}) + a_1(t) + b_1(t) * \dot{x}_1(t) + c_1(t) * \dot{x}_1^2(t) + M_1 g \sin \theta + M_1 \ddot{x}_d(t) - \left( \frac{\beta * p}{q} * \dot{e}_1^{\frac{p-q}{p}} * M_1 * \dot{e}_1 \right) \quad (36a)$$

$$F_{11}(t) = -H_{01} \text{sign}(s_1) \quad (36b)$$

$$F_{21}(t) = -\Omega_1 |F_{01}| \text{sign}(s_1) \quad (36c)$$

$$F_{31}(t) = -k_s \text{sign}(s_1) \quad (36d)$$

The same for the second vehicle becomes;

$$F_{02}(t) = k(\Delta x_{2,1} + \Delta x_{2,3}) + a_2(t) + b_2(t) * \dot{x}_2(t) + M_2 g \sin \theta + M_2 \ddot{x}_d(t) - \left(\frac{\beta * p}{q} * \dot{e}_2^{\frac{p-q}{p}} * M_2 * \dot{e}_2\right) \quad (37a)$$

$$F_{12}(t) = -H_{02} \text{sign}(s_2) \quad (37b)$$

$$F_{22}(t) = -\Omega_2 |F_{02}| \text{sign}(s_2) \quad (37c)$$

$$F_{32}(t) = -k_s \text{sign}(s_2) \quad (37d)$$

As well for the last vehicle the control force becomes;

$$F_{03}(t) = k(\Delta x_{3,2}) + a_3(t) + b_3(t) * \dot{x}_3(t) + M_3 g \sin \theta + M_3 \ddot{x}_d(t) - \left(\frac{\beta * p}{q} * \dot{e}_3^{\frac{p-q}{p}} * M_3 * \dot{e}_3\right) \quad (38a)$$

$$F_{13}(t) = -H_{03} \text{sign}(s_3) \quad (38b)$$

$$F_{23}(t) = -\Omega_3 |F_{03}| \text{sign}(s_3) \quad (38c)$$

$$F_{33}(t) = -k_s \text{sign}(s_3) \quad (38d)$$

Where,  $k_s$  is a positive design parameter designed later,  $H_0$  is the upper bound of  $P$ 's and  $\Omega$  is a compensation term for actuator effectiveness fault and expressed as;

$$H_0 = \frac{H_0^*}{1-\delta} \quad (39)$$

$$\Omega = \frac{\delta}{1-\delta} \quad (40)$$

$$H_0^* \geq b_0 + b_1 |\dot{x}| + b_2 |\dot{x}^2| + b_3 |\ddot{x}| \quad (41)$$

Where,  $\delta$  is upper bound were even if the actuator fails it stile deliver traction force as well as braking force.

**Assumption 4:** In this paper, parameter  $\delta$  is introduced and assumed that  $0 \leq \tau_c < \delta < 1$  satisfied, where  $\delta$  is unknown.

**Remark 3:** in assumption 4,  $0 \leq \tau_c < \delta < 1$  shows that even if the actuator losses its effectiveness, it still can offer required traction force as well braking force to enforce a train to track the specified trajectories. If the actuator fails extremely and the produced forces which is traction or braking force is not sufficient to attain the train tracking function, other actions need to be used. Therefore, Assumption 4 is reasonable.

**Remark 4:** The uncertainty caused by the unknown actuator faults is needed to be compensated. Therefore,  $\Omega$  term is utilized to recompense the uncertainty due to actuator effectiveness fault.

The total tracking control force for the train in (13) is written as;

$$U_i = F_{0i}(t) + F_{1i}(t) + F_{2i}(t) + F_{3i}(t) \quad (42)$$

Where,  $i = 3$  and represents the number of the trains.

In dealing with the reachability condition of the TSM control we utilized a reaching law. The advantage of this reaching law is its straightforwardness. But, the upper bound of uncertainties  $k_s$ , is problematic to obtain accurately in advance for real time uses if  $k_s$  is very minimum, the reaching time will be very large. On the other hand, a very large  $k_s$  value will produce high chattering.

#### 5.2.4. Chattering problem

The control in sliding mode control schemes adopts a switching function, that produces high frequency oscillations in a system states (excite unstable system dynamics), i.e. chattering due to implementation imperfections. The issue of chattering is required to be solved in SMC control framework. A number of techniques have been developed for addressing chattering problem, such as boundary layer method, high-order sliding-mode method, and disturbance estimation method.

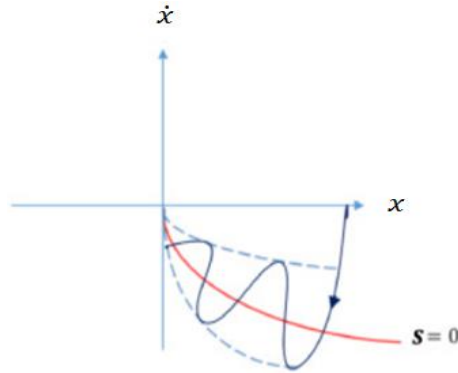


Figure 5.2: Sliding surface with chattering[18]

The high-order sliding-mode technique is to hide the discontinuity of control in its higher derivatives. On the other hand, an adaptive algorithm for determining the bound of lumped uncertainties was proposed to attune the chattering phenomena of the control effort. But, the accumulative implementation of the adaptive algorithm permanently contains a positive value so that the tracking error introduced by any uncertainty, such as sensor error, will cause the estimated bound increase even to infinity with time. This results in the actuator ultimately saturated and the system to be unstable. Therefore, in this paper the boundary layer technique is used to reduce the chattering effect by replacing the discontinuous part of the equation that repeatedly alter the sign of the control force by saturation function.

As, the boundary layer technique is to take account for saturation function approaches. Therefore, the signm function in the equation (36)-(38) is replaced by the saturation function as follows; i.e;

$$\text{sign}(s) = \text{sat}(s) \quad (43)$$

### 5.3. Adaptive Control Design

#### 5.3.1. Definitions and Preliminaries

The Adaptive control is a control technique applied in a controller that need adjust itself to a controlled scheme with variable parameters, or initially uncertain parameters. As an effective control technique for studying uncertain and unpredictable systems, a widely used method in various control systems is an adaptive control. This controller is made by combining an on-line parameter estimator with a control law driven by known parameter conditions. The online parameter estimator offers an approximation of unknown parameter at all times[38][39][40].

**Classification of adaptive control technique:** The collaboration of parameter estimator (also called adaptive law) and control law produces two different adaptive control methods. Such as;

- (i). In-direct adaptive control and
- (ii). Direct adaptive control approaches

**(i). In-direct adaptive control:** in this method controller parameters is calculated from the plant parameters but, in which they primary estimated on-line. This method is referred to as explicit adaptive control, as the strategy is created on explicit plant model.

**(ii). Direct adaptive control:** in this method a plant model is parameterized in relations of a controller parameters that are estimated directly deprived of intermediary calculations concerning plant parameter estimates. This approach is known as implicit adaptive control since the strategy is built on an implicit plant model.

**On-Line Parameter Estimators:** Adaptive control is a control technique utilized by a controller that needs to adjust uncertain parameters within a controlled system. The base of adaptive control is parameter estimation, which is the branch of system identification.

In the collected works of adaptive control the on-line parameter estimator has been called as adaptive law, update law, or adjustment mechanism. In this thesis referred as adaptive law. The vital thing for the stability of the adaptive control is the design of the adaptive law. But, what makes the stability and robustness understanding and analysis of this control more challenging is that the adaptive law can bring a multiplicative nonlinearity which makes a closed-loop plant nonlinear and often time varying.

Some of the main approaches used to design adaptive laws are;

- (i) Sensitivity methods
- (ii) Positivity and Lyapunov design
- (iii) Gradient method and least-squares methods based on estimation error cost criteria

**(i). Sensitivity methods:** in this method the estimated parameters are attuned in a way that reduces a particular operation execution. The adaptive law is calculated as the partial derivative of a performance function respect to estimated parameters is multiplied by an error signal which describes the incongruity among the real and required behavior. This derivative is called as sensitivity function. For most design of adaptive control, the sensitivity function cannot be obtained online because typically it depends on the unknown plant parameters so, it is unattainable. In these circumstances, approximate values are used.

A widely held technique for calculating an estimated sensitivity functions is called MIT rule. Through this rule the unknown parameters that are required to produce the sensitivity functions are substituted by their on-line estimates. Globally, a system based on MIT rule or other approximation may go unstable and this is the main drawbacks of the method.

**(ii). Positivity and Lyapunov design:** This way of formulating adaptive laws is based on the direct technique of Lyapunov and its relationship with positive real functions. The issue of planning the adaptive law is defined as a stability issue where the selected adaptive law differential equation condition is satisfied based on the Lyapunov theory. The adaptive law created is exceptionally comparable to the sensitivity strategy based technique but, the only dissimilarity is that the sensitivity functions in approach (i) is substituted by the online generated ones and moreover, in the Lyapunov-based adaptive control method there is no downsides which are in the MIT rule-based methods.

**(iii). Gradient and least-squares methods:** this technique is established on estimation error cost criteria. A most downside of this strategy is the reduction of the execution cost function takes to sensitivity functions which are not practicable. One method to maintain a strategic distance from this downside is to select a cost function criterion which directs to sensitivity functions that are accessible for determine their values.

### 5.3.2. Adaptive Control Law design

The stated in above proposed controller is based on adaptive control theory. Because, by the utilization of adaptive controller there's no ought to have a priori data regarding the bounds on uncertain parameters and the controller is competent of adapting and adjusting itself to existing situations.

As a nonlinear control scheme for the trains, the Lyapunov direct system is utilized as a central tool for control design. The ordinary method is to take a Lyapunov function to be quadratic in terms of errors and attempt to make its derivative negative definite. This is typically done

through canceling nonlinear terms and adding design parameters fulfilling particular bounds. As mentioned in above the parameters  $a, b, c$  and  $M$  are unknown and vary at certain conditions. Therefore,  $a, b, c$  and  $M$  in equation (36-38) replaced by  $\hat{a}, \hat{b}, \hat{c}$  and  $\hat{M}$ .

Rewriting equation (36-38) by substituting equation (43) and by replacing all the uncertain time varying parameters with their estimate parameters;

$$F_{01}(t) = k(\Delta x_{1,2}) + \hat{a}_1(t) + \hat{b}_1(t) * \dot{x}_1(t) + \hat{c}_1(t) * \dot{x}_1^2(t) + \hat{M}_1 g \sin \theta + \hat{M}_1 \ddot{x}_d(t) - \left( \frac{\beta * p}{q} * \dot{e}_1^{\frac{p-q}{p}} * \hat{M}_1 * \dot{e}_1 \right) \quad (44a)$$

$$F_{11}(t) = -H_{01} \text{sat}(s_1) \quad (44b)$$

$$F_{21}(t) = -\hat{\Omega}_1 |F_{01}| \text{sat}(s_1) \quad (44c)$$

$$F_{31}(t) = -k_s \text{sat}(s_1) \quad (44d)$$

For the second vehicle;

$$F_{02}(t) = k(\Delta x_{2,1} + \Delta x_{2,3}) + \hat{a}_2(t) + \hat{b}_2(t) * \dot{x}_2(t) + \hat{M}_2 g \sin \theta + \hat{M}_2 \ddot{x}_d(t) - \left( \frac{\beta * p}{q} * \dot{e}_2^{\frac{p-q}{p}} * \hat{M}_2 * \dot{e}_2 \right) \quad (45a)$$

$$F_{12}(t) = -H_{02} \text{sat}(s_2) \quad (45b)$$

$$F_{22}(t) = -\hat{\Omega}_2 |F_{02}| \text{sat}(s_2) \quad (45c)$$

$$F_{32}(t) = -k_s \text{sat}(s_2) \quad (45d)$$

For the last vehicle;

$$F_{03}(t) = k(\Delta x_{3,2}) + \hat{a}_3(t) + \hat{b}_3(t) * \dot{x}_3(t) + \hat{M}_3 g \sin \theta + \hat{M}_3 \ddot{x}_d(t) - \left( \frac{\beta * p}{q} * \dot{e}_3^{\frac{p-q}{p}} * \hat{M}_3 * \dot{e}_3 \right) \quad (46a)$$

$$F_{13}(t) = -H_{03} \text{sat}(s_3) \quad (46b)$$

$$F_{23}(t) = -\hat{\Omega}_3 |F_{03}| \text{sat}(s_3) \quad (46c)$$

$$F_{33}(t) = -k_s \text{sat}(s_3) \quad (46d)$$

Where,  $\hat{a}, \hat{b}, \hat{c}, \hat{M}$  and  $\hat{\Omega}$  are the estimates of  $a, b, c, M$  and  $\Omega$  and adaptive control method is proposed for on-line estimation of thus unknown parameters.

Therefore, the control law for the train in (13), which considers the parameter uncertainties, becomes the summation of equation (44a-d) up to (46a-d).

### 5.3.3. Adaptive Updating parameters selection

Therefore, based on Lyapunov function the adaptive updating laws are selected as follows for each uncertain parameter;

$$\dot{\hat{a}}_i = -\gamma_a * s_i * \frac{q}{\beta p} * \dot{e}_i^{\frac{q-p}{p}} \quad (47)$$

$$\dot{\hat{b}}_i = -\gamma_b * s_i * \frac{q}{\beta p} * \dot{e}_i^{\frac{q-p}{p}} * \dot{x}_i(t) \quad (48)$$

$$\dot{\hat{c}}_1 = -\gamma_c * s_1 * \frac{q}{\beta p} * \dot{e}_1^{\frac{q-p}{p}} * \dot{x}_1^2(t) \quad (49)$$

$$\dot{\hat{M}}_i = \gamma_m * s_i * (\dot{e}_i(t) - \frac{q}{\beta p} * \dot{e}_i^{\frac{q-p}{p}} (g \sin \theta + \ddot{x}_d(t))) \quad (50)$$

$$\dot{\hat{\Omega}}_i = \gamma_{\Omega} * |s_i| * |F_{0i}| * \frac{q}{\beta p} * \dot{e}_i^{\frac{q-p}{p}} \quad (51)$$

Where,  $\gamma_a, \gamma_b, \gamma_c, \gamma_m$  and  $\gamma_{\Omega}$  is positive design parameters designed later and  $i = 1..3$ , represents the number of trains and in  $\hat{c}$  remains as 1 because it only acts in the leading train.

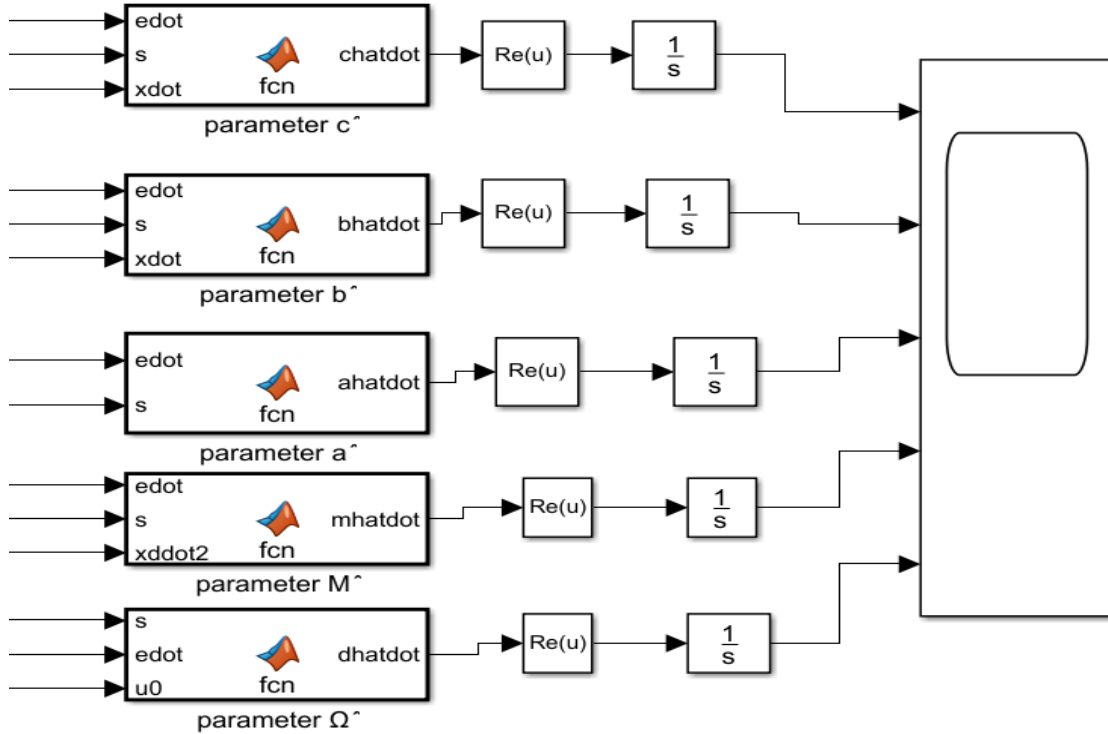


Figure 5.3: adaptive block diagram for online estimated parameters

## Proof of the proposed controller

In common thought, a most reason of a sliding mode control comprises in taking back to the state trajectory towards a sliding surface and to enforce it to slide over this surface until coming to the steadiness point. The sliding mode exists when  $\dot{s} < 0$ . This condition is based on Lyapunov quadratic function. In fact, the effectiveness of Lyapunov based control algorithms for controlling linear and nonlinear systems subject to disturbances is proven. In this way, the existence condition of sliding mode control can be satisfied by the candidate Lyapunov function:  $v = \frac{1}{2}ms^2$  [27].

On this paper, Lyapunov direct method with its relations with positive real functions approach is utilized for creating adaptive laws. In this method, the issue of designing an adaptive law is framed as a stability issue where the differential equation of the adaptive law is selected so that certain stability conditions based on Lyapunov theory are fulfilled.

$$v = \frac{1}{2}ms^2 \quad (52)$$

Defines Lyapunove function as;

$$v_i = \frac{1}{2} * M_i * s_i^2 + \frac{1}{2*\gamma_a} * \tilde{a}_i^2 + \frac{1}{2*\gamma_b} * \tilde{b}_i^2 + \frac{1}{2*\gamma_c} * \tilde{c}_1^2 + \frac{1}{2*\gamma_m} * \tilde{M}_i^2 + \frac{1-\delta}{2*\gamma_\Omega} * \tilde{\Omega}_i^2 \quad (53)$$

Where  $\gamma_a, \gamma_b, \gamma_c, \gamma_m$  and  $\gamma_\Omega$  are positive design parameters to be chosen,  $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{M}$  and  $\tilde{\Omega}$  are error terms and  $i = 1..3$ , represents the number of trains.

The control law uses the estimation parameters rather than the ideal values, because of the difficulty to get the ideal values in operation due to parameter variation on operating conditions. Therefore, introducing error terms to define the difference between the ideal and estimated values as;

$$\tilde{a}_i = a_i - \hat{a}_i \quad (54)$$

$$\tilde{b}_i = b_i - \hat{b}_i \quad (55)$$

$$\tilde{c}_1 = c_1 - \hat{c}_1 \quad (56)$$

$$\tilde{M}_i = M_i - \hat{M}_i \quad (57)$$

$$\tilde{\Omega}_i = \Omega_i - \hat{\Omega}_i \quad (58)$$

Taking the first derivative of  $v$ ;

$$\dot{v}_i = (M_i * s_i * \dot{s}_i) + \frac{1}{\gamma_a} * \tilde{a}_i * \dot{\tilde{a}}_i + \frac{1}{\gamma_b} * \tilde{b}_i * \dot{\tilde{b}}_i + \frac{1}{\gamma_c} * \tilde{c}_1 * \dot{\tilde{c}}_1 + \frac{1}{\gamma_m} * \tilde{M}_i * \dot{\tilde{M}}_i + \frac{1-\delta}{\gamma_\Omega} * \tilde{\Omega}_i * \dot{\tilde{\Omega}}_i \quad (59)$$

By substituting all the error terms equations (54) up to (58) in to (59) yields;

$$\begin{aligned} \dot{v}_i = (M_i * s_i * \dot{s}_i) + \frac{1}{\gamma_a} * \tilde{a}_i * (\dot{a}_i - \hat{\dot{a}}_i) + \frac{1}{\gamma_b} * \tilde{b}_i * (\dot{b}_i - \hat{\dot{b}}_i) + \frac{1}{\gamma_c} * \tilde{c}_1 * (\dot{c}_1 - \hat{\dot{c}}_1) + \frac{1}{\gamma_m} * \tilde{M}_i * \\ (\dot{M}_i - \hat{\dot{M}}_i) + \frac{1-\delta}{\gamma_\Omega} * \tilde{\Omega}_i * (\dot{\Omega}_i - \hat{\dot{\Omega}}_i) \end{aligned} \quad (60)$$

But the derivative of the constant parameters cancelled out and remains;

$$\begin{aligned} \dot{v}_i = (M_i * s_i * \dot{s}_i) - \left( \frac{1}{\gamma_a} * \tilde{a}_i * \hat{\dot{a}}_i \right) - \left( \frac{1}{\gamma_b} * \tilde{b}_i * \hat{\dot{b}}_i \right) - \left( \frac{1}{\gamma_c} * \tilde{c}_1 * \hat{\dot{c}}_1 \right) - \left( \frac{1}{\gamma_m} * \tilde{M}_i * \hat{\dot{M}}_i \right) - \\ \left( \frac{1-\delta}{\gamma_\Omega} * \tilde{\Omega}_i * \hat{\dot{\Omega}}_i \right) \end{aligned} \quad (61)$$

Substituting (32) in to (61) gives;

$$\begin{aligned} \dot{v}_i = s_i \left( M_i \dot{e}_i + \left( \frac{q}{\beta * p} * \dot{e}_i^{\frac{q-p}{p}} \right) \left( (1 - \tau_c) F_i(t) - k\xi - a_i(t) - b_i(t) * \dot{x}_i(t) - c_1(t) * \dot{x}_1^2(t) - M_i g \sin \theta + \right. \right. \\ \left. \left. P_i - M_i \ddot{x}_d \right) \right) - \left( \frac{1}{\gamma_a} * \tilde{a}_i * \hat{\dot{a}}_i \right) - \left( \frac{1}{\gamma_b} * \tilde{b}_i * \hat{\dot{b}}_i \right) - \left( \frac{1}{\gamma_c} * \tilde{c}_1 * \hat{\dot{c}}_1 \right) - \left( \frac{1}{\gamma_m} * \tilde{M}_i * \hat{\dot{M}}_i \right) - \left( \frac{1-\delta}{\gamma_\Omega} * \tilde{\Omega}_i * \hat{\dot{\Omega}}_i \right) \end{aligned} \quad (62)$$

Where,  $\xi = (\Delta x_{1,2}), (\Delta x_{2,1} + \Delta x_{2,3})$  and  $(\Delta x_{3,2})$  for the first, second and third train respectively and utilized accordingly.

Substituting equation (42) in to (62);

$$\begin{aligned} \dot{v}_i = s_i \left( M_i \dot{e}_i + \left( \frac{q}{\beta * p} * \dot{e}_i^{\frac{q-p}{p}} \right) \left( (1 - \tau_c) U_i(t) - k\xi - a_i(t) - b_i(t) * \dot{x}_i(t) - c_1(t) * \dot{x}_1^2(t) - M_i g \sin \theta + \right. \right. \\ \left. \left. P_i - M_i \ddot{x}_d \right) \right) - \left( \frac{1}{\gamma_a} * \tilde{a}_i * \hat{\dot{a}}_i \right) - \left( \frac{1}{\gamma_b} * \tilde{b}_i * \hat{\dot{b}}_i \right) - \left( \frac{1}{\gamma_c} * \tilde{c}_1 * \hat{\dot{c}}_1 \right) - \left( \frac{1}{\gamma_m} * \tilde{M}_i * \hat{\dot{M}}_i \right) - \left( \frac{1-\delta}{\gamma_\Omega} * \tilde{\Omega}_i * \hat{\dot{\Omega}}_i \right) \end{aligned} \quad (63)$$

Now substituting equation (46-48)(( $a_i$ )-(  $d_i$ )) in to (63) gives;

$$\begin{aligned} \dot{v}_i = s_i \left( M_i \dot{e}_i + \left( \frac{q}{\beta * p} * \dot{e}_i^{\frac{q-p}{p}} \right) \left( (1 - \tau_c) (F_{0i}(t) + F_{1i}(t) + F_{2i}(t) + F_{3i}(t)) - k\xi - a_i(t) - b_i(t) * \right. \right. \\ \left. \left. \dot{x}_i(t) - c_1(t) * \dot{x}_1^2(t) - M_i g \sin \theta + P_i - M_i \ddot{x}_d \right) \right) - \left( \frac{1}{\gamma_a} * \tilde{a}_i * \hat{\dot{a}}_i \right) - \left( \frac{1}{\gamma_b} * \tilde{b}_i * \hat{\dot{b}}_i \right) - \left( \frac{1}{\gamma_c} * \tilde{c}_1 * \hat{\dot{c}}_1 \right) - \\ \left( \frac{1}{\gamma_m} * \tilde{M}_i * \hat{\dot{M}}_i \right) - \left( \frac{1-\delta}{\gamma_\Omega} * \tilde{\Omega}_i * \hat{\dot{\Omega}}_i \right) \end{aligned} \quad (64)$$

$$\begin{aligned} \dot{v}_i = s_i \left( M_i \dot{e}_i + \left( \frac{q}{\beta * p} * \dot{e}_i^{\frac{q-p}{p}} \right) \left( F_{0i}(t) - \tau_c F_{0i}(t) + (1 - \tau_c) F_{1i}(t) + (1 - \tau_c) F_{2i}(t) + (1 - \tau_c) F_{3i}(t) \right) - \right. \\ \left. k\xi - a_i(t) - b_i(t) * \dot{x}_i(t) - c_1(t) * \dot{x}_1^2(t) - M_i g \sin \theta + P_i - M_i \ddot{x}_d \right) - \left( \frac{1}{\gamma_a} * \tilde{a}_i * \hat{\dot{a}}_i \right) - \left( \frac{1}{\gamma_b} * \tilde{b}_i * \hat{\dot{b}}_i \right) - \\ \left( \frac{1}{\gamma_c} * \tilde{c}_1 * \hat{\dot{c}}_1 \right) - \left( \frac{1}{\gamma_m} * \tilde{M}_i * \hat{\dot{M}}_i \right) - \left( \frac{1-\delta}{\gamma_\Omega} * \tilde{\Omega}_i * \hat{\dot{\Omega}}_i \right) \end{aligned} \quad (65)$$

$$\begin{aligned}
\dot{v}_i = s_i & \left( M_i \dot{e}_i + \left( \frac{q}{\beta^{*p}} * \dot{e}_i^{\frac{q-p}{p}} \right) \left( k\xi + \hat{a}_i(t) + \hat{b}_i(t) * \dot{x}_i(t) + \hat{c}_1(t) * \dot{x}_1^2(t) + \hat{M}_i g \sin \theta + \hat{M}_i \ddot{x}_d(t) - \right. \right. \\
& \left. \left( \frac{\beta^{*p}}{q} * \dot{e}_i^{\frac{q-p}{p}} * \hat{M}_i * \dot{e}_i \right) - \tau_c F_{0i}(t) - (1 - \tau_c) H_{0i} \text{sign}(s_i) - (1 - \tau_c) \hat{\Omega}_i |F_{0i}| \text{sign}(s_i) - (1 - \right. \\
& \left. \tau_c) k_s \text{sign}(s_i) - k\xi - a_i(t) - b_i(t) * \dot{x}_i(t) - c_1(t) * \dot{x}_1^2(t) - M_i g \sin \theta + P_i - M_i \ddot{x}_d(t) \right) - \\
& \left( \frac{1}{\gamma_a} * \tilde{a}_i * \hat{a}_i \right) - \left( \frac{1}{\gamma_b} * \tilde{b}_i * \hat{b}_i \right) - \left( \frac{1}{\gamma_c} * \tilde{c}_1 * \hat{c}_1 \right) - \left( \frac{1}{\gamma_m} * \tilde{M}_i * \hat{M}_i \right) - \left( \frac{1-\delta}{\gamma_\Omega} * \tilde{\Omega}_i * \hat{\Omega}_i \right) \quad (66)
\end{aligned}$$

As noted in above, the variation among the original parameter value and the estimated parameter values is defined by the error terms in equation from (54) up to (58). Therefore, using thus error term equation in (66) gives;

$$\begin{aligned}
\dot{v}_i \leq s_i & \left( \tilde{M}_i \dot{e}_i + \left( \frac{q}{\beta^{*p}} * \dot{e}_i^{\frac{q-p}{p}} \right) \left( -\tilde{a}_i - \tilde{b}_i * \dot{x}_i(t) - \tilde{c}_1 * \dot{x}_1^2(t) - \tilde{M}_i g \sin \theta - \tilde{M}_i \ddot{x}_d(t) + P_i - \right. \right. \\
& \left. \left( (1 - \tau_c) H_{0i} \text{sign}(s_i) - \tau_c F_{0i}(t) - (1 - \tau_c) \hat{\Omega}_i |F_{0i}| \text{sign}(s_i) - (1 - \tau_c) k_s \text{sign}(s_i) \right) \right) - \\
& \left( \frac{1}{\gamma_a} * \tilde{a}_i * \hat{a}_i \right) - \left( \frac{1}{\gamma_b} * \tilde{b}_i * \hat{b}_i \right) - \left( \frac{1}{\gamma_c} * \tilde{c}_1 * \hat{c}_1 \right) - \left( \frac{1}{\gamma_m} * \tilde{M}_i * \hat{M}_i \right) - \left( \frac{1-\delta}{\gamma_\Omega} * \tilde{\Omega}_i * \hat{\Omega}_i \right) \quad (67)
\end{aligned}$$

Categorizing the same parameters;

$$\begin{aligned}
\dot{v}_i \leq s_i & \left( \left( \frac{q}{\beta^{*p}} * \dot{e}_i^{\frac{q-p}{p}} \right) \left( P_i - (1 - \tau_c) H_{0i} \text{sign}(s_i) - (1 - \tau_c) k_s \text{sign}(s_i) \right) \right) - \tilde{a}_i \left( s_i * \frac{q}{\beta^{*p}} * \dot{e}_i^{\frac{q-p}{p}} + \frac{1}{\gamma_a} * \right. \\
& \hat{a}_i \left. \right) - \tilde{b}_i * \dot{x}_i(t) \left( s_i * \frac{q}{\beta^{*p}} * \dot{e}_i^{\frac{q-p}{p}} + \frac{1}{\gamma_b} * \hat{b}_i \right) - \tilde{c}_1 * \dot{x}_1^2(t) \left( s_i * \frac{q}{\beta^{*p}} * \dot{e}_i^{\frac{q-p}{p}} + \frac{1}{\gamma_c} * \hat{c}_1 \right) + \tilde{M}_i \left( s_i \left( \dot{e}_i - \frac{q}{\beta^{*p}} * \right. \right. \\
& \left. \left. \dot{e}_i^{\frac{q-p}{p}} (g \sin \theta + \ddot{x}_d(t)) \right) - \frac{1}{\gamma_m} * \hat{M}_i \right) + \left( \frac{q}{\beta^{*p}} * \dot{e}_i^{\frac{q-p}{p}} \right) s_i \left( -\tau_c F_{0i}(t) - (1 - \tau_c) \hat{\Omega}_i |F_{0i}| \text{sign}(s_i) \right) - \\
& \left( \frac{1-\delta}{\gamma_\Omega} * \tilde{\Omega}_i * \hat{\Omega}_i \right) \quad (68)
\end{aligned}$$

Substituting the updating laws from (47)-(51) for the first derivative of the estimated parameters and substituting (40) for  $H_0$  and since  $H_0^* \cong P$ ;

$$\begin{aligned}
\dot{v}_i \leq s_i & \left( \left( \frac{q}{\beta^{*p}} * \dot{e}_i^{\frac{q-p}{p}} \right) \left( P_i - (1 - \tau_c) \frac{H_{0i}^*}{(1-\delta)} \text{sign}(s_i) - (1 - \tau_c) k_s \text{sign}(s_i) \right) \right) - \tilde{a}_i(t) \left( s_i * \frac{q}{\beta^{*p}} * \dot{e}_i^{\frac{q-p}{p}} + \frac{1}{\gamma_a} * \right. \\
& \left( -\gamma_a * s_i * \frac{q}{\beta^p} * \dot{e}_i^{\frac{q-p}{p}} \right) \left. \right) - \tilde{b}_i(t) \left( s_i * \frac{q}{\beta^{*p}} * \dot{e}_i^{\frac{q-p}{p}} * \dot{x}_i(t) + \frac{1}{\gamma_b} * \left( -\gamma_b * s_i * \frac{q}{\beta^p} * \dot{e}_i^{\frac{q-p}{p}} * \dot{x}_i(t) \right) \right) - \tilde{c}_1(t) \left( s_i * \frac{q}{\beta^{*p}} * \right. \\
& \left. \dot{e}_1^{\frac{q-p}{p}} * \dot{x}_1^2(t) + \frac{1}{\gamma_c} * \left( -\gamma_c * s_1 * \frac{q}{\beta^p} * \dot{e}_1^{\frac{q-p}{p}} * \dot{x}_1^2(t) \right) \right) + \tilde{M}_i \left( s_i \left( \dot{e}_i(t) - \frac{q}{\beta^{*p}} * \dot{e}_i^{\frac{q-p}{p}} (g \sin \theta + \ddot{x}_d(t)) \right) - \frac{1}{\gamma_m} * \right. \\
& \left. (\gamma_m * s_i * (\dot{e}_i(t) - \frac{q}{\beta^p} * \dot{e}_i^{\frac{q-p}{p}} (g \sin \theta + \ddot{x}_d(t))) \right) + \left( \frac{q}{\beta^{*p}} * \dot{e}_i^{\frac{q-p}{p}} \right) s_i \left( -\tau_c F_{0i}(t) - (1 - \tau_c) \hat{\Omega}_i |F_{0i}| \text{sign}(s_i) \right) + \\
& \left( \frac{1-\delta}{\gamma_\Omega} * \tilde{\Omega}_i * \gamma_\Omega * |s_i| * |F_{0i}| * \frac{q}{\beta^p} * \dot{e}_i^{\frac{q-p}{p}} \right) \quad (69)
\end{aligned}$$

Since, for any real number which is defined as the product of its absolute value and its sign function can be written as;

$$|s| = s \text{sign}(s) \quad (70)$$

Satisfying assumption 3 and equation (40) and canceling out similar terms yields;

$$\dot{v}_i \leq - \left( \left( \frac{q}{\beta * p} * \dot{e}_i^{\frac{q-p}{p}} \right) (1 - \tau_c) k_s |s_i| \right) - \left( \frac{q}{\beta * p} * \dot{e}_i^{\frac{q-p}{p}} \right) \left( \delta |F_{0i}(t)| |s_i| \right) + \left( \frac{q}{\beta * p} * \dot{e}_i^{\frac{q-p}{p}} \right) (1 - \delta) \Omega_i |F_{0i}(t)| |s_i| \quad (71)$$

Substituting (39) for  $\Omega$  in (70) gives;

$$\dot{v}_i \leq - \left( \left( \frac{q}{\beta * p} * \dot{e}_i^{\frac{q-p}{p}} \right) (1 - \tau_c) k_s |s_i| \right) - \left( \frac{q}{\beta * p} * \dot{e}_i^{\frac{q-p}{p}} \right) \left( \delta |F_{0i}(t)| |s_i| \right) + \left( \frac{q}{\beta * p} * \dot{e}_i^{\frac{q-p}{p}} \right) (1 - \delta) \frac{\delta}{1 - \delta} |F_{0i}(t)| |s_i| \quad (72)$$

Canceling out similar terms;

$$\dot{v}_i \leq - \left( \left( \frac{q}{\beta * p} * \dot{e}_i^{\frac{q-p}{p}} \right) (1 - \tau_c) k_s |s_i| \right) \quad (73)$$

$$\dot{v}_i \leq - (1 - \tau_c) \frac{q}{\beta * p} * \dot{e}_i^{\frac{q-p}{p}} * k_s |s_i| < 0 \quad (74)$$

Therefore, we can recognize that  $v$  is a non-increasing function and satisfies  $v \leq 0$ . Consequently,  $v$  is bounded, that indicates a signals  $s, \tilde{a}, \tilde{b}, \tilde{c}$  and  $\tilde{M}$  are bounded. In future, signals  $e, \dot{e}, U, \hat{a}, \hat{b}, \hat{c}$  and  $\hat{M}$  are bounded. According to (59) and (74), we can conclude that  $\dot{s}$  is bounded, that guarantees that  $\ddot{v}$  is bounded based on (74). Accordingly,  $\dot{v}$  is consistently continuous. Based on Barbalat's lemma[41], we can draw a conclusion  $\dot{v} \rightarrow 0$  when  $t \rightarrow \infty$ , which implies  $e \rightarrow 0$  and  $\dot{e} \rightarrow 0$  when  $t \rightarrow \infty$ . The proof is completed.

Subsequently, agreeing to the Lyapunov stability concept, the specified trajectory of a closed-loop system will be certainly convergence to a sliding surface and so, the error  $e$  will goes to zero, then the derivative of the error  $\dot{e}$  will goes zero as well if time is infinity, which implies that the designed controller can ensure the actual trajectory  $\dot{X}$  to follow the given  $\dot{X}_d$ , and the resultant closed-loop system can be globally stable. Therefore, Lyapunove function was satisfied

### 5.5. Reference Speed trajectory generation

A trajectory is the way that an object with mass in motion takes after through space as a function of time. Hence, the train operation function mainly consists five mode of operations[13]. In this paper, a trajectory generated consists of an acceleration phase, constant velocity phase, declaration phase, step velocity increase phase and step velocity decrease phases. The initial and final velocities are zero. The required trajectory was developed on Matlab and then imported to Simulink from the Matlab workspace.

## CHAPTER SIX

### 6. SIMULATION RESULT AND DISCUSSION

In today's world, railway train system is developing with high achievements and plays a significant role in a rapid mass transportation system. To make effective the running process of those vehicles in the rail ATO system were developed and performs automatic control of the train speed by the help of a controller which embed on it. Until now, different control methods were proposed and embed on ATO for the speed tracking of railway trains but, due to influencing factors such as unknown parameters, uncertainties in train dynamical model, external disturbance and actuator effectiveness faults they were limited to give an effective result in their performance.

In this research paper, speed trajectory tracking control problem of a train subject to unknown parameters, model uncertainties, external disturbances and actuator effectiveness faults based on adaptive terminal sliding mode control strategies was conducted by aiming to get a good result in tracking performance with guaranteed stability. To validate the efficiency of the designed control system, the adaptive terminal sliding mode controller is implemented on Matlab/Simulink simulation environment. The key simulation parameters of a train model are labeled as;

| Symbols    | Parameters                             | Values                        | Unit              |
|------------|----------------------------------------|-------------------------------|-------------------|
| M          | Mass                                   | 500                           | ton               |
| a          | Sliding resistance                     | $m \cdot 0.01176$             | N                 |
| b          | Rolling resistance                     | $m \cdot 0.00077616$          | $N \cdot s^2/m$   |
| c          | Aerodynamics drag force                | 8                             | $N \cdot s^2/m^2$ |
| g          | Gravitational constant                 | 9.8                           | $N/Kg$            |
| $\Delta M$ | Uncertainty on Mass                    | $1000 \cdot \text{rand}$      | ton               |
| $\Delta a$ | Uncertainty on Mechanical resistance   | $200 \cdot \text{rand}$       | N                 |
| $\Delta b$ | Uncertainty on basic resistance        | $30 \cdot \text{rand}$        | $N \cdot s^2/m$   |
| $\Delta c$ | Uncertainty on Aerodynamics drag force | $0.2 \cdot \text{rand}$       | $N \cdot s^2/m^2$ |
| $R_t$      | Tunnel resistance                      | $0.00013 l_e Mg$              | $N \cdot m$       |
| $l_e$      | Tunnel length                          | 1000                          | m                 |
| $R_c$      | Curve resistance                       | $10.5 \alpha_s mg / 1000 l_s$ | $N/m$             |
| $R_o$      | Other resistance                       | $1500 \sin(0.002vt)$          | N                 |
| $l_s$      | Curve length                           | 200                           | m                 |
| $\alpha_s$ | Center angle of the curve              | $2\pi/3$                      | Degree            |
| K          | Coupler spring stiffness coefficient   | $1 \cdot 10^5$                | -                 |
| $\theta$   | Ramp angle                             | $\pi/180$                     | degree            |
| -          | Rand                                   | 0.5                           | -                 |

Table 6.1: Parameters of the train

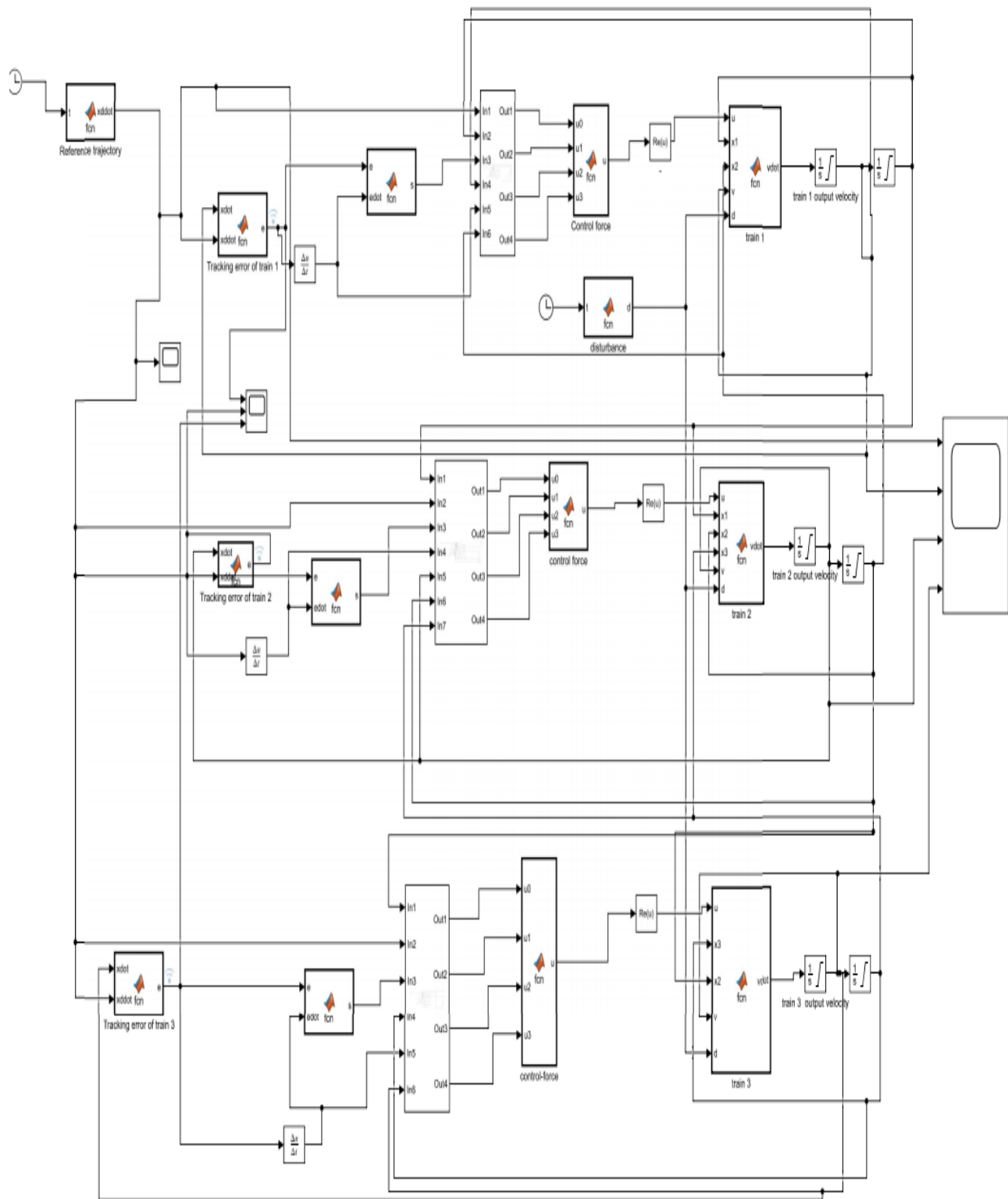


Figure 6.1: Adaptive terminal sliding mode control block diagram

The parameters of the outlined controller were chosen as;

| Parameters      | Values    |
|-----------------|-----------|
| $K_s$           | $10^7$    |
| $H_0$           | $10^5$    |
| $\beta$         | 5         |
| P               | 77        |
| q               | 79        |
| $\gamma_a$      | 10        |
| $\gamma_b$      | 0.1       |
| $\gamma_c$      | 0.01      |
| $\gamma_m$      | 1         |
| $\gamma_\Omega$ | 0.0000001 |
| k               | $10^5$    |

Table 6.2: Controller parameters

Where in practical time operation duration of a train there is a specified speed time curve, which is the reference trajectory that is provided to the train to follow accordingly. In this paper, the trajectory used for the simulation as a reference contains acceleration phase at initial, speed step increase stage, constant speed stage, speed step decrease stage and decelerating stage. As, shown below;

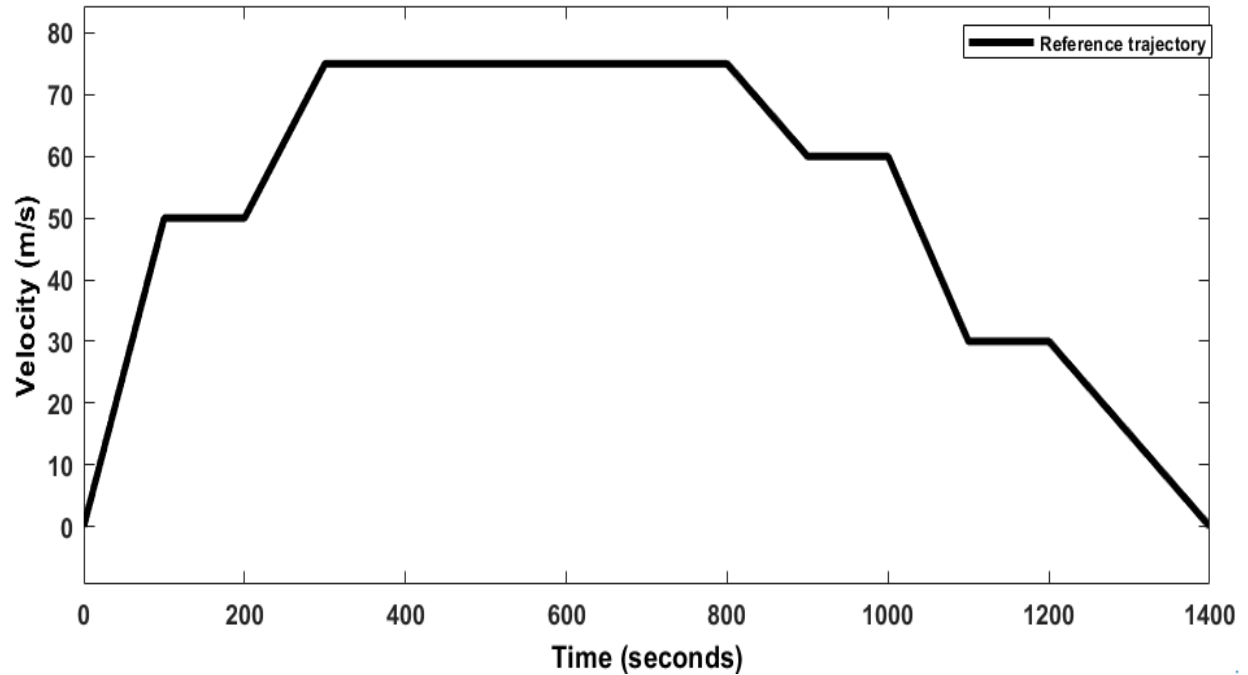


Figure 6.2: A reference trajectory for train operations between two stations

The reference trajectory is generated on the Matlab software based on the above description via the algorithm presented in the appendix.

### 6.1. Result and discussion for actuator fault free case

As control systems analysis and design primary objectives focuses on creating the required transient response, minimizing steady-state errors and attaining stability. Transient response can be settling time. Settling time is the time required for an output to reach and remain within a given error band following some input stimulus. Therefore, to approve the practicability of the offered controller for the trains in actuator fault-free condition, simulation for the trains devoid of actuator faults was carried out. The external disturbance is generated as;

$$d(t) = \begin{cases} R_c & t \in [0, 200] \\ R_t + R_c + R_o & t \in [200, 500] \\ R_t + R_o & t \in [500, 900] \\ R_o & t \in [900, 1400] \end{cases}$$

Which means the train runs in a curved rail for the first 200s therefore, there exist a curve resistance. For the time between 200s-500s the train runs in a curved rail as well in a tunnel with existence of unknown other disturbance as a result, thus three resistances exist. Although, for the time of 500s-900s if tunnel resistance and unknown other disturbance exists. For the last runs only there exist unknown other disturbance.

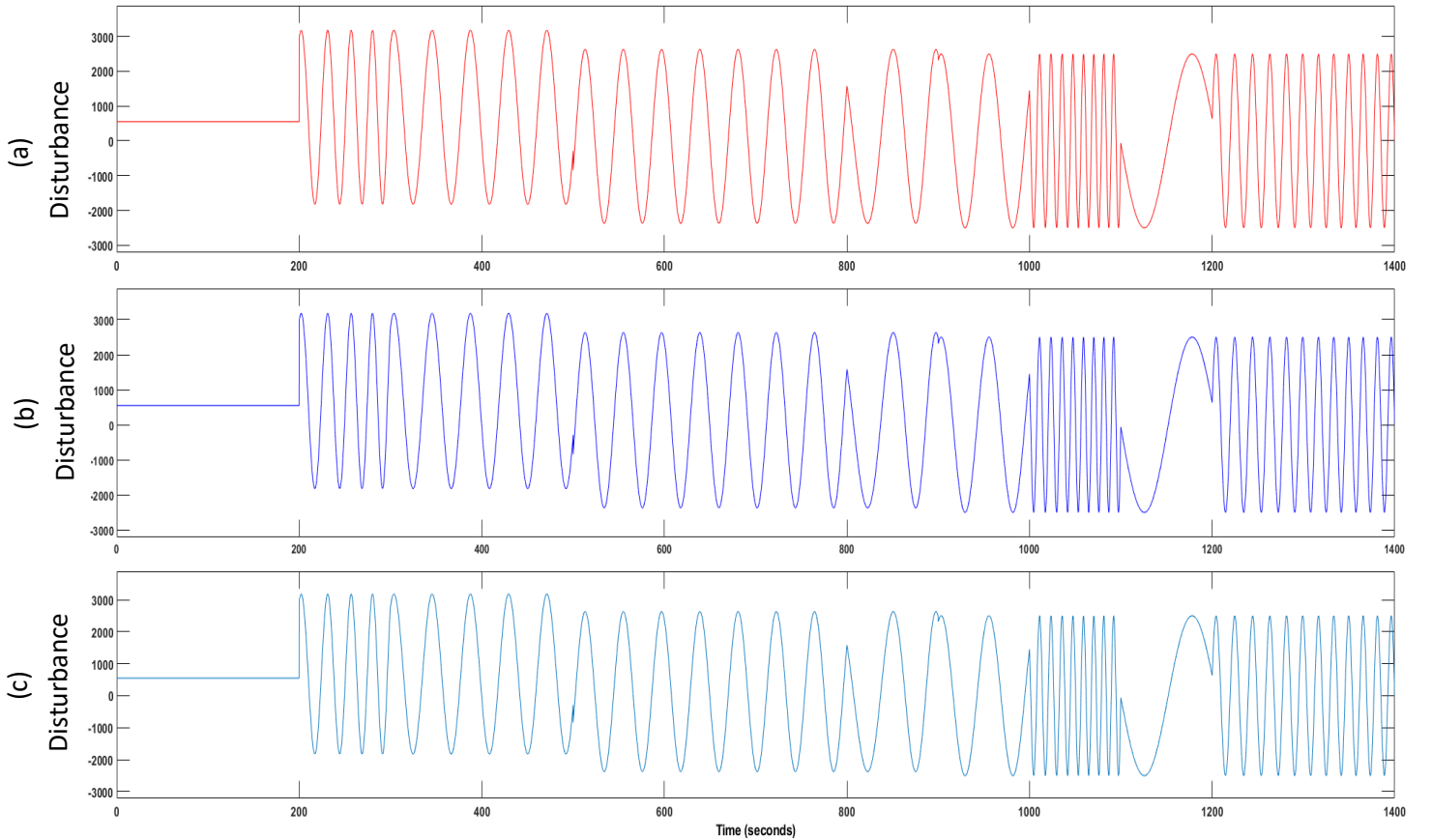


Figure 6.3: Disturbance applied for (a) train 1, (b) train 2 and (c) train 3 respectively.

The simulation results are shown in Figure 6.4 and 6.5.

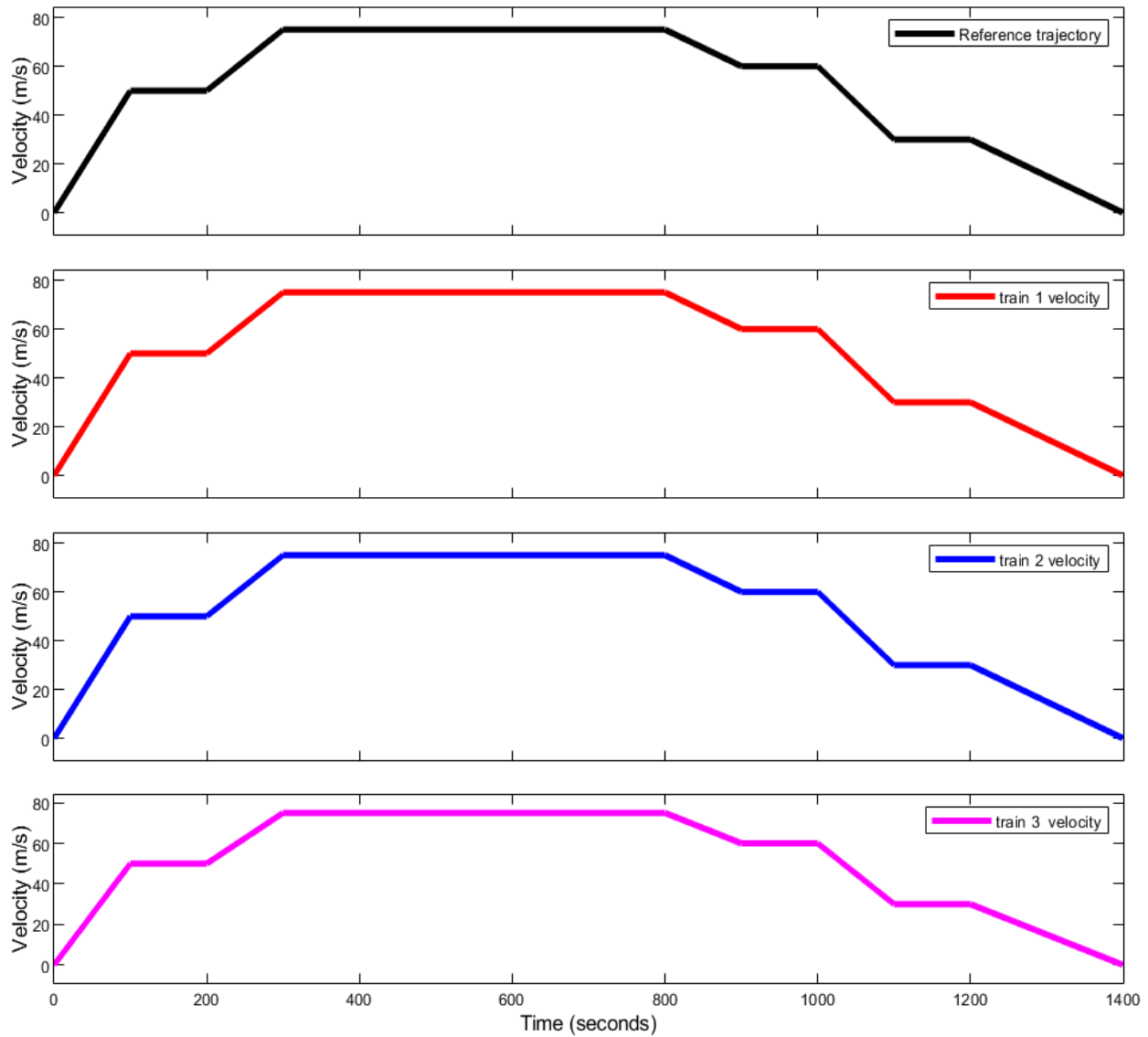


Figure 6.4: Controller output velocity of the trains

Figure 6.4 show that, how closely the output matches the desired reference trajectory and the simulation result shows that, the proposed controller for the tracking problem of the trains has a zero steady state error which determines the accuracy of the control system. Therefore, this shows the proposed controller can guarantee the accurate tracking of the reference speed trajectory and this proves the accuracy of the controller.

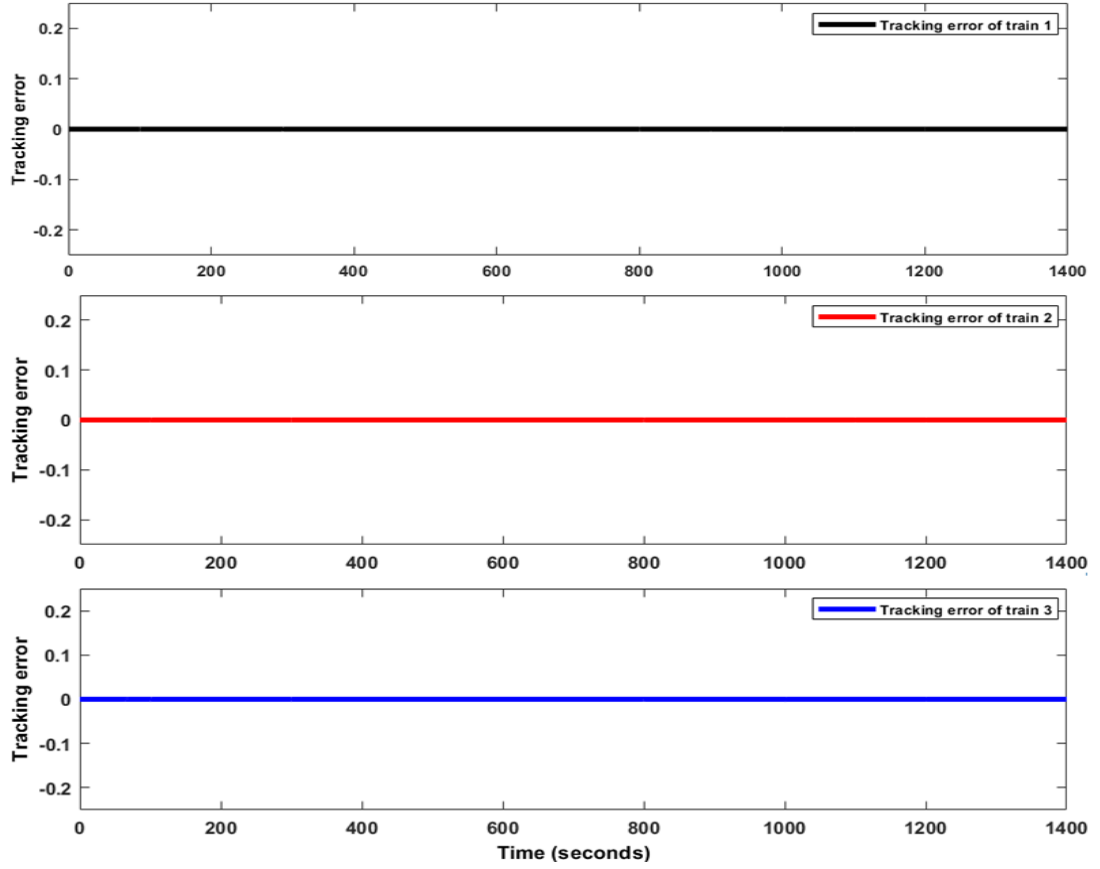


Figure 6.5: Speed tracking error of the trains

Whereas, from control point of view settling time is vital since it influences the speed of the train and impacts human tolerance and consolation and as it is also the main objective of this paper as it guarantees stability in a finite time. Figure 6.5 demonstrates that, the system settled within 3.134ms which means the controller drives the system tracking error to converge to zero within these microseconds. Therefore, as the system settled within short defined time stability in finite time is guaranteed.

In this paper, as saturation function is used to reduce chattering figure 6.6 demonstrates that, the actual control force is smooth and chattering free hence the proposed saturation function is effective in reducing the chattering. Therefore, the feasibility and effectiveness of the proposed controller for the trains despite in presence of unknown model parameters, parameter uncertainties and external disturbance is ensured.

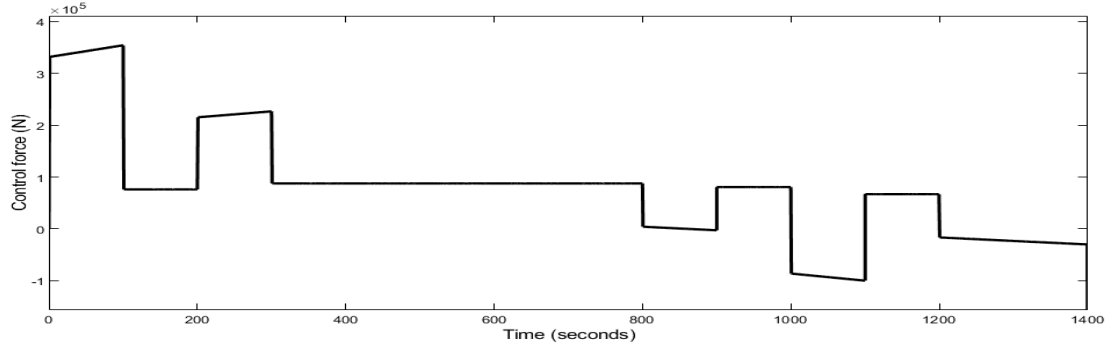


Figure 6.6: Actual control force

Therefore, Figure 6.4 and Figure 6.5 demonstrate the viability and effectiveness of the offered controller for the trains even with input nonlinearities, parameter uncertainties in the train dynamic model and additional disturbance.

## 6.2. Result and discussion for the actuator fault case

To validate the tracking ability of the proposed controller, simulation in distinctive extent of actuator faults are implemented. Different degrees of actuator faults are simulated through varying the rate of actuator adequacy. Assume that the actuator faults happen in 200 seconds and lasts until the end of the simulation. A 10%, 30% and 45% loss of actuator effectiveness, i.e,  $k_c = 0.9$ ,  $k_c = 0.7$  and  $k_c = 0.55$  are demonstrated within the simulation. The simulation outcomes are presented in figure 5.7-5.9.

- i. For the case if the actuator losses 10% of its effectiveness;

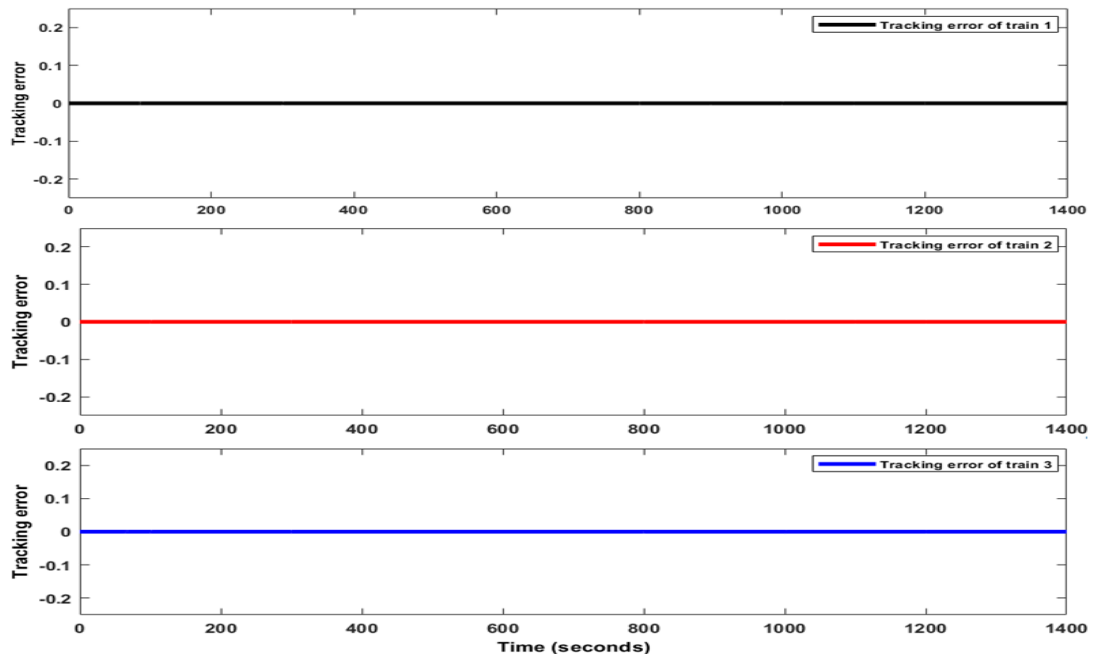


Figure 6.7: Speed tracking error of the trains if the actuator losses 10% of its effectiveness

As seen in figure 6.7, even if the controller faces 10% actuator effectiveness loss, the controller can adjust the train speed state to be close to a desired trajectory effectively with guaranteed finite stability as the same as without actuator faults case.

ii. For the case if the actuator losses 30% of its effectiveness;

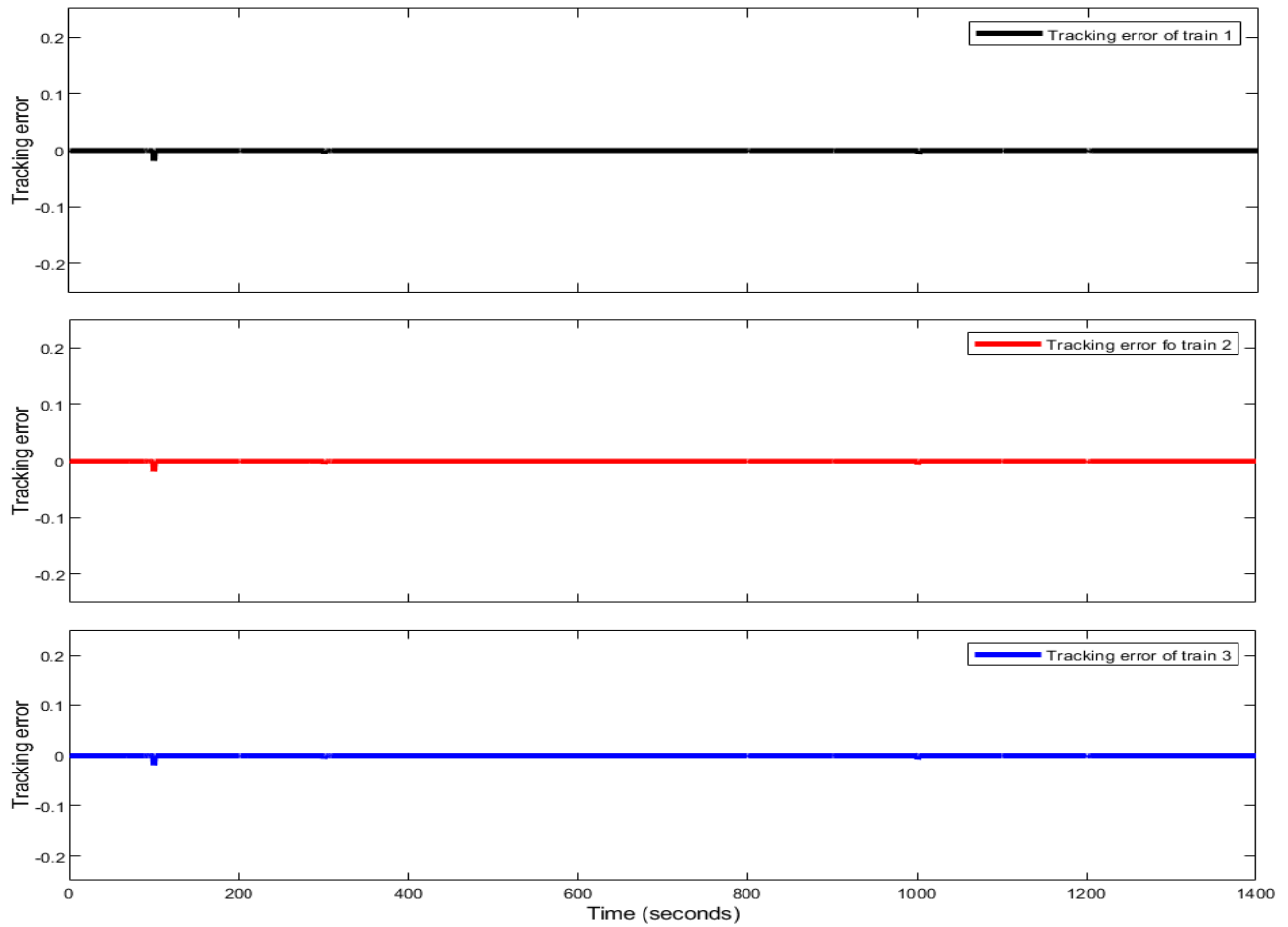


Figure 6.8: Speed tracking error if the actuator losses 30% of its effectiveness

When,  $k_c$  value decreases it means actuator effectiveness loss come to be more and more severe and the effect on train operation is emergent. But, as seen in figure 6.8, even if the controller 30% actuator effectiveness loss, the controller can adjust the train speed state to be close to a desired trajectory effectively. For this case the result shows some transient responses so the system only guarantees asymptotic stability rather than finite time stability. But, for both 10% and 30% fault cases zero steady state error is achieved.

iii. For the case if the actuator losses 45% of its effectiveness;

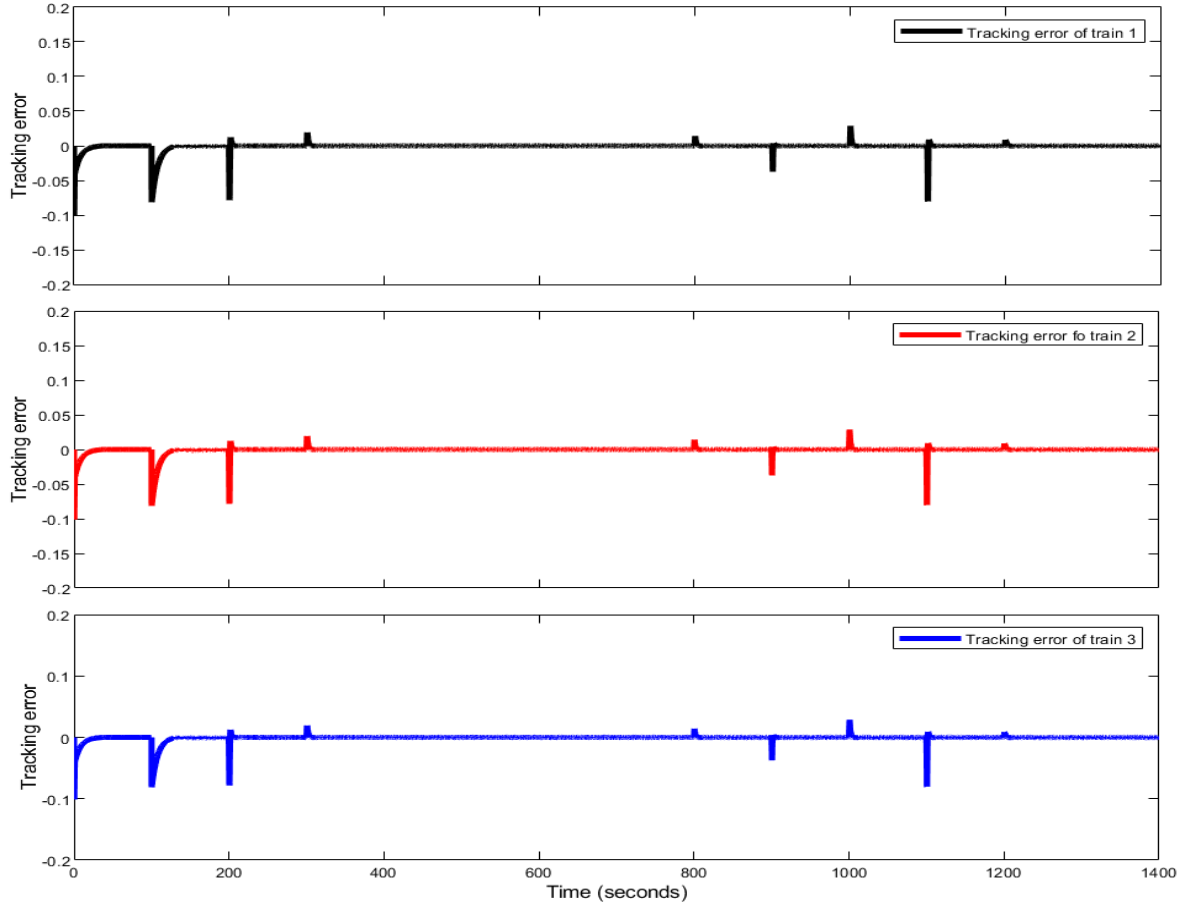


Figure 6.9: Speed tracking error if the actuator losses 45% actuator effectiveness loss

For the case of 45% actuator effectiveness losses, as seen in figure 6.9, the controller tried to regulate the train state to be close to a desired trajectory but there is high transient responses showed. Therefore, for this case the system is unstable and practically some action is required.

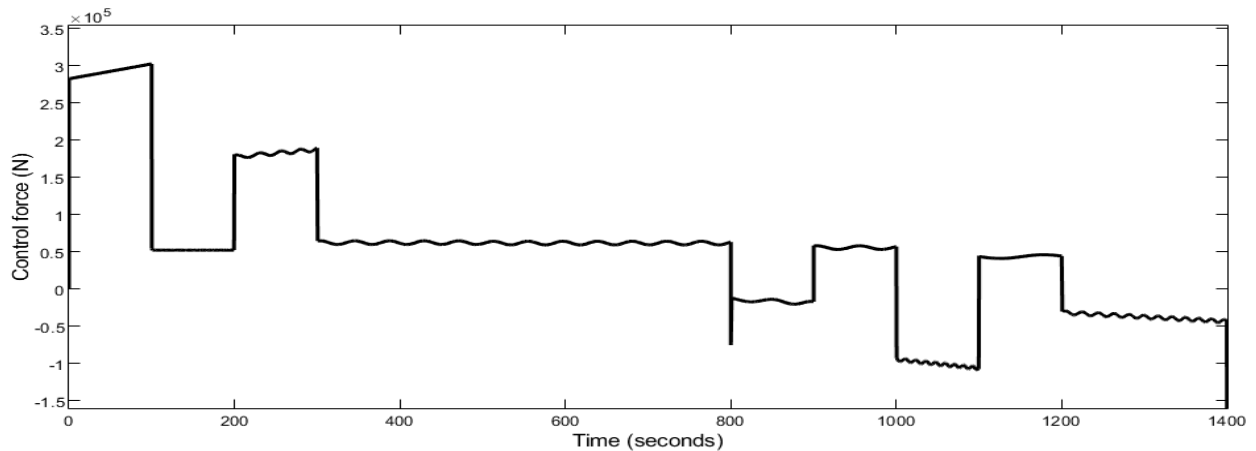


Figure 6.10: The actual control force for case 3.

As, seen in figure 6.10, there is a chattering in a control force of the 45% actuator effectiveness loss which leads the system response to have more transient responses.

As a whole from figure 6.6 and 6.7 demonstrates that, even if the controller faces 10% and 30% actuator effectiveness loss, the controller can control the train speed statuses near to a required trajectory after a few transit responses, as a result effective tracking capability with strong robustness to actuator disruption is accomplished for both cases but for the third case 45% effectiveness loss additional action is required.

### 6.3. Result and discussion for the increased external disturbance and angle of curvature case

In order to verify the proposed controller robustness against increased external disturbance and angle of curvature was investigated. For the case of external disturbance, a 50% and 100% increment was tested and for the second scenario 120 degree of curve angle is increased to 150 degree.

- i. For the case of 50% increased external disturbance with 150 degree curve angle

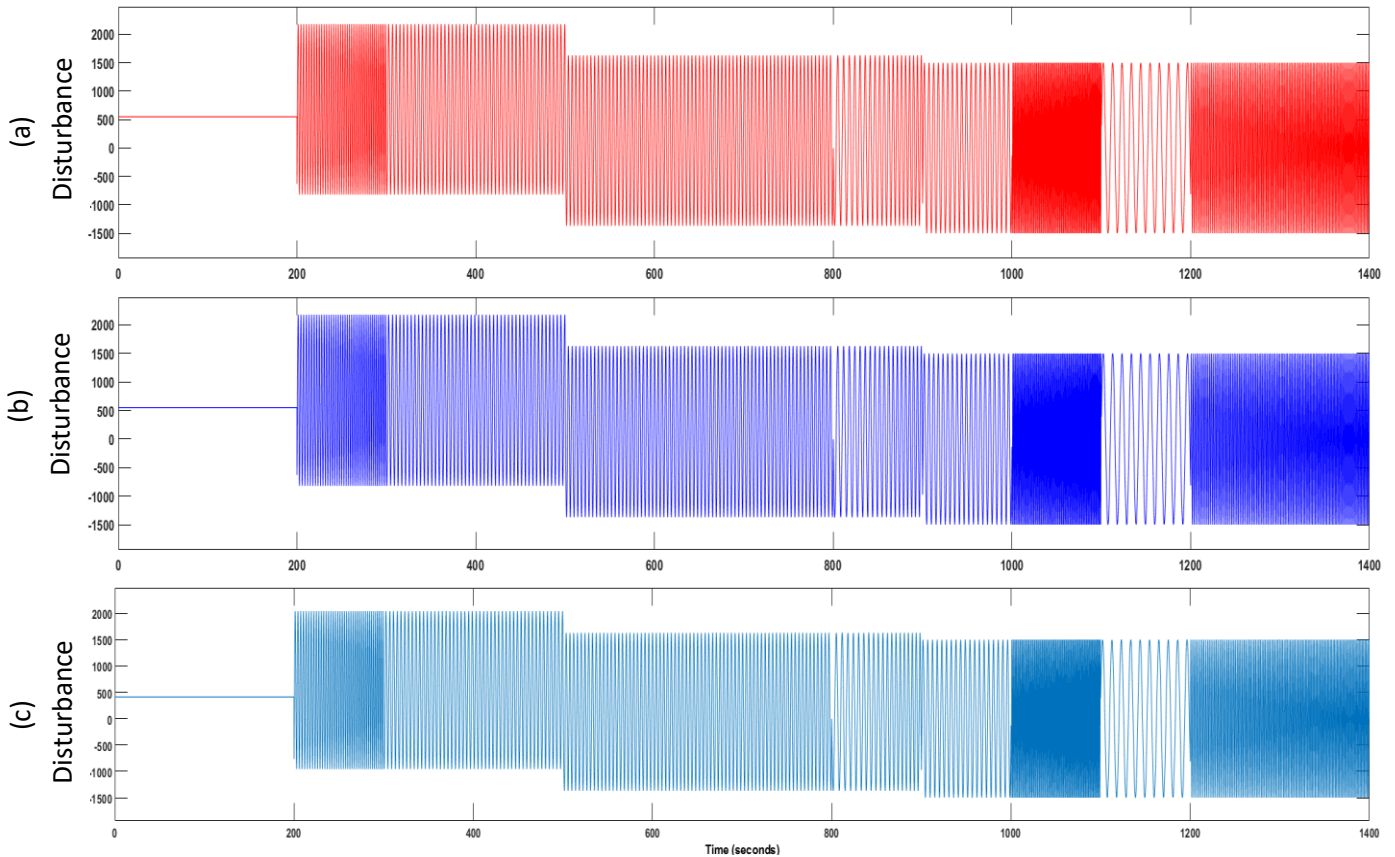


Figure 6.11: 50% increased external disturbance with 150 degree curve angle disturbance applied for (a) train 1, (b) train 2 and (c) train 3 respectively.

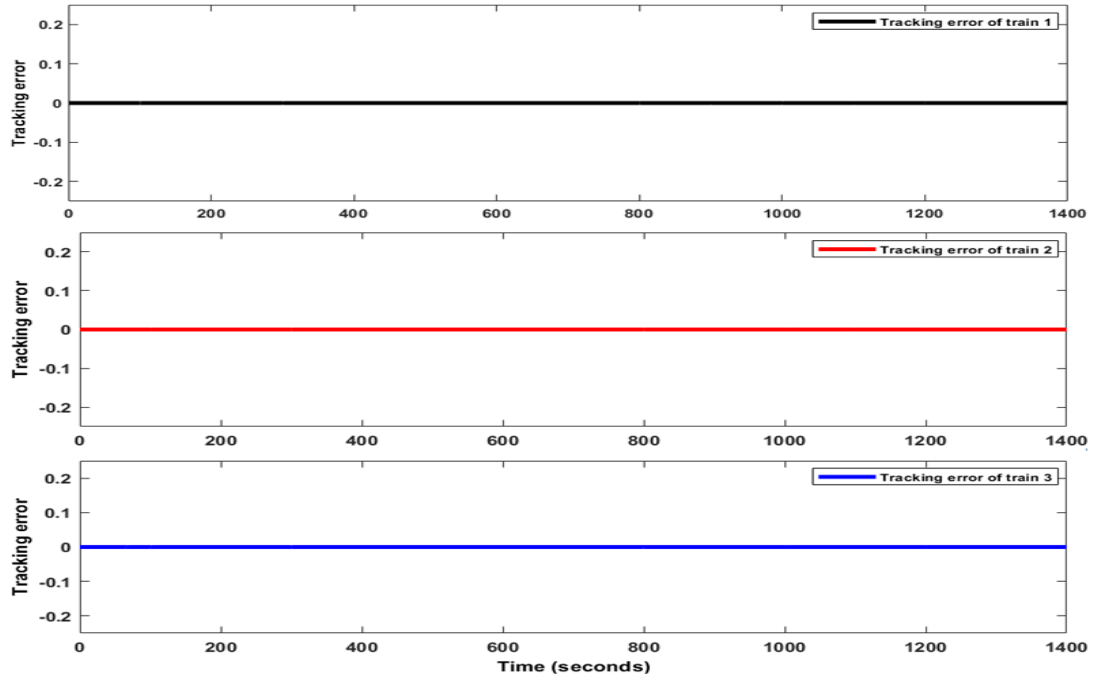


Figure 6.12: Speed tracking error for external disturbance with 150 degree curve angle

For 50% increased parametric a little increase in a settling time which is about 6.432seconds but still guaranteed finite time stability is achieved. This is the same as for fault free case.

ii. For the case of 100% increased external disturbance with 150 degree curve angle

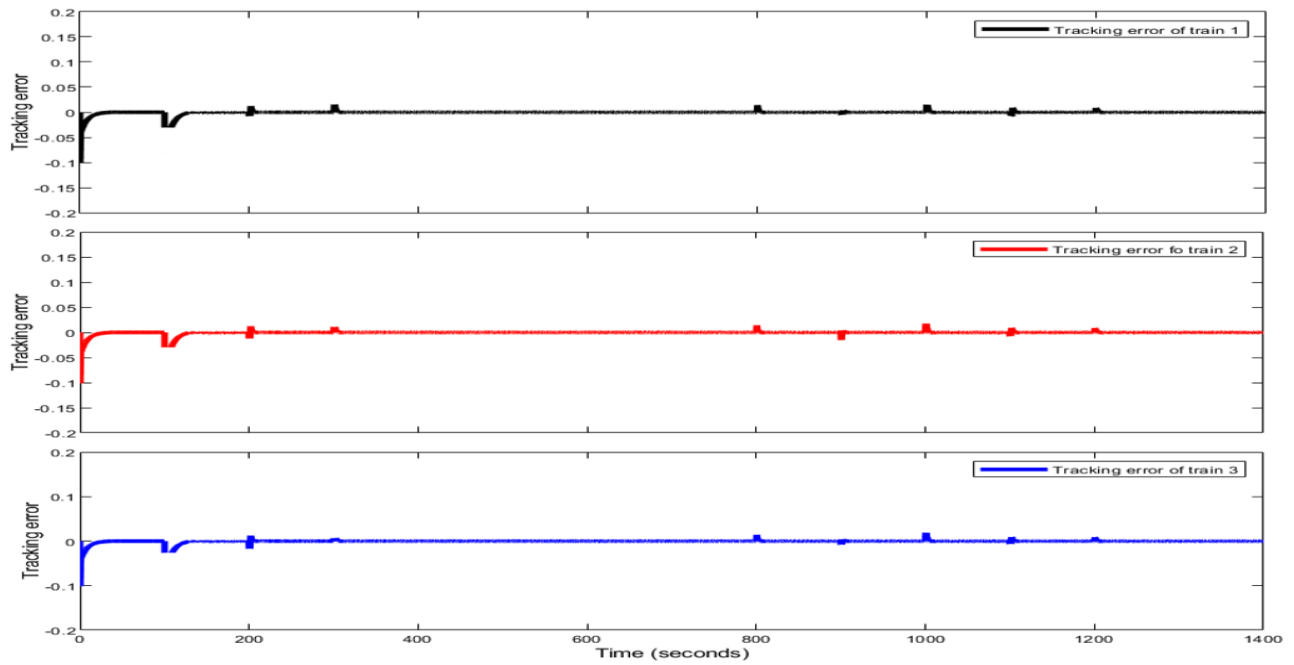


Figure 6.13: Speed tracking error for 100% increased external disturbance with 150 degree curve angle

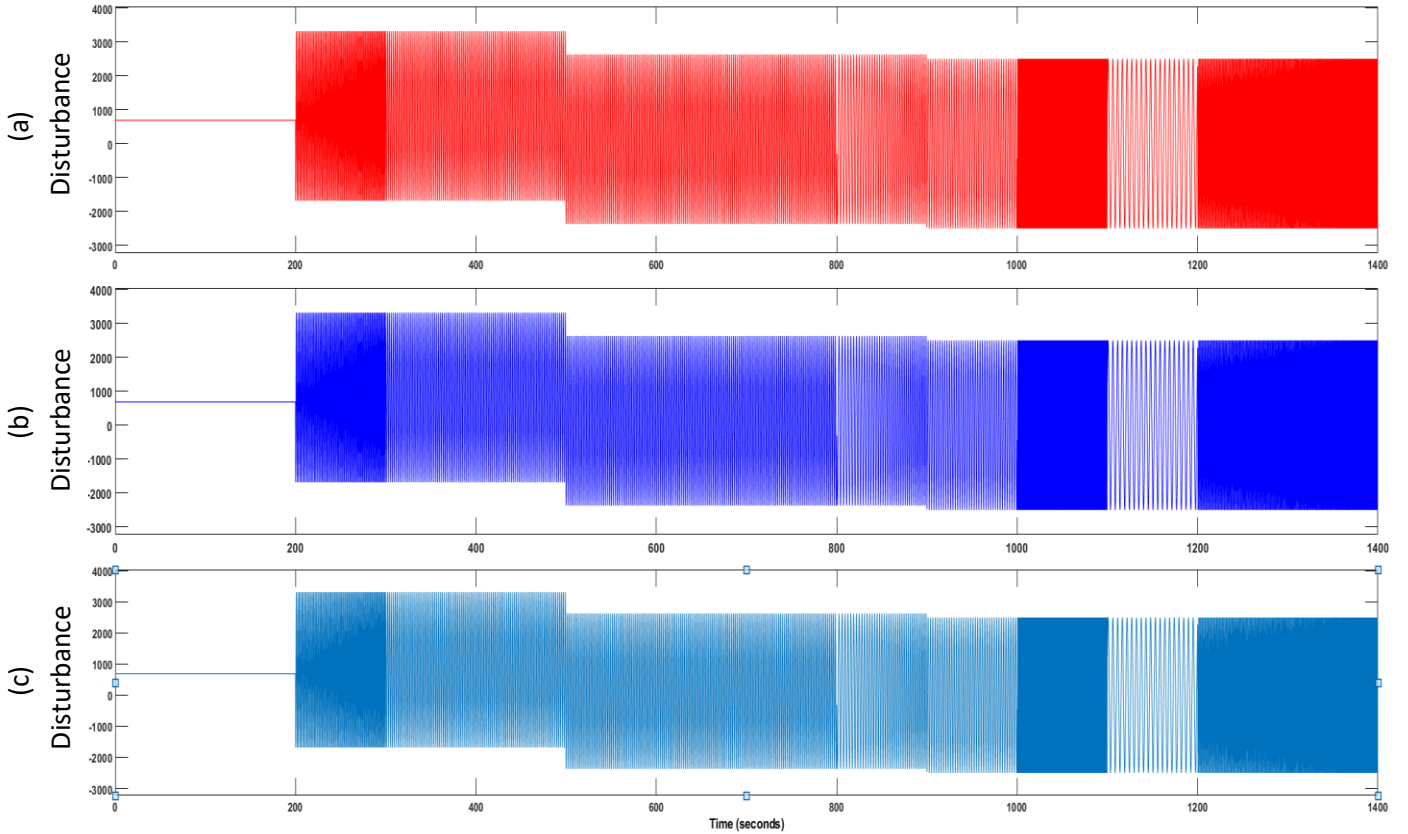


Figure 6.14: 100% increased external disturbance with 150 degree curve angle disturbance applied for (a) train1, (b) train 2 and (c) train3 respectively.

For a 100% increased external disturbance with 150 degree curve angle, the system response has some transient responses but still the system guarantee asymptotic stability.

#### 6.4. Comparative analysis of the result with the state of art

##### i. Actuator fault free case

As a control system evaluates depending on its transient responses, steady state error and foremost most stability a comparison is made on those parameters with what others was found in recent years. Paper [21] and [27] where the last recent papers and had an improved results from which others were found in previous. But, when compared to the simulation result of the proposed controller on this paper on regard to the overshoot and undershoot, settling time, convergence properties and chattering phenomena's even when actuator fault is not considered they get larger values from what is achieved on this paper. That means, the exact settling time of systems response in those papers is not known but for this paper is 3.134ms seconds. Regarding to the convergence of the tracking error they attain asymptotic convergence rather than finite time convergence and in this paper finite time convergence is attained. Concerning to the steady state error all papers attained zero steady state error. Therefore, in general the proposed controller had achieved an improved response from the previous and foremost as finite time stability is the main objective of this paper, objectives is meet.

ii. Actuator fault case

The actuator effectiveness losses produce high overshoot and low convergence rate of consensus that implies the necessity to consider the impact of actuator faults. Therefore, actuator fault is considered on [21] and [27] but, the system response shows increased transient responses, infinite settling time, asymptotic convergence and zero steady state error. Whereas, in the proposed controller in this paper, similar system response with the same settling time as well finite time convergence for 10% actuator fault cases and infinite settling time and asymptotic convergence for 30% cases. Foremost, the same zero steady state value for both cases is achieved. Thus, the proposed controller also has an enhanced response from the previous papers for actuator fault cases as well. For 45% actuator effectiveness loss the system shows high transient response.

iii. Increased external disturbance and angle of curvature case

Simulation for increased external disturbance and angle of curvature was conducted to check the controller robustness against high disturbances. Therefore, paper [27] simulation result shows the same result as actuator fault case and increased chattering in the control force. Whereas, in this paper a little increase in settling time, which is about 6.432seconds with guaranteed finite time stability is achieved for 50% increased parametric uncertainties. For 100% increased parametric uncertainties asymptotic stability. Therefore, improved performance is achieved for this case as well.

## CHAPTER SEVEN

### 7. CONCLUSION, RECOMMENDATION AND FUTURE WORKS

#### 7.1. Conclusion

In this research paper, the speed tracking control problem of the multiple high speed train subject to unknown parameters, model uncertainties, external disturbances, coupler effect and actuator effectiveness faults, based on adaptive terminal sliding mode control strategies has been investigated.

Focusing the longitudinal dynamics, the HST dynamic is modeled as a multi-point mass modeled and an adaptive terminal sliding mode control is designed and introduced to the multiple HST system for tracking the speed trajectory with guaranteed finite time stability. As, the proposed controller is by integrating adaptive and terminal sliding-mode controllers to get a better performance by taking the advantages of thus two controllers, the adaptive controller is for online estimation of uncertain parameters and whereas, the terminal sliding mode control is for fast convergence and for robustness.

Finally, the proposed controller is demonstrated for different cases to prove its robustness for different situations and effective result with guaranteed finite time stability is achieved so, as the objective is accomplished.

#### 7.2. Recommendation

In real time operation of high speed trains (HSTs), stability within finite time interval is desired instead of the asymptotic stability, as it is more physically realizable than concerning infinite time. Due to this, main concerns the automation level of high speed trains are lower than the expected. Since, the proposed controller in this paper have guaranteed finite time stability so, that this controller is recommended to be used to ensure the running automation of high speed trains.

#### 7.3. Future Work

This research work concentrates on the, speed trajectory tracking control problem of the multiple-HST subject to unknown parameters, model uncertainties, external disturbances, a coupler effect and actuator effectiveness faults, based on adaptive TSM control strategies was conducted and an effective result with good stability performance was gained. But, in real time practice of the railway trains actuator saturation is a very serious problem and need to address. Therefore, a further research on train speed tracking control including anti-actuator saturation with good stability is an interesting topic.

## REFERENCES

- [1] J. Yin, T. Tang, L. Yang, J. Xun, Y. Huang, and Z. Gao, "Research and development of automatic train operation for railway transportation systems: A survey," *Transp. Res. Part C Emerg. Technol.*, vol. 85, pp. 548–572, 2017.
- [2] L. Zhou and Z. Shen, "Progress in high-speed train technology around the world," *J. Mod. Transp.*, vol. 19, no. 1, pp. 1–6, 2011.
- [3] A. S. Leblebici and S. Türkay, "An  $H_\infty$  and Skyhook Controller Design for a High Speed Railway Vehicle," *IFAC-PapersOnLine*, vol. 51, no. 9, pp. 156–161, 2018.
- [4] M. Börjesson, "Forecasting demand for high speed rail," *Transp. Res. Part A Policy Pract.*, vol. 70, pp. 81–92, 2014.
- [5] Y. shing Cheng, B. P. Y. Loo, and R. Vickerman, "High-speed rail networks, economic integration and regional specialisation in China and Europe," *Travel Behav. Soc.*, vol. 2, no. 1, pp. 1–14, 2015.
- [6] D. Huang, Y. Chen, D. Meng, S. Member, and P. Sun, "Adaptive Iterative Learning Control for High-Speed Train : A Multi-Agent Approach," *IEEE Trans. Syst. Man, Cybern. Syst.*, vol. PP, pp. 1–11, 2019.
- [7] M. Ganesan, D. Ezhilarasi, and J. Benni, "Hybrid model reference adaptive second order sliding mode controller for automatic train operation," vol. 11, pp. 1222–1233, 2017.
- [8] Y. Wang, M. Zhang, J. Ma, and X. Zhou, "Survey on Driverless Train Operation for Urban Rail Transit Systems," *Urban Rail Transit*, vol. 2, no. 3–4, pp. 106–113, 2016.
- [9] J. M. Cohen, A. S. Barron, R. J. Anderson, and D. J. Graham, "Impacts of unattended train operations on productivity and efficiency in metropolitan railways," *Transp. Res. Rec.*, vol. 2534, no. 2534, pp. 75–83, 2015.
- [10] S. Clark, "( from the Bobby to the Balise )," pp. 6–25.
- [11] M. Faieghi, A. Jalali, S. K. e. ddi. Mousavi Mashhadi, and D. Baleanu, "Passivity-based cruise control of high speed trains," *JVC/Journal Vib. Control*, vol. 24, no. 3, pp. 492–504, 2018.
- [12] J. L. Liu, S. F. Li, and J. Q. Liu, "The study of Automatic Train Operation (ATO) system," *Adv. Mater. Res.*, vol. 159, pp. 562–567, 2011.
- [13] H. Dong, B. Ning, and B. Cai, "Automatic Train Control System Development and Simulation for High-Speed Railways."
- [14] H. Nakamura, "How to Deal with Revolutions in Train Control Systems," *Engineering*, vol. 2, no. 3, pp. 380–386, 2016.
- [15] Y. Li, B. Chen, V. W. Zheng, W. G. Temple, Z. Kalbarczyk, and Y. Wu, "Enhancing Anomaly Diagnosis of Automatic Train Supervision System Based on Operation Log," 2017.

- [16] S. Oh, Y. Yoon, and Y. Kim, “Automatic train protection simulation for radio-based train control system,” *2012 Int. Conf. Inf. Sci. Appl. ICISA 2012*, pp. 5–8, 2012.
- [17] Y. Lin, H. Xiong, X. Wang, Y. Wu, and C. Zhang, “The research of automatic speed control algorithm based on Green CBTC,” *IOP Conf. Ser. Earth Environ. Sci.*, vol. 69, no. 1, 2017.
- [18] Y. Zhou and Z. Zhang, “High-speed train control based on multiple-model adaptive control with second-level adaptation,” *Veh. Syst. Dyn.*, vol. 52, no. 5, pp. 637–652, 2014.
- [19] P. Wu, Q. Y. Wang, and X. Y. Feng, “Automatic train operation based on adaptive terminal sliding mode control,” *Int. J. Autom. Comput.*, vol. 12, no. 2, pp. 142–148, 2015.
- [20] Q. Song and T. Sun, “Neuroadaptive PID-like Fault-Tolerant Control of High Speed Trains with Uncertain Model and Unknown Tracking / Braking,” vol. 2, pp. 318–325, 2017.
- [21] Z. Mao, X. G. Yan, B. Jiang, and M. Chen, “Adaptive Fault-Tolerant Sliding-Mode Control for High-Speed Trains with Actuator Faults and Uncertainties,” *IEEE Trans. Intell. Transp. Syst.*, vol. 21, no. 6, pp. 2449–2460, 2020.
- [22] Y. D. Song, Q. Song, and W. C. Cai, “Fault-tolerant adaptive control of high-speed trains under Traction/Braking failures: A virtual parameter-based approach,” *IEEE Trans. Intell. Transp. Syst.*, vol. 15, no. 2, pp. 737–748, 2014.
- [23] X. G. Guo, J. L. Wang, and F. Liao, “Adaptive fuzzy fault-tolerant control for multiple high-speed trains with proportional and integral-based sliding mode,” *IET Control Theory Appl.*, vol. 11, no. 8, pp. 1234–1244, 2017.
- [24] Z. Li, Z. Hou, and C. Yin, “Iterative learning control for train trajectory tracking under speed constraints with iteration-varying parameter,” *Trans. Inst. Meas. Control*, vol. 37, no. 4, pp. 485–493, 2015.
- [25] H. Ji, Z. Hou, and R. Zhang, “Adaptive iterative learning control for high-speed trains with unknown speed delays and input saturations,” *IEEE Trans. Autom. Sci. Eng.*, vol. 13, no. 1, pp. 260–273, 2016.
- [26] Y. Chen, D. Huang, T. Huang, and N. Qin, “Tracking control via iterative learning for high-speed trains with distributed input constraints,” *IEEE Access*, vol. 7, pp. 84591–84601, 2019.
- [27] C. Xu, X. Chen, and L. Wang, “applied sciences Model-Independent Adaptive Fault-Tolerant Tracking Control for High-Speed Trains with Actuator Saturation,” 2019.
- [28] M. Faieghi, A. Jalali, and S. K. E. D. M. Mashhadi, “Robust adaptive cruise control of high speed trains,” *ISA Trans.*, vol. 53, no. 2, pp. 533–541, 2014.
- [29] X. Yao *et al.*, “Robust Adaptive Nonsingular Terminal Sliding Mode Control for Automatic Train Operation,” pp. 1–10, 2018.

- [30] S. Gao, Y. Hou, H. Dong, S. Stichel, and B. Ning, “High-speed trains automatic operation with protection constraints: A resilient nonlinear gain-based feedback control approach,” *IEEE/CAA J. Autom. Sin.*, vol. 6, no. 4, pp. 992–999, 2019.
- [31] Z. Li, Z. Hou, and C. Yin, “Transactions of the Institute of Measurement and Control,” 2014.
- [32] J. D. Bošković and R. K. Mehra, “Failure Detection, Identification and Reconfiguration in Flight Control,” *Fault Diagnosis Fault Toler. Mechatron. Syst. Adv.*, pp. 129–167, 2007.
- [33] M. Moradi and A. Fekih, “Adaptive PID-sliding-mode fault-tolerant control approach for vehicle suspension systems subject to actuator faults,” *IEEE Trans. Veh. Technol.*, vol. 63, no. 3, pp. 1041–1054, 2014.
- [34] Y. Feng, X. Yu, and Z. Man, “Non-singular terminal sliding mode control of rigid manipulators,” vol. 38, pp. 2159–2167, 2002.
- [35] K. A. Baghaei, H. Ghaffarzadeh, S. A. Hadigheh, and D. Dias-da-costa, “Chattering-Free Sliding Mode Control with a Fuzzy Model for Structural Applications,” 2017.
- [36] S. Amirkhani, S. Mobayen, N. Iliaee, O. Boubaker, and S. H. Hosseinnia, “Fast terminal sliding mode tracking control of nonlinear uncertain mass–spring system with experimental verifications,” *Int. J. Adv. Robot. Syst.*, vol. 16, no. 1, pp. 1–10, 2019.
- [37] K. Park and T. Tsuji, “SECOND-ORDER NONLINEAR UNCERTAIN SYSTEMS,” vol. 780, no. July 1998, pp. 769–780, 1999.
- [38] K. S. Narendra and Z. Han, *Adaptive Control Using Collective Information Obtained from Multiple Models* ★, vol. 44, no. 1. IFAC, 2011.
- [39] G. V Chowdhary and E. N. Johnson, “Theory and Flight-Test Validation of a Concurrent-Learning Adaptive Controller,” vol. 34, no. 2, 2011.
- [40] G. Tao, “Automatica Multivariable adaptive control : A survey ☆,” *Automatica*, vol. 50, no. 11, pp. 2737–2764, 2014.
- [41] M. Morari, *Adaptive Backstepping Control of Uncertain Systems*. 2008.

## APPENDIX

### Code for Reference Trajectory generation

This is a MATLAB code used for the reference trajectory generation. The code was developed for high speed train velocity of 75m/sec, acceleration and deceleration 1.5m/sec<sup>2</sup> and total travel time of 1400seconds as a reference.

```
a=1.5; % acceleration m/sec^2
d=-1.5; % deceleration m/sec^2
f1=50; % speed m/sec
f2=75; % speed m/sec
f3=15; % speed m/sec
f4=30; % speed m/sec
Tt=1400; % total time
Ts=0.1; % sampling period sec
T1=f1/a;
T3=f2/a;
T2=Tt/f1 - (T1+T3)/2;
T4=Tt/f2 - (T1+T2+T3)/3;
T5=-f3/d;
T6=Tt/f3 - (T1+T2+T3+T4+T5)/5;
T7=-f4/d;
T8=Tt/f4 - (T1+T2+T3+T4+T5+T6+T7)/7;
T9=-f4/d;
T10=Tt-T9;
if T1<0 || T2<0 || T3<0 || T4<0 || T5<0 || T6<0 || T7<0 || T8<0 || T9<0 || T10<0 % kinematic
    compatibility conditions
    disp('Error: acceleration, deceleration and travel time are not kinematically compatible.');
```

```
else
    tau1=0:Ts:T1;
    tau2=0:Ts:T2;
```

```

tau3=0:Ts:T3;
tau4=0:Ts:T4;
tau5=0:Ts:T5;
tau6=0:Ts:T6;
tau7=0:Ts:T7;
tau8=0:Ts:T8;
tau9=0:Ts:T9;
tau10=0:Ts:T10;

% pre allocate arrays for speed
sd1=zeros(1,length(tau1));
sd2=zeros(1,length(tau2));
sd3=zeros(1,length(tau3));
sd4=zeros(1,length(tau4));
sd5=zeros(1,length(tau5));
sd6=zeros(1,length(tau6));
sd7=zeros(1,length(tau7));
sd8=zeros(1,length(tau8));
sd9=zeros(1,length(tau9));
sd10=zeros(1,length(tau10));
for index=1:length(tau1)
    sd1(index)=f1*tau1(index)/T1; % speed
end
for index=1:length(tau2)
    sd2(index)=f;
end
for index=1:length(tau3)
    sd3(index)=f2*tau3(index)/T3 + f2;

```

```

end
for index=4:length(tau4)
sd4(index)= f2;
end
for index=5:length(tau5)
sd5(index)=-f3*tau5(index)/T5 + f3;
end
for index=6:length(tau6)
sd6(index)=f3;
end
for index=7:length(tau7)
sd7(index)=-f4*tau7(index)/T7 + f4;
end
for index=8:length(tau8)
sd8(index)=f4;
end
for index=9:length(tau9)
sd9(index)=-f4*tau9(index)/T9 + f4;
end
for index=10:length(tau10)
sd10(index)=0;
end
t1=tau1';
t2=tau2(2:end)' + T1;
t3=tau3(2:end)' + T1 + T2;
t4=tau4(2:end)' + T1 + T2+T3;
t5=tau5(2:end)' + T1 + T2+T3+T4;

```

```

t6=tau6(2:end)' + T1 + T2+T3+T4+T5;
t7=tau7(2:end)' + T1 + T2+T3+T4+T5+T6;
t8=tau8(2:end)' + T1 + T2+T3+T4+T5+T6+T7;
t9=tau9(2:end)' + T1 + T2+T3+T4+T5+T6+T7+T8;
t10=tau10(2:end)' + T1 + T2+T3+T4+T5+T6+T7+T8+T9;
t = [t1;t2;t3;t4;t5;t6;t7;t8;t9;t10]; % concatenation of time vector

sd1=sd1';
sd2=sd2(2:end)';
sd3=sd3(2:end)';
sd4=sd4(2:end)';
sd5=sd5(2:end)';
sd6=sd6(2:end)';
sd7=sd7(2:end)';
sd8=sd8(2:end)';
sd9=sd9(2:end)';
sd10=sd10(2:end)';
sd = [sd1;sd2;sd3; sd4; sd5; sd6; sd7; sd8; sd9; sd10];
plot(t, sd); % plots trajectory versus time
title('Reference Trajectory');
ylabel('speed [m/sec]');
xlabel('Time [sec]')

```