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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

A Study on the Shear Lag Effect and Efficiency of Reinforced Concrete Framed Tube and Trussed Tube Structural System

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A Thesis in Structural Engineering

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A Thesis

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The undersigned have examined the thesis entitled '**A Study on the Shear Lag Effect and Efficiency of Reinforced Concrete Framed Tube and Trussed Tube Structural System**' presented by **MERON YONAS**, a candidate for the degree of **Master of Science** and hereby certify that it is worthy of acceptance.

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UNDERTAKING

I certify that research work titled “A Study on the Shear Lag Effect and Efficiency of Reinforced Concrete Framed Tube and Trussed Tube Structural System” is my own work. The work has not been presented elsewhere for assessment. Where material has been used from other sources it has been properly acknowledged / referred.

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ABSTRACT

The efficiency of the framed tube structural system as a function of shear lag factor and as a function of the ratio of cantilever deflection contribution to the total lateral deflection is investigated in the present study. The distribution of axial force at different height and the lateral deflection components of the structure with varying height for two groups of buildings with different alignment of corner rectangular column are calculated and results are illustrated in graphical format. The result indicates efficiency calculated as a function of shear lag factor is not always directly proportional to efficiency calculated as the contribution of cantilever deflection to that of total lateral sway. Additionally reinforced concrete trussed tube buildings which are formed by filling selected spaces between the columns with shear wall on the exterior of the building is analyzed and the result indicate the addition of multi-story bracing significantly reduce the shear lag effect and lateral deflection of the structure.

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CHAPTER 1 INTRODUCTION

1.1 Background

The frame-tube systems, which constitute the basis of tube systems, can be described as having evolved from rigid frame system and are alternative to shear frame systems. The outstanding structural engineer Fazlur Rahman Khan innovated the framed-tube system. The most significant future of the system, also known as the “vierendeel tube system” or “perforated tube system”, is the closely spaced perimeter/exterior columns, which are usually spaced at 1.5 to 4.5 m centers, connected by deep spandrel beams at floor levels (Gunel and Ilgin, 2014).

The framed tube system combines the behavior of true cantilever, such as shear wall, with that of a rigid frame (Fintel, 1985). The overturning under lateral load is resisted by the tube form causing compression and tension in the columns, while the shear from the lateral load is resisted by bending in columns and beams primarily in the two sides of the building parallel to the direction of the lateral load (Fintel, 1985). From the structural point of view, the framed tube contrasts from the solid tube by many large openings. Its behavior is, of hybrid nature showing characteristics of the pure frame as well as of pure tube. Even though one may assume the framed tube behavior to be more like a tube resisting lateral force through axial forces in the columns, significant moment develop in the columns in the two walls parallel to the wind direction (Fintel, 1985). The study of a number of actual framed tube structures indicates that the lateral deflection, or sway, of these structures due to lateral load is primarily contributed by the frame action of the two walls parallel to the wind force (Fintel, 1985). The general bending behavior of the tubular building is analogous to that of a box girder, nevertheless shear deformations, commonly neglected in box girder, play a significant role in tubular structures (Haji-Kazem and Company, 2020).

Since framed tube buildings have much larger lateral dimensions than the internal shear wall cores, they are more effective in resisting the overturning moments of the lateral load. However, due to flexural and shear flexibilities of the frame members, the basic beam bending actions of the framed tubes are complicated by the existence of shear lag, which could considerably affect the stress distributions in the frame panels by increasing the axial stresses in the corner columns and decreasing those in the inner columns. Shear lag reduce the lateral stiffness of the structures and could produce wrapping of the floor slab and consequently deformation of the secondary structures (Kwan, 1994).

Under uniform load, at the region beyond ¼ the cantilever length from the built-in end, the bending stress near the web is much lesser than that far from the web. This result in opposite to positive shear lag and is called negative shear lag (Chang and Zheng, 1987). Lee et al. (2002) carried out a numerical investigation to determine the origin of negative shear lag. They concluded that the origin of negative shear lag is positive shear lag and positive shear lag creates negative shear lag through the shear lag-aftereffect or compatibility requirement. At any given location, negative shear lag can take place whenever the portion of the shear flow acting along the flange edges, which produces the shear lag-aftereffect, is larger than the remaining portion of the shear flow caused by positive shear lag. Negative shear lag may lead to local buckling on the compression side and cracking on the tension side of the flange frame (Lee et al., 2001).

In tubular design, the rigidity of the structural system against lateral load can be increased with solution such as: closer spacing of the perimeter columns, increasing the depth of the spandrel beams connected to the perimeter columns, adding shear trusses/braces or shear walls to the core, adding an inner tube in place of the core (tube-in-tube), adding a truss (multi-story braces) to the building exterior (trussed-tube), combining more than one tube (bundled tube) (Gunel and Ilgin, 2014). Several researches has been conducted to study the factors affecting shear lag and structural efficiency of tubular structures (Singh et al., 2013; Moghadasi et al., 2017 and Gaur and Goliya, 2015).

According to Sarkisian (2016) efficiency is the ratio of axial column deformation to total deformation in the tower. Eliminating any significant local shear or bending deformations of vertical members, resulting in an essentially 100% efficient structure, a structure that might lead us to expanding further the height limits of the extreme high-rise (Sarkisian, 2016).

The purpose of optimal design of a framed tube is to limit the shear lag effect and aim for more cantilever-type behavior of the structure within reasonable and practical limits (i.e., by achieving a cantilever deflection of 50 to 80 percent of the total lateral sway of the building). More research is needed for exterior structural systems which are technically more efficient (Ali and Moon, 2007).

Alternative way of improving the efficiency of the framed tube, thereby increasing its potential for use for even greater heights as well as allowing greater spacing between the columns, is to add diagonal bracing to the faces of the tube. This arrangement was first used in steel structure in 1969, in Chicago's John Hancock Building, and in reinforced concrete structure in 1985, in New York's 780 Third Avenue Building. In steel tube the bracing traverses the faces of the rigid frames, whereas in the concrete structure the bracing is formed by a diagonal pattern of the concrete window-size panels, poured integrally with the frame (Smith and Coull, 1991).

Trussed-tube system can be utilized in steel, reinforced concrete, and composite construction. By adding multistory diagonal bracings to the face of the tube, the rigidity and efficiency of the framed tube can be improved. Thus the obtained-braced tube system, also known as trussed tube or exterior diagonal-tube system, could be utilized for greater heights, and allows larger spacing between the columns. It offers an excellent solution by utilizing a minimum number of diagonals on each face of the tube intersecting at the same point as the corner columns. In steel buildings, steel diagonals/trusses, are used, while in reinforced concrete buildings, diagonals are created by filling the window openings by reinforced concrete shear walls to achieve the same effect as a diagonal bracing (Gunel and Ilgin, 2007).

Onterie Center, a 58-story complex located on Lake Michigan shore line, near downtown, Chicago, is another example of diagonal tube system, which is generally considered as the "final work" of Fazlur Khan. The structural system for Onterie center utilize a trussed tube system in concrete. Once again, a structural system originally intended for Hancock, a steel tower, is successfully translated in to reinforced concrete. The result is a concrete tube structure that has visible diagonal stiffening braces on the exterior. Although the diagonal bracing of John Hancock Center was comprised of continuous diagonal steel members, that approach was not strictly possible in concrete. Instead, the diagonal bracing is achieved by blocking out the windows along the facades by filling them in with concrete (Ali, 2001).

1.2 Significance of the study

Framed tube structures are recognized as a highly efficient structural system for high rise buildings, however it does not entirely utilize the potential stiffness and strength of the structure because of the shear lag effect. Shear lag is one of the major concerns in tall building design. The aim of optimal design of a framed tube is to limit the shear lag effect and target for more cantilever-type behavior of the structure within reasonable and practical limits. This study investigate the efficiency of framed tube structure as a function of shear lag factor and as a ratio of cantilever deflection contribution to the total lateral deflection and will help to further understand the behavior of framed tube structures. The effect of multi-story shear wall bracing formed by filling selected spaces between the columns on shear lag effect and later deflection is also studied.

Though computer programs which are being used routinely in everyday engineering practice are fast and powerful to numerically analyze framed tube structures, they cannot substitute the theoretical analysis method which may offer better understanding of the structural system. Additionally, a simplified analysis methods used to verify the reasonableness of the computer analysis and for initial assessment of structural behavior during preliminary design stage. This thesis attempts to analyze framed tube structure numerically and analytically and compare their results. The result of this study will help to understand the factors affecting efficiency of framed tube buildings and will help to further understand their behavior.

1.3 Objective of the Study

1.3.1 General objective

The frame tube system is widely accepted as an efficient structural system for tall buildings. However, because of flexural and shear flexibilities of the frame members, the basic beam bending actions of the framed tubes are complicated by the occurrence of shear lag, which could significantly affect the stress distributions in the frame panels and reduce the lateral stiffness’s of the structures. The main objective of this thesis is to investigate and gain understanding about the shear lag effect and efficiency of RC framed tube and trussed tube structures by using both analytical method and structural analysis software.

1.3.2 Specific objective

- To investigate the efficiency of framed tube structural system as a function of shear lag factor and as a ratio of cantilever deflection contribution to the total lateral deflection.
- To study the effect of rectangular corner column alignment and height of the structure on the shear lag and structural behavior of framed tube structure.
- To investigate the effect of reinforced concrete multi-story shear wall bracing formed by filling selected spaces between the columns on the shear lag and lateral deflection of the structure.
- To briefly understand the behavior of framed tube structural systems by using Kwan (1994) simplified method for approximate analysis of framed tube structures and ETABS 2015 v15.0.0 integrated building design software.
- To draw a comment on the analysis results of Kwan (1994) method and those produced by ETABS software.

1.4 Scope of the Study

The main aim of this study is to investigate the behavior of reinforced concrete framed tube and Trussed tube structural system and to assess the factors affecting the efficiency of the structure subjected to static lateral load by using structural analysis software and approximate analysis methods.

The scope of this study is limited to investigating the behavior of framed tube and trussed tube RC structural systems subjected to static lateral load. It is assumed that each member of the structure considered has constant cross section and material properties throughout its height.

The efficiency of the framed tube is considered as a function of shear lag factor and as a function of cantilever deflection contribution to the total lateral deflection of the structure.

1.5 Methodology

In order to meet the objectives of this thesis a comprehensive literature survey about RC tall building structural systems and their behavior was conducted prior to theoretical and numerical analysis of framed tube and braced tube structural systems.

Kwan (1994) simplified method for approximate analysis of framed tube structures is used to theoretically analyze framed tube structures and a well-known computer program ETABS 2015 v15.0.0 integrated building design software is used to numerically analyze framed tube and trussed tube structures. The analysis results are illustrated in graphical format.

To guarantee a better understanding and reasonableness of the computer analysis, a number of different and simplified models are investigated and compared with simplified hand calculations.

1.6 Content of the Thesis

The report consists of five chapters:

Chapter 1: present the introductory part that includes background, significance, objective, scope and methodology of the thesis.

Chapter 2: briefly reviews the behavior of rigid frame, shear wall, frame-shear wall, framed tube and trussed tube structural systems for tall reinforced concrete buildings.

Chapter 3: analysis of framed tube structure by using Kwan (1994) simplified method for approximate analysis of framed tube structures and comparison of results with ETABS analysis outputs are presented in this chapter of the study.

Chapter 4: contains analysis of selected models using ETABS 2015 v15.0.0 integrated building design software and the assessment of analysis results.

Chapter 5: Summarize the conclusion obtained throughout the thesis as well as recommendation for further studies.

CHAPTER 2 LITERATURE REVIEW

2.1 Structural Systems for Tall RC Buildings

According to Fintel (1985) for the structural engineer, a tall building can be defined as one whose structural system must be modified to make it sufficiently economical to resist lateral forces due to wind or earthquake within the prescribed criteria for strength, drift, and comfort of the occupants.

When structural systems are defined for a tower structure, it is usually implied that the system describes to the lateral load-resisting portion of the structure. Practical limits of structural systems in reinforced concrete buildings typically associated with office floors based on the associated floor-to-floor heights which can similarly be applied to other building uses are shown in the following diagram (Sarkisian, 2016).

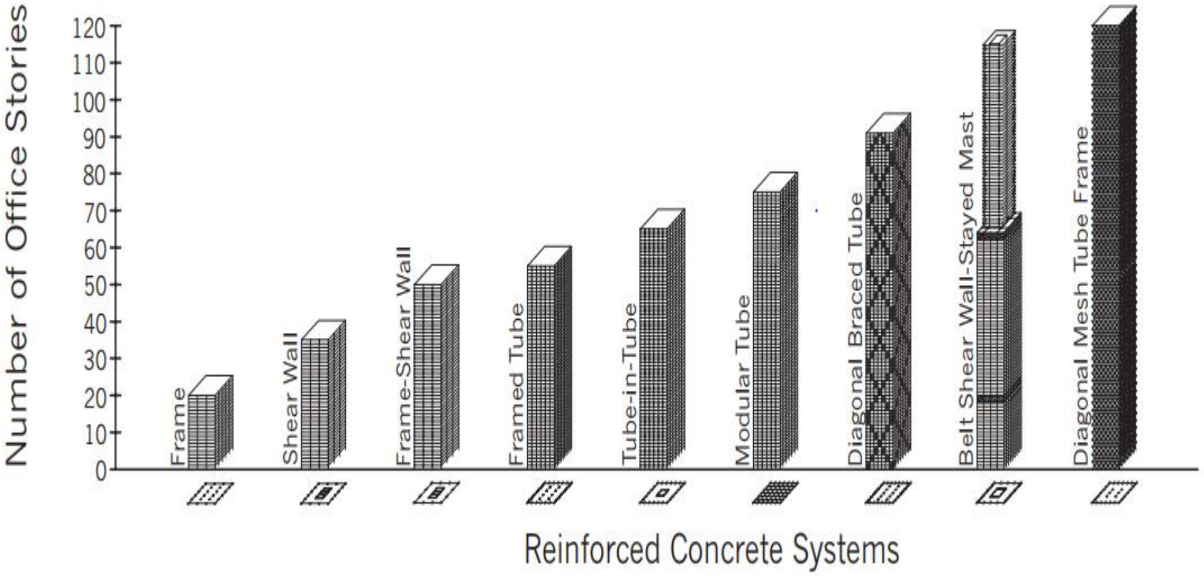


Figure 2-1: Practical height limits of structural systems (Sarkisian, 2016)

Ali (2001) divides structural systems for tall buildings into two broad categories: interior structures and exterior structures. This classification is based on the distribution of the components of the primary lateral load-resisting system over the building. A system is categorized as an interior structure when the major part of the lateral load resisting system is located within the interior of the building. Likewise, if the major part of the lateral load-resisting system is located at the building perimeter, a system is categorized as an exterior structure. However, that any interior structure is likely to have some minor components of the lateral load-resisting system at the building perimeter, and any exterior structure may have some minor components within the interior of the building. One of the most typical exterior structures is the tube, which can be defined as a three-dimensional structural system utilizing the entire building perimeter to resist lateral loads (Ali, 2001).

Since framed tube structural system which is considered to be evolved from rigid frame system and are alternative to shear frame system have a combined behavior of true cantilever similar to shear wall and moment resisting frame, based on the literatures cited below this chapter of the study will briefly describe: rigid frame system whose lateral stiffness of the entire frame is greatly dependent on the bending stiffness of the frame and to lesser extent on the axial stiffness of the columns; shear wall system which deform mainly in flexure and frame-shear wall system which resistance to lateral loading is provided by a combination of shear wall and rigid frame structural systems prior to studying framed tube and trussed/braced tube structural systems.

Since framed tube structural system which is considered to be evolved from rigid frame system and are alternative to shear frame system have a combined behavior of true cantilever similar to shear wall with that of moment resisting frame, based on the literatures cited below this chapter of the study briefly describe rigid frame, shear wall and frame-shear wall structural systems prior to studying framed tube and trussed/braced tube structural systems.

2.1.1 Rigid Frame

A rigid frame high-rise structure typically comprise of orthogonally arranged bents consisting of columns and girders with moment resisting joints. Horizontal loading is resisted by bending resistance of columns, girders and joints. The continuity of the frame also contributes to resisting gravity loading, by reducing moments in the girders. Above 25 stories the rigid frame structures becomes uneconomical because of their drift resistance becomes expensive (Smith and Coull, 1991).

The moment frame cannot displace laterally without bending of columns and beams due to the rigidity of beam column connections. The lateral stiffness of the entire frame is therefore dependent, to large extent, on the bending stiffness of the frame members, and to lesser extent on the axial compression and tension of the columns (Taranath, 2010).

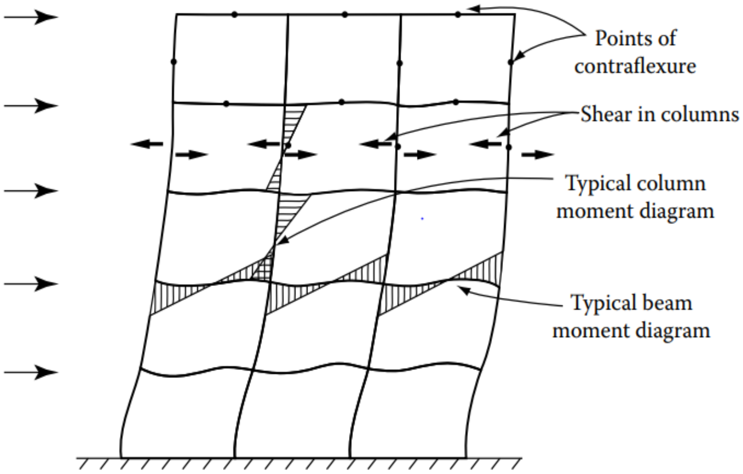


Figure 2-2: Rigid frame: Forces and deformation (Taranath, 2010).

The bending resistance of beams, columns and their connections mainly govern the horizontal stiffness of rigid frame. And for tall frames, the horizontal stiffness is also governed by the axial rigidity of the columns. The accumulated horizontal shear above any story of a rigid frame is resisted by shear in the columns of that story (Figure 2-2). The shear causes the story-height columns to bend in double curvature with point of contra-flexure at approximately mid-story-height levels. The moments applied to a joint from the columns above and below a particular level are resisted by the attached girders, which also bend in double curvature, with point of contra-flexure at approximately mid-span. These rotational deformations of the columns and girders result in shear deformation, often referred to as frame racking, greatly contributing to the horizontal deflection. The deflected shape of a rigid frame due to racking has a shear configuration with concavity up wind that has a maximum inclination near the base and minimum at the top (Taranath, 2010).

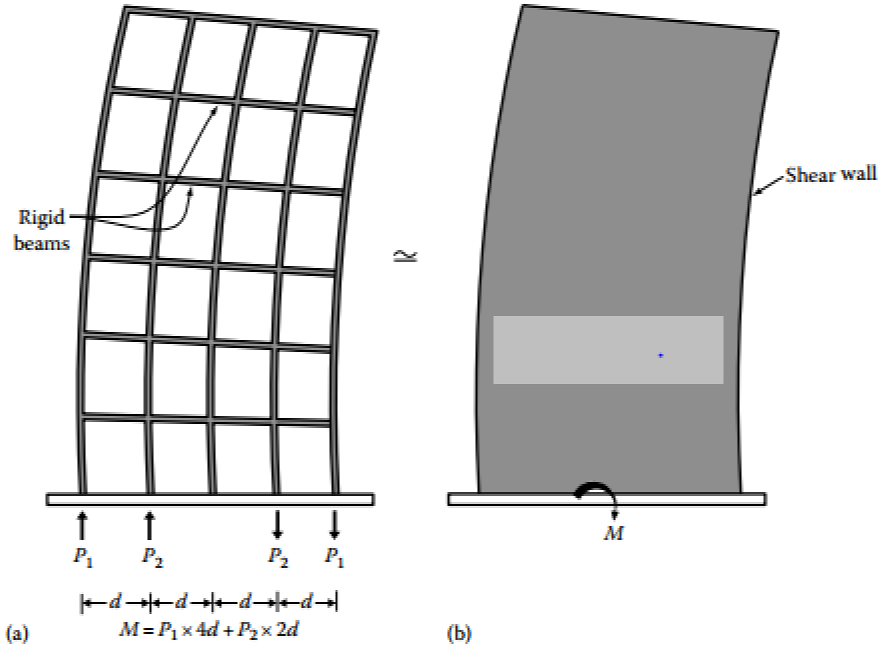


Figure 2-3: Bending deformation of rigid frame: (a) Moment resisted by axial loads in columns. (b) Cantilever bending of shear wall (Taranath, 2010).

The overall external moment is resisted at each floor level by a couple resulting from the axial tensile and compressive forces in the columns on opposite sides of the structure (Figure 2-3). The extension and shortening of the columns cause overall bending and associate horizontal displacements due to curvature of the structure. The story drift due to overall bending increase with height because of the cumulative rotation up the height, while racking deflection tend to decrease. Consequently, the contribution to story drift from overall bending may, in the uppermost stories exceed that from racking. The contribution of overall bending to the total drift, however, will usually be much less, no more than 30% of that of racking (Taranath, 2010).

About 70% of the total lateral deflection of rigid frame is due to shear mode deformation, with the beam flexure contributing about 10%-15%, and the column bending furnishing the remainder. This is because in rigid frame, typically the column stiffness, as measured by I_c/L_c

ratio, is substantially greater than the beam stiffness ratio, I_b/L_b, where I_b, moment of inertia of the beam; Ic moment of inertia of column; L_b length of the beam and L_c length of the column. Therefore, in general, to reduce the lateral deflection, the place to start adding stiffness is in the beams because of the cumulative effect of rotation up the height (Taranath, 2010). The story drift increase with height, while that due to shear racking tends to stay the same up the height. The contribution to story drift due to cantilever bending in the uppermost stories exceeds that of shear racking. However, the bending effect usually does not exceed 10%-20% of that due to shear racking, except in very tall and slender rigid frames. Therefore, the overall deflected shape of a frame for moderately tall building usually has a shear deflection configuration (Taranath, 2010).

As indicated previously the typical proportioning of member sizes in tall rigid frames is such that girder flexure is the major cause of drift, with column flexure a close second. Therefore, the most effective and economical way of correcting excessive drift is usually by increasing the girder bending stiffness (Taranath, 2010).

2.1.2 Shear Wall

Since the late 1940s, when the first shear walls were introduced, their use in high-rise buildings to resist lateral loads has been extensive, in particular to supplement frames that, if unaided, often could not be efficiently designed to satisfy lateral load requirements (Fintel, 1985).

Shear wall systems are used in reinforced concrete buildings. This system consists of reinforced concrete shear walls, which can be perforated (with openings) or solid. Shear wall systems can be thought as a vertical cantilever rigidly fixed at the base, and can resist all vertical and lateral loads on the building without columns (Figure 2-4). Owing to the nature of cantilever behavior, the inter-story drift between adjacent floors is greater in the upper floors than in the other floors. For this reason, in supertall buildings it is difficult to control the lateral drift at the building top (Gunel and Ilgin, 2014).

Their high in-plane stiffness and strength makes them well suited for bracing buildings up to 35 stories, while simultaneously carrying gravity loading.

It is usual to select the placement of shear walls so that they attract an amount of gravity dead loading sufficient to suppress the maximum tensile bending stresses in the wall caused by lateral loading. In this situation, only minimum wall reinforcement is required. The term “shear wall” is in some ways a misnomer because the walls deform predominantly in flexure. Shear wall may be planar, but are often of L-, T-, or U-shaped section to better suit the planning and to increase their flexural stiffness (Smith and Coull, 1991).

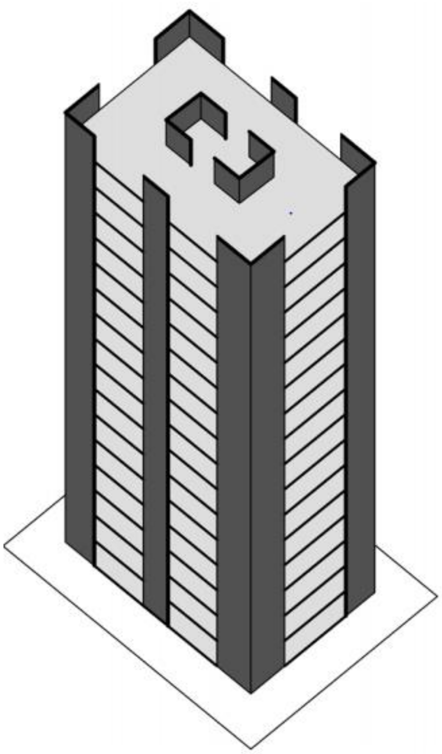


Figure 2-4: Shear wall system (Gunel and Ilgin, 2014)

2.1.3 Shear Wall-Frame

For buildings above 25 stories, rigid frame system cannot be economically sufficient to resist lateral loads because of bending on columns that cause large deformations. In this case, the total stiffness and so the economical height of the building can be increased by adding vertical shear walls to the rigid frame to carry the external shear induced by lateral loads (Figure 2-5) (Gunel and Ilgin, 2014).

The resistance to horizontal loading of this system is provided by a combination of shear walls and rigid frames. The shear walls are often placed around elevator and service cores while the frames with relatively deep spandrels occur at the building perimeter. When lateral load is applied to shear wall-frame buildings, the distinctly deferent deflected forms of the walls and the frames can be quite effective in reducing the lateral deflections to the extent that buildings of up to 50 stories or more are economical (Taranath, 2010). The potential advantages of a wall-frame structure depend on the intensity of horizontal interaction, which is governed by the relative stiffness of the walls and frames, and the height of the structure. The taller the building and the stiffer the frame, the greater interaction (Taranath, 2010).

The classical mode of interaction between a prismatic shear wall and a moment frame is shown in Figure 2-6. The lateral load distribution between the two systems becomes more or less, according to their stiffness if the deflection mode of moment frames and shear walls were similar. However, in general, the two systems deform with their own characteristic shapes. The interaction between the two, particularly at the upper levels of the buildings, results in quite a different lateral load distribution (Taranath, 2010).

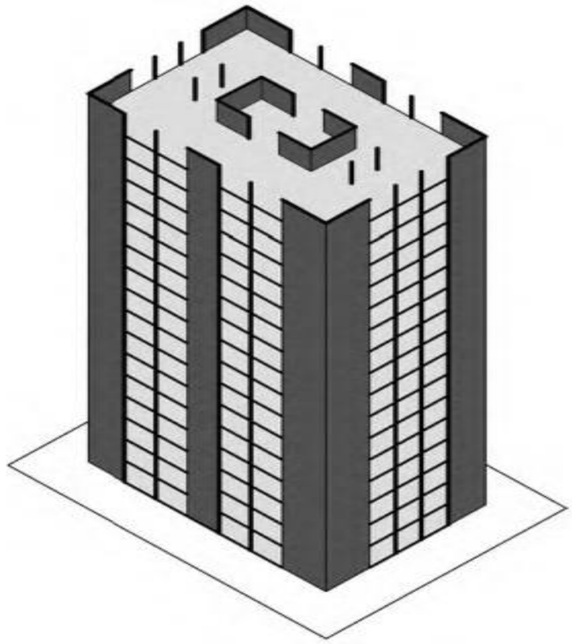


Figure 2-5: Shear walled frame system (Gunel and Ilgin, 2014)

The lateral deflection of a shear wall may be considered as similar to those of a cantilever column. Near the bottom, the shear wall is relatively stiff, and therefore, the floor-to-floor deflections will be less than half the values near the top (Taranath, 2010). From the cumulative, result of wall rotation the deflection increase rapidly at the top floors. Rigid frames, on the other hand, deform predominantly in a shear mode. The relative story deflections depend primarily on the magnitude of the shear applied at each story level. Although the deflections are larger near the bottom and smaller near the top as compared to shear walls, the floor-to-floor deflections can be considered more nearly uniform throughout the height (Taranath, 2010). When the shear wall and moment frame structures are connected by rigid floor diaphragms, a non-uniform shear force develops between them. The resulting interaction typically results in more economical structural system (Taranath, 2010).

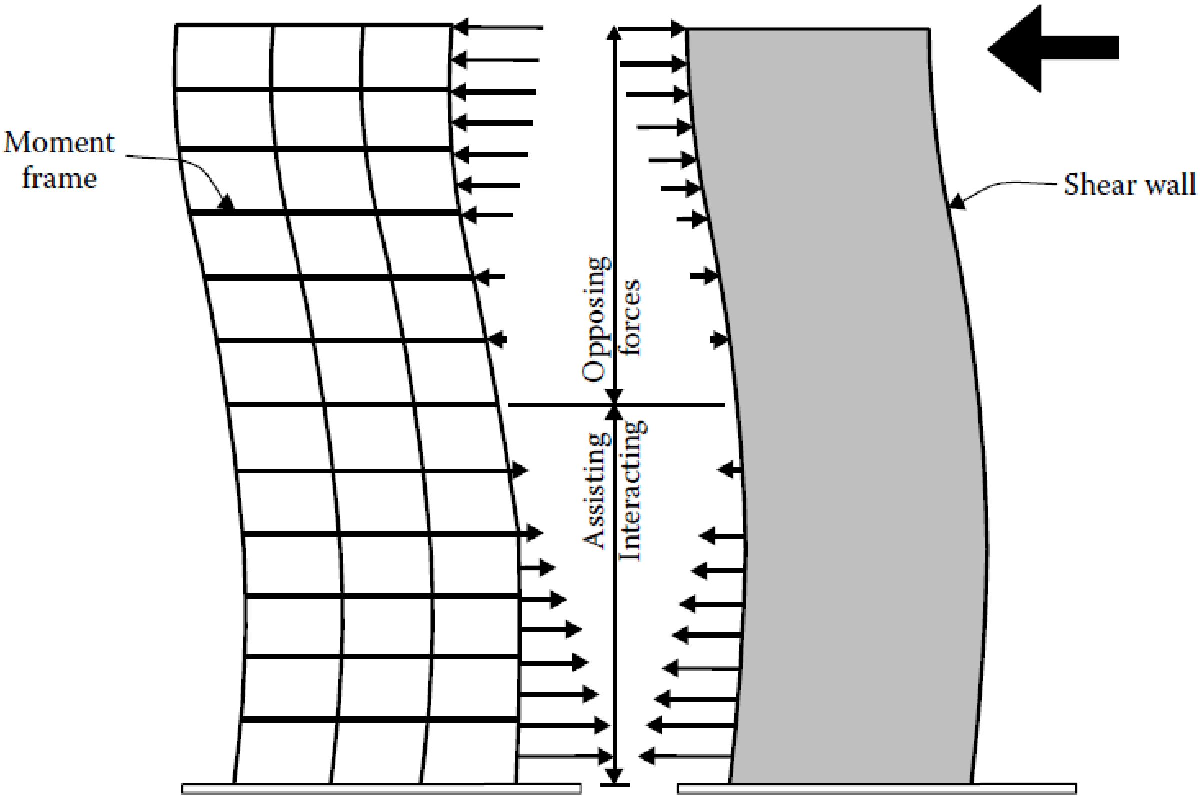


Figure 2-6: Shear wall-frame interaction (Taranath, 2010).

Figure 2-6 shows the deformation patterns of a shear wall and moment frame subjected to lateral loads. Also shown therein are the horizontal shear force between the two, the length of the arrows schematically representing the level of interaction. While the moment frames deform in shear mode with slop greater at the base of the structure where shear is maximum, the shear wall acts as a vertical cantilever column with greater slop at the top. Since the lateral deflection characteristics of the two frames are entirely different, the moment frame tends to pull back the shear wall in the upper levels of the building while pushing it forward in the lower levels. at the upper portion of the building the frame participate more efficiently where lateral shear are relatively small, while the shear wall carries most of the shear in the lower portion of the building. Because of the distinct difference in the deflection characteristics, the two systems tend to help each other a great deal. The frame tends to reduce the lateral deflection of the shear wall at the top, while the shear wall supports the frame near the base (Taranath, 2010).

A frame consisting of closely spaced columns and deep beams, behave more like a shear wall responding predominantly in a bending mode. Correspondingly, a shear wall weakened by large openings behaves more like a frame by deflecting in a shear mode. therefore the combined structural action depend on the relative rigidity of the frame and shear wall and their modes of deflection (Fu, 2018).

2.1.4 Framed Tube

The frame-tube systems, which constitute the basis of tube systems, can be described as having evolved from rigid frame system and are alternative to shear frame systems. The outstanding structural engineer Fazlur Rahman Khan innovated the framed-tube system (Gunel and Ilgin, 2014).

In the early 1950s, the introduction of shear wall type of construction opened up the probability of using concrete in apartment and office buildings as high as 30 stories. Because of shear walls which were mostly used in core were too small to sufficiently and economically provide the stability and stiffness for building over 30 or 40 stories, taller buildings remained economically unattractive. The natural tendency then was to find new system that would utilize the perimeter configuration of such buildings rather than to rely on the core configuration alone. The development of the framed tube system was, therefore, a logical outcome of this challenge (Fintel, 1985).

The framed tube system in its simplest form consists of closely spaced exterior columns tied at each floor level with relatively deep spandrel beams, thereby creating the effect of a hollow concrete tube perforated by opening for windows. Since the system simulated a hollow tube using perimeter closely spaced frame elements, it is referred to as “framed tube” (Fintel, 1985).

The behavior of framed tube combines the characteristics of true cantilever such as shear wall with that of rigid frame. The overturning under lateral load is resisted by the tube form causing compression and tension in the columns, while the shear from the lateral load is resisted by bending in columns and beams primarily in the two sides of the building parallel to the direction of the lateral load (Fintel, 1985).

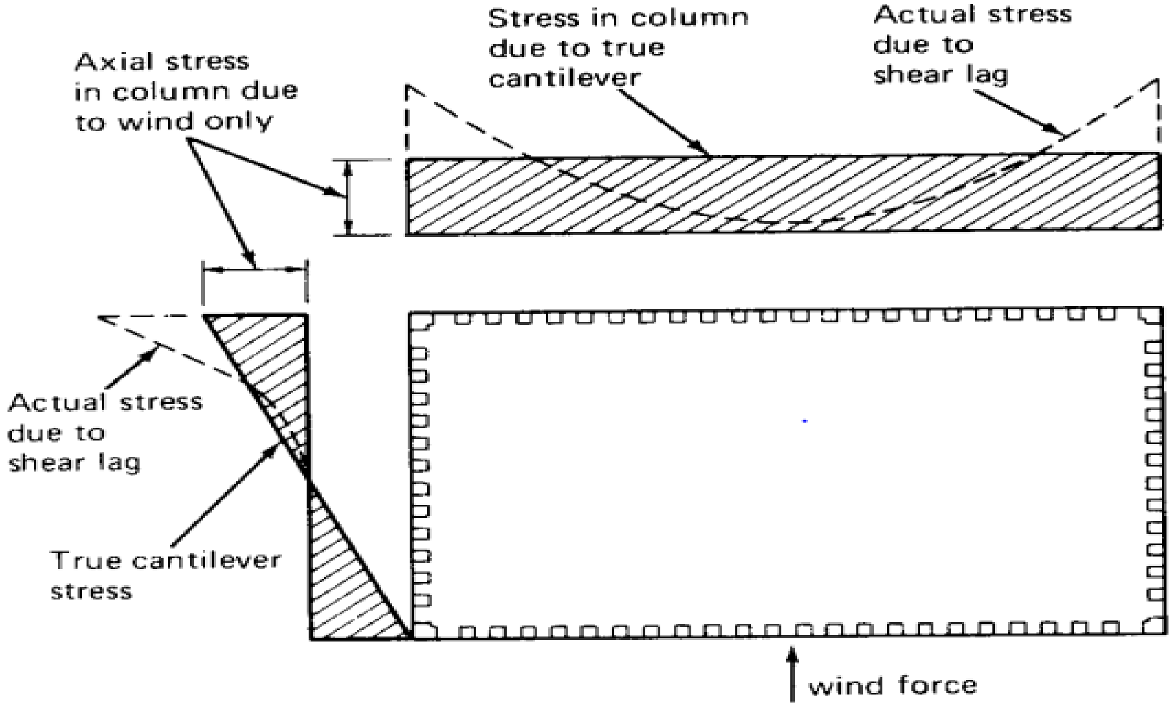


Figure 2-7: Axial stress distribution in the columns due to wind: True cantilever vs actual stress due to shear lag (Fintel, 1985).

When the overturning of the entire structure due to lateral load is considered, the cantilever tube type behavior becomes significant. For analyzing the overturning of the entire framed tube, the exterior column system can be considered as part of a rigidly-diaphragmed hollow tube. However, in recognition of the fact that the webs of the hollow tube that is the two sides parallel to the direction of the lateral force, are not truly solid webs but are, in fact grid frames, one must consider then the effect of loss of efficiency due to the flexibility of this web - frame causing what is known as shear lag, as shown in the Figure 2-7 (Fintel, 1985).

From the structural point of view, because of the several large openings the framed tube structure differ from solid tube. Its characteristics is, therefore, of a hybrid nature showing behavior of the pure frame as well as of the pure tube. Even though one may expect the framed tube behavior to be more like a tube resisting the lateral forces through axial forces in the columns, significant moments develop in the columns in the two walls parallel to the wind direction (Fintel, 1985).

The study of a number of actual framed tube structures indicate that majority of the lateral deflection is contributed by the frame action of the web panels parallel to the wind force. For example, in the 43-story DeWitt-Chestnut Building, out of a total of 7 in. of lateral sway under the Chicago wind loading, approximately 5 in. (about 70%) was contributed by the frame action and only 2 in. was contributed by the overturning moment. This particular case indicate that the efficiency of a framed tube diminishes as the building gets taller. As the height reaches a certain point, the lateral deflection, and not the strength, of the building will control the design of the structural system. Therefore, the structural efficiency producing a premium-free building will gradually diminish with increasing number of stories (Fintel, 1985).

As previously mentioned, the behavior of framed tube consists of: (a) the frame action of the two sides parallel to the direction of the lateral load, and (b) the overturning action of the entire tube causing only tension and compression in the exterior columns (Fintel, 1985).

2.1.4.1 Shear lag effect in framed tube system

Through the axial displacement of the corner column the principal interaction between the web and flange panels occur. When the corner column C, for example, is under compression, it will tend to compress the adjacent column C_1 (Figure 2-8) because the two are connected by spandrel beam. Since the connecting spandrel beam will bend the compressive deformation of C_1, will not be identical to that of the corner column. The axial deformation of C_1 will be less, by an amount depending on the stiffness of the connecting beam. The deformation of column C_1 will, in turn, induce compressive deformation of the next inner column C_2, but the deformation will again be less. Thus, each successive interior column will experience a smaller deformation and hence a lower stress than the outer ones. The stresses in the corner column will be greater than those from a pure tubular action, and those in the inner columns will be less. The stress in the inner columns lag behind those in the corner columns, hence the term shear lag (Taranath 2010).

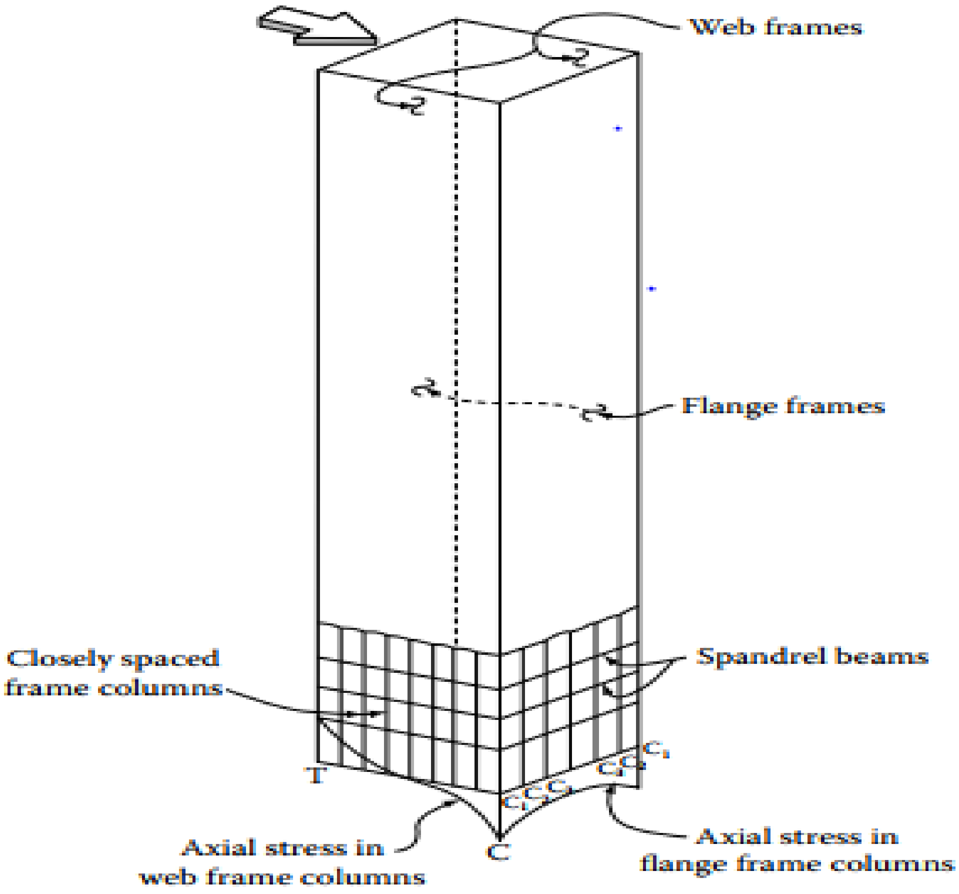


Figure 2-8: Shear lag in framed tube (Taranath, 2010).

The difference between stress distribution as predicted by ordinary beam theory, which assumes that plane sections remain in plane, and the actual distribution due to shear lag is illustrated in Figure 2-8. Since the column stresses distributed less effectively compared to an ideal tube, the moment resistance and the flexural rigidity of a tubular building is much less. Thus, although a framed tube is highly efficient, it does not fully utilize the potential stiffness and strength of the structure because of the effect of shear lag (Taranath 2010).

In addition to the basic bending-beam and shear-beam behaviors, the shear lag phenomenon is also present in framed tubes. This means that modeling the building as an equivalent beam with shear and bending rigidities is not sufficient to capture the true behavior of a framed tube. When a framed tube is loaded laterally, at the lower floors, especially at the ground level (first floor), the force in the corner column is much larger than the force in the center column of the flange frame. On the other hand, the forces in the web frame, instead of growing smaller toward the center linearly, grow smaller much faster. This phenomenon is called “shear lag”. The structure behaves differently to those predicated by the engineering bending theory due to the shear flexibility of the flange frames. The ratio of the stress at the center column to the stress at the corner column is defined as shear-lag factor (Conner and Pouangare 1991).

The stress in all columns becomes approximately equal at certain height of the structure, which results in the shear lag factor of one. Moreover, at some point since the stress at the corner columns are zero the shear lag factor becomes zero. At even higher floors, the corner column stresses change sign, and thus the shear-lag factor becomes negative (Conner and Pouangare 1991).

In framed tube systems, two different modes of shear lag may occur: negative shear lag and positive shear lag. Singh and Negpal (1994) concluded positive shear lag occurs in the bottom portion of the building while negative shear lag occurs at the top portion of the building. Negative shear lag originates from positive shear lag and counteracts it, and is absent when positive shear lag is absent. Both shear lag effects are more significant for buildings with lower stiffness values. Shear lag effects are also higher for buildings with low number of stories to number of bays ratio. As the positive shear lag increases, the negative shear lag also increases and the level of shear lag reversal shifts upward. When the negative shear lag is very large, columns at the corner of the building may experience axial stresses opposite to those in the middle of the panel (Singh and Negpal, 1994).

The shear lag effect will produce bending of the floor slab, since plane cross section no longer remain plane consequently, deformation of interior partitions and secondary structural components occur, which increase cumulatively throughout the height of the building. It is therefore of considerable importance to forecast precisely the structural behavior of the system in order to produce an efficient and acceptable design. (Smith and Coull 1991). Shear lag is one of the major concerns in tall-building design. The designer must make the shear- lag factor at the base as close to unity as possible. With infinite shear rigidity of the flange panels the shear lag factor of one can be achieved; this is exactly what the classical engineering theory of bending assumes. In practice, a shear-lag factor of 0.7 is considered satisfactory (Conner and Pouangare 1991).

2.1.5 Trussed Tube (Braced Tube or Exterior Diagonal Tube)

Because of the shear-lag effect, the effectiveness of framed tube is greatly reduced. Due to this reason, the framed tube has its limitations when used in buildings taller than 50 or 60 stories unless opening for windows are made relatively narrow. The frames parallel to the wind essentially act as multi-bay rigid frames, with the result that bending moments in the columns and spandrel beams become controlling factors in design, requiring either wide columns or an unacceptable high percentage of steel in columns and spandrels. Furthermore, of the total lateral sway, only about 25 percent is due to the cantilever action of the framed tube; the remaining 75 percent comes from frame racking as a direct consequence of shear lag. The columns at the corner of the building take more than their share of load because of the racking of the frame. While columns in between do less work than in an ideal tube. Efficiency is reduced to the extent that the system becomes uneconomical for unusually tall buildings. A method of overcoming this problem is to stiffen the exterior frames with diagonals (Taranath 1988).

Master builder Fazlur Khan of Skidmore, Owings & Merrill envisioned as early as 1972 that it was possible to build high rises in concrete rivaling in height those in structural steel. His quest to find a structural solution for eliminating the shear lag phenomenon directed him to diagonal tube concept. A brilliant manifestation of this principle in steel construction is seen in John Hancock Tower in Chicago (Taranath 1988). Applying similar principles, Khan visualized a concrete version of diagonal truss tube consisting of exterior columns spaced at about 10-ft (3.04-3) centers with blocked out windows at each floor to create a diagonal pattern on the façade. The diagonal could then be designed to carry the shear forces, thus eliminating bending in the tube columns and girders (Taranath 1988).

The diagonals created by filling in the windows serve a dual function. First, they increase the efficiency of the tube by diminishing the shear lag, and second they reduce the differential shortening of exterior columns by redistributing the gravity loads. A stiffer, much more efficient structure is realized with the addition of the diagonals (Taranath 1988).

In buildings with steel or composite trussed-tube system, multi-story braces (diagonal or X-braces) are used on the façade of the building (Figure 2-9a). In the case of buildings with reinforced concrete trussed-tube systems, space between the columns are filled with reinforced concrete shear walls to form multi-story diagonal or X-brace pattern on the exterior of the building (Figure 2-9b) (Gunel and Ilgin 2014).

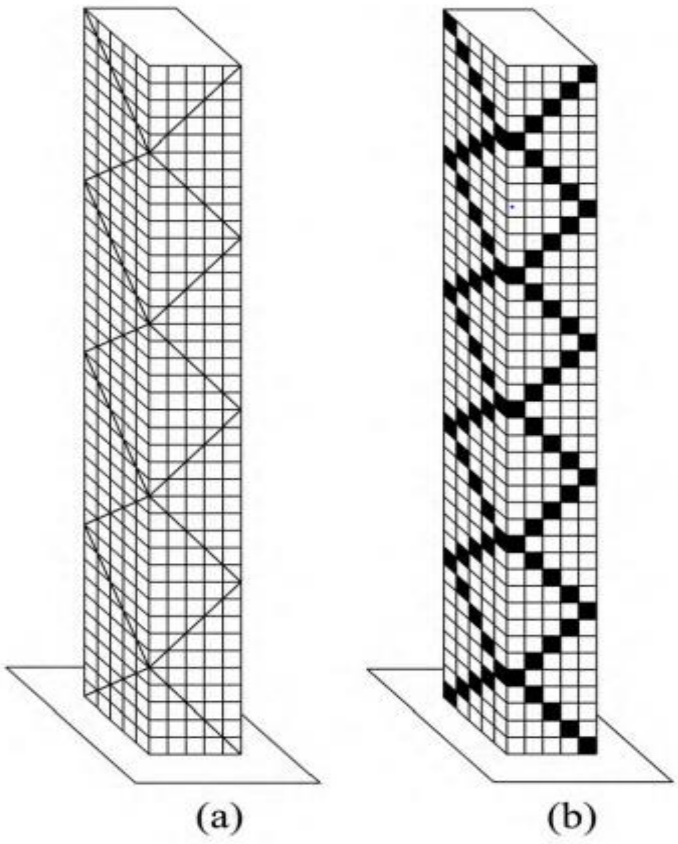


Figure 2-9: Trussed-tube system: (a) Steel or composite, (b) Reinforced concrete (Taranath, 2010).

The 50-story, 174m high 780 Third Avenue Building (New York, 1985) (Figure 2-10) was the first reinforced concrete building in which a trussed-tube system was used (Gunel and Ilgin 2014). Onterie Center, a 58-story complex located on Lake Michigan shore line, near downtown, Chicago, is another example of diagonal tube system. It comprises of a main tower with a tapering auxiliary low-rise building (Figure 2-11). The entire lateral loads are resisted by closely spaced columns and spandrels, as it common in framed tubes. However, to achieve additional lateral stiffness, window space at selected location is infilled with reinforced concrete to form two exterior diagonal channels, one at each end of the tower. Interior columns are designed to carry gravity loads only, thus allowing flexibility in interior space planning. The architectural and structural design is by Skidmore, Owings and Merrill, Chicago (Taranath 2010).

Figure 2-10: 780 Third Avenue building, New York, USA, 1985 (Gunel and Ilgin, 2014).

Figure 2-11: Example of exterior diagonal tube: Onterie Center, Chicago, IL (Taranath, 2010).

CHAPTER 3 APPROXIMATE ANALYSIS METHOD OF FRAMED TUBE STRUCTURES

Sophisticated computer analysis are necessary in reducing the number of imprecisions produced by hand analysis techniques and are being used routinely in everyday engineering practice. Even though such computer analysis may intimidate the structural engineer by virtue of their incredible amount of documentation and output, the judicious engineer will always verify the reasonableness of the computer analysis by using approximate hand-calculated values (Taranath 1988).

While computer programs can be used to numerically analyze the framed tube structures, they cannot substitute theoretical analysis methods, which may offer better understanding of the structural system (Kwan, 1994). Furthermore, in view of the wide application of such system, there is obvious need for a simplified analysis method that can be used during the preliminary design stage to give an initial valuation of the structural behavior and in the final design stage for manual checking of the computer analysis results (Kwan, 1994).

3.1 Orthotropic Membrane Tube Analogy for Simplified Analysis of Framework Panels

Kwan (1994) followed the methodology of modeling the framework panels as equivalent orthotropic membranes so that the framed tubes can be analyzed as continuous structures. He stated that the equivalent elastic properties for the membranes and the method of analyzing the equivalent membrane structures are the two factors that can affect the accuracy of this membrane analogy. His methodology used independent distribution of axial displacements for web and flange panels. Thus, the shear lag in each panel is individually allowed for. The axial displacement distributions are assumed to be cubic in the web panels, and parabolic in the flange panel, and the principle of minimum total potential energy is employed in his formulation.

In this section, the method proposed for approximate analysis of framed tube structures with the shear lag effects taken in to account that is carried out by Kwan (1994) is presented.

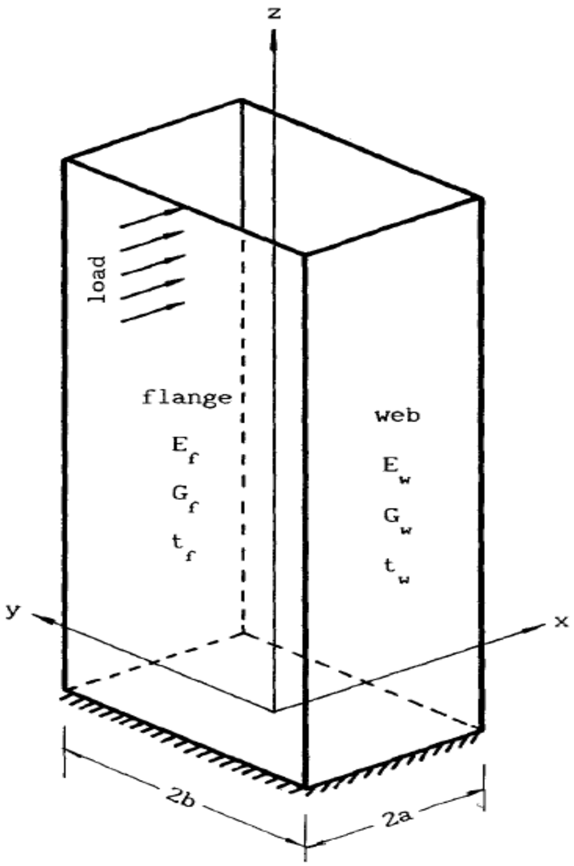


Figure 3-1: Orthotropic membrane tube analogy (Kwan, 1994).

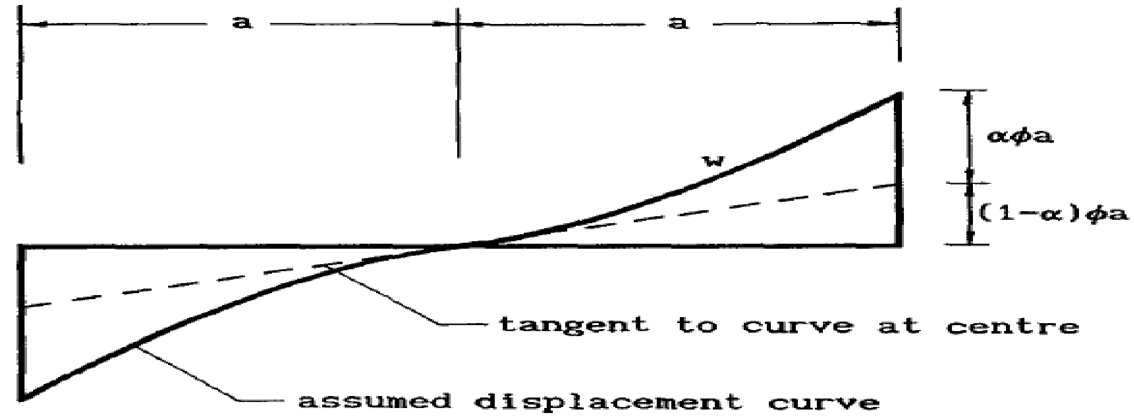
Considering the analogous membrane tube in Figure 3-1, plane sections will no longer remain plane because shear lag when the structure is loaded. The axial displacement in the web and flange panels, denoted respectively by *w* and *w'*, estimated by the following equation (Kwan, 1994):

$$w = \phi a\left[\left(1-\alpha\right)\left(\frac{x}{a}\right)+\alpha\left(\frac{x}{a}\right)^3\right] \tag{3.1}$$

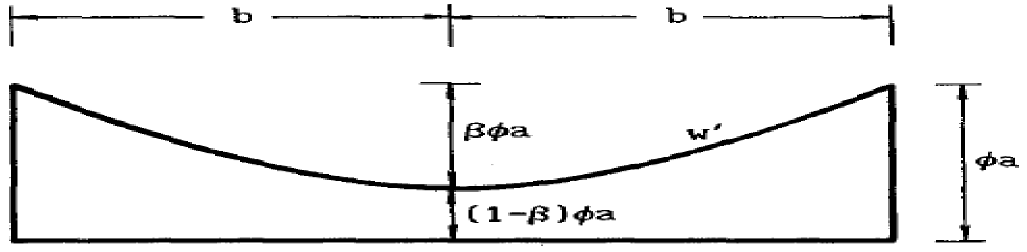
$$w' = \phi a\left[\left(1-\beta\right)+\beta\left(\frac{y}{b}\right)^2\right] \tag{3.2}$$

Which gives cubic and parabolic distributions of axial displacements in the web and flange panels respectively. These anticipated distributions of axial displacements are showed in Figure

3-2. Note that ϕ is the rotation of the plane section joining the four corners of the tubular structure which initially lie on the same horizontal plane, α and β are dimensionless shear lag coefficients representing the degree of shear lag in the web and flange panels, respectively and a & b are half with of web and flange panels respectively (Kwan, 1994).



(a) distribution of axial displacement in web



(b) distribution of axial displacement in flange

Figure 3-2: Assumed distribution of axial displacement (Kwan, 1994).

Axial Stiffness

Under the action of vertical axial forces, the load-deformation relationships for both the frame unit and the equivalent membrane will be equal if;

$$Est = E_m A_c \tag{3.3}$$

Where E is equivalent elastic modulus of the membrane, s is center-to-center spacing of column, t is thickness of the membrane, E_m is elastic modulus of the construction material, and A_c is sectional area of the column (Kwan, 1994). It is normal to fix the value of t such that the area of the membrane is equal to the sectional area of the column (i.e. $st = A_c$) and so that the axial stress in the column and that in the membrane are equal (Kwan, 1994). In such case;

$$t = \frac{A_c}{s} \tag{3.4}$$

$$E = E_m \tag{3.5}$$

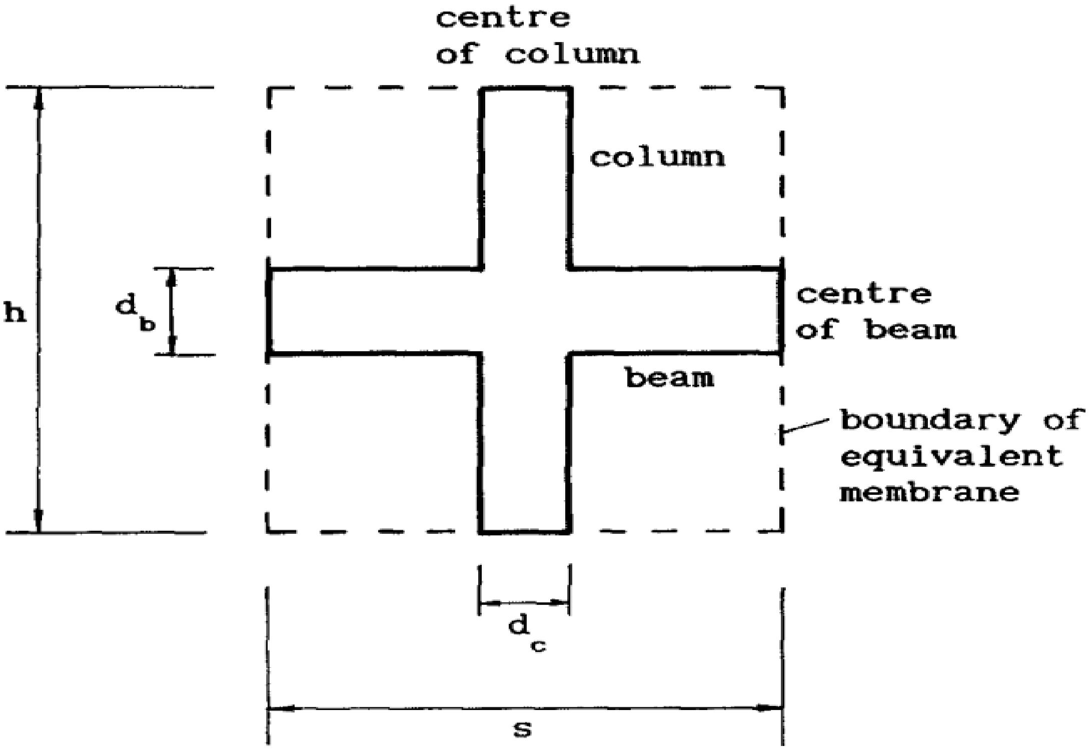


Figure 3-3: Membrane analogy for basic frame unit (Kwan, 1994).

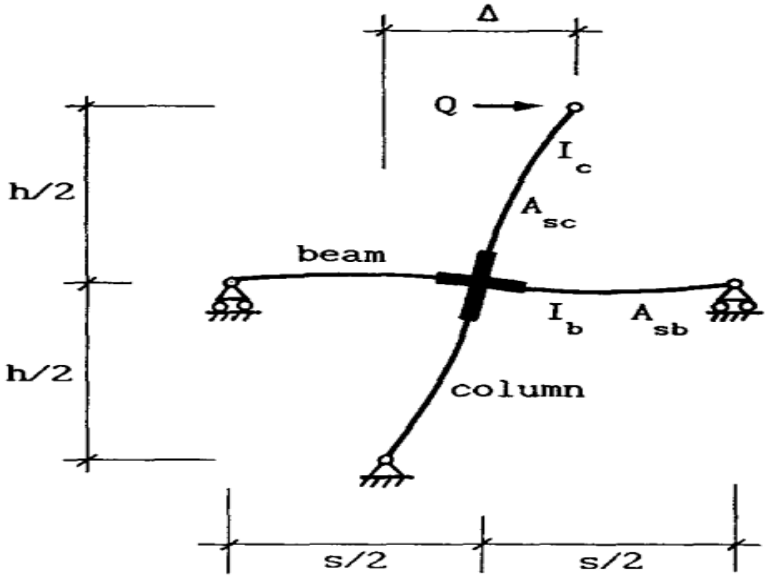


Figure 3-4: Basic Frame unit under lateral shear force (Kwan, 1994).

Shear Stiffness

Consider the case of the frame unit subject to a lateral force Q (Figure 3-4). The lateral deflection may be calculated as the sum of that due to bending Δ_b and due to shear Δ_s (Kwan, 1994). The bending deflection Δ_b is given by;

$$\frac{\Delta_b}{Q} = \frac{(h-d_b)^3}{12E_mI_c} + \left(\frac{h}{s}\right)^2 \frac{(s-d_c)^3}{12E_mI_b} \tag{3.6}$$

Where I_b and I_c are moments of inertia of the beam and the column respectively. On the other hand, the shear deflection Δ_s is given by;

$$\frac{\Delta_s}{Q} = \frac{(h-d_b)}{G_mA_{sc}} + \left(\frac{h}{s}\right)^2 \frac{(s-d_c)}{G_mA_{sb}} \tag{3.7}$$

In which A_{sb} and A_{sc} are effective shear areas of the beam and column respectively; and G_m is shear modulus of the material. Equating the total lateral deflection of the frame unit to the shear deflection of the membrane, the following equation is obtained (Kwan, 1994);

$$Q\frac{h}{Gst}=\Delta_b+\Delta_s \tag{3.8}$$

Where *G* is equivalent shear modulus of the membrane. From this equation, the value of *G* is derived as;

$$G=\frac{\frac{h}{st}}{\frac{\Delta_b}{Q}+\frac{\Delta_s}{Q}} \tag{3.9}$$

In which Δ_b/Q and Δ_s/Q are given by Equation (3.6) and (3.7), respectively.

Axial Stresses

The axial stresses in the web and flange panels can be evaluated by:

$$\sigma_z=E_w\frac{d\phi}{dz}a\left[\left(1-\alpha\right)\frac{x}{a}+\alpha\left(\frac{x}{a}\right)^3\right] \tag{3.10}$$

$$\sigma'_z=E_f\frac{d\phi}{dz}a\left[\left(1-\beta\right)+\beta\left(\frac{y}{b}\right)^2\right] \tag{3.11}$$

Where E_w and E_f are equivalent young’s moduli of web and flange panels, respectively.

$d\phi/dz$ is determined as $d\phi/dz = M/EI$, where *EI* is given by;

$$EI=\frac{4}{3}E_wt_wa^3\left(1-\frac{2}{5}\alpha\right)+4E_ft_fa^2b\left(1-\frac{2}{3}\beta\right)+4E_mA_ka^2 \tag{3.12}$$

Where *M* is overturning moment of lateral load, A_k is sectional area of corner column and t_w & t_f are equivalent thickness of web and flange panels, respectively.

The shear lag coefficients α and β determined by:

$$\alpha=\alpha_1\left(1-\frac{z}{H}\right)^2+\alpha_2\left[2\frac{z}{H}-\left(\frac{z}{H}\right)^2\right] \tag{3. 13}$$

$$\beta = \beta_1\left(1-\frac{z}{H}\right)^2 + \beta_2\left[2\frac{z}{H}-\left(\frac{z}{H}\right)^2\right] \tag{3. 14}$$

Where α_1, α_2, β_1, and β_2 are given in Table 3-1, in which the relative shear stiffness parameters m_w and m_f are defined by;

$$m_w = \frac{G_w H^2}{E_w a^2} \tag{3. 15}$$

$$m_f = \frac{G_f H^2}{E_f b^2} \tag{3. 16}$$

Where G_w & G_f are equivalent shear moduli of web and flange panels, respectively, H is structural height and z height measured from the base of the structure.

Table 3-1: Formulas for α and β (Kwan, 1994).

Load case	α	β
Point load at top	$\alpha_1 = \frac{1.17m_w + 1.00}{m_w^2 + 2.67m_w + 0.57}$ $\alpha_2 = \frac{0.29m_w + 1.00}{m_w^2 + 2.67m_w + 0.57}$	$\beta_1 = \frac{3.50m_f + 12.60}{m_f^2 + 11.20m_f + 10.08}$ $\beta_2 = \frac{0.88m_f + 12.60}{m_f^2 + 11.20m_f + 10.08}$
Uniform distributed load	$\alpha_1 = \frac{2.57m_w + 1.12}{m_w^2 + 2.94m_w + 0.64}$ $\alpha_2 = \frac{0.03m_w + 1.12}{m_w^2 + 2.94m_w + 0.64}$	$\beta_1 = \frac{7.72m_f + 14.15}{m_f^2 + 12.35m_f + 11.32}$ $\beta_2 = \frac{0.08m_f + 14.15}{m_f^2 + 12.35m_f + 11.32}$

Triangular distributed load	$\alpha_1 = \frac{2.22m_w + 1.09}{m_w^2 + 2.86m_w + 0.62}$ $\alpha_2 = \frac{0.10m_w + 1.09}{m_w^2 + 2.86m_w + 0.62}$	$\beta_1 = \frac{6.67m_f + 13.71}{m_f^2 + 12.01m_f + 10.97}$ $\beta_2 = \frac{0.29m_f + 13.71}{m_f^2 + 12.01m_f + 10.97}$

Lateral Deflection

The exact value of *EI* near the top do not really affect the value of ϕ and *u* significantly because of most of bending defamation occurs near the base and the exact values of the variation of *EI* with height may be neglected and the value of *EI* at all height taken as its value at the base (Kwan, 1994). This is equivalent to assuming that the structure behaves like a cantilevered beam with a constant bending stiffness of *EI*. After such simplifications, the formulas for the lateral deflection u become as follows (Kwan, 1994).

Load case 1 – point load at top

$$u = \frac{P}{EI}\left(\frac{1}{2}Hz^2 - \frac{1}{6}z^3\right) + \frac{P}{4G_w t_w a}z \tag{3.17}$$

Load case 2 – uniform load

$$u = \frac{U}{EI}\left(\frac{1}{4}H^2z^2 - \frac{1}{6}Hz^3 + \frac{1}{24}z^4\right) + \frac{U}{4G_w t_w a}\left(Hz - \frac{1}{2}z^2\right) \tag{3.18}$$

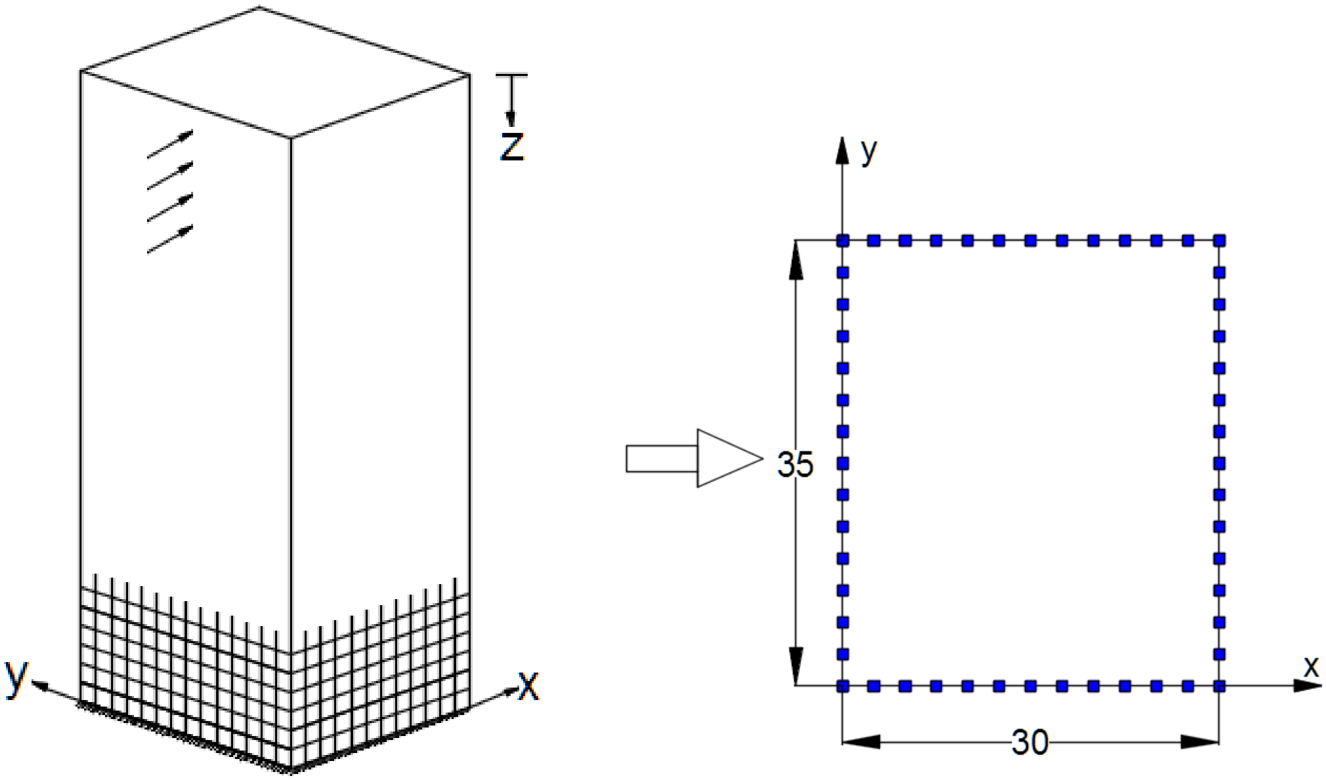
Load case 2 – triangular load

$$u = \frac{T}{EI}\left(\frac{1}{6}H^2z^2 - \frac{1}{12}Hz^3 + \frac{1}{120}\frac{z^5}{H}\right) + \frac{T}{4G_w t_w a}\left(\frac{1}{2}Hz - \frac{1}{6}\frac{z^3}{H}\right) \tag{3.19}$$

3.2 Simplified Analysis of Sample Framed Tube Structures and Comparisons with Computer Analysis

Three reinforced concrete framed tube structures of 40, 50, and 60 stories, as shown in Figure 3-5 are analyzed. All the beam and column members are of sizes 0.8 m x 0.8 m. The height of each story is 3.0 m and the center-to-center spacing of the columns are 2.5 m (Kwan, 1994). The Young's and shear moduli of the material are 20 GPa and 8.0 GPa, respectively. A uniformly distributed lateral load of 120 KN/m is applied to the structures (Kwan, 1994). The results obtained from Kwan (1994) method are compared with those produced by ETABS 2015 v15.0.0 software.

The equivalent elastic properties of the analogous orthotropic membrane tube for 40 story framed tube structure as evaluated by the method discussed in section 3.1 are presented in Table 3-2, similarly shear stiffness parameters and shear lag coefficients of the web, and flange panels are computed and summarized in Table 3-3. Having determining the shear lag coefficients, the axial stresses in the web and flange panels at the base and 10th floor of the structure are computed by Equation (3.10) and (3.11) respectively. The resulting axial force distribution in the web and flange columns of 40, 50 and 60 story buildings obtained from Kwan (1994) method and those produced by ETABS 2015 v15.0.0 integrated building design software at the base and 10th floor of the structure are plotted in Figure 3-6 and Figure 3-7 respectively.



(a) **(b)**

Figure 3-5: Geometry of framed-tube structure: (a) 3D view. (b) Plan view.

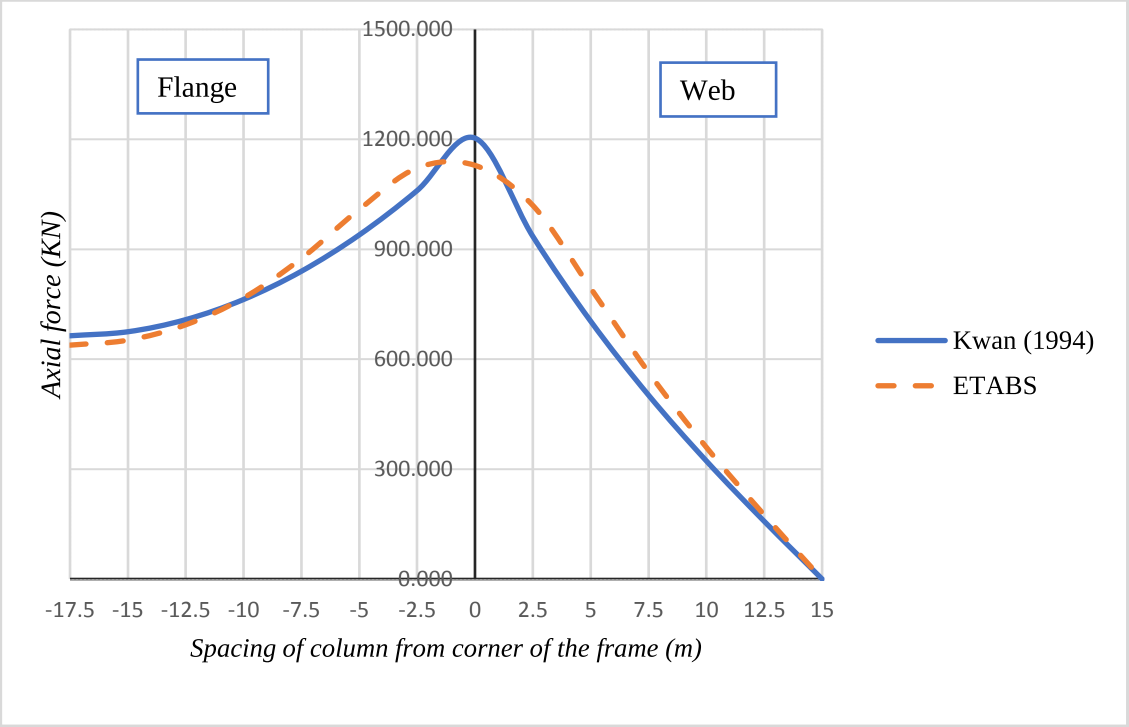
Table 3-2: Equivalent elastic properties of the analogous orthotropic membrane tube

H	120 m
h	3 m
s	2.5 m
a	15 m
b	17.5 m
$d_b = d_c$	0.8 m
$I_c = I_b$	0.034 m^4
$t_f = t_w$	0.256 m
$E = E_m = E_w = E_f$	20.000 GPa

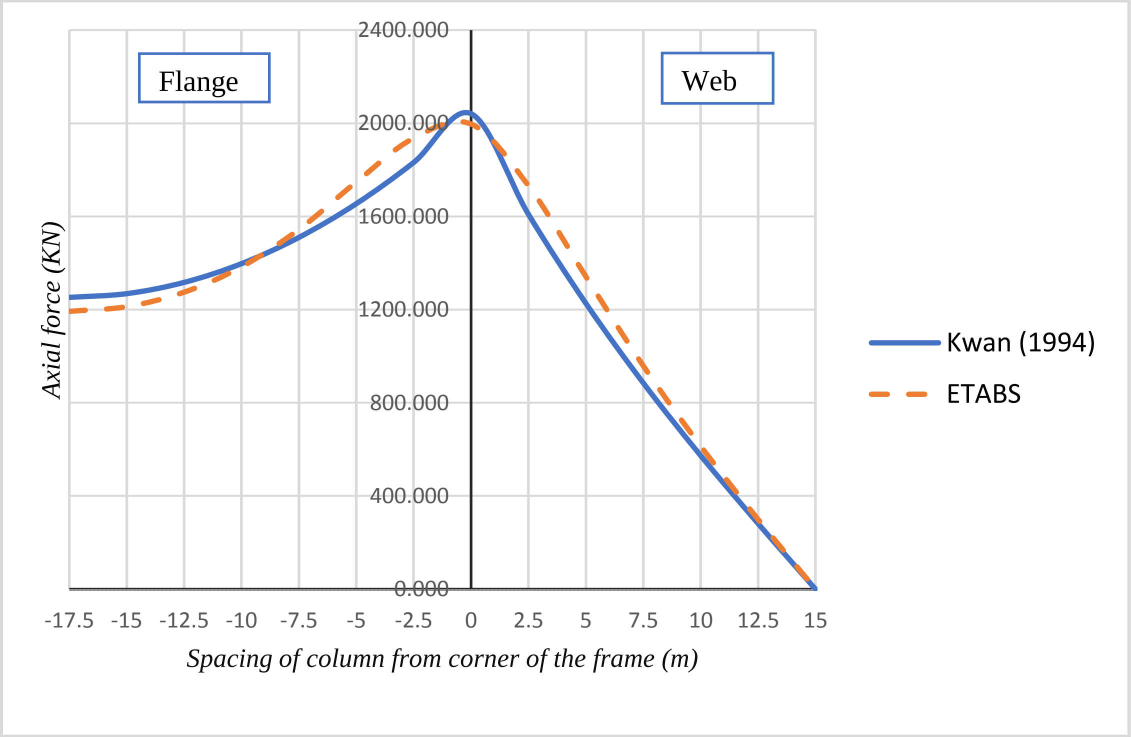
$\frac{\Delta_b}{Q} = \frac{\left(h-d_b\right)^3}{12E_mI_c} + \left(\frac{h}{s}\right)^2 \frac{\left(s-d_c\right)^3}{12E_mI_b}$	2.163 m/10^6KN
$\frac{\Delta_s}{Q} = \frac{\left(h-d_b\right)}{G_mA_{sc}} + \left(\frac{h}{s}\right)^2 \frac{\left(s-d_c\right)}{G_mA_{sb}}$	1.089 m/10^6KN
$\frac{h}{st}$	4.688/m
$G = G_w = G_f = \frac{\frac{h}{st}}{\frac{\Delta_b}{Q} + \frac{\Delta_s}{Q}}$	1.441 GPa

Table 3-3: Shear stiffness parameters and shear lag coefficients of the web, and flange panels

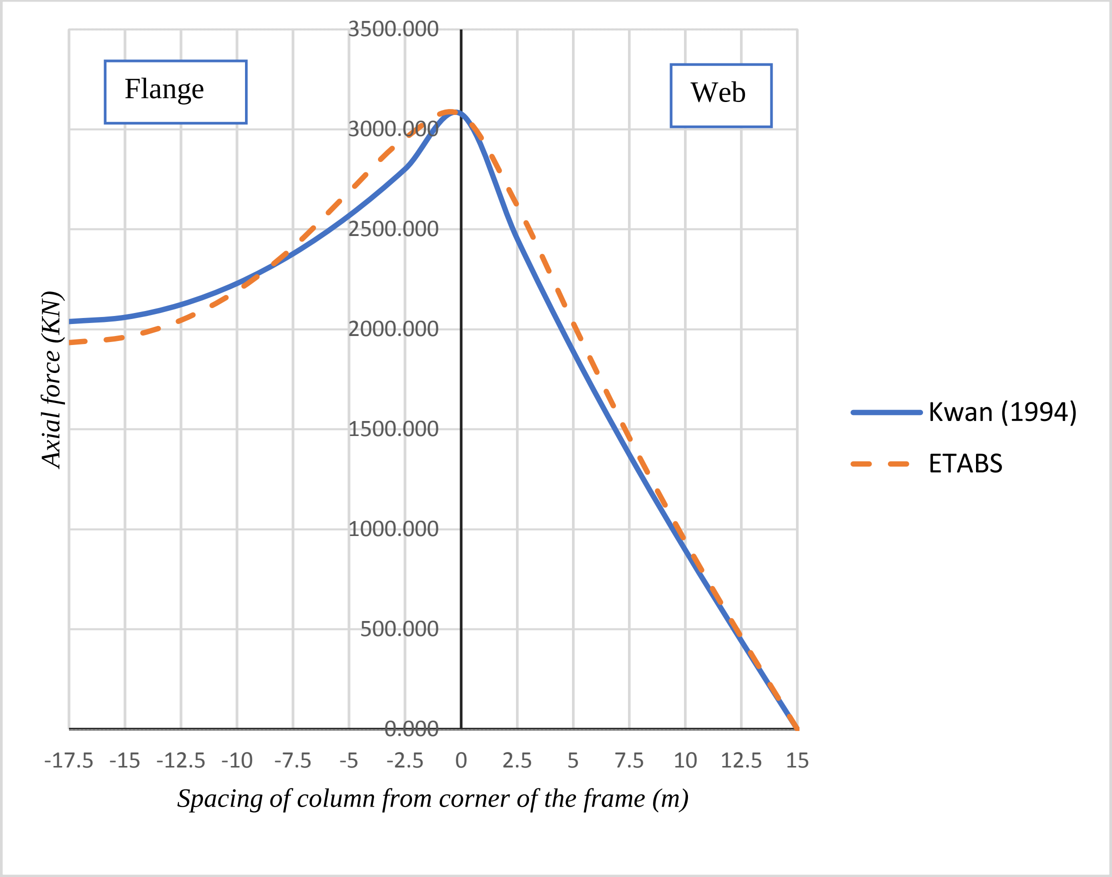
$m_w = \frac{G_wH^2}{E_wa^2}$	4.611
$m_f = \frac{G_fH^2}{E_fb^2}$	3.388
$\alpha_1 = \frac{2.57m_w + 1.12}{m_w^2 + 2.94m_w + 0.64}$	0.366
$\alpha_2 = \frac{0.03m_w + 1.12}{m_w^2 + 2.94m_w + 0.64}$	0.035
$\beta_1 = \frac{7.72m_f + 14.15}{m_f^2 + 12.35m_f + 11.32}$	0.624
$\beta_2 = \frac{0.08m_f + 14.15}{m_f^2 + 12.35m_f + 11.32}$	0.223


Flange
Web
1500.000
1200.000
900.000
600.000
300.000
0.000
Axial force (KN)
-17.5
-15
-12.5
-10
-7.5
-5
-2.5
0
2.5
5
7.5
10
12.5
15
Spacing of column from corner of the frame (m)
Kwan (1994)
ETABS

(a)


Flange
Web
2400.000
2000.000
1600.000
1200.000
800.000
400.000
0.000
Axial force (KN)
-17.5
-15
-12.5
-10
-7.5
-5
-2.5
0
2.5
5
7.5
10
12.5
15
Spacing of column from corner of the frame (m)
Kwan (1994)
ETABS

(b)



(c)

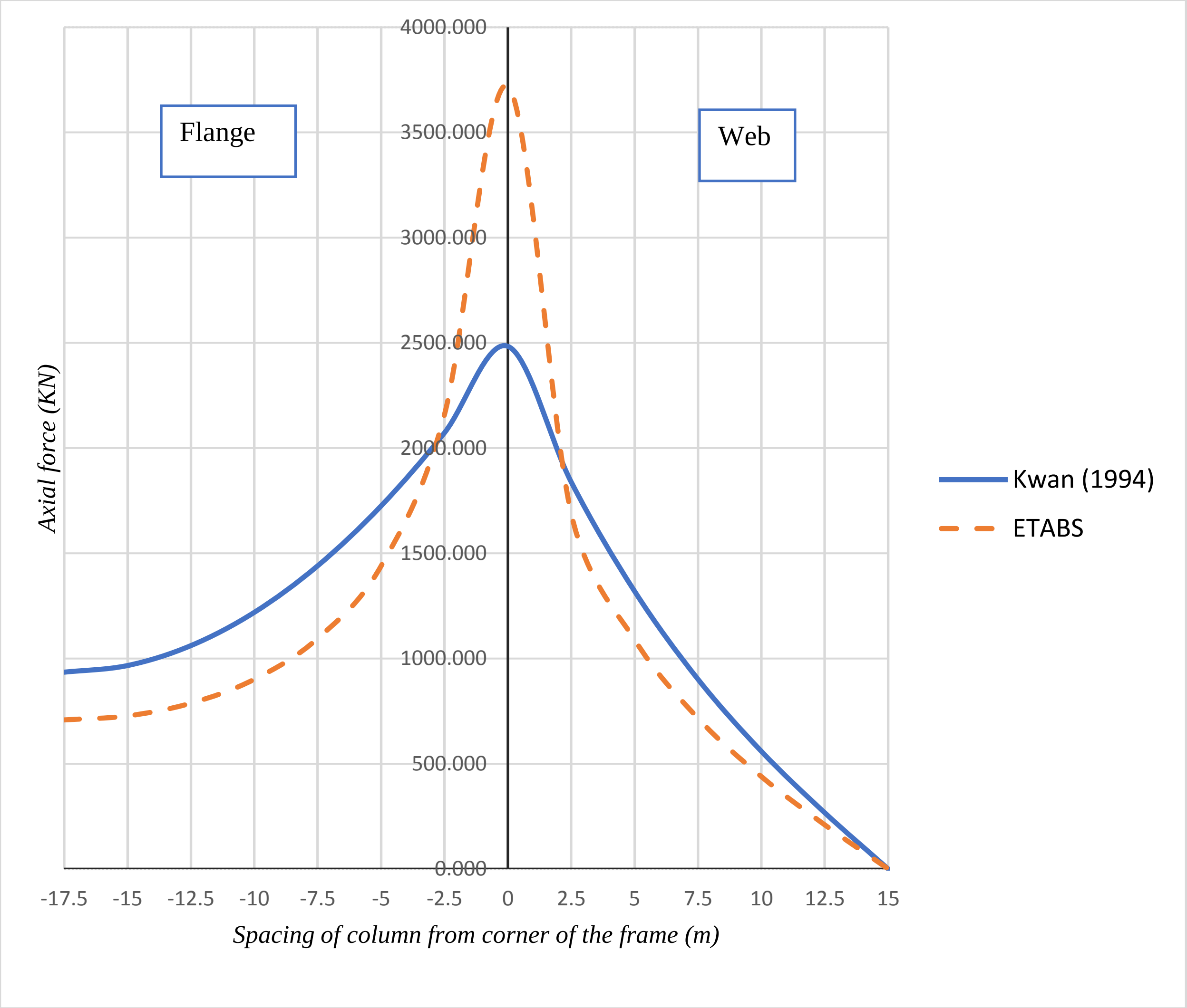
Figure 3-6: Flange and web columns axial force distribution at 10th floor of the structure: (a) 40-story. (b) 50-story. (c) 60-story.

Comparing the total axial force at the 10th floor of the structure computed by Kwan (1994) method and those produced by ETABS software there is only a 0.48% and 5.37% deviation in the flange and web panels of 40-story building respectively. Similarly, the percentage deviation of axial force in the flange and web panels is 0.13% and 4.69% respectively for 50-story building and 0.09% and 4.24% respectively for 60-story building.

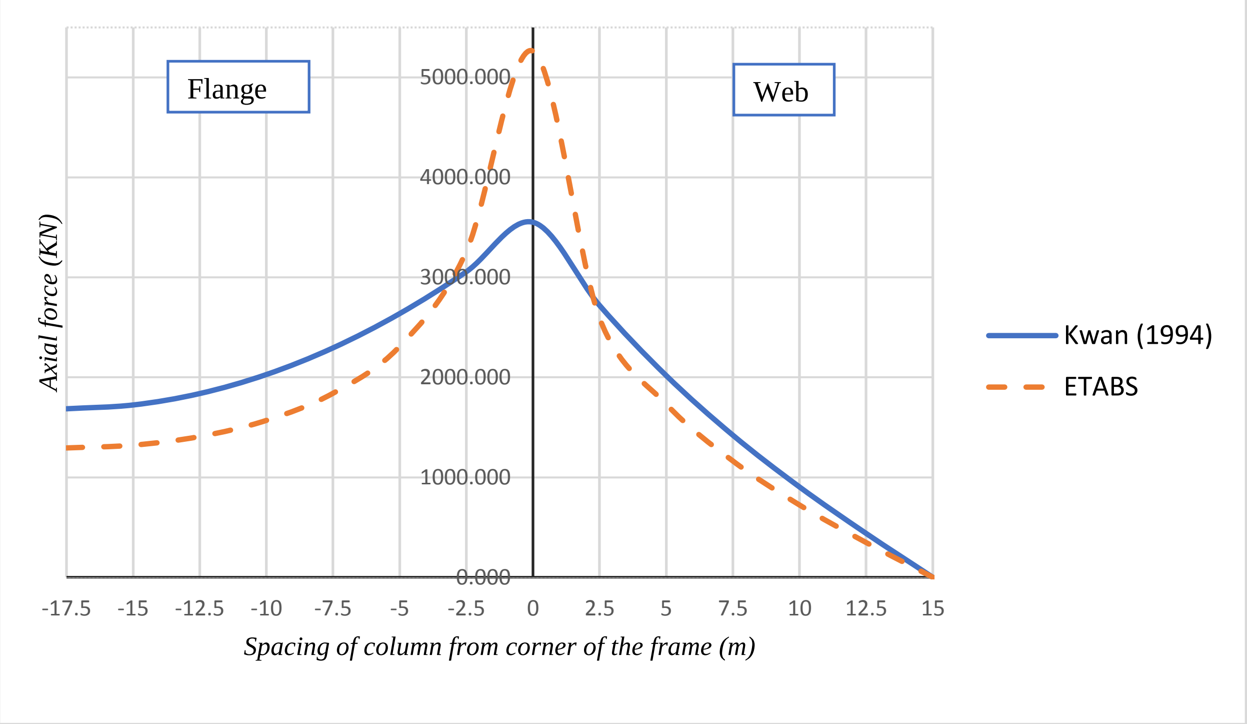
$$Shear\ lag\ factor = \frac{Axial\ stress\ at\ the\ center\ column}{Axial\ stress\ at\ the\ corner\ column}$$

The resulting shear lag factors for 40-story, 50-story and 60-story buildings at the 10th floor of the building are 0.552, 0.614 and 0.663 respectively according to Kwan (1994) method.

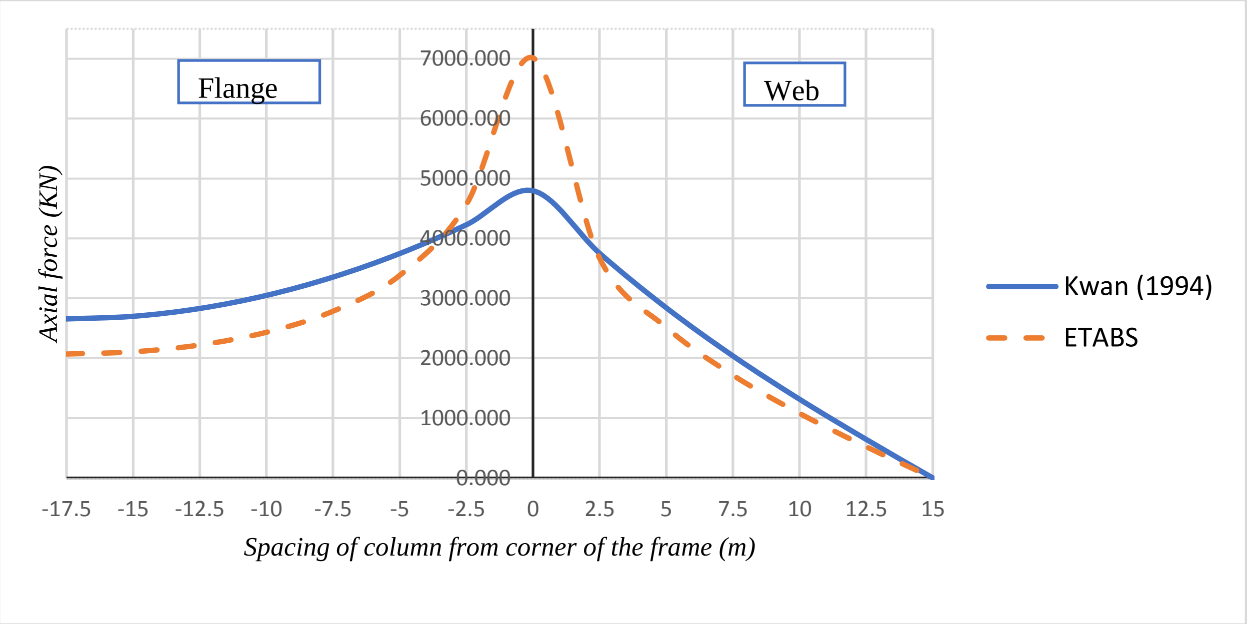
Similarly, the resulting shear lag factor computed from ETABS software analysis result is 0.565, 0.597 and 0.629 respectively for 40-story, 50-story and 60-story buildings at the 10th floor. The deviation in magnitude of axial force at the corner columns of the 40-story, 50-story and 60-story buildings at the 10th floor is only 6.2%, 2.2%, 0.2% respectively.



(a)



(b)



(c)

Figure 3-7: Flange and web columns axial force distribution at the base of the structure: (a) 40-story. (b) 50-story. (c) 60-story.

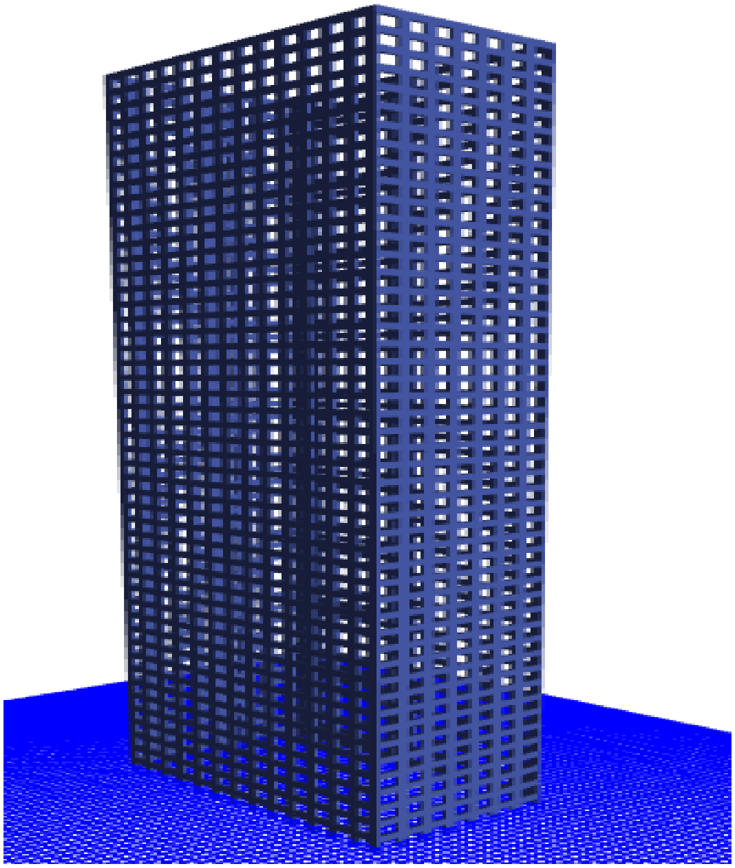
Comparing the total axial force at the base of the structure computed by Kwan (1994) method and those produced by ETABS software there is only a 3.08% and 6.26% deviation in the flange and web panels of 40-story building respectively. Similarly, the percentage deviation of axial force in the flange and web panels is 2.91% and 6.51% respectively for 50-story building and 2.85% and 6.84% respectively for 60-story building.

Compared to ETABS software result, Kwan (1994) method underestimate the shear lag effect at the base of the structure. The resulting shear lag factor for 40-story, 50-story and 60-story buildings at the base are 0.376, 0.475 and 0.554 respectively according to Kwan (1994) method and similarly, the resulting shear lag factor computed from ETABS software analysis result is 0.190, 0.246 and 0.295 respectively for 40-story, 50-story and 60-story buildings. Which results in 33.2%, 32.5% and 31.6% deviation in magnitude of axial force at the corner columns at the base of the structure for 40-story, 50-story and 60-story buildings respectively.

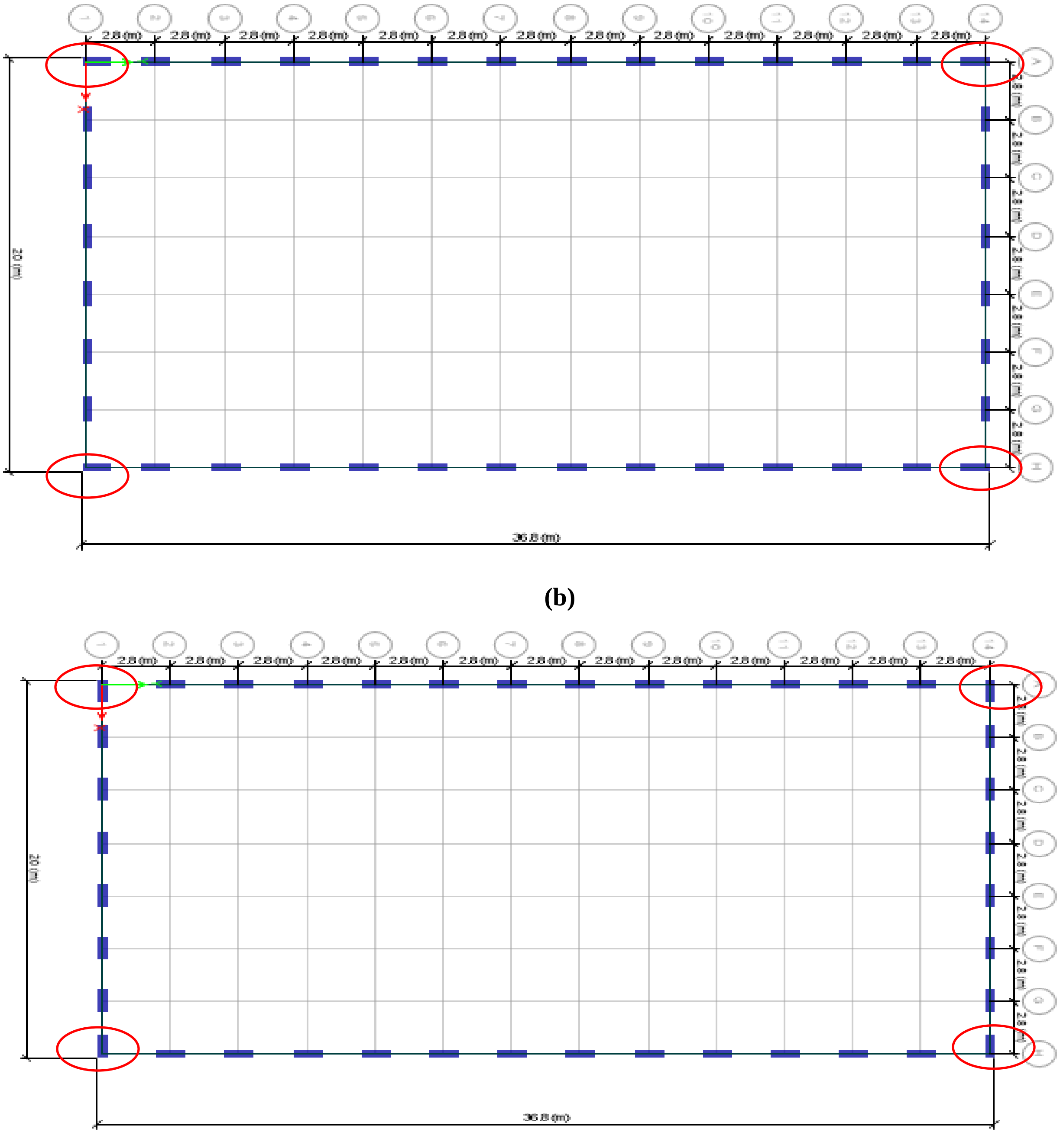
CHAPTER 4 ANALYSIS OF FRAMED TUBE AND TRUSSED TUBE STRUCTURES

4.1 Modeling and Analysis of Framed Tube Buildings

To investigate the shear lag phenomenon and structural behavior of framed tube structural system six reinforced concrete framed tube structures with two groups having 30, 40, and 49 stories with a plan area of 20.0mx36.8m having a story height of 3m and column center to center spacing of 2.8m are modeled, and analyzed using ETABS 2015 v15.0.0 integrated building design software. The wider side of the corner rectangular columns are aligned parallel to the flange panel of the frame for the first group buildings (Figure 4-1b), and for the second group buildings the wider side of the corner rectangular columns are aligned parallel to the web panel of the frame (Figure 4-1c).



(a)



(b)

(c)

Figure 4-1: Framed-tube building model: (a) 3D view (b) Plan view of group 1 structures (c) plan view of group 2 structures

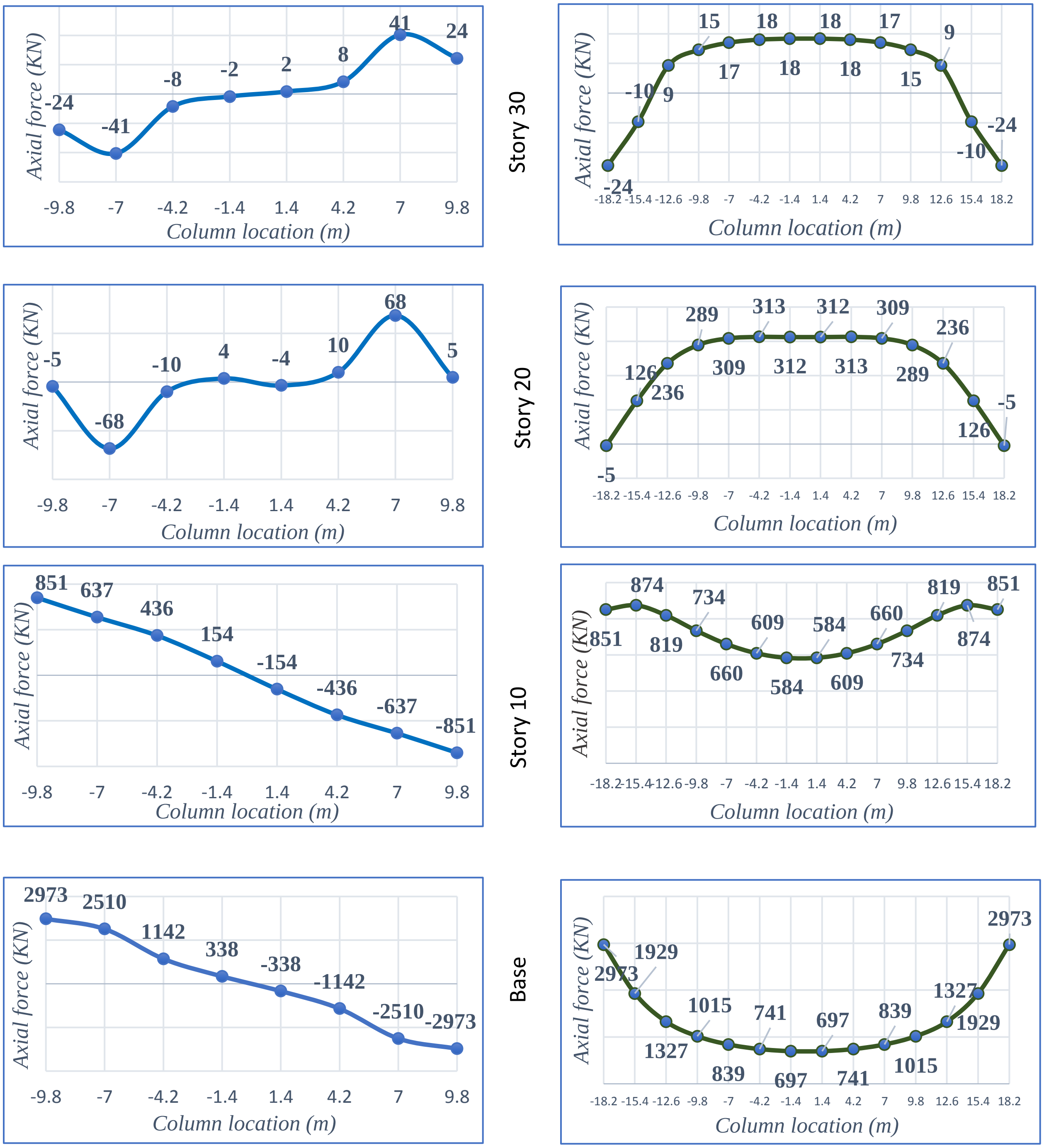
The building is assumed to be loaded with a uniformly distributed lateral load of 120 KN/m along the height of the structure (Kwan, 1994). In each of the six structures, the cross-sectional area of all the beams and columns is 0.48 m^2 (0.4m x 1.2m). The Young's modulus and Poisson's ratio of the material are considered to be 28.33 GPa and 0.2 respectively. For the analysis of the structure, diaphragm constraint is assigned in order to connect all constrained joints to each other by links that are rigid in plane. The size of the structural members, material and spacing are assumed to be uniform throughout the building height.

4.2 Assessment of Analysis Results of Framed Tube Buildings

4.2.1 Shear lag in framed tube buildings

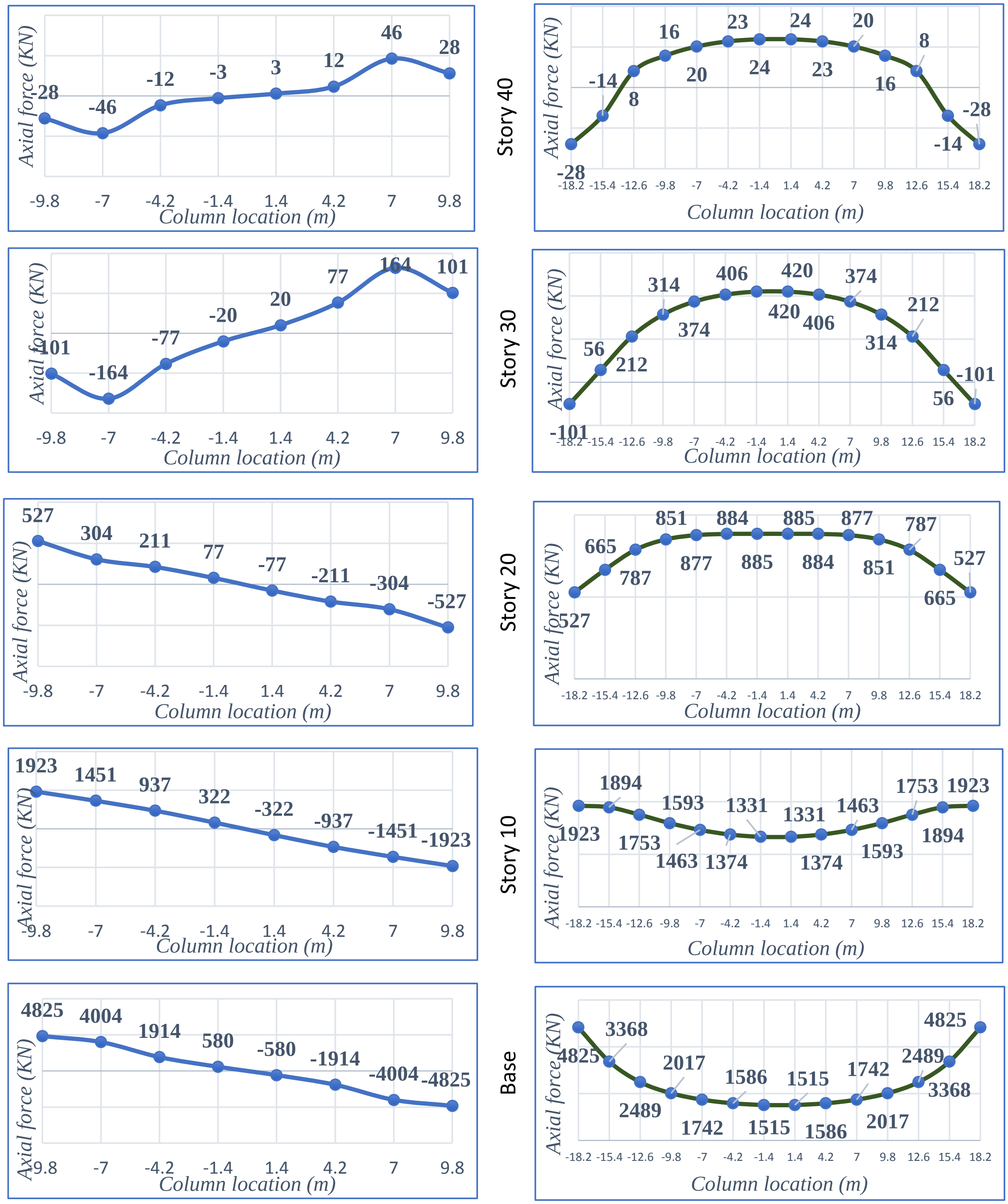
As reported by the literatures discussed in the above sections of the paper, because of the flexibility of spandrel beams, when the framed tube is subjected to lateral loading, columns near the corner of the tube experience the greatest axial force and it spreads non-linearly for the web and flange panels. This effect is called "shear lag" and greatly reduce the effectiveness of the structure.

The flange frames are coaxed to resist the overturning moment due to the continuity of closely spaced columns and spandrels surrounding the corner of the building. Whether the portion or all of the flange columns contributes to the bending resistance of the structure is a function of shear rigidity of the tube. One of the aim of optimal design of framed tube is to limit the shear lag effect. The resulting axial force distribution in the web and flange columns of 30, 40 and 49 story group one buildings are plotted in Figure 4-2, Figure 4-3 & Figure 4-4 respectively.



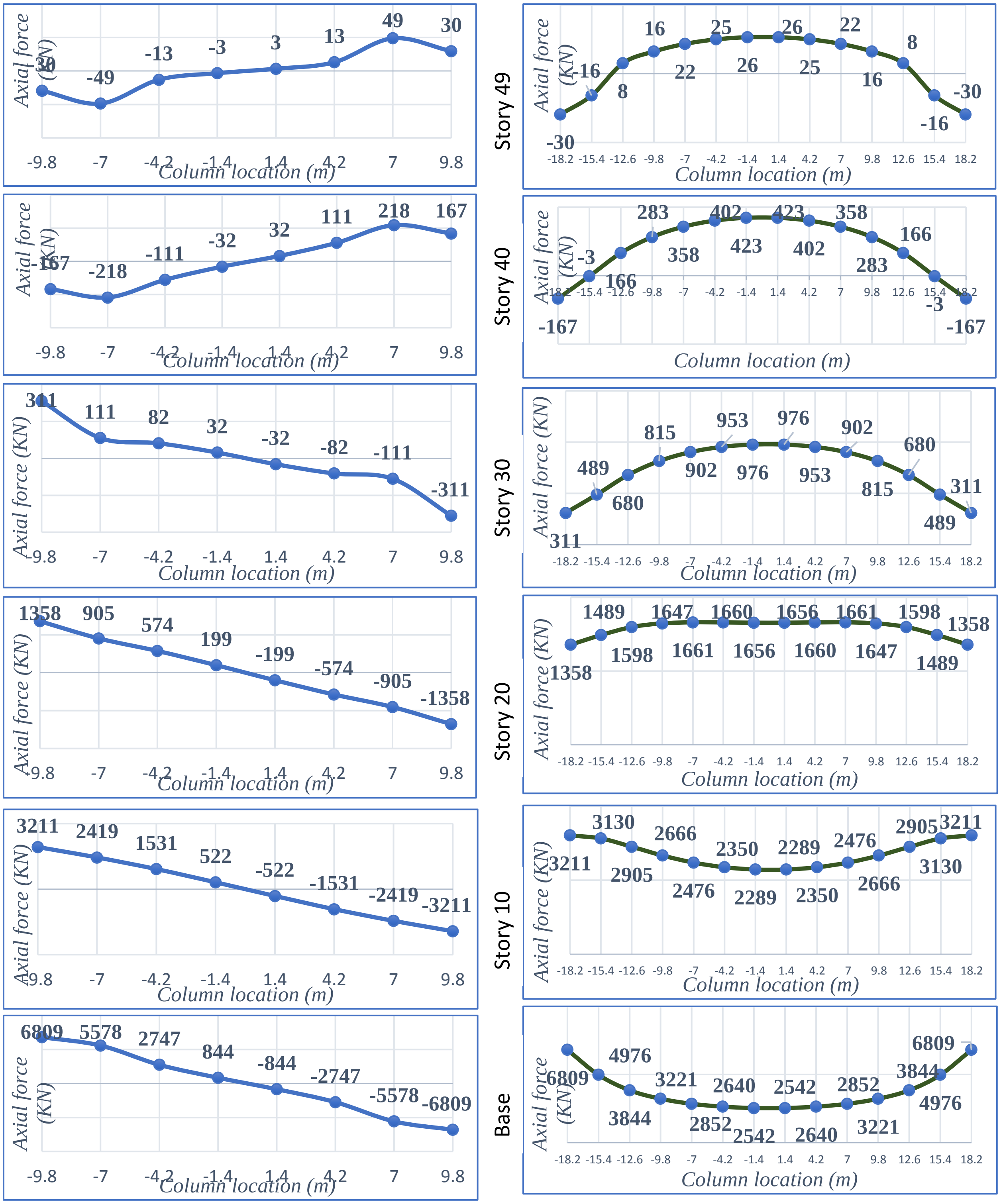
(a) **(b)**

Figure 4-2: Distribution of axial force for 30-story group one building: (a) Web panel. (b) Flange panel.



(a) **(b)**

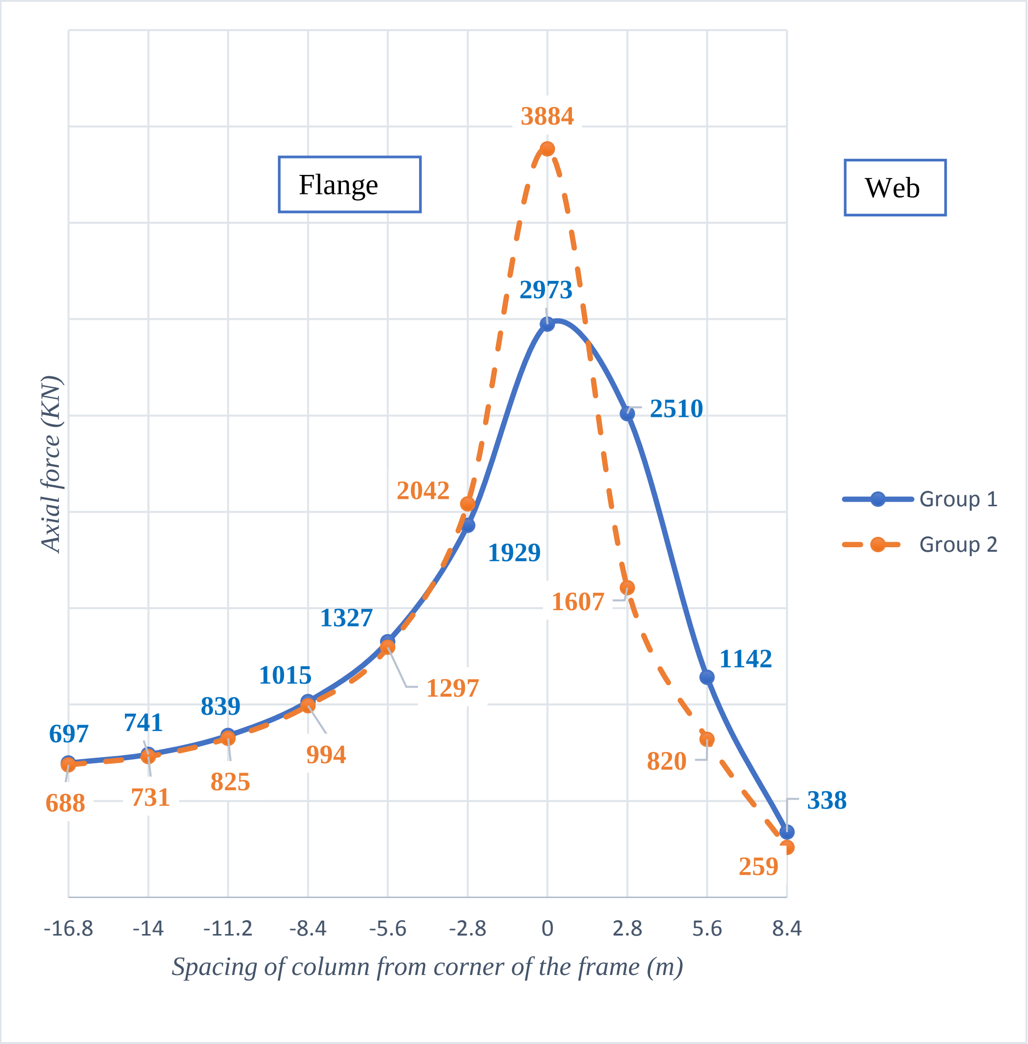
Figure 4-3: Distribution of axial force for 40-story group one building: (a) Web panel. (b) Flange panel.



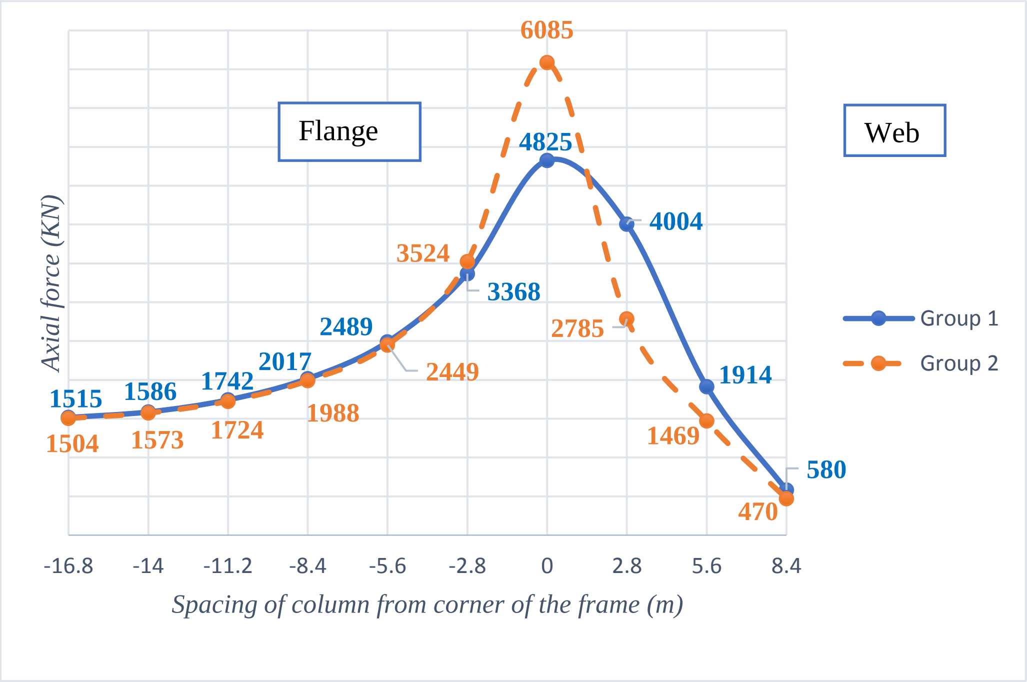
(a) **(b)**

Figure 4-4: Distribution of axial force for 49-story group one building: (a) Web panel. (b) Flange panel.

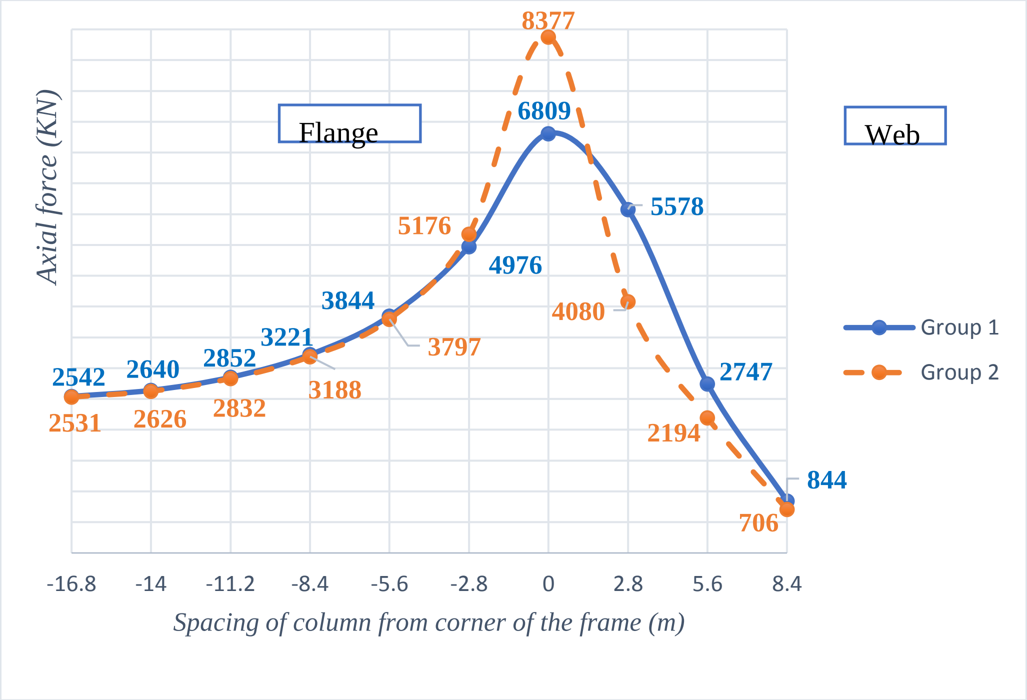
In order to investigate the effect of corner column alignment on the axial force distribution, the resulting axial distribution of group one and two buildings at the base of the structure are compared and plotted in Figure 4-5a, Figure 4-5b & Figure 4-5c respectively for 30, 40 and 49 story buildings. As evident from Figure the second group buildings experience a higher amount of axial force in the corner columns compared to those of first group buildings corner columns for all 30, 40 and 49 story buildings.



(a)



(b)



(c)

Figure 4-5: Distribution of axial force in the flange and web columns at the base of the structure: (a) 30-story. (b) 40-story. (c) 49-story.

The shear lag factor defined as the ratio of the axial stress at the center column to the axial stress at the corner column is evaluated to measure the efficiency of the framed tube system.

At lower floors of the structure a positive shear lag is observed and the value of shear lag factor increased going upward from the base of the structure up to certain height, and at some point the shear lag factor approaches to one. Moreover, at certain height it becomes much more higher since the stress at the corner columns approaches to zero. At even higher floors, the corner column stresses change sign, and thus the shear-lag factor becomes negative. One of the purpose of optimal design of a framed tube is to limit shear lag. The designer must make the shear- lag factor at the base as close to unity as possible. A shear-lag factor of one can only be achieved with infinite shear rigidity of the flange panels; this is exactly what the classical engineering theory of bending assumes. In practice, a shear-lag factor of 0.7 is considered satisfactory (Conner and Pouangare 1991).

From axial force distribution of columns, one can calculate shear lag factor as the ratio of axial force at center column to axial force of the corner column in the same axis. The result clearly show the influence of height of the building and corner column alignment on the shear lag factor. For the first group buildings, the shear lag factor at the base of the structure is 0.23, 0.31 and 0.37 for the 30, 40 & 49 story respectively. Similarly, the shear lag factor of 0.18, 0.25 and 0.30 is observed at the base of the structure for 30, 40 & 49 story buildings. The result indicates aligning the wider dimension of the corner column parallel to flange panel increase the shear lag factor by 32.36%, 27.02% and 23.58% respectively for 30, 40 and 49 story buildings.

4.2.2 Lateral deflection of framed tube buildings

As stated in the literatures, the behavior of framed tube has a hybrid nature of pure frame as well as pure tubes. The overturning under lateral load is resisted by the tube form causing compression and tension in the columns, while the shear from the lateral load is resisted by bending in columns and beams primarily in the two sides of the building parallel to the direction of the lateral load.

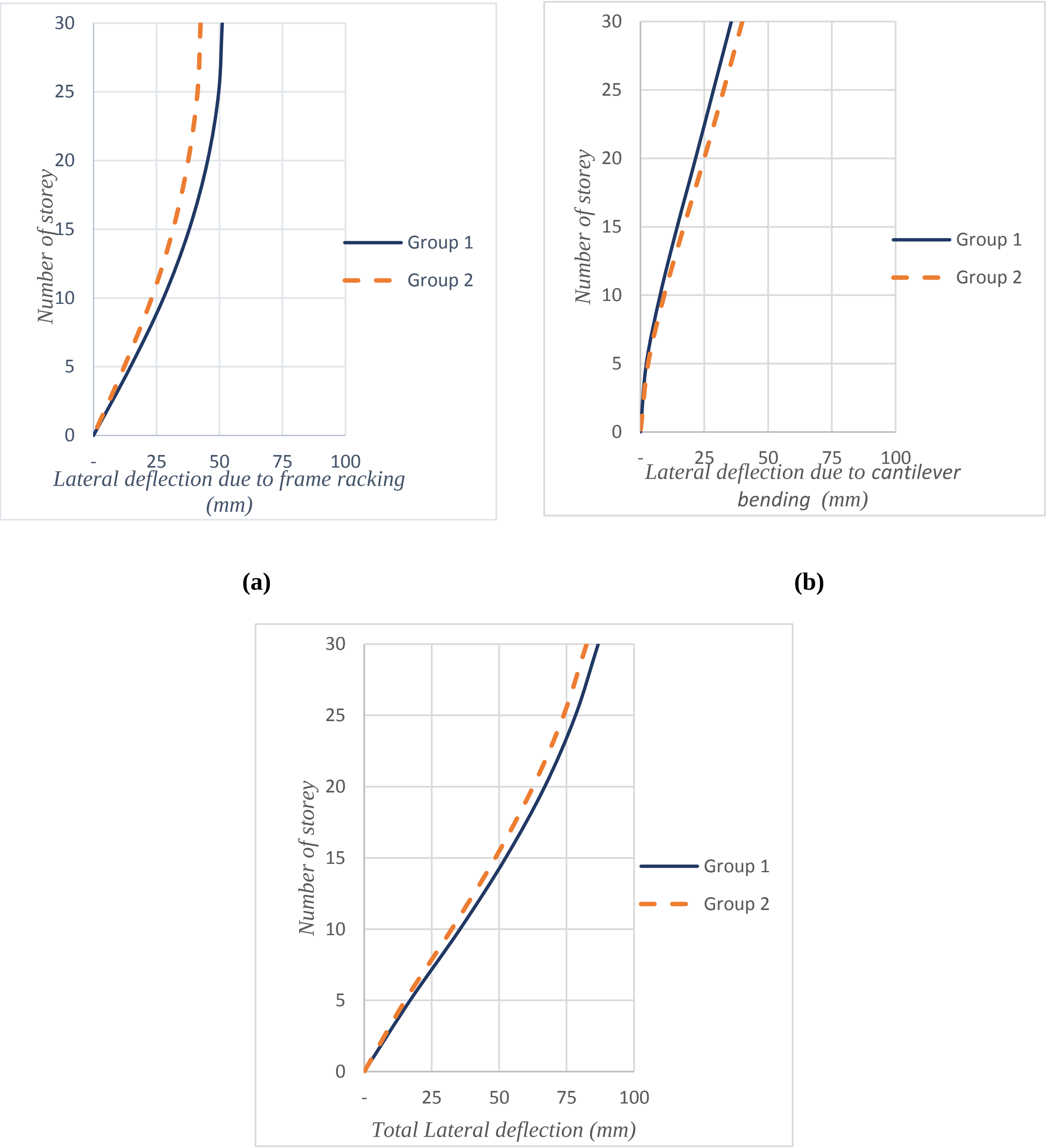
Efficiency is the ratio of axial column deformation to total deformation in the tower. Eliminating any significant local shear or bending deformations of vertical members, resulting in an essentially 100% efficient structure. It is more informative in high-rise buildings to assess the lateral sway as components due to rigid body deformation and racking (shear) deformation.

Since the bending deformation of a frame is a consequence of axial deformation of the columns alone and is independent of the size, location, type and arrangement of the web system, we can find the shear deformation components of the frame from ETABS software by making the axial rigidity of the column infinite. The bending deformation component of the frame can then be determined by deducting the shear deformation from the total lateral deflection of the frame. Lateral deflection out puts from ETABS software is verified by using simplified hand calculation method from Smith and Coull (1991) and Taranath (1988) and the results are presented in APPENDEX A of this thesis.

The resulting lateral deflection due to frame racking (shear deformation), cantilever bending (axial deformation of columns) and the total lateral deflection of the building for the two groups of 40, 50 and 49 story buildings are plotted in Figure 4-6 , Figure 4-7 and Figure 4-8 respectively.

The efficiency of the buildings calculated as the ratio of cantilever deflection to the total lateral sway of the building is calculated. For all the 30, 40 and 49 story buildings, the efficiency of group two buildings is higher than that of the group one buildings by 17.9%, 12.4% and 9.4% respectively. The contribution of cantilever deflection from the total lateral sway of the building for the group one is 41.1%, 52.1% and 60.4% respectively for 30, 40 and 49 story buildings.

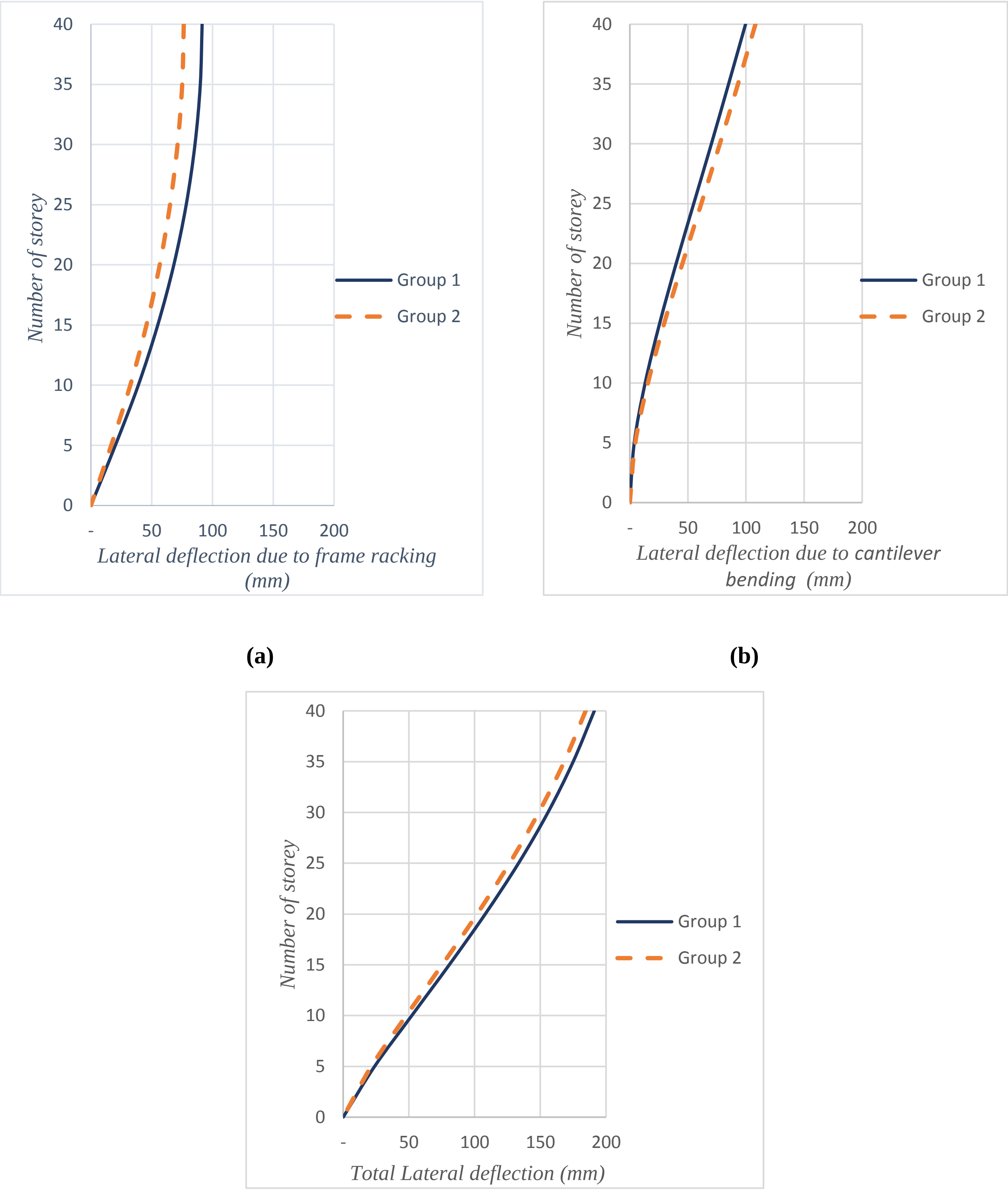
Similarly, 48.4%, 58.6% and 66.1% respectively for the group two of 30, 40 and 49 story buildings.



(a) **(b)**

(c)

Figure 4-6: Lateral deflection of 30 story framed tube structure: (a) Lateral deflection due to frame racking. (b) Lateral deflection due to cantilever bending. (c) Total lateral deflection.

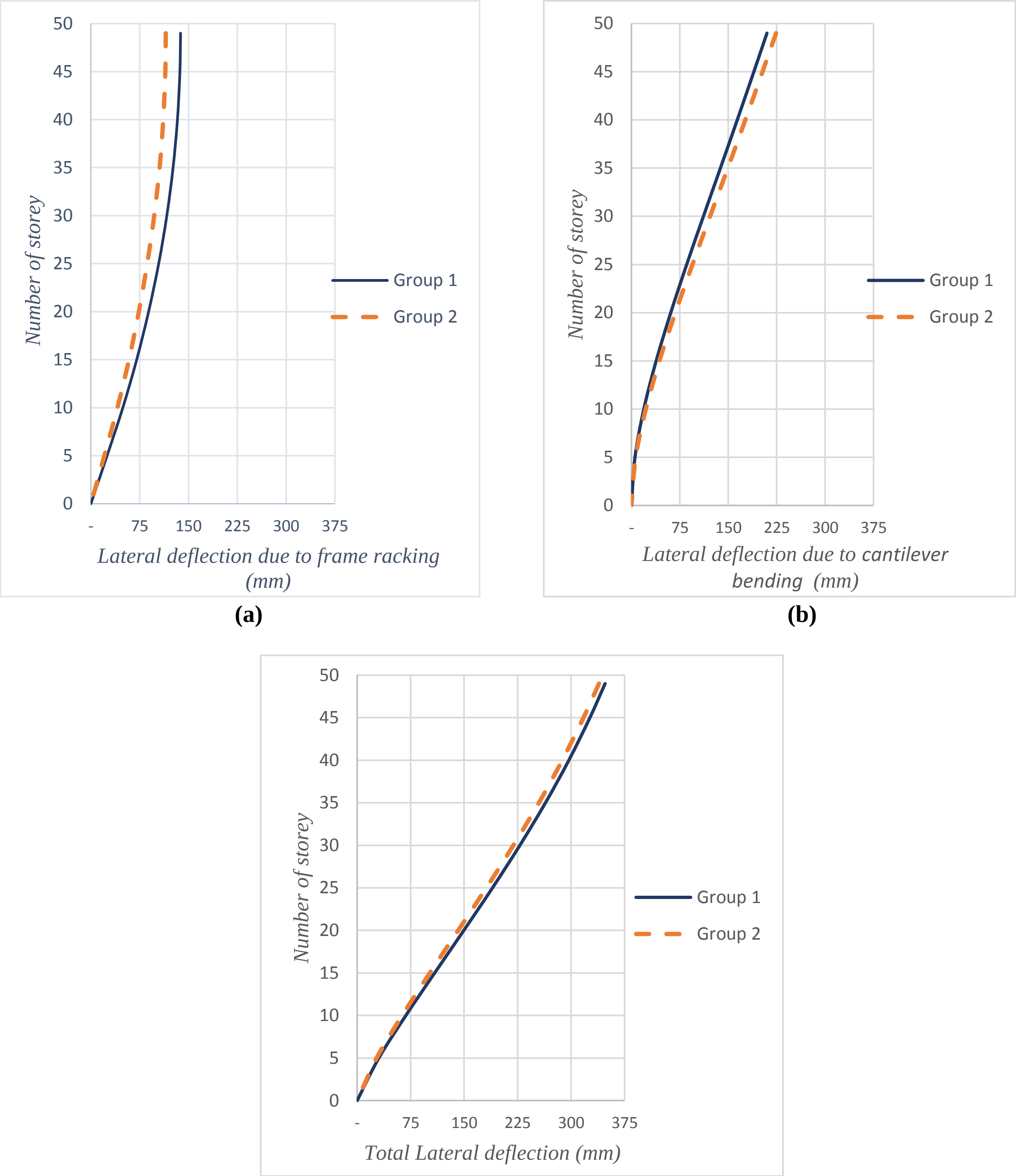


(a)

(b)

(c)

Figure 4-7: Lateral deflection of 40 story framed tube structure: (a) Lateral deflection due to frame racking. (b) Lateral deflection due to cantilever bending. (c) Total lateral deflection.



(a)

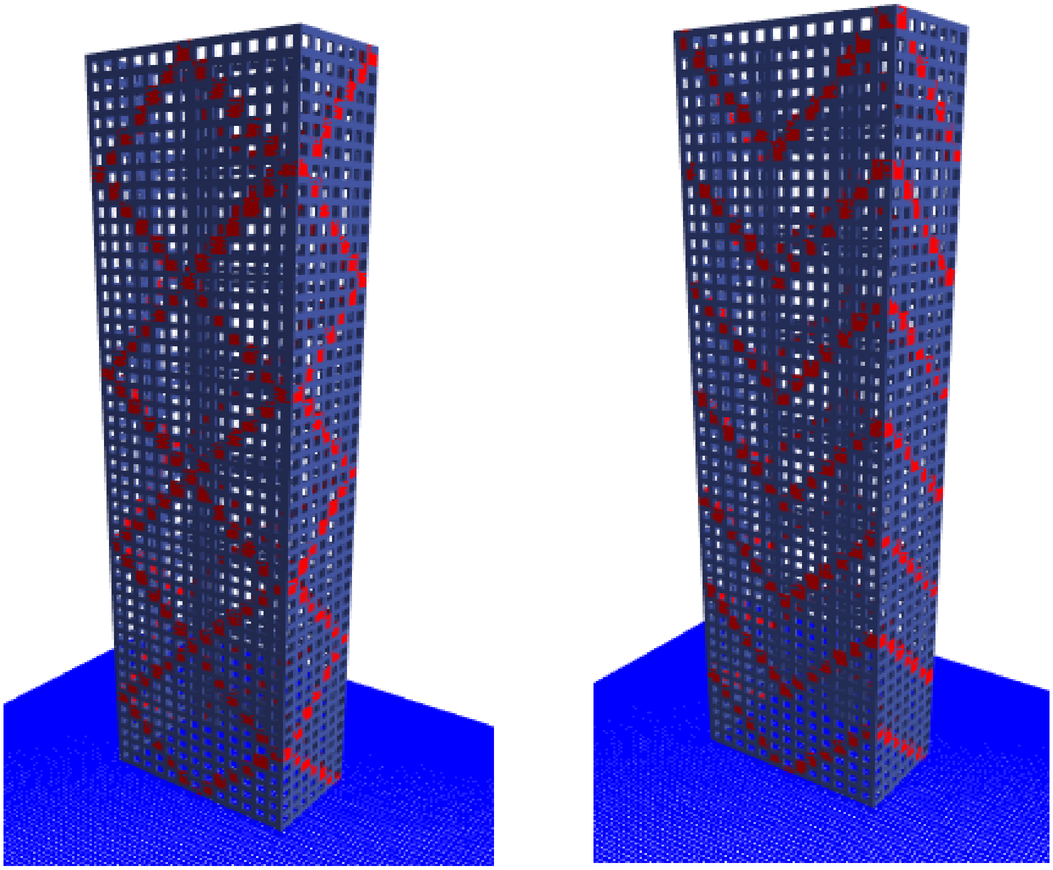
(b)

(c)

Figure 4-8: Lateral deflection of 49 story framed tube structure: (a) Lateral deflection due to frame racking. (b) Lateral deflection due to cantilever bending. (c) Total lateral deflection.

4.3 Modeling and Analysis of Trussed Tube Building

In order to evaluate the effectiveness of trussed tube structure, a 49 story trussed tube building with two bracing patterns as shown in Figure 4-10 is modeled and analyzed using ETABS 2015 v15.0.0 integrated building design software. The plan dimension, number of stories, floor height, spacing of columns, cross section of beam and columns, material and loading of structure is similar to that of 49-story group one framed tube building discussed in section 4.2 of this chapter. The building is braced by shear walls with thickness same as adjacent columns in cross and zigzag pattern in the flange and web panels respectively.



(a) **(b)**

Figure 4-9: 3D Model of trussed tube structure: (a) Model one. (b) Model two.

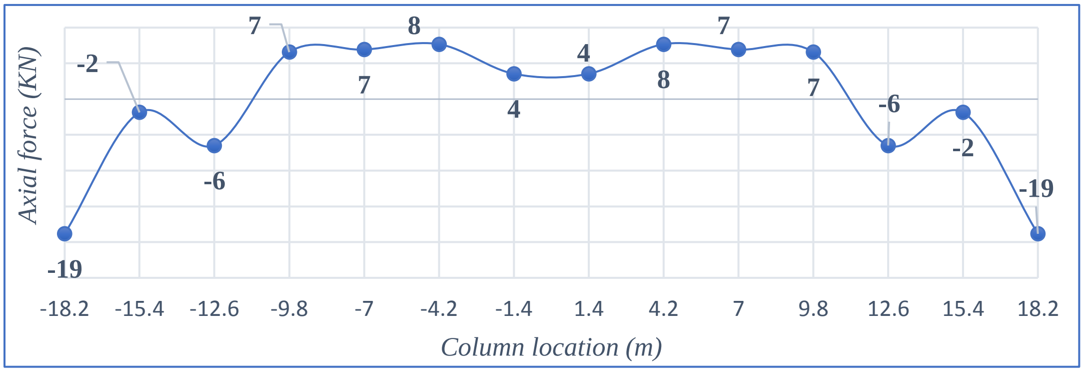
4.4 Assessment of Analysis Results of Trussed Tube Building

4.4.1 Shear lag in trussed tube building

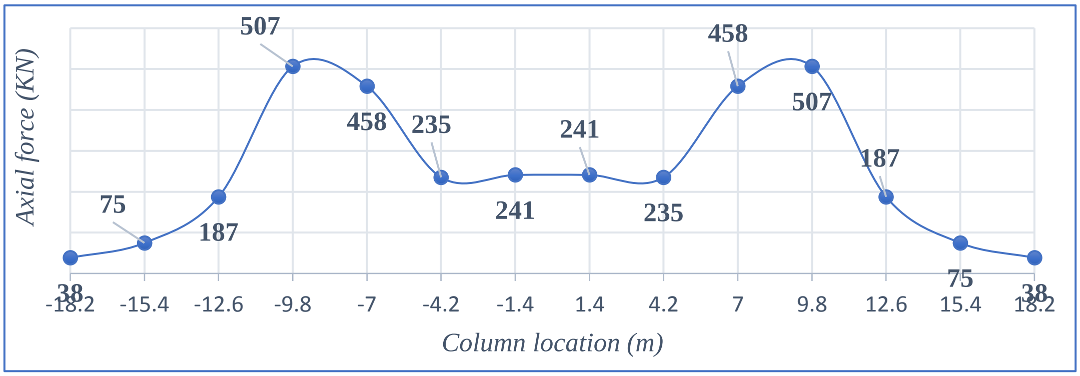
As stated in the literatures, the structural effectiveness of the framed tube is greatly reduced by shear lag effect. The diagonals created by filling selected window openings by reinforced concrete shear wall increase the efficiency of the tube by minimizing shear lag. The shear lag factor calculated as the ratio of the stress at the center column to the stress at the corner column is evaluated at the base of the structure and compared with that of framed tube structure.

The result indicates the first model trussed tube building with cross and zigzag-bracing pattern in the flange and web panels respectively increase the shear lag factor by 39% compared with that of framed tube building. Similarly, the second model trussed tube structure increase the shear lag factor by 52 %. The resulting axial force distribution in the flange columns of model one and model two trussed tube structures are plotted in Figure 4-11.

Story 49



Story 40



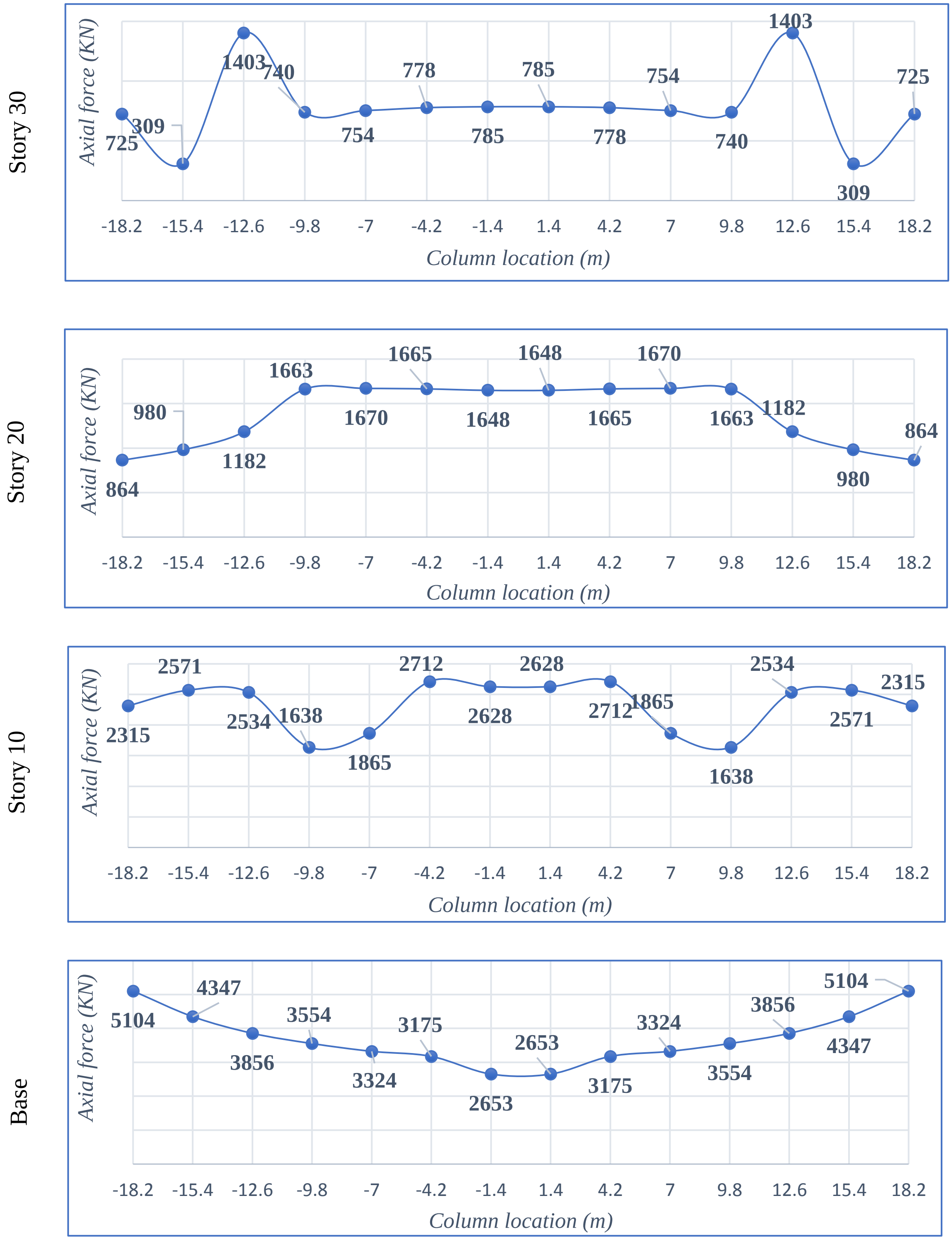


Figure 4-10: Axial force distribution in the flange panel of model one trussed tube building.

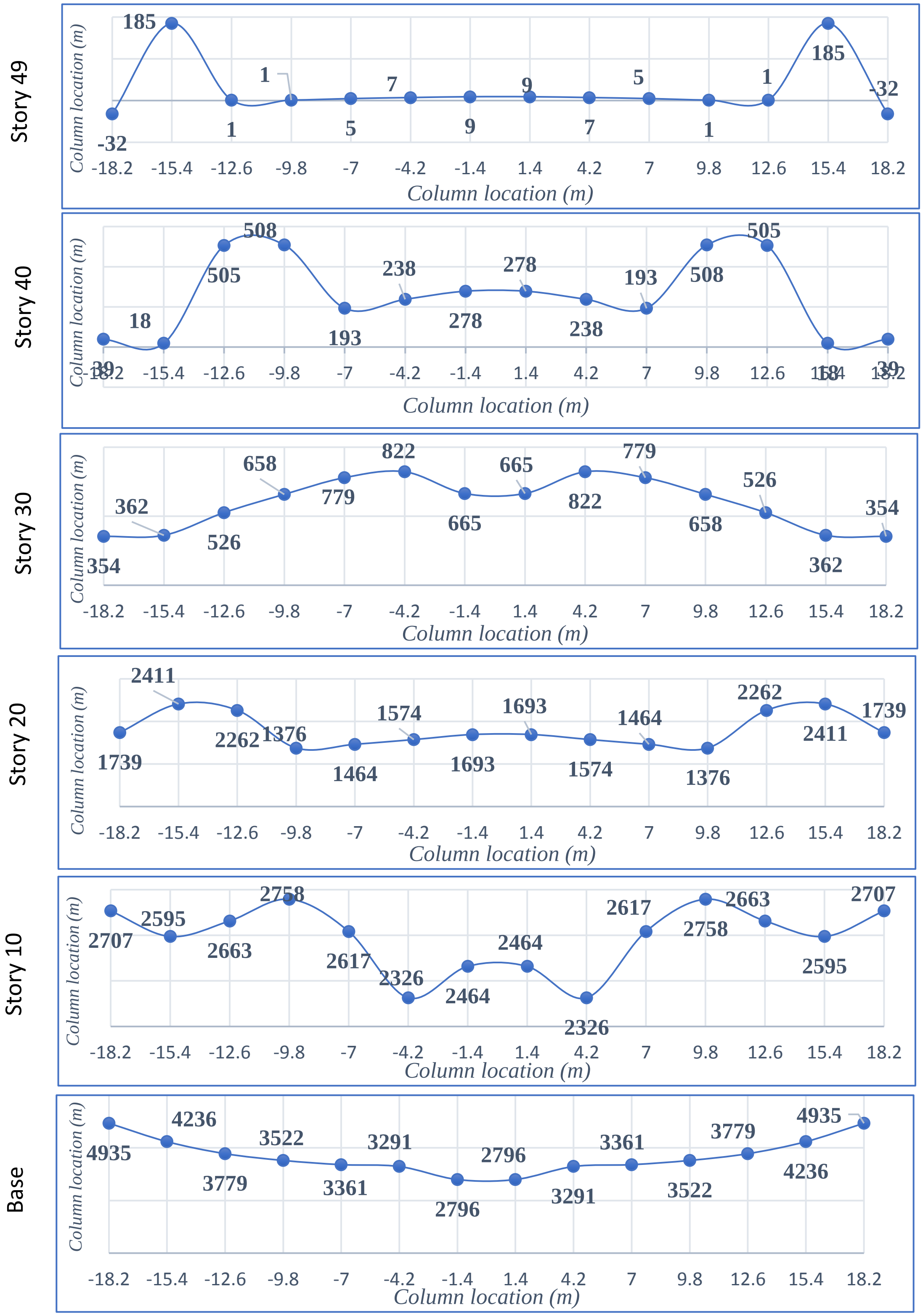
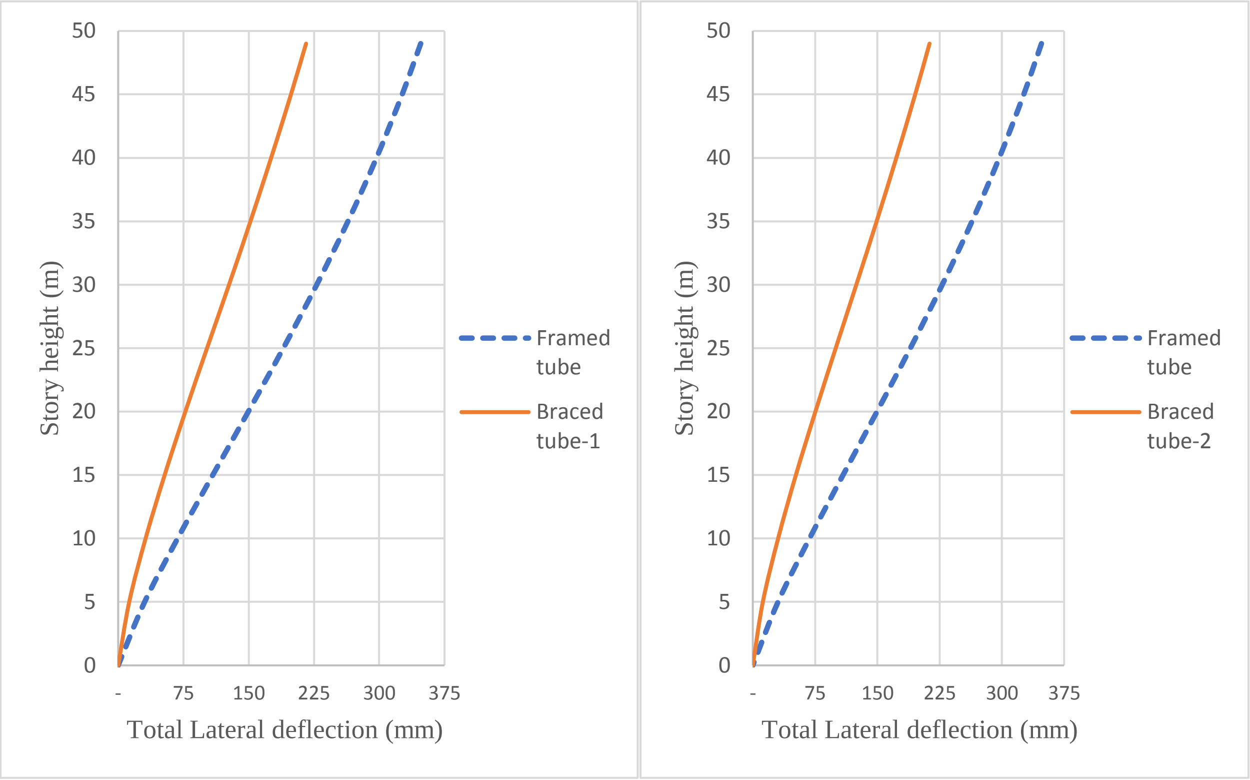


Figure 4-11: Axial force distribution in the flange panel of model two trussed tube building.

4.4.2 Lateral deflection of trussed tube building

The resulting lateral deflection of trussed tube structures compared with framed tube structure is plotted in Figure 4-12. The result indicates the first and second models of trussed tube buildings decreased the lateral deflection of framed tube structure by 38% and 39% respectively.



(a) **(b)**

Figure 4-12: Lateral deflection of: (a) Model one trussed tube structure Vs framed tube structure. (b) Model two trussed tube structure Vs framed tube structure.

CHAPTER 5 CONCLUSION AND RECOMENDATION

5.1 Conclusion

Since, framed-tube system is considered to be evolved from rigid frame system and are alternative to shear frame system, a comprehensive literature survey was performed on rigid frame, shear wall and shear wall-frame system in addition to framed tube and braced tube structural systems to briefly understand their behavior.

Though computer programs which are being used routinely in everyday engineering practice are fast and powerful to numerically analyze framed tube structures, they cannot substitute the theoretical analysis method which may offer better understanding of the structural system. In addition, a simplified analysis methods used to verify the reasonableness of the computer analysis and for initial assessment of structural behavior during preliminary design stage. Three reinforced concrete framed tube structures having 40, 50 and 60 stories are analyzed by the method of orthotropic membrane tube analogy for simplified analysis of framework panels which is carried out by Kwan (1994) and the results are compared with those produced by ETABS 2015 v15.0.0 software. The following conclusions are drawn from the analysis results:

- Comparing the total axial force at the 10th floor of the structure computed by Kwan (1994) method and those produced by ETABS software there is only a 0.48% and 5.37% deviation in the flange and web panels of 40-story building respectively. Similarly, the percentage deviation of axial force in the flange and web panels is only 0.13% and 4.69% respectively for 50-story building and 0.09% and 4.24% respectively for 60-story building.
- The resulting shear lag factors for 40-story, 50-story and 60-story buildings at the 10th floor of the building are 0.552, 0.614 and 0.663 respectively according to Kwan (1994) method. Similarly, the resulting shear lag factor computed from ETABS software analysis result is 0.565, 0.597 and 0.629 respectively for 40-story, 50-story and 60-story buildings at the 10th floor. The deviation in magnitude of axial force at the corner

columns of the 40-story, 50-story and 60-story buildings at the 10th floor is only 6.2%, 2.2%, 0.2% respectively.

- Comparing the total axial force at the base of the structure computed by Kwan (1994) method and those produced by ETABS software there is only a 3.08% and 6.26% deviation in the flange and web panels of 40-story building respectively. Similarly, the percentage deviation of axial force in the flange and web panels is 2.91% and 6.51% respectively for 50-story building and 2.85% and 6.84% respectively for 60-story building.
- Compared to ETABS software result, Kwan (1994) method underestimate the shear lag effect at the base of the structure. The resulting shear lag factor for 40-story, 50-story and 60-story buildings at the base are 0.376, 0.475 and 0.554 respectively according to Kwan (1994) method and similarly, the resulting shear lag factor computed from ETABS software analysis result is 0.190, 0.246 and 0.295 respectively for 40-story, 50-story and 60-story buildings. Which results in 33.2%, 32.5% and 31.6% deviation in magnitude of axial force at the corner columns at the base of the structure for 40-story, 50-story and 60-story buildings respectively.

To investigate the shear lag effect and structural behavior of framed tube structural system six reinforced concrete framed tube structures with two groups is modeled and analyzed using ETABS 2015 v15.0.0 integrated building design software. The effect of rectangular corner column alignment and height of the structure on the shear lag and structural behavior of framed tube structure is studied. The efficiency of framed tube structure as a function of shear lag factor and as a function of the ratio of lateral deflection due to cantilever bending to that of total lateral sway is studied and the following conclusions are drawn:

- The result clearly show the influence of height of the building and corner column alignment on the shear lag factor. For the first group buildings, the shear lag factor at the base of the structure is 0.23, 0.31 and 0.37 for the 30, 40 & 49 story respectively. Similarly, the shear lag factor of 0.18, 0.25 and 0.30 is observed at the base of the structure for 30, 40 & 49 story buildings. The result indicates aligning the wider dimension of the corner column parallel to flange panel increase the shear lag factor by 32.36%, 27.02% and 23.58% respectively for 30, 40 and 49 story buildings.

- Calculating efficiency as the ratio of cantilever deflection contribution to that of total lateral sway, the efficiency of group two buildings are higher than that of the group one buildings by 17.9%, 12.4% and 9.4% respectively for 30, 40 and 49 story buildings. The contribution of cantilever deflection from the total lateral sway for the group one structures is 41.1%, 52.1% and 60.4% respectively for 30, 40 and 49 story buildings. Similarly, 48.4%, 58.6% and 66.1% respectively for the group two of 30, 40 and 49 story buildings.
- As evident from the above result the efficiency of framed tube structure as a function of shear lag factor and as a function of contribution of cantilever deflection to the total lateral sway is not always directly proportional. Considering efficiency as a function of shear lag factor the first group buildings are more efficient than group two buildings. Inversely, group two buildings are more efficient than group one buildings when efficiency is considered as contribution of cantilever deflection to that of total lateral sway is high.

In order to evaluate the effectiveness of trussed tube structure, a 49 story trussed tube building with two bracing patterns as shown in Figure 4-10 is modeled and analyzed using ETABS 2015 v15.0.0 integrated building design software. By comparing the analysis results of framed tube and braced tube structures the following conclusions are drawn:

- The result indicates the first model trussed tube building with cross and zigzag-bracing pattern in the flange and web panels respectively increase the shear lag factor by 39% compared with that of framed tube building. Similarly, the second model trussed tube structure increase the shear lag factor by 52 %.
- The first and second model trussed tube structures decreased the lateral deflection of framed tube building by 38% and 39% respectively.

5.1 Recommendation

As observed from this and previous studies several factors affect shear lag and structural behavior of framed tube structures. The effect of number of story, corner rectangular column alignment and bracing on the efficiency of framed tube structure as a function of shear lag factor and deflection is studied and the above-mentioned results are observed. More researches are needed on exterior structural systems to increase their efficiencies and to further understand the constraints, which affect their efficiency.

On this study, trussed tube system with only two type of bracing patters having rectangular plan configuration is considered. It is recommended to investigate the effect of several bracing patterns and plan configurations on shear lag and efficiency of framed tube structural system.

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APPENDIX A:

Validation of ETABS Lateral Deflection Components Output

The lateral sway of a floor relative to the floor below results from a combination of bending and shear deformation of the bent. The bending deformation is a consequence of axial deformation of the columns independent of the size, type, location and arrangement of the web system. The shear deformation is due to the rotation of the joints in the frame, which cause bending of columns and girders of the frame (Taranath, 1988). The bending deformation contribution to deflection can be obtained by considering the frame as a cantilever fixed at the base.

Deflection due to column rotation (Δ_1): we can consider the contribution of columns by assuming the girders to be infinitely rigid (Taranath, 1988 and Smith and Coull, 1991):

$$\Delta_1 = \frac{Vh^3}{12E\sum I_c}$$

Deflection due to girder rotation (Δ_2): similarly, we can consider the contribution of girders by assuming the columns to be infinitely rigid (Taranath, 1988 and Smith and Coull, 1991):

$$\Delta_2 = \frac{Vh^2}{12E\sum(I_b / L_i)}$$

The resulting total shear deflection becomes (Taranath, 1988):

$$\Delta_s = \Delta_1 + \Delta_2 = \frac{Vh^2}{12}\left\{\frac{h}{\sum\left(EI\right)_{col}} + \frac{1}{\sum\left(\frac{EI}{L}\right)_{beam}}\right\}$$

Where, V is total lateral load shear; h is story height; L is length of beam; E is the modulus of elasticity of the material and I is moment of inertia of the section.

Cantilever deflection (Δ_c): the contribution of cantilever deflection to the total lateral deflection can be obtained by:

At the top of the frame:

$$\Delta_c = \frac{WL^4}{8EI}$$

At any height of the frame:

$$\Delta_c = \frac{W}{24EI}\left(Z^4 - 4LZ^3 + 6L^2Z^2\right)$$

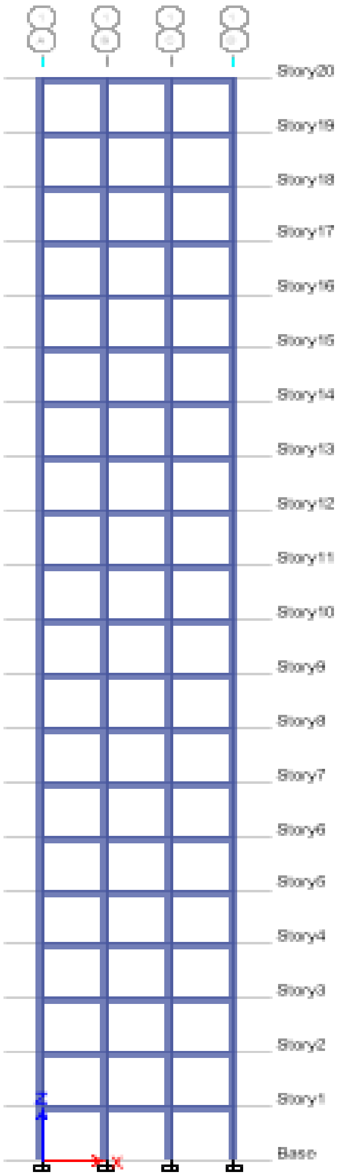
Where I is determined by:

$$I = \sum_1^n a_1 d_1^{\,2}$$

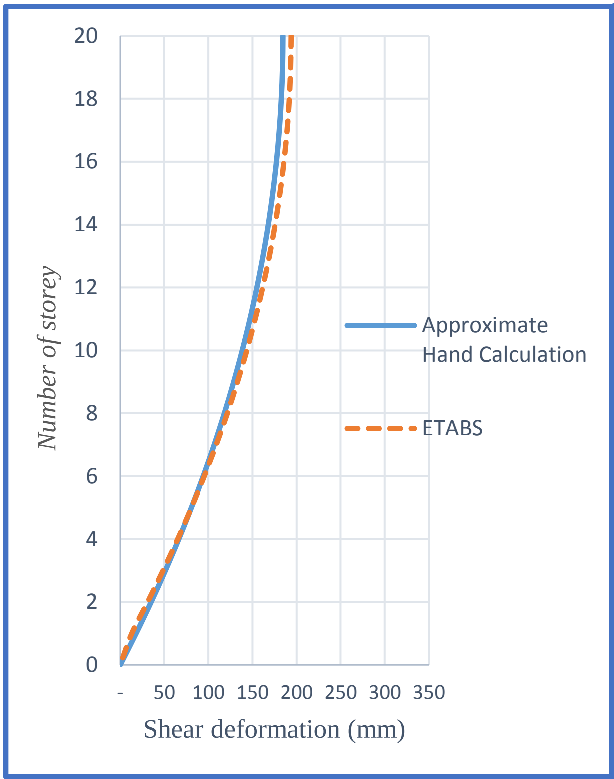
Where W is uniformly distributed horizontal load; Z is the height of the structure from the fixed end; a_1, a_2…,a_n represent the areas of the columns and d_1, d_2…,d_3 represent their corresponding distances from the neutral axis of the frame.

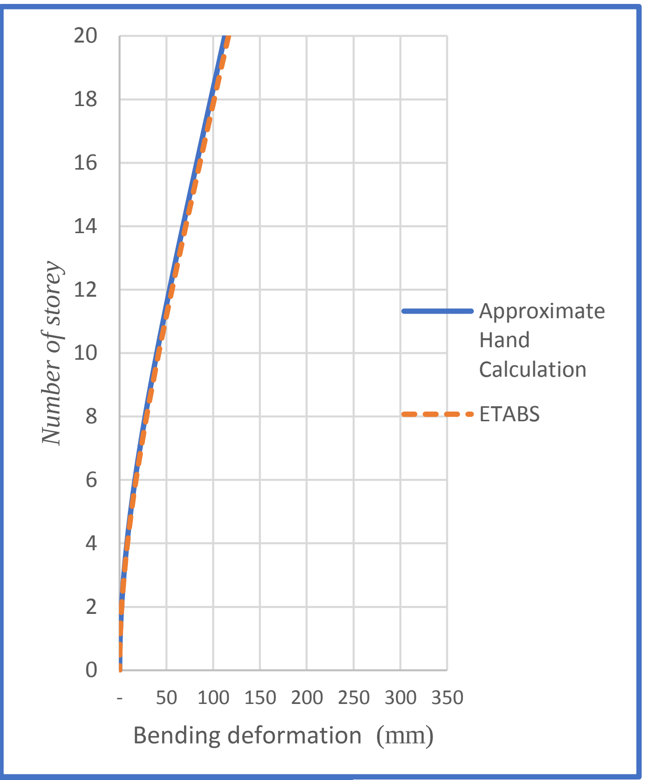
Since the bending deformation of a frame is a consequence of axial deformation of the columns alone and is independent of the size, location, type and arrangement of the web system, we can find the shear deformation components of the frame from ETABS software by making the axial rigidity of the column infinite. The bending deformation component of the frame can then be determined by deducting the shear deformation of the frame from the total lateral deflection of the frame.

A three bay 20-story reinforced concrete rigid frame as shown in the figure below is considered to validate ETABS lateral deflection components. All the beam and column members are of sizes 0.4 m x 0.4 m. the height of each story and the center-to-center spacing of the columns are 3 m. The Young’s moduli of the material are assumed to be 20 GPa and the frame is assumed to be loaded with a uniformly distributed lateral load of 10 KN/m.



The resulting deflection components produced by ETABS software and approximate hand calculation results are compared and illustrated in graphical format as shown below. The result indicates ETABS analysis result for shear deformation, bending deformation and total lateral deflection of the rigid frame is higher than, those estimated by approximate hand calculation by only 4.8%, 3.35% and 4.26 % respectively.


20
18
16
14
12
10
8
6
4
2
0
Number of storey
-
50
100
150
200
250
300
350
Shear deformation (mm)
Approximate Hand Calculation
ETABS


20
18
16
14
12
10
8
6
4
2
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Number of storey
-
50
100
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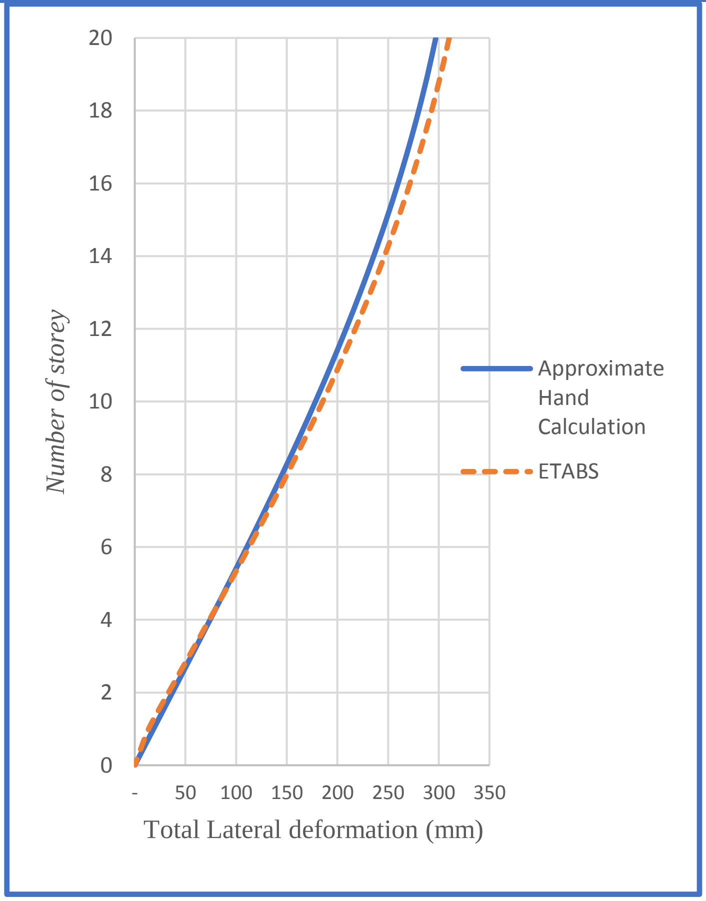
200
250
300
350
Bending deformation (mm)
Approximate Hand Calculation
ETABS


20
18
16
14
12
10
8
6
4
2
0
Number of storey
-
50
100
150
200
250
300
350
Total Lateral deformation (mm)
Approximate Hand Calculation
ETABS

APPENDIX B:

Axial Force Calculation by using Kwan (1994) Simplified Method for Approximate Analysis of Framed Tube Structures

40 Story Framed Tube Building Axial Force Calculation @ the Base of the Structure

a (m)	b (m)	$A_b=A_C$ (m^2)	$A_{sb}=A_{sc}$ (m^2)	S (m)	h (m)	$t_f=t_w$ (m)	$E=E_m=E_w=E_f$ (GPa)	G_m (GPa)	$d_b=d_c$ (m)	I (m^4)	H (m)
15.00	17.50	0.640	0.533	2.50	3.00	0.256	20.00	8.00	0.80	0.034	120.00

$\frac{(h-d_b)^3}{12E_mI_c}$	$\left(\frac{h}{s}\right)^2\frac{(s-d_c)^3}{12E_mI_b}$	$\frac{\Delta_b}{Q}$	$\frac{(h-d_b)}{G_mA_{sc}}$	$\left(\frac{h}{s}\right)^2\frac{(s-d_c)}{G_mA_{sb}}$	$\frac{\Delta_s}{Q}$	$\frac{h}{st}$	$G = G_w = G_f$ (Gpa)
1.300	0.864	2.163	0.516	0.574	1.089	4.688	1.441

m_w	m_f	α_1	α_2	β_1	β_2
4.611	3.388	0.366	0.035	0.624	0.223

At Base (z = 0m)	0.00			
$\alpha = \alpha_1$	$\beta = \beta_1$	EI (KN-m^2)	M (KN-m)	$\frac{d\phi}{dz}(1/m)$
0.366	0.624	66,787,860,488.77	864,000.00	0.00001294

Flange Panel			**Web Panel**		
Y (m)	σ_z (KN/m^2)	Axial force (KN)	X (m)	σ_z' (KN/m^2)	Axial force (KN)
0.0	1,461.05	935.07	0.0	-	-
2.5	1,510.43	966.68	2.5	416.80	266.76
5.0	1,658.59	1,061.50	5.0	873.04	558.75
7.5	1,905.52	1,219.53	7.5	1,408.14	901.21
10.0	2,251.22	1,440.78	10.0	2,061.54	1,319.38
12.5	2,695.69	1,725.24	12.5	2,872.66	1,838.50
15.0	3,238.93	2,072.92	15.0	3,880.94	2,483.80
17.5	3,880.94	2,483.80	-	-	-

40 Story Framed Tube Building Axial Force Calculation @ Z=30m

a (m)	b (m)	A_b=A_C (m^2)	A_{sb}=A_{sc} (m^2)	S (m)	h (m)	t_f=t_w (m)	E=E_m=E_w=E_f (GPa)	G_m (GPa)	d_b=d_c (m)	I (m^4)	H (m)
15.00	17.50	0.640	0.533	2.50	3.00	0.256	20.00	8.00	0.80	0.034	120.00

$\frac{(h-d_b)^3}{12E_mI_c}$	$\left(\frac{h}{s}\right)^2\frac{(s-d_c)^3}{12E_mI_b}$	$\frac{\Delta_b}{Q}$	$\frac{(h-d_b)}{G_mA_{sc}}$	$\left(\frac{h}{s}\right)^2\frac{(s-d_c)}{G_mA_{sb}}$	$\frac{\Delta_s}{Q}$	$\frac{h}{st}$	G = G_w = G_f (Gpa)
1.300	0.864	2.163	0.516	0.574	1.089	4.688	1.441

m_w	m_f	α_1	α_2	β_1	β_2
4.611	3.388	0.366	0.035	0.624	0.223

At z = 30 m	30.00			
α	β	EI (KN-m^2)	M (KN-m)	$\frac{d\phi}{dz}(1/m)$
0.221	0.448	77,537,845,029.51	486,000.00	0.00000627

Flange Panel			Web Panel		
Y (m)	σ_z (KN/m^2)	Axial force (KN)	X (m)	σ_z' (KN/m^2)	Axial force (KN)
0.0	1,037.32	663.89	0.0	-	-
2.5	1,054.53	674.90	2.5	245.98	157.42
5.0	1,106.14	707.93	5.0	503.51	322.25
7.5	1,192.17	762.99	7.5	784.16	501.86
10.0	1,312.60	840.07	10.0	1,099.48	703.67
12.5	1,467.45	939.17	12.5	1,461.03	935.06
15.0	1,656.71	1,060.29	15.0	1,880.37	1,203.44
17.5	1,880.37	1,203.44	-	-	-