ADDIS ABABA UNIVERSITY
GRADUATE STUDIES PROGRAMME
COLLEGE OF COMPUTATIONAL&NATURAL SCIENCES
DEPARTMENT OF STATISTICS

FORECASTING THE CO-VOLATILITY OF COFFEE ARABICA AND CRUDE OIL PRICES: A MULTIVARIATE GARCH APPROACH WITH HIGH FREQUENCY DATA

BY
DAWIT YESHIWAS

A THESIS SUBMITTED TO THE DEPARTMENT OF STATISTICS IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTERS OF SCIENCE (APPLIED STATISTICS)

Addis Ababa University
Addis Ababa, Ethiopia
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ADVISOR: PROFESSOR SOURAFEL GIRMA

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This is to certify that, the thesis prepared by Dawit Yeshiwas, entitled: **Forecasting the co-volatility of coffee Arabica and crude oil prices: A multivariate GARCH approach with high frequency data,** and submitted in partial fulfillment of the requirements for the Degree of Masters of Science in Statistics (Applied Statistics) complies with the regulations of the University and meets the accepted standards with respect to originality and quality.

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DECLARATION

I hereby declare that, the thesis is my original work and all sources and materials used for writing it have been duly acknowledged. I declare that I have not so far submitted this thesis to any other institution anywhere for that award of any academic degree, diploma or certificate.

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This thesis has been submitted for examination with my approval as a University advisor.

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Advisor's name Signature

Forecasting the co-volatility of coffee Arabica and crude oil prices: A multivariate GARCH approach with high frequency data

Dawit Yeshiwas
Addis Ababa University
June, 2017

Abstract

Forecasting the volatility dynamics of asset returns has been the subject of extensive research among academics, practitioners and portfolio managers. This thesis estimates a variety of multivariate GARCH models using weekly closing price (in USD/barrel) of Brent crude oil and weekly closing prices (in USD/pound) of coffee Arabica, and compares the forecasting performance of these models based on a high frequency intra-day data which allows for a more precise realized volatility measurement. The study used weekly price data to explicitly model co-volatility, and employed high-frequency intra-day data to assess model forecasting performance. The analysis points to the conclusion that varying conditional correlation (VCC) model with Student's t distributed innovation terms is the most accurate volatility forecasting model in the context of our empirical setting. We recommend and encourage future researchers studying the forecasting performance of MGARCH models to pay particular attention to the measurement of realized volatility, and employ high-frequency data whenever feasible.

Keywords*: commodity price co-volatility, conditional correlation, forecasting, multivariate GARCH*

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List of Acronyms

ADF	Augmented ducky fuller
AIC	Akaike information criteria
AR	Auto regressive
ARCH	Auto Regressive Conditional Heteroskedasticity
ARMA	Auto regressive moving average
BEKK	Baba-Engle-Kraft-Kroner
BIC	Bayesian information criteria
CCC	Constant conditional correlation
DCC	Dynamic conditional correlation
DVEC	Diagonal vector error correction
FAO	Food and agricultural organization
EGARCH	Exponential generalized auto regressive conditional heteroskedastic
MA	Moving average
MAE	Mean absolute error
MGARCH	Multivariate generalized auto regressive conditional heteroskedastic
LB	Ljung-Box
LDCs	Least developed countries
MLE	Maximum likelihood estimation
PP	Phillips-Perron
QMLE	Quasi- maximum likelihood estimation
UNCTAD	United Nations conference on trade and development
RMSE	Root mean square error
VEC	Vector error correction

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CHAPTER ONE

INTRODUCTION

1.1 Background of the study

The recent global financial crisis has increased the susceptibility of Commodity Dependent Developing Countries (CDDCs) to excessive price volatility in commodity markets. Moreover, structural weaknesses in these countries render their economies more vulnerable to increased commodity market turbulence than developed countries, given their comparatively lower income and high dependence on commodity exports. The World Bank estimates that 119 million additional people have been pushed into hunger as a result of the 2008 food price crisis (World Bank, 2009).

Although supply and demand fundamentals played a significant role in the food price crisis outbreak, many other factors contributed to the economic turmoil. For example, large increases in oil prices have contributed to rising production costs and driving food prices higher as a consequence. According to the World Bank, weakness of the dollar accounted for 15% of the food price increases between 2002 and 2008 (Mitchell, 2008). Furthermore, over the last decade, drought in Russia, freezes in Brazil and heavy rains in Canada and Australia caused major disruptions in agricultural commodities production. Global warming also proves partly responsible for livestock and crops diseases thereby threatening food security and exacerbating food supply problems.

Recently a number of papers have studied the co-movement of oil prices with equities, agricultural commodities and precious metals prices. Prior studies provide evidence on the connectedness between oil and one or more markets. However, the bulk of these studies has focused on price connectedness. Relatively little research has been devoted to volatility co-movement. This is surprising since price volatilities are the main driving forces in options and commodity future markets. This paper seeks to bridge this gap in the literature. In particular it undertakes a systematic analysis of the volatility interconnectedness between two important commodity markets - oil and coffee markets – using multivariate GARCH (MGARCH) family of models with the view of providing some rigorous price co-volatility forecasting models.

Modeling, analyzing, and forecasting volatility has been the subject of extensive research among academics, practitioners and portfolio managers (Baume *et al,* .2003). This has been used in risk management, derivative pricing and hedging, portfolio selection and policy making. Similarly the analysis of volatility spillovers between commodity and asset prices has a profound implication for risk management and portfolios maximization by the government and investors alike (Salisu and Oloko, 2015). In view of the current bearish behavior of oil price in the international markets, it is arguably of special interest to study the relationship between oil and coffee prices (Gil-Alana and Yaya, 2014).

MGARCH models are becoming increasingly popular for modelling and forecasting time-varying correlations between asset prices. One advantage of MGARCH models is that they specify time varying equations of volatility co-movements. This makes them ideal to study the volatility linkages and transmissions, understand commodity market integration, price cycle of booms and busts. MGARCH models also inform financial managers in deciding optimal portfolio selection to achieve maximum asset returns. They can be used to calculate the value at risk of a financial position comprising multiple assets (Tsay, 2005). Volatility linkages and transmissions are also important to understand commodity market integration, price cycles of booms and busts and analysis of market crises.

Naturally, different papers in the literature tend to focus on different commodity pairs when studying volatility interconnectedness. The focus of this work is on the co-volatility between Brent crude oil and Coffee Arabica prices in the commodity futures markets which are markets where one can buy specific quantities of a commodity at a specified price with delivery set at a specified time in the future. The choice of these commodities is motivated by the fact that these are the most actively traded commodities in the world which also happen to be major importing and exporting commodities of Ethiopia. Existing work appears to focus mostly on in-sample modeling of co-volatility of commodity prices giving less attention to model forecasting performance. This work will also seek to bridge this gap in the literature.

1.2 Statement of the problem

It is said that coffee and crude oil futures are 'acting like sisters': the two has color in common. Both have a dark brown hue. But that's about all. One is a liquid mineral that keeps us on the move. The other is a crop, a bean that keeps us awake. So, why futures in crude oil and coffee should have started moving as a pair? The duo have been named as the latest odd couple in commodities, following copper and wheat, and gold and farmland. In London, broker Marex said the correlation between the two assets was "particularly tight". For example, "Crude oil topped on May 2, 2011 and then fell 22%. Coffee topped on May 3, 2011 and then fell 22%. Crude has since put in a bottom on May 23, 2011 and then retraced 8.2% and coffee put in a bottom on May 23, 2011 and then retraced 11.5%". "The correlation between coffee and crude oil even goes down to the hourly charts". It may be that the surge in interest in coffee, even in developing countries, has tied it more closely to world economic sentiment. If so, the crude-coffee coupling may last a little (Agrimoney.com, 2011).

A number of recent papers have investigated the co-volatility of energy and commodity price mainly due to increased interest for understanding the drivers and dynamics of such co-volatilities in these highly volatile times. Alternative estimation approaches are used in the literature, but thus far there is a dearth of work undertaking a thorough "MGARCH" analysis –a statistical framework which is particularly suited for modeling asset prices co-volatility. Moreover existing work appears to focus mostly on in-sample modeling of co-volatility of commodity prices giving less attention to model forecasting performance. Accordingly, this work will seek to contribute to the literature by undertaking a systematic analysis of volatility forecasting performance.

Most of the existing research on volatility spillovers employ statistical models in order to estimate realized volatilities which turned out to be oftentimes poor approximations of true volatilities. An attractive alternative to model-based statistical volatility is to compute realized volatility based on high frequency intra-day or `tick' data. Realized volatility has been found to be more accurate than model based volatilities in predicting latent volatility (Christensen and Prabhala, 1998; Fleming, 1998; Jorion, 1995; Blair et al., 2000). Thus, the results provided by previous studies can be potentially misleading in that they may have underestimated or overestimated the extent of the true volatility.

This thesis avoids this potential pitfall by using realized volatility based on a 30-minute intra-day data. We argue that doing so represents an important contribution to the literature of co-volatility measurement within commodity markets.

1.3 Objective of the study

1.3.1 General Objective

The general objective of this study is to model the co-volatility of the Arabica coffee and crude oil prices using MGARCH type models and perform volatility and co-volatility forecasting using realized variance.

1.3.2 Specific Objectives

The specific objectives of this study are:

1. To fit a univariate GARCH models for each commodity prices separately as reference point for the analysis.
2. To fit and select the best fitting MGARCH models for the two commodities' price co-volatility.
3. To forecast the volatility of the two commodities taking into account their co-volatility.
4. To compare the realized volatility forecasts obtained from weekly return square and a high frequency (a thirty minute interval) data.

1.4 Significance of the study

The work makes the following contributions to the literature.

1. From an applied econometrics stand point, the work provides a robust and systematic investigation of the statistical and econometric properties of the co-volatility of the two important commodities.

2. The direction and magnitude of correlation between the prices of coffee and crude oil which are documented in this work would help the design of a more informed policy making on the part of Ethiopian authorities as well as others stakeholders and market participants interested in foreseeing the volatility of these two important commodities.

The remaining part of the paper is organized as follows. The next chapter includes a brief review of related literatures about commodity volatility and co-volatility modeling. In chapter three we describe the methodology used to model the volatility of financial time series returns, check model adequacy and evaluate relative forecasting performance. Chapter four comprises the descriptive analysis, model selection, the univariate GARCH and the bivariate GARCH model fits, and forecast comparison analyses. The last chapter concludes.

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CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

In this section we review the development of multivariate GARCH models and their wide applications in financial returns. Understanding and predicting volatilities and correlations of asset returns has been the object of much attention, since volatilities and correlations are the two most important elements in financial activities such as asset pricing, asset allocation decisions, portfolio management and risk assessment (Tsay, 2005).

In the last two decades there has been an explosion in volatility research. The "Dynamic Volatility Era" began with the introduction of the Autoregressive Conditional Heteroskedastic (ARCH) model (Engle, 1982). The ARCH (p) model expresses the conditional variance as $p-th$ order weighted average of past (squared) disturbances and thus is able to describe volatility clustering in financial series. Following this, an enormous body of research has focused on extending and generalizing the ARCH model, mainly by providing alternative functional forms for the conditional variance. Some of the most important contributors to the dynamic volatility literature have been Engle, Bollerslev, Nelson and Ding. Bollerslev (1986) proposed the Generalized ARCH (GARCH) model, as a more parsimonious way of modeling volatility dynamics. A limitation of the standard ARCH and the GARCH models is that they only use information on the magnitude of the returns while completely ignoring information on the sign of the returns. To remedy this Nelson (1991) introduced the Exponential GARCH (EGARCH) model, which was the first in the family of asymmetric GARCH models.

The most significant contribution of this extensive volatility research is that now we have a much clearer and better understanding of the probabilistic features of speculative returns data. This is of crucial importance in the search for an overarching framework, which will be able to first capture the empirical regularities adequately and ultimately give us reliable volatility forecasts. There are a number of extensive surveys of the literature including for instance Bollerslev (1990), Chou and Kroner (1992), Engle and Nelson (1994), and Ding and Engle (2001).

During the last few decades, a body of research regarding energy and commodity price volatility has been presented. A large proportion of this research has focused on a univariate approach for studying volatility clustering and volatility persistent and asymmetric effect in the returns. A well-known research among econometricians is the study performed by Andersen & Bollerslev (1998), where they concluded that univariate standard volatility models in fact are able to perform accurate volatility forecasts. But, only a smaller fraction of the literature has concentrated on a multivariate approach for conditional correlation forecasting, and the performance of these models. Empirical literatures on volatility modeling and forecasting performances are discussed in the following two sections separately.

2.2 Modeling commodity prices co-volatility

The surge in oil and grains prices in the last decade has motivated many studies to focus on the co-movement of crude oil and agricultural commodities. A number of studies blame increasing food prices in the last decade on the rise of biofuel market and high oil prices (see, for example, Abott *et al*., 2008; Baffes, 2007; Chang and Su, 2010; Collins, 2008; Mitchell, 2008; Rose grant *et al*., 2008; and Yang *et al*., 2008). For instance, Chang and Su use an EGARCH model to study the volatility transmission between crude oil, corn and soybeans and found significant volatility spillovers across these markets. Choi and Hammoudeh (2010) applied the DCC model to study the time volatility and correlations in the returns of Brent oil, copper, gold and silver and the S&P500 index. They found increasing significant correlations among all the commodities since the 2013 Iraq war but decreasing volatility correlations in to S&P500 index. Kiohos and Sariannidis (2010) explored, in the short run, the effects of oil and financial markets on the gold market. They used a GJR-GARCH model in testing the relationships between them using daily data from January 1, 1999 to August 31, 2009. Their findings showed that the oil market had positive influences on the gold market, and indicated volatility persistence in the gold market.

Noemi and Junior (2011) investigate causality links between the crude oil price and, coffee and cocoa prices in the long-run. It appears that variations in coffee and cocoa price follow oil price with respective four and five year intervals. They also examine the volatility of oil and futures spillover effects on three major tropical commodities: Arabica coffee, Robusta coffee and cocoa applying GARCH models. The study aimed at identifying interactions, similarities, and causalities between coffee and cocoa price on the one hand and, oil and futures price on the other hand.

Their analysis of coffee and cocoa historical prices shows that coffee price volatility has uneven or differing reactions depending on the nature of the market shocks.

Zhang *et al.* (2010) study the causality between oil and gold volatility. Their results indicate that the crude oil price changes Granger cause the volatility of gold but not the other way around. Alom *et al.* (2011) found that the oil price volatility is positively correlated with food price volatility. However, they also found that this relationship varies across countries and time periods. Similarly, Du *et al.* (2011) investigate the volatility linkages between crude oil, wheat and corn using a bivariate stochastic volatility model. Their results show that oil price shocks trigger sharp increases in agricultural commodity markets. They attributed the increase in connections between oil and food to the recent increase in commodity investments. Similar results on variance dependence between oil and corn is provided by Harry and Husdon (2009).

Singh *et al*. (2011) analyzed the time-varying volatility in crude oil, heating oil, and natural gas futures market. They incorporated changes in important macroeconomic variables and major political and wealth-related events into the conditional variance equations. Their results showed that, among the macro variables considered, the spread between the 10-year and 2-year treasury constant maturity rate had a positive relationship with the volatility of all commodities. Ciner *et al*., (2013) investigated return relations among stocks, bonds, gold, oil and exchange rates, taking data from the US and the UK. Their results showed significant relationship between oil and gold, whereas weak relationship was found in the case of gold with both UK and US exchange rates. Owing to this fact, gold was placed as a safe-haven commodity against exchange rate shocks, while it was not placed as a safe-haven commodity against oil price shocks.

Ewing and Malik (2013) considered daily samples between July 1993 and June 2010 for multivariate GARCH models in order to examine the volatility transmission between gold and oil futures by incorporating structural breaks. They found strong evidence of significant volatility transmission between gold and oil price returns in the presence of structural breaks.

Mensi *et al.* (2013) applied VAR-GARCH to investigate the returns and volatility transmission between the S&P500 and commodity price indices for energy, food, gold and beverages for the period 2000- 2011.They found significant volatility transmission between the S&P500 and commodity markets. The results further revealed highest correlation between the S&P500 index and gold prices, and between the S&P500 index and WTI oil price. Shams and Zarshenas (2014) found evidence of co-movements in gold, oil prices and exchange rates based on copular functions and applications of GARCH models.

2.3 Volatility forecasting

Literatures on volatility forecasting are vast. Andersen *et al.* (2006) provide a comprehensive theoretical overview on the topic. An extensive survey of the literature's main findings is provided in Poon and Granger (2003, 2005). Volatility forecasting assessments are commonly structured to hold the test asset and estimation strategy fixed, focusing on model choice. Christian *et al*. (2012) take a more pragmatic approach and consider how much data should be used for estimation, how frequently a model should be re-estimated and what innovation distributions should be used. Their forecast evaluation relies on recent contributions for robust forecast assessment developed in Hansen and Lunde (2005) and Patton (2009). Their study omits volatility prediction models based on high-frequency realized volatility measures and stochastic volatility models.

Important contributions in these areas include Andersen *et al*. (2003), Deo *et al.*(2006), Engle and Gallo (2006), Aït-Sahalia and Mancini (2008), Hansen *et al.* (2010), Corsi (2010),and Ghysels *et al.*(1995). While realized volatility models often demonstrate excellent forecasting performance, there is still much debate concerning optimal approaches. However, recent results reported by Hansen and Lunde (2005) have suggested that, in the context of exchange rate returns, nothing can beat a GARCH (1, 1) model.

David G. McMillan and Alan E. H. Speight (2012) extend the Hansen and Lunde line of research by utilizing intra-day data and obtaining daily volatility forecasts from a range of models applied to the higher-frequency data. They have considered a range of models estimated using daily and intra-day data to inform the choice of the most appropriate model for daily returns volatility for exchange rates. Their paper particularly consider a daily GARCH(1,1) model as in Hansen and Lunde and a GARCH(1,1) model estimated using 5-minute returns as their two baseline models and compare these to a range of models from the extant literature, including both asymmetric and long-memory GARCH models.

The volatility forecasts obtained from these models are assessed using four different measures of 'true' volatility. More specifically, in addition to the usual 'ex post' squared returns measure they also construct realized volatility and two versions of bias-corrected realized volatility. In order to evaluate these forecasts, they utilize the Mincer–Zarnowitz regression test of predictive power, comparing forecasting performance on the basis of the R^2 values obtained.

Their results showed a substantial degree of consistency across the three euro exchange rate series they considered, across the four different measures of 'true' volatility and across the different test regressions and encompassing tests. Namely, the daily GARCH (1, 1) model is largely inferior to all other models, whereas the intra-day GARCH (1, 1) model provides largely superior forecasts compared to all other models.

Following the results they conclude that, while it appears that a daily GARCH(1,1) model can be beaten in obtaining accurate daily volatility forecasts, it appears that an intra-day GARCH(1,1) model performs as well as any alternative model and better than most, and any improvement on it provided by more complex models is marginal. Therefore, it appears to remain the case that a GARCH(1,1) form but utilizing higher-frequency data cannot be beaten, at least on the basis of the data series considered there, though it remains an avenue for further research to determine whether this conclusion continues to hold in other market settings.

CHAPTER THREE

DATA AND METHODOLOGY

3.1 Financial time series and their characteristics

Financial time series analysis is concerned with the theory and practice of asset valuation over time. Most financial studies are often concerned with the analysis of returns of assets, rather than the prices of assets. The main reason for using returns instead of prices is that return series are easier to handle than price series because the former have more attractive statistical properties. Financial data have some peculiar behaviors that are not usually observed in other non-financial types of data. The following are among the main and common characteristics that distinguish return series from prices series (Rossi, 2004).

1. Return of an asset is a scale-free measure of the investment opportunity.
2. Asset prices are generally non stationary. Returns are usually stationary;
3. Return series usually show no or little autocorrelation;
4. Serial independence between the squared values of the series is often rejected;
5. Volatility of the return series appears to be clustered;
6. Normality tends to be rejected in favor of some thick-tailed distribution.

Statisticians assume stationarity for a financial variable under study because non-stationary data, as a rule, are unpredictable and cannot be modeled or forecasted. The results obtained by using non-stationary time series may be spurious in that they may indicate a relationship between two variables where one does not exist. In order to get consistent, reliable results, non-stationary data need to be transformed into stationary data. As mentioned above and econometricians agreed so far, return series are usually found to be stationary. It is thus important to ensure stationarity of the series before analyzing the price data.

Let $p_t, t = 1,2,3$ ..., be a time series of prices of a financial asset, e.g. daily quotes on a share, stock index, currency exchange rate or a commodity price, etc. Instead of analyzing p_t, which often displays unit-root behavior and thus cannot be modeled as stationary, we often analyze log returns on p_t (Fryzlewicz, 2007), i.e. the series

$$y_t = log\frac{p_t}{p_{t-1}} = log p_t - log p_{t-1} \quad (3.1)$$

The log return series (y_t) displays many of the typical behaviors in financial log-return series such as: Absence of autocorrelations, heavy tails, aggregational Gaussianity, volatility clustering, conditional heavy tails, leverage effect, volume/volatility correlation and asymmetry in time scales.

3.2 Stationarity

The concept of stationarity is fundamental in time series analysis. A time series $\{y_t\}$ is said to be *strictly stationary* if the joint distribution of $(y_{t_1,} y_{t_2}, \dots, y_{t_T})$ is identical to that of $(y_{t_1+k}, y_{t_2+k}, \dots, y_{t_T+k})$for all t, where k is an arbitrary positive integer and $(t_1, t_2 \dots, t_T)$ is a collection of T positive integers. In other words, strict stationarity requires that the joint distribution of $(y_{t_1,} y_{t_2} \dots, y_{t_T})$ is invariant under time shift. Strict stationarity imposes a very strong condition and is hard to test empirically. Often the concept of weak stationarity is used. A time series $\{y_t\}$ is weakly stationary if both the mean of y_t and the covariance between y_t and y_{t-l} are time invariant, where l is an arbitrary integer. More specifically, $\{y_t\}$ is weakly stationary if (a) $E(y_t)$=μ, which is a constant and (b) $cov(\gamma_t, \gamma_{t-l})$ =γ_l which only depends on l . In practice, suppose that we have observed n data points $\{y_t \mid t = 1, \dots, n\}$. Weak stationarity implies that the time plot of the data would show that the values fluctuate with constant variation around a fixed level. In application, weak stationarity enables one to make inferences concerning future observations (e.g. Forecasting).The covariance $\gamma_l = cov(\gamma_t, \gamma_{t-l})$ is called the $lag - l$ auto-covariance of y_t . It has two important properties: (a) $\gamma_o = Var(y_t)$ and (b) $\gamma_{-l} = \gamma_l$. In the finance literature, it is necessary to test weakly stationarity of an asset return series. There are three methods of testing stationarity: graphical analysis, unit root test due to Dickey and Fuller (1979) and Phillips-Perron (PP) test.

A. Graphical Analysis of the Series

Before pursuing formal tests, it is always advisable to plot the time series under study. Such a plot gives an initial clue about the likely nature of the time series. For instance, if a line-graph of a time series shows an upward trend, then this suggests perhaps that the mean of the data has been changing. This may be a clue that the series is not stationary. Such an intuitive feel is the starting point of more formal tests of stationarity.

B. Unit Root Test

A test for stationarity that has become widely popular over the past several years is the $unit\ root\ test$.The Augmented Dickey-Fuller (ADF) test is discussed below. To illustrate the use of Dickey-Fuller test, consider first an autoregressive process of order one (AR (1)) process:

$$y_t = \omega + \alpha_1 y_{t-1} + \varepsilon_t \qquad (3.2)$$

where ω and α_1 are parameters and ε_t is assumed to be white noise. y is a stationary series if $-1 < \alpha_1 < 1$. If $\alpha_1 = 1$, y is a non-stationary series (a random walk with drift); if the process is started at some point, the variance of y increases steadily with time and goes to infinity. If the absolute value of α_1 is greater than one, the series is explosive. Therefore, the hypothesis of a stationary series can be evaluated by testing whether the absolute value of α_1 is strictly less than one. The augmented Dickey-Fuller (ADF) test takes the unit root as the null hypothesis, $H_o: \alpha_1 = 0$. Since explosive series do not make much economic sense, this null hypot0hesis is tested against the one-sided alternative $H_\alpha : \alpha_1 < 1$.The test is carried out by estimating an equation with y_{t-1} subtracted from both sides of the equation:

$$\Delta y_t = \omega + \delta y_{t-1} + \varepsilon_t \qquad (3.3)$$

where $\delta = \alpha_1 - 1$, Δ is the first difference operator, and the null and alternative hypotheses are $H_o: \delta = 0$,$H_\alpha: \delta < 0$.

While it may appear that the test can be carried out by performing a $t-test$ on the estimated δ, the $t-statistic$ under the null hypothesis of a unit root does not have the conventional t-distribution. Dickey and Fuller (1979) showed that the distribution under the null hypothesis is nonstandard, and simulated the critical values for selected sample sizes. Recently, MacKinnon (1991) has implemented a much larger set of simulations than those tabulated by Dickey and Fuller. In addition, MacKinnon estimates the response surface using the simulation results, permitting the calculation of Dickey-Fuller critical values for any sample size and for any number of right-hand variables. The simple unit root test described above is valid only if the series is an AR (1) process. If the series is correlated at higher order lags, the assumption of white noise disturbances is violated. The ADF test uses different methods to control for higher-order serial correlation in the series. It makes a parametric correction for higher-order correlation by assuming that the y series follows an AR (p) process and adjusting the test methodology:

$$y_t = \omega + \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \cdots + \alpha_p y_{t-p} + \varepsilon_t \qquad (3.4)$$

The ADF approach controls for higher-order correlation by adding lagged difference terms of the dependent variable to the right-hand side of the regression:

$$\Delta y_t = \omega + \delta y_{t-1} + \gamma_1 \Delta y_{t-1} + \gamma_2 \Delta y_{t-2} + \cdots + \gamma_{p-1} \Delta y_{t-p+1} + \varepsilon_t \qquad (3.5)$$

where $\delta = (\sum_{i=1}^{p} a_i) - 1$,$\gamma_j = -\sum_{k=j+1}^{p} \gamma_k$, Δ is the first difference operator, and the null and alternative hypotheses are $H_o: \delta = 0$,$H_\alpha: \delta < 0$.

An important result obtained by Fuller is that the asymptotic distribution of the t-statistic on δ is independent of the number of lagged first differences included in the ADF regression. Moreover, while the parametric assumption that y follows an autoregressive (AR) process may seem restrictive, Said and Dickey (1984) demonstrate that the ADF test remains valid even when the series has a moving average (MA) component, provided that enough lagged difference terms are augmented to the regression.

Researchers usually face two practical issues in performing the ADF test. First, they have to specify the number of lagged first difference terms to add to the test regression (selecting zero yields the Dickey-Fuller (DF) test; choosing n0umbers greater than zero generate ADF tests). The usual advice is to include lags sufficient to remove any serial correlation in the residuals. Second, they have the choice of including a constant, a constant and a linear time trend, or neither in the test regression. The choice here is important since the asymptotic distribution of the t-statistic under the null hypothesis depends on the assumptions regarding these deterministic terms.

To solve the two problems, one approach would be to run the test with both a constant and a linear trend since the other two cases are just special cases of this more general specification. However, including irrelevant regressors in the regression reduces the power of the test, possibly concluding that there is a unit root when, in fact, there is none. If the series seems to contain a trend one should include both a constant and trend in the test regression.

If the series does not exhibit any trend and has a nonzero mean, one should only include a constant in the regression, while if the series seems to be fluctuating around a zero mean, one should include neither a constant nor a trend in the test regression. The null hypothesis of a unit root is rejected against the one-sided alternative if the t-statistic is less than the critical value.

C. **The Phillips-Perron (PP) Test**

Phillips and Perron (1988) propose an alternative (nonparametric) method of controlling for serial correlation when testing for a unit root. The **PP** method estimates the non-augmented DF test equation and modifies the t -ratio of α coefficient so that serial correlation does not affect the asymptotic distribution of the test statistic. The **PP** test is based on the statistic:

$$\hat{t}_\alpha = t_\alpha \left(\frac{\gamma_0}{f_0}\right)^{1/2} - \frac{T(\gamma_0 - f_0)(s.e(\hat{\alpha}))}{2 f_0^{1/2} * s} \qquad (3.6)$$

where $\hat{\boldsymbol{\alpha}}$ is the estimate of $\boldsymbol{\alpha}$, $\hat{\boldsymbol{t}}_{\boldsymbol{\alpha}}$ is the t -ratio, $\boldsymbol{s.e}(\hat{\boldsymbol{\alpha}})$ is the standard error of $\hat{\boldsymbol{\alpha}}$ and s is the standard deviation of the test regression. In addition, $\boldsymbol{\gamma_0}$ is a consistent estimate of the error variance in Equation (3.2) (calculated as $(t-n)\frac{s^2}{t}$ where n is the number of regressors).The remaining term $\boldsymbol{f_0}$ is an estimator of the residual spectrum at frequency zero. The asymptotic distribution of the PP modified t-ratio is the same as that of the ADF statistic.

3.3 Tests for autocorrelation

A weakly stationary series is not serially correlated (or is a white noise process) if the autocorrelation function is 0 for all k >0. The ACF at lag k, denoted by ρ_k is defined as:

$$\rho_k = \frac{\gamma_k}{\gamma_o} = \frac{Covariance\ at\ lag\ k}{variance} \tag{3.7}$$

Since both covariance and variance are measured in the same units of measurement, ρ_k is unit less, or pure, number. It lies between -1 and 1. If we plot ρ_k against k , the graph we obtain is known as the population correlogram.

In practice it is only possible to compute the sample *autocorrelation function (SACF)*, $\hat{\rho}_k$.To compute this, first computing the sample covariance at lag k, $\hat{\gamma}_k$, and the sample variance, γ_o is a must.

Therefore, the SACF at lag k is

$$\hat{\rho}_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_o} \tag{3.8}$$

A plot of $\hat{\rho}_k$against k is known as the sample *correlogram*.

Financial applications often require to test jointly that several autocorrelations of y_t are zero.Box and Pierce (1970) propose the Portmanteau *statistics* (Q-statistics). The Q-statistic is defined as

$$Q = n\sum_{k=1}^{m} \hat{\rho}_k^2 \tag{3.9}$$

Where n=sample size and m=lag length. The null hypothesis for this test is, all the ρ_k up to certain lags are simultaneously zero. In large samples, the q statistic is approximately distributed as the $chi - square$ distribution with m degree of freedom .In application, if the computed Q exceeds the critical value from the $chi - square$ distribution at the chosen level of significance, the null hypothesis is rejected.

Ljung and Box (1978) modify the above Q-statistic as LB *statistic (Ljung-Box Q-statistic)* which is defined as:

$$LB = n(n+2)\sum_{k=1}^{m}\left(\frac{\hat{\rho}_k^2}{n-k}\right) \sim \chi_m^2 \tag{3.10}$$

Although in large samples both Q and LB statistics follow the chi-square distribution with m degrees of freedom, LB statistic has been found to have better performance in small samples than the Q statistic.

3. 4 Univariate Time Series Analysis

At first sight it would not seem very sensible for economists to pay much attention to univariate analysis. After all, economic theory is rich in suggestions for relationships between variables. Thus, attempting to explain and forecast a series using only information about the history of that series would appear to be an inefficient procedure, because it ignores the optional information in related series. There are two possible rationales: The first is that a priori information about the possible relationships between series may not be well founded. In such case a purely statistical model relating current to previous values may be a useful summary device and may be used to generate reliable short-term forecasts.

Alternatively, if theoretical speculations about the economic structure are well founded, it can be shown that one manifestation of that structure yields autoregressive moving average (ARMA) equations for each of the dependent variable in the structure (Johnston and DiNardo, 1997).

3. 4.1 The univariate GARCH model

First of all let the returns r_t be expressed as the change in logarithmic price over a certain period

$$r_t = log\frac{p_t}{p_{t-1}} \tag{3.11}$$

Consider the following financial return model;

$$\begin{aligned} &r_t = \mu_t + \varepsilon_t \quad , \varepsilon_t = \sigma_t z_t && \text{(i)} \\ &z_t \sim \text{iid N } (0,1) && \text{(ii)} \\ &\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \quad , \ \alpha_0 > 0 \ ,0 \leq \alpha_1 < 1 && \text{(iii)} \end{aligned} \tag{3.12}$$

Where;

r_t –return series at time t,

μ_t-the average return at time t,

ε_t –the innovation term

The autoregressive AR (p), moving average MA (q) and autoregressive moving average ARMA (p, q) models are applicable when the innovation term (ε_t) maintains a constant variance (homoscedasticity). If the error term ε_t is conditionally heteroskedastic, then ARCH/GARCH models are applicable to model the conditional variance of the innovation term. Since the model involves just one lag of $\varepsilon_t^2(i.e\ \varepsilon_{t-1}^2)$, it is known as ARCH(1) model. The generalized ARCH (here after, GARCH) model is a generalization of the ARCH model in that it includes lagged variances in the conditional variance equation. GARCH models have the advantage of capturing long lags in the shocks by using fewer parameters than ARCH models. The GARCH (1, 1) model is defined as:

$$r_t = \mu_t + \varepsilon_t \tag{3.13}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{3.14}$$

The GARCH model with p number of lagged variance term and q number of lagged squared error terms denoted as GARCH (q, p),and can be written as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} a_i \varepsilon_{t-i}^2 + \sum_{\mathrm{j}=1}^{\mathrm{p}} b_j \sigma_{t-j}^2 \qquad \alpha_0 > 0 \ , \ a_i \geq 0 \ , \quad b_j \geq 0 \tag{3.15}$$

Where the volatility term σ_{t-j}^2 denote the variance and j represents the number of lags, and the term ε_{t-i}^2 is the squared error for the period $t-i$. We can write the above equation more compactly:

$$\sigma_t^2 = \omega + \alpha(L)\varepsilon_t^2 + \beta(L)\sigma_t^2$$

where $\alpha(L) = \alpha_1 L + \cdots + \alpha_q L^q$

$\beta(L) = \beta_1 + \cdots + \beta_p L^p$

To ensure that the conditional variance is well defined in a GARCH (p, q) model all the coefficients in the corresponding linear ARCH (∞) should be positive rewriting the GARCH (p, q) modeled as an ARCH (∞):

$$\sigma_t^2 = \left(1 - \sum_{i=1}^{p} \beta_i L_i\right)^{-1} + (\omega + \sum_{j=1}^{q} \alpha_j \, \varepsilon_{t-j}^2$$

$$= \omega^* + \sum_{k=1}^{\infty} \phi_k \, \varepsilon_{t-k-1}^2$$

$\sigma_t^2 \geq 0$, if $\omega^* \geq 0$ and all $\phi_k \geq 0$. The non-negativity of ω^* and ϕ_k is also a necessary condition for the non-negativity of σ_t^2 (Rossi, 2004).

The a coefficients of lagged squared returns are interpreted as how fast the model react to, for example, market events, the b −coefficients of lagged conditional variance determines the degree of persistence in the volatility. A large value of a indicates that the conditional variance decays slowly, and that the volatility is persistent. On the other hand, if the a value is relatively higher than the b value, then the volatility is more extreme. The sum of a_i and b_j ($\sum_{i=1}^{q} a_i + \sum_{\mathrm{j}=1}^{\mathrm{p}} b_j$) should be less than one for the above $GARCH\ (q, p)$ to be stationary. Nevertheless, one regions' volatility has impact on volatility of the other regions. Making estimations that include this contagion effect requires an extended model.

3.5 Multivariate GARCH models

A large part of the literature deals with univariate models. But markets interact, and therefore a generalization from the univariate model to a multivariate one is needed. MGARCH models can be categorized in to four types (Silvennoinen and Teräsvirta, 2008):

• **Models of the conditional covariance matrix**: The conditional covariance is computed in a direct way. For example the vector error correction (VEC) and Baba-Engle-Kraft-Kroner (BEKK) models.

• **Factor models**: The return process is assumed to consist of a small number of unobservable heteroskedastic factors. This approach benefits from that the dimensionality of the problem reduces when the number of factors compared to the dimension of the return vector is small.

• **Models of conditional variances and correlations**: At first the univariate conditional variances and correlations are computed and then used to get the conditional covariance matrix. Some models are for example the Constant Conditional Correlations (CCC) model and the Dynamic Conditional Correlations (DCC) model.

• **Nonparametric and semi parametric approaches**: Models in this class form an alternative to parametric estimation of the conditional covariance structure. The advantage of these models is that they do not impose a particular structure (that can be misspecified) on the data.

This paper uses models of conditional variances and correlations. But before describing these models more profoundly, some definitions are required. This study uses two commodity price series, resulting in a bivariate approach.

$$r_t = \begin{bmatrix} r_{1,t} \\ r_{2,t} \end{bmatrix} \tag{3.16}$$

r_t is the vector of returns which can be decomposed as:

$$\begin{aligned} r_t &= \mu_t + \varepsilon_t \text{ , } \varepsilon_t = H_t^{1/2} z_t \\ \mu_t &= E(\ r_t | I_{t-1}) \\ z_t &\sim iidN(\underline{0}, I_N) \\ var(\underline{r_t} \mid I_{t-1}) &= H_t^{1/2} (H_t^{1/2})' = \Sigma_t \end{aligned} \tag{3. 17}$$

where:

r_t is $N \times 1$ vector of returns

μ_t is $N \times 1$ expected return at time t given the available information set I_{t-1}

$H_t^{1/2}$ $N \times N$ is cholesky factor of time varying conditional covariance matrix Σ_t.

z_t is $N \times 1$ vector of independent zero-mean and unit variance

The variance of the conditional unpredictable component can also be defined as:

$$var(\underline{r_t} \mid I_{t-1}) = \Sigma_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{21,t} & h_{22,t} \end{bmatrix} \tag{3.18}$$

As can be seen in Equation (3.18) the matrix obtained is symmetric. Additionally, the matrix Σ_t has to be positive definite for all t. The approach of multivariate modeling brings complications. Firstly, it may be hard to ensure that Σ_t is positive definite for all t. Secondly, the number of parameters becomes too large to estimate, and at last it may be difficult to obtain the stationarity condition for Σ_t.

The following subsections will describe the theory of the models, where the VEC and DVEC models which are the predecessors to the CCC, VCC and DCC models are first described. The various MGARCH models proposed in the literature differ in how they trade off flexibility and parsimony in their specifications for $\sum_t$. Increased flexibility allows a model to capture more complex $\sum_t$.

3.5.1 The VEC model

The VEC model is a generalization of a univariate GARCH model and developed by Bollerslev, Engle and Wooldridge in 1988. In this model the conditional variances and covariance are not only functions of all lagged conditional variances and covariances, but also of lagged squared returns and the cross products of the returns (Silvennoinen &Teräsvirta, 2008). The general VEC model is defined as:

$$vech(\textstyle\sum_t) = vech(C) + \displaystyle\sum_{i=1}^{p} Ai\ vech(\varepsilon_{t-i}\varepsilon_{t-i}') + \sum_{j=1}^{q} Bj\ vech\textstyle\sum_{t-j} \qquad (3.19)$$

and the case when $p = q = 1$ is defined as

$$vech(\textstyle\sum_t) = vech(C)+Avech(\varepsilon_{t-1}\varepsilon_{t-1}')+Bvech(\sum_{t-1}) \qquad (3.20)$$

The left term is the lower diagonal matrix that is transformed into a $N\frac{(N+1)}{2} \times 1$ vector. For a $VEC(1,1)$ model the left term can be expressed as follows.

$$vech(\textstyle\sum_t) = \begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} \qquad (3.21)$$

Despite the models flexibility, the number of parameters tends to grow rapidly. The rate of growth can be expressed as $N\ (N + 14)\ (N\ (N + 1) + 1)\ /2$.

3.5.2 The DVEC model

The diagonal VEC is a simplified VEC, where A_i and B_j are diagonal matrices in aspiration to obtain a positive definite $\sum_t$. This new version was also introduced by Bollerslev, Engle and Wooldridge in 1988. The $DVEC(p,q)$ model is defined as

$$\textstyle\sum_t = C + \displaystyle\sum_{i=1}^{p} Ai \odot (\varepsilon_{t-i}\,\varepsilon_{t-i}') + \sum_{j=1}^{q} Bj \odot \textstyle\sum_{t-j} \qquad (3.22)$$

where Ai and Bj are diagonal matrices and $\odot$ is the Hadamard (element by element) product of two matrices.

The $DVEC(1,1)$ is

$$\sum_t = C + A\odot(\varepsilon_{t-1}\varepsilon'_{t-1}) + B\odot(\sum_{t-1}) \tag{3.23}$$

The model above can be decomposed into univariate GARCH models of variances and co variances as:

$$\begin{aligned} h_{1t} &= a_{01} + a_1\varepsilon^2_{1t-1} + \beta_1 h_{1t-1} \\ h_{2t} &= a_{02} + a_2\varepsilon^2_{2t-1} + \beta_2 h_{2t-1} \\ h_{21t} &= a_{03} + a_3\varepsilon_{1t-1}\varepsilon_{2t-1} + \beta_3 h_{21t-1} \end{aligned} \tag{3.24}$$

where h_{11}&h_{22} denote the conditional variance of r_1 and r_2 and h_{21} is the covariance between the two returns.

3.5.3 The CCC model

The Constant Conditional Correlation model was suggested by Bollerslev (1990), where the time varying covariance matrix $\sum_t$ at time t is expressed as:

$$\sum_t = D_t R D_t \tag{3.25}$$

The right hand side consists of the conditional correlation matrix R that is time invariant . D is a diagonal matrix of $(h_1, h_2, \dots, h_k)$ such as:

$$D_t = \begin{bmatrix} \sqrt{h_{1,t}} & & & \\ & \ddots & & \\ & & & \sqrt{h_{k,t}} \end{bmatrix}$$

where each $h_{i,t}$ follows a univariate GARCH process. The conditional correlation matrix is given by $R = [\rho_{i,j}]$, and the non-diagonal elements of $\sum_t$ are:

$$h_{ij,t} = h^{1/2}_{i,t} h^{1/2}_{j,t} \rho_{i,j} \quad ,\forall\ i \neq j \tag{3.26}$$

Since the return process $r_{i,t}$ is modeled with a univariate approach, the desired conditional variances can be expressed in vector form

$$\sum_t = C + \sum_{i=1}^{q} A_i \varepsilon_{t-j} \odot \varepsilon'_{t-j} + \sum_{j=1}^{p} B_j \sum_{t-j} \odot \sum'_{t-j} \quad (3.27)$$

The first term C is a vector of the intercepts with a size of $n \times 1$ and the matrices of the coefficients are $n \times n$.

The advantage of the CCC model is that the computational procedure is more easily performed since the correlation matrix $|H|_{i,j}$ is constant. However, this means that the model may be too restrictive (Orskaug, 2009).

3.5. 4 The DCC model

This model was developed by Engle & Sheppard (2011). The dynamic conditional correlation model is

$$\sum_t = Dt\ RtDt \quad (3.28)$$

Where $\sum_t$ is the covariance matrix and Rt is $n \times n$ matrix of the conditional correlation of the returns. The diagonal matrix Dt is expressed as:

$$
D_t = \begin{bmatrix} \sqrt{h_{1t}} & 0 & . & . & . & 0 \\ . & & & & & . \\ . & & & & & . \\ 0 & \sqrt{h_{2t}} & . & . & & . \\ . & . & . & & & 0 \\ . & . & . & & & 0 \\ 0 & & . & . & . \; 0 & \sqrt{h_{kt}} \end{bmatrix}
$$

where each $h_{i,t}$ follows a univariate GARCH process. Furthermore, $\sum_t$ has to be positive definite, which is automatically obtained while, Rt is a correlation matrix that is symmetric by definition. When this matrix is defined, two requirements are needed. Firstly, $\sum_t$ needs to be positive definite, since it is a covariance matrix. Secondly, the parts that belong to Rt need to be less than one. These requirements are met through decomposition:

$$
R_t = diag(q_{iit})^{-1} Q_t diag(q_{iit})^{-1} \quad \text{for, i=1,…,n where}
$$

$$
Q_t = (1 - \pi_1 - \pi_2)\bar{Q} + \pi_1 \varepsilon_{t-1} \varepsilon_{t-1}^{'} + \pi_2 Q_{t-1} \text{ and } \bar{Q} = \text{cov } [\varepsilon_t \varepsilon_t^{'}] = \text{E}[\varepsilon_t \varepsilon_t^{'}].
$$

Where $\bar{Q}$ is the unconditional covariance, and will be finite as the model contains finite parameters and the vector ε_t has finite variance. Additionally, the parameters π_1 and π_2 are scalars and $diag\ (Q)$ is used to rescale the parts of Qt in order to fulfill that $|\rho_{ij}| = \left|\frac{q_{ijt}}{\sqrt{q_{iit} q_{jjt}}}\right| \leq 1$ and where $\sqrt{q_{nnt}}$ is the content of the matrix $diag\ (Q)$. An estimate of $\bar{Q}$ is:

$$
\hat{Q} = \frac{1}{T} \sum_{t=1}^{T} \varepsilon_t \, \varepsilon_t^{'} \tag{3.29}
$$

Moreover, the scalars π_1 and π_2 must be larger than zero, but the sum has to be less than one. One may note that these are conditions of the univariate GARCH to be stationary, but which is applied in the DCC model. (Orskaug, 2009).

3.5.5 The VCC model

Varying conditional correlation (VCC) multivariate generalized autoregressive conditionally heteroskedastic (MGARCH) models in which the conditional variances are modeled as univariate generalized autoregressive conditionally heteroskedastic (GARCH) models and the conditional covariance are modeled as nonlinear functions of the conditional variances. The conditional correlation parameters that weight the nonlinear combinations of the conditional variance follow the GARCH -like process specified in Tse and Tsui (2002). The VCC MGARCH model is about as flexible as the closely related dynamic conditional correlation\ MGARCH model and more flexible than the constant conditional correlation model, and more parsimonious than the diagonal vector error conditionally heteroskedastic (DVECH) models.

MGARCH models differ in the parsimony and flexibility of their specifications for a time varying conditional covariance matrix of the disturbances, denoted by $\sum_t$. In the conditional correlation family of MGARCH models, the diagonal elements of $\sum_t$ are modeled as univariate GARCH models, whereas the off-diagonal elements are modeled as nonlinear functions of the diagonal terms. In the VCC MGARCH model,

$$h_{ij,t} = \rho_{ij,t}\sqrt{h_{ii,t}h_{jj,t}} \tag{3.30}$$

Where the diagonal elements hii,t and hjj,t follow univariate GARCH processes and $\rho ij,t$ follows the dynamic process specified in Tse and Tsui (2002) and discussed below. Because the $\rho\ ij,t$ varies with time, this model is known as the VCC GARCH model.
The VCC-GARCH (1,1) model proposed by Tse and Tsui (2002) can be written as

$$\begin{gathered} r_t = \theta x_t + \epsilon_t \\ \epsilon_t = H_t^{1/2} z_t \\ \textstyle\sum_t = D_t R_t D_t \\ R_t = (1 - \pi_1 - \pi_2)R + \pi_1 \epsilon_{t-1} + \pi_2 R_{t-1} \end{gathered} \tag{3.31}$$

Where

x_t is a k × 1 vector of independent variables, which may contain lags of r_t;

θ is an m × k matrix of parameters;

r_t is an m × 1 vector of dependent variables;

$H_t^{1/2}$ is the Cholesky factor of the time-varying conditional covariance matrix $\sum_t$

z_t is an m × 1 vector of independent and identically distributed innovations;

D_t is a diagonal matrix of conditional standard deviation

$$
\mathrm{D_t} = \begin{bmatrix} \sqrt{\mathrm{h_{1t}}} & 0 & \cdots & 0 \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ 0 & \sqrt{\mathrm{h_{2t}}} & \cdots & \cdot \\ \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & 0 \\ 0 & \cdots & 0 & \sqrt{\mathrm{h_{mt}}} \end{bmatrix}
$$

R_t is a matrix of conditional correlations,

$$
\mathrm{R_t} = \begin{bmatrix} 1 & \rho_{12,t} & \cdots & \rho_{1m,t} \\ \rho_{21,t} & 1 & \cdots & \rho_{2m,t} \\ \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \rho_{m1,t} & \cdots & \rho_{m2,t} & 1 \end{bmatrix}
$$

R is the matrix of means to which the dynamic process in Equation (3.31) reverts; π_1 and π_2 are parameters that govern the dynamics of conditional correlations, and they are nonnegative and satisfy $0 \leq \pi_1 + \pi_2 < 1$.

3.6 The model order selection: AIC and BIC

An important step before making the estimations is to determine the order of the models. Theoretically one can do this using the autocorrelation function, but in practice this may be difficult. A more formal way is to use an information criterion and choose the order that minimizes the criterion value. Two common criteria are the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). The formulas for these are

$$AIC = -2 \times LLF + 2M$$

$$BIC = -2 \times LLF + M \times LN(N)$$

where N is the sample size, and M is the number of parameters. LLF is an abbreviation for log likelihood function. The reason for using both criteria is that the BIC is consistent but inefficient, and the AIC is the opposite, not consistent but efficient. No criteria are superior to others, but an overall assessment is needed based on the results showed by the criteria. For a model to be best it should have the smallest information criteria (Brooks, 2008).

3.7 Estimation Procedure: The Quasi Maximum Likelihood method

A common issue using multivariate data for estimation is that the likelihood function becomes flat. Therefore, the choice of starting values is a crucial aspect of the estimation. Nevertheless, the remedy for the problem is simply to perform the estimation with different start values. The starting values should be set in the way that renders the highest possible likelihood. (Orskaug, 2009).

The majority of empirical papers has concluded that financial data suffer from fat tailed distributions and this is often corrected by assuming the data follow Student-t distribution. However, the maximum likelihood (ML) method relies on the assumed distribution, then appointing an incorrect distribution to the maximum likelihood data then the ML is not consistent. But, by using the quasi maximum likelihood (QMLE) the estimation remains consistent even under misspecification. Furthermore when dealing with models with conditional heteroskedasticity, the estimates are known to be asymptotically normal (Bollerslev&Wooldridge, 1992). Another thing that may be pointed out regarding the estimation procedure is that robust standard errors are used since they allow the sample to contain heteroskedasticity. In other words, the robust standard error better deals with non-constant variance.

Given the MGARCH models in the previous section, the log-likelihood function for $\{\varepsilon_T, \dots, \varepsilon_1\}$ obtained under the assumption of conditional multivariate normality is:

$$logL_T(\epsilon_T, \dots, \epsilon_1; \theta) = -\frac{1}{2}[TNlog(2\pi) + \sum_{t=1}^{T}(log|\Sigma_t| + \epsilon_t' \Sigma_t^{-1} \epsilon_t)] \tag{3.32}$$

- The assumption of conditional normality can be quite restrictive.
- The symmetry imposed under normality is difficult to justify, and the tails of even conditional distributions often seem fatter than that of normal distribution.

Let $\{(r_t, x_t); t = 1,2, \dots\}$ be a sequence of observable random vectors with r_t(N× 1) and x_t(L× 1).

The vector r_t contains the "endogenous" variables and x_t contains contemporaneous "exogenous" variables.

$$w_t = (x_t, r_{t-1}, x_{t-1}, \dots, r_1, x_1)$$

The conditional mean and variance functions are jointly parameterized by a finite dimensional vector $\boldsymbol{\theta}$:

$$\{\mu_t(w_t, \theta), \theta \in \Theta\}$$
$$\{\Sigma_t(w_t, \theta), \theta \in \Theta\}$$

where $\Theta \subset R^P$ and μ_t and Σ_t are known functions of w_t and $\boldsymbol{\theta}$.

The validity of most of the inference procedures is proven under the null hypothesis that the first two conditional moments are correctly specified, for some $\theta_0 \in \Theta$,

$$E(r_t \mid w_t) = \mu_t(w_t, \theta_0)$$

$$var(r_t \mid w_t) = \Sigma_t(w_t, \theta_0)$$

The procedure most often used to estimate θ_0 is the maximization of a likelihood function that is constructed under the assumption that

$$(r_t \mid w_t) \sim N(\mu_t, \Sigma_t)$$

The approach taken here is the same, but the subsequent analysis does not assume that r_t has a conditional normal distribution (Rossi, 2012).

For observation t the quasi-conditional log-likelihood is

$$l_t(\theta, r_t, w_t) = -\frac{N}{2}\ln(2\pi) - {}^1/_2 \; ln|\Sigma_t(w_t, \theta)|$$

$$= - {}^1/_2 \, (r_t - \mu_t(w_t, \theta))' \Sigma_t^{-1}(w_t, \theta)(r_t - \mu_t(w_t, \theta))$$

Letting $\epsilon_t(r_t, w_t, \theta_0) \equiv r_t - \mu_t(w_t, \theta) : (N \times 1)$ denote the residual function

$$l_t(\theta) = -\frac{N}{2}\log(2\pi) - {}^1/_2 \, log|\Sigma_t(\theta)| - \frac{1}{2}\epsilon_t'(\theta)\Sigma_t^{-1}(\theta)\epsilon_t(\theta) \tag{3.33}$$

$$LogL_t(\theta) = \sum_{t=1}^{T} l_t(\theta)$$

3.8 Determining the conditional distribution

When fitting a GARCH-model based on financial data, the conditional distribution of the returns has to be defined. Studies, for example Bollerslev (1987) and Hsieh (1989), illustrate that returns are not normally distributed. Instead, the Student-*t* distribution captures the observed kurtosis in empirical returns in a more sufficient way than the normal distribution. Returns have excess kurtosis and fatter tails than the normal distribution. Therefore, the Student-*t* distribution is more suitable (Reider, 2009). There are three assumptions about the conditional distribution of the error term commonly employed when working with GARCH models: normal (Gaussian) distribution, student's t-distribution, and the generalized error distribution (GED).

3.8.1 Univariate N-GARCH, T-GARCH, and GED- GARCH

For normal (Gaussian) distribution we have:

$$r_t = \mu + \varepsilon_t \,, \quad \varepsilon_t = \sigma_t z_t$$
$$z_t \sim \text{N } (0, 1)$$
$$\sigma_t^2 = a_0 + + a_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{3.34}$$

Many studies show that distributions of financial series have fat-tail, so Nelson (1991) and Hamilton (1994) use GED and t-distributions to adjust the deviation of the tail. For the students distribution the density function is given by:

$$f(x, r) = \frac{\Gamma((v+1)/2)}{[(v-2)\pi]^{1/2}} (1 + x^2/(v-2))^{-(v+1)/2} \,, \; v \text{ is degree of freedom} \tag{3.35}$$

For the GED-GARCH model first we give out the distribution density:

$$f(x \mid \mu, \delta, r) = \frac{r.\Gamma(3/r)^{1/2}}{2\delta.\Gamma(1/r)^{3/2}} exp - \left\{ \frac{\Gamma(3/r)}{\Gamma(1/r)} (\frac{x-\mu}{\delta})^2 \right\}^{r/2} , \quad 0<r\leq\infty \tag{3. 36}$$

Γ(.) is GAMMA function. If a variate x is drawn from this pdf, we can denote x~GED (μ,σ^2,r).we can describe the model below:

$$r_t = \mu + \varepsilon_t, \;\; \varepsilon_t = \sigma_t z_t$$
$$z_t \sim \text{G } (0,1,r)$$
$$\sigma_t^2 = a_0 + + a_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{3.37}$$

3.8.2 Multiple N-GARCH, T-GARCH, GED -GARCH

We just introduce multiple student t-distribution because we are familiar with multiple normal distribution.

A. Multiple student's t-distribution

$$f_Y(y) = c_1|\Sigma|^{-1/2}[v + (y-\mu)'\Sigma^{-1}(y-\mu)]^{v+n/2}, \quad y \epsilon R^n \tag{3. 38}$$

In which, $c_1 = v^{v/2}\boldsymbol{\Gamma}[\frac{\boldsymbol{v+n}}{\boldsymbol{2}}]\left[\boldsymbol{\pi}^{\frac{n}{2}}\boldsymbol{\Gamma}(\frac{\boldsymbol{v}}{\boldsymbol{2}})\right]^{-\mathbf{1}}$, $\boldsymbol{\Gamma}$(.) is GAMMA function, then

$$E(Y) = \mu, \qquad var(Y) = \frac{v}{v-2}\Sigma, v > 2$$

We use $T_n(v, \mu, \Sigma)$ to denote multiple students t-distribution.

B. Multiple generalized error distribution

If a random variable x has a GED with mean zero and unit variance, the pdf of x is given by:

$$df(x \mid \mu, \Sigma, r) = \frac{d^n x}{\sqrt{\pi_n \Sigma}} \frac{\Gamma\left(1+\frac{n}{2}\right)}{\Gamma\left(1+\frac{n}{r}\right)}\left\{\frac{\Gamma(3/r)}{\Gamma(1/r)}\right\}^{n/2} * exp - \left\{\frac{\Gamma\left(\frac{3}{r}\right)}{\Gamma\left(\frac{1}{r}\right)}(x-\mu)^T\Sigma^{-1}(x-\mu)\right\}^{r/2} \tag{3.39}$$

If n dimensional vector x obeys this distribution, it can be denoted as x~ GED (μ, Σ, r). Here μ, Σ, r are parameters, μ is the mode of the distribution, and also is the mean. The relationship between variance-covariance matrix v and Σ can be shown below:

$$v = \Sigma \times \frac{\Gamma(\frac{n+2}{r})\Gamma(1+1/r)}{\Gamma(3/r)\Gamma(1+n/r)} \tag{3.40}$$

3.9 Estimation evaluation

The evaluation is done with several methods. Firstly, the estimations are compared using the method of three measures: MAE, RMSE and the explanatory power from an ordinary least squares regression. Secondly, an attempt to determine the goodness of fit of the residuals is done through univariate approach consisting of an analysis with the Ljung-Box test.

3.9.1The Ljung-Box test

If the estimated output contains auto correlated residuals, this indicates that there is still variation left to be explained by the model. One way to test for autocorrelation in the residuals is the Ljung-

Box test which checks if the autocorrelations are different from zero in a set of data. The test hypothesis with lag length q, is defined as:

$H_O: \gamma_1 = \cdots = \gamma_q = 0$

$H_o: at\ least\ one\ \gamma_i \neq 0, \quad i = 1, \ldots, q$

where $\gamma_i$denotes the autocorrelation for a certain lag i and the test statistic is

$$Q_q = T(T+2)\sum_{i=1}^{q} \frac{\hat{\gamma}_i^2}{T-i} \tag{3.41}$$

where T is the length of the series, and q is the number of lags specified and $\hat{\gamma}_i$ is the estimated autocorrelation for lag i.When n gets large Qq becomes asymptotically χ^2distributed with q degrees of freedom. The null hypothesis for significance level α, is rejected if $Qq > \chi^2_{1-a,q}$ where a rejection means that the hypothesis of random errors is rejected. In practice, several lag lengths are used and compared since different lags may give different results.

The multivariate form of this test was proposed by Hosking (1980) and others. Hosking (1981) demonstrated the equivalence of the several forms in the literature. It is often applied to the residuals of a multivariate regression, such as a VAR (vector auto regression). The null hypotheses of no multivariate autocorrelation of degree q are:

$H_O: \boldsymbol{\gamma_1} = \cdots = \boldsymbol{\gamma_q} = \mathbf{0}$

$H_o: at\ least\ one\ \boldsymbol{\gamma_i} \neq \mathbf{0}, \quad i = 1, \ldots, q$

The test statistic in its multivariate form is defined as the following according to Hosking (1980):

$$LB_q = T(T+2)\sum_{j=1}^{q} \frac{1}{T-q} tr\left[c_{0j} c_{00}^{-1} c_{0j}' c_{00}^{-1}\right] \sim \chi^2_{k^2(q-L)} \tag{3.42}$$

where

$$c_{0j} = \frac{1}{T}\sum_{t=j+1}^{T} \varepsilon_t \varepsilon_{t-j}'$$

Under the null hypothesis (of independence) the Ljung-Box test statistic has approximately a $chi-square$ distribution with $k^2(q-L)$ degrees of freedom. Here T is the length of the series, and q

denotes the order of autocorrelation, L is the length of lags in the multivariate regression. The null hypothesis of the multivariate test is that the autocorrelation functions of all series have no significant elements for lags one through that specified lags. A rejection indicates that at least one series is not white noise.

3.9.2The ARCH LM test

This test was introduced by Engle in 1982, and it investigates whether the conditional Volatilities of the chosen sample of data contain variation. The test can be used first to determine if there is volatility clustering in the data, and then to determine if there is any $ARCH$ −effects left in the residuals. Firstly, the estimated residuals, $\hat{\varepsilon}_t^2$ are regressed as:

$$\hat{\varepsilon}_t^2 = C + \left(\sum_{i=1}^{q} a_i\, \hat{\varepsilon}_{t-i}^2\right) + v_t \qquad (3.43)$$

and secondly, the hypotheses are

H_o:$a_1 = a_2 = \cdots = a_q$=0

H_1:$at\ least\ one\ of\ a_i$≠0

If H_0 is true, then homoscedasticity characterizes the variance indicating no volatility clustering. The test statistic is

$$LM = nR^2 \sim \chi^2 \qquad (3.44)$$

Where n is the number of observations and R^2 is computed from the regression (3.43) using estimated residuals. If n is large, LM is chi-squared distributed with q degrees of freedom.

The multivariate LaGrange Multiplier (LM) test can be used, since we are using two series. The test is based on the following regression:

$$\boldsymbol{\varepsilon_t = C + \rho_1 \varepsilon_{t-1} + \cdots + \rho_q \varepsilon_{t-q} + v_t} \qquad (3.45)$$

where $\boldsymbol{v_t}$ is an error term which is assumed to be white noise. The null and alternative hypotheses of no multivariate autocorrelation of degree $\mathbf{q}$ are:

$\boldsymbol{H_0}$**:** $\boldsymbol{\rho_1 = \rho_2 = \cdots = \rho_q}$**=0**

$\boldsymbol{H_1}$**:** $at\ least\ one\ \boldsymbol{\rho_i}$ **≠0**

This test is defined as follows according to Johansen (1996):

$$LM(q) = \Delta log\left(\frac{|C_R|}{|C_U|}\right) \sim \chi^2_{q*k} \quad (3.46)$$

Where $\Delta = T - (k(k+1) - qk) + \frac{1}{2}(k(q-1) - 1)$

C_R is the estimated variance covariance matrix of the residuals in Equation (3.45) when the null hypothesis is imposed and C_U is the estimated variance covariance matrix of the residuals in Equation (3.45) when null hypothesis is not imposed.

3.10 Forecasting Procedure and Evaluation

3.10.1 The Procedure

The series of conditional correlation forecasts for the period of 2016w1 to 2016w44 is compared to the realized values for the same period. Starting with an initial estimation for the eleven years period of 2005w1 to 2015w52, a forecast is made for the following week $(T + 1)$. The estimation window is moved one week and another forecast is made for $(T + 2)$. This procedure continues throughout the forecast period. More generally expressed, the estimation window reaches between the first observation at time t and the last observation of the first period at time T. This means that the length of the window d is $d = T - t$. The last observation of the evaluation period is denoted T *. The forecast horizon k is set to be constant as $k = 1$. The forecast loop continues until the starting point t and the ending point T of the first period both have moved a number of steps equal to $T * -T - 1$, which results in a series of $T * -(T + 1)$ forecasts.

3.10.2 Defining a proxy

In evaluating volatility forecasts the usual proxy for 'true' volatility is 'ex post' squared returns, or the squared errors. However, as noted by Andersen and Bollerslev (1998) and Andersen *et al.* (1999), although the use of squared returns or the squared errors is justifiable as an unbiased estimate of the volatility process, it provides a very noisy measure due to a large idiosyncratic term. Specifically, the returns innovation can be written as $\varepsilon_t = \sigma_t z_t$ with z_t an independent zero mean and unit variance stochastic process and σ_t is the volatility process. If the model for σ_t^2 is correctly specified, then the conditional expectation, $E(\varepsilon_t^2|I_{t-1}) = E(\sigma_t^2 z_t^2|I_{t-1}) = \sigma_t^2$, and the squared error is an unbiased estimate of the volatility process; however, it still contains the noisy idiosyncratic term, z_t^2. This typically resulted in a poor performance, which instigated a discussion of the practical relevance of volatility models.

However, Andersen and Bollerslev (1998) showed that the 'poor' performance could be explained by the fact that the squared return is a noisy proxy for the conditional variance. By substituting the realized variance (instead of the squared return), Andersen and Bollerslev (1998) showed that volatility models perform quite well.

Hansen and Lunde (2003) provide another important argument for using the realized variance rather than the squared return. They show that substituting the squared returns for the conditional variance can severely distort the comparison, in the sense that the empirical ranking of models may be inconsistent for the true (population) ranking. So an evaluation that is based on squared returns may select an inferior model as the 'best' with a probability that converges to one as the sample size increases. For this reason, our evaluation is based on the realized variance. Building upon this line of research Andersen *et al*. (2003) define the so-called 'realized volatility' on day t as:

$$h_t^{rv} = \sum_{i=1}^{N} r_{t,i}^2 \tag{3.47}$$

Where;

N–is the number of equally spaced intervals within a day,

$r_{t,i}$– is a logarithmic return on day *t* at time interval *i*; with *i* = 1, 2…, *N* and *t* = 1, …, *T*.

Thus realized volatility is the sum of squared intra-day returns. In principle, letting N tend to large, i.e. continuous time sampling, the measure approaches the true integrated volatility of the underlying continuous time process and is theoretically free from measurement error. Further, this measure allows a market participant to essentially treat volatility as an observed variable and to allow direct estimation.

3.11 Forecast evaluation

3.11.1 The MZ regression

A popular way to evaluate volatility models out-of-sample is in terms of the R^2 from a Mincer – Zarnowitz (MZ) regression,

$$\sigma_t^2 = a + bh_t^2 + v_t, \tag{3.48}$$

that is, squared returns are regressed on the model forecasts of σ_t^2 and a constant.

Here:

σ_t^2 -is ex-post volatility (e.g. realized volatility) at time t, h_t^2 is estimated (in-sample) or forecasted (out-of sample) volatility at time *t*,

v_t -independent and identically distributed; $v_i \sim N(0, 1)$. α and b are parameters to be estimated. If the model for conditional variance is well specified, we should have: $a = 0$, $b = 1$. According to specific features of financial data series, the value of R^2 is usually low (even less than 5%) (Anderson *et al*., 2009).

3.11.2 Mean Absolute Error (MAE)

Another way of determining the goodness of the estimations and forecasts is calculating the MAE. The approach is to measure how the received conditional covariance are close to their corresponding realized value. The formula is:

$$MAE = \frac{1}{n}\sum_{t=1}^{n}|\sigma_t^2 - h_t^2| \tag{3.49}$$

Where the proxy is used as σ_t^2 and the estimated conditional covariance is used as h_t^2 .By comparing the MAE between the estimated models, it can give an indication of which model that makes the best estimations.

3.11.3 Root Mean Square Error (RMSE)

The third measure is the Root Mean Square Error (RMSE), which is defined as

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(\sigma_t^2 - h_t^2)^2} \tag{3.50}$$

Using these methods, the estimated models can be compared, and using the same measurements for estimations and forecasts, one can determine if the relatively best estimations model also makes the best forecast.

CHAPTER FOUR
RESULTS AND DISCUSSION

4.1 Data

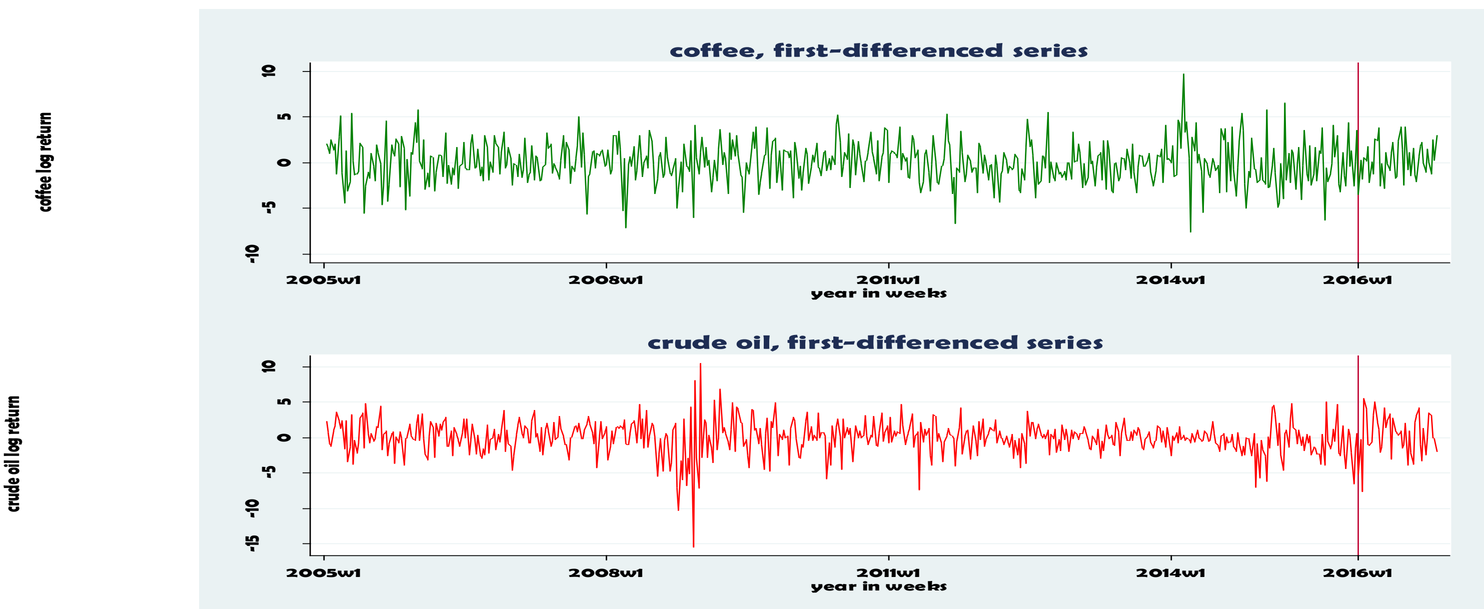
The data considered in this paper were the weekly time series of **Brent crude oil** and **coffee Arabica** futures market closing price, given in US dollars per barrel and US dollars per pound, respectively. Both series contain data spanning between first week of January 2005 and last week of October, 2016 (a total of 616 observations) extracted from Bloomberg database. The full sample is split into two parts: in sample data, in order to estimate the parameters of models and out of sample data, in order to make forecasts. The in-sample period spans from first week of January 2005 up to last week of December, 2015 and the out-of-sample period spans from first week of January, 2016 through last week of October, 2016. For the out of sample period, we have also extracted a 30 minute intra-day data for realized volatility measuring purpose. All calculations were done using **Stata 13**.

4.2 Test of stationarity and features of log return series

Our data consists of two commodity prices; crude oil, and Arabica coffee. **Fig. 4.1** is the trend charts of the two commodity prices in the study period. The two commodity prices present the same trend and direction during the entire study period. Price for both of the commodities have risen form 2005 up to 2008 and fallen from 2009 up to 2010 with a similar pattern. Another impression is that both commodity level series are non-stationary. To visualize the returns series for these two markets, we depict the return time series plots in **Fig.4.2**. The weekly return series display volatility persistence properties, indicating that large changes tend to be followed by large changes of both sign and small changes tend to be followed by small changes, and the differenced series suggests stationarity. The plots further show that the volatilities of these two commodity returns not only have a volatility clustering phenomenon during the selected sample period, but have certain relation on their return volatility processes. That is, when the fluctuation of the Arabica coffee price grew larger, the volatility of crude oil return also became larger for most of the times. This is the main motive for discussing the relationships of coffee price returns and crude oil price returns.

Figure 4.1: Time series plot for level series, Arabica coffee and crude oil

Figure 4.2 Time series plot for differenced series, Arabica coffee and crude oil



The level series plot in Fig 4.1 show that the level prices are non-stationary. Thus, the first logical step is to check the stationarity of these prices using unit root test. In this test the null hypothesis of unit root is rejected if the t-statistic is less than the critical value.

Table 4.1 and **Table 4.2** summarize the DF and PP unit root test results of the two commodity level prices, respectively. As can be seen in both tables, the t-statistics are greater than the critical values, at 1%, 5% and 10% the significance. Hence, the null hypothesis of unit root would not be rejected, that is, there is a unit root problem in each of the data.

Table 4.1: DF Unit root test of stationarity for level prices, with trend

Series	t-statistic	1%critical value	5% critical value	10%critical value
Crude oil	-1.675	-3.960	-3.410	-3.120
Arabica coffee	-2.053	-3.960	-3.410	3.120

Table 4.2: PP Unit root test of stationarity for level prices, with trend

Series	t-statistic	1%critical value	5% critical value	10%critical value
Crude oil	-2.335	-3.960	-3.410	-3.120
Arabica coffee	-2.365	-3.960	-3.410	3.120

If a time series data is non-stationary, it is necessary to look for possible transformations that might bring stationarity. In practice, econometricians usually transform financial prices in to return forms. This is because often return series are found to be stationary such that analysis is possible. The log return series is obtained by:

$$r_t = 52 * (log \frac{p_t}{p_{t-1}})$$

where r_t is the log return series of the real price multiplied by 52 (which is simply a scaling factor), to annualize as we are using weekly data and each year contains 52 weeks, and p_t is the real price series and log is the natural logarithm.

Table 4.3 and **Table 4.4** summarize the DF and PP unit root test of the log return series for each of the commodities. Both tables show that all the t-statistics are less than the critical values at all levels of significance. These indicates that the null hypothesis of unit root would be rejected in both cases. Hence, the log return series are stationary for each of the items as shown by the time series plots.

Table 4.3: DF Unit root test of stationarity for log Returns

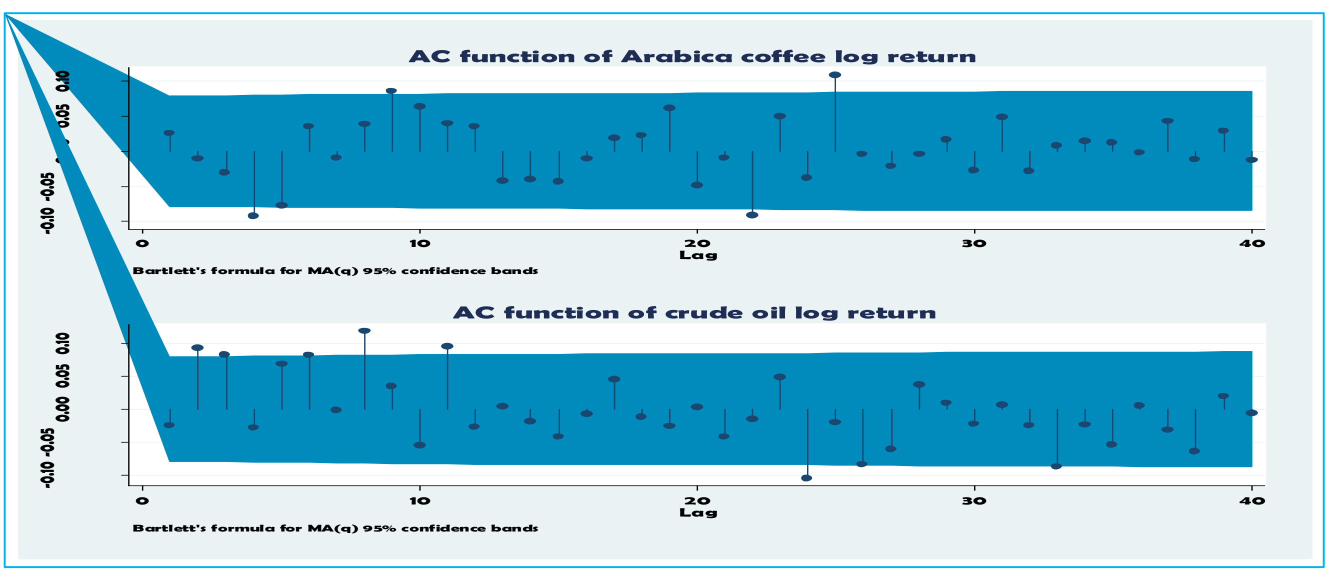
Series	t-statistic	1%critical value	5%critical value	10%critical value
Crude oil	-5.791	-3.430	-2.860	-2.570
Arabica coffee	-5.877	-3.430	-2.860	-2.570

Table 4.4: PP Unit root test of stationarity for log Returns

Series	t-statistic	1%critical value	5%critical value	10%critical value
Crude oil	-24.090	-3.430	-2.860	-2.570
Arabica coffee	-25.400	-3.430	-2.860	-2.570

Another important feature of return series is that usually the series show no or little autocorrelation. **Figure 4.3** shows the plots of the sample autocorrelation functions (ACFs) of the two return series for lags 1 to 40. A sample ACF has the approximate upper and lower confidence bounds. The two sample ACFs suggest that the serial correlations of weekly price log returns are very small except at a few lags. Most of the sample ACFs are with in or near to their confidence limits indicating that autocorrelations that fall inside the limits are not significantly different from zero. The null hypothesis of zero autocorrelation is rejected if the sample autocorrelation lies outside the interval$\left[-1.96 \times {}^{1}/_{\sqrt{T}}, 1.96 \times {}^{1}/_{\sqrt{T}}\right]$, which covers the blue shaded area.

Figure 4.3: Sample autocorrelation functions of weekly coffee and oil level series

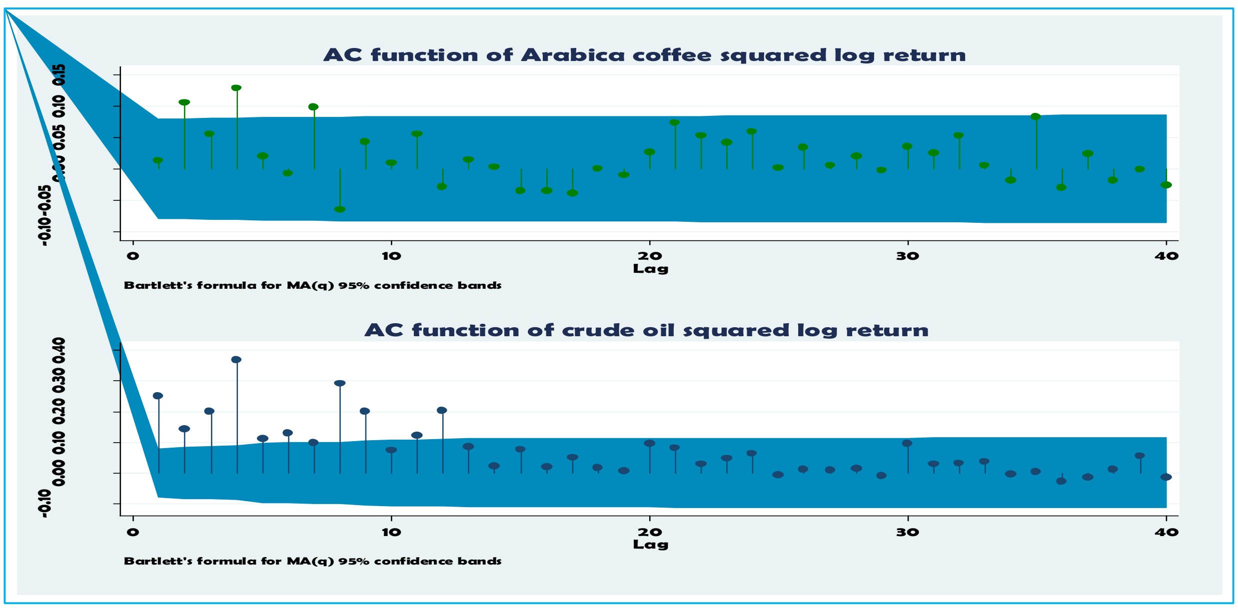


** In each plot, the blue shaded area denote two standard-error limits of the sample ACFs.

Furthermore, squared log return series are usually strongly auto-correlated. **Figure 4.4** represents the plots of the sample ACFs of the squared log returns of the Arabica coffee and crude oil futures prices.

Both sample ACFs suggest that the serial correlations of weekly price squared log returns are very large. Most of the sample ACFs are out of their two standard-error limits (Blue shaded area). This indicates that the sample ACFs of the squared log returns are significantly different from zero. This illustrates the degree of persistence in variance and implies that GARCH models may be used for modeling the variance.

Figure 4.4: sample autocorrelation functions of weekly differenced series, coffee and oil return



** In each plot, the blue shaded area denote two standard-error limits of the sample ACFs

In financial time series analysis it is often required to test that several auto correlation of the return series are jointly zero. For this purpose, first we plot the autocorrelation function. But the plot is an indication and not applicable for absolute decision. Instead we can use the Portmanteau or general statistics of linear dependency Q (m), where m is the lag length. Under the null hypothesis that $\rho_1 = \rho_2 = \cdots = \rho_m = 0$, Q (m) is distributed as chi-squared variable with m degree of freedom. The null hypothesis is rejected if $Q\ (m)$ is greater than the tabulated value of chi-square distribution.

Table 4.5: Portmanteau test for white noise for returns and squared returns series

Return series	Q(52)	Prob.>chi(52)
Coffee return	60.989	0.1841
Oil return	70.1784	0.069
Coffee return squared	80.428	0.0472
Oil return squared	343.54	0.000

From table **4.5**, looking at the p-values at 5% level of significance, we can agree that the returns are not autocorrelated, but the squared returns are highly autocorrelated, but the squared returns are autocorrelated.

It is also typical property of financial log-return series to exhibit skewness and kurtosis. A descriptive analysis was then conducted on the prices, Pt and returns, rt. **Table 4.6** presents a general summary statistics of the two level series and differenced log return series. The table shows that the skewness and kurtosis of the two log return series is different from that of the normal distribution. Both kurtosis exceed three, which is the kurtosis value of the normal distribution, indicating that the log return series data is peaked relative to the normal distribution.

All the skewness values are different from zero, which is the skewness value of the normal distribution, showing the distribution of the log return series is not symmetric around its mean compared to the normal distribution. Positive skewness means that the distribution has a long right tail and negative skewness implies that the distribution has a long left tail. The Jarque-Bera test of normality indicate the rejection of the normality of the log return series in the two commodities.

Looking at the distributions of the univariate series (with the theoretical univariate normal distribution marked with a solid line), figure 1(**appendix B),** they clearly seem to be characterized by leptokurtic distributions which is confirmed by the Jarque-Bera test of normality.

Table 4.6: General Summary Statistics for Arabica coffee and crude oil series

Statistics	Coffee market		Crude oil market	
	Pt	r_t	Pt	r_t
Mean	145.8668	0.0458	80.9327	0.0119
Median	132.7250	0	76.1950	0.1327
Maximum	299.3500	9.6800	144.4900	10.449
Minimum	88.2000	-7.538	28.9400	-15.451
Std.dev.	43.3636	2.229	26.1545	2.3639
Skewness	1.4120	0.0359	0.1347	-0.6890
Kurtosis	4.4545	3.6941	1.7781	7.2470
Jarque-Bera	144.8500 (0.0000)	8.2600 (0.0161)	488.0200 (0.0000)	107.4100 (0.0000)

**Values below the Jarque-Bera statistics are p-values at 5% of significance level

Looking at Table 4.6, the average prices for Arabica coffee (per Pound) and crude oil (per Barrel) were $145.8668 and $80.9327, respectively, over the entire period. The average log-returns were then computed and it is 0.0458 and 0.0119 for coffee and oil, respectively. The average returns for crude oil price were smaller than that of Arabica coffee price return in the sample period considered. One more impression here is that the two commodities return is positive, which is an implication of the increasing trend in the two commodity price. The estimate of the standard deviation is 2.229 for coffee and 2.3639 for oil which is nearly similar. Hence, we can say that they exhibit approximately similar variations in their prices.

4.3 Volatility Modeling

To build a volatility model for the log return series, the first step is specifying the mean equation. Once we specify the mean equation we have to test for ARCH effects using the residuals of the mean equation. If ARCH effects are statistically significant, specifying a volatility model and carrying out a joint estimation of the mean and volatility equations is necessary. The conditional mean specification is, in general, arbitrary for GARCH models of the conditional volatility. Various modifications to the conditional means in GARCH models are possible (see, for example, Asai and McAleer (2003).

4.3.1 Specification of Mean Equation

Based on two statistics, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) we have chosen the temporary mean equation to estimate the joint mean and variance parameters. In most applications, lower order ARMA models, say, ARMA (1, 1), ARMA (1, 2), ARMA (2, 1) and ARMA (2, 2) are used**.** **Table 4.7** gives the alternative ARMA models together with their corresponding AIC and BIC.

Table 4.7: ARMA model selection for the weekly log returns using AIC & BIC

Lag length		Returns			
		Arabica Coffee		Crude Oil	
P	Q	AIC	BIC	AIC	BIC
0	1	2587.192	2600.234	2551.377	2572.674
0	2	2581.203	2598.593	2553.267	2570.656
1	0	2587.093	2600.135	2515.398	2564.44
1	1	2588.146	2605.535	2553.22	2570.609
1	2	2578.419	2600.156	2553.64	2575.377
2	0	2582.588	2599.977	2553.274	2570.664
2	1	2577.982	2606.049	2552.285	2568.674
2	2	2579.96	2599.719	2546.107	2564.419

As can be seen in Table 4.7, the best in-sample results are achieved by ARMA (2, 2) for both commodity log return series. Therefore, ARMA (2, 2) is the best (having minimum AIC & BIC) conditional mean equation for both of the two cases. Hence, the ARMA (2, 2) model given by the following expression is used to model the conditional variance as mean equation of the log return series of prices:

$$r_t = \mu + \rho_1 r_{t-1} + \rho_2 r_{t-2} + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} \qquad (4.1)$$

Where;

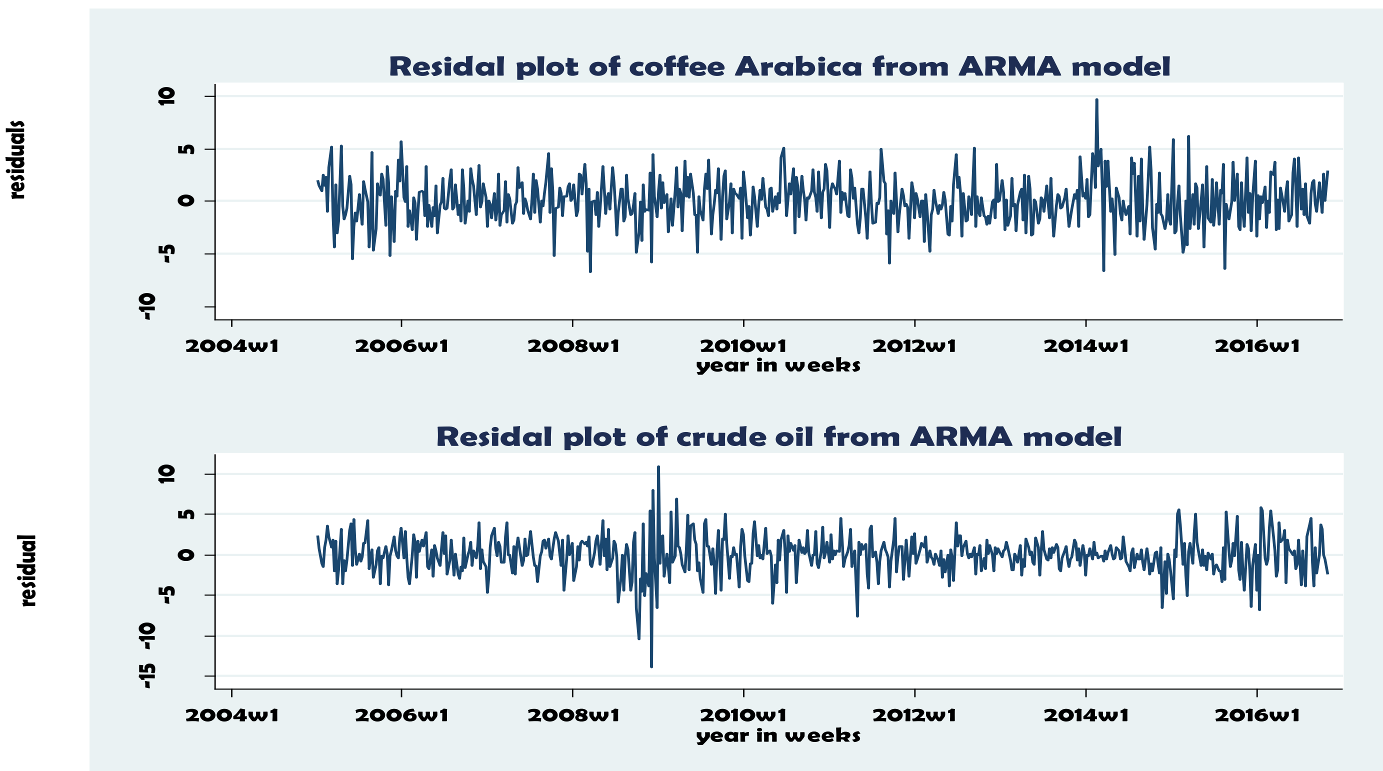
r_t –return series at time t, either of Arabica coffee or crude oil return, ARMA (2, 2) process

ε_t is an innovation term.

4.3.2 Test for ARCH Effects

Considering the above chosen financial returns model i.e. Equation (4.1) and on fitting this model, if there is no volatility clustering in each of the returns, the random disturbance term ε_t should be a white noise process. The standardized residual plot from Equation (4.1) can be an initial insight to judge the heteroskedastic characteristics of the error term. The standardized residual plot from each commodity return series, and is given below **(Figure4.5).** The figure depicts the residual plots of the two commodity return series generated from the mean equation .We see from the figure that for both of the series there is a prolonged period of low volatility and prolonged period of high volatility. For example for Arabica coffee return there is a long period of low volatility from the first week of 2005 to the end of 2008 and also there exist a long periods of high volatility from first week of 2014 to the end week of 2016. Crude oil return also exhibits a prolonged period of low volatility than the coffee series, which is from the first week of 2005 to the first week of 2009. In other words periods of high volatility are followed by periods of high volatility and periods of low volatility tend to be followed by periods of low volatility, which is known as volatility clustering. This suggests that the residuals or error terms are conditionally heteroskedastic and can be represented by GARCH models.

Figure 4.5: Residual plots of the Arabica coffee and crude oil return series from the mean



Based on the residuals from the mean equation chosen above, it is possible to test for the existence of ARCH effect which will allow continuing the analysis using GARCH model.

Table 4.8 shows the results of ARCH LM test for the two commodity returns. The last column of the table includes the p-values that indicate rejection of the null hypothesis that "there is no ARCH effect" up to the fourth lag at 5% level of significance. The results indicate that the two commodities price log return series are volatile and need to be modeled using GARCH models.

Table 4.8: ARCH LM Test Summary Statistics

Series	Obs*R^2	$\chi^2(4)$	Lag(p)	Prob.
Arabica coffee	12.6477	9.488	4	0.0040
Crude oil	105.6419	9.488	4	0.0000

4.3.3 GARCH Model Identification

If an ARCH effect is found to be significant, the PACFs are helpful to determine the order of an ARCH model. This is due to the expectation that is linearly related to in a manner similar to that of AR model. In practice, for ARCH modeling **a long lag** is often needed and this requires estimating a large number of parameters. To reduce the computational burden, a GARCH model with low lags can be helpful. This results in a more parsimonious representation of the conditional variance process.

To estimate and evaluate the forecasts of the competing GARCH models, different p and q values for the standard symmetric GARCH models are tested using different statistics such as Akaike Information Criterion (AIC) and Bayesian information Criterion (BIC) in order to choose the best model based on the in-sample data. The appropriate specification is chosen, and then for this specific p and q, the alternative GARCH models are estimated, tested and finally one GARCH model is chosen based on forecasting performance. For this GARCH (p, q)-model different distributions are evaluated. Namely, normal distribution, student's t-distribution and generalized error distribution (GED).Finally the parameters for the chosen GARCH model is presented. **Table 4.9** summarizes these two statistics computed from different GARCH models. Note that the AIC and BIC of the GARCH models are obtained by estimating the mean return and variance equations simultaneously.

Table 4.9: ARMA (2, 2)-GARCH model selection based on AIC and BIC

Arabica coffee, in sample output

Statistics	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH(2,2)
AIC	3290.974 (1)	3291.862 (2)	3292.442 (3)	3293.363 (4)
BIC	3308.363 (1)	3313.599 (2)	3314.179 (3)	3319.447 (4)
Rank Sum	2	4	6	8

Crude oil, in sample output

Statistics	GARCH(1,1)	GARCH(1,2)	GARCH(2,1)	GARCH(2,2)
AIC	3208.934 (4)	3204.773 (3)	3202.883 (2)	3200.631 (1)
BIC	3226.324 (2)	3226.510 (3)	3224.620 (1)	3226.716 (4)
Rank Sum	6	6	3	5

The row ranks indicate the ranking of the various GARCH models based on the two statistics. As a result, the lower the rank sum is the better the model. The best models for the two commodity series are:

- GARCH(1,1) for Arabica coffee
- GARCH(1,2) for crude oil

That is, the conditional variance processes are modeled by

$$\sigma_t^2 = \omega + a_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \qquad \text{GARCH (1, 1)}$$
$$\sigma_t^2 = \omega + a_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 + \beta_1\sigma_{t-2}^2 \qquad \text{GARCH (1, 2)}$$

4.4 Checking the adequacy of ARMA-GARCH models

In the above tables we have chosen the appropriate lag lengths for both commodity series, ARMA(2,2)-GARCH (1, 1) for the coffee return series and, ARMA(2,2)-GARCH(1,2) for the oil return series, and now it is necessary to check the adequacy of each of the model under the proposed distributions.

To check the adequacy of the mean equation, we test whether the standardized residual is serially correlated or not. The standardized residual is defined as:

$$\hat{v} = \frac{\hat{\varepsilon}_t}{\sqrt{h_t}}$$

If the mean equation is adequate, we expect the standardized residual term to be a white noise process. To check the adequacy of the variance equation, test whether the square of the standardized residual is serially correlated or not. If it is serially correlated, this would suggest that the conditional variance equation is not valid.

Figures 2, 3, 4 and 5 in **Appendix B** comprise the correlogram of the standardized and squared standardized residuals from the GARCH models. They show that the residuals are not autocorrelated for both Arabica coffee and crude oil returns. This fact must be confirmed by a statistical test, which is portmanteau test of white noise, and is given below in **Table 4.10.** All p-values in the table are greater than 0.05. So we cannot reject the null hypothesis of white noise of the error term. Hence all GARCH models under the three distribution assumptions are adequate.

Table 4.10: Portmanteau test for white noise, ARMA-GARCH model

	N-GARCH		T-GARCH		GED-GARCH	
Residuals	Coffee	Oil	Coffee	Oil	Coffee	Oil
Standardized	50.557 (0.1224)	63.56 (0.1306)	50.69 (0.1197)	43.903 (0.3096)	50.6165 (0.1215)	43.36 (0.329)
Squared stand.resid.	29.2764 (0.8944)	50.03 (0.396)	33.97 (0.7373)	41.848 (0.390)	30.5925 (0.858)	41.065 (0.423)

**values in parenthesis are p-values at 5% level of significance

4.4.1 Checking for normally distributed standardized errors

Once the selected model is adequate in modeling the volatility of a financial time series variable it is necessary to check the normality of the residuals. Below the q-q plot of the standardized residuals from each ARMA-GARCH models are given.

Figure 4.6: q-q plot, checking for normally distributed standardized errors, Arabica coffee series

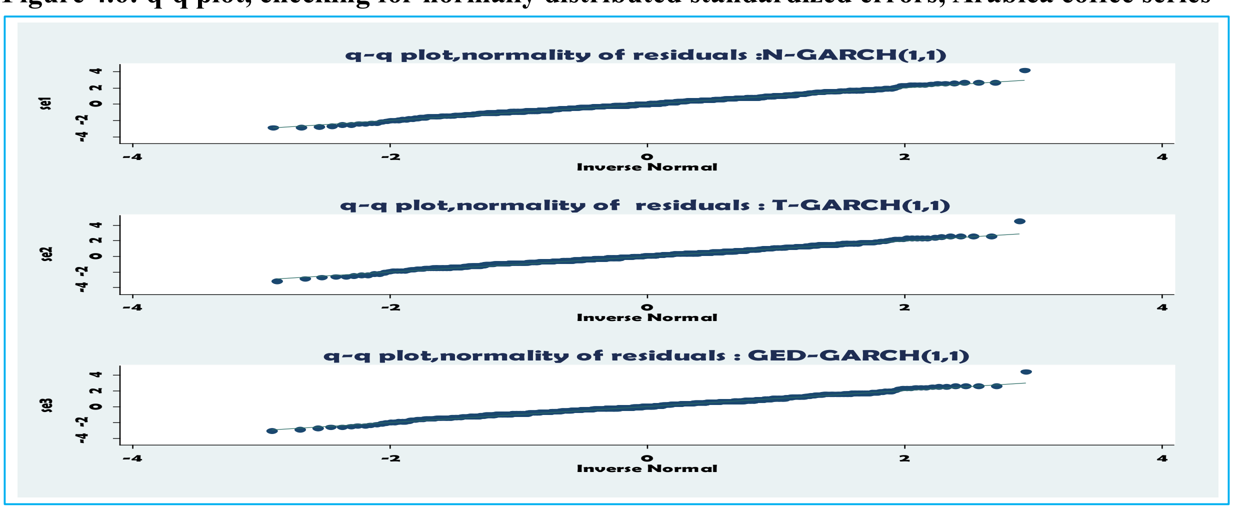
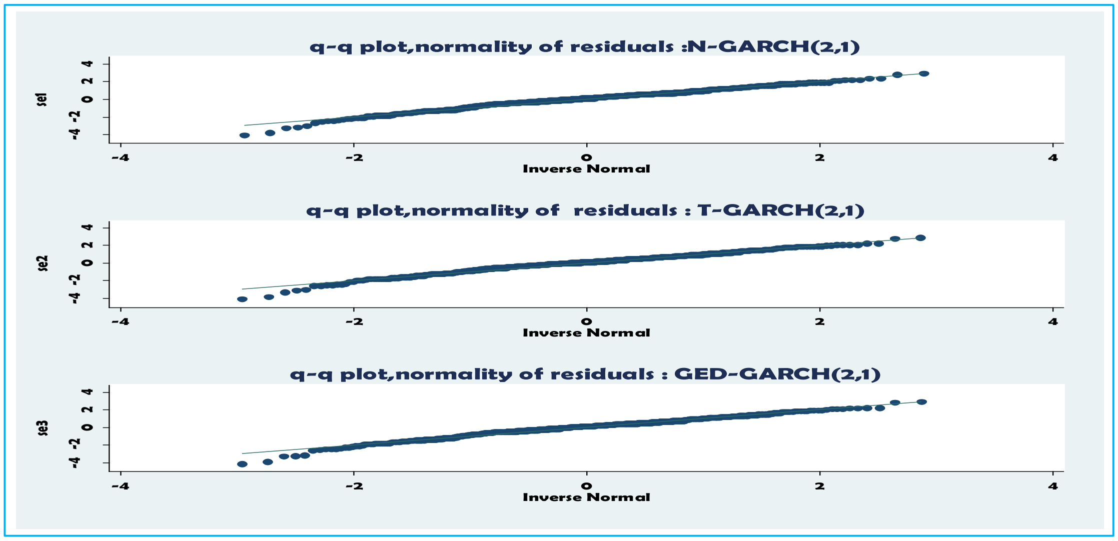


Figure 4.7: Checking for normally distributed standardized errors, Crude oil return series



In all of the plots it appears that we have approximately normally distributed standardized errors, except for deviations at the end tails. This indicates the normality of the residuals

4.4.2 Forecast comparison to realized volatility measures

Below in **Table 4.11** and **Table 4.12** we present different forecast accuracy measures. We compare the realized volatility measures by taking the weekly return square and a high frequency (30-minute intra-day) data as realized volatility measure. We use the mincer-zarnowitz regression to our estimates. If the estimated parameters in this regression are as expected, it means a close to zero and b close to one and R^2 is not lower than 30% from Equation(3.35), indicating that the differences between the compared estimates are not huge and if so both of them are good estimates (Mincer, 1969).

Table 4.11: Forecast error measures, ARMA (2,2)-GARCH (1, 1) models under different distribution and realized volatility, Arabica coffee return.

Statistics	Realized volatility					
	Weekly return square			High frequency		
	N-GARCH	T-GARCH	GED-GARCH	N-GARCH	T-GARCH	GED-GARCH
MZ.reg .R2	0.0667	0.0712	0.010	0.156	0.230	0.155
MSE	21.030	22.6411	21.641	18.44	18.139	18.1755
RMSE	4.586	4.758	4.651	4.2941	4.25894	4.26234
MAE	4.576	4.753	4.581	3.5013	3.553	3.497
AIC	2540.605	2535.28	2536.791	2540.605	2535.28	2536.791
LogL.	-1261.393	-1264.303	-1259.64	-1261.393	-1264.303	-1259.64

Table 4.12: Forecast error measures, ARMA (2,2)-GARCH (1,2) model under different distribution and realized variance, Crude oil return

Statistics	Realized volatility					
	Weekly return square			High frequency		
	N-GARCH	T-GARCH	GED-GARCH	N-GARCH	T-GARCH	GED-GARCH
MZ.reg .R2	0.016	0.022	0.011	0.1763	0.267	0.18
MSE	126.6363	125.6408	125.6408	67.985	64.512	64.56
RMSE	11.2532	11.2089	11.2039	8.2453	8.0319	8.0349
MAE	7.6397	7.5618	7.5817	7.9126	7.739	7.762
AIC	2460.264	2443.797	2443.877	2460.264	2443.797	2443.877
LogL.	-1222.132	-1213.938	-1212.898	-1222.132	-1213.938	-1212.898

Forecast error measures and R^2 values from the Mincer-zarnowitz regression presented in Table 4.11 & Table 4.12 imply that the realized volatility measure from high frequency data is a better measure of unobserved volatility than the commonly used squares of returns (R2 values are the lowest). This is also confirmed by the other forecast error measures having 44 one step ahead forecasts (2016w1-2016w44). The values of the **RMS** and **MAE** should get lower if the computing model is best performing. With regards to the distribution of the error term, the student's t-distribution combined with high frequency data is best performing for both series.

4.5 Estimations of the univariate GARCH

The values of the coefficients of the two commodities are shown below in **Table 4.13**.The estimation was done assuming that the error term comes from student's t-distribution with eight degree of freedom. From the table, it can be seen that some of the coefficients in both of the returns are not significant at 5% level of significance; for instance, the constant term is not significant in both cases.

Non-negativity rules: As we are modeling variance and the estimated variance must be non-negative, there is no easy rule that ensures non-negativity for the general GARCH(p, q) process .But Nelson &Cao(1992) give easy checkable condition for GARCH(1,q) model and (somewhat) manageable conditions for GARCH(2,q) model. The non-negativity condition for GARCH (1, 1) and GARCH(2,1) model are:

$$\sigma_t^2 = \omega + a_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \qquad \text{GARCH (1, 1)}$$

$$\sigma_t^2 = \omega + a_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 + \beta_1\sigma_{t-2}^2 \qquad \text{GARCH (1,2)}$$

For both of the models to be non-negative the following condition should be satisfied:

$\omega > 0, \quad a_1 \geq 0\ ,\ \beta_1 \geq 0\ , \beta_1 + \beta_2 < 1,$ and $\beta_2 \geq 0$

Looking the parameter estimates of ARCH and GARCH terms from the table below they satisfy the non-negativity rule described above for both commodity return series.

Table 4.13: parameter estimates from ARMA(2,2)-GARCH models

Returns	Coffee	Oil
Mean equation		
$\hat{\theta}_1$	-0.5158 (0.000)	-0.7636 (0.000)
$\hat{\theta}_2$	-0.4884 (0.000)	0.6585 (0.000)
$\hat{\rho}_1$	0.5645 (0.000)	0.780 (0.000)
$\hat{\rho}_2$	0.4362 (0.000)	-0.9325 (0.000)
Variance equation		
ω	0.1037 (0.38)	0.0317 (0.160)
$\hat{\alpha}$	0.0249 (0.0097)	0.0391 (0.002)
$\hat{\beta}_1$	0.9556 (0.000)	1.5861 (0.000)
$\hat{\beta}_2$		-0.6194 (0.000)
$\hat{\alpha} + \hat{\beta}$	0.9805	0.9988

***values in parenthesis are p-values at 5% level of significance*

4.6 Multivariate GARCH modeling

At the beginning of reviewing different formulations of MGARCH models, one should consider what specification of an MGARCH model should be imposed in contrast to the univariate case. On the one hand, it should be flexible enough to state the dynamics of the conditional variances and covariance. On the other hand, as the number of parameters in an MGARCH model increases rapidly along with the dimension of the model, the specification should be parsimonious to simplify the model estimation and also reach the purpose of easy interpretation of the model parameters. However, parsimony may reduce the number of parameters, in which situation the relevant dynamics in the covariance matrix cannot be captured.

A number of alternative MGARCH formulations for modeling long-run conditional heteroskedasticity are extensively used in relevant literature. The most widely used are the time varying conditional correlation (VCC), the constant conditional correlation (CCC) (Bollerslev, 1990), and the dynamic conditional correlation (DCC) models (Bollerslev, 1990). A more elaborate discussion of these models is presented in chapter three. One complexity of MGARCH models is that the number of parameters grow rapidly and researchers often use a single ARCH and GARCH term in the model specification ,and often it is enough in capturing the variation (Berben and Jansen ,2009).So in this paper we use bivariate-GARCH(1,1) models.

4.7 Model adequacy checking

Checking the adequacy of MGARCH models is essential in identifying whether a well specified MGARCH model can obtain reliable estimates and inference. Graphical diagnostics for MGARCH models can be done by examining plots of the sample autocorrelation (ACF).To ensure the inference from the estimated parameters in the MGARCH model is enough valid, the residuals should exhibit a set of white noise features like expected zero vector, no autocorrelations, constant variance, and normal distribution of the residuals. The autocorrelation functions for the squared process are shown to be useful in identifying and checking time series behavior in the conditional variance equation of the GARCH form.

4.7.1 Residual analysis

Starting by looking at the residuals they should be following a white noise process. The correlogram for the standardized and squared standardized residuals are given in **Figure**11,12,13 (**Appendix B).** They shows that the residuals are not auto correlated for both Arabica coffee and crude oil returns. By looking at the ACF graphs, it can be determined whether the processed data has any trace of correlation in the residuals. If more than 5 % of the lags in the autocorrelation plot falls outside the confidence interval, the residuals cannot be treated as white noise. In both the estimated CCC, VCC, and DCC models, about less than five out of 100 lags fall outside the confidence level. It can therefore be concluded that the residuals do not suffer from heteroscedasticity. Portmanteau test of white noise confirms the graphical view. It is given in Table 4.14. Since all *p*-values exceed 5 %, the null hypothesis that there is no autocorrelation is not rejected, meaning that the test confirms that the estimations have been successful.

Table 4.14: Portmanteau test of white noise residuals from VCC-GARCH model

Returns	Residuals	Q(52)	Prob.>chi(52)
Arabica coffee	Standardized residuals	59.07	0.2329
	Squared Standardized residuals	54.69	0.3726
Crude oil	Standardized residuals	61.69	0.1783
	Squared Standardized residuals	51.71	0.4851

4.7.2 The ARCH LM test of residuals

The results from the ARCH LM test of the return series (Table 4.8) indicate the presence of volatility clustering in the data. The next model diagnostic step is to determine whether there are still ARCH effects left in the residuals. If there still are unexplained effects left, this means that the chosen model suits the sample poorly since the return series suffer from heteroscedasticity according to the first ARCH LM test. As can be seen in **Equation 3.32**, a desirable outcome for the ARCH LM test of residuals is that all $a_i = 0$ and that it is not possible to reject H_0. **Table 3, 4 , 5, (appendix A)** contains the univariate Lagrange multiplier test and **Table 6, 7 , 8, (appendix A)** contains the multivariate Lagrange multiplier test, both approaches show insignificant *p*-values for all lags. The conclusion of the three all LM tests is that all models capture the volatility clustering successfully.

4.7.3 Normality of standardized residuals

GARCH estimates could be sensitive to the assumption of normally distributed errors. So it is important to check the extent to which this assumption is correct. Since the standardized errors are not directly observable and they have to be estimated, chi-squared based tests such as the skewness and kurtosis tests for normality are not strictly speaking applicable. Instead it is advisable to plot the quantiles of the standardized residuals against the quantiles of the standardized normal distribution (Q-Q plot) and check if they are similar (Sourafel, 2016).

Figure 4.8: Normality (q-q) plot of standardized residuals, VCC-GARCH model

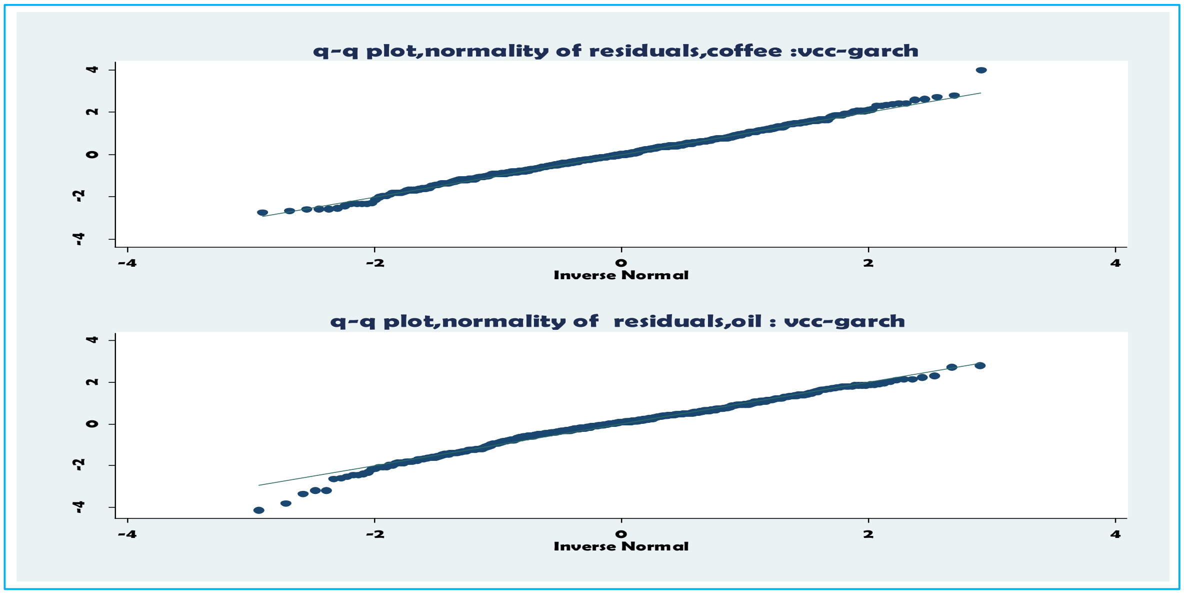


Figure 4.8 is the q-q plots of the standardized residuals from the selected VCC model, and the q-q plot of the residuals from CCC and DCC model are presented in Figure 14 and 15, **Appendix B**. In all of the plots it appears that we have approximately normally distributed standardized errors, except for deviations at the end tails.

One of the major objective of this study is choosing an appropriate multivariate GARCH model from the competing models. Using similar mechanism like the univariate GARCH model selection, that is, using the Mincer-zarnowitz regression and the forecast error measures, a model with minimum forecast error measure value and maximum MZ R^2 value is selected as an appropriate model. We can chose a single bi-variate GARCH model, and then estimate the parameters of the selected model. For the distribution of the innovation term, the multivariate normal distribution and the multivariate version of student's t-distribution with eight degree of freedom are considered. Like the previous analysis, we also consider two volatility measures, that is, the weekly return square and the 30-minute intra-day returns square.

Table 4.15: Bi-variate GARCH model comparison under normal distribution for co-volatility of coffee and oil returns

Statistics	Realized volatility measure					
	Weekly returns squared			30-minutes interval returns based measured (high freq.)		
	CCC	DCC	VCC	CCC	DCC	VCC
MZ.reg .R2	0.0145	0.0037	0.0070	0.2575	0.1740	0.3929
MSE	29.2503	29.9553	29.313629	0.7865	0.9161	0.8590
RMSE	5.1442	5.4730	5.4083	0.8868	0.9571	0.9268
MAE	3.7900	3.7452	3.7307	0.879	0.8517	0.8000
AIC	4965.981	4957.583	4962.655	4965.981	4962.655	4957.583
BIC	5022.474	5022.768	5027.84	5022.474	5022.768	5027.84
LL	-2469.99	-2463.79	-2466.32	-2469.99	-2463.79	-2466.32

Table 4.16: Bi-variate GARCH model comparison under student's t-distribution for co-volatility of coffee and oil returns

Statistics	Realized volatility measure					
	Weekly return square			High frequency		
	CCC	DCC	VCC	CCC	DCC	VCC
MZ.reg .R2	0.0937	0.0747	0.060	0.2372	0.1614	0.4014
MSE	29.36	29.99	29.28	0.7876	0.8877	0.8634
RMSE	5.4184	5.4763	5.4110	0.8874	0.9421	0.9291
MAE	3.7953	3.7532	3.742	0.8819	0.8423	0.8372
AIC	4945.74	4940.492	4943.54	4945.74	4943.54	4940.492
BIC	5002.24	5005.677	5008.725	5002.24	5005.677	5008.725
LL	-2459.84	-2455.24	-2456.77	-2459.84	-2455.24	-2456.77

Looking at **Table 4.15** and **Table 4.16** for each of the models, the R^2 obtained from mincer-zarnowitz regression (R^2) value is larger for high frequency data. Moreover, the R^2 is larger under student t-distribution assumption of the innovation term. Now combining the high frequency data and student t-distribution, the time-varying conditional correlation (VCC) model is better model for the variance covariance estimation of the Arabica coffee return and crude oil returns.

Table 4.17: parameter estimates from time varying conditional correlation (VCC-GARCH) model under t-distribution (v=8)

Parameters	Coefficients	z-statistics	p-value
Mean equation			
$\hat{\varphi}_{coffee}$	0.0515	0.58	0.564
$\hat{\varphi}_{oil}$	0.4915	0.64	0.521
$\hat{\theta}_{coffee}$	0.0634	1.46	0.144
$\hat{\theta}_{oil}$	0.0372	0.84	0.401
$\hat{\theta}_{coffee,oil}$	-0.0005	-0.01	0.992
$\hat{\theta}_{oil,coffee}$	-0.0099	-0.30	0.760
Variance equation			
$\hat{\omega}_{coffee}$	0.1278	1.03	0.302
$\hat{\alpha}_{coffee}$	0.0264	1.66	0.097
$\hat{\beta}_{coffee}$	0.9501	28.33	0.000
$\hat{\omega}_{oil}$	0.0894	1.62	0.105
$\hat{\alpha}_{oil}$	0.0783	3.57	0.000
$\hat{\beta}_{oil}$	0.9067	36.70	0.000
$\hat{\rho}_{coffee,oil}$	0.285	4.14	0.000
$\hat{\pi}_1$	0.0320	1.76	0.031
$\hat{\pi}_2$	0.9125	15.38	0.000
$\hat{\alpha}_{coff.} + \hat{\beta}_{coff.}$	0.985	-	-
$\hat{\alpha}_{oil} + \hat{\beta}_{oil}$	0.9785	-	-

Table 4.17 presents the parameter estimates from the VCC-GARCH (1,1) model with single ARCH and single GARCH term. In the VCC model specified in Equation (3.31), the mean equation contains the following parameters:

$$\Theta=\begin{pmatrix}\theta_{11} & \theta_{12}\\ \theta_{21} & \theta_{22}\end{pmatrix} \equiv \begin{pmatrix}\theta_{coffee} & \theta\ _{coffee,oil}\\ \theta_{oil,coffee} & \theta_{oi}\end{pmatrix}$$

θ_{Coffee} and θ_{oil} are the autoregressive parameters of order one for coffee and oil returns, respectively, and $\theta_{coffee,\ oil,}$ measures the returns spillover from oil market to coffee market, and also $\theta_{oil\ coffee}$ measures the returns spillover from coffee market to oil market. . If both spillover parameters are significant, there is a bidirectional spillover effect between the asset prices. But from the table, the estimated of coefficients from the mean equations for coffee and oil prices were not significant at 5% level, implying that the current returns of oil price did not explain the immediate past returns of oil price and also the reverse is true. If these mean return parameters are not significant, refitting the variance equation by excluding the irrelevant expression makes sense. The variance parameters excluding the mean estimate are presented in **Table 4.18.**

Table 4.18: variance parameter estimates from time varying conditional correlation (VCC-GARCH) model under t-distribution (v=8)

Parameters	Coefficients	z-statistics	p-value
$\widehat{\omega}_{coffee}$	0.1647	1.04	0.300
$\hat{\alpha}_{coffee}$	0.0305	1.60	0.109
$\hat{\beta}_{coffee}$	0.9393	22.16	0.000
$\widehat{\omega}_{oil}$	0.0835	1.60	0.109
$\hat{\alpha}_{oil}$	0.0774	3.60	0.000
$\hat{\beta}_{oil}$	0.9092	38.48	0.000
$\hat{\rho}_{coffee,oil}$	0.2886	1.60	0.000
$\hat{\pi}_1$	0.02919	1.70	0.034
$\hat{\pi}_2$	0.919	15.88	0.000
$\hat{\alpha}_{coff.} + \hat{\beta}_{coff.}$	0.9698	-	-
$\hat{\alpha}_{oil} + \hat{\beta}_{oil}$	0.9866	-	-

The results in Table 4.18 of the variance equations actually indicated volatility effects in the returns series across the sample period considered. With regard to the estimates of the time varying correlation, ($\hat{\rho}_{coffee,oil}$) the correlation was found to be positive and significant. This implies positive relationship between the conditional volatility series of the two commodity series and indicates that the returns on these commodities rise or fall together.

The estimates of the parameters $\hat{\pi}_1$ and $\hat{\pi}_2$, that is the parameters dealing with the effects of previous shocks and previous dynamic conditional correlations on the current dynamic conditional correlation, were significantly different from zero, with $\hat{\pi}_1$=0.0291 and $\hat{\pi}_2$=0.919.Time varying conditional correlation(VCC) model reduces to constant conditional correlation (CCC) model when $\hat{\pi}_{1=}\hat{\pi}_2 = 0$; (Bauwens, Hafner and Laurent, 2012). Hence, employing CCC is too restrictive for these series.

Concerning the estimates of persistence ($\hat{\alpha}_i + \hat{\beta}_{i.}$), the persistence estimates were 0.9698 and 0.9866 at coffee and oil markets, respectively which is closer to one, implying that volatility is persistent in these markets. The sufficient condition for the variance matrix to be positive definite are the usual GARCH restrictions i.e. $\omega, \alpha, \beta > 0$ for the univariate GARCH (1,1) model. Also, this univariate GARCH process needs to be stationary i.e. $1 - \alpha - \beta$>0 .These conditions are satisfied in the above output table.

4.8 Discussion

The aim has been modeling and forecasting the co-volatility dynamics of weekly time series of **Brent crude oil** and **coffee Arabica** futures market closing price using bi-variate GARCH models. From the preliminary analysis over the time period considered, both of the price series show an increasing trend. To determine whether the series are stationary or not, the Augmented Dickey-Fuller (ADF) test was carried out. Often, raw data of commodity prices are non-stationary, which was also the case in this study .For both level time series, the tests indicates to the existence of unit root I(1). The first log difference of each time series was considered as stationary (Table 4.3).

There are some basic features to be fulfilled with the analysis of time series data in addition to stationarity. One main feature is that, the distribution of financial log return series show high skewness and kurtosis compared to the normal distribution, which is a hint of presence of volatility in the time series data. This was true for the two commodity returns (**Table 4.5**). Another characteristic of a stationary time series data is the behavior of autocorrelation in the data. Usually, log return series show no or little autocorrelation but the squares of the log return series exhibit high autocorrelation. Having autocorrelation in the squared log returns helps to use the concept of heteroscedasticity embedded in a GARCH model. As shown in **Figure 4.4**, squared log return series of both commodities are strongly auto correlated which is an indication of heteroscedasticity in the log returns.

As a reference point for the analysis, a standard univariate GARCH models were fitted with ARMA(2,2) mean equation, and it is found that the variance equation for Arabica coffee and crude oil returns were GARCH(1,1) and GARCH(1,2) ,respectively.

In the two commodity returns it appears that the ARCH term coefficients were significant at 5% level of significance. The result is an implication of the presence of volatility clustering, i.e. large changes followed by large changes and small changes followed by small changes. In the case of Arabica coffee our result go in line with the Hansen and Lunde (2005) findings. They conclude that GARCH (1,1) model with high frequency data is enough to capture the volatility of exchange rates return volatility, but in the case of crude oil our finding is something different.

Noemi and Junior (2011) investigate the volatility of two tropical commodities i.e. world coffee and crude oil, and found that the two commodities were volatile, and hence our finding is in line with their analysis. They also conclude that fluctuations in coffee and cocoa price follow the oil price fluctuation. Our results shows that Arabica coffee and crude oil volatilities are correlated (in fact, not highly). In addition the co-variance of Arabica coffee and crude oil is found to be similar to the variance of crude oil indicates that the variation in the Arabica coffee price follows crude oil price, a similar finding with Noemi and Junior.

This study suggests that using high frequency data results in a more accurate forecast values in both the standard univariate GARCH and multivariate GARCH models. This result agrees with David G. McMillan and Alan E. H. Speight (2010) inference in the context of exchange rate returns. In fact, they consider a daily GARCH (1,1) model and 5-minute intra-day return. In addition to the usual 'ex post' squared returns measure, they also construct realized volatility and two versions of bias-corrected realized volatility. In order to evaluate these forecasts they utilize the Mincer–Zarnowitz regression test of predictive power, comparing forecasting performance on the basis of the R^2 values obtained. In our analysis the procedure is similar, but we use a weekly return series and a 30-minute interval intra-day return series and we stretch their conclusion.

Over all, the positive relationship between these commodity returns is consistent with previous researchers on energy and commodity co-volatility (see for example, Choi and Hammoudeh (2010); Du *et al.* (2011) ; Mensi *et al.* (2013).

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

Forecasting the volatility dynamics of asset returns has been the subject of extensive research among academics, practitioners and portfolio managers. This thesis estimates a variety of Multivariate GARCH models using weekly closing price (in USD/barrel) of Brent crude oil and weekly closing prices (in USD/per pound) of coffee Arabica, and compares the forecasting performance of these models based on a high frequency intra-day data which allows for a more precise realized volatility measurement. The analysis points to the conclusion that varying conditional correlation (VCC) model with Student's t distributed innovation terms is the most accurate volatility forecasting model in the context of our empirical setting.

We recommend and encourage future researchers studying the forecasting performance of MGARCH models to pay particular attention to the measurement of realized volatility, and employ high-frequency data whenever feasible.

We also recommend that public policy makers interested in foreseeing the price volatility of these two major commodities in the context of the Ethiopian economy consider using the information documented in this study as input in their deliberations given that it is based on some robust econometric work and highly appropriate data.

The scope of the analysis in this study has been limited to the co-volatility between the two commodities. In order to overcome this limitation and provide a more nuanced analysis, it might be profitable for future researchers to consider incorporating stock, currency and bond markets volatilities into the analysis. The potential complexity of such a research agenda notwithstanding, the results are likely to be rewarding in light of the deeper integration between global financial and commodity markets in recent years, a phenomenon which came to be known as the financialization of commodities.

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APPENDIX

Appendix A: Tables

Table 1: Parameter estimates from DCC model

Parameters	Coefficients	z-statistics	p-value
Mean equation			
$\hat{\varphi}_{coffee}$	0.0517	0.0.58	0.561
$\hat{\varphi}_{oil}$	0.0573	0.75	0.455
$\hat{\theta}_{coffee}$	0.0507	1.19	0.232
$\hat{\theta}_{oil}$	0.0397	-0.91	0.0.362
$\hat{\theta}_{coffee,oil}$	-0.0066	-0.17	0.865
$\hat{\theta}_{oil,coffee}$	-0.0138	-0.42	0.971
Variance equation			
$\hat{\omega}_{coffee}$	0.1318	1.02	0.309
$\hat{\alpha}_{coffee}$	0.0158	1.59	0.112
$\hat{\beta}_{coffee}$	0.950	27.28	0.000
$\hat{\omega}_{oil}$	0.0871	1.58	0.113
$\hat{\alpha}_{oil}$	0.0806	3.62	0.000
$\hat{\beta}_{oil}$	0.9055	36.65	0.000
$\hat{\rho}_{coffee,oil}$	0.24	2.27	0.023
$\hat{\pi}_1$	0.036	2.53	0.011
$\hat{\pi}_2$	0.9379	41.23	0.000
$\hat{\alpha}_{coff.} + \hat{\beta}_{coff.}$	0.9658	-	-
$\hat{\alpha}_{oil} + \hat{\beta}_{oil}$	0.9861	-	-

Table 2: Parameter estimates from CCC model

Parameters	coefficients	z-statistics	p-value
Mean equation			
$\hat{\varphi}_{coffee}$	0.0304	0.34	0.735
$\hat{\varphi}_{oil}$	0.0573	0.75	0.456
$\hat{\theta}_{coffee}$	0.0535	1.23	0.219
$\hat{\theta}_{oil}$	0.02578	0.60	0.550
$\hat{\theta}_{coffee,oil}$	-0.01724	-0.42	0.671
$\hat{\theta}_{oil,coffee}$	-0.0155	-0.47	0.637
Variance equation			
$\hat{\omega}_{coffee}$	0.1540	1.02	0.45
$\hat{\alpha}_{coffee}$	0.0270	1.54	0.123
$\hat{\beta}_{coffee}$	0.9446	23.68	0.000
$\hat{\omega}_{oil}$	0.0944	1.62	0.105
$\hat{\alpha}_{oil}$	0.0761	3.48	0.0000
$\hat{\beta}_{oil}$	0.9072	35.36	0.000
$\hat{\rho}_{coffee,oil}$	0.29	6.09	0.000
$\hat{\alpha}_{coff.} + \hat{\beta}_{coff.}$	0.9716	-	-
$\hat{\alpha}_{oil} + \hat{\beta}_{oil}$	0.9833	-	-

Table 3: ARCH LM test for CCC

	Arabica coffee		Crude oil	
Lags (p)	LM	p-value	LM	p-value
1	0.12	0.7287	0.058	0.8092
2	0.25	0.8830	0.102	0.9505
3	0.73	0.8660	0.123	0.9889
4	1.02	0.1200	0.125	0.9982

Table 4: ARCH LM test for DCC

	Arabica coffee		Crude oil	
Lags (p)	LM	p-value	LM	p-value
1	0.11	0.73 4	0.078	0.7803
2	0.26	0.878	0.115	0.9442
3	0.69	0.875	0.133	0.9876
4	1.05	0.13 0	0.134	0.9978

Table 5: ARCH LM test for CCC

	Arabica coffee		Crude oil	
Lags(p)	LM	p-value	LM	p-value
1	0.107	0.744	0.077	0.78
2	0.238	0.889	0.114	0.9446
3	0.637	0.888	0.132	0.9877
4	1.44	0.100	0.133	0.9979

Table 6: multivariate LM test for , CCC-GARCH

Lags(p)	LM	df	p-value
1	5.2758	4	0.2601
2	1.4475	4	0.8359

Table 7: multivariate LM test for, DCC-GARCH

Lags(p)	LM	Df	p-value
1	5.6559	4	0.22636
2	1.4848	4	0.82934

Table 8: multivariate LM test for, VCC-GARCH

Lags(p)	LM	Df	p-value
1	5.0033	4	0.2870
2	1.4710	4	0.8318

Appendix B: Graphs

Figure 1: Histograms of Arabica coffee and crude oil return series, normality checking

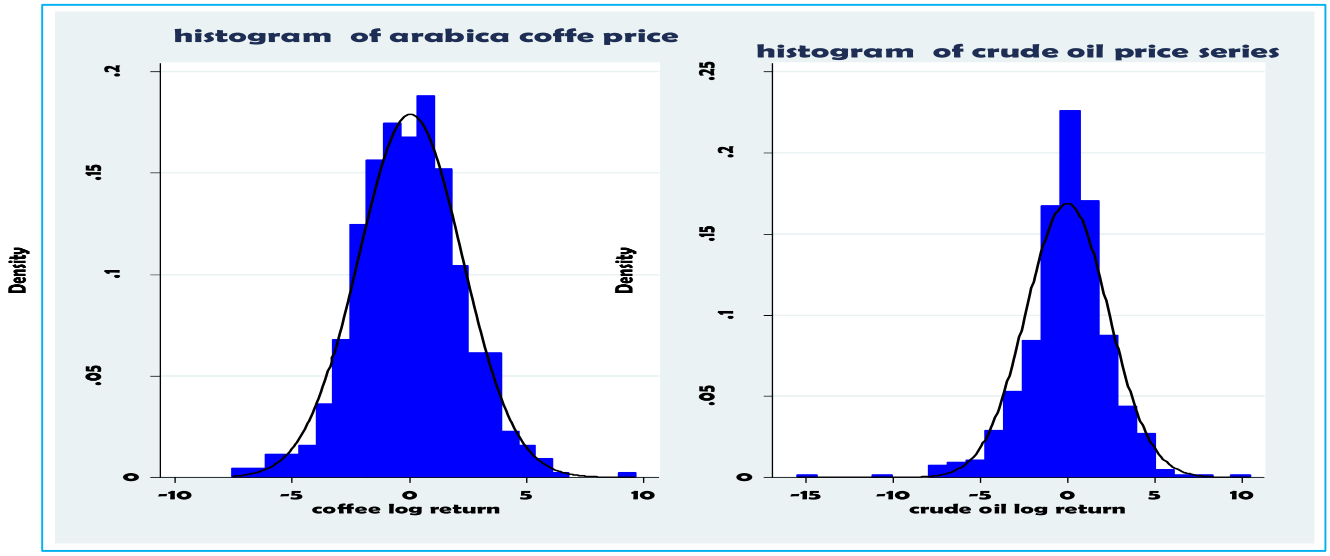

Figure 2: Correlogram of standardized residuals,coffee return

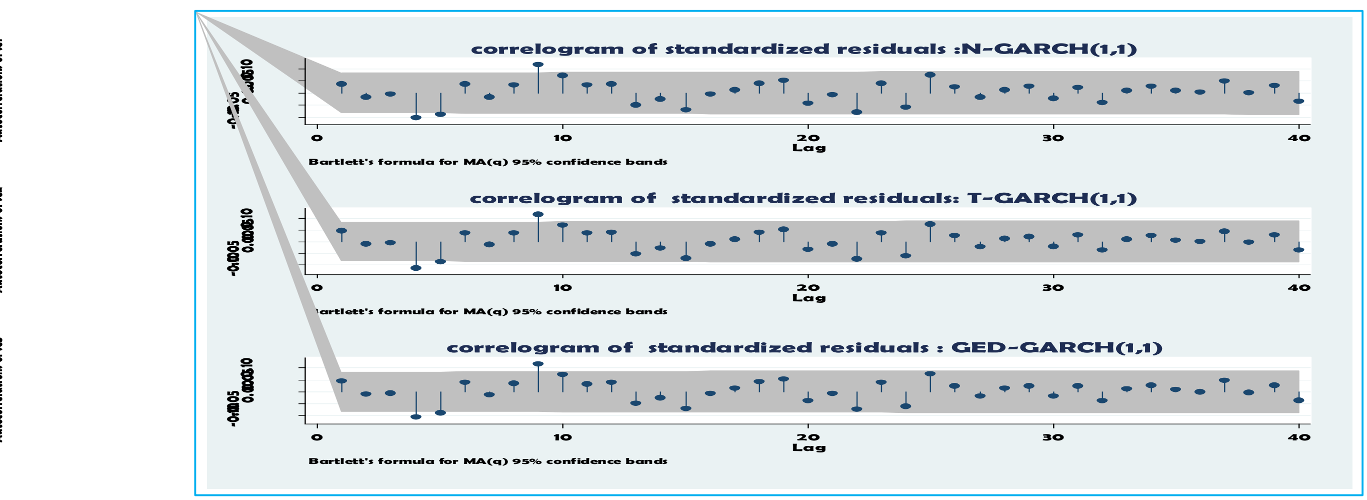

Figure 3 : Correlogram of squared standardized residuals,coffee return

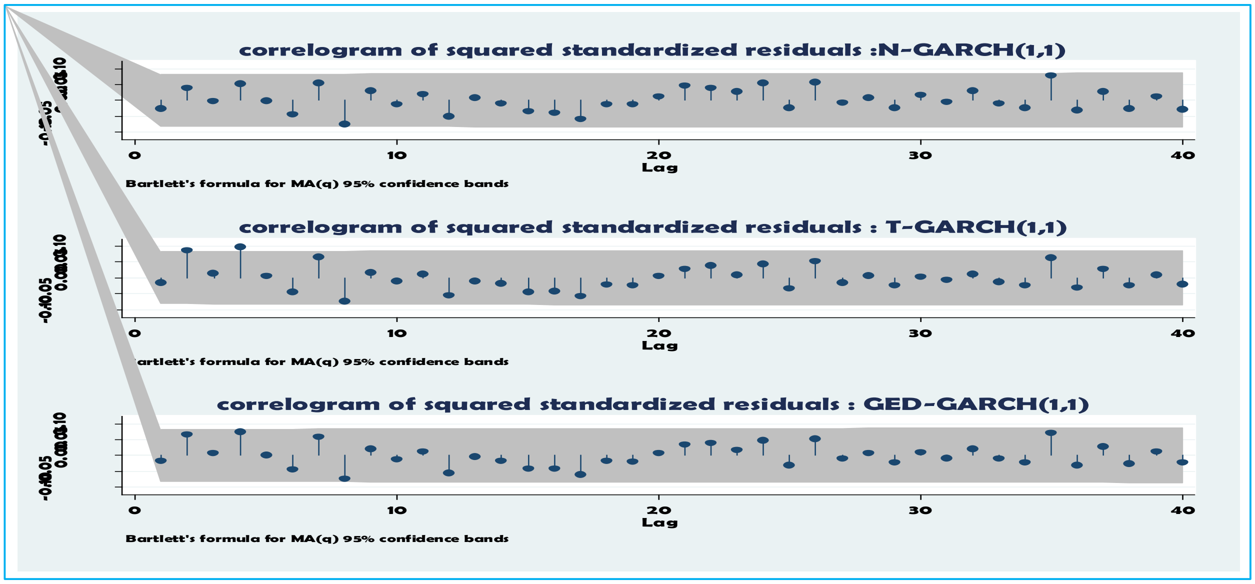

Figure 4 :Correlogram of standardized residuals,oil return

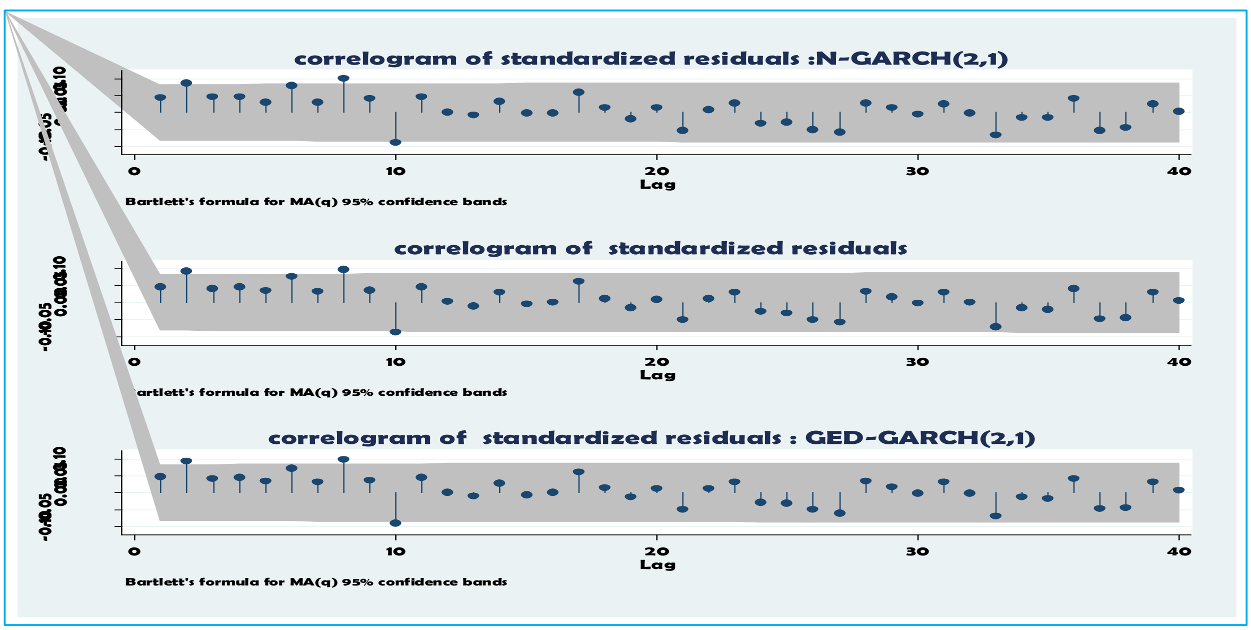

Figure 5: Correlogram of standardized residuals,oil return

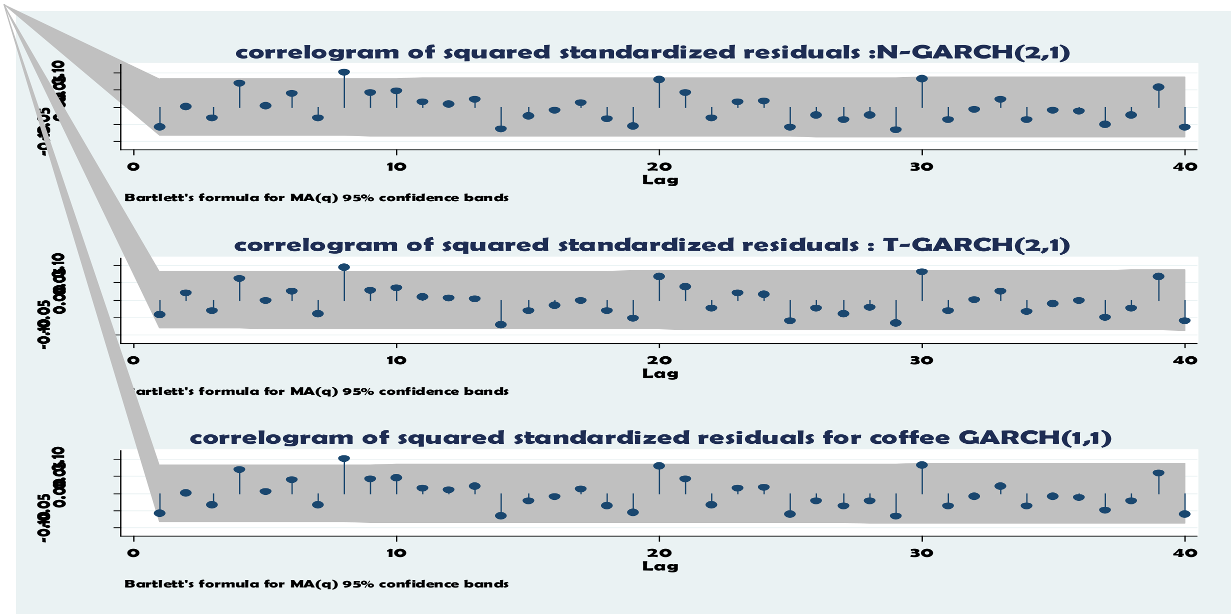

Figure 6:Variance and covariance prediction from GARCH (1,1),Coffee return

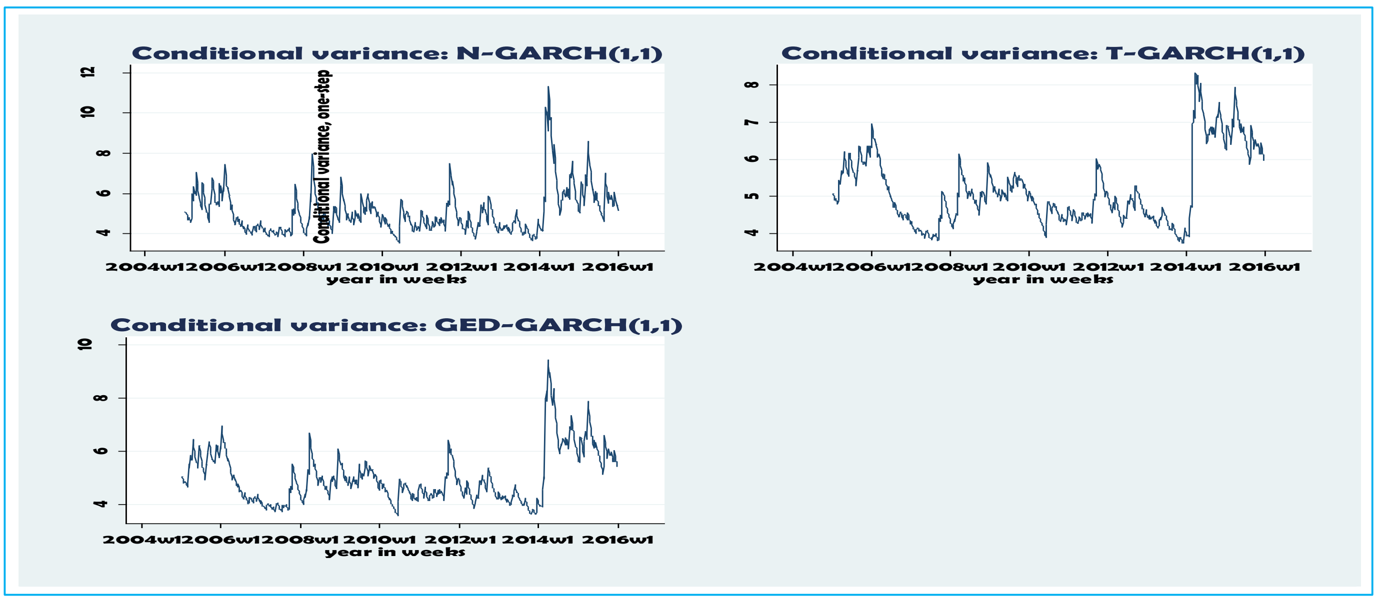

Figure 7: variance and covariance predictions from GARCH (2,1) ,crude oil return

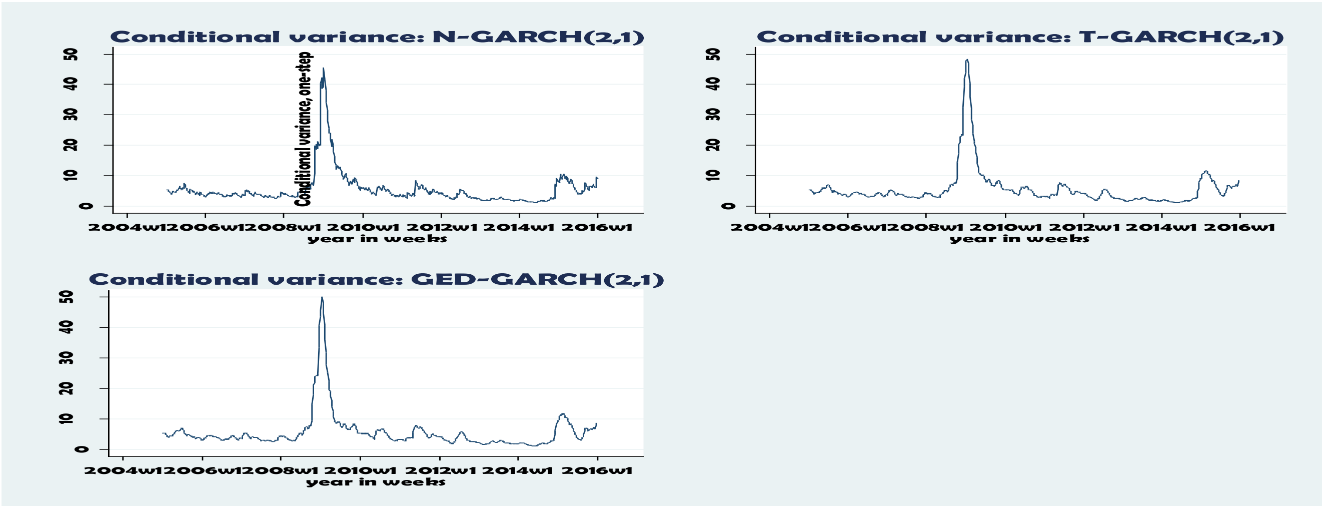

Figure 8: variance and covariance prediction from VCC-GARCH

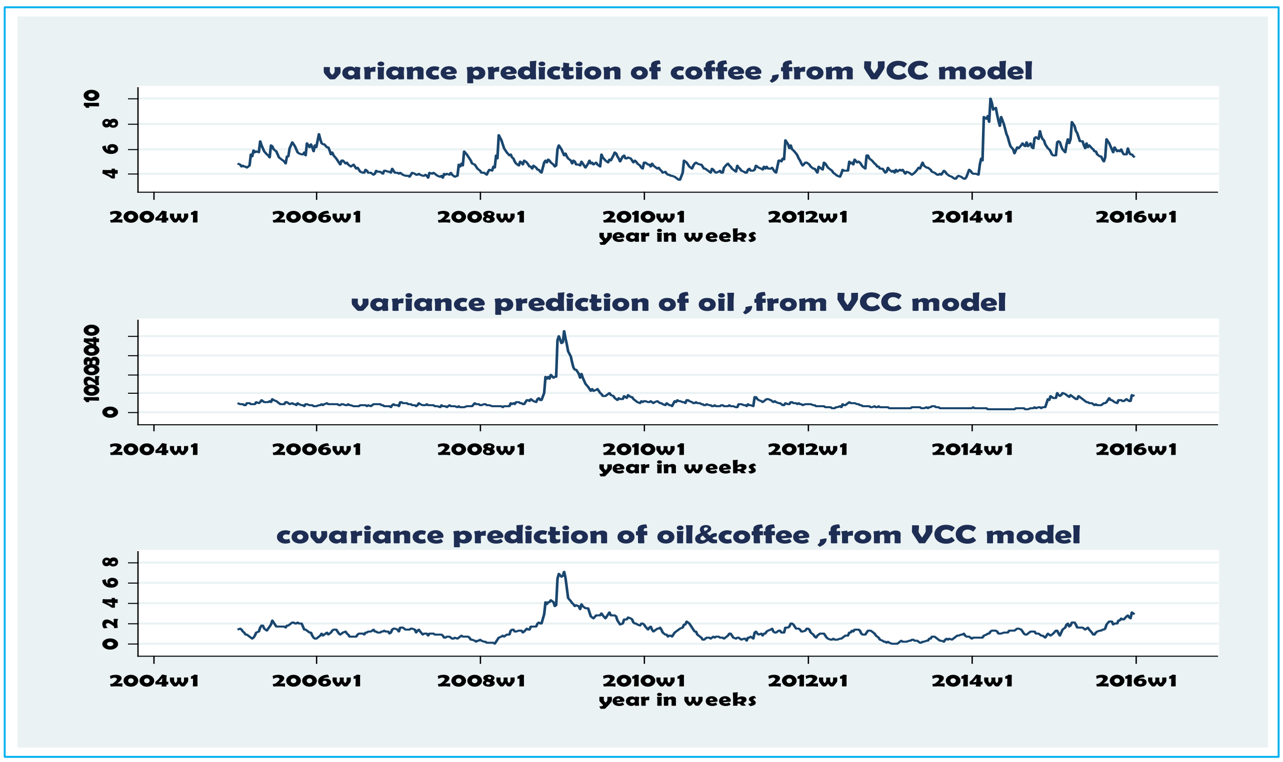

Figure 9: variance and covariance prediction from CCC-GARCH

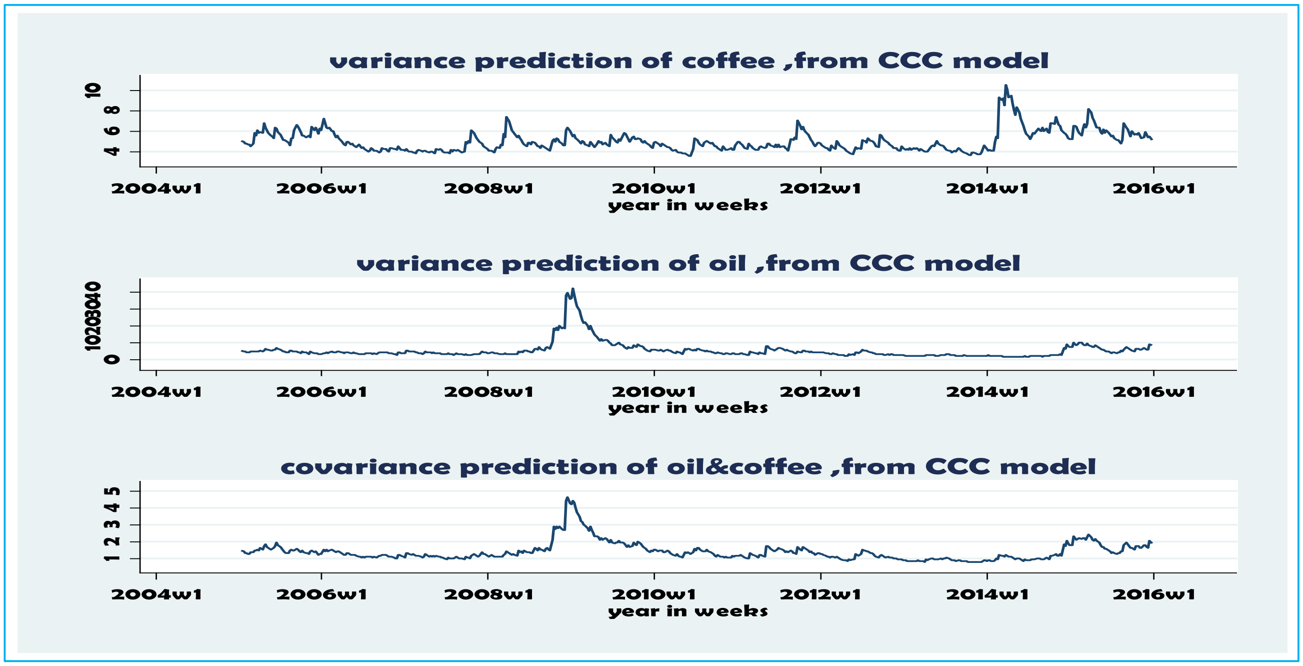

Figure 10: variance and covariance prediction from DCC-GARCH

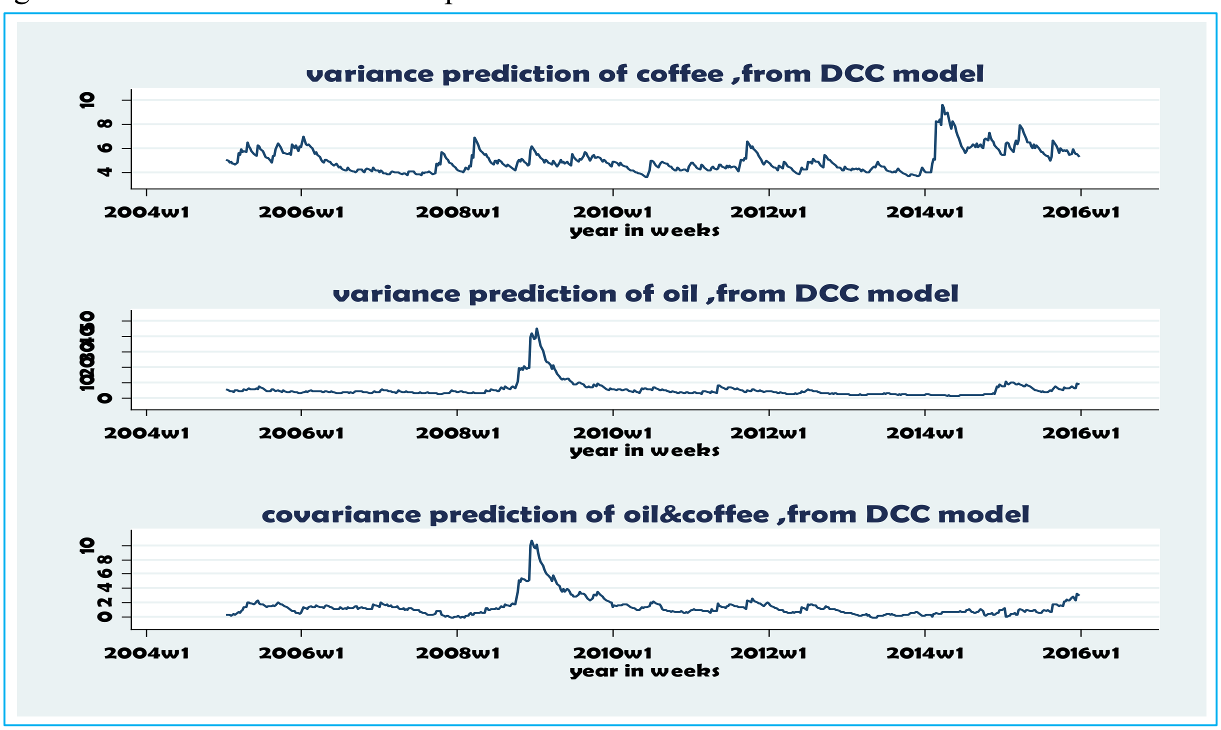

Figure 11: correlogram of squared residuals from VCC-GARCH

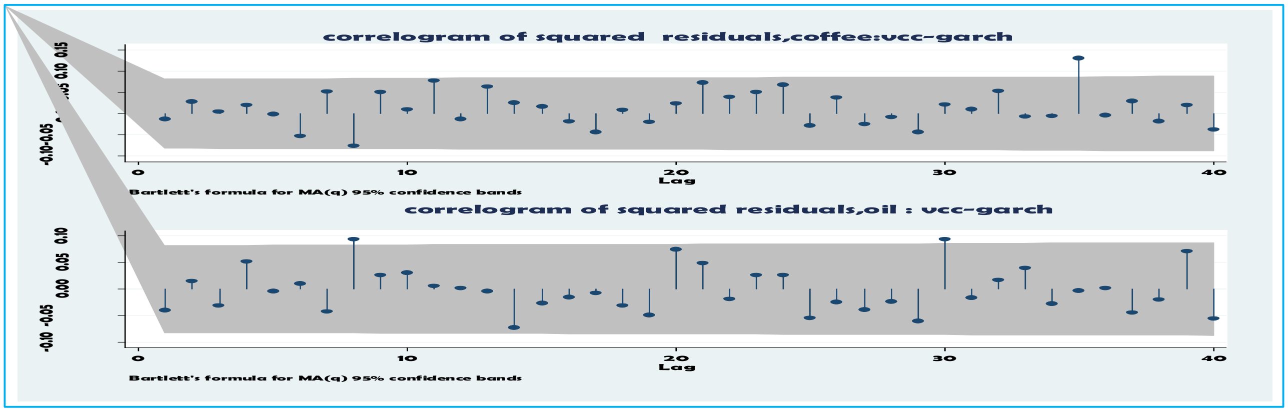

Figure 12: correlogram of squared residuals from CCC-GARCH

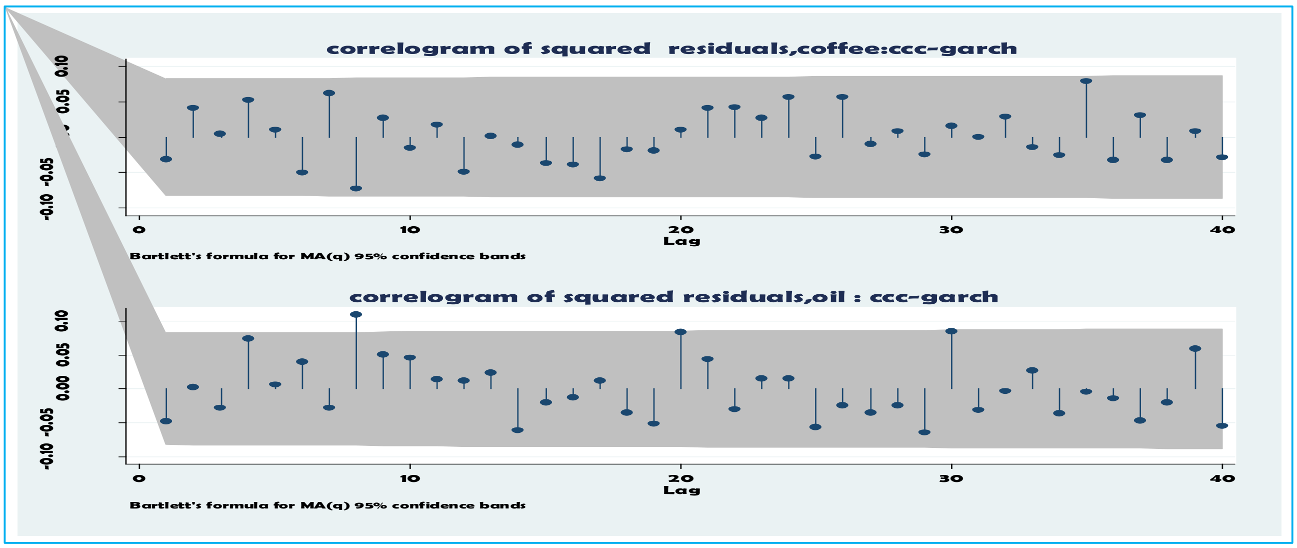

Figure 13: correlogram of squared residuals from DCC-GARCH

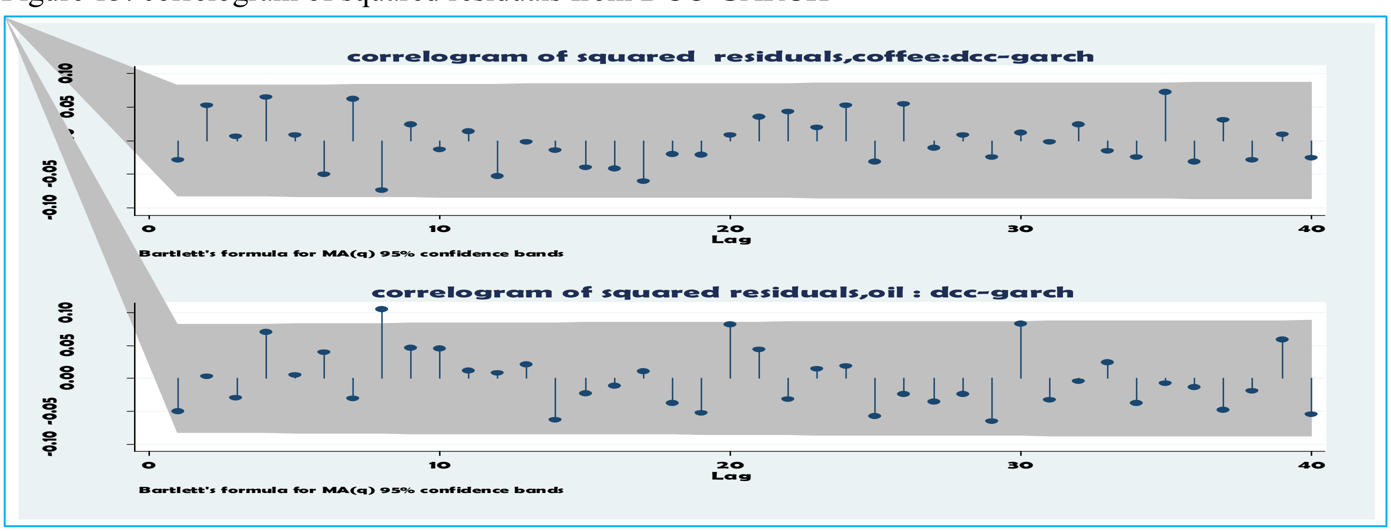

Figure 14: normality checking from the CCC-GARCH

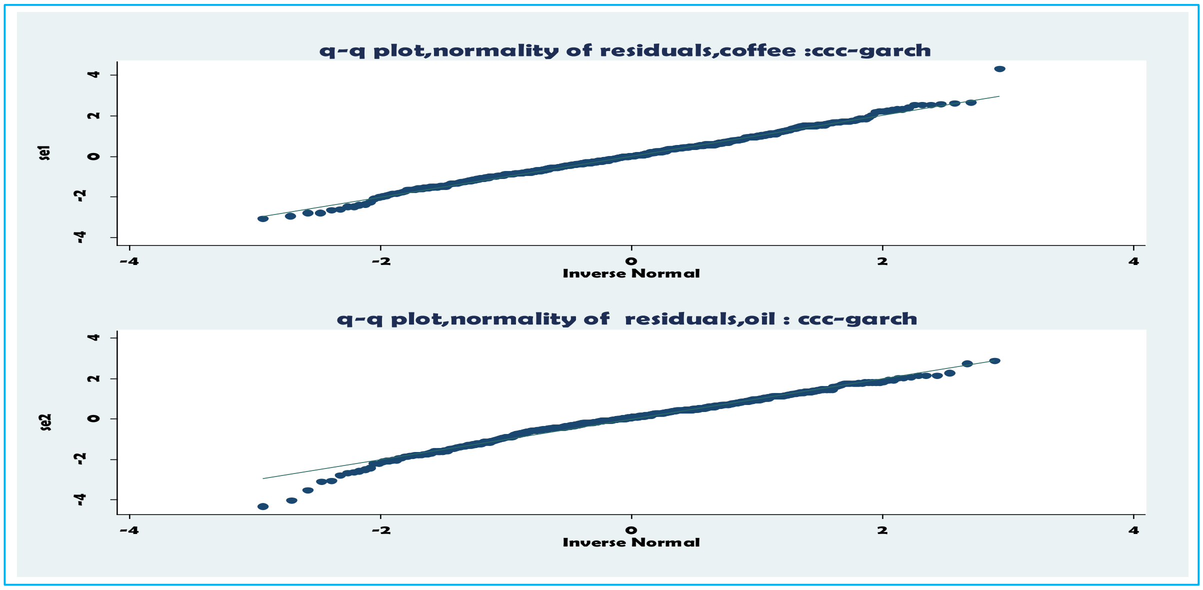

Figure 15: normality checking from the DCC-GARCH