TUNABLE NEGATIVE REFRACTIVE INDEX IN A METAMATERIAL COMPOSED OF MAGNETIZED PLASMA AND FERROMAGNETIC GRAINS

By

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The undersigned hereby certify that they have read and recommended to the Graduate Program of Addis Ababa University for acceptance a graduate project entitled "**Tunable Negative Refractive Index in a Metamaterial Composed of Magnetized Plasma and Ferromagnetic Grains**" by **Yeshi Worku** in partial fulfillment of the requirements for the degree of **Master of Science**.

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DECLARATION

I, the undersigned, certify that all material in this M.Sc. project which is not my own work has been identified and duly acknowledged, and that no material has previously been submitted and approved for the award of a degree by this or any other University.

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This M.Sc. project has been submitted for examination with my approval as university research supervisor.

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This MSc work is dedicated with great affection to

Shiferaw Worku

and

Nigussie Kefeni

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Abstract

Electromagnetic waves (EMWs) propagating in magnetized plasma with ferromagnetic grains (MPFG) in a direction perpendicular to the applied constant magnetic field are investigated theoretically and numerically. The MPFG system consists of conventional magnetized electron-ion plasma and ferromagnetic grains that are embedded uniformly in the plasma. The magnetized plasma forms the electric subsystem while the ferrite grains forms the magnetic subsystem. Being an anisotropic medium, the permittivity, ϵ, and permeability, μ, of the MPFG are second-rank tensors. For certain frequency domain, in the vicinity of the electron cyclotron frequency, both ϵ and μ can be simultaneously negative and the corresponding refractive index $n = \pm\sqrt{\epsilon\mu}$ becomes negative, i.e., the MPFG system behaves as left-handed medium (LHM). Moreover, by tuning the material parameters of the electric and magnetic subsystems, the different regions, namely, the left-handed medium, right-handed medium, and nontransparent frequency domains can be varied. The analysis and results obtained show that the MPFG medium possesses 'tunable' negative index of refraction near the electron cyclotron frequency enabling it to be a left-handed metmaterial. With the current technological advance in the field of plasma system, the MPFG medium may be fabricated in a laboratory, and hence makes the medium a potential candidate for practical applications that employs LHM.

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Chapter 1

Introduction

The electrodynamics of media with simultaneously negative values of the electric permittivity and magnetic permeability is theoretical extensively investigated in 1968 by V.G. Vesalago [1]. His study showed that, EMWs propagating in these media with index of refraction $n = \pm\sqrt{\epsilon\mu}$ is to be negative and referred to as left-handed media (LHM) or negative index media (NIM). The Maxwell equation used in convectional media is not affected by the double negative value of the two parameters ϵ and μ. The LHM and the associated peculiar properties are predicated through the application of Maxwell's equation and it fabricated practically. In 2000, David smith and his colleagues constructed the first LHM, which is based on a periodic array of interspaced conducting nonmagnetic split-ring resonators (SRR) and continuous metallic wires [2]. Therefore these composite structure of SRR and thin wire used to make both the effective electric permittivity and magnetic permeability simultaneously negative, so that EMW propagation is made possible in special direction and special polarization at micro-wave frequencies.

After this initial demonstration of the first left-handed medium at micro-wave frequencies, various theoretical and experimental methods and materials that can be suitable to realize negative index of refraction at higher microwaves and optical frequencies have been reported [3]. Also, it is found out that for a monochromatic uniform plane wave propagating in such a medium the phase velocity is in the opposite direction of the Poynting vector [4]. However, the realization of the simultaneously negative $\epsilon(\omega)$ and $\mu(\omega)$ is limited by the dispersion properties the metamaterials.

Even though most negative index materials have been constructed using artificially made metamaterials, scientists currently turning towards using combinations of naturally occurring materials to achieve NIMs. Recent progresses in the research of electromagnetic metamaterials have been reported by using chiral materials, gyrotropic chiral composite materials and it has been demonstrated that negative index can be exhibited by magnetodielectric spherical particles and super-lattices of natural materials [5]. In addition, bianisotropic media have been considered as the most generalized materials whose properties are characterized by the magneto-electric as well as the permittivity and permeability tensors. Because of these features, bi-anisotropic plays significant role in the pursuit of negative permeability and hence left-handed metamaterials.

A large amount of research works have been done by many research groups around the globe to understand the properties of metamaterials and their potential applications in various fields like wireless and mobile communication, medical application, right from the microwave to the optical frequency range especially in designing of filters. Realization of metamaterials has been increasing attention for various applications due to its several advantages. Some interesting potential applications have been more efficient in wireless power transferring, superlensing and radar-absorbing materials. While the demonstrations of such unique properties and their abilities for improving on previous technologies have been widespread, the adoption of metamaterials in industry has been underwhelming. This is primarily due to the barriers in transition from technology to market due to high-cost of fabrication. Cheaper and simpler manufacturing methods might therefore allow metamaterials to see more widespread adoption [4, 6, 7]. In military application, it is used as microwave absorbers to reduce the radar cross-section of a conducting target and it also helps in reducing the electromagnetic interference among microwave components. The use of artificially structured metamaterials provides the advancement in the absorber technology, which creates a high performance absorber for the microwave and terahertz frequency regime. Typically, absorbers are usually made with metallic backing plates in order to prevent power transmission through the absorbers, which may cause many problems in stealth applications. Various types of metamaterials has been proposed in THz frequencies and applied to the sensing applications. Since the transmission spectrum of the metamaterials are largely effected by the properties of the metamaterials, the metamaterials can be used as high sensitive sensors by measuring the transmittance change when the particles are absorbed to the surface of the metamaterials [8].

In this work, we considered a composite system that consists of conventional magnetized electron-ion plasma and ferromagnetic grains that are embedded homogeneously in the plasma. The resulting medium, which is composed of the electric (magnetized plasma) and magnetic (ferrite grains) subsystems, is named as magnetized plasma with ferromagnetic grains (MPFG). The MPFG medium is assumed to be a homogeneous medium which can be artificially fabricated from magnetized electron-ion plasma with the addition of ferrite grains. Therefore, the MPFG is essentially composite structure where both the permittivity and permeability are second-rank tensors. It allows the propagation of two light modes: the ordinary and extraordinary waves. The MPFG was shown to have left-hand medium properties for the extraordinary wave in certain frequency domain in the microwave regime [9, 10]. That is, if EMWs propagate in the MPFG in a direction perpendicular to the external magnetic field, the refractive index takes the form $n_+ = \sqrt{(\epsilon_1^2 - \epsilon_2^2)/\epsilon_1}$ and $n_- = \sqrt{\epsilon_3(\mu_1^2 - \mu_2^2)/\mu_1}$ for the ordinary and extraordinary waves, respectively. Here,

ϵ_1, ϵ_3, μ_1, and μ_3 are the diagonal components whereas ϵ_2 and μ_2 are the off-diagonal components of the permittivity and the permeability tensors, respectively. It is obvious that for the extraordinary wave, the effective ϵ and μ may be simultaneously negative for certain frequency range in vicinity of the resonant frequencies while the refractive index $n_-(\omega)$ is still real so that EMWs can propagate in the medium in the given frequency range.

In particular, we investigated the propagation of EMWs in a direction perpendicular to the applied external magnetic field as well as how the transparent region of the negative refractive index frequency domain is affected by changes of the parameters of the magnetic subsystem of the MPFG. Moreover, by tuning the parameters of the electric and magnetic subsystems, the MPFG medium is shown to be transparent to the entire frequency domain.

The paper is organized as follows: In Chapter 2, the propagation of electromagnetic waves in a medium are discussed in detail. Media with negative refractive index is presented in Chapter 3. In Chapter 4, the propagation of electromagnetic waves in MPFG system and summary of the project are displayed. In addition, results of the work are given. Lastly, conclusions of the work are described in Chapter 5.

Chapter 2

Propagation of Electromagnetic Waves in a Medium

When an electric charge is moving in space, the region of space surrounding it is affected by the electric field produced by the charge. This field that is created due to the motion of a charge in turn induces a magnetic field. We say a moving charge is surrounded by both electric and magnetic fields. In addition, it is well known that a moving charge produces a magnetic field which varies in space and time and this changing magnetic field produces an electric field that varies in space and time. This mutual generation of electric and magnetic fields results in the propagation of electromagnetic waves. The propagation of electromagnetic waves (EMWs) in a medium is completely described using Maxwell's equations and the material constitutive relations. The electric and magnetic constitutive relations are consequences of the materials responses to electric and magnetic fields.

In this Chapter we discuss the main concepts required to explain the propagation of EMWs in an arbitrary media. In particular, concepts such as the electric and magnetic constitutive relations, Maxwell's equations, the dispersion relation, refractive index, the Poynting vector as well as phase and group velocity are discussed. In addition, left-handed media are introduced and their responses to the propagation of EMWs are compared with that of the 'ordinary' (right-handed) media.

2.1 Maxwell's Equations and the Constitutive Relations

Maxwell's curl equations

The materials response to EMWs is described by Maxwell's equation. The source free Maxwell's curl equations for a linear, isotropic, and homogeneous medium are written as [11]

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{2.1.1}$$

$$\nabla \times \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t} \tag{2.1.2}$$

where the vectors $\vec{E}$, $\vec{H}$, $\vec{D}$, $\vec{B}$ are the electric field, magnetic field, electric displacement vector and the magnetic induction vector, respectively, and c is the speed of light in vacuum.

Permittivity

It is well known that an electric field $\vec{E}$ applied on a dielectric media results to the polarization of charges, a phenomena known as polarization. The corresponding electric displacement vector is given by [12]

$$\vec{D} = \vec{E} + 4\pi\vec{P}, \tag{2.1.3}$$

where $\vec{D}$ is the displacement vector, $\vec{E}$ is electric field, and $\vec{P}$ is the polarization. For a linear and isotropic medium, the polarization is linearly related with the electric field by

$$\vec{P} = \chi_e\vec{E}, \tag{2.1.4}$$

where χ_e is known as the electric susceptibility. Substituting eq. (2.1.4) into (2.1.3), we obtain

$$\vec{D} = (1 + 4\pi\chi_e)\vec{E} = \epsilon\vec{E}, \tag{2.1.5}$$

where we have introduced the electric permittivity, ϵ, of the medium as

$$\epsilon = 1 + 4\pi\chi_e. \tag{2.1.6}$$

Equation (2.1.5) is referred to as the electric constitutive relation.

Permeability

Similarly, when a magnetic field is applied on a magnetic material, it results to the magnetization of the medium. Hence, the magnetic induction vector due to the magnetization of the medium is given by [12]

$$\vec{B} = \vec{H} + 4\pi\vec{M}, \tag{2.1.7}$$

where $\vec{B}$ is the magnetic induction, $\vec{H}$ is the applied magnetic field, and $\vec{M}$ is the magnetization of the medium. In linear and isotropic medium, the magnetization is linearly related with the magnetic field by

$$\vec{M} = \chi_m\vec{H}, \tag{2.1.8}$$

where χ_m is the magnetic susceptibility of the medium. Substituting eq. (2.1.8) into (2.1.7), we get

$$\vec{B} = (1 + 4\pi\chi_m)\vec{H} = \mu\vec{H}, \tag{2.1.9}$$

where the permeability, μ, of the medium is defined by

$$\mu = 1 + 4\pi\chi_m. \tag{2.1.10}$$

Equation (2.1.9) is known as the magnetic constitutive relation. Equations (2.1.5) and (2.1.9) are known as the electromagnetic constitutive relations.

It is worth noting that, in isotropic media, the parameters ϵ and μ that appear in the constitutive relations are scalars. However, in the case of anisotropic medium both the permittivity and permeability are second-rank tensors.

2.2 The Dispersion Relation

The propagation of electromagnetic waves in the medium is controlled by the electric permittivity and magnetic permeability. This reality is because of the only parameters (ϵ and μ) of the material occur in the dispersion relation equation, which is [1]

$$|n^2\epsilon_{il}\mu_{lj} - k^2\delta_{ij} + k_ik_j| = 0. \tag{2.2.1}$$

For the case of isotropic medium, eq. (2.2.1) reduces to

$$k^2 = \frac{\omega^2}{c^2}n^2, \tag{2.2.2}$$

where ω is frequency of a monochromatic wave, k is the magnitude of the wave vector, c is speed of wave in free space, and n is the index of refraction of a medium. From eq. (2.2.2), we find that

$$n^2 = \frac{k^2c^2}{\omega^2}. \tag{2.2.3}$$

Also, using the Maxwell's equations, it can be shown that the refractive index is given by

$$n^2 = \epsilon\mu, \tag{2.2.4}$$

where ϵ and μ are the permittivity and permeability of the medium, respectively.

2.3 Group Velocity and Phase Velocity

Phase velocity is the velocity of individual wave with which it is travelling in the direction of propagation. The phase velocity, v_p, is defined by

$$v_p = \frac{\omega}{k} = \frac{c}{n}, \tag{2.3.1}$$

where ω is the frequency, k is the the magnitude of the wave vector, c is the speed of light in vacuum, and n is the refractive index.

From Maxwell curl equation, we find that the direction of phase velocity is defined by the direction of $\vec{E} \times \vec{B}$, which is also in the direction of vector $\vec{k}$. This direction is away from a source in right-handed media, but towards the source in left-handed media. In a non-dispersive medium and vacuum is the only truly non-dispersive environment, $v_p = \omega/k$ and a plot of ω versus k is a straight line. The frequency and wavelength changes so as to keep v_p constant. All waves of a particular type (e.g., all EM-waves) travel with the same phase speed in a non-dispersive medium. By contrast, in a dispersive medium different components of the wave travel with different speeds and tends to change phase with respect to on another. When a number of different frequency harmonic waves superimpose to form a composite disturbance, the resulting modulation envelope will travel at a speed different from that of the constituent waves. This raises the important motion of the group velocity and its relationship to the phase velocity.

In ordinary medium ω is dependent on λ, or equivalently on k. The particular function $\omega = \omega(k)$ is called the dispersion relation. Therefore, the group velocity, v_g, is equal to the derivative of the dispersion relation. That is,

$$v_g = \frac{\partial \omega}{\partial k}. \tag{2.3.2}$$

In addition, the group velocity in terms of phase velocity is given by [5]:

$$\frac{1}{v_g} = \frac{1}{v_p} + \omega \frac{\partial}{\partial \omega}\left(\frac{1}{v_p}\right). \tag{2.3.3}$$

From eq. (2.3.3), we have three possibilities. These are:

1. Nondispersive: frequency dispersion does not exist and the value of phase velocity equal to group velocity. Here, since $(\partial/\partial\omega)(1/v_p) = 0$, we get $v_g = v_p$.

2. Normal dispersion: this type of dispersion is in which the group velocity less than phase velocity. Since $(\partial/\partial\omega)(1/v_p) > 0$, we obtain $v_g < v_p$.

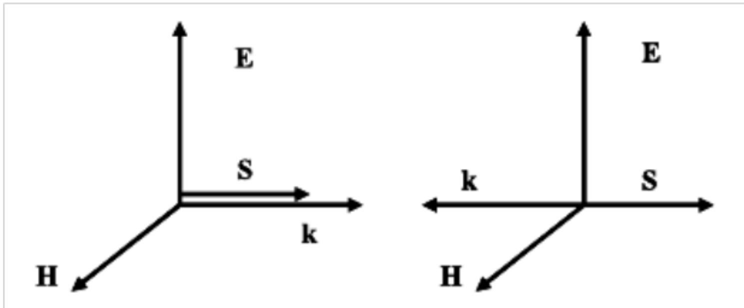

Figure 2.1: Schematics of the electric field, magnetic field, wave vector, and Poynting vector for electromagnetic wave propagation in right- and left-handed media.

3. Anomalous dispersion: in this dispersion, group velocity greater than phase velocity. In this case, $(\partial/\partial\omega)(1/v_p) < 0$, we get $v_g > v_p$.

2.4 The Poynting Vector

The quantity that describes the energy carried by electromagnetic waves is known as the Poynting vector, $\vec{S}$. It is defined by

$$\vec{S} = \frac{c}{4\pi}(\vec{E} \times \vec{H}). \tag{2.4.1}$$

The direction of $\vec{S}$ is always away from the source and this direction is unaffected by the change of sign of index of refraction. Thus, the Poynting vector ($\vec{S}$), electric field ($\vec{E}$), and magnetic field ($\vec{H}$) always form a right-handed triplet in both right- and left-handed medium. However, the phase velocity and group velocity are parallel in right-handed medium ($\epsilon > 0$ and $\mu > 0$) but anti-parallel in the case of left-handed medium ($\epsilon < 0$ and $\mu < 0$). Figures 2.1 display the relative orientation of these vectors in (Left side) RHM and (Right side) LHM. The corresponding directions of $\vec{v}_g$ and $\vec{v}_p$ are in the directions of $\vec{S}$ and $\vec{k}$, respectively.

In 2012, Woodley also explained the type of wave propagation depending on the angle θ, where θ is the angle between the phase velocity ($\vec{v}_p$) and group velocity ($\vec{v}_g$) [13]. He stated that if $\theta = 0^0$, the EMWs propagate forward and the medium in which EMW propagate is referred as RHM. For the second case, if $\theta = 180^0$, $\vec{v}_p$ and $\vec{v}_g$ are perfectly anti-parallel. Then, the wave propagates perfectly backward and the medium is known as isotropic with negative index of refraction. The third case, if θ is not exactly 180^0 and the dot product of $\vec{v}_p$ and $\vec{v}_g$ less than zero ($\vec{v}_p \cdot \vec{v}_g < 0$), then the type of propagation is imperfect backward and the medium is called anisotropic with negative index of refraction.

2.5 Right-Handed and Left-Handed Media

A medium in which EMWs propagating with positive phase velocity is referred as right-handed medium [2-10]. The electric permittivity and magnetic permeability of this medium are simultaneously positive and the wave is forward. But left-handed medium (LHM) is the opposite of RHM. This means that the electric permittivity and magnetic permeability are simultaneously negative and wave vector $\vec{k}$ and phase velocity ($\vec{v}_p$) are backward.

To explain more RHM and LHM the relationship between the electric field vector, $\vec{E}$, magnetic field vector, $\vec{H}$, and wave vector $\vec{k}$ will be given by Maxwell's curl equations, eqs. (2.1.1) and (2.1.2) together with the constitutive relations, eqs. (2.1.5) and (2.1.9). To see this, consider a plane wave propagating through this medium with fields of the form

$$\vec{E},\ \vec{H} \propto e^{-i(\vec{k}\cdot\vec{r}-\omega t)}.$$

Then, eq. (2.1.1) can be written as

$$\vec{k} \times \vec{E} = \frac{\omega}{c}\mu\vec{H}. \tag{2.5.1}$$

Similarly, eq. (2.1.2) can be written as

$$\vec{H} \times \vec{k} = \frac{\omega}{c}\epsilon\vec{E}. \tag{2.5.2}$$

In a medium where ϵ and μ are real (no losses) and positive, (2.5.1) and (2.5.2) describe the typical right-handed relationship between $\vec{E}$, $\vec{H}$ and $\vec{k}$, such a medium is referred to as right-handed medium (RHM) [3, 13]. However if ϵ and μ are real and negative, (2.5.1) and (2.5.2) become:

$$\vec{k} \times \vec{E} = -\frac{\omega}{c}|\mu|\vec{H}, \tag{2.5.3}$$

$$\vec{H} \times \vec{k} = -\frac{\omega}{c}|\epsilon|\vec{E}. \tag{2.5.4}$$

Thus, (2.5.3) and (2.5.4) describes a left-handed relationship among these vectors. Therefore, the medium which exhibit such behavior is called as left-handed medium (LHM) [9].

Chapter 3

Media with Negative Refractive Index

Non-natural media, i.e., metamaterials may exhibit properties that are much more pronounced than those found in natural materials or they may even exhibit properties that were not observed before in natural media. The most prominent example of not previously observed properties are metamaterials with a negative index of refraction, i.e., negative index metamaterials (NIMs) that are also called left-handed materials (LHMs). In order to realize these metamaterials, there are several approaches such as composite structure, anisotropic, bianisotropic, and chiral materials.

3.1 Consequences of Negative Refractive Index

The phenomena associated with electromagnetic wave propagating in materials with ϵ and μ simultaneously negative is almost reversed when compared to the material with ϵ and μ simultaneously positive. For example, in Snell's refraction, the refracted ray will be in the same side of the normal as the incident ray Fig. 3.1 (b); the Doppler shift is reversed, with a light source moving toward an observer or an observer moving toward the source is resulting in a reduced frequency and the Cherenkov radiation from a charge passing through the material is emitted in the opposite direction to the charge's motion rather than in the forward direction.

The origin of this newly predicted behavior can be traced to the distinction between the group velocity, which characterizes the flow of energy, and the phase velocity, which characterizes the movement of the wave fronts. In conventional (regular) materials, the group and phase velocities are parallel. By contrast, the group and phase velocities point in opposite directions when $\epsilon < 0$ and $\mu < 0$. The reversal of phase and group velocity in a material implies a simply stated but very serious consequence: the sign of the refractive index, n, must be taken as negative.

In his original paper, Veselago indicated that media with negative permittivity and negative permeability, i.e., LHM, are expected to have the peculiar phenomena such as negative refraction, reversed Doppler effect, and reversed Vavilov-Cerenkov effect. These are discussed in detail below.

3.1.1 Negative refraction - Snell's law

The propagation of electromagnetic waves in left-handed medium shows peculiar characteristic compared to that in ordinary (or RHM) medium. Among these is the anomalous (negative) refraction. This backward propagation or negative refraction can be understood by investigating the wave vectors of the reflected, incident, and transmitted (refracted) optical rays at the interface between two ordinary media of refractive indices $n_1 > 0$ and $n_2 > 0$ that is shown in Fig. 3.1 (a); and the interface between ordinary and left-handed media of refractive indices $n_1 > 0$ and $n_2 < 0$, respectively, as shown in Fig. 3.1 (b).

The Snell's law supports the wave propagation through left-handed medium [5, 9]. The refracted index of medium two in Fig. 3.1 (a) and (b) are equal in magnitude but have opposite sign and the wave is refracted to the opposite side compared to the ray propagating in right-handed medium. Although the wave bends in the 'wrong'

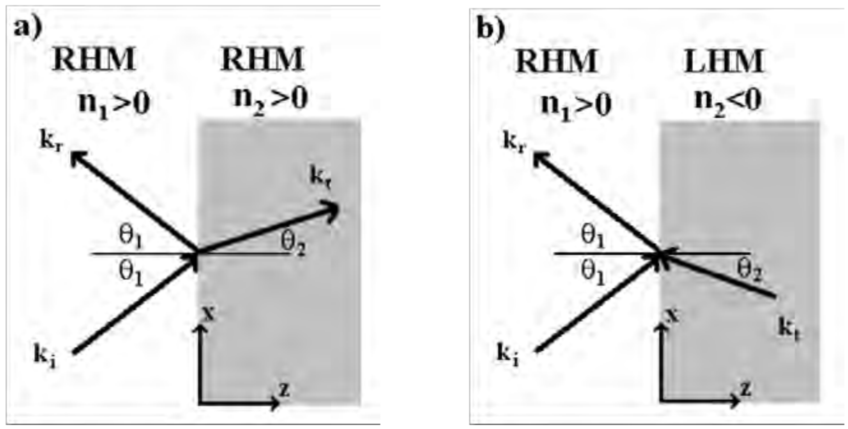

Figure 3.1: Incident, reflected, and transmitted wave vectors at (a) an RHM/RHM interface and (b) an RHM/LHM interface. [13]

way, the Snell's law is satisfied when a negative value of n is substituted and $\theta_2 < 0$ into the equation,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2, \tag{3.1.1}$$

where n_1 is index of refraction of medium one and n_2 is index of refraction of medium two. Therefore, eq. (3.1.1) is valid in both right-handed and left-handed media. Hence the value of index of refraction of medium two in Fig. 3.1 (b) is given by [14]:

$$n = n_2 = -\sqrt{\epsilon\mu}, \tag{3.1.2}$$

where ϵ is electric permittivity and μ is magnetic permeability.

3.1.2 Reversed Doppler effect

Reversed Doppler effect occurs when a detector of radiation from a source moves relative to the source of radiation. The frequency of the detected radiation depends on the relative velocity of the source of radiation and the velocity of the detector. Suppose the receiver and the source are placed in a medium of refractive index n, and the source is at rest while the receiver (detector) moves towards the source. Then, the Doppler effect in the medium is given by [15]:

$$\Delta\omega = \frac{V}{v_p}\omega_0, \tag{3.1.3}$$

where ω_0 is frequency of radiation, $v_p = c/n$ is the wave (phase) velocity, V is the velocity of the receiver and $\Delta\omega$ is the frequency difference between the receiver and source. In ordinary medium, $v_p = c/n > 0$ and $V < 0$, so that $\Delta\omega$ is negative since $n > 0$. On the other hand, for LHM rewriting eq. (3.1.3) in terms of the refractive index, with $n < 0$, we find that

$$\Delta\omega = -|n|\frac{V}{c}\omega_0, \tag{3.1.4}$$

showing that $\Delta\omega$ is positive in LHM since $V < 0$. Hence, from eqs. (3.1.3) and (3.1.4), we observe the sign of the Doppler shift in left-handed media is reversed compared to that in right-handed media.

3.1.3 Reversed Vavilov-Cerenkov effect

The Cerenkov effect is produced when a charged particle moves in a material with a speed faster than the phase velocity of light in that material [14]. The Cerenkov radiation depends on the sign of the index of refraction of that medium. If ϵ and μ are simultaneously positive, the Cerenkov radiation directed forward. In the case of left-handed medium ($\epsilon < 0$ and $\mu < 0$) the Cerenkov radiation is directed backward (opposite to that in RHM). Suppose a particle moves in a medium with velocity $\vec{V}$. Then, it will emit a radiation (having a flux $\vec{S}$) at an angle of θ with respect to $\vec{V}$, as shown in Figs. 3.2. The relation between the direction of the particle's velocity and the cone of the emitted radiation field is given by [1]

$$\cos\theta = \frac{v_p}{V} = \frac{c}{nV}. \tag{3.1.5}$$

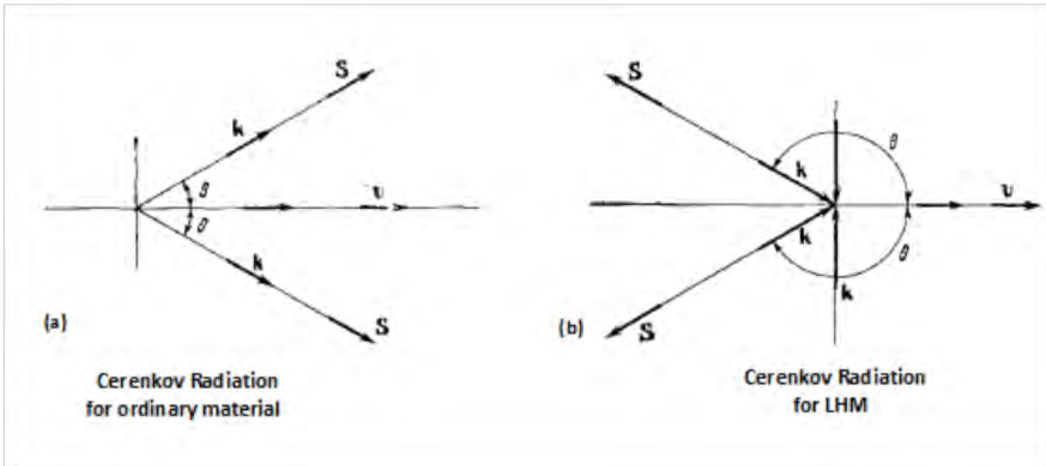

Figure 3.2: Schematics of Cerenkov's radiation in (a) right-handed and (b) left-handed media.

where v_p is phase velocity ($v_p = c/n$) of the radiation, which is in the direction of the Poynting vector $\vec{S}$.

From eq. (3.1.5), it is obvious that for left-handed media $\cos\theta$ is negative, i.e., θ is an obtuse angle between $\vec{V}$ and $\vec{S}$ so that the cone of the radiation will be directed backward relative to the motion of the particle. For RHM $\cos\theta$ is positive and θ corresponds to an acute angle between $\vec{V}$ and $\vec{S}$. Therefore, we can conclude that the cone of Cerenkov's radiation in LHM is reversed compared with that in RHM, as shown in Figs. 3.2.

3.2 Routes in Realizing Left-Handed Media

It is well known that serious investigation on left-handed media began after the pioneering theoretical work of Veselago [1], in 1968. At the time, he speculated that left-handed media with simultaneous negative values of permittivity and permeability, that may have peculiar characteristics to the normal types of material (RHM), are not readily available in nature. So, he proposed the fabrication of 'artificial' materials in order to realize left-handed materials. Subsequently, many scientists have carried out experiments as well as theoretical investigations to attain LHM. Pendry and Smith were the first to identify a practical way to make this new material [2, 16]. Recently, the research of electromagnetic metamaterials have been reported by using chiral materials [5].

3.2.1 Isotropic left-handed media

Nowadays, various materials that are not found in nature are being assembled artificially from naturally available materials. Such man-made structures are called metamaterials. Left-handed metamaterials were first artificially constructed by Smith, et. al in 2000 [2]. They successfully managed to assemble isotropic left-handed medium from two kinds of cell elements: split ring resonators (SRR) that produce negative permeability and a wire array to produce negative permittivity. The details of these cell elements are given below.

Negative permittivity

A material with negative permittivity exists in nature. The good known examples are metals, low-loss plasmas, and semiconductors at optical and infrared frequency. In 1996, Pendry proposed an artificial material which was metallic thin wire medium with negative permittivity at microwave frequency [16]. From the available analytical models, the most common ones that are employed for the the description of the effective permittivity and permeability of left-handed materials are the Drude and Lorentz models [9]. These are given by

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}, \tag{3.2.1}$$

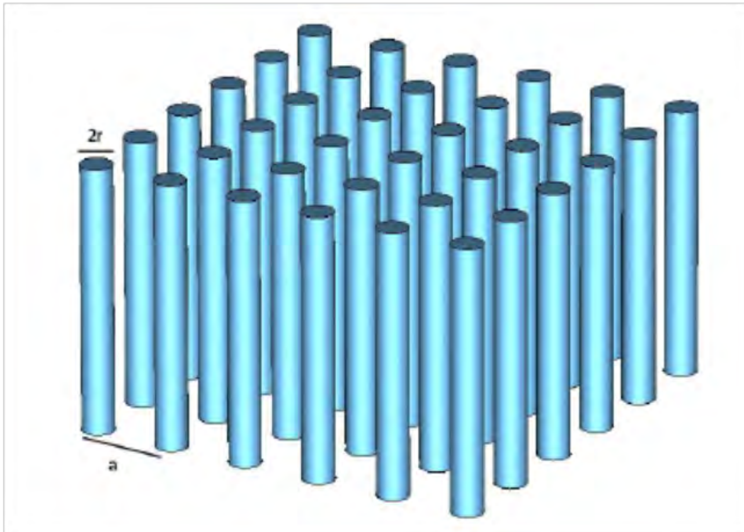

Figure 3.3: Schematic of array of thin metallic wire. The parameter a is the lattice constant and r is the radius of a single wire. [14]

which is the permittivity in the Drude model, and

$$\epsilon(\omega) = 1 - \frac{\omega_p^2 - \omega_0^2}{\omega^2 - \omega_0^2 + i\gamma\omega}, \tag{3.2.2}$$

which is the equation for the permittivity in the Lorentz model. The parameters ω_p, ω_0, γ, and ω are the plasma, resonant, damping, and incident EMW frequencies, respectively.

The array of thin metal wire, illustrated in Fig. 3.3, presents the effective negative permittivity (ϵ_{eff}). If an electric field, $\vec{E}$, is applied along the wire's axes, it induces currents through the wires which is equivalent to the generation of electric dipole moments in the metal arrays. There are two factors affecting the electron movement in a wire radius, r: (i) the average electron density n_{eff} and (ii) the effective mass of electrons by magnetic effects [16]. These are

$$n_{eff} = \frac{n\pi r^2}{a^2}, \tag{3.2.3}$$

where n is the density of electrons in the wire, and a is the lattice constant.

From Ampere's law, a current flowing through a wire produces a magnetic field, whose direction depends on the direction of the current. The effective mass of an electron, m_{eff}, in the wire may be expressed as

$$m_{eff} = \frac{2n\pi e^2 r^2}{c^2} \ln(a/r), \tag{3.2.4}$$

where e is the electron charge. Also, the plasma frequency, which is associated with the fundamental oscillation frequency of the electrons while returning to the equilibrium position is given by

$$\omega_p^2 = \frac{4\pi n_{eff} e^2}{m_{eff}} = \frac{2\pi c^2}{a^2 \ln(a/r)}, \tag{3.2.5}$$

where c is the speed of light in vacuum.

If the metal is lossless and the electric field is along the wire, so in the case of left-handed medium, the Drude equation for the effective permittivity (ϵ_{eff}) can be reduced to:

$$\epsilon_{eff} = 1 - \frac{\omega_p^2}{\omega^2}, \tag{3.2.6}$$

where ω_p is the plasma frequency and ω is the frequency of the propagating electromagnetic wave. From (3.2.6), the effective permittivity is negative for $\omega_p > \omega$ and positive for $\omega_p < \omega$. The structure shown in Fig. 3.3 is demonstrated to have negative permittivity at microwave frequencies.

Negative permeability

The availability of materials with negative permeability are less common in nature due to the weak magnetic interactions in most solid state materials. Only in ferromagnetic and ferrite materials are magnetic interaction strong enough to produce regions of negative magnetic permeability [17]. In 1999, Pendry showed that the split-ring resonators (SRRs) structure possess negative permeability in the microwave frequency

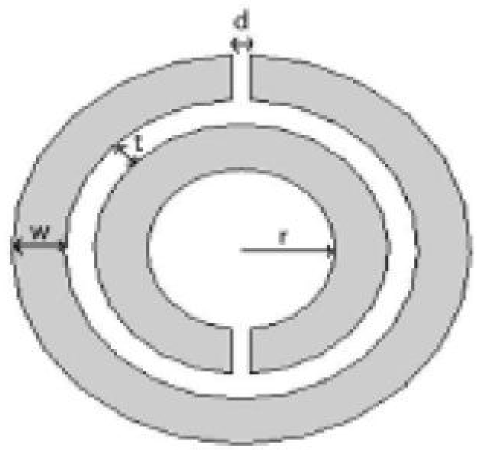

Figure 3.4: The split ring resonator. [17]

range [18]. This artificial medium is made from two concentric (inter placed) conducting rings with one gap in each as shown in Fig. 3.4.

For split ring resonator, if the magnetic field, $\vec{H}$, is perpendicular to the plane of the ring, then the effective permeability as a function of frequency is found to be given by [13]:

$$\mu_{eff}(\omega) = 1 - \frac{\omega_{pm}^2}{\omega^2 - \omega_{0m}^2 + i\gamma\omega}. \tag{3.2.7}$$

In 2000, the combination of thin wire medium and SRRs were investigated by Smith, et.al as potential metamaterial to attain negative permittivity and negative permeability, respectively [2]. Indeed, the combined medium was used to realize Veselago's left-handed medium in certain narrow frequency domain. In 2001, Shelby, et. al also achieved left-handed medium consisting of metal post and SRR experimentally [19]. According to this experiment, the material constructed by combining these two media that are known to exhibit negative permittivity and negative permeability was shown to possess negative refractive index ($n < 0$) in the microwave regions ($f \approx 5\ GHz$). In Fig. 3.5 (right side) the metal posts were substituted by strip wires and (left side)

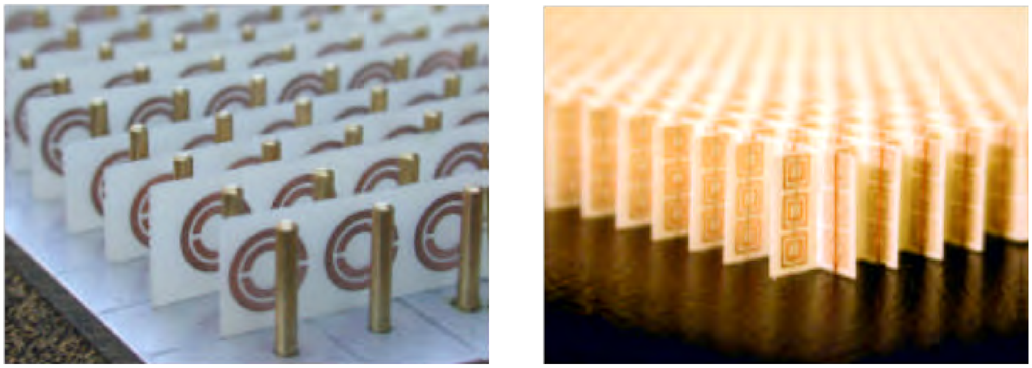

Figure 3.5: The first left-handed (Left) one-dimensional and (Right) two-dimensional system. [19]

the circular SRRs were substituted by square SRRs.

3.2.2 Anisotropic left-handed media

The electrodynamics of electromagnetic waves in anisotropic media was theoretically predicted by Veselago for the first time. Then after, this medium is a topic that has been studied for many years, originally because this type of media can be directly found in nature and later it was realized in the laboratory. Anisotropic medium is a medium in which the electrical and/or magnetic properties of a medium depend upon the directions of field vectors.

The relationships between fields can be written in the form:

$$\vec{D} = \hat{\epsilon}\vec{E}, \tag{3.2.8}$$

$$\vec{B} = \hat{\mu}\vec{H}, \tag{3.2.9}$$

where $\hat{\epsilon}$ and $\hat{\mu}$ are the electric permittivity and permeability tensors, respectively. The good example of anisotropic medium is gyrotropic material [1]. In this material the natural modes of propagation are circularly polarized waves and the permittivity or permeability components for lossless media are:

$$\hat{\epsilon} = \begin{pmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}, \qquad \hat{\mu} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}. \tag{3.2.10}$$

3.2.3 Bi-anisotropic left-handed media

Bi-anisotropic media is a special type of materials whose properties are characterized by the magneto-electric as well as the permittivity and permeability tensors. Due to strong magneto-electric coupling, the wave propagation in bi-anisotropic media is usually described by high order dispersion relations with circularly or elliptically polarized waves. Because of these features, bi-anisotropic plays important role in the pursuit of negative permeability and left-handed metamaterials.

The constitutive relations of a bi-anisotropic medium relate $\vec{D}$ and $\vec{B}$ to both $\vec{E}$ and $\vec{H}$. The constitutive relation in the Telegen model is [20]

$$\vec{D} = \bar{\bar{\epsilon}}\vec{E} + \bar{\bar{\xi}}\vec{H}, \tag{3.2.11}$$

$$\vec{B} = \bar{\bar{\mu}}\vec{H} + \bar{\bar{\zeta}}\vec{E}, \tag{3.2.12}$$

where $\bar{\bar{\epsilon}}$, $\bar{\bar{\mu}}$, $\bar{\bar{\xi}}$, and $\bar{\bar{\zeta}}$ are in general arbitrary tensors representing the properties of the medium and are related to the relative quantities $\bar{\epsilon}$, $\bar{\mu}$, $\bar{\xi}$, and $\bar{\zeta}$ by

$$\bar{\bar{\epsilon}} = \epsilon_0\bar{\epsilon}, \quad \bar{\bar{\mu}} = \mu_0\bar{\mu}, \quad \bar{\bar{\xi}} = \xi_0\bar{\xi}, \quad \text{and} \quad \bar{\bar{\zeta}} = \zeta_0\bar{\zeta}, \tag{3.2.13}$$

In general, the tensor $\bar{\bar{\epsilon}}$ and $\bar{\bar{\mu}}$ have the form:

$$\bar{\bar{\epsilon}} = \epsilon_0 \begin{pmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{21} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{31} & \epsilon_{32} & \epsilon_{33} \end{pmatrix}, \qquad \bar{\bar{\mu}} = \mu_0 \begin{pmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{21} & \mu_{22} & \mu_{23} \\ \mu_{31} & \mu_{32} & \mu_{33} \end{pmatrix}. \tag{3.2.14}$$

3.2.4 Chiral left-handed media

The term chiral is used to describe a material that is non-super imposable on its mirror image. The literal origin of chiral is from the Greek word for hand [14]. Some crystals have the ability to rotate the plane of a linearly polarized light wave passing through them, a property known as optical activity or rotary power. Optical activity is caused by chirality, either of the molecules making up the substance, or in the helical arrangement of the atomic or molecular constituents in a crystal. The quartz crystal rotates the plane of polarization of a linearly polarized light which passes along the crystal optic axis. The chiral media that represent a subject of magnetoelectric materials can exhibit backward wave phenomenon and negative refraction in the condition of chiral nihility (i.e., $\epsilon < 0,\ \mu < 0$ and $\kappa \neq 0$). The nihility in chiral media requires that at least one of the permittivity and permeability is at its resume and both imaginary and real parts are zero, which is not physically realizable in electromagnetic materials [20, 21]. Due to those assumptions, the potential applications of chiral nihility become limited.

The backward waves and left-handed phenomena can be exhibited by magnetoelectric materials and in principle it is realized even if the medium has very weak magnetic properties or no magnetic properties [20]. Thus it appears that using magnetoelectric materials can realize left-handed material in the optical region without creating artificial magnetic materials. Unlike conventional negative index material designs, such as the split-ring resonator type design, the chiral negative index material does not require simultaneously negative permittivity and permeability, and, therefore, the chiral design can offer much simpler geometry and a more efficient way to realize negative refraction index. Obviously chiral materials offer a relatively simpler route to negative refraction and hence, this subject has been constantly approached by researchers [6].

The index of refraction of a chiral media in which the magnetoelectric coupling is present in terms of the chirality takes the form [5]:

$$n_{\pm} = \sqrt{\epsilon\mu} \pm \kappa, \tag{3.2.15}$$

where '+' and '-' refer, respectively, to the right-handed and left-handed circularly polarized eigen states and κ is the chirality parameter. This implies that in principle strong enough chirality is sufficient to achieve negative refraction for one circular polarization. On the other hand the handedness of the medium is given by [20]:

1. when chirality is positive, the polarization is right-handed and the medium is right-handed,

2. when chirality is negative, the right-handed system is reversed to the left-handed system, and

3. when no chirality is present, no magneto-electric couplings or optical activity exists.

Chapter 4

Propagation of EMWs in Magnetized Plasma with Ferromagnetic Grains

A tunable left-handed media is a media with a variable response to an incident electromagnetic wave. This includes remotely controlling how an incident electromagnetic wave interacts with a metamaterial. This means the capability to determine whether the EMW is transmitted, reflected, or absorbed. In general, the lattice structure of the tunable metamaterial is adjustable in real time, making it possible to reconfigure a metamaterial device during operation. It encompasses developments beyond the bandwidth limitations in left-handed materials by constructing various types of metamaterials. [22]

Magnetized plasma with ferrite grains (MPFG) consists of conventional magnetized electron-ion plasma with the addition of the ferrite grains that are embedded uniformly in the plasma. Such a medium may be fabricated in a laboratory. The index of refraction of MPFG can be shown to be negative in certain frequency ranges. Moreover, the parameters of the electric and magnetic subsystems can be tuned in such a way that the LHM, RHM, and nontransparent frequency domains can be varied. In this Chapter, the tunability of the MPFG system is discussed in detail.

4.1 Permittivity and Permeability of MPFG

Let us consider magnetized electron-ion plasma with ferromagnetic grains (MPFG) in an external constant magnetic field, $\vec{H}_0$, which is directed along the positive z-axis. In the model of cold plasma, the plasma in the magnetic field is characterized by permittivity tensor, $\hat{\epsilon}$, given by: [22]

$$\hat{\epsilon} = \begin{pmatrix} \epsilon_1 & i\epsilon_2 & 0 \\ -i\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_3 \end{pmatrix}. \tag{4.1.1}$$

The non-zero components of the permittivity tensor can be shown to have the form [3, 9].

$$\begin{aligned} \epsilon_1 &= 1 - \frac{\omega_p^2}{\omega^2 - \omega_c^2}, \\ \epsilon_2 &= \frac{\omega_c \omega_p^2}{\omega(\omega^2 - \omega_c^2)}, \\ \epsilon_3 &= 1 - \frac{\omega_p^2}{\omega^2}, \end{aligned} \tag{4.1.2}$$

where $\omega_p = (4\pi n e^2/m)^{1/2}$ and $\omega_c = (eH_0)/mc$ are the plasma frequency and the electron cyclotron frequency, respectively, c is the speed of light in a vacuum, n is the density number of the electrons and m is the mass of an electrons. Typical values of n for diffuse laboratory plasma are $n \sim 10^8 - 10^{12}\ cm^{-3}$ and the magnetic field $H_0 \sim 0.5 - 5\ T$ [23]. Consequently, ω_p and ω_c can be adjusted to be $\sim 1\ GHz$ with $\omega_p > \omega_c$.

Moreover, the ferromagnetic grains in a magnetic field is induce dipole moments and this forms magnetic subsystem. Then the interaction between dipole moment with time varying magnetic field enables the magnetic subsystem to have variable magnetization. The permeability tensor of the magnetic subsystem can be derived by analyzing the motion of the ferrite grains, which can be shown to have the form

[22]:

$$\mu = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}. \tag{4.1.3}$$

The nonzero components of permeability tensor, specially, for spherical and homogenously magnetized ferrite grains, can be shown to have the form [9]

$$\begin{aligned} \mu_1 &= 1 - \frac{\omega_c \omega_m}{\omega^2 - \omega_c^2}, \\ \mu_2 &= \frac{\omega \omega_m}{\omega^2 - \omega_c^2}, \\ \mu_3 &= 1, \end{aligned} \tag{4.1.4}$$

where $\omega_m = (16\pi^2 a^3 N_g \chi H_0)/3$, a is grain's radius, N_g is the density of the grains, and χ is static magnetic susceptibility. Those components of the permittivity tensor and permeability tensors comes together in the next section, to explain refractive index of MPFG. Typical values of the parameters are $a \sim 10^{-4}$ cm, $N_g \sim 10^5$ cm^{-3}, and $\chi \sim 10^3$ [9].

4.2 Refractive Index of MPFG

Consider a monochromatic plane electromagnetic wave that propagates in MPEG medium with the applied constant magnetic field, $\vec{H}_0$, being directed parallel to the z-axis. In particular, we may identify two possible directions of propagation directions: (i) parallel propagation, when the wave vector $\vec{k}$ parallel to the z-axis, so that the angle between $\vec{H}_0$ and $\vec{k}$ is $\theta = 0$; and (ii) perpendicular propagation, when the wave vector $\vec{k}$ parallel to the x-axis, so that the angle between $\vec{H}_0$ and $\vec{k}$ is $\theta = \pi/2$. The corresponding refractive indexes of the medium are obtained by employing the dispersion relation [5]. The permittivity, permeability, and refractive index for parallel and perpendicular propagation of EMWs are presented below.

Propagation of EMWs parallel to $\vec{H}_0$

In this case $\theta = 0$. Thus, using the dispersion relation, eq. (2.2.1), the refractive index of the MPFG is found to be:

$$n_{\pm}^2(\omega, 0) = (\epsilon_1 \pm \epsilon_2)(\mu_1 \pm \mu_2), \tag{4.2.1}$$

which gives

$$n_{+}(\omega, 0) = \sqrt{(\epsilon_1 + \epsilon_2)(\mu_1 + \mu_2)}, \tag{4.2.2}$$

and

$$n_{-}(\omega, 0) = \sqrt{(\epsilon_1 - \epsilon_2)(\mu_1 - \mu_2)}. \tag{4.2.3}$$

where eqs. (4.2.2) and (4.2.3) represent the refractive indexes of the ordinary and extraordinary waves, respectively. The propagation of EMWs for parallel propagation has been studied in Refs. [3, 5]. They showed that the MPFG possesses LHM material properties for certain frequency domain for the extraordinary wave. On the other hand, the medium behaves only as RHM for the ordinary wave. Here, we are mainly interested in the case where EMWs propagate in a direction perpendicular to $\vec{H}_0$.

Propagation of EMWs perpendicular to $\vec{H}_0$

This case corresponds to $\theta = \pi/2$. Similarly, using the dispersion relation, eq. (2.2.1), the refractive index of the MPFG is found to be given by

$$n_{+}^2(\omega, \pi/2) = \frac{\epsilon_1^2 - \epsilon_2^2}{\epsilon_1}, \tag{4.2.4}$$

and

$$n_{-}^2(\omega, \pi/2) = \epsilon_3 \left(\frac{\mu_1^2 - \mu_2^2}{\mu_1} \right). \tag{4.2.5}$$

Hence, equations (4.2.4) and (4.2.5) represent ordinary and extraordinary waves, respectively. The negative index of refraction is found only for extraordinary waves

[5, 9, 10] and the positive index of refraction exist for both ordinary and extraordinary waves in MPFG medium.

In the present work, we are interested in the propagation of EMWs in MPFG in a direction perpendicular to the applied magnetic field $\vec{H}_0$. In particular, when EMWs are propagating along x-axis and ($\vec{H}_0$) is along z-axis, then the index of refraction:

$$n_{+}(\omega, \pi/2) = \sqrt{\frac{\epsilon_1^2 - \epsilon_2^2}{\epsilon_1}}, \tag{4.2.6}$$

and

$$n_{-}(\omega, \pi/2) = \pm\sqrt{\epsilon_3\left(\frac{\mu_1^2 - \mu_2^2}{\mu_1}\right)}. \tag{4.2.7}$$

Equations (4.2.6) and (4.2.7) are represents index of refraction of ordinary and extraordinary waves, respectively.

4.3 Negative Refractive Index in MPFG

The negative index of refraction in this medium is obtained using eq. (4.2.7). Letting, the effective permittivity and permeability of the extraordinary wave by

$$\epsilon = \epsilon_3, \qquad \text{and} \qquad \mu = \frac{\mu_1^2 - \mu_2^2}{\mu_1}, \tag{4.3.1}$$

we find that

$$n_{-}(\omega, \pi/2) = \pm\sqrt{\epsilon\mu}, \tag{4.3.2}$$

where ϵ is effective electric permittivity and μ is effective magnetic permeability. Equation (4.3.2) represents $n_{-}(\omega, \pi/2) = +\sqrt{\epsilon\mu}$ where the parameters ϵ and μ are simultaneously positive, and $n_{-}(\omega, \pi/2) = -\sqrt{\epsilon\mu}$ where the parameters ϵ and μ are simultaneously negative.

Next, let us introduce the following dimensionless frequencies:

$$\begin{aligned} x &= \frac{\omega - \omega_c}{\omega_c}, \\ \alpha_e &= \frac{\omega_p}{\omega_c}, \\ \alpha_m &= \frac{\omega_m}{\omega_c}. \end{aligned} \tag{4.3.3}$$

By substituting eq. (4.3.3) into eqs. (4.1.2) and (4.1.4), which means the effective permittivity (ϵ) and effective permeability, i.e., eq. (4.3.1), becomes

$$\epsilon = 1 - \frac{\alpha_e^2}{(x+1)^2}, \tag{4.3.4}$$

and

$$\mu = \frac{(x - \alpha_m)(x^2 + 2 + \alpha_m)}{x^2 + 2x - \alpha_m}. \tag{4.3.5}$$

Also, using eqs. (4.3.4) and (4.3.5) in eq. (4.3.2), we obtain

$$n_-(x, \pi/2) = \pm\sqrt{\left(1 - \frac{\alpha_e^2}{(x+1)^2}\right)\left[\frac{(x - \alpha_m)(x^2 + 2 + \alpha_m)}{x^2 + 2x - \alpha_m}\right]}, \tag{4.3.6}$$

Note that when $\alpha_e^2 > (x+1)^2$, $x < \alpha_m$ and $x^2 + 2x > \alpha_m$, then ϵ and μ are simultaneously negative. Therefore, the '-' sign in eq. (4.3.6) is taken and the medium is considered as left-handed medium (LHM). Hence, EMWs are propagating in MPFG medium with negative index of refraction. But if $\alpha_e^2 < (x+1)^2$, $x > \alpha_m$, then electric permittivity (ϵ) and magnetic permeability (μ) are simultaneously positive. Then, the sign '+' is taken and the medium considered to be right-handed medium (RHM). The MPFG medium is transparent to electromagnetic wave with positive index of refraction. If one of them is negative and the other positive, the square of index of refraction is negative while the index of refraction is imaginary. In this case, EMWs cannot propagate in the MPFG medium.

4.4 Analysis of the Usual MPFG

The MPFG system consists of the electric and magnetic subsystems. The electric subsystem is described by α_e and the magnetic subsystem by α_m. The system is referred as the 'usual' MPFG when α_e and α_m assumes arbitrary values. Next, we depict the graphs ϵ, μ, n_-, and n_-^2 as a function of the dimensionless frequency, x, for $\alpha_e^2 = 1.44$ and $\alpha_m = 0.10, 0.135, 0.17$.

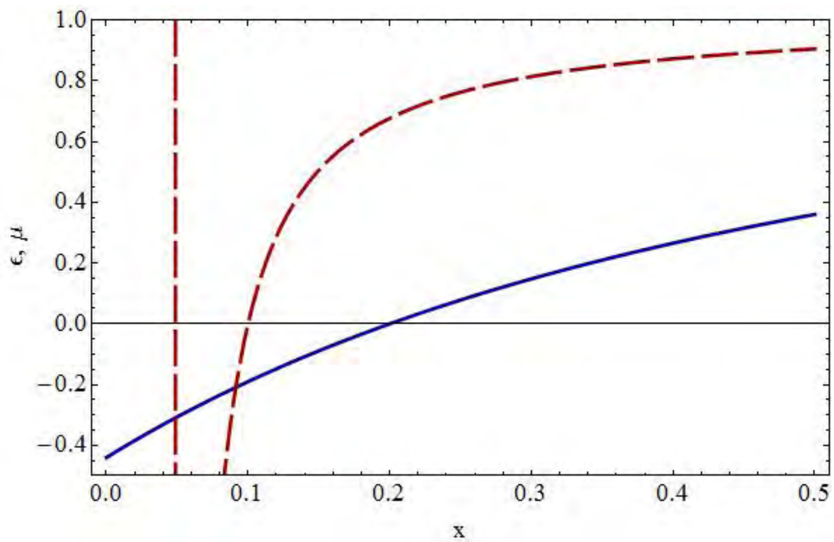

Figure 4.1: The permittivity (solid line) and permeability (dashed line) of MPFG versus dimensionless frequency (x) for $\alpha_e^2 = 1.44$ and $\alpha_m = 0.10$.

(I) $\alpha_e^2 = 1.44$ **and** $\alpha_m = 0.1$

In Fig. 4.1 the permittivity ϵ, eq. (4.3.4), versus the dimensionless frequency x of the MPFG system for $\alpha_e^2 = 1.44$ is plotted in the frequency range $0 < x < 0.50$. It is observed that for the frequency range $0 < x < 0.20$, $\epsilon(x)$ is negative while it is positive for $x > 0.20$. In addition, Fig. 4.1 shows the graph of the permeability μ, eq. (4.3.5), of the MPFG versus the dimensionless frequency x for the parameter

$\alpha_m = 0.10$ in the frequency range $0 < x < 0.50$. In this case, when $0 < x < 0.10$ and $x > 0.20$, the permeability $\mu(x)$ of the MPFG is positive whereas it is negative for the interval of $0.05 < x < 0.10$. Therefore, in interval $0.05 < x < 0.10$, both the permittivity and permeability of the MPFG system are simultaneously negative and consequently the refractive index is negative in that frequency domain, enabling the MPFG to behave as LHM. On the other hand, both the permittivity and permeability are simultaneously positive for $x > 0.20$ and the resulting refractive index becomes positive and the MPFG behaves as RHM.

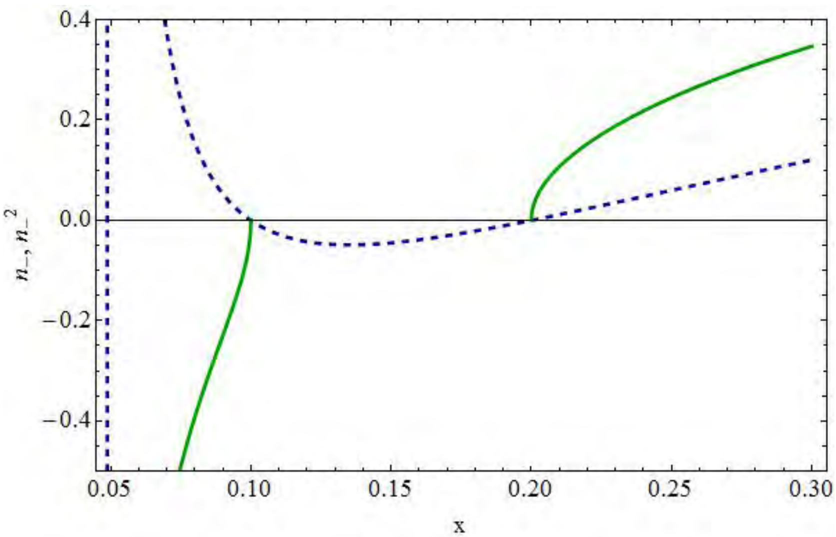

Figure 4.2: The square of the refractive index (n_-^2) and the refractive index (n_-) of the usual MPFG versus the dimensionless frequency (x) for $\alpha_m = 0.10$ and $\alpha_e^2 = 1.44$.

In Fig. 4.2, the refractive index ($n_-(x)$) and the square of the refractive index ($n_-^2(x)$) of MPFG versus the dimensionless frequency (x), plotted using eq. (4.3.6), for $\alpha_m = 0.1$ and $\alpha_e^2 = 1.44$ in the range of $0.05 < x < 0.30$ are depicted. It shows the frequency ranges where the MPFG behaves as RHM, LHM, and nontransparent. In the frequency range $0.1 < x < 0.20$ the square of index of refraction is negative and the corresponding index of refraction is imaginary. As a result, for this frequency domain the medium is nontransparent to electromagnetic waves. For range of $0.05 < x < 0.1$ the square of index of refraction is positive (the product of negative electric permittivity and negative magnetic permeability) and the index of refraction is negative so that the medium behaves as left-handed medium. Note that the 'extent' of the LHM frequency domain is $\Delta x = 0.05$ and that of the nontransparent domain is $\Delta x' = 0.10$. On the other hand, for frequencies $x > 0.2$, both the square of the index of refraction and the index of refraction are positive. Since both ϵ and μ are simultaneously positive in this frequency range (see Figs. 4.1), the MPFG system behaves as right-handed (ordinary) medium.

(II) $\alpha_e^2 = 1.44$ **and** $\alpha_m = 0.135$

Figure 4.3 depicts the graphs of the refractive index and the square of the refractive index of MPFG versus the dimensionless frequency (x) for $\alpha_m = 0.135$ and $\alpha_e^2 = 1.44$, which is plotted according to eq. (4.3.6). Similar to the previous case, it displays the frequency domains at which the MPFG behaves as RHM, LHM, and nontransparent. For the frequency range $0.065 < x < 0.135$ the square of index of refraction is positive and the corresponding index of refraction is negative since both ϵ and μ are simultaneously negative in the given domain. Consequently, for $0.065 < x < 0.135$ the medium behaves as LHM. Here, the extent of the LHM frequency domain is $\triangle x = 0.07$. Compared with the case for $\alpha_m = 0.10$ the extent of the LHM domain increases by $\Delta x = 0.07$ with an increase of α_m. Correspondingly, the nontransparent domain decreases to $\Delta x' = 0.065$. For frequencies $x > 0.2$, both the square of the index of refraction and the index of refraction are positive with both $\epsilon > 0$ and $\mu > 0$ so that the MPFG system behaves as RHM. Finally, the medium is nontransparent to electromagnetic waves in the frequency range $0.135 < x < 0.20$, since the square of index of refraction is negative and the corresponding index of refraction is imaginary.

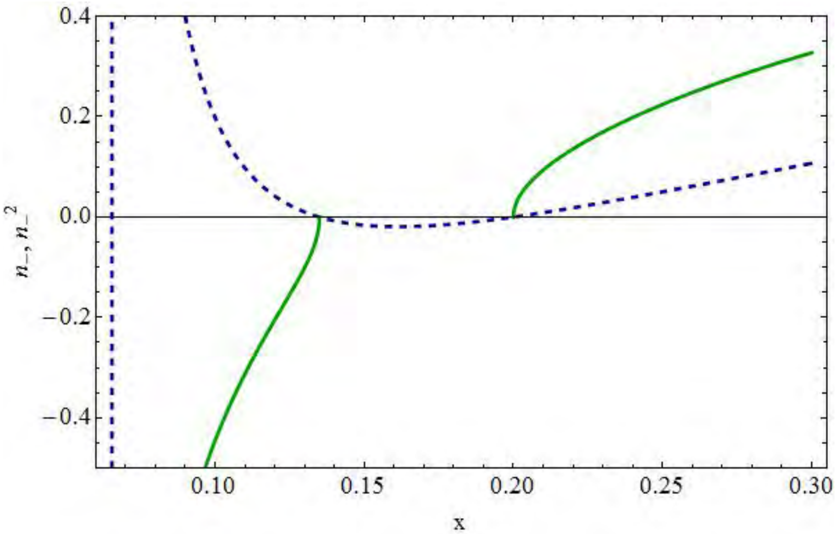

Figure 4.3: The square of the refractive index (n_-^2) and the refractive index (n_-) of the usual MPFG versus the dimensionless frequency (x) for $\alpha_m = 0.135$ and $\alpha_e^2 = 1.44$.

(III) $\alpha_e^2 = 1.44$ **and** $\alpha_m = 0.17$

Figure 4.4 depicts the graphs of $n_-^2(x)$ and $n_-(x)$ of the MPFG system versus x for $\alpha_m = 0.17$ and $\alpha_e^2 = 1.44$. Here, the nontransparent frequency domain is $0.17 < x < 0.20$, the RHM domain is $x > 0.2$, and the LHM domain is $0.08 < x < 0.17$. In this case, the extent of the LHM frequency domain is $\triangle x = 0.09$, which shows that it increases by $\Delta x = 0.04$ as α_m increases from 0.10 to 0.17. Here also, the nontransparent domain decreases by $\Delta x' = 0.03$. Note also that as the 'extent' of the LHM domain is increased with an increase in α_m (for a fixed α_e), the nontransparent frequency domain shrinks. This indicates that for a fixed value of the electric parameter α_e, there is a critical value of α_m for which the nontransparent region observed in Figs. 4.2-4.3 can be completely eliminated. That is, indeed, the case as it is demonstrated

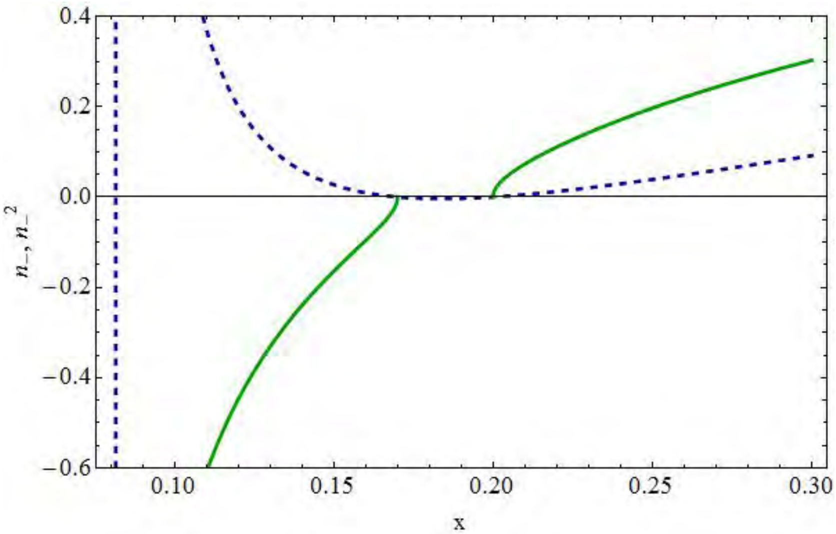

Figure 4.4: The square of the refractive index (n_-^2) and the refractive index (n_-) of the usual MPFG versus the dimensionless frequency (x) for $\alpha_m = 0.17$ and $\alpha_e^2 = 1.44$.

in the next Section for the so-called tuned MPFG.

4.5 Refractive Index of the Tuned MPFG

The MPFG system is a promising composite medium for LHM applications since it allows the possibility of control (tunability) over the materials parameters. That is, the parameter of the electric subsystem, α_e can be tuned by varying the concentration of the electron-ions n and the applied constant magnetic field ($\vec{H}_0$). Similarly, the parameter of the magnetic subsystem, α_m, can be tuned by varying the density number (N_g), the size a of the grains as well as the applied constant magnetic field ($\vec{H}_0$).

Note that the zeros $n_-(x, \pi/2)$ can be tuned in such a way that the nontransparent region that appears for arbitrary values of α_e and α_m. Here we referred to the system as the 'usual MPFG' when α_e and α_m assumes arbitrary values. On the other hand, the system is referred as 'tuned MPFG' when the zeros of $\epsilon(x)$ and $\mu(x)$ coincides. It is seen from eqs. (4.3.4) and (4.3.5) that the zeros of $n_{-}(x, \pi/2)$ are at $x_0 = \alpha_e - 1$ with $\alpha_e > 1$, and $x_0 = \alpha_m$. This means that the zeros of $\epsilon(x)$ and $\mu(x)$ coincide when $x_0 = \alpha_m = \alpha_e - 1$; so that we referred to the system as tuned MPFG. For these particular values eq. (4.3.6) may be written in terms of α_e as follows:

$$n_{-}(x, \pi/2) = \pm\sqrt{\left(1 - \frac{\alpha_e^2}{(x+1)^2}\right)\left[\frac{[x - (\alpha_e - 1)][x^2 + 2 + (\alpha_e - 1)]}{x^2 + 2x - (\alpha_e - 1)}\right]}, \tag{4.5.1}$$

or

$$n_{-}(x, \pi/2) = \pm\sqrt{\left(1 - \frac{\alpha_e^2}{(x+1)^2}\right)\left[\frac{[x - \alpha_e + 1)][x^2 + \alpha_e + 1)]}{(x+1)^2 - \alpha_e}\right]}. \tag{4.5.2}$$

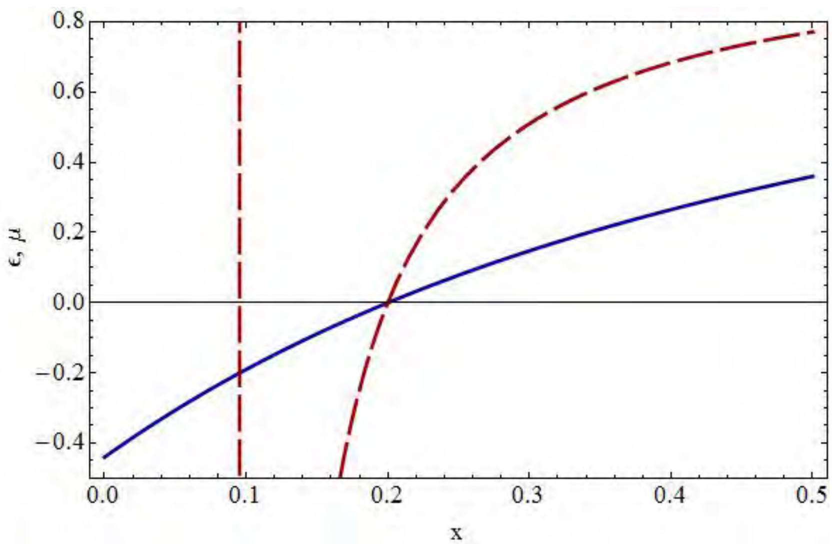

Figure 4.5: The permittivity (solid line) and permeability (dashes line) of the tuned MPFG versus dimensionless frequency (x) for $\alpha_e^2 = 1.44$ and $\alpha_m = 0.20$.

4.6 Analysis of the Tuned MPFG

The system is referred as 'tuned' MPFG when the zeros of $\epsilon(x)$ and $\mu(x)$ coincides. It is seen from eqs. (4.3.4) and (4.3.5) that the zeros of $n_{-}(x, \pi/2)$ are at $x_0 = \alpha_e - 1$ with $\alpha_e > 1$, and $x_0 = \alpha_m$. So, we set the zeros of $\epsilon(x)$ and $\mu(x)$ to be $\alpha_e^2 = 1.44$ and $\alpha_m = 0.20$. The graphs of ϵ, μ, n_{-}, and n_{-}^2 as a function of the dimensionless frequency, x are shown below.

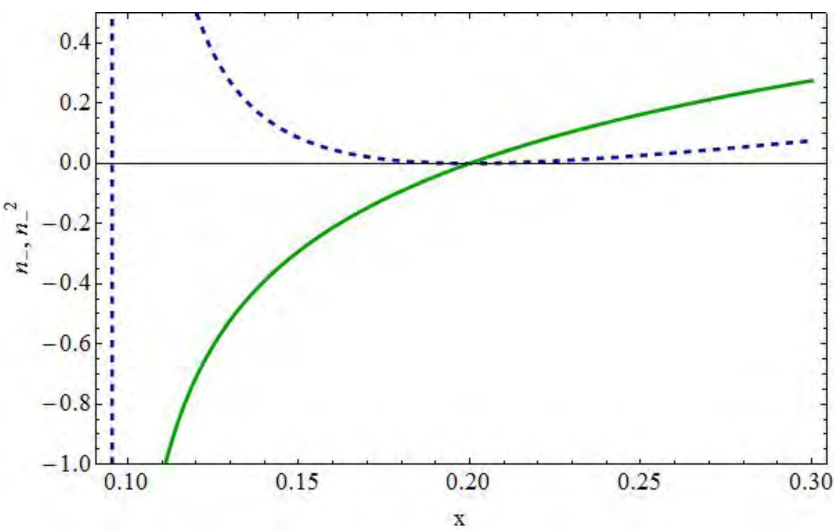

Figure 4.6: The square of the refractive index (n_{-}^2) and the refractive index (n_{-}) of the tuned MPFG versus the dimensionless frequency (x) for $\alpha_m = 0.20$ and $\alpha_e^2 = 1.44$.

In Fig. 4.5, the permittivity ϵ (solid line) and permeability μ (dashes line) of MPFG versus the dimensionless frequency (x) for $\alpha_e^2 = 1.44$ and $\alpha_m = 0.20$ are depicted. As we have seen from the graph, for the frequency domain $0.10 < x < 0.20$ the electric permittivity ϵ as well as the permeability μ are simultaneously negative. On the other hand, for frequencies $x > 0.20$, both ϵ and μ are simultaneously positive. Consequently, the MPFG is expected to behave as LHM for $0.10 < x < 0.20$ and RHM for $x > 0.20$.

MPFG	α_e^2	α_m	**Frequency**	Δx	$\Delta x'$	η_-	η_-^2	**Remark**
Usual	1.44	0.10	$0.05 < x < 0.10$	0.05		-ve	+ve	LHM
			$0.10 < x < 0.20$		0.10	Im	-ve	NT
			$x > 0.20$			+ve	+ve	RHM
Usual	1.44	0.135	$0.065 < x < 0.135$	0.07		-ve	+ve	LHM
			$0.135 < x < 0.20$		0.065	Im	-ve	NT
			$x > 0.20$			+ve	+ve	RHM
Usual	1.44	0.17	$0.08 < x < 0.17$	0.09		-ve	+ve	LHM
			$0.17 < x < 0.20$		0.03	Im	-ve	NT
			$x > 0.20$			+ve	+ve	RHM
Tuned	1.44	0.20	$0.095 < x < 0.20$	0.105		-ve	+ve	LHM
			$x > 0.20$		0	+ve	+ve	RHM

Table 4.1: Summary of the results of the usual and tuned MPFG for different values of α_m and α_e^2, for perpendicular propagation of EMWs. Here, Δx and $\Delta x'$ are the LHM and nontransparent domains, respectively, and $-ve$, $+ve$, Im, and NT mean 'negative', 'positive', 'imaginary', and 'nontransparent', correspondingly.

Figure 4.6 shows the refractive index ($n_-(x)$) and the square of the refractive index ($n_-^2(x)$) of the tuned MPFG versus the dimensionless frequency (x) for $\alpha_m = 0.20$ and $\alpha_e^2 = 1.44$ in the range of $0.09 < x < 0.30$. The zeros of $n_-(x, \pi/2)$ are at $x = \alpha_m$ and $x = \alpha_e - 1$, with $\alpha_e > 1$. Note that the parameters in Fig. 4.6, $\alpha_m = \alpha_e - 1 = 0.20$ so that we referred to the system as tuned MPFG. As we have seen in Fig. 4.6, for frequency of $0.095 < x < 0.20$ the square of index of refraction is positive while the index of refraction is negative. Therefore, the MPFG is transparent and it is said to be left-handed medium (LHM). For the case of $x > 0.20$ both the index of refraction and square of index of refraction ($n_-(x)$) are positive and the material behaves as right-handed medium (RHM). Consequently, in the tuned MPFG the nontransparent region for $x > 0.05$ that appears in the usual MPFG is completely avoided ($\Delta x' = 0$) so that the system becomes transparent for all frequencies $x > 0.095$. In deed, the MPFG behaves as LHM for $0.095 < x < 0.20$ whereas it behaves as RHM for $x > 0.20$.

4.7 Summary

In this Chapter we have plotted the graphs of the permittivity, permeability, refractive index, and the square of the refractive index as a function of the dimensionless frequency for the usual and tuned MPFG. For the usual MPFG, the values of the parameters of the electric and magnetic subsystem used are $\alpha_e^2 = 1.44$ and $\alpha_m = 0.10, 0.135, 0.17$, whereas for the tuned MPFG $\alpha_e^2 = 1.44$ and $\alpha_m = 0.20$ are used. In the case of the usual MPFG, we identified three frequency domains, namely, the LHM, RHM, and nontransparent domains. For the tuned MPFG, the nontransparent frequency domain is eliminated so that the MPFG system becomes transparent over the entire frequency domain of interest with the system showing LHM as well as RHM materials property. The main results obtained are summarized in Table 4.1.

Chapter 5

Conclusions

We investigated the propagation of EMWs in MPFG when the wave is propagating perpendicular to the applied constant magnetic field. We identified two sets of MPFG system, namely the usual MPFG where the parameters α_e and α_m assumes arbitrary values and the tuned MPFG where the parameters $x_0 = \alpha_m = \alpha_e - 1$, with the condition that $\alpha_e > 1$ that corresponds to the zeros of ϵ and μ coincides. In both cases, we showed that the MPFG behaves as LHM and RHM in different frequency ranges.

In the usual MPFG in addition to the LHM and RHM frequency domains, there is a region that is nontransparent to the propagation of EMWs. For a fixed value of $\alpha_e^2 = 1.44$, the nontransparent region becomes narrower and narrower with an increase of α_m from 0.10 to 0.17. Further increase of, i.e., to $\alpha_m = 0.20$, results a complete elimination of the nontransparent region allowing the propagation of EMWs over the entire frequency range. However, the MPFG system behaves as left-handed medium for the frequency domain $0.095 < x < 0.20$, whereas it behaves as right-handed medium for frequencies $x > 0.20$.

The nontransparent region, where the index of refraction is imaginary, is shown to be functions of the electric and magnetic subsystems of the MPFG via the parameters α_m and α_e. This region is avoided by tuning the parameters of α_m and α_e. In our case, the tuned MPFG is obtained when the condition $\alpha_e = 1 + \alpha_m$ is satisfied. In particular, choosing the values $\alpha_m = 0.20$ and $\alpha_e^2 = 1.44$, the index of refraction $(n_-(\omega, \pi/2))$ of the tuned MPFG is negative for the range of $1.095\omega_c < \omega < 1.20\omega_c$ and positive for frequencies above $1.20\omega_c$. Thus, the MPFG becomes transparent for all frequencies $\omega > \omega_c$.

In brief, by tuning the material parameters of the electric and magnetic subsystems, the different regions, namely, the left-handed medium, right-handed medium, and nontransparent frequency domains can be varied. The analysis and results obtained show that the MPFG medium possesses 'tunable' negative index of refraction near the electron cyclotron frequency enabling it to be a left-handed metamaterial. With the current technological advance in the field of plasma systems, the MPFG medium may be fabricated in a laboratory, and hence makes the medium a potential candidate for practical applications that employs LHM.

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