

**VIBRATION ANALYSIS AND DESIGN OF BLOCK-TYPE MACHINE
FOUNDATIONS INTERACTING WITH SOIL**

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LIST OF SYMBOLS

$Q_z, \theta_z, \text{Teta Y, Teta X}$ = Torsional and rocking displacement amplitudes

$C_j (j = z, y, x)$ = Dashpot coefficients in the z, y and x directions

k' = Dynamic modulus of sub-grade reaction

$\bar{K}_j (j = z, y, x)$ = Dynamic stiffness in the z, y and x directions

$\delta_x, \delta_y, \text{Delta X, Delta y}$ = Lateral displacement amplitudes

M_z = Moment around the vertical -axis

U_z = Vertical displacement amplitudes

B = Half the width of the machine foundation

C_c = Critical damping

$C_{rx1}, C_{rx2}, C_{rx3}$ = Rocking radiation dashpot coefficient in the x-direction

$C_{ry1}, C_{ry2}, C_{ry3}$ = Rocking radiation dashpot coefficient in the y-direction

$C_{tz1}, C_{tz2}, C_{tz3}$ = Torsional radiation dashpot coefficient in the z-direction

C_{x1}, C_{x2}, C_{x3} = Radiation dashpot coefficient in the x-direction

C_{y1}, C_{y2}, C_{y3} = Radiation dashpot coefficient in the y-direction

C_{z1}, C_{z2}, C_{z3} = Radiation dashpot coefficient in the z-direction

D = Damping ratio

D =Depth of the foundation

d_1 =Average height of the side wall in good contact with the surrounding soil

E =Young's modulus

f =Frequency of oscillation in cyc./sec.

f_n =Cyclic frequency

f_{nr} =Reduced natural frequency

F_o =Exciting force

G =Shear modulus

I_{ox}, I_{oy} , = Mass moment of inertia in the x, and y-directions

J_z =Moment of inertia of both foundation and machine about z-axis

k =spring constant

$k_{rx1}, k_{rx2}, k_{rx3}$ Rocking dynamic stiffness coefficient around the x-direction

$k_{ry1}, k_{ry2}, k_{ry3}$ = Rocking dynamic stiffness coefficient around the y-direction

k_{t1}, k_{t2}, k_{t3} = Horizontal dynamic stiffness coefficient around the x-direction

k_{y1}, k_{y2}, k_{y3} = Horizontal dynamic stiffness coefficient around the y-direction

k_{z1}, k_{z2}, k_{z3} = Vertical dynamic stiffness coefficient around the z-direction

L =Half the length of the machine foundation

L_R =Measured wavelength

M =Magnification factor

M_c =Constrained modulus

q_0 =Average vertical contact pressure

r =Frequency ratio

t =Time

V_s =Shear wave velocity

Z =Displacement amplitude

Z_{st} =Static displacement

γ =Shear strain

τ =Shear stress

ν =Poisson's ratio

ρ =Mass density

ω =Circular excitation frequency

ω_n =Natural circular frequency

\ddot{O}_x, \ddot{O}_y = Rocking impedance in the x and y directions

\ddot{O}_t = Torsional impedance

$\ddot{O}_x, \ddot{O}_y, \ddot{O}_z$ =Swaying impedances in the z, x and y-directions

$W_{nz}, W_{nx}, W_{ny}, W_{nt}$ =Natural frequencies

D_z = Damping ratio in the z-direction

D_x = Damping ratios in the x-direction

D_r = Damping ratios for rocking mode

D_t = Damping ratios for torsional mode

r_z = Frequency ratios in the z-direction

r_x = Frequency ratios in the x-direction

r_{ry} = Frequency ratios for rocking mode

r_t = Frequency ratios for torsional mode

M_{zz} = Magnification factor in the vertical direction

M_{xx} = Magnification factors in the horizontal direction

M_{ry} = Magnification factors for rocking mode

M_t = Magnification factors for torsional mode

ABSTRACT

Until recently, the design and analysis of block-type machine foundations did not properly consider impedance functions. This thesis aims at incorporating impedance functions by making use of recently compiled closed form expressions and dimensionless graphs for the purpose of determining dynamic stiffness and dashpot coefficients. Based on these expressions and the well-known solutions of the dynamic equations of motions, a computer programme is written in FORTRAN.

The general requirements and criteria to be fulfilled for machine foundations are compiled. The input soil parameters essential for the design including shear modulus, G , Poisson's ratio, ν , damping ratio, D , spring stiffness K , and shear wave velocity, V_s , are reviewed.

The basic concepts in vibration of structures like frequency, free vibration (undamped and damped), forced vibration and foundation vibration are discussed. In addition, the performance requirements and the basic steps employed in the design of a machine foundation are presented. The older approaches of machine foundation analysis are also reviewed.

The basic steps and relations used to calculate the uncoupled vertical and torsional vibration amplitudes as well as the coupled rocking and horizontal vibration amplitudes are provided.

Based on the basic relationships for the vibration amplitudes and the relatively simplified and well-compiled recent works of G. Gazatas [2] for the determination of static stiffness, dynamic

foundation soil stiffness as well as radiation dashpot coefficient, a computer program in FORTRAN is written to analyze block-type machine foundations for the following four conditions in a rational approach.

- (a) Foundation on the surface of a homogeneous half space
- (b) Partially or fully embedded foundation in a homogeneous half space.
- (c) Foundations on the surface of a homogeneous stratum overlying the bedrock.
- (d) Partially or fully embedded foundations in a homogeneous stratum overlying the bedrock.

Finally, practical examples are solved for the above four cases using the programme and the results are checked against each other. The same example is also solved using the classical method even though this method does not appropriately incorporate the impedance functions. A comparison of the results obtained using the classical approach and the more rational method adopted in this work indicates that the latter is a substantial improvement over the former. A comparison of the results obtained showed that embedding a foundation is a very effective way to reduce to the acceptable levels of the anticipated amplitudes of vibration, especially if these amplitudes arise due to rocking or torsion. Such an improvement would be effected mainly by the increase in radiation damping produced by waves emanating from the vertical sidewalls.

1. INTRODUCTION

The design of machine foundations is more complex than that of foundations, which support static load only. Loads acting on such foundations are dynamic in nature. These loads may result from various causes such as vibratory motion of machines, movement of vehicles, impact of hammers, earthquakes, wind waves, nuclear blasts, mine explosions, pile driving etc. It is, therefore, necessary to understand the effects of dynamic forces in the foundation soil.

In general a machine foundation weighs several times as much as the machine it supports. Also a dynamic load associated with the moving parts of a machine is generally small as compared to its static load. However, in machine foundations, dynamic loads act repetitively over a very long period of time. It is therefore necessary that the soil should be elastic, or else deformation will increase with each cycle of loading until the amplitude of deformation becomes larger and out of the acceptable limit. The amplitude of motion of a machine at its operating frequency is the most important parameter to be determined in designing a machine foundation, in addition to determining the system's natural frequency.

There are many kinds of machines that generate different types of time-dependent forces. The three most important categories are:

(1) Reciprocating Machines: Machines that produce periodic unbalanced force (such as compressor and reciprocating engines) belong to this category. The operating speeds of such machines are usually

less than 600 r.p.m. For analysis of their foundations, the unbalanced forces can be considered to vary sinusoidally.

(2) Impact machines: Machines that produce impact loads like forging hammers and punching press are included in this category. Their speeds of operations are usually from 60 to 150 blows per minute. In these machines, the dynamic force attains a peak value in a very short time and then dies out gradually.

(3) Rotary machines: High-speed machines like turbo generators or rotary compressors may have speeds of more than 300 r.p.m and up to 10,000 r.p.m. Foundation

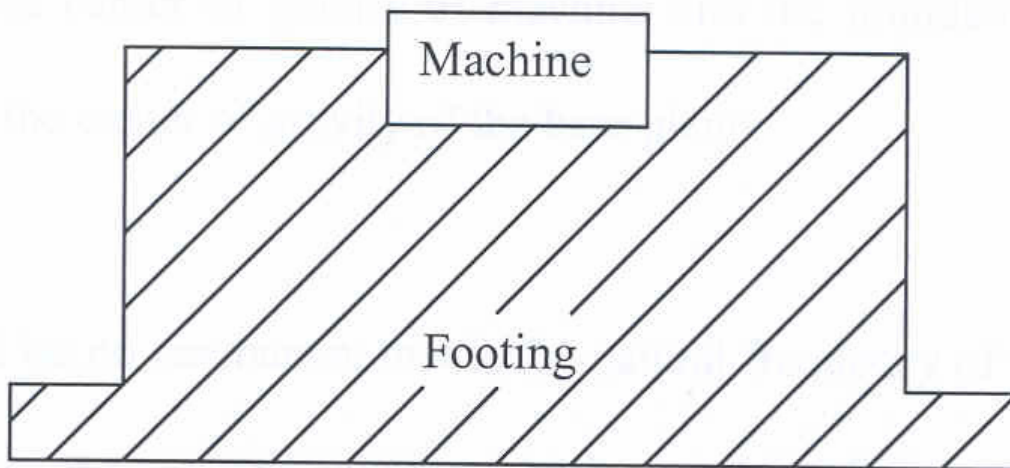


Figure 1.1 Block-type machine foundation

Because this type of foundation is easy for construction and very commonly in use, this thesis is concerned with the analysis of such foundations, but taking into account that they can generally be embedded in a layered formation. Recently developed closed-form expressions for the dynamic stiffness and dashpot coefficients of the underlying soil are employed unlike the conventional approach, which

involve some degree of empiricity to estimate these quantities and add arbitrary magnitude of soil mass in addition to the actual mass of the machinery and its foundation.

1.1 General requirements of machine foundation

The following requirements should be fulfilled from the design point of view of machine foundations [7]

- (a) The foundation should be able to carry the superimposed loads without causing shear or crushing failure of the underlying soil.
- (b) The settlement should be within the permissible limits.
- (c) The combined center of gravity of machine and the foundation should be on the vertical line passing through the center of gravity of the base plane.
- (d) There should be no resonance; that is the natural frequency of the foundation-soil system should be either too large or too small compared to the operating frequency of the machines. For low-speed machine, the natural frequency should be high and vice-versa.
- (e) The amplitude of motion at operating frequencies should not exceed the limiting amplitude, which is generally specified by machine manufacturers. If the computed amplitude is within tolerable limit, but is close to resonance, it is important that this situation be avoided.

(f) Where possible the foundation should be planned in such a manner as to permit a subsequent alteration of natural frequency by changing base area or mass of the foundation as may subsequently be found necessary.

From the practical point of view, the following requirements should be fulfilled [7]

(a) The ground-water level should be as low as possible and ground-water level should be at least deeper by one-fourth of the width of foundation below the base plane. This limits the vibration propagations, ground water being a good conductor of vibration waves, especially P-waves

(b) Machine foundations should be separated from adjacent building components by means of expansion joints.

(c) Any steam or hot air pipes, embedded in the foundation must be properly isolated

(d) Machine foundation should be taken to a level lower than the level of the foundations of adjacent buildings.

(e) The foundation must be protected from machine oil by means of acid resistant coating or suitable chemical treatment.

1.2 Design Data

The specific data required for design vary depending upon the type of machine. The general requirements of data for the design of machine foundations are, however, as follows:

(i) Loading diagram showing the magnitude and position of static and dynamic loads exerted by the machine on its foundation.

- (ii) Power of engine and the operating speed.
- (iii) Diagram showing the embedded parts, openings, grooves for foundation bolts etc.
- (iv) Nature of soil and its static and dynamic properties.

1.3 Permissible Vibration Amplitudes

The manufacturers of the machinery generally define the permissible amplitudes. The permissible amplitude of a machine foundation is governed by the relative importance of the machine and the sensitivity of neighboring structures to vibration.

If manufacturer's data do not contain the permissible amplitude, the values shown in Figure 3.6 are generally taken as a guide for various limits of frequency and amplitude.

2 DESIGN PARAMETERS

2.1 General

In the design of machine foundation, the geometric properties of the foundation block and the physical properties of the underlying soil are the basic properties, which should be determined first.

The geometric properties of the block include center of gravity, moments of inertia and mass moments of inertia. The physical properties of the soil include damping and stiffness characteristics of the supporting soil formation.

2.2 Evaluation of Soil parameters

In order to evaluate the spring stiffness and the corresponding damping ratios required for analysis of dynamic structure-soil interaction problems, it is necessary to determine relevant values for the soil parameters including dynamic shear modulus, G , Poisson's ratio, ν , and mass density, ρ .

2.2.1 Shear modulus (G)

Unbalanced loads in vibrating machinery produce shear strains in the supporting soil that are usually of a much smaller magnitude than the strains produced by static loading. The mechanism governing the stress-strain behavior of soils at small strains involves mainly the stress-deformation characteristics of the soil particle contacts and is not controlled by the relative slippage of particle associated with longer strains. As a consequence, the stress-deformation behavior of soil is much stiffer at very small strain levels than at usual static strain levels. It is therefore inappropriate to obtain a shear modulus directly

from a static stress-deformation test, such as a laboratory triaxial compression test or a field plate-bearing test, unless the stress and strains in the soil can be measured accurately for very small values of strain [5].

Even at very small values of strain, the stress-strain relationship is non-linear. Therefore, it becomes appropriate to define the shear modulus as an equivalent linear modulus having the slope of the line joining the extremities of a closed loop stress-strain curve shown in Figure 2.1.

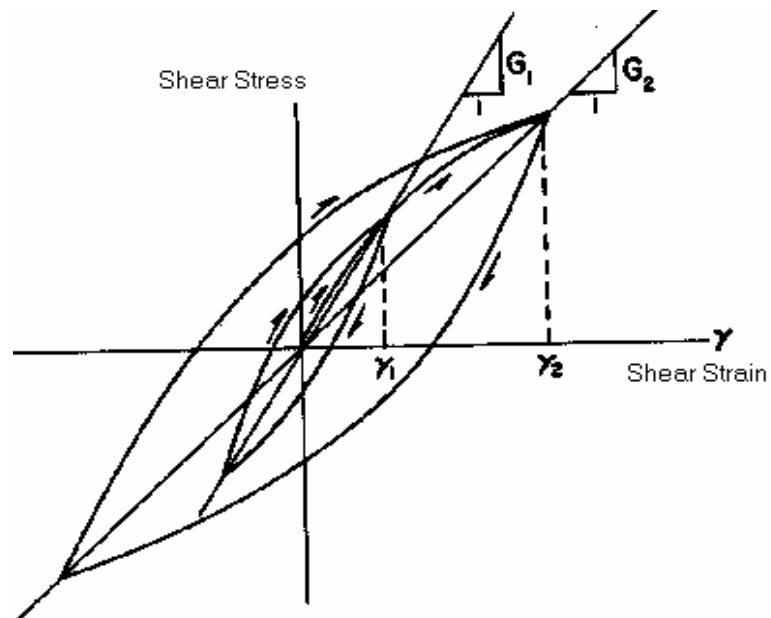


Figure 2.1 Hypothetical shear stress-strain curve for soils [5]

It is obvious that the shear modulus so defined is strain dependent, and that in order to conduct an equivalent linear analysis; it is necessary to know the approximate strain amplitude in the soil. For conditions of controlled applied strain, the ordinates of the shear stress in Figure 2.1 and therefore G , will vary (usually decrease) with number of cycles applied until a stable condition is reached. Hence,

only the stable equivalent linear value of shear modulus will be considered and will be termed the shear modulus.

Richart et al [4] has described at least eight variables that influence the shear modulus of soil. These are:

1. Amplitude of dynamic strain
2. Mean effective stress and length of time since the stress is applied
3. Void ratio
4. Grain characteristics and structure of the soil
5. Stress history
6. Frequency of vibration
7. Degree of saturation
8. Temperature.

Regarding Variable 1, the amplitude of strain due to the dynamic component of loading should be considered when evaluating the shear modulus in normal practice, as superimposed static strain levels have a relatively minor effect on G unless the static strains are of a very large magnitude not usually present in a foundation for vibrating machinery.

The first five variables are considered directly or indirectly in the procedure for obtaining G but the variables from 6 to 8 are generally of secondary importance, although the temperature of the soil may change G considerably near the freezing point of the soil.

Soil shear modulus may be determined by field measurements, laboratory measurements and use of published correlations that relate shear modulus to other more easily measured properties. The field and laboratory methods are described below.

(a) **Field procedures:** Two widely used in in-situ procedures are the steady-state oscillators test and the cross-hole test. From these tests, the shear wave velocity is determined and the shear modulus of the soil is obtained from the basic relation:

$$G_{\max} = \rho V_s^2 = \rho f^2 L_R^2 \quad (2.1)$$

Where G_{\max} = Shear modulus (at very low strain level occurring in the test)

ρ = Mass density of the soil

f = Frequency of oscillation in cycles per second of the shear wave

L_R = Measured wavelength of the shear wave

V_s = Shear wave velocity.

(b) **Laboratory procedures:** The shear modulus can be evaluated from very low-amplitude cyclic or simple shear tests. Resonant column test is also an accurate means of obtaining G_{\max} . It has found generally wide acceptance because of its relative simplicity. In most common type of resonant column tests, a solid cylindrical column of soil is excited either longitudinally or torsionally at low amplitude within a cell in which an appropriate confining pressure has been applied. The exciting frequency is varied and the amplitude of deformation in the soil is monitored at each exciting frequency in order to determine the resonant frequency of the soil column. Simple elasticity equations for vibrating rods are then used to compute G_{\max} .

2.2.2 Damping ratio (D)

Damping in a soil-foundation system consists of a geometric component, which is a measure of energy radiated away from the immediate region of the foundation into the soil, and a material damping within the soil, which is a measure of energy lost as a result of hysteretic effects.

The geometric damping for all modes of oscillations are given in the form of equations and dimensionless graphs in appendix A and B. Material damping is defined in Figure 2.3. It is seen to be proportional to the ratio of A_L , the area of the soil hysteretic loop in simple shear (energy lost), to A_T the hatched area (energy input).

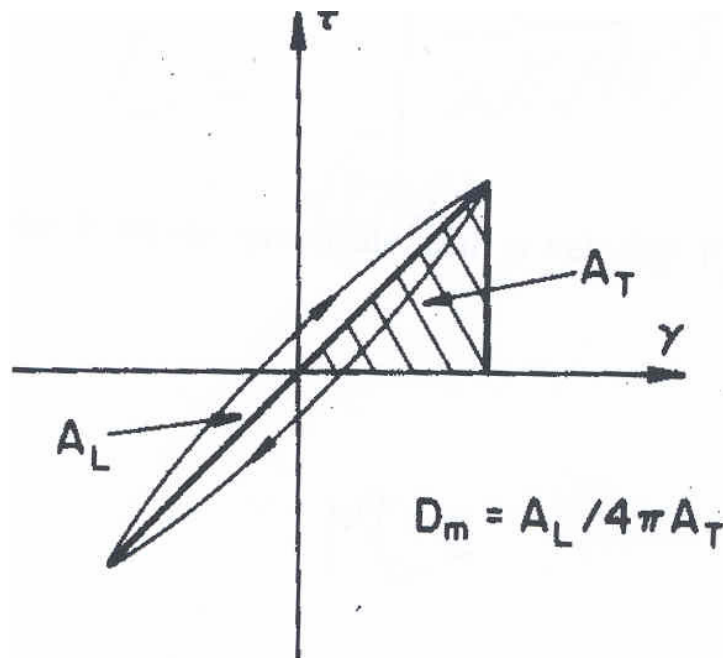


Figure 2.3 Definition of material damping [5]

Material damping ratios can be, for example obtained as a part of resonant column testing. After the soil has been vibrating in a steady-state condition, the exciter is stopped and the soil vibrations are monitored as they decay. The displacement-time relationship is essentially sinusoidal but with the amplitudes decreasing with time. If two successive amplitudes z_1 and z_2 , are measured then the damping ratio can be obtained from the logarithmic decay as:

$$D_m = \left(\ln\left(\frac{z_1}{z_2}\right) \right) \left(4\pi^2 + \left(\ln\left(\frac{z_1}{z_2}\right) \right)^2 \right)^{-0.5} \quad (2.2)$$

2.2.3 Shear Wave Velocity

A vibrating foundation emits shear and dilatational waves into the supporting ground. The shear waves, denoted S- waves, propagate with a velocity V_s , that is controlled by the shearing stiffness G and the mass density ρ of the soil in accordance with

$$V_s = \sqrt{\frac{G}{\rho}} \quad (2.3)$$

Dilatational waves denoted by P waves, propagate with a velocity V_p related to the constrained modulus M_c

$$V_p = \sqrt{\frac{M_c}{\rho}} \quad (2.4)$$

For an elastic material, M_c depends on the shear modulus G and Poisson's ratio ν of the soil so that

$$V_p = V_s \sqrt{\frac{2(1-\nu)}{1-2\nu}} \quad (2.5)$$

Therefore V_s and V_p or G and M_c or G and ν , are the equivalent pairs of soil parameters relevant to wave propagation phenomena. Note that waves other than S and P also arise in the ground under an oscillating foundation, most notably Rayleigh and Love waves. All these other waves, however, also relate to G and ν as they are the outcome of combinations (“interference”) of S and P waves.

Shear waves can propagate only through the material skeleton of a soil (pore water offer no shear resistance). Dilatational (P) waves can propagate through both the mineral skeleton and the pore water. Since water is far less compressible than any soil skeleton, P-waves in fully saturated soils are essentially transmitted solely through the water phase with a velocity V_p that is of the order of or somewhat larger than, $V_w=5000\text{m/sec}$, the velocity of sound wave in water.

On the other hand, the percentage of even small amounts of air in the pores might drastically increase the compressibility of the water-air phase. Only the soil skeleton would then resist the induced dilatation. V_p would be essentially the same as P-wave velocity of a dry, but otherwise identical soil sample [5]

2.2.4 Poisson’s Ratio, ν and Soil Density, ρ

Soil-foundation interaction problems are relatively insensitive to the values chosen for ν and ρ . Generally, Poisson’s ratio can be selected based on the predominant soil type using Table-2.1

Soil Type	ν
Saturated clay	0.45-0.50
Partially saturated clay	0.35-0.45
Dense sand or gravel	0.4-0.5
Medium dense sand or gravel	0.3-0.4
Silt	0.3-0.4

Table 2.4 Typical values for Poisson's ratio [7]

It is also possible to obtain a value for Poisson's ratio by measuring independently the shear modulus (G) in a laboratory torsional resonant column test and Young's modulus (E) in a laboratory longitudinal resonant column test. Assuming isotropy, the Poisson's ratio is then estimated from

$$\nu = \frac{E}{2G} - 1 \quad (2.6)$$

Soil mass density value should always be calculated from the total unit weight rather than the buoyant unit weight, because the density term in the mass and inertia ratio equations always represent soil undergoing vibration.

3 VIBRATION ANALYSIS

3.1 Undamped Free Vibration

Figure 3.1 shows mass (m) resting on a spring of stiffness k . The system has one degree of freedom. Let us assume that the system has been set in motion and it vibrates in the vertical direction.

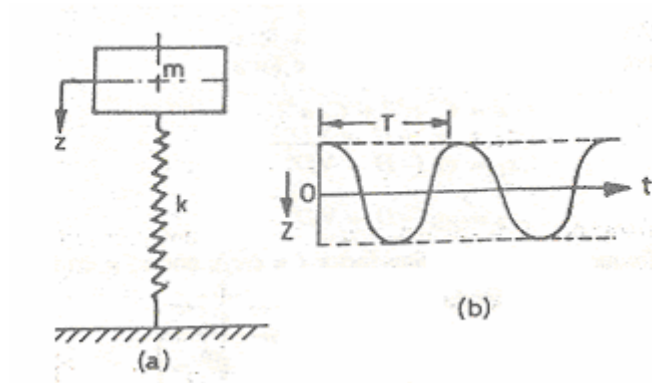


Figure 3.1 Undamped vibration [7]

The equation of motion can be written as:

$$m\ddot{z} = -kz \quad \text{or} \quad m\ddot{z} + kz = 0 \quad (3.1)$$

The solution of equation (3.1) is given by:

$$z = A \cos \omega_n t + B \sin \omega_n t \quad (3.2)$$

Where A and B are arbitrary constants and ω_n is the natural circular frequency (rad./sec.).

The solution can also be written as:

$$z = A \sin(\omega_n t + \alpha) \quad (3.3)$$

Where A and α are constants

Alternatively, the solution can be expressed also as:

$$z = A \cos(\omega_n t - \alpha) \quad (3.4)$$

From equation (3.3),

$$\dot{z} = A \omega_n \cos(\omega_n t + \alpha)$$

$$\text{and } \ddot{z} = -A \omega_n^2 \sin(\omega_n t + \alpha) = -\omega_n^2 z \quad (3.5)$$

Substituting Equation (3.5) into Equation (3.1) gives:

$$\omega_n = \sqrt{\frac{k}{m}} \quad (3.6)$$

This shows us that the greater the mass m, the smaller will be the natural frequency

$$\text{The natural period of vibration is obtained then as, } T = \frac{1}{f_n} = \frac{2\pi}{\sqrt{\frac{k}{m}}} \quad (3.7)$$

Figure 3.1b shows the response curve of the system. It is evident that the cycle repeats after time T (the natural period of vibration).

3.2 Damped Free Vibration

Figure 3.2 shows a rigid mass m resting on a spring of stiffness k and a viscous damper with a damping coefficient c . In this case, there is an additional force due to damping. The equation of motion can be written as:

$$m\ddot{z} + c\dot{z} + kz = 0 \quad (3.8)$$

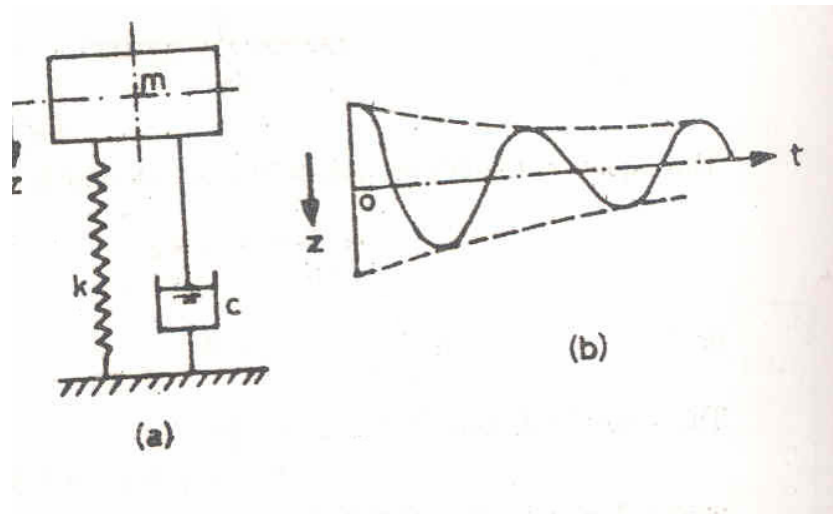


Figure 3.2 : Damped Vibration [7]

A solution of the form $z = e^{st}$ is assumed for Equation (3.8) where s is a constant to be determined and t is the independent time variable: Then $\dot{z} = se^{st}$ and $\ddot{z} = s^2e^{st}$

Substituting these values in Equation (3.8) gives:

$$\left[s^2 + \left(\frac{c}{m} \right) s + \left(\frac{k}{m} \right) \right] e^{st} = 0 \quad (3.9)$$

Since e^{st} is greater than zero for all values of t ,

$$s^2 + \left(\frac{c}{m} \right) s + \frac{k}{m} = 0 \quad (3.10)$$

Equation (3.10) is a quadratic equation having two roots.

$$s_{1,2} = \left(\frac{1}{2m} \right) \left[-c \pm \sqrt{c^2 - 4km} \right] \quad (3.11)$$

Several terms relating various parameters of Equations (3.8) are as follows:

$\omega_n = \sqrt{\frac{k}{m}}$ is called the circular natural frequency of the system in rad./sec.

$c_c = 2\sqrt{km}$ is the critical damping of the system in units of force/velocity:

$D = \frac{c}{c_c} = \frac{c}{2\sqrt{km}}$ is called the damping ratio; and

$\omega_d = \omega_n \sqrt{1 - D^2}$ is named the frequency of oscillation of the system with damping included.

The complete solution of Equation (3.8) is

$$Z = Ae^{s_1 t} + Be^{s_2 t} \quad (3.12)$$

Where A and B are arbitrary constants that depend upon the initial problem conditions.

Equations (3.11) and (3.12) show that the nature of oscillation depends upon the value of c. Four possible cases of c can be considered to illustrate the physical significance of Equation (3.11). Only the two of them are relevant to this work. The critically damped and over-damped conditions.

Case 1: $c^2 = 4km$ (Critically Damped system) The damping corresponding to this case ($c = 2\sqrt{km}$, $c = c_c$, Or $\frac{c}{c_c} = D = 1$) is defined to as critical damping. For this value of $c^2 = 4km$, Equation (3.9) has two equal roots, $s_{1,2} = -\frac{c}{2m}$. In this case, the general solution of the second order differential equation will be:

$$Z = Ae^{-\frac{c}{2m}t} + Bte^{-\frac{c}{2m}t} \quad (3.13)$$

Substituting the value of $\frac{c}{2m} = \frac{2\sqrt{km}}{2m} = \omega_n$ and applying the initial boundary condition, $Z(t=0) = Z_o$ and $\dot{Z}(t=0) = v_o$ in Equation (3.13) gives:

$$Z = \left\{ Z_o + \left[\left(\frac{v_o}{\omega_n} \right) + Z_o \right] \omega_n t \right\} e^{-\omega_n t} \quad (3.14)$$

Case 2 : $c^2 < 4km$ (Underdamped System) In this case, the roots of Equation (3.11) are complex conjugates, and s_1 and s_2 become:

$$s_{1,2} = \omega_n(-D \pm \sqrt{D^2 - 1}) \quad (3.15)$$

Recalling the damping ratio $D = \frac{c}{c_c} = \frac{c}{2\sqrt{km}} = \frac{c}{2\omega_n m}$ and Further substitution of Equation (3.15)

in to Equation (3.12) and converted to a trigonometric form with the aid of Euler's formula

$$e^{\pm i\theta} = \cos \theta \pm i \sin \theta, \text{ gives us:} \quad (3.16)$$

$$Z = e^{-D\omega_n t} (B_1 \sin \omega_n \sqrt{1 - D^2} t + B_2 \cos \omega_n \sqrt{1 - D^2} t) \quad (3.17)$$

Or

$$Z = e^{-D\omega_n t} Y \sin(\omega_n t + \phi) \quad (3.18)$$

Where $B_1 = Y \cos \phi$ and $B_2 = Y \sin \phi$ and $\omega_d = \omega_n \sqrt{1 - D^2}$. The term ω_d is called the damped natural frequency, and Y and ϕ are arbitrary constants to be determined from the initial boundary conditions.

3.3 Forced Vibration

Figure 3.3 shows a rigid mass with a single degree of freedom. The system is damped and subjected to an exciting force $F(t)$. The equation of motion can be written as:

$$m\ddot{z} + c\dot{z} + kz = F(t) \quad (3.19)$$

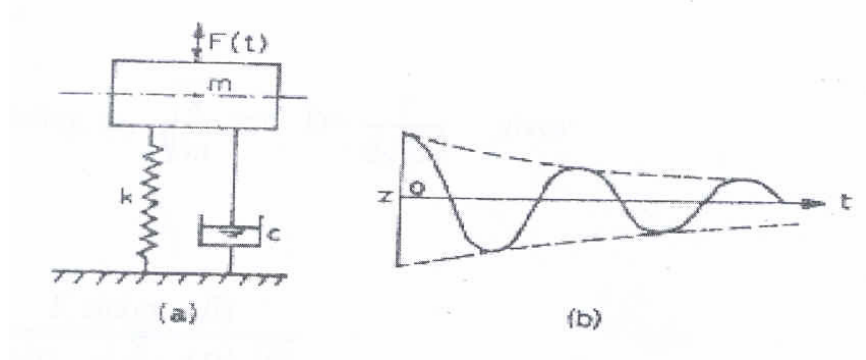


Figure 3.3 : Damped vibration subjected to external force [7]

Assume that the exciting force is sinusoidal of the form $F(t) = F_o \sin \omega t$,

where F_o = Amplitude of the exciting force, and

ω = Circular frequency of exciting force.

$$\text{Thus, } m\ddot{z} + c\dot{z} + kz = F_o \sin \omega t \quad (3.20)$$

The solution of equation (3.20) is

$$Z = e^{-D_{\omega}t} [A \cos(\omega_{nd}t - \alpha)] + \frac{F_o \sin(\omega t - \beta)}{\sqrt{((k - m\omega^2)^2 + c^2\omega^2)}} \quad (3.21)$$

The first part of the solution is transient and dies out after some time. The second part is the steady state response, thus

$$Z = \frac{F_o \sin(\omega t - \beta)}{\sqrt{(k - m\omega^2)^2 + c^2\omega^2}} \quad (3.22)$$

Substituting $\omega_n = \sqrt{\frac{k}{m}}$ and $D = \frac{c}{2\sqrt{km}}$ gives:

$$Z = \frac{F_o \sin(\omega_t - \beta)}{\sqrt{k^2(1-r^2)^2 + 4D^2r^2k^2}} \quad (3.23)$$

where $r = \frac{\omega}{\omega_n}$ is the frequency ratio

For undamped system $c=0$ and $D=0$, Therefore

$$Z = \frac{F_o \sin(\omega_t - \beta)}{k(1-r^2)} \quad (3.24)$$

When $r = 1$, i.e. $\omega = \omega_n$, the displacement amplitude is infinite. This condition is known as resonance. As an ideal undamped system is non-existing, damping always exists and the response is finite. However when the operating frequency, ω , is close to the natural frequency, ω_n , the response is very high. To avoid this condition, the operating frequency should not be close to the natural frequency. For a safe design, the frequency ratio is normally kept outside the critical range of 0.8 to 1.2. The magnitude of displacement for the undamped system is given by:

$$|Z| = \frac{F_o}{k(1-r^2)} = \frac{F_o}{m(\omega_n^2 - \omega^2)} \quad (3.25)$$

For the general case of a damped system,

$$|Z| = \frac{\frac{F_o}{k}}{\sqrt{(1-r^2)^2 + 4D^2r^2}} \quad (3.26)$$

The static displacement under a force F_o is given by:

$$Z_{st} = \frac{F_o}{k} \quad (3.27)$$

The ratio of the magnitude of the steady-state displacement, Z , of a forced system to the static displacement, Z_{st} , is known as magnification factor, M . Thus

$$M = \frac{|Z|}{Z_{st}} = \frac{1}{\sqrt{(1-r^2)^2 + 4D^2r^2}} \quad (3.28)$$

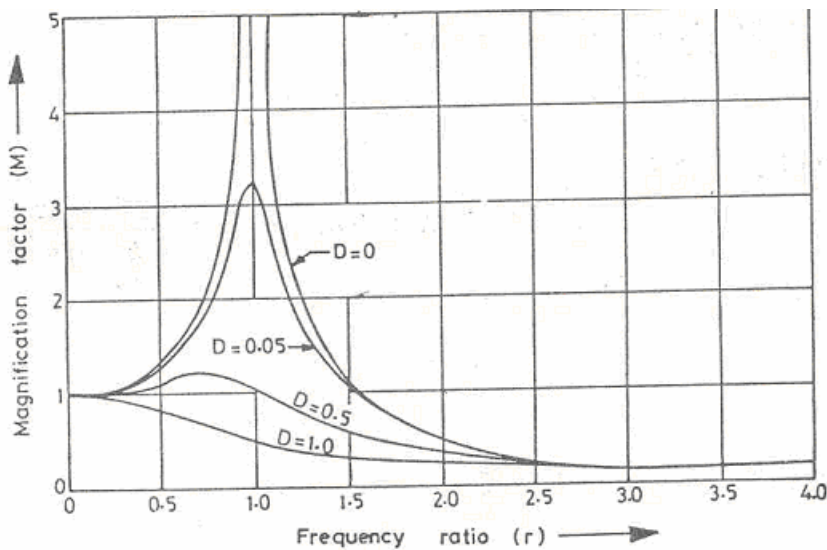


Figure 3.4 : Magnification factor, M , versus frequency ratio for some damping ratio (D) [5]

Figure 3.4 Shows the variation of the magnification factor M with r for different values of D .

It may be noted that the magnification is high for the value of r between 0.4 and 1.5 for the

common values of damping ratios, thus the reason for keeping the frequency ratio outside this range.

3.4 Foundation Vibration

A sketch of a typical rigid block foundation carrying rotatory machinery supported on a generalized layered soil profile is shown in Figure 3.5. The dynamic loading arises from an unbalanced mass m_0 rotating with eccentricity r_0 at the operational circular frequency $\omega=2\pi f$. The forces and moments acting on the soil-foundation interface and transmitted into the ground are of the form $m_0 r_0 \omega^2 \cos \omega t$ or using complex notation, $F_0 \exp[i(\omega t + \phi)]$; that is, they vary harmonically with time. Waves are emitted from the interface and propagate in all directions within the deposit. These waves undergo numerous reflections and refractions, as well as transformation into surface waves. Much of the energy imparted on to the foundation is diffused by such outward spreading waves (geometric damping) while a small portion is dissipated by inelastic action in the soil (hysteretic or material damping).

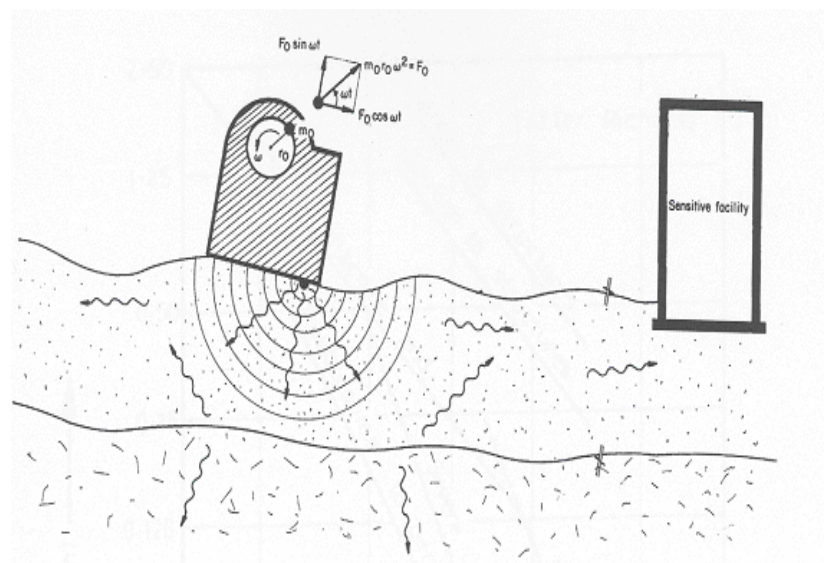


Figure 3.5: The machine foundation problem [2]

As a result, the soil-foundation interface, and with it the foundation block, undergoes harmonic oscillation of the form $F_0 \cos(\omega t + \phi)$ or using complex notation $F_0 \exp[i(\omega t + \phi)]$. The basic goal of the geotechnical design is to limit the amplitudes of all possible modes of oscillation to small enough levels that will neither endanger the satisfactory operation of the machine nor disturb the people working in the immediate vicinity.

Charts as shown below in Figure 3.6 [4] may guide the selection of an appropriate upper limit for satisfactory foundation performance. Manufacturers of machines are however, expected to deliver this information for proper design.

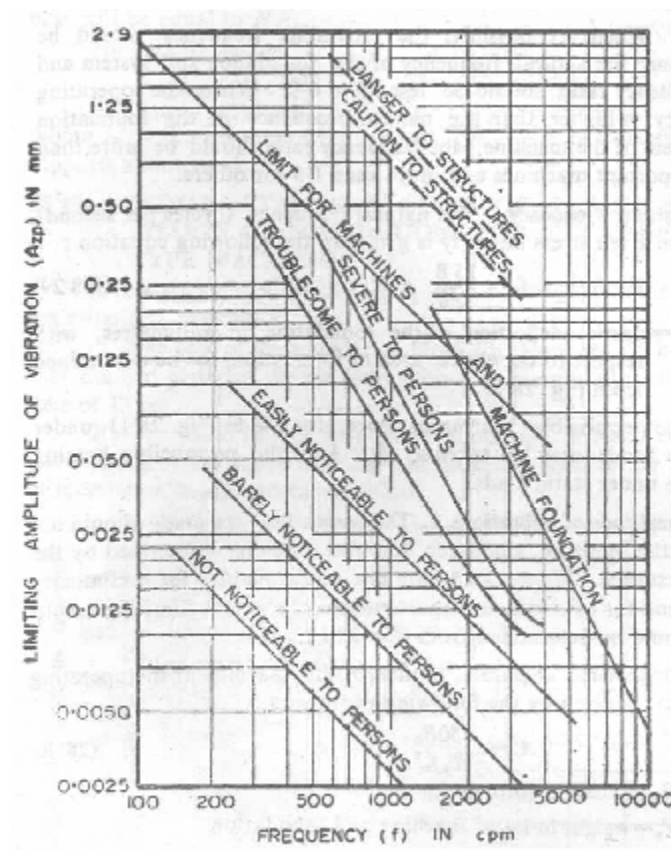


Figure 3.6: Typical performance requirement for machine foundation [4]

Notice that this limiting displacement amplitudes are typically of the order of a hundredth of a centimeter compared to several centimeters specified for foundation settlement under static load.

The design of machine foundations is a trial-and error process involving the following main steps

- (a) Estimate the magnitude and characteristics of the dynamic loads.
- (b) Establish the soil profile and determine the appropriate shear modulus and damping, G and D , for each soil layer
- (c) Guided by experience select the type and trial dimensions of the foundation and establish performance criteria (see Figure 3.6)
- (d) Estimate the dynamic response of this trial foundation subjected to the load of step (a) and supported by the soil deposit established in step (b). This key step of the design process usually starts with simplifying and idealizing soil profile and foundation geometry, and involves selecting the most suitable method of dynamic soil-foundation interaction analysis.
- (e) Check whether the estimated response amplitude of step (d) at the particular operation frequency conforms with the performance criteria given by machine manufacturers or Figure 3.6.
- (f) Monitor the actual motion of the completed foundation and compare with the theoretical prediction of step (d)

(g) Finally, if the actual performance of the constructed foundation does not meet the aforesaid design criteria, step(c), remedial measures must be devised. This may be, change of the mass of the foundation or the location of the machinery; stiffening of the sub-soil through, for example, grouting; increasing the soil-foundation contact surface; etc. Steps (d), (e) and (f) must be repeated until satisfactory design is finally achieved.

4 REVIEW OF THE CLASSICAL METHODS OF MACHINE FOUNDATION ANALYSIS

The great majority of the material presented in this chapter is based on the works of G.

Gazetas [2,3] and Richart et al [4]. These works are greatly acknowledged.

Machine foundations were until recently designed by rules-of thumb without any analysis of the expected vibration amplitudes. For instance one such design rule called for a massive concrete foundation of total weight equal to at least three to five times the weight of the supported machine. This approach is in fact no longer in use since it ignores the effect on the motion of all other variables. Of course, increasing the mass of the foundation decreases the resonant frequency of the system but, perhaps more importantly, reduces its effective damping. Obviously, this is not what those applying the rule had in mind [3].

Following the pioneering experimental studies carried out by German researchers in early 1930s, a number of empirical analysis procedures were developed and used extensively until 1950s. These methods focused on determining only the "natural frequency" of a foundation. In the late 1950's, the concept of "in-phase mass" and "reduced natural frequency" were developed. The former assumes that a certain mass of soil immediately below the footing moves as a rigid body in-phase with the foundation. The latter postulates that the 'natural frequency' is solely a function of the contact area, the soil bearing pressure and the type of soil. From tests and subsequent analysis, the concept of 'in-phase mass', i.e. mass of soil moved with the footing, is illustrated by the zone labeled m_s beneath the footing shown in Figure 4.1 [4].

By working backward from the equation for the resonant frequency, one is able to evaluate m_s for each test if f_n could be determined.

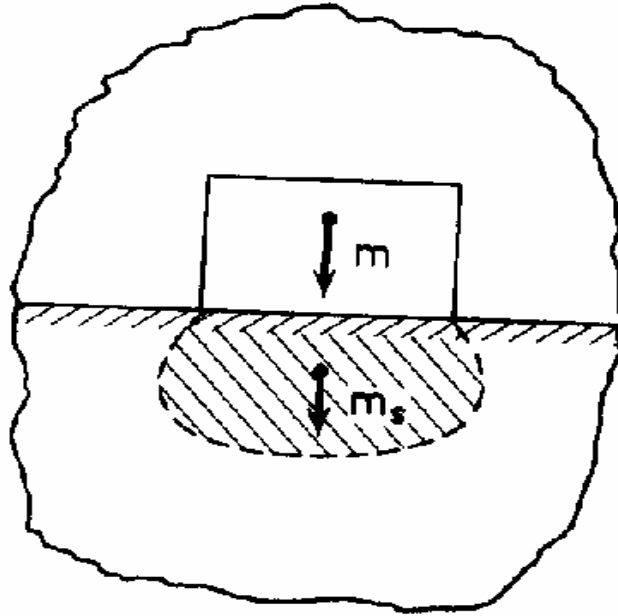


Figure 4.1: In phase mass of soil [4]

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m + m_s}} \quad (4.1)$$

where f_n = natural frequency (cyclic), m is mass of the machine and foundation, m_s is the in-phase mass of soil and k is the spring stiffness.

However, it was found that m_s varied with dead load, exciting force, base plate area, mode of vibration and type of soil on which the oscillator rested. Therefore it is difficult to obtain reliable values of m_s for design purposes.

Even if the " in-phase mass" could be determined satisfactorily, this information would not lead directly to evaluate the amplitude of vibration needed for design purpose.

Physical reality contradicts the concept of 'in-phase mass'. No soil mass moves as rigid body with the foundation. Instead, shear and dilatational waves emanate from the footing-soil interface into the soil, causing oscillating deformations at the surface and carrying away some of the input energy. The factors that have an influence on these phenomena cannot be possibly accommodated through such an artificial concept. In deed, the early attempts to obtain specific values of the 'In-phase mass' were frustrated by the sensitivity of this 'mass' to the foundation weight, mode of vibration, type of exiting force, contact area and nature of the underlying soil.

In an attempt to improve the methods for evaluating the resonant frequency of machine foundations supported by different soils, an expression for a "reduced natural frequency" of the system is developed. Beginning with Equation 4.1 for the resonant frequency of a foundation (including the effect of 'in-phase mass'), the spring constant k was replaced by $k'A$, where k' is the dynamic modulus of sub grade reaction (i.e lb/ft^3) and A is the contact area (ft^2) of the foundation against the soil. With this substitution, Equation 4.1 takes the form:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k'A}{m + m_s}} \quad (4.2)$$

Rearranging the above equation gives

$$f_n = \frac{1}{2\pi} \sqrt{\frac{A}{W}} \sqrt{\frac{k'g}{1 + \frac{m_s}{m}}} = \frac{1}{\sqrt{q_o}} f_{nr} \quad (4.3)$$

In Equation (4.3), $q_o = \frac{W}{A}$ and $f_{nr} = \frac{1}{2\pi} \sqrt{\frac{k'g}{1 + \frac{m_s}{m}}} = f_n \sqrt{q_o}$ (4.4)

In which:

f_n = Natural cyclic frequency

q_o =Average vertical contact pressure between the base of the foundation and the soil, and the remaining terms are lumped together and called "reduced natural frequency" (f_{nr}).

Then from the evaluation of a limited number of case histories available, Tschebotarioff proposed curves, which relate f_{nr} to the base area of the foundation for several soils. This relation appears as a straight line on the log-log plot as shown in Figure 4.2 [4].

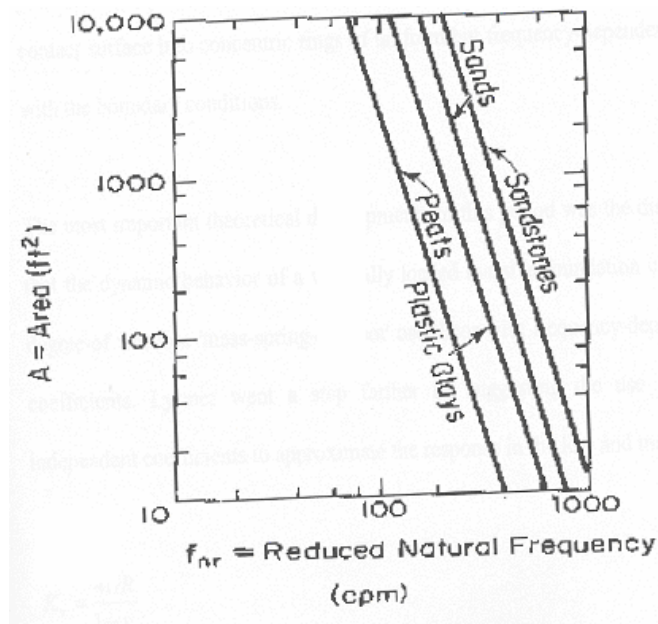


Figure 4.2: Tschebotarioff's "Reduced Natural Frequency"[4]

In order to calculate the resonant frequency for a given footing size on a particular soil, Figure 4.2 gives a value of f_{nr} which then leads to f_n by using Equation 4.4 and of the design values of q_0 .

Although this older method was not without merit, it was often interpreted to mean that the single most important factor in machine foundation design was the soil bearing pressure. Thus, the design was based on soil bearing capacity values taken from local building codes.

In addition to the above-mentioned drawbacks, these old rules were only concerned with the resonant frequency. Providing no information about vibration amplitudes that are primarily needed for design purposes.

In the 1960s Lysmer obtained a solution for the vertical axisymmetric vibration by discretizing the contact surface into concentric rings of uniform but frequency-dependent vertical stresses consistent with the boundary conditions.

The most important theoretical developments of this period was the discovery by Hsieh and Lysmer that the dynamic behavior of a vertically loaded massive foundation can be represented by a single degree-of-freedom 'mass-spring-dashpot' oscillator with frequency-dependent stiffness and damping coefficients. Lysmer went a step farther by suggesting the use of the following frequency-independent coefficients to approximate the response in the low and medium frequency range.

$$K_v = \frac{4GR}{1-\nu} \quad (4.5)$$

$$C_v = \frac{3.4R^2}{1-\nu} \sqrt{G\rho} \quad (4.6)$$

In which: K_v =Spring constant (stiffness), C_v =Dashpot constant (damping), R =Radius of circular rigid loading area, G =Shear modulus of the homogeneous half space (soil), ν =Poisson's ratio and ρ =Mass density of the soil

The success of Lysmer's approximation (often called 'Lysmer's Analog') in reproducing with very good accuracy, the actual response of the system had profound effect on the further development and engineering application of the 'half space' theories.

Richart and Whitman extended Lysmer's Analog by demonstrating that all modes of vibration can be studied by means of lumped-parameter mass-spring dashpot systems having properly selected frequency-independent parameters. The axis symmetric (vertical and torsional) oscillations of a cylindrical foundation can be represented by a 1-degree of freedom system described by

$$m\ddot{x} + c\dot{x} + kx = p(t) \quad (4.7)$$

In which x , \dot{x} , and \ddot{x} are the displacement, velocity and acceleration of the oscillating mass respectively; $p(t)$ = the external dynamic force from the operation of the machine. The lumped parameters are the equivalent mass, m , the effective dumping c , and the effective stiffness k . For torsional oscillation, m should be replaced by I_z , the effective mass polar moment of inertia, and x should be interpreted as the angle of rotation around the vertical axis of symmetry. On the other hand,

the two antisymmetric modes of oscillation (horizontal translation and rocking) of a cylindrical foundation are coupled and can be represented by a two degree of freedom system characterized by the effective mass and mass moment of inertia, the two effective values of damping (for swaying and rocking) , and the two effective values of stiffness (for swaying and rocking)

For each one of these four modes of excitation, different values of inertia, stiffness and damping parameters are needed. Whitman and Richart suggested the choice of stiffness appropriate for low frequencies, and of average damping values over the range of frequencies at which resonance usually occurs. With the intention to obtain a better agreement between the resonant frequencies of the lumped parameters and the actual system, they recommended that a fictitious mass (or mass moment of inertia) be added to the actual foundation mass (or mass moment of inertia).

The need for such recommendation stemmed not from the existence of any identifiable soil mass moving in phase with the foundation, but rather from the fact that in reality the stiffness decrease with increasing frequency, instead of remaining constant and equal to the static stiffnesses as the model assumes. In other words, instead of decreasing k , the lumped-parameter model increases m to keep the resonant frequency ω_r unchanged. Recall that ω_r is proportional to the square root of (k/m) .

Whitman and Richart and later, Richart, Woods and Hall and Whitman [4] presented expressions for these parameters for all four-vibration modes. Table 4.1 displays these expressions, which have enjoyed a significant popularity over decades [3]. Note that in Table 4.1, I_x , and I_z are mass moment of inertia around the horizontal and vertical axis respectively and damping ratio= C/C_{cr} where $C_{cr}=2\sqrt{km}$ or

$C_{cr} = 2\sqrt{kI}$ for torsional or rotational modes of vibration, with I_x or I_z for rocking or torsion, respectively.

Using the Table (4.1), one is able to calculate the dynamic stiffness by directly applying the formula while to calculate the damping ratio of the foundation soil, first the mass ratio should be determined and its value is substituted in the formula given. The value of the fictitious added mass is added with the mass of the foundation to keep the resonant frequency unchanged.

Mode	Vertical	Horizontal	Rocking	Torsional
Stiffness	$\frac{4GR}{1-\nu}$	$\frac{8GR}{2-\nu}$	$\frac{8GR^3}{3(1-\nu)}$	$\frac{16GR^3}{3}$
Mass ratio \bar{m}	$\frac{m(1-\nu)}{4\rho R^3}$	$\frac{m(2-\nu)}{8\rho R^3}$	$\frac{3I_x(1-\nu)}{8\rho R^5}$	$\frac{I_z}{\rho R^5}$
Damping ratio	$\frac{0.425}{\bar{m}^{1/2}}$	$\frac{0.29}{\bar{m}^{1/2}}$	$\frac{0.15}{(1+\bar{m})\bar{m}^{1/2}}$	$\frac{0.5}{1+2\bar{m}}$
Fictitious added mass	$\frac{0.27m}{\bar{m}}$	$\frac{0.095m}{\bar{m}}$	$\frac{0.24I_x}{\bar{m}}$	$\frac{0.24I_z}{\bar{m}}$

Table (4.1): Equivalent lumped parameters for analysis of circular foundation on elastic half space [3]

5 A RATIONAL APPROACH FOR THE DESIGN AND ANALYSIS OF BLOCK -TYPE MACHINE FOUNDATION

5.1 Degrees of Freedom of a Block Foundation

The great majority of the material presented in this chapter is based on the works of G. Gazetas [2]. This works are greatly acknowledged.

A Block foundation is regarded as rigid when compared with the soil over which it rests. Therefore, it shall be assumed that it undergoes only rigid-body motion. Under the action of unbalanced forces, the rigid block foundation thus undergoes displacements and rotations as shown in Figure 5.1

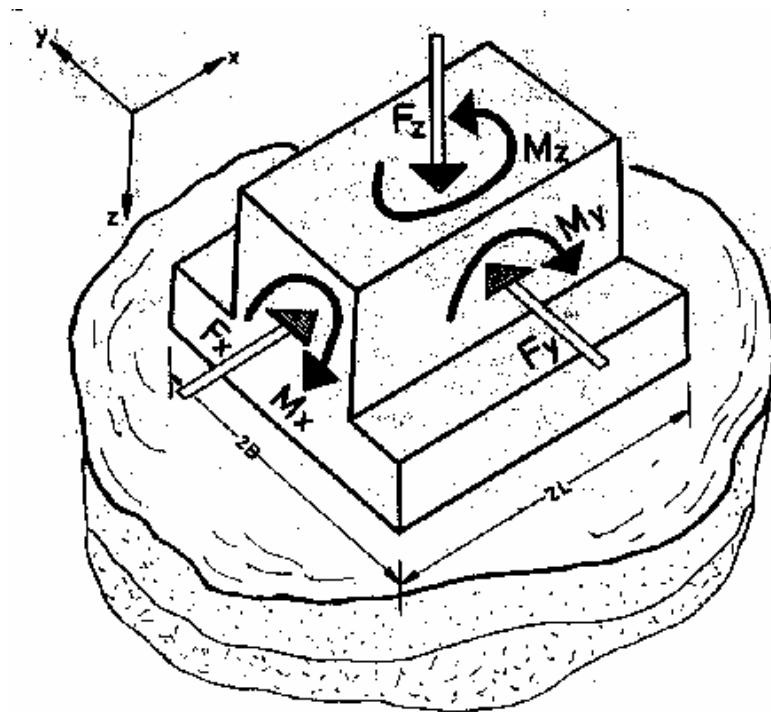


Figure 5.1: Rigid foundation block with six degrees of freedom [2]

Any motion can be resolved into six components of displacements and rotations: three translations along the three axes and three rotations around them.

Of the six components of motion mentioned above, the translation along the z-axis and the rotation about the z-axis occur independently of any other component. However, translation along the x-axis and rotation about y-axis and translation along y-axis and rotation about x-axis are coupled motions. In this section, a general rational method is presented for computing each of these six dynamic displacements and rotations due to steady-state harmonic excitations (forces and moments). The choice of harmonic oscillations is made not only because many machines usually produce unbalanced forces that indeed vary harmonically with time (rotatory or reciprocating engines), but also because non-harmonic forces (such as those produced by punch press and forging hammers) can be decomposed into a large number of sinusoids through Fourier analysis.

5.2 Vertical Vibration

Let us explain the method with the help of the easy-to-visualize case of vertical vibration. Figure 5.2 shows a rigid foundation block of total mass m , assumed to have a vertical axis of symmetry passing through the centroid of the soil foundation interface. The foundation is underlain by a deposit consisting of horizontal soil layers. Subjected to vertical harmonic force $F_z(t)$ along the z-axis, this foundation will experience only a vertical harmonic displacement $u_z(t)$. The question is to determine $u_z(t)$, given $F_z(t)$.

To determine $u_z(t)$, we consider separately the motion of each body: the foundation block and the supporting ground shown in Figure 5.2

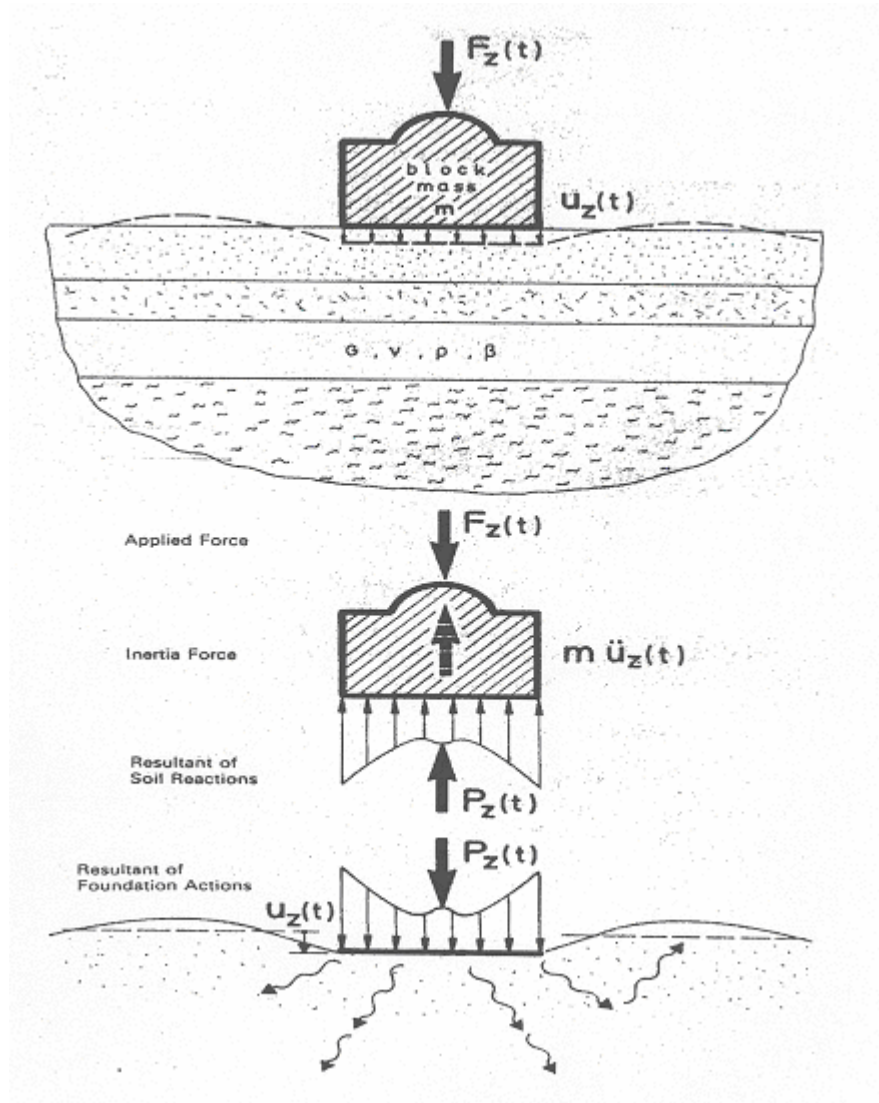


Figure 5.2 :Analysis of the dynamic equilibrium of a vertically vibrating foundation block [2]

The two free-body diagrams are sketched in the figure and the inertial (D’Almbert) forces indicated. The foundation actions on the soil generate equal and opposite reactions, distributed in some unknown way across the interface and having an unknown resultant $P_z(t)$. Furthermore, since in reality the two bodies remain always in contact, their displacements, are identical and equal to the rigid body displacement $u_z(t)$. Thus the dynamic equation of the block takes the form.

$$P_z(t) + m \ddot{u}_z(t) = F_z(t) \quad (5.1)$$

And that of the linearly deforming multi layered ground can be summarized as:

$$P_z(t) = K_z u_z(t) \quad (5.2)$$

In which K_z is defined as the dynamic vertical impedance determined for this particular system.

Combining Equations (5.1) and (5.2) gives:

$$m \ddot{u}_z(t) + K_z u_z(t) - F_z(t) = 0 \quad (5.3)$$

It is evident that the key to solving the problem is the determination of the impedance K_z , which is of the dynamic force-to-displacement ratio according to Equation (5.2). Note also that from structural dynamics, the steady-state solution $u_z(t)$ to Equation (5.3) for a harmonic excitation $F_z(t) = \bar{F}_z \cos \omega t$ is also harmonic with the same frequency ω .

Theoretical and experimental results show that in Equation (5.2), a harmonic action P_z applied on to the ground and the resulting harmonic displacement u_z have the same frequency ω but out of phase.

That is:

$$P_z(t) = \bar{P}_z \cos(\omega t + \alpha) \quad (5.4)$$

Then u_z can be expressed in the following two equivalent ways.

$$u_z(t) = \bar{u}_z \cos(\omega t + \alpha + \varphi) = \bar{u}_1 \cos(\omega t + \alpha) + \bar{u}_2 \sin(\omega t + \alpha) \quad (5.5)$$

Where the amplitude \bar{u}_z and phase angle φ are related to the in phase, \bar{u}_1 , and the 90° out of phase, \bar{u}_2 , components according to

$$\bar{u}_z = \sqrt{\bar{u}_1^2 + \bar{u}_2^2} \quad (5.6)$$

$$\tan \varphi = \frac{\bar{u}_2}{\bar{u}_1}$$

We can rewrite the forgoing expressions in an equivalent but more elegant way using complex number notation.

$$P_z(t) = \bar{P}_z \exp(i\omega t) \quad (5.7)$$

$$u_z(t) = \bar{u}_z \exp(i\omega t) \quad (5.8)$$

Where P_z and u_z are complex quantities that can be expressed as:

$$\bar{P}_z = \bar{P}_{z1} + i\bar{P}_{z2} \quad (5.9)$$

$$\bar{u}_z = \bar{u}_{z1} + i\bar{u}_{z2} \quad (5.10)$$

Equation (5.7) to (5.10) is equivalent to Equation (5.4) to (5.6) with the following relations being valued for the amplitudes.

$$|\bar{P}_z| = \sqrt{\bar{P}_{z1}^2 + \bar{P}_{z2}^2} \quad (5.11)$$

$$|\bar{u}_z| = \sqrt{\bar{u}_{z1}^2 + \bar{u}_{z2}^2} \quad (5.12)$$

The two-phase angles α and φ are properly hidden in the complex forms. With \bar{p}_z and \bar{u}_z being complex numbers (Equation 4.7 to 4.10), the dynamic vertical impedance (force over displacement ratio) becomes.

$$K_z = \frac{P_z(t)}{u_z(t)} = \frac{\bar{P}_z}{\bar{u}_z} = \text{Complex Number} \quad (5.13)$$

Which may be put in the form:

$$K_z = \bar{K}_z + i\omega C_z \quad (5.14)$$

In Equation (5.14), both \bar{K}_z and C_z are functions of frequency ω . They can be interpreted as follows. The real component, \bar{K}_z termed as dynamic stiffness, reflects the stiffness and inertia of the supporting soil; its dependence on frequency is attributed solely to the influence that frequency exerts on inertia, since soil properties are to a good approximation frequency-independent. The imaginary component,

ωC_z is the product of circular frequency and dashpot coefficient C_z , which reflects the two types of damping. Radiation and material damping generated in the system, the former due to energy carried by waves spreading away from the foundation, and the latter due to energy dissipated in soil due to hysteretic action

Equation 5.14 is a (theoretical and experimental) fact for all foundation soil systems. However, the interpretation of \bar{K} and C as dynamic stiffness and dashpot coefficients must be justified. This is easy if we substitute Equation 5.14 in to Equation 5.3 to obtain:

$$m\ddot{u}_z(t) + i\omega C_z u_z(t) + \bar{K}_z u_z(t) = F_z(t) \quad (5.15)$$

We are looking for a harmonic response $\bar{u}_z \exp(i\omega t)$ to the harmonic excitation $\bar{F}_z \exp(i\omega t)$. Straight-forward operations lead to:

$$[(\bar{K}_z - m\omega^2) + i\omega C_z] \bar{u}_z = \bar{F}_z \quad (5.16)$$

Equation 5.15 is the equation of motion of a simple oscillator with mass m spring constant \bar{K}_z and dashpot “constant” C_z . The quotation mark around the word constant is placed deliberately to remind that \bar{K}_z and C_z are not as such constants but vary with the frequency ω of oscillation. Equation 5.15 suggests for the vertical mode of oscillation, an analogy between the actual foundation-soil system and the model shown in Figure 5.3b consisting of the same foundation but supported on a spring and dashpot with characteristic moduli equal to \bar{K}_z and C_z respectively. Note that the load is acting on a mass less rigid foundation.

Once these moduli have been established for a particular excitation frequency ω , \bar{u}_z is obtained from Equation 5.16 as

$$\bar{u}_z = \frac{\bar{F}_z}{(\bar{K}_z - m\omega^2) + i\omega C_z} \quad (5.17)$$

Therefore, the amplitude of oscillation that is of interest is simply:

$$|\bar{u}| = \frac{\bar{F}_z}{\sqrt{(\bar{K}_z - m\omega^2)^2 + \omega^2 C_z^2}} \quad (5.18)$$

In general the soil reaction against a vertically oscillating foundation is fully described by the complex frequency dependent dynamic vertical impedance $K_z(\omega)$ or equivalently, the frequency-dependent spring (stiffness) and dashpot (damping) coefficient, $\bar{K}_z(\omega)$ and $C_z(\omega)$. Once these parameters have been obtained for the particular frequency (or frequencies) of interest, solving the equation of motion yields the desired amplitude of the harmonic vertical displacement.

5.3 Generalization to All Modes of Oscillation.

The definition of dynamic impedance given in Equation 5.13 for vertical excitation-response is also applicable to each of the remaining five modes of vibration. Thus, we define as lateral (swaying) impedance K_y the ratio of the horizontal harmonic force, $P_y(t)$, imposed in the short direction at the base of the foundation to the resulting harmonic displacement, $u_y(t)$, in the same direction.

$$K_y = \frac{P_y(t)}{u_y(t)} = \frac{\bar{P}_y}{\bar{u}_y} \quad (5.19)$$

Similarly:

K_x = the longitudinal (swaying) impedance for horizontal motion in the long direction

K_{rx} = the rocking impedance for rotational motion about the axis parallel to the shorter side of the foundation

K_{ry} = The rocking impedance for rotational motion about the axis parallel to the longer side of the foundation

K_t = The torsional impedance for rotational oscillation about the vertical axis.

Moreover, in embedded foundations and piles, horizontal forces along principal axis induce rotational in addition to transnational oscillation. Hence two more cross-coupling horizontal-rocking impedance exist; K_{xry} and K_{yrx} . They are usually negligibly small in shallow foundations, but their effects may become appreciable for larger depths of embedment, owing to the moments about the base axis produced by horizontal soil reactions against the soil walls.

Note that all the impedance refer to axes passing through the foundation basement-soil interface.

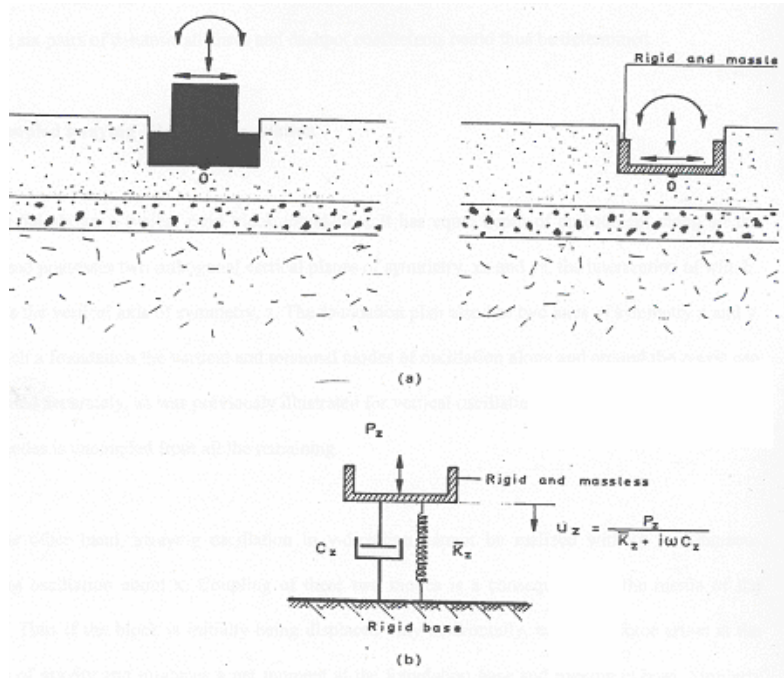


Figure 5.3 (a) A foundation -structure system and the associated rigid and mass less foundation. (b) Physical interpretation of the dynamic stiffness \bar{K}_z and dashpot c_z for a vertically vibrating footing [2]

The six impedances of block foundation turn out to be complex numbers and functions of frequency that can be written in the form of Equation 5.14. Thus, in general, for each mode.

$$K(\omega) = \bar{K}(\omega) + i \omega C(\omega) \tag{5.20}$$

And the analogy suggested in Figure 5.3 extends to all modes. For a particular excitation frequency ω , the six pairs of dynamic stiffness and dashpot coefficients are assumed to be known.

5.4 Coupled swaying –Rocking Oscillation

Figure 5.4 shows a typical rigid block foundation: it has equal depth of embedment along all the sides and possesses two orthogonal vertical planes of symmetry, xz and yz , the intersection of which defines the vertical axis of symmetry, z . The foundation plan also has two axis of symmetry x and y . For such a foundation the vertical and torsional modes of oscillation along and around the z -axis can be treated separately, as was previously illustrated for vertical oscillation. In other words, each of the two modes is uncoupled from all the remaining.

On the other hand, swaying oscillation in y -direction cannot be realized without simultaneous rocking oscillation about x . Coupling of these two modes is a consequence of the inertia of the block. Thus if the block is initially being displaced only horizontally, an inertia force arises at the center of gravity and produces a net moment at the foundation base and rocking is born. Similarly coupled are swaying in the x -direction and rocking around y .

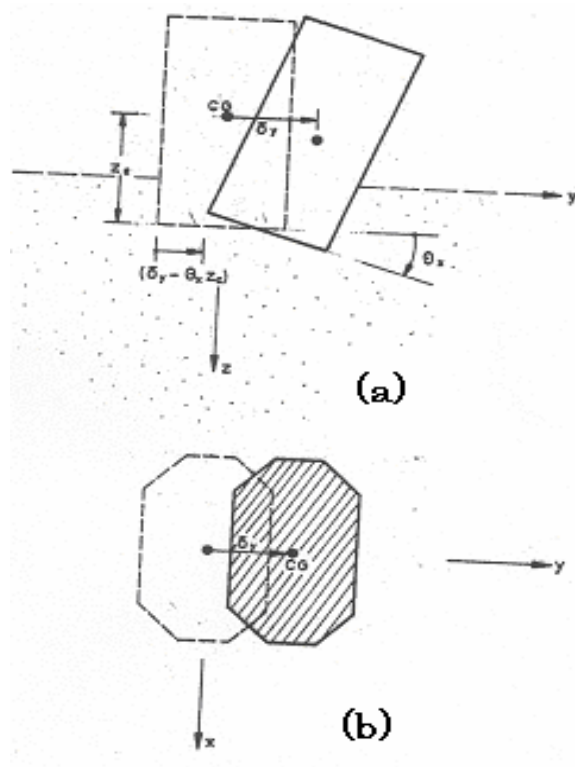


Figure: 5.4 coupled-swaying- rocking oscillation: definition of displacement variables. (a) section and (b), plan.[2]

To study the coupled swaying-rocking oscillation of the block in the zy plane, we call δ_y and θ_x represent the horizontal displacement at the foundation center of gravity and the angle of rotation of the rigid block, respectively. Referring to Figure 5.4 and calling $F_y(t)$ and $M_x(t)$ the excitation force and moment at the block center of gravity, one can write the transnational force and rotational moment dynamic equilibrium as follows.

$$P_y(t) + m\ddot{\delta}_y(t) = F_y(t) \quad (5.21)$$

$$T_x(t) - P_y(t)Z_c + I_{ox}\ddot{\theta}(t) = M_x(t) \quad (5.22)$$

Where m = Total mass (mass of foundation + mass of machinery), I_{ox} = Mass moment of inertia about the principal horizontal axis parallel to x and passing through the block center of gravity, P_y and T_x = Net horizontal force and rocking moment reactions acting from the soil against the foundation during swaying and rocking, and referring to the centroid of the foundation basement and Z_c = Height from soil foundation interface to the center of gravity of the foundation

For a Harmonic excitation:

$$F_y(t) = \bar{F}_y \exp(i\omega t) \quad (5.23)$$

$$M_x(t) = \bar{M}_x \exp(i\omega t) \quad (5.24)$$

In which the amplitudes \bar{F}_y and \bar{M}_x may be either constant, or (more typically) proportional to the square of the operational frequency $\omega = 2\pi f$. \bar{F}_y and \bar{M}_x result from the operation of the machine.

The steady-state harmonic response can be written in the form.

$$\delta_y(t) = \bar{\delta}_y \exp(i\omega t) \quad , \quad \bar{\delta}_y = \delta_{y1} + i \delta_{y2} \quad (5.25)$$

$$\theta_x(t) = \bar{\theta}_x \exp(i\omega t) \quad , \quad \bar{\theta}_x = \theta_{x1} + i\theta_{x2} \quad (5.26)$$

In which $\bar{\delta}_y$ and $\bar{\theta}_x$ are complex frequency-dependent displacement and rotation amplitudes at the center of gravity. Note that equations (5.23) to (5.26) do not imply that the two components of motion and the two components of excitation are all in phase. Instead, the true phase angles are hidden in the complex form of each displacement component.

Using similar argument with regard to the soil reaction, one may set the following without loss of generality.

$$P_y(t) = \bar{P}_y \exp(i\omega t) \quad (5.27)$$

$$T_x(t) = \bar{T}_x \exp(i\omega t) \quad (5.28)$$

The complex amplitudes \bar{P}_y and \bar{T}_x are related to the complex displacement and rotation amplitudes through the corresponding dynamic impedances. Recalling that the latter are referred to the center of the foundation base, rather than the block center of gravity (see Figure 5.4), one can immediately write:

$$\bar{P}_y = K_y(\bar{\delta}_y - Z_c \bar{\theta}_x) + K_{yrx} \bar{\theta}_x \quad (5.29)$$

$$\bar{T}_x = K_{rx} \bar{\theta}_x + K_{yrx}(\bar{\delta}_y - Z_c \bar{\theta}_x) \quad (5.30)$$

Substituting Equations (5.23) to (5.30) into the governing Equation of (5.21) and (5.22), and rearranging one obtains the following equation for the displacement and rotation amplitudes:

$$\bar{\delta}_y (K_y - m\omega^2) + \bar{\theta}_x (K_{yrx} - K_y Z_c) = \bar{F}_y \quad (5.31)$$

$$\bar{\delta}_y (K_{yrx} - K_y Z_c) + \bar{\theta}_x (K_{rx} - 2K_{yrx} Z_c + K_y Z_c^2 - I_{ox} \omega^2) = \bar{M}_x \quad (5.32)$$

$$\text{Let } a = K_y - m\omega^2$$

$$b = K_{yrx} - K_y Z_c$$

$$c = K_{rx} - I_{ox} \omega^2 + K_y Z_c^2 - 2K_{yrx} Z_c$$

$$d = ac - b^2$$

Substituting a, b and c in Equation (5.31) and (5.32) leads to a system of two algebraic equations with two unknowns $\bar{\delta}_y$ and $\bar{\theta}_x$, that is

$$a\bar{\delta}_y + b\bar{\theta}_x = \bar{F}_y \quad (5.33)$$

$$b\bar{\delta}_y + c\bar{\theta}_x = \bar{M}_x \quad (5.34)$$

Or in matrix form:

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{Bmatrix} \bar{\delta}_y \\ \bar{\theta}_x \end{Bmatrix} = \begin{Bmatrix} \bar{F}_y \\ \bar{M}_x \end{Bmatrix}$$

Solving Equation (5.33) and Equation (5.34) simultaneously gives:

$$\bar{\delta}_y = (c\bar{F}_y - b\bar{M}_x)d^{-1} \quad (5.35)$$

$$\bar{\theta}_x = (a\bar{M}_x - b\bar{F}_y)d^{-1} \quad (5.36)$$

For a particular frequency ω , determination of the motions from Equations (5.33) to (4.36) is a straightforward operation once the dynamic impedances K_{ij} (or the corresponding spring and dashpot coefficients, k_{ij} and C_{ij}) are known.

Vibrations in the vertical and torsional mode, each of which is practically uncoupled from all the other modes can be respectively analyzed with Equation (5.18) and its torsional counter part as:

$$|\bar{\theta}_z| = \frac{\bar{M}_z}{\sqrt{(\bar{K}_t - J_z \omega^2)^2 + \omega^2 C_t^2}} \quad (5.37)$$

In which:

\bar{K}_t = The dynamic spring coefficient for torsion

C_t = The dashpot coefficient for torsion

J_z = The moment of inertia of the whole foundation (including)
the machine about the vertical z-axis and

\bar{M}_z = The amplitude of the harmonic external moment around z.

Therefore, using Equation (5.18) one obtains the amplitude of vertical oscillation. Using Equation (5.37), one gets the amplitude of torsional oscillation and using Equation (5.35) and (5.36), one obtains the amplitudes of $\bar{\delta}_y$ and $\bar{\theta}_x$ of the coupled swaying-rocking oscillation in the y-z plane.

5.5 Dynamic Impedances

5.5.1 Methods of Computing Dynamic Impedances

Several alternative computational procedures are available to obtain dynamic impedance functions (Springs and Dashpots coefficients) for each specific machine-foundation problem. The choice among these methods depends on the required accuracy, which in turn is primarily dictated by the size and importance of the particular project. Furthermore, the method to be selected must reflect the key

characteristics of the foundation and the supporting soil. Specifically, one may broadly classify soil-foundation systems according to the following material and geometric characteristics:

- The shape of foundation (circular, strip, rectangular, arbitrary)
- The type of soil profile (Deep uniform deposit, deep multi-layered deposit, shallow stratum on rock)
- The amount of embedment(Surface foundation, embedded foundation, piled foundation)

The various computational methods can be grouped into four categories, each with its own merits and limitations: These methods are: (a) analytical and semi-analytical method; (b) dynamic finite-element method; (c) combined analytical-numerical method; and (d) approximate techniques.

An alternative engineering approach is the development of easy-to-use closed-form expressions and graphs, based on the results of rigorous and approximate formulations. This is the approach adopted in this thesis and discussed in the sections that follow.

5.5.2 Tables and Graphs of Dynamic Impedances

A comprehensive and easy-to-use information for some authors have compiled dynamic spring and dashpot coefficients since recently. The information is in the form of simple algebraic formulas and dimensionless graphs pertaining to all possible translational and rotational modes of oscillation and covering idealized soil profiles and different foundation geometries. These tables as compiled by Gazetas [2], are given in appendix A as Tables A-1 to A-4.

The attached tables and graphs are described as follows:

1. Table A-1 and the accompanying set of graphs refer to foundation of any shape resting on the surface of a homogeneous half space.
2. Table A-2 and the related graphs are for foundations with any shape partially or fully embedded in a homogenous half space.
3. Table A-3 refers mainly to circular foundation on the surface of a homogeneous soil stratum underlain by bedrock.
4. Table A-4 refers to circular foundation partially or fully embedded in a homogeneous stratum underlain by bedrock.

The formulas and graphs given in these tables have been contributed by a number of researchers including Gazetas [2] are sufficient for complete dynamic analysis of a rigid foundation interacting with the underlying soil.

5.5.3 Foundation on the Surface of a Homogeneous Half-Space

For an arbitrary shaped foundation, the circumscribed rectangle with side lengths of $2B$ and $2L$ ($L > B$) is first defined. To compute the impedances in the six modes of vibration from Table A-1, the following values are required:

- Area moment of inertia about x and y axis (I_{bx} and I_{by}) of the base area
- Foundation-soil interface area (A_b)

- Polar moment of inertia about vertical-axis (J_z)
- Half the width and length, B and L, of the circumscribed rectangle
- $\omega = 2\pi f =$ The circular frequency (in radians/second) of the applied force.(i.e. frequency of operation of the machine)
- G and ν or V_s and V_{La} which are shear modulus and Poisson's ratio or shear wave velocity and "Lysmer's analog" wave velocity respectively.

Lysmer's analog wave velocity is the apparent propagation velocity of compression-extension waves under a foundation and is related to V_s according to:

$$V_{La} = \frac{3.4V_s}{\pi(1-\nu)} \quad (5.38)$$

Table A-1 as well as all other tables gives:

- The dynamic stiffness (stiffnesses), $\bar{K} = \bar{K}(\omega)$, as a product of the static stiffness, K, and the dynamic stiffness coefficient $k=k(\omega)$ i.e. $\bar{K} = K * k(\omega)$
- The radiation damping ("dashpot") coefficients $C=C(\omega)$. These coefficients do not include the soil hysteretic damping. To incorporate this damping, simply add the radiation damping coefficient with the corresponding material damping coefficient $\frac{2\bar{K}s}{\omega}$ [2]

$$\text{i.e. Total } C = \text{Radiation } C + \frac{2\bar{K}s}{\omega}$$

Where s is the soil hysteretic damping coefficient

The total dashpot coefficient will be simply the sum of radiation dashpot coefficient and hysteretic damping.

$$\text{i.e. Total } C = \text{radiation } C + \frac{2\bar{K}_s}{\omega} \quad [2] \quad (5.39)$$

5.5.4 Foundations Embedded in a Homogeneous Half -Space

With the formulas of Table A-2 and the accompanying graphs, one can assess the effects of embedment in a variety of realistic situations. The additional parameters that must be known or computed to use Table A-2 are indicated below.

- The depth to the foundation base (D)
- The total area of the actual side wall-soil contact surface (A_w). This area should, in general, be smaller than the nominal area of contact to account for slippage and separation that may occur near the ground surface.
- The average height of the sidewall that is in good contact with the surrounding soil (d1)
- The distance from the ground surface up to the centroid of the foundation (h).

Table A-2 compares the dynamic stiffness and dashpot coefficients of embedded foundations

$\bar{K}_{z,emb} = K_{emb} * k_{emb}$ and C_{emb} , with those of the corresponding surface foundation, $\bar{K}_{sur} = K_{sur} * k_{sur}$ and

C_{sur} .

5.5.5 Foundations on the Surface of a Flexible Layer Overlying the Bedrock

Natural soil deposits are frequently underlain by very stiff material or even bedrock at a shallow depth, rather than extending to infinite depth as the homogeneous half space implies. The proximity of such stiff formation to the oscillating surface modifies the static stiffness K , the dynamic stiffness coefficient $k(\omega)$ and the dashpot coefficient $c(\omega)$. With reference to Table A-3 and the accompanying graphs, we observe the following.

1. The static stiffness in all modes increases with the ratio H/B increases. This is evident from the formulas of Table A-3 in which the values of the stiffness approach to the corresponding half space values when H/R becomes significantly large.
2. The variation of dynamic stiffness coefficients with frequency reveals an equally strong dependence on H/B . On a stratum, $k(\omega)$ is not a smooth function, as with a halfspace, but exhibits undulations (Peaks and valleys) associated with the natural frequencies (in shear and compression) of the stratum (see the graph accompanying Table A-3 i.e. Appendix B). The observed functions are the outcome of resonance phenomena. Waves emanating from the oscillating foundation reflect at the soil-bedrock interface and return back to their source at the surface. As a result, the amplitude of foundation motion may significantly increase at frequencies near the natural frequencies of the deposit. Thus, the dynamic stiffness, being the inverse of displacement exhibits troughs, which are very steep when the hysteretic damping in the soil is small (in fact, in certain cases k would be exactly zero if the soil were ideally elastic).

For the “shearing” mode of vibration (swaying and torsion) the natural fundamental frequency of the stratum, which controls the behavior of $k(\omega)$, is

$$f_s = \frac{V_s}{4H} \quad (5.40)$$

While for “Compressive” modes (Vertical, rocking) the corresponding frequency is

$$f_c = \frac{V_{la}}{4H} = \frac{3.4 f_s}{\pi(1-\nu)} \quad (5.41)$$

3. The variation of the dashpot coefficient with frequency shows a twofold effect of the presence of a rigid base at relatively shallow depth. First, the $c(\omega)$ exhibits undulation (crests and troughs) due to wave reflections at the rigid boundary. Second, and far more important from a practical viewpoint, is that at low frequencies, below the first resonant (“cut-off”) frequency of each mode of vibration, radiation damping is zero or negligible for all shapes of footings and all modes of vibration. This is due to the fact that no surface waves can exist in a soil stratum over bedrock at such low frequencies; and, since the bedrock also prevents waves from propagating downward, the overall radiation of wave energy from the footing is negligible or nonexistent (see appendix B).

Such an elimination of radiation damping may have severe consequences for heavy foundations oscillating vertically or horizontally, which would have enjoyed substantial amounts of damping in a very deep deposit. On the other hand, since the low-frequency values of c in rocking and torsion are

small even in a half space, operation below the cut-off frequencies may not be affected appreciably by the presence of bedrock.

5.5.6 Foundation Embedded in a Flexible Layer Overlying the Bedrock

As shown in Tables A-4, embedding a foundation in a shallow stratum, rather than a half space, has one additional effect over those expressed in Table A-2. That is, the static stiffness tends to increase thanks to decrease in the depth of deforming zone underneath the foundation.

The dynamic stiffness coefficients for surface and embed foundation follow approximately the same pattern on a homogeneous half space. Therefore use the results of the embedded foundation for the dynamic stiffness coefficient for embedded foundation exceed the surface foundation by an amount that depends on the geometry of side wall soil contact and is practically independent of the presence or absence of a rigid base at shallow depth. Therefore, one can use the results of Table A-2 buy C_{sur} corresponding to the layered profile and thus obtained according to Table A-3.

It is evident that the result summarized in Table A-4 can follow from a proper combination of the pertinent results of Table A-2 (embedded foundation in half space) and of Table A-3 (Surface foundation on a stratum).

5.6 Trial Sizing of Block Foundation

The design of a block foundation for a centrifugal or reciprocating machine starts with a preliminary sizing of the block. This initial sizing is based on a number of guidelines that are partially derived from

empirical and practical experience sources. Initial sizing is only primary; it does not constitute a final design.

A block foundation design can only be considered complete when a dynamic analysis and check is performed and the foundation is predicated to behave in an acceptable manner. The following guidelines for initial trial sizing have been found to result in acceptable configuration [5]

(1) A rigid block-type foundation resting on soil should have a mass of two to three times the mass of the supported machine for centrifugal machines. However, when the machine is reciprocating, the mass of the foundation should be three to five times the mass of the machine.

(2) The top of the block is usually kept 30 cm above the finished floor or pavement elevation to prevent damage from surface water run off.

(3) The vertical thickness of the block should not be less than 60cm or as dictated by the length of anchor bolts used.

(4) The foundation should be wide to increase damping in the rocking mode; the width should be at least 1 to 1.5 times the height of the foundation.

(5) Once the thickness and width have been selected, the length is determined according to(1) above, provided sufficient space is available to support the machine.

(6) The length and width of the foundation are adjusted so that the center of gravity of the machine plus equipment coincides with the center of gravity of the foundation.

(7) For large reciprocating machines, it may be desirable to increase the embedded depth in soil such that 50% to 80% of the depth should be embedded in soil. This will increase the lateral restraint and the damping ratios for all modes of vibration.

5.7 Design Checklist

Once the trial sizes of the block foundation is selected and the displacement and rotation amplitudes are determined, the predicted behavior of the proposed structure is checked or compared against certain design requirements. These include:

- (a) The static strength checks against soil structural failures and excessive deformations.
- (b) Comparison to limiting dynamic behaviors including maximum amplitude of vibration, maximum velocity and acceleration, maximum magnification factor and possible resonance conditions with the allowable limit.

5.7.1 Static Conditions

For all type of machine foundation, the following static conditions should be fulfilled.

1. Static bearing Capacity: Proportion footing area for 50% of allowable soil pressure

2. Static Settlement: Settlement must be uniform and should be within the allowable limit.

3. Bearing Capacity: static plus dynamic loads: The magnification factor should preferably be less than 1.5. The sum of static plus dynamic load should not create a bearing pressure greater than 75% of the allowable soil pressure given in the soil report.

4. Settlement Static plus repeated dynamic load: The combined center of gravity of the dynamic load and the static loads should be within 0.5% of the linear dimension from the center of gravity of footing. In the case of rocking motion, the axis of rocking should coincide with principal axis of the footing.

5.7.2 Limiting Dynamic Conditions

The dynamic force generated by the machine will result in vertical, lateral, rocking and torsional oscillation of the foundation. These dynamic displacements of the foundation should be within the tolerable limit listed below.

1. Vibration amplitude at operating frequency: The maximum amplitude of motion for the foundation system as calculated by the programme developed in this thesis should lie in zone A or B of Figure 5.5 for the given excitation frequency. Where unbalanced forces are caused by machines at different frequencies, the total displacement amplitudes to be compared at the lower acting frequency, are taken as the sum of all displacement amplitudes.

2. Velocity: Velocity is equal to the circular frequency ω times the displacement amplitude as determined in (1) above. This velocity should be compared against the limiting values of Table 5.1 and Figure 5.5 at least for the case of "good" operation. If two machines operate at different frequencies, the

resultant velocity is calculated by RMS (root mean square) method $V = [(\omega_1 A_1)^2 + (\omega_2 A_2)^2]^{0.5}$, where V =Resultant velocity, in cm/sec, ω_1, ω_2 = operation frequencies for machines 1 and 2, respectively (rad/sec), and A_1, A_2 = vibration displacement (cm) for machine 1 and 2 respectively.

3. Acceleration: Acceleration is equal to ω^2 times the displacement amplitude as determined in (1) above. This check is only necessary if conditions (1) and (2) are not satisfied. The acceleration should fall in zone A or B of Figure 5.5.

4. Magnification factor: The calculated values of M should be less than 1.5 at resonance frequency.

5 Resonance: The frequency of the machine should have at least $\pm 20\%$ with the resonance frequencies; that is $\omega < 0.8 \omega_n$ or $\omega > 1.2 \omega_n$

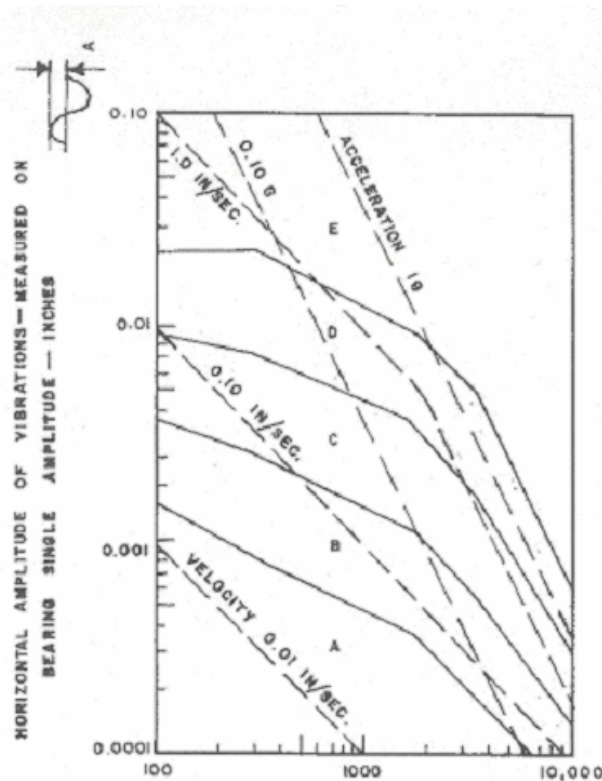


Figure 5.5 Vibration performance of rotating machines [5]

Horizontal peak velocity (cm/sec)	Machine Operation
<0.013	Extremely Smooth
0.013-0.025	Very Smooth
0.025-0.051	Smooth
0.051-0.102	Very Good
0.102-0.203	Good
0.203-0.406	Fair
0.406-0.800	Slightly rough
0.800-1.600	Rough
>1.600	Very Rough

Table 5.1 General machinery-Vibration-Severity data [5]

6 THE COMPUTER PROGRAMME AND ITS APPLICATION

6.1 General

The computer programme in this thesis is based on the closed form expressions and dimensionless graphs for the determination of static stiffness, dynamic springs and dashpot coefficients compiled by G. Gazetas. [2]

The programme enables one to determine the dynamic stiffness and dashpot coefficients of the foundation soil; the amplitudes of vertical, horizontal, rocking and torsional oscillations; the natural frequencies, the damping ratios, frequency ratios and the magnification factors of block-type machine foundations for the following soil and embedment conditions.

- (a) Foundations of any shape resting on the surface of a homogeneous half space.
- (b) Foundations of any shape partially or fully embedded in a homogeneous half space.
- (c) Circular foundations on the surface of a homogeneous stratum underlain by bedrock.
- (d) Circular foundations partially or fully embedded in a homogeneous stratum underlain by bedrock.

The application to the above four conditions are illustrated by example problems with identical machine and soil parameters. The results are controlled based on the design checklist of section 5.7 and a comparison of the results for the above different conditions are performed.

The computer programme enables also the analysis of block-type machine foundations according to the classical methods of chapter 4. The results of the analysis according to the classical methods are compared with those according to the more rational method.

6.2 Numerical Examples and Evaluations

Example Problem 1

A machine foundation for a vertical reciprocating compressor is to be constructed on the surface of a homogeneous half space. Machine manufacturers and soil engineers supply the following machine and soil parameters.

Machine Parameters:

Weight of machine = 250.7 kN

Compressor speed, (Operating) = 585 rpm

Vertical force, $F_z = 8.54$ kN

Vertical moment, $M_z = 47.0$ kN m

Horizontal force, $F_x = 3.29$ kN

Horizontal moment, $M_y = 15.63$ kN m

Soil Parameters

The soil type is medium dense silty sand with gravel

Soil density, $\rho = 1.8$ kN.S²/m⁴

Shear modulus, $G=98431.4 \text{ kN/m}^2$

Poisson's ratio, $\nu=0.35$

Soil internal damping ratio, $S= 0.05$

Static allowable bearing capacity, $q_{\text{all}}=120.0 \text{ kN/m}^2$

Permanent settlement of soil= 0.51 cm at 120.0 kN/m^2

Selection of a foundation Configuration

The concrete block type foundation shown in Figure 6.1 is the first trial with the combined center of gravity of the machine and the footing coinciding in plan with the centroid of the contact area of the footing.

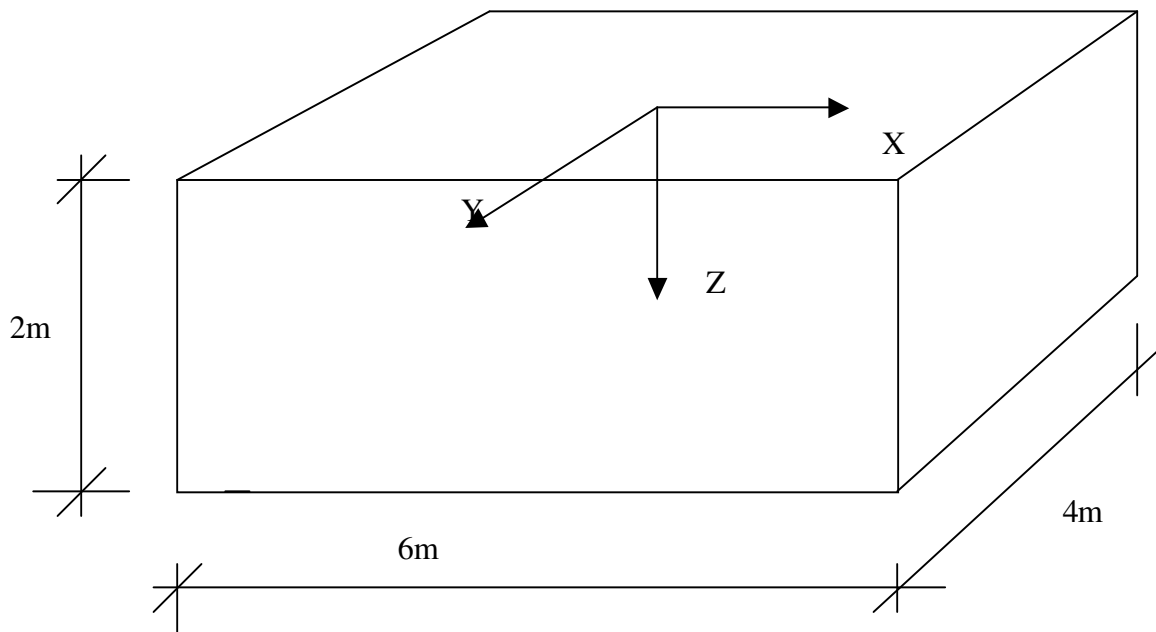


Figure 6.1: Foundation block on a homogeneous half space

Unit weight of concrete = 24 kN/m^3

Weight of the foundation = $6*4*2*24 = 1152.0 \text{ kN}$

Total static load= Weight of machine +weight of foundation

$$= 250.7 \text{ kN} + 1152.0 \text{ kN}$$

$$= 1402.7 \text{ kN}$$

$$\text{Weight of footing/Weight of machine} = \frac{1152}{250.7} = 4.6 > 3 \text{ OK!}$$

Static Conditions

(a) **Static bearing capacity:** Proportion footing area for 50% of allowable soil pressure.

$$\text{Actual soil pressure} = \frac{1402.7}{6m \times 4m} = 58.45 \text{ kN/m}^2 < 0.5 * q_{all} = 60.0 \text{ OK!}$$

(b) **Bearing capacity: Static plus dynamic loads:** The sum of static and modified dynamic loads should not create bearing pressure greater than 75% of the allowable soil pressure given in the static load conditions.

$$\text{Transmitted dynamic vertical force, } P_v = F_z M_{zz} \sqrt{1 + (2D_z r_z)^2} \quad [7]$$

$$= 8.54 * 1.04 \sqrt{1 + (2 * 0.6 * 0.56)^2}$$

$$= 14.4 \text{ kN}$$

$$\text{Total static plus dynamic bearing pressure} = \frac{1402.7 \text{ kN}}{6 * 4 \text{ m}^2} + \frac{14.4 \text{ kN}}{6 * 4 \text{ m}^2}$$

$$= 59.05 \text{ kN/m}^2 < 0.75 * q_{all} = 90.0 \text{ kN/m}^2 \text{ OK!}$$

(c) **Checking for settlement:** The magnitude of the resulting settlement should be less than the permissible deflecting capacity of the soil.

$$\text{Total settlement} = \frac{59.05}{120.0} * 0.51 \text{ cm} = 0.25 \text{ cm} < 0.51 \text{ cm} \text{ OK!}$$

Dynamic Conditions

(a) **Vibration Amplitude:** The maximum amplitude of motion for the foundation is 0.000007755 m = 0.000305 inch in the horizontal direction. This amplitude falls in zone A of Figure 5.5 at the operating frequency of 585 rpm and is therefore acceptable!.

(b) **Velocity:** The maximum amplitude is in the lateral direction

$$V_x = 2\pi f \Delta X = 2\pi \left(\frac{585}{60}\right) * 0.0007755 \text{ cm} = 0.0475 \text{ cm/sec}$$

This velocity falls in a “Smooth” range of Table 5.1 and is therefore acceptable!.

(c) **Magnification Factor:** The magnification factor should be less than 1.5

The maximum magnification factor is $1.22 < 1.5$ OK!

(d) **Resonance:** The operating frequency of the machine should have at least a difference of $\pm 20\%$ with the resonance frequency.

$$1.2 * \omega = 1.2 * 61.23 = 73.48 < \omega_{n_x} = 96.29. \text{OK!}$$

Therefore, resonance cannot occur!

Conclusion: All the static and dynamic design requirements listed under the design checklist are satisfied. Therefore, the assumed foundation block is acceptable for the given machine and soil parameters.

Numerical Example 2

The same type of foundation block, machine and soil parameters as in Example 1, but the foundation is partially embedded in a homogeneous halfspace.

Static Conditions

(a) **Static bearing capacity:** Proportion footing area for 50% of allowable soil pressure.

$$\text{Actual soil pressure} = \frac{1402.7 \text{ kN}}{6 \text{ m} \times 4 \text{ m}} = 58.45 \text{ kN/m}^2 < 0.5 * q_{\text{all}} = 60.0 \text{ OK!}$$

(b) **Bearing capacity: Static plus dynamic loads:** The sum of static and modified dynamic loads should not create bearing pressure greater than 75% of the allowable soil pressure given in the static load conditions.

$$\begin{aligned} \text{Transmitted dynamic vertical force [7], } P_v &= F_z M_{zz} \sqrt{1 + (2D_z r_z)^2} \\ &= 8.54 * 0.79 \sqrt{1 + (2 * 1.01 * 0.5)^2} \\ &= 9.59 \text{ kN} \end{aligned}$$

$$\begin{aligned} \text{Total static plus dynamic bearing pressure} &= \frac{1402.7 \text{ kN}}{6 * 4 \text{ m}^2} + \frac{9.59 \text{ kN}}{6 * 4 \text{ m}^2} \\ &= 58.85 \text{ kN/m}^2 < 0.75 * q_{\text{all}} = 90.0 \text{ kN/m}^2 \text{ OK!} \end{aligned}$$

(c) **Checking for Settlement:** The magnitude of the resulting settlement should be less than the permissible deflecting capacity of the soil.

$$\text{Total settlement} = \frac{58.85}{120.0} * 0.51 \text{ cm} = 0.25 \text{ cm} < 0.51 \text{ cm} \text{ OK!}$$

Dynamic Conditions

(a) **Vibration Amplitude:** The maximum amplitude of motion for the foundation is 0.00000177 m = 0.0000696 inch in the horizontal direction. This amplitude falls outside the range of Figure 5.5. Therefore, we should check for acceleration.

(b) **Velocity:** The maximum amplitude is in the lateral direction

$$V_x = 2\pi f \Delta X = 2\pi \left(\frac{585}{60}\right) * 0.000177 \text{ cm} = 0.01084 \text{ cm/sec}$$

This velocity falls in “Extremely Smooth” range of Table 5.1 and is therefore acceptable!.

(c) **Acceleration:** The maximum acceleration in the lateral direction equals

$$A_x = 4\pi^2 f^2 * \Delta x = 2\pi^2 * \left(\frac{585}{60}\right)^2 * 0.000177 = 0.664 \text{ cm/sec}^2 = 0.068G.$$

This acceleration falls in zone B of Figure 5.5 and is therefore acceptable!

(d) **Magnification Factor:** The magnification factor should be less than 1.5

The maximum magnification factor is 1.03 < 1.5 OK!

(e) **Resonance:** The acting frequency of the machine should have at least a difference of $\pm 20\%$ with the resonance frequency.

$$1.2 * \omega = 1.2 * 61.23 = 73.48 < \omega_{nz} = 123.02. \text{OK!}$$

Therefore, resonance cannot occur!

Conclusion: All the static and dynamic design requirements listed under the design checklist are satisfied. Therefore, the assumed foundation block is acceptable for the given machine and soil parameters.

Numerical Example 3

The same foundation block, machine and soil parameters as in Example 1 but the foundation is constructed on top of a homogeneous soil overlying the bedrock for H=5.

Static Conditions

(a) **Static bearing capacity:** Proportion footing area for 50% of allowable soil pressure.

$$\text{Actual soil pressure} = \frac{1402.7kN}{6m \times 4m} = 58.45 \text{ kN/m}^2 < 0.5 * q_{\text{all}} = 60.0 \text{ OK!}$$

(b) **Bearing capacity: Static plus dynamic loads.** The sum of static and modified dynamic loads should not create bearing pressure greater than 75% of the allowable soil pressure given in the static load conditions.

$$\begin{aligned} \text{Transmitted dynamic vertical force [7], } P_v &= F_z M_{z,z} \sqrt{1 + (2D_z r_z)^2} \\ &= 8.54 * 1.29 \sqrt{1 + (2 * 0.1 * 0.48)^2} \\ &= 11.07 \text{ kN} \end{aligned}$$

$$\text{Total static plus dynamic bearing pressure} = \frac{1402.7kN}{6 * 4m^2} + \frac{11.07kN}{6 * 4m^2}$$

$$=58.91 \text{ kN/m}^2 < 0.75 * q_{all} = 90.0 \text{ kN/m}^2 \text{ OK!}$$

(c) Checking for Settlement: The magnitude of the resulting settlement should be less than the permissible deflecting capacity of the soil.

$$\text{Total settlement} = \frac{58.91}{120.0} * 0.51 \text{ cm} = 0.25 \text{ cm} < 0.51 \text{ cm} \text{ OK!}$$

Dynamic Conditions

(a) Vibration Amplitude: The maximum amplitude of motion for the foundation is $0.00000656 \text{ m} = 0.0002583 \text{ inch}$ in the horizontal direction. This amplitude falls in zone A of Figure 5.5 at the operating frequency of 585 rpm and is therefore acceptable!

(b) Velocity: The maximum amplitude is in the lateral direction

$$V_x = 2\pi f \Delta X = 2\pi \left(\frac{585}{60}\right) * 0.000656 \text{ cm} = 0.0402 \text{ cm/sec}$$

This velocity falls in a “Smooth” range of Table 5.1 and is therefore acceptable!.

(c) Magnification Factor: The magnification factor should be less than 1.5

The maximum magnification factor is $1.33 < 1.5$ and is therefore acceptable!

(d) Resonance: The operating frequency of the machine should have at least a difference of $\pm 20\%$ with the resonance frequency.

$$1.2 * \omega = 1.2 * 61.23 = 73.56 < \omega_{nz} = 105.75 \text{ OK!}$$

Therefore, resonance cannot occur!

Conclusion: All the static and dynamic design requirements listed under design checklist are satisfied. Therefore, the predicted foundation block is acceptable for the given machine and soil parameters.

Numerical Example 4

The same foundation block, machine and soil parameters as in Example 1 but the foundation is partially embedded in a homogeneous soil overlying the bedrock for H=5.

Static Conditions

(a) **Static bearing capacity:** Proportion footing area for 50% of allowable soil pressure.

$$\text{Actual soil pressure} = \frac{1402.7 \text{ kN}}{6 \text{ m} \times 4 \text{ m}} = 58.45 \text{ kN/m}^2 < 0.5 * q_{\text{all}} = 60.0 \text{ OK!}$$

(b) **Bearing capacity: Static plus dynamic loads.** The sum of static and modified dynamic loads should not create bearing pressure greater than 75% of the allowable soil pressure given in the static load conditions.

$$\begin{aligned} \text{Transmitted dynamic vertical force [7], } P_v &= F_z M_{zz} \sqrt{1 + (2D_z r_z)^2} \\ &= 8.54 * 1.06 \sqrt{1 + (2 * 0.46 * 0.34)^2} \\ &= 9.94 \text{ kN} \end{aligned}$$

$$\text{Total static plus dynamic bearing pressure} = \frac{1402.7 \text{ kN}}{6 * 4 \text{ m}^2} + \frac{9.94 \text{ kN}}{6 * 4 \text{ m}^2}$$

$$=58.86 \text{ kN/m}^2 < 0.75 * q_{all} = 90.0 \text{ kN/m}^2 \quad \text{OK!}$$

(c) Checking for Settlement: The magnitude of the resulting settlement should be less than the permissible deflecting capacity of the soil.

$$\text{Total settlement} = \frac{58.86}{120.0} * 0.51 \text{ cm} = 0.25 \text{ cm} < 0.51 \text{ cm} \quad \text{OK!}$$

Dynamic Conditions

(a) Vibration Amplitude: The maximum amplitude of motion for the foundation is $0.000001362 \text{ m} = 0.000053622 \text{ inch}$ in the horizontal direction. This amplitude falls outside the range of Figure 5.5. Therefore, we should check for acceleration.

(b) Velocity: The maximum amplitude is in the lateral direction

$$V_x = 2\pi f \Delta X = 2\pi \left(\frac{585}{60}\right) * 0.0001362 \text{ cm} = 0.0083 \text{ cm/sec}$$

This velocity falls in “Extremely Smooth” range of Table 5.1 and is therefore acceptable!

(c) Acceleration: The maximum acceleration in the lateral direction

$$A_x = 4\pi^2 f^2 * \Delta x = 4\pi^2 * \left(\frac{585}{60}\right)^2 * 0.0001362 = 0.5106 \text{ cm/sec}^2 = 0.052 \text{ G.}$$

This acceleration falls in zone B of Figure 5.5 and is therefore acceptable!

(c) Magnification Factor: The magnification factor should be less than 1.5

The maximum magnification factor is $1.06 < 1.5$ OK!

(d) Resonance: The acting frequency of the machine should have at least a difference of ± 20 with the resonance frequency.

$$1.2 * \omega = 1.2 * 61.23 = 73.48 < \omega_{nz} = 157.50 \text{ OK!}$$

Therefore, resonance cannot occur!

Conclusion: All the static and dynamic design requirements listed under the design checklist are satisfied. Therefore, the assumed foundation block is acceptable for the given machine and soil parameters.

Numerical Example 5

The same foundation block, machine and soil parameters as in Example 1, but the foundation is analyzed using classical method.

Static Conditions

(a) Static bearing capacity: Proportion footing area for 50% of allowable soil pressure.

$$\text{Actual soil pressure} = \frac{1402.7 \text{ kN}}{6 \text{ m} \times 4 \text{ m}} = 58.45 \text{ kN/m}^2 < 0.5 * q_{\text{all}} = 60.0 \text{ OK!}$$

(b) Bearing capacity: Static plus dynamic loads: The sum of static and modified dynamic loads should not create bearing pressure greater than 75% of the allowable soil pressure given for the static load conditions.

$$\text{Transmitted dynamic vertical force [7], } P_v = F_z M_{z_z} \sqrt{1 + (2D_z r_z)^2}$$

$$= 8.54 * 0.38 \sqrt{1 + (2 * 1.56 * .85)^2}$$

$$= 5.16 \text{ kN}$$

$$\text{Total static plus dynamic bearing pressure} = \frac{1402.7 \text{ kN}}{24 \text{ m}^2} + \frac{5.16 \text{ kN}}{24 \text{ m}^2}$$

$$= 58.66 \text{ kN/m}^2 < 0.75 * q_{all} = 90.0 \text{ kN/m}^2 \quad \text{OK!}$$

(c) Checking for Settlement: The magnitude of the resulting settlement should be less than the permissible deflecting capacity of the soil.

$$\text{Total settlement} = \frac{58.66}{130.0} * 0.51 \text{ cm} = 0.25 \text{ cm} < 0.51 \text{ cm} \quad \text{OK!}$$

Dynamic Conditions

(a) Vibration Amplitude: The maximum amplitude of motion for the foundation is 0.000009174 m = 0.000361 inch in the vertical direction This amplitude falls in zone A of Figure 5.5 at the operating frequency of 585 rpm and is therefore acceptable!.

(b) Velocity: The maximum amplitude is in the lateral direction

$$V_x = 2\pi f \Delta X = 2\pi \left(\frac{585}{60} \right) * 0.0009174 \text{ cm} = 0.0562 \text{ cm/sec}$$

This velocity falls in a “Very Good” range of Table 5.1 and is therefore acceptable!.

(c) Magnification Factor: The magnification factor should be less than 1.5

The maximum magnification factor is 1.07 < 1.5 OK!

(d) Resonance: The acting frequency of the machine should have at least a difference of $\pm 20\%$ with the resonance frequency.

$$1.2 * \omega = 1.2 * 61.23 = 73.48 < \omega_{nz} = 139.88. \text{OK!}$$

Therefore, resonance cannot occur!

Conclusion: All the static and dynamic design requirements listed under the design checklist are satisfied. Therefore, the assumed foundation block is acceptable for the given machine and soil parameters.

6.3 Discussions

The dynamic stiffness and damping ratios for the first two numerical examples are given in tabular form as follows.

Mode	\bar{K}_{sur}	\bar{K}_{emb}	$\bar{K}_{emb} / \bar{K}_{sur}$	D_{sur}	D_{emb}	D_{emb} / D_{sur}	$\Delta_{sur} / \Delta_{emb}$
Vertical	1661706	2122955	1.28	60%	100%	1.67	5.58
Lateral	1300611	2436821	1.87	45%	66%	1.47	4.38
Rocking	9430634	22917130	2.43	20%	64%	3.20	3.55
Torsion	11881790	33355200	3.23	17%	47%	2.77	3.81

Table 6.1: Comparison of dynamic stiffness and damping ratios for example 1 and 2

When we consider the surface foundation on a homogeneous half space (Example 1), we observe that for rotational modes (rocking and torsion), the effective damping ratios are low as compared to the translational modes (vertical and lateral oscillations). These differences are a direct consequence of the different amount of radiation damping, which results from geometric spreading of waves generated at the foundation soil interface.

When the foundation undergoes a vertical oscillation, waves are emitted in phase and reach long distance away from the foundation. Therefore, relatively large amount of wave energy are lost due to high radiation damping especially in the vertical direction.

In the numerical example for foundations partially or full embedded in a homogeneous half space, the damping ratios are high as compared to the surface foundations. This is due to the increase in radiation damping produced by waves emanating from the vertical sidewall of the foundation. This high damping ratio leads to the reduction of the four modes of vibration amplitudes.

In general, from the first two numerical examples and Table 6.1, we could observe that the dynamic stiffness and damping ratios of foundations partially or fully embedded in a homogeneous half space increases significantly from that of foundations on the surface of homogeneous half space. This results in the reduction of the displacement and rotation amplitudes of the embedded foundations from 3.55 to 5.58 times that of surface foundation as shown in Table 6.1. In other words, embedding a foundation is a very effective way to reduce to the acceptable levels of the anticipated amplitudes of vibration, especially if these amplitudes arise due to rocking or torsion

The formulas for dynamic stiffness and damping coefficients given in tabular form in Table A-3 and A-4 of appendix A are for circular block type machine foundations on the surface of homogeneous soil over bedrock and embedded foundations in a homogeneous soil over bedrock, respectively. But it is possible to use the formulas given in Table A-3 and A-4 for rectangular block type foundation by converting it into an equivalent circular one under the following conditions.

For translational mode, the base area of the rectangular foundation should be equated to the

base area of the circular foundation i.e., $R = 2\sqrt{\frac{BL}{\pi}}$. For rocking mode, the area moment of

inertia around the horizontal axis of the rectangular foundation should be equated with that of

the circular one i.e $R = \sqrt[4]{\frac{16}{3\pi}LB^3}$ or $R = \sqrt[4]{\frac{16}{3\pi}BL^3}$. For torsional mode, the polar moment of

inertia of the rectangular foundation should be equated with that of the circular foundation.

$$\text{i.e, } R = \sqrt[4]{\frac{8}{3\pi}BL^3} + \sqrt[4]{\frac{8}{3\pi}LB^3}$$

Using the above substitutions, the same example problem of Examples 1 and 2, are analyzed for layered formation, i.e, foundations on the surface of a homogeneous soil over bedrock and partially embedded foundations in a homogeneous soil over bedrock. The results of the dynamic stiffness, damping ratios and the translational and rotational amplitude ratios are summarized in Tables 6.2 to 6.4 for different magnitudes of depth to bedrock H.

Mode	\bar{K}_{sur}	\bar{K}_{emb}	$\bar{K}_{emb}/\bar{K}_{sur}$	D_{sur}	D_{emb}	D_{emb}/D_{sur}	$\Delta_{sur}/\Delta_{emb}$
Vertical	2306572	4627564	2.01	10%	46%	4.6	5.11
Lateral	1568545	3479483	2.22	30%	78%	2.60	5.49
Rocking	10355390	29065860	2.81	19%	62%	4.77	3.96
Torsion	13607180	35815570	2.63	17%	45%	2.65	3.01

Table 6.2: Comparison of dynamic stiffness and damping ratios for example 3 and 4 for H=5

Mode	\bar{K}_{sur}	\bar{K}_{emb}	$\bar{K}_{emb}/\bar{K}_{sur}$	D_{sur}	D_{emb}	D_{emb}/D_{sur}	$\Delta_{sur}/\Delta_{emb}$
Vertical	1823933	2969560	1.63	35%	53%	1.51	5.77
Lateral	1396525	2833399	2.03	44%	93%	2.11	5.13
Rocking	9862369	24825940	2.52	20%	67%	3.35	3.59
Torsion	13019400	34848740	2.68	17%	45%	2.65	3.08

Table 6.3: Comparison of dynamic stiffness and damping ratios for example 3 and 4 for H=10

Mode	\bar{K}_{sur}	\bar{K}_{emb}	$\bar{K}_{emb}/\bar{K}_{sur}$	D_{sur}	D_{emb}	D_{emb}/D_{sur}	$\Delta_{sur}/\Delta_{emb}$
Vertical	1663054	2562506	1.54	49%	85%	1.74	5.71
Lateral	1341851	2632569	1.96	45%	96%	2.13	5.08
Rocking	9698027	23476090	2.42	20%	67%	3.35	3.49
Torsion	12823470	34526470	2.69	17%	45%	2.65	3.41

Table 6.4: Comparison of dynamic stiffness and damping ratios for example 3 and 4 at H=15

From the above three tables (Table 6.2 to 6.4), we observe that:

(a) The dynamic stiffness of the embedded foundations for all modes of oscillations are larger than that of surface foundations.

(b) The dynamic stiffness for all modes of oscillations increases as the depth to bedrock (H), is getting smaller. This is because the static stiffness tends to increase due to the decrease in the depth of deforming zone underneath the foundation.

(c) The damping coefficients increase as the depth to bedrock (H) increases for vertical and lateral oscillations. But the damping coefficients for rocking and torsion are independent of the magnitude of the depth to bedrock (H).

(d) The damping coefficients of the embedded foundations are larger than that of surface foundations. This is due of the geometric spreading of waves away from the vertical sidewalls of the foundation.

From the above four tables, we also observe that:

(1) As the depth to bedrock (H) increases, the magnitudes of the dynamic stiffness and damping coefficient of surface and embedded foundations over bedrock approaches to the magnitude of the dynamic stiffness and damping coefficient of surface and embedded foundations in a homogeneous half space.

(2) The magnitudes of the damping coefficients for rocking and torsion are independent of whether the foundation soil is homogeneous or layered and the magnitude of the depth to bedrock H is larger or smaller.

The dynamic stiffness and damping ratios for the classical method of machine foundation analysis are shown in Table 6.5 below.

Mode	\bar{K}	D
Vertical	1673652	55%
Lateral	1318635	33%
Rocking	6521473	24%
Torsion	12190370	11%

Table 6.4: The dynamic stiffness and damping ratios for the classical method (Example 5)

The classical method of machine foundation analysis has the following drawbacks as compared with the recent methods.

(a) The formulas for dynamic stiffness and damping coefficients are not distinguished between surface and embedded foundation. The same formulae are used for surface and embedded foundations.

(b) The classical method does not appropriately incorporate the impedance functions and the coefficients for dynamic stiffness and damping are not considered and given in the form of dimensionless graphs or any other form. It simply takes one for all modes of oscillations.

Due to the reasons mentioned above, the classical method of machine foundation analysis lacks rationality. But, from the four examples analyzed above, the dynamic stiffness and damping coefficient values of the classical method of analysis seen to be closer to the corresponding values of foundations on the surface of a homogeneous half space.

6.4 Conclusion and Recommendations

Until recently, the methods of machine foundation analysis lacked rationality in the way they incorporate impedance functions. In this work an attempt is made to properly include the impedance functions through the use of recently well-compiled expressions and dimensionless graphs contributed by G. Gazatas for the determination of dynamic stiffness and dashpot coefficients. Based on these expressions, a computer programme in FORTRAN is written. The programme makes the analysis of block-type machine foundation more general, more realistic and faster.

The formulas used for the determination of dynamic stiffness and damping coefficient are independently formulated for foundations on the surface of homogeneous half space, embedded foundations over homogeneous half space, foundations on the surface of homogeneous soil underlain by bedrock and embedded foundations over bedrock in the recent approach of machine foundation analysis. But the formulae used in the classical methods, to determine the dynamic stiffness and damping coefficient do not distinguish among such different foundation conditions. In other words, no distinction is made

between surface and embedded foundations as well as homogeneous layered soil conditions. This is a major drawback of the classical methods as compared with the method employed in this work.

Furthermore, the following conclusions can be drawn which confirm the general observations made in the past.

(a) As the mass or mass moment of inertia of the foundation block increases, the damping ratio and the natural frequency of the foundation gets smaller. Therefore, if small-frequency machines are constructed on a bigger foundation block, the probability of the natural frequency of the foundation and the operational frequency of the machine to coincide is big. In other words, a foundation having big mass or mass moment of inertia could fail with small frequency machines due to resonance conditions.

(b) For surface foundations on homogeneous stratum overlying bedrock, the magnitude of static stiffness are strongly dependent on the ratio of the radius, R , of the foundation to the depth, H , of the bedrock from the ground surface (R/H) for all modes of oscillations. That is, as the depth to the bedrock from the ground surface is getting larger, the static stiffness reduces because of the increase in the deformable zone of the formation.

(c) From the numerical examples and discussions, we can generalize that embedding a foundation, whether it is in a homogeneous half space or homogeneous stratum over bedrock, reduces the amplitudes of all modes of oscillations. In other words, waves generated by the machine are highly damped in the embedded foundations than surface foundations. Therefore, embedding a block-type machine foundation, is always helpful to reduce vibration amplitude.

The formulae written in this thesis for the determination of dynamic stiffness and damping coefficients are all for homogeneous half space and homogeneous soil over bedrock conditions. But due to the variable nature of the underground soil, the probability of the soil formation to be inhomogeneous within short depth interval is very big. However, the soil profiles considered in this work are representative of most practical cases.

TABLE A-1 DYNAMIC STIFFNESS AND DASHPOT COEFFICIENTS FOR ARBITRARILY SHAPED FOUNDATIONS ON THE SURFACE OF A HOMOGENEOUS HALFSPACE. [2]

Vibration Mode	Dynamic stiffness $\bar{K} = K * k(\omega)$		
	Static Stiffness K		Dynamic stiffness Coefficient k (General shape; $0 \leq a_o \leq 2$)
	General shape (Foundation-soil contact surface is of area A_b and has a circumscribed rectangle $2L$ by $2B$; $L > B$)	Square $L=B$	
Vertical, Z	$K_z = \frac{2GL}{1-\nu} (0.73 + 1.54x^{0.75})$ <p>With $x = \frac{A_b}{4L^2}$</p>	$K_z = \frac{4.54GB}{1-\nu}$	$k_z = k_z \left(\frac{L}{B}, \nu, a_o \right)$ <p>is plotted in Graph a</p>
Horizontal, Y (In the lateral Direction)	$K_y = \frac{2GL}{2-\nu} (2 + 2.50x^{0.85})$	$K_y = \frac{9GB}{2-\nu}$	$k_y = k_y \left(\frac{L}{B}; a_o \right)$ <p>is plotted in Graph b</p>
Horizontal, X (In the longitudinal Direction)	$K_x = K_y - \frac{0.2}{0.75-\nu} GL \left(1 - \frac{B}{L} \right)$	$K_x = K_y$	$k_x \cong 1$
Rocking, rx (around Longitudinal X-axis)	$K_{rx} = \frac{G}{1-\nu} I_{bx}^{0.75} \left(\frac{L}{B} \right)^{0.25} * \left(2.4 + 0.5 \frac{B}{L} \right)$ <p>With I_{bx} (I_{by}) area moment of inertia of the foundation-soil Contact surface around the x(y) axis</p>	$K_{rx} = \frac{3.6GB^3}{1-\nu}$	$k_{rx} \cong 1 - 0.20a_o$
Rocking, ry (around lateral axis)	$K_{ry} = \frac{G}{1-\nu} I_{by}^{0.75} \left[3 * \left(\frac{L}{B} \right)^{0.15} \right]$	$K_{ry} = K_{rx}$	$\left\{ \begin{array}{l} \nu < 0.45 : \\ k_{ry} \cong 1 - 0.30a_o \\ \nu \cong 0.50 : \\ k_{ry} \cong 1 - 0.25a_o \end{array} \right.$

Torsional	$K_t = G(J_b)^{0.75} \left[4 + 11 \left(1 - \frac{B}{L} \right)^{10} \right]$ <p>With $J_b = I_{bx} + I_{by}$ being the polar moment of the soil-foundation contact surface</p>	$K_t = 8.3GB^3$	$k_t \cong 1 - 0.14a_o$
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TABLE A-2 DYNAMIC STIFFNESS AND DASHPOT COEFFICIENTS FOR ARBITRARILY SHAPED PARTIALLY OR FULLY EMBEDDED IN A HOMOGENOUS HALFSPACE. [2]

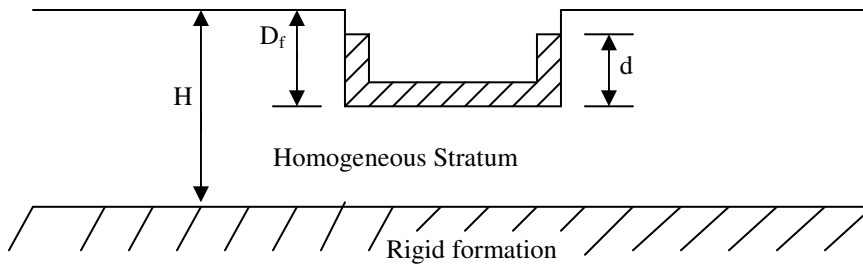
Vibration Mode	Dynamic stiffness $\bar{K}_{emb} = K_{emb} k_{emb}(\omega)$		Radiation Dashpot Coefficient
	Static stiffness K_{emb} For foundations with arbitrarily-shaped basement A_b of circumscribed rectangle 2L by 2B: total sidewall-soil contact area A_w (or constant sidewall-soil contact height d)	Dynamic stiffness Coefficient $k_{emb}(\omega)$ $0 < a_o \leq 2$	General Foundation Shape

<p>Vertical, Z</p>	$K_{z,emb} = K_{z,sur} \left[1 + \frac{D_f}{21B} (1 + 1.3x) \right]$ $* \left[1 + 0.2 \left(\frac{A_w}{A_b} \right)^{2/3} \right]$ <p>$K_{z,sur}$ is obtained from Table A-1</p> <p>A_w = actual sidewall-soil contact area; for constant effective contact height d along the perimeter</p> <p>$A_w = (d) * (\text{perimeter}); x = \frac{A_b}{4L^2}$</p>	<p>For $\nu \leq 0.4$ Fully embedded</p> $k_{z,emb} = k_{z,sur} \left[1 - 0.09 \left(\frac{D_f}{B} \right)^3 \right]$ <p>In a trench;</p> $k_{z,tre} \cong k_{z,sur} \left[1 + 0.09 \left(\frac{D_f}{B} \right)^{3/4} a_o^2 \right]$ <p>For partially embedded, estimate by interpolating between the two</p> <p>For $\nu \approx 0.5$</p> $\begin{cases} \text{Fully - embedded, } L/B = 1 - \\ k_{z,emb} \cong 1 - 0.09(D_f/B)^{3/4} * \\ \text{Fully - embedded, } L/B > 3 \\ k_{z,emb} \cong 1 - 0.35(D_f/B)^{1/2} * \end{cases}$	$C_{z,emb} = C_{z,sur} + \rho V_s A_w$ <p>with $C_{z,sur}$ and \tilde{c}_z according to Table A-1</p>
<p>Horizontal y or x</p>	$K_{y,emb} = K_{y,sur} \left(1 + 0.15 \sqrt{\frac{D_f}{B}} \right)$ $* \left[1 + 0.52 \left(\frac{h A_w}{B L^2} \right)^{0.4} \right]$ <p>$K_{y,sur}$ obtained from Table A-1</p> <p>$K_{x,emb}$ similarly computed from $K_{x,sur}$</p> <p>h is shown in the graph accompanying this table (Appendix-B)</p>	<p>$k_{y,emb}$ and $k_{x,emb}$ will be estimated in terms of $\frac{L}{B}$, $\frac{D_f}{B}$, and $\frac{d}{B}$ for each value of a_o from the graphs accompanying this table.</p>	$C_{y,emb} = C_{y,sur} + \rho V_s A_{ws} + \rho V_{la} A_{wce}$ <p>$A_{ws} = \sum A_{wi}$ = Total effective sidewall area shearing the soil.</p> <p>A_{wce} = Total effective sidewall area compressing the soil.</p> <p>$C_{y,sur}$ Obtained from Table A-1</p> <p>$C_{x,emb}$ Similarly computed from $C_{x,sur}$</p>

<p>Rocking, rx (around the longitudinal axis)</p> <p>Rocking, ry (around the lateral axis)</p>	<p>Expressions valid for any basement shape but constant effective contact height d along the perimeter</p> $K_{rx,emb} = K_{rx,sur} *$ $\left\{ 1 + 1.26 \frac{d}{B} \left[1 + \frac{d}{b} \left(\frac{d}{D_f} \right)^{-0.2} \sqrt{\frac{B}{L}} \right] \right\}$ $K_{ry,emb} = K_{ry,sur} *$ $\left\{ 1 + 0.92 \left(\frac{d}{L} \right)^{0.6} \left[1.5 + \left(\frac{d}{L} \right)^{1.9} * \left(\frac{d}{D_f} \right)^{-0.6} \right] \right\}$	$k_{rx,emb} = k_{rx,sur}$ $k_{ry,emb} = k_{ry,sur}$	$C_{rx,emb} = C_{rx,sur} + \rho V_{la} I_{wce} C_1 + \rho V_s [J_{ws} + \sum_i (A_{wcei} \Delta_i^2)] \tilde{c}_1$ $\tilde{c}_1 = 0.25 + 0.65 \sqrt{a_o} \left(\frac{d}{D_f} \right)^{-0.2} * \left(\frac{D_f}{B} \right)^{-1/4}$ <p>I_{wce} = Total moment of inertia about their base axes parallel to x of all sidewall surfaces effectively compressing the soil.</p> <p>Δ_i = Distance of surface A_{wcei} from the x axis</p> <p>J_{ws} = Polar moment of inertia about their base axes parallel to x for all sidewall surfaces effectively shearing the soil.</p> <p>$C_{ry,emb}$ is similarly evaluated from $C_{ry,sur}$ with Y replacing X and, in the equation for C_1, L replacing B.</p>
<p>Coupling term Swaying-rocking (x,ry)</p> <p>Swaying-rocking (y,rx)</p>	$K_{xry,emb} \cong \frac{1}{3} d K_{x,emb}$ $K_{yrx,emb} \cong \frac{1}{3} d K_{y,emb}$	$k_{xry,emb} \cong k_{yrx,emb} = 1$	$C_{xry,emb} \cong \frac{1}{3} d C_{x',emb}$ $C_{yrx,emb} \cong \frac{1}{3} d C_{y,emb}$

Torsional	$K_{t,emb} = K_{t,sur} * \left[1 + 1.4 \left(1 + \frac{B}{L} \right) \left(\frac{d}{B} \right)^{0.9} \right]$	$k_{t,emb} = k_{t,sur}$	$C_{t,emb} = C_{t,sur} + \rho V_{la} J_{wce} C_2 + \rho V_s \sum (A_{wi} \Delta_{zi}^2) \tilde{c}_2$ $\tilde{c}_2 \approx \left(\frac{d}{D_f} \right)^{-0.5} * \frac{a_o^2}{a_o^2 \frac{1}{2} \left(\frac{L}{B} \right)^{-1.5}}$ <p>J_{wce} = Total moment of inertia of all sidewall surfaces effectively compressing the soil about the projection of the z-axis onto their plane.</p> <p>Δ_{zi} = Distance of surface A_{wi} from the Z axis</p>
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TABLE A-4 FOUNDATIONS EMBEDDED IN HOMOGENEOUS STRATUM OVER BEDROCK [2]

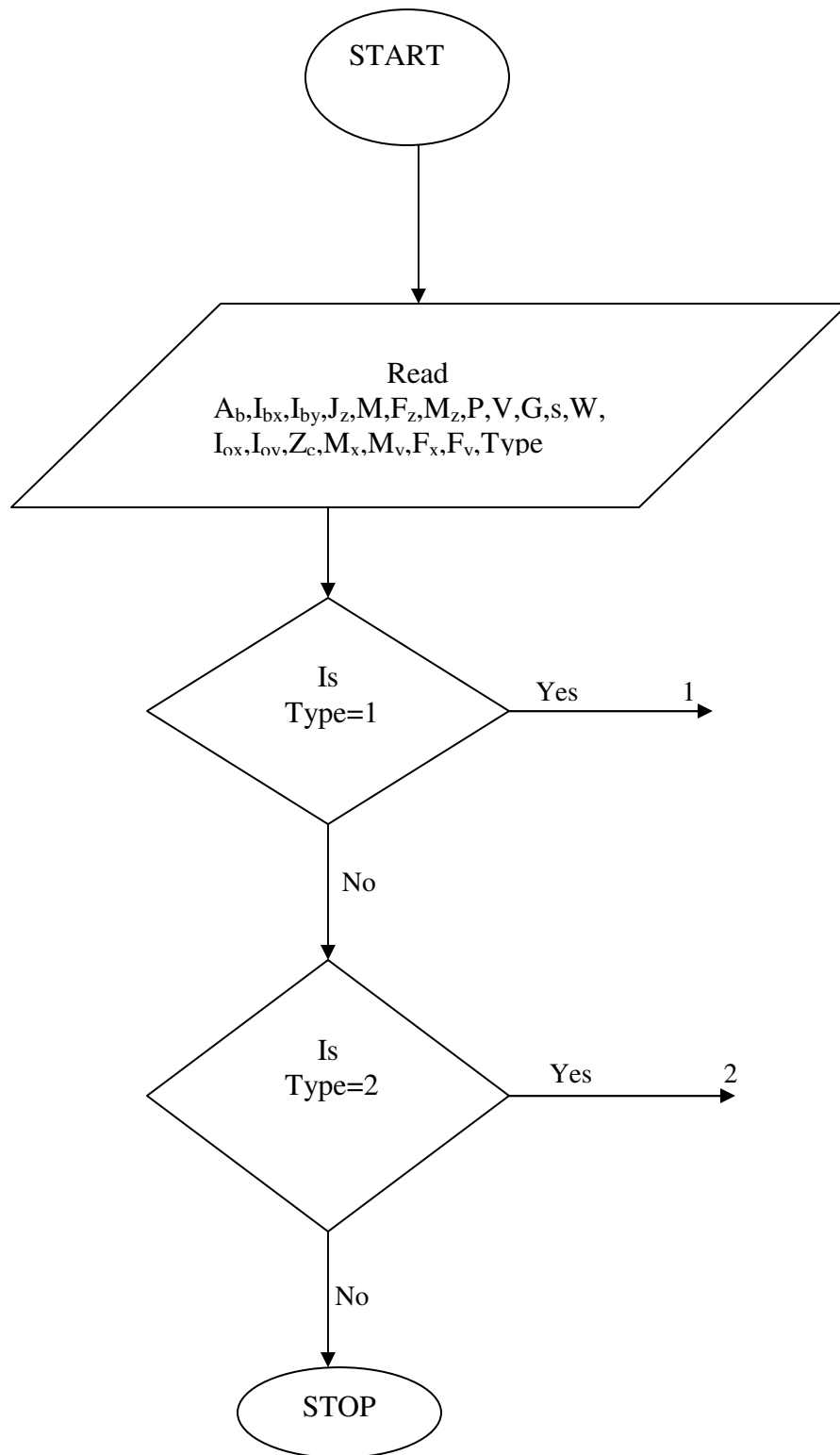


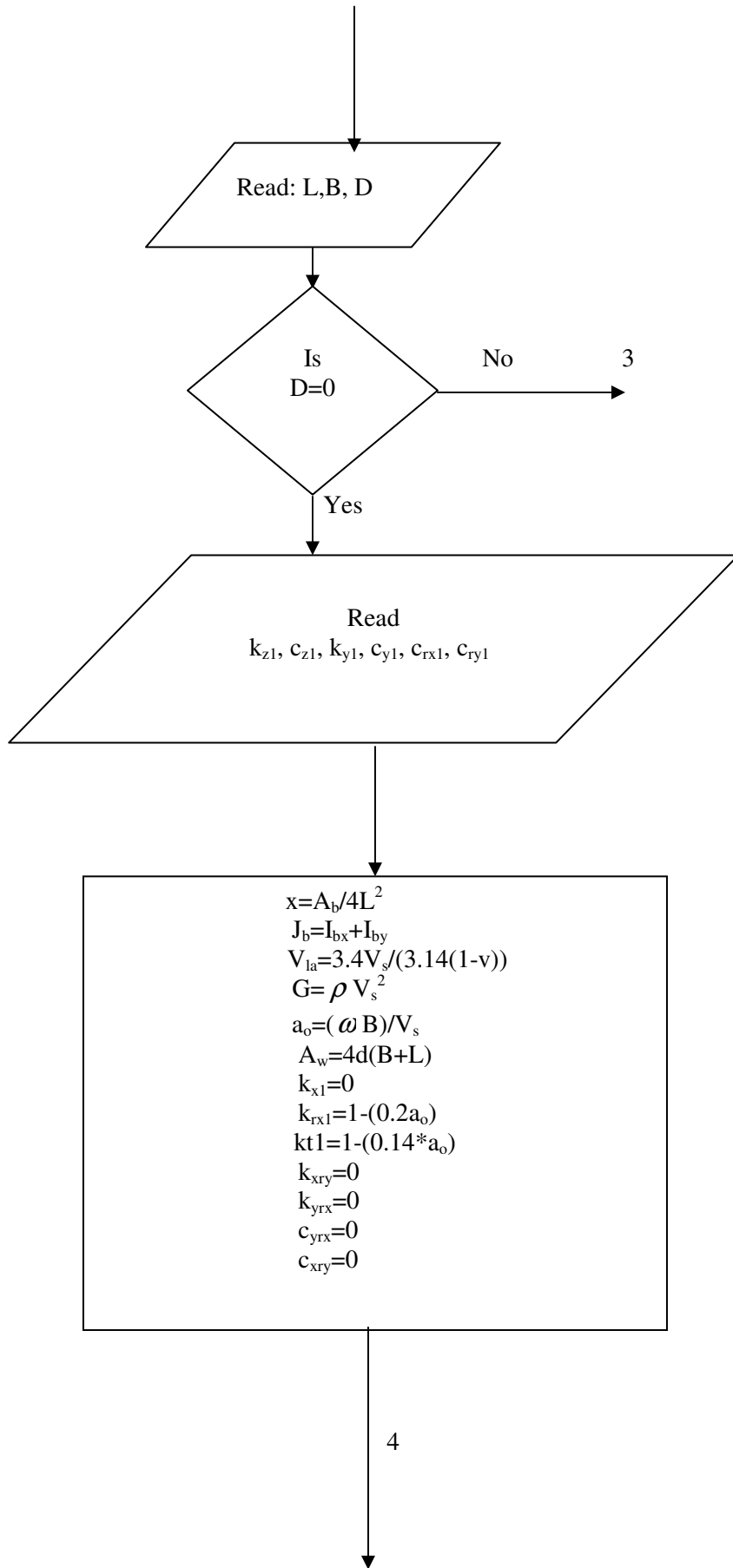
Foundation Shape		Circular Foundation of Radius R
Static Stiffness, K	Vertical, z	$K_{z,emb} \cong K_{z,sur} \left(1 + 0.55 \frac{d}{R} \right) \left[1 + \left(0.85 - 0.28 \frac{D_f}{R} \right) \frac{D_f}{H - D_f} \right]$
	Horizontal, y or x	$K_{y,emb} \cong K_{y,sur} \left(1 + \frac{d}{R} \right) \left(1 + 1.25 \frac{D_f}{H} \right)$
	Rocking rx or ry	$K_{rx,emb} \cong K_{rx,sur} \left(1 + 2 \frac{d}{R} \right) \left(1 + 0.65 \frac{D_f}{H} \right)$

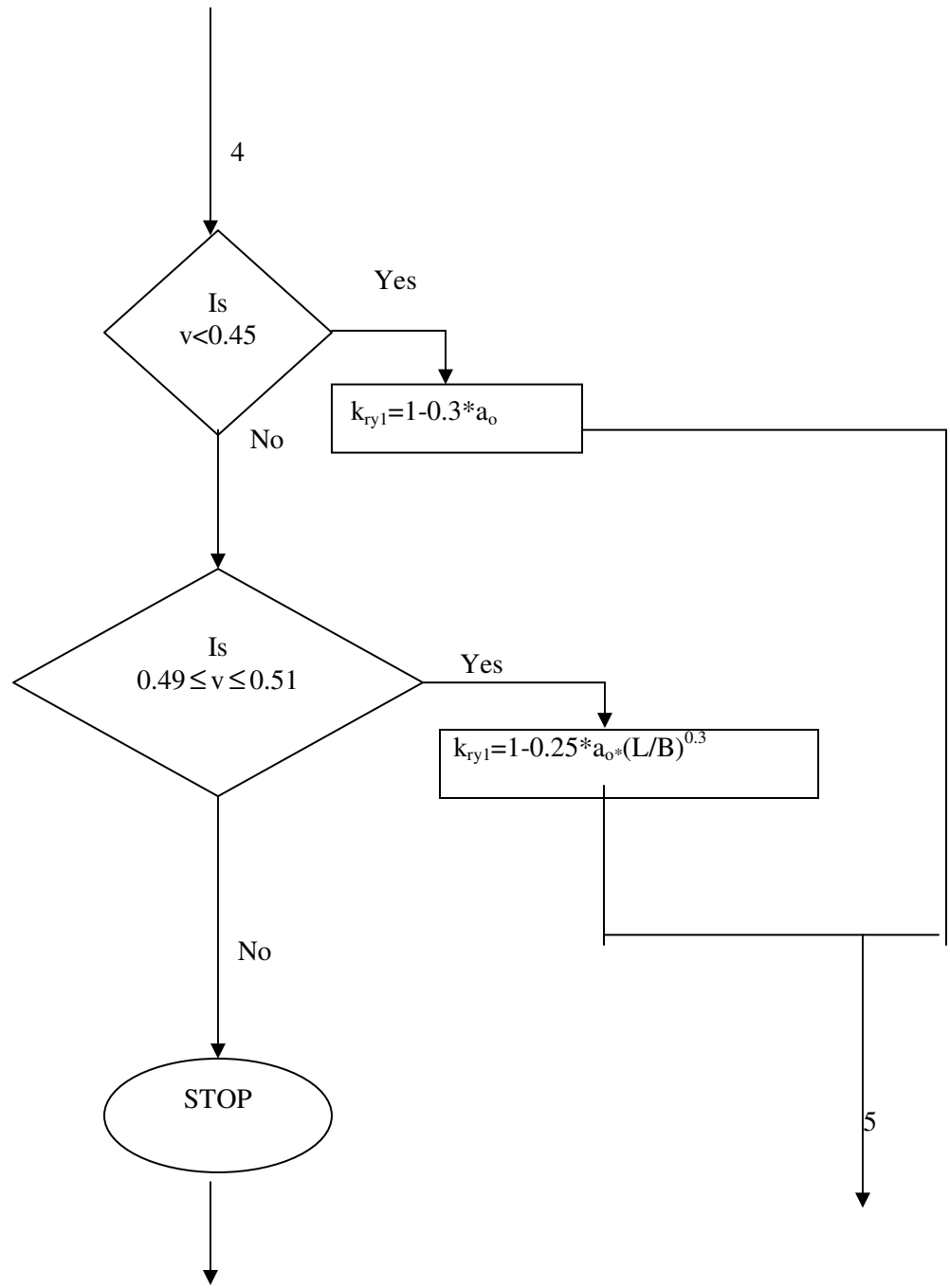
	Coupled Swaying-rocking	$K_{yrx,emb} \cong \frac{1}{3} dK_{y,emb}$
	Torsional	$K_{t,emb} \cong K_{t,sur} \left(1 + 2.67 \frac{d}{R} \right)$
Dynamic Stiffness Coefficients,	$k(\omega)$	The relation between k_{emb} and k_{sur} follow approximately the same pattern as those b/n embedded and surface foundation on a homogeneous half space. Therefore, use the results of Table A-2 as a first approximation.
Radiation dashpot Coefficient	$C(\omega)$	C_{emb} exceeds C_{sur} by an amount that depends on the geometry of the side wall-soil contact surface and is practically independent of the presence or absence of a rigid base at shallow depth. Therefore, use the results of Table A-2 , but with C_{sur} corresponding to the layered profile and thus obtained according to Table A-3.

$K_{z,sur}, K_{y,sur} \dots$ are the stiffness for the corresponding surface foundations, and can be obtained from Table A-3

The Flow Chart For The Main Program



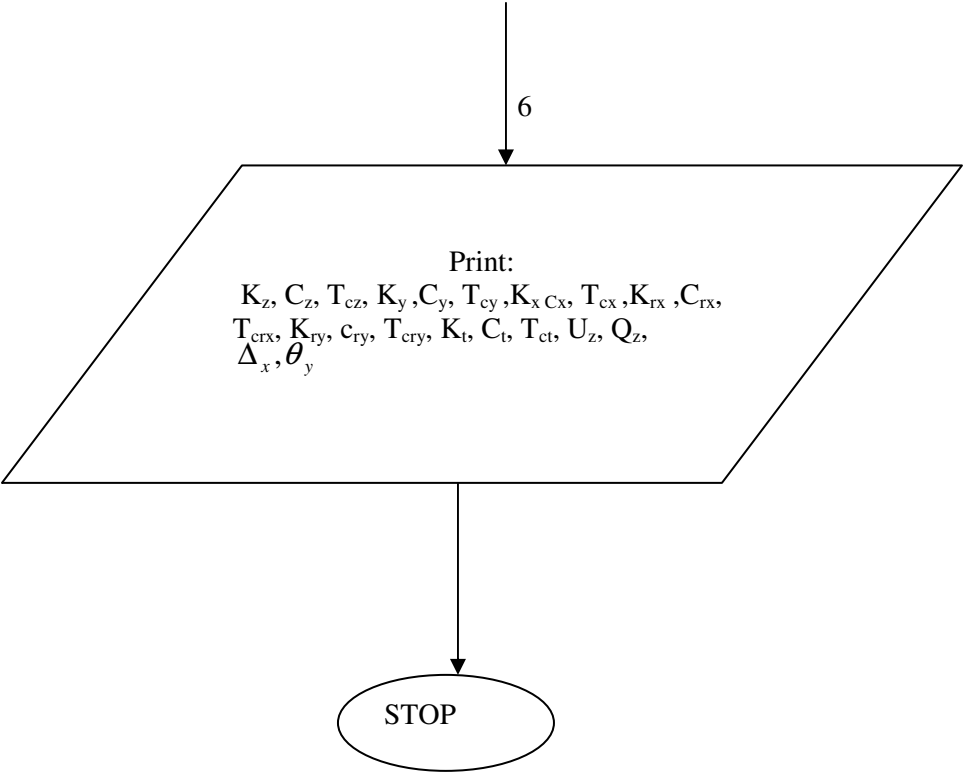


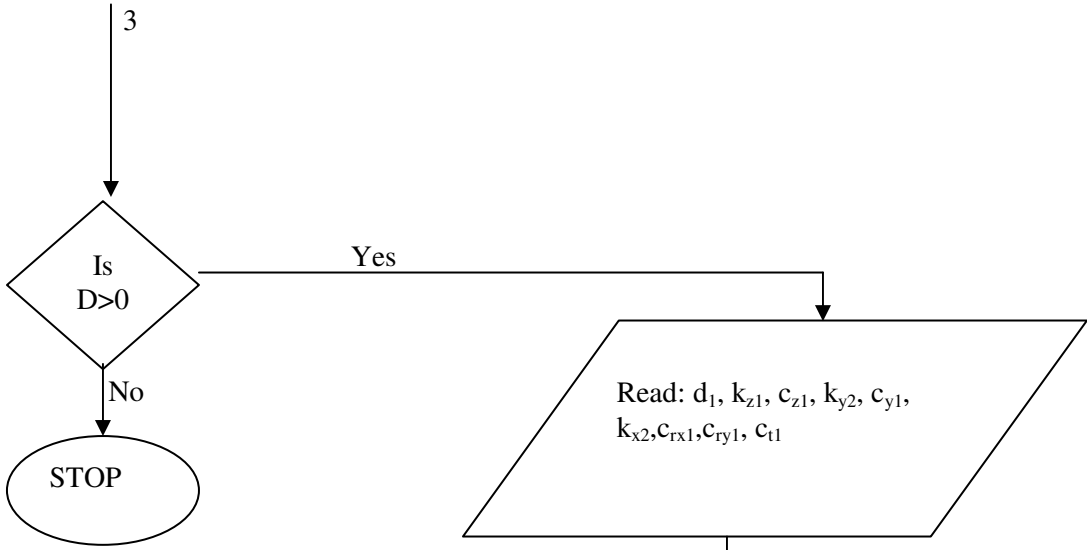


$$\begin{aligned}
K_z &= (2GL(0.73 + 1.54 * 0.75) k_{z1}) / (1 - \nu) \\
C_z &= \rho * V_{la} * c_{z1} \\
T_{cz} &= C_z + (2 * K_z * S) / \omega \\
K_y &= (2 * G * L (2 + 2.5 * X^{0.85}) * k_{y1}) / (2 - \nu) \\
C_y &= \rho * V_s * A_b * c_{y1} \\
T_{cy} &= c_y + (2 * K_y * s) / \omega \\
K_x &= K_y - (0.2 * G * L * (1 - (B/L)) * k_{x1}) / (0.75 - \nu) \\
C_x &= \rho * V_s * A_b \\
T_{cx} &= c_x + (2 * K_x * s) / \omega \\
K_{rx} &= ((G * (I_{bx})^{0.75} / (L/B)^{0.25} * (2.4 + 0.5(B/L))) * k_{rx1}) / (1 - \nu) \\
C_{rx} &= \rho * V_{La} * I_{bx} * c_{rx1} \\
T_{crx} &= c_{rx} + (2 * k_{rx} * s) / \omega \\
K_{ry} &= (3 * G * (I_{by})^{0.75} / (L/B)^{0.15} * k_{ry1}) / (1 - \nu) \\
C_{ry} &= \rho * V_{la} * I_{by} * c_{ry1} \\
T_{cry} &= c_{ry} + (2 * K_{ry} * s) / \omega \\
K_t &= G * (J_b)^{0.75} * (4 + (11 * (1 - B/L)^{10})) * k_{t1} \\
C_t &= \rho * V_s * J_b * c_{t1} \\
T_{ct} &= c_t + (2 * K_t * s) / \omega
\end{aligned}$$

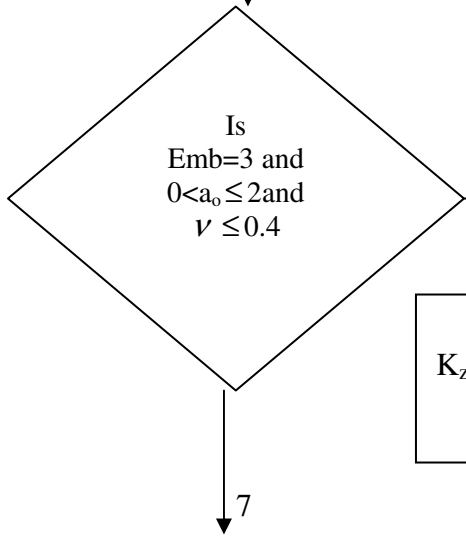
$$\begin{aligned}
U_z &= \frac{F_z}{\sqrt{(K_z - m\omega^2)^2 + \omega^2 T_{cz}^2}} & N &= A_1 * A_3 - A_2^2 \\
Q_z &= \frac{M_z}{\sqrt{(K_t - J_z \omega^2)^2 + \omega^2 T_{ct}^2}} & \theta_y &= \frac{(A_3 * F_y - A_2 * M_x)}{N} \\
A_1 &= \sqrt{(K_y^2 + \omega^2 T_{cy}^2) - m\omega^2} \\
A_2 &= \sqrt{(k_{yrx}^2 + \omega^2 C_{yrx}^2)^2 - (\sqrt{k_y^2 + \omega^2 T_{cy}^2}) * z_c} \\
A_3 &= \sqrt{(K_{rx}^2 + \omega^2 T_{crx}^2 - I_{ox} \omega^2 + (\sqrt{K_y^2 + \omega^2 T_{cy}^2}) * z_c^2} \\
&\quad - \sqrt{K_{yrx}^2 + \omega^2 C_{yrx}^2} * 2z_c} \\
\Delta_x &= \frac{(A_1 * M_x - A_2 * F_y)}{N}
\end{aligned}$$

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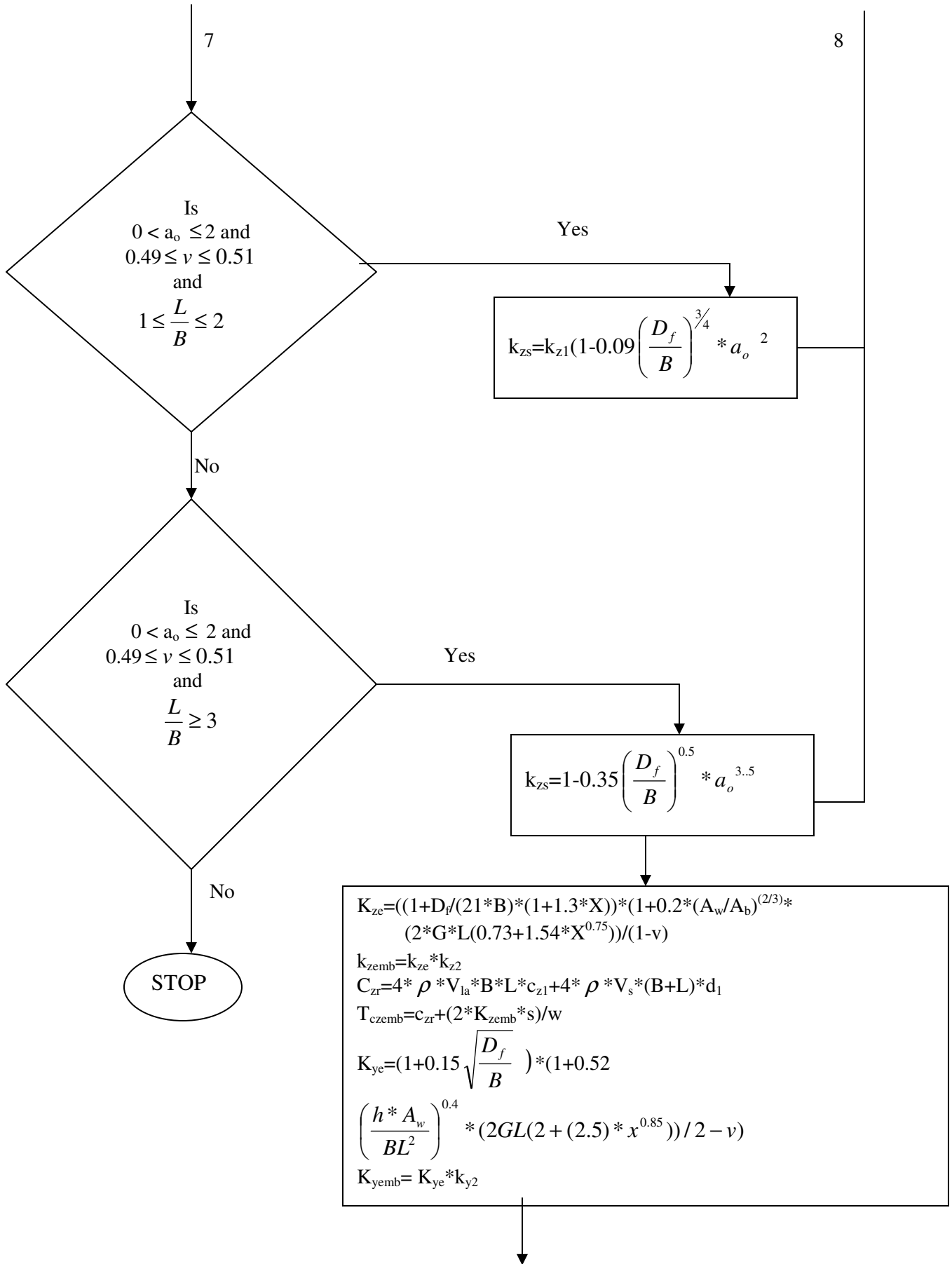




$$\begin{aligned}
 h &= D_f - 0.5 * d_1 & V_{la} &= (3.4 * V_s) / (3.14 * (1 - v)) \\
 A_w &= 2 * d_1 * (L + B) & k_{rx1} &= 1 - 0.2 * a_o \\
 J_b &= I_{bx} + I_{by} & k_{ry1} &= 1 - 0.3 * a_o \\
 V_s &= \sqrt{G/P} & k_{t1} &= 1 - 0.14 * a_o \\
 a_o &= (W * B) / V_s \\
 x &= A_b / 4L^2 \\
 C_1 &= 0.25 + 0.65 \sqrt{a_o} \left(\frac{d_1}{D_f} \right)^{-0.5 a_o} * \left(\frac{D_f}{B} \right)^{-0.25} \\
 C_2 &= 0.25 + 0.65 * \sqrt{a_o} * \left(\frac{d_1}{D_f} \right)^{-0.5 a_o} * \left(\frac{D_f}{L} \right)^{-0.25} \\
 C_3 &= \frac{(d_1 / D)^{-0.5} * a_o^2}{a_o^2 + 0.5 * (L / B)^{-1.5}}
 \end{aligned}$$



$$K_{zs} = (k_{z1} (1 - 0.09 \left(\frac{D_f}{B} \right)^{3/4} * a_o^2) * d_1 + (D_f - d_1) * k_{z1} (1 + 0.09 \left(\frac{D_f}{B} \right)^{3/4} * a_o^2)) / D_f$$



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$$C_{ry} = 4 * \rho * V_s * B * + 4 * \rho * V_s * B * d_1 + 4 * \rho * V_{la} * L * d_1$$

$$T_{cyemb} = C_{ry} + (2 * K_{yemb} * s) / w$$

$$K_{xe} = (1 + 0.15 \sqrt{\frac{D_f}{L}}) * (1 + 0.52 \left(\frac{h * A_w}{BL^2} \right)^{0.4} * (2GL(2 + (2.5) * X^{0.85})) / (2 - \nu) - (0.2 * G * L * (1 - B/L)) / (0.75 - \nu))$$

$$K_{xemb} = K_{xe} * k_{x2}$$

$$C_{xr} = 4 * \rho * V_s * B * L * c_{x1} + 4 * \rho * V_s * L * d_1 + 4 * \rho * V_{la} * B * d_1$$

$$T_{cxemb} = C_{xr} + (2 * K_{xemb} * s) / w$$

$$K_{rxe} = (1 + 1.26) * \left(\frac{d1}{B} \right) * \left(1 + \left(\frac{d1}{B} \right) * \left(\frac{d1}{D_f} \right)^{-0.2} * \sqrt{\frac{B}{L}} \right) * (G * I_{bx}^{0.75} * \left(\frac{L}{B} \right)^{0.25} * (2.4 + 0.5 \left(\frac{B}{L} \right)) / (1 - \nu)$$

$$K_{rxwmb} = K_{rxe} * k_{rx1}$$

$$C_{rx} = \left(\frac{4}{3} \right) * p * V_{la} * B^3 * L * c_{rx1} + \left(\frac{4}{3} \right) * p * V_{la} * d_1^3 * L * C_1 +$$

$$\left(\frac{4}{3} \right) * \rho * V_s * B * d_1 * (B^2 + d_1^2) * C_1 + 4 * \rho * V_s * B^2 * d_1 * L * C_1$$

$$T_{crxemb} = C_{rx} + (2 * K_{rxemb} * s) / w$$

$$K_{rye} = (1 + 0.92 \left(\frac{d1}{L} \right)^{0.6} * (1.5 \left(\frac{d1}{L} \right)^{-0.6})) * (3 * G * (I_{by})^{0.75} * \left(\frac{L}{B} \right)^{0.15} / (1 - \nu)$$

$$K_{ryemb} = K_{rye} * k_{ry1}$$

$$C_{ry} = (4/3) * \rho * V_{la} * L^3 * B * c_{ry1} + (4/3) * \rho * V_{la} * d_1^3 * B * c_z + (4/3) * \rho * V_s * L * d_1 (L^2 + d_1^2) * C_2 + 4 * \rho * V_s * L_2 * d_1 * B * C_2$$

$$T_{cryemb} = C_{ry} + (2 * K_{ryemb} * s) / w$$

$$C_{xry} = d_1 * T_{cxemb} / 3$$

$$C_{yrx} = d_1 * T_{cyemb} / 3$$

$$K_{xry} = d_1 * K_{xemb} / 3$$

$$K_{yrx} = d_1 * K_{yemb} / 3$$

$$K_{temb} = (1 + 1.4 * (1 + (B/L) * (d1/B)^{0.9} * G * (J_b)^{0.75} * (4 + 11 * (1 - (B/L)^{10})))$$

$$C_{tz} = (4/3) * \rho * V_s * B * L * (b^2 + L^2) * c_{t1} + (4/3) * \rho * V_{la} * d_1 * (L^3 + B^3) * C_3 + 4 * \rho * V_s * d_1 * B * L * (B + L) * C_3$$

$$T_{ctemb} = C_{tz} + (2 * K_{temb} * s) / w$$

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$$U_z = F_z / \sqrt{(K_{zemb} - m\omega^2)^2 + \omega^2 T_{ctemb}^2}$$

$$Q_t = M_z / \sqrt{(K_{temb} - J_z \omega^2)^2 + \omega^2 T_{ctemb}^2}$$

$$A_1 = \sqrt{K_{yemb}^2 + \omega^2 T_{cryemb}^2} - M\omega^2$$

$$A_2 = \sqrt{K_{yrx}^2 + \omega^2 C_{yrx}^2} - \sqrt{K_{yemb}^2 + \omega^2 T_{cyemb}^2} * Z_c$$

$$A_3 = \sqrt{K_{rxemb}^2 + \omega^2 T_{crxemb}^2} - I_{ox} * \omega^2 + \sqrt{K_{yemb}^2 + \omega^2 T_{cyemb}^2} * Z_c^2 - \sqrt{K_{yrx}^2 + \omega^2 C_{yrx}^2} * 2 * Z_c$$

$$N = A_1 * A_3 - A_2^2$$

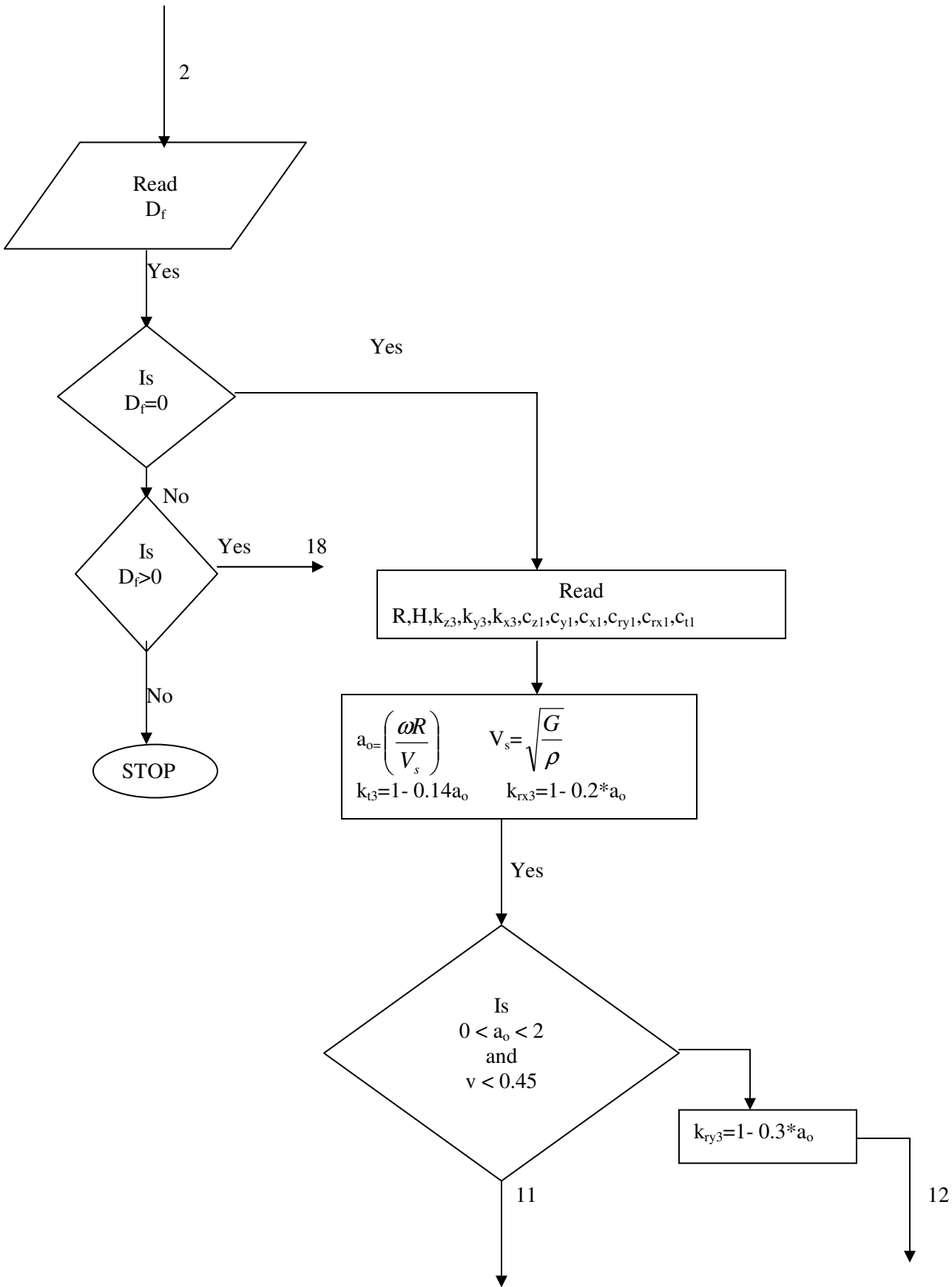
$$\Delta_x = \frac{A_3 * F_y - A_2 * M_x}{N}$$

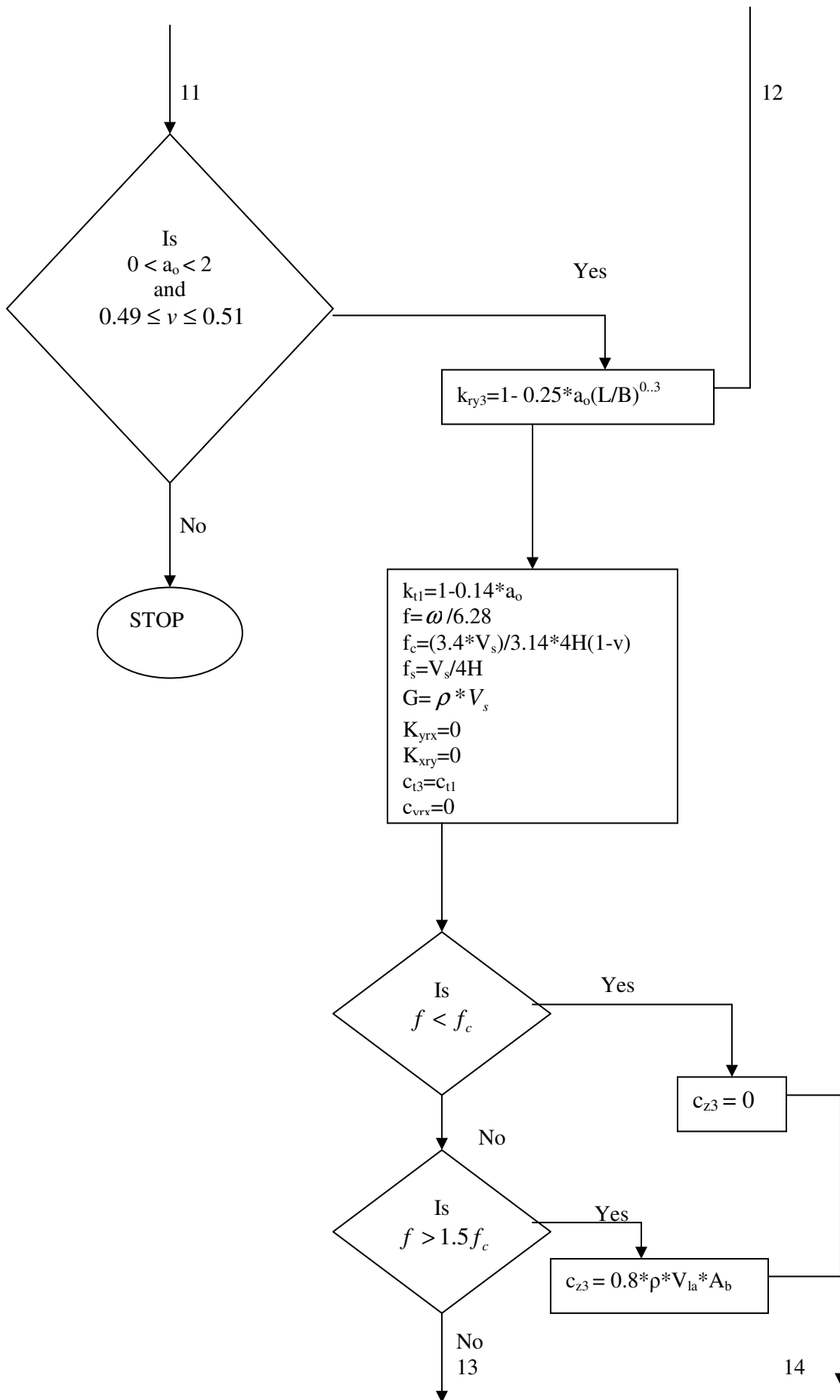
$$\theta_y = \frac{A_1 * M_x - A_2 * F_y}{N}$$

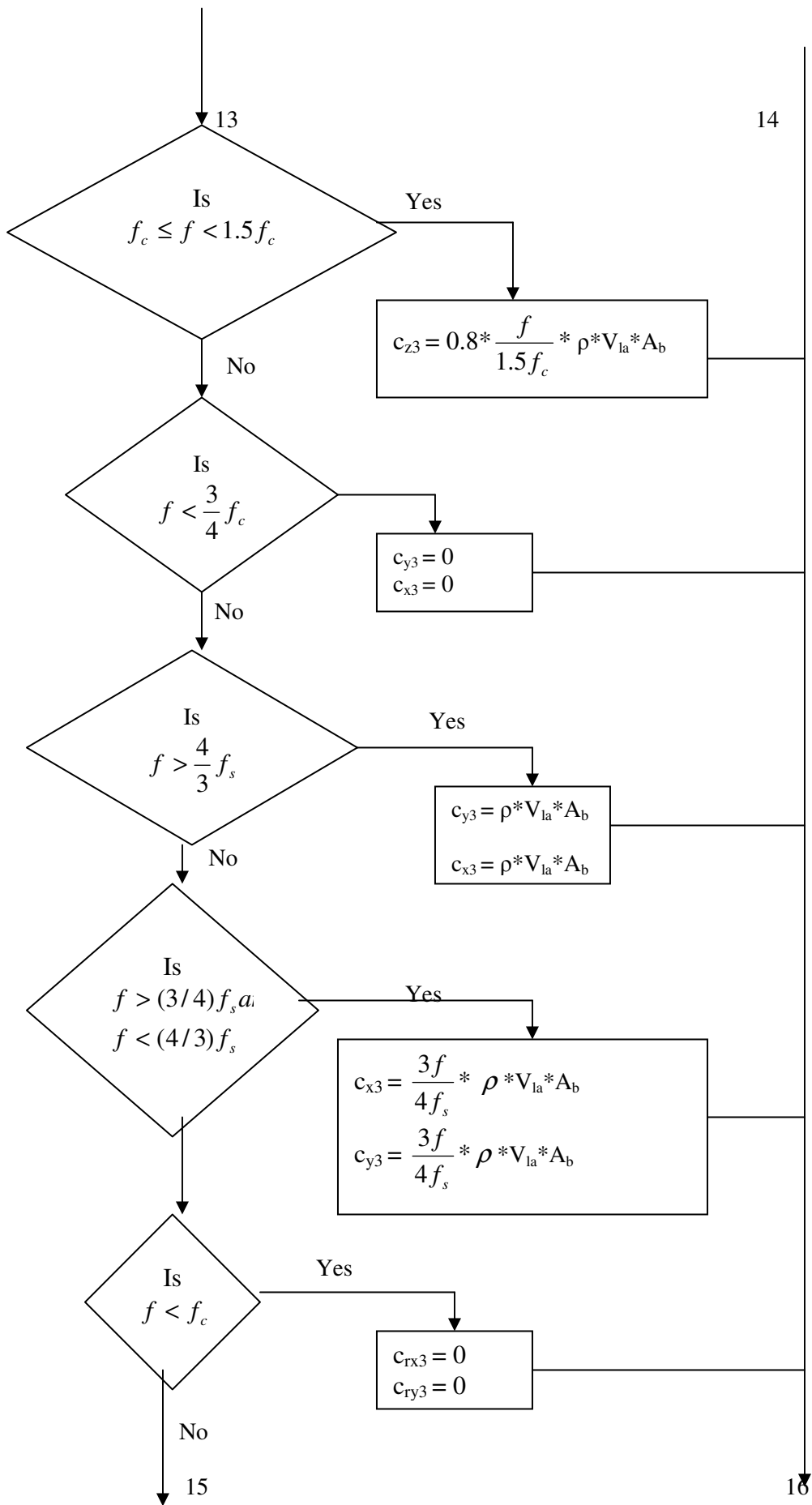
Print:

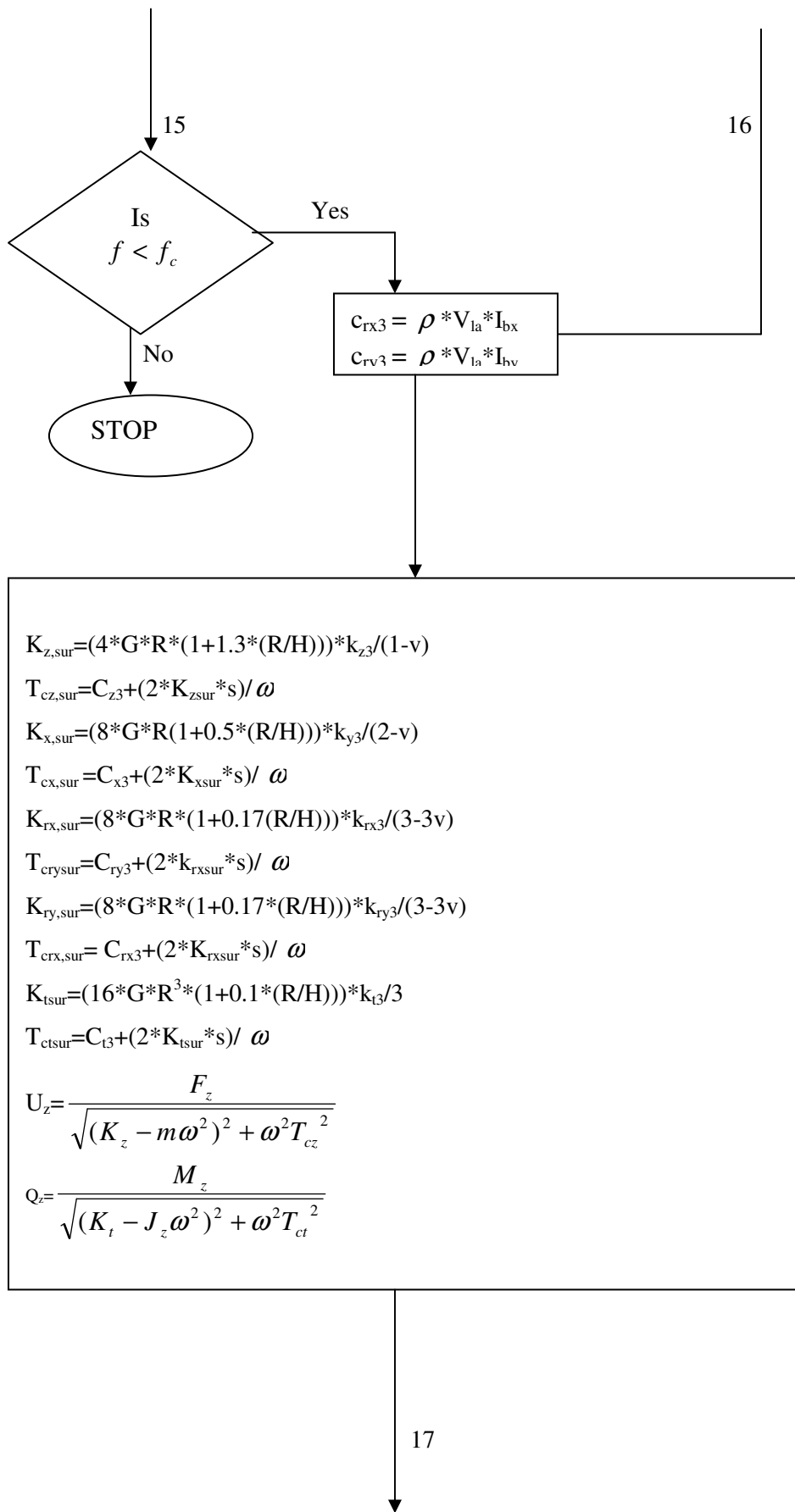
$K_{zemb}, C_{zr}, T_{czemb}, K_{ye}, K_{yemb}, C_{ry}, T_{cyemb}, K_{xe}, K_{xemb}, C_{xr}, T_{cemb}, K_{rx}, K_{rxemb}, C_{rx},$
 $T_{crxemb}, K_{ry}, K_{ryemb}, C_{ry}, T_{cryemb}, K_{temb}, C_{tz}, T_{ctemb}, U_z, Q_z, \Delta_x, \theta_y$

STOP









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$$A_1 = \sqrt{(K_y^2 + \omega^2 T_{cy}^2) - m\omega^2}$$

$$A_2 = \sqrt{(K_{yrx}^2 + \omega^2 C_{yrx}^2)^2} \cdot (\sqrt{K_y^2 + \omega^2 T_{cy}^2}) * z_c$$

$$A_3 = \sqrt{(K_{rx}^2 + \omega^2 T_{crx}^2 - I_{ox}\omega^2 + (\sqrt{K_y^2 + \omega^2 T_{cy}^2}) * z_c^2} \\ - \sqrt{K_{yrx}^2 + \omega^2 C_{yrx}^2} * 2z_c}$$

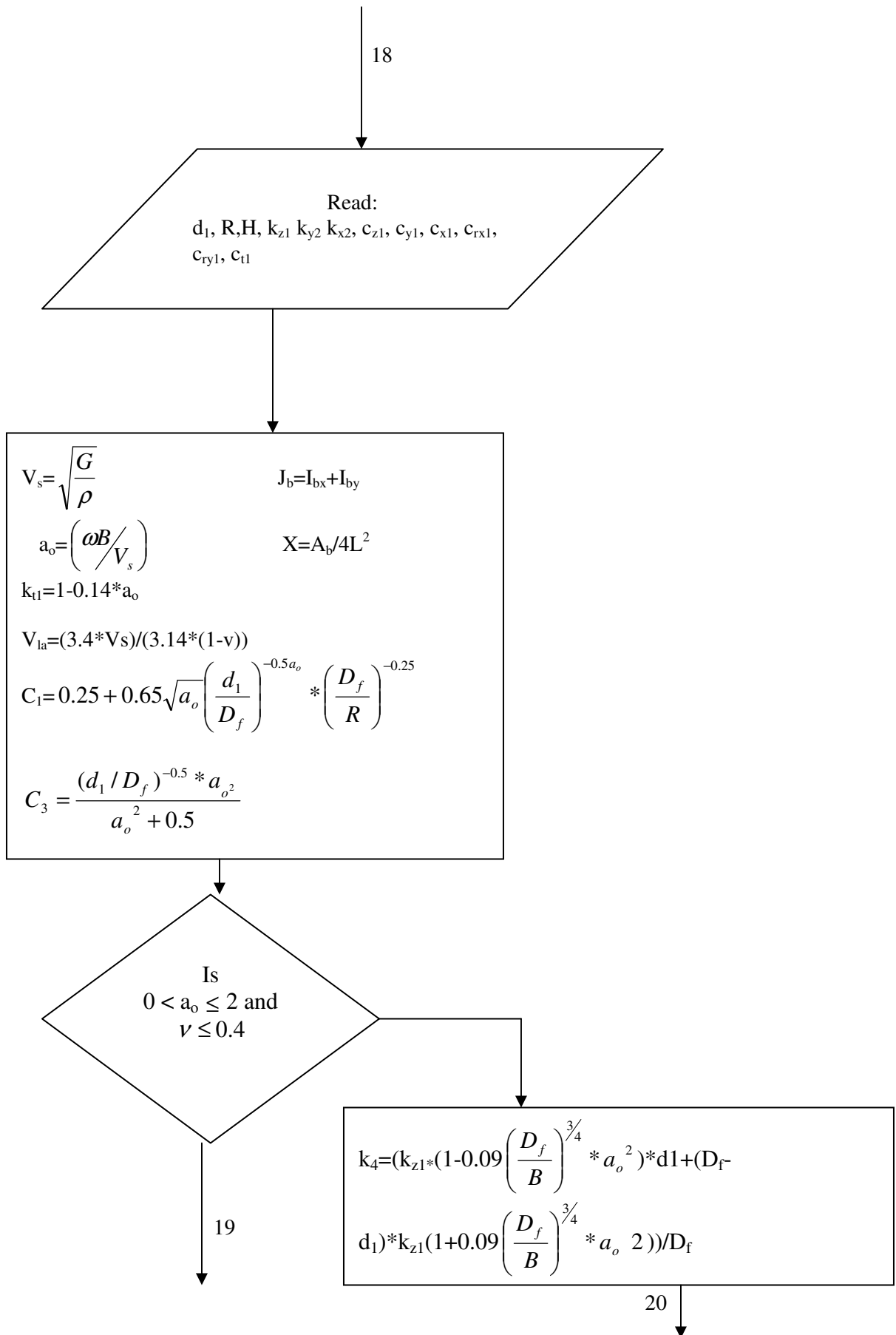
$$N = A_1 * A_3 - A_2^2$$

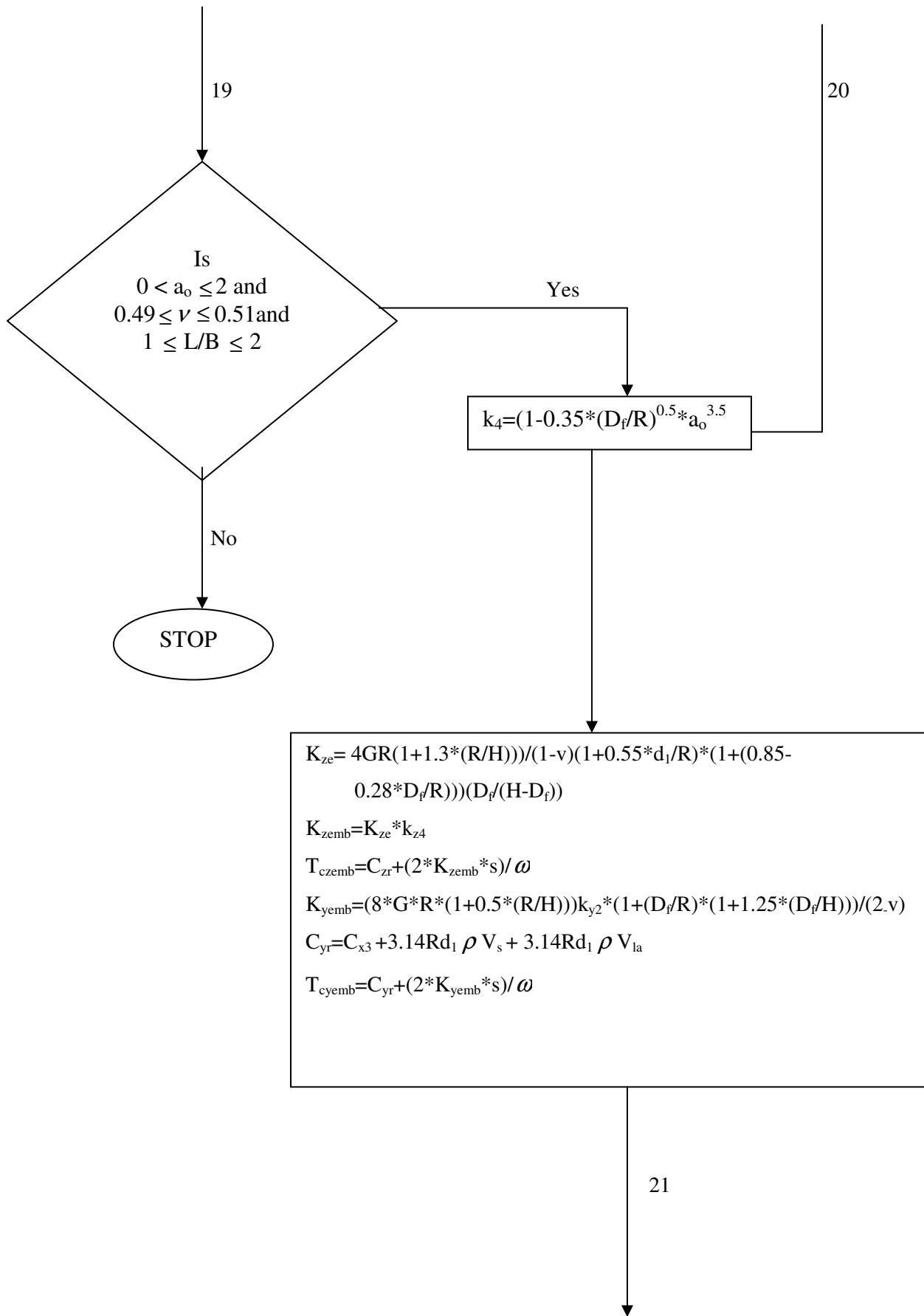
$$\Delta_x = \frac{(A_3 * F_y - A_2 * M_x)}{N}$$

$$\theta_y = \frac{(A_1 * M_x - A_2 * F_y)}{N}$$

Print
 $k_{zsur}, C_{zs}, T_{czsur}, k_{ysur}, C_{ys}, T_{cysur}, k_{xsur}, C_{xs}, T_{cxsur}, k_{rx}$
 $emb, C_{rxs}, T_{crxsur}, k_{ry}, k_{rys}, C_{rys}, T_{crys}, k_{tesur}, C_{ts}, T_{ctsur}$
 $r, U_z, Q_z, \Delta X, \theta Y$

STOP





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$$K_{rxemb} = (8 * G * R * (1 + 0.17 * (R/H))) k_{rx1} / (3 - 3v) * (1 + 2 * d_1 / R) * (1 + 0.65 * (D_f / H))$$

$$C_{rx} = C_{rx3} + 3.14 \rho V_{la} R d_1^3 C_1 + \rho V_s R d_1 (R^2 + d_1^2) C_1 / 3 + 3.14 \rho V_s R^3 d_1 C_1$$

$$T_{crxemb} = C_{rx} + (2 * K_{rxemb} * s) / \omega$$

$$K_{ryemb} = (8 * G * R * (1 + 0.17 * (R/H))) k_{ry1} * (3 - 3v) * (1 + 2 * d_1 / R) * (1 + 0.65 * (D_f / H))$$

$$C_{ry} = C_{ry3} + 3.14 \rho V_{la} R d_1^3 C_1 + \rho V_s R d_1 (R^2 + d_1^2) C_1 / 3 + 3.14 \rho V_s R^3 d_1 C_1 / 2$$

$$T_{cryemb} = C_{ry} + (2 * K_{ryemb} * s) / \omega$$

$$K_{te} = (16/3) * G * R^3 * (1 + 0.1 * (R/H)) * (1 + 2.67 * d_1 / R)$$

$$K_{temb} = K_{te} * k_{t1}$$

$$C_{te} = C_{t1} + 3.14 \rho V_{la} R^3 d_1 C_2 + 6.28 \rho V_s R^3 d_1 C_2$$

$$T_{cryemb} = c_{ry} + (2 * K_{ryemb} * s) / \omega$$

$$C_{xry} = d_1 * T_{crxemb} / 3$$

$$C_{yrx} = d_1 * T_{cryemb} / 3$$

$$K_{xry} = d_1 * K_{xemb} / 3$$

$$K_{yrx} = d_1 * K_{yemb} / 3$$

$$T_{ctemb} = c_{t2} + (2 * K_{temb} * d) / \omega$$

$$U_z = F_z / \sqrt{(K_{zemb} - m \omega^2)^2 + \omega^2 T_{ctemb}^2}$$

$$Q_t = M_z / \sqrt{(K_{temb} - J_z \omega^2)^2 + \omega^2 T_{ctemb}^2}$$

$$A_1 = \sqrt{K_{yemb}^2 + \omega^2 T_{cryemb}^2} - M \omega^2$$

$$A_2 = \sqrt{K_{yrx}^2 + \omega^2 c_{yrx}^2} - \sqrt{K_{yemb}^2 + \omega^2 T_{cryemb}^2} * z_c$$

$$A_3 = \sqrt{K_{rxemb}^2 + \omega^2 T_{crxemb}^2} - I_{ox} * \omega^2 + \sqrt{K_{yemb}^2 + \omega^2 T_{cryemb}^2} * z_c^2 - \sqrt{K_{yrx}^2 + \omega^2 c_{yrx}^2} * 2 * z_c$$

$$N = A_1 * A_3 - A_2^2$$

$$\Delta_x = \frac{A_3 * F_y - A_2 * M_x}{N}$$

$$\theta_y = \frac{A_1 * M_x - A_2 * F_y}{N}$$

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$$U_z = F_z / (\sqrt{K_{zemb} - m\omega^2}^2 + \omega^2 + T_{ctemb}^2)$$

$$Q_z = M_z / (\sqrt{K_{temb} - J_{zx}\omega^2}^2 + \omega^2 * T_{ctemb}^2)$$

$$A_1 = \sqrt{K_{yemb}^2 + \omega^2 * T_{cryemb}^2} - m\omega^2$$

$$A_2 = \sqrt{K_{yrx}^2 + \omega^{2*} * C_{yrx}^2} - \sqrt{K_{yemb}^2 + \omega^{2*} * T_{cyemb}^2 * Zc^2}$$

$$A_3 = \sqrt{K_{rxemb}^2 + \omega^2 * T_{crxemb}^2} - I_{ox} * \omega^2 + \sqrt{K_{yemb}^2 + \omega^{2*} * T_{cyemb}^2 * Zc^2} - \sqrt{K_{yrx}^2 + \omega^2 * cyrx^2} * 2 * Zc$$

$$N = A_1 * A_3 - A_2^2$$

$$\Delta Y = (A_3 * F_y - A_2 * M_x) / N$$

$$\theta Y = (A_1 * M_x - A_2 * F_y) / N$$

Print:
 $k_{zemb}, C_{zr}, T_{czemb}, k_{ye}, k_{yemb}, C_{ry}, T_{cyemb}, k_{xe}, k_{xemb}, C_{xr}, T_{cemb},$
 $k_{rx}, k_{rxemb}, C_{rx},$
 $T_{crxemb}, k_{ry}, k_{ryemb}, C_{ry}, T_{cryemb}, k_{temb}, C_{tz}, T_{ctemb}, U_z, Q_z, \Delta X,$
 θY

STOP

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