VECTOR MESON DECAY CONSTANTS WITH INCLUSION OF BOTH LO AND NLO DIRAC COVARIANTS IN HADRON-QUARK VERTEX

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Abstract

The Bethe-Salpeter equation (BSE) provides a non-perturbativ method of treating the relativistic two-particle bound state problems on the basis of covariant instantaneous ansatz (CIA). On this basis, the structure of the hadron-quark vertex function $\Gamma(\hat{q})$ of vector mesons is derived from a QCD motivated Bethe-salpeter (BS) frame work and it is generalized to include various Dirac covariants (other than $i\gamma.\varepsilon$) from their complete set. They are incorporated in accordance with a naive power counting rule order-by-order in powers of the inverse of the meson mass.

We study vector meson decay constants with inclusion of both the leading order (LO) and the next-to-leading order (NLO) Dirac Covariants, in addition to $i\gamma.\varepsilon$, in the Hadron-Quark vertex function $\Gamma(\hat{q})$ from their complete set on the basis of power counting rule through the frame work of BSE under covariant instantaneous ansatz.

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Chapter 1

Introduction

Many powerful theories in physics are described by Lagrangians which are invariant under certain symmetry transformation groups. Invariance of a Lagrangian under a symmetry transformation leads to a conservation law. The conservation of energy, linear momentum and angular momentum are consequences of space-time symmetries of the Lagrangian. It is believed that all interactions are generated by imposition of local gauge invariant on a given Lagrangian. When they are invariant under a transformation identically performed at every point in the space in which the physical processes occur, they are said to have a global symmetry. The requirement of local symmetry is much more strict than the requirement of global symmetry. In fact, a global symmetry is just a local symmetry whose group's parameters are dependent of space-time co-ordinates. Historically, these ideas were first noticed in the context of classical electromagnetism and later in general relativity. However, the importance of gauge symmetries appeared in quantum field theory. Today, gauge theories have applications in high energy physics, condensed matter and nuclear physics among other subfields.

In Chapter 2 of this thesis we discuss the most widely known gauge theories, quantum field theories, including quantum electrodynamics.

The earliest field theory having a gauge symmetry was Maxwell's formulation of electrodynamics in 1864. The importance of this symmetry remained unnoticed in the earliest formulations. After the development of quantum mechanics, Weyl, Vladimir Fock and Fritz London modified gauge by replacing the scale factor with a complex quantity and turned the scale transformation into a change of phase (a U(1) gauge symmetry). This explained the electromagnetic field effect on the wave function of a charged quantum mechanical particle. This was the first widely recognized gauge theory.

In 1954, attempting to resolve some of the great confusion in elementary particle physics, Chen Ning Yang and Robert Mills introduced non-abelian gauge theories as models to understand the strong interaction holding together nucleons in atomic nuclei. Generalizing the gauge invariance of electromagnetism, they attempted to construct a theory based on the action of the (non-abelian) SU(2) symmetry group on the isospin doublet of protons and neutrons. This is similar to the action of the U(1) group on the spinor fields of quantum electrodynamics. For the theory (SU(2) symmetry group) to succeed there had to exist a massless isotriplet of intermediate spin-1 vector bosons.

This idea later found application in the quantum field theory of the strong and weak forces. The strong force gauge theory, now known as quantum chromodynamics, is a gauge theory with the action of the SU(3) group on the color triplet of quark fields. The Standard Model unifies the description of electromagnetism, weak interactions and strong interactions in the language of gauge theory. The importance of gauge theories for physics stems from the tremendous success of the mathematical formalism in providing a unified framework to describe the quantum field theories of electromagnetism, the weak force and the strong force. This theory, known as the Standard Model, accurately describes experimental results regarding three of the four fundamental forces of nature, and is a gauge theory with the gauge group SU(3)x SU(2)x U(1) [1].

In the scientific investigations, there is a great need for covariant solutions of bound state equations in order to construct realistic Quantum Chromodynamics (QCD) based models of mesons. Furthermore, we ideally need to know the structure of these bound states. This is because mesons are the simplest bound states in QCD. The Bethe-Salpeter formalism is generally accepted to represent the appropriate framework for the description of bound states within relativistic quantum field theory.

In chapter 3 we discuss all about Bethe-salpeter equation. We first make generalization of one particle propagator in external field to two particle propagator With particles in mutual interaction to derive Bethe-Salpeter equation (BSE) . The formulation of BSE under covariant instantaneous ansatz (CIA) is also discussed.
In chapter 4 we study leptonic decays of vector mesons (V-mesons) which proceed via one-photon annihilation of a $q\bar{q}$ pair constituting a V-meson through a quark loop as shown in figure 1.1.

We employ a QCD-oriented framework of the BetheSalpeter equation (BSE) under

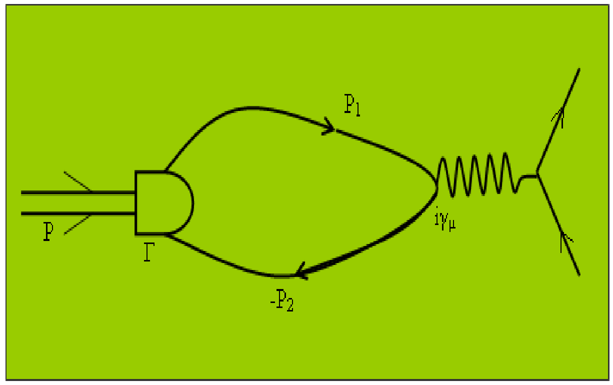

Figure 1.1: Quark loop diagram for leptonic decay of V-meson.

covariant instantaneous ansatz (CIA) [2,3]. CIA is a Lorentz-invariant generalization of instantaneous approximation (IA). For $q\bar{q}$ system, CIA formulation ensures an exact interconnection between 3-D and 4-D forms of the BSE. The 3-D form of BSE serves for making contact with the mass spectra of hadrons, whereas the 4-D form provides the $Hq\bar{q}$ vertex function $\Gamma(\hat{q})$ for the evaluation of transition amplitudes.
Recent studies [2,3] indicated that various mesons have many different covariant structures in their wave functions whose inclusion was also found necessary to obtain quantitatively accurate observable. Hence, this work uses a BS wave function for a hadron, which would incorporate more than a single Dirac structure for the calculation of transition amplitudes for various processes.
Finally We summarize and give conclusion in Chapter 5.

Chapter 2

Gauge theories: QED and QCD

In the current view, all the fundamental interactions are derived from a ”gauge principle” similar to that in electromagnetism. Gauge symmetry lies at the heart of gauge theory. A gauge symmetry differs from an ordinary symmetry in two important respects:

1. Gauge symmetry is a local symmetry rather than a global symmetry. For a local symmetry, the element of the symmetry group (G) that acts on the fields of a theory at a space-time point (x , t) depends on the position x and time t, whereas for a global symmetry a fixed group element acts on fields at different space-time points.

2. A gauge transformation leaves a physical state invariant. Gauge symmetry reflects a redundancy in the variables used to describe a physical state. By contrast, a global symmetry acting on a physical state in general produces a new, distinct physical state.

In section 2.1 we explain the origin of gauge invariance starting from Maxwell equations. In section 2.2, we discuss Quantum Electrodynamics(QED) gauge theory. In section 2.3 we introduce Quantum Chromo-dynamics(QCD). In the last section (section 2.4) we drive the Feynman rules for the Lagrangian in QCD.

2.1 Origin of Gauge invariance

Let us begin by considering the basic laws of classical electromagnetism; the Maxwell equations

$$\vec{\nabla}.\vec{E} = \rho. \tag{2.1.1}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}. \tag{2.1.2}$$

$$\vec{\nabla}.\vec{B} = 0. \tag{2.1.3}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c}\vec{j} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}. \tag{2.1.4}$$

It is convenient to introduce the potentials $\vec{A}(x)$ and $\phi(x)$ in place of the field $\vec{E}$ and $\vec{B}$ and write,

$$\vec{B} = \vec{\nabla} \times \vec{A}. \tag{2.1.5}$$

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}. \tag{2.1.6}$$

The origin of gauge invariance lies in the fact that $\vec{A}$ and ϕ are not unique for given fields $\vec{E}$and $\vec{B}$. The transformations which $\vec{A}$ and ϕ may under go while preserving $\vec{E}$ and $\vec{B}$, and hence the Maxwell equations, are called gauge transformations. These transformations are

$$\vec{A} \rightarrow \vec{A'} = \vec{A} + \vec{\nabla}\Lambda(\vec{x}, t). \tag{2.1.7}$$

And

$$\phi \rightarrow \phi' = \phi - \frac{\partial \Lambda(\vec{x}, t)}{\partial t}. \tag{2.1.8}$$

where $\Lambda(\vec{x}, t)$ is an arbitrary function. Equations (2.2.7) and (2.2.8) can be combined in to a single component equation by introducing the 4- vector potential $A_\mu = (\vec{A}, i\phi)$, thus

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu \Lambda(x). \tag{2.1.9}$$

Maxwell equations' can also be written in a covariant form using the 4- current density $j_\mu = (\vec{j}, i\rho)$. In terms of which we can write these equations as,

$$\partial_\mu F_{\mu\nu} = j_\mu, \tag{2.1.10}$$

where the field strength tensor is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{2.1.11}$$

Under the gauge transformation in (2.1.9) it can be easily checked that

$$F'_{\mu\nu} = F_{\mu\nu}. \tag{2.1.12}$$

This in turn leads to invariance of Maxwell's equation (2.1.10). The gauge invariance is in turn linked to charge conservation. In general, there are two types of gauge transformations. Global gauge transformations, i.e transformations which are the same at all space time points and Local gauge transformations, i.e transformations which vary from point to point in space time. The above gauge transformations (2.1.9) are local gauge transformations on A_μ field. In the physical world all fundamental interactions are believed to originate from a gauge principle [4]. Now, we discuss the gauge theory Quantum Electrodynamics(QED) in the next section.

2.2 QED Gauge Theory

In this section, we begin with the Lagrangian formulation of Quantum Electrodynamics describing the interaction between light and matter.

The free Dirac Lagrangian is described by,

$$\pounds_{Dirac} = -\bar{\psi}(x)(\gamma_\mu \partial_\mu + m)\psi(x). \tag{2.2.1}$$

It can be easily checked that $\pounds_{Dirac}$ is invariant under the global gauge transformation

$$\psi \rightarrow e^{i\theta}\psi, \tag{2.2.2}$$

where ϑ is constant. And hence

$$\pounds'_{DIRAC} = \pounds_{DIRAC}. \tag{2.2.3}$$

Now let us generalize to local gauge transformation on ψ(x)

$$\psi(x) \rightarrow \psi(x)e^{i\theta(x)}, \tag{2.2.4}$$

where θ now depends on space-time variable x. $\bar{\psi}$ Should then transform as,

$$\bar{\psi}(x) \rightarrow e^{-i\theta(x)}\bar{\psi}(x). \tag{2.2.5}$$

Thus

$$\pounds'_{Dirac} = -e^{-i\theta(x)}\bar{\psi}(x)(\gamma_\mu\partial_\mu + m)\psi(x)e^{i\theta(x)}$$

$$= -\bar{\psi}(x)(\gamma_\mu\partial_\mu + m)\psi(x) - i\bar{\psi}(x)\gamma_\mu(\partial_\mu\theta(x))\psi(x). \tag{2.2.6}$$

We see that the extra term $i\bar{\psi}(x)\gamma_\mu(\partial_\mu\theta(x))\psi(x)$ spoils the gauge symmetry. Thus we see that $\pounds_{Dirac}$ is not invariant under local gauge transformation on $\psi(x)$. Pulling a factor of 'e' out of $\theta(x)$, by letting $\theta(x) = e\Lambda(x)$, we can write (2.2.6) as,

$$\pounds'_{Dirac} = \pounds - ie(\bar{\psi}\gamma_\mu\psi)\partial_\mu\Lambda. \tag{2.2.7}$$

Since the free Dirac Lagrangian is not locally gauge invariant, to make it locally invariant we are obliged to add a term $ie(\bar{\psi}\gamma_\mu)\psi A_\mu(x)$ in the Lagrangian in order to soak the extra term in (2.2.7), where $A_\mu(x)$ is the new carrier gauge field introduced. Thus, let us write the lagrangian as

$$\pounds_{Dirac} = -\bar{\psi}(x)(\gamma_\mu\partial_\mu + m)\psi(x) + ie(\bar{\psi}\gamma_\mu)A_\mu(x), \tag{2.2.8}$$

where $A_\mu(x)$ transforms under local gauge transformation according to the rule

$$A_\mu \rightarrow A_\mu(x) + \frac{1}{e}\partial_\mu \theta(x). \tag{2.2.9}$$

This new Lagrangian is now invariant under local gauge transformations.Thus the price we have to pay to make $\pounds_{Dirac}$ locally invariant is the introduction of a new vector field $A_\mu(x)$ that should couple to ψ and $\bar{\psi}$ through the term $ie\bar{\psi}\gamma_\mu\psi A_\mu$ in the Lagrangian. The charge 'e' plays the role of coupling constant between matter field and gauge field. One can also say that to conserve the total charge of the universe, matter field should coupled with gauge field in a certain mathematically self consistent manner through a term $ie\bar{\psi}\gamma_\mu\psi A_\mu$, in the Lagrangian [4]. This in turn Mathematically models the interaction between the charged particle and electromagnetic field.
The full QED Lagrangian must also include a term for the free gauge field $A_\mu(x)$, i.e $-1/4F_{\mu\nu}F_{\mu\nu}$. Thus we write the full QED lagrangian as

$$\pounds_{QED} = -\bar{\psi}(x)(\gamma_\mu\partial_\mu + m)\psi(x) + ie(\bar{\psi}(x)\gamma_\mu\psi(x))A_\mu(x) - \frac{1}{4}F_{\mu\nu}F_{\mu\nu}. \tag{2.2.10}$$

In section 1 of this unit we have seen that $F_{\mu\nu}$ is invariant under local gauge transformation. Thus $-1/4F_{\mu\nu}F_{\mu\nu}$ is invariant under local gauge transformation. It can be seen that a simple prescription for making $\pounds$ locally invariant is to replace ∂_μ by the covariant derivative D_μ,

$$D_\mu = \partial_\mu - ieA_\mu, \tag{2.2.11}$$

so that we can write the locally invariant full QED Lagrangian as,

$$\pounds_{QED} = -\bar{\psi}(x)(\gamma_\mu D_\mu + m)\psi(x) - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} \tag{2.2.12}$$

Let us see the basic qualitative structure of the Lagrangian in (2.2.10). The first and third terms describe the free propagation of Dirac particle and photon respectively.

The second term involves three fields ($\bar{\psi}, \psi$ and A_μ), and these define a vertex in which three lines are joined. An incoming fermion, an outgoing fermion and a photon. To obtain the vertex factor, simply rubout the field variables. The QED vertex factor for this is

$$ie\gamma_\mu \tag{2.2.13}$$

It is to be visualized that a set of local gauge transformations $\{U = e^{i\theta(x)}\}$ form a U(1) group. Where U is a unitary unimodular 1x1 matrix i.e $U^\dagger U = UU^\dagger = \mathbf{1}$, and $\mid U \mid = +1$. The group of all such matrices forms U(1) group. Since element of U(1) commute with each other, QED is an abelian U(1) gauge theory.

2.3 QCD Gauge theory

In this section we infer the structure of Quantum Chromo-dynamics(QCD) from local gauge invariance. QCD is based on the extension of the idea in QED, but with the U(1) gauge group replaced by the SU(3) group of phase transformations on the quark color field. In the colored quark model, each flavor of quark comes in three different colors - Red, Blue and Green. Although the various flavors of quarks carry different masses, the three colors of a given flavor are all supposed to weigh the same. Thus the lagrangian for a particular quark flavor can be expressed as,

$$\pounds_{Dirac} = -\bar{\psi}_R(\gamma_\mu\partial_\mu + m)\psi_R - \bar{\psi}_B(\gamma_\mu\partial_\mu + m)\psi_B - \bar{\psi}_G(\gamma_\mu\partial_\mu + m)\psi_G. \tag{2.3.1}$$

We can simplify the notation by introducing 3 component color matrices $\psi = \begin{pmatrix} \psi_R \\ \psi_B \\ \psi_G \end{pmatrix}$ and $\bar{\psi} = \begin{pmatrix} \bar{\psi}_R & \bar{\psi}_B & \bar{\psi}_G \end{pmatrix}$ where each of ψ_R, ψ_B, ψ_G... is a 4x1 Dirac spinor.

We can write $\pounds_{Dirac}$ compactly as,

$$\pounds_{Dirac} = -\bar{\psi}(\gamma_\mu \partial_\mu + m)\psi, \tag{2.3.2}$$

where $m = \begin{pmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{pmatrix}$ is mass matrix

This looks just like the original Dirac lagrangian, but ψ is now a 3x1 matrix. This lagrangian exhibits SU(3) symmetry. That is to say,it is invariant under transformation of the form

$$\psi \Rightarrow \psi' = U\psi, \tag{2.3.3}$$

where U is any unitary 3x3 matrix, ie $\mid U \mid = \mathbf{1}$ and $U^\dagger U = UU^\dagger = \mathbf{1}$. Any unitary matrix can be written in the form

$$U = e^{iH}, \tag{2.3.4}$$

where H is Hermitian $H^\dagger = H$. Any 3x3 matrix can be expressed as super position of a unit matrix $\mathbf{1}$ and $\lambda_1, \lambda_2, \lambda_3, ...\lambda_8$ which are the generators of SU(3) group, that is

$$H = \theta\mathbf{1} + \lambda^a \theta^a(X); a = 1, 2...8, \tag{2.3.5}$$

where θ and $\theta(x)$ are arbitrary parameters. Thus the unitary 3x3 matrix can be expressed as

$$U = e^{i\theta} e^{i\lambda^a \theta^a}. \tag{2.3.6}$$

Let us consider first a global SU(3) transformation on $\psi(x)$

$$\psi \rightarrow \psi' = e^{i\lambda^a \theta^a}\psi, \tag{2.3.7}$$

where θ's are constants and are the same at all space time points. It can be easily checked that

$$\pounds'_{dirac} = \pounds_{Dirac}.$$

Thus We see that $\pounds_{Dirac}$ is invariant under global SU(3) group transformation on ψ. Now what we are interested in is the invariance of L_{Dirac} under SU(3) local gauge transformations. Consider the following local SU(3) gauge transformation.

$$\psi \rightarrow \psi' = e^{i\lambda^a\theta^a(x)}\psi. \tag{2.3.8}$$

In this case $\theta(x)$ varies from one space time point to another. Under this transformation $\pounds$ transforms as,

$$\pounds \rightarrow \pounds' = -\bar{\psi}U^\dagger(\theta)(\gamma_\mu\partial_\mu + m)U(\theta)\psi$$

$$= -\bar{\psi}(\gamma_\mu\partial_\mu + m)\psi - \bar{\psi}U^\dagger(\theta)\gamma_\mu(\partial_\mu U(\theta))\psi. \tag{2.3.9}$$

where $U = e^{i\lambda^a\theta^a(x)}$. $\pounds$ is not invariant under such a transformation, due to the extra term $-\bar{\psi}U^\dagger(\theta)\gamma_\mu(\partial_\mu U(\theta))\psi$. To make $\pounds$ locally invariant, let us make use of the minimal coupling prescription by replacing

$$\partial_\mu\psi \rightarrow D_\mu\psi, \tag{2.3.10}$$

where

$$D_\mu = \partial_\mu - ig\lambda^a A^a(x).$$

Pulling out 'g' from θ, where g is a coupling constant, equation (2.3.9) becomes

$$\pounds'_{Dirac} = -\bar{\psi}(\gamma_\mu\partial_\mu + m)\psi - ig\bar{\psi}\gamma_\mu\psi\partial_\mu\lambda(x). \tag{2.3.11}$$

If $D_\mu\psi$ is to transform under SU(3) local gauge transformations then,

$$D_\mu\psi \rightarrow D'_\mu\psi' = U(\theta)D_\mu\psi. \tag{2.3.12}$$

This implies that

$$(\partial_\mu - ig\lambda^a A'^a_\mu(x))U(\theta)\psi(x) = U(\theta)(\partial_\mu - ig\lambda^a A^a_\mu(x))\psi(x).$$

Hence

$$\Rightarrow (\partial_\mu U - ig\lambda.A'_\mu(x)U)\psi(x) = -igU\lambda.A_\mu(x)\psi(x); \lambda.A_\mu(x) = \lambda^a A^a_\mu(x).$$

Thus

$$\mathbf{A}_\mu(x) = U\lambda.A_\mu(x)U^{-1} + \frac{1}{ig}(\partial_\mu U)U^{-1}, \tag{2.3.13}$$

where $\mathbf{A}_\mu(x) = \lambda^a A^a_\mu(x)$ is a 3x3 matrix that should transform as

$$\mathbf{A}'_\mu(x) = UA_\mu(x)U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1}. \tag{2.3.14}$$

This is transformation property of non-abelian gauge field $\mathbf{A}_\mu(x)$. U and U^{-1} in the first term of equation (2.3.14) can not brought together, because they do not commute with $A_\mu(x)$. If we add the term $ig\bar{\psi}(x)\lambda^a A^a_\mu(x)\psi(x)$ to the initial Lagrangian, the modified lagrangian

$$\pounds_{Dirac} = -\bar{\psi}(\gamma_\mu\partial_\mu + m)\psi + ig\bar{\psi}\gamma_\mu\lambda^a A^a_\mu\psi. \tag{2.3.15}$$

$\pounds$ is now invariant under local SU(3) gauge transformations. But as usual the cost is introduction of A_μ (eight of them). Since we can write

$$F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]; D_\mu = \partial_\mu - igA_\mu(x), \tag{2.3.16}$$

$$\Rightarrow F_{\mu\nu} = \frac{i}{g}(\partial_\mu - igA_\mu(x))(\partial_\nu - igA_\nu(x)) - (\partial_\nu - igA_\nu(x))(\partial_\mu - igA_\mu(x))$$

$$= \frac{i}{g}\partial_\mu\partial_\nu - \partial_\nu\partial_\mu - ig(A_\mu\partial_\nu - A_\nu\partial_\mu) - ig(\partial_\mu A_\nu - \partial_\nu A_\mu) - g^2(A_\mu A_\nu - A_\nu A_\mu).$$

Thus the non abelian SU(3) gauge field tensor can be expressed as,

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu(x), A_\nu(x)] \tag{2.3.17}$$

The kinetic term for for gauge field lagrangian which corresponds to free gauge field can be written as,

$$\pounds_{gluon} = -\frac{1}{4}Tr(F_{\mu\nu}F_{\mu\nu}). \tag{2.3.18}$$

Thus we write the full QCD Lagrangian as

$$\pounds_{QCD} = -\bar{\psi}(\gamma_\mu\partial_\mu + m)\psi + ig\bar{\psi}\gamma_\mu\mathbf{A}_\mu(x)\psi - \frac{1}{4}Tr(F_{\mu\nu}F_{\mu\nu}), \tag{2.3.19}$$

where $\mathbf{A}_\mu(x)$ are the eight gluon fields coupled to three color fields. $\pounds_{QCD}$ is invariant under local gauge transformation if $\pounds_{gluon} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$ is invariant in it. To show this, let us consider local SU(3) transformation $F^a_{\mu\nu} \rightarrow F'^a_{\mu\nu}$

$$F'_{\mu\nu} = \partial_\mu A'_\nu - \partial_\nu A'_\mu - ig[A'_\mu(x), A'_\nu(x)]$$

$$= \partial_\mu(UA_\nu U^{-1} - \frac{i}{g}(\partial_\nu U)U^{-1}) - \partial_\nu(UA_\mu U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1})$$

$$-ig[(UA_\mu U^{-1} - \frac{i}{g}(\partial_\mu U)U^{-1}), (UA_\nu U^{-1} - \frac{i}{g}(\partial_\nu U)U^{-1})].$$

Using the identity $\partial_\mu U = U(\partial_\mu U^{-1})U$ in the above equation, we can write,

$$F'_{\mu\nu} = UF_{\mu\nu}U^{-1}. \tag{2.3.20}$$

Let under this transformation the gluon field lagrangian transforms as,

$$\pounds'_{gluon} = -\frac{1}{4}Tr(F'_{\mu\nu}F'_{\mu\nu}) = -\frac{1}{4}(UF_{\mu\nu}U^{-1}UF_{\mu\nu}U^{-1}),$$

$$\Rightarrow \pounds'_{gluon} = -\frac{1}{4}(F_{\mu\nu}F_{\mu\nu}) = \pounds_{gluon}.$$

This shows the invariance of $\pounds_{gluon}$ under local SU(3) transformations. Thus $\pounds_{QCD} = -\bar{\psi}(x)(\gamma_\mu\partial_\mu + ig\bar{\psi}(x)A_\mu(x)\psi(x) - \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$ is invariant under local SU(3) gauge transformation. The gluon field tensor is

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu], \tag{2.3.21}$$

where $\mathbf{A}_\mu = \lambda^a A^a_\mu$ can be written as

$$F_{\mu\nu} = (\partial_\mu A^a_\nu)\lambda^a - (\partial_\nu A^a_\mu)\lambda^a - igA^b_\mu A^c_\nu[\lambda^b, \Lambda^c],$$

$$\Rightarrow F_{\mu\nu} = (\partial_\mu A^a_\nu)\lambda^a - (\partial_\nu A^a_\mu)\lambda^a + gf_{abc}\lambda^a A^b_\mu A^c_\nu,$$

where f_{abc} the structure constants of SU(3) group and $f_{abc} = 1$ for even permutation of 'abc' while $f_{abc} = -1$ for odd permutation of 'abc'. Thus we can write the gluon filed Lagrangian as,

$$\pounds_{gluon} = -\frac{1}{4}Tr[(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)] - \frac{1}{4}Tr[gf_{abc}A^b_\mu A^c_\nu(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)]$$
$$-\frac{1}{4}Tr[gf_{ade}A^d_\mu A^e_\nu(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) + g^2 f_{abc}f_{ade}A^b_\mu A^c_\nu A^d_\mu A^e_\nu]. \quad (2.3.22)$$

From this we ca see that, it contains terms describing gluon-gluon self interactions. In the equation (2.3.22) the first term is for free A_μ field propagation, the second and third terms are three gluon vertex ('g' is linear in this gluon-gluon interaction),and the last term is for a four-gluon vertex. The Dirac fields constitute eight color currents

$$J^{\mu a} = ig\bar{\psi}\gamma_\mu\lambda^a, \quad (2.3.23)$$

which act as sources for the color fields(A_μ) in the same way that electric currents act as source for the electromagnetic field. We now derive the Feynman rules for QCD.

2.4 Feynman rules for QCD

The full QCD Lagrangian is

$$\pounds_{QCD} = -\bar{\psi}(\gamma_\mu\partial_\mu + m)\psi + ig\bar{\psi}\gamma_\mu\lambda^a A^a_\mu\psi + \pounds_{gluon}, \quad (2.4.1)$$

where $\pounds_{gluon}$ is given in (2.3.22). The second term $ig\bar{\psi}\gamma_\mu\lambda^a A^a_\mu\psi$ describes the quark-gluon vertex. We just rub out the field variables and obtain the quark-gluon vertex as

$$ig\gamma_\mu\lambda^a. \tag{2.4.2}$$

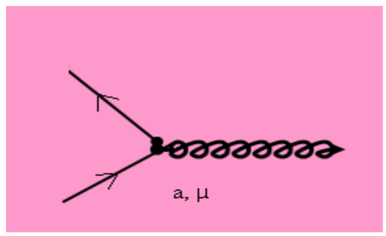

Figure 2.1: quark-gluon vertex.

Now, looking at $\pounds_{gluon}$, it contains not only the free gluon propagation part, $(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)$, but also gluon-gluon interaction terms

$$\begin{aligned}\pounds_{g-g} = &-\frac{1}{4}Tr[gf_{abc}A^b_\mu A^c_\nu(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) + gf_{ade}A^d_\mu A^e_\nu(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)] \\ &-\frac{1}{4}Tr[+g^2 f_{abc}f_{ade}A^b_\mu A^c_\nu A^d_\mu A^e_\nu].\end{aligned} \tag{2.4.3}$$

First and second terms carry three factors of A_μ, and leads to the three-gluon vertex. The third term Carries four factors of A_μ, and leads to the four-gluon vertex. To fond the three gluon vertex let us see fig 2.2.

$$\pounds_{3-gluon} = -\frac{1}{2}f_{abc}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)A^b_\mu A^c_\nu. \tag{2.4.4}$$

Now let us consider all possible permutations of a,b,c in (2.4.4) the non zero terms are

$$\pounds_{3-gluon} = -gf_{abc}(\partial_\mu A^a_\nu)A^b_\mu A^c_\nu - gf_{bac}(\partial_\mu A^b_\nu)A^a_\mu A^c_\nu - gf_{acb}(\partial_\mu A^a_\nu)A^c_\mu A^b_\nu$$

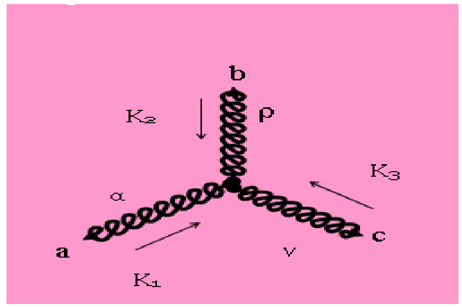

Figure 2.2: 3-gluon vertex.

$$-gf_{bca}(\partial_\mu A^b_\nu)A^c_\mu A^a_\nu - gf_{cab}(\partial_\mu A^c_\nu)A^a_\mu A^b_\nu. \tag{2.4.5}$$

Writing the Fourier transform of the gauge fields in momentum space, we get

$$A^{a,b,c}_\nu(x) = \frac{1}{(2\pi)^4}\int d^4\kappa_{1,2,3}e^{i\kappa_{1,2,3}.x}A^{a,b,c}_\nu(\kappa_{1,2,3}). \tag{2.4.6}$$

Thus making contraction over the indices, $\pounds_{3-gluon}$ can be written as

$$\begin{aligned}\pounds_{3-gluon} &= -gf_{abc}\{-i(\kappa_1.A^b)(A^a.A^c) + i(\kappa_1.A^c)(A^b.A^a) - i(\kappa_2.A^c)(A^a.A^b)\\ &\quad + i(\kappa_2.A^a)(A^b.A^c) + i(\kappa_3.A^b)(A^a.A^c) - i(\kappa_3.A^a)(A^b.A^c)\},\\ \pounds_{3-gluon} &= -gf_{abc}\{(\kappa_3-\kappa_1).A^b(A^a.A^c) + (\kappa_1-\kappa_2).A^c(A^a.A^b)\\ &\quad + (\kappa_2-\kappa_3).A^a(A^b.A^c)\}. \end{aligned}\tag{2.4.7}$$

We can get the 3-gluon vertex factor by rubbing out the gauge field. It is

$$-gf_{abc}\{(\kappa_3-\kappa_1)_\beta\delta_{\gamma\alpha} + (\kappa_1-\kappa_2)_\gamma\delta_{\alpha\beta} + (\kappa_2-\kappa_3)_\alpha\delta_{\beta\gamma}\}. \tag{2.4.8}$$

Now the 4-gluon interaction part of the Lagrangian is

$$\pounds_{4-gluon} = -\frac{1}{4}g^2 f_{abc}f_{ade}A^b_\mu A^c_\nu A^d_\mu A^e_\nu \tag{2.4.9}$$

Consider Figure 2.3 to extract the $\mathcal{L}_{4-gluon}$ vertex factor. Considering all possible permutations os indices a,b,c and a,d,e in (2.4.9), we can write,

$$\mathcal{L}_{4-gluon} = -\frac{1}{4}g^2\{f_{abc}f_{ade}A^b_\mu A^c_\nu A^d_\mu A^e_\nu + f_{abc}f_{aed}A^a_\mu A^b_\nu A^e_\mu A^d_\nu + f_{abd}f_{ace}A^b_\mu A^d_\nu A^c_\mu A^e_\nu$$

$$+f_{abd}f_{aec}A^b_\mu A^d_\nu A^e_\mu A^c_\nu + f_{abe}f_{acd}A^b_\mu A^e_\nu A^c_\mu A^d_\nu + f_{aeb}f_{acd}A^e_\mu A^b_\nu A^c_\mu A^d_\nu\}. \qquad (2.4.10)$$

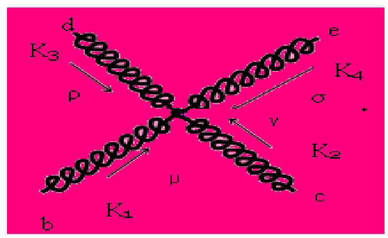

Figure 2.3: 4-gluon vertex.

Let us make contraction over the indices. We get,

$$\mathcal{L}_{4-gluon} = -\frac{1}{4}g^2\{f_{abc}f_{ade}((A^b.A^d)(A^c.A^e) - (A^b.A^e)(A^c.A^d)) + f_{abd}f_{ace}((A^b.A^c)(A^d.A^e)$$

$$-(A^b.A^e)(A^d.A^c)) + f_{abe}f_{acd}((A^b.A^c)(A^d.A^e) - (A^e.A^c)(A^b.A^d))\}. \qquad (2.4.11)$$

Now we rub out the gauge fields to get the 4-gluon interaction vertex factor as,

$$-\frac{1}{4}g^2\{f_{abc}f_{ade}(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho}) + f_{abd}f_{ace}(\delta_{\mu\nu}\delta_{\rho\sigma}$$

$$-\delta_{\mu\sigma}\delta_{\rho\nu}) + f_{abe}f_{acd}(\delta_{\mu\nu}\delta_{\rho\sigma} - \delta_{\nu\sigma}\delta_{\rho\mu})\}. \qquad (2.4.12)$$

The presence of three and four gluon vertices in QCD is due to the fact that gluons themselves carry color charge. This arise on account of the non-ablelian character of SU(3) color group.

Chapter 3

Bethe-Salpeter Equation

The Bethe salpeter equation is a relativistic equation for bound state problems, i.e it describes the bound states between two or more elementary particles on a fully covariant relativistic footing. Examples of two-particle systems described by the Bethe-Salpeter equation are the positronium, bound state of an electron-positron pair; the exciton, bound state of an electron-hole pair meson, a quark-anti quark pair in bound state etc. Since the particle pair described by the Bethe-Salpeter equation is in a bound state, the particles can interact infinitely often. Concurrently, since the number of interactions can be arbitrary, the number of possible Feynman diagrams will quickly exceed feasible calculations.The Bethe-Salpeter equation is then a direct application of the Feynman rules, and a resummation of the infinite set of diagrams by using integral equation. If both the irreducible kernel, and the n-point Green functions are understood as the full (non approximate) expressions, then the Bethe-Salpeter result for any n-point Green function is the exact result of field theory, as far as the full expression for a given Green function is considered, and as far as the existence of a fixed value of the mass of bound system is taken into account.

in this chapter, we first derive the propagator equation for a Dirac particle in an external field. Then we make generalization of one particle propagator in external field to two particle propagator With particles in mutual interaction, making generous use of Feynman propagator formulation to derive Bethe-Salpeter equation which is firmly rooted in field theory. Moreover we formulate the BSE under Covariant instantaneous ansatz (CIA) framework.

3.1 Single particle Dirac propagator in external field

We recall that the Schrodinger equation of a particle with total Hamilton

$$H = -\frac{1}{2m}\vec{\nabla}^2 + V \tag{3.1.1}$$

is

$$(i\frac{\partial}{\partial t} - H)\psi(\vec{x}, t) = 0. \tag{3.1.2}$$

Similarly Schrodinger equation for the propagator is

$$(i\frac{\partial}{\partial t} - H)K(\vec{x}, t; \vec{x}', t') = i\delta^3(\vec{x} - \vec{x}')\delta(t - t'). \tag{3.1.3}$$

If we replace H by the Dirac Hamiltonian

$$H = \vec{\alpha}.\vec{p} + \beta m_0, \tag{3.1.4}$$

Equation (3.1.3) becomes the Dirac equation for the propagator.

In the absence of external field, the Dirac equation for the state function is given by

$$(i\frac{\partial}{\partial t} - \vec{\alpha}.\vec{p} - \beta m_0)\psi(\vec{x}, t) = 0, \tag{3.1.5}$$

with $\vec{p} = -i\vec{\nabla}$. In the presence of external field A_μ, we make use of the minimal coupling prescription:

$$\vec{\nabla} \rightarrow \vec{\nabla} - ie\vec{A}; i\frac{\partial}{\partial t} \rightarrow i\frac{\partial}{\partial t} - e\phi. \tag{3.1.6}$$

Now equation (3.1.5) becomes

$$(i\frac{\partial}{\partial t} - e\phi + \vec{\alpha}.(i\vec{\nabla} + e\vec{A}) - \beta m)\psi(\vec{x}, t) = 0. \tag{3.1.7}$$

Let $K(\vec{x}, t; \vec{x}^{'}, t')$ be the full Dirac propagator; the propagator equation for Dirac particle in the presence of external field A_μ is

$$((i\frac{\partial}{\partial t} - e\phi) + \vec{\alpha}.(i\vec{\nabla} + e\vec{A}) - \beta m)K(\vec{x}, t; \vec{x}^{'}, t') = i\beta\delta^3(\vec{x} - \vec{x}^{'})\delta(t - t'). \tag{3.1.8}$$

Multiplying (3.1.8) by β from the left and use $\beta = \gamma_4, -i\beta\alpha_i = \gamma_i$ it be written in a covariant form as

$$(\gamma_\mu\partial_\mu - i\gamma_\mu eA_\mu(x) + m)K(x, x^{'}) = -i\delta^4(x - x^{'}),$$

$$\Rightarrow (\gamma_\mu\partial_\mu + m)K(x, x^{'}) = -i\delta^4(x - x^{'}) + ie\gamma_\mu A_\mu(x)K(x, x^{'}), \tag{3.1.9}$$

where the propagation is from $x^{'}$ to x. Now let us introduce an intermediate space-time point $x^{''}$, where $t^{'} < t^{''} < t$. Then equation (3.1.9) becomes

$$(\gamma_\mu\partial_\mu + m)K(x, x^{'}) = -i\int d^4x^{''}\delta^4(x - x^{''})\delta^4(x^{''} - x^{'})$$

$$+ie\int d^4x^{''}\delta^4(x - x^{''})\gamma_\mu A_\mu(x^{''})K(x^{''}, x^{'}). \tag{3.1.10}$$

From (3.1.3), we can deduce the equation satisfied by a free propagator $K_0(x, x')$ as,

$$(\gamma_\mu\partial_\mu + m)K_0(x, x^{'}) = -i\delta^4(x - x^{'}). \tag{3.1.11}$$

Equation (3.1.10) can be written as,

$$(\gamma_\mu\partial_\mu + m)K_0(x, x^{'}) = \int d^4x^{''}(\gamma_\mu\partial_\mu + m)K_0(x, x^{''})\delta^4(x^{''} - x^{'})$$

$$-e\int d^4x^{''}(\gamma_\mu\partial_\mu+m)K_0(x,x^{'})\gamma_\mu A_\mu(x^{''})K(x^{''},x^{'}). \tag{3.1.12}$$

Dropping the $(\gamma_\mu\partial_\mu+m)$ term from both sides of (3.1.12), we get

$$K(x,x^{'})=K_0(x,x^{'})-e\int d^4x^{''}K_0(x,x^{''})\gamma_\mu A_\mu(x^{''})K(x^{''},x^{'}). \tag{3.1.13}$$

It is also possible to introduce an intermediate space-time point $x^{'''}$ and write $K(x^{''},x^{'})$ in the same manner, we get

$$K(x^{''},x^{'})=K_0(x^{''},x^{'})-e\int d^4x^{'''}K_0(x^{''},x^{'''})\gamma_\mu A_\mu(x^{'''})K(x^{'''},x^{'}). \tag{3.1.14}$$

Using the symbolic notation $x\to 1, x^{'}\to 2, x^{''}\to 3, x^{'''}\to 4$...the full propagator can be written as

$$K(1,2)=K_0(1,2)-e\int d^43K_0(1,3)\gamma_\mu A_\mu(3)K_0(3,2)+(-e)^2\int d^43\int d^44K_0(1,3)$$
$$\gamma_\mu A_\mu(3)K_0(3,4)\gamma_\mu A_\mu(4)K_0(4,2)+... \tag{3.1.15}$$

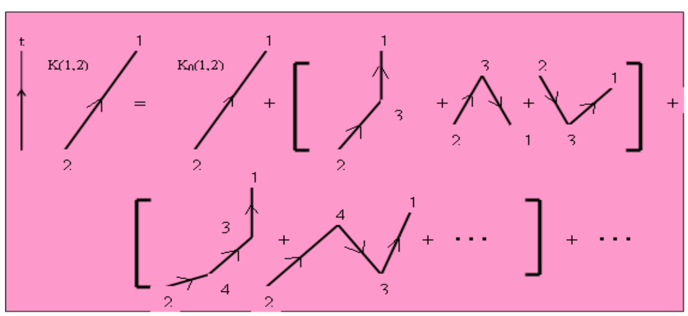

Figure 3.1: Diagrammatical interpretation of one particle propagator.

Since the propagating particle is Dirac particle, both direction of times ($\uparrow e^-$ and $\downarrow e^+$) are meaningful. In the coming subsection we generalize one particle propagator to two particle propagator where the particles are mutually interacting.

3.2 Two particle propagator with particles in mutual interaction

Now we consider the simultaneous propagation of two free Dirac particles. Particle one propagates from point 3 to 1 through path a and particle two propagates from point 4 to 2 through path b.

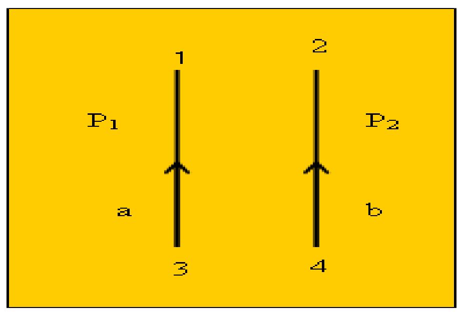

Figure 3.2: Simultaneous propagation of two free particles.

The definition of 2-free particles propagator is

$$K_{0ab}(1,2;3,4) = K_{0a}(1,3)K_{0b}(2,4). \tag{3.2.1}$$

where $K_{0a}(1,3)$ and $K_{0b}(2,4)$ are the one particle free Dirac propagator. Let us consider the two particles interact through single photon exchange.

The simplest possibility is the coulomb interaction between a and b, which is,

$$\frac{e^2}{r_{ab}}, \tag{3.2.2}$$

but it is non covariant. To write in a covariant form, we first write the Fourier transformation in momentum space, we get

$$\frac{e^2}{r_{ab}} \rightarrow \frac{e^2\gamma_4^a\gamma_4^b}{(\vec{p}_a - \vec{p}_a')^2}. \tag{3.2.3}$$

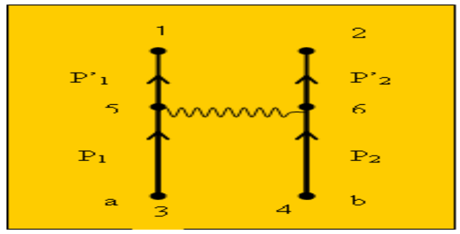

Figure 3.3: simultaneous propagation with mutual interaction through a photon.

Now we can write the covariant generalization of equation (3.2.3) as,

$$\frac{e^2\gamma_4^a\gamma_4^b}{(\vec{p}_a-\vec{p}_a')^2} \Rightarrow \frac{e^2\gamma_\mu^a\gamma_\mu^b}{(\vec{p}_a-\vec{p}_a')^2}. \tag{3.2.4}$$

The right hand side of (3.2.4) is Moller interaction in momentum space in the form,

$$\frac{e^2\gamma_\mu^a\gamma_\mu^b}{(\vec{p}_a-\vec{p}_a')^2} \equiv (ie^2)\gamma_\mu^a(-\frac{-i}{(\vec{p}_1-\vec{p}_1')^2})\gamma_\mu^b. \tag{3.2.5}$$

In two particle case, we further generalize,

$$e\gamma_\mu A_\mu(3) \Rightarrow ie^2\gamma_\mu^a D_F(5,6)\gamma_\mu^b \equiv I(5,6), \tag{3.2.6}$$

where 5 and 6 are intermediate space-time points at which a photon connects to two fermion lines a and b such that $t_3 < t_5 < t_1$ and $t_4 < t_6 < t_2$, and $D_F(5,6)$ is photon propagator joining space time points 5 and 6,

$$D_F(5,6) = \int \frac{d^4q}{(2\pi)^4}(\frac{-i}{q^2})e^{iq.(x_5-x_6)}. \tag{3.2.7}$$

Thus, the first order correction to two particle propagator due to simultaneous interaction of both Dirac particle through exchange of a photon is

$$K_1(1,2;3,4) = \int d^45\int d^46 K_0(1,2;5,6)ie^2\gamma_\mu^a D_F(5,6)\gamma_\mu^b K_0(5,6;3,4)$$

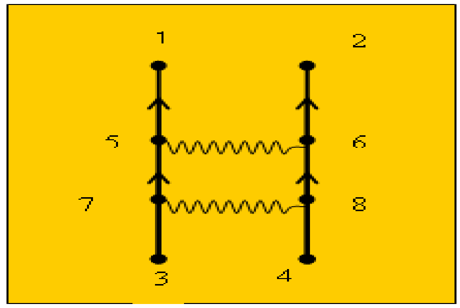

Figure 3.4: simultaneous propagation with mutual interaction through two photons.

$$= \int d^4 5 \int d^4 6 K_0(1,2;5,6) I(5,6) K_0(5,6;3,4). \qquad (3.2.8)$$

For the interaction of the two particles through exchange of two photons,

the second order correction to the two-particle propagator becomes

$$K_2(1,2;3,4) = \int d^4 5 \int d^4 6 \int d^4 7 \int d^4 8 K_0(1,2;5,6) I(5,6) K_0(5,6;7,8) I(7,8) K_0(5,6;3,4). \qquad (3.2.9)$$

If we consider up to n photon interaction, the full propagator for two-particles in a sequential action of Moller interaction in Ladder approximation is,

$$K(1,2;3,4) = K_0(1,2;3,4) + K_1(1,2;3,4) + K_2(1,2;3,4) + ...$$

$$K(1,2;3,4) = K_0(1,2;3,4) + \int d^4 5 \int d^4 6 K_0(1,2;5,6) I(5,6) K(5,6;3,4), \qquad (3.2.10)$$

where K(5,6;3,4) is the full 2-particle kernel for propagation from (3,4)→ (5,6)

In the Ladder approximation limit, we can conclude from (2.3.10) that,

$$K = K_0 + K_0 I K. \qquad (3.2.11)$$

In a single particle case, if the wave function at point 2 is known, we can write $\psi(1)$ as

$$\psi(1) = \int d^3 2 K(1,2) \beta \psi(2), \qquad (3.2.12)$$

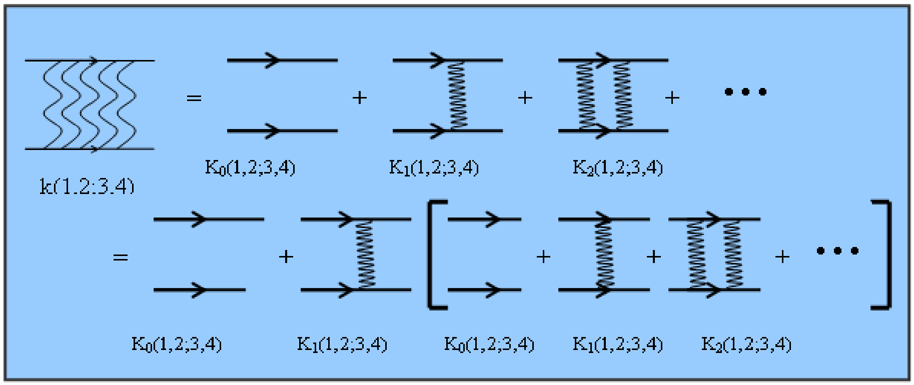

Figure 3.5: Ladder approximation for 2-particle propagation.

where $\beta = \gamma_4$ is the insertion needed to write the equation in covariant form. Now let us use the above case to derive for two particle wave function.

$$\psi(1,2) = \int d^3 3 \int d^3 4 K(1,2;3,4)\beta^a \beta^b \psi(3,4). \tag{3.2.13}$$

If we use equation (3.2.10)to write (3.2.13), we get

$$\psi(1,2) = \psi_0(1,2) + \int d^4 5 \int d^4 6 K_0(1,2;5,6) I(5,6)\psi(5,6). \tag{3.2.14}$$

3.3 Derivation of Bethe-Salpeter Equation(BSE)

Let us operate $(\gamma_\mu \partial_{1\mu a} + m_a)(\gamma_\mu \partial_{2\mu b} + m_b)$ on both sides of (3.2.14) from left, we get

$$(\gamma_\mu \partial_{1\mu a} + m_a)(\gamma_\mu \partial_{2\mu b} + m_b)\psi(1,2) = -I(1,2)\psi(1,2). \tag{3.3.1}$$

Equation (3.3.1) is Bethe-salpeter equation in co-ordinate space, where

$$I(1,2) = ie^2 \gamma_\mu^a D_F(1,2)\gamma_\mu^b \equiv -i\bar{K}(1,2) \tag{3.3.2}$$

is two particle interaction kernel. Let us transform the the variables x_1 and x_1 to new variables $\bar{X}$ and x, where $\bar{X}$ is the center of mass co-ordinate and x is the relative co-ordinate.

$$\bar{X} = x_1 + \frac{x_2}{2}; x = x_1 - \frac{x_2}{2}, \tag{3.3.3}$$

$$\partial_1 = \frac{1}{2}\partial_{\bar{X}} + \partial_x; \partial_2 = \frac{1}{2}\partial_{\bar{X}} - \partial_x. \tag{3.3.4}$$

Thus, equation (3.3.1) becomes

$$(\gamma_\mu \partial_{1\mu} + m_a)(\gamma_\mu \partial_{2\mu} + m_b)\psi(\bar{X}, x) = -I(X)\psi(\bar{X}, x). \tag{3.3.5}$$

Writing the two particle wave function in a variable separable manner

$$\psi(\bar{X}, x) = \psi(\bar{X})\psi(x), \tag{3.3.6}$$

where we have used

$$\psi(\bar{X}) = \frac{1}{\sqrt{V}} e^{iP.\bar{X}}. \tag{3.3.7}$$

$P = p_1 + p_2$ being the total momentum of the composite. We can write,

$$(\gamma_\mu \partial_{1\mu} + m_a)(\gamma_\mu \partial_{2\mu} + m_b)\frac{1}{\sqrt{V}} e^{iP.\bar{X}}\psi(X) = -I(X)\frac{1}{\sqrt{V}} e^{iP.\bar{X}}\psi(X). \tag{3.3.8}$$

If we use Fourier transformation of $\psi(x)$ and I(x) as below

$$\psi(X) = \frac{1}{(2\pi)^4}\int d^4q e^{iq.X}\psi(q)$$

$$I(X) = \frac{1}{(2\pi)^4}\int d^4K e^{iK.X} I(K), \tag{3.3.9}$$

then (3.3.8) becomes

$$(i(\frac{1}{2}\gamma_\mu P_\mu + \gamma_\mu q_\mu) + m_a)(i(\frac{1}{2}\gamma_\mu P_\mu - \gamma_\mu q_\mu) + m_b)\psi(q) = \frac{1}{(2\pi)^4}\int d^4q^{'} K(q, q^{'})\psi(q^{'}), \tag{3.3.10}$$

where $2q = p_1 - p_2$, and $q = K + q^{'}$. The variable q stands for internal momentum or relative momentum and $K(q, q^{'})$ is interaction kernel. If we express it by p_1 and p_2, where

$$p_{1,2} = \hat{m}_{1,2} P_\mu \pm q_\mu \tag{3.3.11}$$

to obtain,

$$(i\gamma^a p_1 + m_1)(i\gamma^b p_{2\mu} + m_b)\psi(q) = \frac{1}{(2\pi)^4}\int d^4q^{'}K(q,q^{'})\psi(q^{'}). \tag{3.3.12}$$

This is BSE in momentum space. Let us define first BS wave function for spin less quarks,

$$\psi(P,q) = (-i\gamma^a.p_1 + m_1)(-i\gamma^b.p_{2\mu} + m_2)\Phi(P,q). \tag{3.3.13}$$

We use the general form of the $q\bar{q}$ interaction kernel, which is taken as a one gluon exchange type [2,3]

$$K(q,q^{'}) = (\frac{1}{2}\vec{\lambda_1}.\frac{1}{2}\vec{\lambda_2})(\gamma^a_\mu\gamma^b_\mu)V(q,q^{'}), \tag{3.3.14}$$

where$(\frac{1}{2}\vec{\lambda_1}.\frac{1}{2}\vec{\lambda_2})$ is the color part of the interaction kernel, $(\gamma^a_\mu\gamma^b_\mu)$ is spin part of the interaction kernel and $V(q,q^{'})$ is the confinement potential [2,3].

$$\bar{K}(q,q^{'}) = (\frac{1}{2}\vec{\lambda_1}.\frac{1}{2}\vec{\lambda_2})\gamma^a_\mu\gamma^b_\mu V(q,q^{'}). \tag{3.3.15}$$

Substituting (3.3.13) into (3.3.12) and using (3.3.15), the BSE in momentum representation becomes

$$\Delta_1\Delta_2\Phi(P,q) = \frac{i}{(2\pi)^4}\int d^4q^{'}\bar{K}(q,q^{'})\Phi(p,q^{'}). \tag{3.3.16}$$

And

$$\Delta_{1,2} = m^2_{1,2} + p^2_{1,2}, \tag{3.3.17}$$

where $\Phi(P,q)$ is BS wave function for spinless quarks and $\Delta_{1,2}$ are the inverse propagators for scalar quarks. Thus, (3.3.16) is four dimensional BSE for spinless quarks. We now give the formulation of BSE under Covariant Instantaneous Ansatz (CIA) [5,6-8] in the next section.

3.4 BSE Under Covariant instantaneous unsatz (CIA)

For simplicity we first outline the CIA framework for scalar quarks. For a bound state of quark and anti quark system with total momentum P, the 4D BSE takes the form

$$\Delta_1\Delta_2\Phi(P,q) = \frac{i}{(2\pi)^4}\int d^4q'\bar{K}(q,q')\Phi(P,q'), \tag{3.4.1}$$

where $\bar{K}(q,q')$ is the effective Kernel which operates on the 4D wave function $\Phi(P,q')$ and $\Delta_{1,2}$ are the inverse propagators of two scalar quarks. The 4-momentum of the constituent quark and anti quark $p_{1,2}$ are related to the internal momentum q_μ of the hadron by

$$p_{1,2} = \hat{m}_{1,2}P_\mu \pm q_\mu, \tag{3.4.2}$$

where $\hat{m}_{1,2} = \frac{1}{2}(1 \pm \frac{(m_1^2+m_\mu^2)}{M^2})$ are the WightmanGarding (WG) definitions [12] of masses of individual quarks. We now use an ansatz on the BS kernel K in equation (3.4.1), which is assumed to depend on 3D variables $\hat{q}, \hat{q}'$, i.e

$$\bar{K}(q,q') \equiv \bar{K}(\hat{q},\hat{q}'), \tag{3.4.3}$$

where

$$\hat{q}_\mu = q_\mu - \frac{q.P}{P^2}P_\mu. \tag{3.4.4}$$

The internal momentum is

$$q_\mu = (\hat{q}_\mu - iM\sigma); \sigma = \frac{q.P}{P^2}. \tag{3.4.5}$$

$\hat{q}$ is observed to be orthogonal to the total 4-momentum P irrespective of whether the individual quarks are on-shell or off-shell.

$$\hat{q}_\mu P_\mu = q_\mu P_\mu - \frac{q.P}{P^2}P^2 = 0. \tag{3.4.6}$$

The longitudinal component of q_μ (i.e $M\sigma$) which does not appear in $\hat{K}(\hat{q},\hat{q}')$ is

$$M\sigma = M\frac{q.P}{P^2}. \tag{3.4.7}$$

To reduce the 4D BSE in to 3D form, we define a 3D wavefunction $\phi(\hat{q})$,

$$\phi(\hat{q}) = \int_{-\infty}^{\infty} Md\sigma\Phi(P,q). \tag{3.4.8}$$

And we have the relation

$$d^4q' = d^3\vec{q}'Md\sigma. \tag{3.4.9}$$

Substituting this in to (3.4.1), we get

$$\Phi(P,q) = \frac{i}{(2\pi)^2\Delta_1\Delta_2}\int d^3\vec{q}'\bar{K}(\hat{q},\hat{q}')\phi(\hat{q}). \tag{3.4.10}$$

Integrating both sides by $Md\sigma$ from $-\infty$ to ∞

$$\int_{-\infty}^{\infty} Md\sigma\Phi(P,q) = -\frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{Md\sigma}{2\pi i\Delta_1\Delta_2}\int d^3\hat{q}'\bar{K}(\hat{q},\hat{q}')\phi(\hat{q}'),$$

we get

$$(2\pi)^3 D(\hat{q})\phi(\hat{q}) = \int d^3\hat{q}'\bar{K}(\hat{q},\hat{q}')\phi(\hat{q}'), \tag{3.4.11}$$

where $D(\hat{q})$ is a 3D denominator function defined by

$$\frac{1}{D(\hat{q})} = \frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{Md\sigma}{\Delta_1\Delta_2}. \tag{3.4.12}$$

We note that the RHS of (3.4.11) is identical to RHS of (3.4.1) by virtual of (3.4.3) and (3.4.8). Thus we get an exact interconnection between 3D wave function $\phi(\hat{q})$ and the 4D wave function $\Phi(P,q)$.

$$\Delta_1\Delta_2\Phi(P,q) = \frac{D(\hat{q})\phi(\hat{q})}{2\pi i} \equiv \Gamma(\hat{q}), \tag{3.4.13}$$

where $\Gamma(\hat{q})$ is the Hadron gluon BS vertex function under CIA. In (3.4.13) the 3D form serves for making contact with the mass spectrum of hadrons, where as the 4D form provides the $Hq\bar{q}$ vertex function $\Gamma(\hat{q})$ which satisfies a 4D BSE with a natural off-shell extension over the entire 4D space (due to the positive definiteness of the quantity $\hat{q}^2 = q^2 - \frac{(P.q)^2}{P^2}$ through out the inter 4D space) and thus provides a fully Lorentz invariant basis for the evaluation of various amplitudes through various quark loop diagrams [2,3]. Due to these properties, this framework can be profitably implied for the evaluation of various transition amplitudes [2,3] all the way from low energies to high energies. To find the value of the 3D denominator function $D(\hat{q})$, we first get the inverse propagators as a function of σ.

$$\Delta_{1,2} = m_{1,2}^2 + p_{1,2}^2, \tag{3.4.14}$$

where $p_{1,2}$ are the momentum of constituent quarks and $m_{1,2}$ are masses of it. In terms σ the 4-momentum of individual quarks can be written as

$$p_{1\mu} = P_\mu(\hat{m}_1 + \sigma) + \hat{q}_\mu; p_{2\mu} = P_\mu(\hat{m}_2 - \sigma) - \hat{q}_\mu. \tag{3.4.15}$$

Inserting (3.4.15) to (3.4.14) and using the orthogonality between $\hat{q}_\mu$ and P_μ we get

$$\Delta_1 = \omega^2 + P_\mu^2(\hat{m}_1 + \sigma)^2,$$

$$\Delta_2 = \omega^2 + P_\mu^2(\hat{m}_2 - \sigma)^2, \tag{3.4.16}$$

where $\omega_{1,2}^2 = m_{1,2}^2 + \hat{q}_\mu^2$. In this ansatz $m_1 \equiv m_2 = \frac{1}{2}$. Using the Wightman-Garding definition of mass and $P_\mu^2 = -M^2$ we can write;

$$\Delta_1 = \omega_1^2 - M^2(\frac{1}{2} + \sigma)^2,$$

$$\Delta_2 = \omega_2^2 - M^2(\frac{1}{2} - \sigma)^2. \tag{3.4.17}$$

Now the value of $D(\hat{q})$ can be worked out using contour integration by noting the positions of the poles in the complex σ-plane. The poles can be identified from

$$\Delta_1 = 0, \Delta_2 = 0. \tag{3.4.18}$$

Inserting (3.4.17) to (3.4.18), we get the poles to be

$$\begin{aligned} \sigma_1^{\pm} &= \pm\frac{\omega_1}{M} - \frac{1}{2}, \\ \sigma^{\pm} &= \pm\frac{\omega_2}{M} + \frac{1}{2}, \end{aligned} \tag{3.4.19}$$

which lie on the real σ-axis. Now we shift the poles by $\pm i\epsilon$ as shown in figure 3.8. Considering semicircular contour of large radius R with center at the origin in the complex σ-plane

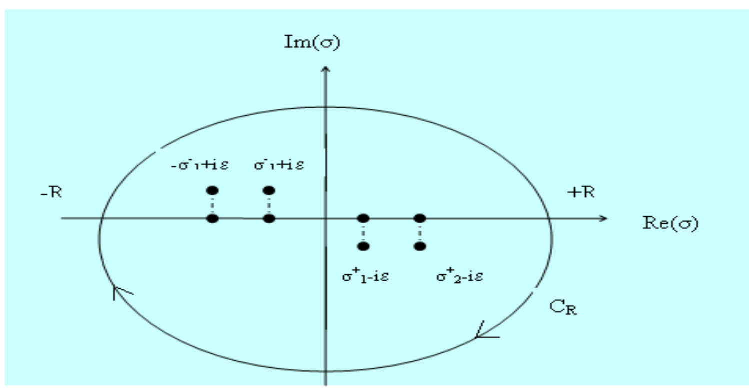

Figure 3.6: Contour integration in σ-plane.

From Cauchy's-residue theorem, let $f(\sigma) = \frac{1}{2\pi i}\frac{M}{\Delta_1\Delta_2}$, then

$$\int_{-R}^{R} f(\sigma)d\sigma = -2\pi i a_{-1}, \tag{3.4.20}$$

where $a_{-1} = a'_{-1} + a''_{-1}$ the sum of the residues at its poles in the lower half plane. The poles lying with in the contour are $\sigma_1^+ - i\epsilon$ and $\sigma_2^+ - i\epsilon$. The residues at $\sigma_1^+ - i\epsilon$

and $\sigma_2^+ - i\epsilon$ can be;

$$a_1' = -\frac{1}{2\pi i}[\frac{1}{2\omega_1(\omega_2-\omega_1+M)(\omega_2+\omega_1-M)}]$$

$$,a_1'' = -\frac{1}{2\pi i}[\frac{1}{2\omega_2(\omega_1-\omega_2-M)(\omega_1+\omega_1+M)}]. \tag{3.4.21}$$

Thus we get,

$$\frac{1}{D(\hat{q})} = -2\pi i(a_1'+a_1''),$$

$$\Rightarrow \frac{1}{D(\hat{q})} = \frac{1}{((\omega_1+\omega_2)^2-M^2}(\frac{1}{2\omega_1}+\frac{1}{2\omega_2}). \tag{3.4.22}$$

Let us define $D_0(\hat{q}) = (\omega_1+\omega_2)^2 - M^2$, then

$$D(\hat{q}) = \frac{D_0(\hat{q})}{(\frac{1}{2\omega_1}+\frac{1}{2\omega_2})}. \tag{3.4.23}$$

We now outline the framework of BSE under CIA for the case of fermionic quarks constituting a particular meson. The scalar propagators Δ_i^{-1} in the above equations are replaced by the proper fermionic propagators S_F . Let us see how to incorporate the relevant Dirac structures in vertex function $\Gamma(\hat{q})$. Recent studies [5, 6 - 8] have revealed that various mesons have many different covariant structures in their wavefunctions and their inclusion was also found necessary to obtain quantitatively accurate observables [5]. Thus, it was introduced by [2] to incorporate the relative Dirac covariants in the structure of $Hq\bar{q}$ vertex function systematically and use the leading-order in their study. This study incorporate the relative Dirac covariants up to the next leading-order so that to get more accurate observables. The transverse $Hq\bar{q}$ vertex function for a V-meson can be expressed as a linear combination of eight Dirac covariants [2, 9 - 13], Γ_i^v (i = 0, . . . , 7), each multiplying a Lorentz scalar amplitude $F_i(q^2, qP, P^2)$. The choice of the Dirac covariants is not unique. Thus in [2,3] a criterion was proposed so as to systematically choose among these eight Dirac covariants. For a V-meson it was noted that [2,3] the covariant $\Gamma_1 = \hat{\gamma}_\mu$ (where $\hat{\gamma} - P_\mu \frac{(P.\gamma}{P^2}$ is the transverse projection of the 4-vector γ_μ) was considered was considered to be the most important one, but in this study we include more terms $(i\gamma.\varepsilon)$ than this. Note that $\gamma.\varepsilon$ is the same as $\hat{\gamma}.\varepsilon$ on account of $P.\varepsilon = 0$ with ε_μ being the polarization vector.

In this study, we generalize the structure of $Hq\hat{q}$ vertex function for a V-meson in [2,8].

$$\Gamma(\hat{q}) = \frac{1}{2\pi i} N_v (i\gamma.\varepsilon) D(\hat{q})\phi(\hat{q}). \tag{3.4.24}$$

For incorporating the Dirac structures in the expression for $\Gamma(\hat{q})$, we take their forms as in [2,3]. The factor $D(\hat{q})\phi(\hat{q})$ multiplying $i\gamma.\varepsilon$ in (3.4.24) is Lorentz-invariant momentum-dependent scalar which depends upon q^2, P^2 and q.P and thus has a certain dimensionality of mass. But the Lorentz scalar amplitudes $\Gamma(q, P)$ (i = 0, 1, 2, . . .) multiplying the various Dirac structures in [2,3] have different dimensionalities of mass. In [2,3], Γ_i(q,P) was made dimensionless by weighing each covariant with an appropriate power of M and rewritten as in equation (3.4.25) [2,3].
We now give the vertex function of V-meson in the framework discussed above associated with a certain power of M. We can express $\Gamma^v_\mu \varepsilon_\mu$ as [2,3]

$$\Gamma^v_\mu \varepsilon_\mu = \Omega_\mu \varepsilon_\mu . \frac{1}{2\pi i} N_v D(\hat{q})\phi(\hat{q}),$$

$$\Omega_\mu \varepsilon_\mu = i(\gamma.\varepsilon)A_0 + (\gamma.\varepsilon)(\gamma.P)\frac{A_1}{M} + [q.\varepsilon - (\gamma.\varepsilon)(\gamma.q)]\frac{A_2}{M} + [(\gamma.\varepsilon)(\gamma.P)(\gamma.q) -$$
$$(\gamma.\varepsilon)(\gamma.q)(\gamma.P) + 2(q.\varepsilon)(\gamma.P)]i\frac{A_3}{M^2} + (q.\varepsilon)\frac{A_4}{M} + i(q.\varepsilon)(\gamma.P)\frac{A_5}{M^2} - i(q.\varepsilon)(\gamma.q)\frac{A_6}{M^2}$$
$$+(q.\varepsilon)[(\gamma.P)(\gamma.q) - (\gamma.q)(\gamma.P)]\frac{A_7}{M^3}, \tag{3.4.25}$$

where A_i (i = 0, ... , 7) are eight dimensionless and constant coefficients to be determined. Now since we use constituent quark masses where the quark mass m is approximately half of the hadron mass M, we can use the ansatz

$$q \ll P \sim M \qquad (3.4.26)$$

in the rest frame of the hadron. Then, each of the eight terms in equation (3.4.25) receives suppression by different powers of $\frac{1}{M}$. Thus, we can arrange these terms as an expansion in powers of $O(\frac{1}{M})$ [2]. We can then see in the expansion of $\Omega_\mu \varepsilon_\mu$ that the structures associated with the coefficients A_0, A_1 have magnitudes $O(\frac{1}{M^0})$ and are of leading order. Those with A_2, A_3, A_4, A_5 are $O(\frac{1}{M^1})$, while those with A_6, A_7 are $O(\frac{1}{M^2})$. This naive power counting rule suggests that the maximum contribution to the calculation of any vector meson observable should come from the Dirac structures $i\gamma.\varepsilon$ and $(\gamma.\varepsilon)(\gamma.P)\frac{1}{M}$ associated with the constant coefficients A_0 and A_1, respectively [2].

In this study we incorporate the terms associated with $A_0, A_1, A_2, A_3, A_4, A_5$ in (3.4.25) and ignoring $O(\frac{1}{M^2})$ terms for the moment and try to calculate the V -meson decay constants. Thus, we take the modified form of the $Hq\bar{q}$ vertex function [2,3],

$$\begin{aligned}\Gamma(\hat{q}) = \{i(\gamma.\varepsilon)A_0 + (\gamma.\varepsilon)(\gamma.P)\frac{A_1}{M} + [q.\varepsilon - (\gamma.\varepsilon)(\gamma.q)]\frac{A_2}{M} \\ +[(\gamma.\varepsilon)(\gamma.P)(\gamma.q) - (\gamma.\varepsilon)(\gamma.q)(\gamma.P) \\ +2(q.\varepsilon)(\gamma.P)]i\frac{A_3}{M^2} + (q.\varepsilon)\frac{A_4}{M} + i(q.\varepsilon)(\gamma.P)\frac{A_5}{M^2}\}.\frac{1}{2\pi i}N_v D(\hat{q})\phi(\hat{q}). \qquad (3.4.27)\end{aligned}$$

The structure of BSE is characterized by a single effective kernel arising out of a four-fermion Lagrangian in the Nambu-Jona Lasino [2,3] sense. The input kernel K(q, q') in BSE is taken as one-gluon exchange like as regards color $(\frac{1}{2}\vec{\lambda}^1.\frac{1}{2}\vec{\lambda}^2)$ and spin $(\gamma^1_\mu.\gamma^2_\mu)$

dependence. The scalar function V(q-q') is a sum of one-gluon exchange V_{OGE} and a confining term V_{conf}. For details see [2,3].

$$K(q,q') = \frac{1}{2}\lambda^1.\frac{1}{2}\lambda^2\gamma_\mu^{(1)}\gamma_\mu^{(2)}V(q-q').$$

The values of parameters interring in to the interaction kernel have been calibrated to fit the meson mass spectra obtained by solving the 3D BSE. Now come to the problem of the 3D BS wavefunction. The ground-state wavefunction $\phi(\hat{q})$ satisfies the 3D BSE, on the surface P.q = 0, which is appropriate for making contact with O(3)-like mass spectrum [2,3]. The ground-state wavefunction $\phi(\hat{q})$ deducible from this equation thus has a Gaussian structure [3,5,12] and is expressible as

$$\phi(\hat{q}) \approx e^{-\frac{\hat{q}^2}{2\beta^2}}. \tag{3.4.28}$$

In the structure of $\phi(\hat{q})$ in equation (3.4.28), the parameter β is the inverse range parameter which incorporates the content of BS dynamics and is dependent of the input kernel K(q, q'). In the next unit we calculate the decay constant of V-mesons using this CIA formulation.

Chapter 4

Leptonic Decay Of Vector Mesons

4.1 V-Meson Decay Constant

Vector meson decay proceeds through the loop diagram shown in figure 1.1. The coupling of a vector meson to the photon is expressed via a dimensionless coupling constant g_v which can be described as [2]

$$\frac{M^2}{g_v}\varepsilon_\mu(P) =< 0|\bar{Q}\hat{\Theta}\gamma_\mu Q|V(P)>, \tag{4.1.1}$$

where Q is the flavour multiplet of quark field and $\hat{\Theta}$ is the quark electromagnetic charge operator. (4.1.1) can in tern be expressed as a loop integral [2,3],

$$\frac{M^2}{g_v}\varepsilon_\mu = \sqrt{3}e_Q \int d^4qTr[\psi_v(P,q)i\gamma_\mu], \tag{4.1.2}$$

where $e_Q^2 = \frac{1}{2}, \frac{1}{9}, \frac{1}{18}$ for $v = \rho, \phi, \omega$ respectively, and the polarization vector of V-meson ε_μ satisfying $\varepsilon.P = 0$. The BetheSalpeter wavefunction $\psi(P,q)$ for a V-meson is expressed as

$$\psi(P,q) = S_F(p_1)\Gamma(\hat{q})S_F(-p_2). \tag{4.1.3}$$

And the fermionic propagators for the two constituent quarks of the hadron are

$$S_F(p_1) = -i\frac{(m_1 - i\gamma.p_1)}{\Delta_1}; S_F(-p_2) = -i\frac{(m_2 + i\gamma.p_2)}{\Delta_2}. \tag{4.1.4}$$

Here $\Gamma(\hat{q})$ is the hadron-quark vertex function in (3.3.27). The non preservative aspect inter through the $\Gamma(\hat{q})$ in the process. Using $\psi(P,q)$ from (4.1.3) and the structure of $Hq\bar{q}$ vertex function $\Gamma(\hat{q})$ from (3.3.27), evaluating the trace over γ-matrices and noting that only the components of $p_{1\mu}$ and $p_{2\mu}$ in the direction of ε_μ will contribute to g_v , we further make the replacements $p_{1\mu} \to (p_1.\varepsilon)\varepsilon_\mu$, $p_{2\mu} \to (p_2.\varepsilon)\varepsilon_\mu$. Now let us first do the trace in (4.1.2), we see that

$$Tr[\psi_v(P,q)i\gamma_\mu] = \frac{N_v}{2\pi i}\frac{D(\hat{q})\phi(\hat{q})}{\Delta_1\Delta_2}\{A_0Tr[-(m_1 - i\gamma.p_1)i(\gamma.\varepsilon)(m_2 + i\gamma.p_2)i\gamma_\mu]$$
$$+\frac{A_1}{M}Tr[-(m_1 - i\gamma.p_1)(\gamma.\varepsilon)(\gamma.P)(m_2 + i\gamma.p_2)i\gamma_\mu]$$
$$+\frac{A_2}{M}Tr[-(m_1 - i\gamma.p_1)(q.\varepsilon - (\gamma.\varepsilon)(\gamma.q))(m_2 + i\gamma.p_2)i\gamma_\mu]$$
$$+\frac{A_3}{M^2}Tr[-(m_1 - i\gamma.p_1)(i(\gamma.\varepsilon)(\gamma.P)(\gamma.q) - i(\gamma.\varepsilon)(\gamma.q)(\gamma.P)$$
$$+2i(q.\varepsilon)(\gamma.P))(m_2 + i\gamma.p_2)i\gamma_\mu] + \frac{A_4}{M}Tr[-(m_1 - i\gamma.p_1)(q.\varepsilon)(m_2 + i\gamma.p_2)]$$
$$+\frac{A_5}{M^2}Tr[-(m_1 - i\gamma.p_1)i(q.\varepsilon)(\gamma.P)(m_2 + i\gamma.p_2)i\gamma_\mu]\}. \tag{4.1.5}$$

Let the term in (4.1.5) with coefficient A_0 be T_0, the term with $\frac{A_2}{M}$ be T_1...term with $\frac{A_5}{M^2}$ be T_5, then

$$T_0 = Tr[-(m_1 - i\gamma.p_1)i(\gamma.\varepsilon)(m_2 + i\gamma.p_2)i\gamma_\mu]$$
$$= Tr[m_1m_2(\gamma.\varepsilon)\gamma_\mu + im_1(\gamma.\varepsilon)(\gamma.p_2)\gamma_\mu$$
$$-im_2(\gamma.p_1)(\gamma.P)(\gamma.\varepsilon)\gamma_\mu + (\gamma.p_1)(\gamma.\varepsilon)(\gamma.p_2)\gamma_\mu]. \tag{4.1.6}$$

But terms with odd number of γ's have zero trace. Thus we can write,

$$T_0 = m_1m_2Tr[(\gamma.\varepsilon)\gamma_\mu] + Tr[(\gamma.p_1)(\gamma.\varepsilon)(\gamma.p_2)\gamma_\mu]$$
$$= 4m_1m_2\varepsilon_\mu + 4((p_1.\varepsilon)p_{2\mu} - (p_1.p_2) + (\varepsilon.p_2)p_{1\mu}). \tag{4.1.7}$$

Then using the replacement $p_{1\mu} \rightarrow (p_1.\varepsilon)\varepsilon_\mu$, $p_{2\mu} \rightarrow (p_2.\varepsilon)\varepsilon_\mu$ and due to the fact that the average value of $< (p_1.\varepsilon)(p_2.\varepsilon) >= \frac{1}{3}(p_1.p_2)$,

$$T_0 = 4m_1 m_2 - \frac{4}{3}(p_1.p_2). \tag{4.1.8}$$

But in terms of σ the 4-momentum of individual quarks is in (3.3.15). So $(p_1.p_2)$ can be expressed as

$$p_1.p_2 = -\hat{m}_1\hat{m}_2 M^2 - \hat{m}_2\sigma M^2 - \hat{q}^2 + M^2\hat{m}_1\sigma = m^2 - \frac{1}{2}(\Delta_1 + \Delta_2 + M^2). \tag{4.1.9}$$

Because the values of $\hat{m}_1$ and $\hat{m}_2$ for the case of two equal mass quarks is $\hat{m}_1 = \hat{m}_2 = \frac{1}{2}$. Similarly

$$p_1.P = \frac{1}{2}(\Delta_1 - \Delta_2 - M^2); And, p_2.P = \frac{1}{2}(-\Delta_1 + \Delta_2 - M^2). \tag{4.1.10}$$

This gives the expression for T_0 as,

$$\Rightarrow T_0 = 4(\frac{1}{6}(\Delta_1 + \Delta_2) + (\frac{1}{6}M^2 + \frac{2}{3}m^2)). \tag{4.1.11}$$

We similarly get,

$$\begin{aligned} T_1 = -4mM^2; T_2 = -\frac{4}{3}m(\Delta_1 - \Delta_2); \\ T_3 = [-M^2(\Delta_1 - \Delta_2) - \frac{7}{3}m^2(\Delta_1 - \Delta_2) - (\Delta_1^2 - \Delta_2^2) - \frac{7}{3}m^2M^2]; \\ T_4 = [\frac{8}{3}m^3 - \frac{4}{3}m(\Delta_1 + \Delta_2) - \frac{2}{3}mM^2]; \end{aligned}$$

$$T_5 = [\frac{4}{3}m^2(\Delta_1 - \Delta_2) - \frac{2}{3}(\Delta_1^2 - \Delta_2^2) - \frac{2}{3}M^2(\Delta_1 - \Delta_2)]. \tag{4.1.12}$$

Now using (4.1.12) and the relation $d^4\hat{q} = d^3\hat{q}Md\sigma$, then we can write (4.1.2) as

$$\frac{M^2}{g_v} = \sqrt{3}e_Q \int d^3\hat{q} N_v \frac{D(\hat{q})\phi(\hat{q})}{2\pi i} I;$$

$$I = \int \frac{Md\sigma}{\Delta_1\Delta_2} 4\{[\frac{1}{6}(\Delta_1+\Delta_2)+(\frac{1}{6}M^2+\frac{2}{3}m^2)]A_0 - [mM]A_1 - [\frac{1}{3}m(\Delta_1-\Delta_2)]\frac{A_2}{M}$$
$$-[\frac{1}{4}M^2(\Delta_1-\Delta_2)+\frac{7}{12}m^2(\Delta_1+\Delta_2)+\frac{1}{4}(\Delta_1^2-\Delta_2^2)+\frac{7}{12}M^2m^2]\frac{A_3}{M^2}$$
$$+[\frac{2}{3}m^3-\frac{1}{3}m(\Delta_1+\Delta_2)-\frac{1}{6}mM^2]\frac{A_4}{M}$$
$$+[\frac{1}{3}m^2(\Delta_1-\Delta_2)-\frac{1}{6}(\Delta_1^2-\Delta_2^2)-\frac{1}{6}M^2(\Delta_1-\Delta_2)]\frac{A_5}{M^2}\}. \tag{4.1.13}$$

The vector meson decay constant is defined as in [2,3], i.e

$$f_v = \frac{M}{e_Q g_v}. \tag{4.1.14}$$

Thus, we can write

$$f_v = \frac{\sqrt{3}}{M} N_v \int d^3\hat{q}\phi(\hat{q})[\frac{D(\hat{q})}{2\pi i} I]. \tag{4.1.15}$$

Let us now carry out the integration over $d\sigma$ by method of contour integration in the complex σ-plane. The integration of Δ_1^2 on the plane is worked out in a similar way as in the denominator function.

$$\int_{-\infty}^{\infty} \frac{M}{2\pi i}\frac{D(\hat{q})}{\Delta_1\Delta_2}\Delta_1^2 d\sigma = \int_{-\infty}^{\infty} \frac{M}{2\pi i}\frac{D(\hat{q})}{[\omega_1^2 - M^2(\frac{1}{2}-\sigma)^2]}[\omega_1^2 - M^2(\frac{1}{2}+\sigma)^2]. \tag{4.1.16}$$

The poles can identified from $\Delta_2 = 0$ and are at

$$\sigma = \pm\frac{\omega_2}{M} + \frac{1}{2}. \tag{4.1.17}$$

Considering semicircular contour of large radius R with center at the origin in the complex σ-plane. If we shift the poles by $\pm i\epsilon$ and use the lower path for the contour, we enclose the pole $+\frac{\omega_2}{M} + \frac{1}{2} - i\epsilon$.

If we let $f(\sigma) = \frac{M}{2\pi i}\frac{D(\hat{q})}{\Delta_1\Delta_2}\Delta_1^2 d\sigma = \int_{-\infty}^{\infty} \frac{M}{2\pi i}\frac{D(\hat{q})}{[\omega_1^2 - M^2(\frac{1}{2}-\sigma)^2]}[\omega_1^2 - M^2(\frac{1}{2}+\sigma)^2]$, using Cauchy-residue theorem

$$\int_{-\infty}^{\infty} f(\sigma) = -2\pi i a_{-1}, \tag{4.1.18}$$

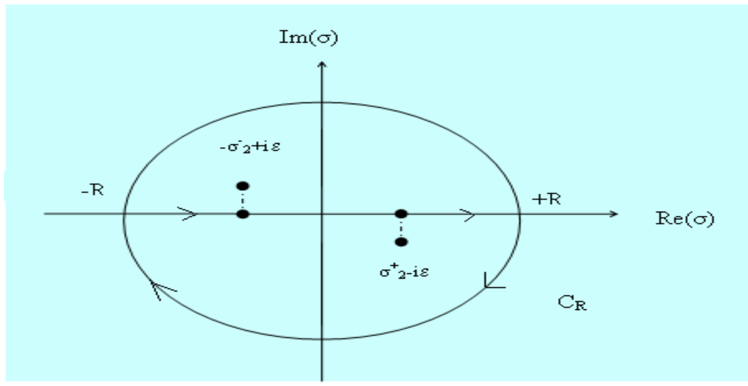

Figure 4.1: Contour integration in σ-plane.

but

$$a_{-1} = lim_{\sigma \to \frac{\omega_2}{M}+\frac{1}{2}} - \frac{D(\hat{q})}{2\pi i} \frac{[\omega_1^2 - M^2(\frac{1}{2}+\sigma)^2]}{[\sigma + \frac{\omega_2}{M} - \frac{1}{2}]}. \quad (4.1.19)$$

Thus we get the result

$$\int_{-\infty}^{\infty} \frac{M}{2\pi i} \frac{D(\hat{q})}{\Delta_1 \Delta_2} \Delta_1^2 d\sigma = \frac{D(\hat{q})}{2\omega_2} [\omega_1^2 - M^2(\frac{\omega_2}{M}+1)^2]. \quad (4.1.20)$$

Using the same way we can get

$$\int_{-\infty}^{\infty} \frac{M}{2\pi i} \frac{D(\hat{q})}{\Delta_1 \Delta_2} (\Delta_1 + \Delta_2) d\sigma = D_0(\hat{q}), \quad (4.1.21)$$

$$\int_{-\infty}^{\infty} \frac{M}{2\pi i} \frac{D(\hat{q})}{\Delta_1 \Delta_2} (\Delta_1 - \Delta_2) d\sigma = D_0(\hat{q}) \frac{(\omega_1 - \omega_2)}{(\omega_1 + \omega_2)}, \quad (4.1.22)$$

$$\int_{-\infty}^{\infty} \frac{M}{2\pi i} \frac{D(\hat{q})}{\Delta_1 \Delta_2} \Delta_2^2 d\sigma = \frac{D(\hat{q})}{2\omega_1} [\omega_1^2 - M^2(\frac{\omega_1}{M}+1)^2]. \quad (4.1.23)$$

Now inserting (4.1.13),(4.1.19) and (4.120) in to (4.1.15) and taking $m_1 = m_2 = m$, so that $\omega_1 = \omega_2 = \omega$, the v-meson decay constant f_v becomes

$$f_v = \frac{4\sqrt{3}}{M} N_v \int d^3\hat{q} \phi(\hat{q}) \{ [\frac{M^2}{6} + \frac{2}{3}m^2 + \frac{D_0(\hat{q})}{6}] A_0 - [mM] A_1 - [\frac{7}{12} M^2 m^2$$

$$-\frac{1}{2} D_0(\hat{q}) M\omega] \frac{A_3}{M^2} + [\frac{2}{3}m^3 - \frac{1}{3} m D_0(\hat{q}) - \frac{1}{6} m M^2] \frac{A_4}{M} + [\frac{D_0(\hat{q})}{3} M\omega] \frac{A_5}{M^2} \}. \quad (4.1.24)$$

The structure of the parameter β in $\phi(\hat{q})$ appearing in (4.1.21) as well as other input parameters ω_0, Λ, c_0 are listed in [2,3].

4.2 The Normalization factor N_v

To calculate the Bethe-salpeter normaliz N_v we use the current conservation condition in [2,11]

$$2iP_\mu = (2\pi)^4 \int d^4q Tr[\bar{\psi}(P,q)(\frac{\partial}{\partial P_\mu} S_F^{-1}(p_1))\psi(P,q)S_F^{-1}(-p_2)] + (1 \leftrightarrow 2), \quad (4.2.1)$$

where the momentum of constituent quarks can be expressed as

$$p_{1,2\mu} = P_\mu(\hat{m}_1 \pm \sigma) \pm \hat{q}_\mu. \quad (4.2.2)$$

The inverse propagators can also be expressed as

$$S_F^{-1}(p_1) = i(m_1 + i\gamma.p_1); S_F^{-1}(-p_2) = i(m_2 - i\gamma.p_2) \quad (4.2.3)$$

Thus the derivative of the inverse propagator with with respect to the total 4-momentum P_μ is

$$\frac{\partial}{\partial P_\mu}(S_F^{-1}(p_1)) = -(\hat{m}_1 + \sigma)\gamma_\mu. \quad (4.2.4)$$

Let $A = \bar{\psi}(P,q)(\frac{\partial}{\partial P_\mu} S_F^{-1}(p_1))\psi(P,q)S_F^{-1}(-p_2)$. Using the expression for $\psi(P,q)$ we can write the trace in (4.2.1) as

$$Tr[A] = \frac{(\hat{m}_1 + \sigma)}{\Delta_1^2 \Delta_2} Tr[i(m_1 + i\gamma.p_1)\gamma_4\Gamma^\dagger\gamma_4 i(m_2 - i\gamma.p_2)\gamma_\mu i(m_1 + i\gamma.p_1)\Gamma]. \quad (4.2.5)$$

Now taking the value of Γ from (3.4.27) we first evaluate the trace over the γ-matrices following the steps used in section 4.1. The trace contains a total of 648 terms. Out of this 324 terms survive on account of trace theorems. The trace is now

$$Tr[A] = i\frac{(\hat{m}_1 + \sigma)}{\Delta_1^2 \Delta_2} D^2(\hat{q}) N_v^2 \phi^2(\hat{q})\{\mathbf{I}_0 + \mathbf{I}_1 + ... + \mathbf{I}_{20}\}; \quad (4.2.6)$$

$$
\begin{aligned}
\mathbf{I}_0 = \frac{2}{3}iA_0^2[-4\frac{\Delta_1^2}{M^2} + 3\frac{\Delta_1\Delta_2}{M^2} + \frac{\Delta_2^2}{M^2} + \frac{1}{M^2}(4m_1^2 + 2M^2)\Delta_2 \\
+(M^2 + 4m) - \frac{1}{M^2}(4m^2 + 3M^2)\Delta_1] \\
\mathbf{I}_1 = \frac{2}{3}iA_1^2[\frac{\Delta_1^2}{M^2} + \frac{3\Delta_1\Delta_2}{M^2} - 4\frac{\Delta_2^2}{M^2} - 8\frac{m^2\Delta_1}{M^2} + \frac{1}{M^2}(8m^2 + M^2)\Delta_2 \\
+2(4m^2 + M^2)], \\
\mathbf{I}_2 = +\frac{1}{3}iA_2^2[\frac{1}{M^2}(1 - 12\frac{m^2}{M^2})\Delta_1^2 + \frac{3}{M^4}(\frac{m^2}{M^2} - 1)\Delta_1\Delta_2 + \frac{1}{M^2}(7m^4 - 3M^2m^2] \\
+\frac{1}{M^2}(M^2 + 13\frac{m^4}{M^2} - 5m^2)\Delta_1 - 13\frac{m^4\Delta_2}{M^4} + 9\frac{m^2\Delta_2^2}{M^4}, \\
\mathbf{I}_3 = \frac{1}{3}iA_3^2[\frac{1}{M^2}(-6\frac{m^4}{M^4} + 9\frac{m^2}{M^2} - 22\frac{m^3}{M^3} - 55\frac{m^2}{2M^2} + 13)\Delta_1^2 + \frac{1}{M^2}(18\frac{m^4}{M^4} \\
-7\frac{m^2}{M^2} + 70\frac{m^2}{M^2} + 12\frac{m^3}{M^3} - 4)\Delta_1\Delta_2 \\
+\frac{1}{M^2}(9\frac{m^4}{M^4} - 67\frac{m^2}{M^2} + 10\frac{m^3}{M^3} + 35 - \frac{9m^2}{2M^2})\Delta_2^2 \\
+\frac{1}{M^2}(13M^2 + 17m^2 + 12\frac{m^4}{M^2} - 3\frac{m^3}{M} - 48\frac{m^4}{M^2} + 16\frac{m^6}{M^4} - 30m^2 - 36\frac{m^5}{M^3} \\
\frac{3}{2}M^2)\Delta_1 + \frac{1}{M^2}(22M^2 + 3m^2 - 12\frac{m^4}{M^2} + 3\frac{m^3}{M} + 48\frac{m^4}{M^2} - 16\frac{m^6}{M^4} - 10m^2 \\
+4\frac{m^5}{M^3})\Delta_2 + \frac{1}{M^2}(-m^2M^2 - 14m^3M + 76m^4 + 16mM^3 + 6M^4 + \frac{5}{2}m^4] \\
\mathbf{I}_4 = \frac{1}{3}iA_4^2[\frac{1}{M^2}(4\frac{m^2}{M^2} + 1)\Delta_1^2 + \frac{7}{2M^2}\Delta_1\Delta_2 + \frac{1}{M^2}(\frac{1}{6} - 4\frac{m^2}{M^2})\Delta_2^2 + (2 - 6\frac{m^4}{M^4} \\
-\frac{m^5}{M^5} - 6\frac{m^2}{M^2} - \frac{m^3}{M^3})\Delta_1 + (\frac{1}{6} + \frac{m^5}{M^5} + 6\frac{m^4}{M^4} - 8\frac{m^2}{M^2})\Delta_2 \\
+(-4m^2 + 8\frac{m^4}{M^2} + \frac{m^5}{M^3} + \frac{1}{3}M^2)], \\
\mathbf{I}_5 = +\frac{1}{3}iA_5^2[-\frac{3}{2M^2}(\frac{1m^4}{3M^4} + \frac{11m^2}{3M^2} + \frac{3}{2})\Delta_1\Delta_2 + \frac{1}{M^2}(-M^2 + \frac{3m^4}{2M^2} \\
+\frac{7m^5}{2M^3} - 2 + \frac{17m^2}{4})\Delta_1 + \frac{1}{M^2}(-\frac{1}{4}M^2 - \frac{3m^5}{2M^3} - \frac{7m^4}{2M^2} + \frac{15m^2}{4})\Delta_2 \\
+\frac{1}{M^2}(-\frac{3}{2}M^4 + \frac{9}{4}m^2M^2 + \frac{7}{2}m^4)] \\
\mathbf{I}_6 = A_0A_1i[8\frac{\Delta_1 m}{M} - 8\frac{\Delta_2 m}{M} - 8mM],
\end{aligned}
$$

$$\mathbf{I}_7 = +A_0A_2i[-2\frac{\Delta_1^2 m}{M^3} + \frac{4m}{3M^3}\Delta_1\Delta_2 + \frac{2m}{3M^3}\triangle_2^2 + \frac{8}{3M}(\frac{m^3}{M^2} - m)\Delta_1$$
$$+\frac{8m^3}{3M^3}\Delta_2 + (\frac{8m^3}{3M} - \frac{2}{3}mM)],$$
$$\mathbf{I}_8 = +A_0A_3i[\frac{1}{M^2}(6\frac{m^2}{M^2} - \frac{34}{3})\Delta_1^2 + \frac{1}{3M^2}(26\frac{m^2}{M^2} - 14)\Delta_2^2 -$$
$$\frac{1}{3M^2}(44\frac{m^2}{M^2} + 20)\Delta_1\Delta_2 + \frac{1}{M^2}(-6M^2 + \frac{8m^4}{3M^2} - 12m^2)\Delta_1$$
$$+\frac{1}{M^2}(-\frac{22M^2}{3} - \frac{8m^4}{3M^2} + \frac{44}{3}m^2)\Delta_2 + \frac{1}{M^2}(14m^2M^2 - \frac{8}{3}m^4 - \frac{10}{3}M^4)],$$
$$\mathbf{I}_9 = A_0A_4i[-\frac{4m}{3M^3}\Delta_1^2 - \frac{8m}{3M^3}\Delta_2^2 + \frac{4m}{M^3}\Delta_1\Delta_2 - \frac{4}{3M}(\frac{4m^3}{M^2} + m)\Delta_1$$
$$+\frac{4}{M}(\frac{4m^3}{3M^2} - m)\Delta_2 + (\frac{16m^3}{3M} - \frac{4}{3}mM)],$$
$$\mathbf{I}_{10} = +A_0A_5i[-\frac{7m^2}{3M^4}\Delta_1^2 + \frac{1}{3M^2}(-\frac{2m^4}{M^4} - 7\frac{m^2}{M^2} + 8)\Delta_2^2 +$$
$$\frac{1}{3M^2}(\frac{2m^2}{M^2} - 8)\Delta_1\Delta_2 + \frac{1}{M^2}(-M^2 - \frac{2m^4}{3M^2} + \frac{8}{3}m^2)\Delta_1 + \frac{1}{M^2}(\frac{4}{3}M^2$$
$$+\frac{2m^4}{3M^2} - \frac{8}{3}m^2)\Delta_2 + \frac{1}{M^2}(-m^2M^2 - 2m^4)],$$
$$\mathbf{I}_{11} = A_1A_2i[\frac{1}{M^2}(\frac{m^2}{M^2} + \frac{2}{3})\Delta_1^2 + \frac{1}{M^2}(\frac{m^2}{M^2} - 4)\Delta_2^2 + \frac{1}{M^2}(-2\frac{m^2}{M^2}$$
$$+\frac{10}{3})\Delta_1\Delta_2 + \frac{1}{M^2}(\frac{8}{3}M^2 - \frac{2}{3}m^2)\Delta_1 + \frac{1}{M^2}(-2M^2 + \frac{2}{3}m^2)\Delta_2 + \frac{5}{3}m^2]$$
$$\mathbf{I}_{12} = +A_1A_3i[\frac{1}{3M^3}(-4\frac{m^3}{M^2} - 34m)\Delta_1^2 + \frac{1}{3M^3}(-4\frac{m^3}{M^2} - 26m)\Delta_2^2 +$$
$$(\frac{8m^2}{M^4} + \frac{20}{M^2})\Delta_1\Delta_2 + \frac{1}{M^3}(-\frac{16m^5}{3M^4} + \frac{8m^3}{3M^2} + 28m)\Delta_2^2 + \frac{1}{M}(\frac{16m^5}{3M^4}$$
$$-\frac{8m^3}{3M^2} - \frac{46}{3}m)\Delta_2 + (-\frac{28m^4}{3M^2} - \frac{8m^3}{3M} - 2mM)],$$
$$\mathbf{I}_{13} = A_1A_4i[\frac{1}{M^2}(-\frac{m^4}{M^4} + \frac{4}{3})\Delta_1^2 + \frac{1}{M^2}(-\frac{m^2}{M^2} + 2)\Delta_2^2 + \frac{1}{M^2}(2\frac{m^2}{M^2}$$
$$-\frac{10}{3})\Delta_1\Delta_2 + \frac{1}{M^2}(2\frac{m^2}{3} + \frac{11}{3}m^2)\Delta_1$$
$$+\frac{1}{M^2}(2m^2 - \frac{11}{3}m^2)\Delta_2 + (-\frac{8}{3}m^2 + \frac{2}{3}M^2)],$$
$$\mathbf{I}_{14} = +A_1A_5i[\frac{5m}{3M^3}\Delta_1^2 - \frac{m}{3M^3}\Delta_2^2 - \frac{4m}{3M^3}\Delta_1\Delta_2 + \frac{1}{M}(-2\frac{m^3}{M^2}$$
$$+\frac{2}{3}m)\Delta_2 + (-2\frac{m^3}{M} + mM)],$$

$$\mathbf{I}_{15} = A_2A_3i[\frac{13m^3}{2M} + \frac{1}{M^3}(-\frac{3m^3}{2M^2} + \frac{7}{3}m)\Delta_1^2 - \frac{1}{M^3}(\frac{m^3}{3M^2} + 6m)\Delta_2^2$$
$$+\frac{1}{M^3}(3\frac{m^3}{M^2} + \frac{11}{3}m)\Delta_1\Delta_2 + \frac{1}{3M}(-25\frac{m^3}{M^2} - 5m)\Delta_1$$
$$+\frac{1}{3M}(25\frac{m^3}{M^2} - 19m)\Delta_2],$$
$$\mathbf{I}_{16} = A_2A_4i[\frac{1}{M^2}(-\frac{3m^2}{2M^2} + 4\frac{m^4}{M^4} + 1)\Delta_1^2 + \frac{1}{M^2}(-\frac{7m^2}{2M^2} - 4\frac{m^4}{3M^4} + \frac{10}{3})\Delta_2^2$$
$$+\frac{1}{M^2}(\frac{7}{6}M^2 + \frac{m^4}{3M^2} + \frac{4}{3}m^2 - \frac{8m^6}{3M^4} - \frac{14}{3}m^2)\Delta_1 + \frac{1}{M^2}(\frac{8}{3}M^2$$
$$-\frac{m^4}{3M^2} + \frac{8m^6}{3M^4} - 8m^2)\Delta_2 + \frac{1}{M^2}(5\frac{m^2}{M^2} - \frac{8m^4}{3M^4} + 2+)\Delta_1\Delta_2$$
$$+(-\frac{23}{6}m^2 + \frac{19m^4}{3M^2} + \frac{8m^6}{3M^4} + \frac{2}{3}M^2)],$$
$$\mathbf{I}_{17} = A_2A_5i[(\frac{7m^3}{3M^5} + \frac{3}{4}\frac{m^2}{M^4} + \frac{5m}{12M^3})\Delta_1^2 + (\frac{7m^3}{3M^5} - \frac{3m^2}{6M^4})\Delta_2^2 + (-\frac{14m^3}{3M^5}$$
$$+\frac{m}{3M^3} - \frac{3}{4}\frac{m^2}{M^4})\Delta_1\Delta_2 + (-\frac{7m^3}{6M^3} + \frac{m}{6M} - \frac{3}{8}\frac{m^4}{M^4} + \frac{3}{8}\frac{m^2}{M^2})\Delta_1$$
$$+(\frac{7m^3}{6M^3} - \frac{m}{M} + \frac{3}{8}\frac{m^4}{M^4})\Delta_2 + (\frac{5m^3}{6M} - \frac{1}{4}mM + \frac{3}{16}\frac{m^4}{M^2})],$$
$$\mathbf{I}_{18} = +A_3A_4i[-\frac{m^3}{M^5}\Delta_1^2 - \frac{m^3}{M^5}\Delta_2^2 + 4\frac{m^3}{M^5}\Delta_1\Delta_2],$$
$$\mathbf{I}_{19} = A_3A_5i[\frac{1}{M^2}(\frac{m^4}{M^4} + \frac{26m^2}{3M^2} - \frac{16m^4}{3M^4} - \frac{14}{3} + \frac{2m^2}{3M^2})\Delta_1^2 + \frac{1}{M^2}(\frac{3m^2}{3M^2} - \frac{8m^4}{M^4}$$
$$+\frac{16m^3}{3M^2} + \frac{16m^4}{3M^4} - \frac{2}{3} - \frac{10m}{3M})\Delta_2^2 + \frac{1}{M^2}(\frac{2}{3}M^2 - \frac{4}{3}m^2\frac{3m^4}{3M^2} - 10\frac{m^3}{M}$$
$$-\frac{2m^4}{3M^2} + \frac{16m^6}{3M^4} + \frac{22}{3}m^2)\Delta_1 + \frac{1}{M^2}(2M^2 - \frac{8}{3}m^2 - \frac{2m^4}{3M^2}$$
$$+\frac{28m^4}{3M^2} - \frac{2m^4}{3M^2} - \frac{16m^6}{3M^4} - \frac{4}{3}m^2)\Delta_2 + \frac{1}{M^2}(-\frac{2m^4}{3M^4} + \frac{6}{M^6} - 14\frac{m^2}{M^2}$$
$$+\frac{4}{3} - \frac{8}{3}\frac{m^2}{M^2})\Delta_1\Delta_2 + \frac{1}{M^2}(-\frac{8}{3}m^2M^2 + \frac{2m^6}{3M^2} - 2m^4 - \frac{10}{3}M^4 - \frac{16}{3}\frac{m^6}{M^2}$$
$$+\frac{2}{3}M^4 - \frac{2}{3}m^2M^2)],$$
$$\mathbf{I}_{20} = +A_4A_5i[\frac{1}{M^3}(-8\frac{m^3}{M^2} - \frac{8}{3}m)\Delta_1^2 + \frac{1}{M^3}(-8\frac{m^3}{M^2} + \frac{16}{3}m)\Delta_2^2 + \frac{1}{M^3}(16\frac{m^3}{M^2}$$
$$-\frac{8}{3}m)\Delta_1\Delta_2 + \frac{1}{M}(8\frac{m^3}{M^2} - \frac{4}{3}m)\Delta_1 + \frac{1}{M}(-8\frac{m^3}{M^2} + \frac{4}{3}m)\Delta_2]. \tag{4.2.7}$$

Now let us multiply both sides of (4.2.1) by -$\frac{P_\mu}{M^2}$ and insert the the trace in (4.2.7) to it, we can write the normalization constant as

$$N_v^{-2} = \frac{(2\pi)^2}{2}\int d^3\hat{q}D^2(\hat{q})\phi^2(\hat{q})\int_{-\infty}^{\infty}\frac{Md\sigma}{\Delta_1^2\Delta_2}(\hat{m}_1+\sigma)\{\mathbf{I}_1+\mathbf{I}_2+...+\mathbf{I}_{20}\}+(1\leftrightarrow 2). \tag{4.2.8}$$

In the above expression, terms like Δ_1^2, Δ_2^2 and $\Delta_1\Delta_2$ give divergent contributions due to the presence of the factor $(\hat{m}_1+\sigma)$ in the integrand. To overcome this problem, one can consider, following [2,3], the charge to be concentrated on one of the quark lines (say p_1). This may amount to taking the derivative with respect to p_1 (instead of P) [2,3] in the expression for N_v^{-2} in (4.2.1). Thus, we can express N_v^{-2} as

$$N_v^{-2} = \frac{(2\pi)^2}{2}\int d^3\hat{q}D^2(\hat{q})\phi^2)(\hat{q})\int_{-\infty}^{\infty}\frac{Md\sigma}{\Delta_1^2\Delta_2}\{\mathbf{I}_0+\mathbf{I}_1+...+\mathbf{I}_{20}\}+(1\leftrightarrow 2). \tag{4.2.9}$$

where we have been able to get rid of the factor $(\hat{m}_1+\sigma)$ and thus the integration can be carried out over all the terms in the above expression. The integration over $d\sigma$ over various terms in (4.2.9) which can be carried out by noting the pole positions in complex σ-plane, in the the same way as we solve for the denominator function, gives us

$$\begin{aligned}
a)&\int_{-\infty}^{\infty}\frac{Md\sigma}{\Delta_1^2\Delta_2}\Delta_1 = 2\pi i[\frac{1}{D(\hat{q})}],\\
b)&\int_{-\infty}^{\infty}\frac{Md\sigma}{\Delta_1^2\Delta_2}\Delta_2 = 2\pi i[\frac{2}{(2\omega)^3}],\\
c)&\int_{-\infty}^{\infty}\frac{Md\sigma}{\Delta_1^2\Delta_2}\Delta_1\Delta_2 = 2\pi i[\frac{1}{2\omega}],\\
d)&\int_{-\infty}^{\infty}\frac{Md\sigma}{\Delta_1^2\Delta_2}\Delta_1^2 = 2\pi i[\frac{1}{2\omega}],\\
e)&\int_{-\infty}^{\infty}\frac{Md\sigma}{\Delta_1^2\Delta_2}\Delta_2^2 = 2\pi i[\frac{\omega^2-\frac{1}{2}M^2}{\omega^3}],\\
f)&\int_{-\infty}^{\infty}\frac{Md\sigma}{\Delta_1^2\Delta_2} = 2\pi i[\frac{M^2-12\omega^2}{4\omega^2(M^2-4\omega^2)^2}].
\end{aligned} \tag{4.2.10}$$

Hence, we get

$$N_v^{-2} = -\frac{(2\pi)^3}{2}\int d^3\hat{q}D^2(\hat{q})\phi^2(\hat{q})\{\mathbf{I}_0 + \mathbf{I}_1 ... + \mathbf{I}_{20}\} + (1 \leftrightarrow 2), \tag{4.2.11}$$

where

$$\mathbf{I}_0 = A_0^2\frac{2}{3}\{-\frac{4m^2+3M^2}{D(\hat{q})M^2} + \frac{4m^2+2M^2}{4M^2\omega^3} - \frac{1}{2M^2\omega}$$
$$+\frac{(4m+M^2)(M^2-12\omega^2)}{4\omega^3(M^2-4\omega^2)^2} + \frac{-\frac{M^2}{2}+\omega^2}{M^2\omega^3}\},$$
$$\mathbf{I}_1 = A_1^2\frac{2}{3}\{-\frac{8m^2}{D(\hat{q})M^2} + \frac{8m^2+M^2}{4M^2\omega^3} + \frac{2}{M^2\omega}$$
$$+\frac{(4m+M^2)(M^2-12\omega^2)}{2\omega^3(M^2-4\omega^2)^2} - \frac{4(-\frac{M^2}{2}+\omega^2)}{M^2\omega^3}\},$$
$$\mathbf{I}_2 = A_2^2\frac{1}{3}\{\frac{-5m^2+\frac{13m^4}{M^2}+M^2}{D(\hat{q})M^2} - \frac{13m^4}{4M^4\omega^3} + \frac{3(-1+\frac{m^2}{M^2})}{2M^4\omega}$$
$$+\frac{1-\frac{12m^2}{M^2}}{2M^2\omega} + \frac{9m^2(-\frac{M^2}{2}+\omega^2)}{M^4\omega^3} + \frac{(7m^4-3m^2M^2)(M^2-12\omega^2)}{4M^2\omega^3(M^2-4\omega^2)^2}\},$$
$$\mathbf{I}_3 = A_3^2\frac{1}{3}\{\frac{1}{D(\hat{q})M^2}(-30m^2-36M^2+\frac{12m^4}{M^2}$$
$$-\frac{3m^4}{M^2} - \frac{48m^4}{M^2} + \frac{16m^6}{M^4} + 13M^2 + 17mM - \frac{3M^2}{2})$$
$$+\frac{1}{M^2\omega^3}(-90m^2 + \frac{36m^4}{M^2} + \frac{3m^{42}}{M^2} + 22M^2$$
$$+3mM) + \frac{1}{2M^2\omega}(9 - 7\frac{m^4}{M^4} + \frac{88m^2}{M^2}$$
$$+\frac{1}{4M^2\omega^3(M^2-4\omega^2)^2} + \frac{12m^4}{M^2} + \frac{55m^2}{2} + \frac{9m^2}{M^2} - \frac{22m^4}{M^2}$$
$$-6M^2)((76m^4 + 16m^6 - m^2M^2 - 14m^4M^2 + 6M^2$$
$$+\frac{5m^5M^4}{2})(M^2-12\omega^2))$$
$$+\frac{(35-\frac{9m^2}{2}+\frac{9m^2}{M^4}-\frac{67m^2}{M^2}+\frac{10m^4}{M^2})(-\frac{M^2}{2}+\omega^2)}{M^2\omega^3}\},$$

$$\mathbf{I}_4 = A_4^2 \frac{1}{3}\{\frac{(\frac{1}{6}-\frac{4m^2}{M^2})(-\frac{M^2}{2}+\omega^2)}{M^2\omega^3} + \frac{\frac{1}{6}+\frac{6m^4}{M^4}+\frac{m^5}{M^5}-\frac{8m^2}{M^2}}{4\omega^3}$$

$$+\frac{1+\frac{4m^2}{M^2}}{2M^2\omega} + \frac{(-4m^2+\frac{8m^4}{M^2}+\frac{m^5}{M^3}+\frac{M^2}{3})(M^2-12\omega^2)}{4\omega^3(M^{2-4\omega^2})^2}$$

$$+\frac{7}{4M^2\omega} + \frac{2-\frac{6m^4}{M^4}-\frac{m^5}{M^5}-\frac{6m^2}{M^2}-\frac{m^3}{M^3}}{D(\hat{q})}\},$$

$$\mathbf{I}_5 = A_5^2 \frac{1}{3}\{\frac{(-\frac{7}{4}+\frac{1}{2}+\frac{25m^2}{4M^2})(-\frac{M^2}{2}+\omega^2)}{M^2\omega^3}$$

$$+\frac{\frac{15m^2}{4}-\frac{3m^4}{2M^2}-\frac{7m^4}{2M^2}-\frac{M^2}{4}}{4M^2\omega^3} - \frac{3(\frac{3}{2}+\frac{1}{3\frac{m^3}{M^3}}+\frac{11m^2}{3M^2})}{4M^2\omega}$$

$$+\frac{(\frac{7m^2}{2}+\frac{7m^4}{2}-\frac{3M^2}{2}+\frac{9m^2M^2}{4})(M^2-12\omega^2)}{4M^2\omega^3(M^2-4\omega^2)^2}$$

$$+\frac{-2M^2+\frac{17m^2}{4}+\frac{3m^4}{2M^2}+\frac{7m^4}{2M^2}-M^2}{D(\hat{q})M^2} - \frac{3(1+\frac{m^2}{3M^2}+\frac{m^2}{2M^2})}{4M^2\omega}\},$$

$$\mathbf{I}_6 = A_0A_1\{\frac{8m}{D(\hat{q})M} - \frac{2m}{M\omega^3} - \frac{2mM(M^2-12\omega^2)}{\omega^3(M^2-4\omega^2)^2}\},$$

$$\mathbf{I}_7 = A_0A_2\{\frac{8(-m+\frac{m^3}{M^2})}{3D(\hat{q})M} + \frac{2m^3}{3M^3\omega^3} - \frac{m\omega}{M^2} + \frac{2}{3}mM^2\omega$$

$$+\frac{(\frac{8m^3}{3M}-\frac{2mM}{3})(M^2-12\omega^2)}{4\omega^3(M^2-4\omega^2)^2} + \frac{2mM^3(-\frac{M^2}{2}+\omega^2)}{3\omega^3}\},$$

$$\mathbf{I}_8 = A_0A_3\{\frac{-12m^2+\frac{8m^4}{3M^2}-6M^2}{D(\hat{q})M^2} + \frac{\frac{44m^2}{3}-\frac{8m^4}{3M^2}-\frac{22M^2}{3}}{4M^2\omega^3}$$

$$-\frac{20+\frac{44m^2}{M^2}}{6M^4\omega} + \frac{(-\frac{8m^4}{3}+14m^2M^2-\frac{10M^4}{3})(M^2-12\omega^2)}{4M^2\omega^3(M^2-4\omega^2)^2}$$

$$+\frac{(-\frac{34}{3}+\frac{6m^2}{M^2})\omega}{2M^2} + \frac{(-14+\frac{26m^2}{M^2})(-\frac{M^2}{2}+\omega^2)}{3M^2\omega^3}\},$$

$$\mathbf{I}_9 = A_0A_4\{-\frac{4(-m+\frac{4m^3}{M^2})}{3D(\hat{q})M} + \frac{-m+\frac{4m^3}{3M^2}}{M\omega^3} + \frac{2m}{M^3\omega} - \frac{2m\omega}{3M^3}$$

$$+\frac{(\frac{16m^3}{3M}-\frac{4mM}{3})(M^2-12\omega^2)}{4\omega(M^2-4\omega^2(M^2-4\omega^2)^2} - \frac{8m(-\frac{M^2}{2}+\omega^2)}{3M^3\omega^3}\},$$

$$\mathbf{I}_{10} = A_0A_5\{-\frac{7m^2}{3M^4} + \frac{\frac{2M^2}{3}+\frac{8m^2}{3}-\frac{2m^4}{3M^2}-M^2}{D(\hat{q})M^2} + \frac{(-8+\frac{2m^2}{M^2})\omega}{6M^2}$$

$$+\frac{\frac{2M^2}{3}-\frac{8m^2}{3}+\frac{2m^4}{3M^2}+\frac{4M^2}{3}}{4M^2\omega^3} + \frac{(-2m^2-m^2M^2)(M^2-12\omega^2)}{12M^2\omega^3(M^2-4\omega^2)^2}$$

$$+\frac{(8-\frac{2}{M^2}-\frac{7m^2}{M^2})(-\frac{M^2}{2}+\omega^2)}{3M^2\omega^3}\},$$

$$\mathbf{I}_{11}=A_1A_2\{\frac{-\frac{2m^2}{3}+\frac{8m^4}{3M^2}}{D(\hat{q})M^2}+\frac{\frac{2m^2}{3}-2M^2}{4M^2\omega^3}+\frac{\frac{10}{3}-\frac{2m^2}{M^2}}{2M^2\omega}+\frac{\frac{2}{3}+\frac{m^2}{M^2}}{2M^2\omega}$$
$$+\frac{5m^2(M^2-12\omega^2)}{12\omega^3(M^2-4\omega^2)^2}+\frac{(-4+\frac{m^2}{M^2})(-\frac{M^2}{2}+\omega^2)}{M^2\omega^3}\},$$
$$\mathbf{I}_{12}=A_1A_3\{\frac{-\frac{46m}{3}+\frac{16m^3}{3M^2}-8\frac{m^3}{M^2}}{4M\omega^3}+\frac{28m-\frac{16m^3}{3M^2}+8\frac{m^3}{3M^2}}{2M\omega}$$
$$+\frac{-34m-\frac{4m^3}{M^2}}{6M^3\omega}+\frac{(-\frac{28m}{3M}-\frac{8m^2}{3M}-2mM)(M^2-12w^2)}{4\omega^3(M^2-4\omega^2)^2}$$
$$+\frac{4m}{3M^5\omega}+\frac{(-26m-\frac{4m}{M^2})(-\frac{M^2}{2}+\omega^2)}{3M^3\omega^3}\},$$
$$\mathbf{I}_{13}=A_1A_4\{-\frac{5m^2(-\frac{M^2}{2}+\omega^2)}{3M^2\omega^3}+\frac{-\frac{10}{3}+\frac{2m^2}{M^2}}{2M^2\omega}$$
$$+\frac{\frac{11m^2}{3}+\frac{2M^2}{3}}{2M^2\omega}+\frac{\frac{4}{3}-\frac{m^4}{M^4}}{2M^2\omega}+\frac{(-\frac{8m^2}{3}+\frac{2M^2}{3})(M^2-12\omega^2)}{4w^3(M^2-4\omega^2)^2}$$
$$+\frac{(2-\frac{m^2}{M^2})(-\frac{M^2}{2}+\omega^2)}{M^2\omega^3}\},$$
$$\mathbf{I}_{14}=A_1A_5\{-\frac{m}{3M^3}+\frac{\frac{4m}{3}+\frac{2m^3}{M^2}}{D(\hat{q})M}+\frac{\frac{2m}{3}-\frac{2m^3}{M^2}}{4M\omega^2}+\frac{m}{6M^3\omega}$$
$$+\frac{(-\frac{2m^2}{M}+mM)(M^2-12\omega^2)}{4\omega^3(M^2-4\omega^2)^2}\},$$
$$\mathbf{I}_{15}=A_2A_3\{\frac{-5m-\frac{25m^3}{M^2}}{3D(\hat{q})M}+\frac{-19m+\frac{25m^3}{M^2}}{12M\omega^3}+\frac{\frac{11m}{3}+\frac{3m^3}{M^2}}{2M^3\omega}$$
$$+\frac{\frac{7m}{3}-\frac{3m^2}{2M^2}}{2M^3\omega}+\frac{13m^3(M^2-12\omega^2)}{8M\omega^3(M^2-4\omega^2)^2}$$
$$+\frac{(-6m+\frac{m}{3M^2})(-\frac{M^2}{2}+\omega^2)}{M^2\omega^3}\},$$
$$\mathbf{I}_{16}=A_2A_4\{\frac{-\frac{7m^2}{2}-\frac{8m^2}{3}+\frac{m^4}{3M^2}+\frac{4M^2}{3}-8\frac{m^6}{3M^2}}{2M^2\omega}$$
$$+\frac{(2+\frac{5m^2}{M^2}-\frac{8m^4}{3M^2}+\frac{8m^6}{3M^2}+\frac{2M^2}{3})(M^2-12\omega^2)}{4M^2\omega^3(M^2-4\omega^2)^2}$$
$$+\frac{1+\frac{4m^4}{M^4}-\frac{3m^3}{2M^3}}{2M^3\omega}+\frac{(\frac{10}{3}-\frac{7m^2}{2M^2}-\frac{4m^4}{3M^2})(-\frac{M^2}{2}+\omega^3)}{M^2\omega^3}$$
$$+\frac{(-8m^2-\frac{8m^4}{3M^2}+\frac{8m^6}{3M^2}+\frac{8M^2}{3})(-\frac{M^2}{2}+\omega^2)}{M^2\omega^3}\},$$

$$
\begin{aligned}
\mathbf{I}_{17} = A_2A_5\{\frac{3m}{8M^3\omega} - \frac{7m^3}{6D(\hat{q})M^3} + \frac{m}{6D(\hat{q})M} - \frac{3m^6M}{8D(\hat{q})} \\
+\frac{3m^4M^3}{8D(\hat{q})} + \frac{3m^6M}{32\omega^3} - \frac{7m^3}{6M^5\omega} \\
+\frac{(\frac{5m^3}{6M} - \frac{mM}{4} + \frac{3m^6M^3}{16})(M^2-12\omega^2)}{4\omega^3(M^2-4\omega^2)^2} \\
+\frac{7m^3}{24M^3\omega^3} - \frac{m}{4M\omega^3} + \frac{7m^3(-\frac{M^2}{2}+\omega^2)}{24M^5\omega^3} - \frac{m^3(-\frac{M^2}{2}+\omega^2)}{16M^3\omega^3}\}, \\
\mathbf{I}_{18} = A_3A_4\{\frac{m^3}{M^5\omega} - \frac{m^3(-\frac{M^2}{2}+\omega^2)}{2M^5\omega^3(M^2-4\omega^2)^2}\}, \\
\mathbf{I}_{19} = A_3A_5\{\frac{1}{D(\hat{q})M^2}(2M^2 + \frac{22m^2}{3} + \frac{4m^2}{3} + \frac{2m^4}{3M^2} - \frac{10m^4}{M^2} \\
+\frac{16m^6}{3M^4} + \frac{2M^2}{3} - \frac{4m^4M^2}{3}) + \frac{1}{4M^2\omega^3}(-\frac{4m^2}{3} + \frac{28m^2}{3} \\
-\frac{2m^4}{3M^2} + \frac{28m^4}{3M^2} + \frac{2m^4}{3M^2} - \frac{16m^6}{3M^4} + 2M^2 - \frac{8m^4}{3M^2}) \\
+\frac{-\frac{14}{3} + \frac{2m^2}{3M^2} + \frac{2m^2}{M^2} + \frac{26m^2}{3M^2}}{2M^2\omega} + \frac{\frac{4M^2}{3} - \frac{8m^2}{3} + \frac{6m^4}{M^2} - \frac{14m^4}{M^2} - \frac{2m^6}{3M^4}}{2M^2\omega} \\
+\frac{1}{4M^2\omega^3(M^2-4\omega^2)^2}(-2m^2 - \frac{10m^2}{3} - \frac{16m^6}{3M^4} + \frac{2m^4}{3M^2} \\
-\frac{8m^4}{3M^2} + \frac{2M^2}{3} - \frac{2m^6}{3M^4})(M^2-12\omega^2) \\
+\frac{1}{M^2w^3}(\frac{2}{3} - \frac{10m^2}{3M^2} - \frac{8m^2}{M^2} + \frac{16m^2}{3M^2} + \frac{m^3}{M^3} + \frac{16m^4}{3M^4})(-\frac{M^2}{\omega^2})\}, \\
\mathbf{I}_{20} = A_4A_5\{\frac{-\frac{4m}{3} + \frac{8m^3}{M^2}}{D(\hat{q})M} + \frac{\frac{4m}{3} - \frac{8m^3}{M^2}}{4M\omega^3} + \frac{-\frac{16m}{3} + \frac{8m^3}{M^2}}{2M^3\omega} \\
+\frac{(\frac{16m}{3} - \frac{8m^3}{M^2})(-\frac{M^2}{2}+\omega^2)}{M^3\omega^3}\}.
\end{aligned}
$$

We have thus evaluated the general expressions for f_v and N_v in the framework of BSE under CIA, with Dirac structure introduced in the $Hq\bar{q}$ vertex function besides $i\gamma.\varepsilon$. We see that so far the results are independent of any model for $\phi(\hat{q})$. However, for calculating the numerical values of these decay constants one needs to know the constant coefficients $A_0, A_1...A_5$. In this frame work, the relative parameters($\frac{A_1}{A_0}, \frac{A_3}{A_2}, \frac{A_4}{A_2}, ...$) are free parameters with out any further knowledge the meson structure. In ref.[2], only the leading order covariants O($\frac{1}{M^0}$) along with their respective coefficients (A_0 and A_1). They vary the parameters $\frac{A_1}{A_0}$ at the lowest order to see the effects of introducing $\frac{(\gamma.\varepsilon)(\gamma.P)}{M}$ on the value of f_v. They showed that the values of f_v under CIA improved when Dirac structure $\frac{(\gamma.\varepsilon)(\gamma.P)}{M}$ was introduced in the vertex function $\Gamma(\hat{q})$ in comparison to the calculated value with only $i\gamma.\varepsilon$. They observed an improvement in the value of f_v for the lightest vector mesons ρ and ω. To get an observable improvement on f_v of the heavier vector mesons, we include the next-to-leading order terms with O($\frac{1}{M^1}$) in the vertex function $\Gamma(\hat{q})$. This is what we expect from incorporation of NLO covariants O($\frac{1}{M^1}$) in the Hadron-quark vertex. Further discussion is relegated to next chapter.

Chapter 5

Summery and Conclusion

The electromagnetic interaction between fermions and photons is described by quantum electrodynamics (QED) which is based on the abelian gauge group U(1). The locally invariant full QED Lagrangian contains the fermions-photon interaction part as $ie\hat{\psi}\gamma_\mu\psi A_\mu$ which shows the coupling of matter field and gauge field with e playing the role of a coupling constant. This coupling confirms conservation of total charge in the universe. QCD is based on analogy of the idea in QED, but with the abelian U(1) gauge group replaced by the non-abelian SU(3) group of phase transformations on the quark color field. We have seen also the strong force between quarks and gluons can be explained by QCD based on the group SU(3).

Quarks can carry one of three possible charges known as "color",Red, Blue and green (this has nothing to do with color in the macroscopic world!); hence, in QCD the strong color force is mediated by particles called gluons, just as photons mediate the electromagnetic force. Unlike photons, though, gluons themselves carry a color charge and, therefore, interact with one another (in addition to quark-gluon interaction). This gluon-self interaction makes QCD a nonlinear theory.

As we know the decays $V \rightarrow e^+e^-$ proceeds through the coupling of $q\bar{q}$ loop to a vector current. The $Hq\bar{q}$ vertex is non-perturbative and an extended vertex through which the non-perturbative aspects of QCD enter into the calculation. The 4-D form of $Hq\bar{q}$ vertex $\Gamma(\hat{q})$ is derived from a QCD motivated Bethe-salpeter equation frame work. The structure of V-meson vertex contains eight Dirac covariants which have been arranged into leading and sub leading covariants on the basis of power counting rule proposed recently [2,3]. According to this result, the leading order (LO) covariants ($\sim O(\frac{1}{M^0})$ such as $i\gamma.\varepsilon$, $(\gamma.\varepsilon)(\gamma.P)\frac{1}{M}$) are expected to give maximum contribution to V- meson observables followed by next-to-leading order (NLO) ($\sim O(\frac{1}{M^1})$) covariants and then followed by next-to-next-to-leading order (NNLO) covariants and so on. The calculation done using this scheme recently on P-meson sector [14] confirmed this power counting rule. This rule in turn gives us the relative importance of various covariants in hadronic calculations at quark level.

With this aim in mind, we worked out the decay constants of equal mass V-mesons using NLO covariants ($[q.\varepsilon-(\gamma.\varepsilon)(\gamma.q)]\frac{A_2}{M}$, $[(\gamma.\varepsilon)(\gamma.P)(\gamma.q)-(\gamma.\varepsilon)(\gamma.q)(\gamma.P)+2(q.\varepsilon)(\gamma.P)]i\frac{A_3}{M^2}$, $(q.\varepsilon)\frac{A_4}{M}$, $i(q.\varepsilon)(\gamma.P)\frac{A_5}{M^2}$) along with LO covariants such as $i\gamma.\varepsilon$ and $(\gamma.\varepsilon)(\gamma.P)\frac{1}{M}$) studied earlier in [2]. The earlier results on LO covariants were close to experiment for lighter V-mesons such as $f_\rho(exp) = 220.9 \pm 1.7$, and $f_\omega(exp) = 194.6 \pm 3.2$. But were no so close to experimental data for heavier V-mesons like $f_\phi(exp) = 228 \pm 2.87$. By this work we do hope that the results for heavier mesons would also substantially improve and thus further validating the power counting rule. Moreover it improves our understanding of the V-meson structure.

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