ELECTRICAL CONDUCTIVITY AND DRIFT MOBILITY IN HEAVILY DOPED n-TYPE SILICON

WITH MIXED SCATTERING

A thesis Submitted to the
School of Graduate studies
Addis Ababa University

In partial fulfillment of the
Requirement for the Degree of
Master of Science in Physics

By

Girma Gizaw

June 2003
Addis Ababa

ADDIS ABABA UNIVERSITY

SCHOOL OF GRADUATE STUDIES

ELECTRICAL CONDUCTIVITY AND DRIFT MOBILITY
IN HEAVILY DOPED n-TYPE SILICON WITH MIXED SCATTERING

By

Girma Gizaw

Department of Physics

Faculty of Science

APPROVED BY THE EXAMINATION COMMITTEE

Name		Signature
1.Prof.S .K. Sharma, Advisor		……………………………
2.Prof. P Singh	Examiner	…………………………….
3.Dr. Tesgera Bedasa	Examiner	……………………………

DECLARATION

I declare that the thesis is my original work, and has not been presented for a degree in any other university and that all sources of materials used for this thesis have been duly acknowledged.

Name: Girma Gizaw

Signature ______________

Place and date of submission: Addis Ababa University

Department of Physics

June, 2003

This thesis has been submitted for examination with my approval as university advisor.

Name: Prof. S. K. Sharma

Signature ______________

Date ___________________

Acknowledgement

My first and foremost heartfelt gratitude should go to my advisor and instructor Prof. S. K. Sharma for his keen encouragements and invaluable guidance throughout the work.

My deep appreciation should also go to Dr. Fesseha Kassahun and Dr. Mulugeta Bekele
for providing me computers and office as a working station without which this work would have been impossible.

.

I would also like to express my thank to the A.A.U physics department that offered me a two years course with greatest encouragements.

Last but not the least, I would like to thank the regional Education Bureau at Awassa who sponsored me throughout my stay in the University.

Table Of Content

List of Tables.

List of figures

Abstract

In this thesis I have calculated the electrical conductivity and drift mobility in heavily doped n-type silicon in the presence of band tail effects considering mixed scattering of ionized impurities and acoustic phonons.

The results of the calculation show that the electrical conductivity can be as much as 58% lower than the value obtained by using parabolic density of states and the drift mobility in heavily doped n-type silicon on the other hand is 40% lower than the value calculated using parabolic density of states.

1. Introduction

It is well known that in heavily doped semiconductors, the density of states are no longer parabolic as the densities of states function in such semiconductors extend from above the conduction band to well into the energy gap. Therefore, any theory of transport coefficient of heavily doped semiconductor must incorporate the density of states with band tails.

Sharma [1] was the first to calculate D/μ, Diffusion mobility ratio in heavily doped n-type silicon. His calculations show that by incorporating the appropriate density of states function, the ratio D/μ can change by as much as 40%. Yitagesu and et al. [2], have calculated various galvanomagnetic transport coefficients viz. drift mobility, Hall coefficient and coefficient of magneto resistance in heavily doped n-type silicon by incorporating the effect of band tails and have also shown that incorporating the effects of band tails can significantly change the calculations of transport coefficients. However, these authors have considered only the electron scattering by ionized impurities and have not included the electron scattering by acoustic phonon.

It can be shown that for heavily doped, concentrations $\leq 10^{18}/\text{cm}^3$ it is important to include electron scattering by acoustic phonons in addition to that by ionized impurities. In this thesis I have calculated the drift mobility and electrical conductivity in heavily doped n-type silicon by taking into account the density of states with band tails for the case when electrons are scattered by ionized impurity scattering as well as acoustic phonon scattering. Our analysis is based on the assumption that (i), the electrons in the semiconductor are described by Fermi-Dirac distribution function. (ii),all the impurities are ionized.(iii),the electrons are scattered by ionized impurities as well as acoustic phonon scattering.

Following the work of Kane [3] we use simplified versions of Slotboom's density of states function [4] to derive the explicit expression for the drift mobility and electrical conductivity. We then use these expressions to calculate their numerical value for various concentrations by computing various integrals on a computer. For the sake of comparison we have also made the corresponding calculations when the effects of band tails is ignored. Our calculation shows that taking into account the effect of band tails can significantly change the values of drift mobility and electrical conductivity of heavily doped n-type silicon.

2.Density Of States In Heavily Doped n-Type Silicon.

The impurity atoms in a heavily doped semiconductor [5,6] are distributed all over the crystal in a random way and disturb the periodicity of the crystal and cause fluctuations in the local electrostatic potential which results in spatially dependent band edges and hence spatially dependent density of states. Thus, we describe the parabolic density of states and modified density of states in the following sub sections

2.1.The Parabolic Density of states.

For an electron gas having the parabolic density of states, the number of states in the ranges E to E + dE is given by [2-4]: -

$$dn = \frac{2^{3/2} 4\pi m_n^{3/2} E^{1/2} dE}{h^3} \qquad \textbf{(2.1.1)} \text{ or}$$

$$\frac{dn}{dE} = \rho(E) = \frac{2^{3/2} 4\pi m_n^{3/2} E^{1/2}}{h^3} \qquad \textbf{(2.1.2)}$$

where,m_n is the density of states effective mass, , $E = \frac{1}{2} m_n u^2$ and others have their usual representations.

The density of state function given by (2.1.2) doesn't incorporate the effects of band tails. Since the impurity distribution in an extrinsic semiconductor is not uniform, the statistical average over the entire lattice of the density of the states function shows "tailing"[5] into the energy gap. Thus, the bands are no longer parabolic. And therefore, as shown in the next section the parabolic density function ρ(E) is replaced by $y(z)$ in accordance with Kan's [4] model.

2.2. Formation Of The Band Tails.

A semiconductor is considered heavily doped, when the impurity band associated with the doped impurity merges with either the conduction or valence band [7]. In heavily doped semiconductor the presence of impurities causes the variation of electrostatic potential [8] in the lattice, which results in the fluctuation of energy in electronic states in both the conduction and valence bands. Hence, all states are affected in the similar manner so that the local energy gap remains unaltered.

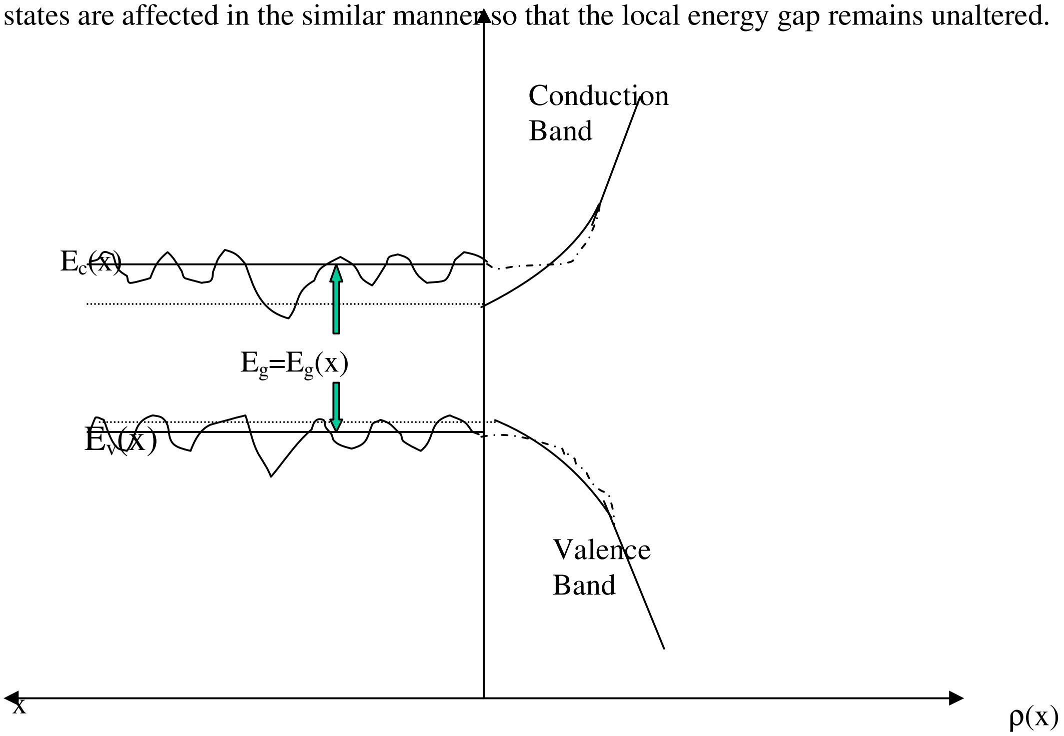

Figure2. 1 Formation of Band Tails due to fluctuation of the electrostatic potential in heavily doped n-type silicon vis-à-vis, the local conduction and valence band edges $E_c(x)$ and $E_v(x)$.

The local energy gap E_g is spatially constant and unaltered by the fluctuation, but the macroscopic or electrical energy gap is effectively reduced and there is an effective change in the macroscopic energy gap because the statistically averaged density of each quantum state with particular energy is different from that of intrinsic silicon. Spatial variation in local donor concentration causes spatial fluctuation in $E_c(x)$ and $E_v(x)$.

In a local region having as higher density of ionized donors than the average, the local electron potential energy is lower than its average value and hence, $E_c(x)$ is lower as shown in figure2.1. Thus, the macroscopic density of states shows tail states below the fundamental conduction band. In a similar manner the local electron potential energy is higher than the average and hence $E_v(x)$ is higher.

Hence, the average macroscopic density of states shows the tail states below the parabolic band and above the valence band.

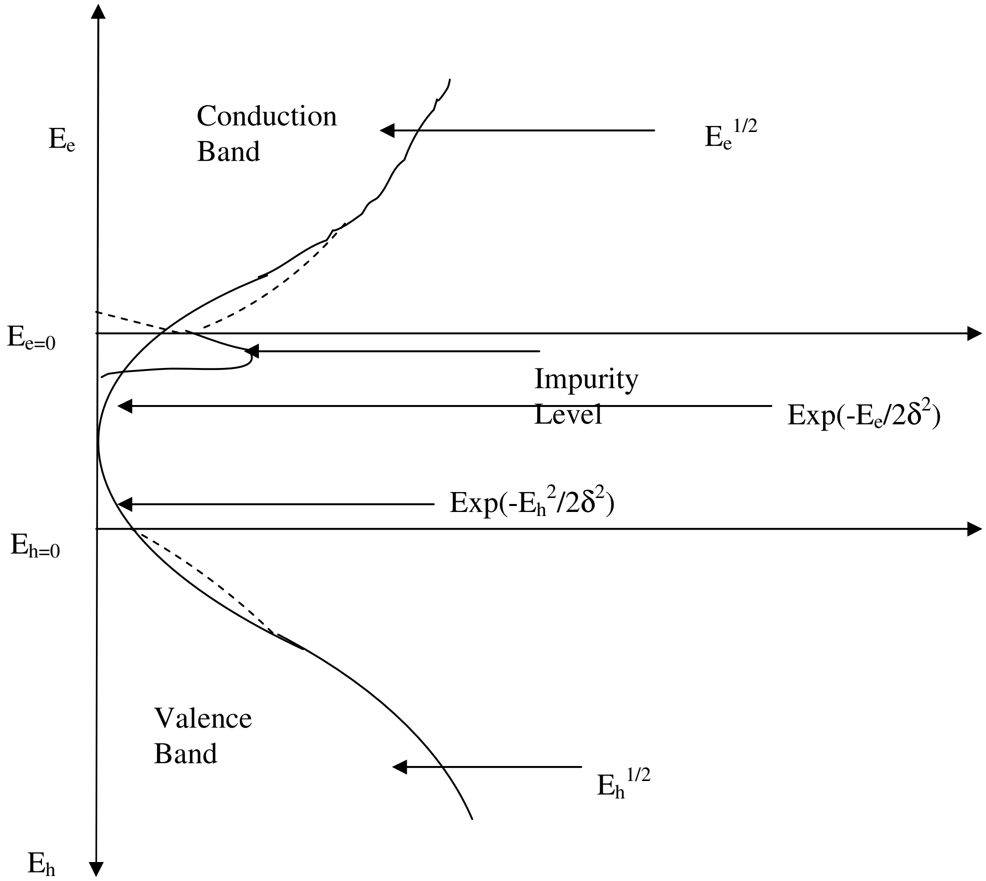

Figure. 2 .2. Superimposed quantum density of states for heavily doped n-type Silicon with conduction and valence band tails.

2.2.1. Density of States having Band Tails.

We have seen that the density of state function given by equation (2.1.2) doesn't include the effect of band tails. Kane [4] has shown that if the variation in local electrostatic potential is sufficiently slow, the average density of states function for conduction band is expressed as: -

$$\rho(z) = \frac{m_n{}^{3/2} 2^{3/4} \delta^{1/2}}{\pi^2 \hbar^3} y(z) \qquad \textbf{(2.2.1)}$$

where we have replaced a new variable $y(z)$ in place of $z^{1/2}$ in equation (2.1.2). The variable $y(z)$ in equation (2.2.1) as forwarded by Kane is given by: -

$$y(z) = \frac{1}{\pi^{1/2}} \int_{-\infty}^{z} (z-\xi)^{1/2} Exp(-\xi) d\xi \qquad \textbf{(2.2.2)}$$

In expression (2.2.2) the variable Z implies the dimensionless energy expressed as: -

$$z = \frac{E}{\sqrt{2}\delta} \qquad \textbf{(2.2.3)}$$

where,δ in equation (2.2.3) is the standard deviation of a Gaussian distribution function for impurity potential energy which is expressed as: -

$$\delta = \left[\frac{2\pi N_d \lambda e^4}{16\pi^2 \varepsilon_0{}^2 k^2} \right]^{1/2} \qquad \textbf{(2.2.2a)}$$

λ is the Thomas-Fermi screening length [4] and k is the dielectric constant of the specimen.

The density of states function given by (2.2.2) is so complicated that one cannot use it to make any useful calculations. To avoid these complications, Slotboom [3] has suggested the following approximation for $y(z)$:

$$y(z) \cong z^{1/2}\left[1 - \frac{1}{16z^2}\right]. \text{ For } z > 0.601, \text{visa-vis}, E > 0.85 \qquad \textbf{(2.2.4)}$$

and

$$y(z) \cong \frac{1}{2\pi^{1/2}} Exp(-z^2)\{0.319 + 0.906 Exp(2z)\}. \text{For } z \le 0.601 \qquad \textbf{(2.2.5)}$$

In heavily doped semiconductor one must integrate equation (2.2.4) and (2.2.5) to obtain the electron concentration n. In our analysis we assumed that, the total mobile electron concentration in conduction band is equal to the donor concentration N_d [8] and that of the number of intrinsic conduction electrons is negligible due to large number of ionized impurities at room temperature. Thus, we have the following expression for electron concentration n.

$$N_d \cong n = \int_0^{+\infty} dn. \tag{2.2.6}$$

$$= \frac{m_n^{3/2} 2^{3/4}}{\pi^2 \hbar^3} \int_0^{\infty} E^{1/2} f_n^{\,0}(E) dE. \tag{2.2.7}$$

$$n = \frac{m_n^{3/2} 2^{5/4} \delta^{3/2}}{\pi^2 \hbar^3} \int_{-\infty}^{+\infty} f_n^{\,0} y(z) dz \tag{2.2.8}$$

$$n = \frac{m_n^{3/2} 2^{5/4} \delta^{3/2}}{\pi^2 \hbar^3} \psi_0 \tag{2.2.9}$$

Where

$$\psi_0 = \frac{1}{2\pi^{1/2}} \int_{-\infty}^{0.601} Exp(-z^2) \left[\frac{0.319 + 0.906 Exp(2z)}{\left(1 + Exp\left\{ \frac{\sqrt{2}\delta}{k_B T} z - \eta \right\}\right)} \right] dz + \int_{0.601}^{+\infty} \frac{z^{1/2}\left(1 - \frac{1}{16z^2}\right)}{\left(1 + Exp\left\{ \frac{\sqrt{2}\delta}{k_B T} z - \eta \right\}\right)} dz \tag{2.2.10}$$

and η is dimensionless Fermi Energy $\frac{E_f}{k_B T}$.

It is more convenient to introduce normalized electron concentration n_n as:

$$n_n = \frac{n}{10^{24} / m^3} = \frac{m_n^{3/2} 2^{5/4} \delta^{3/2}}{10^{24} \pi^2 \hbar^3} \psi_0 \tag{2.2.11}$$

3.Electron Scattering Mechanisms

Among various scattering mechanisms, which are responsible for electrical resistivity of heavily doped silicon in the ranges of room temperature only, ionized impurity scattering and acoustic phonon scattering are important. Therefore, both scattering mechanisms are discussed below. It has been shown that [9] the presence of impurities in a crystal alters the electrostatic potential in the neighborhood and creates periodicity in the potential field with -in the crystal.

In heavily doped n-type silicon with electron concentration equal to or more than $10^{25}/m^3$, ionized impurity scattering is the only important scattering mechanism than the acoustic phonon scattering. However for $n \leq 10^{25}/m^3$, both ionized impurity and acoustic phonon scattering are important.

Thus, it will be appropriate to derive the Conwell-Wesskopf [10] expression for collision frequency due to ionized impurity scattering and weisskopf –Wigner [11] expression for collision frequency due to acoustic phonon scattering.

3.1. Ionized Impurity Scattering.

In the presence of a D.C electric field along x-direction, the Boltzmann equation is written as [12]

$$\frac{-eE_x}{m_n}\frac{\partial f_n}{\partial u_x} = \left[\frac{\partial f_n}{\partial t}\right]_{col.} = b - a \qquad \textbf{(3.1.1)}$$

where m_n is the electron effective mass for the stated direction of field, b is the number of electrons entering the volume element $d\tau$,a is the corresponding number of electrons leaving the volume element per unit time u_x is the velocity component along x-direction and $\left[\frac{\partial f_n}{\partial t}\right]_{col.}$ is the rate of change of the distribution function .

So as to get an expression for the rate of change of distribution function due to ionized impurity scattering, Conwell and weisskopf have made the following assumptions [10]:

(i). The mass of the scattering center is infinite.

(ii). Collision between an electron and ionized impurity atom is perfectly elastic, i.e. only the direction of an electron changes during collision.

(iii). Scattering of an electron by an ionized impurity atom can be treated as independent of all other ions, i.e. an electron gets scattered by a single ion at a time.

If the mean distance between the two nearest impurity ions is 2d,then electron gets scattered by an ionized impurity ion with in a distance d of that electron. Hence, the number of electrons per unit volume scattered per second into a solid angle, $d\Omega$ at polar angles $\theta^{'}$ and $\phi^{'}$ can be expressed as:

$$Nf_n(u,\theta,\phi)\delta(\theta,\theta^{'})ud\Omega \qquad \textbf{(3.1.2)}$$

where N is the number of electrons per unit volume, $Nf_n(u,\theta,\phi)d\Omega$ is the number of electrons per unit volume with in a solid angle and $\delta(\theta,\theta')$ is the Rutherford scattering cross-section given by [11]:

$$\delta(\theta,\theta^{'}) = \left[\frac{Ze^2}{8\pi\varepsilon_0\varepsilon_d m_n u^2}\right]^2 \frac{1}{Sin^4(\frac{\theta-\theta^{'}}{2})} \qquad \textbf{(3.1.3)}$$

The number of electrons leaving the volume element given in (3.1.2) can now be expressed as:

$$a = Nv\int_{\theta^{'}=0}^{\pi}\int_{\phi^{'}=0}^{2\pi} f_n(u,\theta,\phi)\delta(\theta^{'},\theta^{'})d\Omega \qquad \textbf{(3.1.4)}$$

Since in our assumption an electron gets scattered only when it comes within a distance d of an ion, we considered only those collisions in which electron gets scattered through minimum angle $\theta_{\min.}$. The limitation on an impact parameter β is equivalent to a limit on $\theta^{'}$ and can be expressed as:

$$\tan\left(\frac{\theta^{'}-\theta}{2}\right) = \frac{Ze^2}{\varepsilon_d m_n u^2\beta} \qquad \textbf{(3.1.5)}$$

where ε_d is the dielectric constant β is the impact parameter and Z is the atomic number. For largest impact parameter, $\beta_{\max.} = d$ we may have: -

$$(\theta' - \theta)_{\min} = 2\tan^{-1}\left(\frac{Ze^2}{\varepsilon_d m_n u^2 d}\right) \qquad \textbf{(3.1.6)}$$

where $(\theta' - \theta)_{\min.}$ gives the lower limit of integration over θ'. We can choose the axis in such a manner that $\theta = 0$, so that $(\theta' - \theta)_{\min.} = \theta'_{\min.}$ and thus expression given in equation (3.1.4) reduces to

$$a = Nu \int_{\theta'_{\min}}^{\pi} \int_{\phi'=0}^{2\pi} f_n(u,0,\phi)\delta(\theta')d\Omega \qquad \textbf{(3.1.7)}$$

In a similar manner replacing $f_n(u,\theta',\phi')$ in place of $f_n(u,\theta,\phi)$ in equation (3.1.4) we obtain an expression for the number of electrons entering the volume element b as

$$b = Nu \int_{\theta'_{\min}}^{\pi} \int_{\phi'=0}^{2\pi} f_n(u,\theta',\phi')\delta(\theta')d\Omega \qquad \textbf{(3.1.8)}$$

Thus, the total number of electrons transported, (b – a) can be expressed as

$$b - a = Nu \int_{\theta_{\min}'}^{\pi} \int_{\phi'=0}^{2\pi} \left[f_n(u,\theta'\phi') - f_n(u,0,\phi)\right]\delta(\theta')d\Omega \qquad \textbf{(3.1.9)}$$

Here, we have to note that, electrons are transported by collision or scattering interaction and thus

$$b - a = \left(\frac{\partial f_n}{\partial t}\right)_{col.} \quad . \qquad \textbf{(3.1.10)}$$

Following the Lorentz and other workers assumption, the distribution function f_n in the presence of external electric field **E** may be expanded as [1,12,13] where we have assumed a weak field

$$f_n = f_n^{\,0} - u_x \chi(u) \qquad \textbf{(3.1.11)}$$

$$f_n = f_n^{\,0} - u\cos\theta\chi(u) \qquad \textbf{(3.1.12)}$$

and f_n^0 is the isotropic part of the distribution function and $\chi(u)$ is anisotropic part of the distribution of electron velocity. Making use of equation (3.1.12) so as to expand $f_n(u,\theta',\phi')$ and $f_n(u,\theta,\phi)$, we then obtain

$$f_n(u,\theta',\phi') = f_n^{\,0} - u\cos\theta'\chi(u) \qquad \text{and}$$

$$f_n(u,0,\phi) = f_n^{\,0} - v\chi(u) \qquad \textbf{(3.1.13)}$$

$$f_n(u,\theta',\phi') - f_n(u,0,\phi) = u\chi(u)(1-\cos\theta') \qquad \textbf{(3.1.14)}$$

Substituting equation (3.1.14) in equation (3.1.9) we have the following expression.

$$b-a = Nu\int_{\theta_{\min}'}^{\pi}\int_{\phi'=0}^{2\pi}\left[f_n(u,\theta'\phi') - f_n(u,0,\phi)\right]\delta(\theta')d\Omega \qquad \textbf{(3.1.15)}$$

$$b-a = Nu\int_{\theta_{\min}'}^{\pi}\int_{\phi'=0}^{2\pi} u\chi(u)(1-Cos\theta')\left[\frac{Ze^2}{8\pi\varepsilon_0\varepsilon_d m_n u^2}\right]^2 \frac{\sin\theta' d\theta' d\phi'}{\sin^4(\frac{\theta'}{2})}\ . \qquad \textbf{(3.1.16)}$$

After integrating over ϕ' equation (3.1.15) reduces to:

$$b-a = 2\pi Nu^2\chi(u)\left(\frac{Ze^2}{8\pi\varepsilon_0\varepsilon_d m_n u^2}\right)^2 \int_{\theta'_{\min}}^{\pi} \frac{(1-\cos\theta')d(-\cos\theta')}{\sin^4\left(\frac{\theta'}{2}\right)} \qquad \textbf{(3.1.17)}$$

To evaluate the integral in (3.1.17) we make a change of variables from θ' to $x = 1-\cos\theta'$ and from this follows: -

$$b-a = 8\pi Nu^2\chi(u)\left(\frac{Ze^2}{8\pi\varepsilon_0\varepsilon_d m_n u^2}\right)^2 \int_{x_{\min}}^{2} \frac{xdx}{x^2} \qquad \textbf{(3.1.18)}$$

$$= 8\pi Nu^2\chi(u)\left(\frac{Ze^2}{8\pi\varepsilon_0\varepsilon_d m_n u^2}\right)^2 \ln\frac{2}{x_{\min.}} \qquad \textbf{(3.1.19)}$$

where $x_{\min} = 1 - Cos\theta' = 2\left(1+\frac{1}{\tan^2\frac{\theta_{\min}}{2}}\right)^{-1} = 2\left(1+\frac{\varepsilon_d{}^2 m_n{}^2 u^4 d^2}{Z^2 e^4}\right)^{-1}.$

Inserting the expression for $x_{\min.}$ in equation (3.1.19) and after making some rearrangement , we get the following relation for (b-a): -

$$b - a = N\chi(u)\frac{Z^2 e^4}{2\pi\varepsilon_o{}^2 m_e{}^2 u^2}\ln\left(1+\frac{\varepsilon_d{}^2 m_n{}^2 u^4 d^2}{Z^2 e^4}\right). \qquad \textbf{(3.1.20)}$$

Substituting the above relation for (b-a) in equation (3.1.1) and linearizing the resulting equation gives: -

$$\left(\frac{\partial f_n}{\partial t}\right)_{col.} = N\chi(u)\frac{Z^2 e^4}{2\pi\varepsilon_o{}^2 m_n{}^2 u^2}\ln\left(1+\frac{\varepsilon_d{}^2 m_n{}^2 u^4 d^2}{Z^2 e^4}\right). \qquad \textbf{(3.1.21)}$$

In the relaxation time ansatz, the collision term is expressed as: -

$$\left(\frac{\partial f_n}{\partial t}\right)_{col.} = -\nu_I(f_n - f_n{}^0) = \nu_I u\chi(u) \qquad \textbf{(3.1.22)}$$

Comparing equation (3.2.21) and (3.1.22) we obtain the following expression for collision frequency due to ionized impurity scattering.

$$\nu_I = \frac{NZ^2 e^4}{2\pi\varepsilon_o{}^2 m_n{}^2 u^3}\ln\left(1+\frac{\varepsilon_d{}^2 m_n{}^2 u^4 d^2}{Z^2 e^4}\right). \qquad \textbf{(3.1.23)}$$

Equation (3.1.23) is called the Conwell-weisskopf formula for collision frequency due to ionized impurity scattering. Equation (3.1.23) can be written in a more compact form as:

$$\nu_I = \nu_{0i} \in^{-3/2} \qquad \textbf{(3.1.24)}$$

Where $\nu_{0i} = \frac{NZ^2 e^4}{2^{5/2}\pi m_n{}^{1/2}\varepsilon_0 (K_B T)^{3/2}}\ln\left(1+\frac{\varepsilon_d{}^2 m_n{}^2 u^4 d^2}{Z^2 e^4}\right).$ and $\in = \frac{m_{.n} u^2}{2K_B T}$ is the dimensionless energy of electron

3.2.Acoustic Phonon Scattering.

The atoms in the crystal are vibrating in a random fashion [13]. Thermal vibrations of atoms distort the periodic potential field in which the electron moves. At low temperatures, the most important lattice scattering process is the scattering of electrons by acoustic mode of lattice vibrations or phonons because the thermal energy required to excite optical mode of lattice vibration is not available at lower temperatures. Therefore, the effect of transverse mode is negligible and in this thesis we consider only the acoustic phonon scattering of electrons by longitudinal mode vibrations.

The quantum mechanical derivation of collision is based on Weisskopf and Wigner [10] treatment of radiation broadening. Weisskopf and Wigner have calculated that the mean free path λ_n between every collision is given by

$$\lambda_n = \frac{\pi\hbar^4 C_{ll}}{{E_d}^2 K_B T {m_n}^2} \qquad \textbf{(3.2.1)}$$

where,C_{ll} is the average longitudinal elastic constant and E_d is the deformation potential constant that represents the shift of the conduction band edge per unit dilatational strain. If we introduce an average velocity of phonon [14] as

$$u_s = \left(\frac{C_{ll}}{\rho'}\right)^{1/2}$$

$$C_{ll} = {u_s}^2 \rho'. \qquad \textbf{(3.2.2)}$$

where ρ' is the density of the specimen.

For the case where the mean free path λ_n is independent of velocity u then the mean free time τ_A may be expressed as

$$\tau_A = \frac{\lambda_n}{u} \qquad \textbf{(3.2.3)}$$

Using equation (3.2.1), and equation (3.2.3) expression for τ_A is given by

$$\tau_A = \frac{\pi\hbar^4 {u_s}^2 \rho'}{{E_d}^2 K_B T {m_n}^2 u} \qquad \textbf{(3.2.4)}$$

But, the collision frequency due to acoustic phonon scattering is given as

$$\nu_A=1/\tau_A. \qquad \textbf{(3.2.5)}$$

Hence, the general expression for collision frequency ν_A becomes

$$\nu_A = \frac{{E_d}^2 K_B T {m_n}^2 u}{\pi\hbar^4 {u_s}^2 \rho'} \quad . \qquad \textbf{(3.2.6)}$$

We can put expression (3.2.6) in more compact form as

$$\nu_A = \nu_{0A}\left(\frac{E}{K_B T}\right)^{1/2}$$

$$\nu_A = \nu_{0A} \in^{1/2} \qquad \textbf{(3.2.7)}$$

where $\nu_{0A} = \frac{\sqrt{2}}{{E_d}^2 \pi\hbar^4 \rho' {u_s}^2}\left(m_m k_B T\right)^{3/2}$ and $E = \frac{1}{2} m_n u^2$.

3.3.Mixed scattering By Ionized impurities And Acoustic Phonons.

For two independent scattering mechanisms to operate simultaneously thermalzing the electron energy distribution function there will be two collision frequencies ν_I and ν_A. Therefore, for simultaneous effects, the two collision frequencies should be added and the total collision frequency of electron is: -

$$\nu = \nu_I + \nu_A \qquad \textbf{(3.2.8)}$$

In the next chapter we shall use the above given expression of the collision frequency in our derivation of the electrical conductivity and drift mobility of heavily doped n-type silicon.

4.Derivation Of Electrical Conductivity and Drift Mobility With Mixed Scattering.

The simultaneous electron scattering due to acoustic phonons and ionized impurities significantly affects the electrical conductivity and drift mobility [15]. In order to calculate electrical conductivity and drift mobility we need to calculate anisotropic part of electron velocity in the presence of the external field and the collision process, which is obtained by solving the Boltzmann, transport equation.

In this section we shall drive an expression for electrical conductivity and drift mobility in heavily doped silicon by solving the Boltzmann transport equation in the presence of an external D.C electric field when the electron velocity distribution function is in a steady state condition.

4.1. The Boltzmann Transport Equation.

Systems in thermal equilibrium are expressed using either Fermi-Dirac or Boltzmann distribution function [8]. But, the states of a system, which are not in equilibrium, are expressed by taking account of some small perturbations. Naturally, the non-equilibrium distribution function must approach the appropriate equilibrium distribution.

If $f_n(x, y, z, u_x, u_y, u_z)$ is the distribution function [12] (which expresses the number of particles per unit energy/momentum range), then the change df_n which occurs during time dt due to a force field acting on the particle is given by Boltzmann equation.

The Boltzmann transport equation in the presence of external electric field along $\vec{E} = E_x\hat{\imath}$, in the absence of magnetic field and with in a gradient independent medium, may be put in the form of: -

$$\frac{-eE_x}{m_n}\frac{\partial f_n}{\partial u_x} = -\nu(f_n - f_n{}^o) \qquad \textbf{(4.1.1)}$$

where f_n is the total distribution function discussed in the preceding section (3.1.11) and the gradient term vanishes due to temperature independent medium.Therefore,we may expanded the total distribution function as

$$f_n = f_n{}^0 + \left(\frac{u_x}{u}\right) f_x$$

$f_n{}^o$ is the Fermi-Dirac distribution function and ν is the total collision frequency due to various processes and $\left(\frac{u_x}{u}\right) f_x$ is anisotropic part of the distribution function.

Neglecting second order and higher terms on the left hand side of expanded total distribution function f_n and equating the linear terms in u_x , we obtain the following expression for anisotropic part of distribution function f_x as

$$\frac{-eE_x}{m_n}\frac{\partial f_n}{\partial u_x} \approx \frac{-eE_x}{m_n}\frac{\partial f_n{}^0}{\partial u}\frac{u_x}{u} \qquad \textbf{(4.1.2)}$$

This gives: -

$$f_x = \frac{1}{\nu}\left(\frac{eE_x}{m_n}\frac{\partial f_n{}^o}{\partial u}\right) \qquad \textbf{(4.1.3)}$$

4.2 Electrical Conductivity and Drift Mobility for Parabolic Density of States.

4.2.1. The electrical current density.

The general expression for electrical current density **J** in n-type semiconductors is given by [15]:

$$\mathrm{J} = \frac{-2m_n{}^3 e}{h^3}\int u f_n d^3u \qquad \mathbf{(4.2.1)}$$

Since we have applied the electric field along the x-direction, only the x-component of electric current density is finite and is

$$J_x = \frac{-2m_n{}^3 e}{h^3}\int u_x f_n d^3u \qquad \mathbf{(4.2.2)}$$

Inserting the expansions for f_n from equation (3.1.11) in the above expression for the electrical current density we have

$$J_x = \frac{-2m_n{}^3 e}{h^3}\int u_x (f_n{}^0 + \frac{u_x}{u} f_x) du_x du_y du_z . \qquad \mathbf{(4.2.3)}$$

Since, $f_n{}^0$ term does not contribute to the current density the expression for current density reduces to

$$J_x = \frac{-2m_n{}^3 e}{h^3}\int \frac{u_x{}^2}{u} f_x du_x du_y du_z \qquad \mathbf{(4.2.4)}$$

Making transformation from Cartesian to spherical coordinate in velocity space gives

$$J_x = \frac{-4\pi m_n{}^3 e}{h^3}\int_0^\infty \int_{+1}^{-1} u^3 du f_x Cos^2\theta d(-Cos\theta) \qquad \mathbf{(4.2.4a)}$$

After integrating over θ, we obtain

$$J_x = \frac{-8\pi m_n{}^3 e}{3\pi h^3}\int_0^\infty u^3 f_x du \qquad \mathbf{(4.2.5)}$$

Substituting for f_x from equation (4.1.3) in equation (4.2.5) for the current density gives

$$J_x = \frac{-m_n^{3} e^2}{3\pi^2\hbar^3}\int_0^\infty \frac{u^3}{\nu}\frac{\partial f_n^{\,0}}{\partial u}E_x du\,. \qquad \textbf{(4.2.6)}$$

After changing the variables of integration from u to energy$\in$ where $\in\ = \frac{m_n u^2}{2K_B T}$ we obtain

$$J_x = \frac{-m_n^{2} e^2 u_0^{3} E_x}{3\pi^2\hbar^3}\int_0^\infty \frac{\in^{3/2}}{\nu}\frac{\partial f_n^{\,0}}{\partial \in}d\in \qquad \textbf{(4.2.7)}$$

where, $u_0 = \left(\frac{2K_B T}{m_n}\right)^{1/2}$.

4.2.2. The electrical conductivity for parabolic density of states.

The electrical conductivity σ is a material constant which connects the component of electrical current density $\mathbf{J_x}$ with the electrical field strength $\mathbf{E_x}$ is given by [15,16]: -

$$\mathbf{J_x = \sigma_{xx}\, E_x} \qquad \textbf{(4.2.8)}$$

Actually, the conductivity σ is not strictly constant but tit rather depends on different parameters like temperature and the impurity concentrations.

Since in the present case both electric field **E** and current density **J** are along x-direction only, the σ_{xx} component of electrical conductivity which is relevant is written as

$$\boldsymbol{\sigma_{xx}} = \mathbf{J_x/E_x} \qquad \textbf{(4.2.9)}$$

Substituting for J_x from equation (4.2.9) in equation (4.2.7) we have a relation for conductivity

$$\sigma_{xx} = \frac{J_x}{E_x} = \frac{-m_n^{2} e^2 u_0^{2}}{3\pi^2\hbar^3}\int_0^\infty \frac{\in^{3/2}}{\nu}\frac{\partial f_n^{\,0}}{\partial \in}d\in \qquad \textbf{(4.2 10)}$$

The collision frequency ν in equation (4.2.10) is the sum of collision frequencies due to electron scattering by ionized impurities and acoustic phonons.

Or

$$\nu = \nu_I + \nu_A$$

$$\nu = \nu_{0A} \in^{-3/2} \left(\in^2 + \frac{\nu_{0I}}{\nu_{0A}} \right) \tag{4.2.11}$$

Substituting equation (4.2.11) for ν in equation (4.2.10) we have an expression for electrical conductivity as

$$\sigma_{xx} = \frac{{m_n}^{1/2} e^2 (2K_B T)^{3/2}}{3\pi^2 \hbar^3 \nu_{0A}} \int_0^\infty \frac{\in^3}{\left(\in^2 + \frac{\nu_{0I}}{\nu_{0A}} \right)} \left(-\frac{\partial {f_n}^0}{\partial \in} \right) d\in \tag{4.2.12}$$

Now the electron concentration n is

$$n = \frac{8\pi {m_n}^3}{2h^3} \left(\frac{2K_B T}{m_n} \right)^{3/2} \int_0^\infty \in^{1/2} {f_n}^0 d\in \tag{2.2.9}$$

$$n = \frac{8\pi {m_n}^3}{2h^3} \left(\frac{2K_B T}{m_n} \right)^{3/2} F_{1/2}(\eta) \tag{4.2.13}$$

where $F_{1/2}(\eta)$ is the Fermi-Dirac integral of order 1/2[17].

Dividing equation (4.2.12) by $\sigma_0 = \frac{e^2 n}{m \nu_{0A}}$ equation (4.2.13) gives the following expression of a dimensionless normalized electrical conductivity σ_n as

$$\sigma_n = \frac{\sigma_{xx}}{\sigma_0}$$

$$\sigma_n = \frac{\sigma_{xx} m_n \nu_{0A}}{e^2 n} = \frac{2}{3} \left\{ \frac{\int_0^\infty \frac{\in^3}{\left(\in^2 + \frac{\nu_{0I}}{\nu_{0A}} \right)} \left(-\frac{\partial {f_n}^0}{\partial \in} \right) d\in}{\int_0^\infty \in^{1/2} {f_n}^0 d\in} \right\} \tag{4.2.14}$$

Equation (4.2.14) is an expression for normalized dimensionless electrical conductivity with mixed scattering of acoustic phonons and ionized impurities using parabolic density of states.

4.2.3.The drift mobility for parabolic density of states.

We define the mobility μ as [15]

$$\mu = \boldsymbol{\sigma_{xx}/ne}. \qquad \textbf{(4.2.15)}$$

where **n** is the electron concentration per unit volume and σ is the scalar electrical conductivity. From equation (4.2.14) we derived the electrical conductivity$\boldsymbol{\sigma_{xx}}$ as: -

$$\sigma_{xx} = \frac{2e^2 n}{3m_n \nu_{0A}} \left\{ \frac{\int_0^\infty \frac{\in^3}{\left(\in^2 + \frac{\nu_{0I}}{\nu_{0A}}\right)} \left(-\frac{\partial f_n{}^0}{\partial \in}\right) d\in}{\int_0^\infty \in^{1/2} f_n{}^0 d\in} \right\}$$

Using equation (4.2.12) for $\boldsymbol{\sigma_{xx}}$ substituting it in equation (4.2.15) gives the following expression for mobility as: -

$$\mu = \frac{\sigma_{xx}}{ne} = \frac{2e}{3m_n \nu_{0A}} \left\{ \frac{\int_0^\infty \frac{\in^3}{\left(\in^2 + \frac{\nu_{0I}}{\nu_{0A}}\right)} \left(-\frac{\partial f_n{}^0}{\partial \in}\right) d\in}{\int_0^\infty \in^{1/2} f_n{}^0 d\in} \right\} \qquad \textbf{(4.2.16)}$$

Equation (4.216) is electron drift mobility with mixed scattering for parabolic density of states.

4.3.Electrical Conductivity and Drift mobility for Modified Density Of States With Band Tails.

As mentioned in the preceding sections we can obtain expression for electrical conductivity and electron drift mobility based on modified density of states from the corresponding expressions of electrical conductivity and electron drift mobility i.e. equations (4.2.12) and (4.2.16) respectively derived by using parabolic density of states. To obtain the expression for conductivity and mobility based on the modified density of states we do the following: -

1.we change the variable of integration E in the expressions for conductivity and mobility, which were derived by using parabolic density of states in equation (4.2.14) and (4.2.16) to a new variable $z = \frac{E}{\sqrt{2}\delta}$ where δ is as defined by equation (2.2.3).

2.We extend the range of integration $z = 0$ to $z = \infty$ in equations (4.2.14) and (4.2.16) from $-\infty$ to $+\infty$.

3.We break the entire range of integration from $-\infty$ to $+\infty$ into two ranges, one from $z = -\infty$ to $z = 0.601$ and the other from $z = 0.601$ to $z = +\infty$. And then substitute:

(i). $z^{1/2}\left(1-\frac{1}{16z^2}\right)$ for $z^{1/2}$ in the range of integration from $0.601 \le z < +\infty$ and

(ii). $\frac{1}{2\pi^{1/2}}\left(0.319+0.906Exp[2z]\right)Exp[-z^2]$ for $z^{1/2}$ in the integration range $-\infty < z \le 0.601$ in the expression of conductivity and mobility derived by using parabolic density of states.

After implementing the changes mentioned in steps 1 to 3 we obtain the following expressions of electrical conductivity and mobility for n-type silicon, which are based on the density of states with band tails.

Thus, from equations (4.2.12) we obtain the following expressions for electrical conductivity which is valid for heavily doped n-type silicon:

$$\sigma_{xx} = \frac{2e^2 n}{3m_n \nu_{0A}} \left(\frac{\sqrt{2}\delta}{K_B T} \right)^{5/2} \frac{\psi_{5/2}}{\psi_0} \qquad \textbf{(4.3.1)}$$

where $\psi_{5/2}$ and ψ_0 are to be described below:

$$\psi_{5/2} = \frac{1}{2\pi^{1/2}} \int_{-\infty}^{0.601} \frac{|z|^{5/2}\left(0.319 + 0.906 Exp[2z]\right) Exp[\frac{\sqrt{2}\delta z}{K_B T} - \eta - z^2]}{\left(\left(\frac{\sqrt{2}\delta z}{K_B T} \right)^2 + \frac{\nu_{0I}}{\nu_{0A}} \right) \left(1 + Exp[\frac{\sqrt{2}\delta z}{K_B T} - \eta] \right)^2} dz$$

$$+ \int_{0.601}^{\infty} \frac{z^3 \left(1 - \frac{1}{16z^2} \right) Exp[\frac{\sqrt{2}\delta z}{K_B T} - \eta]}{\left(\left(\frac{\sqrt{2}\delta z}{K_B T} \right)^2 + \frac{\nu_{0I}}{\nu_{0A}} \right) \left(1 + Exp[\frac{\sqrt{2}\delta z}{K_B T} - \eta] \right)^2} dz \qquad \textbf{(4.3.2)}$$

And

$$\psi_0 = \frac{1}{2\pi^{1/2}} \int_{-\infty}^{0.601} \frac{\left(0.319 + 0.906 Exp[2z]\right) Exp[-z^2]}{\left(1 + Exp[\frac{\sqrt{2}\delta z}{K_B T} - \eta] \right)} dz$$

$$+ \int_{0.601}^{\infty} \frac{z^{1/2} \left(1 - \frac{1}{16z^2} \right)}{\left(1 + Exp[\frac{\sqrt{2}\delta z}{K_B T} - \eta] \right)} dz \qquad \textbf{(4.3.3)}$$

It is more convenient to express the electrical conductivity in the Normalized form

as: -

$$\sigma_n = \frac{\sigma_{xx}}{\frac{2}{3}\left(\frac{0.5227x10^{18}e^2}{m_n v_{0A}}\right)} = (n_n)^{49/25}\frac{\psi_{5/2}}{\psi_0} \qquad \textbf{(4.3.4)}$$

where n_n is a dimensionless normalized electron concentration $n/10^{19}$ /cm^3 and the numerical value 0.5227 is introduced for a room temperature T = 300K.

Similarly, after implementing steps 1 to 3 we obtain from equation (4.2.6 the following expression for drift mobility, which is valid for heavily doped n-type silicon having density of states with band, tails: -

$$\mu = \frac{\sigma_{xx}}{ne}$$

$$\mu = \frac{2e}{3m_n v_{0A}}\left(\frac{\sqrt{2}\delta}{K_B T}\right)^{5/2}\frac{\psi_{5/2}}{\psi_0} \qquad \textbf{(4.3.5)}$$

Therefore, dimensionless normalized electron drift mobility, which incorporates the effect of band tails, is given by: -

$$\mu_n = \frac{\mu}{\frac{2}{3}\left(\frac{0.5227e}{m_n v_{0A}}\right)} = (n_n)^{25/24}\frac{\psi_{5/2}}{\psi_0} \qquad \textbf{(4.3.6)}$$

Yitagesu and et.al [19] have calculated the electrical conductivity and electron drift mobility when the sole mechanism of scattering is ionized impurity scattering having density of states with band tails.

Consequently, for the dominant ionized impurity scattering, the simplified forms of the normalized dimensionless electrical conductivity and drift mobility may be written as: -

$$\sigma_n = 2.1696(n_n)^{49/25}\frac{\psi_{5/2}}{\psi_0} \quad \text{And}$$

$$\mu_n = 2.1696(n_n)^{25/24}\frac{\psi_{5/2}}{\psi_0}$$

5.Numerical calculations of electrical conductivity and drift mobility.

For numerical calculations of the electrical conductivity and drift mobility given by equations (4.2.14) and (4.2.16) we need to know the numerical values for the dimensionless Fermi energy η, which is related to n_n and temperature trough equation (2.2.11). As can be seen from equation (2.2.11) it is impossible to calculate η analytically for a given concentration n_n . However it is possible to calculate η using the iterative method as follows: For a given a carrier concentration n_n we choose an arbitrary value of η as a first approximation and using that η, we compute the RHS of equation (2.2.11) which obviously will be different from n_n. Next we choose another value of η by increasing the value of first approximation of η and then we recalculate the RHS of equation (2.2.11) by using the second approximation of η. If the value of RHS of equation (2.2.11) is closer to the n_n in comparison to the value of the RHS of equation (2.2.11) obtained by using first approximation of η, then we continue to change η in the same direction until the two sides of equation (2.2.11) are exactly equal. However, if the second approximation for η yields a value of RHS of equation (2.2.11), which is farther away from n_n then, we change the value of η in the opposite direction until the value of RHS is equal to n_n. The value of η for which the RHS is equal to n_n is the dimensionless Fermi energy for the given electron concentration. We use this value of η to calculate the electrical conductivity and electron drift mobility based on modified density of states. This is done by computing various values of definite integrals occurring in the expressions of electrical conductivity and drift mobility i.e. equations (4.3.4) and (4.3.6) for the case of mixed scattering. These numerically calculated values of dimensionless electrical conductivity, drift mobility and various definite integrals are tabulated on pages (26) – (30) i.e. tables 5.1,5.2,5.3 and 5.4.

In our numerical calculation we have used the following values of various parameters for n-type silicon: $\varepsilon_d = 11.7$,m_n=1.18m_e, T = 300K, and ρ =2.42g/cm^3

Table 5.1 Numerically calculated values of Fermi-energy and various definite integrals with parabolic density of states and scattering ratios with mixed scattering in Heavily Doped n-Type Silicon. Where, (n_n) is Normalized doping concentration,η is dimensionless Fermi energy, $\frac{v_{0I}}{v_{0A}}$ is electron scattering ratios and others have their representations as described in this paper.

	(n_n)	η	$\frac{v_{0I}}{v_{0A}}$	ψ_0	$\psi_{5/2}$	$\frac{\psi_{5/2}}{\psi_0}$
1	1.0	-3.65	6.23632	0.0336851	0.025238	0.749233
2	2.0	-2.65	10.3354	0.0584743	0.016483	0.281884
3	4.0	-1.9	16.6626	0.0782206	0.00816085	0.104331
4	8.0	-1.15	26.0283	0.102074	0.0039451	0.0386494
5	10.0	-0.90	29.8303	0.111158	0.00313276	0.0281828
6	15.0	-0.45	37.8598	0.127458	0.00203946	0.016001
7	25.0	0.17	50.2426	0.152809	0.00122881	0.00804146
8	35.0	0.675	59.9119	0.179353	0.000943619	0.00526122
9	50.0	1.18	71.5814	0.200506	0.000681879	0.00340079
10	100.0	2.45	98.832	0.259191	0.000414757	0.001600198

Table 5.2. Numerically calculated Values For Normalized Dimensionless Electrical Conductivity σ_n and normalized Dimensionless Electron Drift Mobility μ_n for parabolic density of states in heavily doped n-Type Silicon.

	(n_n)	η	$\frac{\nu_{0I}}{\nu_{0A}}$	σ_n	μ_{nt}	$\frac{\psi_{5/2}}{\psi_0}$
1	1.0	-3.65	6.23632	0.749233	0.749233	0.749233
2	2.0	-2.65	10.3354	1.160575327	0.580288	0.281884
3	4.0	-1.9	16.6626	1.76855751	0.4421394	0.104331
4	8.0	-1.15	26.0283	2.6974381	0.3371798	0.0386494
5	10.0	-0.90	29.8303	3.1020644	0.3102064	0.0281828
6	15.0	-0.45	37.8598	4.0302635	0.268684	0.016001
7	25.0	0.17	50.2426	5.747281	0.22989124	0.00804146
8	35.0	0.675	59.9119	7.474097	0.21354562	0.00526122
9	50.0	1.18	71.5814	10.00715	0.2001429	0.00340079
10	100.0	2.45	98.832	19.38684	0.19386413	0.001600198

Table 5.3. Numerically calculated values for Dimensionless Fermi energy η, Dimensionless electron doping concentration (n_n) and various definite integrals for density of sates with band tails in heavily doped n-Type Silicon with mixed scattering.

	(n_n)	η	$\frac{\nu_{0I}}{\nu_{0A}}$	ψ_0	$\psi_{5/2}$	$\frac{\psi_{5/2}}{\psi_0}$
1	0.5	-4.256454	3.67582	0.031012	0.0552338	1.781046047
2	1.0	-3.62391	6.23632	0.0402174	0.0260908	0.648744075
3	2.0	-3.032384	10.3354	0.0521554	0.0116688	0.223731387
4	4.0	-2.496676	16.6626	0.0676373	0.00498453	0.073694987
5	8.0	-1.871945	26.0283	0.098473	0.00229372	0.023292882
6	15.0	-1.592437	37.8598	0.111032	0.0008802	0.007927444
7	25.0	-1.22347	50.2426	0.134475	0.00043396	0.003227038
8	35.0	-0.946475	59.9119	0.152559	0.00027203	0.001783113
9	45.0	-0.711575	67.9765	0.167636	0.000193465	0.001154077
10	50.0	-0.60399	71.5814	0.174392	0.00016843	0.000965829
11	60.0	-0.402661	78.1432	0.186733	0.00013377	0.000716365
12	70.0	-0.215264	84.0203	0.197845	0.000111467	0.000563405
13	80.0	-0.038318	89.3631	0.208004	0.000096319	0.000463063
14	90.0	+0.13039	94.2753	0.217397	0.00008561	0.000393791
15	100.0	+0.29238	98.832	0.226159	0.00007781	0.000344041

Table 5.4 Numerical Values For Normalize Dimensionless Electrical Conductivity σ_n and normalized Dimensionless Electron Drift Mobility μ_n in heavily doped n-Type Silicon for density of states having band tails with mixed scattering.

	(n_n)	η	$\frac{\psi_{5/2}}{\psi_0}$	σ_n	μ_n
1	0.5	-4.256454	1.781046047	0.432638452	0.865171559
2	1.0	-3.62391	0.648744075	0.648744073	0.648744073
3	2.0	-3.032384	0.223731387	0.921148868	0.460574432
4	4.0	-2.496676	0.073694987	1.249233903	0.312308475
5	8.0	-1.871945	0.023292882	1.625668346	0.203208541
6	15.0	-1.592437	0.007927444	1.996730736	0.133115381
7	25.0	-1.22347	0.003227038	2.306383896	0.092255354
8	35.0	-0.946475	0.001783113	2.533092934	0.072383193
9	45.0	-0.711575	0.001154077	2.738694994	0.060859887
10	50.0	-0.60399	0.000965829	2.842043144	0.056840862
11	60.0	-0.402661	0.000716365	3.058626839	0.051375139
12	70.0	-0.215264	0.000563405	3.295306856	0.047075811
13	80.0	-0.038318	0.000463063	3.557258873	0.044465735
14	90.0	+0.13039	0.000393791	3.847490678	0.042749895
15	100.0	+0.29238	0.000344041	4.168151872	0.041681517

Table 5.5 Numerical calculations for fractional changes in electrical conductivity and Electron drift mobility in comparison to parabolic density of states with modified density of states.

	n_n	$\Delta\sigma/\sigma$	$\Delta\mu/\mu$
1	1	0.154905	0.13413
2	2	0.25992	0.20631
3	4	0.415718	0.29364
4	8	0.659278	0.39732
5	15	1.018432	0.50455
6	25	1.491905	0.59868
7	35	1.950585	0.66106
8	50	2.521115	0.716
9	100	3.651186	0.785

6. Discussion and Results.

In the preceding section we described the procedure to compute numerical values of the electrical conductivity σ and drift mobility μ in heavily doped n-type silicon having band tail effects in the conduction band for various electron concentrations. The values of the Fermi energy electrical conductivity and drift mobility based on density of states with band tails which are tabulated on pages 26 to 29 have also been plotted as a function of n_n in figures 6.1 to 6.3 and is shown by solid rectangular dot. For the sake of comparison, we also plot the variation of electrical conductivity and drift mobility by using the parabolic density of states with mixed scattering. This is shown in figures 6.2-6.3 by solid circular dot.

Figure 6.1 shows the variation of Fermi energy with electron concentrations. It can be deduced from it that the Fermi energy calculated by using density of states having band tails is lower than the Fermi energy obtained by ignoring the effect of band tails. The gap between the two curves increases as the electron concentrations increases. In other words, taking into account the band tails at higher electron concentrations results significant changes in the calculation of Fermi energy.

Figure 6.2 and 6.3 show the variation of electrical conductivity and drift mobility with electron concentration in heavily doped n-type silicon respectively. The circular dot, in Fig.6.2 shows the variation of electrical conductivity calculated by using parabolic density of states for mixed scattering whereas the rectangular dots depict the variation of electrical conductivity for mixed scattering obtained by incorporating the density of states having band tails. It can be seen from figure 6.2 that at lower electron concentrations the electrical conductivity obtained by using the parabolic density of states and those obtained by incorporating the density of states having band tails have no significant difference. But, the effect of band tails on the calculation becomes significant as the electron concentration increases, for $n_n \geq 60$ and calculations based on density of

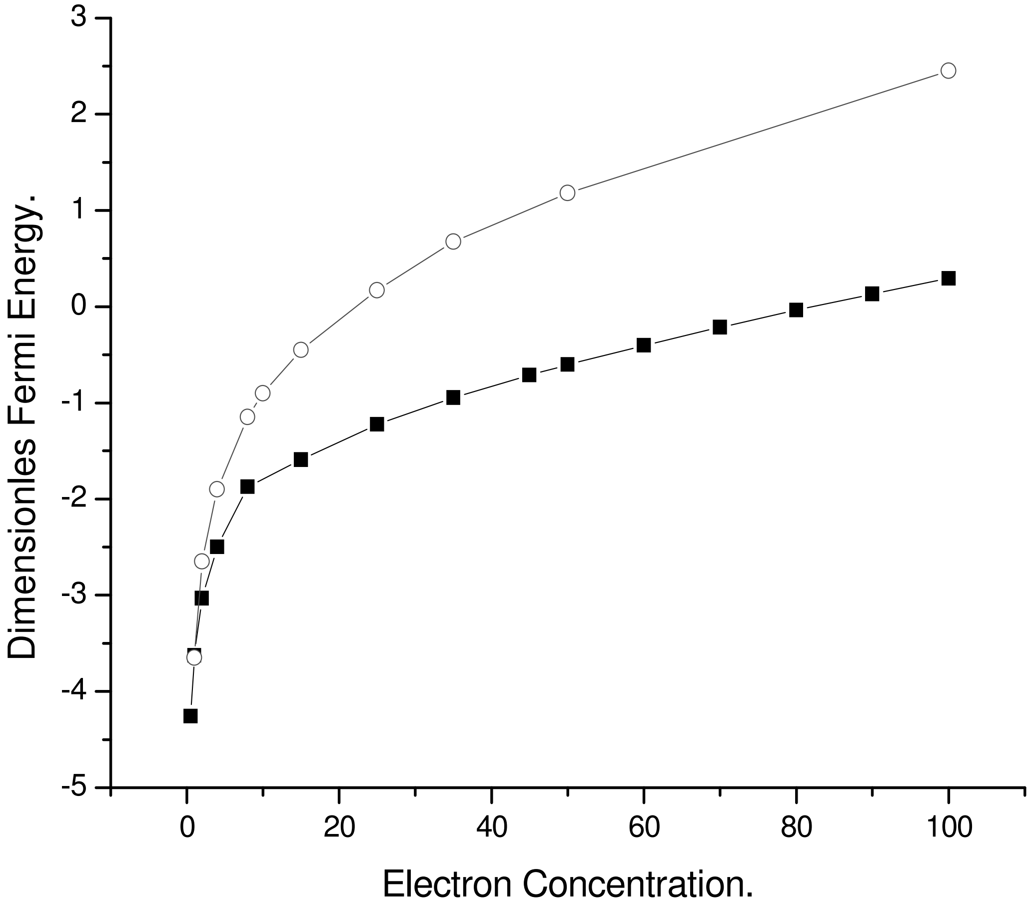

Figure 6.1. Normalized electron concentration versus dimensionless Fermi energy. The solid curve formed by rectangular dots is for electron distribution as a function of Fermi energy when band tail effect is taken into account. The circular dotes are for parabolic density of states.

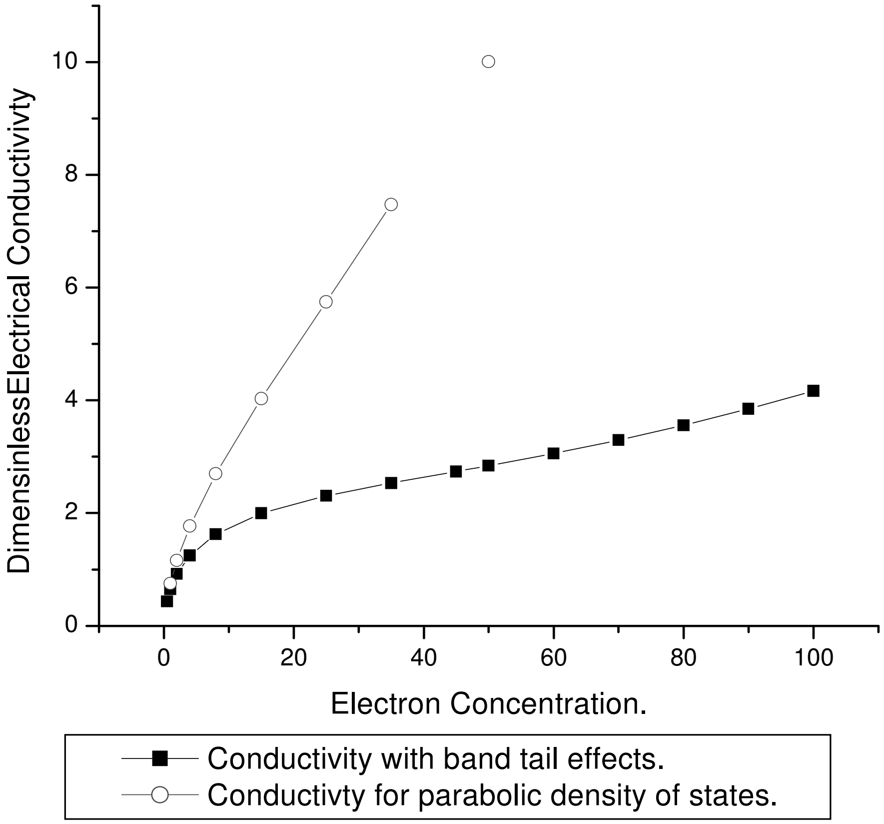

Figure 6.2.Electrical conductivity in n-type silicon for parabolic and modified density of states having band tail effects with mixed scattering.

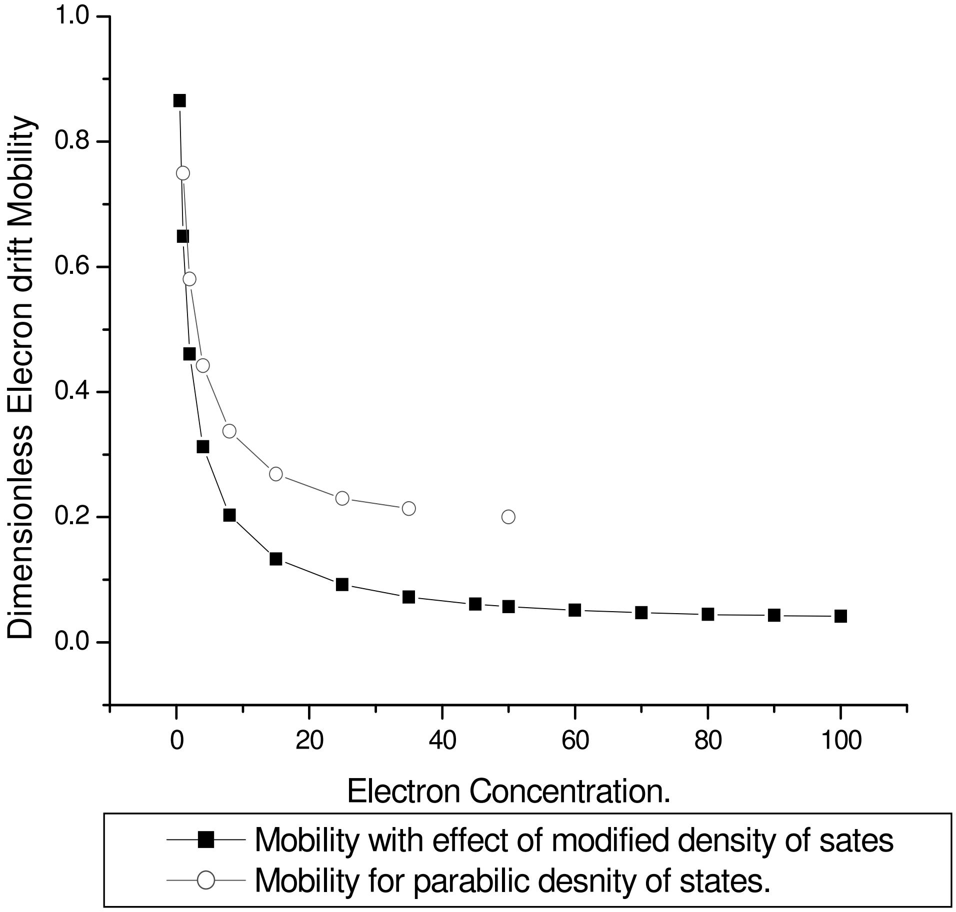

Figure 6.3.Electron Drift Mobility in n-type silicon for parabolic density of states with mixed scattering, and for modified density of states with mixed scattering

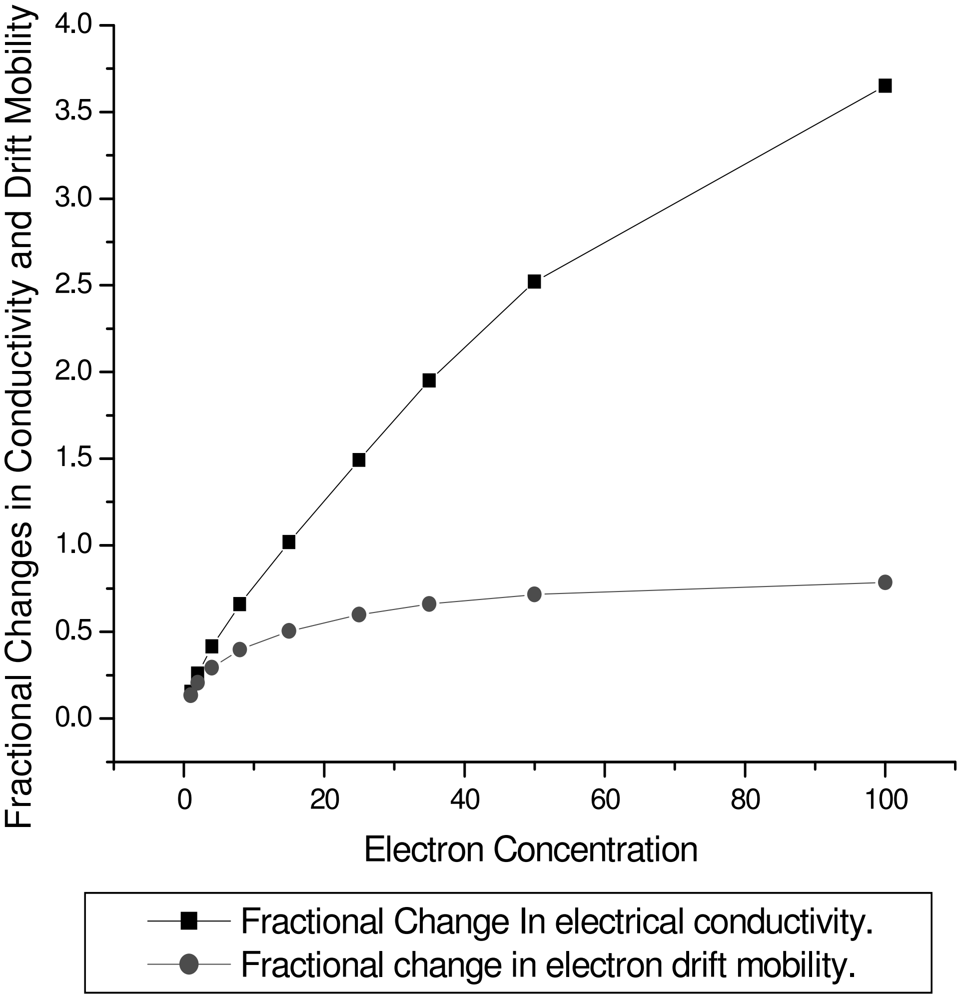

Figure 6.4 Fractional changes in electrical conductivity electron drift mobility in comparison to the parabolic density of states

states with band tails and the calculations made by using the parabolic density of sate increases with increasing electron concentration.

The variation of electron drift mobility with electron concentration for the case of mixed scattering is plotted in figure 6.3. The curve shown by circular dots depicts the variation of drift mobility for parabolic density states whereas; the curve shown by rectangular dots depicts its variation when we incorporate the band tail effect. At lower concentrations calculations of drift mobility based on parabolic and modified densities of states are quite close but as the electron concentration increases the drift mobility obtained by using modified density of states decreases rapidly than the calculation obtained from parabolic density of states.

Conclusion.

The result of this thesis shows that it is very important to incorporate the effect of band tails with mixed scattering in the intermediate concentration while making any calculations of electrical conductivity and drift mobility in heavily doped silicon at a temperature of 300K.

References.

[1]. S. K. Sharma "Effect Of Energy Band Distortion On *D/μ* In Heavily Doped n-Type Silicon." Canada, J. Phys. **45**. 2386, 1967.

[2]. Y.Elfagd, B. Workalemahu, S. K. Sharma. " Some Transport Coefficients In Highly Doped n-Type Silicon," Phys, Stat. Sol. (**b**) 227 pp549-555, 2001.

[3]. J.W.Slotboom, "The pn Product In Silicon." Solid State Electron Vol. **20.** Pp2 79-283, 1977

[4]. E.O.Kane, "Thomas-Fermi Approach To Impure Semiconductor Band Structure," Phys. Rev. Vol. **131** pp79-88, 1963.

[5]. D.S. Lee and J.G.Fossum, " Energy-Band Distortion In highly Doped Silicon," IEE Trans**,** electron Devices **Ed** 30. Pp 626-632, June 1983.

[6]. C.J.Hwang. " Calculations Of Fermi Energy and Band Tail Parameters in Heavily Doped and Degenerate n-Type GaAs," J. Appl. Phys. Vol. **41** pp 2668-2674,1970.

[7]. J. POORTMANS, S.C.JIAN, D.H.TOLLER, M.CAYMAX and et al. "Solid State electron." **36**,1763,199

[8] John. P.Mckelvey, " Solid State and Semiconductor Physics.'' Robert E.Krieger Publishing Company Malabara, Florida. Pp 267-317.

[9]. R.B.Adler and et al. " Introduction to Semiconductor Physics." John Wiley & Sons, Inc, New York. EEEC Vol.**1.** pp 17-55, 1953.

[10]. E.Conwell and V.F.Weisskopf, " Theory of Impurity Scattering in Semiconductors," Phys. Rev. Vol**. 77.** Pp.388-390, 1950.

[11]. W.Shockely, " Electrons and Holes in Semiconductors," D.Van Nostrand Company, Inc. New York. Pp 258-269, 1959.

[12]. F.Seitz, " The Modern Theory Of Solids," McGraw-Hill Book Company Inc. New York Pp 168-175. 1940.

[13]. Albert Haug, “ Theoretical Solid State Physics,” Pergamon Press Oxford. Vol. **2** pp 216-232,1972.

[14]. C.Kittle, “ Introduction to Solid State Physics,” John Wiley and Sons, Inc. New York. 7th edition.pp 85-90.

[15]. N.Cusack, “ The Electrical and Magnetic Properties of solids,” An Introductory Textbook Longmans, Green and Co.LTD. London. Pp 200-218, 1958.

[16]. S.K.Sharma, “ Some Transport Coefficients in n-Type Degenerate Semiconductors,” IEEE Trans. Devices **36.** 768, 1989.

[17]. J.Mcdougall and E.C. Stower, Phil. Royal Soc. London A **237**, 350, 1938.

[18]. W.Eisenberg, Ann. Phys**. 7,** 131(1974)

[19]. Mulugeta Habte and et.al. “Weidman-Franz Ratio in Heavily Doped n-Type Silicon.” M.Sc. Thesis to be communicated, pp30-31, 2002.