Addis Ababa University

Addis Ababa Institute of Technology (AAiT)

African Railway Center of Excellence

Modeling Passengers' Attitude and Analysis of Travel Experience Attributes:

A case of Addis Ababa Light Rail Transit

By

Gizachew Kefelew

A Thesis Submitted to the School of Graduate Studies of Addis Ababa University

In partial fulfillment of the requirements for the degree of Master of Science in Railway Engineering (Civil Infrastructure)

Advisor: Fitsum Teklu (PhD)

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BOARD OF EXAMINERS

We, the undersigned, approve that the thesis entitled "**Modeling Passengers' Attitude and Analysis of Travel Experience Attributes: A case of Addis Ababa Light Rail Transit** "was presented by Gizachew Kefelew. The authors also confirm that the thesis complies with the requirements of Addis Ababa University and with the standards regarding originality and quality.

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DECLARATION

I hereby declare that the thesis report entitled "**Modeling Passengers' Attitude and Analysis of Travel Experience Attributes: A case of Addis Ababa Light Rail Transit**" submitted to the school of graduate studies of Addis Ababa University as a partial fulfillment of the requirements for the degree of Master of Science in Railway Engineering (Civil Infrastructure) under the supervision and guidance of my advisor Dr. Fitsum Teklu is my original work and has not been submitted to any other university or institution for an award of any type of work. All works of others used in this Thesis are duly acknowledged.

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ABSTRACT

The importance of public transportation in developing countries lies in its ability to promote sustainable urban development, support economic growth, and enhance the overall well-being of communities. Light rail transit systems are particularly essential for alleviating traffic congestion issues on roads by offering an alternative mode of transportation. Therefore, it is essential to assess the performance of light rail transit services from passengers' perspective to pinpoint both strengths and areas needing improvement within the transit network. Accordingly, the primary goal of this thesis research is to evaluate the light rail transit service performance from the perspective of AA-LRT's passengers and recommend optimal improvement strategies that enhance travel experience of passengers. Through a satisfaction survey conducted via paper-based questionnaire, the authors determined the passengers' satisfaction index on AA-LRT's service through HPSI. The calculated overall passenger satisfaction index of **2.66** indicates that passengers remain below the "fairly satisfied" threshold (score=3). The authors also proposed an NDS compartmental model to explore trends of passenger satisfaction levels. The model categorizes passengers into "N" for less or quite satisfied, "D" for dissatisfied, and "S" for satisfied. The model was evaluated for existence and uniqueness of solutions, as well as the positivity and boundedness of solutions. Local stability was analyzed for both dissatisfaction-free and dissatisfaction-persistent steady states. An optimal control problem was formulated and analyzed to investigate strategies for minimizing passenger dissatisfaction. Numerical findings indicate that the efforts related to providing train services that adhere to high levels of reliability and punctuality for arrivals, and related to trains operate with excellent safety and security standards effectively reduces dissatisfaction levels. The graphical representation of numerical results also indicates the importance of the order of the fractional derivative in representing the evolution of passengers' satisfaction over time with consideration of memory effects.

Keywords: Passengers' attitude; Mathematical model; Satisfaction index; travel experience attributes; optimal control

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ACRONYMS

ARCE	African Railway Center of Excellence
TCRP	Transit Cooperative Research Program
HPSI	Heterogeneous passengers' satisfaction index
PSI	Passengers' satisfaction index
IPA	Importance-performance analysis
AA-LRT	Addis Ababa Light Railway Transit

CHAPTER ONE

INTRODUCTION

1.1 Background of the study

The Addis Ababa Light Rail Transit (AA-LRT) has become an integral part of the city's transportation system, serving for thousands of passengers daily. However, the transit system could not provide a dedicated mass transit solution in order to alleviate the chronic transportation problem in the city. This is evident through the presence of passengers who express complaints about disruptions of the service operations, as well as congestion and prolonged passenger waiting times, with an average wait of 17 minutes, are among the main problems faced by passengers of AA-LRT (Debebe & Asteray, 2022). These transportation related problems (see Figure 1.1) can impact the efficient operation and pose a great challenge to planners and policy makers (Shen et al., 2016).

Figure 1-1:Travel characteristics of AA-LRT

Passengers' attitudes towards the service quality of the rail transport system can be influenced by various factors. Positive attitudes may be influenced by factors such as punctuality, cleanliness, comfort, and courtesy.

On the other hand, negative attitudes may stem from issues like frequent delays, overcrowding, lack of accessibility, and poor customer service. Additionally, passengers who are less or quite satisfied might develop neutral attitudes. This may result from a lack of strong positive or negative experiences, or a perception of the service being neither particularly good nor bad.

In transport planning and the development of advanced transport systems, predicting passengers' satisfaction plays a crucial role (Stelzer et al., 2016). It is critically important to understand how passengers' attitude evolves in the future and whether specific control measures by the transit operator will enhance passengers' satisfaction and can reduce dissatisfied passengers. UK train operators' contracts include provisions that link payments and penalties to their ability to meet certain thresholds for customer satisfaction. In developing countries, light rail transit systems could benefit from adopting a similar approach to the UK train operators' contracts. By including provisions that tie payments and penalties to meeting specific customer satisfaction thresholds, AA-LRT system would be incentivized to prioritize and improve the overall passenger experience, and passengers would receive compensation from the transit system if it fails to meet the minimum service standards. This approach could lead to an increased focus on service reliability, cleanliness, safety, and customer service, ultimately enhancing the quality of public transportation and fostering greater satisfaction among passengers. Furthermore, it could encourage accountability and a commitment to continuous improvement within the light rail transit system, ultimately contributing to their long-term sustainability and success.

A series of surveys in Great Britain have been conducted to gather feedback from passengers about their experiences with rail travel and their level of satisfaction with their most recent train journey. The findings indicate that satisfaction with different aspects of rail journeys changes over time (Rail User Survey – Edition 22 - Transport Focus, 2022).

The shift in passengers' satisfaction as a result of experiencing different travel attributes over time is a complex and dynamic process that can be analyzed and understood through mathematical modeling. By capturing the various factors that contribute to passengers' satisfaction, such as travel comfort, service quality, and efficiency, mathematical models can provide valuable insights into how these elements interact and influence overall satisfaction levels. These models can help urban rail transit system identify key areas for improvement and optimization, leading to a better overall travel experience for passengers. By studying this dynamic system through mathematical modeling, researchers and industry experts can gain a deeper understanding of the complex relationships between different travel attributes and passenger satisfaction, ultimately leading to more informed decision-making and enhanced customer experiences.

Mathematical modeling has been a central part in describing and analyzing various aspects of real-world problems in various disciplines including science and engineering. This can be observed, for instance, by examining its application in various contexts (Din et al., 2022; Ghosh et al., 2021; Sene, 2022). The authors discussed how mathematical modeling can help identify key factors that contribute to the dynamics of real-life situations by analyzing the stability of steady states.

Modeling passengers' attitude can be defined as the creation of a mathematical model to represent the satisfaction expressed by rail passengers regarding their encountered travel experience attributes in order to predict how passengers with similar attitude will behave under similar circumstances. This approach allows for the systematic analysis and interpretation of how passengers move or transition between different levels of satisfaction. By dividing passengers into distinct compartments based on their satisfaction levels, it becomes easier to understand the dynamics of satisfaction within the population and identify factors that influence changes in satisfaction over time. This modeling approach provides a structured way to study and predict passengers' evolution of satisfaction, which can be valuable for improving the overall quality of service.

In the context of rail passengers, memory effects can play a significant role in shaping their attitudes. These effects refer to the influence of past experiences from encountered incidences and include persistence of negative experience, recency bias, confirmation bias, social influence and expectation formation. For example, if passengers have had persistently unfavorable experiences in the past, it can create a pessimistic outlook and low expectations for future trips. This anticipation of experiences potentially recurring in the future can further contribute to how passengers will perceive their travel experience.

The study aims to refine an integer order model representing the dynamics of passengers' level of satisfaction with railway transportation by integrating fractional order derivatives, which offers a more accurate representation of the model in capturing the memory effects (Barros et al., 2021), providing greater flexibility in adjusting system responses to align with real world observations and experimental data.

By formulating an appropriate objective function in terms of costs related to the intervention strategies, the study seeks to determine optimal improvement strategies that can enhance passenger experiences, and contribute to the effort to improve overall railway operations. This aspect of the proposed research aims to translating the theoretical insights into practical interventions.

In summary, the study presents a new approach to modeling and analyzing passengers' attitude towards the transit service in order to predict the passengers' evolution of satisfaction and develop effective strategies that improve passengers' overall travel experience.

1.2 Statement of the problem

There has been an extensive discussion about evaluating passengers' satisfaction within the transit industry, with several indicators and measures established and implemented in different ways to align with the goals and objectives of transit agencies.

These measures typically assess operational performance in either historical or current contexts. Similar to transit agencies worldwide, the AA-LRT system undertakes routine inspections of its facilities and services to ensure that all aspects of the travel experience meet passengers' expectations in terms of quality. However, the transit system has been lacking passenger-oriented measures that can accurately identify which service aspects are the most dissatisfying or satisfying for passengers.

Moreover, there have been no initiatives to introduce cost-efficient improvement strategies that focus on enhancing passengers' travel experiences in accordance with their perceptions of travel attributes.

Presently, there are passengers of AA-LRT who consistently express their dissatisfaction with the transit's service, leading to the formation of negative attitudes resulting from unaddressed aspects of travel experience. Previous research (Bogale & Gizat, 2021) revealed that the overall service quality and customers' satisfaction in AA-LRT falls below moderate threshold. Media outlets frequently broadcast passengers' feedback to assess levels of satisfaction with the LRT service, and the passengers have highlighted issues such as lengthy queues and delayed wait times, overcrowded trains, limited accessibility for elderly and disabled individuals, and safety and security concerns as recurring challenges they encounter. These constraints contribute to the reduced public perception of the transit service, leading to growing concerns as passenger dissatisfaction escalates. Furthermore, while previous studies have explored passengers' satisfaction with AA-LRT system, none have specifically explored into the evolution of passengers' satisfaction from the users' perspective. Therefore, it is imperative to identify key travel experience attributes that influence passengers' satisfaction by evaluating AA-LRT's past performance in critical areas of the service significant to passengers, and to track the development of passengers' satisfaction trends.

1.3 Research questions

The thesis research aims to address the following interrelated questions.

- What is the overall passenger satisfaction level with AA-LRT services analyzing the travel experience attributes?
- How does the attitude of passengers evolve in the future?
- What are the cost effective improvement strategies that enhance passengers' satisfaction?

1.4 Research Objectives

1.4.1 General Objective

The general objective of the study is to evaluate the light rail transit service performance from the perspective of AA-LRT's passengers and recommend optimal improvement strategies that enhance travel experience of passengers.

1.4.2 Specific Objectives

The specific objectives of the study include

Objective 1 To identify and analyze travel experience attributes contributing to passengers' satisfaction,

Objective 2 To predict the future trend of satisfaction/dissatisfaction level, and

Objective 3 To develop optimal improvement strategies by exploring different scenarios, that lead to increased satisfaction.

1.5 Significance of the study

Policy

This study provides valuable insights for policymakers by highlighting key attributes of the travel experience that significantly affect passenger satisfaction.

By identifying areas where satisfaction is low despite high importance, policymakers can prioritize improvements and allocate resources more effectively. This research aids in shaping transit policies that enhance the overall quality of public transport, ultimately promoting a more sustainable and user-friendly transit system.

Theory

From a theoretical standpoint, this study contributes to the existing literature on customer satisfaction by integrating satisfaction-importance analysis with mathematical modeling. By formulating a compartmental model and an optimal control problem, the study advances theoretical frameworks that link passenger satisfaction with dynamic system optimization. This integration offers a new lens through which to understand and predict passenger behavior, paving the way for future research in transport theory and systems dynamics.

Methodological

This study demonstrates the effectiveness of using a customer satisfaction index to quantitatively assess passenger satisfaction levels. The satisfaction-importance analysis serves as a robust framework for identifying critical attributes of the travel experience, while the mathematical modeling component introduces a systematic approach to optimize control strategies. This methodological approach can be replicated in other transportation studies, enhancing the rigor and reliability of passenger satisfaction assessments.

Practical

Practically, the findings of this study provide actionable recommendations for transit agencies and operators. By pinpointing the travel experience attributes that require improvement, transit authorities can implement targeted interventions that directly address passenger concerns.

The study's formulation of optimal control strategies offers a practical roadmap for enhancing service delivery, ultimately leading to increased passenger satisfaction and retention. This research serves as a guide for transit managers seeking to create a more positive travel experience for their users.

1.6 Research design

The main objective of the study is to ascertain the level of satisfaction and how passengers' satisfaction with various aspects of travel experience on AA-LRT service has changed over time, and suggest effective improvement strategies. This involves utilizing measures of satisfaction and formulating a compartmental model. The study utilizes ODE45 solver in MATLAB, to depict the evolution of passengers' satisfaction levels. Optimal control theory is used to determine cost-effecting improvement strategies that minimize the cost of implementing the measures to enhance passengers' satisfaction, taking into account system dynamics and operational constraints, including initial passengers' satisfaction levels. The optimal control model considers factors such as the reliability of service operations, comfort at stations and on trains, and security and safety issues. The research process includes defining the problem, setting objectives, conducting a literature review, surveying passengers' satisfaction, modeling satisfaction dynamics, checking wellposedness of the model, developing improvement strategies and finally offering conclusion and recommendations.

The research is pursued through the following processes.

1. Statement of the problem: This provides an overview of the problem, its importance, and the rationale for its investigation.
2. Objectives: These outline what the research seeks to achieve, highlighting the desired outcomes, purpose, and focus of the investigation.
3. Literature review: This step aims to collate existing knowledge about the topic and contextualize new findings within the field.

It involves exploring various tools and models to assess and evaluate passenger satisfaction, developing detailed research questions addressing current gaps, and identifying appropriate research methodologies, theoretical frameworks, and analytical approaches. Travel experience attributes are determined from established literature resources.

4. Passenger satisfaction survey: This involves assessing the current status of passenger satisfaction through a survey.
5. Satisfaction index: This involves exploring various measures of performance of public transportation.
6. Trend analysis of passenger satisfaction: This involves depicting the trajectories of the state variables and conducting stability analysis.
7. Modeling passengers' satisfaction dynamics: This step focuses on developing a mathematical model to gain insights about the development of passengers' satisfaction.
8. Solution of the model: This involves using analytical methods to show that the solution exists, is unique, positive and bounded.
9. Development of improvement strategies: This step involves defining optimal control problem corresponding to the model formulated in step (7) and applies Pontryagin's maximum principle to develop optimal improvement strategies.
10. Conclusion and recommendations: The study concludes with developed recommendations.

A concise overview of the necessary activities in the course of the study were identified and presented in the flow chart below.

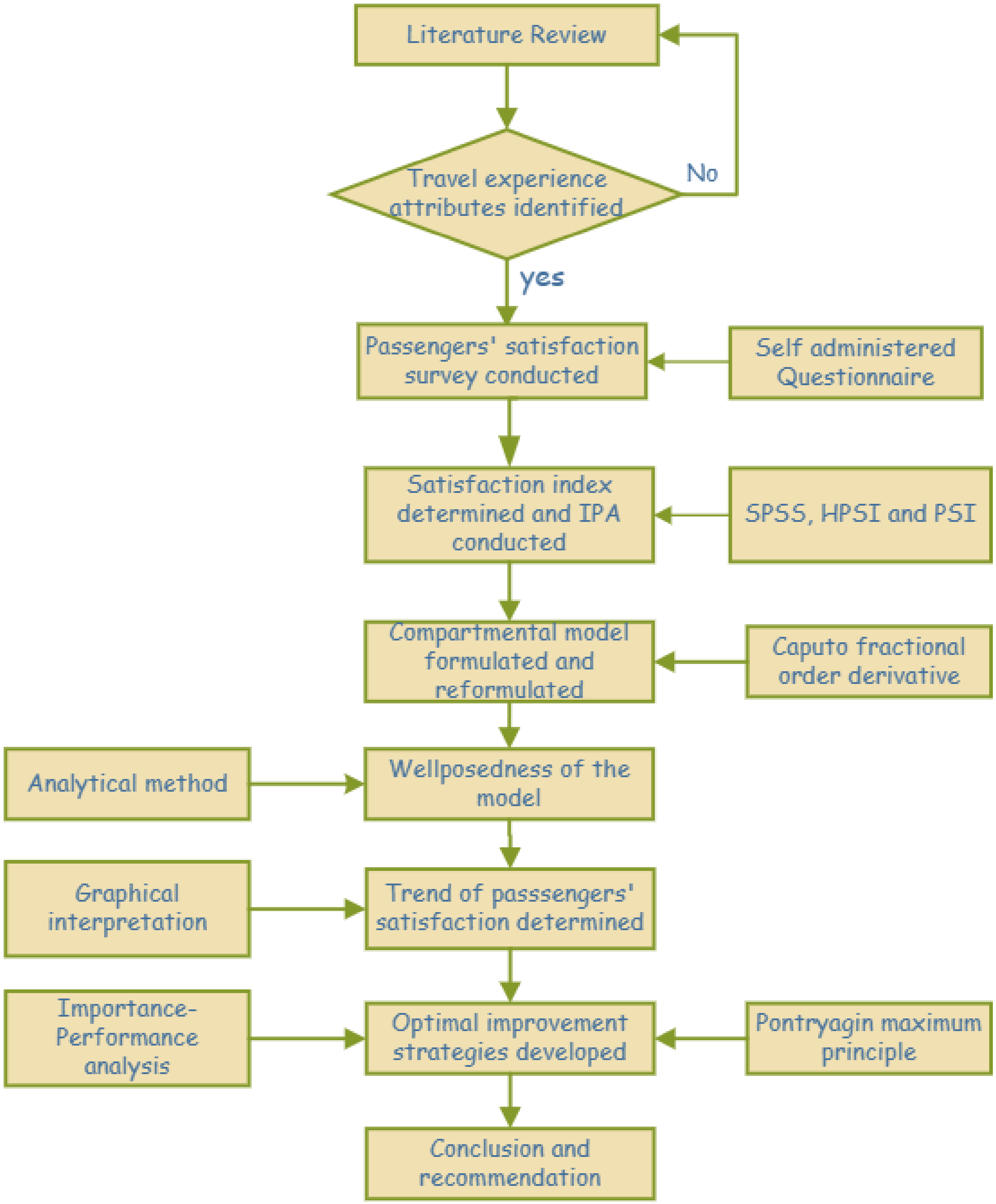

Figure 1-2: Outline of research design

1.7 Scope and limitations

This thesis research was intended to assess the performance and identify travel experience attributes that contribute to the satisfaction/dissatisfaction of passengers with AA-LRT service based on attributes for which satisfaction levels are low but the perceived importance of the service is relatively high, considering the variation of satisfaction and importance ratings provided by the transit users. The study also aims to predict the pattern of passengers' attitudes towards travel experience attributes of key aspects of quality service. To do this, the researchers formulated a satisfaction model which considered only the time variant nature of passengers' state of satisfaction levels and analyzed it to demonstrate the trajectories of the state variables of the model, showing the pattern of passengers' attitudes in the future. The compartmental model used in this study might simplify the fuzzy dynamics of passengers' attitude, incurring formulation error.

Like other studies, this study's findings may be subject to self-reporting biases inherent in survey research, potentially affecting the accuracy and reliability of the data.

Therefore, it is important to interpret the findings within the defined scope and limitations of this study.

1.8 Thesis organization

The rest of the thesis report is structured as follows: Chapter two introduces measures of quality of rail transit service, customer satisfaction models, some fundamental concepts, definitions, and propositions related to fractional-order derivatives used in formulating the NDS-model. In chapter three, the authors present salient features of AA-LRT, the data collection methods, measures of passengers' satisfaction, model formulation, optimal control problem and numerical analysis. The graphical representation of numerical results is discussed in chapter four. The report concludes with chapter five, which presents concluding remarks, recommendations and future work.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

In the realm of urban rail transit systems, understanding and enhancing passengers' travel experiences is critical area of study. This literature review begins by defining key aspects of passenger travel experience attributes within the context of urban rail transit systems, with a specific focus on the AA-LRT system. Furthermore, it reviews a number of metrics for measuring passenger satisfaction, identifying their strengths and shortcomings. It also illustrates how the heterogeneous passenger satisfaction index overcomes these shortcomings. The review also examines the use of mathematical models by researchers to tackle intricate real-world problems. Lastly, it discusses the application of optimal control theory in the literature to identify and implement effective optimal control strategies.

2.2 Quality attributes of passengers' travel experience

The urban transportation system that utilizes electric technology offers various advantages such as saving time for passengers and contributing to pollution reduction. Additionally, it plays a crucial role in supporting the urban transportation system.

To evaluate the effectiveness of light rail transit as a mass transit transportation service, factors such as the service reliability, convenience, safety, comfort, accessibility, and affordability should be taken into account (Imam, 2014). Moreover, Friman and Fellesson (2009) conducted a thorough review of existing literature , highlighting that factors such as reliability, frequency of service, travel time and cost, comfort and cleanliness, network coverage proximity, and safety considerations are crucial aspects for customers when assessing the quality of public transportation services.

In this study, reliability is defined as the consistency and dependability of the urban rail transit system in providing scheduled services without disruptions. This pertains to the capability of the AA-LRT system to consistently and dependably operate, without encountering frequent disruptions or delays. In other words, a reliable transit system can be counted on to maintain a high level of operational consistency, thereby meeting the expectations and needs of its passengers. According to Habib et al. (2011), reliability is a fundamental aspect of transit service quality that influences passenger satisfaction and ridership. Reliability pertinent to scheduled services has been used to measure of the reliability of transportation .

Service characteristics encompass the tangible features of the transit system such as frequency and accessibility as well as intangible features such as convenience. According to a study by Soltanpour et al. (2018), these service characteristics significantly impact the perceived quality of urban rail transit services among passengers, affecting their overall experience and willingness to use the system.

Cleanliness pertains to the physical condition and hygiene of the transit facilities, trains, and stations. Passengers may interpret the presence of litter on trains or at stations as an indication that the less visible aspects of public transportation services are neglected. Research by Diab et al. (2021) emphasizes the role of cleanliness as a key determinant of passenger satisfaction and perceptions of the urban rail transit system.

Providing a comfortable environment for passengers enhances their experience and encourages them to use the service again (Nordin et al., 2016). Comfort in urban rail transit systems includes factors such as seating accessibility and convenience, ergonomics (ease of movement), air quality and temperature, passengers' behavior, and train crowding.

Information availability and accuracy, including real-time updates and signage, are integral to the quality of urban rail transit services.

The work of Nathanail (2008) underscores the significance of clear and timely information in enhancing passenger perception and satisfaction with the transit system.

Customer service care relates to the interactions between passengers and transit staff, ability of the rail operator in delivering services in crowded stations as well as the responsiveness to passenger inquiries or concerns. According to Yilmaz & Ari (2017), effective customer service positively influences the overall image and reputation of urban rail transit systems.

Urban light rail transit operators need to evaluate how passengers perceive the measures taken to keep them safe from the risk of injury and crime, and make sure that a safe environment is maintained for passengers using the transit. Research conducted by (L. Xu et al., 2018; X. Xu et al., 2020) indicates that safety and security play a vital role in building trust and confidence of passengers using urban rail transportation services.

The TCRP report 100 found that elements such as service reliability, frequency of service, passenger behavior, terminal shelters and lighting, and the availability of seats are crucial factors in determining passenger satisfaction (Kittelson & Associates et al., 2003). Other important factors mentioned include safety from criminal activities, cleanliness, courteous staff, and the comfort of seats.

The review of the existing literature demonstrates that various travel experience attributes influence passenger satisfaction and the overall efficiency of the transit system. In order to ensure the accuracy of the satisfaction evaluation, this study considers various service aspects in passengers' satisfaction surveys to uncover their perspectives. As a result, 32 travel experience attributes that characterize AA-LRT service were identified from the existing literature, as outlined in Table 2.1.

Table 2-1: Travel Experience Attributes

Travel Experience Attributes	
(1) Reliability of train departures	(18) Quietness of the train
(2) Reliability of train arrivals	(19) Ease of boarding and alighting the train
(3) Punctuality of train departures	(20) Train crowding
(4) Punctuality of train arrivals	(17) Availability of schedules and maps on train
(5) Frequency of trains on the route	(21) Provision of information for disabled and elderly rail passengers at stations
(6) Operational speed of trains on the route	(22) Information made available at the ticket office
(7) Availability of transit personnel at the station	(23) Provision of information for disabled and elderly rail passengers on trains
(8) Availability of services on weekends/holidays	(24) Provision of schedules and maps at stations
(9) Short wait time for transfers	(25) Efficiency of service at the ticket office
(10) Daily service time	(26) Administration of complaints
(11) Cleanliness of interior and seats on the trains	(27) Administration of crowded stations during rush hours
(12) Cleanliness and hygiene on the stations	(28) Train operators' behavior
(13) Cleanliness and hygiene of the transit facilities	(29) Ability of AA-LRT system to avoid or reduce safety related issues at stations
(14) Accessibility and convenience of seat and shelter on the stations	(30) Ability of AA-LRT system to avoid or reduce safety related issues on train
(15) Comfort of seats on the trains	(31) The effort that AA-LRT system to avoid or reduce security related issues at stations
(16) Smoothness of ride and stops	(32) Ability of AA-LRT system to avoid or reduce security related issues on train

2.3 Customer Satisfaction indices

In marketing literature, ensuring customer satisfaction is a key element in any service industry, including the rail transit service sector. It refers to the extent to which customers are happy with the products and services provided by a company or organization. In the context of rail transit services, customer satisfaction plays a vital role in determining the quality of service delivery. Customer satisfaction survey can not only pinpoint areas where passengers are satisfied or dissatisfied, but also reveal the extent to which specific factors impact customer satisfaction (Kittelson & Associates et al., 2003).

It is significant because it directly impacts the reputation and success of rail transit service providers, and help to identify areas where transit service providers can improve their services to enhance users' loyalty and revenue.

Over the years, numerous customer satisfaction techniques have been developed to assess and improve the quality of services offered by rail transit companies. ServQual method as an attributed based service quality measurement method is the most popular and widely recognized (Chou et al., 2011; Gilbert & Wong, 2003; Lupo, 2013; Tsang & Qu, 2000). This method aims to measure the gap between customers' expectations and perceptions of service quality across multiple dimensions. The ServQual method defines five service quality dimensions, namely reliability, responsiveness, assurance, empathy, and tangibles, each of which includes several items for measuring service quality (Parasuraman et al., 1991). These dimensions and items are used to evaluate the various aspects of service delivery and customer experience within the rail transit industry. However, despite its widespread use, the ServQual method has some disadvantages, such as the small number of attributes: five attributes may be insufficient to fully represent the range of variability within each dimension of service quality (Daniel & Berinyuy, 2010). Other attributes together with the service quality dimensions employed in the ServQual method influence passengers' perceptions and expectations. These attributes encompass the lack of fundamental amenities such as shelter at the stations, availability and standard of sanitation, insufficient safety and security measures, ineffective handling of service-related complaints and availability of information like schedules and maps on the train and the conduct of passengers.

Various techniques for assessing customer satisfaction have been created and refined, including the European Customer Satisfaction Index (ECSI), Swedish Customer Satisfaction Barometer (SCSB), the American Customer Satisfaction Index (ACSI), and the Norwegian Customer Satisfaction Barometer (NCSB).

These indices provide alternative approaches to measuring and understanding customer satisfaction within the rail transit service sector. This can be observed by examining specific benchmarks, methodologies, and indicators tailored to their respective regions and industries (Eboli & Mazzulla, 2007; Nurmahdi, 2019).

One of the most recent developments among these indices is the European Customer Satisfaction Index (ECSI), which offers a more comprehensive and standardized approach to measuring customer satisfaction across European countries (Fornell et al., 1996). This index is increasingly recognized as a more appropriate measure of service quality due to its ability to capture the diverse needs and expectations of customers in different cultural and market contexts (Fornell et al., 1996).

Girma & Woldetensae (2022) used customer satisfaction index to measure the satisfaction level regarding the quality of bus services provided in Addis Ababa. While the authors provide valuable insights into overall service quality, they do not fully account for the heterogeneities among user judgments. In a rail transit service, passengers evaluate travel experience attributes based on their own perspective. For instance, while some passengers may prioritize comfort and cleanliness, others may place a high value on reliability of service operations. Additionally, some passengers may emphasize the service characteristics such as daily service time, while others may be more concerned with on-time performance. This diversity in priorities among passengers illustrates the heterogeneity in how passengers evaluate their travel experiences. This limitation highlights the need for a more nuanced and comprehensive understanding of customer satisfaction within the rail transit service industry, taking into consideration diverse travel experience attributes. To overcome the variations, Eboli & Mazzulla (Eboli & Mazzulla, 2009) suggested a new customer satisfaction index to evaluate satisfaction level by correcting the satisfaction and importance ratings according to the dispersion of the ratings from the average value.

Our research applies this customer satisfaction index recommended by Eboli and Mazzulla to determine passengers' satisfaction index. This approach is a more thorough and detailed examination by concentrating on specific attributes, providing valuable insights into enhancing passengers' satisfaction levels.

By incorporating this approach, our study aims to build upon the existing literature and contribute to a deeper understanding of the drivers of passenger satisfaction in the transportation industry.

2.4 Customers' satisfaction models

In the literature, numerous studies have been conducted on the topic of transit service quality. Some of them are reported in the literature ((Eboli & Mazzulla, 2007), (Iseki & Taylor, 2010),(Joewono & Kubota, 2007), and (Tyrinopoulos & Antoniou, 2008)). Primarily centered on soliciting service users' feedback and viewpoints through surveys, these studies fall short in developing a predictive model that can adapt the evolving trends in passengers' satisfaction.

While our research is valuable in understanding the determinants influencing satisfaction, it also provides a quantitative model for predicting passengers' level of satisfaction. Eboli & Mazzulla ((2007) employed a structural equation model to identify the impact of various service attributes on overall satisfaction and analyze the correlation between service quality attributes. Observing the variance of satisfaction rates, the study fails to account for the differences in user perceptions.

Hailu and Teklu (2024) employed compartmental modeling approach that entails classifying passengers into various compartments according to their vulnerability, exposure to negative perceptions, and acquisition of negative attitudes stemming from safety concerns and unreliable train services, or recovery from negative attitudes.

These compartments are given distinct labels and are interconnected based on changes in perception as passengers undergo different dynamic travel experiences. Consequently, individuals can move between these compartments.

Obsie et al. (2020) employed factor analysis to reduce the dimensionality of data by identifying underlying factors that contribute to overall satisfaction levels among passengers in the case of AA-LRT. Furthermore, they also used the ordered logit model in analyzing the relationship between these identified factors and the level of passenger satisfaction, providing a framework for understanding how different variables influence satisfaction levels.

De Oña (2016) employed a regression model to demonstrate the key factors influencing the overall satisfaction of transit users in Algiers. In the case of train travel, the regression analysis indicated that passengers' satisfaction was primarily influenced by "safety and security" ($\beta = 0.453$) and "comfort on the vehicle"($\beta = 0.262$). Additionally, factors such as "information", "accessibility", and "comfort at the stations" were also found to have significant impacts on passengers' satisfaction. The regression model can be used to make predictions or recommendations for improving overall satisfaction based of those specific attributes. Unlike the regression model which introduces randomness through the error term in predicting the behavior of the model, deterministic model is predictable and does not involve randomness.

2.3 The proposed fractional order model

Bidah et al. (2020) introduced a mathematical model that captures agree-disagree opinions. Drawing inspiration from this model, our research has leveraged it as a motivation to devise a satisfaction-dissatisfaction model tailored specifically for understanding how passenger satisfaction changes in response to important service features, as well as interactions with staff and administration. By incorporating insights from Bidah et al.'s work, the authors aim to enhance our understanding of the dynamic nature of passenger satisfaction within the context of AA-LRT system.

This model for understanding satisfaction and dissatisfaction is a mathematical framework explained using ordinary differential equations involving fractional derivatives. Fractional (fractional order) derivative is a generalization of integer order derivative. An equation which contains fractional derivatives is known as a fractional differential equation. A fractional order model means a representation of a system described by a fractional differential equation or a system of such equations (Podlubny, 1999).

The classic autonomous ordinary differential equations representing dynamical systems have no memory, as their solution is independent of the previous instant (Barros et al., 2021). Fractional calculus has been extensively studied over the past two or three decades by mathematicians including Grunwald-Letnikov, Riemann-Liouville and Caputo (Bonilla et al., 2007). The Riemann-Liouville method results in initial conditions that include the limiting values of the Riemann-Liouville fractional derivatives at the lower bound $t = a$, and these types of initial conditions lack a recognized physical interpretation. Applied engineering problems require formulation of fractional order model with the use of physically meaningful initial conditions, such as $x(a), x'(a), x''(a)$ and so on. In this study, $x(a)$ represents the current level of satisfaction with the rail transit service rendered, while $x'(a)$ reflects the rate at which passengers' satisfactions towards the service are evolving. This could illustrate the rate at which the level of satisfaction is either improving or declining. Additionally, $x''(a)$ could signify the rate at which the change in attitude is itself changing. M. Caputo's approach enables the formulation of initial conditions for fractional order differential equations at the initial state $t = a$ (Podlubny, 1999), and is easy and simple instead of other approaches as its fractional derivatives do not involve in defining initial conditions. Further, the Caputo fractional order model can help detect the evolution of passengers' satisfaction earlier than the conventional models.

It accounts for long-term memory effects, such as the influence of social interactions that may influence passengers' state of satisfaction level, allowing for more rapid implementation of control measures and improvement strategies.

2.5 Optimal control techniques

The utilization of optimal improvement strategies in transportation systems has been widely explored in the literature, with a focus on enhancing passenger satisfaction, and improving overall system performance. By incorporating mathematical models and control theory, researchers have been able to develop efficient improvement strategies to optimize passengers' journey experiences within transit systems.

A recent research conducted by Luan & Corman (2022) demonstrated that the use of optimization techniques can help enhance passengers' comfort and minimize the impact of delays on train timetable . Additionally, the study revealed that incorporating flexibility into train schedules can offer more opportunities to better serve rail passengers. These findings provide valuable insights that can be applied to our study. Specifically, the research suggests that incorporating optimization techniques may contribute to enhancing passengers' experiences and attitudes towards the Addis Ababa Light Rail Transit system.

One of the seminal works in this area is presented by Rosa et al. (2018), who introduced the concept of optimal control of customer dynamics, which can be applied to transportation systems to improve passenger experience and system reliability. The study emphasized the evolution of the number of customers in some firm by considering an objective functional to show the effectiveness of the optimal control strategy.

Furthermore, the existence and formulation of optimal control problems have been extensively discussed in the work of (Evans, 2005; Levine, 1972; Van Long, 2015), providing a comprehensive foundation for understanding the theoretical underpinnings of optimal control. Their rigorous analysis of optimal control problems has contributed significantly to the development of systematic methodologies for formulating and solving optimal control problems in various domains.

Graphical representation of numerical results is essential for identifying and evaluating the most appropriate control strategy to implement within transportation systems.

Notable contributions in this regard include the work of Bryson et al. (2008) who demonstrated the application of numerical optimization techniques to analyze and compare different improvement strategies. Their research provided valuable insights into the practical implementation of optimal control techniques through numerical analysis, enabling researchers and practitioners to assess the effectiveness of various improvement strategies in real-world transportation scenarios.

Hailu and Teklu (2024b) applied optimal control theory to identify the most effective strategies for improving passenger satisfaction. They specified performance criterion and utilized the Pontryagin's maximum principle to evaluate the performance of a time varying system representing the dynamics of passengers' attitude.

In the context of transit systems, the study by Yuan et al. (2020) presented a comprehensive framework for modeling and optimizing passenger flow using advanced optimization software. Their research highlighted the potential benefits of optimal improvement strategies in addressing congestion and improving overall system performance, laying the groundwork for applying similar approaches to enhance passengers' travel experience within urban transit networks.

In summary, the literature on optimal control techniques provides a solid foundation for formulating and analyzing improvement strategies aimed at improving passengers' travel experience attributes within transit systems.

By leveraging mathematical models, rigorous theoretical frameworks, and numerical results, the authors opted to use Pontryagin's maximum principle to develop and evaluate optimal improvement strategies that can enhance the overall quality of service and operational efficiency in transportation networks.

CHAPTER THREE

MATERIALS AND RESEARCH METHODOLOGY

3.1 Introduction

The main aim of this research is to evaluate the service performance of AA-LRT by measuring passengers' satisfaction levels and forecasting satisfaction trends using compartmental modeling with optimal control methods. This section presents the basic preliminaries in the theory of differential equations and fractional calculus and describes the approaches used for collecting and analyzing data and creating a structured framework for modeling and analysis. It also explains the process of developing a compartmental model which includes defining assumptions and descriptions. Additionally, it includes analytical techniques to ensure that the model developed is well posed.

3.2 Preliminaries

This is to present the basic preliminaries in the theory of differential equations and fractional calculus that serve as a foundation in the use of differential equations to model passengers' evolution of satisfaction as time-varying system.

Definition 3.2.1 Let $\boldsymbol{x}$ be the state of a dynamical system. Then a generalized deterministic model involving $\boldsymbol{x}$ is given by

$$\frac{d\boldsymbol{x}}{dt} = f(\boldsymbol{x}, t; \mathfrak{p})$$

where $\boldsymbol{x} \in \mathbb{R}^n$, t represents time, and $\mathfrak{p} \in \mathbb{R}^m$ represents the parameters upon which the evolution of the system depends.

Definition 3.2.2 Let n be a positive integer such that $n-1 < \kappa \leq n$. The Caputo fractional order derivative of order κ for a function $f: [0, \infty) \to \mathbb{R}$ is defined as (Petrás, 2011)

$$^{C}D^{\kappa}f(t) = \frac{1}{\Gamma(n-\kappa)}\int_0^t f^n(\zeta)\frac{(t-\zeta)^{n-\kappa}}{t-\zeta}d\zeta, \forall t > 0 \quad (1)$$

Where $\Gamma(.)$ is the gamma function defined as $\Gamma(z) = \int_0^{\infty} t^{z-1}e^{-t}dt$ for complex numbers z with positive real part.

Definition 3.2.3 The Caputo fractional order integral of order κ for a function $f: [0, \infty) \to \mathbb{R}$ is defined as (Petrás, 2011)

$$^{C}I^{\kappa}f(t) = \frac{1}{\Gamma(\kappa)}\int_0^t f(\zeta)\frac{(t-\zeta)^{\kappa}}{t-\zeta}d\zeta, \forall t > 0$$

(2)

Definition 3.2.4 The Mittag–Leffler function with two parameters $\alpha > 0$ and $\beta > 0$ is provided as (Podlubny, 1999)

$$E_{\alpha,\beta}(z) = \sum_{i=1}^{\infty}\frac{z^i}{\Gamma(\alpha i+\beta)}, \forall z \in \mathbb{C} \quad (3)$$

The Mittag–Leffler function with one parameter (when $\beta = 1$) is the function defined as

$$E_{\alpha}(t) = \sum_{i=1}^{\infty}\frac{z^i}{\Gamma(\alpha i+1)}$$

Definition 3.2.5 A value x^* is identified as a steady state of the Caputo-fractional order model when (Biala & Khaliq, 2021; Li et al., 2009)

$$^{C}D^{\kappa}x(t) = f\big(t, x(t)\big), \kappa \in (0,1] \text{ if and only if } f(t, x^*) = 0 \text{ for all } t \geq 0$$

Definition 3.2.6 A steady state x_e is considered stable when initial conditions close to that point result in trajectories (time evolutions of x) that approach the steady state. On the contrary, if these trajectories move away from x_e , the steady state is unstable.

Definition 3.2.7 The reproduction number $\Re_{eff}$ is defined to be the average number of new passengers who transitioned from the less or quite satisfied to dissatisfaction compartment due to the impact of a dissatisfied passenger introduced in a population with a less or quite satisfied, during the period in which he or she was in that sense of perception.

Proposition 3.2.1 The Laplace transform operator for the Caputo fractional order derivative with order κ, $n-1<\kappa\le n,\ n\in\mathbb{N}$ of the function f is given by the following expressions

$$\mathcal{L}({}^{C}D^{\kappa}f)(s)=s^{\kappa}F(s)-\sum_{i=0}^{n-1}s^{\kappa-i-1}f^{i}(0) \text{ where } F(s)=\mathcal{L}(f(t)).$$

Proposition 3.2.2 The Laplace transform of two parameters function of Mittag Leffler is given by

$$\mathcal{L}\left(t^{\beta-1}E_{\alpha,\beta}(\lambda t^{\alpha})\right)=\frac{s^{\alpha-\beta}}{s^{\alpha}-\lambda}$$

Proposition 3.2.3 (Generalized Mean Value theorem). Suppose $h(t)\in\mathcal{L}[0,T_f]$ and ${}^{C}D^{\kappa}h(t)\in\mathcal{L}[0,T_f]$ for $\kappa\in(0,1]$ then the theorem states that $h(t)=h(0)+\frac{1}{\Gamma(\kappa)}$ ${}^{C}D^{\kappa}h(\zeta)t^{\kappa}$ where $\zeta\in[0,t]$, for each t such that $0<t\le T_f$.

The results presented below are based on Proposition 3.2.3.

a) The function h is non-decreasing for all $t\in[0,T_f]$, if ${}^{C}D^{\kappa}h(t)\ge 0$.

b) The function h is non-increasing for all $t\in[0,T_f]$, if ${}^{C}D^{\kappa}h(t)\le 0$.

Proposition 3.2.4 Suppose $g(t)\in L^{\infty}(\mathbb{R})\cap\mathcal{F}(\mathbb{R})$ and $\kappa\in\mathbb{R}$, $n-1<\kappa\le n, n\in N$ then the following conditions hold

a) $({}^{C}D^{\kappa}I^{\kappa}g)(t)=g(t)$.

b) $({}^{C}I^{\kappa}\,{}^{C}D^{\kappa}g)(t)=g(t)-\sum_{k=0}^{n-1}\frac{t^k}{k!}g^k(0)$.

c) Specifically, if $0<\kappa<1$, then $({}^{C}I^{\kappa}\,{}^{C}D^{\kappa}g)(t)=g(t)-g(0)$.

d) For a constant function $g(t)=b$ then ${}^{C}D^{\kappa}(b)=0$.

3.3 Description of study area

This thesis research is based on the AA-LRT system, which began operations in 2015 by Ethiopia Railways Corporation (ERC). It was the first light rail and rapid transit in eastern and sub-Saharan Africa at that time. The AA-LRT uses electric power, and the network consists of two routes, one running the East-West route from Ayat to Tor-Hailoch, and the other running the North-South route from Minilik II square to Akaki/Kality, as indicated in Figure 3.1.

The two routes share 2.7km of common track from Lideta to Meskel Square. These routes span a total length of 34.25km (North-South: 16.9km, East-West: 17.35km). The system was accommodating the daily transportation of approximately 34,689 passengers on average, playing a vital role in the city's transportation system at the time of this study. The route has a standard gauge width of 1.435m. The design speed is 80km/hr, with an operational speed of 70km/hr. It travels underground, through tunnels and bridges, and includes pedestrian and car crossings. There are a total of 39 stations, with 27 located at the ground level, 9 on the bridge, 2 on the partial bridge and one in the tunnel.

The light rail system features 39 ticket offices and 18 power stations, along with 22 elevators and 12 escalators. The transit system was operating with an average of 12 single tramcars per day when the survey was carried out. On average, passengers experience an average waiting time of 17 minutes (Debebe & Asteray, 2022). The figure below illustrates the routes of the light rail transit system.

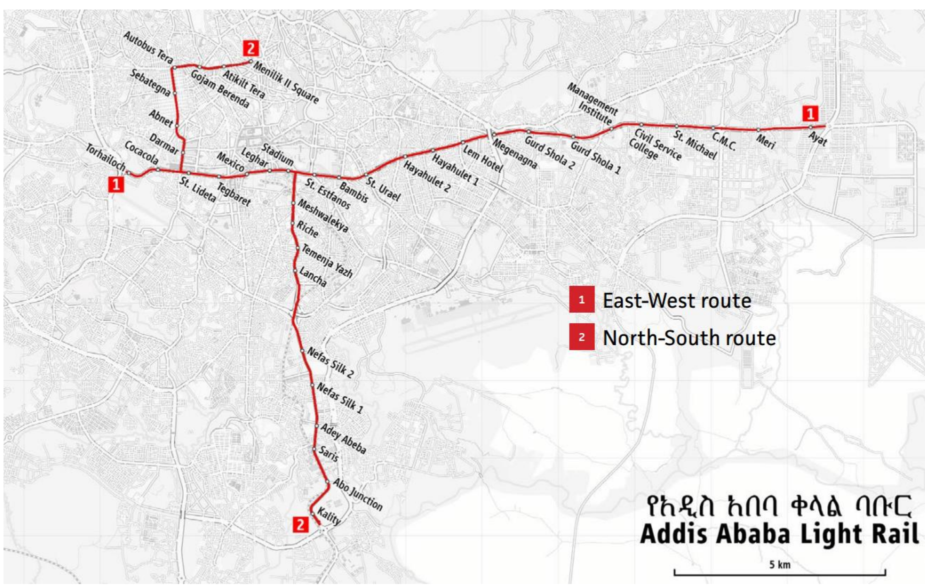

Figure 3-1: Addis Ababa light rail transit routes

3.4 Research method

The methodology used in this study is designed to address the research questions. The essential characteristics of passengers' travel experience are identified by reviewing existing literature and conducting focus group discussions (FGDs). The FGD involved individuals who had first-hand experience using the light rail transit in Addis Ababa. As a result, 15 AA-LRT service users (comprising 4 women, 3 men and 8 male youths) took part in the FGDs. Regarding their overall experience, the participants in the focus group discussion mainly shared their thoughts on the AA-LRT service in response to the questions posed to them.

They experienced incidences such as trains arriving late and not allowing passengers to board, as well as trains stopping mid-trip and later informing passengers that electric power was the cause.

Enquiries made during focus group deliberations include

1. Can you share your overall experience when using urban rail services?
2. What specific aspects of the travel experience do you find most satisfying or enjoyable?
3. Are there any particular features or amenities that improve your overall satisfaction with urban rail travel?
4. In what ways do you think the urban rail service could be improved to better meet your needs and expectations?
5. How do you feel about the cleanliness and maintenance of urban rail facilities?
6. Is there anything about the urban rail service that frustrates you or causes dissatisfaction?
7. Have you encountered any challenges with ticketing, information availability, or customer service during your urban rail travels?

8. How important do you consider factors such as punctuality, safety, comfort, and accessibility in your overall satisfaction with urban rail services?
9. Do you have any suggestions for enhancing the passenger experience on urban rail services?

Subsequently, a survey questionnaire (refer to Appendix A) is developed using the attributes identified from existing literatures and inputs from participants who took part in the FGD.

This questionnaire covered basic travel and demographic information, as well as various aspects of the train service such as reliability, service characteristics, cleanliness, comfort, information, customer service, safety, and security. Not all the selected travel attributes will have equal importance ratings. Some attributes will have a much bigger influence on whether a passenger is satisfied with the overall journey than others. To counter the variability response rates, respondents were requested to express ratings of importance and satisfaction for each service attribute using a Likert scale, with options ranging from "Very satisfied" to "Very Dissatisfied" for satisfaction ratings and from "Unimportant" to "Very important" for importance ratings.

The collected survey data is then analyzed using passenger-specific overall level of satisfaction to segment the total passengers into different compartments, showing distinct levels of satisfaction. These compartments include neutral(less or quite satisfied), satisfaction, and dissatisfaction. To effectively capture the dynamics of passengers' satisfaction and gain valuable insights, a compartmental model is formulated. There are two motivations for choosing compartmental modeling. First, compartmental modeling is appropriate when predicting the future trend of the system is desired.

Second, compartmental modeling approach involves assumptions and approximations that give researchers autonomy to test the model and refine the solution until it effectively addresses the problem ("GAIMME — Guidel. Assess. Instr. Math. Model. Educ.," 2019). Analytical methods were utilized to check some qualitative properties such as the non-negativity and boundedness of the solutions, as well as to establish the existence and uniqueness of solutions of the model formulated. Additionally, numerical techniques are applied to track satisfaction trends, and optimal improvement strategies are identified through the use of optimal control theory.

3.4.1 Data sampling and collection

The population of this study consists of the entire passengers of AA-LRT. To obtain a representative sample from the entire passengers, the number of passengers per day was first identified from 10/2023 to 02/2024 records of passengers' flow of AA-LRT.

The total number of passengers required for the survey was determined for both lines, and then distributed to specific stations. A combination of simple random (for the survey) and purposive (for the station) sampling techniques was employed. Out of the 39 stations available, 13 stations were selected, comprising five stations from East-West route, three communal stations and five stations from North-South route, as depicted in Figure 3.2. The selected stations offer a good representation of all stations, providing a comprehensive overview of the corridor's characteristics.

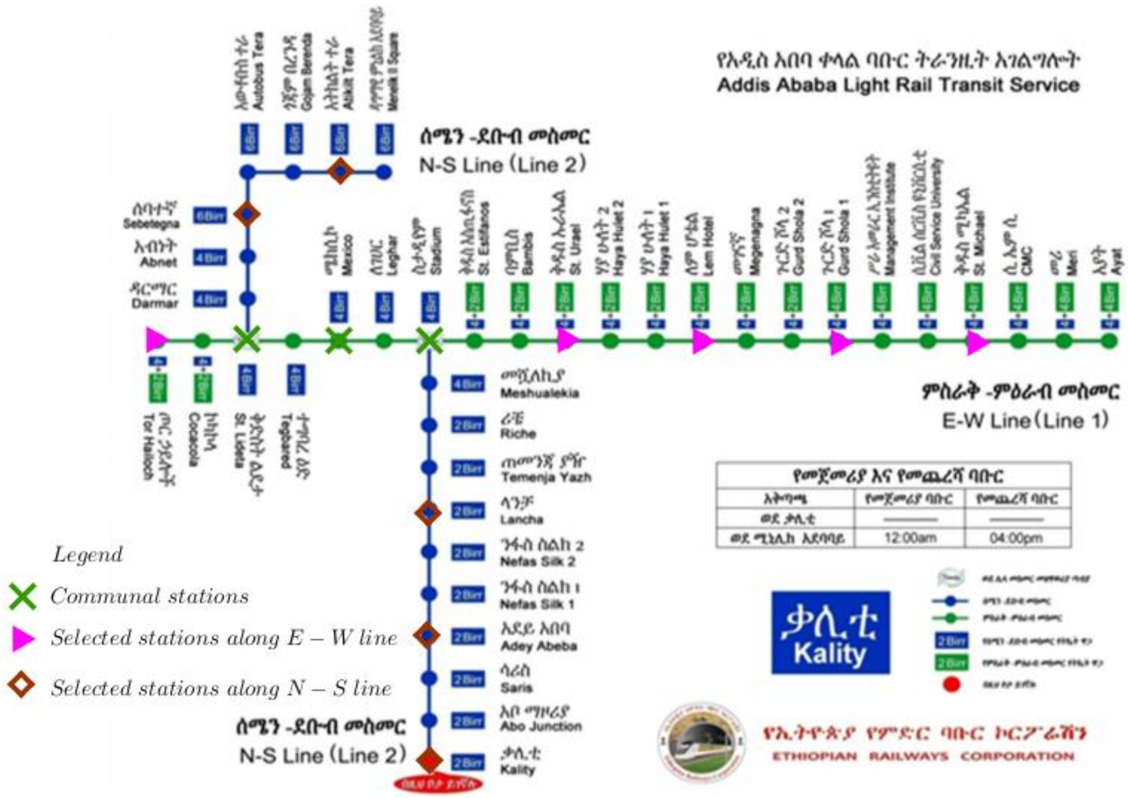

Figure 3-2: Selected stations along the AA-LRT route map

The first step involved calculating the sample size of the total population of LRT users and then distributing it across both lines to determine the sample size at each station. The Cochran's sample size formula, a commonly used formula in survey sampling, was utilized to calculate the sample size required for obtaining a representative sample of the population with 95% level of confidence and ±5% desired margin of error.

AA-LRT's documents from transport center showed that the numbers of passengers were recorded on a monthly basis. The mean daily passenger count was calculated for the period spanning from October to February in the year 2023/2024. A simple random sampling technique was used to approach passengers waiting for the train at selected stations.

Table 3-1: Average number of passengers

Month	*Average number of passengers per day*		*Average number of passengers per day*
	N-S line	*E-W line*	
October	15570	21923	37493
November	8431	23341	31772
December	15461	22590	38051
January	14779	19918	34697
February	13380	18053	31433
Estimated average number of passengers per day	13524	21165	34689

The data from the AA-LRT's transport center indicated that the estimated average number of passengers per day is **34689.**

Cochran's sample size formula:

It is recommended to be used for studies with infinite populations (Cochran 1977).

$$n_0 = \frac{z^2 \times p \times (1-p)}{e^2}$$

n: sample size, $e = 0.05$, desired level of precision, the margin of error, $p = 05$, the fraction of the population that displays the attribute, $z = 1.96$, the z-value, extracted from a z-table.

The Cochran's sample size, $n_0 = 384.146$

Cochran's modified formula for finite populations

The sample size for the population size can be computed using the formula

$$n = \frac{n_0}{1 + \frac{n_0 - 1}{N}}$$

where $n_0 = 384.146$ is the Cochran's sample size and $N = 34689$ is the size of the population.

The sample size for the study is 381 passengers.

The authors proportionally divided the **381** samples into **150** for the N-S route, **81** for the communal stations and **150** for E-W route.

The questionnaires were offered to the selected passengers about to board a train at stations, and this takes place between 02 April and 08 April 2024.

3.4.2 Satisfaction Index (SI) and Importance Performance Analysis (IPS)

The satisfaction index is a measure used to assess the level of satisfaction attained by the transportation service provider in relation to the overall experience of passengers during their journey. The satisfaction index is calculated by averaging the satisfaction and importance ratings in terms of various attributes of travel experience. This approach allows for a systematic evaluation of passengers' satisfaction, but it does not consider variations in user perceptions.

The evaluation of satisfaction index and the analysis of importance and performance are performed to assess the satisfaction levels of passengers and highlight the key areas that need immediate attention from transit operators.

Due to the varying importance of attributes for passengers, a satisfaction index that relies solely on satisfaction and importance ratings may not be appropriate to account for such discrepancies. A new customer satisfaction index suggested by L. Eboli and G. Mazzulla (Eboli & Mazzulla, 2009) was used to identify passengers' travel experience attributes critical to enhancing passengers' travel experience.

This attribute-weight is calculated by the following formula:

$$HPSI_j = S_j^* W_j$$

where S_j is the mean of satisfaction rates on the $j-$ attribute expressed by passengers corrected according to the deviation of the rates from the average value. S_j is calculated by the following formula:

$$S_j^* = S_j \frac{\frac{s_j}{\delta_j^2}}{\sum_{j=1}^{32} \frac{s_j}{\delta_j^2}} \times 32, \text{ where } N = 32 \text{ is the number of attributes}$$

W_j is the mean of the importance rates on the $j -$ attribute expressed by passengers corrected according to the variance of the importance rates.

W_j is calculated according to the formula

$$W_j = \frac{\frac{I_j}{\delta_j^2}}{\sum_{j=1}^{32} \frac{I_j}{\delta_j^2}}$$

This attribute-weight helps the researchers to identify which attributes are likely to have least impacts on the overall satisfaction of passengers in the AA-LRT for the month of February.

Additionally, the overall passenger satisfaction index (HPSI), with 'H' representing heterogeneous, is computed using the following formula

$$HPSI = \sum_{j=1}^{32} S_j W_j$$

Importance-Performance analysis (IPA), first proposed by Martilla & James (1977), provides valuable information on the service quality attributes that a company should prioritize to enhance customer satisfaction. Typically, satisfaction survey data is employed to create a matrix with two dimensions: The average score for importance is represented on the x-axis, while performance (average satisfaction) is shown on the y-axis. Many studies have utilized IPA. For instance, Aghajanzadeh et al. (2022) conducted a study to assess attributes influencing user satisfaction with metro services.

The average satisfaction level for each attribute is determined by the formula: $\overline{x_i} = \frac{\sum_{i=1}^{N} x_i}{N}$, where x_i represents the satisfaction rating given by the i^{th}participant, and N is the total number of participants. Similarly, the average importance level for each attribute is computed as $\overline{y_i} = \frac{\sum_{i=1}^{N} y_i}{N}$, where y_i is the importance rating provided by the i^{th}respondent, and N is the total number of respondents.

The average of satisfaction and importance mean ratings for the whole attributes are provided by $\overline{\overline{S_i}} = \frac{\sum_{i=1}^{N} \bar{x}_i}{N}$ and $\overline{\overline{I_i}} = \frac{\sum_{i=1}^{N} \bar{y}_i}{N}$, respectively.

3.4.3 Assumptions and description of the models

The intricacy of a system model can be evaluated based on the underlying assumptions. Listed below are the underlying assumptions that are reasonable about the missing information.

1. The daily operation of the AA-LRT system is performed under the same condition.
2. The targeted population is thoroughly mixed; that is, the less or quite satisfied individuals are evenly distributed across the entire population. Consequently, a random sample of passengers can be considered. The authors need these assumptions for our model to be useful in the classification process.
3. The number of passengers in each compartment changes only due to the transfer of passengers in or out of the compartment.
4. All selected passengers are either "less or quite satisfied", "dissatisfied", or "satisfied". There are no other types of passengers to be considered.
5. The number of new passengers entering the system is equal to the number of passengers leaving off the system.
6. In the survey conducted to measure satisfaction with AA-LRT service, many of the attributes the authors identified are found to be key attributes.

The reason behind making these assumptions is primarily driven to formulate a mathematical model that is tractable in a deterministic framework using ordinary differential equations, and inspired by Agree-Disagree model that represents public opinion (Bidah et al., 2020).

3.4.4 Descriptions of the models

Because there are various studies where satisfaction ratings that show the present status of passengers' satisfaction towards AA-LRT service are involved, our model explains how satisfaction changes over time in relation to the travel experience features of the AA-LRT service and their interactions. The population the authors are interested in is divided into three distinct compartments, each of which is explained below.

1. "Neutral" "N" individuals refer to the number of passengers who have had mixed travel experiences resulting in feelings of reduced or moderate satisfaction. They exhibit weak or indifferent attitudes towards their overall travel experience.

2. "Dissatisfied" "D" individuals are individuals who express dissatisfaction. They have encountered notable problems or inconveniences during their rail travel experience. This category has unfavorable opinions about what they encountered during their travels.
3. "Satisfied" "S" individuals represent passengers who may have encountered positive experiences, witnessed improvements in the rail system, or received satisfactory resolutions to their complaints. Satisfied passengers will likely to continue to use the service. This category has a favorable outlook on their travel experiences.

If $P(\text{t})$ is the total number of passengers at a given time t,

$$P(t) = N(t) + D(t) + S(t)$$

Each passenger evaluates the various aspects of his/her travel experience based on his/her own perspective, and passengers may be influenced by the satisfaction or dissatisfaction expressed by others. Accordingly, each passenger is categorized as "satisfied," "dissatisfied," or "neutral." Passengers who have no strong feelings (either positive or negative) about their journey on the LRT can be classified as neutral. Passengers can transition from one compartment to another by their interactions/influences with other passengers. Every passenger has also developed positive or negative attitude due to his/her personal travel experience.

When neutral individuals interact with satisfied ones and exchange their travel experiences, the neutral individuals can be influenced and begin to perceive the service as having satisfying travel elements and subsequently become satisfied at a rate α_1. Similarly, the neutral individuals can also be influenced by passengers experiencing dissatisfaction with the transit service at a rate α_2. A passenger experiencing satisfaction with the service may be influenced by someone with a feeling of dissatisfaction at a rate of β_2, or a passenger with a feeling of dissatisfaction may be persuaded by satisfied passengers' experience of improvements at a rate of γ_2.

Satisfied passengers may have experienced both positive and negative aspects of travel experience attributes, which places them in the neutral category at a rate of β_1. And, dissatisfied passengers become neutral at a rate of γ_1. Despite the presence of other contributing factors such as family dynamics, biases, media, economic status, and occupations that contribute to the development of passenger satisfaction, the authors opted to focus solely on developing a simplified mathematical model by disregarding these additional factors.

A flowchart schematically outlining the interactions between the different categories within the system is depicted in Figure 3.3. Each circle in the flowchart represents a compartment labeled with the name of the class. Arrows show the transition of individuals among the compartments.

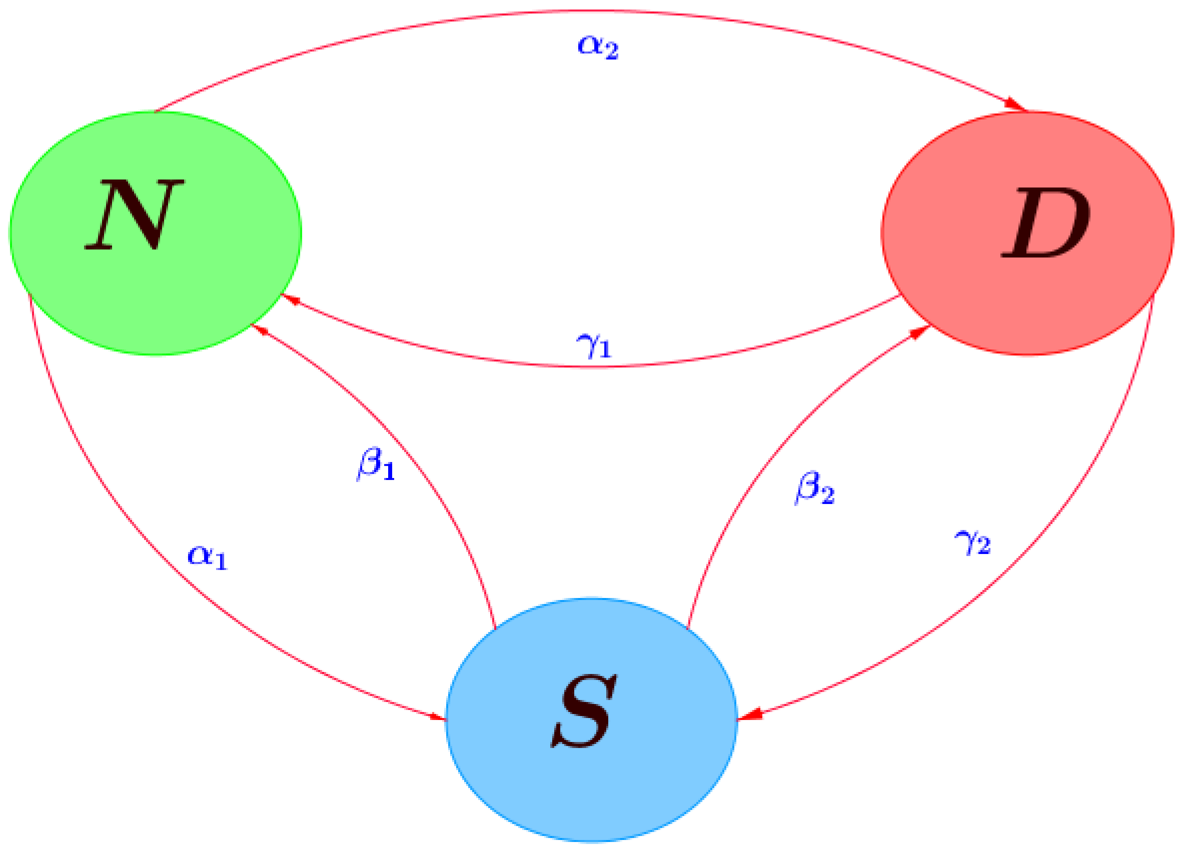

Figure 3-3: Flowchart of NDS model

All the parameters have the unit of inverse time.

3.4.5 The fractional order model

A popular construct in mathematical modeling is that of a compartment. In its simplest form, a compartment consists of an aggregate, possibly of individuals, and different compartments can interact through exchange, or controlling each other's inputs or outputs.

The change in the number of passengers in each compartment equals the difference between what transfers in and what transfers out from the compartment. The net rate of change, which is the difference between input and output, is the basis for writing the governing differential equation.

The deterministic integer order model that represents the time varying system of passengers' satisfaction level is a system of nonlinear integer-order ordinary differential equations shown below

$$\begin{aligned}
\frac{dN(t)}{dt} &= -\alpha_1 \frac{S(t)N(t)}{P(t)} - \alpha_2 \frac{D(t)N(t)}{P(t)} + \gamma_1 D(t) + \beta_1 S(t) \\
\frac{dD(t)}{dt} &= \alpha_2 \frac{D(t)N(t)}{P(t)} + \beta_2 \frac{S(t)D(t)}{P(t)} - \gamma_2 \frac{S(t)D(t)}{P(t)} - \gamma_1 D(t) \\
\frac{dS(t)}{dt} &= \alpha_1 \frac{S(t)N(t)}{P(t)} + \gamma_2 \frac{S(t)D(t)}{P(t)} - \beta_2 \frac{S(t)D(t)}{P(t)} - \beta_1 S(t)
\end{aligned} \tag{1}$$

subject to the constraints $N(0) \geq 0$, $D(0) \geq 0, S(0) \geq 0$.

In the system dynamics, N, D and S are the state variables that characterize the levels of satisfaction of passengers overtime.

Here, the authors reformulated the classic model which represents how passengers' satisfaction changes overtime. This formulation involves utilizing Caputo fractional order derivative, which enables us to account for memory effects and enhance our insights into the evolution of passengers' satisfaction over time. The fractional order model where κ represents the Caputo fractional order within the range of [0,1] can be observed in Equation (2).

$$\begin{aligned}
{}^{C}D^{\kappa}(N(t)) &= -\alpha_1^{\kappa} \frac{S(t)N(t)}{P(t)} - \alpha_2^{\kappa} \frac{D(t)N(t)}{P(t)} + \gamma_1^{\kappa} D(t) + \beta_1^{\kappa} S(t) \\
{}^{C}D^{\kappa}\big(D(t)\big) &= \alpha_2^{\kappa} \frac{D(t)N(t)}{P(t)} + \beta_2^{\kappa} \frac{S(t)D(t)}{P(t)} - \gamma_2^{\kappa} \frac{S(t)D(t)}{P(t)} - \gamma_1^{\kappa} D(t) \\
{}^{C}D^{\kappa}(S(t)) &= \alpha_1^{\kappa} \frac{S(t)N(t)}{P(t)} + \gamma_2^{\kappa} \frac{S(t)D(t)}{P(t)} - \beta_2^{\kappa} \frac{S(t)D(t)}{P(t)} - \beta_1^{\kappa} S(t)
\end{aligned} \tag{2}$$

where $N(0) \geq 0, D(0) \geq 0$, and $S(0) \geq 0$ (3)

The operator ${}^{C}D^{\kappa}$ for fractional derivative has a dimension of $(time)^{-\kappa}$ instead of $(time)^{-1}$. Therefore, in accordance with dimensional analysis, all the parameters present on the right side of the fractional order model must have power κ. As the fractional order model is nonlinear, there are no techniques available to determine the analytical solution. Therefore, the solution is provided through numerical methods.

To simplify the analysis, the authors consider the proportions of passengers who were neutral, dissatisfied, and satisfied.

If the fractions of the N, D and S are represented in the population as $n = N/P$, $d = \frac{D}{P}$ and $s = \frac{S}{P}$, respectively, and replace N by n, D by d and S by s, the model equation (2) can be written as

$$\begin{aligned} {}^{C}D^{\kappa}\big(n(t)\big) &= -\alpha_1^{\kappa} sn - \alpha_2^{\kappa} dn + \gamma_1^{\kappa} d + \beta_1^{\kappa} s \\ {}^{C}D^{\kappa}\big(d(t)\big) &= \alpha_2^{\kappa} dn - \gamma_1^{\kappa} d - \gamma_2^{\kappa} sd + \beta_2^{\kappa}\, ds \\ {}^{C}D^{\kappa}\big(s(t)\big) &= \alpha_1^{\kappa} sn + \gamma_2^{\kappa} ds - \beta_1^{\kappa} s - \beta_2^{\kappa} ds \end{aligned} \tag{4}$$

where $n(0) \geq 0, d(0) \geq 0$, and $s(0) \geq 0$. (5)

Since $P(t) = N(t) + D(t) + S(t)$, $n(t) + d(t) + s(t) = 1$.

The system of equations in model (4) can be simplified into the following differential equations.

$$\begin{aligned} {}^{C}D^{\kappa}\big(d(t)\big) &= \alpha_2^{\kappa} d(1 - s - d) - \gamma_1^{\kappa} d - \gamma_2^{\kappa} sd + \beta_2^{\kappa} ds \\ {}^{C}D^{\kappa}\big(s(t)\big) &= \alpha_1^{\kappa} s(1 - s - d) + \gamma_2^{\kappa} ds - \beta_1^{\kappa} s - \beta_2^{\kappa} ds \end{aligned}$$

which can be simplified to

$$\begin{aligned} {}^{C}D^{\kappa}\big(d(t)\big) &= \alpha_2^{\kappa} d(1 - s - d) - \chi sd - \gamma_1^{\kappa} d \\ {}^{C}D^{\kappa}\big(s(t)\big) &= \alpha_1^{\kappa} s(1 - s - d) + \chi sd - \beta_1^{\kappa} s \end{aligned}$$

where $\chi = \gamma_2^{\kappa} - \beta_2^{\kappa}$.

As non-negative and bounded values of populations are relevant, the authors establish some qualitative properties namely the non-negativity and boundedness of solutions of the fractional order model. Additionally, the authors analyze the existence and uniqueness of the solutions of the Caputo fractional order model given in Equation (2) with initial conditions given in Equation (3).

3.4.6 Well posedness of the model

Theorem 3.4.6.1 *The solutions of the system (shown in Equation (2)) are non-negative for non-negative initial conditions (shown in Equation (3)). That is, the solutions* $(n(t), d(t), s(t))$ *for the model (Equation (2)) is positive invariant in* $\mathbb{R}_+^3$.

Proof: *Appendix B*

Theorem 3.4.6.2 The solutions of the system (shown in Equation (2)) are uniformly bounded at all times for non-negative initial conditions (shown in Equation (3)).

Proof: *Appendix B*

To show the existence and uniqueness of solution of the model, rewrite the fractional order model (Equation (2))) together with initial conditions (Equation (3))) in the vector form

$$\begin{cases} {}^{C}D^{\kappa}X(t) = C\big(t, X(t)\big), 0 < t < T < \infty \\ X = X_0 \end{cases} \tag{6}$$

where $X(t) = \big(N(t), D(t), S(t)\big)^{t}$ and

$$G\big(t, Y(t)\big) = \begin{bmatrix} C_1(t, N(t)) \\ C_2(t, D(t)) \\ C_3(t, S(t)) \end{bmatrix} = \begin{bmatrix} -\alpha_1^{\kappa}\frac{S(t)N(t)}{P(t)} - \alpha_2^{\kappa}\frac{D(t)N(t)}{P(t)} + \gamma_1^{\kappa}D(t) + \beta_1^{\kappa}S(t) \\ \alpha_2^{\kappa}\frac{D(t)N(t)}{P(t)} + \beta_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \gamma_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \gamma_1^{\kappa}D(t) \\ \alpha_1^{\kappa}\frac{S(t)N(t)}{P(t)} + \gamma_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \beta_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \beta_1^{\kappa}S(t) \end{bmatrix} \tag{7}$$

Using condition (c) of proposition 3.2.4, the authors establish the subsequent integral equations

$$\begin{aligned} N(t) - N(0) &= {}^{C}I^{\kappa}\left(-\alpha_1^{\kappa}\frac{S(t)N(t)}{P(t)} - \alpha_2^{\kappa}\frac{D(t)N(t)}{P(t)} + \gamma_1^{\kappa}D(t) + \beta_1^{\kappa}S(t)\right) \\ D(t) - D(0) &= {}^{C}I^{\kappa}\left(\alpha_2^{\kappa}\frac{D(t)N(t)}{P(t)} + \beta_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \gamma_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \gamma_1^{\kappa}D(t)\right) \\ S(t) - S(0) &= {}^{C}I^{\kappa}\left(\alpha_1^{\kappa}\frac{S(t)N(t)}{P(t)} + \gamma_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \beta_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \beta_1^{\kappa}S(t)\right) \end{aligned} \tag{8}$$

From Equation (8), the authors determine the following

$$\begin{aligned} N(t) &= N(0) + \frac{1}{\Gamma(\kappa)}\int_0^t (t-\xi)^{\kappa-1}\, C_1\big(\xi, N(\xi)\big)d\xi \\ D(t) &= D(0) + \frac{1}{\Gamma(\kappa)}\int_0^t (t-\xi)^{\kappa-1}\, C_2\big(\xi, D(\xi)\big)d\xi \\ S(t) &= S(0) + \frac{1}{\Gamma(\kappa)}\int_0^t (t-\xi)^{\kappa-1}\, C_3\big(\xi, S(\xi)\big)d\xi \end{aligned} \tag{9}$$

Employing the Picard iterative method numerically, the authors obtain the following

$$
\begin{aligned}
N_n(t) &= \frac{1}{\Gamma(\kappa)}\int_0^t (t-\xi)^{\kappa-1}\, C_1\big(\xi, N_{n-1}(\xi)\big)d\xi \\
D_n(t) &= \frac{1}{\Gamma(\kappa)}\int_0^t (t-\xi)^{\kappa-1}\, C_2\big(s, D_{n-1}(\xi)\big)d\xi \\
S_n(t) &= \frac{1}{\Gamma(\kappa)}\int_0^t (t-\xi)^{\kappa-1}\, C_3\big(\xi, S_{n-1}(\xi)\big)d\xi
\end{aligned} \tag{10}
$$

Ultimately, the initial value problem (10) is transformed to

$$X(t) = X(0) + \frac{1}{\Gamma(\kappa)}\int_0^t C(\xi, X(\xi))(t-\xi)^{\kappa-1}\, d\xi$$

Then the authors establish the following Lemmas to prove existence uniqueness theorem.

Lemma 3.4.6.1 The vector function $C(t,X)$ is Lipschitz in the variable X on a set $[0,T]\times\mathbb{R}_+^3$ with some Lipschitz constant $\mathfrak{K}$.

***Proof**: Appendix B*

Theorem 3.4.6.3 (Existence-Uniqueness theorem) The initial value problem that constitute Equation (2) and *initial states $N(0)\geq 0, A(0)\geq 0,$ and $S(0)\geq 0$ has a unique solution $X(t)\in\Omega(I)$, the set of bounded continuous functions on I with norm $\|X\| = sup\{|X(t)|: t\in I\}$.*

***Proof**: Appendix B*

Therefore, the fractional order model (Equations (2) and (2)) is well posed.

3.4.7 Qualitative analysis

Qualitative analysis of a compartmental model plays a crucial role in understanding the behavior and dynamics of complex systems. By studying the steady states and their local stability, the authors can gain valuable insights into the long-term behavior of the system.

3.4.7.1 Steady states

This section focuses on establishing certain properties of the solutions to the NDS model. Quantitatively analyzing the steady states (or simply the equilibria) of the model allows us to understand the long-term behavior of passengers' attitudes and identify potential states of stability. The steady states are determined as the values $\mathrm{x_e}$ of x which will remain unchanged for all $t > t_0$ at a specific time $\mathrm{t_0}$.

This means that all the model variables will remain steady, and as a result, will equal zero. This condition can be used to determine the steady states of the system.

It is crucial to observe that the fractional order model described in Equation (2) has two types of steady-state solutions: a dissatisfaction-free steady state (DFSS) and a dissatisfaction-persistent steady state (DPSS), denoted by $E^0 = (N^0, D^0, S^0)$ and $E^* = (N^*, D^*, S^*)$ respectively. The dissatisfaction-free steady state is the solution of

$$^{C}D^{\kappa}N(t) = \ ^{C}D^{\kappa}D(t) = \ ^{C}D^{\kappa}S(t) = 0, \text{for each } t \geq 0$$

In a state of equilibrium without dissatisfaction, $D^0 = 0$.

This yields

$$N^0 = \frac{\beta_1^{\kappa}}{\alpha_1^{\kappa}} P$$

$$S^0 = \frac{P(\alpha_1^{\kappa} - \beta_1^{\kappa})}{\alpha_1^{\kappa}}$$

where N^0 and S^0 represent the number of neutral and satisfied passengers, respectively, in the absence of dissatisfied passengers.

Thus, for the evolutionary system governed by Equation (2), the dissatisfaction-free steady state is

$$E^0 = \left(\frac{\beta_1^{\kappa}}{\alpha_1^{\kappa}} P, 0, \frac{P(\alpha_1^{\kappa} - \beta_1^{\kappa})}{\alpha_1^{\kappa}}\right) \tag{11}$$

In terms of $n = N/P$, $d = \frac{D}{P}$ and $s = \frac{S}{P}$, the dissatisfaction-free steady state is

$E^0 = \left(\frac{\beta_1^{\kappa}}{\alpha_1^{\kappa}}, 0, 1 - \frac{\beta_1^{\kappa}}{\alpha_1^{\kappa}}\right)$, which exists provided that $\alpha_1^{\kappa} > \beta_1^{\kappa}$ holds.

To evaluate the dissatisfaction-persistent steady state denoted by $E^* = (d^*, s^*)$, the authors set $^{C}D^{\kappa}\big(n(t)\big) = 0$, $^{C}D^{\kappa}\big(d(t)\big) = 0$ and $^{C}D^{\kappa}\big(s(t)\big) = 0$ and simplify the resulting equations. Then, the authors obtain the following $d^* = -\frac{\chi\alpha_1^\kappa - \chi\beta_1^\kappa + \alpha_1^\kappa\gamma_1^\kappa - \alpha_2^\kappa\beta_1^\kappa}{\chi(\chi - \alpha_1^\kappa + \alpha_2^\kappa)}$ and $s^* = \frac{\chi\alpha_2^\kappa - \chi\gamma_1^\kappa + \alpha_1^\kappa\gamma_1^\kappa - \alpha_2^\kappa\beta_1^\kappa}{\chi(\chi - \alpha_1^\kappa + \alpha_2^\kappa)}$.

Therefore, for the system governed by Equation (2), the dissatisfaction-persistent equilibrium is $E^* = (n^*, d^*, s^*)$, where $n^* = 1 - d^* - s^*$.

3.4.7.2 Local stability of dissatisfaction steady state

The local stability of the dissatisfaction-free steady state is examined by its effective reproduction number denoted by $\Re_{eff}$, which is determined by using the next generation matrix (Van Den Driessche & Watmough, 2002). The number $\Re_{eff}$ indicates the potential for dissatisfaction to develop within the population. When $\Re_{eff}$ is less than unity, a small number of dissatisfied passengers introduced into a neutral population will generally influence others to develop a feeling of dissatisfaction. Conversely, if $\Re_{eff}$ exceeds unity, the number of dissatisfied passengers will grow, and dissatisfaction will evolve throughout the population. The "dissatisfaction" compartment, the only compartment that contains passengers with a feeling of dissatisfaction, will be decoupled from the system. The equation representing this compartment is

$$
\begin{aligned}
{}^{C}D^{\kappa}\big(d(t)\big) = {}^{C}D^{\kappa}\big(d(t)\big) &= \alpha_2^\kappa dn - \gamma_1^\kappa d - \gamma_2^\kappa sd + \beta_2^\kappa\, s\, d \\
&= (\alpha_2^\kappa n + \beta_2^\kappa\, s\,)d - (\gamma_1^\kappa + \gamma_2^\kappa s)d
\end{aligned}
$$

which is of the form ${}^{C}D^{\kappa}\big(d(t)\big) = (F - V)d(t)$, where $F(\boldsymbol{x}, \boldsymbol{y}) = \alpha_2^\kappa n + \beta_2^\kappa\, s$ and $V(\boldsymbol{x}, \boldsymbol{y}) = \gamma_1^\kappa + \gamma_2^\kappa s$.

Evaluating F and V at the dissatisfaction free steady state $E^0 = \left(\frac{\beta_1^\kappa}{\alpha_1^\kappa}, 0, 1 - \frac{\beta_1^\kappa}{\alpha_1^\kappa}\right)$, the authors obtain $F = \frac{\alpha_2^\kappa\beta_1^\kappa + \beta_2^\kappa\alpha_1^\kappa - \beta_1^\kappa\beta_2^\kappa}{\alpha_1^\kappa}$ and $V = \frac{\alpha_1^\kappa\gamma_1^\kappa + \gamma_2^\kappa\alpha_1^\kappa - \gamma_2^\kappa\beta_1^\kappa}{\alpha_1^\kappa}$.

The next generation matrix, $FV^{-1} = \frac{\alpha_2^\kappa\beta_1^\kappa + \beta_2^\kappa\alpha_1^\kappa - \beta_1^\kappa\beta_2^\kappa}{\alpha_1^\kappa\gamma_1^\kappa + \gamma_2^\kappa\alpha_1^\kappa - \gamma_2^\kappa\beta_1^\kappa}$, and the effective reproduction number, $\Re_{eff} = \frac{\alpha_2^\kappa\beta_1^\kappa + \beta_2^\kappa\alpha_1^\kappa - \beta_1^\kappa\beta_2^\kappa}{\alpha_1^\kappa\gamma_1^\kappa + \gamma_2^\kappa\alpha_1^\kappa - \gamma_2^\kappa\beta_1^\kappa}$.

Theorem 3.3.7.2.1 The dissatisfaction-free steady state of the model (Equation (2)) is locally asymptotically stable if $\Re_{eff}$ *is less than unity. Otherwise, it is unstable.*

Proof: *Appendix B*

Theorem 3.3.7.2.2 The dissatisfaction-persistent steady state of the model (Equation (2)) is locally asymptotically stable if $\Re_{eff}$ *is greater than unity. Otherwise, it is unstable.*

Proof: *Appendix B*

3.5 Optimal control problem (OCP)

Through the application of optimal control techniques, this research aims to provide insights into the optimization of passengers' travel experience within the transit system. The authors formulate the optimal control problem associated with the dynamics of passengers' satisfaction level within the transit system.

- Formulation of the Optimal Control Problem:

The initial step involved formulating the optimal control problem corresponding to the dynamics of passengers' satisfaction level within the transportation system. This included defining the state variables, control variables, constraints, and objective function to be optimized.

- Selection of Improvement Strategies:

Improvement strategies were selected based on the analysis of the importance-performance analysis conducted to identify potential enhancements within the transit system. These strategies aim to improve passengers' travel experience by addressing key performance indicators.

- Definition of Performance Criterion:

Performance criterion, in the form of a cost function, was defined to quantitatively measure the impact of each improvement strategy on enhancing passengers' satisfaction levels. This criterion serves as the basis for evaluating the effectiveness of the proposed enhancements.

- Application of Pontryagin’s Maximum Principle:

The Pontryagin’s maximum principle, a fundamental optimization tool in optimal control theory, was employed to determine the optimal enhancement strategies that maximize passengers' satisfaction levels while considering system dynamics and constraints. By applying this principle, the most effective control strategies were identified.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Demographic and travel characteristics of sample

Although the population is evenly spread between male and female, the majority of the respondents is female (64% of the sample). Most of the respondents were younger than 21 years (67%) while 13% were over 46 years of age. The most frequent purpose of respondents was to travel to work in the private or public sector (66%) while 25% to school/university. In terms of education level, about 95% of the respondents were holders of Bachelor degree and lower. Figure 4.1 depicts the summary of demographic and travel information.

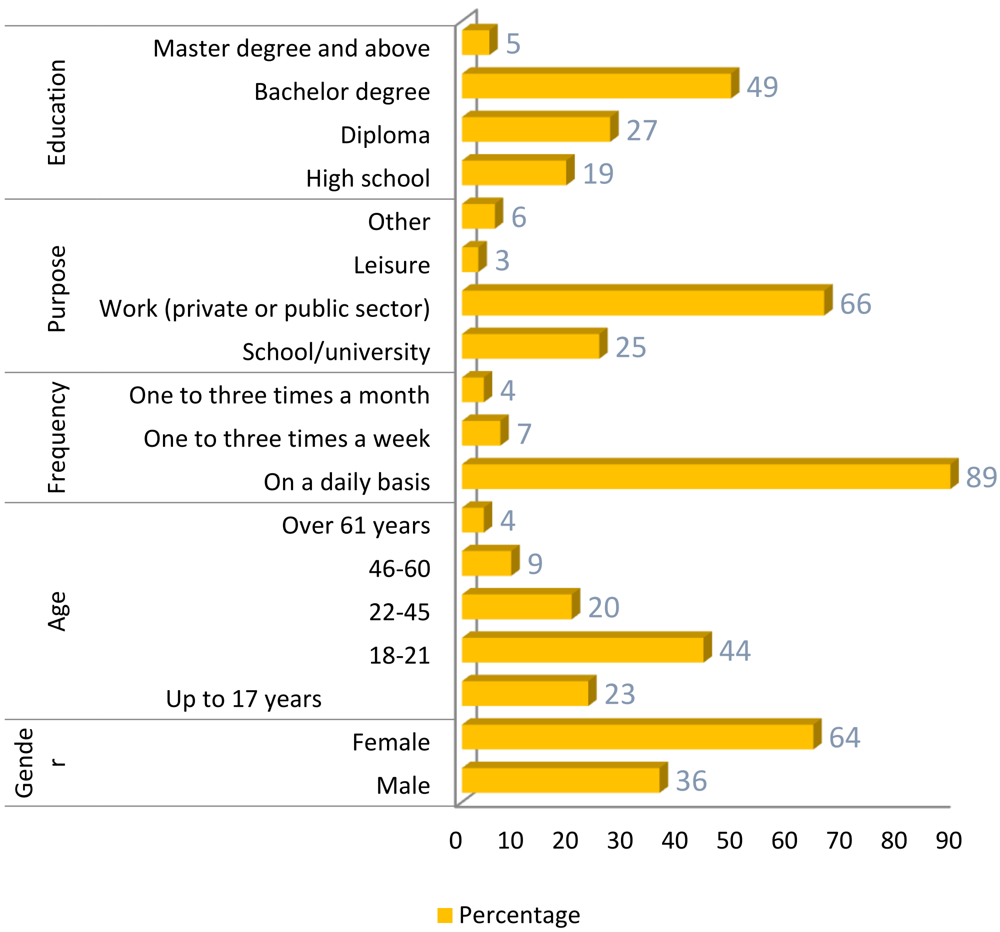

Figure 4-1: Demographic information and travel characteristics

4.2 Reliability test

The reliability test of Cronbache's alpha value is the most commonly used test for assessing the internal consistency of a questionnaire. Thus, Cronbache's alpha value yields a coefficient ranging from 0 to 1. Cronbache's alpha value above 0.7 is considered the minimum acceptable value. The questionnaire's reliability was assessed using Cronbache's alpha, and Table 4.1 shows the details of the reliability values for each aspect of the travel experience. Consequently, the questionnaire was deemed reliable and can be used to measure the passengers' satisfaction.

Table 4-1: Cronbache's alpha values

Aspect	*Number of attributes*	*Cronbache's' alpha*	
		Satisfaction ratings	*Importance ratings*
Reliability	*4*	*0.794*	*0.847*
Service characteristics	***6***	*0.743*	*0.718*
Cleanliness	*3*	*0.746*	*0.715*
Comfort	*6*	*0.758*	*0.773*
Information	*5*	*0.771*	*0.744*
Customer service	*4*	*0.740*	*0.734*
Safety and security	*4*	*0.858*	*0.754*
Overall	*32*	*0.759*	*0.702*

Table 4.2 presents the mean score and variability of satisfaction and importance ratings. These measures help to take into account the heterogeneity among passengers when assessing the quality of service provided by AA-LRT. By observing passengers' evaluations of expected quality (average importance ratings), it seems that passengers find the service attributes to be significant. The majority of service attributes demonstrate greater consistency when compared to the evaluations of perceived quality, as indicated by the average satisfaction ratings. Further, Table 4.2 presents data indicating that more than 21 attributes are rated as important by passengers, with average importance ratings exceeding 2.5. Notably, availability of schedules and maps on train, smoothness of ride and stops, and provision of information for disabled and elderly rail passengers at stations receive an average importance rating below 2.0.

Through variance analysis of satisfaction and importance ratings, the study identified passenger heterogeneity in evaluating service quality.

Table 4-2: Mean and Variance-Satisfaction and Importance

Service aspect	Attributes	ID	Satisfaction		Importance	
			$\bar{S_j}$	δ_j^2	$\bar{I_j}$	δ_j^2
Reliability	Reliability of train departures	R1	2.57	1.76	3.95	1.26
	Reliability of train arrivals	R2	1.81	0.60	3.91	1.47
	Punctuality of train departures	R3	2.71	1.50	4.71	0.62
	Punctuality of train arrivals	R4	1.35	0.31	3.82	1.01
Service characteristics	Frequency of trains on the route	SC1	2.52	2.28	4.21	1.22
	Operational speed of trains on the route	SC2	3.67	1.17	3.90	1.28
	Availability of transit personnel at the station	SC3	4.23	0.87	3.57	1.56
	Availability of services on weekends/holidays	SC4	3.07	2.05	2.85	1.15
	Short wait time for transfers	SC5	2.54	1.04	2.93	1.07
	Daily service time	SC6	2.64	1.04	4.66	0.30
Cleanliness	Cleanliness of interior and seats on the trains	CL1	3.24	2.50	2.36	0.41
	Cleanliness and hygiene on the stations	CL2	2.51	0.44	2.05	0.21
	Cleanliness and hygiene of the transit facilities	CL3	3.38	2.14	1.88	0.14
Comfort	Accessibility and convenience of seat and shelter on the stations	CO1	2.41	1.85	2.04	0.74
	Comfort of seats on the trains	CO2	2.77	1.78	3.81	0.70
	Smoothness of ride and stops	CO3	2.82	1.76	1.18	0.20
	Quietness of the train	CO4	2.21	0.17	2.28	1.02
	Ease of boarding and alighting the train	CO5	1.67	0.88	2.55	1.72
	Train crowding	CO6	1.86	1.34	4.52	0.55
Information	Availability of schedules and maps on train	I1	2.23	0.82	1.92	0.50
	Provision of information for disabled and elderly rail passengers at stations	I2	2.38	1.16	1.65	0.55
	Information made available at the ticket office	I3	2.23	0.89	2.53	1.30
	Provision of information for disabled and elderly rail passengers on trains	I4	1.40	0.49	2.97	1.74
	Provision of schedules and maps at stations	I5	2.00	0.82	3.41	2.25
Customer Care	Efficiency of service at the ticket office	CS1	2.85	1.96	1.45	0.50
	Administration of complaints	CS2	2.01	0.42	2.63	1.75
	Administration of crowded stations during rush hours	CS3	2.57	1.50	1.75	0.74
	Train operators' behavior	CS4	3.87	2.13	1.52	0.37
Safety and security	Ability of AA-LRT system to avoid or reduce safety related issues at stations	SS1	2.34	0.72	3.38	0.76
	Ability of AA-LRT system to avoid or reduce safety related issues on train	SS2	2.09	0.64	3.89	0.69
	The effort that AA-LRT system to avoid or reduce security related issues at stations	SS3	2.90	1.81	3.14	0.63
	Ability of AA-LRT system to avoid or reduce security related issues on train	SS4	2.23	0.63	3.29	0.55
	Mean		2.53		2.93	

Source: Field survey, 2023/24

The satisfaction and importance ratings provided by the transit passengers were instrumental in calculating the attribute weight, as demonstrated in Table 4.3. The final column displays the impact of each attribute on the overall HPSI.

By observing the corrected importance weight, it appears that all the attributes are found to be important by the passengers. Weight scores below 33% can be identified as critical service attributes.

Table 4-3: Determining PSI and HPSI

No	Attributes	Attribute wght (PSI)	Corrected Satis Wght	Corrected import wght	Attribute Weight (HPSI)
1	Reliability of train departures	0.11	1.364	0.024	0.03
2	Reliability of train arrivals	0.07	1.985	0.020	0.04
3	Punctuality of train departures	0.13	1.780	0.058	0.10
4	Punctuality of train arrivals	0.05	2.137	0.029	0.06
5	Frequency of trains on the route	0.11	1.013	0.026	0.03
6	Operational speed of trains on the route	0.15	4.185	0.023	0.10
7	Availability of transit personnel at the station	0.16	7.477	0.017	0.13
8	Availability of services on weekends/holidays	0.09	1.671	0.019	0.03
9	Short wait time for transfers	0.08	2.255	0.021	0.05
10	Daily service time	0.13	2.436	0.118	0.29
11	Cleanliness of interior and seats on the trains	0.08	1.527	0.044	0.07
12	Cleanliness and hygiene on the stations	0.05	5.206	0.074	0.39
13	Cleanliness and hygiene of the transit facilities	0.07	1.941	0.102	0.20
14	Accessibility and convenience of seat and shelter on the stations	0.05	1.141	0.021	0.02
15	Comfort of seats on the trains	0.11	1.567	0.041	0.06
16	Smoothness of ride and stops	0.04	1.643	0.045	0.07
17	Quietness of the train	0.05	10.445	0.017	0.18
18	Ease of boarding and alighting the train	0.04	1.152	0.011	0.01
19	Train crowding	0.09	0.939	0.062	0.06
20	Availability of schedules and maps on train	0.05	2.205	0.029	0.06
21	Provision of information for disabled and elderly rail passengers at stations	0.04	1.775	0.023	0.04
22	Information made available at the ticket office	0.06	2.031	0.015	0.03
23	Provision of information for disabled and elderly rail passengers on trains	0.04	1.454	0.013	0.02
24	Provision of schedules and maps at stations	0.07	1.773	0.011	0.02
25	Efficiency of service at the ticket office	0.04	1.507	0.022	0.03
26	Administration of complaints	0.06	3.497	0.011	0.04
27	Administration of crowded stations during rush hours	0.05	1.601	0.018	0.03
28	Train operators' behavior	0.06	2.556	0.031	0.08
29	Ability of AA-LRT system to avoid or reduce safety related issues at stations	0.08	2.765	0.034	0.09
30	Ability of AA-LRT system to avoid or reduce safety related issues on train	0.09	2.481	0.043	0.11
31	The effort that AA-LRT system to avoid or reduce security related issues at stations	0.10	1.689	0.038	0.06
32	Ability of AA-LRT system to avoid or reduce security related issues on train	0.08	2.870	0.045	0.13
	Overall passengers' satisfaction index(OPSI)	**2.50(50%)**	Overall heterogeneous passengers' satisfaction index (OHPSI)		2.66(53.2%)

The OPSI is measured at 2.50. When the authors correct the satisfaction and importance ratings based on their variations from the mean, the OHPSI is calculated to be 2.66. This index demonstrates consistency with Bogale & Gizat, 2021 findings, where respondents' mean evaluation of service quality ($M \sim 2.9$) similarly fell below moderate satisfaction thresholds. This convergence across independent studies substantiates the index's validity as a robust measure of transit service quality assessment.

The discrepancy between the OPS and OHPS indices arises from the varying impacts of each service attribute on the indices. If the importance ratings have consistent variances across all attributes and, at the same time, the satisfaction ratings also exhibit equal variances across all attributes, the HCSI and CSI values will be identical.

When the authors examine the attribute weights related to PSI, it is evident that the attributes with the greatest impact on overall satisfaction are daily service time, operational speed of trains on the route, availability of transit personnel at the station, and punctuality of train departures. The weighted scores that vary from 0.04 to 0.16 (as indicated in Table 4.3) indicate that other attributes also play a significant role. Despite a 0.16 difference between the two indices, the weighted scores for PSI differ notably from those of HPSI, which range from 0.01 to 0.39. The attribute with the highest weighted score for HPSI pertains to cleanliness and hygiene at the stations and daily service time.

Furthermore, certain service attributes contribute minimally to both indexes. The system's ability to address safety concerns at stations (0.08 to OPSI and 0.09 to OHPSI) and the punctuality of train arrivals (0.05 for OPSI and 0.06 for OHPSI) rank among the least influential factors. These findings help identify potential areas for improvement that may have been over prioritized or require targeted interventions.

HPSI is a valuable tool for assessing transit service quality and tracking transit agency performance by taking into account differences in user evaluations and adjusting importance and satisfaction levels based on deviations from the average. Hence, the overall HPSI score indicates that the AA-LRT service is 53.2% successful in satisfying its passengers.

4.3 Importance-Performance Analysis (IPA)

The IPA is founded on the idea that transit operators can enhance overall customer satisfaction by focusing on areas where the level of satisfaction is relatively low and the perceived importance of the service is relatively high. The importance-performance analysis involves an examination of a matrix divided into four quadrants.

Each quadrant categorizes attributes as either "Require consistent focus," "Surpass what was expected," "Potential opportunities for improvement," or "Less emphasis." This analysis helps identify priority areas for action based on the perception of importance and performance of specific attributes.

- Require consistent focus (Above average importance and above average satisfaction)

This section indicates the area in which the transit service provider is successfully meeting the expectations of passengers. Factors within this category play a vital role in determining the overall satisfaction level of passengers. It is recommended that the service provider continues to prioritize or slightly enhance the focus on these attributes.

- Surpass what was expected (Below average importance and above average satisfaction)

This section highlights where the transit operator is exceeding passengers' expectations by a considerable margin. The attributes in this category do not have a significant impact on the overall level of satisfaction of passengers with the transit services. It is advisable for the transit operator to sustain the current emphasis on these attributes or slightly reduce it.

- Potential opportunities for improvement (Above average importance and below average satisfaction)

This section indicates attributes for which satisfaction levels are low but the perceived importance of the service is relatively high. This section also indicates where the transit operator is falling short of passengers' expectations. These critical attributes of the service encompass safety and security issues, train crowdedness, and the reliability of train operations, and they can be considered for improving passenger satisfaction. And, they have a major influence on passenger satisfaction, and it is highly recommended for the transit operator to definitely enhance the focus on attributes in this area.

- Less emphasis (Below average importance and below average satisfaction)

This section highlights where the transit operator's performance is weaker compared to other areas, but it is deemed less critical to passengers. The attributes in this area do not greatly impact overall satisfaction with transit services because they are considered less important to passengers. Therefore, the transit operator should keep the current emphasis on attributes in this area unchanged.

Figure 4-2: Importance-satisfaction evaluation matrix

As shown in Figure 4.2, the potential opportunities for improving the service include attributes such as:

SS4: Ability of AA-LRT system to avoid or reduce security related issues on train

SS2: Ability of AA-LRT system to avoid or reduce safety related issues on train

I5: Provision of information for disabled and elderly rail passengers at stations

I4: Provision of information for disabled and elderly rail passengers on trains

R2: Reliability of train arrivals

R4: Punctuality of train arrivals

CO6: Train crowding

The attributes listed under "Require consistent focus" signify the service's strength. Hence, it is crucial for operators to uphold a high level of standard for these attributes in order to gain a competitive edge. These attributes encompass

SC3: Availability of transit personnel at the station

SC2: Operational speed of trains on the route

SS3: The effort that AA-LRT system to avoid or reduce security related issues at stations

SC5: Short wait time for transfers

SS1: Ability of AA-LRT system to avoid or reduce safety related issues at stations

CO2: Comfort of seats on the trains

SC1: Frequency of trains on the route

R1: Reliability of train departures

R3: Punctuality of train departures

SC6: Daily service time

The service attributes outlined under "Surpass what was expected" can be viewed as features that may exceed what is necessary. Hence, the transit operator should not focus on these areas of their service in order to improve service quality. These attributes encompass

CS4: Availability of services on weekends/holidays

CL3: Cleanliness and hygiene of the transit facilities

CL1: Cleanliness of interior and seats on the trains

SC4: Availability of services on weekends/holidays

CO3: Smoothness of ride and stops

CS1: Efficiency of service at the ticket office

CS3: Administration of crowded stations during rush hours

I2: Provision of information for disabled and elderly rail passengers at stations

CL2: Cleanliness and hygiene on the stations

CO1: Accessibility and convenience of seat and shelter on the stations

Finally, the attributes categorized as “Less emphasis” can be viewed less critical by passengers. These are travel experience attributes of the service that are neither meeting expectations nor perceived as particularly significant in improving the overall service experience. These services include

I1: Availability of schedules and maps on train

CO4: Quietness of the train

I3: Information made available at the ticket office

CS2: Administration of complaints

CO5: Ease of boarding and alighting the train

4.4 Optimal improvement strategies

This section explores methods for enhancing service quality that could potentially be used to intervene in order to change the systems’ behavior in a way that its performance is optimized. The importance-performance analysis discussed in section 4.3 indicated that critical factors affecting passengers’ satisfaction include safety and security issues within the train, and reliability and punctuality for arrivals.

The authors examine the most effective enhancement strategies for the NDS model outlined in Equation 2 by utilizing the initial conditions specified in Equation 3 to identify the best control strategies that reduce the number of passengers experiencing dissatisfaction.

The objective is to identify the optimal intervention strategies $u^* = (u_1^*, u_2^*)$ based on the control strategies $u = (u_1, u_2)$ such that the resulting state trajectories N^*, D^* and S^* satisfy the model outlined in Equation (2).

The importance-satisfaction evaluation matrix reveals distinct improvement priorities across service attributes. While most "Comfort" attributes exceed expectations (requiring minimal intervention) and "Information" aspects need limited emphasis, the analysis identifies "Reliability" and "Safety/Security" attributes as critical opportunities.

These domains exhibit consistent gaps between passenger expectations and perceived performance, justifying their selection for targeted improvement strategies.

The control $u_1(t)$ pertains to the actions related to providing train services that adhere to high levels of reliability and punctuality for arrivals, the control $u_2(t)$ represents to the effort related to trains operate with excellent safety and security standards. The control problem involves reducing the numbers of passengers experiencing dissatisfaction.

It is assumed that the number of individuals holding a neutral attitude, possibly due to not having strong positive or negative experiences, decreases by the factor $1 - u_1$ as a result of the provision of train services that adhere to high levels of reliability and punctuality for arrivals. Similarly, the number of passengers experiencing dissatisfaction decreases by the factor $1 - u_2$ through the enactment of excellent safety and security standards. The optimal control problem corresponding to the Caputo fractional order model given in Equation 2 is derived by involving the control variables mentioned earlier.

$$\begin{aligned} {}^{C}D^{\kappa}(N(t)) &= -(1-u_1)\alpha_1^{\kappa}\frac{S(t)N(t)}{P(t)} - \alpha_2^{\kappa}\frac{D(t)N(t)}{P(t)} + \gamma_1^{\kappa}D(t) + \beta_1^{\kappa}S(t) \\ {}^{C}D^{\kappa}\big(D(t)\big) &= \alpha_2^{\kappa}\frac{D(t)N(t)}{P(t)} + \beta_2^{\kappa}\frac{S(t)D(t)}{P(t)} - (1-u_2)\gamma_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \gamma_1^{\kappa}D(t) \\ {}^{C}D^{\kappa}(S(t)) &= (1-u_1)\alpha_1^{\kappa}\frac{S(t)N(t)}{P(t)} + (1-u_2)\gamma_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \beta_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \beta_1^{\kappa}S(t) \end{aligned} \tag{12}$$

with initial conditions given in Equation (3), and $\Omega = \{(u_1(t), u_2(t)) : 0 \le u_1(t), u_2(t) \le 1, t \in [0, t_f]\}$.

The system described in Equation (12) can be written in state-space form as

$$\boldsymbol{x}'(t) = f\big(t, \boldsymbol{x}(t), \boldsymbol{u}(t)\big); \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \tag{13}$$

The vector $\boldsymbol{u}(t)$ signifies the control variables at specific time t, while the vector $\boldsymbol{x}(t)$ represents the state variables that characterizes the evolution of the level of passengers' satisfaction in response to the control $\boldsymbol{u}(t)$ at the initial condition $\boldsymbol{x}(t_0) = \boldsymbol{x}_0$. Equation (13) fully determines the evolution of the state variables overtime.

The authors make the assumption that f is a continuous function of the variables $t, \boldsymbol{x}, \boldsymbol{u}$ and is continuously differentiable with respect to $\boldsymbol{x}$. This assumption guarantees the existence and uniqueness (at least locally) of a solution to the equation described by Equation (13).

Setting the controls to zero means no improvement strategies are being taken to enhance passengers' travel experience. Conversely, when the controls are set to their maximum value of 1, it suggests that the intervention is fully implemented even though this does not align with reality. In the case of AA-LRT, this value corresponds to what operationally feasible and align with the transit existing infrastructure.

For AA-LRT system, the authors calibrated these control values to reflect realistic, achievable levels of implementation. The selected values (between 0 and 1) were determined by three critical factors:

(1) Operational feasibility within the current organizational and technical framework,

(2) Compatibility with the existing transit infrastructure's physical and operational characteristics, and

(3) The minimum intervention level required until passenger-perceived service satisfaction.

This approach ensures our proposed strategies maintain practical relevance while still demonstrating measurable potential improvements.

The performance criterion $\mathfrak{J}: \mathfrak{U}[0, T] \rightarrow \mathbb{R}$ can be formulated in what is known as the Lagrange form

$$\Im(u_1, u_2) = \int_0^{t_f} (\mathfrak{h}_1 N + \mathfrak{h}_2 D + \frac{\mathfrak{y}_1}{2} u_1^2 + \frac{\mathfrak{y}_2}{2} u_2^2) dt$$

The coefficients $\mathfrak{h}_1$and $\mathfrak{h}_2$ represent positive weighting factors and $\frac{\mathfrak{y}_1}{2}$ and $\frac{\mathfrak{y}_2}{2}$ indicate the relative costs linked to the intervention strategies u_1 and u_2, respectively. These factors aid in balancing the units of integrand. In the cost functional, the term $\mathfrak{h}_1 N$ refer to the running cost associated with providing train services that adhere to high levels of reliability and punctuality for arrivals, while the term $\mathfrak{h}_2 D$ signifies the running cost associated with operating trains with excellent safety and security standards.

The existence of an optimal control is significantly important, as it would be futile to seek a solution if no such solution is available.

Due to the complexity of most real-world problems, analytical solutions are usually not feasible, making it necessary to rely on computational algorithms to solve optimal control problems.

Pontryagin's Maximum Principle

Consider the optimal control problem

$$\text{minimize: } \Im(u_1, u_2);\ x'(t) = f\big(t, x(t), u(t)\big); x(t_0) = x_0 \tag{14}$$

Suppose there is a (local) minimizer $u^* = (u_1^*, u_2^*) \in (C[0, t_f])^2$ for the problem, and the corresponding state of satisfaction vector $(N^*, D^*, S^*) \in (C[0, t_f])^3$. Then, there is a vector function $\lambda^* \in (C[0, t_f])^3$ such that the ordered triple (u^*, x^*, λ^*) satisfies the Euler Lagrange equations:

$$x'(t) = \mathcal{H}_\lambda; x(t_0) = x_0$$

$$\lambda'(t) = -\mathcal{H}_x;\ \lambda(t_f) = 0$$

$$\mathcal{H}_u = 0$$

where $\mathcal{H}: \mathbb{R} \times \mathbb{R}^3 \times \mathbb{R}^2 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$ is the Hamiltonian function, defined as outlined in

$$\mathcal{H}(t, x(t), u(t), \lambda) = l(t, x(t), u(t)) + \lambda^T f(t, x(t), u(t))$$

$$\begin{aligned}
&\mathcal{H}(t, x(t), u(t), \lambda) \\
&\quad = \mathfrak{h}_1 N + \mathfrak{h}_2 D + \frac{\mathfrak{y}_1}{2} u_1^2 + \frac{\mathfrak{y}_2}{2} u_2^2 + \lambda_1(-(1-u_1)\alpha_1^\kappa \frac{S(t)N(t)}{P(t)} - \alpha_2^\kappa \frac{D(t)N(t)}{P(t)} \\
&\quad + \gamma_1^\kappa D(t) + \beta_1^\kappa S(t)) + \lambda_2(\alpha_2^\kappa \frac{D(t)N(t)}{P(t)} + \beta_2^\kappa \frac{S(t)D(t)}{P(t)} - (1-u_2)\gamma_2^\kappa \frac{S(t)D(t)}{P(t)} \\
&\quad - \gamma_1^\kappa D(t)) + \lambda_3((1-u_1)\alpha_1^\kappa \frac{S(t)N(t)}{P(t)} + (1-u_2)\gamma_2^\kappa \frac{S(t)D(t)}{P(t)} - \beta_2^\kappa \frac{S(t)D(t)}{P(t)} \\
&\quad - \beta_1^\kappa S(t))
\end{aligned}$$

where λ_1, λ_2 and λ_3 are the co-state variables.

For all t, $0 \leq t \leq t_f$, and all admissible controls u, $\mathcal{H}(t, x(t), u(t), \lambda) \leq \mathcal{H}(t, u^*, x^*, \lambda^*)$. This implies that optimal control u^* maximizes the Hamiltonian function $\mathcal{H}(t, x(t), u(t), \lambda)$.

By the same procedure applied in (Hailu & Teklu, 2024)

$-\frac{d\lambda_1}{dt} = \frac{\partial \mathcal{H}}{\partial N} = \mathfrak{h}_1 N + \lambda_1(-(1-u_1)\alpha_1^\kappa \frac{S(t)}{P(t)} - \alpha_2^\kappa \frac{D(t)}{P(t)}) + \lambda_2 \alpha_2^\kappa \frac{D(t)}{P(t)} + \lambda_3(1-u_1)\alpha_1^\kappa \frac{S(t)}{P(t)}$, $-\frac{d\lambda_2}{dt} = \frac{\partial \mathcal{H}}{\partial D} = \mathfrak{h}_2 + \lambda_1(-\alpha_2^\kappa \frac{N(t)}{P(t)} + \gamma_1^\kappa) + \lambda_2(\alpha_2^\kappa \frac{N(t)}{P(t)} + \beta_2^\kappa \frac{S(t)}{P(t)} - (1-u_2)\gamma_2^\kappa \frac{S(t)}{P(t)} - \gamma_1^\kappa) + \lambda_3((1-u_2)\gamma_2^\kappa \frac{S(t)}{P(t)} - \beta_2^\kappa \frac{S(t)}{P(t)})$, $-\frac{d\lambda_3}{dt} = \frac{\partial \mathcal{H}}{\partial S} = \lambda_1(-(1-u_1)\alpha_1^\kappa \frac{N(t)}{P(t)} + \beta_1^\kappa) + \lambda_2(\beta_2^\kappa \frac{D(t)}{P(t)} - (1-u_2)\gamma_2^\kappa \frac{D(t)}{P(t)}) + \lambda_3((1-u_1)\alpha_1^\kappa \frac{N(t)}{P(t)} + (1-u_2)\gamma_2^\kappa \frac{D(t)}{P(t)} - \beta_2^\kappa \frac{D(t)}{P(t)} - \beta_1^\kappa)$ (15)

The conditions for transversality are

$$\lambda_i^*(t_f) = 0, i = 1,2,3 \tag{16}$$

For u_i such that $0 < u_i < 1, i = 1,2$, the subsequent equation hold true:

$\frac{\partial \mathcal{H}}{\partial u_1} = \mathfrak{y}_1 u_1 + \lambda_1 \alpha_1^\kappa \frac{S(t)N(t)}{P(t)} - \lambda_3 \alpha_1^\kappa \frac{S(t)N(t)}{P(t)} = 0$, and

$\frac{\partial \mathcal{H}}{\partial u_2} = \mathfrak{y}_2 u_2 + \lambda_2 \gamma_2^\kappa \frac{S(t)D(t)}{P(t)} + \lambda_3 \gamma_2^\kappa \frac{S(t)D(t)}{P(t)} = 0$.

Solving for u_1 and u_2, the authors obtain

$$u_1 = \frac{\lambda_3 \alpha_1^{\kappa} \frac{S(t)N(t)}{P(t)} - \lambda_1 \alpha_1^{\kappa} \frac{S(t)N(t)}{P(t)}}{\eta_1}, \text{ and } u_2 = -\frac{\lambda_2 \gamma_2^{\kappa} \frac{S(t)D(t)}{P(t)} + \lambda_3 \gamma_2^{\kappa} \frac{S(t)D(t)}{P(t)}}{\eta_2}.$$

Theorem: The optimal control strategies (u_1^*, u_2^*) that minimizes $J(u_1, u_2)$ over Ω_c can be found as follows:

$$u_1^*(t) = Max\left\{0, \min\left(1, \frac{\lambda_3 \alpha_1^{\kappa} \frac{S(t)N(t)}{P(t)} - \lambda_1 \alpha_1^{\kappa} \frac{S(t)N(t)}{P(t)}}{\eta_1}\right)\right\} \quad \text{and}$$

$$u_2^*(t) = Max\left\{0, \min\left(1, -\frac{\lambda_2 \gamma_2^{\kappa} \frac{S(t)D(t)}{P(t)} + \lambda_3 \gamma_2^{\kappa} \frac{S(t)D(t)}{P(t)}}{\eta_2}\right)\right\},$$

where $\lambda_1(t)$ and $\lambda_2(t)$ are the co-state variables satisfying Eq. 7 and the transversality conditions described in Eq. 8.

Then, the authors obtain the conditions given below

$$u_i^* = \begin{cases} 0, u_i \leq 0 \\ u_i, 0 < u_i < 1 \\ 1, u_i \geq 1 \end{cases} \text{ for } i = 1,2$$

Proof: Applying the Pontryagin's maximal principle as described in (Baba & Bilgehan, 2021), the authors obtain the following results

${}^{C}D_t^{\kappa}\lambda_1 = \frac{\partial \mathcal{H}}{\partial N}$, ${}^{C}D_t^{\kappa}\lambda_2 = \frac{\partial \mathcal{H}}{\partial D}$, and ${}^{C}D_t^{\kappa}\lambda_3 = \frac{\partial \mathcal{H}}{\partial S}$, with conditions of transversality $\lambda_i^*(T) = 0, i = 1,2,3$

The optimality conditions are established by calculating the partial derivatives of the Hamiltonian $\mathcal{H}$ with respect to the control variables u_1 and u_2 leading to $\frac{\partial \mathcal{H}}{\partial u_1} = 0$ and $\frac{\partial \mathcal{H}}{\partial u_2} = 0$.

4.5 Numerical Results

In this section, the numerical results to the optimal control problem are achieved through the use of a fourth order Runge-Kutta iterative method. Utilizing MATLAB programming language and ordinary differential equation solvers (ODE45), the authors illustrate certain fundamental qualitative findings of the NDS model through numerical representations. These representations utilizes the initial passenger satisfaction levels, while the optimal system parameters are based on assumptions due to a lack of a database documenting passengers' satisfaction levels, as outlined in Table 4.4. The numerical representations are to show the behavior of the transit system over a period of three years. In the representations of the optimal control model given in Equation (12), the authors investigate the impacts of implementing actions focused on delivering train services with enhanced reliability and punctuality for arrivals ($\boldsymbol{u_1}$), operating trains with upgraded safety and security measures ($\boldsymbol{u_2}$), and a combination of both $\boldsymbol{u_1}$ and $\boldsymbol{u_2}$.

Table 4-4: Value and interpretation of parameters

Parameters	***Values***	***Interpretation***	***Source***
$N(0)$	*201 passengers*	*Initial neutral(less satisfied or quite satisfied) individuals*	*Estimated*
$D(0)$	*93 passengers*	*Initial dissatisfied individuals*	*Estimated*
$S(0)$	*87 passengers*	*Initial satisfied individuals*	*Estimated*
α_1	*0.17/month*	*N to S transition rate*	*Assumed*
β_1	*0.28/month*	*S to N transition rate*	*Assumed*
γ_1	*0.3/month*	*D to N transition rate*	*Assumed*
α_2	*0.45/month*	*N to D transition rate*	*Assumed*
β_2	*0.17/month*	*S to D transition rate*	*Assumed*
γ_2	*0.28/month*	*D to S transition rate*	*Assumed*

Figure 4.3 illustrates the trajectories of the model given in Equation 2 showing the projected trends of passengers' satisfaction levels over time. The graphical representation of the numerical results suggests the stability of dissatisfaction persistent steady state. In our model, the estimated $\Re_{eff}$ is approximately 2.66, indicating a likelihood of persistent passengers' dissatisfaction.

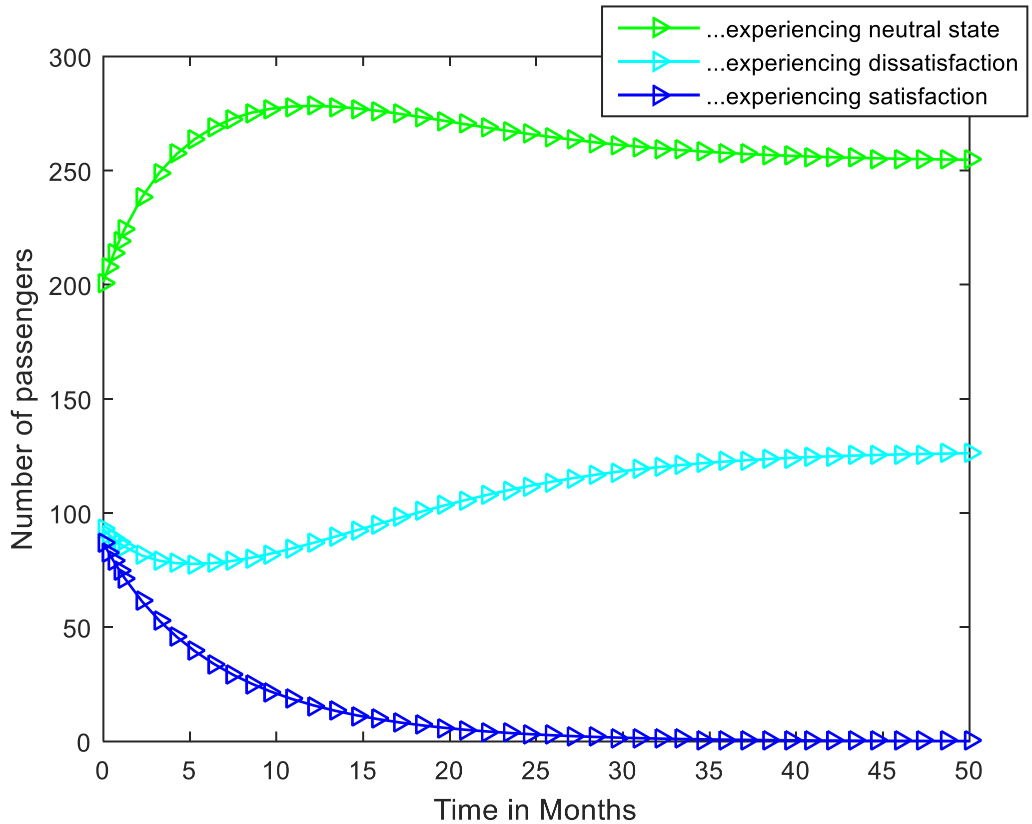

Figure 4-3: Stability of dissatisfaction-persistent steady state

Figure 4.4 illustrates how the effort related to providing train services that adhere to high levels of reliability and punctuality for arrivals has impacted passenger dissatisfaction over a period of three years. The data reveals that this effort plays a significant role in reducing passenger dissatisfaction. While the exact passenger dissatisfaction numbers at specific time points cannot be precisely determined from implicit solutions, the time series plot clearly demonstrates the number of passengers experiencing dissatisfaction decreases when AA-LRT provides train services that adhere to high levels of reliability and punctuality for arrivals are operated with top-notch safety and security standards compared to those without such measures.

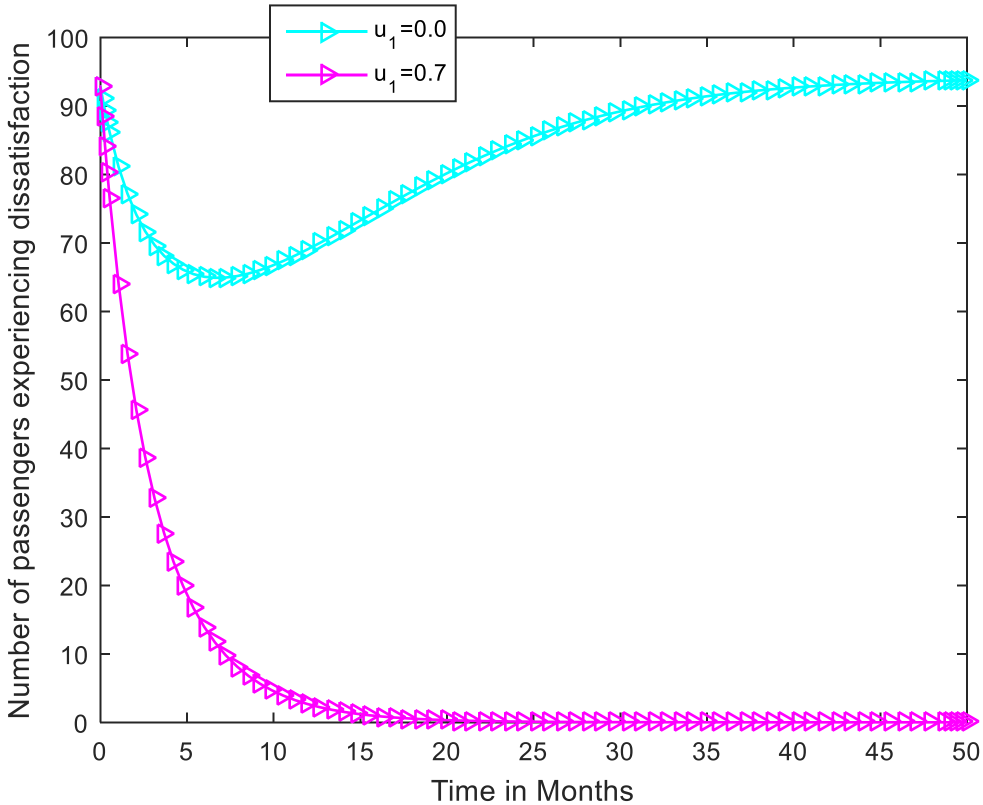

Figure 4-4: The effect of the effort related to providing train services that adhere to high levels of reliability and punctuality for arrivals

Figure 4.5 illustrates the effect of implementing safety and security measures in train services on the level of passenger dissatisfaction over a three-year period. The graph shows that when these actions are taken, the number of dissatisfied passengers is lower compared to situations where no such measures are in place.

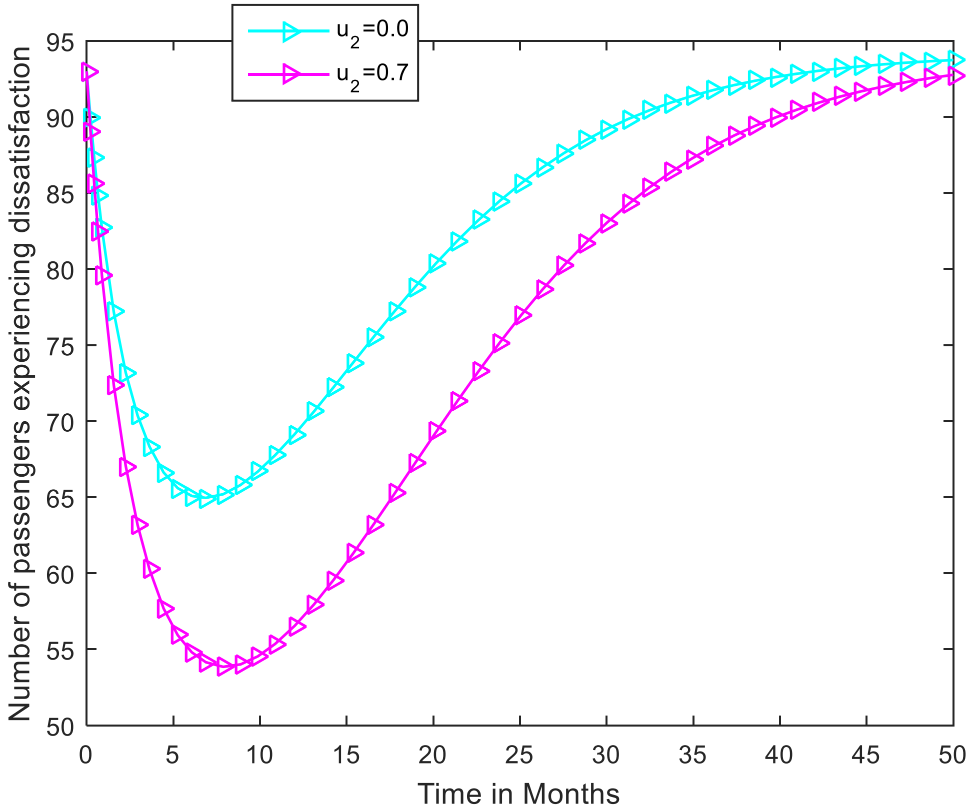

Figure 4-5: The effect of actions related to providing train services that operate with enhanced safety and security standards

Figure 4.6 illustrates the effect of enhanced security and safety procedures, as well as the operation of trains adhering to providing train services that adhere to high levels of reliability and punctuality for arrivals, on the level of passengers' dissatisfaction over a span of three years. In addressing passengers' dissatisfaction, the implementation of such controls correlates with a reduction in dissatisfied passengers, whereas their non-existence corresponds with an escalation. Therefore, the adoption of these measures emerges as the optimal approach for enhancing the general satisfaction levels of passengers.

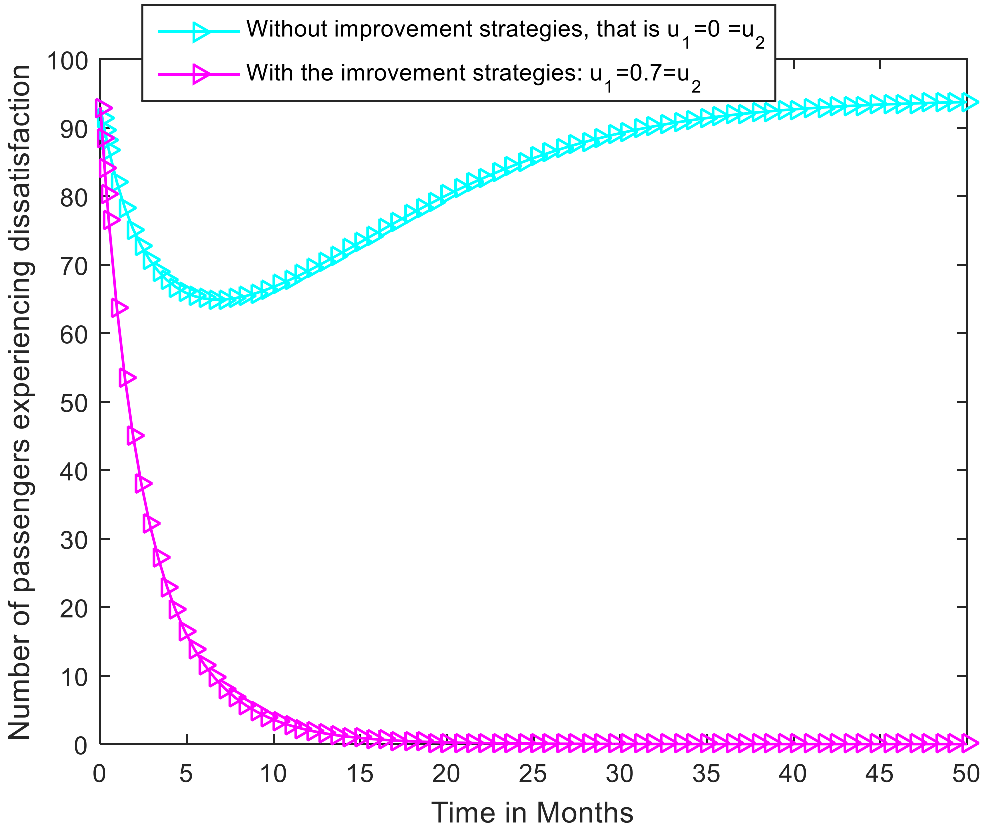

Figure 4-6: The effect of proposed optimal improvement strategies on passengers experiencing dissatisfaction

Figure 4.7 demonstrates the effects of proposed enhancement techniques on passengers with low or quite satisfaction levels. The reduction of the number of passengers experiencing dissatisfaction shown in Figure 4.6 is counteracted by an increase in passengers experiencing less or quite level of satisfaction. This suggests that dissatisfied passengers shift towards feeling less of quite satisfied as a result of improvements in security, safety measures, and the increased reliability and punctuality of train services.

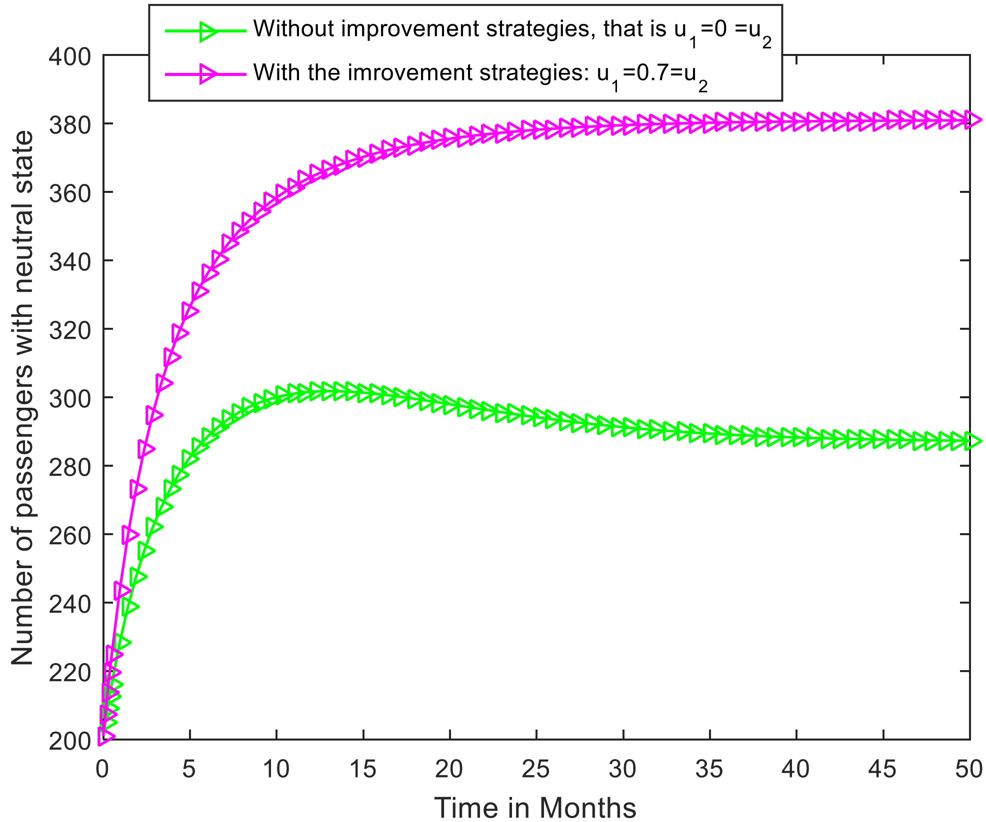

Figure 4-7: The effect of proposed optimal improvement strategies on passengers with low or moderate satisfaction

Figure 4.8 illustrates the combined impact of the suggested enhancement strategies on passenger satisfaction. The graph indicates a slight increase in the number of satisfied passengers following the implementation of these improvement strategies. This suggests that enhancements in security, safety measures, and the overall reliability and punctuality of train services did not lead to a significant change in the number of satisfied passengers.

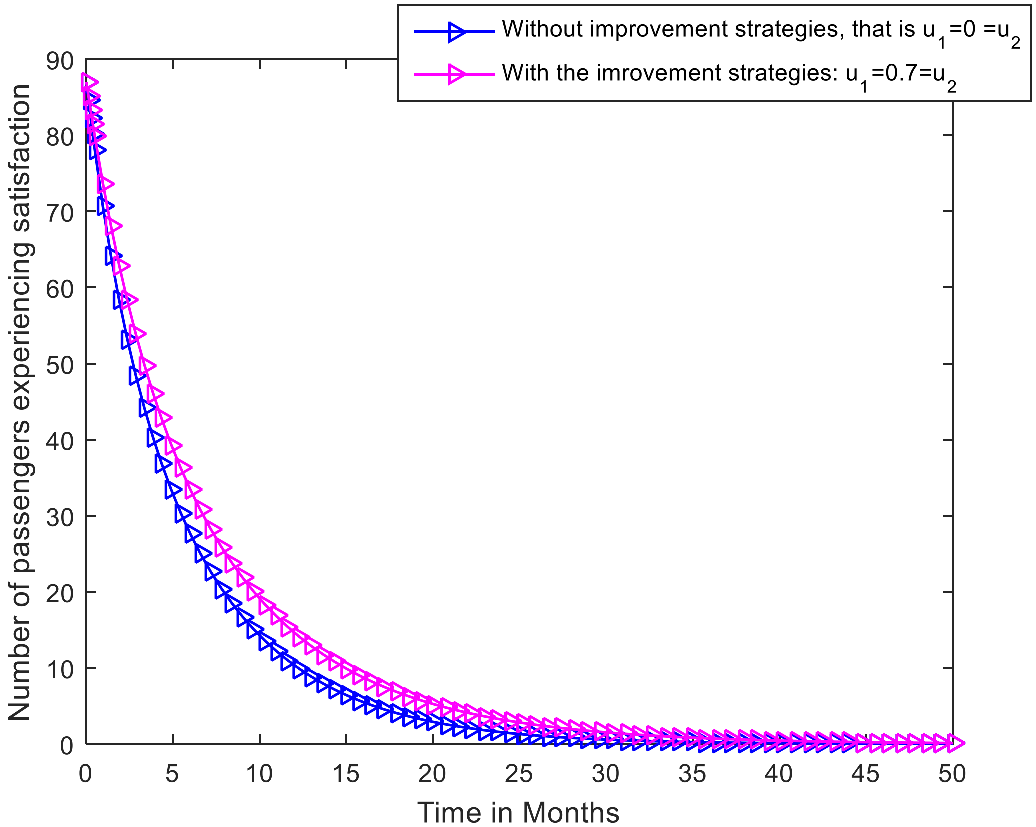

Figure 4-8: The effect of proposed optimal improvement strategies on passengers experiencing satisfaction

The numerical results depicted in Figures 4.9 and 4.10 show that the memory effect can influence passengers' perceptions of satisfaction and dissatisfaction based on how they recall and evaluate past experiences which can impact their overall satisfaction levels with the service. The authors can see that a decrease in the fractional order leads to a reduction in the number of passengers experiencing dissatisfaction, as the memory effect becomes increasingly pronounced. Conversely, a decrease in the fractional order corresponds to an increase in the number of passengers feeling less or slightly satisfied. This may be explained by the ideal that passengers may become accustomed to certain service shortcomings or issues, leading them to lower their expectations and perceive their experiences as less dissatisfying.

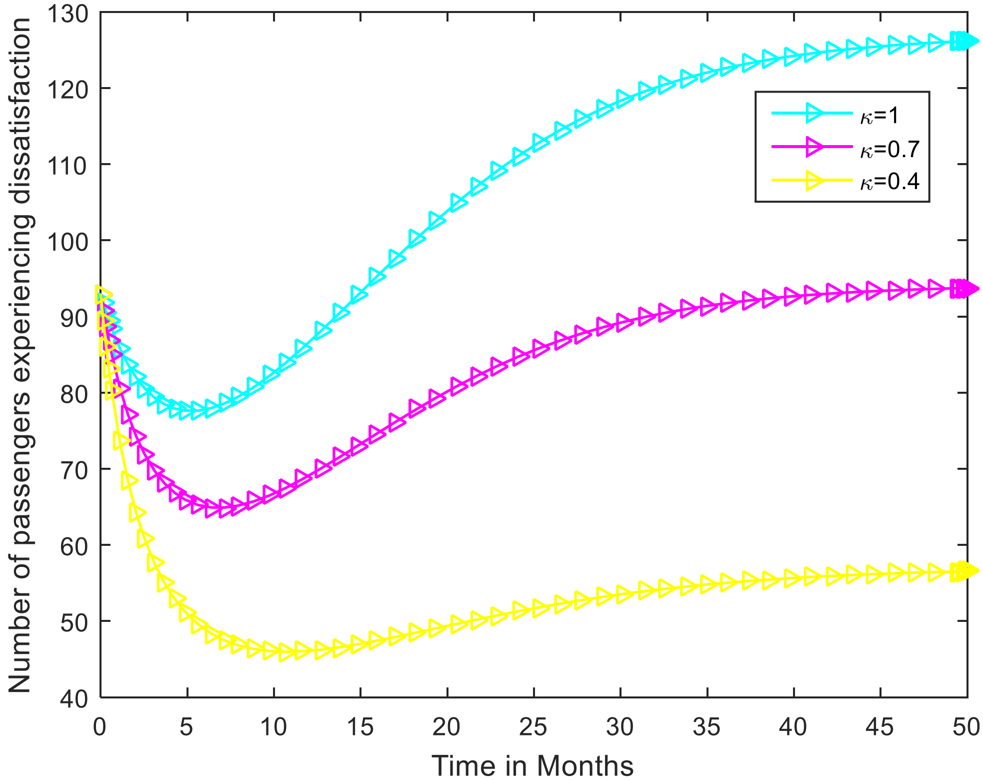

Figure 4-9: Effect of memory on passengers experiencing dissatisfaction

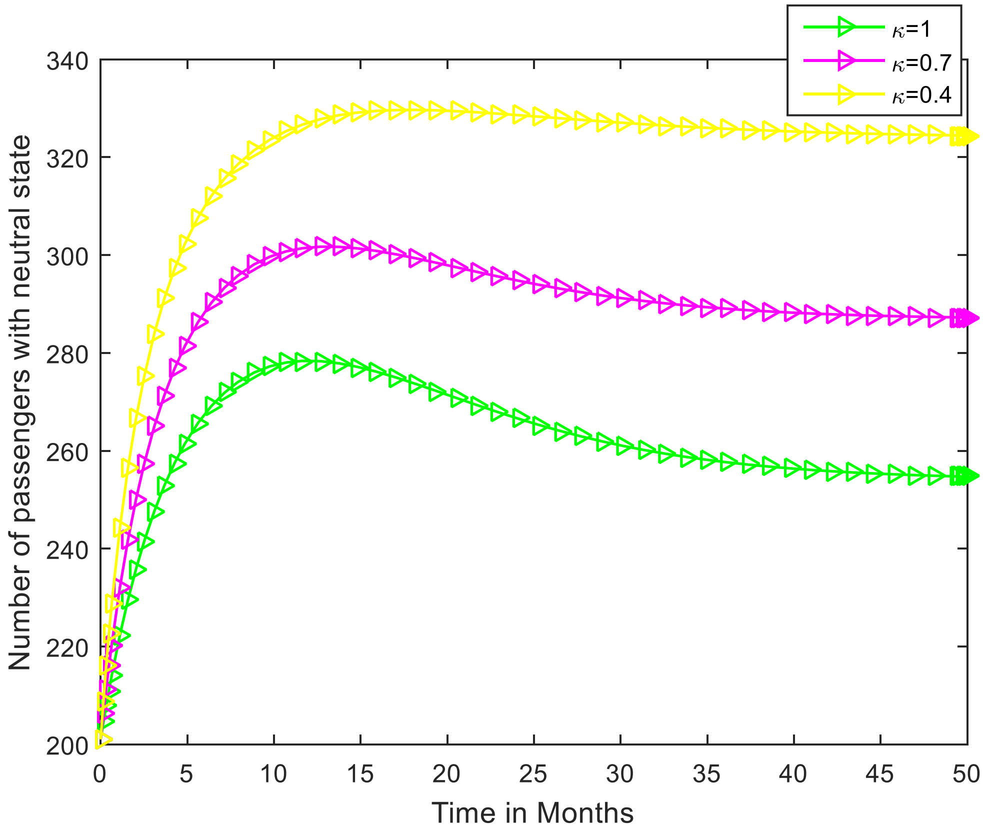

Figure 4-10: Effect of memory on number of passengers with neutral attitude

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

Urban light rail transport service delivery by AA-LRT has been scrutinized in this study. Heterogeneous passenger satisfaction index was used to measure passenger satisfaction considering travel experience attributes. The calculated overall passenger satisfaction index of 2.66 indicates that passengers remain below the “fairly satisfied” threshold (score=3). This demonstrates that there are unmet expectations in critical service dimensions. An Importance-Performance Analysis is also conducted to identify travel attributes that are considered highly important but receive relatively low satisfaction levels. These attributes include the ability of the AA-LRT system to prevent or reduce security-related issues on trains, the effort related to providing train services that adhere to high levels of reliability and punctuality for arrivals, the provision of information for disabled and elderly rail passengers at stations, the provision of information for disabled and elderly rail passengers on train. Additionally, through IPS analysis attributes for which the effort made by AA-LRT surpass what is necessary has been identified.

In this research, the authors develop a compartmental model, by segmenting passengers into "Neutral," "Dissatisfied," and "Satisfied" compartments, that provides valuable insights into the dynamics of passengers' satisfaction. The authors check the wellposedness and analyze the stability of steady states of the model. Furthermore, through the application of the Pontryagin’s maximum principle and numerical results, optimal improvement strategies have been identified to enhance overall passenger satisfaction and improve the quality of service delivery.

The optimal improvement strategies are the action related to providing train services that adhere to high levels of reliability and punctuality for arrivals, and the effort related to trains operate with excellent safety and security standards, which our IPA identifies as primary contributors of passenger dissatisfaction.

The study also examined the memory effect by converting the integer order model into a fractional order model. The graphical representation of numerical results indicates the importance of the order of the fractional derivative in representing the evolution of passengers' satisfaction over time with consideration of memory effects.

5.2 Recommendations and future work

Based on the research findings, the authors recommend that the transit operators and concerned stakeholders focus on enhancing the reliability and punctuality of train arrivals, and operating trains with strict security and safety standards to improve passengers' satisfaction levels. These attributes have been identified as optimal improvement strategies that can positively impact passenger experience, thereby meeting the expectations and needs of its passengers. Implementing these measures can instill confidence and create a positive perception of the rail operation among passengers.

This may involve reallocating resources and efforts from features that may exceed what is necessary in enhancing passengers' satisfaction to the proposed optimal improvement strategies. The efforts made by AA-LRT related to features such as availability of services on weekends/holidays, cleanliness and hygiene of the transit facilities, cleanliness of interior and seats on the trains, availability of services on weekends/holidays, smoothness of ride and stops, efficiency of service at the ticket office, and administration of crowded stations during rush hours are may exceed what is necessary.

Since this study is not exhaustive, other researchers with the same interest could modify the model by introducing new compartments or the division of existing compartments, thereby improving the model's ability to accurately represent the dynamism of passengers’ evolution of satisfaction. For example, by introducing a "fairly dissatisfied" compartment through the division of the existing "dissatisfied" compartment, the model can provide insights into the experiences of fairly dissatisfied passengers, thereby contributing to more accurate predictions of passenger satisfaction evolution.

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Annexes

Appendix A Questionnaire (in English)

Addis Ababa University
Science and Technology Institute
African Railway Center of Excellence
This survey will take you no more than 7 minutes

Date of distribution፡ ______________________Date of collection፡ _________ Station፡______________

Name of data collector፡__________________________

Dear respondents,

This questionnaire pertains to the thesis research entitled with" **Modeling Passengers' Attitude and Analysis of Travel Experience Attributes: A case of Addis Ababa Light Rail Transit**" and conducted by a graduate student from Addis Ababa University, African Railway Center of Excellence. The feedback based on travel experiences and perceptions of different aspects of services provided by Addis Ababa light rail transit in March, 2024 is extremely valuable to provide insights and recommendations for improving the service provided and enhancing passengers' experience and satisfaction. The data collected will remain confidential and be used solely for academic purpose.

Part I: Please provide the following information by placing an "x" mark in the squares.

1. Sex: Male ☐ Female ☐
2. Age: Up to 17 years ☐ 18-21 ☐ 22-45 ☐ 46-60 ☐ Over 61 years ☐
3. How often do you travel by trains?
   On a daily basis ☐ one to three times/week ☐ one to three times/month ☐
4. What is the most frequent purpose of your rail trip?
   Travelling to school/university ☐ work ☐ Leisure ☐ Other ☐
5. Station: Initial _________& final __________
6. Education
   High school ☐ Diploma ☐ Bachelor degree ☐ Master's degree and above ☐
7. Overall satisfaction with the service. Highly satisfied ☐ Fairly satisfied ☐ Neutral ☐ Fairly dissatisfied ☐ Highly dissatisfied ☐
8. Do you recommend the service to others? Yes ☐ No ☐

Part II: Kindly respond to each service attribute based on your experience travelling by train, by putting an "x" in the spaces that best reflects your level of satisfaction, and indicate how important each of these attributes is to you.

Rating scale:

a) Highly dissatisfied "1", Fairly dissatisfied "2", Neutral "3", Fairly satisfied "4", and Highly satisfied "5"

b) Unimportant "1", Of little importance "2", Moderately important "3", Important "4" and Very important "5"

Service aspect	Attributes	ID	Level of satisfaction					Level of importance				
			1	2	3	4	5	1	2	3	4	5
Reliability	Reliability of train departures	R1										
	Reliability of train arrivals	R2										
	Punctuality of train departures	R3										
	Punctuality of train arrivals	R4										
Service characteristics	Frequency of trains on the route	SC1										
	Operational speed of trains on the route	SC2										
	Availability of transit personnel at the station	SC3										
	Availability of services on weekends/holidays	SC4										
	Short wait time for transfers	SC5										
	Daily service time	SC6										
Cleanliness	Cleanliness of interior and seats on the trains	CL1										
	Cleanliness and hygiene on the stations	CL2										
	Cleanliness and hygiene of the transit facilities	CL3										
Comfort	Accessibility and convenience of seat and shelter on the stations	CO1										
	Comfort of seats on the trains	CO2										
	Smoothness of ride and stops	CO3										
	Quietness of the train	CO4										
	Ease of boarding and alighting the train	CO5										
	Train crowding	CO6										
Information	Availability of schedules and maps on train	I1										
	Provision of information for disabled and elderly rail passengers at stations	I2										
	Information made available at the ticket office	I3										
	Provision of information for disabled and elderly rail passengers on trains	I4										
	Provision of schedules and maps at stations	I5										
Customer Care	Efficiency of service at the ticket office	CS1										
	Administration of complaints	CS2										
	Administration of crowded stations during rush hours	CS3										
	Train operators' behavior	CS4										
Safety and security	Ability of AA-LRT system to avoid or reduce safety related issues at stations	SS1										
	Ability of AA-LRT system to avoid or reduce safety related issues on train	SS2										
	The effort that AA-LRT system to avoid or reduce security related issues at stations	SS3										
	Ability of AA-LRT system to avoid or reduce security related issues on train	SS4										

Thank you very much!!!

Appendix B Questionnaire (in Amharic)

አዲስ አበባ ዩኒቨርሲቲ ቴክኖሎጂ እንስቲቲዩት
የአፍሪካ የባቡር የልህቀት ማዕከል

የተሰጠበት ቀን፡ ________________የተሰበሰበት ቀን፡ ________የባቡር መቆሚያ ፌርማታ፡________

የተከበራችሁ የጥናቱ ተሳታፊዎች

የዚህ መጠይቅ አላማ በአዲስ አበባ ዩኒቨርስቲ የአፍሪካ የባቡር የልህቀት ማዕከል (**African Railway Center of Excellence**) የድህር ምረቃ ትምህርት ሟሟያ በአዲስ አበባ ቀላል ባቡር በደንበኞች አገልግሎት እርካታ ላይ ለማካሂደው ጥናት መረጃ ማሰባሰብ ነው፡፡ ጠቃሚ ምክረ ሃሳቦችን ለሚመለከተው አካል በማድረስ የባቡር አገልግሎቱ እንዲሻሻል እና የጥናቱ አላማ እንዲሳካ ምላሻችሁ እጅግ ጠቃሚ ግብአት ስለሆነ ሁሉንም ጥያቄዎች በጥንቃቄ ትመልሱ ዘንድ በአክብሮት እጠይቃለሁ::

ብቻ እንደሚውል አረጋግጥላችኋለሁ:: ምርጫዎትን ለማሳየት ይህን ምልክት "x" ይጠቀሙ

ክፍል 1፡- አጠቃላይ መረጃ

1. ጾታ ሴት ☐ ወንድ ☐
2. እድሜ እስከ 17 ዓመት ☐ ከ18-21 ☐ ከ22-45 ☐ ከ46-60 ☐ ከ61 ዓመት በላይ ☐
3. ምን ያህል ጊዜ የባቡር ተጠቃሚ ነዎት

ሁልጊዜ ☐ በሳምንት ከ1 እስከ 3 ጊዜ ☐ በወር ከ1 እስከ 3 ጊዜ ☐

4. የተለመደ የባቡር ጉዞዎትን አላማ ቢጠቅሱ
   ለትምህርት ☐ ለስራ ☐ ለመዝናናት ☐ ለሌላ ☐
5. ባቡር ጣቢያ መነሻ____________________ መድረሻ ________________
6. የትምህርት ደረጃ

ሁለተኛ ደረጃ ት/ት ያጠናቀቀ ☐ ዲፕሎማ ☐ የመጀመሪያ ድግሪ ☐ ሁለተኛ ድግሪ እና ከዚያ በላይ ☐

7. በቀላል ባቡሩ አገልግሎት ያለዎት እርካታ

በጣም ረክቻለሁ ☐ በመጠኑ ረክቻለሁ ☐ ምንም ማለት አልችልም ☐ አልረካሁበትም ☐
ፈፅሞ አልረካሁበትም ☐

8. የቀላል ባቡር አገልግሎቱን ሌሎች እንዲጠቀሙበት ይመክራሉ? እመክራለሁ ☐ አልመክርም ☐

ክፍል 2፡- የቀላል ባቡር አገልግሎት ሲጠቀሙ በመነሻ እና በመድረሻ ጣቢያዎች እንዲሁም በጉዞ ላይ ከሚያጋጥማችሁ ነገሮች በመነሳት የእርካታ መጠናችሁንና የመለኪያዎቹን አስፈላጊነት በተገቢው ቦታ ላይ የ" x" ምልክት በመጠቀም ይግለፁ፡፡

የእርካታ መለኪያዎች፡ ፈፅሞ አልረካሁበትም "**1**" አልረካሁበትም "**2**" ምንም ማለት አልችልም "**3**"
በመጠኑ እረክቻለሁ "**4**" በጣም እረክቻለሁ "**5**"
የአስፈላጊነት መለኪያዎች፡ በጣም ዝቅተኛ "**1**" ዝቅተኛ "**2**" መጠነኛ "**3**" ከፍተኛ "**4**" በጣም ከፍተኛ "**5**"

የአገልግሎት መገለጫ መለኪያዎች	**እርካታ**					**አስፈላጊነት**				
	1	**2**	**3**	**4**	**5**	**1**	**2**	**3**	**4**	**5**
1. የአገልግሎት አስተማማኝነት										
1.1 በጉዞ መነሻ ቦታዎች የባቡሮች አዘውትረው መድረስ										
1.2 በጉዞ መድረሻ ቦታዎች የባቡሮች አዘውትረው መድረስ										
1.3 በጉዞ መነሻ ቦታዎች የባቡሮች ሰዓት አከባበር										
1.4 በጉዞ መድረሻ ቦታዎች የባቡሮች ሰዓት አከባበር										

2. የአገልግሎት ፀባይ										
2.1 በመስመር ላይ ያሉ የባቡሮች ምልልስ										
2.2 የባቡሮች ፍጥነት										
2.3 የባቡር ጣቢያው ሠራተኞች በስራ ገበታቸው ላይ መገኘት										
2.4 በእረፍትና በበዓል ቀን በባቡር አገልግሎት አሰጣጥ										
2.5 ካባቡር ወደ ባቡር ለመተላለፍ የሚወስደው ጊዜ										
2.6 የቀኑ አገልግሎት ሰዓት										
3. ንፅህና										
3.1 በባቡር ውስጥ ንፅህና										
3.2 በባቡር ፌርማታ ንፅህና										
3.3 የአገልግሎት መስጫ ፋሲሊቲዎች (እንደ መፀዳጃ ቤት እና የመሳሰሉት) ንፅህና										
4. ምቾት										
4.1 በባቡር ፌርማታ ከፀሃይና ከዝናብ መጠለያ፣ በቂ መቀመጫ እና የመፀዳጃ ቤት አገልግሎት መኖር										
4.2 በባቡር መቀመጫ ምቾት										
4.3 መግቢያና መውጫ በሮች ላይ ባለው መጨናነቅ										
4.4 በጉዞ እና በመዳረሻ ቦታዎች የጉዞ ላይ ምቾት (smoothness of ride)										
4.5 በተሳፋሪዎች ባህሪ										
4.6 በባቡር ጉዞ ላይ ባለው መጨናነቅ										
5. መረጃ										
5.1 በጉዞ ላይ ባሉ የጉዞ መረጃ										
5.2 በባቡር መድረሻ በታዎች ላይ ለአቅመ ደካሞችና ለአካል ጉዳተኞች በሚሰጥ መረጃ										
5.3 ከትኬት ቆራጮች በሚገኝ መረጃ										
5.4 በባቡር ጉዞ ላይ ለአቅመ ደካሞችና ለአካል ጉዳተኞች በሚሰጥ መረጃ										
5.5 በባቡር መነሻ እና መድረሻ ስፍራዎች ላይ በሚገኙ የጉዞ መረጃዎች										
6. የደንበኞች አገልግሎት										
6.1 በትኬት አገልግሎት ስርዓትና ቅልጥፍና										
6.2 በባቡር ጣቢያው የደንበኞች ቅሬታ መስተናገጃ ስርዓት በአግባቡ ስራ ላይ እየዋለ ስለመሆኑ										
6.3 ተሳፋሪ በሚበዛባቸው የሥራ መግቢያ እና መውጫ ሰዓታት በአንድ ጊዜ በርካታ ተሳፋሪዎችን ለማጓጓዝ በሚደረግ ተግባር										
6.4 የአዲስ አበባ ቀላል ባቡር ትራንስፖርት አገልግሎት ሰራተኞች ፀባይ አግባብነት										
7. ደህንነትና ከአደጋ መጠበቅ										
7.1 የባቡር ጣቢያው ላይ የሚያጋጥሙ የጤና መታወኮችን ለማስቀረት ወይም ለመቀነስ የሚያደርገው ጥረት										
7.2 በጉዞ ላይ የሚያጋጥሙ የጤና መታወኮችን ለማስቀረት ወይም ለመቀነስ በሚያደርገው ጥረት										
7.3 በባቡር ማቆሚያ ስፍራ ላይ የሚያጋጥሙ የደህንነት ችግሮችን ለማስቀረት ወይም ለመቀነስ በሚያደርገው ጥረት										
7.4 በባቡር ጉዞ ላይ ባለው የደህንነት ስሜት										

አመሰግናለሁ!!!

Appendix C Proofs of theorems

Proposition 3.2.1 *The Laplace transform operator for the Caputo fractional order derivative with order* $\kappa, n-1 < \kappa \leq n,\ n \in N$ *of the function* f *is given by the following expressions*

$$\mathcal{L}(\,^{C}D^{\kappa} f)(s) = s^{\kappa} F(s) - \sum_{i=0}^{n-1} s^{\kappa-i-1} f^{i}(0) \textit{ where } F(s) = \mathcal{L}(f(t)).$$

Proof:

$$\mathcal{L}\left(\frac{1}{\Gamma(\kappa)}\int_0^t f(s)\frac{(t-s)^{\kappa-1}}{t-s}ds\right) = \frac{1}{\Gamma(\kappa)}\mathcal{L}(t^{\kappa-1} * f(t)) = \frac{1}{\Gamma(\kappa)}\mathcal{L}(t^{\kappa-1})\mathcal{L}(f(t)) = \frac{1}{\Gamma(\kappa)}\frac{\Gamma(\kappa)}{s^{\kappa}}F(s)$$

$(\dots\dots\mathcal{L}(t^{n}) = \frac{\Gamma(n+1)}{s^{n+1}})$

$$= s^{-\alpha}F(s)$$

Thus, $\mathcal{L}\left(\frac{1}{\Gamma(n-\kappa)}\int_0^t f^{n}(s)\frac{(t-s)^{\kappa-1}}{t-s}ds\right) = s^{-(n-\kappa)}G(s)$ where $G(s) = \mathcal{L}(g(t)) = \mathcal{L}(f^{n}(t))$.

$$= s^{-(n-\kappa)}G(s) = s^{\kappa-n}\left(s^{n}F(s) - \sum_{i=0}^{n-1} s^{n-i-1} f^{i}(0)\right) = s^{\kappa}F(s) - \sum_{i=0}^{n-1} s^{\kappa-i-1} f^{i}(0).$$

Proposition 3.2.2 *The Laplace transform of two parameters function of Mittag Leffler is given by*

$$\mathcal{L}\left(t^{\beta-1}E_{\alpha,\beta}(\lambda t^{\alpha})\right) = \frac{s^{\alpha-\beta}}{s^{\alpha}-\lambda}$$

Proof: Laplace transform of Mittag Leffler $E_{\alpha,\beta}(t) = \sum_{k=0}^{\infty}\frac{t^{k}}{\Gamma(\alpha k+\beta)}$

$$\mathcal{L}(f(t)) = \int_0^{\infty} e^{-st} f(t)dt = F(s), s > 0$$

$$\mathcal{L}(t^{n}) = \frac{\Gamma(n+1)}{s^{n+1}}, s > 0$$

$$\mathcal{L}(f * g)(x) = \int_0^{t} f(x-u)g(u)\,du = \mathcal{L}(f).\mathcal{L}(g) = F(s).G(s)$$

$$\mathcal{L}\left(t^{\beta-1}E_{\alpha,\beta}(\lambda t^{\alpha})\right) = \frac{s^{\alpha-\beta}}{s^{\alpha}-\lambda}$$

$$\mathcal{L}\left(t^{\beta-1}\sum_{k=0}^{\infty}\frac{(\lambda t^{\alpha})^k}{\Gamma(\alpha k+\beta)}\right)=\int_0^{\infty} e^{-st}t^{\beta-1}\sum_{k=0}^{\infty}\frac{(\lambda t^{\alpha})^k}{\Gamma(\alpha k+\beta)}dt=\sum_{k=0}^{\infty}\frac{\lambda^k}{\Gamma(\alpha k+\beta)}\int_0^{\infty} e^{-st}t^{\beta-1+\alpha k}dt$$
$$=\sum_{k=0}^{\infty}\frac{\lambda^k}{\Gamma(\alpha k+\beta)}\int_0^{\infty} e^{-st}t^{\beta+\alpha k-1}dt=\sum_{k=0}^{\infty}\frac{\lambda^k}{\Gamma(\alpha k+\beta)}\frac{\Gamma(\beta+\alpha k)}{s^{\beta+\alpha k}}=\sum_{k=0}^{\infty}\frac{\lambda^k}{s^{\beta+\alpha k}}$$
$$=\frac{1}{s^{\beta}}\sum_{k=0}^{\infty}\left(\frac{\lambda}{s^{\alpha}}\right)^k=\frac{1}{s^{\beta}}\frac{1}{1-\frac{\lambda}{s^{\alpha}}}=\frac{1}{s^{\beta}}\frac{s^{\alpha}}{s^{\alpha}-\lambda}==\frac{s^{\alpha-\beta}}{s^{\alpha}-\lambda}$$

Theorem 3.3.6.1 *The solutions of the system (shown in Equation (2)) are non-negative for non-negative initial conditions (shown in Equation (3)). That is, the solutions* $(N(t), D(t), S(t))$ *for the model (Equation (2)) is positive invariant in* $\mathbb{R}_+^3$.

Proof:

The system of equations

$\boldsymbol{x}' = f(\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_n}); \boldsymbol{x}(0) = \boldsymbol{x_0} \geq 0$, is a positive system if and only if

$x_i' = f_i(x_1 \geq 0, x_2 \geq 0, \dots, x_i = 0, \dots, x_k \geq 0) \geq 0.$

Thus, for the fractional order model (Equation (2)),

when $N = 0, D = 0 \; and \; S = 0$, ${}^{C}D^{\kappa}N \geq 0$, ${}^{C}D^{\kappa}D \geq 0 \; and \; {}^{C}D^{\kappa}S \geq 0$ respectively.

Therefore all the solutions of the model are non-negative.

Theorem 3.3.6.2 *The solutions of the system (shown in Equation (2)) are uniformly bounded at all times for non-negative initial conditions (shown in Equation (3)).*

Proof:

Note that ${}^{C}D^{\kappa}\big(P(t)\big) = {}^{C}D^{\kappa}\big(N(t)\big) + {}^{C}D^{\kappa}\big(D(t)\big) + {}^{C}D^{\kappa}\big(S(t)\big) = 0$. This shows that $P(t) = N(t) + D(t) + S(t)$ is a constant, and then $N \leq P, D \leq P$, and $S \leq P$.

Thus, the total number of passengers $P(t)$ is bounded.

Hence the solutions of the fractional order model (Equation 2) are bounded in the closed positively invariant feasible set given by

$$\mathfrak{H} = \{(N, D, S) \in \mathbb{R}_+^3 : N + D + S = P\}$$

Lemma 3.3.6.1 The vector function $C(t, X)$ is Lipschitz in the variable X on a set $[0, T] \times \mathbb{R}_+^3$ with some Lipschitz constant $\mathfrak{K}$.

Proof: the authors can observe that

$$\begin{aligned}
&\left\|C_1\big(t, N_1(t)\big) - C_1\big(t, N_2(t)\big)\right\| \\
&\qquad = \left\|\left(-\alpha_1^\kappa \frac{S(t)N_1(t)}{P(t)} - \alpha_2^\kappa \frac{D(t)N_1(t)}{P(t)}\right) - \left(-\alpha_1^\kappa \frac{S(t)N_2(t)}{P(t)} - \alpha_2^\kappa \frac{D(t)N_2(t)}{P(t)}\right)\right\| \\
&\qquad = \left\|\left(\alpha_1^\kappa \frac{S(t)}{P(t)} + \alpha_2^\kappa \frac{D(t)}{P(t)}\right)\big(N_1(t) - N_2(t)\big)\right\| \\
&\qquad \le Max\left\{\alpha_1^\kappa \frac{S(t)}{P(t)} + \alpha_2^\kappa \frac{D(t)}{P(t)}\right\}\left\|\big(N_1(t) - N_2(t)\big)\right\|
\end{aligned}$$

$$\left\|C_2\big(t, D_1(t)\big) - C_2\big(t, D_2(t)\big)\right\| = \left\|\left(\alpha_2^\kappa \frac{D_1(t)N(t)}{P(t)} + \beta_2^\kappa \frac{S(t)D_1(t)}{P(t)} - \gamma_2^\kappa \frac{S(t)D_1(t)}{P(t)} - \gamma_1^\kappa D_1(t)\right) - \left(\alpha_2^\kappa \frac{D_2(t)N(t)}{P(t)} + \beta_2^\kappa \frac{S(t)D_2(t)}{P(t)} - \gamma_2^\kappa \frac{S(t)D_2(t)}{P(t)} - \gamma_1^\kappa D_2(t)\right)\right\|$$

$$= \left\|\left(\alpha_2^\kappa \frac{N(t)}{P(t)} + \beta_2^\kappa \frac{S(t)}{P(t)} - \gamma_2^\kappa \frac{S(t)}{P(t)} - \gamma_1^\kappa\right)(D_1 - D_2)\right\|$$

$$\le Max\left\{\alpha_2^\kappa \frac{N(t)}{P(t)} + \eta^\kappa + \beta_2^\kappa \frac{S(t)}{P(t)} - \gamma_2^\kappa \frac{S(t)}{P(t)} - \gamma_1^\kappa\right\}\left\|\big(D_1(t) - D_2(t)\big)\right\|$$

$$\left\|C_3\big(t, S_1(t)\big) - C_3\big(t, S_2(t)\big)\right\| = \left\|\left(\alpha_1^\kappa \frac{S_1(t)N(t)}{P(t)} + \gamma_2^\kappa \frac{S_1(t)D(t)}{P(t)} - \beta_2^\kappa \frac{S_1(t)D(t)}{P(t)} - \beta_1^\kappa S_1(t)\right) - \left(\alpha_1^\kappa \frac{S_2(t)N(t)}{P(t)} + \gamma_2^\kappa \frac{S_2(t)D(t)}{P(t)} - \beta_2^\kappa \frac{S_2(t)D(t)}{P(t)} - \beta_1^\kappa S_2(t)\right)\right\| = \left\|\left(\alpha_1^\kappa \frac{N(t)}{P(t)} + \gamma_2^\kappa \frac{D(t)}{P(t)} - \beta_2^\kappa \frac{D(t)}{P(t)} - \beta_1^\kappa\right)(S_1 - S_2)\right\| \le Max\left\{\left(\alpha_1^\kappa \frac{N(t)}{P(t)} + \gamma_2^\kappa \frac{D(t)}{P(t)} - \beta_2^\kappa \frac{D(t)}{P(t)} - \beta_1^\kappa\right)\right\}\left\|\big(S_1(t) - S_2(t)\big)\right\|$$

Thus, the authors conclude that

$$\left\|C\big(t, X_1(t)\big) - C\big(t, X_2(t)\big)\right\| \le \mathfrak{K}\|X_1 - X_2\|$$

with

$$\mathfrak{K} = max\left(Max\left\{\alpha_1^\kappa \frac{S(t)}{P(t)} + \alpha_2^\kappa \frac{D(t)}{P(t)}\right\},\ Max\left\{\alpha_2^\kappa \frac{N(t)}{P(t)} + \eta^\kappa + \beta_2^\kappa \frac{S(t)}{P(t)} - \gamma_2^\kappa \frac{S(t)}{P(t)} - \gamma_1^\kappa\right\},\ Max\left\{\left(\alpha_1^\kappa \frac{N(t)}{P(t)} + \gamma_2^\kappa \frac{D(t)}{P(t)} - \beta_2^\kappa \frac{D(t)}{P(t)} - \beta_1^\kappa\right)\right\}\right).$$

Theorem 3.3.3.1 *(Existence-Uniqueness theorem) The initial value problem*

$$
\begin{aligned}
{}^{C}D^{\kappa}(N(t)) &= -\alpha_1^{\kappa}\frac{S(t)N(t)}{P(t)} - \alpha_2^{\kappa}\frac{D(t)N(t)}{P(t)} + \gamma_1^{\kappa}D(t) + \beta_1^{\kappa}S(t)\\
{}^{C}D^{\kappa}\big(D(t)\big) &= \alpha_2^{\kappa}\frac{D(t)N(t)}{P(t)} + \beta_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \gamma_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \gamma_1^{\kappa}D(t)\\
{}^{C}D^{\kappa}(S(t)) &= \alpha_1^{\kappa}\frac{S(t)N(t)}{P(t)} + \gamma_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \beta_2^{\kappa}\frac{S(t)D(t)}{P(t)} - \beta_1^{\kappa}S(t)
\end{aligned}
$$

with initial states $N(0) \geq 0, D(0) \geq 0, and\ S(0) \geq 0$ *has a unique solution* $X(t) \in \Omega(I)$, *the set of bounded continuous functions on* I *with norm* $\|X\| = sup\{|X(t)|: t \in I\}$.

Proof: The IVP

$X(t) = X(0) + \frac{1}{\Gamma(\kappa)}\int_0^t C(s, X(s))(t-s)^{\kappa-1}\,ds$ can be written as $X(t) = F\big(X(t)\big)$where $F = \Omega([0,T], \mathbb{R}_+^3) \to \Omega([0,T], \mathbb{R}_+^3)$ defined by $F\big(X(t)\big) = X(0) + \frac{1}{\Gamma(\kappa)}\int_0^t C(s, X(s))(t-s)^{\kappa-1}ds$.

Further, the authors have $\big\|F\big(X_1(t)\big) - F\big(X_2(t)\big)\big\| = \Big\|\frac{1}{\Gamma(\kappa)}\int_0^t (t-s)^{\kappa-1} \times [C\big(s, X_1(s)\big) - C\big(s, X_2(s)\big)]\Big\|\,ds \leq \frac{1}{\Gamma(\kappa)}\int_0^t (t-s)^{\kappa-1}\,ds \times \big\|C\big(s, X_1(s)\big) - C\big(s, X_2(s)\big)\big\| \leq \frac{\mathfrak{K}}{\Gamma(\kappa)}\int_0^t (t-s)^{\kappa-1}\,ds \leq \frac{\mathfrak{K}}{\kappa\Gamma(\kappa)}T$.

This shows that F is a contraction, and hence, the initial value problem has a unique solution.

Theorem 3.3.7.2.1 The dissatisfaction-free steady state of the model (Equation (2)) is locally asymptotically stable if $\mathfrak{R}_{eff}$ *is less than unity. Otherwise, it is unstable.*

Proof: The authors notice that the Jacobian matrix of the system evaluated at the steady state where there is no dissatisfaction described in Equation (5) is given by

$$
J = \begin{pmatrix} \beta_1^{\kappa} - \alpha_1^{\kappa} & (\chi - \alpha_1^{\kappa})(\alpha_1^{\kappa} - \beta_1^{\kappa}) \\ 0 & \chi\left(\frac{\beta_1^{\kappa}}{\alpha_1^{\kappa}} - 1\right) - \gamma_1^{\kappa} + \frac{\alpha_2^{\kappa}\beta_1^{\kappa}}{\alpha_1^{\kappa}} \end{pmatrix}
$$

As the matrix is a lower triangular matrix, the eigenvalues are simply the diagonal elements of the matrix.

Thus,

$$\lambda_1 = \beta_1^\kappa - \alpha_1^\kappa$$

And

$\lambda_2 = \chi\left(\frac{\beta_1^\kappa}{\alpha_1^\kappa} - 1\right) - \gamma_1^\kappa + \frac{\alpha_2^\kappa\beta_1^\kappa}{\alpha_1^\kappa} = -\frac{\chi\alpha_1^\kappa - \chi\beta_1^\kappa + \alpha_1^\kappa\gamma_1^\kappa - \alpha_2^\kappa\beta_1^\kappa}{\alpha_1^\kappa}.$

It is clear that $\beta_1^\kappa - \alpha_1^\kappa < 0$. Otherwise, the steady state does not exist. Therefore, the stability of the steady state relies on the eigenvalue λ_2. By a simple calculation, the authors can see that $\lambda_2 = -\frac{\chi\alpha_1^\kappa - \chi\beta_1^\kappa + \alpha_1^\kappa\gamma_1^\kappa - \alpha_2^\kappa\beta_1^\kappa}{\alpha_1^\kappa} < 0$ if and only if $\Re_{eff} < 1$. This implies that if $\Re_{eff}$ is less than unity the dissatisfaction-free steady state is stable.

Theorem 3.3.7.2.2 The dissatisfaction-persistent steady state of the model (Equation (2)) is locally asymptotically stable if $\Re_{eff}$ is greater than unity.

Proof: The authors follow the same approach as the previous theorem. The authors compute the Jacobian matrix as the authors did in the previous result, and then evaluate it at the dissatisfaction persistent steady state. The eigenvalues of the matrix satisfy the characteristic equation.

$J(E^*) = J\left(-\frac{\chi\alpha_1^\kappa - \chi\beta_1^\kappa + \alpha_1^\kappa\gamma_1^\kappa - \alpha_2^\kappa\beta_1^\kappa}{\chi(\chi - \alpha_1^\kappa + \alpha_2^\kappa)}, \frac{\chi\alpha_2^\kappa - \chi\gamma_1^\kappa + \alpha_1^\kappa\gamma_1^\kappa - \alpha_2^\kappa\beta_1^\kappa}{\chi(\chi - \alpha_1^\kappa + \alpha_2^\kappa)}\right) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ where

$a_{11} = -\frac{\alpha_1^\kappa(\chi\alpha_2^\kappa - \chi\gamma_1^\kappa + \alpha_1^\kappa\gamma_1^\kappa - \alpha_2^\kappa\beta_1^\kappa)}{\chi(\chi - \alpha_1^\kappa + \alpha_2^\kappa)}$, $a_{12} = \frac{(\chi - \alpha_1^\kappa)(\chi\alpha_2^\kappa - \chi\gamma_1^\kappa + \alpha_1^\kappa\gamma_1^\kappa - \alpha_2^\kappa\beta_1^\kappa)}{\chi(\chi - \alpha_1^\kappa + \alpha_2^\kappa)}$,

$a_{21} = \frac{(\chi + \alpha_2^\kappa)(\chi\alpha_1^\kappa - \chi\beta_1^\kappa + \alpha_1^\kappa\gamma_1^\kappa - \alpha_2^\kappa\beta_1^\kappa)}{\chi(\chi - \alpha_1^\kappa + \alpha_2^\kappa)}$, and $a_{22} = \frac{\alpha_2^\kappa(\chi\alpha_1^\kappa - \chi\beta_1^\kappa + \alpha_1^\kappa\gamma_1^\kappa - \alpha_2^\kappa\beta_1^\kappa)}{\chi(\chi - \alpha_1^\kappa + \alpha_2^\kappa)}$.

The polynomial that defines the characteristics of the Jacobian matrix is

$\lambda^2 + f\lambda + g = 0$ where

$f = -\frac{\alpha_1^\kappa \gamma_1^\kappa - \alpha_2^\kappa \beta_1^\kappa}{\chi}$, and $g = \frac{-1}{\chi(\chi - \alpha_1^\kappa + \alpha_2^\kappa)} \times (\chi\alpha_1^\kappa - \chi\beta_1^\kappa + \alpha_1^\kappa\gamma_1^\kappa - \alpha_2^\kappa\beta_1^\kappa)(\chi\alpha_2^\kappa - \chi\gamma_1^\kappa + \alpha_1^\kappa\gamma_1^\kappa - \alpha_2^\kappa\beta_1^\kappa)$

Set the conditions $\alpha_1^\kappa < \beta_1^\kappa, \gamma_1^\kappa < \alpha_2^\kappa$, and $\chi > 0$. Then if $\Re_{eff} > 1$, $\chi\alpha_1^\kappa - \chi\beta_1^\kappa + \alpha_1^\kappa\gamma_1^\kappa - \alpha_2^\kappa\beta_1^\kappa < 0$. Thus, the authors have $x, y > 0$. Using the Routh-Hurwitz Stability Criterion, the dissatisfaction-persistent steady state is locally asymptotically stable.

Appendix C MATLAB CODES

```
function SafnSecuStan
format short
alpha(1)=0.17;alpha(2)=0.45;
betta(1)=0.28; betta(2)=0.17;
gamma(1)=0.30;gamma(2)=0.28;
tspan=[0 50];
vintial=[201 93 87];
function dy= SafnSecuStan(t,y)
dy = zeros(3,1);
P=y(1)+y(2)+y(3);

dy(1)=-alpha(1)*y(3)*y(1)/P-alpha(2)*y(2)*y(1)/P+gamma(1)*y(2)+betta(1)*y(3);
dy(2)=alpha(2)*y(2)*y(1)/P+betta(2)*y(3)*y(2)/P-gamma(2)*y(3)*y(2)/P-
gamma(1)*y(2);
dy(3)=alpha(1)*y(1)*y(3)/P+gamma(2)*y(2)*y(3)/P-betta(2)*y(2)*y(3)/P-
betta(1)*y(3);

end
[t,y] =ode45(@SafnSecuStan,tspan,vintial);
size(y)
plot(t,y(:,1),'g->',t,y(:,2),'c->',t,y(:,3),'b->','LineWidth',1);
xlabel('Time in Months')
ylabel('Number of passengers' )
legend('...experiencing neutral state', '...experiencing dissatisfaction',
'...experiencing satisfaction')
box on
end
%===========================================================================

function OptimalImproStrategies
format short
alpha(1)=0.17;alpha(2)=0.45;
betta(1)=0.28; betta(2)=0.17;
gamma(1)=0.30;gamma(2)=0.28;
u1=0.7;
u2=0.0;
kapa=0.7
tspan=[0 50];
vintial=[201 93 87 201 93 87];
function dy= control(t,y)
dy = zeros(6,1);
P=y(1)+y(2)+y(3);
P=y(4)+y(5)+y(6);

dy(1)=-alpha(1)^kapa*y(3)*y(1)/P-
alpha(2)^kapa*y(2)*y(1)/P+gamma(1)^kapa*y(2)+betta(1)^kapa*y(3);
dy(2)=alpha(2)^kapa*y(2)*y(1)/P+betta(2)^kapa*y(3)*y(2)/P-
gamma(2)^kapa*y(3)*y(2)/P-gamma(1)^kapa*y(2);
dy(3)=alpha(1)^kapa*y(1)*y(3)/P+gamma(2)^kapa*y(2)*y(3)/P-
betta(2)^kapa*y(2)*y(3)/P-betta(1)^kapa*y(3);
```

```
dy(4)=-alpha(1)^kapa*y(6)*y(4)/P-(1-
u1)*alpha(2)^kapa*y(5)*y(4)/P+gamma(1)^kapa*y(5)+betta(1)^kapa*y(6);
dy(5)=(1-u1)*alpha(2)^kapa*y(5)*y(4)/P+(1-u2)*betta(2)^kapa*y(6)*y(5)/P-
gamma(2)^kapa*y(6)*y(5)/P-gamma(1)^kapa*y(5);
dy(6)=alpha(1)^kapa*y(4)*y(6)/P+gamma(2)^kapa*y(5)*y(6)/P-(1-
u2)*betta(2)^kapa*y(5)*y(6)/P-betta(1)^kapa*y(6);
end
[t,y] =ode45(@control,tspan,vintial);
size(y)
plot(t,y(:,2),'c->',t,y(:,5),'m->','LineWidth',1);
xlabel('Time in Months')
ylabel('Number of passengers experiencing dissatisfaction')
legend('u_1=0.0','u_1=0.7')
box on
end
%==========================================================================

function OptimalImproStrategies
format short
alpha(1)=0.17;alpha(2)=0.45;
betta(1)=0.28; betta(2)=0.17;
gamma(1)=0.30;gamma(2)=0.28;
u1=0.0;
u2=0.7;
kapa=0.7;
tspan=[0 50];
vintial=[201 93 87 201 93 87];
function dy= control(t,y)
dy = zeros(6,1);
P=y(1)+y(2)+y(3);
P=y(4)+y(5)+y(6);

dy(1)=-alpha(1)^kapa*y(3)*y(1)/P-
alpha(2)^kapa*y(2)*y(1)/P+gamma(1)^kapa*y(2)+betta(1)^kapa*y(3);
dy(2)=alpha(2)^kapa*y(2)*y(1)/P+betta(2)^kapa*y(3)*y(2)/P-
gamma(2)^kapa*y(3)*y(2)/P-gamma(1)^kapa*y(2);
dy(3)=alpha(1)^kapa*y(1)*y(3)/P+gamma(2)^kapa*y(2)*y(3)/P-
betta(2)^kapa*y(2)*y(3)/P-betta(1)^kapa*y(3);


dy(4)=-alpha(1)^kapa*y(6)*y(4)/P-(1-
u1)*alpha(2)^kapa*y(5)*y(4)/P+gamma(1)^kapa*y(5)+betta(1)^kapa*y(6);
dy(5)=(1-u1)*alpha(2)^kapa*y(5)*y(4)/P+(1-u2)*betta(2)^kapa*y(6)*y(5)/P-
gamma(2)^kapa*y(6)*y(5)/P-gamma(1)^kapa*y(5);
dy(6)=alpha(1)^kapa*y(4)*y(6)/P+gamma(2)^kapa*y(5)*y(6)/P-(1-
u2)*betta(2)^kapa*y(5)*y(6)/P-betta(1)^kapa*y(6);
end
[t,y] =ode45(@control,tspan,vintial);
size(y)
plot(t,y(:,2),'c->',t,y(:,5),'m->','LineWidth',1);
xlabel('Time in Months')
ylabel('Number of passengers experiencing dissatisfaction')
legend('u_2=0.0','u_2=0.7')
box on
end
```

%===

```
function OptimalImproStrategies
format short
alpha(1)=0.17;alpha(2)=0.45;
betta(1)=0.28; betta(2)=0.17;
gamma(1)=0.30;gamma(2)=0.28;
u1=0.7;
u2=0.7;
kapa=0.7;
tspan=[0 50];
vintial=[201 93 87 201 93 87];
function dy= control(t,y)
dy = zeros(6,1);
P=y(1)+y(2)+y(3);
P=y(4)+y(5)+y(6);

dy(1)=-alpha(1)^kapa*y(3)*y(1)/P-
alpha(2)^kapa*y(2)*y(1)/P+gamma(1)^kapa*y(2)+betta(1)^kapa*y(3);
dy(2)=alpha(2)^kapa*y(2)*y(1)/P+betta(2)^kapa*y(3)*y(2)/P-
gamma(2)^kapa*y(3)*y(2)/P-gamma(1)^kapa*y(2);
dy(3)=alpha(1)^kapa*y(1)*y(3)/P+gamma(2)^kapa*y(2)*y(3)/P-
betta(2)^kapa*y(2)*y(3)/P-betta(1)^kapa*y(3);


dy(4)=-alpha(1)^kapa*y(6)*y(4)/P-(1-
u1)*alpha(2)^kapa*y(5)*y(4)/P+gamma(1)^kapa*y(5)+betta(1)^kapa*y(6);
dy(5)=(1-u1)*alpha(2)^kapa*y(5)*y(4)/P+(1-u2)*betta(2)^kapa*y(6)*y(5)/P-
gamma(2)^kapa*y(6)*y(5)/P-gamma(1)^kapa*y(5);
dy(6)=alpha(1)^kapa*y(4)*y(6)/P+gamma(2)^kapa*y(5)*y(6)/P-(1-
u2)*betta(2)^kapa*y(5)*y(6)/P-betta(1)^kapa*y(6);
end
[t,y] =ode45(@control,tspan,vintial);
size(y)
plot(t,y(:,2),'c->',t,y(:,5),'m->','LineWidth',1);
xlabel('Time in Months')
ylabel('Number of passengers experiencing dissatisfaction')
legend('Without improvement strategies, that is u_1=0 =u_2','With the
imrovement strategies: u_1=0.7=u_2')
box on
end
```

%==

```
function OptimalImproStrategies
format short
alpha(1)=0.17;alpha(2)=0.45;
betta(1)=0.28; betta(2)=0.17;
gamma(1)=0.30;gamma(2)=0.28;
u1=0.7;
u2=0.7;
kapa=0.7;
tspan=[0 50];
vintial=[201 93 87 201 93 87];
function dy= control(t,y)
dy = zeros(6,1);
P=y(1)+y(2)+y(3);
```

```
P=y(4)+y(5)+y(6);

dy(1)=-alpha(1)^kapa*y(3)*y(1)/P-
alpha(2)^kapa*y(2)*y(1)/P+gamma(1)^kapa*y(2)+betta(1)^kapa*y(3);
dy(2)=alpha(2)^kapa*y(2)*y(1)/P+betta(2)^kapa*y(3)*y(2)/P-
gamma(2)^kapa*y(3)*y(2)/P-gamma(1)^kapa*y(2);
dy(3)=alpha(1)^kapa*y(1)*y(3)/P+gamma(2)^kapa*y(2)*y(3)/P-
betta(2)^kapa*y(2)*y(3)/P-betta(1)^kapa*y(3);


dy(4)=-alpha(1)^kapa*y(6)*y(4)/P-(1-
u1)*alpha(2)^kapa*y(5)*y(4)/P+gamma(1)^kapa*y(5)+betta(1)^kapa*y(6);
dy(5)=(1-u1)*alpha(2)^kapa*y(5)*y(4)/P+(1-u2)*betta(2)^kapa*y(6)*y(5)/P-
gamma(2)^kapa*y(6)*y(5)/P-gamma(1)^kapa*y(5);
dy(6)=alpha(1)^kapa*y(4)*y(6)/P+gamma(2)^kapa*y(5)*y(6)/P-(1-
u2)*betta(2)^kapa*y(5)*y(6)/P-betta(1)^kapa*y(6);
end
[t,y] =ode45(@control,tspan,vintial);
size(y)
plot(t,y(:,1),'g->',t,y(:,4),'m->','LineWidth',1);
xlabel('Time in Months')
ylabel('Number of passengers with neutral state')
legend('Without improvement strategies, that is u_1=0 =u_2','With the
imrovement strategies: u_1=0.7=u_2')
box on
end
%=========================================================================

function OptimalImproStrategies
format short
alpha(1)=0.17;alpha(2)=0.45;
betta(1)=0.28; betta(2)=0.17;
gamma(1)=0.30;gamma(2)=0.28;
u1=0.7;
u2=0.7;
kapa=0.7;
tspan=[0 50];
vintial=[201 93 87 201 93 87];
function dy= control(t,y)
dy = zeros(6,1);
P=y(1)+y(2)+y(3);
P=y(4)+y(5)+y(6);

dy(1)=-alpha(1)^kapa*y(3)*y(1)/P-
alpha(2)^kapa*y(2)*y(1)/P+gamma(1)^kapa*y(2)+betta(1)^kapa*y(3);
dy(2)=alpha(2)^kapa*y(2)*y(1)/P+betta(2)^kapa*y(3)*y(2)/P-
gamma(2)^kapa*y(3)*y(2)/P-gamma(1)^kapa*y(2);
dy(3)=alpha(1)^kapa*y(1)*y(3)/P+gamma(2)^kapa*y(2)*y(3)/P-
betta(2)^kapa*y(2)*y(3)/P-betta(1)^kapa*y(3);


dy(4)=-alpha(1)^kapa*y(6)*y(4)/P-(1-
u1)*alpha(2)^kapa*y(5)*y(4)/P+gamma(1)^kapa*y(5)+betta(1)^kapa*y(6);
dy(5)=(1-u1)*alpha(2)^kapa*y(5)*y(4)/P+(1-u2)*betta(2)^kapa*y(6)*y(5)/P-
gamma(2)^kapa*y(6)*y(5)/P-gamma(1)^kapa*y(5);
```

```
dy(6)=alpha(1)^kapa*y(4)*y(6)/P+gamma(2)^kapa*y(5)*y(6)/P-(1-
u2)*betta(2)^kapa*y(5)*y(6)/P-betta(1)^kapa*y(6);
end
[t,y] =ode45(@control,tspan,vintial);
size(y)
plot(t,y(:,3),'b->',t,y(:,6),'m->','LineWidth',1);
xlabel('Time in Months')
ylabel('Number of passengers experiencing satisfaction')
legend('Without improvement strategies, that is u_1=0 =u_2','With the
imrovement strategies: u_1=0.7=u_2')
box on
end
%=============================================================================

function dissatisfied
format short
alpha(1)=0.17;alpha(2)=0.45;
betta(1)=0.28; betta(2)=0.17;
gamma(1)=0.30;gamma(2)=0.28;
u1=0.7;
u2=0.7;
kapa(1)=0.7;
kapa(2)=0.4;
tspan=[0 50];
vintial=[201 93 87 201 93 87  201 93 87];
function dy= dissatisfied(~,y)
dy = zeros(9,1);
P=y(1)+y(2)+y(3);
P=y(4)+y(5)+y(6);
P=y(7)+y(8)+y(9);

dy(1)=-alpha(1)*y(3)*y(1)/P-alpha(2)*y(2)*y(1)/P+gamma(1)*y(2)+betta(1)*y(3);
dy(2)=alpha(2)*y(2)*y(1)/P+betta(2)*y(3)*y(2)/P-gamma(2)*y(3)*y(2)/P-
gamma(1)*y(2);
dy(3)=alpha(1)*y(1)*y(3)/P+gamma(2)*y(2)*y(3)/P-betta(2)*y(2)*y(3)/P-
betta(1)*y(3);

dy(4)=-alpha(1)^kapa(1)*y(6)*y(4)/P-
alpha(2)^kapa(1)*y(5)*y(4)/P+gamma(1)^kapa(1)*y(5)+betta(1)^kapa(1)*y(6);
dy(5)=alpha(2)^kapa(1)*y(5)*y(4)/P+betta(2)^kapa(1)*y(6)*y(5)/P-
gamma(2)^kapa(1)*y(6)*y(5)/P-gamma(1)^kapa(1)*y(5);
dy(6)=alpha(1)^kapa(1)*y(4)*y(6)/P+gamma(2)^kapa(1)*y(5)*y(6)/P-
betta(2)^kapa(1)*y(5)*y(6)/P-betta(1)^kapa(1)*y(6);

dy(7)=-alpha(1)^kapa(2)*y(9)*y(7)/P-
alpha(2)^kapa(2)*y(8)*y(7)/P+gamma(1)^kapa(2)*y(8)+betta(1)^kapa(2)*y(9);
dy(8)=alpha(2)^kapa(2)*y(8)*y(7)/P+betta(2)^kapa(2)*y(9)*y(8)/P-
gamma(2)^kapa(2)*y(9)*y(8)/P-gamma(1)^kapa(2)*y(8);
dy(9)=alpha(1)^kapa(2)*y(7)*y(9)/P+gamma(2)^kapa(2)*y(8)*y(9)/P-
betta(2)^kapa(2)*y(8)*y(9)/P-betta(1)^kapa(2)*y(9);


end
[t,y] =ode45(@dissatisfied,tspan,vintial);
size(y)
```

```
plot(t,y(:,2),'c->', t,y(:,5),'m->',t, y(:,8),'y->','LineWidth',1);
xlabel('Time in Months')
ylabel('Number of passengers experiencing dissatisfaction')
legend('\kappa=1','\kappa=0.7','\kappa=0.4')
grid off
box on
end
%===========================================================================

function neutral
format short
alpha(1)=0.17;alpha(2)=0.45;
betta(1)=0.28; betta(2)=0.17;
gamma(1)=0.30;gamma(2)=0.28;
u1=0.7;
u2=0.7;
kapa(1)=0.7;
kapa(2)=0.4;
tspan=[0 50];
vintial=[201 93 87 201 93 87  201 93 87];
function dy= neutral(t,y)
dy = zeros(9,1);
P=y(1)+y(2)+y(3);
P=y(4)+y(5)+y(6);
P=y(7)+y(8)+y(9);

dy(1)=-alpha(1)*y(3)*y(1)/P-alpha(2)*y(2)*y(1)/P+gamma(1)*y(2)+betta(1)*y(3);
dy(2)=alpha(2)*y(2)*y(1)/P+betta(2)*y(3)*y(2)/P-gamma(2)*y(3)*y(2)/P-
gamma(1)*y(2);
dy(3)=alpha(1)*y(1)*y(3)/P+gamma(2)*y(2)*y(3)/P-betta(2)*y(2)*y(3)/P-
betta(1)*y(3);

dy(4)=-alpha(1)^kapa(1)*y(6)*y(4)/P-
alpha(2)^kapa(1)*y(5)*y(4)/P+gamma(1)^kapa(1)*y(5)+betta(1)^kapa(1)*y(6);
dy(5)=alpha(2)^kapa(1)*y(5)*y(4)/P+betta(2)^kapa(1)*y(6)*y(5)/P-
gamma(2)^kapa(1)*y(6)*y(5)/P-gamma(1)^kapa(1)*y(5);
dy(6)=alpha(1)^kapa(1)*y(4)*y(6)/P+gamma(2)^kapa(1)*y(5)*y(6)/P-
betta(2)^kapa(1)*y(5)*y(6)/P-betta(1)^kapa(1)*y(6);

dy(7)=-alpha(1)^kapa(2)*y(9)*y(7)/P-
alpha(2)^kapa(2)*y(8)*y(7)/P+gamma(1)^kapa(2)*y(8)+betta(1)^kapa(2)*y(9);
dy(8)=alpha(2)^kapa(2)*y(8)*y(7)/P+betta(2)^kapa(2)*y(9)*y(8)/P-
gamma(2)^kapa(2)*y(9)*y(8)/P-gamma(1)^kapa(2)*y(8);
dy(9)=alpha(1)^kapa(2)*y(7)*y(9)/P+gamma(2)^kapa(2)*y(8)*y(9)/P-
betta(2)^kapa(2)*y(8)*y(9)/P-betta(1)^kapa(2)*y(9);
end
 [t,y] =ode45(@neutral,tspan,vintial);
size(y)
plot(t,y(:,1),'g->', t,y(:,4),'m->',t,y(:,7),'y->','LineWidth',1);
xlabel('Time in Months')
ylabel('Number of passengers with neutral state')
legend('\kappa=1','\kappa=0.7','\kappa=0.4')
grid off
box on
end
```