ADDIS ABABA UNIVERSITY
SCHOOL OF GRADUATE STUDIES

A STUDY ON
THE EFFECT OF
SOIL-STRUCTURE INTERACTION
ON THE
DYNAMIC RESPONSE OF SYMMETRICAL
REINFORCED CONCRETE BUILDINGS

By Abdulwasi Usmail Yousuf

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Approved by the board of examiners

Dr Ing Asrat Worku.	____________________
Adviser	
Dr Messele Haile	____________________
External Examiner	
Dr Ing Adil Zekaria	____________________
Internal Examiner	
Dr Lakeselese Abebe	____________________
Chairman	

Declaration

The thesis is my original work, has not been presented for a degree in any other university and that all sources of material used for the thesis have been duly acknowledged.

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ABSTRACT

The effect of soil-structure interaction on the dynamic response of reinforced concrete buildings of regular and symmetrical geometry is considered in this study. The structures are presumed to be generally embedded in a homogenous soil formation underlain by very stiff material or bedrock. The structure-foundation–soil system is excited at the base by an earthquake ground motion.

The superstructure is idealized as a system with lumped masses concentrated at the floor levels, and coupled with the substructure. The substructure system, which comprises of the foundation and soil, is represented and replaced by springs and dashpots.

Frequency-dependent impedances of the foundation system are incorporated in the discrete model in terms of the springs and dashpots coefficients. The excitation applied to the model is field ground motions of actual earthquake records.

Modal superposition principle is employed to transform the equations of motion in geometrical coordinates to modal coordinates. However, the modal equations remain coupled with respect to damping terms due to the difference in damping mechanisms of the superstructure and the soil. Hence, proportional damping for the coupled structural system may not be assumed.

An iterative approach developed by Worku [13,14] is adopted and programmed to solve the system of coupled equations of motion in modal coordinates to obtain the displacement responses of the system.

Parametric studies for responses of building structures with regular and symmetric plans of different structural properties and heights are made for fixed and flexible base conditions, for different soil conditions encountered in Addis Ababa.

Soil borehole log data of three representative sites in Addis Ababa were used for the computation of the stiffness and damping of the soil.

The displacement, base shear and base overturning moments are used in the comparison of different types of structures for various foundation embedment depths, site conditions and height of structures. These values are compared against those of fixed base structure.

The study shows that the flexible base structures, generally exhibit different responses from those structures with fixed base. Basically, the natural circular frequencies, the base shears and the inter-story displacements for the flexible base are less than those of the fixed base structures. This trend is particularly evident when the flexible soil has large thickness. In contrast, the trend becomes less predictable, when the thickness of the flexible soil decreases. Moreover, in the latter case, the iteration undulates significantly making the prediction difficult. This is attributed to the highly jagged nature of the impedance functions of frequencies for such formations. In this particular case, it is difficult to conclude whether the conventional fixed-base approach yields conservative design forces, as is the case for soil formations of large thickness.

1. INTRODUCTION

In customary design practice, it has been assumed that the foundation medium is very stiff and that the seismic motions applied at the structure support points are the same as the free field earthquake motions at these locations. In other words, the effect of the soil structure interaction (SSI), have been neglected. In actuality, however, the structure always interacts with the soil to some extent during earthquakes, imposing soil deformations that cause the motion at the soil structure interface to differ from those that would have been observed in the free field. This effect is commonly known as kinematic interaction and is mostly insignificant.

The SSI influences also the response of the structure itself, an effect known as inertial interaction. Depending on a number of factors, this effect can be significant and is the subject of this study.

The nature and amount of this interaction effect is found to be dependent on the type of the soil around the foundation, the thickness of deformable soil strata, foundation type, the mass and stiffness properties of the superstructure and the depth of embedment of the foundation [12].

The changes in the response, are in most of the cases amplifications of the response, which might be due to the fact that [1]:

1. The flexibly supported structures have more degrees of freedom and hence posses different dynamic behavior than rigidly mounted structures,

2. In contrary to a structure fixed at base, a significant amount of the vibration energy of the flexibly supported structure may be dissipated by radiation of waves into the supporting medium and by hysteretic damping in the foundation material.

1.1 METHODS OF ANALYSIS OF SOIL STRUCTURE INTERACTION RESPONSE

Among available methods of analysis of soil structure interaction is the direct analysis concept of the combined soil-structure system, in which the soil underlying the structure is represented as a "bounded" finite-element model. Unfortunately this direct approach has the major deficiency that the bounded soil model does not allow the dissipiation of vibration energy in the structure and soil and thus ignores an effective damping mechanism. For this reason, a bounded soil layer model should be used only in case where the soil supporting the structure is underlain by a very stiff rock layer. Another deficiency of such a modeling is that the earthquake excitation is applied at the base of the soil layer, whereas the seismic input usually is expressed in terms of accelerograms recorded at the free field soil interface.

Another method of analysis for soil structure interaction is the substructure approach in which the foundation and the structure are represented as two independent mathematical models or substructures. The connection between them is provided by interaction forces of equal amplitude but acting in opposite directions on the two substructures. The total motions developed at the interface are the sum of the free field motions at the interface of the soil without the added superstructure plus the additional motions resulting from the interaction, [12]. Modal analysis is, therefore employed to solve the coupled differential equations of motion.

Conceptually, The substructure method is probably the easiest way to analyze soil-structure interaction for seismic excitation. As described above, it is to model a significant amount of the soil around the base of the foundation and to apply the free-field motion at the fictitious boundary system. This direct procedure would even allow certain non-linear material laws of the soil to be considered. However the degrees of freedom in the soil region is high, resulting in a large computational requirement. As the law of superposition has to be implicitly assumed to be valid in a soil- structure interaction analysis, it is computationally more efficient to use the substructure method [7,14].

Hence, in the substructure method, the unbounded soil is first examined and represented by generalized springs and dashpots for all possible degrees of freedom, to account for the stiffness and damping to be provided by the soil.

The dynamic equilibrium equations of a discrete system could be used in the dynamic analysis of soil-structure model, by implementing the stiffness method. In this study, the parameters, which determine the soil system, are partially predetermined and used in the computation of the soil stiffness and damping, and some of them are iteratively computed in the course of the analysis of the soil structure system.

It must be recalled that linearity is a necessary condition for modal coordinate uncoupling; this result will be achieved only if the system is proportionally damped; for any other type of damping the modal coordinate equations of motion will remain coupled through the modal damping coefficients, despite modal transformation

To carry out such an analysis, the elements of the damping matrix should be determined explicitly. The fact that the damping matrix should be defined explicitly rather than the modal damping ratios may be advantageous in that it increases the generality of the approach. Hence, the construction of the damping matrix for the coupled system can be constructed easily.

A computer program is developed based on the detailed procedure, developed by [14], which is used to analyze the coupled dynamic equations of motions in modal coordinates.

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1.2 INPUT GROUND MOTIONS

Of the many types of external load due to natural hazards that must be considered in the design of structures, the most important by far in terms of its potential for disastrous consequences is the earthquake.

For the purpose of this study, the N-S component of Imperial Valley earthquake recorded at El Centro with a maximum acceleration of 0.32g is used as input motion for all the building models. The recorded time history for this component is shown in Figure 1.1

It is important to mention that, only the horizontal earthquake excitation of the system is treated in this study as this causes the most important effect on the building structure.

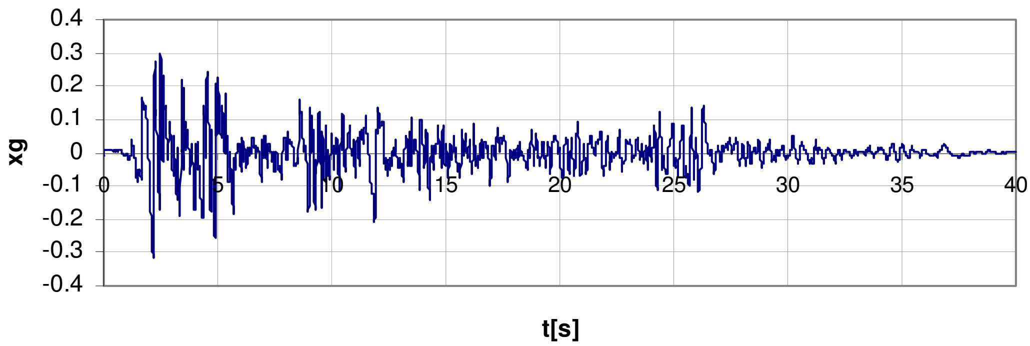

Figure 1.1 North-South component of horizontal ground acceleration recorded at Imperial Valley, El Centro, California, May 18,1940. [5]

2. MODELING OF THE SYSTEMS USING THE SUBSTRUCTURE APPROACH

In the substructure approach, building structures are considered to be composed of two basic different parts. These are the soil-foundation system and the superstructure. The soil-foundation system includes the soil around the foundation to be included in the analysis of the model. The basic element in such an analysis is to model the soil around the foundation when subjected to dynamic loadings.

The superstructure may be assumed as a discrete mass model, where the masses are concentrated at the floor levels.

2.1 THE FOUNDATION MODEL

When subjected to dynamic loads, foundation of structures oscillate in a way that depends on the nature and deformability of the supporting ground, the geometry and inertia of the foundation and super structure, and the nature of dynamic excitation [18].

As previously mentioned, the key step in the response analysis of such systems is to model the soil formation around the foundation, and estimate the dynamic “spring” and “dashpot” coefficients of flexibly supported foundations, represented by the global stiffness and damping of the supporting soil.

The determination of the frequency dependant dynamic stiffness coefficients $K_j(\omega)$ and of the damping coefficients $C_j(\omega)$ for different modes of vibration, foundation shapes, soil profiles, and degrees of embedment has been the subject of many research works for a long time [14]. Gazetas, has compiled these works and provided in [17] formulas and charts for surface and embedded foundations of arbitrary shapes for elastic half-space, and as well for foundations underlined by shallow bedrocks.

A rigid foundation on a flexible soil in general possese six degrees of freedom. Only the two degrees of freedom, the horizontal and rocking, are important for and considered in this study. Figure 2.1(a) shows these motions on a two dimensional planar model of the foundation, in which the remaining degrees of freedom are omitted. Figure 2.1(b) shows the frequency dependant dashpots and springs, replacing the flexible soil around the foundation.

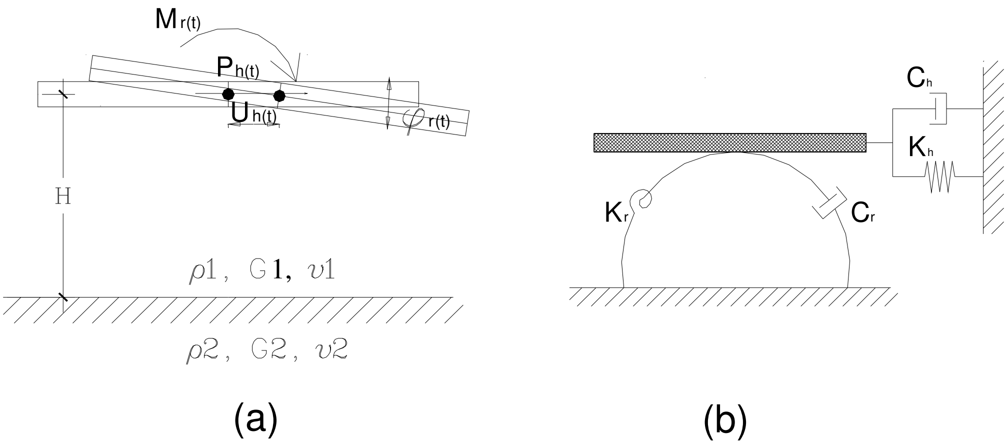

Figure 2.1 (a) Rigid foundation under harmonic excitation;
(b) Model for the soil- foundation system.

The foundation is idealized as rigid and arbitrarily shaped. Both the foundation mass and the mass of the structure are assumed to be uniformly distributed over the foundation area.

2.2 THE BUILDING MODEL

In contrast to procedures often used in determining the static response, it is in general not possible to perform the dynamic analysis in steps, calculating one part of the structure after the other, or to carry the analysis of the superstructure and the substructure separately. The total dynamic system with a correct representation of the stiffness, the damping and the mass has to be modeled. This, however, does not mean a very complicated model is always necessary.

Economical considerations also demand the use of a simple model for a dynamic analysis, as the computational effort for the latter can be an order of magnitude larger than that of a static analysis working with the same discritization [7]. Procedures to establish such models in a direct way are described in many references [5,7,12].

The following model with reference to the multistory framed building structure shown in Figure 2.2(a) is employed that is embedded in a flexible soil layer of mass density ρ_1 shear modulus of rigidity G_1 and Poisson's ratio υ_{1i} which overlies the half space of corresponding parameters ρ_2, G_2, and υ_2.

The superstructure is modeled as shown in figure 2.2(a) that consists of a simple linear structure comprising of lumped floor mass, m_i, stiffness, k_i, and coefficient of damping, c_i. This model includes the discrete foundation mass, m_o embedded in a homogeneous, linearly elastic half space, whose parameters are already described in Figure 2.2(a).

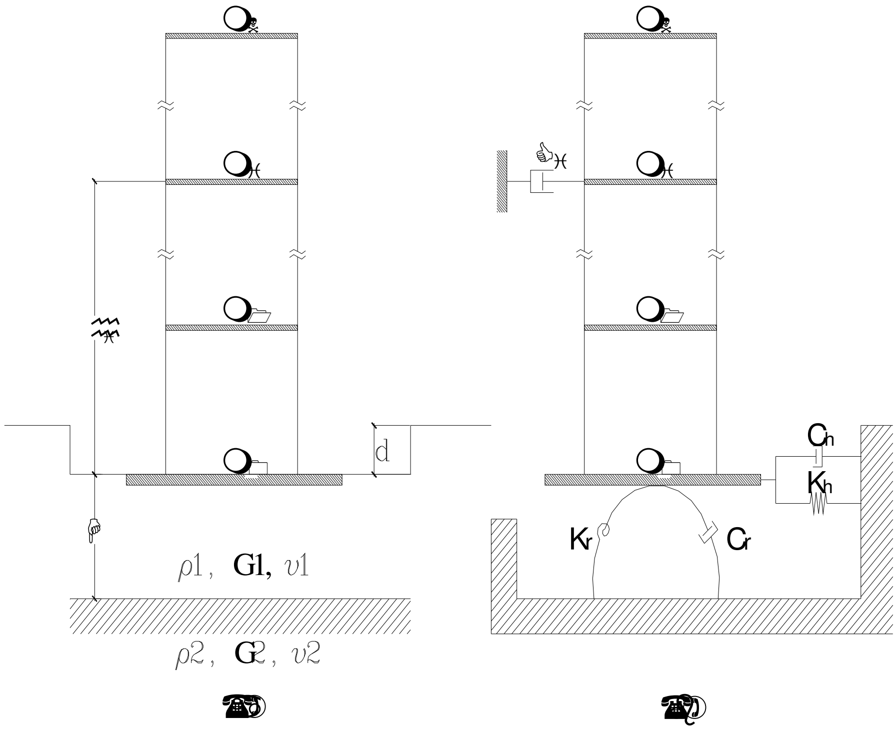

Figure 2.2 (a) Multistory building resting on the top of soil
(b) The kinetics of the system [14]

The idealized structure is viewed as a discrete model of multi-story building, where the masses are concentrated at the floors. The vibration configuration of the superstructure and the foundation system is based on the configuration of the constructed multi-degreed of freedom-coupled model.

The free field motion of the ground surface specifies the base excitation. This is the surface motion, which would have been recorded at the site in the absence of the structure.

2.3 THE KINETICS AND KINEMATICS OF THE SYSTEM

As already mentioned, in the analysis of soil-structure interaction problems, frequency dependant soil parameters are involved in the system. To determine the frequency dependant soil parameters the natural frequencies of the system should be estimated to determine the initial stiffness of the system.

In this study the static stiffness coefficients of the soil are incorporated in the determination of the initial stiffness matrix, which is updated iteratively by determining the fundamental natural circular frequency of the coupled system.

The damping matrix for the fixed base case is established as Rayleigh damping [5,12]. This damping matrix for the superstructure is assembled with that of the foundation to establish the damping matrix of the coupled system.

The lumped mass model of the structure is shown in Figure 2.3(a), in which the mass of the structure is assumed to be concentrated at the foundation and at the overlying floor levels and denoted by m_o, m_1,m_2, etc. The columns and the soil deform elastically giving rise to rigid movements of the masses, viscous dashpots with damping coefficients C_i represent the velocity proportional damping in the superstructure, whereas a pair of spring and dashpot with spring coefficient K_j, and damping coefficient C_j, respectively, represents the dynamic stiffness and damping of the soil corresponding to each degree of freedom j of the foundation [14].

The structure is subjected to the horizontal component of the two possible planar components of free ground acceleration a_h and a_r of an earthquake motion in the, horizontal and rotational (rocking) direction, respectively.

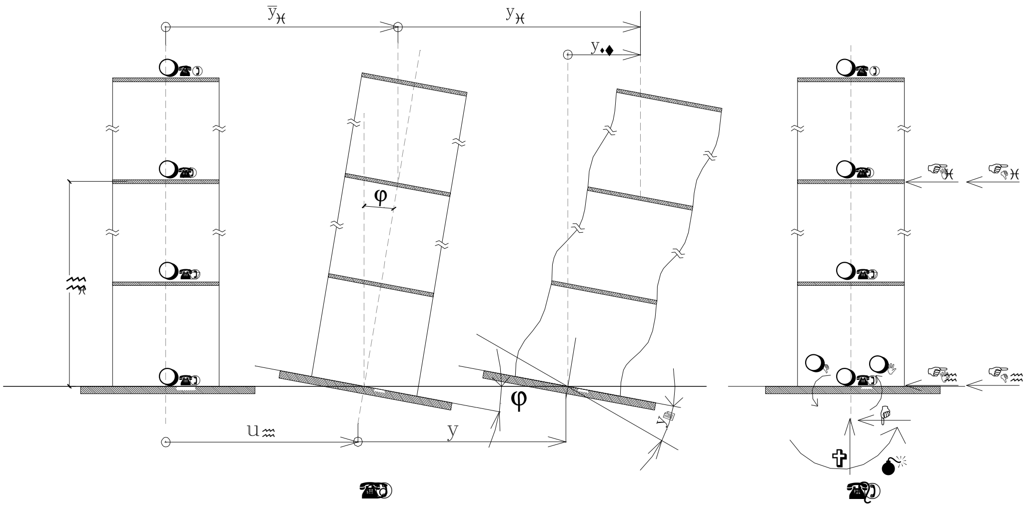

Figure 2.3 (a) the kinematics of the system

(b) the kinetics of the system [14]

The kinematics are shown in Figure 2.3(a) where the free ground displacement $u_h(t)$ in the horizontal direction and the free ground rotation $\varphi(t)$ corresponding, respectively, to the free ground accelerations a_h, and a_r, are indicated.

The rigid body movements of the foundation and of the superstructure for each degree of freedom *i* may be expressed in vector forms in terms of these quantities as [14]

$$\{\bar{y}(t)\} = \{\alpha\} u_h(t) + \{\gamma\}\varphi(t) \qquad (2.1)$$

Where α_i, γ_i, are displacement in the corresponding degree of freedom due to unit movements of the free ground surface u_h, and u_r respectively. It is easy to see that these influence numbers of rigid body motion take the following values for the considered structure and its model: $\alpha_i=1;\gamma_i=h_i$, for a horizontal degree of freedom and $\alpha_i=0$, $\gamma_i=1$, for a rotational degree of freedom.

The elastic deformations are denoted by y_i, whereby the two of these y_1, y_2 represent the horizontal, and rotational deformations, respectively, of the elastic soil. The deformations y_i of the superstructure are interrelated to the soil deformations of the soil as shown in Figure 2.3(a).

Figure 2.3(b) shows the inertia and damping forces and moments of the structure for each degree of freedom. The resultants of the inertia and damping forces corresponding to the two degrees of freedom of the foundation are expressed as:

$$F_h = F_{Ih} + F_{Dh} = m_o(\ddot{\bar{y}}_1 + \ddot{y}_1) + C_h \dot{y}_1 \qquad (2.2a)$$

$$M_r = M_I + F_D = I_t(\ddot{\bar{y}}_2 + \ddot{y}_2) + C_r \dot{y}_2 \qquad (2.2b)$$

Where I_t, is the total mass moment of inertia of the whole structure about the center of the soil- foundation interface.

Corresponding to each degree of freedom in the superstructure the following resultant force of resistance is available:

$$F_i = F_{Ii} + F_{Di} = m_{(i)}(\ddot{\bar{y}}_{i1} + \ddot{y}_{i1}) + C_i \dot{y}_{i1} \qquad (2.3)$$

The round brackets at the index of the mass in Equation (2.3) are introduced to indicate that the numberings of the masses and those of the degrees of freedom are different.

The spring forces and the spring moment of the soil are considered to act at the soil-foundation interface and they may be expressed as [14]:

$$H = K_h y_1 \qquad M = K_r y_2 \qquad (2.4)$$

These reaction forces as indicated in Figure 2.3(b) are dependant on the displacements of the structure

3. FOUNDATION IMPEDANCES

The dynamic stiffness and damping coefficients of the soil which are frequency dependant are determined by relating vector {P(t)} of a set of dynamically applied harmonic excitations on the soil to the resulting response vector {U(t)} as expressed by

$$[K]\ \{U\} = \{P\} \tag{3.1}$$

The elements of the response vector {U} and those of the excitation vector {P} are the complex amplitudes of $U_j(t) = U_j e^{i\omega t}$ and $P_j(t) = P_j e^{i\omega t}$, respectively. The matrix [K] is commonly referred to as the impedance matrix. Due to the phase difference between the loading and the response, the elements of the impedance matrix, which are also called impedance functions, are also complex quantities, which, for the planar vibration modes considered involving the horizontal and rocking motions only, may be given as [14]:

$$[K] = \begin{bmatrix} K^{st}{}_{h}(k_h + i a_0 c_h) & K^{st}{}_{hm}(k_{hm} + i a_0 c_{hm}) \\ K^{st}{}_{mh}(k_{mh} + i a_0 c_{mh}) & K^{st}{}_{m}(k_m + i a_0 c_m) \end{bmatrix} \tag{3.2}$$

Where K_j^{st} are the static stiffness of the soil; k_j and c_j are the dynamic stiffness and damping coefficients which depend on the frequency parameter a_o, on the Poisson's ratio ν of the underlying soil, on the foundation shape and on the layering of the soil. The dimensionless frequency parameter a_o is given by

$$a_o = \frac{\omega B}{V_{sl}} \tag{3.3}$$

where, ω is the execution frequency of the foundation,B is half the least width of the foundation and

$$V_{sl} = \sqrt{\frac{G_1}{\rho_1}} \tag{3.4}$$

V_{sl} is the shear wave velocity of the top soil layer; G_1 is the share modulus of elasticity and ρ_1 is its mass density.

The dynamic stiffness K_{hm} and K_{mh} in Equation (3.2), which couple the horizontal and rocking motions are equal and relatively small for surface and very shallow foundations and may be neglected [14].

The static stiffnesses of the soil are available in the literature for all modes of excitation and for almost all practical shapes of foundations and cases of soil profiles. They can be taken, for example, from Gazetas[18]. A compilation from this and other references is also available in [13].

Similarly the frequency dependant dynamic stiffness coefficients k_j, and the damping coefficients c_j, for the different modes of vibration shapes, soil profiles, and degrees of embedment could be obtained from [18,14]

Gazetas[18] provided formulas and charts for surface and embedded Foundations of arbitrary shapes for elastic half-space, but for long period excitations $a_o \leq 2$.

The availability of such data, which are needed in this area of research, encourages the development of simple methods of dynamic analysis of structures that account for soil-structure interaction effects.

Neglecting the coupling terms in the impendence matrix of Equation (3.2) each decoupled Equation in (3.1) may be written as

$$\frac{P_j}{U_j} = K_j^{st}\left(k_j + i\,a_o c_j\right) \tag{3.5}$$

Where the subscript *j* stands for the considered degree of freedom of the foundation.

It is possible to obtain the spring stiffness and the dashpot coefficients from expressions and charts supplied by [18,13].

One possible expression for determining the foundation impedances without considering the material damping are given as follows [14]

$$K_j(\omega) = K_j^{st}\, k_j(a_o) \tag{3.6}$$

$$C_j(\omega) = \frac{a_o}{\omega} K_j^{st}\, c_j(a_o) \tag{3.7}$$

The relations of Equation (3.6) and (3.7) are obtained by comparing Equation (3.5) with the corresponding equation of motion of Figure (2.1b) subjected to harmonic forces of $P_j(t) = P_j e^{i\omega}$, described in Equation (3.1).

The spring and dashpot coefficients for a viscoelastic soil are given as follows, [14] :

$$K_j^{ve} = K_j - 2\omega\xi\, C_j \tag{3.8}$$

$$C_j^{ve} = C_j + \frac{2\xi}{\omega} K_j \tag{3.9}$$

Equations (3.8) and (3.9) enable one to incorporate both the material (hysteretic) and the geometric damping of the soil as important additional means of energy dissipation for vibrating building structures interacting with the soil.

4. FORMULATION AND SOLUTION OF EQUATIONS OF MOTION

The formulation of the equations of motion using Green's influence numbers and the solution of coupled equations of motion is briefly explained in this chapter.

4.1 FORMULATION OF EQUATIONS OF MOTION

The formulation of the equations of motion using Green's influence numbers used by [14,3] is adopted in this paper to arrive at the equations of motion. The elastic deformation y_i of discrete structural model, including the soil deformations is caused by the resistance forces F_i of Equation (2.2), acting in their reverse directions. Considering the elastic response of the materials, it is possible to express the elastic deformation y_i in a given degree of freedom i as a linear superposition of the individual contributions of all these forces as:

$$y_i(t) = \sum_k (-F_k \delta_{ik}) \tag{4.1}$$

Where, δ_{ik} is the displacement or rotation in the degree of freedom *i* due to a unit force or moment acting in the degree of freedom *k*. These quantities are known as the Green's influence numbers, and are computed from

$$\delta_{ik} = \int_l \frac{M_i M_k}{EI} + \frac{H_i H_k}{K_h(\omega)} + \frac{M_i M_k}{K_r(\omega)} \tag{4.2}$$

In which the contributions of the column shear are neglected, and *E* and *I* are the modulus of elasticity, and the moment of inertia of the columns respectively.

In the above equation,
M_i, and M_k, are the moment distributions in the columns due to a unit force or moment in the degrees of freedom *i* and k,

H_i, Mi and H_k, M_k are reaction forces and moments at the soil-foundation interface due to a unit force or moment in the degrees of freedom *i* and k,

$K_h(\omega)$, and $K_r(\omega)$ are the frequency dependant dynamic spring stiffness of the underlying soil for the horizontal displacement and rotation of foundation.

Substituting for the rigid body motions from Equation (2.1) in Equation (2.2) and the resulting expressions further in Equation (4.1), collecting the elastic deformation terms on the left hand side of the equation, and noting that the second time derivatives of the free ground displacements and rotation are equal, respectively to the free ground accelerations a_h, and a_r, results in the equations of motion.

$$[\Delta][M]\{\ddot{y}\}+[\Delta][C]\{\dot{y}\}+\{y\}=\Delta(a_h\{\eta_h\}+a_r\{\eta_r\}) \qquad (4.3)$$

Premultiplying the matrix Equation (4.3) by the inverse of [Δ], yields finally the equation of motion in the conventional form:

$$[M]\{\ddot{y}\}+[C]\{\dot{y}\}+[K]\{y\}=a_h\{\eta_h\}+a_r\{\eta_r\} \qquad (4.4)$$

Where, [K] is the stiffness matrix of the system obtained by inverting, [Δ], the matrix of Green's influence numbers, [M] is the diagonal lumped mass matrix; [C] is the non proportional damping matrix; $\{\eta_j\}$ are the participation vectors of the free ground acceleration components which could be expressed as follows:

$$\{\eta_h\}=-[M]\{\alpha\}, \qquad \{\eta_r\}=-[M]\{\gamma\} \qquad (4.5)$$

The vectors, {α} and {γ} are the vectors of influence numbers of rigid body motion whose elements are described in Equation (2.1). Vector {y} is the response vector.

4.2 FREE VIBRATION

The free vibration problem is obtained from Equation (4.4), when the right hand side is set to zero yielding:

$$[M]\{\ddot{y}\}+[K]\{y\}=\{0\} \qquad (4.6)$$

The solution of the second-order homogeneous differential equation is in the form of

$$\{y\}=\{\Phi\}e^{i\omega t} \qquad (4.7)$$

Substituting Equation (4.7) into Equation (4.6) results

$$([K]-\omega_n^2[M])\{\Phi\}=\{0\} \qquad (4.8)$$

where, [K] is the stiffness matrix of the combined system, ω_n is the natural circular frequency.

For non-trivial solution of Equation (4.8), the determinant of the coefficient matrix is set to zero, which yields an equation for the determination of the natural frequencies.

$$\det\left|[K]-\omega_n^2[M]\right|=0 \qquad (4.9)$$

It is very important at this stage to note that the stiffness matrix itself is frequency dependant. Hence a direct solution of Equation (4.9) is not possible.

To seek solution to this problem, it is suggested first to obtain the fundamental natural circular frequency of the fundamental mode of the same system considering only the static stiffness of the soil-foundation system.

Next, iterative is made with respect to the fundamental frequency using Equation (4.2) and (4.9) until a reasonable agreement is achieved. The last obtained values of the natural frequencies may then be retained to determine the flexibility matrix and hence the stiffness matrix of the coupled system. Substituting each of the frequencies so obtained in Equation (4.8) and normalizing, one obtains the corresponding eigenvectors, $\{\phi_n\}$

4.3 SOLUTION BY MODAL SUPERPOSITION METHOD

The standard method of modal superposition is a procedure used to solve the differential equations of motion for linear dynamic system with classical damping. This approach is explained in detail by many authors [3,12]. In this study, it is intended to solve the set of coupled differential equations of motion given by Equation (4.4), with non-classical damping in the geometrical coordinates by the method of modal superposition in which a transformation is made to modal coordinates. In this method the response vector $\{y(t)\}$is expressed as a linear combination of the modal forms as:

$$\{y(t)\} = \sum_n \{\phi_n\} q_n(t) \tag{4.10}$$

where:

$q_n(t)$ are the displacement amplitude, of the n^{th} mode, yet to be determined and

$\{\phi_n\}$ are the eigenforms or the natural modes

This transformation of coordinate yields;

$$\ddot{q}_m + \frac{\sum_n q_n C^*_{mn}}{M^*_m} + \omega^2_m q_m = f(\tau) \tag{4.11}$$

The excitation function, $f(\tau)$, is given by:

$$f(\tau) = \frac{1}{M^*_n}\left[L_{hm} a_h + L_{rm} a_r\right] \tag{4.12}$$

where the matrix of the generalized damping, C^*_{mn} and the generalized mass, M^*_m for the m[th] mode are given respectively as follows.

$$C^*_{mn} = \sum_i c_i (\phi_m)_i (\phi_n)_i \tag{4.13}$$

$$M^*_m = \sum_i m_i (\phi_m)^2_i \tag{4.14}$$

The modal participation factors of the free ground acceleration components in Equation (4.12) are given by the following Equations (4.15) and (4.16).

$$L_{hm} = -\sum_i m_i (\phi_m)_i \alpha_i \tag{4.15}$$

$$L_{vr} = -\sum_i m_i (\phi_m)_i \gamma_i \tag{4.16}$$

It is to be observed from Equations (4.11) and (4.12) that the direct solution of the coupled equations is not possible. This is due to the fact that the damping matrix, through the velocity component, couples the differential equations with each other.

One way to obtain direct solution of the equations of motion given by Equation (4.12) is by using the customary assumption that the structure is fixed in a very stiff soil or rock. In such a case uncoupling of the differential equations is possible, as the off-diagonal terms of the generalized damping matrix are zero.

However, structures that are constructed on flexible soils exhibit a totally different form of energy dissipation mechanism from that of the superstructure. Thus, the assumption of classical damping leads to an unrealistic solution.

A possible approach to overcome this problem is to take the off-diagonal terms in the summation, of the left hand side of Equation (4.11) to the right side leaving the diagonal terms corresponding to n=m at the previous position and introducing the Lehr's modal damping ratio, $\xi_{m,}$ to obtain

$$\ddot{q}_m + 2\omega_n \xi_m \dot{q}_m + \omega_m^2 q_m = f(t) \tag{4.17}$$

Where the excitation function is modified as given below

$$f(t) = \frac{1}{M_n^*}\left[L_{hm} a_h + L_{rm} a_r - \sum_{n=n\neq m} q_n C_{mn} \right] \tag{4.18}$$

The Lehr's modal damping ratio in Equation (4.17) is given by

$$\xi_m = \frac{C_{mm}^*}{2M_n^*}\omega_n \tag{4.19}$$

Where the diagonal elements of the generalized damping matrix are given by

$$C_{mm}^* = \sum_i C_i (\phi_m^*)_i^2 \tag{4.20}$$

The excitation function contains coupled damping terms, which are considered as dynamic loadings, and the left hand side is the customary expression for a single degree of freedom system.

As already mentioned, an explicit solution of Equation (4.17) does not exist. This is because its right hand side depends on the required solution itself. As suggested in [14], an iterative method of computation can be used to solve this equation effectively.

4.4 ITERATIVE APPROACH

Ground acceleration during earthquakes varies irregularly to such an extent, (see Figure 1.1) that analytical solution of the equation of motion must be ruled out. Therefore, step-by-step numerical methods are used to determine the structural response. Any of the methods as given by [5,12] could be used for systems with classical damping.

Response in this study are obtained by an iterative approach, whose essence is based on the assumption that the free ground acceleration is assumed to vary linearly between two discrete time points, say , $\triangle t = 0.02$.

An iterative approach developed by Worku[14] is adopted to solve the ordinary differential equations of (4.17), which are still coupled with respect to damping. The flowchart is given at Appendix 2. The approach is briefly presented as follows.

Earthquake records are available digitized in small fractions of a second, like 0.02sec or 0.01sec. The right hand side of Equation. (4.17) may be assumed to vary linearly within such time intervals. At time τ within such an interval of time Δt Equation (4.17) may be written as

$$\ddot{q}_m + 2\omega_n \xi_m \dot{q}_m + \omega_m^2 q_m = f(\tau) = a + b\tau \qquad (4.21)$$

Where the excitation vector is defined at the beginning and at the end of the time interval, dt. It is to be noted that the right hand side includes the coupling damping forces shown in Eqn (4.18)

Figure 4.1 depicts the assumed linear variation of the excitation vector.

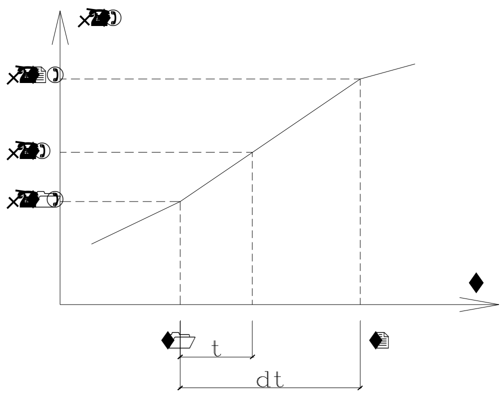

Figure (4.1), Linear variation of the excitation vector, adapted from [14]

The particular solution of the ordinary differential equation of Equation (4.21) is obviously a linear polynomial , which can be easily found and added to the well known homogenous solution, Equation (4.22), to give the general solution.

$$q_{mh}(t) = e^{-\xi_m \omega_m t}[A_m \cos(\omega_{dm} t) + B_m \sin(\omega_{dm} t)] + \frac{1}{\omega^2}\left(a - \frac{2\xi}{\omega} b + bt\right) \quad (4.22)$$

where the constants a and b are given as

$a = f(t_1)$, and $b = \frac{f(t_2) - f(t_1)}{dt}$

and ω_{dm} is the damped circular frequency of the m[th] mode given by

$$\omega_{dm} = \omega_m \sqrt{1 - \xi_m^2} \quad (4.23)$$

The coefficients A_m and B_m are determined from the initial conditions, which are to be substituted in Equation (4.22) and in its derivative [13].

The modal displacement and velocity amplitudes at discrete time, t of the earthquake record are computed directly from Equation (4.22) and its derivative.

The generalized mass M_m, the elements C_{mn} of the generalized damping matrix, and the modal participation factors of the free ground acceleration components L_{hm} and L_{rm} may be calculated in advance for each mode considered just after the determination of the modes.

The required values of $q_m(t)$ and $\dot{q}_m(t)$ can then be computed simultaneously from Equation (4.22) and its derivate [13].

It is possible to start the computation by first setting the damping term in the excitation function, $f(t)$, to zero and successively updating the result simultaneously by iteration

Subsequent iterations are made by always using the $q_m(t)$ and $\dot{q}_m(t)$ terms from the previous step.

As briefly described above, a suitable method for solving the coupled modal differential equations, which is suitable for computer programming, is adopted [14]. The next step is the calculation of the response for each degree of freedom of the system.

4.5 MODE DISPLACEMENT METHOD

The calculation of the response by mode displacement method is based on the modal substitution of Equation (4.10), which is rewritten as:

$$\{y(t)\} = \sum_{n=1}^{N} \{\phi_n\} q_n(t) \qquad (4.24)$$

In which the amplitudes $q_m(t)$ of each mode is obtained from Equation (4.21). Inclusion of the first few modes in Equation (4.24) is mostly sufficient, with the remaining higher modes contributions truncated. This is known as the mode-displacement method.

5. PARAMETRIC STUDY

In the previous chapters, we reviewed the background information on the modeling aspect and the solution approaches of soil structure interaction problems.

In this chapter, parametric studies are carried out to demonstrate numerically the effect of soil-structure interaction on the elastic behavior of symmetrical reinforced concrete structures founded in flexible soil formations.

Some of the major factors, which affect the response, are the nature of the soil, the mass and stiffness of the superstructure, the foundation type and the embedment conditions of the foundations and the foundation types.

Among these parameters, an effort is made to study the effect of the nature of the superstructure, the nature of the soil formation, and the foundation embedment conditions in modifying the responses of the coupled model with flexible base condition.

Time history analysis of the different types of building in fixed base, flexible base and with different embedment conditions is carried. Story displacements and base shear and base overturning values are computed.

The foundation models considered in the analysis of different cases are shown in Figures 5.1 to 5.4.

The acceleration record of the Imperial Valley earthquake, North-South component of El Centro is used as input motion at the base of the structure. The peak ground acceleration for this record is 0.32g.

5.1 SOIL CONDITIONS

The soil formation in Addis Ababa varies from site to site and some times it even shows great variation in the same site. Sites where deposits of flexible soil exist are selected for this study.

5.1.1 SELECTION OF SITES

Three sites are selected and used in modeling the sub structure. Borehole log data for these specific selected sites in Addis Ababa, the Capital city of Ethiopia is used establish the soil model for analysis. These sites are selected on the basis that flexible soil formations occur, important construction works exist in these areas, and on the availability of borehole data.

Site1: The Addis Ababa International airport project site is selected as Site 1. This is important site in Addis Ababa, where a number of expansion projects of the airport have been carried out recently.

Site 2: The Loli Building construction site is selected as Site 2. This construction site is located around the Mekanissa area, next to the Gabriel Church. This building is located in an area where a number of medium-rise buildings are currently under construction.

Site 3: The NISCO Head Office Building located on the Bole Road, in front of Denbel City Center, is selected as the third site. This is an area of high urban expansion.

5.1.2 BOREHOLE LOG DATA

Site 1: There are fifteen borehole log data available for this site. From observation of the logs, it is found that certain bore hole data do not indicate the depth to hard stratum or rocky interface. Therefore, such data are excluded in the determining the thickness and shear wave velocity of the site. Furthermore, some of the data are not considered when the rock formation is very near or exposed to the surface.

Site 2: There are only four borehole log data available for this site. As it was observed from the geological cross-section provided, their exists a basaltic formation at about 13m from the natural ground level.

Site 3: Six borehole log data are available for this site. The geological profiles indicate that the site is erratic in nature. There exists a highly weathered vesicular basaltic formation at shallow depths, which varies from a depth of4m to 8.9m from the natural ground level. The basaltic formation which is located near to the surface, and which is discontinuous over the site is discarded as the interface surface.

5.1.3 DETERMINATION OF SHEAR WAVE VELOCITY OF THE SOIL

Available standard penetration test values are used to determine the shear wave velocities for each layer of the soil formation.

The shear wave velocities are determined using the empirical relations suggested by Ohta and Goto(1976) [15] as presented in Equation(5.1) . Appendix 1, shows the variation of shear wave velocity for the three different sites.

$$V_s = 68.79 * N^{0.171} * H^{0.199} * E * F \qquad (5.1)$$

where, V_s = S-wave velocity (m/s),

E(Epoch) is 1.000 for alluvium deposit and 1.303 and diluvium

F(Facies) is 1.000 for clay,1.086 for fine sand,1.066 for medium sand, 1.135 for course sand, 1.153 sandy gravel and 1.448 for gravel.

It is observed from the bore log data that even in a specific site the type of soil formation, the depth of layers, the depth and existence of the bedrock shows significant variation, rendering the profile highly erratic.

Observation of the borehole data is made to identify the existence of a defined interface such as bedrock underlying the soil formation for all the sites. Some data are discarded which do not show clear soil-rock interface. Weighted average of the shear wave velocity is employed. The averaging parameter is the thickness of the specific soil layer. It is assumed that such means of averaging the soil parameters in relation to the depth of the soil is better than direct averaging of the parameters directly. This will provide representative values of soil thickness and shear wave velocity values. The calculated depths and shear wave velocity for the sites are shown in Table 5.5a through Table 5.5c.

It is observed from Tables 5.5a and 5.5b that the shear wave velocity and the depth of the flexible stratum computed do not show significant difference. Hence Site 1 and Site 3 are used in the modeling of the soil-foundation.

5.2 BUILDING MODELS

To carry out the parametric study, four types of building structures are established. The buildings are assumed to cover the same plan area, but with different columns and concrete wall arrangements. The size of columns and the number of concrete walls is increased realistically as the number of stories is increased. The detail of each building is model discussed as follows.

Building Model 1: The first model is a five-story regular reinforced concrete building, whose plan is shown in Figure 5.1a. The column size assumed in this building is 40 x 40 cm. It is assumed that the columns are rigidly connected to the floor slabs. The floor height is taken 3.0 meters throughout. There are no concrete walls used for this building model

The typical frame in the y-direction and the lumped parameter model assumed in the analysis are shown by Figures 5.1b and 5.1c respectively.

A mat foundation is used for this building. The plan size of the mat is 15 x 35 m, which is equal to the typical floor area of the building. The thickness of the mat is assumed to be 70cm.

Some of the properties of the building are summarized below.

Mass of foundation	9,188 [KN]/g
Mass of story 1-4	8,400 [KN]/g
Mass of story 5	5,250 [KN]/g
Typical floor stiffness (columns)	752,450[KN/m]
Modulus of elasticity of concrete	2.48e+7[KN/m^2]

Table 5.6 Summery of structural properties

and g is the gravitational acceleration,

$$g=9.81\ [m/s^2]$$

Building Model 2: Building model 2 is a regular ten-story reinforced concrete building, whose plan shown in Figure 5.2a. The column size assumed in this building is 65 x 65 cm. It is assumed that the columns as well as the concrete walls are rigidly connected to the floor slabs. The stiffness for the individual columns and walls is determined from Equation 5.1 respectively.

$$K = \frac{12EI_c}{h^3} \quad \text{and} \quad K = \frac{3EI_w}{h^3} \tag{5.1}$$

where, E is the modulus of elasticity of concrete and I_c and I_w are the moment of inertia of the columns and walls respectively.

The floor height is taken as 3.0 meters throughout. There are also two concrete walls used for this building model. The size of the walls is 0.2 x 4.2m. As shown on the plan the concrete walls ate arranged symmetrically, so that torsional effects are not included in the building.

The typical frame in the y-direction and the lumped parameter model assumed in the analysis are shown be Figures 5.2b and 5.2c respectively.

A mat foundation is used for this building. The plan size of the mat is 15 x 35 m, which is equal to the typical floor area of the building. The thickness of the mat is assumed to be 150cm.The mat foundation is assumed to be structurally rigid.

Some of the properties of the building are summarized below.

Mass of foundation	19,688 [KN]/g
Mass of story 1-9	8400 [KN]/g
Mass of story 10	5250 [KN]/g
Typical floor stiffness (columns)	5,246,761[KN/m]
Typical floor stiffness (walls)	6,805,120[KN/m]
Typical floor stiffness (Total)	12,051,881 [KN/m]
Modulus of elasticity of concrete	2.48e+7[KN/m^2]

Table 5.7 Summery of structural properties

Building Model 3: Building model 3 is a regular twenty-story reinforced concrete building, whose plan, is shown in Figure 5.3a. The column size assumed in this building is 65 x 65 cm. It is assumed that the columns as well as the concrete walls are rigidly connected to the floor slabs. The stiffness for the individual columns and walls is determined from Equation 5.1 respectively.

The floor height is taken as 3.0 meters throughout. There are also two concrete walls used for this building model. The size of the walls is 0.2 x 4.2m. As shown on the plan the concrete walls ate arranged symmetrically, so that torsional effects are not included in the building.

The typical frame in the y-direction and the lumped parameter model assumed in the analysis are shown be Figures 5.3b and 5.3c respectively.

A mat foundation is used for this building. The plan size of the mat is 15 x 35 m, which is equal to the typical floor area of the building. The thickness of the mat is assumed to be 150cm.The mat foundation is assumed to be structurally rigid.

Some of the properties of the building are summarized below.

Mass of foundation	19,688 [KN]/g
Mass of story 1-19	8400 [KN]/g
Mass of story 20	5250 [KN]/g
Typical floor stiffness (columns)	5,246,761[KN/m]
Typical floor stiffness (walls)	6,805,120[KN/m]
Typical floor stiffness (Total)	12,051,881 [KN/m]
Modulus of elasticity of concrete	2.48e+7[KN/m^2]

Table 5.8 Summery of structural properties

Building Model 4: Building model 4 is a regular thirty-story reinforced concrete building, whose plan, is shown in Figure 5.4a. The column size assumed in this building is 90 x 90 cm. It is assumed that the columns as well as the concrete walls are rigidly connected to the floor slabs. The stiffness for the individual columns and walls is determined from Equation 5.1.

The floor height is taken as 3.0 meters throughout. There are also six concrete walls used for this building model. The size of the walls is 0.2 x 4.2m. As shown on the plan the concrete walls ate arranged symmetrically, so that torsional effects are not included in the building.

The typical frame in the y-direction and the lumped parameter model assumed in the analysis are shown be Figures 5.4b and 5.4c respectively.

A mat foundation is used for this building. The plan size of the mat is 15 x 35 m, which is equal to the typical floor area of the building. The thickness of the mat is assumed to be 200cm.The mat foundation is assumed to be structurally rigid.

Some of the properties of the building are summarized below.

Mass of foundation	26,250 [KN]/g
Mass of story 1-29	8,400 [KN]/g
Mass of story 30	5,250 [KN]/g
Typical floor stiffness (columns)	19,248,480[KN/m]
Typical floor stiffness (walls)	20,415,360[KN/m]
Typical floor stiffness (Total)	39,699,840[KN/m]
Modulus of elasticity of concrete	2.48e+7[KN/m^2]

Table 5.9. Summery of structural properties

5.3 CASE STUDY

Case 1: The first case considered is the analysis of the five-story building (Model 1), on site 1, where the structure is assumed fixed at its base.

The results of the analysis, including the natural circular frequencies, story displacements relative to the foundation, the maximum base shear and the base overturning moments, are presented in Table 5.5a, Table 5.5b and Table 5.5c respectively.

Case 2: The Second case considered is the analysis of the five-story building (Model 1), on site 1, where the foundation is placed at the surface of elastic half space. Substructure condition is expressed schematically as shown by Figure 5.1, on page 24.

It is assumed that the shear wave velocity computed is adopted as the representative value for the half space model. Other parameters, which are required to determine the spring and dashpot constants, such as the mass density of the soil, ρ, and the poisons ratio, ν are assumed values of 2400 Kg/m^3, and 0.4 respectively. The shear stiffness of the soil is determined according to Equation (5.1), using the shear wave velocity of 118.9m/s, for Site1.The computed value of the shear stiffness, G is 3.4e+4 KPa.

The computed dynamic spring and dashpot values according to the tables and charts provided by Gazetas[7], are determined for a frequency of $7.0s^{-1}$. Note that this frequency value is obtained after a number of iterations, so that this is nearly equal to the fundamental circular frequency of the coupled system.

The computed spring constants are:
For lateral horizontal mode, K_y=2.63e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 1.6e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=1.79e+5 $KN.s.m^{-1}$
For the rocking mode rx (around the longitudinal axis) C_{rx} = 12.54e+6 KN.s.m

The analysis of the coupled system is carried out for this case, by making use of the parameters expressed above and those structural properties previously determined for the superstructure and the foundation.

The results of the analysis, including the natural circular frequencies, story displacements relative to the foundation, the maximum base shear and the base overturning moments, are presented in Table 5.5a, Table 5.5b and Table 5.5c respectively.

Case 3: For the third case, the analysis of the five-story building (Model 1) is considered, where the foundation is embedded at 2m below the surface of elastic half space, with the properties described in case 2.The dynamic spring and dashpot values are determined for a frequency of $8.5s^{-1}$, as obtained by iteration.

The computed spring constants are:
For lateral horizontal mode, K_y=3.4e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 2.01e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=2.7e+5 $KN.s.m^{-1}$
For the rocking mode rx (around the longitudinal axis) C_{rx} = 6.05e+6 KN.s.m

The results of the analysis are presented in Table 5.5a, Table 5.5b and Table 5.5c .

Case 4: For this case, the analysis of the five-story building (Model 1) is considered to be embedded 4m below the surface of elastic half space, for Site 1.

The dynamic spring and dashpot values are determined for a frequency of $8.7s^{-1}$, as obtained by iteration

The computed spring constants are:
For lateral horizontal mode, K_y=3.9e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 2.74e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:

For lateral horizontal mode, C_y=3.6e+5 KN.s.m^{-1}

For the rocking mode rx (around the longitudinal axis) C_{rx} = 9.55e+6 KN.s.m

The results of the analysis are presented in Table 5.5a, Table 5.5b and Table 5.5c.

Case 5: For this case, the analysis of the five-story building (Model 1) is considered to be constructed at the surface of a homogenous stratum over bedrock. The stratum is modeled by using the parameters computed for Ste 1. The upper layer has an average thickness of 7.0m above the hard stratum.

The dynamic spring and dashpot values are determined for excitation frequency of 7.5s^{-1} obtained by iteration.

The computed spring constants are:

For lateral horizontal mode, K_y=3.16e+6 KN/m

For the rocking mode rx (around the longitudinal axis) K_{rx} = 2.53e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:

For lateral horizontal mode, C_y=3.72e+4 KN.s.m^{-1}

For the rocking mode rx (around the longitudinal axis) C_{rx} = 2.98e+4 KN.s.m

At this point, it is worth to note that, when the bedrock is at shallow depth, the dynamic stiffness values show moderate increase, while the damping of the dashpot decreases substantially, as compared to case1, when the foundation is placed on the surface of elastic half space of the same soil type. This decrease in damping is particularly significant in rocking mode.

The analysis of the coupled system is carried out for this case, by making use of the parameters expressed above and those structural properties previously determined for the superstructure and the foundation.

The results of the analysis are presented in Table 5.5a, Table 5.5b and Table 5.5c.

Case 6: For this case, the analysis of the five-story building (Model 1) is considered to be embedded 2.0m below the surface of homogenous stratum over bedrock. The stratum is modeled by using the parameters computed for Ste 1.

The dynamic spring and dashpot values are determined for excitation frequency of $8.0s^{-1}$ obtained by iteration.

The computed spring constants are:
For lateral horizontal mode, K_y=8.5e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 5.39e+6 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For Lateral horizontal mode, C_y=3.2e+5 KN.s.m^{-1}
For The rocking mode rx (around the longitudinal axis) C_{rx} = 3.2e+6 KN.s.m

Note that, when the bedrock is at shallow depth and the foundation is embedded for 2m , the dynamic stiffness values and the damping increase, as compared to when the foundation is placed on the surface of the same stratum, but not for) $K_{rx.}$

The results of the analysis are presented in Table 5.5a, Table 5.5b and Table 5.5c.

Case 7: For this case, the analysis of the five-story building (Model 1) is considered to be constructed embedded 4.0m below the surface of homogenous stratum over bedrock. The stratum is modeled by using the parameters computed for Site 1.

This is the last case considered using Model1 and Site1 with different foundation embedment conditions.

The dynamic spring and dashpot values are determined for excitation frequency of $8.5s^{-1}$ obtained by iteration.

The computed spring constants are:
For lateral horizontal mode, K_y=1.26e+7 KN/m

For the rocking mode rx (around the longitudinal axis) K_{rx} = 8.19e+6 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=4.13e+5 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 5.6e+6 KN.s.m

The results of the analysis for case1 to case7 are summarized in Tables 5.1a, Table 5.5b and Table 5.5c.

Mode	**Case1**	**Case2**	**Case3**	**Case4**	**Case5**	**Case6**	**Case7**
Mode 1	9.05	8.572	8.681	8.728	7.362	8.301	8.57
Mode 2	26.27	24.974	25.282	25.414	13.825	17.96	21.144
Mode 3	40.937	39.301	39.745	39.927	25.718	26.861	27.527
Mode 4	51.665	50.346	50.795	50.958	39.62	40.782	40.981
Mode 5	57.637	57.15	57.367	57.426	50.554	51.478	51.585
Mode 6		63.069	68.448	72.008	57.257	57.569	57.595
Mode 7		96.113	107.489	125.359	65.286	99.73	119.628

Table 5.1a Summary of computed natural circular frequencies, [s^{-1}]

Floor Level	**Case1**	**Case2**	**Case3**	**Case4**	**Case5**	**Case6**	**Case7**
5	0.08423	0.07275	0.07707	0.08298	0.08493	0.04064	0.04913
4	0.07781	0.06800	0.07208	0.07829	0.08077	0.03801	0.04721
3	0.06303	0.05682	0.06029	0.06707	0.07082	0.03205	0.04176
2	0.04313	0.04291	0.04557	0.05073	0.05753	0.02459	0.03251
1	0.02276	0.02586	0.02715	0.03043	0.04122	0.01797	0.02006
0	0.00000	0.00691	0.00657	0.00704	0.00518	0.00083	0.00086

Table 5.1b Summary of story displacements relative to the foundation, [m]

	Case1	**Case2**	**Case3**	**Case4**	**Case5**	**Case6**	**Case7**
Base Shear [KN]	27,431.3	14,476.4	16,097.8	18,310.2	27,383.9	13,269.4	14,505.0
Overturning Moment [KNm]	41,147.0	21,714.6	24,146.7	27,465.3	41,075.9	19,904.1	21,757.5

Table 5.1c Summary of computed maximum base shear, and base overturning moment

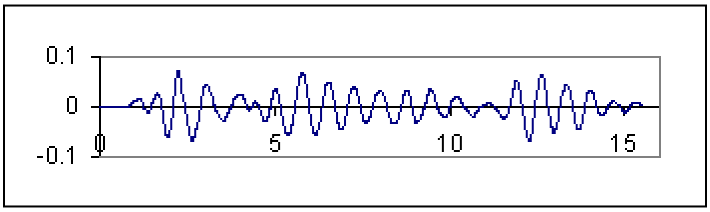

Figure 5.1 Displacement history of m(5) , Maximum response = 0.07275m

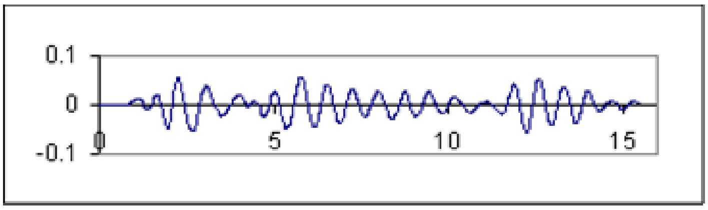

Figure 5.2 Displacement history of m(3) , Maximum response = 0.05682m

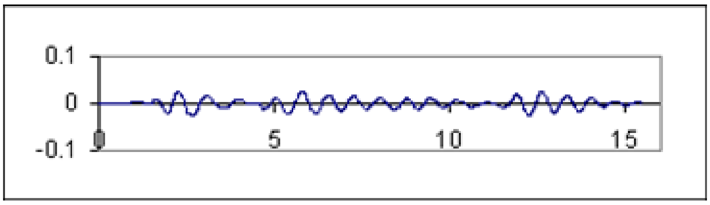

Figure 5.3 Displacement history of m(0) , Maximum response = 0.00691m

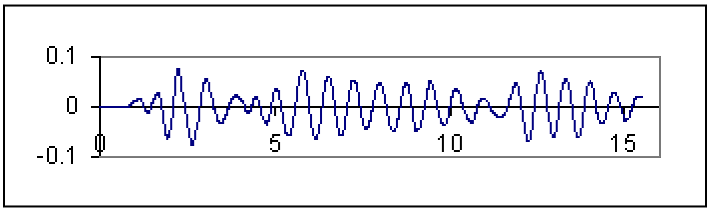

Figure 5.4 Displacement history of m(5) , Maximum response = 0.07707m

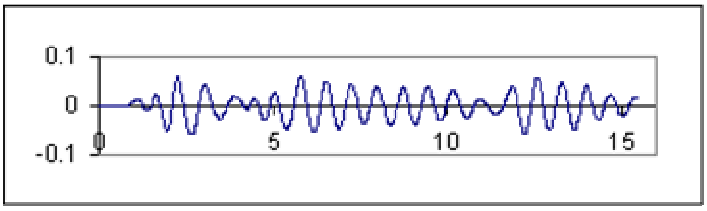

Figure 5.5 Displacement history of m(3) , Maximum response = 0.06029m

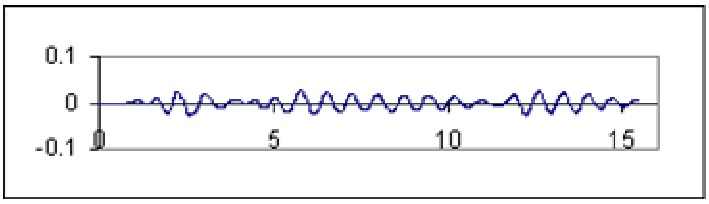

Figure 5.6 Displacement history of m(1) , Maximum response = 0.02715m

The plots of displacement response the time history at foundation level, mid story and at the roof level for case2 and case3 are given by Fig 5.1 to 5.6 respectively

It is observed from cases 1 to 7, which are based on Building Model 1 and site 1 but with different substructure conditions, that the response of the fixed base and the flexible base structure are different

It is observed from Table 5.1a that the natural circular frequencies of all the modes reduce from their respective values of the fixed base structure. For the half space soil model, gentle increase of the natural frequencies is observes as the foundation embedment is increased.

The story displacements relative to the foundation show gradual decrease and increase from one case to another, but the inter-story displacements generally decrease as from the fixed base condition, case1. Due to this decrease of the inter story displacements, the internal forces developed in the structure, such as the story shear and the story moments generally decrease, when compared to the fixed base values.

Like the natural frequencies, the base shear and the base turning moments tend to decrease. Slight increase of theses values is shown for the half elastic model from case2 to case 4, as the stiffness of the soil increase.

Case 8: This case is similar to case 1, but for Building Model 2, which is a ten-story building, on Site 1, where the foundation is assumed fixed.

The results of the analysis, including the natural circular frequencies, Maximum story displacements, the maximum base shear and the base overturning moments, are presented in Table 5.2a, Table 5.2b and Table 5.2c respectively.

Case 9: This case is the analysis of the ten-story building, Mode2, on Site 1, where the foundation is placed at the surface of elastic half space.

It is assumed that the shear wave velocity computed is adopted as the representative value for the half space model. Other parameters, which are required to determine the spring and dashpot constants, such as the mass density of the soil, ρ, and the poisons ratio, ν shear wave velocity of 118.9m/s, for Site1. The computed value of the shear stiffness, G is 3.4e+4 KPa.

The computed dynamic spring and dashpot values according to the tables and charts provided by Gazetas[7], are determined for excitation frequency of $12.0s^{-1}$. Note that this frequency is obtained by iteration.

The computed spring constants are:
For lateral horizontal mode, K_y=2.63e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 1.53e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=1.72e+5 $KN.s.m^{-1}$
For the rocking mode rx (around the longitudinal axis) C_{rx} = 2.54e+6 KN.s.m

The results of the analysis are presented in Table 5.2a, Table 5.2b and Table 5.2c.

<u>Case 10:</u> Case for the analysis of the ten-story building, Model 2 is considered, where the foundation is embedded at 2m below the surface of elastic half space of Site 1.

The dynamic spring and dashpot values are determined for frequency of $13.0s^{-1}$

The computed spring constants are:
For lateral horizontal mode, K_y=3.1e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 1.81e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=267e+5 $KN.s.m^{-1}$
For the rocking mode rx (around the longitudinal axis) C_{rx} = 5.87e+6 KN.s.m

The results of the analysis are presented in Table 5.2a, Table 5.2b and Table 5.2c.

<u>Case 11:</u> For this case, the analysis of the ten-story building, Model 2, is considered to be constructed embedded 4m below the surface of elastic half space, for Site 1.

The dynamic spring and dashpot values are determined for frequency of $13.0s^{-1}$

The computed spring constants are:
For lateral horizontal mode, K_y=3.6e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 2.48e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For Lateral horizontal mode, C_y=3.5e+5 KN.s.m^{-1}
For The rocking mode rx (around the longitudinal axis) C_{rx} = 9.3e+6 KN.s.m

<u>Case 12:</u> For this case, the analysis of the ten-story building, Model 2 is considered to be constructed at the surface of homogenous stratum over bedrock. The stratum is modeled by using the parameters computed for Site 1.

The dynamic spring and dashpot values are determined for frequency of $6.0s^{-1}$

The computed spring constants are:
For lateral horizontal mode, K_y=2.17e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 2.6e+6 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=4.52e+4 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 4.33e+4 KN.s.m

The results of the analysis are presented in Table 5.2a, Table 5.2b and Table 5.2c.

<u>Case 13:</u> The analysis of the ten-story building, Model 2, is considered to be constructed embedded 2.0m below the surface of homogenous stratum over bedrock. The stratum is modeled by using the parameters computed for Site 1.The dynamic spring and dashpot values are determined for frequency of $8.0s^{-1}$

The computed spring constants are:

For lateral horizontal mode, K_y=7.15e+6 KN/m

For the rocking mode rx (around the longitudinal axis) K_{rx} = 4.98e+6 KN.m

And the computed dashpot constants, which are frequency dependant, are:

For lateral horizontal mode, C_y=3.29e+5 $KN.s.m^{-1}$

For the rocking mode rx (around the longitudinal axis) C_{rx} = 3.28e+6 KN.s.m

Note that, when the bedrock is at shallow depth and the foundation is embedded for 2m, the dynamic stiffness values and the damping increase, as compared to when the foundation is placed on the surface of the same stratum. The computation of the foundation stiffness and damping values has been found to be difficult because of the undulations of the stiffness of the soil, which occurs as a result of reflection of, weaves emanated at the foundation. The results computed are not stable, i.e. with slight change of the excitation frequency, the stiffness and damping values change much.

The results of the analysis are presented in Table 5.2a, Table 5.2b and Table 5.2c.

Mode	**Case 8**	**Case 9**	**Case 10**	**Case 11**	**Case 12**	**Case 13**
Mode 1	18.389	12.502	13.046	13.638	5.927	8.714
Mode 2	54.717	37.806	38.801	39.803	14.875	20.041
Mode 3	89.699	60.237	63.556	66.154	36.022	45.87
Mode 4	122.479	68.88	71.453	80.245	64.118	71.639
Mode 5	152.254	97.918	98.291	98.964	95.652	98.714
Mode 6	178.3	128.126	128.227	127.988	125.425	128.224
Mode 7	199.994	156.092	156.222	155.19	155.018	155.617
Mode 8	216.85	180.704	181.024	179.232	180.394	182.11
Mode 9	228.579	201.708	201.951	199.894	200.693	202.063
Mode 10	235.23	218.488	218.453	216.695	219.634	218.067
Mode 11		230.04	229.943	228.729	230.529	228.264
Mode 12		235.94	235.962	235.398	236.204	236.134

Table 5.2a Summary of computed Natural frequencies, [s^{-1}]

Floor level	Case 8	Case 9	Case 10	Case 11	Case 12	Case 13
10	0.02891	0.02203	0.01713	0.01348	0.03061	0.03830
9	0.02842	0.02186	0.01697	0.01337	0.03049	0.03818
8	0.02718	0.02140	0.01657	0.01314	0.03018	0.03784
7	0.02520	0.02064	0.01592	0.01274	0.02969	0.03735
6	0.02258	0.01963	0.01507	0.01219	0.02895	0.03667
5	0.01988	0.01837	0.01404	0.01149	0.02814	0.03581
4	0.01665	0.01689	0.01286	0.01063	0.02720	0.03467
3	0.01296	0.01523	0.01156	0.00961	0.02608	0.03339
2	0.00889	0.01341	0.01014	0.00844	0.02509	0.03192
1	0.00454	0.01147	0.00861	0.00714	0.02405	0.03026
0		0.00909	0.00676	0.00588	0.01345	0.00324

Table 5.2b Summary of story displacement relative to the foundation, [m]

	Case 8	Case 9	Case 10	Case 11	Case 12	Case 13
Base Shear [KN]	62,996	20,632	23,041	30,107	246,537	325,641
Overturning Moment[KNm]	94,494.0	30,948.0	34,561.5	45,160.5	369,805.5	488,461.5

Table 5.2c Summery of computed Maximum Base shear and base overturning moment

It is observed from cases 8 to 13, which are based on Building Model 2 and Site 1 but with different substructure conditions that the response of the structure when the structure is assumed fixed and that when flexible foundation is introduced are different

It is observed from table 5.2a that the natural circular frequencies of all the modes reduce from their respective values of the fixed base structure. Gentle increase of the natural frequencies is observes as the foundation embedment increase, for the half space model. For example, for Mode 1, the natural frequency is 12.502 s^{-1} for case 9, and increase to a value of 13.638 s^{-1}, for case 11.When the bedrock is at shallow depth as for cases12 to 13 though generally the natural frequency decreases there is no clear trend of decreasing or increase.

The maximum story displacements show gradual decrease, for the elastic half space model, but the inter-story displacements generally decrease as from the fixed base condition, case8. Due to this decrease of the inter story displacements, the internal forces developed in the structure, such as the story shear and the story moments generally decrease, when compared to the fixed base values.

Like the natural frequencies, the base shear and the base turning moments tend to decrease. Slight increase of theses values is shown for the half elastic model from case11 to case 9, as the stiffness of the soil increase.

The plots of displacement response the time history at foundation level, mid story and at the roof level for case9 and case10 are given by Fig 5.7 to 5.12 respectively

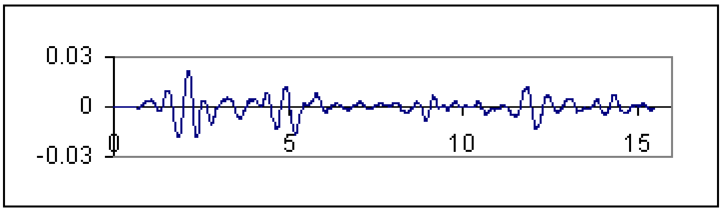

Figure 5.7 Displacement history of m(10) , Maximum response = 0.02203m

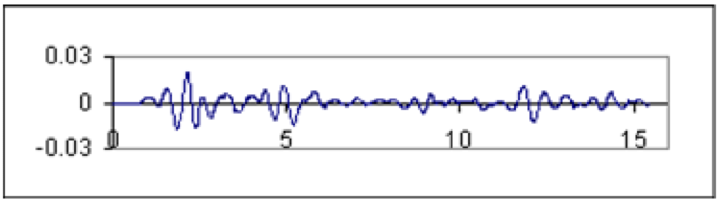

Figure 5.8 Displacement history of m(6) , Maximum response = 0.01963m

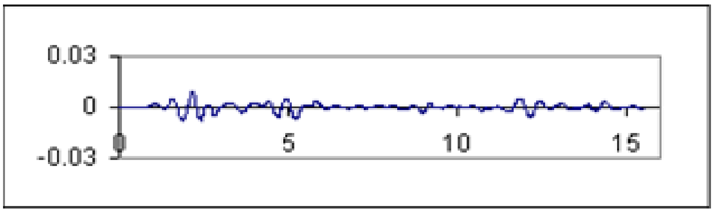

Figure 5.9 Displacement history of m(0) , Maximum response = 0.00909m

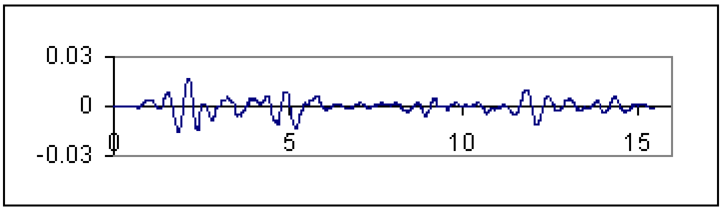

Figure 5.10 Displacement history of m(10) , Maximum response = 0.01713m

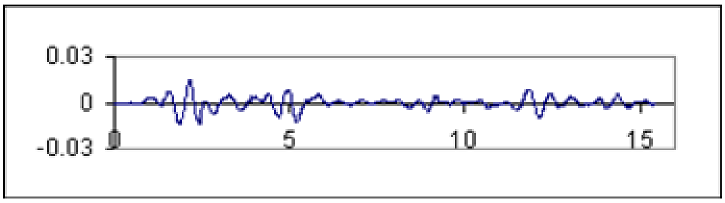

Figure 5.11 Displacement history of m(6) , Maximum response = 0.01507m

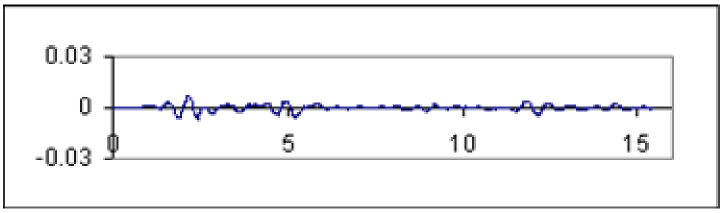

Figure 5.12 Displacement history of m(0) , Maximum response = 0.00676m

<u>Case 14:</u> This case is similar to case 1and case 8, but for building Model 3, which is a twenty-story building, on Site 1, where the foundation is assumed fixed.

The results of the analysis, which are the natural circular frequencies, story displacements relative to the foundation, the maximum base shear and the base overturning moments, are presented in Table 5.3a, Table 5.3b and Table 5.3c respectively.

<u>Case 15:</u> This case is the analysis of the twenty-story building, Mode3, on Site 1, where the foundation is placed at the surface of elastic half space.

The dynamic spring and dashpot values are determined for excitation frequency of 7.0s^{-1} obtained by iteration.

The computed spring constants are:
For lateral horizontal mode, K_y=2.63e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 1.65e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=1.8e+5 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 3.11e+6 KN.s.m

The results of the analysis are presented in Table 5.3a to Table 5.3c.

Case 16: Case for the analysis of the twenty-story building, Model 3 is considered, where the foundation is embedded at the 2m below the surface of elastic half space of Site 1.

The dynamic spring and dashpot values are determined for excitation frequency of $7.0s^{-1}$ obtained by iteration.

The computed spring constants are:
For lateral horizontal mode, K_y=3.4e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 2.09e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=2.8e+5 $KN.s.m^{-1}$
For the rocking mode rx (around the longitudinal axis) C_{rx} = 6.64e+6 KN.s.m

The results of the analysis are presented in Table 5.3a to Table 5.3c.

Case 17: For this case, the analysis of the twenty-story Building, Model 3, is considered to be constructed embedded 4m below the surface of elastic half space, for Site 1.
The dynamic spring and dashpot values are determined for frequency of $7.0s^{-1}$

The computed spring constants are:
For lateral horizontal mode, K_y=3.9e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 2.86e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=3.7e+5 $KN.s.m^{-1}$
For the rocking mode rx (around the longitudinal axis) C_{rx} = 1.03e+7 KN.s.m

Case 18: For this case, the analysis of the twenty-story Bilding, Model 3, is considered to be constructed at the surface of homogenous stratum over bedrock. The stratum is modeled by using the parameters computed for Site 1.

The dynamic spring and dashpot values are determined for excitation frequency of 8.0s^{-1}

The computed spring constants are:

For lateral horizontal mode, K_y=2.0e+6 KN/m

For the rocking mode rx (around the longitudinal axis) K_{rx} = 1.4e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:

For lateral horizontal mode, C_y=1.22e+4 KN.s.m^{-1}

For the rocking mode rx (around the longitudinal axis) C_{rx} = 3.11e+4 KN.s.m

The results of the analysis for cases 14 to 18 are presented in Table 5.3a to Table 5.3c.

Mode	**Case 14**	**Case 15**	**Case 16**	**Case 17**	**Case 18**
Mode 1	9.4	7.1	7.5	7.5	8.0
Mode 2	28.1	22.4	23.3	23.3	23.9
Mode 3	46.7	38.0	38.8	38.9	39.7
Mode 4	65.0	53.8	54.6	54.5	55.2
Mode 5	82.8	64.0	69.1	70.8	70.4
Mode 6	100.2	72.7	75.1	83.5	85.2
Mode 7	116.9	87.8	88.1	90.8	99.4
Mode 8	132.9	104.4	104.1	104.9	113.0
Mode 9	148.1	120.1	120.1	120.6	125.9
Mode 10	162.4	136.4	135.8	135.5	138.1
Mode 11	175.7	149.2	149.4	151.7	149.4
Mode 12	187.9	164.2	164.1	164.5	159.7
Mode 13	199.0	176.6	177.6	177.6	169.1
Mode 14	208.8	188.8	190.0	188.4	177.5
Mode 15	217.3	198.4	199.5	201.3	184.7
Mode 16	224.5	209.2	210.2	209.5	190.8
Mode 17	230.3	217.4	218.5	218.7	195.8
Mode 18	234.8	223.4	224.5	224.1	199.5
Mode 19	237.8	230.3	230.0	231.7	202.1
Mode 20	239.4	235.5	235.3	236.5	203.5
Mode 21		238.1	237.6	238.4	238.1
Mode 22		240.1	240.0	240.5	240.1

Table 5.2a Summary of computed Natural frequencies, [s^{-1}]

Level	Case 14	Case 15	Case 16	Case 17	Case 18
20	0.0871	0.0583	0.0615	0.0571	0.06856
19	0.0867	0.0582	0.0614	0.0570	0.06840
18	0.0857	0.0578	0.0609	0.0566	0.06790
47	0.0841	0.0572	0.0602	0.0559	0.06707
16	0.0819	0.0564	0.0592	0.0549	0.06588
15	0.0791	0.0553	0.0579	0.0535	0.06420
14	0.0758	0.0539	0.0562	0.0521	0.06247
13	0.0720	0.0524	0.0545	0.0504	0.06048
12	0.0678	0.0507	0.0525	0.0485	0.05822
11	0.0632	0.0488	0.0503	0.0464	0.05570
10	0.0583	0.0467	0.0479	0.0439	0.05262
9	0.0530	0.0441	0.0451	0.0415	0.04980
8	0.0475	0.0417	0.0422	0.0389	0.04667
7	0.0418	0.0391	0.0392	0.0361	0.04334
6	0.0359	0.0363	0.0361	0.0332	0.03985
5	0.0299	0.0334	0.0329	0.0302	0.03619
4	0.0238	0.0304	0.0295	0.0270	0.03239
3	0.0177	0.0273	0.0261	0.0238	0.02851
2	0.0119	0.0241	0.0225	0.0205	0.02458
1	0.0060	0.0209	0.0189	0.0171	0.02057
0		0.0171	0.0147	0.0134	0.01607

Table 5.2b Summary of computed story displacement relative to the foundation, [m]

	Case 14	Case 15	Case 16	Case 17	Case 18
Base Shear[KN]	82,787.0	52,326.0	59,092.0	54,546.0	62,791.2
Overturning Moment[KNm]	124,180.5	78,489.0	88,638.0	81,819.0	94,186.8

Table 5.2c Summary of computed Maximum Base shear and base moment

It is observed from cases 14 to 18, which are based on building Model 3 and Site 1 but with different substructure conditions, that the response of the structure when the structure is assumed fixed and that when flexible foundation is introduced are different.

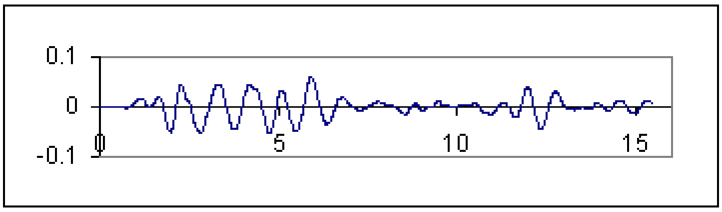

Figure 5.13 Displacement history of m(20) , Maximum response = 0.05830m

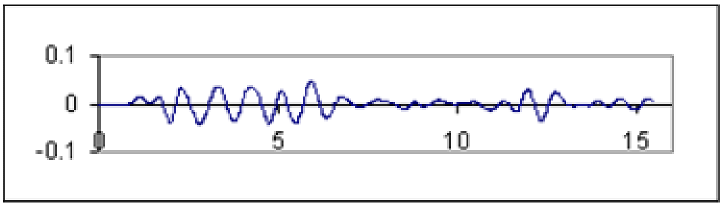

Figure 5.14 Displacement history of m(10) , Maximum response = 0.0488m

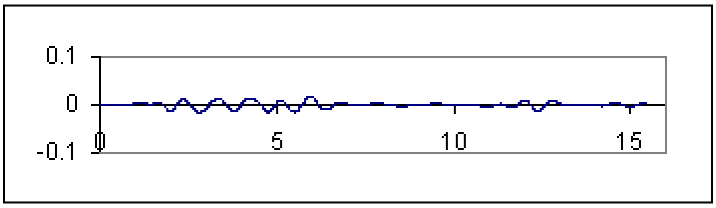

Figure 5.15 Displacement history of m(1) , Maximum response = 0.0171m

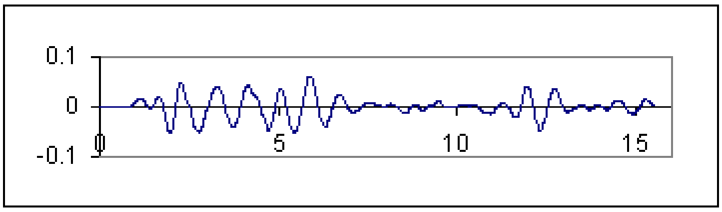

Figure 5.16 Displacement history of m(20) , Maximum response = 0.0615m

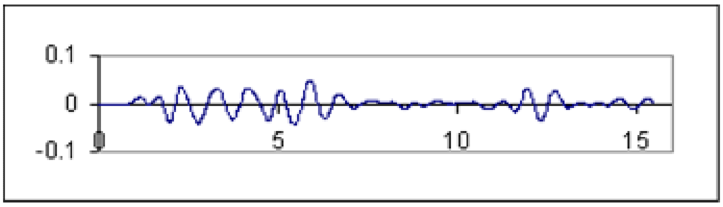

Figure 5.17 Displacement history of m(10) , Maximum response = 0.0479m

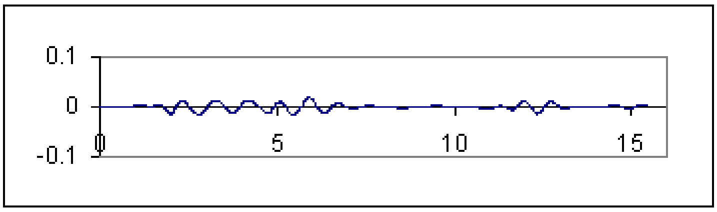

Figure 5.18 Displacement history of m(1) , Maximum response = 0.0147m

The plots of displacement response the time history at foundation level, mid story and at the roof level for case15and case16 are given by Fig 5.13 to 5.18 respectively

It is observed from Table 5.2a that the natural circular frequencies of all the modes reduce from their respective values of the fixed base structure. Gentle increase of the natural frequencies is observes as the foundation embedment increase, for the half space model.

The maximum story displacements show gradual decrease, for the elastic half space model, but the inter-story displacements generally decrease as from the fixed base condition. Due to this decrease of the inter story displacements, the internal forces developed in the structure, such as the story shear and the story moments generally decrease, when compared to the fixed base values.

Like the natural frequencies, the base shear and the base turning moments tend to decrease from the respective fixed base case values.

Case 19: This case is similar to case 1 and case 8 and 14, but for building Model 4, which is a thirty-story building, on Site 1, where the foundation is assumed fixed.

The analysis of the coupled system is carried for this case, by making use of the parameters previously determined for the supper structure and the foundation for Model 4.

The results of the analysis, which are the natural circular frequencies, Maximum story displacements, the maximum base shear and the base overturning moments, are presented in Table 5.4a, Table 5.4b and Table 5.4c respectively.

Case 20: This case is the analysis of the thirty-story building, Mode 4, on Site 1, where the foundation is placed at the surface of elastic half space.

The computed dynamic spring and dashpot values according to the tables and charts provided by Gazetas[7], are determined for excitation frequency of $7.5s^{-1}$. Note that this frequency value is obtained by iteration.

The computed spring constants are:
For lateral horizontal mode, K_y=2.63e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 1.65e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=1.87e+5 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 3.11e+6 KN.s.m

The results of the analysis are presented in Table 5.4a to Table 5.4c.

Case 21: Case for the analysis of the thirty-story building, Model 4 is considered, where the foundation is embedded at 2m below the surface of elastic half space of Site 1.
The dynamic spring and dashpot values are determined for excitation frequency of 7.5s^{-1}

The computed spring constants are:
For lateral horizontal mode, K_y=3.1e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 2.09e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=2.09e+5 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 6.60e+6 KN.s.m

Case 22: For this case, the analysis of the thirty-story building, Model 4, is considered to be constructed embedded 4m below the surface of elastic half space, for Site 1.

The dynamic spring and dashpot values are determined for frequency of 8.0s^{-1}

The computed spring constants are:
For lateral horizontal mode, K_y=3.7e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 2.86e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=3.91e+5 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 1.13e+7 KN.s.m

Case 23: For this case, the analysis of the thirty-story building, Model 4, is considered to be constructed at the surface of homogenous stratum over bedrock. The stratum is modeled by using the parameters computed for Site 1.

The dynamic spring and dashpot values are determined for frequency of $8.0s^{-1}$

The computed spring constants are:
For lateral horizontal mode, K_y=2.0e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 1.4e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=1.22e+4 $KN.s.m^{-1}$
For the rocking mode rx (around the longitudinal axis) C_{rx} = 3.11e+4 KN.s.m

In a similar manner, the results of the analysis are presented in Table 5.4a, Table 5.4b and Table 5.4c.

Mode	Case 19	Case 20	Case 21	Case 22	Case 23
Mode 1	10.609	7.011	7.482	7.957	8.487
Mode 2	31.802	23.117	23.756	23.852	25.442
Mode 3	52.907	41.226	41.775	39.680	42.326
Mode 4	73.865	54.901	59.095	55.399	59.092
Mode 5	94.622	61.882	64.587	70.967	75.698
Mode 6	115.120	80.736	81.251	86.340	92.096
Mode 7	135.302	100.240	100.650	101.477	108.242
Mode 8	155.113	119.856	119.802	116.335	124.090
Mode 9	174.499	139.133	139.664	130.874	139.599
Mode 10	193.407	158.793	158.745	145.055	154.726
Mode 11	211.785	177.876	177.769	158.839	169.428
Mode 12	229.582	195.823	196.623	172.187	183.666
Mode 13	246.750	213.235	215.157	185.063	197.400
Mode 14	263.242	230.771	233.341	197.432	210.594
Mode 15	279.013	249.910	249.772	209.260	223.210
Mode 16	294.018	266.650	265.166	220.514	235.214
Mode 17	308.219	277.592	278.751	231.164	246.575
Mode 18	321.574	289.659	292.225	241.181	257.259
Mode 19	334.049	305.864	306.387	250.537	267.239
Mode 20	345.608	321.532	320.788	259.206	276.486
Mode 21	356.221	336.577	332.600	267.166	284.977
Mode 22	365.858	348.833	345.798	274.394	292.686
Mode 23	374.494	358.458	357.237	280.871	299.595
Mode 24	382.106	366.331	367.350	286.580	305.685
Mode 25	388.674	374.992	375.970	291.506	310.939
Mode 26	394.183	384.045	382.561	295.637	315.346
Mode 27	398.624	390.064	390.395	298.968	318.899
Mode 28	401.997	395.582	395.126	301.498	321.598
Mode 29	404.322	399.609	399.496	303.242	323.458
Mode 30	405.653	403.039	402.968	304.240	324.522
Mode 31		405.313	405.255	405.350	406.320
Mode 32		411.419	408.564	409.300	409.900

Table 5.4a Summary of computed Natural frequencies, [s^{-1}]

Floor	Case 19	Case 20	Case 21	Case 22	Case 23
30	0.08605	0.03749	0.04413	0.05296	0.06884
29	0.08591	0.03745	0.04408	0.05290	0.06876
28	0.08553	0.03738	0.04400	0.05280	0.06864
27	0.08491	0.03721	0.04383	0.05260	0.06837
26	0.08405	0.03705	0.04360	0.05232	0.06802
25	0.08295	0.03675	0.04331	0.05197	0.06756
24	0.08164	0.03652	0.04295	0.05154	0.06700
23	0.08009	0.03611	0.04253	0.05104	0.06635
22	0.07833	0.03571	0.04204	0.05045	0.06558
21	0.07637	0.03526	0.04147	0.04976	0.06469
20	0.07420	0.03476	0.04085	0.04902	0.06373
19	0.07183	0.03422	0.04017	0.04820	0.06267
18	0.06927	0.03361	0.03943	0.04732	0.06151
17	0.06654	0.03299	0.03863	0.04636	0.06026
16	0.06362	0.03232	0.03774	0.04529	0.05887
15	0.06052	0.03161	0.03686	0.04423	0.05750
14	0.05726	0.03095	0.03594	0.04313	0.05607
13	0.05384	0.03020	0.03497	0.04196	0.05455
12	0.05028	0.02944	0.03395	0.04074	0.05296
11	0.04656	0.02864	0.03284	0.03941	0.05123
10	0.04271	0.02781	0.03174	0.03809	0.04951
9	0.03875	0.02693	0.03058	0.03670	0.04770
8	0.03467	0.02597	0.02938	0.03526	0.04583
7	0.03050	0.02502	0.02815	0.03378	0.04391
6	0.02626	0.02404	0.02684	0.03221	0.04187
5	0.02194	0.02305	0.02554	0.03065	0.03984
4	0.01758	0.02207	0.02421	0.02905	0.03777
3	0.01319	0.02108	0.02286	0.02743	0.03566
2	0.00883	0.02001	0.02147	0.02576	0.03349
1	0.00444	0.01896	0.02007	0.02408	0.03131
0		0.01729	0.01802	0.02162	0.02811

Table 5.4b Summary of story displacement s relative to the foundation, [m]

	Case 19	Case 20	Case 21	Case 22	Case 23
Base Shear[KN]	176,371.0	69,283.0	81,580.0	122,370.0	211,645.2
Overturning Moment[KNm]	264,556.5	103,924.5	122,370.0	183,555.0	317,467.8

Table 5.4c Summery of computed Maximum Base shear and moment

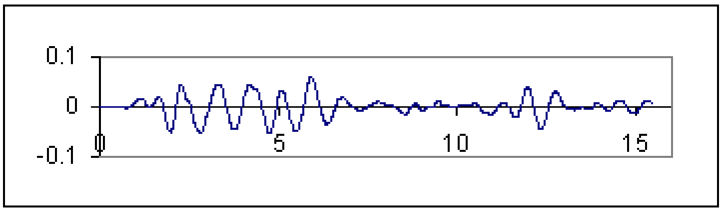

Figure 5.19 Displacement history of m(30) , Maximum response = 0.03749m

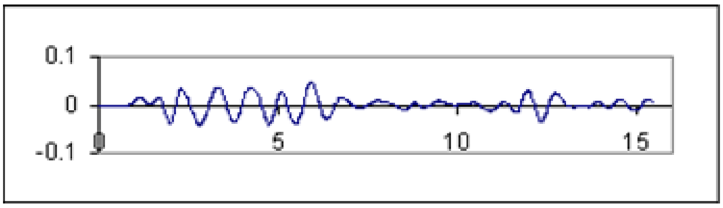

Figure 5.20 Displacement history of m(15) , Maximum response = 0.03161m

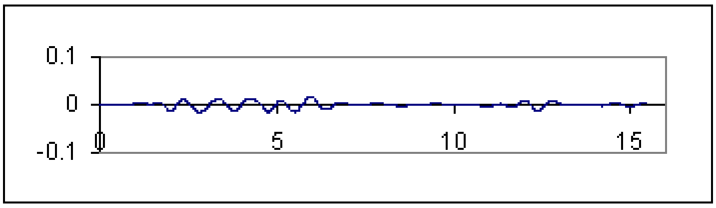

Figure 5.21 Displacement history of m(1) , Maximum response = 0.01729m

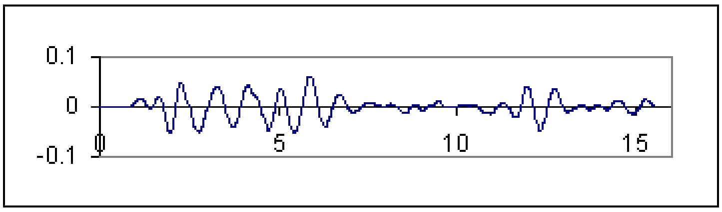

Figure 5.22 Displacement history of m(30) , Maximum response = 0.04413m

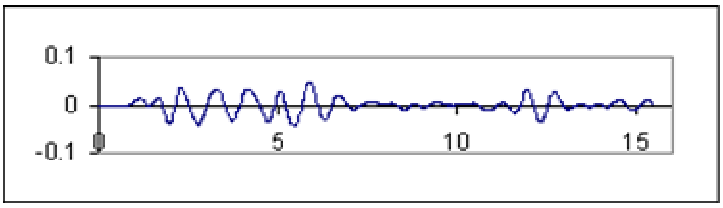

Figure 5.23 Displacement history of m(15) , Maximum response = 0.03686m

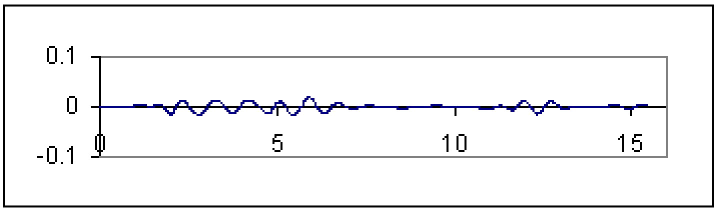

Figure 5.24 Displacement history of m(1) , Maximum response = 0.01802m

The plots of displacement response the time history at foundation level, mid story and at the roof level for case20 and case 21 are given by Fig 5.19 to 5.24 respectively

It is observed from cases 19 to 23, which are based on Building Model 1 and site 1 but with different substructure conditions, that the response of the fixed base and the flexible base structure are different

It is observed from Table 5.4a that the natural circular frequencies of all the modes reduce from their respective values of the fixed base structure. Gentle increase of the natural frequencies is observed as the foundation embedment increases, for the half space model.

The maximum story displacements show gradual decrease, for the elastic half space model, but the inter-story displacements generally decrease as from the fixed base condition, case4. Due to this decrease of the inter story displacements, the internal forces developed in the structure, such as the story shear and the story moments generally decrease, when compared to the fixed base values.

The base shear and the base turning moments tend to decrease from the respective fixed base cases.

Case 24: This case considers the analysis of the five-story building Model, on Site 3, where the foundation is placed at the surface of elastic half space.

The mass density of the soil, ρ, and the poisons ratio, ν are assumed values of 2400Kg/m^3, shear wave velocity of 166.5m/s, for Site2.The computed value of the shear stiffness, G is 6.65e+4 KPa.

The computed dynamic spring and dashpot values according to the tables and charts provided by Gazetas[7], are determined for frequency of 8.5s^{-1} .

The computed spring constants are:
For lateral horizontal mode, K_y=4.65e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 5.61e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=2.95e+5 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 4.15e+6 KN.s.m

The results of the analysis, which are the natural circular frequencies, story displacements, relative to the foundation, the maximum base shear and the base overturning moments, are presented in Table 5.5a, Table 5.5b and Table 5.5c respectively.

Case 25: The analysis of the five-story building is considered, where the foundation is embedded at the 2m below the surface of elastic half space of site 3, schematically shown by Figure 5.2.

The dynamic spring and dashpot values are determined for frequency of 8.5s^{-1}

The computed spring constants are:
For lateral horizontal mode, K_y=7.4e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 6.52e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=4.7e+5 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 7.05e+6 KN.s.m

The results of the analysis, which are the natural circular frequencies, story displacements relative to the foundation, the maximum base shear and the base overturning moments, are presented in Table 5.5a, Table 5.5b and Table 5.5c respectively.

<u>**Case 26:**</u> For this case, the analysis of the five-story building (Model 1) is considered to be constructed embedded 4m below the surface of elastic half space, for Site 3.

The dynamic spring and dashpot values are determined for frequency of 8.5s^{-1}

The computed spring constants are:
For lateral horizontal mode, K_y=6.5e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 7.95e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=4.2e+5 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 9.7e+6 KN.s.m

The results of the analysis are presented in Table 5.5a, Table 5.5b and Table 5.5c.

<u>**Case 27:**</u> For this case, the analysis of the five-story building (Model 1) is considered to be constructed at the surface of homogenous stratum over bedrock. The stratum is modeled by using the parameters computed for Site 3.

The dynamic spring and dashpot values are determined for frequency of 7.5s^{-1}

The computed spring constants are:
For lateral horizontal mode, K_y=5.16e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 3.93e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:

For lateral horizontal mode, C_y=4.71e+4 KN.s.m^{-1}

For the rocking mode rx (around the longitudinal axis) C_{rx} = 5.98e+4 KN.s.m

The results of the analysis for case24 to case27 are presented in Table 5.5a, Table 5.5b and Table 5.5c.

Mode	Case24	Case25	Case26	Case27
Mode 1	8.744	8.855	8.903	7.510
Mode 2	25.476	25.790	25.925	14.103
Mode 3	40.091	40.544	40.730	26.235
Mode 4	51.358	51.562	51.605	40.416
Mode 5	58.087	58.520	57.939	51.570
Mode 6	64.337	69.824	73.455	58.408
Mode 7	98.045	7.640	127.879	66.598

Table 5.5a Summery of computed Natural circular frequencies, [s^{-1}]

Floor Level	Case24	Case25	Case26	Case27
5	0.07868	0.08335	0.08974	0.09185
4	0.07354	0.07795	0.08467	0.08735
3	0.06145	0.06520	0.07254	0.07659
2	0.04641	0.04928	0.05486	0.06222
1	0.02797	0.02936	0.03291	0.04458
0	0.00747	0.00711	0.00761	0.00560

Table 5.5b Summary of Maximum story displacements relative to the foundation, [m]

	Case24	Case25	Case26	Case27
Base Shear [KN]	16,720.24	18,592.96	21,148.28	31,628.40
Overturning Moment [KNm]	25,080.36	27,889.44	31,722.42	47,442.61

Table 5.5c Summary of computed Maximum Base shear, and base overturning moment

It is observed from Table 5.5a that the natural circular frequencies of all the modes reduce from their respective values of the fixed base structure. For the half space soil model, smooth increase of the natural frequencies is observes as the foundation embedment increase.

The story displacements relative to the foundation show gradual variation, some times increase and decrease, but the inter-story displacements generally decrease as from the fixed base condition, case1. Due to this decrease of the inter story displacements, the internal forces developed in the structure, such as the story shear and the story moments generally decrease, when compared to the fixed base values.

Like the natural frequencies, the base shear and the base turning moments tend to decrease. Slight increase of theses values is shown for the half elastic model from case24 to case 27, as the stiffness of the soil increase.

Case 28: This case is the analysis of the ten-story Building, Mode2, on Site 3, where the foundation is placed at surface of elastic half space.

The computed dynamic spring and dashpot values according to the tables and charts provided by Gazetas[7], are determined for frequency of $13.0s^{-1}$.

The computed spring constants are:
For lateral horizontal mode, K_y=4.93e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 6.53e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=5.72e+5 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 5.54e+6 KN.s.m

The results of the analysis are presented in Table 5.6a, Table 5.6b and Table 5.6c.

Case 29: Case for the analysis of ten-story building, Model 2 is considered, where the foundation is embedded at the 2m below the surface of elastic half space of Site 3.

The dynamic spring and dashpot values are determined for frequency of $13.0s^{-1}$

The computed spring constants are:
For lateral horizontal mode, K_y=6.1e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 3.81e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=3.99e+5 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 7.87e+6 KN.s.m

Case 30: For this case, the analysis of the ten-story building, Model 2, is considered to be constructed embedded 4m below the surface of elastic half space, for Site 3.

The dynamic spring and dashpot values are determined for frequency of $13.0s^{-1}$

The computed spring constants are:

For lateral horizontal mode, K_y=8.6e+6 KN/m

For the rocking mode rx (around the longitudinal axis) K_{rx} = 7.48e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:

For lateral horizontal mode, C_y=7.5e+5 KN.s.m^{-1}

For the rocking mode rx (around the longitudinal axis) C_{rx} = 1.05e+7 KN.s.m

<u>Case 31:</u> For this case, the analysis of the ten-story building, Model 2 is considered to be constructed at the surface of homogenous stratum over bedrock. The stratum is modeled by using the parameters computed for Site 3.

The dynamic spring and dashpot values are determined for frequency of 7.0s^{-1}

The computed spring constants are:

For lateral horizontal mode, K_y=6.82e+6 KN/m

For the rocking mode rx (around the longitudinal axis) K_{rx} = 4.85e+6 KN.m

And the computed dashpot constants, which are frequency dependant, are:

For lateral horizontal mode, C_y=5.11e+4 KN.s.m^{-1}

For the rocking mode rx (around the longitudinal axis) C_{rx} = 6.35e+4 KN.s.m

The results of the analysis are summarized in Table 5.6a, Table 5.6b and Table 5.6c.

Mode	Case 28	Case 29	Case 30	Case 31
Mode 1	14.849	15.495	16.198	7.039
Mode 2	44.902	46.084	47.274	17.667
Mode 3	71.543	75.485	78.571	42.783
Mode 4	81.809	84.865	95.307	76.153
Mode 5	116.297	116.740	117.540	113.606
Mode 6	152.175	152.295	152.011	148.967
Mode 7	185.390	185.545	184.319	184.115
Mode 8	214.622	215.002	212.874	214.254
Mode 9	239.569	239.857	237.414	238.363
Mode 10	259.498	259.457	257.369	260.859
Mode 11	273.219	273.103	271.661	273.799
Mode 12	280.226	280.252	279.582	280.539

Table 5.6a Summery of computed Natural frequencies, [s^{-1}]

Floor level	Case 28	Case 29	Case 30	Case 31
10	0.02406	0.01871	0.01472	0.03343
9	0.02387	0.01853	0.01460	0.03330
8	0.02337	0.01809	0.01435	0.03296
7	0.02254	0.01738	0.01391	0.03242
6	0.02144	0.01646	0.01331	0.03161
5	0.02006	0.01533	0.01255	0.03073
4	0.01844	0.01404	0.01161	0.02970
3	0.01663	0.01262	0.01049	0.02848
2	0.01464	0.01107	0.00922	0.02740
1	0.01253	0.00940	0.00780	0.02626
0	0.00993	0.00738	0.00642	0.01469

Table 5.6b Summery of Maximum story displacement relative to the foundation, [m]

	Case 28	Case 29	Case 30	Case 31
Base Shear [KN]	35,767	27,373	24,511	292,886
Overturning Moment[KNm]	53,651	41,059	36,766	439,329

Table 5.6c Summery of computed Maximum Base shear and base overturning moment

It is observed from table 5.6a that the natural circular frequencies of all the modes reduce from their respective values of the fixed base structure. Gentle increase of the natural frequencies is observes as the foundation embedment increase, for the half space model.
When the bedrock is at shallow depth as for cases31 the natural frequency decreases to further to lesser values.

Like the natural frequencies, the base shear and the base turning moments tend to decrease. Slight increase of theses values is shown for the half elastic model from case28 to case 30, as the stiffness of the soil increase. The results of case31 cannot be explained.

Case 32: This case is the analysis of the twenty-story building, Mode3, on Site 3, where the foundation is placed at the surface of elastic half space.

The computed dynamic spring and dashpot values according to the tables and charts provided by Gazetas[7], are determined for excitation frequency of $7.0s^{-1}$

The computed spring constants are:
For lateral horizontal mode, K_y=5.63e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 6.65e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=5.28e+5 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 6.10e+6 KN.s.m

The results of the analysis are presented in Table 5.7a, Table 5.7b and Table 5.7c.

Case 33: Case for the analysis of the twenty-story building, Model 3 is considered, where the foundation is embedded at 2m below the surface of elastic half space of Site 3.

The dynamic spring and dashpot values are determined for frequency of $7.0s^{-1}$

The computed spring constants are:
For lateral horizontal mode, K_y=7.4e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 4.94e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=6.8e+5 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 7.54e+6 KN.s.m

Case 34: For this case, the analysis of the twenty-story building, Model 3, is considered to be constructed embedded 4m below the surface of elastic half space, for Site 3.

The dynamic spring and dashpot values are determined for frequency of $7.0s^{-1}$

The computed spring constants are:

For lateral horizontal mode, K_y=7.51e+6 KN/m

For the rocking mode rx (around the longitudinal axis) K_{rx} = 6.22e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:

For lateral horizontal mode, C_y=6.71e+5 KN.s.m^{-1}

For the rocking mode rx (around the longitudinal axis) C_{rx} = 7.2e+7 KN.s.m

Case 35: For this case, the analysis of the twenty-story building, Model 3, is considered to be constructed at the surface of homogenous stratum over bedrock. The stratum is modeled by using the parameters computed for site 3.

The dynamic spring and dashpot values are determined for excitation frequency of 8.0s^{-1}

The computed spring constants are:

For lateral horizontal mode, K_y=5.0e+6 KN/m

For the rocking mode rx (around the longitudinal axis) K_{rx} = 6.44e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:

For lateral horizontal mode, C_y=5.14e+4 KN.s.m^{-1}

For the rocking mode rx (around the longitudinal axis) C_{rx} = 6.21e+4 KN.s.m

The results of the analysis for cases 32 to 35 are presented in Table 5.7a, Table 5.7b and Table 5.7c.

Mode	Case 32	Case 33	Case 34	Case 35
Mode 1	7.5	7.9	7.9	8.4
Mode 2	23.6	24.6	24.6	25.2
Mode 3	40.0	40.9	41.0	41.8
Mode 4	56.7	57.5	57.4	58.2
Mode 5	67.4	72.8	74.6	74.2
Mode 6	76.6	79.1	88.0	89.8
Mode 7	92.5	92.8	95.7	104.7
Mode 8	110.0	109.7	110.5	119.1
Mode 9	126.5	126.5	127.1	132.7
Mode 10	143.7	143.1	142.8	145.5
Mode 11	157.2	157.4	159.8	157.4
Mode 12	173.0	172.9	173.3	168.3
Mode 13	183.7	184.8	184.8	175.9
Mode 14	196.4	197.7	196.0	184.7
Mode 15	206.4	207.5	209.4	192.1
Mode 16	217.6	218.7	217.9	198.5
Mode 17	225.0	226.2	226.4	202.7
Mode 18	231.3	232.4	232.0	206.5
Mode 19	238.4	238.1	237.2	209.2
Mode 20	240.8	240.3	244.8	210.7
Mode 21	246.5	246.0	246.8	246.5
Mode 22	248.5	248.4	249.0	248.5

Table 5.7a Summery of computed Natural frequencies, [s^{-1}]

Floor level	Case 32	Case 33	Case 34	Case 35
20	0.06407	0.06758	0.06275	0.07534
19	0.06396	0.06747	0.06264	0.07516
18	0.06352	0.06692	0.06220	0.07461
17	0.06286	0.06615	0.06143	0.07370
16	0.06198	0.06505	0.06033	0.07239
15	0.06077	0.06363	0.05879	0.07055
14	0.05923	0.06176	0.05725	0.06865
13	0.05758	0.05989	0.05538	0.06646
12	0.05571	0.05769	0.05330	0.06398
11	0.05363	0.05527	0.05099	0.06121
10	0.05132	0.05264	0.04824	0.05782
9	0.04846	0.04956	0.04560	0.05472
8	0.04582	0.04637	0.04275	0.05129
7	0.04297	0.04308	0.03967	0.04763
6	0.03989	0.03967	0.03648	0.04379
5	0.03670	0.03615	0.03319	0.03977
4	0.03341	0.03242	0.02967	0.03559
3	0.03000	0.02868	0.02615	0.03133
2	0.02648	0.02473	0.02253	0.02701
1	0.02297	0.02077	0.01879	0.02260
0	0.01879	0.01615	0.01473	0.01766

Table 5.7b Summary of story displacement relative to the foundation, [m]

	Case 32	Case 33	Case 34	Case 35
Base Shear[KN]	64,784.28	70,183.57	62,147.59	74,577.11
Overturning Moment[KNm]	97,176.43	105,275.35	93,221.39	111,865.66

Table 5.7c Summery of computed Maximum Base shear and base moment

It is observed from table 5.7a that the natural circular frequencies of all the modes reduce from their respective values of the fixed base structure. Gentle increase of the natural frequencies is observes as the foundation embedment increase, for the half space model.

The maximum story displacements show gradual decrease, for the elastic half space model, but the inter-story displacements generally decrease as from the fixed base condition. Due to this decrease of the inter story displacements, the internal forces developed in the structure, such as the story shear and the story moments generally decrease, when compared to the fixed base values.

Like the natural frequencies, the base shear and the base turning moments tend to decrease from the respective fixed base case values.

Case 36: This case is the analysis of the thirty-story building, Mode 4, on Site 3, where the foundation is placed at the surface of elastic half space.

The computed dynamic spring and dashpot values according to the tables and charts provided by Gazetas[7], are determined for excitation frequency of $8.5s^{-1}$.

The computed spring constants are:
For lateral horizontal mode, K_y=7.63e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 4.55e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=4.58e+5 $KN.s.m^{-1}$
For the rocking mode rx (around the longitudinal axis) C_{rx} = 5.11e+6 KN.s.m

The results of the analysis are presented in Table 5.8a, Table 5.8b and Table 5.8c.

Case 37: Case for the analysis of the thirty-story building, Model 4 is considered, where the foundation is embedded at the 2m below the surface of elastic half space of Site 3.

The dynamic spring and dashpot values are determined for frequency of $8.5s^{-1}$

The computed spring constants are:
For lateral horizontal mode, K_y=5.9e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 4.99e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=5.89e+5 $KN.s.m^{-1}$
For the rocking mode rx (around the longitudinal axis) C_{rx} = 8.60e+6 KN.s.m

In a similar manner, the results of the analysis are presented in Table 5.8a, Table 5.8b and Table 5.8c.

Case 38: For this case, the analysis of the thirty-story building, Model 4, is considered to be constructed embedded 4m below the surface of elastic half space, for Site 3.

The dynamic spring and dashpot values are determined for excitation frequency of $9.0s^{-1}$

The computed spring constants are:
For lateral horizontal mode, K_y=5.7e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 4.96e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=6.91e+5 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 7.13e+7 KN.s.m

Case 39: For this case, the analysis of the thirty-story building, Model 4, is considered to be constructed at the surface of homogenous stratum over bedrock. The stratum is modeled by using the parameters computed for Site 3.

The dynamic spring and dashpot values are determined for excitation frequency of $9.0s^{-1}$

The computed spring constants are:
For lateral horizontal mode, K_y=6.55e+6 KN/m
For the rocking mode rx (around the longitudinal axis) K_{rx} = 3.54e+8 KN.m

And the computed dashpot constants, which are frequency dependant, are:
For lateral horizontal mode, C_y=4.42e+4 KN.s.m^{-1}
For the rocking mode rx (around the longitudinal axis) C_{rx} = 6.87e+4 KN.s.m

The results of the analysis are presented in Table 5.8a, Table 5.8b and Table 5.8c.

Mode	Case36	Case 37	Case 38	Case 39
Mode 1	7.6546	8.1688	8.6875	9.2661
Mode 2	25.2391	25.9368	26.0416	27.7776
Mode 3	45.0105	45.6099	43.3226	46.2115
Mode 4	59.9409	64.5199	60.4846	64.5166
Mode 5	67.5628	70.5161	77.4818	82.6471
Mode 6	88.1476	88.7098	94.2660	100.5504
Mode 7	109.4420	109.8897	110.7926	118.1786
Mode 8	130.8588	130.7998	127.0146	135.4815
Mode 9	151.9054	152.4852	142.8882	152.4142
Mode 10	173.3702	173.3178	158.3710	168.9298
Mode 11	194.2050	194.0882	173.4204	184.9815
Mode 12	213.7996	214.6730	187.9938	200.5265
Mode 13	232.8100	234.9084	202.0518	215.5213
Mode 14	251.9558	254.7617	215.5563	229.9265
Mode 15	272.8517	272.7011	228.4701	243.7007
Mode 16	291.1285	289.5082	240.7572	256.8066
Mode 17	303.0749	304.3403	252.3849	269.2106
Mode 18	316.2497	319.0513	263.3214	280.8754
Mode 19	333.9423	334.5133	273.5363	291.7715
Mode 20	341.1133	340.3240	274.9916	293.3240
Mode 21	357.0745	352.8553	283.4364	302.3321
Mode 22	370.0769	366.8571	291.1046	310.5106
Mode 23	374.7499	373.4734	293.6366	313.2116
Mode 24	375.5442	376.5889	293.7875	313.3730
Mode 25	384.4230	385.4256	298.8374	318.7591
Mode 26	393.7037	392.1824	303.0723	323.2770
Mode 27	399.8741	400.2134	306.4870	326.9193
Mode 28	405.5309	405.0634	309.0807	329.6862
Mode 29	409.6592	409.5433	310.8685	331.5930
Mode 30	413.1754	413.1026	311.8916	332.6837
Mode 31	415.5066	415.4472	415.5446	416.5389
Mode 32	421.7662	418.8394	419.5939	420.2090

Table 5.8a Summary of computed Natural frequencies, [s^{-1}]

Floor level	Case36	Case 37	Case 38	Case 39
30	0.04373	0.05147	0.06177	0.08029
29	0.04368	0.05141	0.06170	0.08020
28	0.04360	0.05132	0.06159	0.08006
27	0.04340	0.05112	0.06135	0.07975
26	0.04322	0.05086	0.06103	0.07934
25	0.04287	0.05052	0.06062	0.07880
24	0.04260	0.05010	0.06012	0.07815
23	0.04212	0.04961	0.05953	0.07739
22	0.04165	0.04904	0.05884	0.07649
21	0.04113	0.04837	0.05804	0.07545
20	0.04054	0.04765	0.05718	0.07433
19	0.03991	0.04685	0.05622	0.07310
18	0.03920	0.04599	0.05519	0.07175
47	0.03848	0.04506	0.05407	0.07029
16	0.03770	0.04402	0.05283	0.06867
15	0.03687	0.04299	0.05159	0.06707
14	0.03610	0.04192	0.05031	0.06540
13	0.03523	0.04079	0.04894	0.06363
12	0.03434	0.03960	0.04752	0.06177
11	0.03341	0.03830	0.04597	0.05975
10	0.03244	0.03702	0.04443	0.05775
9	0.03141	0.03567	0.04281	0.05564
8	0.03029	0.03427	0.04113	0.05346
7	0.02918	0.03283	0.03940	0.05122
6	0.02804	0.03131	0.03757	0.04884
5	0.02689	0.02979	0.03575	0.04647
4	0.02574	0.02824	0.03388	0.04405
3	0.02459	0.02666	0.03199	0.04159
2	0.02334	0.02504	0.03005	0.03906
1	0.02211	0.02341	0.02809	0.03652
0	0.02017	0.02102	0.02522	0.03279

Table 5.8b Summery of story displacement relative to the foundation [m]

	Case36	Case 37	Case 38	Case 39
Base Shear[KN]	87,643.0	103,198.7	154,798.1	267,731.2
Overturning Moment[KNm]	131,464.5	154,798.1	232,197.1	401,596.8

Table 5.8c Summery of computed Maximum Base shear and moment

It is observed from cases 36 to 39 , which are based on Building Model 4 and Site 3, but with different substructure conditions, that the response of the structure fixed at the foundation and that when flexible are different

It is observed from Table 5.8a that the natural circular frequencies of all the modes reduce from their respective values of the fixed base structure. Gentle increase of the natural frequencies is observes as the foundation embedment increase, for the half space model.

Once again, like the natural frequencies, the base shear and the base turning moments tend to decrease from the respective fixed base case values.

10. CONCLUSIONS

It has been observed from this study that, structures that are routinely analyzed for earthquake excitations assuming that they are fixed at their bases and that a small amount of viscous damping exists in the superstructure as the only means of energy dissipation leads to results which are different from the reality.

The results from the parametric study (Tables 5.1a to 5.8a), shows that, the natural circular frequencies for all modes decrease when the foundation of the structure is assumed flexible, for all conditions of embedment considered. From Table 5.2a, we can observe that the value of the natural frequency of the fundamental mode is 18.39 s^{-1} which reduces to 12.05s^{-1}, when the structure is placed on the surface of elastic half space, this value shows slight increase with embedment. Similar pattern of decrement of the natural frequency is shown for all cases considered when the foundation model considered is elastic half space.

For structures founded on flexible base, the inter story displacements generally decrease from that of the fixed-base structures. This leads to the reduction of the internal actions such as story shear and story moments developed in the structure.

It is also observed that as the height of the structures increases, the flexibility of the structure decreases, hence the effect of soil structure interaction is minimized.

When the thickness of the flexible soil is large, a uniform trend is observed for the building models analyzed. In contrast, this trend becomes less predictable, when the thickness of the flexible soil decreases. In the latter case, the iteration undulates significantly making the prediction difficult. This is attributed to the highly jagged nature of the impedance functions of frequencies for layered formations. As the thickness of the flexible soil layer reduces, it has been found difficult to conclude whether the conventional fixed-base approach yields conservative design forces, as is the case for soil formations of large thickness.

The base shear and overturning moments as shown by Tables5.1c to 5.8c, decrease all building models constructed on the surface and as well embedded in the homogenous half space model as compared to fixed base structures. This may be due to the fact that large flexible soils, dissipate much energy of the earthquake excitation as a form of radiation of waves in the soil and the material damping of the soil.

For the majority of models analyzed, for structures founded on flexible soils, modeled as half space, the stiffness and the damping of the soil progressively increases as the embedment of the structure in to the soil increases. This is because the waves emanated from the foundations during earthquake excitations, radiate in to the soil for long distances, with out being reflected to the foundations.

The computer program developed, which is based on the procedures developed in [14], have been found very important to further investigate the effects of the soil- structure interaction through parametric studies. It can also serve as a good tool for the dynamic analysis of real structures that accounts for the effect of soil-structure interaction.

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```
!-------------------------------------------------------
!     PROGRAM TO EVALUATE THE DISPLACEMENT RESPONSE
!     OF LUMPED MASS SYSTEM COUPLED WITH FOUNDATION
!     CONSIDERING NON-PROPORTIONAL DAMPING
!-------------------------------------------------------

      PROGRAM Time_history_analysis

!---- sub programs  --------------

        USE GAUSS
        USE MULTIPLY
        USE BALANCE
        USE HESSENBERG
        USE EIGEN
        USE MODES
        USE MODESOL
        USE SPECTRAL
        USE DAMPING
        USE NORMALIZE
        USE DRESPONSE
        USE TRANSPOSE

      INTEGER   NP,MP
      PARAMETER (NP=50)
      REAL      a(NP,NP),b(NP,NP),x(NP),h(np,np)
      REAL      f(NP,NP),cc(NP,NP),CM(NP,NP)
      REAL      nf(NP),wi(NP),SP(NP,NP),ai(NP,NP)
      REAL      wr(NP),dm(NP,NP),dr,am(NP,NP)
        REAL      cmt(NP,NP),cmn(np,np),nfd(np),mn(np)
        REAL      uf(1000,np)
        REAL      a1(np,np),a11,a12,a21,a22,b11,b12,b21,b22
        REAL            Mng(np)
        REAL      sum,nmt(np,np)
        REAL      MM(np,np),d0(np),Lhm(np),Lrm(np),Ud(2000,np,np)
        REAL            alfa(np,np),gama(np,np),tt,zz(1000,np)
        REAL            v1,v2,v3,v4,v5,v6,v7,rm1,rm2
        REAL            zv22(np)
        REAL            zv1(np),zv2(np),zu1(np),zu2(np),ah1,ah2,ar1
        REAL      c1,c2,c3,c4,c5
        REAL      Ac,Bc,gg
      INTEGER   j,k,l,m,n,nmax,jj,j1,count
      INTEGER   n1,n2,n3
      CHARACTER title1*50,title2*30,title3*30
      CHARACTER fname*20
        CHARACTER (50) name

        REAL            ag(1000)

!-----Read from input data file --------

      open(15,file='output.dat')
      open(16,file='dres.dat')
        open(17,file='output0.dat')
        open(18,file='out_temp.dat')
        open(14,file='output_sdof.dat')
        open(40,file='disp.tmp')
        open(50,file='momt.tmp')
        open(60,file='sher.tmp')
        open(70,file='basic1.tmp')
        open(71,file='index.tmp')
```

```
        h(1,1)=0.0

!     open(19,file='data
!       Write(*,*)' Input file Name (with ".dat":)'
!       read (*,*)  fname

!---------------------------------------------------------
        fname='ex1-1.txt'
      write(15,*)'Data file  :  ',fname
!---------------------------------------------------------

      open(7,file=fname,status='old')
      read(7,*)title1
      read(7,*)
        m=1
        x(np)=1
      read(7,*) n, dr, dt ,gg

      read(7,*) title2
      read(7,*) ((a(k,l), l=1,n), k=1,n)

      read(7,*) title3

      read(7,*) ((b(k,l), l=1,n), k=1,n)
      read(7,*)
      read(7,*) ((dm(k,l),l=1,n), k=1,n)
      close(7)

        write(14,'(i5)')n
        write(14,'(f5.3)')dr
        write(40,'(i5)')n
        write(50,'(i5)')n
        write(60,'(i5)')n
        write(70,*)'NO OF DEGREES OF FREEDOM'
        write(70,'(i5)')n
        write(71,'(i4)')n

!------- Save to output file      ---------------

      write(15,*)title1
      write(15,*)title2
      write(15,*)
        write(70,*)'MASS MATRIX'
        do 52 i=1,n
        write(15,'(1x,50f15.4)') ((a(i,j)), j=1,n)
        write(70,'(1x,50f15.4)') ((a(i,j)), j=1,n)
52      continue

        write(70,*)'STIFFNESS MATRIX'

      write(15,*)title3
      write(15,*)
        do 53 i=1,n
        write(15,'(1x,50f15.4)') ((b(i,j)), j=1,n)
        write(70,'(1x,50f15.4)') ((b(i,j)), j=1,n)
53      continue
        write(70,*)'NO OF CONSIDERED MODES'
        write(70,'(i5)')n

!     save original mass matrices for later use
```

```
      do 13 l=1,n
        do 11 k=1,n
          ai(k,l)=a(k,l)
11      continue
13    continue

      do 73 l=1,n
        do 71 k=1,n
          am(k,l)=a(k,l)
71      continue
73    continue

!--- Inverse of mass matrix   ---------------------

      call gaussj(ai,n,NP,x,m,MP)
        write(17,*) 'Inverse of Mass Matrix'
      do 14 k=1,n
      write(17,'(1h ,(50f12.8))')(ai(k,l),l=1,n)
14    continue

!       Multiply the mass inverse with K
        n1=n
        n2=n
        n3=n
        call mult(n1,n2,n3,ai,b,F)

! Compute the eigen values for matrix f
! Compute the NATURAL FREQUENCIES.

        call balanc(f,n,NP)               ! module Balance
        call elmhes(f,n,NP)             ! module Hessenberg
        call hqr(f,n,NP,wr,wi)            ! module Eigen

        write(17,'(/1x,a)')                 'Eigen                Values
++++++++++++++++++++++++++++++++++++++++++++++++'
        do 12 i=1,n
        write(17,'(5x,2F12.3)') wr(i),wi(i)
12      continue

      call SPECTRAL_MAT(n,np,wr,sp,nf,nfd,dr) ! module spectral1

! Compute ALL POSSIBLE MODES

      call getmodes(a,b,cc,n,np,sp)           ! Module modes
      call Modeshapes(a,n,np,cm)                ! Module modesol
      call Normalizing(n,am,cm,cmt,cmn,mn)    ! Module Normalize

!     call DAMPING_MAT(n,np,am,b,nf,dr,dm)  ! Module damping

        do 214 i=1,n1      !no of columns
        do 215 j=1,n         !no of raws
        nmt(i,j)=cmn(j,i)
215   continue
214   continue


!-----Analysis of coupled system  -----------
        write(15,*)'<<<<<<<<<<<<<<<<<V>>>>>>>>>>>>>>>>>'

!     Mass matrix(input data)
        write(15,*)'>>>Mass matrix'
```

```
          do 304 i=1,n
        write(15,'(1x,50f16.5)') ((am(i,j)), j=1,n)
304     continue

          write(15,*)'>>>Normalized Nodes'
          do 300 i=1,n
        write(15,'(1x,50f16.5)') ((nmt(i,j)), j=1,n)
300     continue

          write(15,*)'>>>Transpose of normalized modes'
          do 301 i=1,n
        write(15,'(1x,50f16.5)') ((cmn(i,j)), j=1,n)
301     continue

!         The generalized mass
        do 90 i=1,n
          sum=0.0
          do 91 j=1,n
          sum=sum+(am(j,j)*nmt(j,i)*nmt(j,i))
91      continue
          Mng(i)=sum
90      continue

!         The generalized mass
          call mult(n,n,n,cmn,am,a1)
          call mult(n,n,n,a1,nmt,MM)
          write(15,*)'>>>Generalized Mass matrix'
          do 307 i=1,n
          write(15,'(1x,50f16.5)') ((MM(i,j)), j=1,n)
307       continue


!         The generalized damping  original
        call mult(n,n,n,cmn,dm,a1)
          call mult(n,n,n,a1,nmt,CC)

          write(15,*)'>>>Generalized damping matrix'
          do 308 i=1,n
          write(15,'(1x,50f16.5)') ((CC(i,j)), j=1,n)
308       continue

          write(15,*)'>>>Natural frequency'
          do 311 i=1,n
          write(15,'(1x,50f16.5)') nf(i)
311       continue

!         The Lehar's  Modal Damping ratio
          do 309 i=1,n
          d0(i)= CC(i,i)/(2.0*MM(i,i)*nf(i))
          if  (d0(i) .gt. 0.90) then
          d0(i)=0.90
          endif

309       continue

          write(15,*)'>>>Lehars damping ratio'
          do 310 i=1,n
          write(15,'(1x,50f16.5)') d0(i)
310       continue

          do 409 i=1,n
```

```
        nfd(i)=nf(i)*sqrt(1.0-(d0(i)*d0(i)))
409     continue

      write(15,*) '>>>Damped natural frequency '
        do 410 i=1,n
        write(15,'(1x,50f16.5)') nfd(i)
410     continue


!=====modal part. factors of the free ground acc=====

        open(88,file='alfa.txt')
      read(88,*) ((alfa(k,l), l=1,n), k=1,n)
      read(88,*) ((gama(k,l), l=1,n), k=1,n)
        write(15,*)'>>>Alpha'
        do 440 i=1,n
      write(15,'(1x,50f8.3)') ((alfa(i,j)), j=1,n)
440   continue

        write(15,*)'>>>Gama'
        do 441 i=1,n
      write(15,'(1x,50f8.3)') ((gama(i,j)), j=1,n)
441   continue

        do 312 i=1,n
        sum=0.0
        do 313 j=1,n
        sum=sum+(am(j,j)*nmt(j,i)*alfa(j,i))
313   continue
        Lhm(i)=sum
312   continue
        write(15,*)'>>>Lhm'
        do 314 i=1,n
        write(15,'(1x,6f8.3)') Lhm(i)
314     continue


        do 317 i=1,n
        sum=0.0
        do 318 j=1,n
        sum=sum+(am(j,j)*nmt(j,i)*gama(j,i))
318   continue
        Lrm(i)=sum
317   continue
        write(15,*)'>>>Lrm'


        do 319 i=1,n
        write(15,'(1x,6f8.3)') Lrm(i)
319     continue


!       Constants aij & bij

!=================================================================
        write(*,*)'Computation in progress, Please wait...'
!=================================================================

        ah1=0.0
        ar1=0.0
        rm1=0.0
```

```
        do 51 i=1,n
        zu1(i)=0.0
        zv1(i)=0.0
        zv2(i)=0.0
        zv22(i)=0.0
51      continue

        zv2(0)=0.0

        name = 'file4.dat'                    !create file to store each
floor data
        open(unit=16,file=name)


!EQEQEQEQEQEQEQEQEQQEQEQEQEQEQEQEQEQEQQEQEQEQEQEQEQEQEQEQEQEQQEQEQEQEQEQ
EQEQEQEQEQE


      !read earthquake, ground accelarations

            open(90,file='elcentro.dat')  !get input ground motion
            read(90,*) nmax

            do 702 j1=1,nmax                  !number time steps
            read(90,*) tt,ah2
            ag(j1)=gg*ah2                           !multiplied      by
grav.acc.

702         continue
            close(90)

    !---------------------------------------------------
>>>>>>>>>>>>>>>>>>>>>>>>>>

   do 701 j1=1,nmax                     !number time steps

        ah2=ag(j1)
        count=0

1000  do 857 i=1,n                 !number of modes

        sum=0.0
        do 914 jj=1,n
        if (jj.ne.i) then
             sum=sum+zv1(jj)*cc(i,jj)     ! velocity at (t-dt)
        endif


914   continue
        rm1=1.0*sum

        sum=0.0
        do 514 jj=1,n
        if (jj.ne.i) then
             sum=sum+zv22(jj)*cc(i,jj)    ! assumed velocity
        endif

514   continue
        rm2=1.0*sum
```

```
          aj= (-1.0/mm(i,i))*((Lhm(i)*ah1)+rm1)
          bj= (-1.0/mm(i,i))*((Lhm(i)*ah2)+rm2)

          c1= (d0(i)*nf(i))/nfd(i)
          c2=1.0/(nf(i)*nf(i))
          c3=nf(i)*nf(i)
          c4=(2.0*d0(i))/nf(i)
          c5=1.0/nfd(i)

        v1=exp(-1.0*d0(i)*nf(i)*dt)
          v2=cos(nfd(i)*dt)
          v3=sin(nfd(i)*dt)

          v4=1.0/(dt*nf(i)*nf(i)*nf(i))
          v5=(2.0*d0(i))/(dt*nf(i))
          v6=1.0/(dt*nf(i)*nf(i))
          v7=0

          a11=v1*(v2+c1*v3)
          a12=c5*v1*v3
          a21=-1*c3*a12
          a22=a11+ c4*a21

          b11=2*d0(i)*(v4-(a11*c2)*(1+v5))+a12*v6
          b12=(c2*(1-a11))-b11
          b21=(v6*(a11-1))+a12
          b22=a12-b21

          zu2(i)=a11*zu1(i)+a12*zv1(i)+b11*aj+b12*bj
          zv2(i)=a21*zu1(i)+a22*zv1(i)+b21*aj+b22*bj

          zu2(i)=zu1(i)+ 0.5*(zu2(i)-zu1(i))

857       continue       ! next mode

          do 233 j4=1,n

          if (abs(abs(zv22(j4))-abs(zv2(j4))).gt. 1e-4) then

          do 524 j3=1,n
          zv22(j3)=zv2(j3)
524     continue
          count=count+1
          goto 1000
          end if

233       continue

          do 563 j2=1,n
          zv1(j2)=zv2(j2)
          zu1(j2)=zu2(j2)
563       continue

          rm1=rm2
          ah1=ah2

        write(16,'(1x,50f16.8)') (zu2(j),j=1,n)
701       continue      ! next time step

        close(16)
```

```
      open(unit=30,file='file4.dat')
      do 902 j=1,nmax   !number discrete time points
    read(30,*) (zz(j,i),i=1,n)
902   continue
      close(30)
      tt=0.0
      open(unit=33,file='out.txt')
      do 201 j=1,nmax   !number discrete time points
      do 202 i=1,n      !number of modes
      do 203 l=1,n      !number of masses
      ud(j,l,i)=nmt(l,i)*zz(j,i)
203   continue
202   continue
    tt=tt+dt
    write(33,'(f6.2,50f14.10)') tt,((ud(j,l,i),l=1,n),i=1,n)
201   continue
      close(33)

!<<<<<<<Total displacement for each mass >>>>>>>>>>>>>>.
      tt=0.0
    open(unit=56,file='displacement.txt')
      do 551 j=1,nmax    !number of discrete time points

      do 552 i=1,n       !number of masses
      sum=0.0
      do 553 l=1,n         !summing the contribution of all modes
      sum=sum+ud(j,i,l)
553   continue
      uf(j,i)=sum
552   continue
      tt=tt+dt
    write(56,'(f6.2,50f16.8)')tt,(uf(j,i),i=1,n)
551   continue
      close(56)

      close(16)    ! output file
      close(15)    ! output file
      close(17)    ! output file
      close(18)    ! output file
      close(14)    ! output file
      close(40)    ! output file
      close(50)    ! output file
      close(70)    ! output file

      END        ! END OF PROGRAM
```

```
!----------------------------------------------------------------
!     Given an n by n matrix stored in an array of physical
!     Dimensions np by np, this routine replaces it by a
!     Balanced matrix   with identical eigen values.
!----------------------------------------------------------------

      MODULE BALANCE

      CONTAINS
      SUBROUTINE balanc(a,n,np)
      INTEGER n,np
      REAL a(np,np),RADIX,SQRDX
            PARAMETER (RADIX=2.,SQRDX=RADIX**2)
            INTEGER i,j,last
            REAL c,f,g,r,s
1           continue
        last=1
        do 14 i=1,n
          c=0.
          r=0.
          do 11 j=1,n
            if(j.ne.i)then
              c=c+abs(a(j,i))
              r=r+abs(a(i,j))
            endif
11        continue
          if(c.ne.0..and.r.ne.0.)then
            g=r/RADIX
            f=1.
            s=c+r
2           if(c.lt.g)then
              f=f*RADIX
              c=c*SQRDX
            goto 2
            endif
            g=r*RADIX
3           if(c.gt.g)then
              f=f/RADIX
              c=c/SQRDX
            goto 3
            endif
            if((c+r)/f.lt.0.95*s)then
              last=0
              g=1./f
              do 12 j=1,n
                a(i,j)=a(i,j)*g
12            continue
              do 13 j=1,n
                a(j,i)=a(j,i)*f
13            continue
            endif
          endif
14      continue
      if(last.eq.0)goto 1
      return
      END    SUBROUTINE balanc
      END   MODULE BALANCE
```

```
!-----------------------------------------------------------------
!     This module establishes the proportional damping matrix of
!     a structural system by computing the Releigh constants
!-----------------------------------------------------------------

      MODULE damping

      CONTAINS

      SUBROUTINE DAMPING_MAT(n,np,am,b,nf,dr,dm)
      INTEGER n,np
      REAL nf(np),dm(np,np),am(np,np),b(np,np)
      REAL a0,a1,dr

      write(*,*)  '------>  Computing Releigh constants: '
      write(*,*) nf(1)
      write(*,*) nf(n)
      write(15,*)  ' Releigh constants: '
      a0=dr*(2*nf(1)*nf(n))/(nf(1)+nf(n))
      a1=dr*2/(nf(1)+nf(n))

      write(15,30) a0 , a1
30    format(' a0=',f10.4, 3x,'a1=',f10.4)

      write(*,*)  '------>  Conputing the damping matrix : '

      do 52 i=1,n
      write(*,'(1x,6f16.6)') ((am(i,j)), j=1,n)
52    continue

      do 53 i=1,n
      write(*,'(1x,10f16.6)') ((b(i,j)), j=1,n)
53    continue

      do 40 i=1,n
      do 39 j=1,n
      dm(i,j)=a0*am(i,j)+a1*b(i,j)
39    continue
40    continue

      write(15,*)
      write(15,*) '------ Damping Matrix------'
      write(15,*)
      write(70,*)'DAMPING MATRICS'

      do 42 i=1,n
      write(15,'(1x,6f12.4)') ((dm(i,j)), j=1,n)
      write(70,'(1x,6f12.4)') ((dm(i,j)), j=1,n)

42    continue

      return
      END     SUBROUTINE
      END   MODULE damping
```

```
!-----------------------------------------------------------------
!     This module computes the displacement responce  of MDOF
!     system from predetermined modal values.
!----------------------------------------------------------------
      MODULE  DRESPONSE

      USE MULTIPLY
      USE TRANSPOSE

      CONTAINS

      SUBROUTINE DISPLACEMENT_R(n,np,cmn,am,u0,v0,nf,nfd)
      INTEGER n,np,i,j,n1,l
      REAL cmn(np,np),am(np,np),ax(np,np),v1(np)
      REAL q1(np),v0(np,np),v(np,np),ut(20)
      REAL f(np,np),q(np,np),u0(np,np),cmnt(np,np)
      REAL xx1,xx2,xx3,t,dt,nf(np),nfd(np)
      REAL ux(np,np)

!----------------------------------------------------

      open(30,file='udisp.dat')

      write(*,*)  '------>  Computing displacement  ..... '

      n1=1
      call TRANSPOSING(n,cmn,cmnt)

      do 53 i=1,n
      do 54 j=1,n
      ax(1,j)=cmnt(j,i)
54    continue

      call MULT(n1,n,n,ax,am,f)
      call MULT(n1,n,n1,f,u0,q)

      q1(i)=q(1,1)
      write(*,'(1x,6f16.6)') ((     q(1,1)), l=1,1)
53    continue

      write(15,*)  '-------------qn(0)----------------- '
      write(15,'(1x,6f16.6)') ((q1(i)), i=1,n)

      do 55 i=1,n
      do 56 j=1,n
      ax(1,j)=cmnt(j,i)
56    continue

      call MULT(n1,n,n,ax,am,f)
      call MULT(n1,n,n1,f,v0,v)

      v1(i)=v(1,1)
55    continue

      write(15,*)  '--------------q''n(0)----------------- '
      write(15,'(1x,6f16.6)') ((v1(i)), i=1,n)
!-------------------------------------------------------------

!      Calculation of displacement responses
      dt=1.0
      dr=.05
```

```
      t=0.0
      do 9  i=1,1000,1  ! time loop
      do 10 k=1,n     ! mode loop

      xx1=((v1(k)+(dr*nf(k)*q1(k)))*sin(nfd(k)*t))/nfd(k)
      xx2=q1(k)*cos(nfd(k)*t)
      xx3=exp(-1*dr*nf(k)*t)
      xx0=xx3*(xx2+xx1)

      do 11 j=1,n     ! number of degrees of freedom
      ux(j,k)=cmn(j,k)*xx0
11    continue
10    continue

!................sum all modes................
      do 7 i1=1,n
      sum=0.0
      do 8 i2=1,n
      sum=sum+ux(i1,i2)
 8    continue
      ut(i1)=sum
 7    continue

      write(30,47) t, (ut(k),k=1,n)
47    format(f10.4,3x,6f16.4)
      t=t+dt
 9    continue

      close(30)
      RETURN
      END     SUBROUTINE  DISPLACEMENT_R
      END   MODULE  DRESPONSE
```

```
!-----------------------------------------------------------------
!     This routine solves the eigen values of pre assembled
!     matrix to obtain a vector of natural frequencies
!-----------------------------------------------------------------

      MODULE EIGEN

      CONTAINS

      SUBROUTINE hqr(a,n,np,wr,wi)
      INTEGER n,np
      REAL a(np,np),wi(np),wr(np)
      INTEGER i,its,j,k,l,m,nn
      REAL anorm,p,q,r,s,t,u,v,w,x,y,z

      anorm=0.
      do 12 i=1,n
         do 11 j=max(i-1,1),n
           anorm=anorm+abs(a(i,j))
11       continue
12    continue

      nn=n
      t=0.
1     if(nn.ge.1)then
        its=0
2       do 13 l=nn,2,-1
          s=abs(a(l-1,l-1))+abs(a(l,l))
          if(s.eq.0.)s=anorm
          if(abs(a(l,l-1))+s.eq.s)goto 3
13      continue
        l=1
3       x=a(nn,nn)
        if(l.eq.nn)then

          wr(nn)=x+t
          wi(nn)=0.
          nn=nn-1
        else
          y=a(nn-1,nn-1)
          w=a(nn,nn-1)*a(nn-1,nn)
          if(l.eq.nn-1)then
            p=0.5*(y-x)
            q=p**2+w
            z=sqrt(abs(q))
            x=x+t
            if(q.ge.0.)then
              z=p+sign(z,p)
              wr(nn)=x+z
              wr(nn-1)=wr(nn)
              if(z.ne.0.)wr(nn)=x-w/z
              wi(nn)=0.
              wi(nn-1)=0.
            else

              wr(nn)=x+p
              wr(nn-1)=wr(nn)
              wi(nn)=z
              wi(nn-1)=-z
            endif
            nn=nn-2
```

```
          else
            if(its.eq.30)pause 'too many iterations in hqr'
            if(its.eq.10.or.its.eq.20)then
              t=t+x
              do 14 i=1,nn
                a(i,i)=a(i,i)-x
14            continue
              s=abs(a(nn,nn-1))+abs(a(nn-1,nn-2))
              x=0.75*s
              y=x
              w=-0.4375*s**2
            endif
            its=its+1

            do 15 m=nn-2,l,-1
              z=a(m,m)
              r=x-z
              s=y-z
              p=(r*s-w)/a(m+1,m)+a(m,m+1)
              q=a(m+1,m+1)-z-r-s
              r=a(m+2,m+1)
              s=abs(p)+abs(q)+abs(r)
              p=p/s
              q=q/s
              r=r/s
              if(m.eq.l)goto 4
              u=abs(a(m,m-1))*(abs(q)+abs(r))
              v=abs(p)*(abs(a(m-1,m-1))+abs(z)+abs(a(m+1,m+1)))
              if(u+v.eq.v)goto 4

15          continue
4           do 16 i=m+2,nn
              a(i,i-2)=0.
              if (i.ne.m+2) a(i,i-3)=0.
16          continue
            do 19 k=m,nn-1
              if(k.ne.m)then
                p=a(k,k-1)
                q=a(k+1,k-1)
                r=0.
                if(k.ne.nn-1)r=a(k+2,k-1)
                x=abs(p)+abs(q)+abs(r)
                if(x.ne.0.)then
                  p=p/x
                  q=q/x
                  r=r/x
                endif
              endif
              s=sign(sqrt(p**2+q**2+r**2),p)

              if(s.ne.0.)then
                if(k.eq.m)then
                  if(l.ne.m)a(k,k-1)=-a(k,k-1)
                else
                  a(k,k-1)=-s*x
                endif
                p=p+s
                x=p/s
                y=q/s
                z=r/s
                q=q/p
```

```
                r=r/p
                do 17 j=k,nn
                  p=a(k,j)+q*a(k+1,j)
                  if(k.ne.nn-1)then
                    p=p+r*a(k+2,j)
                    a(k+2,j)=a(k+2,j)-p*z

                  endif
                  a(k+1,j)=a(k+1,j)-p*y
                  a(k,j)=a(k,j)-p*x
17              continue
                do 18 i=l,min(nn,k+3)
                  p=x*a(i,k)+y*a(i,k+1)
                  if(k.ne.nn-1)then
                    p=p+z*a(i,k+2)
                    a(i,k+2)=a(i,k+2)-p*r
                  endif
                  a(i,k+1)=a(i,k+1)-p*q
                  a(i,k)=a(i,k)-p
18              continue
              endif
19          continue
            goto 2
          endif

        endif
      goto 1
      endif
      return

      END     SUBROUTINE
      END   MODULE EIGEN
```

```
!-------------------------------------------------------------------
!     Module to solve set of linear simultaneous equations
!     by the Gauss Jordan method. This module is used to calculate
!     the mode shapes for each mode.
!-------------------------------------------------------------------
        MODULE GAUSS

        CONTAINS

      SUBROUTINE gaussj(a,n,np,b,m,mp)
      INTEGER m,mp,n,np,NMAX
      REAL a(np,np),b(np,mp)
      PARAMETER (NMAX=50)
      INTEGER i,icol,irow,j,k,l,ll,indxc(NMAX),indxr(NMAX),ipiv(NMAX)
      REAL big,dum,pivinv
      do 11 j=1,n
        ipiv(j)=0
11    continue
      do 22 i=1,n
        big=0.
        do 13 j=1,n
          if(ipiv(j).ne.1)then
            do 12 k=1,n
              if (ipiv(k).eq.0) then
                if (abs(a(j,k)).ge.big)then
                  big=abs(a(j,k))
                  irow=j
                  icol=k
                endif

              else if (ipiv(k).gt.1) then
                pause 'singular matrix in gaussj'
              endif
12          continue
          endif
13      continue
        ipiv(icol)=ipiv(icol)+1
        if (irow.ne.icol) then
          do 14 l=1,n
            dum=a(irow,l)
            a(irow,l)=a(icol,l)
            a(icol,l)=dum
14        continue
          do 15 l=1,m
            dum=b(irow,l)
            b(irow,l)=b(icol,l)
            b(icol,l)=dum
15        continue
        endif
        indxr(i)=irow
        indxc(i)=icol
        if (a(icol,icol).eq.0.) pause 'singular matrix in gaussj'

        pivinv=1./a(icol,icol)
        a(icol,icol)=1.
        do 16 l=1,n
          a(icol,l)=a(icol,l)*pivinv
16      continue
        do 17 l=1,m
          b(icol,l)=b(icol,l)*pivinv
17      continue
```

```
        do 21 ll=1,n
          if(ll.ne.icol)then
            dum=a(ll,icol)
            a(ll,icol)=0.
            do 18 l=1,n
              a(ll,l)=a(ll,l)-a(icol,l)*dum
18          continue
            do 19 l=1,m
              b(ll,l)=b(ll,l)-b(icol,l)*dum
19          continue
          endif
21      continue
22    continue
      do 24 l=n,1,-1
        if(indxr(l).ne.indxc(l))then

          do 23 k=1,n
            dum=a(k,indxr(l))
            a(k,indxr(l))=a(k,indxc(l))
            a(k,indxc(l))=dum
23        continue
        endif
24    continue
      return

      END  SUBROUTINE gaussj
        END  MODULE GAUSS
```

```
!-------------------------------------------------------------
!       Module to generate group of simeltaneous equations
!       to solve for the modal shape values for all modes
!-------------------------------------------------------------

        MODULE  MODES

        CONTAINS

      SUBROUTINE getmodes(a,b,cc,n,np,sp)

      INTEGER n,n1
      REAL a(np,np),b(np,np),cc(np,np),x(np)
        REAL  sp(np,np)
      INTEGER i,j,k,k1

        n1=n

        do 28 k1=1,n  ! Set solution vector to zero
      x(k1)=0.0
28    continue


        OPEN(UNIT=22,FILE='modal.dat')

        WRITE(22,*) 'Equations generated to evaluate the modal matrix '
        WRITE(22,92) n,',',1
92      FORMAT(i1,a1,i1)

        do 12 i=1,n  ! degrees of freedom
      do 11 j=1,n
      do 10 k=1,n

      cc(j,k)=b(j,k)-sp(i,i)*a(j,k)
10    continue
11    continue


        do 53 i1=1,n1
        write(22,'(6f20.4)') ((cc(i1,j1)), j1=1,n1)
53      continue
        write(22,'(6f20.4)') ((x(k1)), k1=1,n1)
12    continue

        WRITE(22,*) 'END'
        close(22)

      END SUBROUTINE  getmodes

        END  MODULE  MODES
```

```
!------------------------------------------------------------
!     Module to perform the multiplication of two
!     matrices
!------------------------------------------------------------

          MODULE MULTIPLY

          CONTAINS

            SUBROUTINE MULT(N1,N2,N3,A,B,F)
            DIMENSION A(20,20),B(20,20),F(20,20)
      !     INTEGER     N1,N2,N3

!           MULTIPLICATION OF MATRIX 'A' BY MATRIX 'B'
            DO 700I=1,N2
            DO 700J=1,N1
            DO 700M=1,N3
                  SUM=0.0
                  DO 700K=1,N2
                        F(J,M)=A(J,K)*B(K,M)
                        SUM= SUM + F(J,M)
                        F(J,M)=SUM
700         CONTINUE
            RETURN
            END    SUBROUTINE MULT
      END  MODULE MULTIPLY
```

```
!------------------------------------------------------------
!     Module to perform the inverse of a matrix with
!     physical dimension np by np
!------------------------------------------------------------

      MODULE TRANSPOSE

      CONTAINS

      SUBROUTINE TRANSPOSING(n,cm,cmt)
      DIMENSION cm(20,20),cmt(20,20)
      INTEGER      n,i,j
!     transposing a square matrix a(n,n) to b(n,n)

      DO 80i=1,n
      DO 70j=1,n
            cmt(i,j)=cm(j,i)
70    continue
80    continue

      do 42 i=1,n
!     write(*,'(1x,6f16.4)') ((cm(i,j)), j=1,n)
42    continue

      do 43 i=1,n
!     write(*,'(1x,6f16.4)') ((cmt(i,j)), j=1,n)
43    continue

      RETURN
      END    SUBROUTINE TRANSPOSING

      END  MODULE TRANSPOSE
```

```
!-----------------------------------------------------------
!     Module to arrenge precomputed eigen values,
!     form the specttral matrix, compute the natural
!     frequencies and natural periods for each mode
!-----------------------------------------------------------

      MODULE SPECTRAL

      CONTAINS

      SUBROUTINE SPECTRAL_MAT(n,np,wr,sp,nf,nfd,dr)

      INTEGER n,np,m
      REAL wr(np),sp(np,np),nf(np),nfd(np)
      REAL value1,dr,Tn(np)


      do 16 k=1,n
!     write(*,*)wr(k)
16    continue


!------Rearrangement of computed eigen values-----
      do 13 j=1,n-1
      value1=wr(j)
      m=j
      do 12 i=j+1,n
      if (wr(i).lt.value1) then
      value1=wr(i)
      m=i
      else
      endif
12    continue
      wr(m)=wr(j)
      wr(j)=value1
13    continue

!------form matrix of rearrenged eigen values-----

      do 40 i=1,n
        do 39 j=1,n
          sp(i,j)=0.0
39      continue
40    continue

      do 41 i=1,n
          sp(i,i)=wr(i)
41    continue
 !     write(*,*)  '------>  Rearranging Spectral Matrix : '
      write(15,*) '------ Spectral Matrix ------'

      do 42 i=1,n
      write(15,'(1x,6f16.4)') ((sp(i,j)), j=1,n)
42    continue

 !     write(*,*)  '------>  Computing natural frequencies : '
      do 43 j=1,n
      nf(j)=sqrt(sp(j,j))
43    continue

      write(15,*) '------ Natural frequencies ------'
```

```
      write(70,*)'NATURAL FREQUENCIES'
      do 44 i=1,n
      write(15,47) i, nf(i)
      write(14,'(f7.2)')nf(i)
      write(70,'(f7.2)')nf(i)
47    format('Mode',i4,3x,6f12.3)
44    continue

      write(15,*) '------ Natural Period ------'
      do 74 i=1,n
      Tn(i)=(2.0*3.141592654)/nf(i)
      write(15,49) i, Tn(i)
49    format('Mode',i4,3x,6f12.3)
74    continue


!     Damping ratio assumed 5%
!     dr=.05

!      nfd = Natural Frequency value, Damped

      write(15,*) '- Natural frequencies with classical damping -'
      do 94 i=1,n
      nfd(i)=nf(i)*sqrt(1-(dr*dr))

      write(15,93) i, nfd(i)
93    format('Mode',i4,3x,6f16.4)
94    continue

      return
      END     SUBROUTINE
      END   MODULE SPECTRAL
```

```
!------------------------------------------------------------
!     Module to normalize pre computed modes
!     of the discretized system.
!------------------------------------------------------------

      MODULE NORMALIZE

      USE TRANSPOSE
      USE MULTIPLY

      CONTAINS

      SUBROUTINE Normalizing(n,am,cm,cmt,cmn,mn)

      DIMENSION  cm(20,20),cmt(20,20),am(20,20)
      dimension  cmtf(20,20),f(20,20),cmn(20,20)
      REAL       mn(20)
      INTEGER        n,i,j


!      Normalization of modes

      call TRANSPOSING(n,cm,cmt)
      call MULT(n,n,n,cmt,am,F)
      call MULT(n,n,n,f,cm,cmtf)


      write(17,*) 'Normalizing so that Mn=I.. '

      write(15,*) ' mass..... '
      do 41 i=1,n
      write(17,'(1x,10f12.3)') ((am(i,j)), j=1,n)
41    continue

      write(17,*) ' modal matrix /without transposing... '
      do 46 i=1,n
      write(17,'(1x,10f12.3)') ((cm( i,j)), j=1,n)
46    continue

      write(17,*) ' modal matrix T / transposd..... '
      do 47 i=1,n
      write(17,'(1x,10f12.3)') ((cmt( i,j)), j=1,n)
47    continue

      write(17,*) ' modal T * mass matrix ..... '
      do 42 i=1,n
      write(17,'(1x,10f12.3)') ((f(i,j)), j=1,n)
42    continue

      write(17,*) ' modal T * mass matrix  * modal ... '
      do 43 i=1,n
      write(17,'(1x,10f12.3)') ((cmtf(i,j)), j=1,n)
43    continue

!     Orthonormalization relative to the mass

      do 71i=1,n
      mn(i)=cmtf(i,i)
71    continue

      write(17,*) 'Normalization factors, Generalized Mass Matrix'
```

```
      write(17,'(1x,10f12.3)') ((mn(j)), j=1,n)

!     nor=sqrt(1/cmtf(1,1))

!     Normalized modal matrix
      do 80 i=1,n
      do 70 j=1,n
      cmn(i,j)=cm(j,i)/SQRT(mn(i))
70    continue
80    continue

      write(15,*) ' Normalized modal matrix '
      write(16,*) ' Normalized modal matrix '

      do 53 i=1,n
      write(15,'(3x,10f16.6)') ((cmn(j,i)), j=1,n)
53    continue

      do 63 i=1,n
      write(16,'(3x,10f16.6)') ((cmn(j,i)), j=1,n)
63    continue

      do 64 i=1,n
      write(16,'(3x,10f10.3)') ((am(j,i)), j=1,n)
64    continue

      RETURN

      END    SUBROUTINE Normalizing

      END  MODULE NORMALIZE
```

```
!-------------------------------------------------------------
!     This module which is originally LUD - back substitution
!     is modified to account for the normalization of modes,
!     if not zero solution will result as mode shape
!-------------------------------------------------------------

        MODULE LUDBACK

        CONTAINS

      SUBROUTINE lubksb(a,n,np,indx,b)
      INTEGER n,np,indx(n)
      REAL a(np,np),b(n)
      INTEGER i,ii,j,ll
      REAL sum

      ii=0
      do 12 i=1,n
        ll=indx(i)
        sum=b(ll)
        b(ll)=b(i)
        if (ii.ne.0)then
          do 11 j=ii,i-1
            sum=sum-a(i,j)*b(j)
11        continue
        else if (sum.ne.0.) then
          ii=i
        endif
        b(i)=sum
12    continue
      do 14 i=n,1,-1
        sum=b(i)
        do 13 j=i+1,n
          sum=sum-a(i,j)*b(j)
13      continue
        b(i)=sum/a(i,i)

!   IMPORTANT NOTE
!   following line included to normalize the modes
!   for free vibration only
!   the lower most mode is normalized as it is
!   feasible in this analysis procedure.
!   which will be adjusted in the main program
!   by ( 1/x )and reversing the order

     b(n)=1.0

14    continue
      return
      END  SUBROUTINE lubksb

      END  MODULE LUDBACK
```

```
!------------------------------------------------------------
!     This module is used to perform the LUD - decomposition
!     of a square matrix with a physical dimension np by np
!------------------------------------------------------------

        MODULE  LUDCOMP

        CONTAINS

      SUBROUTINE ludcmp(a,n,np,indx,d)
      INTEGER n,np,indx(n),NMAX
      REAL d,a(np,np),TINY
      PARAMETER (NMAX=500,TINY=1.0e-20)
      INTEGER i,imax,j,k
      REAL aamax,dum,sum,vv(NMAX)
      d=1.
      do 12 i=1,n
        aamax=0.
        do 11 j=1,n
          if (abs(a(i,j)).gt.aamax) aamax=abs(a(i,j))
11      continue
        if (aamax.eq.0.) pause 'singular matrix in ludcmp'
        vv(i)=1./aamax
12    continue
      do 19 j=1,n
        do 14 i=1,j-1
          sum=a(i,j)
          do 13 k=1,i-1
            sum=sum-a(i,k)*a(k,j)
13        continue
          a(i,j)=sum
14      continue
        aamax=0.
        do 16 i=j,n
          sum=a(i,j)
          do 15 k=1,j-1
            sum=sum-a(i,k)*a(k,j)
15        continue
          a(i,j)=sum
          dum=vv(i)*abs(sum)
          if (dum.ge.aamax) then
            imax=i
            aamax=dum
          endif
16      continue
        if (j.ne.imax)then
          do 17 k=1,n
            dum=a(imax,k)
            a(imax,k)=a(j,k)
            a(j,k)=dum
17        continue
          d=-d
          vv(imax)=vv(j)
        endif
        indx(j)=imax
        if(a(j,j).eq.0.)a(j,j)=TINY
        if(j.ne.n)then
          dum=1./a(j,j)
          do 18 i=j+1,n
            a(i,j)=a(i,j)*dum
18        continue
```

```
        endif
19    continue

      return
      END SUBROUTINE ludcmp

      END  MODULE  LUDCOMP
```

```
!-----------------------------------------------------------------
!     Reduction to Hessenberg form by the ellimination method
!     The real non-symetric n by n matrix is replaced by upper
!     Hessenberg matrix with identical eigen values.
!-----------------------------------------------------------------

      MODULE  HESSENBERG

      CONTAINS

      SUBROUTINE elmhes(a,n,np)
      INTEGER n,np
      REAL a(np,np)
      INTEGER i,j,m
      REAL x,y
      do 17 m=2,n-1
        x=0.
        i=m
        do 11 j=m,n
          if(abs(a(j,m-1)).gt.abs(x))then
            x=a(j,m-1)
            i=j
          endif
11      continue
        if(i.ne.m)then
          do 12 j=m-1,n
            y=a(i,j)
            a(i,j)=a(m,j)
            a(m,j)=y
12        continue
          do 13 j=1,n
            y=a(j,i)
            a(j,i)=a(j,m)
            a(j,m)=y
13        continue
        endif
        if(x.ne.0.)then

          do 16 i=m+1,n
            y=a(i,m-1)
            if(y.ne.0.)then
              y=y/x
              a(i,m-1)=y
              do 14 j=m,n
                a(i,j)=a(i,j)-y*a(m,j)
14            continue
              do 15 j=1,n
                a(j,m)=a(j,m)+y*a(j,i)
15            continue
            endif
16        continue
        endif
17    continue
      return
      END    SUBROUTINE elmhes

      END   MODULE  HESSENBERG
```

```
!--------------------------------------------------
!     Module to solve for the mode shapes
!     from pre assembled set of linear equations
!--------------------------------------------------

      MODULE MODESOL

        USE LUDCOMP
        USE LUDBACK

        CONTAINS

        SUBROUTINE  Modeshapes(a,n,np,cm)

      INTEGER NP
      REAL p,a(NP,NP),b(NP,NP),c(NP,NP),x(NP)
      REAL cm(NP,NP)
      INTEGER j,k,l,m,n,indx(NP),lx

      open(7,file='modal.DAT',status='old')
        read(7,*)
        read(7,*) n,m

      do 100 lx=1,n
      read(7,*) ((a(k,l), l=1,n), k=1,n)
      write(17,*) '------- Input matrix  -------'
        do 52 i=1,n
        write(17,'(1x,6f18.8)') ((a(i,j)), j=1,n)
52      continue

      read(7,*) ((b(k,l), k=1,n), l=1,m)
      write(17,*) '--------Right-hand vector ------'
        do 53 i=1,m
        write(17,'(1x,6f18.8)') ((b(i,j)), j=1,n)
53      continue

!     save matrix a for later testing
      do 12 l=1,n
        do 11 k=1,n
          c(k,l)=a(k,l)
11      continue
12    continue

!     do LU decomposition
      call ludcmp(c,n,NP,indx,p)

!     solve equations for each right-hand vector
      do 16 k=1,m
        do 13 l=1,n
          x(l)=b(l,k)
13      continue
        call lubksb(c,n,NP,indx,x)

        write(17,*) '------ solution vector ------'

!     normalization of the solution vector for
!     convenience. The amplitude of the first
!     degree of freedom is set to unity.

          write(17,'(1x,6f8.3)') (x(l)/x(1),l=1,n)
            do 200 l=1,n
```

```
           cm(l,lx)=x(l)/x(1)
200          continue

!      test results with original matrix

         write(17,*) 'Right-hand side vector:'
         write(17,'(1x,6f12.6)') (b(l,k), l=1,n)
         write(17,*) 'Result of matrix applied to sol''n vector'
         do 15 l=1,n
           b(l,k)=0.0
           do 14 j=1,n
             b(l,k)=b(l,k)+a(l,j)*x(j)
14         continue
15       continue
         write(17,'(1x,6f12.6)') (b(l,k), l=1,n)

16     continue

100      continue
       close(7)
        write(15,*)
        write(15,*) '------ * MODAL MATRIX * --------'
        write(15,*)
         do 75 i=1,n
         write(15,'(1x,10f12.3)') ((cm(i,j)), j=1,n)
75       continue

        END SUBROUTINE  Modeshapes
      END  MODULE  MODESOL
```