



ADDIS ABABA UNIVERSITY
ADDIS ABABA INSTITUTE OF TECHNOLOGY
SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

***Design of Prestressed Concrete Sleeper Based on Ultimate Limit State
Approach***

A Thesis Submitted to the School of Civil and Environmental Engineering in
Partial Fulfillment of the Requirement for the Degree of Master of Science in
Civil Engineering (Railway Engineering)

By

Ketemaw Amare

Advisor

Dr.Ing Adil Zekaria

Addis Ababa, Ethiopia
December 2017

Addis Ababa University
Addis Ababa Institute of Technology
School of Civil and Environmental Engineering

This is to certify that the thesis prepared by **Ketemaw Amare**, entitled: **Design of Prestressed Concrete Sleeper Based on Ultimate Limit State Approach** and submitted in partial fulfillment of the requirement for the degree of Master of Science (Civil and Environmental Engineering) complies with the regulation of the University and meets the accepted standards with respect to the originality and quality.

Approved by the Examining committee:

External Examiner _____ Signature _____ Date _____

Internal Examiner _____ Signature _____ Date _____

Advisor _____ Signature _____ Date _____

School or Center Chair Person

ABSTRACT

Railway prestressed concrete sleepers are amongst the vital components of the rail track superstructure, and play significant role in providing integrity, and safe operation of the tracks. Due to the combined effect of the weight burden from the train, and the dynamic load as a result of the train motion, sleepers are subjected to both static and dynamic loading conditions. To achieve the desired service life of railway system, sleepers can be designed to carry and transfer the wheel loads from the rails to the track foundation.

Current design philosophy for prestressed concrete sleeper is based on permissible stress principle taking into account only the static and quasi-static loads, which are unrealistic to the actual dynamic loads on tracks. However, there still shortcomings when viewed from the limit state design perspective. Henceforth, limit state function needs to be included in the current design approach as an improvement. There is a need to improve the current understanding of the design of prestressed concrete sleepers by reliability based design method by formulating limit state function. The limit state function depends on reliability index, and load and resistance factors determined through calibration based on available statistical or scholastic data.

This paper, tries to follow the ultimate limit state design approach for axial, flexure, and shear design of sleeper by taking into account the compressive and tensile stresses at both initial and final stages of concrete and prestress wires.

Key words: Prestressed concrete sleeper, Ultimate limit state, permissible stress, axle load, dynamic load, axial, shear, flexure.

ACKNOWLEDGEMENTS

This work could not have been completed without the support of a number people to whom I am grateful.

I would like to express deepest appreciation to my Advisor Dr.Ing Adil Zekaria, for his valuable guidance and encouragement. His constructive advice was crucial for this study to maintain proper direction towards completion

My co-advisor Mr. Yisshak Taddesse has made a relentless effort through his appreciated consultation and encouragement for the betterment of this work. His professional insight on every part of the work progress is truly invaluable. I am very grateful for the help I had received.

I am also very thankful for the Ethiopian railway corporation, and Addis Ababa institute of technology for the privilege I have been given to work on this study, and for all the necessary arrangements they have made throughout my study.

Special thanks to Dr. Abrham Gebre, instructor at Addis Ababa University, school of civil and environmental Engineering for his professional advice.

Finally, I would like to express my sincere appreciation for my friends, and family for their kindness and support throughout the study period.

TABLE OF CONTENTS

ABSTRACT	I
ACKNOWLEDGEMENTS.....	II
TABLE OF CONTENTS	III
LIST OF FIGURES	V
LIST OF TABLES.....	VII
LIST OF ABBREVIATION AND SYMBOLS	VIII
CHAPTER ONE	1
1. INTRODUCTION	1
1.1. Background	1
1.2. Statement of the Problem.....	3
1.3. Objective of the Research	4
1.3.1. General objective	4
1.4. Methodology of the Research	4
1.5. Structure/Outline of the Research	5
CHAPTER TWO	7
2. LITERATURE REVIEW	7
2.1. Ballasted Track	7
2.2. Concrete Sleepers.....	7
2.3. Development of prestressed concrete sleeper	8
2.4. Structural behavior of prestressed concrete sleeper	9
2.4.1. Prestressed as the Preferred for Concrete Sleeper	9
2.4.2. Static and Dynamic Behaviour of PC Sleeper	11
2.4.2.1. Static Behaviour of PC Sleeper.....	11
2.4.2.2. Dynamic Behaviour of PC Sleeper	14
2.5. Loading of Railway PC Sleepers	15
2.6. Load Assessment on PC Sleepers	15
2.7. Concept of Sleeper Design.....	18
2.7.1. Current Practice to Design Sleeper	18
2.7.2. Limit State Design of Sleeper	19
2.8. Structural Reliability	21
CHAPTER THREE	24

3. RELIABILITY ASSESSMENT OF PC SLEEPER	24
3.1. Reliability Analysis.....	26
3.1.1. Ultimate Capacity of Sleeper	28
3.1.2. Static and Dynamic Loads on the Sleeper	28
3.1.2.1. Static Load	28
3.1.2.2. Dynamic load	29
3.1.3. Reliability Index Analysis.....	30
3.2. Load Factor	31
3.2.1. Static Load Factor	33
3.2.2. Dynamic Load Factor.....	33
3.3. Strength Reduction Factor	34
CHAPTER FOUR	35
4. DESIGN OF PRESTRESSED CONCRETE SLEEPER.....	35
4.1. Design Forces Acting on the Sleeper	37
4.2. Parametric Optimization of Prestressed Concrete Sleeper	38
4.2.1. Sleeper Dimension	38
4.2.1.1. Selection of Best Proposed Sleeper	40
4.2.2. Concrete Grade and Prestressing Wires.....	42
4.3. Design Bending Moments of Selected Sleeper.....	47
4.4. Detail Design of Selected Sleeper.....	51
4.4.2.1. Bending Strength of Selected Sleeper.....	56
4.4.3. Design for Shear.....	64
CHAPTER FIVE	67
5. CONCLUSION AND FUTURE RECOMMENDATION	67
5.1. Conclusion	67
5.2. Future recommendation	68
REFERENCES	69
APPENDICES	72
Appendix A: COMREL V9.0 of reliability index analysis result.....	72
Appendix B: Geometrical model result of trial 11 sleeper by SAFE software.....	73
Appendix C: Reinforcement and concrete grade selection	74
Appendix D: Pre-calculated graphs for evaluation of $M_{c,ng,100}$	81

LIST OF FIGURES

Figure 2.1: Typical ballasted track	7
Figure 2.2: Types of concrete sleepers	8
Figure 2.3: Stages of pre-tensioning	11
Figure 2.4: Estimated distribution of loads.....	13
Figure 2.5: Contact pressure distribution patterns	14
Figure 2.6: Frequency-Load Reliability Diagrams	22
Figure 2.7: Relationship of safety index and probability of failure.....	23
Figure 4.1: Chart flow design PC sleeper by concept of limit state	36
Figure 4.2: Selected sleeper cross section	41
Figure 4.3: Design model for $M_{d.r.pos}$	48
Figure 4.4: Conceptual diagrams for calculating bending strength of selected sleeper	57
Figure 4.5: Shear diagram of selected sleeper	65

LIST OF TABLES

Table 2.1: Hypothetical bearing pressure distribution of sleeper	18
Table 3.1: Sleeper ultimate load capacity lab test results	28
Table 3.2: Statistical parameters of static load	29
Table 3.3: Statistical parameters of dynamic load.....	29
Table 4.1: Possible dimension of PC sleeper.....	39
Table 4.2: Allowable stress of C-50 concrete grade.....	52
Table 4.3: Bending moments of selected sleeper	64
Table 4.4: Resistance shear at different width of sleeper	66

LIST OF ABBREVIATION AND SYMBOLS

R_n	nominal resistance load
f_{pe}	effective prestress in the tendon after all losses
f_{pk}	characteristic strength of prestressing steel
b_w	breadth of the section,
M_0	moment initiating a flexural crack
M_d	design moment at the design section
f_{ctd}	tensile strength of concrete
f_{cp}	compressive stress in concrete at CGC due to the prestress
V_{Rd}	shear resistance of section
M_c	concrete compression moment;
M_{tn}	tension moment from the n^{th} steel wires
A_s	cross section area of a single steel wire;
A_t	area of trapezium shape of sleeper cross section;
A_{sg}	gross area of all cross sections of steel wires
A_c	net area of concrete,
$M(g+q)$	moment due to axial force and self-weight
P_e	effective prestress after long term losses
e	eccentricity of prestressing force from center of section
M_d	moment due to self-weight,
P_t	the prestress force at transfer after short –term loss.
P_e	allowable prestress force at transfer

f_{ct}	allowable stress of concrete under compression
f_{tt}	allowable stress of concrete under tension
$M_{d,c,pos}$	positive bending moment at the center
$M_{d,c,neg}$	negative bending moment at the center part
$M_{d,r,neg}$	negative bending moment at rail seat
$M_{d,r,pos}$	positive bending moment at rail seat
L	sleeper length
Z_t	top section modulus,
Z_b	bottom section modulus
P_i	initial prestress force at end of tendon
Q	dynamic load
S	static load
k_s	static load factors
k_d	dynamic load factor
DF	distribution factor
ERC	Ethiopian Railway Corporation
I	moment of inertia
ϵ_{pt}	strain in prestressing steel at transfer
ϵ_{cu}	ultimate strain in concrete
ϵ_{ct}	strain in concrete at transfer
β	reliability index
$\emptyset$	resistance reduction factor

CHAPTER ONE

1. INTRODUCTION

1.1. Background

Railway sector is an outstanding mode of transport among the most essential and broadly utilized methods for transportation, conveying cargo, passengers, minerals, grains, and so forth. Railway track structures often experience impact loading conditions due to wheel/rail interactions associated with abnormalities in either a wheel or a rail. As a result, its structural components performance must be repetitively quantified in order to identify any potential safety hazards (Kaewunruen, 2008).

Railway sleepers are the main structural elements of railway track. As well as pressure distribution and load transfer to the underlying layers, railway sleepers are in duty to maintain track gauge, grantee for lateral stability of the track and contribute in better geometrical conditions of the track (RE, 1968). Vertical, lateral and axial forces are applied to rail sleepers. These forces should be transferred to the underlying ballast layer within the admissible stress range with minimum disturbance of track quality and permanent deformation (Grassie, 1985).

The sleepers can be manufactured using timber, concrete, steel or other engineered materials. Concrete sleepers were initially installed on track for many decades ago and at present are being used in almost everywhere in the world, and become the most popular choice everywhere in the world because of its cost efficiency, durability, maintainability and positive benefits in track stability. Current design philosophy for prestressed concrete sleepers is based on permissible stress principle taking into account only the static and quasi-static loads, which are unrealistic to the actual dynamic loads on tracks. In order to devise a new limit states design concept, the research efforts are required to perform comprehensive studies of the loading conditions, the static behaviour, the dynamic response, and the impact resistance of the prestressed concrete sleepers (Remennikov, 2007).

A prestressed concrete sleeper (PC) is designed mainly in consideration of an impact wheel load during the train running. The impact load is generally caused by the wheel flat, the rail joint, the welded joint irregularity of the rail and the railhead corrugation. Among those defects, the wheel flat and the rail joint have the greatest influence on the dynamic response of the PC sleeper. At the beginning of the development of the PC sleeper, it had been thought that the enormous impact wheel load was mainly induced by the wheel flat. For this reason, a large number of researches into the wheel flat have been conducted. Through the running test, and the impact load examination of the PC sleepers, these researches made clear that the design load of the PC sleepers shall be the four times larger than the static load in order to allow no tensile stress (full prestressing) under the assumed maximum impact load by wheel flat. Recently, various kinds of research and development have been promoted from the viewpoint of the optimization of the sleeper (S. Kaewunruen, 2007).

Sukontasukkul (2004) found that concrete strength under impact loading shows different behavior from that under static loading. Considerably, the concrete material behaves in a more brittle manner, and increases in strength, toughness, and modulus of elasticity were found as the rate of loading increased. This is because the impact cracks tend to propagate through rather than around aggregate granular, resulting in an increase in strength and toughness, and a decrease in the non-linear portion of stress-strain curve.

Under static loading, mixed shear-flexure failure modes can be observed, whilst under impact loading the identical specimens failed only in shear failure type. Apart from the impact material property testing, the studies of impact behaviour of concrete members mostly dealt with flexural members. The failure of such experiments was simply represented by the flexural toughness of such specimens (Banthia et al., 1987).

Regarding to railway sleepers; Wang (1996) investigated the resistance of concrete railroad ties to impact loading. Their study focused on the effect of material uses on the ultimate capacity of prestressed concrete sleepers. However, it was unclear whether strain rate has an effect on the behaviors of concrete sleepers or not, and whether there could be a simplified prediction for the ultimate capacity of concrete sleepers. So far, the ultimate behaviors of

prestressed concrete sleepers under impacts are currently unclear and there is no method to predict the ultimate moment capacity under impact loading.

The study focused on the limit state design of pre-stressed concrete sleepers using the reliability design concept. This design approach is used in order to give evidence based insight into the structural design helping improve safety, and reliability of railway structures.

The design basis of this PC sleeper is with reference of AS 1085.14-2003, AREMA, 2010, and EN 13230-6, 2015.

1.2. Statement of the Problem

Most of Ethiopia's railway projects are conducted by foreign experts. These professionals tend to use standards they are used to despite where the project resides. For instance, the Awash-Woldiya project being undertaken by a Turkish Company named Yapi Merkezi uses European standards, and the Addis Ababa – Djibouti project constructed by a Chinese company (CCECC) was carried out according to the Chinese standard.

When sleeper standards differ, stiffness variation will occur at the junction point. In the current economic environment, suitable design of sleeper is important for railway organizations to be as competitive as possible. The major task for a railway track engineer is often analyzing, and determining the economic benefit of an organization by increasing axle load, vehicle speed, and decreasing maintenance period of railway track.

The government of Ethiopia has planned to construct more than 5000 Kilometers railway line from all direction of our country including Addis Ababa light railway transit (AALRT). With this in mind, a uniform standard that takes in to account local conditions needs to be formulated and followed in design and construction of PC sleepers. Such standard should also provide economical dimensions for enhancing resistance of impact loads in addition to static ones.

Major problems of the research are:-

- lack of statistical data recording from field

- lack of uniform production, and design method of prestressed concrete sleeper
- difficulty of determining the accurate capacity of prestressed concrete sleeper in the current construction practice.

1.3. Objective of the Research

1.3.1. General objective

The main objective of this thesis is to study the design of mono block prestressed concrete sleeper by limit state approach for ballasted railway track. The practical aim behind this objective is to adopt ultimate limit state design approach by concept of structural reliability obtained by reliability index, load and resistance factor. This thesis targets to acquire an accurate and efficient method that would allow choosing parameters of suitable load combination for static and dynamic load of the railway system so that, its stability is guaranteed at operational train speeds, and service life of railway track.

1.3.2. Specific Objective

Having ultimate limit state method adopted, this thesis aims to:-

- get economical dimension of PC sleeper
- enhance the knowledge in design of PC sleepers by limit state, and reliability concept
- determine the suitable load factors, reduction factor, and safety index in sleeper design

1.4. Methodology of the Research

Addis Ababa- Djibouti railway line is under trial session to fulfill international standard requirements for a period of six months. The rail way has not yet began its full service and hence does not give the exact flow of traffic. This has limited the researcher from being able to collect data from the line related to its actual traffic flow. For that reason, probabilistic method of estimation has been used for evaluation of reliability index, reduction factor and

load factors to design the sleepers. The reliability index has further been determined using the analysis software, Comrel V9.0.

The main reasons for ultimate design of sleeper is to assign economical dimension of PC sleeper to decrease sleeper bending strength, and increase performance by accounting factors for each load types and enhance structural service life of it.

For ultimate limit state design methodology of railway PC sleeper, the following information is required

1. Probabilistic data from previous studies, lab tests, and local conditions need to be developed
2. The reliability index, reduction factor, and load factors must be determined
3. Ultimate limit state equation for the design of PC sleepers has to be adopted

1.5. Structure/Outline of the Research

This thesis consists of five chapters in the following framework

The first chapter introduces the background, problem statement, objectives and methodology of the research.

The literature review is presented in the second chapter. It discusses about previous studies and basic inputs for this project to develop probability data. The review encompasses ballasted track, concrete sleepers, development of prestressed concrete sleeper, structural behavior of prestressed concrete sleeper, loading of railway PC sleepers, load assessment on PC sleepers, concept of sleeper design, structural reliability, and safety concept. All Theories and previously developed concepts used as a basis to this project are briefly discussed under this chapter.

Reliability assessment of PC sleepers is discussed under the third chapter. The chapter describes how to obtain the reliability index, load factors and strength reduction factors to drive final ultimate limit state design equation of PC sleeper.

The design of prestressed concrete sleeper based on calculation of design rail-seat load is discussed in the fourth chapter. Optimization of PC sleeper depending of ballast distribution beneath sleeper, and allowable stress of concrete and steel for axial, flexural, and shear design of selected sleeper after determining the design load is carried out under this portion of the thesis.

The final chapter provides the main conclusion of the thesis based on major findings and provides recommendation for further research.

CHAPTER TWO

2. LITERATURE REVIEW

2.1. Ballasted Track

As explained by S.Kaewunruen (2008), the most obvious parts of the ballasted track as which include the rails, rail pads, concrete sleepers, and fastening systems are referred to as the superstructure while the substructure is associated with a geotechnical system consisting of ballast, sub-ballast and subgrade (formation). Both superstructure and substructure are mutually important in ensuring the safety, and comfort of passengers and quality of the ride. This research is focused solely on sleepers, which can be classified according to the material used in their construction in the following way:

- 1) Wooden sleepers
- 2) Metal sleepers
- 3) Concrete sleepers

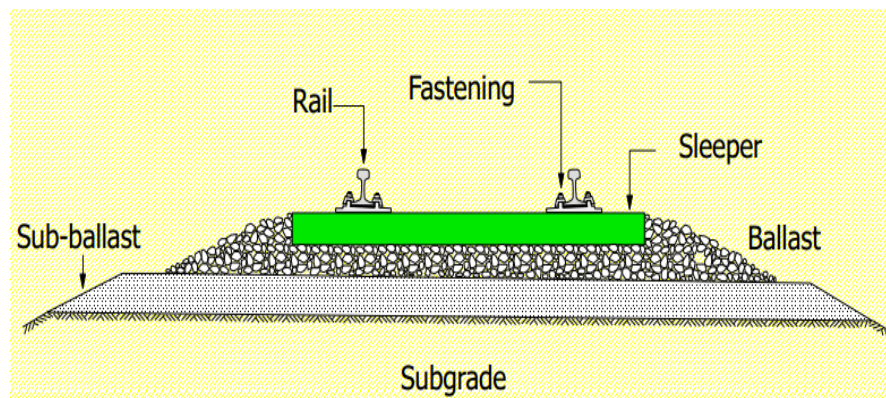


Figure 2.1: Typical ballasted track (Esveld, 2001)

2.2. Concrete Sleepers

Sleepers are transverse beams resting on ballast. Depending on their orientation they can be classified as mono-block and twin-block as illustrated in Figure 2.2. Due to the durable

nature of concrete in numerous environmental conditions, concrete sleepers are widely chosen. This thesis focuses on the design of mono-block prestressed concrete sleepers.



a) Mono block concrete sleeper b) Twin-block concrete sleeper

Figure 2.2: Types of concrete sleepers (Esveld, 2001)

Sleepers serve the following purposes in a railway line (Esveld, 2001):

- They hold rails together at the correct gauge length
- They maintain proper level of transverse tilt between rails
- They act as an elastic medium between the ballast and rails
- They distribute loads from rails to the index area beneath
- They provide means to rectify track geometry during the railway line life service
- To support the rails at a proper level in straight tracks and at proper super elevation on curves.

2.3. Development of prestressed concrete sleeper

A prestressed concrete sleeper is a concrete beam with prestressing steel wires inside of it. As described by Taylor (1993), the need for replacement of timber sleepers emerged in the United Kingdom at the start of World War 2. This is mainly due to the scarcity of the material for mass production of sleepers in new railway lines, the introduction of continuous welded rail track, the advancement of concrete technology, and pre-stressing techniques.

In 1941, two designs of reinforced concrete sleepers were manufactured and tested in branch line near Derby. In the following year, 100 of them were laid in the Mainline near Watford and survived for just 10 days. This testing was valuable for providing the experience in strain measurement and highlighting the difficulties of carrying out research on the real track (Taylor, 1993).

On 21 February 1943, the first prestressed concrete sleepers were laid in the west coast mainline at Cheddington. The durability of the prestressed concrete sleepers was found to be better than reinforced concrete sleepers. This is the starting point to the use of prestressed concrete as the preferred sleepers in railway lines across the world (Taylor, 1993).

2.4. Structural behavior of prestressed concrete sleeper

2.4.1. Prestressed as the Preferred for Concrete Sleeper

Pre-tensioning is widely used in concrete railway sleepers, where tension is applied to tendons before casting of concrete, which results in a prestressed concrete member. The difference between the behavior of prestressed concrete, and reinforced concrete sleepers lead to different basis in design. The reinforced concrete sleeper is very heavy and deep. Cracking is inevitable in reinforced concrete members. The design approach in such members is based on limiting the rapidity of crack formation under repeated loading in order to prolong the service life of the member. In contrast, no crack should occur during the service life of a prestressed member, hence the major difference in their design.

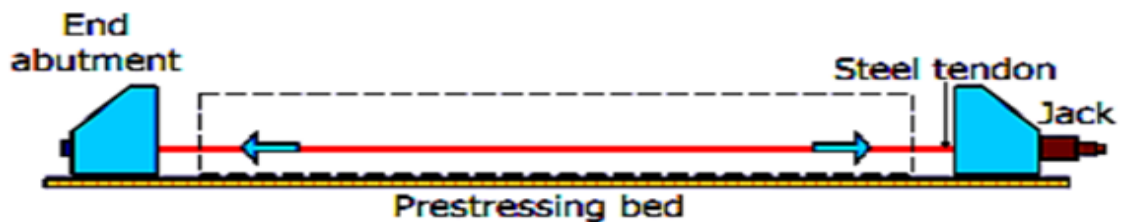
In pre-tensioning system, the high strength tendons are stretched to the required tension amount between two end abutments prior to the casting of concrete. The tendons are anchored to the ends of the mold for the concrete. The concrete is cast around the tensioned tendons, once the concrete reaches its desired strength; the tendons are cut from the abutments. The force in the tendon is transferred to the concrete by the bond between the tendon and concrete. The bond between the stretch tendon and the concrete will prevent the wire from regaining to its original length thus inducing pre-stressed force (Amlan, 2008).

In discussing the work by Thomas (1944), Dr. Mautner shares the design basis of the first prestressed concrete sleeper. The bond between concrete and steel bars in reinforced concrete sleepers was very sensitive to repeated loading. This is principally due to the inability of the steel bars to response to the plasticity of the concrete in tension. As a result, slipping would occur between the steel bars and the concrete and a considerable degree of cracking in the concrete takes place without actually leading to failure.

On the contrary, apart from the development length of the prestressing strands, discrepancy was non-existent between the steel and concrete strains in the prestressed concrete as long as no crack developed. Thus, the concrete is only in compression state instead of tension and both steel and concrete will identically elongates under loading. Prestressed concrete sleepers are more durable under repeated static load hence they are preferred over reinforced concrete sleepers.

The procedure of pretensioning is summarized as follows:

1. Tendons are anchored on end abutments
2. Jacks are put in place
3. Tension is then applied to the tendons
4. Concrete is casted
5. Tendons are cut at the proper location

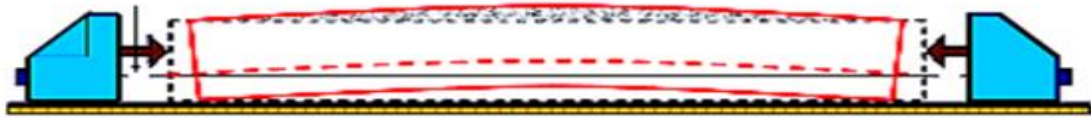


a) Applying tension to tendon



b) Casting of concrete

Cutting of tendon



c) Transferring of prestress

Figure 2.3: Stages of pre-tensioning (Amlan, 2008)

The basic principle of prestressed concrete tendons is to transfer the prestress to the concrete. The mechanism of transfer between the pre-tensioned and post-tensioned members is different for each other. In this study, only pre-tensioned is focus. For a pre-tensioned member without any anchorage at the ends, the prestress is transferred by the bond between the concrete and the tendons.

2.4.2. Static and Dynamic Behaviour of PC Sleeper

Railway sleeper is a structural element subjected to cyclic static and dynamic loading during its entire service life. These cyclic loadings cause fatigue of concrete sleeper, which results in permanent progressive changes in the structure. These changes can cause crack, or micro-crack propagation which consequently reduces stiffness of the structural behaviour leading to a fatigue failure.

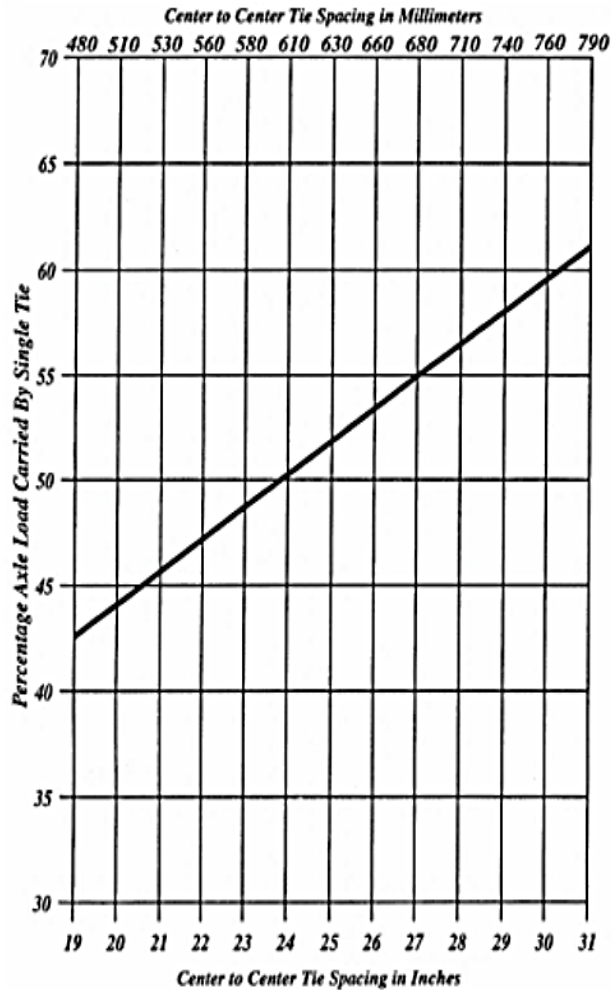
2.4.2.1. Static Behaviour of PC Sleeper

Most of the static behavior of PC sleeper is on the basis of the static sectional capacity depending on designed strength, and ductility nature of itself. Theoretical concepts of strength, ductility, stability and fracture refer to static behaviour of prestressed concrete sleeper. The static load of a sleeper is the load undertaken by the sleeper when a vehicle axle at standstill is situated exactly above the location of the sleeper. The static load is further analyzed into individual component loads; which represents the mean weight of the train vehicles, unchanged over a reasonably long period of time. The load transfer mechanism from the rail to the sleeper and then to the underlying layer affects the static behaviour of sleeper (Kaewunruen, 2006).

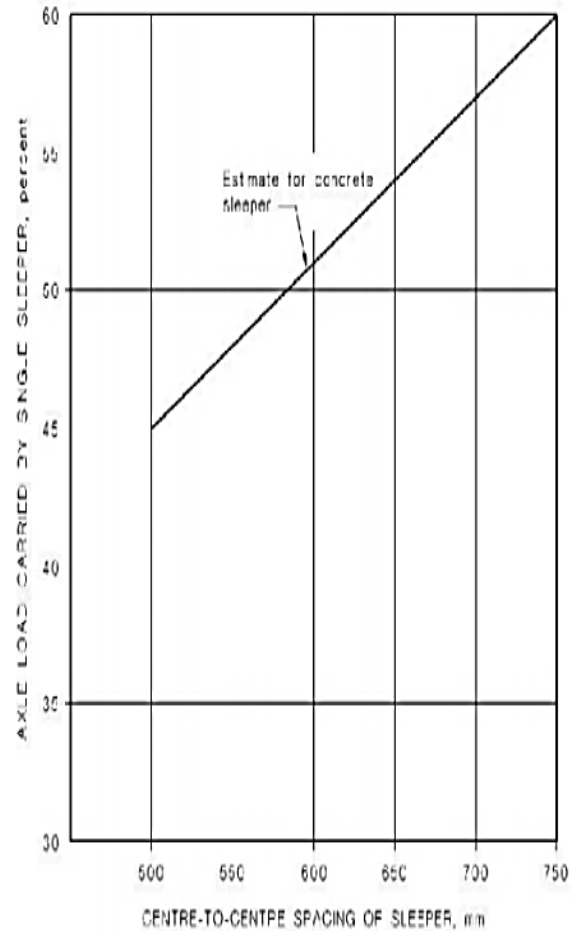
2.4.2.1.1. Rail-Seat Load

The AREMA (2010) and AS 1085.14 (2003) manuals present methods for sleeper to rail-seat load information with varying center-to-center crosstie spacing values, because it is believed that this spacing value will have the largest effect on track deflections under constant sleeper, ballast, and subgrade conditions.

Increasing sleeper spacing not only lowers the overall value of track modulus for a given section, but also places a higher stress on individual sleepers. Figure 2.4, is an estimated distribution of loads for concrete sleeper track. The purpose of this chart is to determine the percentage of an axle load that a single sleeper must carry under varying sleeper spacing. The rail seat load on individual sleeper is dependent on sleeper spacing, fastening system, rail stiffness, and ballast and sub-grade conditions with sleeper spacing having the largest effect. Typical track design with concrete sleeper utilizes sleeper spacing of 60cm which correlates with 50% of the applied axle load being carried by an individual sleeper, (AREMA, 2010 & AS 1085.14, 2003).



a) AREMA, 2010



b) AS 1085.14-2003

Figure 2.4: Estimated distribution of loads

2.4.2.1.2. Sleeper Support Load

Static load on a sleeper increases the bond with the ballast to stiffen the whole system. The exact contact pressure distribution between the sleeper, and the ballast and its variation with time, will be of importance in the structural design of sleepers. When track is freshly tamped the contact area between the sleeper, and the ballast occurs below each rail-seat. After the tracks have been in service the contact pressure distribution between the sleeper and the ballast tends towards a uniform pressure distribution. This condition is associated

with a gap between the sleeper, and the ballast surface below the rail seat. The most accepted contact pressure distribution patterns between sleeper and ballast are presented in Figure 2.5 (J.Sadeghi, 2012).

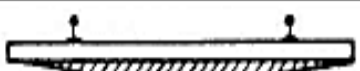
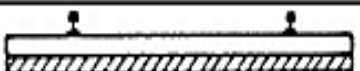
Pressure Distribution	Remarks
	Laboratory test
	Principal bearing on rails
	Tamped either side of rail
	Maximum intensity in middle
	Uniform pressure

Figure 2.5: Contact pressure distribution patterns (J. Sadeghi, 2012)

2.4.2.2. Dynamic Behaviour of PC Sleeper

The transient dynamic loading is the dynamic loading that occurs over a very short period of time. The causes of transient loading are usually from collisions, sudden loading, shock, etc. Particularly in modern railway tracks, the transient loading is usually caused by the abnormalities in wheel/rail interaction. The magnitude of the forces is very high within the very short impulse duration. The effects of impact forces are very significant in the design and utilization of concrete sleepers as parts of the railway track structures (Remennikov, 2011).

The energy from impact loads between wheel and rail is absorbed by track component and train. Rail pads, sleepers and ballast bed play an extreme role in energy dissipation behaviour of the track. The concrete sleepers dissipate the imparted energy by vibration,

deformation and fracture. The energy absorption mechanism of PCSs under dynamic loading has been investigated by Kaewunruen and Remennikov (2011) using the impact testing machine. Kaewunruen and Remennikov (2011) have imparted impact forces to the specimens and measured the accelerations and displacements. They concluded that the energy balance theory can be applied to concrete sleepers and railway track. The results indicate that around 45% of the impact energy would be absorbed by the concrete sleeper as bending curvature and fracture. The remaining portion would accelerate the cracks in sleepers, break the ballast gravel, degrade the rail pad and damage the wheel.

2.5. Loading of Railway PC Sleepers

The whole railway track is subjected to static and dynamic load, which is caused by train transportation. The design static wheel load per rail-seat for standard railway tracks is 125kN. But it should be noted that the railway tracks often suffer from extreme loading conditions. The extreme loading is attributed to the wheel and rail abnormalities, which are, for example, the flat wheels, wheel corrugation, out-of-round wheels, dipped rails, etc. These defects can cause loading of a very high magnitude but short duration and the occurrence of such loading is of low probability during the design life of the sleeper. The magnitude of the dynamic impact loads per wheel seat over individual sleeper varies from 200kN up to 750kN. These forces may cause cracking and failure of the sleeper (J.Sykorova, 2012).

2.6. Load Assessment on PC Sleepers

As described by Taylor (1993), the American Railway Engineering Association (AREA) and the American Society of Civil Engineers (ASCE) had set up special committees under the chairmanship of Professor A.N. Talbot in 1913 to initiate a scientific study on the deformations and stresses in railway track. A consistent research between 1918 and 1940 has produced seven important reports through the development of data and the establishment of procedures for laboratory tests and track trials.

The Talbot Committee had used fixed reference locations to measure the rail displacement beneath different wheel configuration - single wheels and pairs of wheels of normal passenger coach and complete locos. Valuable data on the distribution of wheel loads along the rail to the adjacent sleepers was obtained. The Talbot Committee also measured the sleeper's deformation to determine the ballast's bearing pressure distributions on sleeper soffits and at depth.

Laboratory test of sleeper on ballast bed was carried out by measuring the pressure on the ballast through embedment of pressure capsules in the ballast. Then, the scientific study by the Talbot Committee went on to consider stresses in rails and joints as well as determining the effect of flat spots on wheels and the consequential imposed stresses on rails.

Johansen (1944) described the work by the Research Department of London Midland and Scottish Railway on reinforced concrete sleepers laid at Cheddington main line from 1942 as well as timber sleepers. The experiments were conducted by fitting the strain gauges on different position of the sleepers and placed in track for monitoring.

One of the conclusions recognizes ballast as support to the sleeper is the most influential variable in affecting sleeper stresses. It appears that sleeper stresses are linearly correlated in general with load and speed under identical sleeper design, type and spacing. It also appears that concrete sleepers had three times as much variation in stresses as compared to wood sleepers. This is due to the greater stiffness of the concrete sleepers and ductility of the wooden sleeper to flex to the contour of the ballast surface to the extent that any increase in a given load would cause relatively further increase in stress of the wooden sleeper (Thomas, 1944).

Further tests were carried out at Cheddington by Thomas (1944). The focus was to study the forces to which a concrete sleeper is subjected rather than the effects of these forces. This includes investigation of the chair reaction and the distribution of ballast pressure reaction under the sleeper using load cells and 'ball sandwich' respectively.

'Ball sandwich' is formed by laying out ball bearings regularly between two separated metal plates. It was fixed to the soffit of the sleeper to be in contact with the ballast. The ballast

pressure distribution was estimated by measuring the width of the balls indentation into the outer steel leaves of the sandwich and used with the information from the calibration. It appears that in any particular day, there was usually no great variation between a sleeper and the other. However, the local variation in the intensity of the ballast's bearing pressure distribution underneath a sleeper was considerable. This is due to the slight rearrangement of the ballast which leads to points of high pressure as the sleeper lifts a little on the approach of the train. Although the ballast aggregate that causing the local pressure increase were soon pushed down, others were continually being displaced.

Thomas (1944) also suggested that the method of packing the ballast beneath the sleeper contribute to the difference in the ballast's bearing pressure distribution. It is interesting to note that ballast's bearing pressure distribution in the middle of the sleeper was occasionally observed due to the initial conditions of the ballast during laying of the sleeper. As time from laying increases, the ballast's bearing pressure distribution in the center of the sleeper was slowly reduced to zero as the continual packing under the rails lead to transfer of the ballast' bearing pressure distribution nearer to the rail.

Sadeghi (2005) has developed a finite element model to investigate the stress distribution patterns under the sleeper and the deflection of the sleeper. Spring supports that represent the ballast stiffness have been adopted in the finite element modeling.

After decades of research on understanding the interaction between sleeper and ballast, Sadeghi (2005) compiled the work of researchers and international standards on the proposed hypothetical bearing pressure distribution under railway sleepers as shown in Table 2.1.

Table 2.1: Hypothetical bearing pressure distribution of sleeper (Sadeghi, 2005)

Item No.	Distribution of bearing pressure	Developers	Remarks
1		ORE, Talbot	Laboratory test
2		ORE, Talbot, Bartlett Clarke	Tamped either side of rail
3		ORE, Talbot	Principal bearing on rails
4		ORE, Talbot	Maximum intensity at the ends
5		Talbot	Maximum intensity in the middle
6		Talbot	Center bound
7		Talbot	Flexure of sleeper produces variations form
8		ORE, Talbot, Kerr, Schramm	Well tamped sides
9		ORE, Talbot	Stabilized rail seat and sides
10		AREA, Raymond, Talbot	Uniform pressure

2.7. Concept of Sleeper Design

2.7.1. Current Practice to Design Sleeper

Zakeri and Sadeghi (2007) summarize four main steps applied in current practices of the analysis and design of railway sleepers.

First, the design vertical wheel load is considered. This is a product of static wheel load and combined vertical design load factor of not less than 2.5. The combined vertical design load factor is established to make allowance for the effects of static load at speed and the effects of dynamic load in addition to the static wheel load (AS 1085.14-2003). As mentioned by Kaewunruen and Remennikov (2006), the influence of the dead load for prestressed concrete sleepers is negligible and the design vertical load can be expressed by wheel load alone.

Second, the load transferred to the sleepers from the rails is defined as a percentage of the design vertical wheel load. This percentage is linearly correlated with the center-to-center spacing of the sleeper. AS 1085.14-2003 recommended for rails of equal to or heavier than 47 kg/m, center-to-center spacing of the sleeper between 500 mm and 750 mm is used and the distribution factor to be adopted on a single sleeper are respectively ranging from 45 to 60 percent of the rail load. Product of design vertical wheel load and the distribution factor is the design rail seat load.

Third, pressure distribution pattern of the ballast beneath the sleeper is considered. AS 1085.14, (2003) considers uniform pressure distribution beneath each rail seat which is dependent on the track gauge and sleeper length. However, the pressure distribution under the sleeper is very dependent on the condition of the sleeper-ballast interaction.

Fourth, after assuming loading pattern on rail-sleeper interaction and pressure distribution on sleeper-ballast interaction, bending moment at the rail seat and at the center of the sleeper are calculated. Thus, capacity of the sleeper will be designed according to this calculated bending moment, which is resulted from the action of vertical static load from the rail onto the sleeper.

It is to be noted that only flexural stresses is checked through calculation if the design is complied with all clauses in AS 1085.14 (2003), specifically related to shape and dimension of the prestressed concrete sleeper. However, prestressed concrete sleepers capacity to carry shear and principal tension could be measured by carrying out load test.

In addition to its static load capacity, the performance of sleepers is also measured in terms of its capability to withstand lateral and longitudinal loading. The contributing factors to these include the sleeper size, shape, surface geometry, weight and spacing (Doyle, 1980).

2.7.2. Limit State Design of Sleeper

The parameters in limit design that need to be addressed to ensure satisfactory margins of safety for the elements in-service are illustrated in the well-known limit state of strength design equation;

$$\phi R_u \geq R_d \quad (2.1)$$

Where:- $R_d = \sum LF_i \times L_i$

ϕ = the capacity reduction factor,

R_d = the factored applied design load,

LF_i = load factor applied to L_i type of load.

L_i = Applied characteristic loads

To apply Equation 2.1 to the design of sleepers, one must determine the parameters on the strength side of the equation, namely R_u and the associated reduction factor (ϕ), and on the right side of the equation one must determine the type and magnitude of the loads sustained by the sleeper and the factors to be applied to those loads (Murray, 2012).

According to Martin H. Murray (2012) limit states mainly have two aspects:

2.7.2.1. Ultimate Limit State

The ultimate limit state is caused by a single once-off event such as a severe wheel flat that generates an impulsive load capable of failing a single concrete sleeper. Failure under such a severe event would fit within failure definitions causing severe cracking at the rail seat or at the mid-span. The single once-off event will be based on the probabilistic analysis of train load spectrums recorded over several years or for a suitable period (generally at least a year as to obtain the good representative of track forces over its lifetime under various train/track operational conditions). The load magnitude for ultimate limit state design of sleepers depends on the significance or importance level of the railway track and statistical operational service data.

2.7.2 .2. Serviceability Limit State

This limit state defines a condition where sleeper failure is beginning to impose some restrictions or tolerances on the operational capacity of the track, for example, prestressing losses, sleeper deformations (shortening and camber), track stiffness, etc. The

failure of a single sleeper (in track system) is rarely if ever a cause of a speed restriction or a line closure. However, when there is failure of a cluster of sleepers, an operational restriction is usually applied until the problem is rectified. Recently, this serviceability limit state has extensively applied to the methodology for retrofit and replacement of sleepers made of different material properties in the existing aged track systems.

2.8. Structural Reliability

As discussed by S.Kaewunruen (2012), the errors and uncertainties involved the estimation of the loading action and the capacity of a structure may be allowed for in strength design using load factors to increase the nominal loads and using capacity factors to decrease the available structural strength. The purpose of using any factors is to ensure that the probability of failure under the most adverse conditions of structural overload remains very small, which may be implicit or explicit in the rules written in a code. In outdated structural design codes that employed the traditional working stress design (e.g. AS 1250-1981 Steel Structures, and in the current AS1085.14 sleeper code, safe design was achieved using factors of safety to reduce the failure stress to permissible working stress values, but ultimately the purpose was to limit the likelihood of failure under normal services.

AS1085.14 (2003) prescribes the maximum allowed stresses, which are expressions of the ultimate strengths of isolated members divided by the safety factors (SFs).

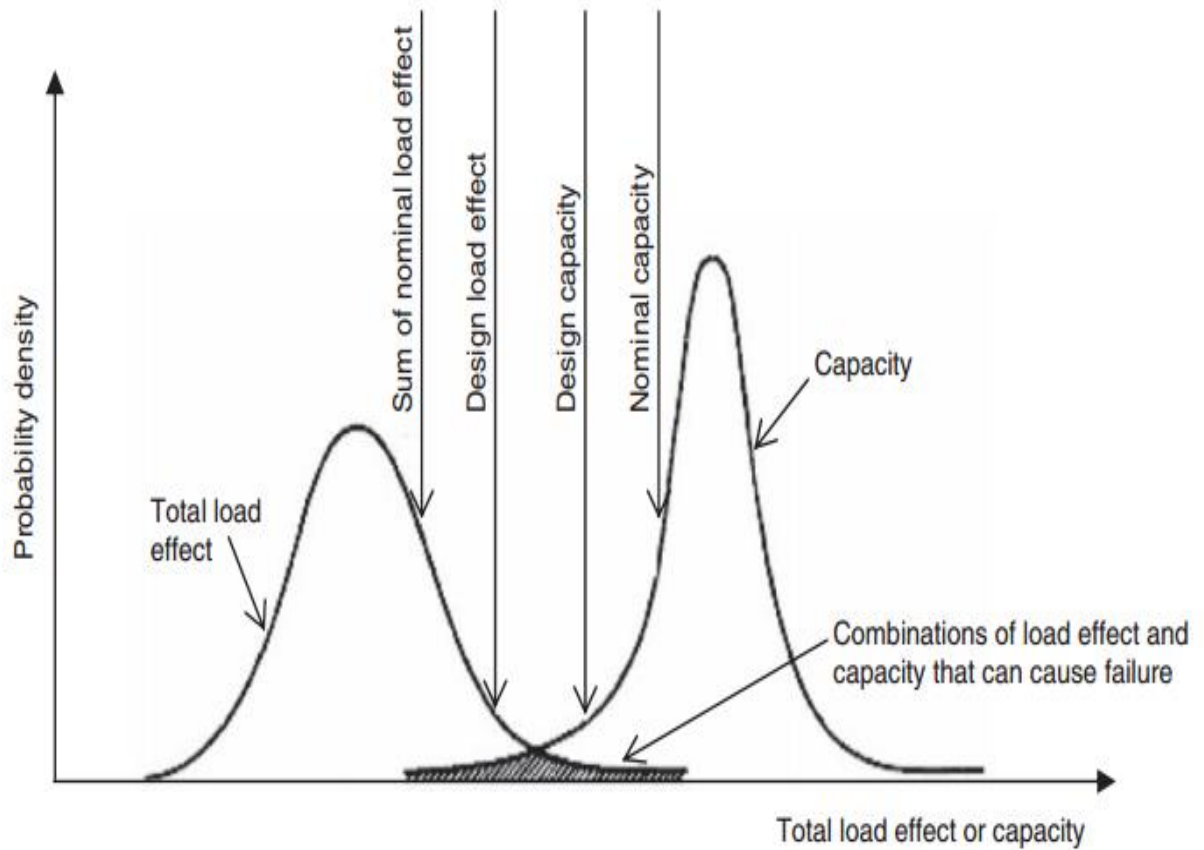


Figure 2.6: Frequency-Load Reliability Diagrams (S.Kaewunruen, 2012))

In Australia, all structural design codes except AS1085.14 have been amended to a limit state design approach. Limit state deems that the strength of a structure is satisfactory if its calculated nominal capacity (resistance), reduced by an appropriate capacity factor (ϕ), exceeds the sum of the nominal load effects multiplied by various load factors γ , so that.

$$\sum(\gamma \times (\text{nominal load effects})) \leq \phi \times \text{nominal capacity} \quad (2.2)$$

Where the nominal load effects are the appropriate bending moments, axial forces, or shear forces, determined from the nominal applied loads by an appropriate method of structural analysis (static or dynamic). Even though the limit states are described in a deterministic form, the load and capacity factors involved are usually derived from probabilistic models based on statistical distributions of the loads and the capacities, as depicted in Fig. 2.7. The probability of failure P_F is indicated by the region for which the load distribution exceeds

that for the structural capacity. In limit state codes, the probability of failure P_F is usually related to a parameter, safety index or reliability index, by the transformation (Zakeri, 2007).

$$\Phi(-\beta) = P_F \quad (2.3)$$

Where: Φ is the standard normal distribution (zero mean and unit variance),

P_F is probability of failure

β is reliability index

The relationship between reliability index and probability of failure (P_F) shown in Fig. 2.7 indicates that an increase of 0.5 in implies a decrease in the probability of failure by approximately one order of magnitude.

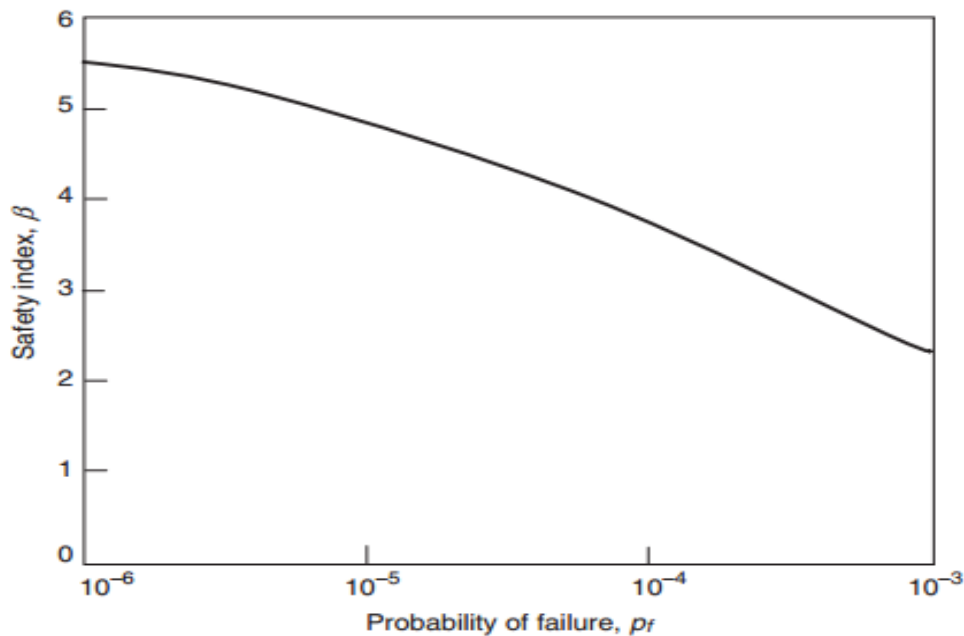


Figure 2.7: Relationship of safety index and probability of failure (Sadeghi, 2007)

After getting load related data from field, previous studies, and laboratory test can be evaluate reliability index by reliability analysis software. Such as:- Comrel, Systrel, Costrel, and strurel. For conducting this research work Comrel V9.0 would adopt; it is suitable for time invariant and time variant data.

CHAPTER THREE

3. RELIABILITY ASSESSMENT OF PC SLEEPER

This chapter presents reliability analysis procedure. Its main objective is to determine necessary parameters that can be used for adopting limit state design approach for PC sleepers. In order to achieve this, obtaining suitable combination of static and dynamic loads is necessary. This helps evaluate the ability of systems or components to remain safe and operational during their lifecycle. For this research work; statistical properties of related parameters are adopted from previous studies, standards, and local conditions such as: axle load, sleeper spacing, dynamic load, distribution factor, etc.

In the reliability analysis of the pre-stressed concrete sleepers, the maximum stress is considered to be present in either the rail seat or the mid-span respectively. These are the areas of the sleeper where the maximum positive and negative bending moments are exist. In this chapter, the relevant functions are formulated taking into account the load factors, safety index, and strength reduction factor to find reliable design load of sleeper. The effects of static and dynamic loads in determining the maximum positive and negative bending moment at these two locations has been considered.

The structural reliability theory is concerned with rational treatment of uncertainties in structures, and with the method for assessing the safety, and serviceability in structures. Uncertainties arise from variations in loads, material properties, dimensions, natural and manmade hazard, insufficient knowledge, and human errors in design and construction. The uncertain nature of the data makes structural reliability theory a logical and powerful tool for performing quantitative performance assessments of existing structures. Moreover, probability-based limit states analysis provides a clear link between theoretical research, and in-service experience, and also provides a theoretical basis for utilizing in situ data in structural evaluation process. (Naiyu Wang, 2010).

As mentioned in the work of Emilio (2015), reliability methods have been established to take into account, in a rigorous manner, the uncertainties involved in the analysis of an engineering problem. The failure probability and the reliability index are used to quantify

risks and therefore evaluate the consequences of failure. In this approach, the governing parameters of the problem are modeled as random variables. Random variables can be grouped in a random vector X where $f_X(x)$ is the joint probability density function (PDF). Probabilistic concepts are used in the reliability analysis and reliability based design of structures. Using the principles of structural reliability, the level of reliability of an existing structure, which is designed as per the existing structural standards, can also be evaluated. It can also be used for developing a reliability based design criteria, in the form of code calibration to compute the partial safety factors for an accepted level of reliability.

The overall aim of structural reliability analysis is to quantify the reliability of structures under consideration of the uncertainties associated with the resistances and loads. The structural performance is assessed by means of models based on physical understanding and empirical data. Due to idealizations, inherent physical uncertainties and inadequate or insufficient data the models themselves and the parameters entering the models such as material parameters and load characteristics are uncertain. Structural reliability theory takes basis in the probabilistic modeling of these uncertainties and provides methods for the quantification of the probability that the structures do not fulfil the performance criteria.

In the simplest case, the performance function $G(X)$ is expressed as the difference between the ultimate capacity (resistance) $R(X)$ and the demand on the system $S(X)$ i.e (Emilio, 2015).

$$G(x) = R(x) - S(x) \quad (3.1)$$

The failure probability (P_f), is determined by:

$$P_f = P[G(X) \leq 0] = \int_{G(X) \leq 0} f_X(x) dx_1 \dots dx_n \quad (3.2)$$

Geometrically, the limit state equation is an n -dimensional space. One side of the failure surface is the safe state, $G(X) > 0$ whereas the other side of the failure surface is the failure state, $G(X) < 0$.

$$G(x_1, x_2, \dots, x_n) = \begin{cases} > 0 \text{ safe state} \\ 0 \text{ limit state} \\ < 0 \text{ failure state} \end{cases}$$

In general, solving reliability integrals equation is a complicated process, and can be done in closed form only for a simple case. Therefore, some alternative methods are needed in order to evaluate the probability of failure. These methods can be either analytical or numerical. Numerical method evaluation of reliability index is directly through simulations, such as COMREL, as it is the case for this research.

3.1. Reliability Analysis

To obtain the reliability index of the sleepers, the first step is to accurately define the limit state function as well as all the corresponding variables. Also, the statistical properties that influence the limit state function need to be determined. Therefore, before doing analyses to calculate reliability index, the nominal flexural capacity part of the limit state function had to be derived.

As expressed in equation 3.1 the limit state function can be shown as.

$$G = R - (Q + S) \quad (3.3)$$

Where:-

G- is the difference of sleeper ultimate load capacity, and static and dynamic load

R - Sleeper ultimate load capacity

S - Static load, and

Q - Dynamic load

To perform calibration using reliability analysis, the mean, standard deviation, and coefficient of variation (COV) as well as the type of distribution that best fits the data (i.e.,

normal, lognormal, Weibull, etc.) must be determined for each data to considered in the limit state function.

Mean, standard deviation, and coefficient of variation of the data are the most important points for analyzing of safety index by COMREL software. They are calculated from developed (collected) data by using the formulas as stated below.

Mean:- Mean is the average of a set of numbers in a collected data. To find the mean of a data set, the summation of all values is dividing by the number of values in the set.

$$X_m = \frac{\sum X_i}{N} \quad (3.4)$$

Standard deviation:- Standard deviation is a measure of the dispersion of a set of data from its mean. If the data points are further from the mean, there is higher deviation within the data set. It is calculated as the square root of variance by determining the variation between each data point relative to the mean.

$$\delta = \sqrt{\frac{\sum (X_i - X_{i(\text{mean})})^2}{N+1}} \quad (3.5)$$

Coefficient of variation:- The coefficient of variation is a measure of spread that describes the amount of variability relative to the mean. It can be used in place of standard deviation to compare the spread of data sets that have different units or different means; which is the ratio of standard deviation of data by mean of the data

$$V_i = \frac{\delta}{X_m} \quad (3.6)$$

Where: V_i is coefficient of variation

δ is standard deviation

x_m is mean of the data

3.1.1. Ultimate Capacity of Sleeper

To evaluate the performance of prestressed concrete (PC) sleepers, an experimental work has been carried out by Turkish company (Yapiray railway construction systems industry and Trading Inc., on Awash –Woldiya railway project) at Combolcha sleeper production site. A series of positive load tests at rail-seat section was performed by this company in order to assess ultimate capacity values of produced sleeper. For this research; the tested results as shown in table 3.1; can be used as an input to determine safety index, and resistance factors for ultimate limit state design of PC sleeper.

Table 3.1: Sleeper ultimate load capacity lab taste results

Teste No	1	2	3	4	5	6	7	8	9
Load(kN)	266.9	178.1	245.2	244.6	281.2	237.3	203.4	189.8	203.5
Teste No	10	11	12	13	14	15	16	17	18
Load(kN)	207.9	178.2	188.2	254.3	180.2	283.4	208.2	202	226.9

Based on the above table and equation 3.4-3.6; mean, standard deviation, and coefficient of variation value of sleeper performance data are 221.07, 35.04, and 0.16 respectively.

3.1.2. Static and Dynamic Loads on the Sleeper

After calculating of wheel load; distribution of load is based on the knowledge that wheel loads applied to the rail will be distributed by the rail to several sleepers. The distribution of load is depends upon the sleeper properties, spacing, ballast and subgrade reaction, rail fastening system and rail stiffness. According to AS 1085.14 (2003) the percentage load transferring of 50% is consider for the sleeper directly below the applied point of wheel load with corresponding sleeper spacing of 0.6m for design of PC.

3.1.2.1. Static Load

The load that comes from weight of train is considered as static load over the railway track. The value of static load related parameters are taken from ERC feasibility study and previous studies as shown Table 3.2 below.

Table 3.2: Statistical parameters of static load

Parameters	Unit	Value	Source
Axle load	kN	250	ERC feasibility study
Distribution factor	%	50	AS 1085.14 2003
Wheel Load	kN	125	50% of axle load
Rail-seat load	kN	62.5	50% of wheel load
Coefficient of variation	%	25	Sakdirat K., 2007
Standard deviation	kN	15.63	$0.25 \times 62.5 = 15.63$

3.1.2.2. Dynamic load

The most severe impact forces are most likely from wheel flats, because such flats strike the rail head every revolution of the wheel, and severe flats have the potential to cause damage of railway track.

Australian Railway Association (2002); prescribes a maximum allowed impact force of 230kN to be applied to the rail head by passing train wheels. Despite the code of practice requirements, there is little published data able to be found showing the actual range, and peak values of impact for normal operation of trains. Therefore, the value of 230kN is the desired upper limit rather than a measure of real maximum forces encountered on track (Sakdirat K., 2007).

Table 3.3: Statistical parameters of dynamic load

Parameters	Unit	Value	Source
Dynamic wheel load	kN	230	Sakdirat K., 2007
Distribution factor	%	50	AS 1085.14, 2003
Rail-seat load	kN	115	50% of wheel load
Coefficient of variation	%	25	Sakdirat K., 2007
Standard deviation	kN	28.75	$0.25 \times 115 = 28.75$

3.1.3. Reliability Index Analysis

Method 1:- Incorporating of the above mentioned, and calculated parameters can carry out simulation using COMREL V9.0 (it was mentioned at the end of section 2.8.) by setting those mentioned load related parameters in the symbolic expression tab under job control window. This is the most well-known, and simple to apply kind of software for structural reliability analysis. Reliability index analysis from the package, can calculate by accounting the means, standard deviations, and distributions in stochastic model tab. Under licensed version of this software; plentiful functions are found for analysis of reliability related parameters. For this study, reliability index is the necessary and the most expected output value from the software for computation of resistance factor for the purpose of adopting recent approach.

Method 2:- mathematical calculation of reliability index can be estimated using equation 3.7 based on works of Nowak and Collins (2000).

$$\beta = \frac{X_R - \sum X_Q}{\sqrt{\delta_r^2 + \sum \delta_q^2}} \quad (3.7)$$

Where: X_R = mean of capacity (resistance) load

$\sum X_Q$ = summation of mean of static and dynamic load

δ_r = standard deviation for capacity load

$\sum \delta_q$ = summation of standard deviation of static and dynamic load

Then, reliability index can be calculated as follows using equation 3.7

$$\beta = \frac{221.07 - (115 + 62.5)}{\sqrt{35.04^2 + 15.63^2 + 28.75^2}} = 0.909$$

As shown from software result of COMREL software (depict in Appendix A), and mathematical calculation, the value of safety index is 0.909.

Establishment of reliability index value (β) for a given limit state, and structure component being designed depends on the redundancy inherent in the system. At present, no specific guidance is available for the selection of β for design of concrete structure but it must not be less than 1. The final selection of target reliability index (β_T) value to use for a given limit state must take into account the range of β values implied by past successful design and construction practice, and consistency with the β_T values used for design of structures in general, with consideration of the redundancy inherent in the structural. As experience is gained in the application of LRFD to design, the role of past successful allowable stress design practice will become less important, and consistency with the value of β used for structural design in general will become more important, lessening the need for the application of judgment to make the final selection of the β_T (Richard, 2005).

As described from the work of MacGregor et al. (1997) (H.Murray, 2012); the value of target reliability index (β_T) is 4.0 for structures that have serious consequences of failure, such as individual concrete bridge girders.

The value of β that found from reliability analysis software COMREL, and mathematical equation of 3.7 from the given data is less target reliability index value. Sleepers are providing services for moving loads on it like bridge girders. For this study take the specified target reliability index of bridge girder (β_T) = 4 for calculation of resistance factor.

3.2. Load Factor

For design of PC sleeper by concept of structural reliability work in order to achieve the objective of this research work; its prestressed performance when subjected to static and impact loading can be assessed by considering vertical components of those types of loads for ultimate limit state design. The magnitude and frequency content of these two types of forces act independently of each other, because they have different origins.

From equation 2.1; limit states design equation of concrete sleeper can be written as below.

$$\phi R_u \geq R_d = k_s S + k_d Q \quad (3.8)$$

Where: R_d - design rail-seat load

k_s - static load factor

S - characteristics component of static load,

k_d - dynamic load factor, and

Q - characteristics component of impact load.

R_u - resistance load

ϕ - Reduction factor

According to work of Richard (2005), before beginning the final calibration process, an estimate of the load factor should be made. There are many combinations of load and resistance factors that will yield the desired probability of failure P_f , or reliability index β , for a given limit state and set of statistics for the random variables involved. It is desirable to set the load factors so that they are greater than 1.0 and the resistance factors are less than 1.0 for LRFD.

The following equation can be used as a starting point to estimate the load factor:

$$k_n = \delta_R(1 + n_\sigma V_i) \quad (3.9)$$

Where:-

k_n = the load factor;

V_i = Coefficient of variation

n_σ = probability of exceedance. In the development of the Ontario Highway Bridge Design Code and AASHTO LRFD Bridge Design specifications, $n_\sigma = 2$ for strength limit state.

δ_R = bias factor. Bias coefficient is simply the mean of the distribution divided by nominal value, as expressed below.

$$\delta_R = \frac{X_{i(\text{mean})}}{X_n} \quad (3.10)$$

As suggested by ACI 318 Committee (2005) (J.Bian, 2012); nominal load (X_n) can be estimated using equation 3.11 from relationship of nominal load, and mean load as shown below.

$$X_n = X_m + 1.65\delta \quad (3.11)$$

Where:-

X_n = nominal load

X_m = mean load

δ = Standard deviation

3.2.1. Static Load Factor

The static component of vertical design load can be determined directly from the static axle load of trains. Which is a source of static bending moment of sleeper, but its effect is small as compared from dynamic load when the drop height of wheel is increase. Static load factor can be calculated as follows by using equation 3.9 based on the data presented in section 3.1.2.1.

Using equation 3.11, nominal load for static load (X_n) = $62.5 + 1.65 \times 15.63 = 88.29$

Bias coefficient (δ_R) = $62.5 / 88.29 = 0.708$

Static load coefficient (k_s) = $0.708(1 + 2 \times 0.25) = 1.06$

3.2.2. Dynamic Load Factor

Variable component of railway track is dynamic load that comes from the motion of train. This load component is an important part for design life of sleeper, and it needs further study to determine the exact value of its impact on railway system as whole. Use equation 3.9 to determine the load factors of dynamic load as shown below.

Using equation 3.11, nominal load for dynamic load (X_n) = $115 + 1.65 \times 28.75 = 162.44$

Bias coefficient (δ_R) = $115/162.44 = 0.708$

Dynamic load coefficient (k_d) = $0.708(1 + 2 \times 0.25) = 1.06$

3.3. Strength Reduction Factor

Design codes propose to restrict the nominal probability of failure within specific target structural reliability levels using a load factor, and strength reduction factor. Once the load factor and reliability index is evaluated, the resistance factor can be estimated by using equation 3.12. In the current ACI318 Code (2005), the strength reduction factor varies from 0.65 to 0.90. As seen from resistance formula, this is related to the reliability index and ultimate properties of structure.

Resistance factor can be estimated as expressed below based on AASHTO, LRFD (2007).

$$\phi = \frac{R_m}{R_n} e^{-0.55\beta V} \quad (3.12)$$

Where: R_m = Mean value of resistance load

R_n = Resistance nominal load

β = Reliability index

V = Coefficient of variation of resistance

$R_n = 221.07 + 1.65 \times 35.04 = 278.89 \text{ kN}$

Then, resistance factor (ϕ) = $\frac{221.07}{278.89} e^{-0.55 \times 4 \times 0.16} = 0.759 > 0.65$, and < 0.9 ok!

CHAPTER FOUR

4. DESIGN OF PRESTRESSED CONCRETE SLEEPER

Currently, many organizations (e.g. American Railway Engineering and Maintenance-of-Way Association ((AREMA, 2010), Australia Standards (AS 1085.14, 2003), European standard (EN 13230, 2015), and International Union of Railways (UIC, 2005)) address the design methodology of railway sleepers. Most of them prescribe a design methodology for concrete sleepers using permissible stress design concept.

Limit states design concept has been adopted for structural concrete worldwide; its use in prestressed concrete sleepers is limited. The proposed methodology for the calculation of the design rail-seat load, and the design approach of the ultimate limit states concept for concrete sleeper strength is concurrence with generic design standards for concrete structures of European codes.

As mentioned in the work of S.Kaewunruen (2008); there are three main steps in designing the concrete sleepers on the basis of the new limit states design concept:

1. Determination of design load (P_d)
2. Analysis of design moment
3. Structural design and optimization of concrete sleeper.

For good performance of sleepers to fulfill the functions (presented in section 2.2) an ideal sleeper should possess the following characteristics:

- i. The sleepers to be used should be economical, i.e., they should have minimum possible initial and maintenance costs.
- ii. The weight of sleepers should not be too heavy or excessively light, i.e., they should have moderate weight, for ease of handling.
- iii. The design of sleepers should be such that the gauge, alignment of track and levels of the rails can be easily adjusted and maintained.
- iv. The sleeper design should be such as to facilitate easy removal and replacement of ballast.

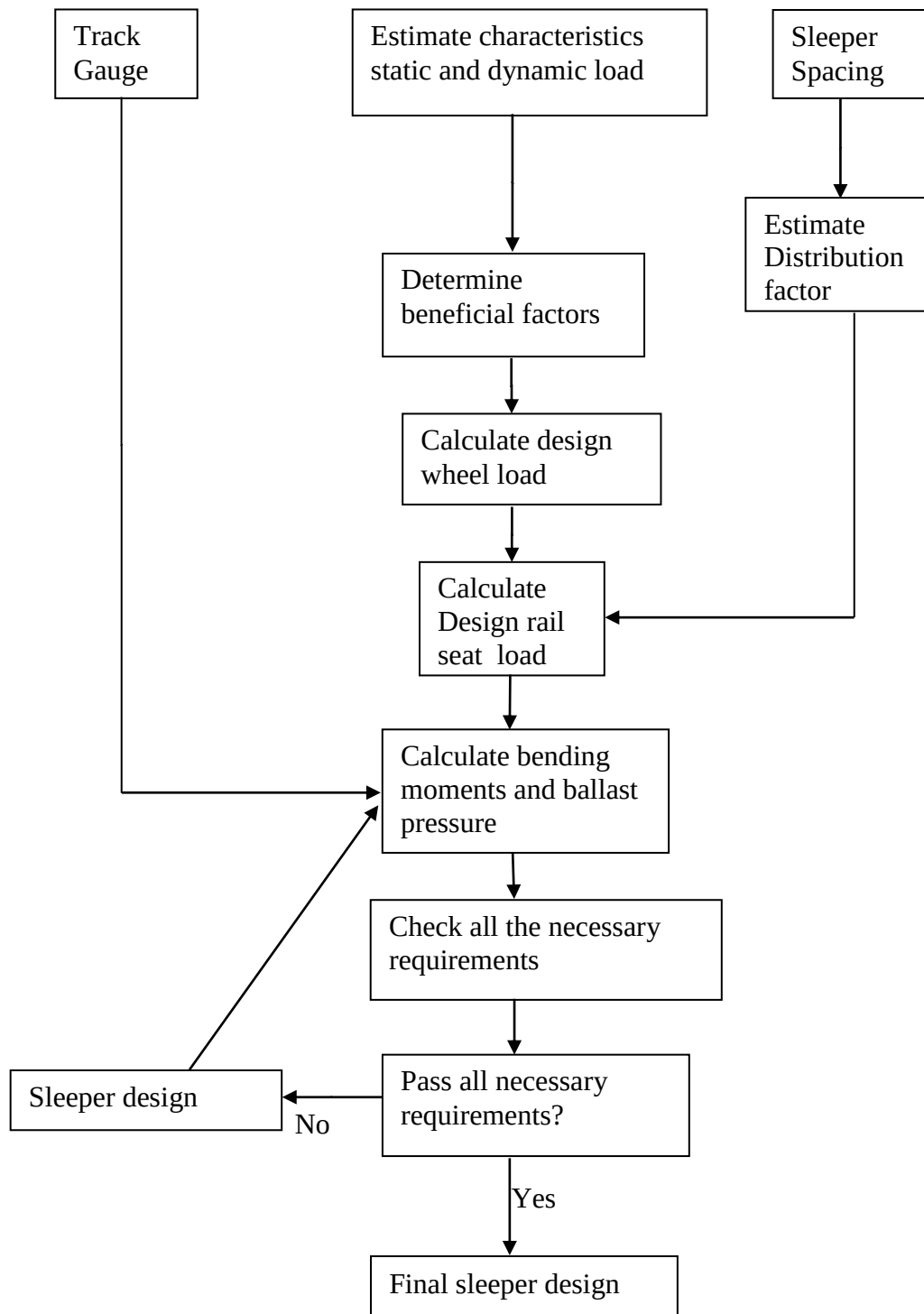


Figure 4.1: Chart flow design PC sleeper by concept of limit state

4.1. Design Forces Acting on the Sleeper

In terms of the standard design of prestressed concrete sleeper, action forces (wheel load & impact force), and ballast pressure are primary basic variables.

4.1.1. Rail-seat Load

Railway standards have their own assumptions to find rail-seat load to design economical dimension of PC sleeper from their own country perspective. For the purposes of this research, EN 13230 (2015) is used to determine the rail-seat load for design of PC sleeper by limit state approach. In the limit states method, the design loads are calculated from the characteristic loads by multiplying them with load factor. For design of sleeper, especially in case of this study, two types of loads are considered to act together under selected load combination.

The demands in a member for these two types of loads are obtained from weight of train and motion of train due to effect of wheel flange, and sleeper subjected to the characteristic value of the loads. The demands are then combined under the load combinations based on the principle of superposition. This load combination has the appropriate weighed load factors as explained in chapter three, section 3.2. Once the forces are combined under the load combinations, the total force demand for design of sleeper member using ultimate limit state approach can be known. The member can be designed either for the service loads or for the ultimate loads. This study encompasses design for ultimate loads only.

Limit state equation for design of PC sleeper can be proposed as obvious limit state design of concrete design by taking into account vertical component of both static (S) and dynamic (Q) rail-seat loads with corresponding load coefficient for assessment of its performance as expressed by equation 4.1

$$R_d = k_s * S + k_d * Q \quad (4.1)$$

Under this situation the value of static and dynamic load is 62.5kN (from feasibility study of ERC) and 115kN (from code of practice of Australian railway association) respectively.

$$R_d = 1.06 \times 62.5 + 1.06 \times 115 = 188.15 \text{ kN}$$

4.1.2. Ballast Pressure

According to the AREMA (2010) standard; sleeper-to-ballast pressure is not uniformly distributed across or along the bottom of a cross sleeper, however an approximate calculation can be made of average pressure at the bottom of the sleeper. The average pressure at the sleeper bottom is equal to the axle load modified by distribution and impact factors, and divided by the bearing area of the sleeper. Will briefly discuss in, section 4.3.4 about design ballast pressure; it depends on the sleeper cross section.

4.2. Parametric Optimization of Prestressed Concrete Sleeper

The new limit states concept permits a sleeper design with a reduced depth, and weight that is beneficial to any railway corridor, especially in our country Ethiopia. In addition to cost saving, the use of the new design method has a positive potential gearing to environment, and sustainability in a railway corridor over its life cycle. In this thesis work, geometry of the sleeper, concrete grade, and prestressing wire are the main inputs for optimization of the sleeper to improve the existing used sleeper of Ethiopia railway corporation (ERC) to be economical.

4.2.1. Sleeper Dimension

In a non-optimized design, the use of sleepers with unnecessary large cross-section and/or stiffness has the same effect as reducing the spacing between sleepers below the optimum amount. Their effect in reducing the track stress or increasing the truck modulus is nil. Hence, reaching the optimum sleeper dimension is vital in every sleeper design.

The length of sleeper has a great influence on the sleeper bending moment. In general, it has been observed from different literatures, the absolute value of negative bending moment at a sleeper center decreases as the sleeper's length increases. Therefore, a PC sleeper length should be determined so that the bending moment of sleeper center balances that of sleeper

right under the rail. In this paper, a case study on a single sleeper shall be carried out to a preferred length of 2.5m with varying width and height.

Sleeper height and width are necessary to increase a sleeper weight and a longitudinal and lateral resistance within ballast. However, the individual effect increases the sleeper prices due an increase in the concrete amount, and the corresponding necessary amount of ballast, therefore the overall design is prone to be uneconomical. Moreover, when the height of it increases, it becomes easy to cause a torsional failure, whereas, larger width at the rail-seat reduces ballast pressure.

In this study, twelve possible dimensions of sleeper are selected by putting bench-mark dimension of the currently used sleeper of ERC. They have different dimensions as shown Table 4.1 below (except sleeper length). After investigation of each sleeper responses by accounting design load of rail-seat, and ballast pressure; the well again in load carrying capacity, and ballast stress distribution, one has been selected as best for construction of ballasted track for our country.

Table 4.1: Possible dimension of PC sleeper

No	Length(mm)	Rail seat load (kN)	Height(mm)		Width(mm)			
					Real-seat		Center	
			Rail seat	Center	base	top	base	top
1	2500	188.15	200	160	280	170	250	200
2	2500	188.15	170	160	250	180	230	200
3	2500	188.15	175	150	300	180	250	190
4	2500	188.15	220	220	220	220	220	220
5	2500	188.15	180	160	280	170	250	200
6	2500	188.15	180	160	280	180	240	190
7	2500	188.15	200	160	254	175	230	170
8	2500	188.15	200	160	270	175	240	180
9	2500	188.15	190	160	254	175	230	190
10	2500	188.15	220	175	254	180	230	190
11	2500	188.15	200	160	261	185	240	195
12	2500	188.15	210	170	255	180	230	190

4.2.1.1. Selection of Best Proposed Sleeper

Selection of best geometrical fit sleeper for ballasted track is based on the ballast distribution along the length of sleeper, and bending moment of the sleeper. For easy identification of the effect of ballast stress beneath the sleeper, SAFE12 software was used for this study. Analysis procedure of concrete sleeper in SAFE follows a similar manner as that of combined footing modeling.

From SAFE software analysis as shown in Appendix B, a criterion can be set for the nominal optimum sleeper dimension from the other proposed sleepers based on output results of moment, and ballast stress distribution beneath the sleeper.

As described in the work of Babae (2006) the following points are the leading factors for selection of best sleeper.

1. Position of maximum and minimum pressure beneath the sleeper

The most suitable position of the maximum ballast pressure is at the support point (around rail seat position). Hence the distance range of 40 to 70 centimeters from the end sides of the sleeper is the best place for the maximum ballast pressure to occur. The appropriate position of the minimum ballast pressure is at the center; in order to decrease negative bending moment of the sleeper. The positions of the maximum and the minimum ballast pressure in the models are presented in Appendix B.

2. Symmetric ballast stress distribution beneath the sleeper

Ballast pressure distribution along beneath of the sleeper is the leading factor for design of sleeper. However, ballast pressure distribution may not be equal or nearly equal at the same point of sleeper which has equal distance from both ends. In this study appropriate distribution of stresses along the length of sleeper used as the criteria to select best sleeper as compared from the other to increase safety, and service life of railway system. Ballast stress distributions of models are presented in Appendix B.

3. The maximum positive bending moment and the maximum negative bending moment

Position of occurrence with corresponding values of maximum negative and positive bending moments are regarded as the most important factors in the design of the sleeper..

4. Differences between positive bending moment and negative bending moment

Maximum positive moment, and maximum negative moment interval is the main criteria for this study for selection of best sleeper to accomplish the objective of the research. A larger difference of maximum positive bending moments and maximum negative bending moments indicates that the better condition of the sleeper.

According to the above four requirements, the more satisfying the criterion the better the sleeper dimension. From model results of finite element software SAFE, the dimensions of trail 11 fulfill the criterion as compared from the others. The section properties of selected sleeper are presented below.

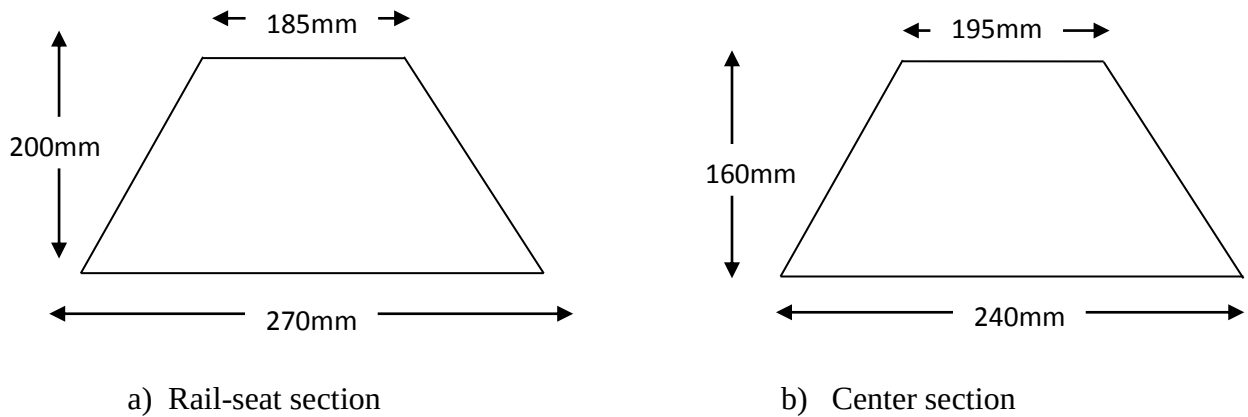


Figure 4.2: Selected sleeper cross section

At rail seat:

$$\text{Area} = A_r = 0.5 \cdot (270 + 185) \cdot 200 = 45500 \text{ mm}^2$$

$$\text{Center of gravity from bottom} = y_b = (2 \cdot 185 + 270) / (185 + 270) \cdot 200 / 3 = 93.77 \text{ mm}$$

$$\text{Center of gravity from top} = y_t = (2 \cdot 270 + 185) / (270 + 185) \cdot 200 / 3 = 106.23 \text{ mm}$$

$$\text{Moment of inertia} = I_r = (200^3 / 36) \{ 185^2 + 270^2 + 4 \cdot 185 \cdot 270 \} / (185 + 270) = 149.9 \cdot 10^6 \text{ mm}^4$$

$$\text{Section modulus from bottom} = Z_b = I_r / y_b = 149.9 / 93.77 = 1.6 \cdot 10^6 \text{ mm}^3$$

$$\text{Section modulus from top} = Z_t = I_r / y_t = 149.9 / 106.23 = 1.41 \times 10^6 \text{ mm}^3$$

At center:

$$\text{Area} = A_c = 0.5 \times (240 + 195) \times 160 = 34800 \text{ mm}^2$$

$$\text{Center of gravity from bottom} = y_b = (2 \times 160 + 240) / (160 + 240) \times 160 / 3 = 74.67 \text{ mm}$$

$$\text{Center of gravity from top} = y_t = (2 \times 240 + 160) / (240 + 160) \times 160 / 3 = 85.33 \text{ mm}$$

$$\text{Moment of inertia} = I_c = (160^3 / 36) \{ 160^2 + 240^2 + 4 \times 160 \times 240 \} / (160 + 240) = 67.4 \times 10^6 \text{ mm}^4$$

$$\text{Section modulus from bottom} = Z_b = I_c / y_b = 67.4 / 74.67 = 0.902 \times 10^6 \text{ mm}^3$$

$$\text{Section modulus from top} = Z_t = I_c / y_t = 67.4 / 85.33 = 0.789 \times 10^6 \text{ mm}^3$$

4.2.2. Concrete Grade and Prestressing Wires

After selecting the best sleeper geometry; evaluation of engineering properties of different concrete grades and diameter of wires is required. This is crucial for choice of economical combination of concrete grade and number of tendons (with diameter) for suggested dimension of sleeper satisfying the allowable stresses under service loads and at transfer both for concrete and steel.

4.2.2.1. Concrete Grade

As mentioned in most of concrete design standards including EBCS the minimum grade of concrete for prestressing is 40MPa. For the purpose of this study, concrete grade within the range of 40MPa to 70MPa can be used to obtain best fit concrete grade for selected sleeper from safety and economical aspect based on the allowable value. According to EN 13230-6 (2015) the minimum concrete cover for prestressing tendons shall be 30 mm from the bottom surface, and 20 mm from the other surfaces except at the ends of sleeper.

4.2.2.2. Prestressing Wires

High strength wires are used for production of pre-tensioned sleeper in order to increase tensile capacity of the member, because prestressed concrete is influenced by the output tensile strength of steel.

In practical case 1770MPa and 1860 MPa ultimate strength are used for pre-stressing. The type of wires used for optimal design includes the EN10138-BS5896 7-wire strands having 9.3mm & 8mm diameter and 1860 MPa ultimate strength, 7mm diameter and 2060MPa ultimate strength, 3-wire strands 5.2mm and 6.5mm diameter and 1960MPa and 1860MPa ultimate strength respectively, (Desalew F. 2014).

For optimization of parameters for this study; use 7mm, 8mm, and 9.3mm diameter having 7- strands of tendon with corresponding characteristics strength of 1860MPa by varying its position over the height of sleeper.

4.2.2.3. Selection of Concrete Grade and Prestressing Wires

For selection of concrete grade and prestressing strands the following formulas are used.

The force applied to a tendon, P_{max} (i.e. the force at the active end during tensioning) (EN 1992-1-1, 2004).

$$P_{max}=0.8f_{pk}A_p \quad (4.2)$$

At transfer

The value of the initial prestress force $P_t(x)$ (at time $t= t_0$) applied to the concrete immediately after tensioning and anchoring (post-tensioning) or after transfer of prestressing (pre-tensioning) is obtained by subtracting the immediate losses from the force at tensioning P_{max} (EN 1992-1-1, 2004).

$$P_t = 0.75f_{pk}A_p \quad (4.3)$$

$$P_t \approx 0.9P_{jack} \quad (4.4)$$

At service

At a given time t and distance x (or arc length) from the active end of the tendon the prestress force $P_e(x)$ is equal to the maximum force P_{max} imposed at the active end, minus the immediate losses and the time dependent losses (EN1992-1-1, 2004l).

For this study; assume total loss due to the immediate losses and the time dependent losses of prestress is 20%.

$$P_e = 0.8 \cdot 0.8 f_{pk} A_p \quad (4.5)$$

Here: P_t is prestressing force at transfer

P_e is prestressing force at service

P_{jack} is jacking force

A_p is area of tendons

f_{pk} is characteristics strength of tendon

Allowable stresses in concrete

At transfer

Compression

$$f_{ct} = 0.6 f_{ck}(t) \quad (4.6)$$

Where: $f_{ck}(t)$ = compressive strength of concrete at time 't'. Take 7 days for fully transfer

$$= f_{cm}(t) - 8 \text{ (MPa)}, \text{ for } 3 < t < 28 \text{ days or } f_{ck}, \text{ for } t \geq 28 \text{ days}$$

Here:

$$f_{cm}(t) = \beta_{cc}(t) f_{cm}$$

f_{cm} = mean compressive strength at 28 days

s = coefficient which depends on the type of cement, assume class R, $s = 0.25$

$\beta_{cc}(t)$ = coefficient which depends on the age of the concrete t

$$= \exp[s(1-(28/t)^{1/2})] = 0.78$$

Tension

$$f_{ctt}(t) = (\beta_{cc}(t))^{\alpha} \cdot f_{ctm} \quad (4.7)$$

Where: $f_{ctm}(t)$ is tensile strength of concrete at time 't'

f_{ctm} is tensile strength of concrete at 28 days

$\alpha = 1$ for $t < 28$ or $\alpha = 2/3$ for $t \geq 28$

At service

Compression

$$f_{cw} = 0.45f_{ck} \quad (4.8)$$

Tension

$$f_{ctw} = \begin{cases} 0.30(f_{ck})^{2/3} & \text{for } f_{ck} \leq 50/60 \\ 2.12 \ln(1.8 + 0.1f_{ck}) & \text{for } f_{ck} > 50/60 \end{cases} \quad (4.9)$$

Allowable stresses in tendon

According to Euro code 2, (EN 1992-1-1, 2004); Unacceptable cracking or deformation may be assumed to be avoided if, under the characteristic combination of loads, the tensile stress in the reinforcement does not exceed $0.8f_{yk}$. Where the stress is caused by an imposed deformation, the tensile stress should not exceed f_{yk} . The mean value of the stress in prestressing tendons should not exceed $0.75f_{pk}$.

The stresses at top and bottom fiber are calculated using the following general equations (Anthony R., 2008)

Concrete stresses at transfer:

Top

$$f_{ut} = \frac{P_t}{A_c} - \frac{P_t \cdot e}{Z_t} + \frac{M_g}{Z_t} \leq f_{ct} \text{ or } \geq -f_{ctt}(t) \quad (4.10)$$

Bottom

$$f_{ub} = \frac{P_t}{A_c} + \frac{P_t \cdot e}{Z_b} - \frac{M_g}{Z_b} \leq f_{ct} \text{ or } \geq -f_{ctt}(t) \quad (4.11)$$

Where P_t = pre stressing force immediately after short term loss

A_C = area of concrete

Z_t = section modulus top,

Z_b = section modulus bottom

M_g = bending moment caused by the own weight of sleeper.

f_{ut} & f_{ub} = Stress at top and bottom respectively

Concrete stresses at service

Top

$$f_{wt} = \frac{P_e}{A_c} - \frac{P_e \cdot e}{Z_t} + \frac{M_{(g+q)}}{Z_t} \leq f_{cw} \text{ or } \geq -f_{ctw} \quad (4.12)$$

Bottom

$$f_{wb} = \frac{P_e}{A_c} + \frac{P_e \cdot e}{Z_b} - \frac{M_{(g+q)}}{Z_b} \leq f_{cw} \text{ or } \geq -f_{ctw} \quad (4.13)$$

Where: P_e = pre stressing after all losses

A_C = area of concrete

Z_t = section modulus top,

Z_b = section modulus bottom

$M_{(g+q)}$ = bending moment caused by maximum total load.

f_{wt} & f_{wb} = working stress at top and bottom respectively

For preliminary calculation for parametric selection purpose; assuming the area of concrete is almost equal to the total area, means $A_c = A_g$.

As shown in appendix C, from six trials using applied stress formulas; trial 5 is more convenient for economical design of PC sleeper; means that concrete grade 50 (C-50), and 8mm diameter (8 numbers) tendon having 7-wires of strands is selected for final design of selected member based on the allowable stress values of both steel and concrete. Once concrete grade and number (with diameter) of wires are selected from preliminary design, can proceed to final design of nominated sleeper.

4.3. Design Bending Moments of Selected Sleeper

4.3.1. Bending Moment due to External Load

For this study; EN 13230-6 (2015) is used to calculate bending capacity of sleeper due to rail-seat load, and ballast pressure. For the calculation of the bending moments this method assumes that there is uneven contact pressure distribution between sleeper and ballast; it produces positive flexure at rail seat and negative flexure in the middle of sleeper. Information regarding the bending moment calculation is describing in the following sections based on European standard (EN 13230-6, 2015).

4.3.1. Bending Moments at Rail-seat

Wheel loads generate positive and negative bending moments under the rail seat. The bending resistance at the end of the required service life time under the rail-seat is determined from bending moment.

Positive bending moment at rail seat ($M_{d,r,pos}$): The required flexural strength under the rail-seat is derived from the bending moment induced by the design load. The value of this

bending moment ($M_{d,r,pos}$) is based on the length of the ballast support beneath each rail-seat.

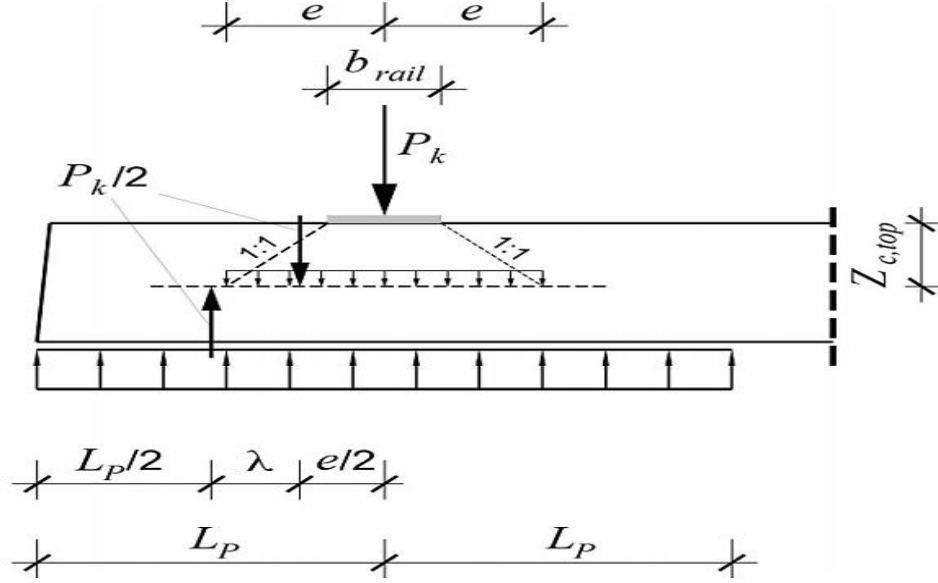


Figure 4.3: Design model for $M_{d,r,pos}$ (EN13230-6, 2015)

For calculation of positive bending moment ($M_{d,r,pos}$); the simplified design model shown in Figure 4.3 is used. A load distribution in the sleeper according to Figure 4.3 and a constant ballast reaction over the length $2L_p$ may be assumed. Load distribution from the rail foot to axis of inertia of the cross section is calculated as below.

$$2e = b_{rail} + (2 \times z_{c,top}) \quad (4.14)$$

Where: b_{rail} is the width of the rail foot

$z_{c,top}$ is the distance to the axis of inertia from top surface of sleeper

Width of rail foot (b_{rail}) and distance from sleeper surface to axis of inertia (y_t) (from section properties of selected sleeper) is 0.132m and 106.23mm respectively.

$$2e = 0.132 + 2 \times 0.10623 = 0.323, \text{ then } e = 0.32/2 = 0.16$$

The length of ballast pressure (L_p) is:

$$L_p = \frac{L - c}{2} \quad (4.15)$$

Where: c is the rail-seat center spacing

L is the sleeper length

$$L_p = (2.5-1.51)/2 = 0.495\text{m}$$

Lever length(λ) of resulting internal forces

$$\lambda = \frac{L_p - e}{2} \quad (4.16)$$

$$\lambda = (0.495-0.16)/2 = 0.168\text{m}$$

$$M_{d,r,pos} = k_{ir} \cdot \lambda \cdot \frac{R_d}{2} \quad (4.17)$$

The simplified design model may be used for sleepers with $0.35\text{m} \leq L_p \leq 0.55\text{m}$ in combination with $k_{ir} = 1.6$.

$$M_{d,r,pos} = 0.5 \cdot 1.6 \cdot 0.168 \cdot 188.15 = 25.29\text{kN-m}$$

Negative bending moment at rail seat ($M_{d,r,neg}$): Negative bending moments under the rail-seat can arise from vertical movement of the track. The characteristic negative bending moment for 1435 mm gauge sleepers with a length $2.50\text{ m} < L < 2.60\text{ m}$ is:

$$M_{d,r,neg} = 0.5 \cdot M_{d,r,pos} \quad (4.18)$$

$$M_{d,r,neg} = 0.5 \cdot 25.29 = 12.65\text{kN-m}$$

4.3.2. Bending Moments at the Center of Sleeper

Wheel loads generate positive and negative bending over the central length of the sleeper. The required flexural strength over the central part of the sleeper is determined from the bending moment induced by the rail seat load and depends on geometry of sleeper.

Negative bending moment at the center part ($M_{d,c,neg}$): The maximum negative bending moment shall be taken to occur at the center of the sleeper, under partially or totally center bound conditions producing tensile stress at the top and compressive stress at the underside of the sleeper. Using simplified method the characteristic value of the negative bending moment for sleepers with variable stiffness and different bottom width calculated as:

$$M_{d,c,neg} = k_{ic} \cdot M_{c,neg,100} \cdot \frac{R_d}{100} \quad (4.19)$$

The value of k_{ic} for sleeper length of 2.5m is 0.75

The values of $M_{c,neg,100}$ are given by diagrams in Appendix D (figure 2) for 1435 mm gauge. The diagrams are pre-calculated graphs using an elastic beam on elastic foundation and a unit load of 100 kN. The input values for the diagrams are explained in Appendix D (figure 1). The general pattern of $M_{c, neg, 100}$ is depending on the relations of sleeper inertia and variable bottom width.

$M_{c,neg,100}$ obtained from Appendix D (figure 2) as shown above, using the ratio value of I_c/I_r and B_c/B_r as follows.

$$I_c/I_r = 67.4 \times 10^6 / 149.9 \times 10^6 = 0.45 \text{ and } B_c/B_r = 240/270 = 0.8$$

After finding of B_c/B_r and I_c/I_r , $M_{c,neg,100}$ can be estimated using interpolation which is found on graph $B_c/B_r = 0.8$, $M_{c,neg,100} = 7.5 \text{ kN-m}$.

$$M_{d,c,neg} = 0.75 \times 7.5 \times 188.15 / 100 = 10.58 \text{ kN-m}$$

Positive bending moment at the center ($M_{d,c,pos}$): The designed positive bending moment of the sleeper center is based on negative bending moment of sleeper center. The value for the positive bending moment for 1435 mm gauge sleepers with a length $2.20 \text{ m} \leq L \leq 2.60 \text{ m}$ is calculated as follow as:

$$M_{d,c,pos} = 0.7 \times M_{d,c,neg} \quad (4.20)$$

$$M_{d,c,pos} = 0.7 \times 10.58 = 7.41 \text{ kN-m}$$

4.3.2. Bending Moment due to Self-weight

Bending moment due to self-weight of sleeper is very small as compared from bending moment that comes from external load but stress due to bending have some effect on concrete stress property.

$$\text{Self-weight load} = \text{Area} \times \text{density of concrete} \quad (4.21)$$

$$\text{Bending moment} = M_d = \frac{wx}{2} (L - x) \quad (4.22)$$

Take average height and width of sleeper for calculation of self-weight load.

$$P_d = 0.255 \times 0.18 \times 25 = 1.15 \text{ kN/m}$$

Using equation (4.22)

Moment at rail-seat = $M_r = 0.5 \times 1.15 \times 0.945 \times (2.5 - 0.495) = 1.09 \text{ kN-m}$

Moment at center = $M_c = 0.5 \times 1.15 \times 1.25 \times (2.5 - 1.25) = 0.898 \text{ kN-m}$

4.4. Detail Design of Selected Sleeper

For design of prestressed concrete beam member by limit states method, first, the force demand in a member under the design loads is determined from a structural analysis. Next, the member is designed to meet the demand. If necessary, another cycle of analysis and design is performed. For design of PC sleeper; the concept of simply supported prestressed concrete beam can be adopted.

For ultimate limit state design of sleeper; three possible designs are considered in this study for checking the effectiveness of a selected member under the combined effect of design loads. Under this subsection, design for axial tension is first discussed and then design for flexure and shear are elaborated afterwards.

4.4.1. Design for Axial

For design of member for axial load the first thing is to study the calculation of demand in a member, and selection of its appropriate dimension. Here, already determined the load demand and cross-section of the member in section 4.2 and 4.3 respectively. Directly going to the axial tension design, the design of the prestressing force and then the analysis of ultimate strength are conducted.

For preliminary iteration of parameters, it can be assumed that the area of concrete is equal to gross area of the section. But for final design, the concrete cross-section is broken up into two components: the net area of the section with concrete (A_c), and the area of prestressing steel (A_p).

$$A = A_c + A_p \Rightarrow A_c = A_g - A_p \quad (4.23)$$

From preliminary design of trial five, $A_p = 401.92 \text{ mm}^2$

Design of prestressed concrete sleeper based on ultimate limit state approach

Taking the average width, and height of the sleeper to determine the net area of concrete in order to estimate prestress force at transfer after short-term loss.

$$A_g = 180 \times 255 = 45900 \text{ mm}^2$$

$$\text{Then, } A_c = 45900 - 401.92 = 45498.08 \text{ mm}^2$$

Once the net area of concrete is known the prestressing force at transfer can be estimate as the expression below (Amlan).

$$P_t = f_{ct} \cdot A_c \quad (4.24)$$

Where, P_t is allowable prestress force

f_{ct} is allowable stress of concrete under compression

A_c is net area of concrete

$$P_t = 22.68 \times 45498.08 = 1031896.45 \text{ N} > 0.75 \times 401.92 \times 1860 = 560678.4 \text{ N, not ok!}$$

$$\text{Take } P_t = 560678.4 \text{ N}$$

Prestressing force at service is determined as follows by assuming 20% of total loss is occurred.

$$P_e = 0.8 \cdot P_{\max}$$

$$\text{Where: } P_{\max} = 0.8 \times 401.92 \times 1860 = 598056.96 \text{ N}$$

$$P_e = 0.8 \times 474969.6 = 478445.57 \text{ N}$$

Table 4.2: Allowable stress of C-50 concrete grade

At transfer		At service	
Compression(Mpa)	Tension(Mpa)	Compression(Mpa)	Tension(Mpa)
22.34	-3.18	22.5	-4.07

At transfer, the stress in concrete is given as follows:

$$A_c = 45500 - 401.92 = 45098.08 \text{ mm}^2$$

At rail-seat

Top

$$f_{ut} = \frac{P_t}{A_c} - \frac{P_{t,e}}{Z_t} + \frac{M_g}{Z_t} = \frac{560678.4}{45098.08} - \frac{560678.4 \cdot 27.52}{1410000} + \frac{1090000}{1410000} = 2.26 \text{MPa} > -3.18 \quad \text{ok!}$$

Bottom

$$f_{ub} = \frac{P_t}{A_c} + \frac{P_{t,e}}{Z_b} - \frac{M_g}{Z_b} = \frac{560678.4}{45098.08} + \frac{560678.4 \cdot 27.52}{1600000} - \frac{1090000}{1600000} = 21.39 \text{MPa} < 22.34 \quad \text{ok!}$$

At center

$$A_c = 34800 - 401.92 = 34398.08 \text{mm}^2$$

Top

$$f_{ut} = \frac{P_t}{A_c} - \frac{P_{t,e}}{Z_t} + \frac{M_g}{Z_t} = \frac{560678.4}{34398.08} - \frac{560678.4 \cdot 8.42}{789000} + \frac{898000}{789000} = 11.45 \text{MPa} > -3.18 \quad \text{ok!}$$

Bottom

$$f_{ub} = \frac{P_t}{A_c} + \frac{P_{t,e}}{Z_b} - \frac{M_g}{Z_b} = \frac{560678.4}{34398.08} + \frac{560678.4 \cdot 8.42}{902000} - \frac{898000}{902000} = 20.54 \text{MPa} < 22.34 \quad \text{ok!}$$

Next, the analysis of stresses for service loads. At service, the stress in concrete can be calculated as follows:

Due to positive moment

$$A_c = 45500 - 401.92 = 45098.08 \text{mm}^2$$

At rail-seat

Top

$$f_{wt} = \frac{P_e}{A_c} - \frac{P_{e,e}}{Z_t} + \frac{M_{(g+q)}}{Z_t} = \frac{478445.57}{45098.08} - \frac{478445.57 \cdot 27.52}{1410000} + \frac{26380000}{1410000} = 19.98 \text{MPa} < 22.5 \quad \text{ok!}$$

Bottom

Design of prestressed concrete sleeper based on ultimate limit state approach

$$f_{wb} = \frac{P_e}{A_c} + \frac{P_e \cdot e}{Z_b} - \frac{M_{(g+q)}}{Z_b} = \frac{478445.57}{45098.08} + \frac{478445.57 \cdot 27.52}{1600000} - \frac{26380000}{1600000} = 2.35 \text{MPa} > -4.07 \quad \text{ok!}$$

Due to negative moment

Top

$$f_{wt} = \frac{P_e}{A_c} - \frac{P_e \cdot e}{Z_t} + \frac{M_{(g+q)}}{Z_t} = \frac{478445.57}{45098.08} - \frac{478445.57 \cdot 27.52}{1410000} + \frac{13710000}{1410000} = 10.99 \text{MPa} < 22.5 \quad \text{ok!}$$

Bottom

$$f_{wb} = \frac{P_e}{A_c} + \frac{P_e \cdot e}{Z_b} - \frac{M_{(g+q)}}{Z_b} = \frac{478445.57}{45098.08} + \frac{478445.57 \cdot 27.52}{1600000} - \frac{13710000}{1600000} = 10.27 \text{MPa} > -4.07 \quad \text{ok!}$$

At center

$$A_c = 34800 - 401.92 = 34398.08 \text{mm}^2$$

Due to positive moment

Top

$$f_{wt} = \frac{P_e}{A_c} - \frac{P_e \cdot e}{Z_t} + \frac{M_{(g+q)}}{Z_t} = \frac{478445.57}{34398.08} - \frac{478445.57 \cdot 8.42}{789000} + \frac{8310000}{789000} = 19.33 \text{MPa} < 22.5 \quad \text{ok!}$$

Bottom

$$f_{wb} = \frac{P_e}{A_c} + \frac{P_e \cdot e}{Z_b} - \frac{M_{(g+q)}}{Z_b} = \frac{478445.57}{34398.08} + \frac{478445.57 \cdot 8.42}{902000} - \frac{8310000}{902000} = 9.16 \text{MPa} > -4.07 \quad \text{ok!}$$

Due to negative moment

Top

$$f_{wt} = \frac{P_e}{A_c} - \frac{P_e \cdot e}{Z_t} + \frac{M_{(g+q)}}{Z_t} = \frac{478445.57}{34398.08} - \frac{478445.57 \cdot 8.42}{789000} + \frac{11480000}{789000} = 22.15 \text{MPa} < 22.5 \quad \text{ok!}$$

Bottom

$$f_{wb} = \frac{P_e}{A_c} + \frac{P_e \cdot e}{Z_b} - \frac{M_{(g+q)}}{Z_b} = \frac{478445.57}{34398.08} + \frac{478445.57 \cdot 8.42}{902000} - \frac{11480000}{902000} = 5.65 \text{MPa} > -4.07 \quad \text{ok!}$$

Once the member is designed for the stresses at service and at transfer cases satisfying the allowable stress limits, ultimate limit state analysis can be carried out next. For better service of the selected section; factored strength of the member must be greater than the demand load. The ultimate tensile strength of a section is calculated as per Amlan work as shown below.

$$\text{Resistance load } (R_n) = f_{pk} A_p \quad (4.25)$$

$$R_n = 1860 * 401.92 = 747.57 \text{ kN}$$

$$\text{Factored resistance load } (R_f) = \Phi f_{pk} A_p \quad (4.26)$$

$$R_f = 0.759 * 593.71 = 567.41 \text{ kN} > 188.15 \text{ kN} \quad \text{ok!}$$

4.4.2. Design for Flexure

Flexural design of sleeper is just like design of simply supported beam. By taking maximum moment at the critical section using beam theory, the sleeper can be designed for that moment value. The design is done for the critical section, that is the section which has the highest positive moment or negative moment. For a prestressed concrete sleeper, the critical sections are at rail-seat and center of the member.

For design of PC sleeper section for flexure, there are two stages. First, the member is designed for the service loads making sure that the allowable stresses at transfer and at service are satisfied; next, analysis is done for bending strength of the selected sleeper in order to make sure that the factored bending strength is greater than the demand moment due to applied load.

Applied stress analysis under axial design, and under flexural design has the same procedure, and the same value as presented in section 4.4. If the stress is within the allowable value, no need to proceed to another step. In case; the area selected for the trial section does not satisfy this requirement, the section needs to be redesigned.

Next, the bending strength of the sleeper is by comparing applied moment at critical sections, and factored resistance moments.

4.4.2.1. Bending Strength of Selected Sleeper

As described in the following section calculation of ultimate bending strength of selected sleeper depends on strain compatibility method.

4.4.2.1.1. Compression Force of Concrete

The actual distribution of the compressive stress in a prestressed concrete member has the form of a rising parabola. It is time consuming to evaluate the volume of the compressive stress block if it has a parabolic shape. In this study, an equivalent rectangular stress block was used as a substitute for the rising parabola to calculate the compressive force. This equivalent stress block has 'a' depth and an average compressive strength of ηf_{cd} .

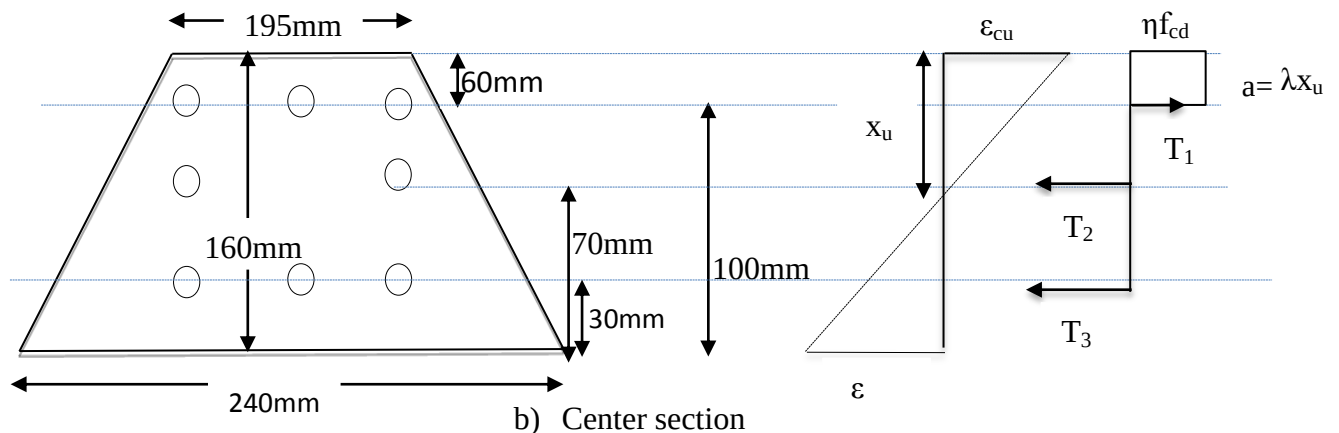
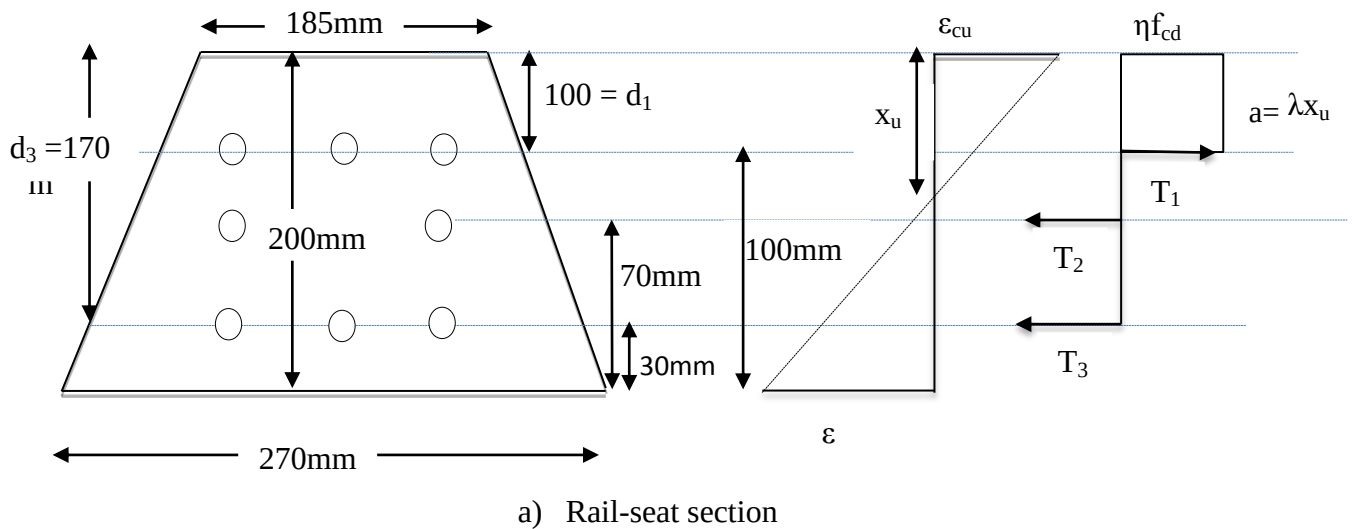


Figure 4.4: Conceptual diagrams for calculating bending strength of selected sleeper

Where: ϵ represents the strains in the diagram

T_1 to T_3 are the forces in steel wires

a is the depth of compression, $a = \eta * x_u$;

x_u is the depth between the top line and neutral axis can be calculated by centroid method as follows(Jian B., 2012).

$$Y_s = \frac{\sum n_i A_s}{\sum A_s} \quad (4.27)$$

$$Y_c = \frac{\frac{(B_b - B_t)h^2}{3} + \frac{B_t h^2}{2} - A_p \sum n_i h_i}{A - A_p} \quad (4.28)$$

$$x_u = h - \left(\frac{Y_c A + Y_s A_{sg}}{A + A_p} \right) \quad (4.29)$$

Where, A_p is the cross section area of a single steel wire

A is the area of trapezium shape of sleeper cross section

A_{sg} is the gross area of all cross sections of steel wires.

B_b and B_t is width of bottom and top of sleeper respectively

At rail-seat

$$Y_s = \frac{20836.98}{319.2} = 66.25\text{mm}$$

$$Y_c = \frac{\frac{(270-185)200^2}{3} + 0.5*185*200^2 - 50.24*530}{45500 - 401.92} = 106.58\text{mm}$$

$$x_u = 200 - \left(\frac{106.58*45500 + 66.25*401.92}{45500 + 401.92} \right) = 93.77\text{mm}$$

At center

$$Y_s = \frac{20836.98}{319.2} = 66.25\text{mm}$$

$$Y_c = \frac{\frac{(240-195)*160^2}{3} + 0.5*195*160^2 - 50.24*530}{34800 - 401.92} = 82.95\text{mm}$$

$$x_u = 160 - \left(\frac{82.95*34800 + 66.25*401.92}{34800 + 401.92} \right) = 77.24\text{mm}$$

For rectangular stress distribution, the factor λ , defining the effective height of the compression zone and the factor η , defining the effective strength, are as expressed below (prEN1992-1-1, 2002).

$$\lambda = \begin{cases} 0.8 & \text{for } f_{ck} \leq 50\text{MPa} \\ 0.8 - \frac{(f_{ck}-50)}{400} & \text{for } 50 < f_{ck} \leq 90\text{MPa} \end{cases} \quad \lambda = 0.8$$

$$\eta = \begin{cases} 1 & \text{for } f_{ck} \leq 50\text{MPa} \\ 1 - \frac{(f_{ck}-50)}{200} & \text{for } 50 < f_{ck} \leq 90\text{MPa} \end{cases} \quad \eta = 1$$

Depth of the compression zone is:

At rail-seat

$$a = \lambda * x_u = 0.8 * 93.77 = 75.02\text{mm}$$

At center

$$a = \lambda * x_u = 0.8 * 77.24 = 61.79\text{mm}$$

The compressive force of simplified block stress-strain distribution is expressed by:

$$C = \alpha_c f_{cd} b d_{\max} \quad (4.30)$$

Where: $\alpha_c = 0.8k_x$

$$= 0.8x_u/d$$

$d_{\max}$ = maximum distance at decompression reinforcement

b = width of section

f_{cd} = design compressive strength of concrete

$$= \frac{\alpha_{cc} f_{ck}}{\gamma_c} = \frac{0.85 \cdot 50}{1.5} = 28.33 \text{ MPa}$$

Take average value of width of sleeper for computation of compressive force

At rail-seat

$$k_x = x_u/d = 93.77/170 = 0.552, \alpha_c = 0.8 \cdot 0.552 = 0.442$$

$$C = 0.442 \cdot 227.5 \cdot 170 \cdot 28.33 = 483.77 \text{ kN}$$

At center

$$k_x = x_u/d = 77.24/130 = 0.594, \alpha_c = 0.8 \cdot 0.594 = 0.475$$

$$C = 0.475 \cdot 217.5 \cdot 130 \cdot 28.33 = 380.49 \text{ kN}$$

4.4.2.1.2. Tension forces of steel wires

The tension forces from steel wires can be calculated by (Jian, 2012):

$$T_n = \epsilon_{tn} E_p A_p n_i \quad (4.31)$$

Where: ϵ_{tn} is the total strain of single steel wire in n^{th} row

E is Young's modulus of steel wire

A_p is the area of steel wires

n_i is the number of steel wires in n^{th} row.

Strain in prestressing steel at transfer

$$\epsilon_{pt} = \frac{f_{pt}}{E_p} \quad (4.32)$$

Where: f_{pt} is allowable stress of steel and E_p is young's modulus of steel

Strain in concrete at transfer

$$\epsilon_{ct} = \frac{1}{E_c} \left(\frac{P_t}{A} + \frac{P_t \cdot (d_{max} - y_t)^2}{I} \right) \quad (4.33)$$

Where: P_t is prestressing force at transfer

A is area of the cross-section

d_{max} lowest level of steel in the decompression zone

y_t is distance of centroid from top or bottom

E_c is young's modulus of concrete

I is moment of inertia

The stress at failure corresponding to the total strain is:

$$f_p = \begin{cases} E_p * \epsilon_{pu}, & \text{if } \epsilon_{pu} \leq 0.0076 \\ 1860 \text{MPa} - \frac{0.3 \text{MPa}}{\epsilon_{pu} - 0.0064} & \text{if } \epsilon_{pu} > 0.0076 \end{cases} \quad (4.34)$$

Apply similarity of triangle to find the strains

At rail-seat

$$\epsilon_{pt} = \frac{0.75 * 1860}{200000} = 0.007$$

From top of sleeper

$$\epsilon_{ct} = \frac{1}{37000} \left(\frac{560678.45}{45500} + \frac{560678.4 * (170 - 106.23)^2}{149.9 * 10^6} \right) = 0.0006$$

$$\epsilon_{ct1} = \frac{\epsilon_{ct}(y_t - d_1)}{d_{max} - y_t} = \frac{0.0006(106.23 - 100)}{170 - 106.23} = 0.00005$$

$$\epsilon_{ct2} = \frac{\epsilon_{ct}(y_t - d_2)}{d_{max} - y_t} = \frac{0.0006(106.23 - 130)}{170 - 106.23} = -0.00022$$

$$\epsilon_{ct3} = \frac{\epsilon_{ct}(y_t - d_3)}{d_{max} - y_t} = \frac{0.0006(106.23 - 170)}{170 - 106.23} = -0.0006$$

Assume concrete strain (ϵ_{cu}) = 0.0035

$$\varepsilon_{cpu1} = \varepsilon_{cu} \left(\frac{d_1 - x_u}{x_u} \right) = 0.0035 \left(\frac{100 - 75.02}{75.02} \right) = 0.0012$$

$$\varepsilon_{cpu2} = \varepsilon_{cu} \left(\frac{d_2 - x_u}{x_u} \right) = 0.0035 \left(\frac{130 - 75.02}{75.02} \right) = 0.0026$$

$$\varepsilon_{cpu3} = \varepsilon_{cu} \left(\frac{d_3 - x_u}{x_u} \right) = 0.0035 \left(\frac{170 - 75.02}{75.02} \right) = 0.0044$$

Summation of strain components

$$\varepsilon_{pu1} = \varepsilon_{pt} + \varepsilon_{ct1} + \varepsilon_{cpu1} = 0.007 + 0.00005 + 0.0012 = 0.0083$$

$$\varepsilon_{pu2} = \varepsilon_{pt} + \varepsilon_{ct2} + \varepsilon_{cpu2} = 0.007 - 0.00022 + 0.0026 = 0.0094$$

$$\varepsilon_{pu3} = \varepsilon_{pt} + \varepsilon_{ct3} + \varepsilon_{cpu3} = 0.007 - 0.0006 + 0.0044 = 0.0108$$

The stress at failure corresponding to the total strain is:

$$f_{p1} = 1860 - \frac{0.3}{0.0083 - 0.0064} = 1702.11 \text{ MPa}$$

$$f_{p2} = 1860 - \frac{0.3}{0.0094 - 0.0064} = 1760 \text{ MPa}$$

$$f_{p3} = 1860 - \frac{0.3}{0.0108 - 0.0064} = 1791.82 \text{ MPa}$$

The corresponding tensile forces are:

$$T_1 = A_{p1} * f_{p1} = 150.72 * 1702.11 = 256.54 \text{ kN}$$

$$T_2 = A_{p2} * f_{p2} = 100.48 * 1760 = 176.84 \text{ kN}$$

$$T_3 = A_{p3} * f_{p3} = 150.72 * 1791.82 = 270.01 \text{ kN}$$

$$T = T_1 + T_2 + T_3 = 256.54 + 176.84 + 270.01 = 703.39 \text{ kN}$$

At Center

$$\varepsilon_{pt} = 0.007$$

$$\varepsilon_{ct} = \frac{1}{37000} \left(\frac{445284}{34800} + \frac{445284 * (130 - 85.33)^2}{67.4 * 10^6} \right) = 0.0007$$

$$\epsilon_{ct1} = \frac{\epsilon_{ct}(y_t - d_1)}{d_{max} - y_t} = \frac{0.0007(85.33 - 60)}{130 - 85.33} = 0.0004$$

$$\epsilon_{ct2} = \frac{\epsilon_{ct}(y_t - d_2)}{d_{max} - y_t} = \frac{0.0007(85.33 - 90)}{130 - 85.33} = -0.00007$$

$$\epsilon_{ct3} = \frac{\epsilon_{ct}(y_t - d_3)}{d_{max} - y_t} = \frac{0.0007(85.33 - 130)}{130 - 85.33} = -0.0007$$

$$\epsilon_{cu} = 0.0035$$

$$\epsilon_{cpu1} = \epsilon_{cu} \left(\frac{d_1 - x_u}{x_u} \right) = 0.0035 \left(\frac{60 - 61.79}{61.79} \right) = -0.0001$$

$$\epsilon_{cpu2} = \epsilon_{cu} \left(\frac{d_2 - x_u}{x_u} \right) = 0.0035 \left(\frac{90 - 61.79}{61.79} \right) = 0.0016$$

$$\epsilon_{cpu3} = \epsilon_{cu} \left(\frac{d_3 - x_u}{x_u} \right) = 0.0035 \left(\frac{130 - 61.79}{61.79} \right) = 0.0039$$

Summation of strain components

$$\epsilon_{pu1} = \epsilon_{pt} + \epsilon_{ct1} + \epsilon_{cpu1} = 0.007 + 0.0004 - 0.0001 = 0.0073$$

$$\epsilon_{pu2} = \epsilon_{pt} + \epsilon_{ct2} + \epsilon_{cpu2} = 0.007 - 0.00007 + 0.0016 = 0.0085$$

$$\epsilon_{pu3} = \epsilon_{pt} + \epsilon_{ct3} + \epsilon_{cpu3} = 0.007 - 0.0007 + 0.0039 = 0.0039$$

The stress at failure corresponding to the total strain is:

$$f_{p1} = E_p * \epsilon_{pu1} = 200000 * 0.0073 = 1460 \text{ MPa}$$

$$f_{p2} = 1860 - \frac{0.3}{0.0085 - 0.0064} = 1717 \text{ MPa}$$

$$f_{p2} = E_p * \epsilon_{pu2} = 200000 * 0.0039 = 780 \text{ MPa}$$

Tensile forces are calculated as follows:

$$T = A_p * f_p$$

$$T_1 = A_{p1} * f_{p1} = 150.72 * 1460 = 220.1 \text{ kN}$$

$$T_2 = A_{p2} \cdot f_{p2} = 100.48 \cdot 1717 = 172.52 \text{ kN}$$

$$T_3 = A_{p3} \cdot f_{p3} = 150.72 \cdot 780 = 117.56 \text{ kN}$$

$$T = T_1 + T_2 + T_3 = 220.1 + 172.52 + 117.56 = 510.18 \text{ kN}$$

4.4.2.1.3. Bending Moment

After calculation of compressive force in the concrete and the tensile forces in the steel wires; ultimate moments of the compression and tension forces are expressed by:

$$M_c = C \cdot (y_t - a/2) \quad (4.35)$$

$$M_{tn} = T_n y_n \quad (4.36)$$

Where, M_c is concrete compression moment;

M_{tn} is tension moment from the n^{th} steel wires.

The moments produced by each layer of pre-stressing are calculated as follows:

At rail-seat

Top

$$M_{u1} = -T_1 (y_t - d_1) = -256.54 \cdot (106.23 - 100) = -1.60 \text{ kN-m}$$

$$M_{u2} = -T_2 (y_t - d_2) = -176.84 \cdot (106.23 - 130) = 4.20 \text{ kN-m}$$

$$M_{u3} = -T_3 (y_t - d_3) = -270.01 \cdot (106.23 - 170) = 17.22 \text{ kN-m}$$

Total ultimate bending moment and factored moment is:

$$M_u = M_c + M_{tn} = M_{u1} + M_{u2} + M_{u3} + C (x_u - a/2)$$

$$M_u = 483.77 \cdot (93.77 - 0.5 \cdot 75.02) + 17.22 + 4.2 - 1.6 = 47.04 \text{ kN-m}$$

$$MR = 0.759 \cdot 47.04 = 35.70 \text{ kN} > 25.29 \text{ kN} \quad \text{ok!}$$

As the same calculation of moments at top of rail-seat; bending moments for all critical section of the sleeper can be summarized as follows.

Table 4.3: Bending moments of selected sleeper

Moment	At rail-seat		At center	
	Top	Bottom	Top	Bottom
Ultimate moment	47.04	27.19	25.29	9.88
Factored moment	35.7	20.64	19.2	7.5
Applied moment	25.29	12.65	10.58	7.41

4.4.3. Design for Shear

The objective of this design step is to provide ultimate resistance for shear and should be greater than the shear demand under ultimate loads. The shear is studied based on the capacity of a section; which is the limit state of collapse. The value of concrete shear is depends on whether the section is cracked or uncracked due to flexure. For in case of this research assume no crack is take place in service life of PC sleeper.

The sleeper is fully supported, and has uniform ballast pressure distribution along the length of the sleeper. According to AREMA (2010) (Desalew F., 2014) uniform load of the sleeper is:

$$W = 2 \cdot R_d / L = 2 \cdot 189.21 / 2.5 = 151.37 \text{ kN/m}$$

Next, the maximum design shear is determined:

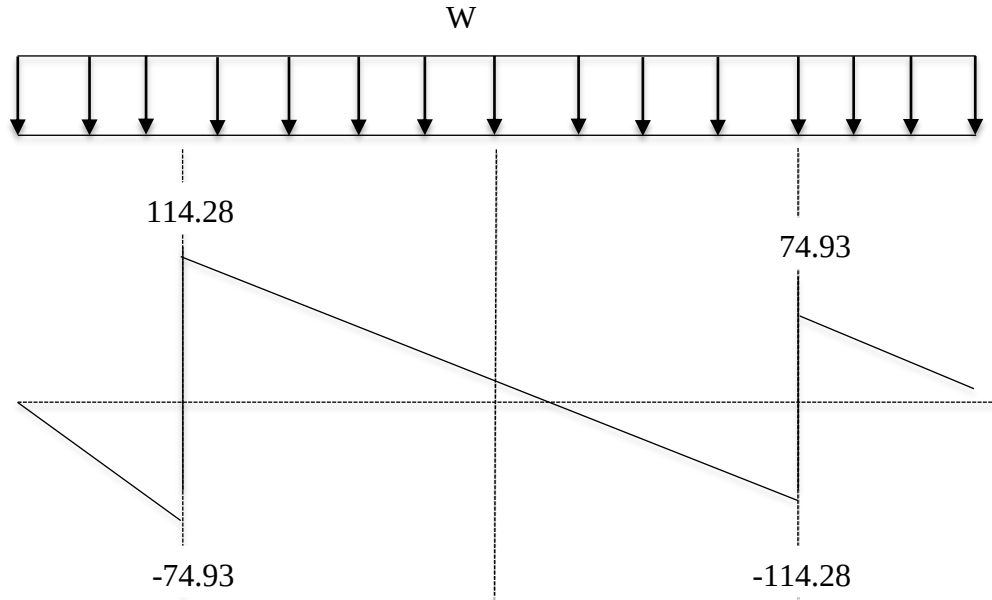


Figure 4.5: Shear diagram of selected sleeper

As shown from Figure 4.5; maximum design shear of the section is 114.28kN, at rail-seat of the sleeper.

In prestressed single span members without shear reinforcement, the shear resistance of the regions of uncracked in bending; the shear resistance should be limited by the tensile strength of the concrete. In these regions the shear resistance is given by equation 4.37. For cross-sections where the width varies over the height, the maximum principal stress may occur on an axis other than the centroidal axis. In such a case the minimum value of the shear resistance should be found by calculating V_{Rd} at various widths in the cross-section over its height (prEN1992-1-1, 2000).

$$V_{Rd} = \frac{I \cdot b_w}{s} \sqrt{(f_{ctd})^2 + \alpha_I \sigma_{cp} f_{cd}} \quad (4.37)$$

I = second moment of inertia of the section

f_{ctd} = design tensile strength of concrete

f_{cd} = design compressive strength of concrete

b_w = the width of cross section at centroidal axis,

S = first moment of area above and about the centroidal axis of the section

$\alpha_I = I_x/I_{p2} \leq 1$ for pretensioned tendons, take $\alpha_I = 1$

σ_{cp} = concrete compressive stress at the centroidal axis due to prestressing

$$= P_t/A_c$$

Table 4.4: Resistance shear at different width of sleeper

Width(mm)	185	195	205	215	225	235	245	255	270
Shear(kN)	153.2	161.5	170	178.1	186	194.6	203	211.2	224

As shown from Table 4.4 and figure 4.5; selected sleeper is adequate for shear, means that minimum shear resistance of the section is greater than design shear of the section (153.2kN > 114.28kN).

CHAPTER FIVE

5. CONCLUSION AND FUTURE RECOMMENDATION

5.1. Conclusion

As it has been discussed in the first few chapters of this paper work, design of PC sleepers by adopting the ultimate limit state design approach using suitable load factors and reduction factors for an economic design was among the major aims of this thesis.

It has been well discussed that knowing the structural behavior of PC sleepers and reliability works are crucial for design of sleepers using the limit state approach. Even though an understanding of their dynamic behavior due to effect of wheel flat on sleeper performance is far from complete, a promising effort has been made in the limit state design formulation.

From the reliability work carried out to obtain necessary parameters for the new formulated load combination in the ultimate limit state design, considering failure safety improvement, load factors, safety index and reduction factors obtained are summarized below.

Table 5.1: Major findings

Parameters	Values(factors)
Static	1.06
Dynamic	1.06
Reduction	0.759
Safety index	4.0

The procedures presented in this thesis on the design of sleepers by parametric selection using input values stated in the corresponding chapter shades light on the current practice of design that only depends on permissible stress concept.

In conclusion, the limit state principles that allow the design of structural element based on the probability of occurrences of loads and defining the structural failure can be applied for

PC sleepers. This thesis tries to present an adoptable approach towards such design practice that gives more accurate and economical result.

Even though most railway standards follow the allowable stress methodology, it is the research's conclusion that, with the presented design procedures and formulated factors, a better design approach can be adopted for PC sleeper design.

5.2. Future recommendation

Based on the study, it can be recommended that further study regarding design of prestress concrete sleeper by limit state approach should be carried out.

Ultimate limit state design of sleeper presented in this thesis has the potential to be further improved in the following areas:

- ✓ Data that were used for calculation of load factors and simulation of reliability index were developed from secondary sources. With a more compressive collection of real data from field over a long period of time, a much more economical and general design method can be formulated
- ✓ This thesis covered vertical load due to static and dynamic load caused by weight of train, and motion of train due to wheel flat without considering lateral loads at curved and turnouts of railway lines. Therefore, there is a need for further research on lateral load effect in limit state design approach of PC sleepers.
- ✓ This study only considered the ultimate limit state condition in limit state approach. Leaving an open space in service limit state design to be carried out as further research area. This limit state defines a condition where sleeper failure is beginning to impose some restrictions on the operational capacity of the track and service life of sleeper. Serviceability limit state design can be done by checking the reliability of prestressed concrete sleepers based on:
 - the design under serviceability limit state using allowable stresses method;
 - the development length for both tendons;
 - the total loss of prestress force.

REFERENCES

1. ACI Committee 318 (2005). Building code requirements for structural concrete. American Concrete Institute, Michigan, USA.
2. Amlan K. S. and Devdas M. (2008). Prestressed concrete structures, Indian Institute of Technology, Madras.
3. Amlan K. Sengupta. Prestressed concrete structures design for axial and flexure. Department of Civil Engineering, Indian Institute of Technology.
textofvideo.nptel.iitm.in
4. Anthony R. (2008). Analysis and design of prestress concrete members to Eurocodes (EC2) considering temperature. Dissertation, Napier University.
5. AREMA, (2010). American Railway Engineering and Maintenance-of-Way Association. Manual for Railway Engineering, Chapter 30 Ties, USA.
6. A.Remennikov, M. Murray, and S.Kaewunruen (2008). Dynamic design guidelines for prestressed concrete sleepers. In 20th Australasian Conference on the Mechanics of Structures, and Materials (ACMSM20), Queensland, Australia.
7. AS1085.14 (2003). Australia standard, Railway track materials, Part 14: Railway prestressed concrete sleepers, Australia.
8. David M. Steffens (2005). Identification and development of a model of railway track dynamic behaviour. Dissertation, Queensland University of technology, Australia.
9. Desalew F. (2014). Analysis and design of prestressed concrete sleeper. Dissertation, Addis Ababa University, Ethiopia.
10. Doyle, N.F. (1980). Railway track design: A review of current Practice. Occasional Paper No. 35, Bureau of transport economics, common wealth of Canberra, Australia.
11. Emilio B., Abdel S. (2015). Reliability Analysis Methods. University of Nantes, Nantes, France.
12. Esveld, C. (2001). Modern Railway Track. 2nd edition, Netherland.
13. Ethiopian Railways Corporation (2012). Sebeta-Djibouti/ Railway Feasibility Study, Part I General Specification, Executive Edition, Ethiopia.

14. Grassie S.L, and Cox S. J. (1985). The dynamic response of Railway Track with Unsupported sleepers. Proceedings of Institution of Mechanical Engineers (part D), 199(2): 123~135
15. H.J.Taylor, (1993).The railway sleeper: 50 years of pretension, prestressed concrete. The Structural Engineer, Volume 71, No 16/17.
16. Javad Sadeghi (2012). New advances in design of railway track system. Available from:http://www.intechopen.com/books/reliability-and-safety-in-railway/new_advances-in-design-of-railway-tracksystem.
17. J. Bian (2012). Ultimate flexural limit states analysis of prestressed concrete sleeper. Dissertation, Queensland University of Technology, Australia.
18. J.M.Sadeghi, and A.Babaei (2006). Structural optimization of b-70 railway prestressed concrete sleepers. Shiraz University, Iranian Journal of Science & Technology, Transaction Engineering, Vol. 30, No. B4.
19. J.Sykorova, J.Bartova, and P.Stemberk (2012). Prestressed concrete sleeper under extreme loading conditions. 18th international conference, Svratka, Czech Republic.
20. Jungyoul, Choi (2014). Qualitative analysis for dynamic behavior of railway ballasted track. Department of track and railway operations, Berlin, Germany.
21. Kaewunruen S., and Remennikov, A. M. (2009). Impact fatigue responses of railway prestressed concrete sleepers.doi:10.1080/19373260802479377
22. Kaewunruen S., and Remennikov A.M. (2007). Effect of improper ballast packing/tamping on dynamic behaviors of on-track railway concrete sleeper. International Journal of structural stability and dynamics: pp167-177.
23. LRFD Bridge Design (2007). AASHTO, LRFD Bridge design specifications, loading and General Information, 4th Edition. University of Cincinnati, Dept of Civil & Env. Engineering 765 Baldwin Hall Cincinnati, OH 45221-0071.
24. Martin H. & B.Jian (2012). Ultimate limit states design of concrete railway sleepers. Proceedings of the ICE - Transport, 165 (3), pp. 215-223.
25. Naiyu Wang (2010). Reliability-based condition assessment of existing highway bridges. Dissertation, School of Civil and Environmental Engineering, Georgia Institute of Technology.

26. EN 1992-1-1 (2004). European standard, Design of concrete structures, General rules and rules for buildings.
27. EN132302 (2015). Railway applications: Track Concrete sleepers and bearers Part 6: Design of prestressed mono-block sleepers.
28. Sadeghi, J. (2005). Investigation on the accuracy of current practices in analysis of railway track sleepers. *International Journal of Civil Engineering*; 3(1):34-51.
29. S.Kaewunruen, A.Remennikov, and H.Murray (2012). Development of reliability-based design for railway prestressed concrete sleepers. Queensland University of Technology, Australia.
30. S.Kaewunruen, A., and M. Remennikov (2008). Reliability assessment of railway prestressed concrete sleepers. University of Wollongong, Australia.
31. S.Kaewunruen and A. Remennikov (2006). Rotational capacity of railway prestressed concrete sleeper under static hogging moment. Asia Institute of technology, Bangkok, Thailand.
32. Sukontasukkul P, Mindess S, and Banthia N. (2004). Effect of loading rate on damage of concrete. *Cement and concrete research* 34: 2127-2134.
33. M. Allen, Andrzej, S. Nowak, and Richard J. (2005). Calibration to determine load and resistance factors for geotechnical and structural Design. General Structures Committee Transportation Research Board, Washington.
34. Wang, N. (1996). Resistance of concrete railroad ties to impact loading. University of British Columbia, Canada.
35. Wakui, and H.Okuda, H. (1999). A study on limit-state design method for prestressed concrete sleepers. *Concrete Library of JSCE*, Vol. 33, 1999, pp1-25.
36. Vincent G. (2001). Modal analysis and numerical modeling of a concrete railway sleeper. Dissertation, Department of Structural Engineering, Chalmers University of Technology, Sweden.

APPENDICES

Appendix A: COMREL V9.0 of reliability index analysis result

File Function Model Job View Window Help

Reliability Job - No comment

- Symbolic Expressions
 - Limit State Functions
 - FLIM(1)
- Stochastic Variables
 - R - Variables
- Correlations
- Deterministic Parameter Stud
- Characteristic Values
- Multiple Runs
- Starting Solution

Job Control

Symbolic Expressions Stochastic Model Correlations Multiple Runs Results Plots

Numerical Results

```

*****
----- Comrel-TI (Version 9) -----
---- (c) Copyright: RCP GmbH (1989-2015) ----
*****

Job name ..... : k
Failure criterion no. : 1
Comment : Ultimate Capacity
Transformation type : Rosenblatt
Optimization algorithm: RFLS

-----
Iteration No. 1; CPU-seconds(cumulative): 0.016
Scaled St.F(U) = 0.9564E-09; BETA = 0.0000; BETA/||U|| = 0.0000
Multipl.= 1.652 ; Step-length= 1.0000; State Func.calls: 5
-----
Iteration No. 2; CPU-seconds(cumulative): 0.016
Scaled St.F(U) = 0.6523E-15; BETA = 0.9088; BETA/||U|| = 1.0000
Multipl.= 1.652 ; Step-length= 1.0000; State Func.calls: 9

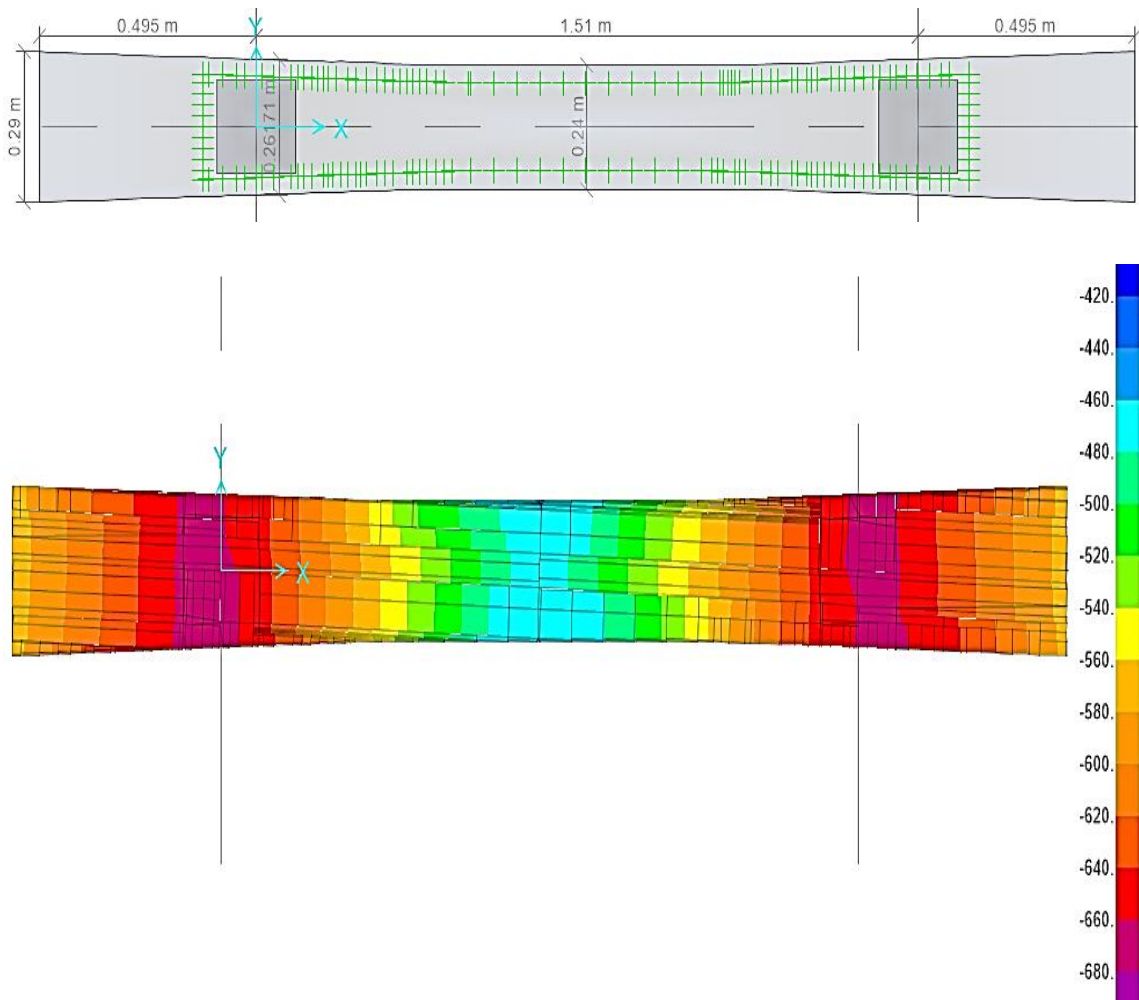
FORM-beta= 0.909; SORM-beta= -- ; beta(Sampling)= -- (IER= 0)
FORM-Pf= 0.18 ; SORM-Pf= -- ; Pf(Sampling)= --

----- Statistics after COMREL-TI -----
State Function calls = 10
State Funct. gradient evaluations = 2
Total computation time (CPU-secs.)= 0.03
The error indicator (IER) was = 0
*****

Reliability analysis is finished
    
```

Appendix B: Geometrical model result of trial 11 sleeper by SAFE software

Values are in kN/m².



Appendix C: Reinforcement and concrete grade selection

Section properties of selected sleeper

Section properties	At rail seat	At center
$A(\text{mm}^2)$	45500	34800
$y_b(\text{mm})$	93.77	74.67
$I(\text{mm}^4)$	149.9×10^6	67.4×10^6
$Y_t(\text{mm})$	106.23	85.33
$Z_b(\text{mm}^3)$	1600000	902000
$Z_t(\text{mm}^3)$	1410000	789000

Allowable stresses for concrete and steel

Allowable stress(MPa)								
Concrete grade	Concrete							Prestress ing tendon
	f _{cm}	f _{cm} (t)	f _{ck} (t)	At transfer		At service		
				Compression	Tension	Compression	Tension	Tension
40	48	37.4	29.4	17.664	-2.74	18	-3.51	1395
45	53	41.3	33.3	20.004	-2.96	20.25	-3.80	1395
50	58	45.2	37.2	22.344	-3.18	22.5	-4.07	1395
55	63	49.1	41.1	24.684	-3.29	24.75	-4.21	1395
60	68	53.0	45.0	27.024	-3.40	27	-4.35	1395
70	78	60.8	52.8	31.704	-3.60	31.5	-4.61	1395

Note: f_{cm} - mean compressive strength of concrete for 28 days

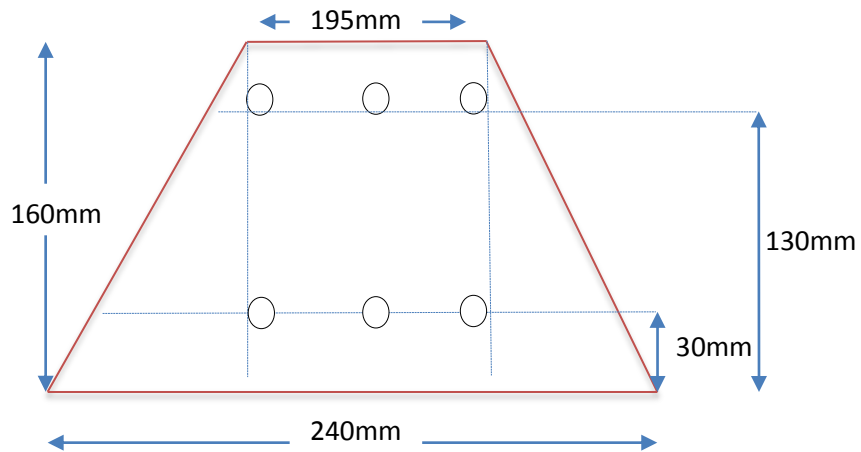
$f_{cm}(t)$ -mean compressive strength of concrete at time of 7 days

$f_{ck}(t)$ - compressive strength of concrete at time of 7 days

Bending moment due to self-weight and external force

Bending moment				
Moment	At Rail-seat(kN-m)		At center(kN-m)	
	M+	M-	M+	M-
External	25.29	12.65	7.41	10.58
Self-weight	1.09	1.09	0.90	0.90
Total	26.38	13.71	8.31	11.48

Design of prestressed concrete sleeper based on ultimate limit state approach



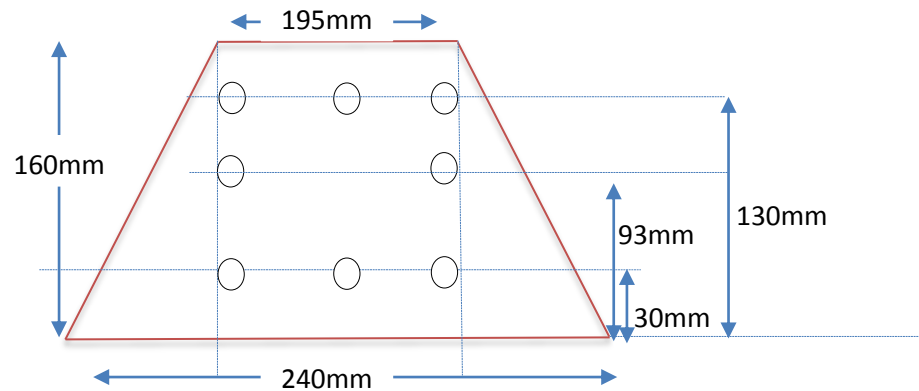
Pre-stressing wires				
Nominal diameter(mm)	9.3			
Layer	Quantity	Area(mm ²)	y(mm)	A*y
1	3	203.68	30	6110.52
2	0	0		0
3	3	203.68	130	26478.91
Total	6	407.3679		32589.43
Centroid(mm)				80
Eccentricity(mm)	Rail-seat			13.77
	Center			-5.33

Pre-stressing force	
Initial(N)	606163.44
At transfer(N)	568278.22
At service (N)	484930.75

Applied stresses in concrete(MPa)				
Stresses in concrete (MPa)	Rail-seat		Center	
	Top	Bottom	Top	Bottom
At transfer	7.71	16.70	21.31	12.97
At service(M+)	24.73	-1.74	24.51	1.81
At service(M-)	15.50	6.39	31.83	-1.72

Trial 1

Design of prestressed concrete sleeper based on ultimate limit state approach



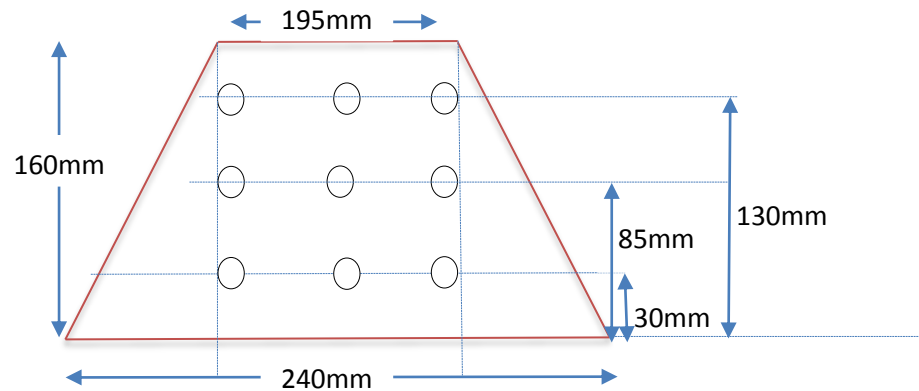
Pre-stressing wires				
Nominal diameter(mm)	7			
Layers	Quantity	Area(mm ²)	y(mm)	A*y
1	3	115.40	30	3461.85
2	2	76.93	93	7154.49
3	3	115.40	130	15001.35
Total	8	307.72		25617.69
Centroid(mm)				83.25
Eccentricity(mm)	Rail-seat			10.52
	Center			-8.58

Pre-stressing force	
Initial(N)	368139.11
At transfer(N)	345130.42
At service (N)	294511.29

Applied stresses in concrete(MPa)				
Stresses in concrete(MPa)	Rail-seat		Center	
	Top	Bottom	Top	Bottom
At transfer	7.00	11.58	18.14	8.25
At service(M+)	24.13	-6.12	22.61	-2.21
At service(M-)	14.90	2.02	29.13	-5.75

Trial 2

Design of prestressed concrete sleeper based on ultimate limit state approach



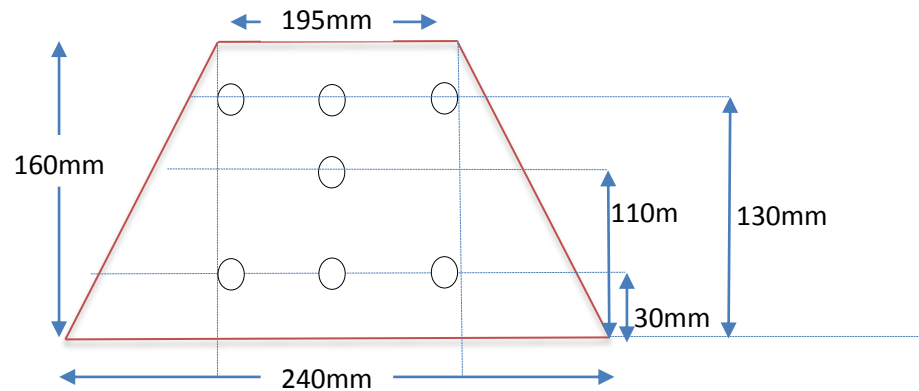
Pre-stressing wires				
Nominal diameter(mm)	7			
Layers	Quantity	Area(mm ²)	y(mm)	A*y
1	3	115.40	30	3461.85
2	3	115.40	85	9808.58
3	3	115.40	130	15001.35
Total	9	346.19		28271.77
Centroid(mm)				81.67
Eccentricity(mm)	Rail-seat			12.10
	Center			-7.00

Pre-stressing forces	
Initial (N)	380500.91
At transfer(N)	356719.60
At service (N)	304400.73

Applied stresses in concrete(MPa)				
Stresses in concrete(MPa)	Rail-seat		Center	
	Top	Bottom	Top	Bottom
At transfer	7.24	13.59	19.30	10.13
At service(M+)	24.33	-4.40	23.29	-0.61
At service(M-)	15.10	3.73	30.12	-4.15

Trial 3

Design of prestressed concrete sleeper based on ultimate limit state approach



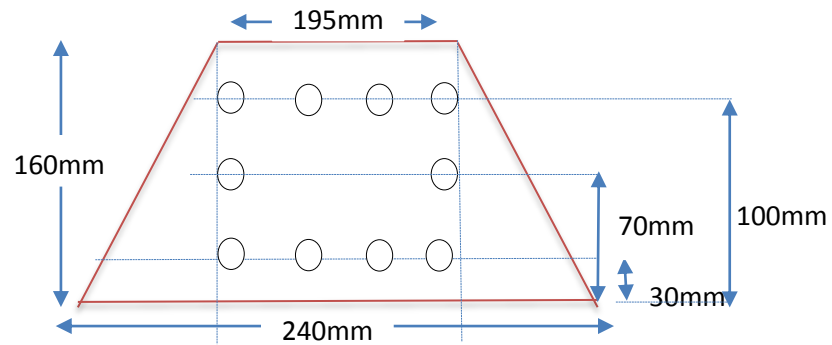
Pre-stressing wires				
Nominal diameter(mm)	8			
Layers	Quantity	Area(mm ²)	y(mm)	A*y
1	3	150.72	30	4521.60
2	1	50.24	110	5526.4
3	3	150.72	130	19593.60
Total	7	351.68		29641.60
Centroid(mm)				84.29
Eccentricity(mm)	Rail-seat			9.48
	Center			-6.62

Pre-stressing force	
Initial (N)	461758.04
At transfer(N)	432898.16
At service (N)	369406.43

Applied stresses in concrete(MPa)				
Stresses in concrete(MPa)	Rail-seat		Center	
	Top	Bottom	Top	Bottom
At transfer	8.26	13.01	21.21	8.87
At service(M+)	25.19	-4.89	24.88	-1.69
At service(M-)	15.97	3.24	31.79	-5.22

Trial 4

Design of prestressed concrete sleeper based on ultimate limit state approach



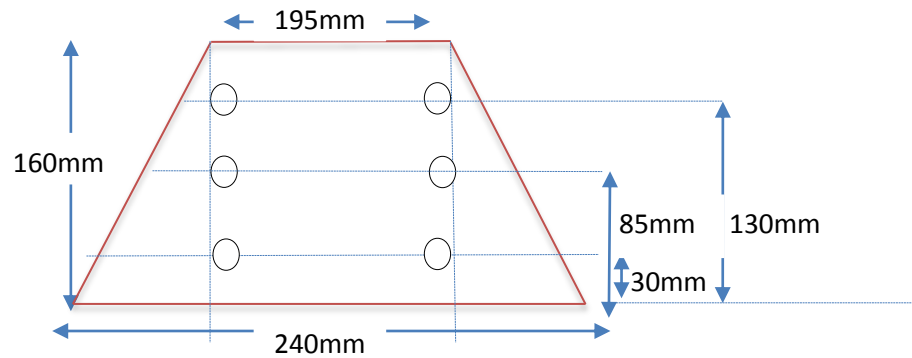
Pre-stressing wires				
Nominal diameter(mm)	8			
Layers	Quantity	Area(mm ²)	y(mm)	A*y
1	3	150.72	30	4521.60
2	2	100.48	70	7033.60
3	3	150.72	100	15072.00
Total	8	401.92		26627.2
Centroid(mm)				66.25
Eccentricity(mm)	Rail-seat			27.52
	Center			8.42

Pre-stressing force	
Initial (N)	474973.35
At transfer(N)	445287.52
At service (N)	379978.68

Applied stresses in concrete(MPa)				
Stresses in concrete(MPa)	Rail-seat		Center	
	Top	Bottom	Top	Bottom
At transfer	2.15	21.29	11.27	21.34
At service(M+)	19.99	2.17	15.99	8.96
At service(M-)	10.76	10.30	22.42	5.42

Trial 5

Design of prestressed concrete sleeper based on ultimate limit state approach



Pre-stressing wires				
Nominal diameter(mm)	9.3			
Layers	Quantity	Area(mm ²)	y(mm)	A*y
1	2	135.79	30	4073.68
2	2	135.79	85	11542.09
3	2	135.79	130	17652.61
Total	6	407.37		33268.38
Centroid(mm)				81.67
Eccentricity(mm)	Rail-seat			12.10
	Center			-7.00

Pre-stressing force	
Initial (N)	432606.28
At transfer(N)	405568.39
At service (N)	346085.02

Applied stresses in concrete(MPa)				
Stresses in concrete(MPa)	Rail-seat		Center	
	Top	Bottom	Top	Bottom
At transfer	8.38	16.11	22.51	11.92
At service(M+)	25.30	-2.25	25.54	-0.92
At service(M-)	16.08	5.88	32.86	-2.62

Trial 6

Appendix D: Pre-calculated graphs for evaluation of $M_{c,ng,100}$

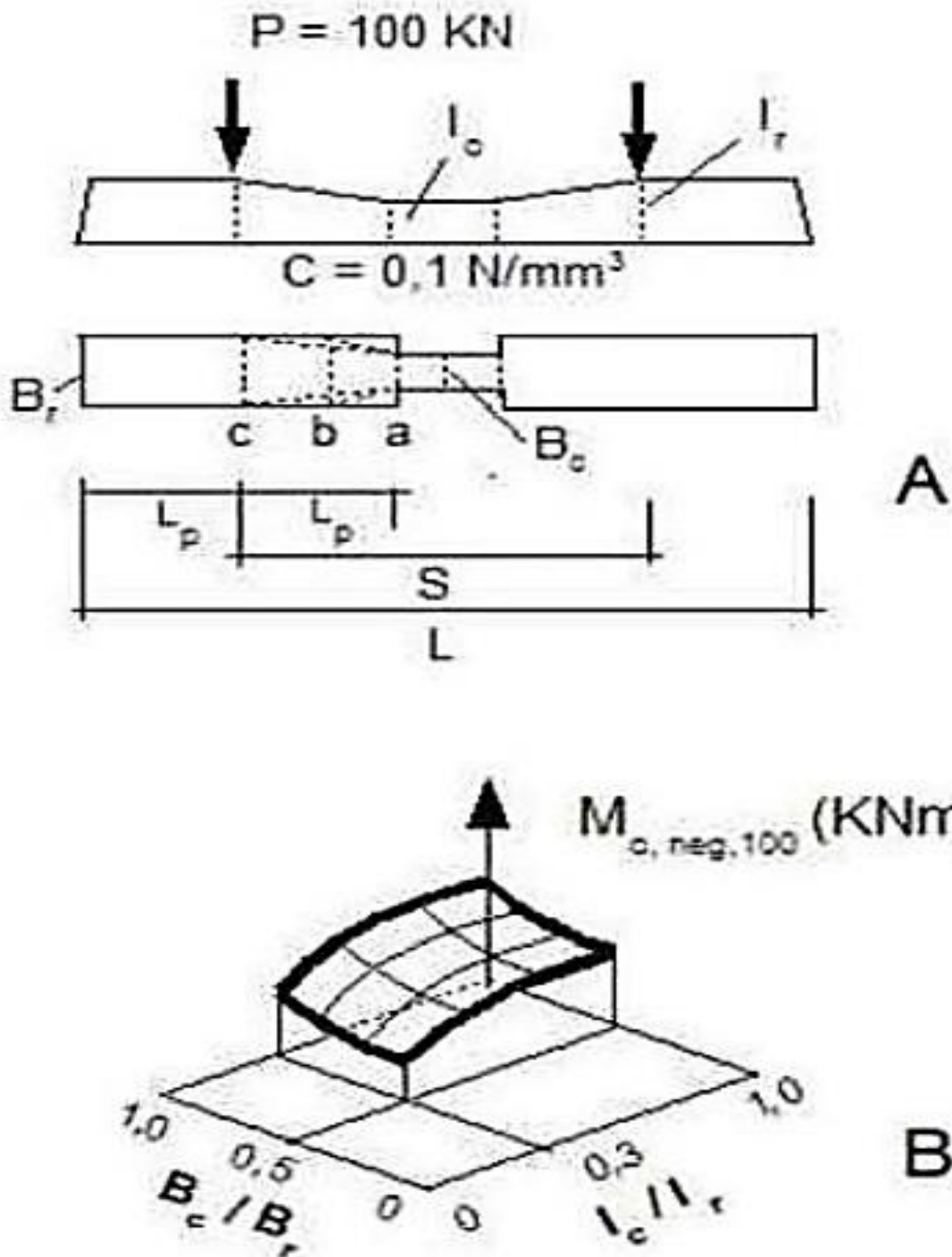


Figure 1: Determination of center bending moment $M_{c, neg, 100}$ (in kNm) for 1435 mm gauge sleepers (EN13230-6, 2015)

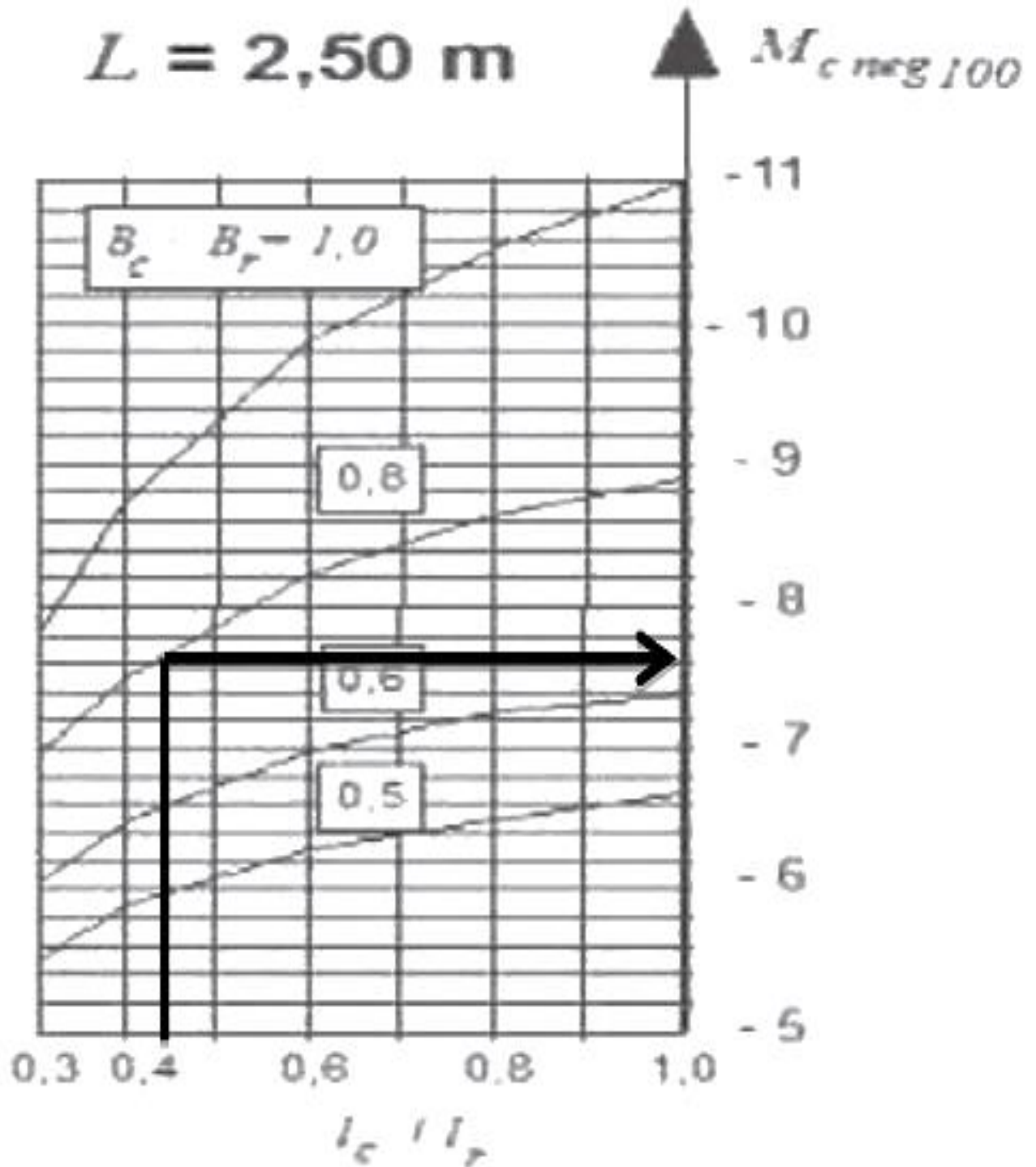


Figure 2: Pre-calculated curves for center bending moment $M_{c, neg, 100}$ (in kN-m) for standard 1435 gauge sleepers (EN13230-6, 2015)